

Computer algebra independent integration tests

1-Algebraic-functions/1.3-Miscellaneous/1.3.2-Algebraic-functions

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Chapter 1

Introduction

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This report gives the result of running the computer algebra independent integration test. Elementary Algebraic integrals version.

The download section below contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [714]. This is test number [38].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.3.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.45 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)
9. IntegrateAlgebraic under Mathematica 12.3.1 on windows 10. https://github.com/stblake/algebraic_integration. September 15, 2021 version.

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	98.74 (705)	1.26 (9)
Mathematica	97.90 (699)	2.10 (15)
Fricas	96.22 (687)	3.78 (27)
Maple	87.82 (627)	12.18 (87)
IntegrateAlgebraic	87.25 (623)	12.75 (91)
Giac	66.39 (474)	33.61 (240)
Mupad	57.00 (407)	43.00 (307)
Maxima	52.52 (375)	47.48 (339)
Sympy	28.71 (205)	% 71.29 (509)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

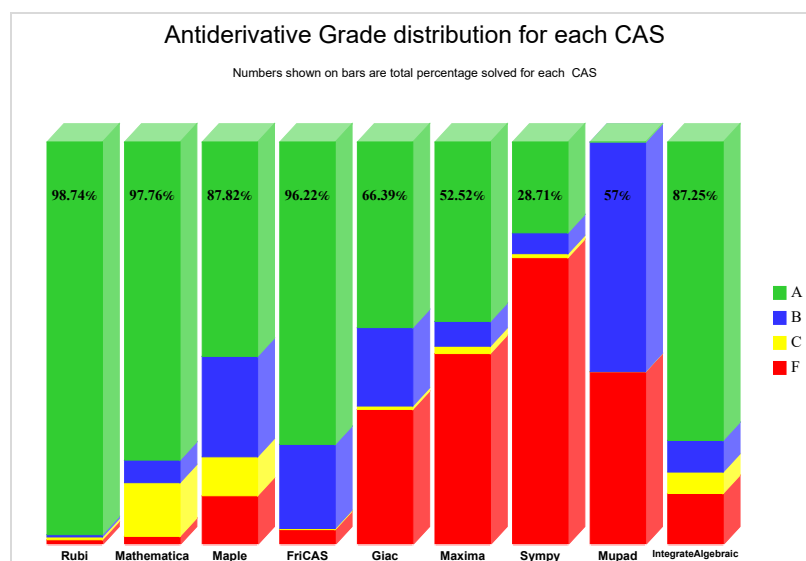
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

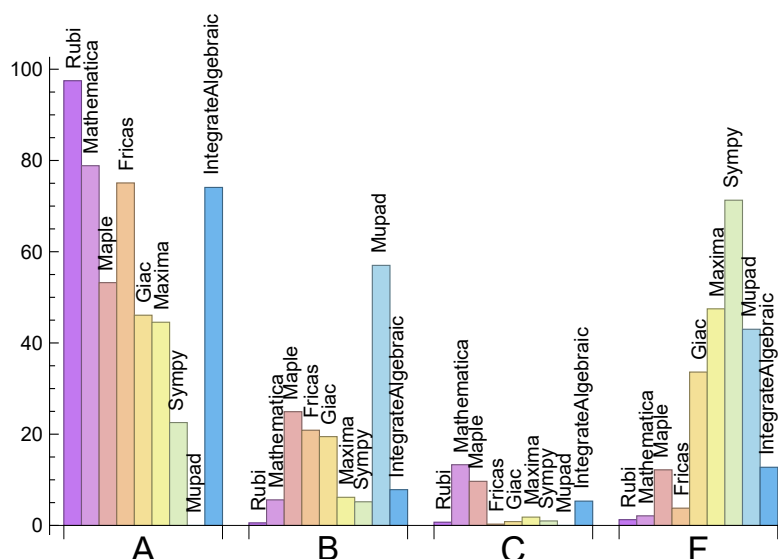
System	% A grade	% B grade	% C grade	% F grade
Rubi	97.48	0.56	0.70	1.26
Mathematica	78.85	5.60	13.31	2.10
Fricas	75.07	20.87	0.28	3.78
IntegrateAlgebraic	74.09	7.84	5.32	12.75
Maple	53.22	24.93	9.66	12.18
Giac	46.08	19.47	0.84	33.61
Maxima	44.54	6.16	1.82	47.48
Sympy	22.55	5.18	0.98	71.29
Mupad	N/A	57.00	0.00	43.00

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This is the typical normal failure **F**.

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	9	100.00 %	0.00 %	0.00 %
Mathematica	15	100.00 %	0.00 %	0.00 %
Maple	87	97.70 %	0.00 %	2.30 %
Fricas	27	0.00 %	92.59 %	7.41 %
IntegrateAlgebraic	91	92.31 %	7.69 %	0.00 %
Giac	240	60.00 %	11.67 %	28.33 %
Maxima	339	94.40 %	0.29 %	5.31 %
Sympy	509	83.69 %	16.31 %	0.00 %
Mupad	307	94.14 %	5.86 %	0.00 %

Table 1.4: Failure statistics for each CAS

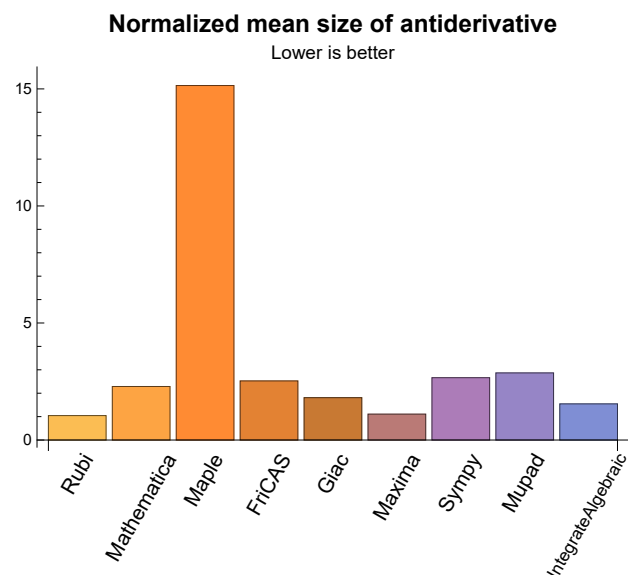
1.3 Performance

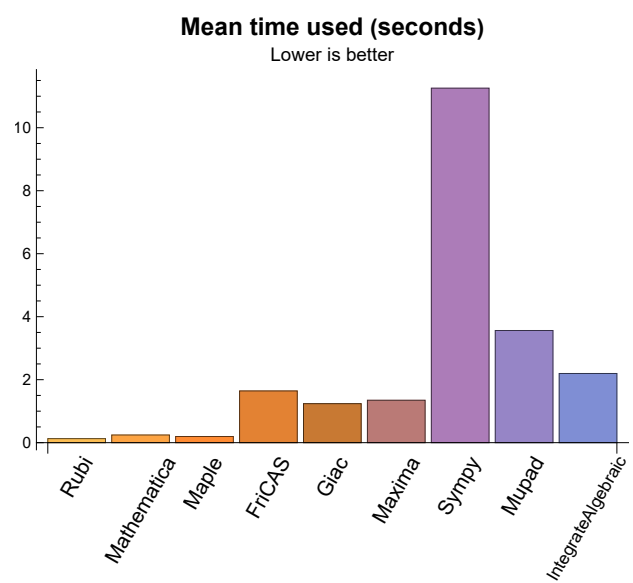
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	87.57	1.04	54.00	1.00
Mathematica	0.24	159.62	2.29	61.00	1.00
Maple	0.20	1708.13	15.14	60.00	1.23
Maxima	1.35	97.59	1.11	44.00	0.93
Fricas	1.64	232.38	2.53	76.00	1.42
Sympy	11.26	197.56	2.66	49.00	1.16
Giac	1.24	203.91	1.81	52.50	1.13
Mupad	3.56	233.18	2.87	45.00	1.06
IntegrateAlgebraic	2.20	130.59	1.54	62.00	1.08

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

IntegrateAlgebraic {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {470}

Mathematica {6, 7, 8, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 63, 64, 65, 66, 150, 228, 229, 498, 499, 587, 698, 699, 707, 713, 714}

IntegrateAlgebraic {}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user slelievre at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

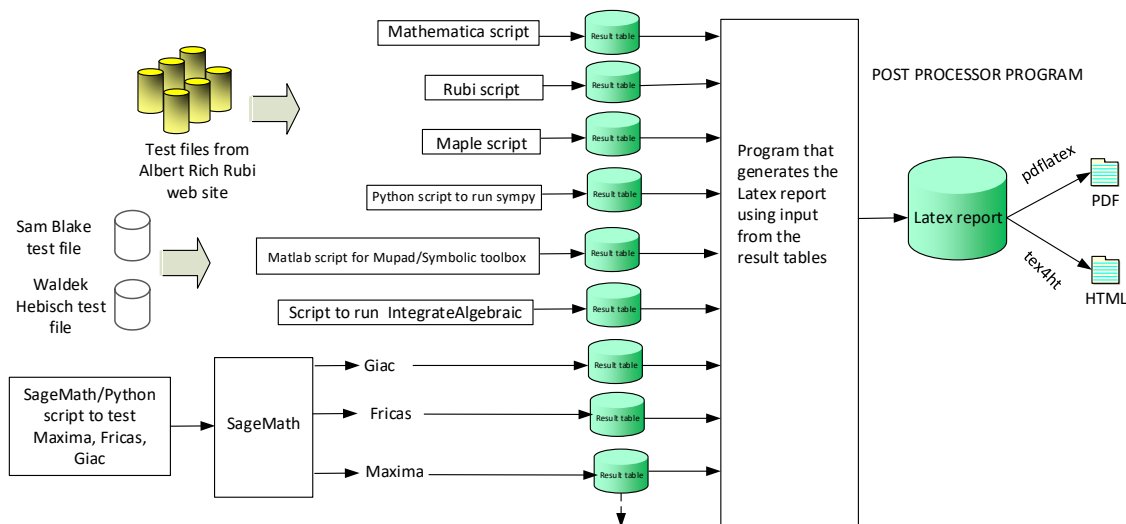
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^{2/2}$

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
The following field present only in Rubi and Mathematica Tables
13. integer. 1 if result was verified or 0 if not verified.
The following fields present only in Rubi Tables
14. integer. Number of rules used.
15. integer. Integrand leaf size.
16. real number. Ratio of field 14 over field 15
17. integer. 1 if result was verified or 0 if not verified.
18. String of form "{n,n,...}" which is list of the rules used by Rubi

High level overview of the CAS independent integration test build system

Chapter 2

detailed summary tables of results

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2.1 List of integrals sorted by grade for each CAS

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2.1.1 Rubi

A grade: { 1, 2, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 708, 709, 713 }

B grade: { 190, 470, 689, 714 }

C grade: { 174, 633, 707, 710, 711 }

F grade: { 3, 4, 5, 361, 362, 687, 688, 706, 712 }

2.1.2 Mathematica

A grade: { 15, 16, 17, 18, 19, 20, 21, 22, 23, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 85, 87, 89, 90, 91, 92, 93, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 291, 292, 295, 296, 299, 300, 301, 302, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 326, 327, 329, 330, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 408, 409, 410, 411, 414, 415, 416, 417, 418, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479,

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B grade: { 96, 134, 185, 190, 285, 368, 369, 425, 426, 481, 483, 484, 549, 550, 555, 556, 558, 560, 564, 572, 574, 576, 578, 610, 611, 618, 619, 653, 677, 678, 679, 680, 681, 682, 688, 700, 701, 702, 703, 704 }

C grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 59, 60, 61, 62, 63, 64, 65, 66, 81, 82, 86, 88, 94, 106, 126, 150, 152, 208, 219, 286, 287, 288, 293, 294, 297, 298, 325, 328, 331, 334, 348, 406, 407, 412, 413, 419, 420, 421, 438, 455, 503, 504, 505, 516, 568, 570, 608, 609, 634, 676, 690, 694, 695, 696, 698, 699, 707, 708, 710, 711, 713, 714 }

F grade: { 1, 2, 3, 4, 5, 48, 49, 289, 290, 303, 304, 605, 606, 687, 706 }

2.1.3 Maple

A grade: { 52, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 111, 112, 113, 114, 116, 117, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 202, 203, 204, 205, 206, 207, 208, 215, 216, 217, 218, 221, 222, 223, 225, 227, 228, 230, 231, 232, 249, 250, 291, 292, 299, 300, 319, 320, 321, 323, 324, 325, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 381, 382, 384, 385, 386, 388, 389, 391, 392, 393, 394, 395, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 426, 429, 430, 431, 433, 435, 436, 437, 438, 439, 440, 442, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 471, 472, 477, 479, 480, 482, 485, 486, 487, 488, 492, 493, 494, 495, 496, 500, 501, 502, 506, 507, 508, 509, 510, 511, 512, 513, 516, 517, 518, 520, 521, 522, 523, 524, 525, 526, 527, 528, 531, 532, 533, 536, 537, 538, 539, 540, 542, 544, 545, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 567, 568, 570, 571, 573, 575, 577, 579, 581, 584, 585, 586, 587, 588, 589, 590, 591, 592, 594, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 659, 661, 662, 663, 664, 667, 671, 672, 673, 674, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 696, 697, 709, 712 }

B grade: { 50, 51, 53, 54, 55, 56, 57, 58, 72, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 159, 185, 190, 199, 201, 209, 210, 219, 220, 224, 226, 229, 233, 234, 235, 248, 251, 252, 253, 262, 285, 286, 289, 290, 295, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 322, 326, 374, 376, 380, 383, 387, 390, 396, 397, 425, 427, 428, 432, 434, 441, 443, 444, 453, 470, 473, 474, 475, 476, 478, 481, 483, 484, 489, 490, 491, 497, 498, 499, 503, 504, 505, 514, 515, 519, 529, 530, 534, 535, 543, 546, 549, 558, 560, 562, 564, 565, 566, 572, 574, 576, 578, 580, 582, 583, 593, 610, 611, 612, 613, 657, 658, 660, 665, 666, 668, 669, 670, 675, 676, 689, 690, 704, 707 }

C grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 18, 23, 24, 25, 26, 27, 36, 37, 38, 39, 59, 60, 61, 62, 63, 64, 65, 66, 115, 172, 183, 184, 186, 187, 211, 212, 213, 214, 244, 245, 246, 260, 261, 287, 288, 293, 294, 297, 298, 313, 314, 315, 316, 317, 541, 569, 622, 646, 694, 695, 698, 699, 705, 706, 708, 710, 711, 713, 714 }

F grade: { 1, 2, 3, 10, 11, 12, 13, 19, 20, 21, 22, 28, 29, 30, 31, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 93, 94, 236, 237, 238, 239, 240, 241, 242, 243, 247, 254, 255, 256, 257, 258, 259, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 318, 398, 399, 400, 401, 603, 604, 605, 606, 634, 687, 688, 691, 692, 693, 700, 701, 702, 703 }

2.1.4 Maxima

A grade: { 50, 51, 52, 54, 55, 67, 68, 69, 70, 71, 73, 74, 75, 76, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 112, 115, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 137, 138, 139, 140, 141, 142, 144, 145, 147, 148, 150, 151, 152, 153, 154, 160, 161, 162, 163, 164, 165, 166, 168, 170, 171, 174, 175, 176, 177, 196, 197, 198, 199, 200, 201, 202, 220, 221, 222, 223, 224, 225, 226, 227, 230, 231, 232, 291, 292, 299, 300, 305, 306, 307, 313, 314, 315, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 391, 392, 393, 394, 399, 400, 401, 414, 420, 421, 423, 424, 425, 427, 428, 429, 430, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 468, 469, 477, 478, 479, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 506, 507, 508, 511, 512, 513, 517, 518, 519, 520, 521, 522, 523, 525, 527, 528, 536, 538, 539, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 555, 557, 559, 560, 561, 562, 563, 564, 565, 567, 568, 569, 570, 571, 573, 575, 577, 585, 586, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 627, 628, 629, 633, 634, 635, 636, 637, 638, 639, 641, 642, 643, 644, 645, 646, 651, 652, 653, 655, 656, 663, 664, 667, 669, 671, 672, 673, 674, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 697, 709, 712 } }

B grade: { 53, 56, 57, 58, 72, 78, 132, 135, 136, 143, 146, 149, 155, 159, 318, 320, 328, 334, 357, 358, 359, 360, 361, 362, 390, 398, 419, 422, 473, 474, 475, 476, 480, 490, 494, 532, 582, 640, 649, 650, 666, 668, 670, 704 } }

C grade: { 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 558 } }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 59, 60, 61, 62, 63, 64, 65, 66, 77, 79, 80, 81, 82, 83, 85, 95, 96, 111, 113, 114, 116, 117, 156, 157, 158, 167, 169, 172, 173, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 228, 229, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 308, 309, 310, 311, 312, 316, 317, 374, 375, 376, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 426, 431, 432, 433, 434, 435, 436, 445, 446, 447, 448, 449, 450, 454, 464, 465, 467, 470, 471, 472, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 509, 510, 514, 515, 516, 524, 526, 529, 530, 531, 533, 534, 535, 537, 540, 541, 553, 554, 556, 566, 572, 574, 576, 578, 579, 580, 581, 583, 584, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 613, 622, 623, 624, 625, 626, 630, 631, 632, 647, 648, 654, 657, 658, 659, 660, 661, 662, 665, 675, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 710, 711, 713, 714 } }

2.1.5 FriCAS

A grade: { 18, 26, 27, 28, 30, 32, 34, 36, 37, 40, 42, 44, 46, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 149, 150, 151, 154, 156, 159, 160, 161, 162, 163, 166, 167, 168, 169, 170, 171, 173, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 192, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 236, 237, 238, 239, 240, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 254, 255, 256, 257, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 289, 290, 291, 292, 295, 296, 299, 300, 301, 302, 303, 304, 305, 308, 309, 310, 312, 313, 314, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 391, 392, 393, 394, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 429, 430, 435, 436, 437, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 455, 456, 457, 458, 459, 460, 461, 462, 463, 466, 467, 468, 469, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 495, 496, 498, 499, 500, 501, 502, 504, 505, 506, 507, 508, 509, 510, 511, 512, }

513, 514, 515, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 538, 539, 540, 541, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 565, 570, 575, 577, 579, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 623, 624, 625, 626, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 655, 656, 657, 658, 659, 661, 662, 663, 664, 667, 669, 671, 672, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 694, 695, 697, 698, 699, 700, 701, 702, 703, 707, 708, 709, 710, 711, 712, 713, 714 }

B grade: { 4, 5, 6, 7, 8, 9, 14, 15, 16, 17, 23, 24, 25, 29, 31, 33, 35, 38, 39, 41, 43, 45, 47, 50, 51, 52, 53, 54, 55, 56, 57, 58, 66, 72, 78, 127, 133, 134, 147, 148, 152, 153, 155, 157, 158, 164, 165, 172, 185, 190, 199, 224, 234, 235, 241, 252, 253, 258, 259, 287, 288, 293, 294, 297, 298, 306, 307, 311, 315, 320, 328, 334, 344, 374, 375, 376, 389, 390, 395, 396, 397, 398, 399, 400, 407, 425, 426, 427, 428, 431, 432, 433, 434, 438, 454, 464, 465, 470, 473, 474, 476, 494, 497, 503, 516, 535, 536, 537, 542, 543, 544, 545, 546, 547, 548, 549, 550, 562, 564, 566, 567, 568, 569, 571, 572, 573, 574, 576, 578, 580, 581, 610, 611, 627, 649, 650, 660, 665, 666, 668, 670, 673, 689, 690, 691, 696, 704, 705, 706 }

C grade: { 558, 560 }

F grade: { 1, 2, 3, 10, 11, 12, 13, 19, 20, 21, 22, 48, 49, 93, 94, 174, 358, 359, 360, 361, 362, 605, 606, 622, 688, 692, 693 }

2.1.6 Sympy

A grade: { 50, 51, 52, 55, 84, 85, 90, 91, 92, 111, 112, 168, 172, 180, 185, 189, 190, 198, 199, 220, 223, 224, 229, 230, 231, 232, 273, 285, 286, 288, 292, 298, 300, 306, 307, 308, 318, 319, 321, 323, 324, 325, 327, 329, 330, 331, 334, 347, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 379, 380, 381, 386, 387, 388, 402, 403, 406, 407, 408, 409, 412, 413, 414, 420, 421, 422, 423, 424, 425, 426, 427, 428, 430, 431, 432, 433, 438, 440, 441, 442, 443, 444, 451, 452, 453, 455, 466, 468, 469, 470, 473, 475, 479, 509, 511, 512, 513, 514, 515, 517, 518, 520, 521, 522, 523, 532, 536, 538, 539, 542, 544, 551, 552, 553, 555, 557, 559, 561, 563, 565, 571, 573, 575, 577, 580, 581, 607, 608, 609, 616, 620, 628, 629, 631, 635, 636, 637, 638, 640, 641, 642, 644, 645, 646, 658, 669, 676, 709, 712 }

B grade: { 193, 194, 195, 244, 245, 246, 262, 269, 287, 291, 293, 294, 297, 299, 320, 348, 349, 350, 368, 429, 439, 458, 506, 507, 540, 543, 546, 549, 579, 625, 649, 650, 651, 655, 656, 659, 697 }

C grade: { 437, 633, 634, 643, 657, 663, 664 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 53, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 181, 182, 183, 184, 186, 187, 188, 191, 192, 196, 197, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 222, 225, 226, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 289, 290, 295, 296, 301, 302, 303, 304, 305, 309, 310, 311, 312, 313, 314, 315, 316, 317, 322, 326, 328, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 358, 359, 360, 361, 362, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 382, 383, 384, 385, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 404, 405, 410, 411, 415, 416, 417, 418, 419, 434, 435, 436, 445, 446, 447, 448, 449, 450, 454, 456, 457, 459, 460, 461, 462, 463, 464, 465, 467, 471, 472, 474, 476, 477, 478, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 508, 510, 516, 519, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 537, 541, 545, 547, 548, 550, 554, 556, 558, 560, 562, 564, 566, 567, 568, 569, 570, 572, 574, 576, 578, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 610, 611, 612, 613, 614, 615, 617, 618, 619, 621, 622, 623, 624, 626, 627, 630, 632, 639, 647, 648, 652, 653, 654, 660, 661, 662, 665, 666, 667, 668, 670, 671, 672, 673, 674, 675, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 710, 711, 713, 714 }

2.1.7 Giac

A grade: { 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 111, 112, 113, 115, 117, 130, 131, 144, 145, 157, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 175, 176, 177, 180, 186, 193, 194, 195, 229, 230, 231, 232, 248, 249, 250, 291, 292, 299, 300, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 347, 348, 349, 350, 351, 352, 353, 354, 355, 363, 364, 365, 367, 368, 369, 374, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 414, 419, 420, 421, 425, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 460, 462, 464, 465, 466, 467, 468, 469, 471, 472, 475, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 492, 493, 499, 500, 506, 507, 509, 510, 511, 512, 513, 517, 519, 520, 521, 522, 523, 524, 525, 526, 533, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 547, 548, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 571, 575, 577, 579, 584, 585, 595, 596, 597, 598, 599, 600, 602, 607, 608, 609, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 651, 652, 653, 654, 655, 656, 657, 658, 659, 662, 663, 664, 665, 666, 667, 668, 669, 671, 672, 673, 674, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 697, 709 }

B grade: { 50, 51, 52, 53, 54, 55, 57, 58, 67, 101, 102, 103, 132, 134, 135, 136, 137, 138, 139, 146, 148, 149, 150, 151, 152, 158, 172, 178, 179, 181, 182, 183, 184, 185, 187, 188, 189, 190, 191, 192, 196, 197, 198, 199, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 220, 221, 222, 223, 224, 226, 320, 336, 337, 338, 339, 340, 366, 370, 371, 372, 373, 375, 376, 396, 397, 398, 399, 400, 401, 422, 423, 424, 426, 458, 459, 463, 470, 473, 474, 476, 484, 490, 491, 494, 495, 497, 498, 501, 502, 503, 504, 505, 516, 518, 529, 530, 531, 534, 535, 546, 549, 570, 573, 580, 581, 582, 583, 610, 611, 649, 650, 660, 661, 670, 675, 689, 690, 696, 704, 710, 711, 712 }

C grade: { 116, 342, 346, 527, 528, 594 }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 56, 59, 60, 61, 62, 63, 64, 65, 66, 100, 104, 105, 106, 107, 108, 109, 110, 114, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 133, 140, 141, 142, 143, 147, 153, 154, 155, 156, 160, 174, 200, 202, 219, 225, 227, 228, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 293, 294, 295, 296, 297, 298, 301, 302, 303, 304, 310, 335, 341, 343, 344, 345, 356, 357, 358, 359, 360, 361, 362, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 461, 496, 508, 514, 515, 532, 566, 567, 569, 572, 574, 576, 578, 586, 587, 588, 589, 590, 591, 592, 593, 601, 603, 604, 605, 606, 612, 613, 633, 634, 687, 688, 691, 692, 693, 694, 695, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 713, 714 }

2.1.8 Mupad

A grade: { }

B grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 70, 72, 78, 84, 85, 87, 89, 90, 91, 92, 111, 112, 113, 114, 115, 132, 139, 146, 152, 159, 160, 168, 170, 171, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 231, 232, 269, 273, 280, 281, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 297, 298, 299, 300, 305, 306, 307, 308, 309, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 373, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 394, 401, 414, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 451, 452, 453, 455, 456, 466, 468, 469, 470, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 494, 495, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 527, 528, 529, 530, 531, 533, 534, 536, 538, 539, 540, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 557, 558, 559, 560, 561, 562, 563, 564, 565, 568, 570, 571, 573, 575, 577, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590, 591, 592, 602, 607, 608, 609, 610, 611, 614, 615, 616, 617, 620, 622, 623, 625, 627, 628, 629, 631, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 649,

650, 651, 652, 655, 656, 657, 658, 659, 660, 661, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684, 685, 686, 689, 697, 704, 709, 710, 711, 712 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 68, 69, 71, 73, 74, 75, 76, 77, 79, 80, 81, 82, 83, 86, 88, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 165, 166, 167, 169, 172, 173, 230, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 282, 283, 284, 295, 296, 301, 302, 303, 304, 310, 311, 312, 344, 358, 359, 370, 371, 372, 374, 375, 376, 391, 392, 393, 395, 396, 397, 398, 399, 400, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 415, 416, 417, 418, 445, 446, 447, 448, 449, 450, 454, 457, 458, 459, 460, 461, 462, 463, 464, 465, 467, 471, 472, 493, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 520, 521, 522, 523, 524, 525, 526, 532, 535, 537, 541, 553, 554, 555, 556, 566, 567, 569, 572, 574, 576, 578, 584, 585, 593, 594, 595, 596, 597, 598, 599, 600, 601, 603, 604, 605, 606, 612, 613, 618, 619, 621, 624, 626, 630, 632, 647, 648, 653, 654, 662, 687, 688, 690, 691, 692, 693, 694, 695, 696, 698, 699, 700, 701, 702, 703, 705, 706, 707, 708, 713, 714 }

2.1.9 IntegrateAlgebraic

A grade: { 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 178, 179, 180, 183, 184, 185, 186, 187, 188, 189, 190, 193, 194, 195, 203, 204, 205, 209, 210, 216, 229, 230, 231, 232, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 254, 256, 257, 258, 259, 269, 273, 280, 281, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 336, 337, 338, 339, 340, 342, 343, 345, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 533, 534, 535, 536, 537, 538, 539, 540, 541, 545, 546, 547, 551, 552, 553, 554, 555, 556, 557, 559, 561, 563, 565, 568, 571, 575, 577, 579, 580, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 607, 608, 609, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 649, 650, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 666, 667, 668, 669, 670, 671, 672, 674, 675, 677, 678, 679, 680, 681, 682, 686, 688, 689, 692, 693, 694, 695, 696, 697, 698, 700, 702, 704, 708, 713, 714 }

B grade: { 72, 76, 93, 127, 181, 182, 191, 192, 196, 198, 206, 207, 211, 212, 213, 215, 221, 223, 233, 234, 235, 248, 249, 250, 251, 252, 255, 425, 426, 473, 543, 548, 549, 550, 558, 560, 562, 564, 570, 573, 581, 610, 611, 647, 648, 665, 673, 683, 684, 685, 687, 701, 703, 705, 706, 709 }

C grade: { 1, 95, 96, 197, 199, 200, 201, 202, 220, 222, 224, 225, 226, 227, 228, 328, 334, 508, 542, 544, 567, 569, 582, 583, 584, 585, 586, 587, 603, 604, 605, 606, 690, 691, 699, 707, 710, 711 }

F grade: { 2, 3, 4, 5, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 67, 94, 174, 175, 176, 177, 208, 214, 217, 218, 219, 253, 260, 261, 262, 263, 264, 265, 266, 267, 268, 270, 271, 272, 274, 275, 276, 277, 278, 279, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 335, 341, 344, 346, 358, 359, 360, 361, 362, 398, 399, 400, 401, 496, 532, 566, 572, 574, 576, 578, 634, 651, 676, 712 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column N.S. in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, Mathematica was abbreviated to MMA and IntegrateAlgebraic to I.A.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	139	139	0	0	0	0	0	0	-1	385
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	2.77
time (sec)	N/A	0.070	0.076	0.075	0.000	0.000	0.000	0.000	0.000	2.893
Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	186	186	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.204	0.084	0.061	0.000	0.000	0.000	0.000	0.000	6.708
Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	F(-1)	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	187	0	0	0	0	0	0	0	-1	0
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.106	0.064	0.059	0.000	0.000	0.000	0.000	0.000	6.960
Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	147	0	0	3064	0	712	0	0	-1	0
N.S.	1	0.00	0.00	20.84	0.00	4.84	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.085	0.073	42.609	0.000	19.239	0.000	0.000	0.000	6.362
Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	F
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	159	0	0	3250	0	720	0	0	-1	0
N.S.	1	0.00	0.00	20.44	0.00	4.53	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.102	0.085	41.368	0.000	14.366	0.000	0.000	0.000	7.276

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	37	37	326	258	0	75	0	0	70	39
N.S.	1	1.00	8.81	6.97	0.00	2.03	0.00	0.00	1.89	1.05
time (sec)	N/A	0.105	0.439	0.069	0.000	2.046	0.000	0.000	3.593	1.906

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	40	40	327	253	0	76	0	0	74	41
N.S.	1	1.00	8.18	6.32	0.00	1.90	0.00	0.00	1.85	1.02
time (sec)	N/A	0.123	0.372	0.061	0.000	1.951	0.000	0.000	3.631	1.959

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	38	38	325	262	0	238	0	0	62	39
N.S.	1	1.00	8.55	6.89	0.00	6.26	0.00	0.00	1.63	1.03
time (sec)	N/A	0.112	0.304	0.059	0.000	1.994	0.000	0.000	2.861	1.930

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	39	39	328	249	0	241	0	0	63	41
N.S.	1	1.00	8.41	6.38	0.00	6.18	0.00	0.00	1.62	1.05
time (sec)	N/A	0.114	0.291	0.056	0.000	1.820	0.000	0.000	2.843	1.883

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	63	63	325	0	0	0	0	0	106	65
N.S.	1	1.00	5.16	0.00	0.00	0.00	0.00	0.00	1.68	1.03
time (sec)	N/A	0.179	1.116	0.368	0.000	0.000	0.000	0.000	5.812	9.722

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	65	65	336	0	0	0	0	0	107	67
N.S.	1	1.00	5.17	0.00	0.00	0.00	0.00	0.00	1.65	1.03
time (sec)	N/A	0.199	1.135	0.353	0.000	0.000	0.000	0.000	5.851	9.732

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	66	66	390	0	0	0	0	0	102	68
N.S.	1	1.00	5.91	0.00	0.00	0.00	0.00	0.00	1.55	1.03
time (sec)	N/A	0.201	0.473	0.153	0.000	0.000	0.000	0.000	3.677	9.712
Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	66	66	375	0	0	0	0	0	103	68
N.S.	1	1.00	5.68	0.00	0.00	0.00	0.00	0.00	1.56	1.03
time (sec)	N/A	0.193	0.681	0.156	0.000	0.000	0.000	0.000	3.647	9.681
Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	49	49	373	889	0	300	0	0	95	55
N.S.	1	1.00	7.61	18.14	0.00	6.12	0.00	0.00	1.94	1.12
time (sec)	N/A	0.123	1.097	0.073	0.000	2.121	0.000	0.000	4.731	1.246
Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	46	240	0	44	0	0	205	31
N.S.	1	1.00	2.00	10.43	0.00	1.91	0.00	0.00	8.91	1.35
time (sec)	N/A	0.059	0.009	0.045	0.000	2.022	0.000	0.000	0.225	0.876
Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	54	240	0	47	0	0	221	33
N.S.	1	1.00	2.00	8.89	0.00	1.74	0.00	0.00	8.19	1.22
time (sec)	N/A	0.065	0.012	0.038	0.000	1.446	0.000	0.000	0.184	0.880
Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	240	0	40	0	0	205	21
N.S.	1	1.00	1.00	9.60	0.00	1.60	0.00	0.00	8.20	0.84
time (sec)	N/A	0.059	0.008	0.042	0.000	1.663	0.000	0.000	2.551	0.873

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	240	0	38	0	0	221	23
N.S.	1	1.00	1.00	9.60	0.00	1.52	0.00	0.00	8.84	0.92
time (sec)	N/A	0.067	0.009	0.037	0.000	1.454	0.000	0.000	2.529	0.879
Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	51	0	0	0	0	0	65	69
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.30	1.38
time (sec)	N/A	0.132	0.024	0.132	0.000	0.000	0.000	0.000	3.444	6.621
Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	53	0	0	0	0	0	67	70
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.29	1.35
time (sec)	N/A	0.138	0.028	0.128	0.000	0.000	0.000	0.000	3.591	6.793
Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	54	0	0	0	0	0	74	65
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.40	1.23
time (sec)	N/A	0.142	0.027	0.119	0.000	0.000	0.000	0.000	5.453	5.815
Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F(-1)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	54	0	0	0	0	0	78	66
N.S.	1	1.00	1.02	0.00	0.00	0.00	0.00	0.00	1.47	1.25
time (sec)	N/A	0.141	0.023	0.122	0.000	0.000	0.000	0.000	5.373	5.621
Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	650	0	294	0	0	67	44
N.S.	1	1.00	1.00	14.13	0.00	6.39	0.00	0.00	1.46	0.96
time (sec)	N/A	0.116	0.038	0.195	0.000	1.763	0.000	0.000	3.126	1.294

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	42	42	267	245	0	205	0	0	-1	53
N.S.	1	1.00	6.36	5.83	0.00	4.88	0.00	0.00	-0.02	1.26
time (sec)	N/A	0.114	0.446	0.085	0.000	1.727	0.000	0.000	0.000	2.035
Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	46	46	269	243	0	207	0	0	-1	53
N.S.	1	1.00	5.85	5.28	0.00	4.50	0.00	0.00	-0.02	1.15
time (sec)	N/A	0.114	0.489	0.075	0.000	1.571	0.000	0.000	0.000	2.152
Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	44	267	245	0	50	0	0	-1	51
N.S.	1	1.00	6.07	5.57	0.00	1.14	0.00	0.00	-0.02	1.16
time (sec)	N/A	0.104	0.379	0.063	0.000	1.412	0.000	0.000	0.000	2.096
Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	44	269	247	0	59	0	0	-1	55
N.S.	1	1.00	6.11	5.61	0.00	1.34	0.00	0.00	-0.02	1.25
time (sec)	N/A	0.094	0.377	0.070	0.000	0.933	0.000	0.000	0.000	2.095
Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	69	322	0	0	1240	0	0	-1	115
N.S.	1	1.00	4.67	0.00	0.00	17.97	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.204	0.631	0.223	0.000	4.523	0.000	0.000	0.000	12.677
Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	446	0	0	1294	0	0	-1	115
N.S.	1	1.00	6.28	0.00	0.00	18.23	0.00	0.00	-0.01	1.62
time (sec)	N/A	0.192	1.437	0.219	0.000	4.322	0.000	0.000	0.000	12.590

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	447	0	0	1245	0	0	-1	116
N.S.	1	1.00	6.21	0.00	0.00	17.29	0.00	0.00	-0.01	1.61
time (sec)	N/A	0.185	0.432	0.138	0.000	4.247	0.000	0.000	0.000	12.448
Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	72	325	0	0	1303	0	0	-1	118
N.S.	1	1.00	4.51	0.00	0.00	18.10	0.00	0.00	-0.01	1.64
time (sec)	N/A	0.169	0.530	0.145	0.000	3.776	0.000	0.000	0.000	12.390
Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	73	73	663	0	0	1273	0	0	-1	90
N.S.	1	1.00	9.08	0.00	0.00	17.44	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.199	1.276	0.195	0.000	2.658	0.000	0.000	0.000	12.791
Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	648	0	0	1330	0	0	-1	92
N.S.	1	1.00	8.64	0.00	0.00	17.73	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.201	1.176	0.186	0.000	2.846	0.000	0.000	0.000	12.624
Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	76	649	0	0	1278	0	0	-1	93
N.S.	1	1.00	8.54	0.00	0.00	16.82	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.192	0.594	0.127	0.000	2.920	0.000	0.000	0.000	12.584
Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	76	666	0	0	1339	0	0	-1	93
N.S.	1	1.00	8.76	0.00	0.00	17.62	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.180	0.781	0.136	0.000	3.170	0.000	0.000	0.000	12.606

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	42	42	269	245	0	50	0	0	-1	53
N.S.	1	1.00	6.40	5.83	0.00	1.19	0.00	0.00	-0.02	1.26
time (sec)	N/A	0.090	0.435	0.033	0.000	0.912	0.000	0.000	0.000	2.121
Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	46	46	267	247	0	59	0	0	-1	53
N.S.	1	1.00	5.80	5.37	0.00	1.28	0.00	0.00	-0.02	1.15
time (sec)	N/A	0.102	0.487	0.037	0.000	1.464	0.000	0.000	0.000	2.064
Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	44	265	245	0	204	0	0	-1	51
N.S.	1	1.00	6.02	5.57	0.00	4.64	0.00	0.00	-0.02	1.16
time (sec)	N/A	0.090	0.298	0.034	0.000	1.448	0.000	0.000	0.000	2.005
Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	F	F(-2)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	44	44	271	243	0	206	0	0	-1	55
N.S.	1	1.00	6.16	5.52	0.00	4.68	0.00	0.00	-0.02	1.25
time (sec)	N/A	0.087	0.349	0.028	0.000	0.926	0.000	0.000	0.000	2.143
Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	69	69	320	0	0	1236	0	0	-1	115
N.S.	1	1.00	4.64	0.00	0.00	17.91	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.176	0.594	0.187	0.000	4.776	0.000	0.000	0.000	12.510
Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	71	71	329	0	0	1288	0	0	-1	115
N.S.	1	1.00	4.63	0.00	0.00	18.14	0.00	0.00	-0.01	1.62
time (sec)	N/A	0.185	0.817	0.189	0.000	3.809	0.000	0.000	0.000	12.455

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	72	330	0	0	1239	0	0	-1	116
N.S.	1	1.00	4.58	0.00	0.00	17.21	0.00	0.00	-0.01	1.61
time (sec)	N/A	0.188	0.360	0.129	0.000	4.056	0.000	0.000	0.000	12.480
Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-1)	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	72	72	323	0	0	1299	0	0	-1	118
N.S.	1	1.00	4.49	0.00	0.00	18.04	0.00	0.00	-0.01	1.64
time (sec)	N/A	0.168	0.489	0.136	0.000	4.431	0.000	0.000	0.000	12.473
Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	73	73	667	0	0	1270	0	0	-1	90
N.S.	1	1.00	9.14	0.00	0.00	17.40	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.176	1.215	0.174	0.000	2.978	0.000	0.000	0.000	12.604
Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	75	75	649	0	0	1324	0	0	-1	92
N.S.	1	1.00	8.65	0.00	0.00	17.65	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.185	1.153	0.179	0.000	3.230	0.000	0.000	0.000	12.661
Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	76	650	0	0	1273	0	0	-1	93
N.S.	1	1.00	8.55	0.00	0.00	16.75	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.185	0.643	0.125	0.000	2.634	0.000	0.000	0.000	12.759
Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	76	76	670	0	0	1335	0	0	-1	93
N.S.	1	1.00	8.82	0.00	0.00	17.57	0.00	0.00	-0.01	1.22
time (sec)	N/A	0.174	0.716	0.129	0.000	2.609	0.000	0.000	0.000	12.634

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-2)	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	95	95	0	0	0	0	0	0	-1	159
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01	1.67
time (sec)	N/A	0.125	0.143	0.070	0.000	0.000	0.000	0.000	0.000	1.417
Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	234	234	0	0	0	0	0	0	-1	0
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.220	0.184	0.066	0.000	0.000	0.000	0.000	0.000	15.819
Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	133	451	253	490	6397	835	495	0
N.S.	1	1.00	0.83	2.82	1.58	3.06	39.98	5.22	3.09	0.00
time (sec)	N/A	0.106	0.124	0.010	0.984	1.248	7.628	0.398	3.204	0.064
Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	104	283	184	348	3704	577	363	0
N.S.	1	1.00	0.83	2.25	1.46	2.76	29.40	4.58	2.88	0.00
time (sec)	N/A	0.068	0.080	0.007	1.020	1.618	4.856	0.379	2.950	0.056
Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	94	167	122	222	1906	361	247	0
N.S.	1	1.00	1.00	1.78	1.30	2.36	20.28	3.84	2.63	0.00
time (sec)	N/A	0.046	0.071	0.004	0.956	0.787	2.786	0.327	2.946	0.055
Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	294	294	252	1565	601	1565	0	2660	1410	0
N.S.	1	1.00	0.86	5.32	2.04	5.32	0.00	9.05	4.80	0.00
time (sec)	N/A	0.202	0.270	0.021	0.810	0.997	0.000	0.613	3.734	0.103

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	248	248	211	1142	474	1216	0	2034	1136	0
N.S.	1	1.00	0.85	4.60	1.91	4.90	0.00	8.20	4.58	0.00
time (sec)	N/A	0.148	0.205	0.018	0.984	0.855	0.000	0.614	3.392	0.074
Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	203	203	172	793	359	893	11851	1477	878	0
N.S.	1	1.00	0.85	3.91	1.77	4.40	58.38	7.28	4.33	0.00
time (sec)	N/A	0.115	0.170	0.013	0.663	0.648	14.440	0.498	3.192	0.075
Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	459	459	402	3780	1153	3564	0	0	2896	0
N.S.	1	1.00	0.88	8.24	2.51	7.76	0.00	0.00	6.31	0.00
time (sec)	N/A	0.317	0.471	0.056	0.901	0.942	0.000	0.000	7.140	0.115
Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	396	396	345	2972	953	2919	0	4934	2436	0
N.S.	1	1.00	0.87	7.51	2.41	7.37	0.00	12.46	6.15	0.00
time (sec)	N/A	0.264	0.381	0.037	0.853	0.974	0.000	0.717	5.598	0.104
Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	337	337	290	2280	770	2313	0	3874	2001	0
N.S.	1	1.00	0.86	6.77	2.28	6.86	0.00	11.50	5.94	0.00
time (sec)	N/A	0.209	0.359	0.028	0.785	0.837	0.000	0.665	4.338	0.101
Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	296	1640	0	19	0	0	273	23
N.S.	1	1.00	18.50	102.50	0.00	1.19	0.00	0.00	17.06	1.44
time (sec)	N/A	0.077	0.869	0.072	0.000	0.805	0.000	0.000	0.201	1.030

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	280	732	0	28	0	0	292	23
N.S.	1	1.00	14.00	36.60	0.00	1.40	0.00	0.00	14.60	1.15
time (sec)	N/A	0.085	0.727	0.062	0.000	0.828	0.000	0.000	2.831	1.031
Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	278	1656	0	25	0	0	276	21
N.S.	1	1.00	15.44	92.00	0.00	1.39	0.00	0.00	15.33	1.17
time (sec)	N/A	0.079	0.234	0.063	0.000	0.721	0.000	0.000	2.771	1.029
Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	298	724	0	28	0	0	289	25
N.S.	1	1.00	16.56	40.22	0.00	1.56	0.00	0.00	16.06	1.39
time (sec)	N/A	0.083	0.559	0.061	0.000	0.821	0.000	0.000	0.106	1.040
Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	30	30	424	4397	0	181	0	0	632	37
N.S.	1	1.00	14.13	146.57	0.00	6.03	0.00	0.00	21.07	1.23
time (sec)	N/A	0.093	1.356	0.071	0.000	0.957	0.000	0.000	2.822	2.306
Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	38	38	427	1908	0	191	0	0	677	37
N.S.	1	1.00	11.24	50.21	0.00	5.03	0.00	0.00	17.82	0.97
time (sec)	N/A	0.108	1.485	0.071	0.000	0.977	0.000	0.000	0.141	2.452
Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	A	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	36	36	425	4437	0	187	0	0	629	35
N.S.	1	1.00	11.81	123.25	0.00	5.19	0.00	0.00	17.47	0.97
time (sec)	N/A	0.093	0.457	0.052	0.000	0.941	0.000	0.000	2.801	2.316

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F(-2)	B	F	F	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	32	32	426	1888	0	185	0	0	680	39
N.S.	1	1.00	13.31	59.00	0.00	5.78	0.00	0.00	21.25	1.22
time (sec)	N/A	0.093	0.546	0.059	0.000	1.056	0.000	0.000	0.120	2.359
Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	205	132	200	119	233	0	355	234	0
N.S.	1	1.27	0.82	1.24	0.74	1.45	0.00	2.20	1.45	0.00
time (sec)	N/A	0.128	0.104	0.009	0.985	0.708	0.000	0.382	2.973	0.832
Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	134	63	60	136	74	0	72	-1	74
N.S.	1	0.94	0.44	0.42	0.95	0.52	0.00	0.50	-0.01	0.52
time (sec)	N/A	0.162	0.024	0.009	1.033	0.673	0.000	0.299	0.000	0.819
Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	187	63	60	47	74	0	72	-1	74
N.S.	1	1.31	0.44	0.42	0.33	0.52	0.00	0.50	-0.01	0.52
time (sec)	N/A	0.105	0.023	0.008	1.216	0.865	0.000	0.287	0.000	0.074
Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	78	63	60	98	74	0	48	50	74
N.S.	1	1.18	0.95	0.91	1.48	1.12	0.00	0.73	0.76	1.12
time (sec)	N/A	0.113	0.023	0.006	1.042	0.920	0.000	0.300	2.831	0.758
Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	143	187	63	60	47	74	0	72	-1	74
N.S.	1	1.31	0.44	0.42	0.33	0.52	0.00	0.50	-0.01	0.52
time (sec)	N/A	0.102	0.020	0.008	1.197	0.955	0.000	0.268	0.000	0.069

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	29	59	60	73	0	25	40	73
N.S.	1	1.00	0.91	1.84	1.88	2.28	0.00	0.78	1.25	2.28
time (sec)	N/A	0.022	0.013	0.007	0.932	0.804	0.000	0.248	2.835	0.700
Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	175	61	58	43	72	0	46	-1	72
N.S.	1	1.30	0.45	0.43	0.32	0.53	0.00	0.34	-0.01	0.53
time (sec)	N/A	0.053	0.016	0.003	0.960	0.666	0.000	0.327	0.000	0.040
Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	139	183	62	59	171	73	0	73	-1	263
N.S.	1	1.32	0.45	0.42	1.23	0.53	0.00	0.53	-0.01	1.89
time (sec)	N/A	0.103	0.024	0.023	0.977	0.723	0.000	0.346	0.000	0.567
Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	178	62	60	48	72	0	69	-1	73
N.S.	1	1.33	0.46	0.45	0.36	0.54	0.00	0.51	-0.01	0.54
time (sec)	N/A	0.093	0.026	0.008	0.970	0.484	0.000	0.244	0.000	0.102
Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	184	65	61	176	76	0	91	-1	324
N.S.	1	1.31	0.46	0.44	1.26	0.54	0.00	0.65	-0.01	2.31
time (sec)	N/A	0.105	0.027	0.017	0.986	0.685	0.000	0.294	0.000	1.016
Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	253	254	143	236	0	433	0	177	-1	174
N.S.	1	1.00	0.57	0.93	0.00	1.71	0.00	0.70	-0.00	0.69
time (sec)	N/A	0.246	0.153	0.036	0.000	0.836	0.000	0.507	0.000	10.955

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	29	26	70	87	0	28	62	29
N.S.	1	1.00	0.91	0.81	2.19	2.72	0.00	0.88	1.94	0.91
time (sec)	N/A	0.027	0.017	0.006	1.081	0.638	0.000	0.306	2.708	13.181
Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	207	208	132	205	0	402	0	153	-1	155
N.S.	1	1.00	0.64	0.99	0.00	1.94	0.00	0.74	-0.00	0.75
time (sec)	N/A	0.073	0.113	0.018	0.000	0.618	0.000	0.330	0.000	14.688
Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	192	194	111	221	0	391	0	185	-1	146
N.S.	1	1.01	0.58	1.15	0.00	2.04	0.00	0.96	-0.01	0.76
time (sec)	N/A	0.217	0.076	0.019	0.000	0.711	0.000	0.357	0.000	16.444
Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	209	65	215	0	396	0	185	-1	157
N.S.	1	1.00	0.31	1.03	0.00	1.90	0.00	0.89	-0.00	0.75
time (sec)	N/A	0.189	0.017	0.019	0.000	0.731	0.000	0.544	0.000	19.200
Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	204	48	238	0	411	0	204	-1	152
N.S.	1	1.01	0.24	1.18	0.00	2.03	0.00	1.01	-0.00	0.75
time (sec)	N/A	0.216	0.019	0.021	0.000	0.812	0.000	0.394	0.000	20.095
Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	75	89	60	0	141	0	71	-1	88
N.S.	1	0.97	1.16	0.78	0.00	1.83	0.00	0.92	-0.01	1.14
time (sec)	N/A	0.143	0.067	0.013	0.000	0.640	0.000	0.459	0.000	5.722

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	26	19	19	53	28	19	21
N.S.	1	1.00	1.00	1.24	0.90	0.90	2.52	1.33	0.90	1.00
time (sec)	N/A	0.018	0.005	0.005	1.063	0.897	1.507	0.391	2.644	0.028
Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	26	0	19	66	28	19	27
N.S.	1	1.00	1.00	1.24	0.00	0.90	3.14	1.33	0.90	1.29
time (sec)	N/A	0.017	0.008	0.002	0.000	0.497	1.463	0.483	2.739	5.622
Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	73	38	64	80	138	0	59	-1	60
N.S.	1	1.03	0.54	0.90	1.13	1.94	0.00	0.83	-0.01	0.85
time (sec)	N/A	0.136	0.009	0.010	1.953	0.684	0.000	0.246	0.000	0.079
Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	32	37	46	32	0	81	54	38
N.S.	1	1.00	0.67	0.77	0.96	0.67	0.00	1.69	1.12	0.79
time (sec)	N/A	0.114	0.009	0.006	0.965	0.427	0.000	0.343	2.830	5.647
Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	112	40	81	121	175	0	103	-1	77
N.S.	1	1.08	0.38	0.78	1.16	1.68	0.00	0.99	-0.01	0.74
time (sec)	N/A	0.153	0.011	0.010	1.990	0.698	0.000	0.328	0.000	0.129
Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	152	63	58	85	75	0	137	109	83
N.S.	1	1.10	0.46	0.42	0.62	0.54	0.00	0.99	0.79	0.60
time (sec)	N/A	0.188	0.038	0.008	0.984	0.921	0.000	0.336	2.965	19.492

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	102	113	52	47	64	63	116	109	88	71
N.S.	1	1.11	0.51	0.46	0.63	0.62	1.14	1.07	0.86	0.70
time (sec)	N/A	0.157	0.026	0.007	0.932	0.717	88.605	0.242	2.901	25.559
Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	74	41	36	43	51	87	81	67	59
N.S.	1	1.12	0.62	0.55	0.65	0.77	1.32	1.23	1.02	0.89
time (sec)	N/A	0.137	0.021	0.007	0.932	0.651	38.304	0.301	2.885	12.360
Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	31	26	25	37	58	17	28	45
N.S.	1	1.00	0.86	0.72	0.69	1.03	1.61	0.47	0.78	1.25
time (sec)	N/A	0.020	0.007	0.004	0.909	1.078	14.889	0.270	2.769	9.747
Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	F(-1)	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	141	96	0	118	0	0	190	-1	593
N.S.	1	1.21	0.82	0.00	1.01	0.00	0.00	1.62	-0.01	5.07
time (sec)	N/A	0.158	0.038	0.033	1.968	0.000	0.000	0.407	0.000	72.028
Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	F(-1)	F	A	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	151	50	0	138	0	0	212	-1	0
N.S.	1	1.14	0.38	0.00	1.04	0.00	0.00	1.59	-0.01	0.00
time (sec)	N/A	0.163	0.013	0.015	2.404	0.000	0.000	0.398	0.000	180.004
Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	106	122	0	80	0	61	-1	82
N.S.	1	1.00	1.49	1.72	0.00	1.13	0.00	0.86	-0.01	1.15
time (sec)	N/A	0.025	0.162	0.015	0.000	0.734	0.000	0.287	0.000	0.292

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F(-2)	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	72	28	0	43	0	22	-1	34
N.S.	1	1.00	2.25	0.88	0.00	1.34	0.00	0.69	-0.03	1.06
time (sec)	N/A	0.013	0.027	0.007	0.000	0.891	0.000	0.426	0.000	0.281
Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	198	527	414	541	0	243	-1	225
N.S.	1	1.00	0.81	2.16	1.70	2.22	0.00	1.00	-0.00	0.92
time (sec)	N/A	0.333	0.523	0.067	2.279	0.956	0.000	0.726	0.000	0.482
Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	161	161	149	342	269	407	0	185	-1	154
N.S.	1	1.00	0.93	2.12	1.67	2.53	0.00	1.15	-0.01	0.96
time (sec)	N/A	0.163	0.381	0.038	2.246	0.898	0.000	0.817	0.000	0.281
Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	143	200	145	313	0	149	-1	107
N.S.	1	1.00	1.39	1.94	1.41	3.04	0.00	1.45	-0.01	1.04
time (sec)	N/A	0.070	0.284	0.025	2.254	0.684	0.000	0.821	0.000	0.158
Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	173	179	149	865	0	0	-1	116
N.S.	1	1.00	1.54	1.60	1.33	7.72	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.128	0.179	0.042	2.242	1.342	0.000	0.000	0.000	0.149
Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	127	127	133	326	145	333	0	269	-1	113
N.S.	1	1.00	1.05	2.57	1.14	2.62	0.00	2.12	-0.01	0.89
time (sec)	N/A	0.087	0.102	0.053	2.234	1.200	0.000	0.676	0.000	0.161

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	208	208	174	558	265	427	0	598	-1	161
N.S.	1	1.00	0.84	2.68	1.27	2.05	0.00	2.88	-0.00	0.77
time (sec)	N/A	0.170	0.114	0.066	2.171	2.320	0.000	0.823	0.000	0.272
Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	318	318	222	849	410	561	0	1076	-1	234
N.S.	1	1.00	0.70	2.67	1.29	1.76	0.00	3.38	-0.00	0.74
time (sec)	N/A	0.312	0.176	0.077	2.291	6.169	0.000	0.976	0.000	0.472
Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	282	282	294	1027	454	553	0	0	-1	233
N.S.	1	1.00	1.04	3.64	1.61	1.96	0.00	0.00	-0.00	0.83
time (sec)	N/A	0.383	0.541	0.076	2.118	3.456	0.000	0.000	0.000	0.478
Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	191	679	303	417	0	0	-1	157
N.S.	1	1.00	0.96	3.41	1.52	2.10	0.00	0.00	-0.01	0.79
time (sec)	N/A	0.221	0.616	0.057	2.289	1.933	0.000	0.000	0.000	0.367
Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	96	432	189	328	0	0	-1	120
N.S.	1	1.00	0.68	3.06	1.34	2.33	0.00	0.00	-0.01	0.85
time (sec)	N/A	0.090	0.059	0.048	2.210	1.328	0.000	0.000	0.000	0.197
Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	193	401	197	1049	0	0	-1	156
N.S.	1	1.00	1.28	2.66	1.30	6.95	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.192	1.278	0.060	2.271	2.463	0.000	0.000	0.000	0.258

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	146	641	189	350	0	0	-1	127
N.S.	1	1.00	0.88	3.88	1.15	2.12	0.00	0.00	-0.01	0.77
time (sec)	N/A	0.104	0.091	0.076	2.265	2.572	0.000	0.000	0.000	0.209
Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	256	256	186	1042	303	435	0	0	-1	159
N.S.	1	1.00	0.73	4.07	1.18	1.70	0.00	0.00	-0.00	0.62
time (sec)	N/A	0.218	0.129	0.066	2.477	7.435	0.000	0.000	0.000	0.291
Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	366	366	245	1498	453	573	0	0	-1	234
N.S.	1	1.00	0.67	4.09	1.24	1.57	0.00	0.00	-0.00	0.64
time (sec)	N/A	0.369	0.191	0.095	3.038	20.910	0.000	0.000	0.000	0.486
Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	86	52	0	55	39	18	55	51
N.S.	1	1.00	1.69	1.02	0.00	1.08	0.76	0.35	1.08	1.00
time (sec)	N/A	0.023	0.032	0.025	0.000	0.725	21.541	0.285	2.666	0.046
Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	104	78	76	77	66	30	88	72
N.S.	1	1.00	1.44	1.08	1.06	1.07	0.92	0.42	1.22	1.00
time (sec)	N/A	0.032	0.048	0.028	2.055	0.659	66.857	0.353	0.210	0.073
Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	86	68	0	55	0	22	56	53
N.S.	1	1.00	1.62	1.28	0.00	1.04	0.00	0.42	1.06	1.00
time (sec)	N/A	0.029	0.031	0.101	0.000	0.746	0.000	0.291	2.674	0.045

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	113	113	98	80	0	65	0	0	101	63
N.S.	1	1.00	0.87	0.71	0.00	0.58	0.00	0.00	0.89	0.56
time (sec)	N/A	0.064	0.036	0.069	0.000	0.784	0.000	0.000	2.658	0.067
Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	109	130	121	82	0	47	134	77
N.S.	1	1.00	1.03	1.23	1.14	0.77	0.00	0.44	1.26	0.73
time (sec)	N/A	0.056	0.050	0.282	1.993	0.805	0.000	0.366	2.994	0.093
Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	42	0	32	0	40	-1	29
N.S.	1	1.00	0.94	0.81	0.00	0.62	0.00	0.77	-0.02	0.56
time (sec)	N/A	0.099	0.017	0.016	0.000	0.552	0.000	0.420	0.000	0.183
Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	65	60	0	42	0	61	-1	35
N.S.	1	1.00	0.96	0.88	0.00	0.62	0.00	0.90	-0.01	0.51
time (sec)	N/A	0.191	0.025	0.040	0.000	0.636	0.000	0.453	0.000	0.339
Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	281	281	224	527	413	545	0	0	-1	226
N.S.	1	1.00	0.80	1.88	1.47	1.94	0.00	0.00	-0.00	0.80
time (sec)	N/A	0.290	0.386	0.052	2.269	0.640	0.000	0.000	0.000	0.435
Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	169	169	172	342	268	413	0	0	-1	158
N.S.	1	1.00	1.02	2.02	1.59	2.44	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.135	0.342	0.040	2.231	0.782	0.000	0.000	0.000	0.270

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	106	106	152	200	153	313	0	0	-1	110
N.S.	1	1.00	1.43	1.89	1.44	2.95	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.068	0.132	0.026	2.180	0.663	0.000	0.000	0.000	0.160
Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	112	112	190	179	155	881	0	0	-1	116
N.S.	1	1.00	1.70	1.60	1.38	7.87	0.00	0.00	-0.01	1.04
time (sec)	N/A	0.106	0.235	0.043	1.960	0.986	0.000	0.000	0.000	0.187
Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	133	326	153	333	0	0	-1	116
N.S.	1	1.00	1.02	2.51	1.18	2.56	0.00	0.00	-0.01	0.89
time (sec)	N/A	0.084	0.123	0.053	1.520	1.028	0.000	0.000	0.000	0.174
Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	218	218	173	558	271	443	0	0	-1	164
N.S.	1	1.00	0.79	2.56	1.24	2.03	0.00	0.00	-0.00	0.75
time (sec)	N/A	0.135	0.159	0.064	1.755	2.245	0.000	0.000	0.000	0.285
Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	354	348	247	1027	465	781	0	0	-1	293
N.S.	1	0.98	0.70	2.90	1.31	2.21	0.00	0.00	-0.00	0.83
time (sec)	N/A	0.379	0.475	0.074	1.831	2.846	0.000	0.000	0.000	0.587
Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	190	679	311	585	0	0	-1	203
N.S.	1	1.00	0.94	3.36	1.54	2.90	0.00	0.00	-0.00	1.00
time (sec)	N/A	0.239	0.317	0.062	1.813	1.936	0.000	0.000	0.000	0.344

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	146	86	432	199	443	0	0	-1	153
N.S.	1	1.00	0.59	2.96	1.36	3.03	0.00	0.00	-0.01	1.05
time (sec)	N/A	0.098	0.067	0.050	1.739	1.011	0.000	0.000	0.000	0.249
Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F(-1)	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	152	152	253	401	204	1293	0	0	-1	419
N.S.	1	1.00	1.66	2.64	1.34	8.51	0.00	0.00	-0.01	2.76
time (sec)	N/A	0.196	0.349	0.062	1.915	2.147	0.000	0.000	0.000	0.730
Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	170	170	148	641	197	469	0	0	-1	159
N.S.	1	1.00	0.87	3.77	1.16	2.76	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.111	0.087	0.072	1.972	3.107	0.000	0.000	0.000	0.266
Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	255	255	189	1042	311	613	0	0	-1	209
N.S.	1	1.00	0.74	4.09	1.22	2.40	0.00	0.00	-0.00	0.82
time (sec)	N/A	0.228	0.120	0.081	1.956	10.114	0.000	0.000	0.000	0.368
Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	216	259	137	533	328	423	0	219	-1	161
N.S.	1	1.20	0.63	2.47	1.52	1.96	0.00	1.01	-0.00	0.75
time (sec)	N/A	0.620	0.319	0.063	1.724	0.969	0.000	0.489	0.000	0.275
Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	181	97	354	218	325	0	159	-1	116
N.S.	1	1.28	0.69	2.51	1.55	2.30	0.00	1.13	-0.01	0.82
time (sec)	N/A	0.467	0.180	0.036	1.752	0.733	0.000	0.438	0.000	0.192

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	77	180	126	267	0	127	120	85
N.S.	1	1.00	1.12	2.61	1.83	3.87	0.00	1.84	1.74	1.23
time (sec)	N/A	0.054	0.089	0.024	1.644	0.756	0.000	0.511	3.007	0.118
Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	184	80	235	159	927	0	0	-1	108
N.S.	1	1.92	0.83	2.45	1.66	9.66	0.00	0.00	-0.01	1.12
time (sec)	N/A	0.433	0.136	0.042	1.459	0.917	0.000	0.000	0.000	0.181
Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	104	140	212	454	156	433	0	281	-1	120
N.S.	1	1.35	2.04	4.37	1.50	4.16	0.00	2.70	-0.01	1.15
time (sec)	N/A	0.386	0.469	0.050	1.484	0.643	0.000	0.465	0.000	0.192
Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	218	278	923	322	577	0	713	-1	168
N.S.	1	1.25	1.60	5.30	1.85	3.32	0.00	4.10	-0.01	0.97
time (sec)	N/A	0.507	0.482	0.062	1.532	0.931	0.000	0.584	0.000	0.333
Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	265	271	245	1518	557	755	0	1414	-1	248
N.S.	1	1.02	0.92	5.73	2.10	2.85	0.00	5.34	-0.00	0.94
time (sec)	N/A	0.607	0.501	0.076	1.910	1.529	0.000	0.772	0.000	0.516
Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	249	311	142	1018	368	427	0	527	-1	163
N.S.	1	1.25	0.57	4.09	1.48	1.71	0.00	2.12	-0.00	0.65
time (sec)	N/A	0.730	0.409	0.069	1.709	0.853	0.000	2.104	0.000	0.282

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	172	222	104	593	247	335	0	438	-1	115
N.S.	1	1.29	0.60	3.45	1.44	1.95	0.00	2.55	-0.01	0.67
time (sec)	N/A	0.536	0.244	0.056	1.712	0.794	0.000	1.992	0.000	0.181
Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	79	336	156	269	0	387	61	91
N.S.	1	1.00	0.84	3.57	1.66	2.86	0.00	4.12	0.65	0.97
time (sec)	N/A	0.066	0.102	0.046	2.238	0.614	0.000	1.820	3.739	0.130
Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	206	118	652	201	1073	0	0	-1	139
N.S.	1	1.63	0.94	5.17	1.60	8.52	0.00	0.00	-0.01	1.10
time (sec)	N/A	0.490	0.226	0.061	2.111	1.023	0.000	0.000	0.000	0.213
Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	170	256	820	202	404	0	0	-1	136
N.S.	1	1.23	1.86	5.94	1.46	2.93	0.00	0.00	-0.01	0.99
time (sec)	N/A	0.526	0.580	0.071	2.412	0.882	0.000	0.000	0.000	0.195
Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	205	260	190	1653	313	557	0	0	-1	162
N.S.	1	1.27	0.93	8.06	1.53	2.72	0.00	0.00	-0.00	0.79
time (sec)	N/A	0.591	0.326	0.078	2.422	1.702	0.000	0.000	0.000	0.273
Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	292	287	245	2605	534	733	0	0	-1	248
N.S.	1	0.98	0.84	8.92	1.83	2.51	0.00	0.00	-0.00	0.85
time (sec)	N/A	0.725	0.519	0.085	2.492	2.312	0.000	0.000	0.000	0.445

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	267	140	533	340	425	0	274	-1	163
N.S.	1	1.19	0.62	2.37	1.51	1.89	0.00	1.22	-0.00	0.72
time (sec)	N/A	0.622	0.282	0.052	2.277	0.772	0.000	0.566	0.000	0.273
Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	148	189	101	354	223	333	0	191	-1	118
N.S.	1	1.28	0.68	2.39	1.51	2.25	0.00	1.29	-0.01	0.80
time (sec)	N/A	0.465	0.168	0.038	2.490	0.535	0.000	0.524	0.000	0.186
Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	70	184	129	267	0	138	111	88
N.S.	1	1.00	0.97	2.56	1.79	3.71	0.00	1.92	1.54	1.22
time (sec)	N/A	0.052	0.079	0.027	2.809	0.742	0.000	0.507	3.342	0.112
Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	96	184	80	312	155	972	0	0	-1	109
N.S.	1	1.92	0.83	3.25	1.61	10.12	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.414	0.122	0.042	3.338	0.770	0.000	0.000	0.000	0.153
Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	148	210	452	173	451	0	315	-1	131
N.S.	1	1.37	1.94	4.19	1.60	4.18	0.00	2.92	-0.01	1.21
time (sec)	N/A	0.387	0.300	0.043	2.784	0.890	0.000	0.697	0.000	0.199
Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	177	218	269	922	359	593	0	815	-1	174
N.S.	1	1.23	1.52	5.21	2.03	3.35	0.00	4.60	-0.01	0.98
time (sec)	N/A	0.472	0.479	0.054	2.371	1.022	0.000	10.392	0.000	0.333

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	310	323	1215	1240	389	675	0	597	-1	253
N.S.	1	1.04	3.92	4.00	1.25	2.18	0.00	1.93	-0.00	0.82
time (sec)	N/A	0.783	11.647	0.073	2.359	0.842	0.000	1.670	0.000	0.379
Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	242	133	783	262	541	0	510	-1	189
N.S.	1	1.29	0.71	4.19	1.40	2.89	0.00	2.73	-0.01	1.01
time (sec)	N/A	0.574	0.266	0.063	2.309	0.842	0.000	1.574	0.000	0.239
Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	100	104	50	478	161	395	0	449	61	127
N.S.	1	1.04	0.50	4.78	1.61	3.95	0.00	4.49	0.61	1.27
time (sec)	N/A	0.073	0.056	0.047	2.217	0.892	0.000	1.548	3.930	0.185
Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	214	110	1015	201	1477	0	0	-1	169
N.S.	1	1.60	0.82	7.57	1.50	11.02	0.00	0.00	-0.01	1.26
time (sec)	N/A	0.514	0.465	0.059	2.234	1.016	0.000	0.000	0.000	0.301
Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	146	174	229	1088	247	599	0	0	-1	183
N.S.	1	1.19	1.57	7.45	1.69	4.10	0.00	0.00	-0.01	1.25
time (sec)	N/A	0.449	0.418	0.068	2.342	0.774	0.000	0.000	0.000	0.316
Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	212	246	281	1947	450	961	0	0	-1	259
N.S.	1	1.16	1.33	9.18	2.12	4.53	0.00	0.00	-0.00	1.22
time (sec)	N/A	0.584	0.568	0.081	2.347	1.428	0.000	0.000	0.000	0.413

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	49	64	0	169	0	0	-1	84
N.S.	1	1.00	0.65	0.85	0.00	2.25	0.00	0.00	-0.01	1.12
time (sec)	N/A	0.017	0.021	0.072	0.000	0.803	0.000	0.000	0.000	1.433
Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	42	57	0	153	0	68	-1	69
N.S.	1	1.00	0.84	1.14	0.00	3.06	0.00	1.36	-0.02	1.38
time (sec)	N/A	0.012	0.011	0.061	0.000	1.086	0.000	0.315	0.000	0.325
Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	17	0	98	0	58	-1	35
N.S.	1	1.00	1.00	0.71	0.00	4.08	0.00	2.42	-0.04	1.46
time (sec)	N/A	0.008	0.006	0.051	0.000	0.828	0.000	0.191	0.000	0.327
Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	37	41	17	0	28	17	28
N.S.	1	1.00	1.00	1.61	1.78	0.74	0.00	1.22	0.74	1.22
time (sec)	N/A	0.004	0.004	0.005	2.217	0.601	0.000	0.240	2.693	14.875
Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	30	44	50	25	0	0	29	35
N.S.	1	1.00	0.61	0.90	1.02	0.51	0.00	0.00	0.59	0.71
time (sec)	N/A	0.009	0.006	0.004	2.867	0.523	0.000	0.000	2.672	9.883
Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	26	29	0	29	-1	29
N.S.	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03	0.78
time (sec)	N/A	0.008	0.014	0.013	2.525	0.726	0.000	0.185	0.000	0.060

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	33	28	26	29	0	29	-1	29
N.S.	1	1.00	0.89	0.76	0.70	0.78	0.00	0.78	-0.03	0.78
time (sec)	N/A	0.013	0.005	0.012	2.077	0.552	0.000	0.197	0.000	0.048
Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	44	38	40	41	0	42	-1	43
N.S.	1	1.00	0.62	0.54	0.56	0.58	0.00	0.59	-0.01	0.61
time (sec)	N/A	0.015	0.015	0.012	2.095	0.656	0.000	0.175	0.000	0.071
Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	29	37	42	256	0	48	-1	48
N.S.	1	1.00	0.59	0.76	0.86	5.22	0.00	0.98	-0.02	0.98
time (sec)	N/A	0.013	0.061	0.008	2.704	0.652	0.000	0.182	0.000	0.067
Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	29	37	42	256	0	48	-1	48
N.S.	1	1.00	0.59	0.76	0.86	5.22	0.00	0.98	-0.02	0.98
time (sec)	N/A	0.018	0.016	0.008	2.038	0.671	0.000	0.194	0.000	0.051
Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	30	43	32	127	0	43	-1	51
N.S.	1	1.00	0.68	0.98	0.73	2.89	0.00	0.98	-0.02	1.16
time (sec)	N/A	0.015	0.010	0.015	2.175	0.699	0.000	0.212	0.000	0.078
Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	32	27	0	42	0	27	-1	61
N.S.	1	1.00	0.73	0.61	0.00	0.95	0.00	0.61	-0.02	1.39
time (sec)	N/A	0.006	0.009	0.008	0.000	0.547	0.000	0.194	0.000	0.088

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	19	18	20	19	19	21
N.S.	1	1.00	1.00	0.86	0.86	0.82	0.91	0.86	0.86	0.95
time (sec)	N/A	0.003	0.005	0.003	1.903	0.516	0.482	0.158	2.674	0.033
Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	0	76	0	30	-1	22
N.S.	1	1.00	1.00	0.86	0.00	3.45	0.00	1.36	-0.05	1.00
time (sec)	N/A	0.009	0.004	0.008	0.000	0.716	0.000	0.212	0.000	0.046
Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	18	23	30	0	22	18	21
N.S.	1	1.00	1.00	0.86	1.10	1.43	0.00	1.05	0.86	1.00
time (sec)	N/A	0.004	0.004	0.004	1.970	0.494	0.000	0.179	2.870	0.067
Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	31	28	19	0	12	20	26
N.S.	1	1.00	1.00	1.24	1.12	0.76	0.00	0.48	0.80	1.04
time (sec)	N/A	0.004	0.004	0.006	2.260	0.579	0.000	0.168	2.912	0.034
Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	22	321	0	85	14	35	-1	44
N.S.	1	1.00	0.96	13.96	0.00	3.70	0.61	1.52	-0.04	1.91
time (sec)	N/A	0.015	0.005	0.137	0.000	0.769	1.152	0.189	0.000	7.386
Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	19	0	68	0	31	-1	24
N.S.	1	1.00	1.00	0.79	0.00	2.83	0.00	1.29	-0.04	1.00
time (sec)	N/A	0.009	0.004	0.008	0.000	0.790	0.000	0.201	0.000	0.047

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	A	A	F(-2)	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	80	33	30	18	0	0	0	43	0
N.S.	1	2.35	0.97	0.88	0.53	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	0.030	0.029	0.046	1.513	0.000	0.000	0.000	2.888	1.674
Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	17	17	20	0	20	16	0
N.S.	1	1.00	1.00	1.06	1.06	1.25	0.00	1.25	1.00	0.00
time (sec)	N/A	0.005	0.003	0.002	1.480	0.664	0.000	0.208	3.400	0.020
Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	27	32	32	0	32	26	0
N.S.	1	1.00	1.00	1.04	1.23	1.23	0.00	1.23	1.00	0.00
time (sec)	N/A	0.009	0.007	0.002	1.080	0.645	0.000	0.236	2.918	0.044
Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	36	37	44	44	0	44	36	0
N.S.	1	1.00	1.00	1.03	1.22	1.22	0.00	1.22	1.00	0.00
time (sec)	N/A	0.016	0.010	0.003	1.266	0.522	0.000	0.245	3.103	0.070
Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	140	90	0	94	0	390	179	157
N.S.	1	1.00	0.95	0.61	0.00	0.64	0.00	2.65	1.22	1.07
time (sec)	N/A	0.132	0.166	0.005	0.000	0.582	0.000	0.268	2.954	0.654
Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	95	66	0	70	0	206	129	101
N.S.	1	1.00	1.00	0.69	0.00	0.74	0.00	2.17	1.36	1.06
time (sec)	N/A	0.081	0.098	0.005	0.000	0.536	0.000	0.208	2.703	0.574

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	35	40	0	29	136	75	79	47
N.S.	1	1.00	0.74	0.85	0.00	0.62	2.89	1.60	1.68	1.00
time (sec)	N/A	0.047	0.049	0.003	0.000	0.713	0.712	0.208	2.706	0.354
Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	75	73	0	318	0	1016	2983	275
N.S.	1	1.00	0.77	0.75	0.00	3.28	0.00	10.47	30.75	2.84
time (sec)	N/A	0.104	0.074	0.006	0.000	0.607	0.000	0.859	18.080	0.892
Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	99	88	0	399	0	1190	2642	301
N.S.	1	1.00	0.96	0.85	0.00	3.87	0.00	11.55	25.65	2.92
time (sec)	N/A	0.102	0.268	0.016	0.000	0.592	0.000	10.214	18.884	1.370
Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	228	228	361	604	0	196	0	797	1358	291
N.S.	1	1.00	1.58	2.65	0.00	0.86	0.00	3.50	5.96	1.28
time (sec)	N/A	0.373	1.291	0.023	0.000	0.629	0.000	0.389	81.167	0.600
Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	165	165	229	431	0	149	0	445	1012	183
N.S.	1	1.00	1.39	2.61	0.00	0.90	0.00	2.70	6.13	1.11
time (sec)	N/A	0.210	0.876	0.016	0.000	0.756	0.000	0.271	37.521	0.406
Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	63	114	179	377	0	103	388	189	110	122
N.S.	1	1.81	2.84	5.98	0.00	1.63	6.16	3.00	1.75	1.94
time (sec)	N/A	0.099	0.559	0.009	0.000	0.709	1.042	0.238	0.239	0.272

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	195	258	0	290	0	194	524	144
N.S.	1	1.00	1.47	1.94	0.00	2.18	0.00	1.46	3.94	1.08
time (sec)	N/A	0.230	1.068	0.020	0.000	0.831	0.000	0.673	11.141	0.378
Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	205	274	0	367	0	311	7637	221
N.S.	1	1.00	1.45	1.94	0.00	2.60	0.00	2.21	54.16	1.57
time (sec)	N/A	0.216	0.960	0.017	0.000	0.662	0.000	1.890	28.822	0.754
Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	375	375	282	294	0	208	0	1447	529	267
N.S.	1	1.00	0.75	0.78	0.00	0.55	0.00	3.86	1.41	0.71
time (sec)	N/A	0.372	0.405	0.006	0.000	0.549	0.000	0.807	3.303	0.820
Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	214	222	0	159	942	866	385	180
N.S.	1	1.00	0.82	0.85	0.00	0.61	3.61	3.32	1.48	0.69
time (sec)	N/A	0.236	0.260	0.004	0.000	0.713	2.701	0.378	3.211	0.641
Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	B	B	F	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	151	151	146	0	106	384	427	252	110
N.S.	1	2.36	2.36	2.28	0.00	1.66	6.00	6.67	3.94	1.72
time (sec)	N/A	0.095	0.151	0.006	0.000	0.634	1.821	0.302	2.995	0.530
Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	157	142	181	0	516	0	2652	4060	320
N.S.	1	1.00	0.90	1.15	0.00	3.29	0.00	16.89	25.86	2.04
time (sec)	N/A	0.233	0.194	0.006	0.000	0.748	0.000	2.909	27.722	1.062

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	162	223	187	252	0	675	0	2594	4681	8385
N.S.	1	1.38	1.15	1.56	0.00	4.17	0.00	16.01	28.90	51.76
time (sec)	N/A	0.273	0.591	0.018	0.000	0.914	0.000	39.981	33.215	105.521
Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	14	0	13	63	13	21	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00	1.00
time (sec)	N/A	0.006	0.019	0.003	0.000	0.636	0.940	0.230	2.968	0.102
Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	14	0	13	63	13	21	21
N.S.	1	1.00	1.00	0.67	0.00	0.62	3.00	0.62	1.00	1.00
time (sec)	N/A	0.007	0.017	0.003	0.000	0.674	0.388	0.193	2.937	0.107
Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	16	0	15	51	15	15	23
N.S.	1	1.00	1.00	0.70	0.00	0.65	2.22	0.65	0.65	1.00
time (sec)	N/A	0.023	0.020	0.003	0.000	0.727	0.406	0.190	2.838	0.177
Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	33	31	32	0	77	45	80
N.S.	1	1.00	1.00	0.87	0.82	0.84	0.00	2.03	1.18	2.11
time (sec)	N/A	0.114	0.053	0.006	1.306	0.810	0.000	0.223	3.021	0.197
Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	42	59	34	51	0	76	563	101
N.S.	1	1.00	0.88	1.23	0.71	1.06	0.00	1.58	11.73	2.10
time (sec)	N/A	0.090	0.042	0.008	1.395	0.662	0.000	0.232	14.086	0.183

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	24	15	23	110	51	33	41
N.S.	1	1.00	1.00	1.26	0.79	1.21	5.79	2.68	1.74	2.16
time (sec)	N/A	0.054	0.022	0.003	1.342	0.614	106.392	0.250	2.977	0.143
Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	18	58	17	40	44	48	206	59
N.S.	1	1.00	0.95	3.05	0.89	2.11	2.32	2.53	10.84	3.11
time (sec)	N/A	0.025	0.014	0.006	1.418	0.592	31.434	0.204	7.725	0.137
Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	51	41	41	0	0	122	75
N.S.	1	1.00	1.00	1.59	1.28	1.28	0.00	0.00	3.81	2.34
time (sec)	N/A	0.089	0.040	0.010	1.526	0.466	0.000	0.000	4.098	0.160
Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	22	50	24	44	0	149	120	53
N.S.	1	1.00	0.85	1.92	0.92	1.69	0.00	5.73	4.62	2.04
time (sec)	N/A	0.080	0.028	0.015	1.261	0.543	0.000	0.264	3.795	0.158
Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	45	58	54	44	0	0	189	54
N.S.	1	1.00	1.32	1.71	1.59	1.29	0.00	0.00	5.56	1.59
time (sec)	N/A	0.091	0.042	0.017	1.522	0.396	0.000	0.000	4.879	0.237
Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	147	90	0	122	0	451	179	208
N.S.	1	1.00	1.00	0.61	0.00	0.83	0.00	3.07	1.22	1.41
time (sec)	N/A	0.122	0.223	0.004	0.000	0.414	0.000	0.430	2.920	1.571

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	70	66	0	92	0	255	129	135
N.S.	1	1.00	0.74	0.69	0.00	0.97	0.00	2.68	1.36	1.42
time (sec)	N/A	0.101	0.194	0.005	0.000	0.395	0.000	0.390	2.862	1.038
Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	39	40	0	50	0	107	79	80
N.S.	1	1.00	0.83	0.85	0.00	1.06	0.00	2.28	1.68	1.70
time (sec)	N/A	0.055	0.087	0.004	0.000	0.411	0.000	0.316	2.906	0.711
Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	75	73	0	158	0	1093	213	372
N.S.	1	1.00	0.77	0.75	0.00	1.63	0.00	11.27	2.20	3.84
time (sec)	N/A	0.071	0.047	0.006	0.000	0.424	0.000	0.998	4.330	1.920
Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	135	88	0	182	0	1402	1637	4583
N.S.	1	1.00	1.31	0.85	0.00	1.77	0.00	13.61	15.89	44.50
time (sec)	N/A	0.095	0.211	0.010	0.000	0.417	0.000	12.671	10.926	81.011
Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	75	120	0	243	0	1895	1610	0
N.S.	1	1.00	0.44	0.70	0.00	1.42	0.00	11.08	9.42	0.00
time (sec)	N/A	0.112	0.090	0.013	0.000	0.451	0.000	37.261	11.847	180.005
Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	238	517	0	479	0	511	1107	255
N.S.	1	1.00	1.22	2.65	0.00	2.46	0.00	2.62	5.68	1.31
time (sec)	N/A	0.349	0.666	0.021	0.000	0.427	0.000	3.472	18.151	1.649

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	142	142	177	385	0	372	0	272	129	197
N.S.	1	1.00	1.25	2.71	0.00	2.62	0.00	1.92	0.91	1.39
time (sec)	N/A	0.228	0.561	0.008	0.000	0.435	0.000	3.277	0.246	0.939
Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	135	135	195	266	0	346	0	306	5098	434
N.S.	1	1.00	1.44	1.97	0.00	2.56	0.00	2.27	37.76	3.21
time (sec)	N/A	0.181	0.859	0.021	0.000	0.447	0.000	3.526	19.761	1.367
Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	178	272	0	317	0	438	4285	378
N.S.	1	1.00	1.29	1.97	0.00	2.30	0.00	3.17	31.05	2.74
time (sec)	N/A	0.114	0.713	0.017	0.000	0.442	0.000	4.497	17.444	1.888
Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	109	313	0	126	0	532	787	250
N.S.	1	1.00	0.89	2.54	0.00	1.02	0.00	4.33	6.40	2.03
time (sec)	N/A	0.201	0.145	0.014	0.000	0.407	0.000	12.251	12.320	2.963
Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	174	174	153	457	0	182	0	802	1290	0
N.S.	1	1.00	0.88	2.63	0.00	1.05	0.00	4.61	7.41	0.00
time (sec)	N/A	0.223	0.181	0.018	0.000	0.411	0.000	16.937	18.735	180.014
Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	277	277	271	246	0	225	0	932	429	7868
N.S.	1	1.00	0.98	0.89	0.00	0.81	0.00	3.36	1.55	28.40
time (sec)	N/A	0.319	0.393	0.006	0.000	0.423	0.000	5.577	3.342	61.516

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	120	172	0	167	0	480	268	197
N.S.	1	1.00	0.74	1.06	0.00	1.02	0.00	2.94	1.64	1.21
time (sec)	N/A	0.218	0.443	0.004	0.000	0.393	0.000	5.226	3.360	1.363
Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	155	155	119	148	0	321	0	2374	762	0
N.S.	1	1.00	0.77	0.95	0.00	2.07	0.00	15.32	4.92	0.00
time (sec)	N/A	0.202	0.271	0.005	0.000	0.420	0.000	10.122	7.023	180.401
Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	157	223	192	237	0	260	0	2318	559	0
N.S.	1	1.42	1.22	1.51	0.00	1.66	0.00	14.76	3.56	0.00
time (sec)	N/A	0.217	0.883	0.014	0.000	0.418	0.000	78.698	7.488	180.006
Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	275	182	300	0	297	0	0	287	0
N.S.	1	1.68	1.11	1.83	0.00	1.81	0.00	0.00	1.75	0.00
time (sec)	N/A	0.178	0.266	0.018	0.000	0.448	0.000	0.000	5.743	180.030
Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	63	23	44	48	54	209	74
N.S.	1	1.00	1.00	2.03	0.74	1.42	1.55	1.74	6.74	2.39
time (sec)	N/A	0.046	0.015	0.005	1.494	0.400	3.073	0.613	8.122	0.143
Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	38	33	31	32	0	77	42	82
N.S.	1	1.00	1.00	0.87	0.82	0.84	0.00	2.03	1.11	2.16
time (sec)	N/A	0.325	0.040	0.006	1.385	0.395	0.000	0.539	3.065	0.198

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	48	48	43	59	34	51	0	76	381	99
N.S.	1	1.00	0.90	1.23	0.71	1.06	0.00	1.58	7.94	2.06
time (sec)	N/A	0.242	0.037	0.004	1.496	0.394	0.000	0.469	10.394	0.195
Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	26	17	25	110	54	25	43
N.S.	1	1.00	1.00	1.24	0.81	1.19	5.24	2.57	1.19	2.05
time (sec)	N/A	0.113	0.020	0.003	1.472	0.416	101.264	0.391	3.061	0.150
Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	21	59	20	41	46	49	205	59
N.S.	1	1.00	0.95	2.68	0.91	1.86	2.09	2.23	9.32	2.68
time (sec)	N/A	0.055	0.014	0.005	1.389	0.392	37.873	0.368	3.714	0.139
Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	51	41	41	0	0	122	75
N.S.	1	1.00	1.00	1.59	1.28	1.28	0.00	0.00	3.81	2.34
time (sec)	N/A	0.196	0.029	0.007	1.953	0.399	0.000	0.000	4.127	0.167
Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	22	50	24	44	0	149	118	53
N.S.	1	1.00	0.85	1.92	0.92	1.69	0.00	5.73	4.54	2.04
time (sec)	N/A	0.205	0.026	0.013	1.896	0.409	0.000	0.461	3.787	0.166
Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	46	57	51	43	0	0	186	51
N.S.	1	1.00	1.39	1.73	1.55	1.30	0.00	0.00	5.64	1.55
time (sec)	N/A	0.215	0.039	0.015	2.017	0.400	0.000	0.000	4.777	0.252

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	28	28	48	48	0	36	0	0	93	74
N.S.	1	1.00	1.71	1.71	0.00	1.29	0.00	0.00	3.32	2.64
time (sec)	N/A	0.321	0.156	0.007	0.000	0.382	0.000	0.000	4.493	0.184
Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	33	33	58	62	0	37	226	41	200	64
N.S.	1	1.00	1.76	1.88	0.00	1.12	6.85	1.24	6.06	1.94
time (sec)	N/A	0.143	0.162	0.007	0.000	0.415	31.387	0.381	10.853	0.137
Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	175	175	158	175	266	161	279	163	-1	179
N.S.	1	1.00	0.90	1.00	1.52	0.92	1.59	0.93	-0.01	1.02
time (sec)	N/A	0.133	0.320	0.013	0.936	0.415	10.464	0.450	0.000	0.840
Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	136	136	128	126	99	114	116	103	210	132
N.S.	1	1.00	0.94	0.93	0.73	0.84	0.85	0.76	1.54	0.97
time (sec)	N/A	0.103	0.198	0.007	0.889	0.398	4.481	0.521	4.657	0.601
Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	81	75	46	74	54	65	136	91
N.S.	1	1.00	1.19	1.10	0.68	1.09	0.79	0.96	2.00	1.34
time (sec)	N/A	0.034	0.047	0.004	0.873	0.401	2.205	0.345	3.888	0.277
Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	117	117	109	1325	0	187	0	0	-1	395
N.S.	1	1.00	0.93	11.32	0.00	1.60	0.00	0.00	-0.01	3.38
time (sec)	N/A	0.094	0.145	0.039	0.000	0.456	0.000	0.000	0.000	1.017

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	141	4136	0	284	0	0	-1	433
N.S.	1	1.00	0.93	27.39	0.00	1.88	0.00	0.00	-0.01	2.87
time (sec)	N/A	0.113	0.290	0.038	0.000	0.468	0.000	0.000	0.000	1.696
Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	193	193	180	9721	0	536	0	0	-1	528
N.S.	1	1.00	0.93	50.37	0.00	2.78	0.00	0.00	-0.01	2.74
time (sec)	N/A	0.127	0.575	0.052	0.000	0.709	0.000	0.000	0.000	2.536
Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	225	225	213	0	0	416	0	0	-1	277
N.S.	1	1.00	0.95	0.00	0.00	1.85	0.00	0.00	-0.00	1.23
time (sec)	N/A	0.185	0.338	0.050	0.000	0.497	0.000	0.000	0.000	1.521
Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	183	183	175	0	0	337	0	0	-1	222
N.S.	1	1.00	0.96	0.00	0.00	1.84	0.00	0.00	-0.01	1.21
time (sec)	N/A	0.150	0.235	0.021	0.000	0.494	0.000	0.000	0.000	1.367
Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	139	0	0	301	0	0	-1	184
N.S.	1	1.00	0.95	0.00	0.00	2.05	0.00	0.00	-0.01	1.25
time (sec)	N/A	0.122	0.334	0.011	0.000	0.498	0.000	0.000	0.000	0.888
Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	143	0	0	298	0	0	-1	171
N.S.	1	1.00	0.97	0.00	0.00	2.03	0.00	0.00	-0.01	1.16
time (sec)	N/A	0.109	0.250	0.020	0.000	0.508	0.000	0.000	0.000	0.868

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	158	158	167	0	0	487	0	0	-1	194
N.S.	1	1.00	1.06	0.00	0.00	3.08	0.00	0.00	-0.01	1.23
time (sec)	N/A	0.156	0.425	0.013	0.000	0.520	0.000	0.000	0.000	1.306
Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	199	199	186	0	0	812	0	0	-1	226
N.S.	1	1.00	0.93	0.00	0.00	4.08	0.00	0.00	-0.01	1.14
time (sec)	N/A	0.177	0.602	0.015	0.000	0.544	0.000	0.000	0.000	1.406
Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	40	0	0	26	0	0	-1	41
N.S.	1	1.00	0.98	0.00	0.00	0.63	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.016	0.015	0.079	0.000	0.395	0.000	0.000	0.000	0.053
Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	67	0	0	59	0	0	-1	69
N.S.	1	1.00	0.97	0.00	0.00	0.86	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.057	0.058	0.045	0.000	0.415	0.000	0.000	0.000	0.663
Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	35	60	0	34	418	0	-1	56
N.S.	1	1.00	0.78	1.33	0.00	0.76	9.29	0.00	-0.02	1.24
time (sec)	N/A	0.010	0.102	0.039	0.000	0.421	1.277	0.000	0.000	0.075
Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	44	55	0	28	197	0	-1	50
N.S.	1	1.00	1.07	1.34	0.00	0.68	4.80	0.00	-0.02	1.22
time (sec)	N/A	0.007	0.060	0.028	0.000	0.446	1.180	0.000	0.000	0.082

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	B	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	44	64	0	30	197	0	-1	50
N.S.	1	1.00	1.07	1.56	0.00	0.73	4.80	0.00	-0.02	1.22
time (sec)	N/A	0.007	0.062	0.025	0.000	0.426	1.222	0.000	0.000	0.061
Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	66	55	0	0	70	0	0	-1	109
N.S.	1	1.00	0.83	0.00	0.00	1.06	0.00	0.00	-0.02	1.65
time (sec)	N/A	0.033	0.226	0.044	0.000	0.496	0.000	0.000	0.000	6.787
Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	303	303	276	685	0	345	0	373	-1	782
N.S.	1	1.00	0.91	2.26	0.00	1.14	0.00	1.23	-0.00	2.58
time (sec)	N/A	0.384	0.552	0.015	0.000	0.814	0.000	0.370	0.000	10.571
Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	237	237	213	409	0	219	0	224	-1	624
N.S.	1	1.00	0.90	1.73	0.00	0.92	0.00	0.95	-0.00	2.63
time (sec)	N/A	0.239	0.318	0.009	0.000	0.638	0.000	0.296	0.000	24.457
Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	118	118	120	173	0	123	0	111	-1	339
N.S.	1	1.00	1.02	1.47	0.00	1.04	0.00	0.94	-0.01	2.87
time (sec)	N/A	0.063	0.207	0.006	0.000	0.611	0.000	0.217	0.000	4.352
Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	215	215	187	4918	0	371	0	0	-1	658
N.S.	1	1.00	0.87	22.87	0.00	1.73	0.00	0.00	-0.00	3.06
time (sec)	N/A	0.193	0.205	0.065	0.000	8.119	0.000	0.000	0.000	2.439

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	266	266	237	58067	0	826	0	0	-1	614
N.S.	1	1.00	0.89	218.30	0.00	3.11	0.00	0.00	-0.00	2.31
time (sec)	N/A	0.232	0.321	0.052	0.000	3.686	0.000	0.000	0.000	5.246

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	330	330	300	295147	0	1954	0	0	-1	0
N.S.	1	1.00	0.91	894.38	0.00	5.92	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.289	0.689	0.164	0.000	32.344	0.000	0.000	0.000	180.263

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	370	370	357	0	0	923	0	0	-1	421
N.S.	1	1.00	0.96	0.00	0.00	2.49	0.00	0.00	-0.00	1.14
time (sec)	N/A	0.600	1.061	0.054	0.000	1.052	0.000	0.000	0.000	5.047

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	302	302	291	0	0	657	0	0	-1	725
N.S.	1	1.00	0.96	0.00	0.00	2.18	0.00	0.00	-0.00	2.40
time (sec)	N/A	0.415	0.568	0.022	0.000	0.830	0.000	0.000	0.000	3.684

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	223	0	0	692	0	0	-1	404
N.S.	1	1.00	0.96	0.00	0.00	2.97	0.00	0.00	-0.00	1.73
time (sec)	N/A	0.305	0.393	0.012	0.000	0.975	0.000	0.000	0.000	1.833

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	244	244	238	0	0	716	0	0	-1	434
N.S.	1	1.00	0.98	0.00	0.00	2.93	0.00	0.00	-0.00	1.78
time (sec)	N/A	0.292	0.572	0.021	0.000	1.140	0.000	0.000	0.000	1.710

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	269	269	257	0	0	1456	0	0	-1	458
N.S.	1	1.00	0.96	0.00	0.00	5.41	0.00	0.00	-0.00	1.70
time (sec)	N/A	0.368	0.486	0.014	0.000	1.418	0.000	0.000	0.000	3.050
Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	335	335	315	0	0	2514	0	0	-1	605
N.S.	1	1.00	0.94	0.00	0.00	7.50	0.00	0.00	-0.00	1.81
time (sec)	N/A	0.500	0.861	0.015	0.000	2.087	0.000	0.000	0.000	4.042
Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	164	164	138	216	0	158	0	0	-1	0
N.S.	1	1.00	0.84	1.32	0.00	0.96	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.110	0.383	0.086	0.000	0.737	0.000	0.000	0.000	0.284
Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	92	167	0	78	0	0	-1	0
N.S.	1	1.00	0.85	1.55	0.00	0.72	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.063	0.140	0.020	0.000	0.677	0.000	0.000	0.000	0.239
Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	43	120	0	32	2147	0	-1	0
N.S.	1	1.00	0.83	2.31	0.00	0.62	41.29	0.00	-0.02	0.00
time (sec)	N/A	0.021	0.034	0.017	0.000	0.531	2.749	0.000	0.000	0.175
Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	176	176	150	0	0	159	0	0	-1	0
N.S.	1	1.00	0.85	0.00	0.00	0.90	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.109	0.392	0.056	0.000	0.534	0.000	0.000	0.000	0.357

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	100	0	0	79	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	0.68	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.064	0.135	0.049	0.000	0.684	0.000	0.000	0.000	0.289
Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	50	0	0	33	0	0	-1	0
N.S.	1	1.00	0.89	0.00	0.00	0.59	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.023	0.061	0.049	0.000	0.582	0.000	0.000	0.000	0.197
Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	187	187	157	0	0	201	0	0	-1	0
N.S.	1	1.00	0.84	0.00	0.00	1.07	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.117	0.379	0.052	0.000	0.753	0.000	0.000	0.000	0.262
Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	111	0	0	110	0	0	-1	0
N.S.	1	1.00	0.85	0.00	0.00	0.84	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.094	0.231	0.051	0.000	0.540	0.000	0.000	0.000	0.156
Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	65	0	0	48	0	0	-1	0
N.S.	1	1.00	0.87	0.00	0.00	0.64	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.076	0.073	0.052	0.000	0.862	0.000	0.000	0.000	0.288
Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	B	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	0	0	15	311	0	15	17
N.S.	1	1.00	1.00	0.00	0.00	0.88	18.29	0.00	0.88	1.00
time (sec)	N/A	0.054	0.007	0.050	0.000	0.659	2.641	0.000	3.000	0.044

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	173	0	0	204	0	0	-1	0
N.S.	1	1.00	0.86	0.00	0.00	1.01	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.112	0.486	0.060	0.000	0.817	0.000	0.000	0.000	0.293
Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	141	141	123	0	0	113	0	0	-1	0
N.S.	1	1.00	0.87	0.00	0.00	0.80	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.095	0.253	0.059	0.000	0.568	0.000	0.000	0.000	0.142
Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	73	0	0	51	0	0	-1	0
N.S.	1	1.00	0.90	0.00	0.00	0.63	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.075	0.073	0.061	0.000	0.826	0.000	0.000	0.000	0.245
Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	0	0	18	36	0	18	20
N.S.	1	1.00	1.00	0.00	0.00	0.90	1.80	0.00	0.90	1.00
time (sec)	N/A	0.057	0.008	0.059	0.000	0.596	1.568	0.000	3.193	0.042
Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	365	365	280	0	0	654	0	0	-1	0
N.S.	1	1.00	0.77	0.00	0.00	1.79	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.474	2.913	0.112	0.000	0.659	0.000	0.000	0.000	4.003
Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	239	239	186	0	0	239	0	0	-1	0
N.S.	1	1.00	0.78	0.00	0.00	1.00	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.250	0.612	0.011	0.000	0.711	0.000	0.000	0.000	2.801

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	89	0	0	80	0	0	-1	0
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.089	0.392	0.010	0.000	0.786	0.000	0.000	0.000	1.395
Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	89	0	0	80	0	0	-1	0
N.S.	1	1.00	0.83	0.00	0.00	0.75	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.133	0.041	0.010	0.000	0.701	0.000	0.000	0.000	0.894
Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	297	297	228	0	0	377	0	0	-1	0
N.S.	1	1.00	0.77	0.00	0.00	1.27	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.419	1.200	0.109	0.000	0.705	0.000	0.000	0.000	7.974
Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	171	171	135	0	0	122	0	0	-1	0
N.S.	1	1.00	0.79	0.00	0.00	0.71	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.325	0.351	0.109	0.000	0.776	0.000	0.000	0.000	3.829
Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	36	0	0	41	0	0	41	43
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	1.00	1.05
time (sec)	N/A	0.254	0.075	0.107	0.000	0.723	0.000	0.000	3.084	0.700
Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	36	0	0	41	0	0	39	43
N.S.	1	1.00	0.88	0.00	0.00	1.00	0.00	0.00	0.95	1.05
time (sec)	N/A	0.444	0.034	0.108	0.000	0.566	0.000	0.000	3.076	0.569

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	327	327	175	0	0	231	0	0	-1	0
N.S.	1	1.00	0.54	0.00	0.00	0.71	0.00	0.00	-0.00	0.00
time (sec)	N/A	0.617	0.234	0.110	0.000	0.753	0.000	0.000	0.000	0.901
Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	76	0	0	117	0	0	-1	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.529	0.100	0.107	0.000	0.668	0.000	0.000	0.000	0.928
Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	76	0	0	117	0	0	-1	0
N.S.	1	1.00	0.82	0.00	0.00	1.26	0.00	0.00	-0.01	0.00
time (sec)	N/A	0.735	0.038	0.107	0.000	0.658	0.000	0.000	0.000	0.869
Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	191	394	0	141	70	0	50	0
N.S.	1	1.00	2.36	4.86	0.00	1.74	0.86	0.00	0.62	0.00
time (sec)	N/A	0.058	0.117	0.053	0.000	0.479	0.561	0.000	3.113	0.001
Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	233	394	0	146	63	0	28	0
N.S.	1	1.00	3.19	5.40	0.00	2.00	0.86	0.00	0.38	0.00
time (sec)	N/A	0.045	0.136	0.054	0.000	0.651	0.579	0.000	3.141	0.001
Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	87	70	0	153	70	0	54	0
N.S.	1	1.00	2.29	1.84	0.00	4.03	1.84	0.00	1.42	0.00
time (sec)	N/A	0.063	0.060	0.010	0.000	0.663	0.747	0.000	3.315	0.001

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	87	70	0	155	63	0	67	0
N.S.	1	1.00	2.29	1.84	0.00	4.08	1.66	0.00	1.76	0.00
time (sec)	N/A	0.062	0.059	0.010	0.000	0.610	0.740	0.000	3.118	0.001
Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	B	F	A	F	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	38	38	0	78	0	144	0	0	196	0
N.S.	1	1.00	0.00	2.05	0.00	3.79	0.00	0.00	5.16	0.00
time (sec)	N/A	0.096	0.267	0.069	0.000	0.753	0.000	0.000	3.419	0.140
Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	B	F	A	F	F	B	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	38	38	0	72	0	144	0	0	139	0
N.S.	1	1.00	0.00	1.89	0.00	3.79	0.00	0.00	3.66	0.00
time (sec)	N/A	0.095	0.271	0.068	0.000	0.418	0.000	0.000	5.302	0.135
Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	42	36	155	78	38	42	0
N.S.	1	1.00	1.00	1.00	0.86	3.69	1.86	0.90	1.00	0.00
time (sec)	N/A	0.066	0.020	0.005	2.171	0.393	0.623	10.576	3.100	0.001
Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	46	42	67	168	75	41	199	0
N.S.	1	1.00	1.05	0.95	1.52	3.82	1.70	0.93	4.52	0.00
time (sec)	N/A	0.067	0.022	0.003	1.872	0.410	0.659	10.053	3.107	0.001
Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	85	74	0	208	90	0	278	0
N.S.	1	1.00	2.12	1.85	0.00	5.20	2.25	0.00	6.95	0.00
time (sec)	N/A	0.130	0.049	0.152	0.000	0.407	1.133	0.000	3.209	0.001

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	85	77	0	213	80	0	30	0
N.S.	1	1.00	2.12	1.92	0.00	5.32	2.00	0.00	0.75	0.00
time (sec)	N/A	0.128	0.052	0.141	0.000	0.409	1.124	0.000	3.143	0.001
Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	61	42	78	0	146	0	0	-1	0
N.S.	1	1.45	1.00	1.86	0.00	3.48	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.219	0.315	0.088	0.000	0.440	0.000	0.000	0.000	0.384
Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	61	42	74	0	146	0	0	-1	0
N.S.	1	1.45	1.00	1.76	0.00	3.48	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.218	0.042	0.088	0.000	0.432	0.000	0.000	0.000	0.387
Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	86	74	0	153	73	0	233	0
N.S.	1	1.00	2.15	1.85	0.00	3.82	1.82	0.00	5.82	0.00
time (sec)	N/A	0.089	0.049	0.010	0.000	0.436	1.117	0.000	3.488	0.001
Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	86	74	0	155	66	0	67	0
N.S.	1	1.00	2.15	1.85	0.00	3.88	1.65	0.00	1.68	0.00
time (sec)	N/A	0.089	0.047	0.011	0.000	0.424	1.124	0.000	3.381	0.001
Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	42	36	155	78	38	42	0
N.S.	1	1.00	1.00	1.00	0.86	3.69	1.86	0.90	1.00	0.00
time (sec)	N/A	0.059	0.020	0.004	1.421	0.409	0.793	10.128	3.433	0.001

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	46	42	67	168	75	41	923	0
N.S.	1	1.00	1.05	0.95	1.52	3.82	1.70	0.93	20.98	0.00
time (sec)	N/A	0.062	0.022	0.005	1.537	0.411	0.846	9.217	3.527	0.001
Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	78	0	146	0	0	-1	0
N.S.	1	1.00	1.00	1.86	0.00	3.48	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.223	0.317	0.052	0.000	0.449	0.000	0.000	0.000	0.568
Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	74	0	146	0	0	-1	0
N.S.	1	1.00	1.00	1.76	0.00	3.48	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.216	0.045	0.055	0.000	0.517	0.000	0.000	0.000	0.570
Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	B	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	42	42	0	84	0	165	0	0	-1	0
N.S.	1	1.00	0.00	2.00	0.00	3.93	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.249	0.474	0.089	0.000	1.111	0.000	0.000	0.000	0.227
Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	B	F	A	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	42	42	0	78	0	165	0	0	-1	0
N.S.	1	1.00	0.00	1.86	0.00	3.93	0.00	0.00	-0.02	0.00
time (sec)	N/A	0.243	0.503	0.084	0.000	0.433	0.000	0.000	0.000	0.223
Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	134	134	126	4947	125	233	0	155	167	142
N.S.	1	1.00	0.94	36.92	0.93	1.74	0.00	1.16	1.25	1.06
time (sec)	N/A	0.373	0.215	0.092	0.652	0.611	0.000	0.350	3.644	0.166

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	65	3410	62	161	88	72	123	71
N.S.	1	1.00	0.94	49.42	0.90	2.33	1.28	1.04	1.78	1.03
time (sec)	N/A	0.215	0.091	0.040	0.686	0.671	6.510	0.405	3.498	0.075
Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	1931	21	105	29	22	45	26
N.S.	1	1.00	1.00	83.96	0.91	4.57	1.26	0.96	1.96	1.13
time (sec)	N/A	0.086	0.024	0.037	0.707	0.658	4.479	0.323	3.469	0.042
Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	88	88	107	2175	0	316	88	94	1270	95
N.S.	1	1.00	1.22	24.72	0.00	3.59	1.00	1.07	14.43	1.08
time (sec)	N/A	0.247	0.129	0.042	0.000	0.875	10.498	0.336	3.948	0.102
Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	291	2459	0	530	0	210	4602	172
N.S.	1	1.00	1.93	16.28	0.00	3.51	0.00	1.39	30.48	1.14
time (sec)	N/A	0.352	1.082	0.054	0.000	1.796	0.000	0.372	5.563	0.256
Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	147	147	157	3485	0	1168	0	0	-1	136
N.S.	1	1.00	1.07	23.71	0.00	7.95	0.00	0.00	-0.01	0.93
time (sec)	N/A	0.239	0.266	0.044	0.000	0.988	0.000	0.000	0.000	0.389
Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	103	103	83	1995	0	510	0	107	-1	93
N.S.	1	1.00	0.81	19.37	0.00	4.95	0.00	1.04	-0.01	0.90
time (sec)	N/A	0.067	0.127	0.040	0.000	0.653	0.000	0.364	0.000	0.327

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	139	2289	0	581	0	211	-1	148
N.S.	1	1.00	0.87	14.31	0.00	3.63	0.00	1.32	-0.01	0.92
time (sec)	N/A	0.240	0.338	0.048	0.000	0.745	0.000	0.461	0.000	0.517
Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	140	140	126	1473	125	191	0	156	200	145
N.S.	1	1.00	0.90	10.52	0.89	1.36	0.00	1.11	1.43	1.04
time (sec)	N/A	0.299	0.198	0.182	0.782	0.540	0.000	0.391	3.740	0.146
Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	63	943	62	118	0	72	119	77
N.S.	1	1.00	0.86	12.92	0.85	1.62	0.00	0.99	1.63	1.05
time (sec)	N/A	0.202	0.070	0.026	0.582	0.621	0.000	0.331	3.564	0.086
Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	455	22	61	0	23	60	29
N.S.	1	1.00	1.00	17.50	0.85	2.35	0.00	0.88	2.31	1.12
time (sec)	N/A	0.111	0.029	0.027	0.559	0.719	0.000	0.320	3.508	0.047
Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	107	636	0	232	0	94	156	100
N.S.	1	1.00	1.15	6.84	0.00	2.49	0.00	1.01	1.68	1.08
time (sec)	N/A	0.223	0.111	0.059	0.000	0.926	0.000	0.354	4.308	0.102
Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	154	154	307	863	0	445	0	211	248	175
N.S.	1	1.00	1.99	5.60	0.00	2.89	0.00	1.37	1.61	1.14
time (sec)	N/A	0.298	0.673	0.073	0.000	0.798	0.000	0.345	5.463	0.216

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	0	61	25	32	26	25	32
N.S.	1	1.00	1.00	0.00	2.26	0.93	1.19	0.96	0.93	1.19
time (sec)	N/A	0.110	0.064	0.024	0.646	0.696	33.632	0.370	3.346	0.076
Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	8
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	1.00
time (sec)	N/A	0.006	0.003	0.005	1.282	0.552	0.222	0.318	0.047	0.015
Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	21	21	26	22	9	13
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69	1.00
time (sec)	N/A	0.008	0.004	0.007	1.408	0.684	0.397	0.345	3.080	0.031
Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	20	19	19	22	20	19	25
N.S.	1	1.00	1.00	0.74	0.70	0.70	0.81	0.74	0.70	0.93
time (sec)	N/A	0.012	0.010	0.014	0.647	0.478	0.241	0.340	3.103	0.025
Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	92	24	24	0	24	24	33
N.S.	1	1.00	1.00	2.88	0.75	0.75	0.00	0.75	0.75	1.03
time (sec)	N/A	0.014	0.012	0.030	0.699	0.707	0.000	0.326	0.030	0.032
Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	20	19	19	22	19	19	25
N.S.	1	1.00	1.00	0.80	0.76	0.76	0.88	0.76	0.76	1.00
time (sec)	N/A	0.011	0.009	0.013	0.695	0.575	0.238	0.344	0.032	0.021

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	15	14	14	15	15	14	18
N.S.	1	1.00	0.90	0.75	0.70	0.70	0.75	0.75	0.70	0.90
time (sec)	N/A	0.009	0.006	0.002	0.699	0.689	0.162	0.427	0.077	0.015
Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	24	46	45	47	68	45	73	65
N.S.	1	1.00	0.39	0.74	0.73	0.76	1.10	0.73	1.18	1.05
time (sec)	N/A	0.037	0.007	0.010	1.533	0.637	0.641	0.335	3.076	0.049
Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	73	73	73	173	49	49	0	49	49	72
N.S.	1	1.00	1.00	2.37	0.67	0.67	0.00	0.67	0.67	0.99
time (sec)	N/A	0.028	0.024	0.122	0.703	0.614	0.000	0.321	0.036	0.043
Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	130	83	82	76	121	82	82	117
N.S.	1	1.00	1.00	0.64	0.63	0.58	0.93	0.63	0.63	0.90
time (sec)	N/A	0.046	0.035	0.004	0.734	0.793	3.081	0.356	0.153	0.054
Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	B	F	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	200	200	22	175	272	638	0	139	223	126
N.S.	1	1.00	0.11	0.88	1.36	3.19	0.00	0.70	1.12	0.63
time (sec)	N/A	0.396	0.007	0.045	1.390	2.678	0.000	1.118	0.122	0.071
Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	7	6	6	7	6	6	8
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.88	0.75	0.75	1.00
time (sec)	N/A	0.005	0.002	0.003	1.997	0.637	0.310	0.286	0.139	0.017

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	17	15	15	24
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79	1.26
time (sec)	N/A	0.014	0.009	0.003	0.869	0.904	0.168	0.386	0.042	0.015
Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	24	71	83	120	110	83	42	69
N.S.	1	1.00	0.22	0.66	0.77	1.11	1.02	0.77	0.39	0.64
time (sec)	N/A	0.083	0.007	0.005	1.956	0.625	2.182	0.438	0.085	0.104
Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	83	61	60	62	0	60	78	87
N.S.	1	1.00	1.09	0.80	0.79	0.82	0.00	0.79	1.03	1.14
time (sec)	N/A	0.034	0.018	0.010	1.357	0.719	0.000	0.340	3.080	0.083
Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	119	119	119	76	75	71	0	75	75	110
N.S.	1	1.00	1.00	0.64	0.63	0.60	0.00	0.63	0.63	0.92
time (sec)	N/A	0.049	0.032	0.004	0.527	0.571	0.000	0.464	0.152	0.065
Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	201	201	29	242	293	547	311	140	208	127
N.S.	1	1.00	0.14	1.20	1.46	2.72	1.55	0.70	1.03	0.63
time (sec)	N/A	0.224	0.006	0.030	1.519	2.828	24.147	1.204	0.061	0.083
Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	35	36	15	44	0	0	32	0
N.S.	1	1.00	0.97	1.00	0.42	1.22	0.00	0.00	0.89	0.00
time (sec)	N/A	0.037	0.022	0.006	0.706	0.615	0.000	0.000	3.257	1.729

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	27	5	35	0	76	23	32
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	2.62	0.79	1.10
time (sec)	N/A	0.034	0.014	0.004	0.814	0.702	0.000	0.376	3.096	10.331
Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	27	5	35	0	56	23	30
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	1.93	0.79	1.03
time (sec)	N/A	0.024	0.012	0.001	0.734	0.562	0.000	0.492	3.064	7.218
Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	25	5	33	0	51	21	27
N.S.	1	1.00	1.00	1.00	0.20	1.32	0.00	2.04	0.84	1.08
time (sec)	N/A	0.014	0.011	0.004	0.670	0.770	0.000	0.400	3.044	5.475
Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	24	5	32	0	42	20	27
N.S.	1	1.00	1.00	1.00	0.21	1.33	0.00	1.75	0.83	1.12
time (sec)	N/A	0.034	0.013	0.003	0.858	0.629	0.000	0.431	3.079	2.616
Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	29	27	5	35	0	60	23	29
N.S.	1	1.00	1.00	0.93	0.17	1.21	0.00	2.07	0.79	1.00
time (sec)	N/A	0.033	0.011	0.001	0.836	0.718	0.000	0.506	3.102	4.631
Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	35	8	44	0	0	29	0
N.S.	1	1.00	1.00	1.06	0.24	1.33	0.00	0.00	0.88	0.00
time (sec)	N/A	0.031	0.018	0.005	1.068	0.882	0.000	0.000	3.358	0.489

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	31	5	41	0	15	25	43
N.S.	1	1.00	1.00	1.00	0.16	1.32	0.00	0.48	0.81	1.39
time (sec)	N/A	0.035	0.010	0.004	1.019	0.549	0.000	0.328	3.137	0.123
Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	42	7	41	0	0	27	26
N.S.	1	1.00	1.00	1.50	0.25	1.46	0.00	0.00	0.96	0.93
time (sec)	N/A	0.026	0.008	0.014	1.124	0.665	0.000	0.000	3.105	0.273
Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	B	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	42	4	51	0	0	-1	0
N.S.	1	1.00	1.00	1.50	0.14	1.82	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.019	0.008	0.014	1.039	0.666	0.000	0.000	0.000	0.929
Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	28	7	41	0	0	22	26
N.S.	1	1.00	1.00	1.08	0.27	1.58	0.00	0.00	0.85	1.00
time (sec)	N/A	0.032	0.008	0.006	1.115	0.559	0.000	0.000	3.345	0.273
Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	C	A	F	C	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	31	5	44	0	5	25	0
N.S.	1	1.00	1.00	1.00	0.16	1.42	0.00	0.16	0.81	0.00
time (sec)	N/A	0.034	0.007	0.005	1.058	0.855	0.000	0.366	3.465	0.936
Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	11	11	12	11	11	15
N.S.	1	1.00	1.00	1.20	0.73	0.73	0.80	0.73	0.73	1.00
time (sec)	N/A	0.009	0.025	0.007	0.888	0.651	0.203	0.319	3.519	0.027

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	72	20	13	20	31	13	19	17
N.S.	1	1.00	4.24	1.18	0.76	1.18	1.82	0.76	1.12	1.00
time (sec)	N/A	0.009	0.045	0.004	0.880	0.618	0.266	0.326	3.163	0.028
Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	11	11	31	11	11	15
N.S.	1	1.00	1.00	1.20	0.73	0.73	2.07	0.73	0.73	1.00
time (sec)	N/A	0.008	0.028	0.006	0.869	0.463	0.263	0.398	3.098	0.020
Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	15	18	11	11	31	11	11	15
N.S.	1	1.00	1.00	1.20	0.73	0.73	2.07	0.73	0.73	1.00
time (sec)	N/A	0.008	0.034	0.005	0.870	0.692	0.264	0.405	3.108	0.021
Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	21	11	11	10	11	10	13
N.S.	1	1.00	1.00	1.62	0.85	0.85	0.77	0.85	0.77	1.00
time (sec)	N/A	0.010	0.009	0.003	0.884	0.692	0.160	0.359	3.232	0.023
Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	34	33	32	39	33	32	54
N.S.	1	1.00	1.00	0.77	0.75	0.73	0.89	0.75	0.73	1.23
time (sec)	N/A	0.024	0.021	0.003	0.842	0.519	0.198	0.482	0.111	0.029
Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	20	16	15	15	17	16	15	24
N.S.	1	1.00	0.95	0.76	0.71	0.71	0.81	0.76	0.71	1.14
time (sec)	N/A	0.013	0.008	0.003	0.884	0.582	0.152	0.307	3.005	0.017

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	27	26	25	27	26	25	41
N.S.	1	1.00	1.00	0.82	0.79	0.76	0.82	0.79	0.76	1.24
time (sec)	N/A	0.022	0.013	0.006	0.882	0.702	0.170	0.333	3.095	0.024
Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	35	30	29	42	38	29	44
N.S.	1	1.00	1.00	1.06	0.91	0.88	1.27	1.15	0.88	1.33
time (sec)	N/A	0.021	0.015	0.004	1.049	0.598	0.436	0.361	3.024	0.043
Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	17	17	13	10	10	32	0	10	10
N.S.	1	1.70	1.70	1.30	1.00	1.00	3.20	0.00	1.00	1.00
time (sec)	N/A	0.052	0.028	0.017	0.868	0.692	6.963	0.000	3.273	0.053
Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	13	86	10	8	0	39	33
N.S.	1	1.00	1.00	0.76	5.06	0.59	0.47	0.00	2.29	1.94
time (sec)	N/A	0.038	0.016	0.027	0.947	0.711	53.072	0.000	3.368	0.077
Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	34	38	97	0	0	0	-1	0
N.S.	1	1.00	0.92	1.03	2.62	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.092	0.349	0.031	1.760	0.000	0.000	0.000	0.000	1.574
Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	32	36	95	0	0	0	-1	0
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.097	0.331	0.030	1.731	0.000	0.000	0.000	0.000	1.540

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	31	35	92	0	0	0	148	0
N.S.	1	1.00	0.91	1.03	2.71	0.00	0.00	0.00	4.35	0.00
time (sec)	N/A	0.117	0.313	0.031	1.705	0.000	0.000	0.000	9.778	1.241
Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	B	F(-1)	F(-1)	F	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	0	34	38	95	0	0	0	37	0
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	3.374	0.808	0.033	1.758	0.000	0.000	0.000	9.680	1.541
Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	B	F(-1)	F(-1)	F(-1)	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	0	34	38	95	0	0	0	146	0
N.S.	1	0.00	0.92	1.03	2.57	0.00	0.00	0.00	3.95	0.00
time (sec)	N/A	2.998	1.154	0.032	1.810	0.000	0.000	0.000	9.216	2.374
Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	185	185	88	78	151	94	139	151	167	171
N.S.	1	1.00	0.48	0.42	0.82	0.51	0.75	0.82	0.90	0.92
time (sec)	N/A	0.255	0.307	0.003	0.879	1.141	7.020	0.407	3.370	0.067
Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	138	138	77	66	112	81	110	127	124	125
N.S.	1	1.00	0.56	0.48	0.81	0.59	0.80	0.92	0.90	0.91
time (sec)	N/A	0.166	0.187	0.004	0.878	1.213	5.966	0.428	0.063	0.060
Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	63	54	72	67	94	131	79	79
N.S.	1	1.00	0.71	0.61	0.81	0.75	1.06	1.47	0.89	0.89
time (sec)	N/A	0.090	0.105	0.003	0.876	1.256	4.767	0.512	0.030	0.047

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	40	35	35	49	68	82	36	41
N.S.	1	1.00	0.98	0.85	0.85	1.20	1.66	2.00	0.88	1.00
time (sec)	N/A	0.030	0.026	0.001	0.826	0.755	0.210	0.357	0.046	0.024
Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	57	57	79	51	70	118	65	59	130	67
N.S.	1	1.00	1.39	0.89	1.23	2.07	1.14	1.04	2.28	1.18
time (sec)	N/A	0.066	0.162	0.006	1.931	1.003	27.295	0.424	0.088	0.059
Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	161	60	73	147	139	80	131	64
N.S.	1	1.00	2.98	1.11	1.35	2.72	2.57	1.48	2.43	1.19
time (sec)	N/A	0.066	0.200	0.015	1.968	1.000	87.775	0.437	0.124	0.154
Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	221	81	113	181	0	105	80	93
N.S.	1	1.00	2.76	1.01	1.41	2.26	0.00	1.31	1.00	1.16
time (sec)	N/A	0.074	0.348	0.016	1.958	0.854	0.000	0.363	3.400	0.234
Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	326	326	232	383	268	286	0	915	-1	313
N.S.	1	1.00	0.71	1.17	0.82	0.88	0.00	2.81	-0.00	0.96
time (sec)	N/A	0.243	0.417	0.006	0.961	1.164	0.000	0.630	0.000	0.213
Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	147	183	167	184	0	549	-1	185
N.S.	1	1.00	0.66	0.82	0.75	0.82	0.00	2.45	-0.00	0.83
time (sec)	N/A	0.156	0.237	0.003	0.944	1.014	0.000	0.463	0.000	0.128

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	84	94	93	103	0	279	-1	95
N.S.	1	1.00	0.63	0.71	0.70	0.77	0.00	2.10	-0.01	0.71
time (sec)	N/A	0.096	0.139	0.004	0.941	0.912	0.000	0.368	0.000	0.072
Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	43	41	43	50	0	99	44	56
N.S.	1	1.00	0.77	0.73	0.77	0.89	0.00	1.77	0.79	1.00
time (sec)	N/A	0.033	0.028	0.003	0.907	1.017	0.000	0.356	3.395	0.043
Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	116	221	0	194	0	150	-1	147
N.S.	1	1.00	1.00	1.91	0.00	1.67	0.00	1.29	-0.01	1.27
time (sec)	N/A	0.156	0.131	0.037	0.000	0.957	0.000	0.494	0.000	0.326
Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	137	137	181	151	0	1003	0	232	-1	168
N.S.	1	1.00	1.32	1.10	0.00	7.32	0.00	1.69	-0.01	1.23
time (sec)	N/A	0.169	0.176	0.026	0.000	1.077	0.000	0.656	0.000	0.507
Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	224	224	258	784	0	2856	0	895	-1	310
N.S.	1	1.00	1.15	3.50	0.00	12.75	0.00	4.00	-0.00	1.38
time (sec)	N/A	0.429	0.422	0.037	0.000	1.314	0.000	1.103	0.000	3.084
Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	230	230	213	394	243	228	0	341	317	268
N.S.	1	1.00	0.93	1.71	1.06	0.99	0.00	1.48	1.38	1.17
time (sec)	N/A	0.259	0.201	0.007	0.904	0.870	0.000	0.374	0.069	0.152

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	151	151	138	235	148	138	0	198	184	160
N.S.	1	1.00	0.91	1.56	0.98	0.91	0.00	1.31	1.22	1.06
time (sec)	N/A	0.157	0.133	0.006	0.882	0.834	0.000	0.411	3.213	0.130
Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	82	116	81	71	109	105	89	88
N.S.	1	1.00	0.91	1.29	0.90	0.79	1.21	1.17	0.99	0.98
time (sec)	N/A	0.080	0.067	0.004	0.880	0.922	4.722	0.332	0.054	0.070
Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	39	87	35	33	49	38	33	47
N.S.	1	1.00	0.95	2.12	0.85	0.80	1.20	0.93	0.80	1.15
time (sec)	N/A	0.024	0.019	0.010	0.886	0.812	0.549	0.351	0.053	0.030
Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	61	77	95	125	85	88	181	86
N.S.	1	1.00	0.74	0.94	1.16	1.52	1.04	1.07	2.21	1.05
time (sec)	N/A	0.079	0.090	0.010	2.026	0.650	13.006	0.343	3.282	0.076
Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	130	130	144	216	191	283	0	191	220	135
N.S.	1	1.00	1.11	1.66	1.47	2.18	0.00	1.47	1.69	1.04
time (sec)	N/A	0.178	0.194	0.016	2.035	0.857	0.000	0.456	3.591	0.266
Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	204	204	228	459	367	534	0	375	1094	237
N.S.	1	1.00	1.12	2.25	1.80	2.62	0.00	1.84	5.36	1.16
time (sec)	N/A	0.278	0.429	0.018	2.081	2.148	0.000	0.458	5.009	0.438

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	240	240	273	416	251	392	0	324	461	341
N.S.	1	1.00	1.14	1.73	1.05	1.63	0.00	1.35	1.92	1.42
time (sec)	N/A	0.280	0.272	0.013	0.930	0.927	0.000	0.484	0.094	0.168
Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	166	166	185	253	158	269	0	191	197	211
N.S.	1	1.00	1.11	1.52	0.95	1.62	0.00	1.15	1.19	1.27
time (sec)	N/A	0.172	0.169	0.013	0.929	0.843	0.000	0.372	3.197	0.140
Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	112	125	90	163	131	102	98	109
N.S.	1	1.00	1.18	1.32	0.95	1.72	1.38	1.07	1.03	1.15
time (sec)	N/A	0.090	0.088	0.010	1.049	0.965	42.382	0.406	0.062	0.082
Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	40	142	43	75	124	44	43	53
N.S.	1	1.00	0.85	3.02	0.91	1.60	2.64	0.94	0.91	1.13
time (sec)	N/A	0.033	0.034	0.020	0.873	0.825	1.091	0.335	0.047	0.037
Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	129	129	164	161	176	444	153	174	125	132
N.S.	1	1.00	1.27	1.25	1.36	3.44	1.19	1.35	0.97	1.02
time (sec)	N/A	0.119	0.277	0.014	1.990	1.043	50.510	0.410	3.511	0.152
Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	202	202	230	312	367	854	0	311	275	212
N.S.	1	1.00	1.14	1.54	1.82	4.23	0.00	1.54	1.36	1.05
time (sec)	N/A	0.245	0.758	0.018	2.014	1.630	0.000	0.413	0.731	0.385

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	306	306	401	610	659	1252	0	521	1441	358
N.S.	1	1.00	1.31	1.99	2.15	4.09	0.00	1.70	4.71	1.17
time (sec)	N/A	0.403	0.826	0.023	2.086	3.959	0.000	0.505	5.964	0.818
Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	324	324	232	383	268	231	0	409	-1	313
N.S.	1	1.00	0.72	1.18	0.83	0.71	0.00	1.26	-0.00	0.97
time (sec)	N/A	0.230	0.349	0.003	0.942	1.061	0.000	0.417	0.000	0.191
Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	222	222	147	183	167	140	0	238	-1	185
N.S.	1	1.00	0.66	0.82	0.75	0.63	0.00	1.07	-0.00	0.83
time (sec)	N/A	0.158	0.170	0.003	0.922	1.013	0.000	0.325	0.000	0.123
Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	131	131	84	94	93	71	0	115	-1	95
N.S.	1	1.00	0.64	0.72	0.71	0.54	0.00	0.88	-0.01	0.73
time (sec)	N/A	0.094	0.098	0.004	0.927	1.024	0.000	0.366	0.000	0.062
Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	42	41	42	34	0	38	44	43
N.S.	1	1.00	0.78	0.76	0.78	0.63	0.00	0.70	0.81	0.80
time (sec)	N/A	0.031	0.022	0.006	0.856	1.024	0.000	0.438	3.258	0.038
Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	97	97	97	92	0	743	0	140	-1	128
N.S.	1	1.00	1.00	0.95	0.00	7.66	0.00	1.44	-0.01	1.32
time (sec)	N/A	0.085	0.103	0.018	0.000	1.112	0.000	0.502	0.000	0.217

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	163	163	216	265	0	2493	0	654	-1	200
N.S.	1	1.00	1.33	1.63	0.00	15.29	0.00	4.01	-0.01	1.23
time (sec)	N/A	0.203	0.253	0.027	0.000	1.174	0.000	0.757	0.000	0.656
Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	261	261	281	840	0	4390	0	1303	-1	332
N.S.	1	1.00	1.08	3.22	0.00	16.82	0.00	4.99	-0.00	1.27
time (sec)	N/A	0.481	0.750	0.102	0.000	2.415	0.000	1.271	0.000	2.010
Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	B	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	350	350	555	0	728	1416	0	5699	-1	0
N.S.	1	1.00	1.59	0.00	2.08	4.05	0.00	16.28	-0.00	0.00
time (sec)	N/A	0.279	0.852	0.007	1.126	1.326	0.000	0.968	0.000	0.203
Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	242	242	284	0	402	712	0	2511	-1	0
N.S.	1	1.00	1.17	0.00	1.66	2.94	0.00	10.38	-0.00	0.00
time (sec)	N/A	0.182	0.374	0.005	1.063	1.170	0.000	0.587	0.000	0.197
Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	B	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	145	145	128	0	187	294	0	806	-1	0
N.S.	1	1.00	0.88	0.00	1.29	2.03	0.00	5.56	-0.01	0.00
time (sec)	N/A	0.108	0.163	0.005	0.988	1.040	0.000	0.444	0.000	0.165
Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	A	A	F	B	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	53	0	60	81	0	129	146	0
N.S.	1	1.00	0.85	0.00	0.97	1.31	0.00	2.08	2.35	0.00
time (sec)	N/A	0.040	0.036	0.006	0.933	0.986	0.000	0.439	3.583	0.124

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	93	93	77	70	0	164	122	0	-1	88
N.S.	1	1.00	0.83	0.75	0.00	1.76	1.31	0.00	-0.01	0.95
time (sec)	N/A	0.075	0.075	0.008	0.000	1.015	92.656	0.000	0.000	0.322
Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	61	54	0	130	102	0	-1	68
N.S.	1	1.00	0.87	0.77	0.00	1.86	1.46	0.00	-0.01	0.97
time (sec)	N/A	0.057	0.040	0.003	0.000	0.971	66.154	0.000	0.000	0.299
Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	47	40	0	103	0	0	-1	49
N.S.	1	1.00	0.96	0.82	0.00	2.10	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.042	0.022	0.003	0.000	0.940	0.000	0.000	0.000	0.276
Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	0	78	0	0	-1	30
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.032	0.018	0.006	0.000	0.716	0.000	0.000	0.000	0.267
Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	41	43	0	164	48	0	-1	52
N.S.	1	1.00	0.79	0.83	0.00	3.15	0.92	0.00	-0.02	1.00
time (sec)	N/A	0.046	0.028	0.008	0.000	0.890	11.425	0.000	0.000	0.311
Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	43	59	0	262	70	0	-1	67
N.S.	1	1.00	0.57	0.79	0.00	3.49	0.93	0.00	-0.01	0.89
time (sec)	N/A	0.064	0.034	0.011	0.000	0.925	16.790	0.000	0.000	0.345

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	81	86	0	169	114	0	-1	96
N.S.	1	1.00	0.80	0.85	0.00	1.67	1.13	0.00	-0.01	0.95
time (sec)	N/A	0.073	0.079	0.010	0.000	0.925	85.592	0.000	0.000	0.328
Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	66	65	0	135	95	0	-1	76
N.S.	1	1.00	0.87	0.86	0.00	1.78	1.25	0.00	-0.01	1.00
time (sec)	N/A	0.057	0.047	0.007	0.000	0.934	70.064	0.000	0.000	0.293
Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	51	46	0	110	0	0	-1	53
N.S.	1	1.00	0.96	0.87	0.00	2.08	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.041	0.023	0.005	0.000	0.973	0.000	0.000	0.000	0.282
Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	27	0	80	0	0	-1	32
N.S.	1	1.00	1.00	0.84	0.00	2.50	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.031	0.021	0.007	0.000	0.654	0.000	0.000	0.000	0.272
Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	44	49	0	175	44	0	-1	56
N.S.	1	1.00	0.79	0.88	0.00	3.12	0.79	0.00	-0.02	1.00
time (sec)	N/A	0.044	0.030	0.011	0.000	0.842	15.942	0.000	0.000	0.282
Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	81	81	46	70	0	277	63	0	-1	73
N.S.	1	1.00	0.57	0.86	0.00	3.42	0.78	0.00	-0.01	0.90
time (sec)	N/A	0.061	0.037	0.010	0.000	0.959	15.629	0.000	0.000	0.336

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	32	56	24	21	17	23
N.S.	1	1.00	1.00	0.78	1.39	2.43	1.04	0.91	0.74	1.00
time (sec)	N/A	0.007	0.005	0.004	1.968	0.874	1.012	0.407	3.108	0.001
Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	30	30	30	25	0	78	0	0	-1	30
N.S.	1	1.00	1.00	0.83	0.00	2.60	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.031	0.032	0.007	0.000	0.991	0.000	0.000	0.000	0.271
Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	32	0	116	0	0	-1	37
N.S.	1	1.00	1.00	0.86	0.00	3.14	0.00	0.00	-0.03	1.00
time (sec)	N/A	0.167	0.065	0.012	0.000	0.841	0.000	0.000	0.000	0.410
Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	44	39	0	147	0	0	-1	44
N.S.	1	1.00	1.00	0.89	0.00	3.34	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.373	0.146	0.019	0.000	0.764	0.000	0.000	0.000	0.670
Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-1)	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	51	46	0	182	0	0	-1	51
N.S.	1	1.00	1.00	0.90	0.00	3.57	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.656	0.260	0.024	0.000	0.719	0.000	0.000	0.000	0.901
Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	B	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	34	83	120	55	0	77	54	51
N.S.	1	1.00	0.45	1.09	1.58	0.72	0.00	1.01	0.71	0.67
time (sec)	N/A	0.026	0.010	0.019	2.074	0.715	0.000	0.391	3.502	0.073

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	35	69	67	50	60	67	44	46
N.S.	1	1.00	0.58	1.15	1.12	0.83	1.00	1.12	0.73	0.77
time (sec)	N/A	0.020	0.007	0.014	2.001	0.589	118.561	0.425	3.355	0.077
Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	34	55	30	43	39	57	30	39
N.S.	1	1.00	0.77	1.25	0.68	0.98	0.89	1.30	0.68	0.89
time (sec)	N/A	0.014	0.007	0.013	1.988	0.583	43.654	0.384	3.575	0.053
Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	13	16	12	8	37	14	15
N.S.	1	1.00	1.00	1.44	1.78	1.33	0.89	4.11	1.56	1.67
time (sec)	N/A	0.004	0.003	0.005	0.579	0.578	2.246	0.400	3.176	0.029
Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	24	29	30	28	20	58	25	30
N.S.	1	1.00	1.14	1.38	1.43	1.33	0.95	2.76	1.19	1.43
time (sec)	N/A	0.011	0.006	0.007	0.727	0.531	3.847	0.382	3.107	0.038
Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	32	34	38	38	34	68	28	38
N.S.	1	1.00	0.94	1.00	1.12	1.12	1.00	2.00	0.82	1.12
time (sec)	N/A	0.014	0.007	0.005	0.641	0.543	5.124	0.436	3.093	0.041
Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	20	23	11	28	8	11	18	24
N.S.	1	1.00	2.22	2.56	1.22	3.11	0.89	1.22	2.00	2.67
time (sec)	N/A	0.004	0.006	0.006	1.397	0.504	2.831	0.337	3.108	0.032

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	20	12	0	28	10	20	18	24
N.S.	1	1.00	2.22	1.33	0.00	3.11	1.11	2.22	2.00	2.67
time (sec)	N/A	0.004	0.004	0.004	0.000	0.680	3.171	0.333	3.097	0.033
Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	1059	16	67	14	16	26	20
N.S.	1	1.00	1.00	58.83	0.89	3.72	0.78	0.89	1.44	1.11
time (sec)	N/A	0.072	0.042	0.056	0.571	0.575	3.387	0.339	3.381	0.039
Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	16	16	16	70	21	32	19	16	10	29
N.S.	1	1.00	1.00	4.38	1.31	2.00	1.19	1.00	0.62	1.81
time (sec)	N/A	0.061	0.026	0.046	0.638	0.807	0.216	0.468	3.345	0.025
Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	44	30	27	49	36	94	29	26	40
N.S.	1	1.00	0.68	0.61	1.11	0.82	2.14	0.66	0.59	0.91
time (sec)	N/A	0.031	0.013	0.007	1.469	0.654	0.652	0.345	0.090	0.278
Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	57	58	89	53	133	138	124	57
N.S.	1	1.00	0.47	0.48	0.74	0.44	1.10	1.14	1.02	0.47
time (sec)	N/A	0.111	0.043	0.013	1.545	0.527	12.715	0.368	3.273	0.041
Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	71	42	0	268	49	41	42	57
N.S.	1	1.00	1.42	0.84	0.00	5.36	0.98	0.82	0.84	1.14
time (sec)	N/A	0.043	0.030	0.028	0.000	0.786	3.711	0.355	3.761	0.079

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	50	50	71	133	0	268	37	41	96	57
N.S.	1	1.00	1.42	2.66	0.00	5.36	0.74	0.82	1.92	1.14
time (sec)	N/A	0.064	0.010	0.023	0.000	0.764	79.828	0.381	4.516	0.066
Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	63	56	0	382	97	55	56	70
N.S.	1	1.00	0.93	0.82	0.00	5.62	1.43	0.81	0.82	1.03
time (sec)	N/A	0.042	0.190	0.023	0.000	0.942	4.393	0.347	3.688	0.133
Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	63	314	0	382	0	52	50	70
N.S.	1	1.00	0.93	4.62	0.00	5.62	0.00	0.76	0.74	1.03
time (sec)	N/A	0.509	0.072	0.027	0.000	0.695	0.000	0.459	3.899	0.084
Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	34	39	44	0	39	0	27	39	37
N.S.	1	1.00	1.15	1.29	0.00	1.15	0.00	0.79	1.15	1.09
time (sec)	N/A	0.031	0.033	0.012	0.000	1.112	0.000	0.368	3.329	0.107
Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	51	42	0	49	0	43	27	50
N.S.	1	1.00	0.69	0.57	0.00	0.66	0.00	0.58	0.36	0.68
time (sec)	N/A	0.045	0.041	0.005	0.000	1.896	0.000	0.452	3.162	0.139
Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	14	13	16	14	16	13	18
N.S.	1	1.00	1.00	0.74	0.68	0.84	0.74	0.84	0.68	0.95
time (sec)	N/A	0.005	0.008	0.002	0.675	0.632	0.193	0.307	0.030	0.014

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	107	52	67	86	100	55	162	52
N.S.	1	1.00	1.98	0.96	1.24	1.59	1.85	1.02	3.00	0.96
time (sec)	N/A	0.083	0.075	0.005	1.331	0.739	1.453	0.485	0.095	0.120
Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	18	12	12	32	12	12	14
N.S.	1	1.00	1.00	1.29	0.86	0.86	2.29	0.86	0.86	1.00
time (sec)	N/A	0.019	0.005	0.013	0.657	1.118	4.230	0.306	0.188	0.014
Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	58	91	46	63	94	50	71	59
N.S.	1	1.00	0.95	1.49	0.75	1.03	1.54	0.82	1.16	0.97
time (sec)	N/A	0.044	0.042	0.009	1.483	0.970	2.554	0.328	3.238	0.078
Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	93	30	32	39	30	32	40
N.S.	1	1.00	1.00	2.51	0.81	0.86	1.05	0.81	0.86	1.08
time (sec)	N/A	0.038	0.015	0.013	1.726	0.613	2.305	0.373	0.066	0.050
Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	58	91	45	64	92	49	71	59
N.S.	1	1.00	0.95	1.49	0.74	1.05	1.51	0.80	1.16	0.97
time (sec)	N/A	0.031	0.024	0.007	1.511	0.919	2.213	0.364	0.119	0.055
Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	54	21	21	36	22	25	29
N.S.	1	1.00	1.00	1.74	0.68	0.68	1.16	0.71	0.81	0.94
time (sec)	N/A	0.021	0.010	0.016	0.591	0.627	2.428	0.387	3.083	0.019

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	62	101	51	65	92	54	79	63
N.S.	1	1.00	0.95	1.55	0.78	1.00	1.42	0.83	1.22	0.97
time (sec)	N/A	0.037	0.027	0.007	1.318	0.967	2.307	0.353	3.158	0.052
Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	62	62	49	42	0	51	0	45	-1	57
N.S.	1	1.00	0.79	0.68	0.00	0.82	0.00	0.73	-0.02	0.92
time (sec)	N/A	0.025	0.018	0.007	0.000	1.281	0.000	0.404	0.000	0.126
Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	75	75	62	55	0	61	0	55	-1	62
N.S.	1	1.00	0.83	0.73	0.00	0.81	0.00	0.73	-0.01	0.83
time (sec)	N/A	0.044	0.053	0.007	0.000	1.466	0.000	0.408	0.000	0.135
Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	54	48	0	59	0	53	-1	65
N.S.	1	1.00	0.79	0.71	0.00	0.87	0.00	0.78	-0.01	0.96
time (sec)	N/A	0.042	0.025	0.007	0.000	1.524	0.000	0.396	0.000	0.131
Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	62	60	0	73	0	67	-1	79
N.S.	1	1.00	0.78	0.75	0.00	0.91	0.00	0.84	-0.01	0.99
time (sec)	N/A	0.039	0.030	0.007	0.000	1.394	0.000	0.447	0.000	0.128
Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	109	109	65	67	0	101	0	88	-1	118
N.S.	1	1.00	0.60	0.61	0.00	0.93	0.00	0.81	-0.01	1.08
time (sec)	N/A	0.070	0.051	0.010	0.000	3.104	0.000	0.471	0.000	0.320

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	47	47	32	0	47	0	33	-1	43
N.S.	1	1.00	1.00	0.68	0.00	1.00	0.00	0.70	-0.02	0.91
time (sec)	N/A	0.030	0.013	0.010	0.000	2.119	0.000	0.428	0.000	0.093
Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	56	52	49	48	58	49	102	85
N.S.	1	1.00	0.84	0.78	0.73	0.72	0.87	0.73	1.52	1.27
time (sec)	N/A	0.130	0.070	0.007	2.239	0.629	38.422	0.376	3.090	0.121
Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	71	71	60	54	51	59	60	51	118	87
N.S.	1	1.00	0.85	0.76	0.72	0.83	0.85	0.72	1.66	1.23
time (sec)	N/A	0.110	0.038	0.005	2.077	0.771	73.416	0.328	0.033	0.098
Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	52	97	46	23	44	46	43	39
N.S.	1	1.00	1.00	1.87	0.88	0.44	0.85	0.88	0.83	0.75
time (sec)	N/A	0.053	0.024	0.010	1.811	0.732	0.770	0.309	3.170	0.062
Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	18	14	0	37	0	25	-1	31
N.S.	1	1.00	0.90	0.70	0.00	1.85	0.00	1.25	-0.05	1.55
time (sec)	N/A	0.105	0.037	0.008	0.000	1.295	0.000	0.374	0.000	0.110
Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	31	67	34	29	29	36	30	53	39
N.S.	1	0.72	1.56	0.79	0.67	0.67	0.84	0.70	1.23	0.91
time (sec)	N/A	0.068	0.029	0.008	1.804	0.698	2.431	0.340	3.105	0.038

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	116	116	127	90	89	76	0	89	88	98
N.S.	1	1.00	1.09	0.78	0.77	0.66	0.00	0.77	0.76	0.84
time (sec)	N/A	0.138	0.079	0.005	0.899	0.641	0.000	0.390	0.127	16.083
Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	58	54	53	35	0	82	-1	72
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01	0.87
time (sec)	N/A	0.059	0.055	0.011	0.896	0.997	0.000	3.457	0.000	0.071
Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	64	64	43	41	40	39	216	268	-1	75
N.S.	1	1.00	0.67	0.64	0.62	0.61	3.38	4.19	-0.02	1.17
time (sec)	N/A	0.049	0.029	0.010	0.883	0.652	2.510	7.653	0.000	0.076
Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	57	59	58	57	0	474	-1	103
N.S.	1	1.00	0.70	0.72	0.71	0.70	0.00	5.78	-0.01	1.26
time (sec)	N/A	0.080	0.047	0.013	0.874	0.709	0.000	7.170	0.000	0.093
Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	58	54	53	35	0	82	-1	72
N.S.	1	1.00	0.70	0.65	0.64	0.42	0.00	0.99	-0.01	0.87
time (sec)	N/A	0.047	0.021	0.006	0.884	0.910	0.000	3.180	0.000	0.001
Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	190	190	135	121	120	76	0	0	-1	195
N.S.	1	1.00	0.71	0.64	0.63	0.40	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.366	0.102	0.014	0.919	0.829	0.000	0.000	0.000	0.236

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	233	233	183	154	153	85	0	271	-1	209
N.S.	1	1.00	0.79	0.66	0.66	0.36	0.00	1.16	-0.00	0.90
time (sec)	N/A	0.381	0.131	0.020	0.935	0.491	0.000	51.246	0.000	0.210
Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	160	160	103	107	106	62	0	859	-1	117
N.S.	1	1.00	0.64	0.67	0.66	0.39	0.00	5.37	-0.01	0.73
time (sec)	N/A	0.276	0.088	0.013	0.889	0.834	0.000	15.544	0.000	0.119
Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	20	16	0	35	0	25	-1	31
N.S.	1	1.00	1.00	0.80	0.00	1.75	0.00	1.25	-0.05	1.55
time (sec)	N/A	0.078	0.019	0.005	0.000	1.309	0.000	0.318	0.000	0.126
Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	44	52	44	38	0	85	0	49	-1	67
N.S.	1	1.18	1.00	0.86	0.00	1.93	0.00	1.11	-0.02	1.52
time (sec)	N/A	0.035	0.018	0.010	0.000	1.792	0.000	0.304	0.000	0.143
Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	50	80	58	45	99	61	50	58
N.S.	1	1.00	0.93	1.48	1.07	0.83	1.83	1.13	0.93	1.07
time (sec)	N/A	0.381	0.081	0.004	0.881	0.589	32.898	0.324	3.100	0.055
Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	53	50	0	84	0	44	-1	64
N.S.	1	1.00	0.76	0.71	0.00	1.20	0.00	0.63	-0.01	0.91
time (sec)	N/A	0.035	0.028	0.006	0.000	2.059	0.000	0.346	0.000	0.142

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	16	15	15	17	15	15	24
N.S.	1	1.00	1.00	0.84	0.79	0.79	0.89	0.79	0.79	1.26
time (sec)	N/A	0.021	0.010	0.002	0.876	1.192	0.166	0.392	3.043	0.018
Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	77	77	66	50	49	49	71	50	47	67
N.S.	1	1.00	0.86	0.65	0.64	0.64	0.92	0.65	0.61	0.87
time (sec)	N/A	0.066	0.052	0.005	0.885	0.591	2.323	0.353	0.048	0.031
Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	A	B	F	B	A	B	B	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	224	68	443	0	302	56	171	255	89
N.S.	1	2.80	0.85	5.54	0.00	3.78	0.70	2.14	3.19	1.11
time (sec)	N/A	0.286	0.103	0.049	0.000	0.599	12.153	1.711	0.104	0.152
Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	89	89	89	130	0	87	0	81	-1	81
N.S.	1	1.00	1.00	1.46	0.00	0.98	0.00	0.91	-0.01	0.91
time (sec)	N/A	0.263	0.059	0.017	0.000	6.642	0.000	1.240	0.000	0.195
Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	68	0	62	0	65	-1	55
N.S.	1	1.00	1.00	1.11	0.00	1.02	0.00	1.07	-0.02	0.90
time (sec)	N/A	0.513	0.048	0.019	0.000	4.390	0.000	1.163	0.000	0.251
Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	28	27	18	26	14	14	18
N.S.	1	1.00	1.00	3.50	3.38	2.25	3.25	1.75	1.75	2.25
time (sec)	N/A	0.002	0.002	0.005	0.872	0.665	0.953	0.252	0.158	0.033

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	8	32	27	27	0	22	12	14
N.S.	1	1.00	1.00	4.00	3.38	3.38	0.00	2.75	1.50	1.75
time (sec)	N/A	0.011	0.005	0.016	0.884	0.648	0.000	0.402	0.064	0.023
Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	42	39	49	28	60	22	26	30
N.S.	1	1.00	1.91	1.77	2.23	1.27	2.73	1.00	1.18	1.36
time (sec)	N/A	0.003	0.016	0.004	0.867	0.614	1.484	0.385	3.719	0.045
Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	42	45	51	42	0	35	35	30
N.S.	1	1.00	1.91	2.05	2.32	1.91	0.00	1.59	1.59	1.36
time (sec)	N/A	0.005	0.002	0.007	0.823	0.752	0.000	0.439	3.124	0.041
Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	50	43	20	39	0	42	138	41
N.S.	1	1.00	1.39	1.19	0.56	1.08	0.00	1.17	3.83	1.14
time (sec)	N/A	0.004	0.018	0.017	1.975	0.787	0.000	0.334	5.090	0.174
Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	50	59	41	36	0	51	41	45
N.S.	1	1.00	1.39	1.64	1.14	1.00	0.00	1.42	1.14	1.25
time (sec)	N/A	0.014	0.005	0.019	1.946	0.787	0.000	0.410	0.060	0.037
Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	76	76	55	46	83	47	473	71
N.S.	1	1.00	1.10	1.10	0.80	0.67	1.20	0.68	6.86	1.03
time (sec)	N/A	0.013	0.071	0.013	0.883	0.988	13.845	0.330	12.863	0.157

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	69	69	76	79	138	64	0	62	119	85
N.S.	1	1.00	1.10	1.14	2.00	0.93	0.00	0.90	1.72	1.23
time (sec)	N/A	0.025	0.037	0.012	0.846	0.705	0.000	0.341	0.050	0.099
Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	32	33	13	13	0	20	13	15
N.S.	1	1.00	2.13	2.20	0.87	0.87	0.00	1.33	0.87	1.00
time (sec)	N/A	0.012	0.018	0.016	1.948	2.000	0.000	0.301	0.179	0.026
Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	34	30	15	15	0	16	15	18
N.S.	1	1.00	1.89	1.67	0.83	0.83	0.00	0.89	0.83	1.00
time (sec)	N/A	0.021	0.019	0.015	2.538	0.914	0.000	0.312	3.139	0.032
Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	93	85	24	24	0	41	36	24
N.S.	1	1.00	3.88	3.54	1.00	1.00	0.00	1.71	1.50	1.00
time (sec)	N/A	0.060	0.229	0.033	1.936	0.620	0.000	0.497	0.180	0.097
Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	97	80	59	105	0	74	31	41
N.S.	1	1.00	2.37	1.95	1.44	2.56	0.00	1.80	0.76	1.00
time (sec)	N/A	0.067	0.075	0.026	1.733	2.265	0.000	0.493	0.204	0.079
Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	43	46	37	28	0	36	37	32
N.S.	1	1.00	1.34	1.44	1.16	0.88	0.00	1.12	1.16	1.00
time (sec)	N/A	0.012	0.013	0.007	1.800	0.595	0.000	0.354	3.127	0.030

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	38	38	67	39	43	32	0	29	43	38
N.S.	1	1.00	1.76	1.03	1.13	0.84	0.00	0.76	1.13	1.00
time (sec)	N/A	0.014	0.021	0.008	1.851	0.516	0.000	0.396	0.030	0.030
Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	83	64	49	38	0	36	49	41
N.S.	1	1.00	1.98	1.52	1.17	0.90	0.00	0.86	1.17	0.98
time (sec)	N/A	0.016	0.052	0.017	2.015	0.698	0.000	0.446	3.171	0.040
Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	78	60	70	58	0	40	51	39
N.S.	1	1.00	1.90	1.46	1.71	1.41	0.00	0.98	1.24	0.95
time (sec)	N/A	0.019	0.068	0.013	0.906	0.662	0.000	0.310	0.050	0.058
Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	123	152	118	180	0	119	90	75
N.S.	1	1.00	1.62	2.00	1.55	2.37	0.00	1.57	1.18	0.99
time (sec)	N/A	0.039	0.272	0.010	1.235	0.623	0.000	0.570	0.273	0.126
Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	49	49	76	76	80	54	0	74	57	56
N.S.	1	1.00	1.55	1.55	1.63	1.10	0.00	1.51	1.16	1.14
time (sec)	N/A	0.012	0.041	0.013	1.395	0.639	0.000	0.398	3.164	0.086
Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	79	103	53	46	0	114	74	47
N.S.	1	1.00	1.72	2.24	1.15	1.00	0.00	2.48	1.61	1.02
time (sec)	N/A	0.023	0.034	0.025	1.425	0.485	0.000	0.404	3.239	0.039

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	19	17	27	17	0	29	17	32
N.S.	1	1.00	0.95	0.85	1.35	0.85	0.00	1.45	0.85	1.60
time (sec)	N/A	0.056	0.009	0.004	0.611	0.598	0.000	0.415	0.057	0.034
Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	17	17	16	17	0	29	-1	21
N.S.	1	1.00	0.94	0.94	0.89	0.94	0.00	1.61	-0.06	1.17
time (sec)	N/A	0.031	0.006	0.003	0.610	0.510	0.000	0.259	0.000	0.055
Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	106	81	103	83	0	107	62	72
N.S.	1	1.00	1.96	1.50	1.91	1.54	0.00	1.98	1.15	1.33
time (sec)	N/A	0.057	0.073	0.023	1.219	0.637	0.000	0.298	0.089	0.101
Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	18	0	17	0	23	15	19
N.S.	1	1.00	1.00	1.64	0.00	1.55	0.00	2.09	1.36	1.73
time (sec)	N/A	0.003	0.005	0.006	0.000	0.782	0.000	0.293	3.125	0.036
Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	41	40	0	34	0	0	-1	0
N.S.	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.012	0.227	0.023	0.000	0.760	0.000	0.000	0.000	2.540
Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	180	180	197	551	0	372	0	287	-1	116
N.S.	1	1.00	1.09	3.06	0.00	2.07	0.00	1.59	-0.01	0.64
time (sec)	N/A	0.195	0.384	0.072	0.000	0.813	0.000	0.517	0.000	0.397

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	172	172	306	1066	0	171	0	350	-1	109
N.S.	1	1.00	1.78	6.20	0.00	0.99	0.00	2.03	-0.01	0.63
time (sec)	N/A	0.144	0.452	0.049	0.000	0.731	0.000	0.477	0.000	0.387
Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	307	307	333	5984	0	223	0	452	-1	129
N.S.	1	1.00	1.08	19.49	0.00	0.73	0.00	1.47	-0.00	0.42
time (sec)	N/A	0.251	1.109	0.067	0.000	0.808	0.000	0.559	0.000	0.506
Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	59	71	0	77	0	81	-1	84
N.S.	1	1.00	0.91	1.09	0.00	1.18	0.00	1.25	-0.02	1.29
time (sec)	N/A	0.030	0.028	0.006	0.000	0.718	0.000	0.370	0.000	0.312
Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	79	118	0	97	0	143	-1	73
N.S.	1	1.00	0.95	1.42	0.00	1.17	0.00	1.72	-0.01	0.88
time (sec)	N/A	0.033	0.034	0.021	0.000	0.850	0.000	0.512	0.000	0.330
Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	101	101	97	146	0	129	0	184	-1	86
N.S.	1	1.00	0.96	1.45	0.00	1.28	0.00	1.82	-0.01	0.85
time (sec)	N/A	0.037	0.051	0.027	0.000	0.638	0.000	0.423	0.000	0.354
Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	108	108	187	370	0	187	0	197	-1	86
N.S.	1	1.00	1.73	3.43	0.00	1.73	0.00	1.82	-0.01	0.80
time (sec)	N/A	0.102	0.422	0.031	0.000	0.625	0.000	0.453	0.000	0.338

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	87	87	881	2407	0	121	0	263	-1	95
N.S.	1	1.00	10.13	27.67	0.00	1.39	0.00	3.02	-0.01	1.09
time (sec)	N/A	0.065	1.666	0.089	0.000	0.696	0.000	0.463	0.000	0.369
Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	A	F(-1)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	149	149	914	14529	0	171	0	367	-1	120
N.S.	1	1.00	6.13	97.51	0.00	1.15	0.00	2.46	-0.01	0.81
time (sec)	N/A	0.095	2.462	0.278	0.000	0.710	0.000	0.453	0.000	0.449
Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	59	62	51	59	58	182	58	38	42
N.S.	1	1.40	1.48	1.21	1.40	1.38	4.33	1.38	0.90	1.00
time (sec)	N/A	0.215	0.368	0.009	0.832	0.677	0.702	0.329	0.183	0.084
Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	59	62	51	59	58	182	58	51	30
N.S.	1	1.40	1.48	1.21	1.40	1.38	4.33	1.38	1.21	0.71
time (sec)	N/A	0.239	0.369	0.007	1.039	0.641	10.179	0.387	3.421	0.037
Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	56	56	57	76	57	81	0	0	158	278
N.S.	1	1.00	1.02	1.36	1.02	1.45	0.00	0.00	2.82	4.96
time (sec)	N/A	0.211	0.108	0.021	0.964	0.624	0.000	0.000	7.906	0.579
Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	65	56	0	84	63	89	55	71
N.S.	1	1.00	1.00	0.86	0.00	1.29	0.97	1.37	0.85	1.09
time (sec)	N/A	0.159	0.076	0.007	0.000	0.766	4.510	0.420	0.043	0.316

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	101	88	58	0	78	0	80	52	72
N.S.	1	1.91	1.66	1.09	0.00	1.47	0.00	1.51	0.98	1.36
time (sec)	N/A	0.215	0.150	0.006	0.000	0.602	0.000	0.439	0.045	0.188
Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	14	13	12	12	12	12	12	14
N.S.	1	1.00	1.00	0.93	0.86	0.86	0.86	0.86	0.86	1.00
time (sec)	N/A	0.117	0.006	0.003	0.982	0.613	0.157	0.396	3.387	0.018
Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	61	46	45	43	60	43	43	56
N.S.	1	1.00	1.00	0.75	0.74	0.70	0.98	0.70	0.70	0.92
time (sec)	N/A	0.166	0.048	0.001	0.438	0.672	2.419	0.430	3.360	0.036
Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	114	114	114	86	85	91	116	84	84	103
N.S.	1	1.00	1.00	0.75	0.75	0.80	1.02	0.74	0.74	0.90
time (sec)	N/A	0.193	0.087	0.003	0.438	1.544	3.453	0.345	0.060	0.052
Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	100	102	0	180	70	0	46	68
N.S.	1	1.00	1.72	1.76	0.00	3.10	1.21	0.00	0.79	1.17
time (sec)	N/A	0.057	0.063	0.024	0.000	1.284	3.183	0.000	4.083	0.088
Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	A	A	F(-2)	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	58	58	100	102	0	180	70	0	62	68
N.S.	1	1.00	1.72	1.76	0.00	3.10	1.21	0.00	1.07	1.17
time (sec)	N/A	0.137	0.007	0.017	0.000	0.868	7.026	0.000	4.045	0.095

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	64	39	0	182	0	123	67	73
N.S.	1	1.00	1.21	0.74	0.00	3.43	0.00	2.32	1.26	1.38
time (sec)	N/A	0.031	0.044	0.023	0.000	0.556	0.000	0.436	3.588	0.654
Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	18	17	22	27	18	23	30
N.S.	1	1.00	1.00	0.78	0.74	0.96	1.17	0.78	1.00	1.30
time (sec)	N/A	0.007	0.011	0.002	0.429	0.780	0.194	0.329	0.030	0.020
Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	23	26	17	25	105	54	35	46
N.S.	1	1.00	1.00	1.13	0.74	1.09	4.57	2.35	1.52	2.00
time (sec)	N/A	0.008	0.004	0.003	0.966	0.724	92.595	0.451	3.695	0.136
Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	33	58	36	37	0	39	180	51
N.S.	1	1.00	1.00	1.76	1.09	1.12	0.00	1.18	5.45	1.55
time (sec)	N/A	0.015	0.012	0.014	0.432	0.731	0.000	0.385	7.557	0.139
Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	27	35	2	2	2	31	-1	57
N.S.	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02	1.27
time (sec)	N/A	0.158	0.071	0.009	0.963	1.064	0.106	0.399	0.000	1.250
Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	27	35	2	2	2	31	-1	57
N.S.	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02	1.27
time (sec)	N/A	0.055	0.015	0.008	0.969	0.825	0.102	0.390	0.000	1.296

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	27	35	2	2	2	31	-1	57
N.S.	1	1.00	0.60	0.78	0.04	0.04	0.04	0.69	-0.02	1.27
time (sec)	N/A	0.098	0.013	0.010	0.980	1.417	0.102	0.363	0.000	1.271
Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	27	27	2	2	2	12	-1	57
N.S.	1	1.00	0.60	0.60	0.04	0.04	0.04	0.27	-0.02	1.27
time (sec)	N/A	0.133	0.035	0.008	0.958	0.471	0.099	0.370	0.000	1.338
Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	41	0	6	0	6	-1	39
N.S.	1	1.00	0.94	0.79	0.00	0.12	0.00	0.12	-0.02	0.75
time (sec)	N/A	0.183	0.062	0.010	0.000	0.811	0.000	0.319	0.000	2.776
Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	52	52	49	30	6	6	0	6	-1	39
N.S.	1	1.00	0.94	0.58	0.12	0.12	0.00	0.12	-0.02	0.75
time (sec)	N/A	0.126	0.029	0.009	0.987	1.414	0.000	0.318	0.000	2.577
Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	68	68	64	40	0	8	0	8	-1	67
N.S.	1	1.00	0.94	0.59	0.00	0.12	0.00	0.12	-0.01	0.99
time (sec)	N/A	0.155	0.165	0.012	0.000	0.497	0.000	0.414	0.000	2.925
Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	23	32	27	25	0	54	19	31
N.S.	1	1.00	0.74	1.03	0.87	0.81	0.00	1.74	0.61	1.00
time (sec)	N/A	0.015	0.189	0.012	0.964	0.653	0.000	0.443	3.327	0.285

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	C	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	23	32	25	25	0	54	19	31
N.S.	1	1.00	0.74	1.03	0.81	0.81	0.00	1.74	0.61	1.00
time (sec)	N/A	0.288	0.146	0.010	0.606	1.094	0.000	0.589	0.036	0.288
Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	23	81	0	25	0	68	19	31
N.S.	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61	1.00
time (sec)	N/A	0.140	0.092	0.035	0.000	0.819	0.000	0.500	0.065	0.284
Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	23	81	0	25	0	68	19	31
N.S.	1	1.00	0.74	2.61	0.00	0.81	0.00	2.19	0.61	1.00
time (sec)	N/A	0.125	0.062	0.036	0.000	0.825	0.000	0.366	0.053	0.281
Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	23	32	0	25	0	68	19	31
N.S.	1	1.00	0.74	1.03	0.00	0.81	0.00	2.19	0.61	1.00
time (sec)	N/A	0.654	0.184	0.013	0.000	0.749	0.000	0.616	3.324	0.291
Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	38	73	85	44	212	0	-1	0
N.S.	1	1.00	0.88	1.70	1.98	1.02	4.93	0.00	-0.02	0.00
time (sec)	N/A	0.067	0.041	0.018	0.450	0.762	10.184	0.000	0.000	0.169
Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	45	0	44	0	57	31	60
N.S.	1	1.00	1.00	1.07	0.00	1.05	0.00	1.36	0.74	1.43
time (sec)	N/A	0.130	0.039	0.008	0.000	0.842	0.000	0.456	3.388	0.179

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	54	262	0	80	0	133	60	58
N.S.	1	1.00	0.83	4.03	0.00	1.23	0.00	2.05	0.92	0.89
time (sec)	N/A	0.133	0.051	0.048	0.000	1.040	0.000	0.567	3.444	0.266
Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F(-2)	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	195	195	240	1407	0	1174	0	536	-1	239
N.S.	1	1.00	1.23	7.22	0.00	6.02	0.00	2.75	-0.01	1.23
time (sec)	N/A	0.207	0.395	0.033	0.000	0.598	0.000	0.492	0.000	1.432
Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	34	52	37	34	20	28
N.S.	1	1.00	1.00	1.04	1.21	1.86	1.32	1.21	0.71	1.00
time (sec)	N/A	0.014	0.017	0.032	0.964	0.748	22.797	0.382	3.850	0.036
Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	28	29	0	52	0	34	-1	28
N.S.	1	1.00	1.00	1.04	0.00	1.86	0.00	1.21	-0.04	1.00
time (sec)	N/A	0.058	0.006	0.034	0.000	0.759	0.000	0.495	0.000	0.040
Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	18	17	18	18	22
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82	1.00
time (sec)	N/A	0.005	0.010	0.003	0.581	0.530	0.148	0.340	0.044	0.266
Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	19	18	18	17	18	18	22
N.S.	1	1.00	1.00	0.86	0.82	0.82	0.77	0.82	0.82	1.00
time (sec)	N/A	0.082	0.003	0.004	0.816	0.643	1.170	0.567	0.029	0.256

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	0	11	36	11	11	17
N.S.	1	1.00	1.00	0.71	0.00	0.65	2.12	0.65	0.65	1.00
time (sec)	N/A	0.061	0.023	0.008	0.000	0.573	1.273	0.371	3.700	0.036
Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	125	0	11	0	11	-1	17
N.S.	1	1.00	1.00	7.35	0.00	0.65	0.00	0.65	-0.06	1.00
time (sec)	N/A	0.047	0.035	0.112	0.000	0.591	0.000	0.326	0.000	0.043
Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	7	6	19	7	6	6	24
N.S.	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60	2.40
time (sec)	N/A	0.001	0.005	0.003	1.957	0.700	0.150	0.364	0.010	0.033
Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	34	6	25	51	12	32	24
N.S.	1	1.00	1.00	3.40	0.60	2.50	5.10	1.20	3.20	2.40
time (sec)	N/A	0.002	0.006	0.007	1.931	0.608	1.042	0.366	0.148	0.047
Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	10	10	10	7	6	19	7	6	6	24
N.S.	1	1.00	1.00	0.70	0.60	1.90	0.70	0.60	0.60	2.40
time (sec)	N/A	0.004	0.005	0.009	1.956	0.857	1.393	0.395	0.012	0.037
Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	7	8	29	0	6	6	23
N.S.	1	1.00	1.00	0.58	0.67	2.42	0.00	0.50	0.50	1.92
time (sec)	N/A	0.007	0.008	0.004	1.970	0.738	0.000	0.420	3.125	0.118

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	21	31	8	29	41	13	30	20
N.S.	1	1.00	1.75	2.58	0.67	2.42	3.42	1.08	2.50	1.67
time (sec)	N/A	0.007	0.018	0.006	1.962	0.626	1.023	0.326	3.428	0.045
Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	21	7	8	29	0	6	6	23
N.S.	1	1.00	1.75	0.58	0.67	2.42	0.00	0.50	0.50	1.92
time (sec)	N/A	0.009	0.003	0.008	1.971	0.591	0.000	0.345	3.370	0.111
Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	5	8	29	0	4	4	23
N.S.	1	1.00	1.00	1.25	2.00	7.25	0.00	1.00	1.00	5.75
time (sec)	N/A	0.005	0.007	0.004	1.989	0.768	0.000	0.342	3.185	0.113
Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	18	29	8	29	41	13	33	20
N.S.	1	1.00	4.50	7.25	2.00	7.25	10.25	3.25	8.25	5.00
time (sec)	N/A	0.006	0.014	0.007	1.972	0.577	1.022	0.316	0.082	0.040
Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	18	5	8	29	0	4	4	23
N.S.	1	1.00	4.50	1.25	2.00	7.25	0.00	1.00	1.00	5.75
time (sec)	N/A	0.007	0.003	0.007	1.950	0.949	0.000	0.388	3.362	0.109
Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	8	7	7	8	7	7	15
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64	1.36
time (sec)	N/A	0.001	0.002	0.002	0.898	0.624	0.059	0.361	0.026	0.007

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	8	7	7	8	7	7	15
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64	1.36
time (sec)	N/A	0.008	0.001	0.002	0.913	0.626	0.151	0.332	0.026	0.010
Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	30	0	26	7	10	-1	38
N.S.	1	1.00	1.00	1.11	0.00	0.96	0.26	0.37	-0.04	1.41
time (sec)	N/A	0.014	0.008	0.006	0.000	0.537	1.014	0.422	0.000	3.948
Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	27	30	0	26	0	10	-1	38
N.S.	1	1.00	1.00	1.11	0.00	0.96	0.00	0.37	-0.04	1.41
time (sec)	N/A	0.021	0.006	0.007	0.000	0.627	0.000	0.337	0.000	4.463
Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	33	56	28	14	14	15	15	-1	36
N.S.	1	1.32	2.24	1.12	0.56	0.56	0.60	0.60	-0.04	1.44
time (sec)	N/A	0.014	0.026	0.007	0.881	0.686	1.519	0.360	0.000	3.959
Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	33	56	28	0	14	0	21	-1	36
N.S.	1	1.32	2.24	1.12	0.00	0.56	0.00	0.84	-0.04	1.44
time (sec)	N/A	0.022	0.003	0.007	0.000	0.623	0.000	0.317	0.000	4.385
Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	10	9	9	8	9	9	11
N.S.	1	1.00	1.00	0.91	0.82	0.82	0.73	0.82	0.82	1.00
time (sec)	N/A	0.001	0.003	0.003	0.890	0.575	0.061	0.334	0.193	0.008

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	C	C	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	23	20	12	16	0	15	16	26
N.S.	1	1.00	2.09	1.82	1.09	1.45	0.00	1.36	1.45	2.36
time (sec)	N/A	0.001	0.023	0.002	0.920	0.765	0.000	0.321	3.557	0.063
Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	9	8	7	7	7	7	7	9
N.S.	1	1.00	1.00	0.89	0.78	0.78	0.78	0.78	0.78	1.00
time (sec)	N/A	0.001	0.002	0.003	0.884	0.657	0.056	0.341	0.095	0.005
Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	C	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	9	9	25	22	7	23	0	13	18	32
N.S.	1	1.00	2.78	2.44	0.78	2.56	0.00	1.44	2.00	3.56
time (sec)	N/A	0.001	0.023	0.003	0.899	0.876	0.000	0.426	3.635	0.055
Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	9	12	10	9	9	13
N.S.	1	1.00	1.00	0.77	0.69	0.92	0.77	0.69	0.69	1.00
time (sec)	N/A	0.001	0.003	0.004	0.808	0.563	0.058	0.345	3.507	0.007
Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	25	20	12	19	0	15	20	28
N.S.	1	1.00	1.92	1.54	0.92	1.46	0.00	1.15	1.54	2.15
time (sec)	N/A	0.001	0.021	0.002	0.831	0.711	0.000	0.334	3.523	0.058
Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	8	7	7	8	7	7	11
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64	1.00
time (sec)	N/A	0.001	0.002	0.003	0.882	0.580	0.057	0.318	3.419	0.007

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	27	22	7	26	0	13	22	34
N.S.	1	1.00	2.45	2.00	0.64	2.36	0.00	1.18	2.00	3.09
time (sec)	N/A	0.001	0.022	0.002	0.919	0.838	0.000	0.334	3.491	0.058
Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	49	67	41	52	97	39	172	53
N.S.	1	1.00	1.40	1.91	1.17	1.49	2.77	1.11	4.91	1.51
time (sec)	N/A	0.007	0.019	0.007	1.968	0.863	1.619	0.352	6.136	0.148
Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	49	86	0	96	0	0	-1	0
N.S.	1	1.00	1.40	2.46	0.00	2.74	0.00	0.00	-0.03	0.00
time (sec)	N/A	0.007	0.008	0.013	0.000	0.901	0.000	0.000	0.000	2.623
Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	43	43	61	70	53	74	0	0	-1	77
N.S.	1	1.00	1.42	1.63	1.23	1.72	0.00	0.00	-0.02	1.79
time (sec)	N/A	0.013	0.039	0.018	1.987	0.710	0.000	0.000	0.000	0.230
Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	47	41	53	63	0	44	37	59
N.S.	1	1.00	1.34	1.17	1.51	1.80	0.00	1.26	1.06	1.69
time (sec)	N/A	0.061	0.023	0.010	1.987	0.787	0.000	0.395	3.204	0.253
Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	B	F	F(-2)	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	72	134	65	93	0	0	-1	90
N.S.	1	1.00	1.41	2.63	1.27	1.82	0.00	0.00	-0.02	1.76
time (sec)	N/A	0.024	0.060	0.041	2.504	0.827	0.000	0.000	0.000	0.236

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	45	45	59	75	65	82	0	87	82	97
N.S.	1	1.00	1.31	1.67	1.44	1.82	0.00	1.93	1.82	2.16
time (sec)	N/A	0.091	0.038	0.014	2.076	0.948	0.000	0.543	3.491	0.413
Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	18	2	2	2	2
N.S.	1	1.00	1.00	1.50	1.00	9.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.001	0.004	0.002	1.675	0.630	0.141	0.344	0.006	0.031
Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	32	29	0	27	0	0	-1	0
N.S.	1	1.00	16.00	14.50	0.00	13.50	0.00	0.00	-0.50	0.00
time (sec)	N/A	0.001	0.026	0.018	0.000	0.834	0.000	0.000	0.000	0.788
Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	2	3	2	14	2	14	2	16
N.S.	1	1.00	1.00	1.50	1.00	7.00	1.00	7.00	1.00	8.00
time (sec)	N/A	0.001	0.003	0.004	2.000	0.836	0.137	0.428	0.028	0.019
Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	2	2	42	29	0	81	0	0	-1	0
N.S.	1	1.00	21.00	14.50	0.00	40.50	0.00	0.00	-0.50	0.00
time (sec)	N/A	0.001	0.024	0.011	0.000	0.561	0.000	0.000	0.000	0.787
Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	20	18	17	31	15	17	17	37
N.S.	1	1.00	0.87	0.78	0.74	1.35	0.65	0.74	0.74	1.61
time (sec)	N/A	0.003	0.006	0.004	1.933	0.621	0.195	0.357	0.031	0.058

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	23	23	50	42	0	60	0	0	-1	0
N.S.	1	1.00	2.17	1.83	0.00	2.61	0.00	0.00	-0.04	0.00
time (sec)	N/A	0.003	0.041	0.011	0.000	0.515	0.000	0.000	0.000	0.850
Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	18	16	15	25	15	25	15	33
N.S.	1	1.00	0.86	0.76	0.71	1.19	0.71	1.19	0.71	1.57
time (sec)	N/A	0.002	0.004	0.003	1.950	0.590	0.194	0.372	0.028	0.028
Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	70	47	0	120	0	0	-1	0
N.S.	1	1.00	3.33	2.24	0.00	5.71	0.00	0.00	-0.05	0.00
time (sec)	N/A	0.003	0.056	0.010	0.000	0.560	0.000	0.000	0.000	0.869
Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	38	21	0	30	58	30	20	40
N.S.	1	1.00	1.36	0.75	0.00	1.07	2.07	1.07	0.71	1.43
time (sec)	N/A	0.009	0.012	0.004	0.000	0.516	0.354	0.336	0.034	0.035
Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	A	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	37	175	0	84	17	140	105	39
N.S.	1	1.00	1.00	4.73	0.00	2.27	0.46	3.78	2.84	1.05
time (sec)	N/A	0.042	0.023	0.046	0.000	0.598	0.169	0.438	0.143	0.074
Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	74	33	0	90	27	88	57	85
N.S.	1	1.00	1.85	0.82	0.00	2.25	0.68	2.20	1.42	2.12
time (sec)	N/A	0.043	0.040	0.011	0.000	0.549	0.208	0.409	3.501	0.142

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	77	111	94	67	0	117	86	72
N.S.	1	1.00	1.43	2.06	1.74	1.24	0.00	2.17	1.59	1.33
time (sec)	N/A	0.128	0.057	0.015	2.029	0.579	0.000	0.441	3.328	0.105
Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	60	60	77	111	0	67	0	117	86	72
N.S.	1	1.00	1.28	1.85	0.00	1.12	0.00	1.95	1.43	1.20
time (sec)	N/A	0.300	0.087	0.017	0.000	0.632	0.000	0.514	3.377	0.096
Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	38	0	66	0	50	-1	58
N.S.	1	1.00	0.76	0.75	0.00	1.29	0.00	0.98	-0.02	1.14
time (sec)	N/A	0.111	0.052	0.005	0.000	0.418	0.000	0.426	0.000	0.276
Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	38	54	66	0	50	-1	58
N.S.	1	1.00	0.76	0.75	1.06	1.29	0.00	0.98	-0.02	1.14
time (sec)	N/A	0.097	0.046	0.006	1.984	0.396	0.000	0.484	0.000	0.217
Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	F(-2)	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	51	51	39	51	54	64	0	0	56	76
N.S.	1	1.00	0.76	1.00	1.06	1.25	0.00	0.00	1.10	1.49
time (sec)	N/A	0.148	0.023	0.011	1.973	0.400	0.000	0.000	4.782	0.115
Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F(-2)	B	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	54	54	82	51	0	64	0	0	56	76
N.S.	1	1.00	1.52	0.94	0.00	1.19	0.00	0.00	1.04	1.41
time (sec)	N/A	0.049	0.142	0.010	0.000	0.394	0.000	0.000	0.061	0.117

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	27	27	23	20	0	20	0	0	20	24
N.S.	1	1.00	0.85	0.74	0.00	0.74	0.00	0.00	0.74	0.89
time (sec)	N/A	0.030	0.011	0.003	0.000	0.398	0.000	0.000	3.419	3.343
Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	14	14	12	13	0	16	0	0	138	18
N.S.	1	1.00	0.86	0.93	0.00	1.14	0.00	0.00	9.86	1.29
time (sec)	N/A	0.149	0.024	0.007	0.000	0.482	0.000	0.000	3.380	0.116
Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	12	11	0	16	0	0	10	18
N.S.	1	1.00	1.00	0.92	0.00	1.33	0.00	0.00	0.83	1.50
time (sec)	N/A	0.070	0.011	0.004	0.000	0.400	0.000	0.000	0.051	0.110
Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	36	36	29	34	0	30	0	0	48	29
N.S.	1	1.00	0.81	0.94	0.00	0.83	0.00	0.00	1.33	0.81
time (sec)	N/A	0.140	0.019	0.007	0.000	0.401	0.000	0.000	3.492	10.534
Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	29	34	0	30	0	0	43	29
N.S.	1	1.00	0.88	1.03	0.00	0.91	0.00	0.00	1.30	0.88
time (sec)	N/A	0.193	0.012	0.006	0.000	0.471	0.000	0.000	3.494	10.112
Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	70	70	105	117	0	208	0	0	-1	68
N.S.	1	1.00	1.50	1.67	0.00	2.97	0.00	0.00	-0.01	0.97
time (sec)	N/A	0.068	0.131	0.053	0.000	0.464	0.000	0.000	0.000	0.726

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	C	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	83	83	52	63	0	38	0	46	-1	42
N.S.	1	1.00	0.63	0.76	0.00	0.46	0.00	0.55	-0.01	0.51
time (sec)	N/A	0.154	0.064	0.032	0.000	0.434	0.000	0.359	0.000	0.087
Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	47	73	91	63	0	36	0	18	-1	44
N.S.	1	1.55	1.94	1.34	0.00	0.77	0.00	0.38	-0.02	0.94
time (sec)	N/A	0.460	0.118	0.026	0.000	0.399	0.000	0.340	0.000	5.571
Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F(-1)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	123	123	70	75	0	74	0	39	-1	71
N.S.	1	1.00	0.57	0.61	0.00	0.60	0.00	0.32	-0.01	0.58
time (sec)	N/A	0.286	0.025	0.028	0.000	0.410	0.000	0.414	0.000	11.120
Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	133	133	64	62	0	117	0	86	-1	80
N.S.	1	1.00	0.48	0.47	0.00	0.88	0.00	0.65	-0.01	0.60
time (sec)	N/A	0.074	0.100	0.023	0.000	0.401	0.000	0.469	0.000	6.909
Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	90	44	49	0	83	0	67	-1	65
N.S.	1	1.00	0.49	0.54	0.00	0.92	0.00	0.74	-0.01	0.72
time (sec)	N/A	0.048	0.039	0.016	0.000	0.397	0.000	0.372	0.000	6.405
Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	40	42	0	75	0	49	-1	56
N.S.	1	1.00	0.66	0.69	0.00	1.23	0.00	0.80	-0.02	0.92
time (sec)	N/A	0.030	0.018	0.010	0.000	0.402	0.000	0.480	0.000	5.314

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	F	F	F	F(-1)	F	F	F	C
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	268	268	0	0	0	0	0	0	-1	1609
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00	6.00
time (sec)	N/A	0.311	0.100	0.073	0.000	0.000	0.000	0.000	0.000	5.170
Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	28	27	25	37	27	27	42
N.S.	1	1.00	1.00	0.68	0.66	0.61	0.90	0.66	0.66	1.02
time (sec)	N/A	0.045	0.026	0.004	2.342	1.070	11.870	0.397	3.367	0.038
Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	38	21	20	20	24	20	22	26
N.S.	1	1.00	1.46	0.81	0.77	0.77	0.92	0.77	0.85	1.00
time (sec)	N/A	0.040	0.025	0.006	2.291	0.859	3.662	0.350	3.361	0.048
Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	54	48	30	30	39	30	42	42
N.S.	1	1.00	1.29	1.14	0.71	0.71	0.93	0.71	1.00	1.00
time (sec)	N/A	0.154	0.023	0.015	2.390	1.257	23.130	0.339	0.026	2.306
Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	48	50	0	75	0	44	17	48
N.S.	1	1.00	2.40	2.50	0.00	3.75	0.00	2.20	0.85	2.40
time (sec)	N/A	0.008	0.014	0.013	0.000	1.224	0.000	0.351	3.460	3.517
Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	F	B	F	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	52	62	0	84	0	40	21	52
N.S.	1	1.00	2.60	3.10	0.00	4.20	0.00	2.00	1.05	2.60
time (sec)	N/A	0.009	0.014	0.012	0.000	1.080	0.000	0.416	3.470	3.578

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	121	121	136	244	0	1532	0	0	-1	205
N.S.	1	1.00	1.12	2.02	0.00	12.66	0.00	0.00	-0.01	1.69
time (sec)	N/A	0.165	0.102	0.033	0.000	2.518	0.000	0.000	0.000	4.586
Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	F(-2)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	181	181	189	397	0	2411	0	0	-1	244
N.S.	1	1.00	1.04	2.19	0.00	13.32	0.00	0.00	-0.01	1.35
time (sec)	N/A	0.273	0.281	0.041	0.000	76.216	0.000	0.000	0.000	6.523
Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F(-1)	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	26	17	16	16	0	11	16	26
N.S.	1	1.00	1.00	0.65	0.62	0.62	0.00	0.42	0.62	1.00
time (sec)	N/A	0.006	0.013	0.004	0.964	0.407	0.000	0.511	3.541	6.639
Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	26	26	27	23	22	32	0	22	22	36
N.S.	1	1.00	1.04	0.88	0.85	1.23	0.00	0.85	0.85	1.38
time (sec)	N/A	0.011	0.031	0.007	1.932	0.411	0.000	0.614	3.529	0.200
Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	13	16	11	11	12	11	9	13
N.S.	1	1.00	0.87	1.07	0.73	0.73	0.80	0.73	0.60	0.87
time (sec)	N/A	0.003	0.012	0.005	0.863	0.438	0.158	0.427	3.565	0.018
Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	19	23	28	27	0	27	18	30
N.S.	1	1.00	0.86	1.05	1.27	1.23	0.00	1.23	0.82	1.36
time (sec)	N/A	0.005	0.013	0.004	0.884	0.404	0.000	0.522	3.504	0.223

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	37	13	9	17	0	17	-1	18
N.S.	1	1.00	3.08	1.08	0.75	1.42	0.00	1.42	-0.08	1.50
time (sec)	N/A	0.007	0.015	0.006	1.945	0.410	0.000	0.589	0.000	0.185
Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	37	15	11	17	0	17	-1	16
N.S.	1	1.00	3.08	1.25	0.92	1.42	0.00	1.42	-0.08	1.33
time (sec)	N/A	0.008	0.017	0.007	1.930	0.409	0.000	0.616	0.000	0.206
Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	12	17	13	13	10	13	10	15
N.S.	1	1.00	0.80	1.13	0.87	0.87	0.67	0.87	0.67	1.00
time (sec)	N/A	0.004	0.008	0.003	0.874	0.403	0.140	0.597	3.640	0.019
Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	95	54	42	49	0	53	-1	58
N.S.	1	1.00	1.76	1.00	0.78	0.91	0.00	0.98	-0.02	1.07
time (sec)	N/A	0.040	0.078	0.008	1.907	0.421	0.000	0.505	0.000	0.213
Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	F(-1)	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	57	342	0	0	0	45	27	47
N.S.	1	1.00	0.97	5.80	0.00	0.00	0.00	0.76	0.46	0.80
time (sec)	N/A	0.069	0.036	0.102	0.000	0.000	0.000	4.566	3.532	0.428
Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	59	59	39	28	0	30	0	33	27	39
N.S.	1	1.00	0.66	0.47	0.00	0.51	0.00	0.56	0.46	0.66
time (sec)	N/A	0.052	0.020	0.008	0.000	0.849	0.000	0.395	3.544	0.031

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	94	94	51	38	0	40	0	51	-1	51
N.S.	1	1.00	0.54	0.40	0.00	0.43	0.00	0.54	-0.01	0.54
time (sec)	N/A	0.091	0.030	0.003	0.000	0.872	0.000	0.383	0.000	0.039
Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	18	18	18	20	0	20	26	18	19	23
N.S.	1	1.00	1.00	1.11	0.00	1.11	1.44	1.00	1.06	1.28
time (sec)	N/A	0.049	0.010	0.007	0.000	1.009	0.497	0.480	3.499	0.052
Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	107	107	84	81	0	62	0	60	-1	63
N.S.	1	1.00	0.79	0.76	0.00	0.58	0.00	0.56	-0.01	0.59
time (sec)	N/A	0.029	0.028	0.007	0.000	0.597	0.000	0.430	0.000	0.185
Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	25	24	14	64	0	14	25	41
N.S.	1	1.00	1.00	0.96	0.56	2.56	0.00	0.56	1.00	1.64
time (sec)	N/A	0.016	0.019	0.005	0.897	0.804	0.000	0.424	3.508	0.055
Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	42	42	34	33	35	49	33	35	45
N.S.	1	1.00	1.00	0.81	0.79	0.83	1.17	0.79	0.83	1.07
time (sec)	N/A	0.029	0.011	0.004	2.011	0.754	0.247	0.391	3.448	0.038
Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	32	32	32	26	25	27	37	25	27	40
N.S.	1	1.00	1.00	0.81	0.78	0.84	1.16	0.78	0.84	1.25
time (sec)	N/A	0.023	0.010	0.004	2.130	0.800	0.230	0.363	0.035	0.031

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	76	76	49	53	0	95	0	45	-1	49
N.S.	1	1.00	0.64	0.70	0.00	1.25	0.00	0.59	-0.01	0.64
time (sec)	N/A	0.023	0.016	0.003	0.000	0.924	0.000	0.427	0.000	0.316
Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	46	53	0	64	48	79	46	41
N.S.	1	1.00	1.00	1.15	0.00	1.39	1.04	1.72	1.00	0.89
time (sec)	N/A	0.086	0.034	0.004	0.000	0.707	2.984	0.432	0.036	0.186
Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	23	18	0	14	0	13	-1	26
N.S.	1	1.00	1.15	0.90	0.00	0.70	0.00	0.65	-0.05	1.30
time (sec)	N/A	0.006	0.027	0.004	0.000	0.735	0.000	0.360	0.000	0.401
Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	A	A	A	A	C	F	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	36	18	35	38	19	68	0	14	18
N.S.	1	1.03	0.51	1.00	1.09	0.54	1.94	0.00	0.40	0.51
time (sec)	N/A	0.011	0.007	0.007	2.153	0.686	1.137	0.000	3.591	0.195
Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	F	A	A	C	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	15	15	111	0	13	16	100	0	13	0
N.S.	1	1.00	7.40	0.00	0.87	1.07	6.67	0.00	0.87	0.00
time (sec)	N/A	0.018	0.179	0.092	1.128	0.591	98.052	0.000	3.684	0.003
Problem 635	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	28	25	37	24	46	37	37	40
N.S.	1	1.00	0.53	0.47	0.70	0.45	0.87	0.70	0.70	0.75
time (sec)	N/A	0.021	0.026	0.004	0.578	0.623	11.728	0.337	0.043	0.027

Problem 636	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	26	25	25	27	25	25	31
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.87	0.81	0.81	1.00
time (sec)	N/A	0.015	0.012	0.015	0.654	0.594	0.240	0.321	0.069	0.028
Problem 637	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	14	9	9	8	9	9	11
N.S.	1	1.00	1.00	1.27	0.82	0.82	0.73	0.82	0.82	1.00
time (sec)	N/A	0.003	0.006	0.004	0.625	0.697	0.139	0.365	3.516	0.015
Problem 638	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	6	6	6	5	4	4	5	4	4	6
N.S.	1	1.00	1.00	0.83	0.67	0.67	0.83	0.67	0.67	1.00
time (sec)	N/A	0.002	0.003	0.004	1.388	0.883	0.211	0.332	0.143	0.014
Problem 639	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	17	17	16	16	0	25	16	20
N.S.	1	1.00	0.85	0.85	0.80	0.80	0.00	1.25	0.80	1.00
time (sec)	N/A	0.005	0.006	0.002	1.432	0.458	0.000	0.414	3.508	0.104
Problem 640	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	17	17	17	12	26	11	12	11	11	18
N.S.	1	1.00	1.00	0.71	1.53	0.65	0.71	0.65	0.65	1.06
time (sec)	N/A	0.003	0.002	0.002	0.730	0.430	0.138	0.349	0.024	0.010
Problem 641	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	19	12	11	11	15	11	12	19
N.S.	1	1.00	1.00	0.63	0.58	0.58	0.79	0.58	0.63	1.00
time (sec)	N/A	0.003	0.004	0.001	0.483	0.506	1.529	0.366	0.025	0.023

Problem 642	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	41	28	27	25	37	27	27	42
N.S.	1	1.00	1.00	0.68	0.66	0.61	0.90	0.66	0.66	1.02
time (sec)	N/A	0.010	0.008	0.004	1.625	0.449	3.363	0.380	0.027	0.033
Problem 643	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	67	67	88	64	63	65	39	64	73	89
N.S.	1	1.00	1.31	0.96	0.94	0.97	0.58	0.96	1.09	1.33
time (sec)	N/A	0.030	0.020	0.010	1.723	0.464	1.055	0.632	3.832	0.052
Problem 644	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	8	7	7	8	7	7	15
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64	1.36
time (sec)	N/A	0.001	0.001	0.001	0.884	0.437	0.058	0.331	0.002	0.000
Problem 645	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	8	7	7	8	7	7	15
N.S.	1	1.00	1.00	0.73	0.64	0.64	0.73	0.64	0.64	1.36
time (sec)	N/A	0.001	0.002	0.002	0.623	0.432	0.058	0.347	0.023	0.008
Problem 646	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	11	11	11	44	7	7	8	7	7	15
N.S.	1	1.00	1.00	4.00	0.64	0.64	0.73	0.64	0.64	1.36
time (sec)	N/A	0.002	0.001	0.013	0.607	0.432	4.355	0.354	0.026	0.016
Problem 647	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	61	61	95	49	0	192	0	68	-1	189
N.S.	1	1.00	1.56	0.80	0.00	3.15	0.00	1.11	-0.02	3.10
time (sec)	N/A	0.025	0.065	0.008	0.000	0.436	0.000	0.721	0.000	0.442

Problem 648	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F(-2)	A	F	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	94	55	0	202	0	59	-1	198
N.S.	1	1.00	1.45	0.85	0.00	3.11	0.00	0.91	-0.02	3.05
time (sec)	N/A	0.025	0.077	0.013	0.000	0.437	0.000	0.740	0.000	0.426
Problem 649	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	21	21	26	22	9	13
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69	1.00
time (sec)	N/A	0.007	0.004	0.007	1.011	0.416	0.386	0.392	3.339	0.028
Problem 650	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	B	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	13	10	21	21	26	22	9	13
N.S.	1	1.00	1.00	0.77	1.62	1.62	2.00	1.69	0.69	1.00
time (sec)	N/A	0.011	0.003	0.006	1.006	0.416	0.535	0.303	0.027	0.027
Problem 651	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	72	72	72	82	77	100	202	80	233	0
N.S.	1	1.00	1.00	1.14	1.07	1.39	2.81	1.11	3.24	0.00
time (sec)	N/A	0.104	0.096	0.022	0.997	0.427	1.547	0.428	4.234	0.001
Problem 652	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	23	23	15	22	0	48	22	23
N.S.	1	1.00	0.62	0.62	0.41	0.59	0.00	1.30	0.59	0.62
time (sec)	N/A	0.027	0.009	0.003	0.451	0.399	0.000	0.383	3.513	0.033
Problem 653	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	12	12	37	13	9	17	0	17	-1	18
N.S.	1	1.00	3.08	1.08	0.75	1.42	0.00	1.42	-0.08	1.50
time (sec)	N/A	0.007	0.008	0.004	0.974	0.401	0.000	0.465	0.000	0.001

Problem 654	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	95	95	60	67	0	89	0	51	-1	71
N.S.	1	1.00	0.63	0.71	0.00	0.94	0.00	0.54	-0.01	0.75
time (sec)	N/A	0.054	0.037	0.007	0.000	1.533	0.000	0.386	0.000	0.193
Problem 655	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	35	35	28	24	23	21	184	23	16	28
N.S.	1	1.00	0.80	0.69	0.66	0.60	5.26	0.66	0.46	0.80
time (sec)	N/A	0.011	0.011	0.002	0.434	0.419	1.166	0.324	3.506	0.018
Problem 656	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	37	37	30	28	27	22	265	27	24	30
N.S.	1	1.00	0.81	0.76	0.73	0.59	7.16	0.73	0.65	0.81
time (sec)	N/A	0.013	0.011	0.005	0.432	0.429	1.155	0.341	3.578	0.019
Problem 657	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	26	48	0	33	32	29	39	40
N.S.	1	1.00	0.90	1.66	0.00	1.14	1.10	1.00	1.34	1.38
time (sec)	N/A	0.026	0.028	0.008	0.000	0.443	1.795	0.408	3.897	0.176
Problem 658	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	25	25	26	48	0	36	87	32	40	42
N.S.	1	1.00	1.04	1.92	0.00	1.44	3.48	1.28	1.60	1.68
time (sec)	N/A	0.025	0.031	0.007	0.000	0.421	3.773	0.402	3.648	0.176
Problem 659	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	21	21	21	16	0	15	56	15	22	21
N.S.	1	1.00	1.00	0.76	0.00	0.71	2.67	0.71	1.05	1.00
time (sec)	N/A	0.023	0.025	0.003	0.000	0.406	0.370	0.341	0.041	0.102

Problem 660	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	65	65	54	175	0	97	0	105	127	53
N.S.	1	1.00	0.83	2.69	0.00	1.49	0.00	1.62	1.95	0.82
time (sec)	N/A	0.054	0.054	0.019	0.000	0.404	0.000	0.572	3.499	0.197
Problem 661	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	31	31	31	28	0	41	0	63	64	62
N.S.	1	1.00	1.00	0.90	0.00	1.32	0.00	2.03	2.06	2.00
time (sec)	N/A	0.042	0.036	0.010	0.000	0.417	0.000	0.464	0.188	0.261
Problem 662	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	F	A	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	82	82	58	54	0	54	0	50	-1	57
N.S.	1	1.00	0.71	0.66	0.00	0.66	0.00	0.61	-0.01	0.70
time (sec)	N/A	0.044	0.067	0.004	0.000	0.976	0.000	0.442	0.000	0.149
Problem 663	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	74	74	74	56	55	57	155	55	95	80
N.S.	1	1.00	1.00	0.76	0.74	0.77	2.09	0.74	1.28	1.08
time (sec)	N/A	0.111	0.038	0.011	0.970	0.420	3.777	0.464	3.412	0.060
Problem 664	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	C	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	115	115	123	81	80	80	221	80	130	115
N.S.	1	1.00	1.07	0.70	0.70	0.70	1.92	0.70	1.13	1.00
time (sec)	N/A	0.150	0.087	0.011	0.960	0.419	5.432	0.431	0.087	0.063
Problem 665	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	4	4	4	51	0	20	0	4	4	22
N.S.	1	1.00	1.00	12.75	0.00	5.00	0.00	1.00	1.00	5.50
time (sec)	N/A	0.043	0.063	0.016	0.000	0.396	0.000	0.333	0.032	0.284

Problem 666	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	41	50	40	0	31	18	26
N.S.	1	1.00	1.00	1.86	2.27	1.82	0.00	1.41	0.82	1.18
time (sec)	N/A	0.010	0.010	0.005	0.439	0.430	0.000	0.390	3.387	0.045
Problem 667	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	40	37	26	0	28	20	28
N.S.	1	1.00	1.00	1.67	1.54	1.08	0.00	1.17	0.83	1.17
time (sec)	N/A	0.009	0.008	0.006	0.972	0.412	0.000	0.396	0.037	0.036
Problem 668	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	28	38	45	51	40	0	35	24	28
N.S.	1	1.17	1.58	1.88	2.12	1.67	0.00	1.46	1.00	1.17
time (sec)	N/A	0.011	0.016	0.007	0.433	0.424	0.000	0.380	0.030	0.043
Problem 669	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	A	A	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	24	60	38	38	32	38	20	28
N.S.	1	1.00	1.00	2.50	1.58	1.58	1.33	1.58	0.83	1.17
time (sec)	N/A	0.017	0.007	0.008	0.435	0.422	3.228	0.380	0.037	0.026
Problem 670	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	B	B	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	42	43	51	42	0	35	35	30
N.S.	1	1.00	1.91	1.95	2.32	1.91	0.00	1.59	1.59	1.36
time (sec)	N/A	0.005	0.020	0.008	0.441	0.409	0.000	0.386	0.040	0.001
Problem 671	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	29	29	43	44	35	25	0	35	23	31
N.S.	1	1.00	1.48	1.52	1.21	0.86	0.00	1.21	0.79	1.07
time (sec)	N/A	0.012	0.013	0.007	0.974	0.438	0.000	0.459	3.367	0.030

Problem 672	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	33	33	32	28	36	35	0	25	26	41
N.S.	1	1.00	0.97	0.85	1.09	1.06	0.00	0.76	0.79	1.24
time (sec)	N/A	0.012	0.051	0.008	1.017	0.408	0.000	0.386	3.468	0.116
Problem 673	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	8	8	12	7	6	16	0	6	6	18
N.S.	1	1.00	1.50	0.88	0.75	2.00	0.00	0.75	0.75	2.25
time (sec)	N/A	0.005	0.008	0.006	0.963	0.412	0.000	0.388	0.014	0.097
Problem 674	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	13	13	11	15	11	18	0	16	13	17
N.S.	1	1.00	0.85	1.15	0.85	1.38	0.00	1.23	1.00	1.31
time (sec)	N/A	0.013	0.004	0.005	0.426	0.456	0.000	0.328	3.518	0.154
Problem 675	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	B	F	A	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	22	41	0	33	0	73	17	24
N.S.	1	1.00	1.00	1.86	0.00	1.50	0.00	3.32	0.77	1.09
time (sec)	N/A	0.032	0.028	0.019	0.000	0.425	0.000	0.534	3.583	0.067
Problem 676	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	A	A	A	A	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	24	24	39	32	11	11	15	13	45	0
N.S.	1	1.00	1.62	1.33	0.46	0.46	0.62	0.54	1.88	0.00
time (sec)	N/A	0.027	0.024	0.027	0.976	0.480	0.579	0.361	0.099	0.001
Problem 677	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	34
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.21
time (sec)	N/A	0.010	0.022	0.004	0.439	0.445	0.000	0.466	3.472	0.149

Problem 678	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	34
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.21
time (sec)	N/A	0.011	0.004	0.007	0.434	0.456	0.000	0.415	3.529	0.002
Problem 679	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	55
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.96
time (sec)	N/A	0.012	0.004	0.007	0.446	0.430	0.000	0.422	3.593	0.037
Problem 680	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	55
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.96
time (sec)	N/A	0.013	0.004	0.006	0.455	0.437	0.000	0.497	3.529	0.037
Problem 681	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	55
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.96
time (sec)	N/A	0.012	0.004	0.006	0.440	0.457	0.000	0.560	3.524	0.037
Problem 682	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	28	28	57	29	27	62	0	35	28	55
N.S.	1	1.00	2.04	1.04	0.96	2.21	0.00	1.25	1.00	1.96
time (sec)	N/A	0.013	0.004	0.007	0.436	0.439	0.000	0.474	3.515	0.034
Problem 683	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	58	37	36	87	0	50	33	133
N.S.	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82	3.32
time (sec)	N/A	0.017	0.017	0.006	0.443	0.434	0.000	0.496	3.903	0.258

Problem 684	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	58	37	36	87	0	50	33	133
N.S.	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82	3.32
time (sec)	N/A	0.017	0.004	0.009	0.445	0.468	0.000	0.570	3.557	0.242
Problem 685	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	58	37	36	87	0	50	33	133
N.S.	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82	3.32
time (sec)	N/A	0.017	0.004	0.006	0.456	0.449	0.000	0.491	3.475	0.236
Problem 686	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	F	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	58	37	36	87	0	50	33	56
N.S.	1	1.00	1.45	0.92	0.90	2.18	0.00	1.25	0.82	1.40
time (sec)	N/A	0.019	0.004	0.006	0.449	0.429	0.000	0.515	3.654	0.108
Problem 687	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	F	F	A	F	F	F	B
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	63	0	0	0	0	68	0	0	-1	133
N.S.	1	0.00	0.00	0.00	0.00	1.08	0.00	0.00	-0.02	2.11
time (sec)	N/A	0.027	0.027	0.024	0.000	0.978	0.000	0.000	0.000	0.945
Problem 688	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	B	F	F	F(-1)	F	F	F	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	66	0	180	0	0	0	0	0	-1	130
N.S.	1	0.00	2.73	0.00	0.00	0.00	0.00	0.00	-0.02	1.97
time (sec)	N/A	0.139	0.525	0.089	0.000	0.000	0.000	0.000	0.000	0.999
Problem 689	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	F	B	F	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	78	319	34	438	0	383	0	218	649	105
N.S.	1	4.09	0.44	5.62	0.00	4.91	0.00	2.79	8.32	1.35
time (sec)	N/A	0.568	0.427	0.199	0.000	0.944	0.000	0.862	4.380	0.670

Problem 690	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	B	F	B	F	B	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	126	126	87	753	0	770	0	444	-1	97
N.S.	1	1.00	0.69	5.98	0.00	6.11	0.00	3.52	-0.01	0.77
time (sec)	N/A	0.157	0.036	0.106	0.000	0.732	0.000	0.720	0.000	0.244
Problem 691	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	B	F	F	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	22	22	24	0	0	62	0	0	-1	96
N.S.	1	1.00	1.09	0.00	0.00	2.82	0.00	0.00	-0.05	4.36
time (sec)	N/A	0.062	1.058	0.084	0.000	1.502	0.000	0.000	0.000	0.353
Problem 692	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	40	40	50	0	0	0	0	0	-1	42
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.02	1.05
time (sec)	N/A	0.135	0.489	0.090	0.000	0.000	0.000	0.000	0.000	1.023
Problem 693	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	F	F	F(-1)	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	41	41	47	0	0	0	0	0	-1	44
N.S.	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.02	1.07
time (sec)	N/A	0.142	0.564	0.083	0.000	0.000	0.000	0.000	0.000	1.033
Problem 694	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	54	54	419	514	0	305	0	0	-1	54
N.S.	1	1.00	7.76	9.52	0.00	5.65	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.253	1.964	0.093	0.000	37.188	0.000	0.000	0.000	22.782
Problem 695	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	53	53	416	517	0	304	0	0	-1	53
N.S.	1	1.00	7.85	9.75	0.00	5.74	0.00	0.00	-0.02	1.00
time (sec)	N/A	0.260	1.450	0.076	0.000	35.747	0.000	0.000	0.000	22.673

Problem 696	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	A	F	B	F	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	84	84	160	69	0	154	0	164	-1	119
N.S.	1	1.00	1.90	0.82	0.00	1.83	0.00	1.95	-0.01	1.42
time (sec)	N/A	0.122	0.332	0.032	0.000	0.976	0.000	0.591	0.000	0.462
Problem 697	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	20	20	17	23	7	22	36	15	13	20
N.S.	1	1.00	0.85	1.15	0.35	1.10	1.80	0.75	0.65	1.00
time (sec)	N/A	0.004	0.005	0.004	1.084	0.990	0.442	0.438	3.413	4.615
Problem 698	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	88	88	13884	242984	0	324	0	0	-1	83
N.S.	1	1.00	157.77	2761.18	0.00	3.68	0.00	0.00	-0.01	0.94
time (sec)	N/A	0.249	6.518	0.165	0.000	9.007	0.000	0.000	0.000	0.861
Problem 699	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	88	88	15147	269221	0	331	0	0	-1	536
N.S.	1	1.00	172.12	3059.33	0.00	3.76	0.00	0.00	-0.01	6.09
time (sec)	N/A	0.329	6.546	0.164	0.000	9.269	0.000	0.000	0.000	15.358
Problem 700	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	148	0	0	161	0	0	-1	88
N.S.	1	1.00	3.22	0.00	0.00	3.50	0.00	0.00	-0.02	1.91
time (sec)	N/A	0.622	1.118	0.089	0.000	25.050	0.000	0.000	0.000	3.476
Problem 701	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	161	0	0	161	0	0	-1	135
N.S.	1	1.00	3.50	0.00	0.00	3.50	0.00	0.00	-0.02	2.93
time (sec)	N/A	0.624	1.186	0.089	0.000	22.677	0.000	0.000	0.000	13.762

Problem 702	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	148	0	0	161	0	0	-1	87
N.S.	1	1.00	3.22	0.00	0.00	3.50	0.00	0.00	-0.02	1.89
time (sec)	N/A	1.171	0.139	0.068	0.000	24.441	0.000	0.000	0.000	3.450
Problem 703	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	F	F	A	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	46	46	161	0	0	161	0	0	-1	135
N.S.	1	1.00	3.50	0.00	0.00	3.50	0.00	0.00	-0.02	2.93
time (sec)	N/A	1.168	0.188	0.068	0.000	22.630	0.000	0.000	0.000	12.501
Problem 704	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	B	B	B	B	F(-1)	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	19	19	75	147	94	96	0	58	132	27
N.S.	1	1.00	3.95	7.74	4.95	5.05	0.00	3.05	6.95	1.42
time (sec)	N/A	0.557	1.376	0.056	0.654	0.878	0.000	0.622	6.098	0.433
Problem 705	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	C	F	B	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	90	123	120	2515	0	458	0	0	-1	189
N.S.	1	1.37	1.33	27.94	0.00	5.09	0.00	0.00	-0.01	2.10
time (sec)	N/A	0.114	0.159	13.667	0.000	20.991	0.000	0.000	0.000	0.449
Problem 706	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	F	C	F	B	F	F	F	B
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD	N/A
size	103	0	0	1026	0	289	0	0	-1	214
N.S.	1	0.00	0.00	9.96	0.00	2.81	0.00	0.00	-0.01	2.08
time (sec)	N/A	0.534	0.186	8.030	0.000	11.268	0.000	0.000	0.000	1.291
Problem 707	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	B	F	A	F	F	F	C
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	49	47	46	88	0	51	0	0	-1	53
N.S.	1	0.96	0.94	1.80	0.00	1.04	0.00	0.00	-0.02	1.08
time (sec)	N/A	0.120	0.021	0.026	0.000	1.297	0.000	0.000	0.000	0.267

Problem 708	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	80	80	383	555	0	472	0	0	-1	80
N.S.	1	1.00	4.79	6.94	0.00	5.90	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.449	0.914	0.086	0.000	120.386	0.000	0.000	0.000	25.007
Problem 709	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	1	1	1	2	1	1	0	1	1	19
N.S.	1	1.00	1.00	2.00	1.00	1.00	0.00	1.00	1.00	19.00
time (sec)	N/A	0.001	0.000	0.001	0.460	0.783	0.058	0.352	0.005	0.026
Problem 710	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	122	1932	234	0	73	0	193	549	74
N.S.	1	2.90	46.00	5.57	0.00	1.74	0.00	4.60	13.07	1.76
time (sec)	N/A	0.406	3.961	0.042	0.000	0.400	0.000	0.417	3.925	0.327
Problem 711	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	C	C	C	F	A	F	B	B	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	42	149	1910	234	0	73	0	193	549	74
N.S.	1	3.55	45.48	5.57	0.00	1.74	0.00	4.60	13.07	1.76
time (sec)	N/A	0.357	0.925	0.064	0.000	0.401	0.000	0.583	3.895	0.290
Problem 712	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	F	A	A	A	A	A	B	B	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD	Yes
size	34	0	31	32	16	31	73	81	31	0
N.S.	1	0.00	0.91	0.94	0.47	0.91	2.15	2.38	0.91	0.00
time (sec)	N/A	0.065	0.040	0.003	1.086	0.744	72.393	0.457	3.452	0.112
Problem 713	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	A	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	177	177	1671	1597	0	164	0	0	-1	182
N.S.	1	1.00	9.44	9.02	0.00	0.93	0.00	0.00	-0.01	1.03
time (sec)	N/A	0.089	6.130	0.743	0.000	1.331	0.000	0.000	0.000	7.110

Problem 714	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad	I.A.
grade	A	B	C	C	F	A	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD	NO
size	100	243	2787	2992	0	97	0	0	-1	100
N.S.	1	2.43	27.87	29.92	0.00	0.97	0.00	0.00	-0.01	1.00
time (sec)	N/A	0.139	6.102	1.123	0.000	1.177	0.000	0.000	0.000	5.887

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [325] had the largest ratio of [.6923]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	1	1	1.00	25	0.040
2	A	3	3	1.00	25	0.120
3	F	0	0	N/A	0	N/A
4	F	0	0	N/A	0	N/A
5	F	0	0	N/A	0	N/A
6	A	2	2	1.00	28	0.071
7	A	2	2	1.00	32	0.062
8	A	2	2	1.00	30	0.067
9	A	2	2	1.00	30	0.067
10	A	2	2	1.00	53	0.038
11	A	2	2	1.00	55	0.036
12	A	2	2	1.00	56	0.036
13	A	2	2	1.00	56	0.036
14	A	2	2	1.00	30	0.067
15	A	2	2	1.00	20	0.100
16	A	2	2	1.00	22	0.091
17	A	2	2	1.00	20	0.100
18	A	2	2	1.00	22	0.091
19	A	2	2	1.00	43	0.047
20	A	2	2	1.00	44	0.045
21	A	2	2	1.00	45	0.044
22	A	2	2	1.00	46	0.043
23	A	2	2	1.00	30	0.067
24	A	2	2	1.00	30	0.067
25	A	2	2	1.00	36	0.056
26	A	2	2	1.00	34	0.059
27	A	2	2	1.00	32	0.062
28	A	2	2	1.00	58	0.034
29	A	2	2	1.00	61	0.033
30	A	2	2	1.00	62	0.032
31	A	2	2	1.00	61	0.033
32	A	2	2	1.00	52	0.038
33	A	2	2	1.00	55	0.036
34	A	2	2	1.00	56	0.036
35	A	2	2	1.00	55	0.036
36	A	2	2	1.00	30	0.067
37	A	2	2	1.00	36	0.056
38	A	2	2	1.00	34	0.059
39	A	2	2	1.00	32	0.062

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
40	A	2	2	1.00	58	0.034
41	A	2	2	1.00	61	0.033
42	A	2	2	1.00	62	0.032
43	A	2	2	1.00	61	0.033
44	A	2	2	1.00	52	0.038
45	A	2	2	1.00	55	0.036
46	A	2	2	1.00	56	0.036
47	A	2	2	1.00	55	0.036
48	A	1	1	1.00	31	0.032
49	A	3	3	1.00	30	0.100
50	A	2	1	1.00	18	0.056
51	A	2	1	1.00	16	0.062
52	A	2	1	1.00	15	0.067
53	A	2	1	1.00	20	0.050
54	A	2	1	1.00	18	0.056
55	A	2	1	1.00	17	0.059
56	A	2	1	1.00	20	0.050
57	A	2	1	1.00	18	0.056
58	A	2	1	1.00	17	0.059
59	A	2	2	1.00	27	0.074
60	A	2	2	1.00	29	0.069
61	A	2	2	1.00	27	0.074
62	A	2	2	1.00	29	0.069
63	A	2	2	1.00	31	0.065
64	A	2	2	1.00	35	0.057
65	A	2	2	1.00	33	0.061
66	A	2	2	1.00	33	0.061
67	A	4	3	1.27	19	0.158
68	A	4	3	0.94	19	0.158
69	A	4	3	1.31	19	0.158
70	A	4	4	1.18	19	0.210
71	A	4	3	1.31	19	0.158
72	A	3	3	1.00	17	0.176
73	A	4	3	1.30	15	0.200
74	A	5	4	1.32	19	0.210
75	A	4	3	1.33	19	0.158
76	A	5	4	1.31	19	0.210
77	A	9	5	1.00	19	0.263
78	A	3	3	1.00	17	0.176
79	A	8	4	1.00	15	0.267
80	A	9	5	1.01	19	0.263
81	A	8	5	1.00	19	0.263
82	A	9	6	1.01	19	0.316
83	A	4	4	0.97	19	0.210
84	A	3	3	1.00	17	0.176
85	A	2	2	1.00	15	0.133
86	A	5	5	1.03	19	0.263
87	A	3	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	6	5	1.08	19	0.263
89	A	4	3	1.10	21	0.143
90	A	4	3	1.11	21	0.143
91	A	4	3	1.12	21	0.143
92	A	3	3	1.00	19	0.158
93	A	7	7	1.21	21	0.333
94	A	7	7	1.14	21	0.333
95	A	4	4	1.00	15	0.267
96	A	3	3	1.00	15	0.200
97	A	5	5	1.00	26	0.192
98	A	4	4	1.00	26	0.154
99	A	3	3	1.00	24	0.125
100	A	4	3	1.00	26	0.115
101	A	3	3	1.00	26	0.115
102	A	4	4	1.00	26	0.154
103	A	5	5	1.00	26	0.192
104	A	6	6	1.00	26	0.231
105	A	5	5	1.00	26	0.192
106	A	4	4	1.00	24	0.167
107	A	5	4	1.00	26	0.154
108	A	4	4	1.00	26	0.154
109	A	5	5	1.00	26	0.192
110	A	6	6	1.00	26	0.231
111	A	3	3	1.00	21	0.143
112	A	3	3	1.00	23	0.130
113	A	3	3	1.00	23	0.130
114	A	5	5	1.00	23	0.217
115	A	4	4	1.00	25	0.160
116	A	4	4	1.00	23	0.174
117	A	4	4	1.00	28	0.143
118	A	5	5	1.00	26	0.192
119	A	4	4	1.00	26	0.154
120	A	3	3	1.00	24	0.125
121	A	4	3	1.00	26	0.115
122	A	3	3	1.00	26	0.115
123	A	4	4	1.00	26	0.154
124	A	6	5	0.98	26	0.192
125	A	5	4	1.00	26	0.154
126	A	4	4	1.00	24	0.167
127	A	5	4	1.00	26	0.154
128	A	4	4	1.00	26	0.154
129	A	5	4	1.00	26	0.154
130	A	9	9	1.20	21	0.429
131	A	8	8	1.28	21	0.381
132	A	5	5	1.00	19	0.263
133	A	9	9	1.92	21	0.429
134	A	6	6	1.35	21	0.286
135	A	7	7	1.25	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
136	A	9	8	1.02	21	0.381
137	A	10	9	1.25	21	0.429
138	A	9	8	1.29	21	0.381
139	A	6	6	1.00	19	0.316
140	A	10	10	1.63	21	0.476
141	A	7	6	1.23	21	0.286
142	A	8	7	1.27	21	0.333
143	A	10	9	0.98	21	0.429
144	A	9	9	1.19	21	0.429
145	A	8	8	1.28	21	0.381
146	A	5	5	1.00	19	0.263
147	A	9	9	1.92	21	0.429
148	A	6	6	1.37	21	0.286
149	A	7	7	1.23	21	0.333
150	A	10	9	1.04	21	0.429
151	A	9	8	1.29	21	0.381
152	A	6	5	1.04	19	0.263
153	A	10	10	1.60	21	0.476
154	A	7	6	1.19	21	0.286
155	A	8	7	1.16	21	0.333
156	A	6	5	1.00	19	0.263
157	A	5	5	1.00	19	0.263
158	A	4	4	1.00	19	0.210
159	A	2	2	1.00	19	0.105
160	A	3	3	1.00	19	0.158
161	A	4	4	1.00	22	0.182
162	A	5	5	1.00	19	0.263
163	A	6	5	1.00	22	0.227
164	A	8	5	1.00	33	0.152
165	A	9	6	1.00	30	0.200
166	A	6	6	1.00	19	0.316
167	A	3	3	1.00	19	0.158
168	A	2	2	1.00	19	0.105
169	A	4	4	1.00	19	0.210
170	A	2	2	1.00	19	0.105
171	A	2	2	1.00	19	0.105
172	A	3	3	1.00	17	0.176
173	A	4	4	1.00	19	0.210
174	C	5	3	2.35	54	0.056
175	A	2	2	1.00	7	0.286
176	A	3	2	1.00	15	0.133
177	A	4	2	1.00	22	0.091
178	A	5	2	1.00	25	0.080
179	A	5	2	1.00	23	0.087
180	A	2	1	1.00	21	0.048
181	A	7	4	1.00	25	0.160
182	A	7	4	1.00	25	0.160
183	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
184	A	8	6	1.00	23	0.261
185	A	7	5	1.81	21	0.238
186	A	9	8	1.00	25	0.320
187	A	9	8	1.00	25	0.320
188	A	10	2	1.00	25	0.080
189	A	10	2	1.00	23	0.087
190	B	6	2	2.36	21	0.095
191	A	8	4	1.00	25	0.160
192	A	14	5	1.38	25	0.200
193	A	3	3	1.00	15	0.200
194	A	3	3	1.00	15	0.200
195	A	2	1	1.00	17	0.059
196	A	5	3	1.00	23	0.130
197	A	5	4	1.00	23	0.174
198	A	3	2	1.00	21	0.095
199	A	4	3	1.00	19	0.158
200	A	6	5	1.00	23	0.217
201	A	4	3	1.00	23	0.130
202	A	6	5	1.00	23	0.217
203	A	5	2	1.00	25	0.080
204	A	5	2	1.00	25	0.080
205	A	3	2	1.00	23	0.087
206	A	8	4	1.00	21	0.190
207	A	7	4	1.00	25	0.160
208	A	9	5	1.00	25	0.200
209	A	8	6	1.00	25	0.240
210	A	7	5	1.00	25	0.200
211	A	9	8	1.00	23	0.348
212	A	9	8	1.00	21	0.381
213	A	6	4	1.00	25	0.160
214	A	7	5	1.00	25	0.200
215	A	10	2	1.00	25	0.080
216	A	6	2	1.00	25	0.080
217	A	8	4	1.00	25	0.160
218	A	14	5	1.42	23	0.217
219	A	16	5	1.68	21	0.238
220	A	4	3	1.00	27	0.111
221	A	6	4	1.00	42	0.095
222	A	6	5	1.00	42	0.119
223	A	4	3	1.00	40	0.075
224	A	5	4	1.00	39	0.103
225	A	7	6	1.00	42	0.143
226	A	5	4	1.00	42	0.095
227	A	7	6	1.00	42	0.143
228	A	15	7	1.00	39	0.180
229	A	9	4	1.00	35	0.114
230	A	3	2	1.00	25	0.080
231	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
232	A	4	3	1.00	23	0.130
233	A	3	2	1.00	25	0.080
234	A	3	2	1.00	25	0.080
235	A	3	2	1.00	25	0.080
236	A	6	5	1.00	27	0.185
237	A	6	5	1.00	27	0.185
238	A	6	5	1.00	27	0.185
239	A	5	5	1.00	27	0.185
240	A	5	5	1.00	27	0.185
241	A	6	5	1.00	27	0.185
242	A	3	2	1.00	17	0.118
243	A	3	2	1.00	26	0.077
244	A	1	1	1.00	17	0.059
245	A	1	1	1.00	15	0.067
246	A	1	1	1.00	15	0.067
247	A	1	1	1.00	25	0.040
248	A	3	2	1.00	28	0.071
249	A	3	2	1.00	28	0.071
250	A	4	3	1.00	26	0.115
251	A	3	2	1.00	28	0.071
252	A	3	2	1.00	28	0.071
253	A	3	2	1.00	28	0.071
254	A	6	5	1.00	30	0.167
255	A	6	5	1.00	30	0.167
256	A	6	5	1.00	30	0.167
257	A	5	5	1.00	30	0.167
258	A	5	5	1.00	30	0.167
259	A	6	5	1.00	30	0.167
260	A	3	2	1.00	21	0.095
261	A	3	2	1.00	19	0.105
262	A	3	2	1.00	13	0.154
263	A	3	2	1.00	23	0.087
264	A	3	2	1.00	21	0.095
265	A	3	2	1.00	15	0.133
266	A	3	2	1.00	23	0.087
267	A	3	2	1.00	23	0.087
268	A	3	2	1.00	23	0.087
269	A	2	2	1.00	23	0.087
270	A	3	2	1.00	25	0.080
271	A	3	2	1.00	25	0.080
272	A	3	2	1.00	25	0.080
273	A	2	2	1.00	25	0.080
274	A	4	3	1.00	56	0.054
275	A	4	3	1.00	54	0.056
276	A	4	3	1.00	33	0.091
277	A	5	4	1.00	33	0.121
278	A	4	3	1.00	58	0.052
279	A	4	3	1.00	58	0.052

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
280	A	3	3	1.00	58	0.052
281	A	4	4	1.00	58	0.069
282	A	5	4	1.00	62	0.065
283	A	4	4	1.00	62	0.065
284	A	5	5	1.00	60	0.083
285	A	4	3	1.00	37	0.081
286	A	4	3	1.00	37	0.081
287	A	2	2	1.00	37	0.054
288	A	2	2	1.00	37	0.054
289	A	2	2	1.00	42	0.048
290	A	2	2	1.00	42	0.048
291	A	4	4	1.00	30	0.133
292	A	4	4	1.00	30	0.133
293	A	2	2	1.00	42	0.048
294	A	2	2	1.00	42	0.048
295	A	2	2	1.45	51	0.039
296	A	2	2	1.45	51	0.039
297	A	2	2	1.00	40	0.050
298	A	2	2	1.00	40	0.050
299	A	4	4	1.00	32	0.125
300	A	4	4	1.00	32	0.125
301	A	2	2	1.00	51	0.039
302	A	2	2	1.00	51	0.039
303	A	2	2	1.00	56	0.036
304	A	2	2	1.00	56	0.036
305	A	4	2	1.00	29	0.069
306	A	4	2	1.00	29	0.069
307	A	3	2	1.00	27	0.074
308	A	7	6	1.00	29	0.207
309	A	8	6	1.00	29	0.207
310	A	8	7	1.00	29	0.241
311	A	4	3	1.00	25	0.120
312	A	7	6	1.00	29	0.207
313	A	4	2	1.00	29	0.069
314	A	4	2	1.00	29	0.069
315	A	3	2	1.00	29	0.069
316	A	7	6	1.00	29	0.207
317	A	8	6	1.00	29	0.207
318	A	3	2	1.00	31	0.065
319	A	3	3	1.00	15	0.200
320	A	5	5	1.00	15	0.333
321	A	4	3	1.00	15	0.200
322	A	4	3	1.00	13	0.231
323	A	4	3	1.00	13	0.231
324	A	4	3	1.00	15	0.200
325	A	9	9	1.00	13	0.692
326	A	4	3	1.00	13	0.231
327	A	4	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
328	A	9	9	1.00	15	0.600
329	A	3	3	1.00	13	0.231
330	A	4	3	1.00	13	0.231
331	A	13	10	1.00	15	0.667
332	A	10	7	1.00	19	0.368
333	A	4	3	1.00	19	0.158
334	A	10	9	1.00	21	0.429
335	A	3	3	1.00	26	0.115
336	A	3	3	1.00	26	0.115
337	A	3	3	1.00	24	0.125
338	A	3	3	1.00	23	0.130
339	A	3	3	1.00	26	0.115
340	A	3	3	1.00	26	0.115
341	A	3	3	1.00	28	0.107
342	A	3	3	1.00	28	0.107
343	A	3	3	1.00	26	0.115
344	A	3	3	1.00	25	0.120
345	A	3	3	1.00	28	0.107
346	A	3	3	1.00	28	0.107
347	A	1	1	1.00	17	0.059
348	A	1	1	1.00	21	0.048
349	A	1	1	1.00	17	0.059
350	A	1	1	1.00	19	0.053
351	A	1	1	1.00	21	0.048
352	A	4	3	1.00	25	0.120
353	A	3	2	1.00	17	0.118
354	A	4	3	1.00	27	0.111
355	A	4	3	1.00	25	0.120
356	A	5	4	1.70	22	0.182
357	A	4	3	1.00	29	0.103
358	A	1	1	1.00	176	0.006
359	A	1	1	1.00	174	0.006
360	A	1	1	1.00	164	0.006
361	F	0	0	N/A	0	N/A
362	F	0	0	N/A	0	N/A
363	A	4	3	1.00	19	0.158
364	A	4	3	1.00	19	0.158
365	A	4	3	1.00	17	0.176
366	A	4	3	1.00	15	0.200
367	A	7	6	1.00	19	0.316
368	A	6	6	1.00	19	0.316
369	A	6	6	1.00	19	0.316
370	A	4	3	1.00	21	0.143
371	A	4	3	1.00	21	0.143
372	A	4	3	1.00	19	0.158
373	A	4	3	1.00	17	0.176
374	A	7	6	1.00	21	0.286
375	A	8	7	1.00	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	9	8	1.00	21	0.381
377	A	4	3	1.00	19	0.158
378	A	4	3	1.00	19	0.158
379	A	4	3	1.00	17	0.176
380	A	4	3	1.00	15	0.200
381	A	7	6	1.00	19	0.316
382	A	8	7	1.00	19	0.368
383	A	9	7	1.00	19	0.368
384	A	4	3	1.00	19	0.158
385	A	4	3	1.00	19	0.158
386	A	4	3	1.00	17	0.176
387	A	4	3	1.00	15	0.200
388	A	7	6	1.00	19	0.316
389	A	8	7	1.00	19	0.368
390	A	9	7	1.00	19	0.368
391	A	4	3	1.00	21	0.143
392	A	4	3	1.00	21	0.143
393	A	4	3	1.00	19	0.158
394	A	4	3	1.00	17	0.176
395	A	6	5	1.00	21	0.238
396	A	7	6	1.00	21	0.286
397	A	8	6	1.00	21	0.286
398	A	4	3	1.00	19	0.158
399	A	4	3	1.00	19	0.158
400	A	4	3	1.00	17	0.176
401	A	4	3	1.00	15	0.200
402	A	8	6	1.00	17	0.353
403	A	7	6	1.00	17	0.353
404	A	6	6	1.00	17	0.353
405	A	5	5	1.00	17	0.294
406	A	6	6	1.00	17	0.353
407	A	7	6	1.00	17	0.353
408	A	8	6	1.00	19	0.316
409	A	7	6	1.00	19	0.316
410	A	6	6	1.00	19	0.316
411	A	5	5	1.00	19	0.263
412	A	6	6	1.00	19	0.316
413	A	7	6	1.00	19	0.316
414	A	2	2	1.00	13	0.154
415	A	5	5	1.00	17	0.294
416	A	6	5	1.00	21	0.238
417	A	7	5	1.00	25	0.200
418	A	8	5	1.00	29	0.172
419	A	8	6	1.00	20	0.300
420	A	7	6	1.00	20	0.300
421	A	6	6	1.00	18	0.333
422	A	2	2	1.00	20	0.100
423	A	4	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
424	A	4	3	1.00	20	0.150
425	A	2	2	1.00	18	0.111
426	A	2	2	1.00	20	0.100
427	A	3	2	1.00	22	0.091
428	A	3	3	1.00	17	0.176
429	A	3	3	1.00	23	0.130
430	A	3	2	1.00	22	0.091
431	A	5	4	1.00	34	0.118
432	A	5	5	1.00	31	0.161
433	A	6	4	1.00	47	0.085
434	A	9	5	1.00	58	0.086
435	A	4	4	1.00	11	0.364
436	A	6	6	1.00	11	0.546
437	A	2	1	1.00	17	0.059
438	A	8	6	1.00	13	0.462
439	A	2	1	1.00	16	0.062
440	A	4	2	1.00	14	0.143
441	A	5	4	1.00	12	0.333
442	A	4	2	1.00	13	0.154
443	A	4	2	1.00	13	0.154
444	A	4	2	1.00	15	0.133
445	A	5	5	1.00	12	0.417
446	A	6	5	1.00	14	0.357
447	A	5	4	1.00	13	0.308
448	A	5	4	1.00	17	0.235
449	A	5	4	1.00	17	0.235
450	A	4	3	1.00	13	0.231
451	A	7	5	1.00	18	0.278
452	A	7	5	1.00	20	0.250
453	A	8	5	1.00	18	0.278
454	A	4	3	1.00	26	0.115
455	A	10	6	0.72	17	0.353
456	A	3	1	1.00	27	0.037
457	A	5	3	1.00	17	0.176
458	A	5	3	1.00	17	0.176
459	A	5	3	1.00	23	0.130
460	A	5	3	1.00	17	0.176
461	A	6	2	1.00	23	0.087
462	A	5	1	1.00	25	0.040
463	A	5	2	1.00	21	0.095
464	A	3	2	1.00	23	0.087
465	A	4	3	1.18	16	0.188
466	A	3	1	1.00	28	0.036
467	A	5	5	1.00	16	0.312
468	A	4	3	1.00	22	0.136
469	A	8	4	1.00	21	0.190
470	B	16	10	2.80	20	0.500
471	A	9	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
472	A	6	4	1.00	35	0.114
473	A	2	2	1.00	13	0.154
474	A	3	3	1.00	15	0.200
475	A	3	3	1.00	13	0.231
476	A	4	4	1.00	11	0.364
477	A	3	3	1.00	18	0.167
478	A	4	4	1.00	17	0.235
479	A	4	4	1.00	18	0.222
480	A	5	5	1.00	17	0.294
481	A	2	2	1.00	16	0.125
482	A	2	2	1.00	21	0.095
483	A	3	3	1.00	26	0.115
484	A	3	3	1.00	25	0.120
485	A	3	3	1.00	12	0.250
486	A	3	3	1.00	15	0.200
487	A	3	3	1.00	15	0.200
488	A	3	3	1.00	15	0.200
489	A	3	3	1.00	17	0.176
490	A	4	4	1.00	15	0.267
491	A	4	4	1.00	21	0.190
492	A	3	3	1.00	22	0.136
493	A	4	4	1.00	20	0.200
494	A	7	7	1.00	20	0.350
495	A	2	2	1.00	15	0.133
496	A	5	5	1.00	21	0.238
497	A	8	6	1.00	18	0.333
498	A	5	4	1.00	18	0.222
499	A	6	4	1.00	18	0.222
500	A	3	2	1.00	16	0.125
501	A	3	2	1.00	16	0.125
502	A	3	2	1.00	16	0.125
503	A	10	9	1.00	18	0.500
504	A	5	4	1.00	18	0.222
505	A	6	5	1.00	18	0.278
506	A	5	5	1.40	35	0.143
507	A	6	6	1.40	28	0.214
508	A	11	10	1.00	27	0.370
509	A	14	7	1.00	20	0.350
510	A	12	6	1.91	21	0.286
511	A	2	1	1.00	22	0.045
512	A	4	2	1.00	18	0.111
513	A	4	2	1.00	18	0.111
514	A	5	5	1.00	23	0.217
515	A	6	6	1.00	22	0.273
516	A	6	6	1.00	20	0.300
517	A	3	2	1.00	15	0.133
518	A	3	2	1.00	21	0.095
519	A	5	4	1.00	19	0.210

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
520	A	9	6	1.00	24	0.250
521	A	9	6	1.00	24	0.250
522	A	10	7	1.00	21	0.333
523	A	11	8	1.00	13	0.615
524	A	12	8	1.00	25	0.320
525	A	13	9	1.00	15	0.600
526	A	14	9	1.00	19	0.474
527	A	2	1	1.00	35	0.029
528	A	8	5	1.00	37	0.135
529	A	16	8	1.00	29	0.276
530	A	16	8	1.00	36	0.222
531	A	31	13	1.00	43	0.302
532	A	5	4	1.00	21	0.190
533	A	6	4	1.00	27	0.148
534	A	8	6	1.00	27	0.222
535	A	4	4	1.00	27	0.148
536	A	4	4	1.00	20	0.200
537	A	6	6	1.00	29	0.207
538	A	2	1	1.00	20	0.050
539	A	3	2	1.00	25	0.080
540	A	2	2	1.00	28	0.071
541	A	3	3	1.00	26	0.115
542	A	1	1	1.00	11	0.091
543	A	2	2	1.00	19	0.105
544	A	2	2	1.00	15	0.133
545	A	2	2	1.00	14	0.143
546	A	3	3	1.00	17	0.176
547	A	3	3	1.00	13	0.231
548	A	2	2	1.00	14	0.143
549	A	3	3	1.00	17	0.176
550	A	3	3	1.00	13	0.231
551	A	1	0	1.00	9	0.000
552	A	4	3	1.00	15	0.200
553	A	2	2	1.00	13	0.154
554	A	3	3	1.00	19	0.158
555	A	3	3	1.32	11	0.273
556	A	4	4	1.32	17	0.235
557	A	1	1	1.00	9	0.111
558	A	2	2	1.00	19	0.105
559	A	1	1	1.00	7	0.143
560	A	2	2	1.00	21	0.095
561	A	1	1	1.00	9	0.111
562	A	2	2	1.00	19	0.105
563	A	1	1	1.00	7	0.143
564	A	2	2	1.00	21	0.095
565	A	3	3	1.00	17	0.176
566	A	4	4	1.00	30	0.133
567	A	7	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
568	A	6	6	1.00	20	0.300
569	A	7	7	1.00	23	0.304
570	A	6	6	1.00	25	0.240
571	A	1	1	1.00	11	0.091
572	A	2	2	1.00	21	0.095
573	A	1	1	1.00	9	0.111
574	A	2	2	1.00	23	0.087
575	A	2	2	1.00	11	0.182
576	A	3	3	1.00	21	0.143
577	A	2	2	1.00	9	0.222
578	A	3	3	1.00	23	0.130
579	A	4	4	1.00	15	0.267
580	A	7	6	1.00	17	0.353
581	A	7	6	1.00	17	0.353
582	A	10	8	1.00	34	0.235
583	A	12	7	1.00	30	0.233
584	A	5	4	1.00	21	0.190
585	A	7	5	1.00	23	0.217
586	A	9	7	1.00	25	0.280
587	A	7	6	1.00	25	0.240
588	A	3	3	1.00	13	0.231
589	A	2	2	1.00	24	0.083
590	A	2	2	1.00	22	0.091
591	A	2	2	1.00	30	0.067
592	A	3	3	1.00	27	0.111
593	A	5	5	1.00	23	0.217
594	A	7	5	1.00	28	0.179
595	A	13	10	1.55	25	0.400
596	A	8	6	1.00	29	0.207
597	A	6	6	1.00	16	0.375
598	A	6	6	1.00	16	0.375
599	A	4	4	1.00	16	0.250
600	A	7	7	1.00	16	0.438
601	A	8	8	1.00	16	0.500
602	A	3	3	1.00	24	0.125
603	A	2	2	1.00	37	0.054
604	A	2	2	1.00	38	0.053
605	A	5	4	1.00	40	0.100
606	A	7	5	1.00	40	0.125
607	A	6	5	1.00	18	0.278
608	A	7	6	1.00	21	0.286
609	A	8	6	1.00	22	0.273
610	A	3	3	1.00	21	0.143
611	A	3	3	1.00	24	0.125
612	A	11	10	1.00	19	0.526
613	A	10	7	1.00	24	0.292
614	A	4	3	1.00	19	0.158
615	A	3	3	1.00	17	0.176

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
616	A	1	1	1.00	15	0.067
617	A	1	1	1.00	17	0.059
618	A	2	2	1.00	17	0.118
619	A	2	2	1.00	17	0.118
620	A	1	1	1.00	17	0.059
621	A	6	6	1.00	17	0.353
622	A	5	5	1.00	11	0.454
623	A	3	3	1.00	11	0.273
624	A	5	3	1.00	13	0.231
625	A	2	2	1.00	21	0.095
626	A	5	5	1.00	16	0.312
627	A	2	2	1.00	13	0.154
628	A	6	6	1.00	16	0.375
629	A	5	4	1.00	12	0.333
630	A	4	3	1.00	18	0.167
631	A	10	6	1.00	17	0.353
632	A	1	1	1.00	17	0.059
633	C	3	2	1.03	27	0.074
634	A	2	2	1.00	37	0.054
635	A	3	2	1.00	17	0.118
636	A	5	4	1.00	17	0.235
637	A	1	1	1.00	15	0.067
638	A	3	3	1.00	14	0.214
639	A	1	1	1.00	17	0.059
640	A	2	1	1.00	13	0.077
641	A	2	1	1.00	15	0.067
642	A	7	4	1.00	15	0.267
643	A	6	6	1.00	15	0.400
644	A	1	0	1.00	9	0.000
645	A	1	0	1.00	9	0.000
646	A	2	1	1.00	19	0.053
647	A	3	3	1.00	15	0.200
648	A	3	3	1.00	16	0.188
649	A	4	4	1.00	15	0.267
650	A	5	5	1.00	15	0.333
651	A	4	4	1.00	25	0.160
652	A	2	2	1.00	11	0.182
653	A	2	2	1.00	17	0.118
654	A	6	6	1.00	22	0.273
655	A	4	3	1.00	13	0.231
656	A	4	3	1.00	15	0.200
657	A	4	4	1.00	19	0.210
658	A	4	4	1.00	21	0.190
659	A	3	3	1.00	17	0.176
660	A	7	7	1.00	19	0.368
661	A	7	6	1.00	19	0.316
662	A	6	6	1.00	17	0.353
663	A	10	8	1.00	17	0.471

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
664	A	11	9	1.00	17	0.529
665	A	3	3	1.00	22	0.136
666	A	5	5	1.00	11	0.454
667	A	5	5	1.00	13	0.385
668	A	5	5	1.17	11	0.454
669	A	5	5	1.00	15	0.333
670	A	4	4	1.00	11	0.364
671	A	5	5	1.00	13	0.385
672	A	4	4	1.00	11	0.364
673	A	3	3	1.00	11	0.273
674	A	2	2	1.00	11	0.182
675	A	5	5	1.00	19	0.263
676	A	2	2	1.00	23	0.087
677	A	2	2	1.00	13	0.154
678	A	3	3	1.00	11	0.273
679	A	3	3	1.00	15	0.200
680	A	3	3	1.00	19	0.158
681	A	3	3	1.00	19	0.158
682	A	3	3	1.00	19	0.158
683	A	2	2	1.00	15	0.133
684	A	3	3	1.00	15	0.200
685	A	3	3	1.00	12	0.250
686	A	3	3	1.00	16	0.188
687	F	0	0	N/A	0	N/A
688	F	0	0	N/A	0	N/A
689	B	25	12	4.09	31	0.387
690	A	5	4	1.00	25	0.160
691	A	2	2	1.00	27	0.074
692	A	2	2	1.00	33	0.061
693	A	2	2	1.00	34	0.059
694	A	2	2	1.00	43	0.047
695	A	2	2	1.00	44	0.045
696	A	9	8	1.00	20	0.400
697	A	3	3	1.00	15	0.200
698	A	1	1	1.00	52	0.019
699	A	1	1	1.00	57	0.018
700	A	2	2	1.00	59	0.034
701	A	2	2	1.00	58	0.034
702	A	3	3	1.00	58	0.052
703	A	3	3	1.00	57	0.053
704	A	3	3	1.00	66	0.045
705	A	9	9	1.37	31	0.290
706	F	0	0	N/A	0	N/A
707	C	9	6	0.96	20	0.300
708	A	2	2	1.00	46	0.043
709	A	1	0	1.00	15	0.000
710	C	12	9	2.90	17	0.529
711	C	13	10	3.55	33	0.303

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
712	F	0	0	N/A	0	N/A
713	A	1	1	1.00	38	0.026
714	B	2	2	2.43	30	0.067

Chapter 3

Listing of integrals

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3.32	$\int \frac{1+\sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{(1-\sqrt{3} + \sqrt[3]{\frac{b}{a}}x)\sqrt{a+bx^3}} dx$	264
3.33	$\int \frac{1+\sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{(1-\sqrt{3} - \sqrt[3]{\frac{b}{a}}x)\sqrt{a-bx^3}} dx$	268
3.34	$\int \frac{1+\sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{(1-\sqrt{3} - \sqrt[3]{\frac{b}{a}}x)\sqrt{-a+bx^3}} dx$	272
3.35	$\int \frac{1+\sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{(1-\sqrt{3} + \sqrt[3]{\frac{b}{a}}x)\sqrt{-a-bx^3}} dx$	276
3.36	$\int \frac{1-\sqrt{3}+x}{(1+\sqrt{3}+x)\sqrt{1+x^3}} dx$	280
3.37	$\int \frac{1-\sqrt{3}-x}{(1+\sqrt{3}-x)\sqrt{1-x^3}} dx$	283
3.38	$\int \frac{1-\sqrt{3}-x}{(1+\sqrt{3}-x)\sqrt{-1+x^3}} dx$	286
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3.40	$\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a+bx^3}} dx$	292
3.41	$\int \frac{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{a-bx^3}} dx$	296
3.42	$\int \frac{(1-\sqrt{3})\sqrt[3]{a} - \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{-a+bx^3}} dx$	300
3.43	$\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx$	304
3.44	$\int \frac{1-\sqrt{3} + \sqrt[3]{\frac{b}{a}}x}{(1+\sqrt{3} + \sqrt[3]{\frac{b}{a}}x)\sqrt{a+bx^3}} dx$	308
3.45	$\int \frac{1-\sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{(1+\sqrt{3} - \sqrt[3]{\frac{b}{a}}x)\sqrt{a-bx^3}} dx$	312

3.46	$\int \frac{1-\sqrt{3}-\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}-\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a+bx^3}} dx \dots\dots\dots$	316
3.47	$\int \frac{1-\sqrt{3}+\sqrt[3]{\frac{b}{a}}x}{\left(1+\sqrt{3}+\sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a-bx^3}} dx \dots\dots\dots$	320
3.48	$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx \dots\dots\dots$	324
3.49	$\int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx \dots\dots\dots$	327
3.50	$\int x^2(a+bx)^n(c+dx^3) dx \dots\dots\dots$	330
3.51	$\int x(a+bx)^n(c+dx^3) dx \dots\dots\dots$	336
3.52	$\int (a+bx)^n(c+dx^3) dx \dots\dots\dots$	340
3.53	$\int x^2(a+bx)^n(c+dx^3)^2 dx \dots\dots\dots$	344
3.54	$\int x(a+bx)^n(c+dx^3)^2 dx \dots\dots\dots$	350
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3.64	$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}} dx \dots\dots\dots$	406
3.65	$\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx \dots\dots\dots$	410
3.66	$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}} dx \dots\dots\dots$	415
3.67	$\int x^m \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	419
3.68	$\int x^5 \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	422
3.69	$\int x^4 \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	425
3.70	$\int x^3 \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	428
3.71	$\int x^2 \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	431
3.72	$\int x \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	434
3.73	$\int \left(c(a+bx^2)^2\right)^{3/2} dx \dots\dots\dots$	437
3.74	$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x} dx \dots\dots\dots$	440
3.75	$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2} dx \dots\dots\dots$	443
3.76	$\int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^3} dx \dots\dots\dots$	446

3.77	$\int x^2 \left(c(a+bx^2)^3 \right)^{3/2} dx$	449
3.78	$\int x \left(c(a+bx^2)^3 \right)^{3/2} dx$	453
3.79	$\int \left(c(a+bx^2)^3 \right)^{3/2} dx$	456
3.80	$\int \frac{(c(a+bx^2)^3)^{3/2}}{x} dx$	460
3.81	$\int \frac{(c(a+bx^2)^3)^{3/2}}{x^2} dx$	464
3.82	$\int \frac{(c(a+bx^2)^3)^{3/2}}{x^3} dx$	468
3.83	$\int x^2 \left(\frac{c}{a+bx^2} \right)^{3/2} dx$	472
3.84	$\int x \left(\frac{c}{a+bx^2} \right)^{3/2} dx$	475
3.85	$\int \left(\frac{c}{a+bx^2} \right)^{3/2} dx$	478
3.86	$\int \frac{\left(\frac{c}{a+bx^2} \right)^{3/2}}{x} dx$	481
3.87	$\int \frac{\left(\frac{c}{a+bx^2} \right)^{3/2}}{x^2} dx$	484
3.88	$\int \frac{\left(\frac{c}{a+bx^2} \right)^{3/2}}{x^3} dx$	487
3.89	$\int x^7 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$	491
3.90	$\int x^5 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$	494
3.91	$\int x^3 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$	497
3.92	$\int x \left(c\sqrt{a+bx^2} \right)^{3/2} dx$	500
3.93	$\int \frac{(c\sqrt{a+bx^2})^{3/2}}{x} dx$	503
3.94	$\int \frac{(c\sqrt{a+bx^2})^{3/2}}{x^3} dx$	507
3.95	$\int \sqrt{(b-x)(-a+x)} dx$	511
3.96	$\int \frac{1}{\sqrt{(b-x)(-a+x)}} dx$	514
3.97	$\int x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$	517
3.98	$\int x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$	521
3.99	$\int x \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$	525
3.100	$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx$	529
3.101	$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} dx$	533
3.102	$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5} dx$	537
3.103	$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx$	541
3.104	$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx$	546

3.105	$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx$	551
3.106	$\int x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx$	555
3.107	$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{x} dx$	559
3.108	$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{x^3} dx$	563
3.109	$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{x^5} dx$	567
3.110	$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{x^7} dx$	572
3.111	$\int x \sqrt{\frac{1-x^2}{1+x^2}} dx$	577
3.112	$\int x \sqrt{\frac{5-7x^2}{7+5x^2}} dx$	580
3.113	$\int x^2 \sqrt{\frac{1-x^3}{1+x^3}} dx$	583
3.114	$\int x^8 \sqrt{\frac{1-x^3}{1+x^3}} dx$	586
3.115	$\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}} dx$	589
3.116	$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx$	592
3.117	$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx$	595
3.118	$\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	598
3.119	$\int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	602
3.120	$\int \frac{1}{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	606
3.121	$\int \frac{1}{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	610
3.122	$\int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	614
3.123	$\int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$	618
3.124	$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	622
3.125	$\int \frac{1}{x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	627
3.126	$\int \frac{1}{x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	631
3.127	$\int \frac{1}{x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	635
3.128	$\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	639
3.129	$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$	643

3.130	$\int x^5 \sqrt{a + \frac{b}{c+dx^2}} dx$	647
3.131	$\int x^3 \sqrt{a + \frac{b}{c+dx^2}} dx$	652
3.132	$\int x \sqrt{a + \frac{b}{c+dx^2}} dx$	657
3.133	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x} dx$	661
3.134	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3} dx$	666
3.135	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^5} dx$	670
3.136	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^7} dx$	675
3.137	$\int x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$	681
3.138	$\int x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$	686
3.139	$\int x \left(a + \frac{b}{c+dx^2}\right)^{3/2} dx$	691
3.140	$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x} dx$	695
3.141	$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$	700
3.142	$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^5} dx$	704
3.143	$\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx$	709
3.144	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^5} dx$	715
3.145	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3} dx$	720
3.146	$\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x} dx$	725
3.147	$\int \frac{1}{x \sqrt{a + \frac{b}{c+dx^2}}} dx$	729
3.148	$\int \frac{1}{x^3 \sqrt{a + \frac{b}{c+dx^2}}} dx$	734
3.149	$\int \frac{1}{x^5 \sqrt{a + \frac{b}{c+dx^2}}} dx$	738
3.150	$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	743
3.151	$\int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	749
3.152	$\int \frac{1}{x \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	754
3.153	$\int \frac{1}{x \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	758
3.154	$\int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	764
3.155	$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$	768

3.156	$\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$	774
3.157	$\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$	777
3.158	$\int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$	780
3.159	$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} dx$	783
3.160	$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} dx$	786
3.161	$\int \frac{\sqrt{ax^6}}{x(1-x^4)} dx$	789
3.162	$\int \frac{\sqrt{ax^6}}{x-x^5} dx$	792
3.163	$\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx$	795
3.164	$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$	798
3.165	$\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$	801
3.166	$\int \frac{\sqrt{ax^3}}{x-x^3} dx$	804
3.167	$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} dx$	807
3.168	$\int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} dx$	810
3.169	$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} dx$	812
3.170	$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}} dx$	815
3.171	$\int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}} dx$	818
3.172	$\int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx$	821
3.173	$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx$	824
3.174	$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}} \right) dx$	827
3.175	$\int (ax^m)^r dx$	830
3.176	$\int (ax^m)^r (bx^n)^s dx$	832
3.177	$\int (ax^m)^r (bx^n)^s (cx^p)^t dx$	834
3.178	$\int \frac{x^2}{\sqrt{a+bx} + \sqrt{c+bx}} dx$	837
3.179	$\int \frac{\frac{x}{\sqrt{a+bx} + \sqrt{c+bx}}}{\sqrt{a+bx} + \sqrt{c+bx}} dx$	840
3.180	$\int \frac{1}{\sqrt{a+bx} + \sqrt{c+bx}} dx$	843
3.181	$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})} dx$	846
3.182	$\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})} dx$	851
3.183	$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$	856
3.184	$\int \frac{\frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^2}}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$	861
3.185	$\int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$	865
3.186	$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$	869
3.187	$\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$	873

3.188	$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$	881
3.189	$\int \frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$	885
3.190	$\int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$	889
3.191	$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$	892
3.192	$\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$	898
3.193	$\int \frac{1}{\sqrt{x} + \sqrt{1+x}} dx$	905
3.194	$\int \frac{1}{\sqrt{-1+x} + \sqrt{x}} dx$	908
3.195	$\int \frac{1}{\sqrt{-1+x} + \sqrt{1+x}} dx$	911
3.196	$\int x^3 (\sqrt{1-x} + \sqrt{1+x})^2 dx$	913
3.197	$\int x^2 (\sqrt{1-x} + \sqrt{1+x})^2 dx$	916
3.198	$\int x (\sqrt{1-x} + \sqrt{1+x})^2 dx$	919
3.199	$\int (\sqrt{1-x} + \sqrt{1+x})^2 dx$	922
3.200	$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx$	925
3.201	$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^2} dx$	928
3.202	$\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx$	931
3.203	$\int \frac{x^3}{\sqrt{a+bx} + \sqrt{a+cx}} dx$	934
3.204	$\int \frac{x^2}{\sqrt{a+bx} + \sqrt{a+cx}} dx$	937
3.205	$\int \frac{x}{\sqrt{a+bx} + \sqrt{a+cx}} dx$	940
3.206	$\int \frac{1}{\sqrt{a+bx} + \sqrt{a+cx}} dx$	943
3.207	$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})} dx$	947
3.208	$\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{a+cx})} dx$	953
3.209	$\int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	958
3.210	$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	963
3.211	$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	967
3.212	$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	973
3.213	$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	979
3.214	$\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$	983
3.215	$\int \frac{x^4}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$	987
3.216	$\int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$	990
3.217	$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$	993
3.218	$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$	998
3.219	$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$	1002

3.220	$\int \sqrt{1-x} (\sqrt{1-x} + \sqrt{1+x}) dx$	1006
3.221	$\int x^3 (-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x}) dx$	1009
3.222	$\int x^2 (-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x}) dx$	1012
3.223	$\int x (-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x}) dx$	1015
3.224	$\int (-\sqrt{1-x} - \sqrt{1+x}) (\sqrt{1-x} + \sqrt{1+x}) dx$	1018
3.225	$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x} dx$	1021
3.226	$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x^2} dx$	1024
3.227	$\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x^3} dx$	1027
3.228	$\int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx$	1031
3.229	$\int \frac{-\sqrt{-1+x} + \sqrt{1+x}}{\sqrt{-1+x} + \sqrt{1+x}} dx$	1035
3.230	$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3 dx$	1038
3.231	$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2 dx$	1041
3.232	$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right) dx$	1044
3.233	$\int \frac{1}{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} dx$	1048
3.234	$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2} dx$	1051
3.235	$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3} dx$	1056
3.236	$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$	1060
3.237	$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$	1064
3.238	$\int \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} dx$	1068
3.239	$\int \frac{1}{\sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} dx$	1072
3.240	$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$	1076
3.241	$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx$	1081
3.242	$\int \sqrt{x - \sqrt{-4 + x^2}} dx$	1085
3.243	$\int \sqrt{ax + b \sqrt{c + \frac{a^2 x^2}{b^2}}} dx$	1088
3.244	$\int \sqrt{1 + \sqrt{1 - x^2}} dx$	1091
3.245	$\int \sqrt{1 + \sqrt{1 + x^2}} dx$	1094
3.246	$\int \sqrt{5 + \sqrt{25 + x^2}} dx$	1097
3.247	$\int \sqrt{a + b \sqrt{\frac{a^2}{b^2} + cx^2}} dx$	1100
3.248	$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3 dx$	1103

- 3.249 $\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2 dx \dots\dots\dots 1107$
- 3.250 $\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right) dx \dots\dots\dots 1111$
- 3.251 $\int \frac{1}{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx \dots\dots\dots 1115$
- 3.252 $\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2} dx \dots\dots\dots 1120$
- 3.253 $\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3} dx \dots\dots\dots 1124$
- 3.254 $\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx \dots\dots\dots 1128$
- 3.255 $\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx \dots\dots\dots 1132$
- 3.256 $\int \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx \dots\dots\dots 1136$
- 3.257 $\int \frac{1}{\sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}} dx \dots\dots\dots 1140$
- 3.258 $\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx \dots\dots\dots 1144$
- 3.259 $\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{5/2}} dx \dots\dots\dots 1149$
- 3.260 $\int (a + x^2)^2 \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1154$
- 3.261 $\int (a + x^2) \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1157$
- 3.262 $\int \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1160$
- 3.263 $\int (a + x^2)^2 \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1164$
- 3.264 $\int (a + x^2) \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1167$
- 3.265 $\int \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1170$
- 3.266 $\int (a + x^2)^{5/2} \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1173$
- 3.267 $\int (a + x^2)^{3/2} \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1176$
- 3.268 $\int \sqrt{a + x^2} \left(x + \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1179$
- 3.269 $\int \frac{\left(x + \sqrt{a + x^2} \right)^n}{\sqrt{a + x^2}} dx \dots\dots\dots 1182$
- 3.270 $\int (a + x^2)^{5/2} \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1185$
- 3.271 $\int (a + x^2)^{3/2} \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1188$
- 3.272 $\int \sqrt{a + x^2} \left(x - \sqrt{a + x^2} \right)^n dx \dots\dots\dots 1191$
- 3.273 $\int \frac{\left(x - \sqrt{a + x^2} \right)^n}{\sqrt{a + x^2}} dx \dots\dots\dots 1194$
- 3.274 $\int \left(a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2} \right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx \dots\dots\dots 1197$

- 3.275 $\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \dots\dots\dots 1201$
- 3.276 $\int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \dots\dots\dots 1205$
- 3.277 $\int \left(d + ex + f \sqrt{\frac{af^2+ex(2d+ex)}{f^2}} \right)^n dx \dots\dots\dots 1208$
- 3.278 $\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^{3/2} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \dots\dots\dots 1211$
- 3.279 $\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \dots\dots\dots 1215$
- 3.280 $\int \frac{\left(d+ex+f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}} dx \dots\dots\dots 1219$
- 3.281 $\int \frac{\left(d+ex+f \sqrt{\frac{af^2+ex(2d+ex)}{f^2}} \right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}} dx \dots\dots\dots 1222$
- 3.282 $\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \dots\dots\dots 1225$
- 3.283 $\int \frac{\left(d+ex+f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} dx \dots\dots\dots 1229$
- 3.284 $\int \frac{\left(d+ex+f \sqrt{\frac{af^2+ex(2d+ex)}{f^2}} \right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}} dx \dots\dots\dots 1233$
- 3.285 $\int \frac{e^{-2fx^2}}{e^2+4dfx^2+4efx^2+4f^2x^4} dx \dots\dots\dots 1237$
- 3.286 $\int \frac{e^{-2fx^2}}{e^2-4dfx^2+4efx^2+4f^2x^4} dx \dots\dots\dots 1240$
- 3.287 $\int \frac{e^{-4fx^3}}{e^2+4dfx^2+4efx^3+4f^2x^6} dx \dots\dots\dots 1243$
- 3.288 $\int \frac{e^{-4fx^3}}{e^2-4dfx^2+4efx^3+4f^2x^6} dx \dots\dots\dots 1246$
- 3.289 $\int \frac{e^{-2f(-1+n)x^n}}{e^2+4dfx^2+4efx^n+4f^2x^{2n}} dx \dots\dots\dots 1249$
- 3.290 $\int \frac{e^{-2f(-1+n)x^n}}{e^2-4dfx^2+4efx^n+4f^2x^{2n}} dx \dots\dots\dots 1252$
- 3.291 $\int \frac{x}{e^2+4efx^2+4dfx^4+4f^2x^4} dx \dots\dots\dots 1255$
- 3.292 $\int \frac{x}{e^2+4efx^2-4dfx^4+4f^2x^4} dx \dots\dots\dots 1258$
- 3.293 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6} dx \dots\dots\dots 1261$
- 3.294 $\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx \dots\dots\dots 1264$
- 3.295 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4+4dfx^{2+2m}} dx \dots\dots\dots 1267$
- 3.296 $\int \frac{x^m(e(1+m)+2f(-1+m)x^2)}{e^2+4efx^2+4f^2x^4-4dfx^{2+2m}} dx \dots\dots\dots 1270$
- 3.297 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx \dots\dots\dots 1273$
- 3.298 $\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx \dots\dots\dots 1276$
- 3.299 $\int \frac{x^2}{e^2+4efx^3+4dfx^6+4f^2x^6} dx \dots\dots\dots 1279$
- 3.300 $\int \frac{x^2}{e^2+4efx^3-4dfx^6+4f^2x^6} dx \dots\dots\dots 1282$
- 3.301 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6+4dfx^{2+2m}} dx \dots\dots\dots 1285$
- 3.302 $\int \frac{x^m(e(1+m)+2f(-2+m)x^3)}{e^2+4efx^3+4f^2x^6-4dfx^{2+2m}} dx \dots\dots\dots 1288$

- 3.303 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2+4dfx^{2+2m}+4efx^n+4f^2x^{2n}} dx \dots\dots\dots 1291$
- 3.304 $\int \frac{x^m(e(1+m)+2f(1+m-n)x^n)}{e^2-4dfx^{2+2m}+4efx^n+4f^2x^{2n}} dx \dots\dots\dots 1294$
- 3.305 $\int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx \dots\dots\dots 1297$
- 3.306 $\int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}} dx \dots\dots\dots 1302$
- 3.307 $\int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}} dx \dots\dots\dots 1306$
- 3.308 $\int \frac{1}{x(ac+bcx^2+d\sqrt{a+bx^2})} dx \dots\dots\dots 1309$
- 3.309 $\int \frac{1}{x^3(ac+bcx^2+d\sqrt{a+bx^2})} dx \dots\dots\dots 1314$
- 3.310 $\int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}} dx \dots\dots\dots 1320$
- 3.311 $\int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}} dx \dots\dots\dots 1325$
- 3.312 $\int \frac{1}{x^2(ac+bcx^2+d\sqrt{a+bx^2})} dx \dots\dots\dots 1329$
- 3.313 $\int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}} dx \dots\dots\dots 1333$
- 3.314 $\int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}} dx \dots\dots\dots 1337$
- 3.315 $\int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx \dots\dots\dots 1340$
- 3.316 $\int \frac{1}{x(ac+bcx^3+d\sqrt{a+bx^3})} dx \dots\dots\dots 1343$
- 3.317 $\int \frac{1}{x^4(ac+bcx^3+d\sqrt{a+bx^3})} dx \dots\dots\dots 1347$
- 3.318 $\int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}} dx \dots\dots\dots 1351$
- 3.319 $\int \frac{1}{\sqrt{x}+4x^{3/2}} dx \dots\dots\dots 1354$
- 3.320 $\int \frac{1}{\sqrt{x}-x^{5/2}} dx \dots\dots\dots 1356$
- 3.321 $\int \frac{1}{-\sqrt[4]{x}+\sqrt{x}} dx \dots\dots\dots 1359$
- 3.322 $\int \frac{1}{\sqrt[3]{x}+\sqrt{x}} dx \dots\dots\dots 1362$
- 3.323 $\int \frac{1}{\sqrt[4]{x}+\sqrt{x}} dx \dots\dots\dots 1365$
- 3.324 $\int \frac{1}{-\sqrt[3]{x}+x^{2/3}} dx \dots\dots\dots 1368$
- 3.325 $\int \frac{1}{\frac{1}{\sqrt[4]{x}}+\sqrt{x}} dx \dots\dots\dots 1371$
- 3.326 $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}} dx \dots\dots\dots 1375$
- 3.327 $\int \frac{1}{\frac{1}{\sqrt[3]{x}}+\frac{1}{\sqrt[4]{x}}} dx \dots\dots\dots 1378$
- 3.328 $\int \frac{1}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} dx \dots\dots\dots 1381$
- 3.329 $\int \frac{\sqrt{x}}{x+x^2} dx \dots\dots\dots 1385$
- 3.330 $\int \frac{x}{4\sqrt{x}+x} dx \dots\dots\dots 1387$
- 3.331 $\int \frac{\sqrt{x}}{\sqrt[3]{x}+x} dx \dots\dots\dots 1389$
- 3.332 $\int \frac{\sqrt[3]{x}}{\sqrt[4]{x}+\sqrt{x}} dx \dots\dots\dots 1393$
- 3.333 $\int \frac{\sqrt{x}}{\sqrt[4]{x}+\sqrt[3]{x}} dx \dots\dots\dots 1397$
- 3.334 $\int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}}+\sqrt{x}} dx \dots\dots\dots 1400$

- 3.335 $\int \frac{\sqrt{b-\frac{a}{x}} x^m}{\sqrt{a-bx}} dx \dots\dots\dots 1404$
- 3.336 $\int \frac{\sqrt{b-\frac{a}{x}} x^2}{\sqrt{a-bx}} dx \dots\dots\dots 1407$
- 3.337 $\int \frac{\sqrt{b-\frac{a}{x}} x}{\sqrt{a-bx}} dx \dots\dots\dots 1410$
- 3.338 $\int \frac{\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}} dx \dots\dots\dots 1413$
- 3.339 $\int \frac{\sqrt{b-\frac{a}{x}}}{x\sqrt{a-bx}} dx \dots\dots\dots 1416$
- 3.340 $\int \frac{\sqrt{b-\frac{a}{x}}}{x^2\sqrt{a-bx}} dx \dots\dots\dots 1419$
- 3.341 $\int \frac{\sqrt{b-\frac{a}{x^2}} x^m}{\sqrt{a-bx^2}} dx \dots\dots\dots 1422$
- 3.342 $\int \frac{\sqrt{b-\frac{a}{x^2}} x^2}{\sqrt{a-bx^2}} dx \dots\dots\dots 1425$
- 3.343 $\int \frac{\sqrt{b-\frac{a}{x^2}} x}{\sqrt{a-bx^2}} dx \dots\dots\dots 1428$
- 3.344 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{\sqrt{a-bx^2}} dx \dots\dots\dots 1431$
- 3.345 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x\sqrt{a-bx^2}} dx \dots\dots\dots 1434$
- 3.346 $\int \frac{\sqrt{b-\frac{a}{x^2}}}{x^2\sqrt{a-bx^2}} dx \dots\dots\dots 1437$
- 3.347 $\int \frac{-1+x^3}{(-4x+x^4)^{2/3}} dx \dots\dots\dots 1440$
- 3.348 $\int (2-x^2) \sqrt[4]{6x-x^3} dx \dots\dots\dots 1442$
- 3.349 $\int (1+x^4) \sqrt{5x+x^5} dx \dots\dots\dots 1444$
- 3.350 $\int (2+5x^4) \sqrt{2x+x^5} dx \dots\dots\dots 1446$
- 3.351 $\int \frac{x+3x^2}{\sqrt{x^2+2x^3}} dx \dots\dots\dots 1448$
- 3.352 $\int \frac{2+\sqrt[3]{1-5x}}{3+\sqrt[3]{1-5x}} dx \dots\dots\dots 1450$
- 3.353 $\int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx \dots\dots\dots 1453$
- 3.354 $\int \frac{1-\sqrt{2+3x}}{1+\sqrt{2+3x}} dx \dots\dots\dots 1455$
- 3.355 $\int \frac{-1+\sqrt{a+bx}}{1+\sqrt{a+bx}} dx \dots\dots\dots 1458$
- 3.356 $\int \frac{a+bnx^{-1+n}}{ax+bx^n} dx \dots\dots\dots 1461$
- 3.357 $\int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx \dots\dots\dots 1464$
- 3.358 $\int x(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (2ad+(3bd+3ae+bdm+aen)x+(4cd+4be+4af+2c$
- 3.359 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (ad+(2bd+2ae+bdm+aen)x+(3cd+3be+3af+2cdm$
- 3.360 $\int (a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (bd+ae+bdm+aen+(2cd+2be+2af+2cdm+bem+b$
- 3.361 $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bf$
- 3.362 $\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bf$
- 3.363 $\int x^3 (a+b\sqrt{c+dx})^2 dx \dots\dots\dots 1482$
- 3.364 $\int x^2 (a+b\sqrt{c+dx})^2 dx \dots\dots\dots 1485$
- 3.365 $\int x (a+b\sqrt{c+dx})^2 dx \dots\dots\dots 1488$
- 3.366 $\int (a+b\sqrt{c+dx})^2 dx \dots\dots\dots 1491$

3.367	$\int \frac{(a+b\sqrt{c+dx})^2}{x} dx$	1494
3.368	$\int \frac{(a+b\sqrt{c+dx})^2}{x^2} dx$	1497
3.369	$\int \frac{(a+b\sqrt{c+dx})^2}{x^3} dx$	1501
3.370	$\int x^3 \sqrt{a+b\sqrt{c+dx}} dx$	1505
3.371	$\int x^2 \sqrt{a+b\sqrt{c+dx}} dx$	1509
3.372	$\int x \sqrt{a+b\sqrt{c+dx}} dx$	1512
3.373	$\int \sqrt{a+b\sqrt{c+dx}} dx$	1515
3.374	$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} dx$	1518
3.375	$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2} dx$	1522
3.376	$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3} dx$	1526
3.377	$\int \frac{x^3}{a+b\sqrt{c+dx}} dx$	1532
3.378	$\int \frac{x^2}{a+b\sqrt{c+dx}} dx$	1535
3.379	$\int \frac{x}{a+b\sqrt{c+dx}} dx$	1538
3.380	$\int \frac{1}{a+b\sqrt{c+dx}} dx$	1541
3.381	$\int \frac{1}{x(a+b\sqrt{c+dx})} dx$	1544
3.382	$\int \frac{1}{x^2(a+b\sqrt{c+dx})} dx$	1548
3.383	$\int \frac{1}{x^3(a+b\sqrt{c+dx})} dx$	1552
3.384	$\int \frac{x^3}{(a+b\sqrt{c+dx})^2} dx$	1557
3.385	$\int \frac{x^2}{(a+b\sqrt{c+dx})^2} dx$	1561
3.386	$\int \frac{x}{(a+b\sqrt{c+dx})^2} dx$	1565
3.387	$\int \frac{1}{(a+b\sqrt{c+dx})^2} dx$	1568
3.388	$\int \frac{1}{x(a+b\sqrt{c+dx})^2} dx$	1571
3.389	$\int \frac{1}{x^2(a+b\sqrt{c+dx})^2} dx$	1575
3.390	$\int \frac{1}{x^3(a+b\sqrt{c+dx})^2} dx$	1579
3.391	$\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}} dx$	1584
3.392	$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}} dx$	1588
3.393	$\int \frac{x}{\sqrt{a+b\sqrt{c+dx}}} dx$	1591
3.394	$\int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} dx$	1594
3.395	$\int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}} dx$	1597
3.396	$\int \frac{1}{x^2\sqrt{a+b\sqrt{c+dx}}} dx$	1601
3.397	$\int \frac{1}{x^3\sqrt{a+b\sqrt{c+dx}}} dx$	1606

3.398	$\int x^3 (a + b\sqrt{c + dx})^p dx$	1612
3.399	$\int x^2 (a + b\sqrt{c + dx})^p dx$	1618
3.400	$\int x (a + b\sqrt{c + dx})^p dx$	1622
3.401	$\int (a + b\sqrt{c + dx})^p dx$	1625
3.402	$\int \frac{(a+b(cx)^n)^{5/2}}{x} dx$	1628
3.403	$\int \frac{(a+b(cx)^n)^{3/2}}{x} dx$	1632
3.404	$\int \frac{\sqrt{a+b(cx)^n}}{x} dx$	1636
3.405	$\int \frac{1}{x\sqrt{a+b(cx)^n}} dx$	1639
3.406	$\int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$	1642
3.407	$\int \frac{1}{x(a+b(cx)^n)^{5/2}} dx$	1645
3.408	$\int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$	1649
3.409	$\int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$	1653
3.410	$\int \frac{\sqrt{-a+b(cx)^n}}{x} dx$	1657
3.411	$\int \frac{1}{x\sqrt{-a+b(cx)^n}} dx$	1660
3.412	$\int \frac{1}{x(-a+b(cx)^n)^{3/2}} dx$	1663
3.413	$\int \frac{1}{x(-a+b(cx)^n)^{5/2}} dx$	1666
3.414	$\int \frac{1}{x\sqrt{a+bx}} dx$	1670
3.415	$\int \frac{1}{x\sqrt{a+b(cx)^m}} dx$	1673
3.416	$\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}} dx$	1676
3.417	$\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}^p} dx$	1679
3.418	$\int \frac{1}{x\sqrt{a+b(c(d(e(fx)^m)^n)^p}^q} dx$	1682
3.419	$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} dx$	1686
3.420	$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^2}{x} dx$	1690
3.421	$\int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)}{x} dx$	1694
3.422	$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)} dx$	1697
3.423	$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^2} dx$	1700
3.424	$\int \frac{\sqrt{-1+\frac{1}{x^2}}}{x(-1+x^2)^3} dx$	1703
3.425	$\int \frac{\sqrt{1+\frac{1}{x^2}}x}{(1+x^2)^2} dx$	1706
3.426	$\int \frac{1}{\sqrt{1+\frac{1}{x^2}}x(1+x^2)} dx$	1709
3.427	$\int \frac{x}{a+bx^2+\sqrt{a+bx^2}} dx$	1712
3.428	$\int \frac{x}{x^2-\sqrt[3]{x^2}} dx$	1715

- 3.429 $\int x(1+x^2)^3 \sqrt{2+2x^2+x^4} dx \dots\dots\dots 1718$
- 3.430 $\int x^5 \sqrt{1-x^3} (1+x^9)^2 dx \dots\dots\dots 1721$
- 3.431 $\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx \dots\dots\dots 1724$
- 3.432 $\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}} dx \dots\dots\dots 1727$
- 3.433 $\int \left(\frac{x}{(a+bx^2)^{5/2}} + \frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx \dots\dots\dots 1731$
- 3.434 $\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} dx \dots\dots\dots 1735$
- 3.435 $\int \frac{1}{\sqrt{\sqrt{x}+x}} dx \dots\dots\dots 1739$
- 3.436 $\int \sqrt{\sqrt{x}+x} dx \dots\dots\dots 1742$
- 3.437 $\int \sqrt{-x} (\sqrt{-x}+x) dx \dots\dots\dots 1745$
- 3.438 $\int \frac{5+\sqrt[4]{x}}{-6+x} dx \dots\dots\dots 1747$
- 3.439 $\int \frac{1}{4+\sqrt{4-x-x}} dx \dots\dots\dots 1750$
- 3.440 $\int \frac{1}{1+x-\sqrt{2+x}} dx \dots\dots\dots 1752$
- 3.441 $\int \frac{1}{4+x+\sqrt{1+x}} dx \dots\dots\dots 1755$
- 3.442 $\int \frac{1}{x-\sqrt{1+x}} dx \dots\dots\dots 1758$
- 3.443 $\int \frac{1}{x-\sqrt{2+x}} dx \dots\dots\dots 1761$
- 3.444 $\int \frac{1}{-\sqrt{1-x+x}} dx \dots\dots\dots 1764$
- 3.445 $\int \sqrt{1+\sqrt{x}+x} dx \dots\dots\dots 1767$
- 3.446 $\int \sqrt{1+x+\sqrt{1+x}} dx \dots\dots\dots 1770$
- 3.447 $\int \sqrt{\sqrt{-1+x}+x} dx \dots\dots\dots 1773$
- 3.448 $\int \sqrt{2x+\sqrt{-1+2x}} dx \dots\dots\dots 1776$
- 3.449 $\int \sqrt{3x+\sqrt{-7+8x}} dx \dots\dots\dots 1779$
- 3.450 $\int \frac{1}{\sqrt{x+\sqrt{1+x}}} dx \dots\dots\dots 1782$
- 3.451 $\int \frac{1+x}{4+x+\sqrt{-9+6x}} dx \dots\dots\dots 1785$
- 3.452 $\int \frac{12-x}{4+x+\sqrt{-9+6x}} dx \dots\dots\dots 1788$
- 3.453 $\int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx \dots\dots\dots 1791$
- 3.454 $\int \frac{1}{2\sqrt{-1+x}\sqrt{-\sqrt{-1+x}+x}} dx \dots\dots\dots 1794$
- 3.455 $\int \frac{1+x^{7/2}}{1-x^2} dx \dots\dots\dots 1797$
- 3.456 $\int \frac{4+2x}{\sqrt[3]{-1+2x}\sqrt{-1+2x}} dx \dots\dots\dots 1800$
- 3.457 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx \dots\dots\dots 1803$
- 3.458 $\int \sqrt{2+\sqrt{4+\sqrt{x}}} dx \dots\dots\dots 1806$
- 3.459 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}} dx \dots\dots\dots 1809$
- 3.460 $\int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx \dots\dots\dots 1812$

3.461	$\int \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}} dx$	1815
3.462	$\int \sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}} dx$	1818
3.463	$\int \sqrt{1 + \sqrt{1 + \sqrt{-1 + x}}} x dx$	1821
3.464	$\int \frac{1}{\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx$	1824
3.465	$\int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx$	1827
3.466	$\int \frac{q+px}{\sqrt{b+ax}(f+\sqrt{b+ax})} dx$	1830
3.467	$\int \sqrt{1 - \sqrt{x} - x} dx$	1833
3.468	$\int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} dx$	1836
3.469	$\int \frac{6-8x^{7/2}}{5-9\sqrt{x}} dx$	1839
3.470	$\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2} dx$	1842
3.471	$\int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}} dx$	1847
3.472	$\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}} dx$	1850
3.473	$\int \frac{1}{\sqrt{x}\sqrt{1+x}} dx$	1853
3.474	$\int \sqrt{\frac{x}{1+x}} dx$	1855
3.475	$\int \frac{x}{\sqrt{x}\sqrt{1+x}} dx$	1858
3.476	$\int \sqrt{\frac{x}{1+x}} dx$	1861
3.477	$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}} dx$	1864
3.478	$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$	1867
3.479	$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}} dx$	1870
3.480	$\int x^3 \sqrt{\frac{-1+x}{1+x}} dx$	1873
3.481	$\int \sqrt{\frac{-x}{1+x}} dx$	1876
3.482	$\int \sqrt{\frac{\frac{x}{1-x}}{1+x}} dx$	1879
3.483	$\int \sqrt{\frac{c-bx}{a+bx}} dx$	1882
3.484	$\int \sqrt{\frac{\frac{a+bx}{c+dx}}{a+bx}} dx$	1885
3.485	$\int \sqrt{\frac{-x}{1+x}} dx$	1888
3.486	$\int \sqrt{\frac{1-x}{1+x}} dx$	1891
3.487	$\int \sqrt{\frac{a+x}{a-x}} dx$	1894
3.488	$\int \sqrt{\frac{-a+x}{a+x}} dx$	1897
3.489	$\int \sqrt{\frac{a+bx}{c+dx}} dx$	1900
3.490	$\int \sqrt{\frac{-1+x}{5+3x}} dx$	1903
3.491	$\int \sqrt{\frac{-1+5x}{1+7x}} dx$	1906

3.492	$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)} dx$	1909
3.493	$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}} dx$	1912
3.494	$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx$	1915
3.495	$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx$	1919
3.496	$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} dx$	1922
3.497	$\int \frac{1}{x+\sqrt{3-2x-x^2}} dx$	1925
3.498	$\int \frac{1}{(x+\sqrt{3-2x-x^2})^2} dx$	1929
3.499	$\int \frac{1}{(x+\sqrt{3-2x-x^2})^3} dx$	1933
3.500	$\int \frac{1}{x+\sqrt{-3-2x+x^2}} dx$	1937
3.501	$\int \frac{1}{(x+\sqrt{-3-2x+x^2})^2} dx$	1940
3.502	$\int \frac{1}{(x+\sqrt{-3-2x+x^2})^3} dx$	1943
3.503	$\int \frac{1}{x+\sqrt{-3-4x-x^2}} dx$	1946
3.504	$\int \frac{1}{(x+\sqrt{-3-4x-x^2})^2} dx$	1950
3.505	$\int \frac{1}{(x+\sqrt{-3-4x-x^2})^3} dx$	1955
3.506	$\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4} dx$	1959
3.507	$\int (1+2x)(x+x^2)^3\sqrt{1-(x+x^2)^2} dx$	1962
3.508	$\int \frac{(2\sqrt{3-x}+\frac{3}{\sqrt{1+x}})^2}{x} dx$	1965
3.509	$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} dx$	1969
3.510	$\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}} dx$	1972
3.511	$\int \frac{2\sqrt{-1+x+x}}{\sqrt{-1+xx}} dx$	1975
3.512	$\int (a+c\sqrt{x}+bx^{2/3})^2 dx$	1977
3.513	$\int (a+c\sqrt{x}+bx^{2/3})^3 dx$	1980
3.514	$\int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}x^3}} dx$	1983
3.515	$\int \frac{-1+x^2}{\sqrt{a+b(-1+\frac{1}{x^2})x^3}} dx$	1987
3.516	$\int \frac{1+x}{(4+x^2)\sqrt{9+x^2}} dx$	1991
3.517	$\int x(1+\sqrt{1-x^2}) dx$	1994
3.518	$\int x(1+\sqrt{1-x}\sqrt{1+x}) dx$	1996
3.519	$\int x(1+\frac{1}{\sqrt{2+x}\sqrt{3+x}}) dx$	1999
3.520	$\int \frac{x-\sqrt{x^6}}{x(1-x^4)} dx$	2002
3.521	$\int \frac{1-\frac{\sqrt{x^6}}{x}}{1-x^4} dx$	2005
3.522	$\int \frac{x-\sqrt{x^6}}{x-x^5} dx$	2008

3.523	$\int \frac{x}{x+\sqrt{x^6}} dx$	2011
3.524	$\int \frac{\sqrt{x}-\sqrt{x^3}}{x-x^3} dx$	2014
3.525	$\int \frac{1}{\sqrt{x}+\sqrt{x^3}} dx$	2017
3.526	$\int \frac{1}{\sqrt{-1+x}+\sqrt{(-1+x)^3}} dx$	2021
3.527	$\int \left(-\frac{3}{(4+5x)^2} - \frac{5+4x}{(4+5x)^2\sqrt{1-x^2}} \right) dx$	2025
3.528	$\int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}} dx$	2028
3.529	$\int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)} dx$	2031
3.530	$\int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}} dx$	2034
3.531	$\int \frac{-1+\sqrt{1-x^2}}{\sqrt{1-x^2}(2+x-2\sqrt{1-x^2})^2} dx$	2038
3.532	$\int \frac{a+bx^{-1+n}}{cx+dx^n} dx$	2043
3.533	$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}} dx$	2046
3.534	$\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}} dx$	2049
3.535	$\int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{-1+x^2}} dx$	2053
3.536	$\int \frac{1+2x^8}{x(1+x^8)^{3/2}} dx$	2057
3.537	$\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx$	2060
3.538	$\int \left(1 - 9x^2 + \frac{x}{\sqrt{1-9x^2}} \right) dx$	2063
3.539	$\int \frac{x+(1-9x^2)^{3/2}}{\sqrt{1-9x^2}} dx$	2065
3.540	$\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx$	2067
3.541	$\int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}} dx$	2070
3.542	$\int \frac{1}{\sqrt{4-9x^2}} dx$	2073
3.543	$\int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}} dx$	2075
3.544	$\int \frac{1}{\sqrt{(2-3x)(2+3x)}} dx$	2078
3.545	$\int \frac{1}{\sqrt{15-2x-x^2}} dx$	2080
3.546	$\int \frac{1}{\sqrt{3-x}\sqrt{5+x}} dx$	2082
3.547	$\int \frac{1}{\sqrt{(3-x)(5+x)}} dx$	2085
3.548	$\int \frac{1}{\sqrt{-15-8x-x^2}} dx$	2088
3.549	$\int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx$	2090
3.550	$\int \frac{1}{\sqrt{(-3-x)(5+x)}} dx$	2093
3.551	$\int (1 - \sqrt{x}) dx$	2095
3.552	$\int \frac{1-x}{1+\sqrt{x}} dx$	2097
3.553	$\int \sqrt{\frac{1}{1-x^2}} dx$	2100
3.554	$\int \sqrt{\frac{1+x^2}{1-x^4}} dx$	2102
3.555	$\int \sqrt{\frac{1}{-1+x^2}} dx$	2105
3.556	$\int \sqrt{\frac{1+x^2}{-1+x^4}} dx$	2108

3.557	$\int \frac{1}{\sqrt{1-x}} dx$	2111
3.558	$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx$	2113
3.559	$\int \frac{1}{\sqrt{1+x}} dx$	2115
3.560	$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx$	2117
3.561	$\int \sqrt{1-x} dx$	2119
3.562	$\int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} dx$	2121
3.563	$\int \sqrt{1+x} dx$	2123
3.564	$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx$	2125
3.565	$\int \frac{\sqrt{2+3x}}{\sqrt{1+x}} dx$	2127
3.566	$\int \frac{\sqrt{1-x} \sqrt{2+3x}}{\sqrt{1-x^2}} dx$	2130
3.567	$\int \frac{(1+x)^{3/2}}{(1-x)^{3/2} x} dx$	2133
3.568	$\int \frac{(1+x)^3}{x(1-x^2)^{3/2}} dx$	2137
3.569	$\int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} dx$	2140
3.570	$\int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}} dx$	2144
3.571	$\int \frac{1}{\sqrt{1-x^2}} dx$	2147
3.572	$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$	2149
3.573	$\int \frac{1}{\sqrt{1+x^2}} dx$	2151
3.574	$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$	2153
3.575	$\int \sqrt{1-x^2} dx$	2155
3.576	$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$	2157
3.577	$\int \sqrt{1+x^2} dx$	2160
3.578	$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$	2162
3.579	$\int \frac{1}{x-\sqrt{1+x^2}} dx$	2165
3.580	$\int \frac{1}{x-\sqrt{1-x^2}} dx$	2168
3.581	$\int \frac{1}{x-\sqrt{1+2x^2}} dx$	2171
3.582	$\int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2} dx$	2174
3.583	$\int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}} dx$	2178
3.584	$\int \frac{1}{-x+\sqrt{2x-x^2}} dx$	2181
3.585	$\int \frac{x+\sqrt{2x-x^2}}{2-2x} dx$	2184
3.586	$\int \frac{\sqrt{2-x} \sqrt{x+x}}{2-2x} dx$	2187
3.587	$\int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx$	2191
3.588	$\int \frac{1}{((1+x)(-1+x^2))^{2/3}} dx$	2194
3.589	$\int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}} dx$	2197
3.590	$\int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} dx$	2200

- 3.591 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx \dots\dots\dots 2203$
- 3.592 $\int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2} dx \dots\dots\dots 2206$
- 3.593 $\int \frac{1}{\sqrt{a+\frac{b}{x^2}} \sqrt{c+dx^2}} dx \dots\dots\dots 2209$
- 3.594 $\int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)} dx \dots\dots\dots 2214$
- 3.595 $\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2} dx \dots\dots\dots 2217$
- 3.596 $\int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2} dx \dots\dots\dots 2222$
- 3.597 $\int \left(1 + \frac{2x}{1+x^2}\right)^{5/2} dx \dots\dots\dots 2226$
- 3.598 $\int \left(1 + \frac{2x}{1+x^2}\right)^{3/2} dx \dots\dots\dots 2230$
- 3.599 $\int \sqrt{1 + \frac{2x}{1+x^2}} dx \dots\dots\dots 2234$
- 3.600 $\int \frac{1}{\sqrt{1+\frac{2x}{1+x^2}}} dx \dots\dots\dots 2237$
- 3.601 $\int \frac{1}{\left(1+\frac{2x}{1+x^2}\right)^{3/2}} dx \dots\dots\dots 2241$
- 3.602 $\int \frac{\sqrt{1+\frac{2x}{1+x^2}}}{1+x^2} dx \dots\dots\dots 2245$
- 3.603 $\int \frac{\sqrt{bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx \dots\dots\dots 2248$
- 3.604 $\int \frac{\sqrt{-bx^2+\sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx \dots\dots\dots 2251$
- 3.605 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}} dx \dots\dots\dots 2254$
- 3.606 $\int \frac{\sqrt{2x^2+\sqrt{3+4x^4}}}{(c+dx)^2\sqrt{3+4x^4}} dx \dots\dots\dots 2257$
- 3.607 $\int \frac{-4+x}{(1+\sqrt[3]{x})\sqrt{x}} dx \dots\dots\dots 2261$
- 3.608 $\int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} dx \dots\dots\dots 2264$
- 3.609 $\int \frac{1+\sqrt{x}}{(1+\sqrt[3]{x})\sqrt{x}} dx \dots\dots\dots 2267$
- 3.610 $\int \frac{\sqrt{2+\frac{b}{x^2}}}{b+2x^2} dx \dots\dots\dots 2270$
- 3.611 $\int \frac{\sqrt{2-\frac{b}{x^2}}}{-b+2x^2} dx \dots\dots\dots 2273$
- 3.612 $\int \frac{\sqrt{a+\frac{c}{x^2}}}{d+ex} dx \dots\dots\dots 2276$
- 3.613 $\int \frac{\sqrt{a+\frac{c}{x^2}+\frac{b}{x}}}{d+ex} dx \dots\dots\dots 2281$
- 3.614 $\int \frac{\sqrt[6]{x}+\sqrt[5]{x^3}}{\sqrt{x}} dx \dots\dots\dots 2286$
- 3.615 $\int \frac{2+x}{\sqrt{4x-x^2}} dx \dots\dots\dots 2289$
- 3.616 $\int \frac{3+x}{\sqrt[3]{6x+x^2}} dx \dots\dots\dots 2292$
- 3.617 $\int \frac{4+x}{(6x-x^2)^{3/2}} dx \dots\dots\dots 2294$

3.618	$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$	2296
3.619	$\int \frac{1}{(1+2x)\sqrt{x+x^2}} dx$	2298
3.620	$\int \frac{-1+x}{\sqrt{2x-x^2}} dx$	2301
3.621	$\int \frac{\sqrt{x-x^2}}{1+x} dx$	2303
3.622	$\int \sqrt{\sqrt[4]{x} + x} dx$	2306
3.623	$\int \sqrt{x + x^{3/2}} dx$	2309
3.624	$\int x\sqrt{x + x^{3/2}} dx$	2312
3.625	$\int (1-x^2)\sqrt{\frac{1}{2-x^2}} dx$	2315
3.626	$\int \sqrt{x^2 + x^3 - x^4} dx$	2318
3.627	$\int \frac{1}{\sqrt{(a^2+x^2)^3}} dx$	2321
3.628	$\int \frac{\sqrt{x}}{1+\sqrt{x}+x} dx$	2324
3.629	$\int \frac{x}{1+\sqrt{x}+x} dx$	2327
3.630	$\int \frac{1}{\sqrt{x}(1+\sqrt{x}+x)^{7/2}} dx$	2330
3.631	$\int \frac{-1+x}{1+\sqrt{1+x^2}} dx$	2333
3.632	$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}} dx$	2336
3.633	$\int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx$	2338
3.634	$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}} dx$	2341
3.635	$\int \frac{x-2x^3}{\sqrt{2+3x}} dx$	2344
3.636	$\int \frac{1}{\sqrt[4]{1+x} + \sqrt{1+x}} dx$	2347
3.637	$\int \frac{1+2x}{\sqrt{x+x^2}} dx$	2350
3.638	$\int \frac{1}{2\sqrt{x}(1+x)} dx$	2352
3.639	$\int \frac{1}{x\sqrt{6x-x^2}} dx$	2354
3.640	$\int (1+\sqrt{x})\sqrt{x} dx$	2356
3.641	$\int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx$	2358
3.642	$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$	2360
3.643	$\int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx$	2363
3.644	$\int (1-\sqrt{x}) dx$	2367
3.645	$\int (1-\sqrt[4]{x}) dx$	2369
3.646	$\int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx$	2371
3.647	$\int \frac{1}{\sqrt{(a+bx)(c+dx)}} dx$	2373
3.648	$\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$	2376
3.649	$\int \frac{1}{\sqrt{x}(1-x^2)} dx$	2379
3.650	$\int \frac{\sqrt{x}}{x-x^3} dx$	2382
3.651	$\int \frac{x}{2-\sqrt{3}+(1+\sqrt{3})x+x^2} dx$	2385
3.652	$\int \sqrt{x^2 + x^3} dx$	2389
3.653	$\int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$	2392

3.654	$\int \sqrt{1 - \sqrt{x} - x} \sqrt{x} dx$	2394
3.655	$\int \sqrt[3]{1 + \sqrt{-3 + x}} dx$	2397
3.656	$\int \frac{1}{\sqrt{3 + \sqrt{-1 + 2x}}} dx$	2400
3.657	$\int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx$	2403
3.658	$\int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx$	2406
3.659	$\int \frac{x}{x-\sqrt{1+x^2}} dx$	2409
3.660	$\int \frac{x}{x-\sqrt{1-x^2}} dx$	2412
3.661	$\int \frac{x}{x-\sqrt{1+2x^2}} dx$	2416
3.662	$\int \sqrt{x} \sqrt{\sqrt{x} + x} dx$	2419
3.663	$\int \frac{1+\sqrt[3]{x}}{1+\sqrt{x}} dx$	2422
3.664	$\int \frac{1+\sqrt[3]{x}}{1+\sqrt[4]{x}} dx$	2426
3.665	$\int \frac{x^2}{-1+x^2+\sqrt{1-x^2}} dx$	2430
3.666	$\int \sqrt{\frac{1+x}{x}} dx$	2432
3.667	$\int \sqrt{\frac{1-x}{x}} dx$	2435
3.668	$\int \sqrt{\frac{-1+x}{x}} dx$	2438
3.669	$\int \sqrt{\frac{1+x}{x}} dx$	2441
3.670	$\int \sqrt{\frac{x}{1+x}} dx$	2444
3.671	$\int \frac{1}{\sqrt{\frac{-1-x}{x}}} dx$	2447
3.672	$\int \sqrt{(4-x)x} dx$	2450
3.673	$\int \frac{1}{\sqrt{(1-x)x}} dx$	2453
3.674	$\int \frac{x}{(x(2+x))^{3/2}} dx$	2455
3.675	$\int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$	2457
3.676	$\int \frac{1}{1+\sqrt{5-x^2}+\sqrt{5}x^2} dx$	2460
3.677	$\int \frac{1}{\sqrt{ax+bx^2}} dx$	2463
3.678	$\int \frac{1}{\sqrt{x(ax+bx^2)}} dx$	2466
3.679	$\int \frac{1}{\sqrt{(b+\frac{a}{x})x^2}} dx$	2469
3.680	$\int \frac{1}{\sqrt{(\frac{a}{x^2}+\frac{b}{x})x^3}} dx$	2472
3.681	$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx$	2475
3.682	$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx$	2478
3.683	$\int \frac{1}{\sqrt{acx+bcx^2}} dx$	2481
3.684	$\int \frac{1}{\sqrt{c(ax+bx^2)}} dx$	2484
3.685	$\int \frac{1}{\sqrt{cx(ax+bx)}} dx$	2487
3.686	$\int \frac{1}{\sqrt{c(b+\frac{a}{x})x^2}} dx$	2490

- 3.687 $\int \sqrt{1-x^2+x}\sqrt{-1+x^2} dx \dots\dots\dots 2493$
- 3.688 $\int \frac{\sqrt{-x+\sqrt{x}}\sqrt{1+x}}{\sqrt{1+x}} dx \dots\dots\dots 2495$
- 3.689 $\int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx \dots\dots\dots 2497$
- 3.690 $\int \frac{1}{(1+x^2)\sqrt{2+2x+x^2}} dx \dots\dots\dots 2502$
- 3.691 $\int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} dx \dots\dots\dots 2506$
- 3.692 $\int \frac{1}{(a+bx^4)\sqrt{cx^2+d}\sqrt{a+bx^4}} dx \dots\dots\dots 2509$
- 3.693 $\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d}\sqrt{a+bx^4}} dx \dots\dots\dots 2512$
- 3.694 $\int \frac{a-cx^4}{\sqrt{a+bx^2+cx^4}(ad+aex^2+cdx^4)} dx \dots\dots\dots 2515$
- 3.695 $\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4}(ad+aex^2+cdx^4)} dx \dots\dots\dots 2518$
- 3.696 $\int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)} dx \dots\dots\dots 2521$
- 3.697 $\int \sqrt{\frac{x^2}{1+x^2}} dx \dots\dots\dots 2525$
- 3.698 $\int \frac{ef-efx^2}{(ad+bdx+adx^2)\sqrt{a+bx+cx^2+bx^3+ax^4}} dx \dots\dots\dots 2528$
- 3.699 $\int \frac{ef-efx^2}{(-ad+bdx-adx^2)\sqrt{-a+bx+cx^2+bx^3-ax^4}} dx \dots\dots\dots 2531$
- 3.700 $\int \frac{\sqrt{ax^2+bx}\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx \dots\dots\dots 2534$
- 3.701 $\int \frac{\sqrt{-ax^2+bx}\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx \dots\dots\dots 2537$
- 3.702 $\int \frac{\sqrt{x\left(ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx \dots\dots\dots 2541$
- 3.703 $\int \frac{\sqrt{x\left(-ax+b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}\right)}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx \dots\dots\dots 2545$
- 3.704 $\int \frac{-\sqrt{-4+x}-4\sqrt{-1+x}+\sqrt{-4+x}x+\sqrt{-1+x}x}{(1+\sqrt{-4+x}+\sqrt{-1+x})(4-5x+x^2)} dx \dots\dots\dots 2549$
- 3.705 $\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}} dx \dots\dots\dots 2552$
- 3.706 $\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx \dots\dots\dots 2557$
- 3.707 $\int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)} dx \dots\dots\dots 2560$
- 3.708 $\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}} dx \dots\dots\dots 2563$
- 3.709 $\int \left(x + \frac{1-x^2}{1+x}\right) dx \dots\dots\dots 2566$
- 3.710 $\int \frac{1}{\frac{1}{x}+\sqrt{1-x^2}} dx \dots\dots\dots 2568$
- 3.711 $\int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx \dots\dots\dots 2573$
- 3.712 $\int (1+x+x^2+x^3)^{-n} (1-x^4)^n dx \dots\dots\dots 2578$
- 3.713 $\int \frac{1}{\sqrt{-44375b^4+576000b^3cx+576000b^2c^2x^2+5308416c^4x^4}} dx \dots\dots\dots 2580$

3.714 $\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} dx \dots\dots\dots 2585$

$$3.1 \quad \int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx$$

Optimal. Leaf size=139

$$-\frac{3 \log \left(2^{2/3} d \sqrt[3]{d^3 x^3 - c^3} + d(c - dx) \right)}{4 \sqrt[3]{2} cd} + \frac{\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3 x^3 - c^3}}}{\sqrt{3}} \right)}{2 \sqrt[3]{2} cd} + \frac{\log \left((c - dx)(c + dx)^2 \right)}{4 \sqrt[3]{2} cd}$$

Rubi [A] time = 0.07, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2148}

$$-\frac{3 \log \left(2^{2/3} d \sqrt[3]{d^3 x^3 - c^3} + d(c - dx) \right)}{4 \sqrt[3]{2} cd} + \frac{\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3 x^3 - c^3}}}{\sqrt{3}} \right)}{2 \sqrt[3]{2} cd} + \frac{\log \left((c - dx)(c + dx)^2 \right)}{4 \sqrt[3]{2} cd}$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]

[Out] (Sqrt[3]*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*2^(1/3)*c*d) + Log[(c - d*x)*(c + d*x)^2]/(4*2^(1/3)*c*d) - (3*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d)

Rule 2148

Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3))]/Sqrt[3])]/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^(1/3)])/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 + a*d^3, 0]

Rubi steps

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx = \frac{\sqrt{3} \tan^{-1} \left(\frac{1 - \frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3+d^3x^3}}}{\sqrt{3}} \right)}{2 \sqrt[3]{2} cd} + \frac{\log \left((c - dx)(c + dx)^2 \right)}{4 \sqrt[3]{2} cd} - \frac{3 \log \left(d(c - dx) + 2^{2/3} d \sqrt[3]{-c^3} \right)}{4 \sqrt[3]{2} cd}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]

[Out] Integrate[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]

IntegrateAlgebraic [C] time = 2.89, size = 385, normalized size = 2.77

$$\frac{(-1)^{5/6} \sqrt{3} \tanh^{-1} \left(\frac{\sqrt[3]{2} \sqrt[3]{-c^3 + d^3 x^3} - \sqrt[3]{2} \sqrt[3]{-c^3 + d^3 x^3} - \sqrt[3]{2} \sqrt[3]{-c^3 + d^3 x^3}}{\sqrt[3]{2} \sqrt[3]{-c^3 + d^3 x^3}} \right)}{2 \sqrt[3]{2} cd} + \frac{\sqrt[3]{2} \log \left(i \sqrt{3} c^{3/2} \sqrt{d} - c^{3/2} \sqrt{d} + 2^{2/3} \sqrt{c} \sqrt{d} \sqrt[3]{d^3 x^3 - c^3} + \sqrt{c} d^{3/2} (x - i \sqrt{3} x) \right)}{2cd} - \frac{\sqrt[3]{2} \log \left(4 \sqrt[3]{2} cd (d^3 x^3 - c^3)^{2/3} - i \sqrt{3} c^2 d - c^2 d + 2i \sqrt{3} c^2 d x + 2c^2 d^2 x + (2(-2)^{2/3} cd x - 2(-2)^{2/3} d^2 d) \sqrt[3]{d^3 x^3 - c^3} - i \sqrt{3} cd^3 x^2 - cd^3 x^2 \right)}{4cd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)),x]

[Out] $((-1)^{5/6} \sqrt{3} \operatorname{ArcTanh}[\frac{(Ic + \sqrt{3}c)}{2^{2/3} \sqrt{3}}] + (((-I)d - \sqrt{3}d)x) / (2^{2/3} \sqrt{3}) + (I(-c^3 + d^3x^3)^{1/3}) / \sqrt{3}) / (-c^3 + d^3x^3)^{1/3}) / (2 \cdot 2^{1/3} cd) + ((-1/2)^{1/3} \operatorname{Log}[-(c^{3/2} \sqrt{d})] + I \sqrt{3} c^{3/2} \sqrt{d} + \sqrt{c} d^{3/2} (x - I \sqrt{3} x) + 2 \cdot 2^{2/3} \sqrt{c} \sqrt{d} (-c^3 + d^3x^3)^{1/3}) / (2cd) - ((-1/2)^{1/3} \operatorname{Log}[-(c^3 d - I \sqrt{3} c^3 d + 2c^2 d^2 x + (2I) \sqrt{3} c^2 d^2 x - cd^3 x^2 - I \sqrt{3} cd^3 x^2 + (-2(-2)^{2/3} c^2 d + 2(-2)^{2/3} cd^2 x) (-c^3 + d^3x^3)^{1/3} + 4 \cdot 2^{1/3} cd (-c^3 + d^3x^3)^{2/3})] / (4cd)$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 - c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="giac")

[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(d^3x^3 - c^3)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)

[Out] int(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 - c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(d^3x^3 - c^3)^{1/3}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)`

[Out] `int(1/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{(-c + dx)(c^2 + cdx + d^2x^2)}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d*x+c)/(d**3*x**3-c**3)**(1/3), x)`

[Out] `Integral(1/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)`

$$3.2 \quad \int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx$$

Optimal. Leaf size=186

$$\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd}$$

Rubi [A] time = 0.20, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2149, 239, 2151}

$$\frac{\log\left(\sqrt[3]{2c^3+d^3x^3}-dx\right)}{4cd} + \frac{3\log\left(d(2c+dx)-d\sqrt[3]{2c^3+d^3x^3}\right)}{4cd} + \frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd}$$

Antiderivative was successfully verified.

[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

[Out] ArcTan[(1 + (2*d*x)/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*Sqrt[3]*c*d) - (Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/(2*c*d) - Log[c + d*x]/(2*c*d) - Log[-(d*x) + (2*c^3 + d^3*x^3)^(1/3)]/(4*c*d) + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(4*c*d)

Rule 239

Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] :> Simp[ArcTan[(1 + (2*Rt[b, 3]*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 2149

Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Dist[1/(2*c), Int[1/(a + b*x^3)^(1/3), x], x] + Dist[1/(2*c), Int[(c - d*x)/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[2*b*c^3 - a*d^3, 0]

Rule 2151

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] :> Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2*b*c^3 - a*d^3, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx &= \frac{\int \frac{1}{\sqrt[3]{2c^3+d^3x^3}} dx}{2c} + \frac{\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx}{2c} \\ &= \frac{\tan^{-1}\left(\frac{1+\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2\sqrt{3}cd} - \frac{\sqrt{3}\tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{2cd} - \frac{\log(c+dx)}{2cd} - \frac{\log\left(-dx + \sqrt[3]{2c^3+d^3x^3}\right)}{4cd} \end{aligned}$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(c + dx)\sqrt[3]{2c^3 + d^3x^3}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

IntegrateAlgebraic [F] time = 6.71, size = 0, normalized size = 0.00

$$\int \frac{1}{(c + dx)\sqrt[3]{2c^3 + d^3x^3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

[Out] Defer[IntegrateAlgebraic][1/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 + 2c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3), x, algorithm="giac")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(d^3x^3 + 2c^3)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3), x)

[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 + 2c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2c^3 + d^3x^3)^{1/3} (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)),x)

[Out] int(1/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c + dx) \sqrt[3]{2c^3 + d^3x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(1/3),x)

[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(1/3)), x)

$$3.3 \quad \int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Optimal. Leaf size=187

$$\frac{\frac{\log(c+dx)}{2c^2d} - \frac{\log(dx - \sqrt[3]{2c^3+d^3x^3})}{4c^2d} + \frac{3 \log(d(2c+dx) - d\sqrt[3]{2c^3+d^3x^3})}{4c^2d} - \frac{\tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{2\sqrt{3}c^2d} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\frac{2dx}{\sqrt[3]{2c^3+d^3x^3}}-1}{\sqrt{3}}\right)}{2c^2d}}$$

Rubi [F] time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Int[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

[Out] Defer[Int][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

Rubi steps

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx = \int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Mathematica [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

[Out] Integrate[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

IntegrateAlgebraic [F] time = 6.96, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(2c^3+d^3x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

[Out] Defer[IntegrateAlgebraic][1/((c + d*x)*(2*c^3 + d^3*x^3)^(2/3)), x]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 + 2c^3)^{\frac{2}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="giac")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx + c)(d^3x^3 + 2c^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)

[Out] int(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d^3x^3 + 2c^3)^{\frac{2}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d^3*x^3+2*c^3)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((d^3*x^3 + 2*c^3)^(2/3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(2c^3 + d^3x^3)^{\frac{2}{3}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)),x)

[Out] int(1/((2*c^3 + d^3*x^3)^(2/3)*(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(c + dx)(2c^3 + d^3x^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d*x+c)/(d**3*x**3+2*c**3)**(2/3),x)

[Out] Integral(1/((c + d*x)*(2*c**3 + d**3*x**3)**(2/3)), x)

$$3.4 \quad \int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx$$

Optimal. Leaf size=147

$$\frac{\log\left(x - \sqrt[3]{x^3+1}\right)}{2 \cdot 2^{2/3}} + \frac{3 \log\left(-\sqrt[3]{2} \sqrt[3]{x^3+1} + \sqrt[3]{2}x + 2\right)}{2 \cdot 2^{2/3}} - \frac{\tan^{-1}\left(\frac{\frac{2x}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}\sqrt{3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\frac{2(x+2^{2/3})}{\sqrt[3]{x^3+1}}+1}{\sqrt{3}}\right)}{2^{2/3}} - \frac{\log\left(\sqrt[3]{2}x + 1\right)}{2^{2/3}}$$

Rubi [F] time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Int[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

[Out] Defer[Int][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

Rubi steps

$$\int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx = \int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx$$

Mathematica [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

[Out] Integrate[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

IntegrateAlgebraic [F] time = 6.36, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 + \sqrt[3]{2}x)(1+x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

[Out] Defer[IntegrateAlgebraic][1/((1 + 2^(1/3)*x)*(1 + x^3)^(2/3)), x]

fricas [B] time = 19.24, size = 712, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3), x, algorithm="fricas")

[Out] $\frac{1}{6}\sqrt{3} \cdot 2^{1/3} \cdot \arctan\left(-\frac{1}{3} \cdot (13910019318573948542\sqrt{3}) \cdot (44297109310930172741433829405399636654451725916403400759596345420183x^{16} + 469911753877577297266687493361266274298219751726156511748796788210304x^{13} - 168603219036433260440647021325346295645242325246375460547582960409424x^{10} - 1978806301182376573938292954227792627373330283397876582611558332893440x^7 - 144090891687177581422918763089301968602581036872213084389912370301872x^4 - 2^{2/3}) \cdot (52271077453125107612995923977654758349394876922885552819209999866413x^{15} + 590674547854548577293285820788340778493299281255213360593997994805172x^{12} + 3063142612229314316198873829666304230648222176902796253391978577817900x^9 + 7331049558697577809008352571597039403457968857066730277786114959327080x^6 + 7723244806756290443759770546780872971739444750173519635544186114816064x^3 + 2911680898783900921956348574183551415589190446015106452608070501424800) + 6 \cdot 2^{1/3} \cdot (12601355996216322093314748679149120543302140685677058235520929344665x^{14} - 55586906300196651392462719491921267847820798890019850227115938089718x^{11} - 450398920105320599307639536027883986131793624729303407436233610788504x^8 - 721888705880948261432517052670394106238338943844373553906510879866584x^5 - 338668158068684373436309273067849464405691360751378507442472921774544x^2) - 62367643045453979229021701235594440425380660140976292433240780519680x) \cdot (x^3 + 1)^{2/3} - 13910019318573948542\sqrt{3} \cdot (20244151386762728582873176440916642276036913846721964342570319874272x^{17} + 741146137078834990968958694956953525786968216162791369141561079231342x^{14} + 2179843197271775401147438396101666875537043663345199103065290718350660x^{11} + 2111024935028444803027635033172373996998638870275081528835019029426808x^8 + 690583979302212649541846671752323578671762361564987198532372077617072x^5 + 42560446719395994043503690929493089250376947849898596094387069196992x^2 + 2^{2/3}) \cdot (58175953016441250552894129028785848895343146706912452780410096144857x^{16} + 603329123440225928459512442880846367498086340467210508410170807919392x^{13} + 993217724421160514640802924970216148872138006799356417482692017634440x^{10} - 315373668616978600368729679828820826067145203897860799345951918357208x^7 - 1535989781175898454904009764080477698123439140009523257833795294171024x^4 - 774581653994506522185065060515457999562469670838035710700279100960480x) - 2 \cdot 2^{1/3} \cdot (44250337395862623641308432146105265591584981692216944246872622437586x^{15} + 93730331994553087914588193029404165015738145719370012253256237142833x^{12} + 1321854131659545520395638093435834861993288285254840631143087754453816x^9 + 4247705767701746889589213825725278162202431773760010908121531655858240x^6 + 4593245463688643634993735851341621838359838170188285500151733185855040x^3 + 1615883737614789297142910770786922880950970969890530541101538638738800) \cdot (x^3 + 1)^{1/3} + \sqrt{3} \cdot (580845856624814138058536658925035752422341023745042657144110018434133971171392378653765x^{18} + 8512850211201658596320322423507979436745037061604662252288106173984889011398391939493844x^{15} + 46037674634299399876464933353333393651798714498861959697684952859181279514449172348801132x^{12} + 100016348353366812357999723948540966952435611836580420294833827058766585456463611215562912x^9 + 91397758625366807679053421068886729440495107689602121025455736534255642370122935700628112x^6 + 27679206471222147818932348914707271406554121216141734785863966451139338545569046396842944x^3 - 13910019318573948542 \cdot 2^{2/3}) \cdot (3844366680114123938578119587438413410802428820066154040455085354797x^{17} - 493131971154919078063173195983280278594703770406004388326552124793591x^{14} - 2263656329733750526575239788393341804272268328404078377386979655411628x^{11} - 3603296088959643040065882606156977332942778368970867958841266275405688x^8 - 2375143924145462474790789297643082581023352457583644433698318090272160x^5 - 538527827084536759298395164308728360347336217790784309877024260129712x^2) + 166920231822887382504 \cdot 2^{1/3} \cdot (13595892044042828366275982006708049395032909698880004129949511339226x^{16} + 135133384885158250377179048595991346450771199327236207956421113461903x^{13} + 402245899028058436823068109521885840258775610614711826343657868879359x^{10} + 547258710149879334691832999834525308297790387563356879645468036532966x^7 + 36367419970364096388496012124387263106254909521640663154302302116404x^4 + 97123895740704644005292055222464498011501842944639406026020532340120x) - 180077408083879446119265$

```
3903259802591850188394016866170707655609076236167687893936558400))/(4912705
745775473374655778624996785809194682896822406415994000025418186301732995555
53387*x^18 + 10277776658535231887928963830517649364075160462302952752368573
529738604577075128345830496*x^15 + 3805307460404116495559861338250665758871
8033800015428848687354515819408113275280820067228*x^12 + 104552977375786496
056156515228686393360634250389206134816652347595105200990156089430013680*x^
9 + 19378977878621710892383256210067417618988113173228069450235805883107523
1461508817660387440*x^6 + 1762507736152141132702163647685459405315437312825
77338989973916134409349945587251955701568*x^3 + 587295173581501937080873224
84283950706773934182867349449322904141070201590185330889048000)) + 1/12*2^(
1/3)*log((6048*x^16 + 6048*x^13 - 9072*x^10 - 12204*x^7 - 2808*x^4 + 2^(2/3
)*(352*x^18 - 5136*x^15 - 10632*x^12 - 3224*x^9 + 3390*x^6 + 1434*x^3 - 35)
+ 3*(2032*x^14 + 752*x^11 - 3000*x^8 - 1576*x^5 + 172*x^2 + 2^(2/3)*(112*x^
^16 - 1760*x^13 - 2228*x^10 + 356*x^7 + 707*x^4 - 22*x) - 2*2^(1/3)*(352*x^
15 - 728*x^12 - 1736*x^9 - 451*x^6 + 215*x^3 - 1))*(x^3 + 1)^(2/3) - 18*2^(
1/3)*(112*x^17 - 192*x^14 - 820*x^11 - 586*x^8 - 21*x^5 + 49*x^2) + 3*(2096
*x^15 + 1664*x^12 - 2680*x^9 - 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 - 176
0*x^14 - 2996*x^11 - 472*x^8 + 779*x^5 + 125*x^2) - 2*2^(1/3)*(336*x^16 - 6
64*x^13 - 2132*x^10 - 1107*x^7 + 55*x^4 + 29*x) + 16)*(x^3 + 1)^(1/3) + 324
*x)/(64*x^18 + 192*x^15 + 240*x^12 + 160*x^9 + 60*x^6 + 12*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)^{\frac{2}{3}} \left(2^{\frac{1}{3}}x + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="giac")

[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)

maple [C] time = 42.61, size = 3064, normalized size = 20.84

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x)

```
[Out] -1/6*ln(-(-15559137585059152-12498127505504256*2^(1/3)*(x^3+1)^(1/3)-160495
4020235328*2^(1/3)*x^4-1604954020235328*2^(2/3)*x^2-23004340956706368*2^(1/
3)*x-936223178470608*x^6+14712078518823840*x^3-4279877387294208*x^5*2^(2/3)
+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^
2-11688730639030284*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x
+6471910353179844*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^3
+1203809884289286*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^4
-3218487773589102*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^5
-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^4
-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x
^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)
*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/
3)*x^3-10498622607665136*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1
/3)*x^4-7759251414704196*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2
/3)*x-3531674097632562*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)
)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1
/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+64346838756483
36*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2-5504178119120758*RootOf(2^(2/3)+2^(
1/3)*_Z+_Z^2)*2^(2/3)-10391133689698608*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+
7959206999356368*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-10107087250606332*(x^3
```


$$\begin{aligned}
& +1)^{(2/3)} * 2^{(2/3)} - 9125490357912936 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ \\
& _Z^2) - 2960082830251008 * (x^3+1)^{(1/3)} * x^5 - 32590199706036744 * (x^3+1)^{(2/3)} * x - \\
& 12827025597754368 * (x^3+1)^{(1/3)} * x^2 + 3712807561447224 * (x^3+1)^{(2/3)} * x^4 + 7959 \\
& 206999356368 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(1/3)} * x^2 + 5665414413224496 * \text{R} \\
& \text{oof}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(1/3)} * x^5 + 2321435374812274 * \text{RootOf}(2^{(2/3)}+ \\
& 2^{(1/3)} * _Z+ _Z^2) * 2^{(2/3)} * x^6 + 3532767618003008 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^ \\
& 2)^2 * 2^{(1/3)} * x^3 + 10197714008127436 * 2^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * \\
& x^3 + 1876782797064098 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(1/3)} * x^6 + 26138868 \\
& 55325640 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x^3 + 28842279448706 \\
& 16 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * x^2 - 8123294120973864 * (x^3+ \\
& 1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * x^3 + 3217341937824168 * \text{RootOf}(2^{(2/3} \\
&) + 2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(2/3)} * x^4 + 2081809489180344 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z \\
& + _Z^2)^2 * 2^{(2/3)} * x - 11138422684341672 * (x^3+1)^{(2/3)} * 2^{(1/3)} * x^2 + 391907464819 \\
& 4292 * (x^3+1)^{(2/3)} * 2^{(2/3)} * x^3 - 1315592369000448 * (x^3+1)^{(1/3)} * 2^{(2/3)} * x^4 - 6 \\
& 964190009986188 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x^4 + 3288980 \\
& 922501120 * (x^3+1)^{(1/3)} * 2^{(1/3)} * x^3 - 20062783627256832 * (x^3+1)^{(1/3)} * 2^{(2/3)} \\
& * x - 7115580883942020 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(1/3)} + 2 \\
& 161300347926748 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x) / (1+2^{(1/ \\
& 3)} * x)^6 * 2^{(1/3)} - 1/6 * \ln(-(-15559137585059152 - 12498127505504256 * 2^{(1/3)} * (x^3 \\
& +1)^{(1/3)} - 1604954020235328 * 2^{(1/3)} * x^4 - 1604954020235328 * 2^{(2/3)} * x^2 - 2300434 \\
& 0956706368 * 2^{(1/3)} * x - 936223178470608 * x^6 + 14712078518823840 * x^3 - 427987738729 \\
& 4208 * x^5 * 2^{(2/3)} + 840505690860402 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z \\
& ^2)^2 * 2^{(1/3)} * x^2 - 11688730639030284 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z \\
& + _Z^2) * 2^{(1/3)} * x + 6471910353179844 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ \\
& _Z^2) * 2^{(1/3)} * x^3 + 1203809884289286 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ \\
& _Z^2) * 2^{(2/3)} * x^4 - 3218487773589102 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ \\
& _Z^2) * 2^{(2/3)} * x^5 - 72607968203490 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^ \\
& 2)^2 * 2^{(1/3)} * x^4 - 1560939140169318 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ \\
& _Z^2)^2 * 2^{(1/3)} * x^5 + 3775614346581480 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z \\
& + _Z^2)^2 * 2^{(2/3)} * x^2 - 3842311729647552 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * \\
& _Z+ _Z^2)^2 * 2^{(2/3)} * x^3 - 10498622607665136 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/ \\
& 3)} * _Z+ _Z^2) * 2^{(1/3)} * x^4 - 7759251414704196 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/ \\
& 3)} * _Z+ _Z^2) * 2^{(2/3)} * x - 3531674097632562 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} \\
& * _Z+ _Z^2) * 2^{(2/3)} * x^2 + 2613886855325640 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} \\
& * _Z+ _Z^2)^2 * 2^{(1/3)} * x + 6150800763487380 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x^ \\
& 5 + 6434683875648336 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x^2 - 5504178119120758 * \text{R} \\
& \text{oof}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(2/3)} - 10391133689698608 * \text{RootOf}(2^{(2/3)}+2^{(\\
& 1/3)} * _Z+ _Z^2) * x + 7959206999356368 * x^4 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) - 101070 \\
& 87250606332 * (x^3+1)^{(2/3)} * 2^{(2/3)} - 9125490357912936 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(\\
& 2/3)}+2^{(1/3)} * _Z+ _Z^2) - 2960082830251008 * (x^3+1)^{(1/3)} * x^5 - 32590199706036744 * \\
& (x^3+1)^{(2/3)} * x - 12827025597754368 * (x^3+1)^{(1/3)} * x^2 + 3712807561447224 * (x^3+1 \\
&)^{(2/3)} * x^4 + 7959206999356368 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(1/3)} * x^2 + 56 \\
& 65414413224496 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(1/3)} * x^5 + 2321435374812274 \\
& * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * 2^{(2/3)} * x^6 + 3532767618003008 * \text{RootOf}(2^{(2/3} \\
&) + 2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(1/3)} * x^3 + 10197714008127436 * 2^{(2/3)} * \text{RootOf}(2^{(2/3)}+2 \\
& ^{(1/3)} * _Z+ _Z^2) * x^3 + 1876782797064098 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(1 \\
& /3)} * x^6 + 2613886855325640 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * x^ \\
& 3 + 2884227944870616 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * x^2 - 812329 \\
& 4120973864 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) * x^3 + 32173419378241 \\
& 68 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(2/3)} * x^4 + 2081809489180344 * \text{RootOf}(2^{ \\
& (2/3)}+2^{(1/3)} * _Z+ _Z^2)^2 * 2^{(2/3)} * x - 11138422684341672 * (x^3+1)^{(2/3)} * 2^{(1/3)} * \\
& x^2 + 3919074648194292 * (x^3+1)^{(2/3)} * 2^{(2/3)} * x^3 - 1315592369000448 * (x^3+1)^{(1/ \\
& 3)} * 2^{(2/3)} * x^4 - 6964190009986188 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^ \\
& 2)^2 * x^4 + 3288980922501120 * (x^3+1)^{(1/3)} * 2^{(1/3)} * x^3 - 20062783627256832 * (x^3+ \\
& 1)^{(1/3)} * 2^{(2/3)} * x - 7115580883942020 * (x^3+1)^{(2/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z \\
& + _Z^2) * 2^{(1/3)} + 2161300347926748 * (x^3+1)^{(1/3)} * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^ \\
& 2)^2 * x) / (1+2^{(1/3)} * x)^6 * \text{RootOf}(2^{(2/3)}+2^{(1/3)} * _Z+ _Z^2) + 1/6 * \text{RootOf}(2^{(2/3)} \\
& + 2^{(1/3)} * _Z+ _Z^2) * \ln(-(-4550781346817636 - 3372637147591320 * 2^{(1/3)} * (x^3+1)^{(\\
& 1/3)} - 3129477143943360 * 2^{(1/3)} * x^4 - 3129477143943360 * 2^{(2/3)} * x^2 - 844958828864
\end{aligned}$$

7072*2^(1/3)*x-1825528333966960*x^6+1382185738574984*x^3-3794491037031324*x^5*2^(2/3)+840505690860402*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^2+16011331334883780*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x-1244136642528564*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^3-1349025820696266*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^4+96609493250466*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^5-72607968203490*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^4-1560939140169318*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^5+3775614346581480*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^2-3842311729647552*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^3-3429757412307240*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^4+12987025125355476*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x+5212685479353366*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^2+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x+6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+5504178119120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)+18718371646419984*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)-2991506366664312*(x^3+1)^(2/3)*2^(2/3)+9125490357912936*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)+355014436588560*(x^3+1)^(1/3)*x^5-11843923165977072*(x^3+1)^(2/3)*x-4082666020768440*(x^3+1)^(1/3)*x^2+1159971856461672*(x^3+1)^(2/3)*x^4+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2+6636187113750264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6+3532767618003008*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^3-3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+2613886855325640*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^3+12218229441455304*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^2-7245952797616344*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*x^3+3217341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4+2081809489180344*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-6471421936049328*(x^3+1)^(2/3)*2^(1/3)*x^2+61051150340088*(x^3+1)^(2/3)*2^(2/3)*x^3+2218840228678500*(x^3+1)^(1/3)*2^(2/3)*x^4-6964190009986188*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^4+3727651584179880*(x^3+1)^(1/3)*2^(1/3)*x^3-6212752640299800*(x^3+1)^(1/3)*2^(2/3)*x+7115580883942020*(x^3+1)^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)+2161300347926748*(x^3+1)^(1/3)*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x)/(1+2^(1/3)*x)^6)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 1)^{\frac{2}{3}} \left(2^{\frac{1}{3}}x + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2^(1/3)*x)/(x^3+1)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 + 1)^{\frac{2}{3}} \left(2^{\frac{1}{3}}x + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)),x)

[Out] int(1/((x^3 + 1)^(2/3)*(2^(1/3)*x + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x+1)(x^2-x+1))^{\frac{2}{3}}(\sqrt[3]{2x+1})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2**(1/3)*x)/(x**3+1)**(2/3),x)

[Out] Integral(1/(((x + 1)*(x**2 - x + 1))**(2/3)*(2**(1/3)*x + 1)), x)

$$3.5 \quad \int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx$$

Optimal. Leaf size=159

$$\frac{\log\left(-\sqrt[3]{1-x^3}-x\right)}{2 \cdot 2^{2/3}} - \frac{3 \log\left(\sqrt[3]{2} \sqrt[3]{1-x^3} + \sqrt[3]{2}x - 2\right)}{2 \cdot 2^{2/3}} - \frac{\sqrt{3} \tan^{-1}\left(\frac{2 \cdot 2^{2/3} - 2x + 1}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}} + \frac{\tan^{-1}\left(\frac{1 - \frac{2x}{\sqrt[3]{1-x^3}}}{\sqrt{3}}\right)}{2^{2/3} \sqrt{3}} + \frac{\log\left(1 - \sqrt[3]{2}x\right)}{2^{2/3}}$$

Rubi [F] time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Int[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

[Out] Defer[Int][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

Rubi steps

$$\int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx = \int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx$$

Mathematica [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

[Out] Integrate[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

IntegrateAlgebraic [F] time = 7.28, size = 0, normalized size = 0.00

$$\int \frac{1}{(1 - \sqrt[3]{2}x)(1 - x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

[Out] Defer[IntegrateAlgebraic][1/((1 - 2^(1/3)*x)*(1 - x^3)^(2/3)), x]

fricas [B] time = 14.37, size = 720, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3), x, algorithm="fricas")

[Out] $1/6*\sqrt{3}*2^{(1/3)}*\arctan(1/3*(13910019318573948542*\sqrt{3}*(44297109310930172741433829405399636654451725916403400759596345420183*x^{16} - 469911753877577297266687493361266274298219751726156511748796788210304*x^{13} - 168603219036433260440647021325346295645242325246375460547582960409424*x^{10} + 1978806301182376573938292954227792627373330283397876582611558332893440*x^7 - 1440090891687177581422918763089301968602581036872213084389912370301872*x^4 + 2^{(2/3)}*(52271077453125107612995923977654758349394876922885552819209999866413*x^{15} - 590674547854548577293285820788340778493299281255213360593997994805172*x^{12} + 3063142612229314316198873829666304230648222176902796253391978577817900*x^9 - 7331049558697577809008352571597039403457968857066730277786114959327080*x^6 + 7723244806756290443759770546780872971739444750173519635544186114816064*x^3 - 2911680898783900921956348574183551415589190446015106452608070501424800) + 6*2^{(1/3)}*(12601355996216322093314748679149120543302140685677058235520929344665*x^{14} + 55586906300196651392462719491921267847820798890019850227115938089718*x^{11} - 450398920105320599307639536027883986131793624729303407436233610788504*x^8 + 721888705880948261432517052670394106238338943844373553906510879866584*x^5 - 338668158068684373436309273067849464405691360751378507442472921774544*x^2) + 62367643045453979229021701235594440425380660140976292433240780519680*x)*(-x^3 + 1)^{(2/3)} + 13910019318573948542*\sqrt{3}*(20244151386762728582873176440916642276036913846721964342570319874272*x^{17} - 741146137078834990968958694956953525786968216162791369141561079231342*x^{14} + 2179843197271775401147438396101666875537043663345199103065290718350660*x^{11} - 2111024935028444803027635033172373996998638870275081528835019029426808*x^8 + 690583979302212649541846671752323578671762361564987198532372077617072*x^5 - 42560446719395994043503690929493089250376947849898596094387069196992*x^2 - 2^{(2/3)}*(58175953016441250552894129028785848895343146706912452780410096144857*x^{16} - 603329123440225928459512442880846367498086340467210508410170807919392*x^{13} + 993217724421160514640802924970216148872138006799356417482692017634440*x^{10} + 315373668616978600368729679828820826067145203897860799345951918357208*x^7 - 1535989781175898454904009764080477698123439140009523257833795294171024*x^4 + 774581653994506522185065060515457999562469670838035710700279100960480*x) - 2*2^{(1/3)}*(44250337395862623641308432146105265591584981692216944246872622437586*x^{15} - 93730331994553087914588193029404165015738145719370012253256237142833*x^{12} + 1321854131659545520395638093435834861993288285254840631143087754453816*x^9 - 4247705767701746889589213825725278162202431773760010908121531655858240*x^6 + 4593245463688643634993735851341621838359838170188285500151733185855040*x^3 - 1615883737614789297142910770786922880950970969890530541101538638738800))*(-x^3 + 1)^{(1/3)} + \sqrt{3}*(580845856624814138058536658925035752422341023745042657144110018434133971171392378653765*x^{18} - 8512850211201658596320322423507979436745037061604662252288106173984889011398391939493844*x^{15} + 46037674634299399876464933353333393651798714498861959697684952859181279514449172348801132*x^{12} - 100016348353366812357999723948540966952435611836580420294833827058766585456463611215562912*x^9 + 91397758625366807679053421068886729440495107689602121025455736534255642370122935700628112*x^6 - 27679206471222147818932348914707271406554121216141734785863966451139338545569046396842944*x^3 + 13910019318573948542*2^{(2/3)}*(3844366680114123938578119587438413410802428820066154040455085354797*x^{17} + 493131971154919078063173195983280278594703770406004388326552124793591*x^{14} - 2263656329733750526575239788393341804272268328404078377386979655411628*x^{11} + 3603296088959643040065882606156977332942778368970867958841266275405688*x^8 - 2375143924145462474790789297643082581023352457583644433698318090272160*x^5 + 538527827084536759298395164308728360347336217790784309877024260129712*x^2) + 166920231822887382504*2^{(1/3)}*(13595892044042828366275982006708049395032909698880004129949511339226*x^{16} - 135133384885158250377179048595991346450771199327236207956421113461903*x^{13} + 402245899028058436823068109521885840258775610614711826343657868879359*x^{10} - 547258710149879334691832999834525308297790387563356879645468036532966*x^7 + 363674199703640963884960012124387263106254909521640663154302302116404*x^4 - 97123895740704644005292055222464498011501842944639406026020532340120*x) - 18007740808387944611926$

```
53903259802591850188394016866170707655609076236167687893936558400))/(491270
574577547337465577862499678580919468289682240641599400002541818630173299555
553387*x^18 - 1027777665853523188792896383051764936407516046230295275236857
3529738604577075128345830496*x^15 + 380530746040411649555986133825066575887
18033800015428848687354515819408113275280820067228*x^12 - 10455297737578649
6056156515228686393360634250389206134816652347595105200990156089430013680*x
^9 + 1937897787862171089238325621006741761898811317322806945023580588310752
31461508817660387440*x^6 - 176250773615214113270216364768545940531543731282
577338989973916134409349945587251955701568*x^3 + 58729517358150193708087322
484283950706773934182867349449322904141070201590185330889048000)) - 1/12*2^
(1/3)*log((6048*x^16 - 6048*x^13 - 9072*x^10 + 12204*x^7 - 2808*x^4 + 2^(2/
3)*(352*x^18 + 5136*x^15 - 10632*x^12 + 3224*x^9 + 3390*x^6 - 1434*x^3 - 35
) + 3*(2032*x^14 - 752*x^11 - 3000*x^8 + 1576*x^5 + 172*x^2 + 2^(2/3)*(112*x
^16 + 1760*x^13 - 2228*x^10 - 356*x^7 + 707*x^4 + 22*x) + 2*2^(1/3)*(352*x
^15 + 728*x^12 - 1736*x^9 + 451*x^6 + 215*x^3 + 1))*(-x^3 + 1)^(2/3) + 18*2
^(1/3)*(112*x^17 + 192*x^14 - 820*x^11 + 586*x^8 - 21*x^5 - 49*x^2) - 3*(20
96*x^15 - 1664*x^12 - 2680*x^9 + 2492*x^6 - 224*x^3 + 2^(2/3)*(112*x^17 + 1
760*x^14 - 2996*x^11 + 472*x^8 + 779*x^5 - 125*x^2) + 2*2^(1/3)*(336*x^16 +
664*x^13 - 2132*x^10 + 1107*x^7 + 55*x^4 - 29*x) - 16))*(-x^3 + 1)^(1/3) -
324*x)/(64*x^18 - 192*x^15 + 240*x^12 - 160*x^9 + 60*x^6 - 12*x^3 + 1))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{(-x^3 + 1)^{\frac{2}{3}} \left(2^{\frac{1}{3}}x - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="giac")

[Out] integrate(-1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)

maple [C] time = 41.37, size = 3250, normalized size = 20.44

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x)

```
[Out] -1/6*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*ln(-(-4550781346817636-337263714759132
0*2^(1/3)*(-x^3+1)^(1/3)-3129477143943360*2^(1/3)*x^4-3129477143943360*2^(2
/3)*x^2+8449588288647072*2^(1/3)*x-1825528333966960*x^6-1382185738574984*x^
3+3794491037031324*x^5*2^(2/3)-6150800763487380*RootOf(2^(2/3)+2^(1/3)*_Z+_
Z^2)^2*x^5+6434683875648336*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*x^2+550417811
9120758*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)-18718371646419984*RootOf(2^
(2/3)+2^(1/3)*_Z+_Z^2)*x+4910160751940304*x^4*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^
2)+4910160751940304*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^2-66361871137
50264*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*2^(1/3)*x^5+1432130219315922*RootOf(2
^(2/3)+2^(1/3)*_Z+_Z^2)*2^(2/3)*x^6-3532767618003008*RootOf(2^(2/3)+2^(1/3)
*_Z+_Z^2)^2*2^(1/3)*x^3+3132178772121420*2^(2/3)*RootOf(2^(2/3)+2^(1/3)*_Z+
_Z^2)*x^3+1876782797064098*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(1/3)*x^6+32
17341937824168*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x^4-20818094891803
44*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)^2*2^(2/3)*x-2991506366664312*2^(2/3)*(-x
^3+1)^(2/3)+9125490357912936*RootOf(2^(2/3)+2^(1/3)*_Z+_Z^2)*(-x^3+1)^(1/3)
+1159971856461672*(-x^3+1)^(2/3)*x^4-355014436588560*(-x^3+1)^(1/3)*x^5+118
43923165977072*(-x^3+1)^(2/3)*x-4082666020768440*(-x^3+1)^(1/3)*x^2-6471421
936049328*2^(1/3)*(-x^3+1)^(2/3)*x^2-61051150340088*2^(2/3)*(-x^3+1)^(2/3)*
x^3+2218840228678500*2^(2/3)*(-x^3+1)^(1/3)*x^4-6964190009986188*RootOf(2^(
2/3)+2^(1/3)*_Z+_Z^2)^2*(-x^3+1)^(1/3)*x^4-3727651584179880*2^(1/3)*(-x^3+1
```

$$\begin{aligned}
&)^{(1/3)} * x^3 + 7245952797616344 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * (-x^3 + 1)^{(1/3)} \\
&* x^3 + 6212752640299800 * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x + 7115580883942020 * \text{RootOf}(2^{(2/3)} \\
&/ + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} - 2161300347926748 * \text{RootOf}(2^{(2/3)} \\
&/ + 2^{(1/3)} * _Z + _Z^2)^2 * (-x^3 + 1)^{(1/3)} * x - 2613886855325640 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} \\
&/ * _Z + _Z^2)^2 * (-x^3 + 1)^{(2/3)} * x^3 + 12218229441455304 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * (-x^3 + 1)^{(2/3)} * x^2 + 3775614346581480 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 \\
&/ * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x^2 + 3842311729647552 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) \\
&/ ^2 * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^3 - 3429757412307240 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + \\
&/ _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^4 - 12987025125355476 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x + 5212685479353366 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^2 - 2613886855325640 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x + 840505690860402 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^2 - 16011331334883780 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x + 1244136642528564 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x^3 - 1349025820696266 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x^4 - 96609493250466 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^5 - 72607968203490 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x^4 + 1560939140169318 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^5) / (2^{(1/3)} * x - 1)^6 + 1/6 * \ln(-(-15559 \\
&/ 137585059152 - 12498127505504256 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} - 1604954020235328 * 2^{(1/3)} \\
&/ * x^4 - 1604954020235328 * 2^{(2/3)} * x^2 + 23004340956706368 * 2^{(1/3)} * x - 9362231784 \\
&/ 70608 * x^6 - 14712078518823840 * x^3 + 4279877387294208 * x^5 * 2^{(2/3)} - 61508007634873 \\
&/ 80 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * x^5 + 6434683875648336 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2)^2 * x^2 - 5504178119120758 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} \\
&/ + 10391133689698608 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * x + 7959206999356368 * x^4 \\
&/ * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) + 7959206999356368 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z \\
&/ + _Z^2) * 2^{(1/3)} * x^2 - 5665414413224496 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} \\
&/ * x^5 + 2321435374812274 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} * x^6 - 353276761 \\
&/ 8003008 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(1/3)} * x^3 - 10197714008127436 * 2^{(2/3)} \\
&/ * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * x^3 + 1876782797064098 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) \\
&/ ^2 * 2^{(1/3)} * x^6 + 3217341937824168 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) \\
&/ ^2 * 2^{(2/3)} * x^4 - 2081809489180344 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(2/3)} \\
&/ * x - 10107087250606332 * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} - 9125490357912936 * \text{RootOf}(2^{(2/3)} \\
&/ + 2^{(1/3)} * _Z + _Z^2) * (-x^3 + 1)^{(1/3)} + 3712807561447224 * (-x^3 + 1)^{(2/3)} * x^4 + 296008 \\
&/ 2830251008 * (-x^3 + 1)^{(1/3)} * x^5 + 32590199706036744 * (-x^3 + 1)^{(2/3)} * x - 1282702559 \\
&/ 7754368 * (-x^3 + 1)^{(1/3)} * x^2 - 11138422684341672 * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x^2 - 391 \\
&/ 9074648194292 * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x^3 - 1315592369000448 * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} \\
&/ * x^4 - 6964190009986188 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * (-x^3 + 1)^{(1/3)} \\
&/ * x^4 - 3288980922501120 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^3 + 8123294120973864 * \text{RootOf}(2^{(2/3)} \\
&/ + 2^{(1/3)} * _Z + _Z^2) * (-x^3 + 1)^{(1/3)} * x^3 + 20062783627256832 * 2^{(2/3)} * (-x^3 + 1) \\
&/ ^{(1/3)} * x - 7115580883942020 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} * (-x^3 + 1) \\
&/ ^{(2/3)} - 2161300347926748 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * (-x^3 + 1)^{(1/3)} * x - \\
&/ 2613886855325640 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * (-x^3 + 1)^{(2/3)} * x^3 + 28842 \\
&/ 27944870616 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * (-x^3 + 1)^{(2/3)} * x^2 + 377561434658 \\
&/ 1480 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x^2 + 384231172 \\
&/ 9647552 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^3 - 104986 \\
&/ 22607665136 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^4 + 7759 \\
&/ 251414704196 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x - 35316 \\
&/ 74097632562 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^2 - 2613 \\
&/ 886855325640 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x + 840 \\
&/ 505690860402 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x^2 + 1 \\
&/ 1688730639030284 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} * x - 6 \\
&/ 471910353179844 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x^3 + \\
&/ 1203809884289286 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(2/3)} * x^4 \\
&/ + 3218487773589102 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2) * 2^{(2/3)} * (-x^3 + 1)^{(1/3)} * x^5 \\
&/ - 72607968203490 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(2/3)} * x \\
&/ ^4 + 1560939140169318 * \text{RootOf}(2^{(2/3)} + 2^{(1/3)} * _Z + _Z^2)^2 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} \\
&/ * x^5) / (2^{(1/3)} * x - 1)^6 * 2^{(1/3)} + 1/6 * \ln(-(-15559137585059152 - 124981275055042 \\
&/ 56 * 2^{(1/3)} * (-x^3 + 1)^{(1/3)} - 1604954020235328 * 2^{(1/3)} * x^4 - 1604954020235328 * 2^{(1/3)} *
\end{aligned}$$

$2/3)*x^2+23004340956706368*2^{(1/3)}*x-936223178470608*x^6-14712078518823840*$
 $x^3+4279877387294208*x^5*2^{(2/3)}-6150800763487380*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z$
 $+_Z^2)^2*x^5+6434683875648336*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*x^2-5504178$
 $119120758*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*2^{(2/3)}+10391133689698608*\text{RootOf}($
 $2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*x+7959206999356368*x^4*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+$
 $_Z^2)+7959206999356368*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*x^2-566541441$
 $3224496*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*x^5+2321435374812274*\text{RootOf}$
 $(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*2^{(2/3)}*x^6-3532767618003008*\text{RootOf}(2^{(2/3)}+2^{(1/$
 $3)**_Z+_Z^2)^2*2^{(1/3)}*x^3-10197714008127436*2^{(2/3)}*\text{RootOf}(2^{(2/3)}+2^{(1/3)*}$
 $_Z+_Z^2)*x^3+1876782797064098*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*2^{(1/3)}*x^6$
 $+3217341937824168*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*2^{(2/3)}*x^4-20818094891$
 $80344*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*2^{(2/3)}*x-10107087250606332*2^{(2/3)}$
 $*(-x^3+1)^{(2/3)}-9125490357912936*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*(-x^3+1)^{($
 $1/3)}+3712807561447224*(-x^3+1)^{(2/3)}*x^4+2960082830251008*(-x^3+1)^{(1/3)}*x^$
 $5+32590199706036744*(-x^3+1)^{(2/3)}*x-12827025597754368*(-x^3+1)^{(1/3)}*x^2-1$
 $1138422684341672*2^{(1/3)}*(-x^3+1)^{(2/3)}*x^2-3919074648194292*2^{(2/3)}*(-x^3+$
 $1)^{(2/3)}*x^3-1315592369000448*2^{(2/3)}*(-x^3+1)^{(1/3)}*x^4-6964190009986188*R$
 $\text{ootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*(-x^3+1)^{(1/3)}*x^4-3288980922501120*2^{(1/3)}$
 $)*(-x^3+1)^{(1/3)}*x^3+8123294120973864*\text{RootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*(-x^3$
 $+1)^{(1/3)}*x^3+20062783627256832*2^{(2/3)}*(-x^3+1)^{(1/3)}*x-7115580883942020*R$
 $\text{ootOf}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*(-x^3+1)^{(2/3)}-2161300347926748*\text{Root}$
 $\text{Of}(2^{(2/3)}+2^{(1/3)}*_Z+_Z^2)^2*(-x^3+1)^{(1/3)}*x-2613886855325640*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)^2*(-x^3+1)^{(2/3)}*x^3+2884227944870616*\text{RootOf}(2^{(2/3)+2}$
 $^{(1/3)}*_Z+_Z^2)*(-x^3+1)^{(2/3)}*x^2+3775614346581480*\text{RootOf}(2^{(2/3)+2^{(1/3)*}$
 $_Z+_Z^2)^2*2^{(2/3)}*(-x^3+1)^{(2/3)}*x^2+3842311729647552*\text{RootOf}(2^{(2/3)+2^{(1/$
 $3)**_Z+_Z^2)^2*2^{(2/3)}*(-x^3+1)^{(1/3)}*x^3-10498622607665136*\text{RootOf}(2^{(2/3)+2}$
 $^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*(-x^3+1)^{(1/3)}*x^4+7759251414704196*\text{RootOf}(2^{(2/3)+}$
 $2^{(1/3)}*_Z+_Z^2)*2^{(2/3)}*(-x^3+1)^{(2/3)}*x-3531674097632562*\text{RootOf}(2^{(2/3)+2}$
 $^{(1/3)}*_Z+_Z^2)*2^{(2/3)}*(-x^3+1)^{(1/3)}*x^2-2613886855325640*\text{RootOf}(2^{(2/3)+}$
 $2^{(1/3)}*_Z+_Z^2)^2*2^{(1/3)}*(-x^3+1)^{(2/3)}*x+840505690860402*\text{RootOf}(2^{(2/3)+}$
 $2^{(1/3)}*_Z+_Z^2)^2*2^{(1/3)}*(-x^3+1)^{(1/3)}*x^2+11688730639030284*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*(-x^3+1)^{(1/3)}*x-6471910353179844*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)*2^{(1/3)}*(-x^3+1)^{(2/3)}*x^3+1203809884289286*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)*2^{(2/3)}*(-x^3+1)^{(2/3)}*x^4+3218487773589102*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)*2^{(2/3)}*(-x^3+1)^{(1/3)}*x^5-72607968203490*\text{RootOf}(2^{(2/$
 $3)+2^{(1/3)}*_Z+_Z^2)^2*2^{(1/3)}*(-x^3+1)^{(2/3)}*x^4+1560939140169318*\text{RootOf}(2$
 $^{(2/3)+2^{(1/3)}*_Z+_Z^2)^2*2^{(1/3)}*(-x^3+1)^{(1/3)}*x^5)/(2^{(1/3)}*x-1)^6)*\text{Root}$
 $\text{Of}(2^{(2/3)+2^{(1/3)}*_Z+_Z^2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{(-x^3+1)^{\frac{2}{3}}(2^{\frac{1}{3}}x-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-2^(1/3)*x)/(-x^3+1)^(2/3),x, algorithm="maxima")

[Out] -integrate(1/((-x^3 + 1)^(2/3)*(2^(1/3)*x - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{1}{(1-x^3)^{2/3}(2^{1/3}x-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)),x)

[Out] -int(1/((1 - x^3)^(2/3)*(2^(1/3)*x - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sqrt[3]{2}x(1-x^3)^{\frac{2}{3}} - (1-x^3)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-2**(1/3)*x)/(-x**3+1)**(2/3),x)

[Out] -Integral(1/(2**(1/3)*x*(1 - x**3)**(2/3) - (1 - x**3)**(2/3)), x)

$$3.6 \quad \int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=37

$$\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(\sqrt[3]{2}x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2137, 203}

$$\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(\sqrt[3]{2}x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]

[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[1 + x^3]])/Sqrt[3]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{2^{2/3} - 2x}{(2^{2/3} + x)\sqrt{1+x^3}} dx &= (2 \cdot 2^{2/3}) \text{Subst}\left(\int \frac{1}{1+3x^2} dx, x, \frac{1 + \sqrt[3]{2}x}{\sqrt{1+x^3}}\right) \\ &= \frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(1 + \sqrt[3]{2}x)}{\sqrt{1+x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.44, size = 326, normalized size = 8.81

$$\frac{4\sqrt[4]{2}\sqrt{\frac{3(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left((-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right),\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}(1+2^{2/3}-i\sqrt{3})\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]),x]

[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])])*(Sqrt[-I + Sqrt[3] + (2*I)*x] *(6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*S

$\text{qrt}[3] + 2^{(1/3)} * \text{Sqrt}[3]) * x) * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[I + \text{Sqrt}[3] - (2*I)*x] / (\text{Sqrt}[2]*3^{(1/4)})], (2*\text{Sqrt}[3]) / (3*I + \text{Sqrt}[3])] - (6*I)*\text{Sqrt}[3]*\text{Sqrt}[I + \text{Sqrt}[3] - (2*I)*x] * \text{Sqrt}[1 - x + x^2] * \text{EllipticPi}[(2*\text{Sqrt}[3]) / (I + (2*I)*2^{(2/3)} + \text{Sqrt}[3]), \text{ArcSin}[\text{Sqrt}[I + \text{Sqrt}[3] - (2*I)*x] / (\text{Sqrt}[2]*3^{(1/4)})], (2*\text{Sqrt}[3]) / (3*I + \text{Sqrt}[3])]) / (\text{Sqrt}[3]*(1 + 2*2^{(2/3)} - I*\text{Sqrt}[3])*\text{Sqrt}[I + \text{Sqrt}[3] - (2*I)*x] * \text{Sqrt}[1 + x^3])$

IntegrateAlgebraic [A] time = 1.91, size = 39, normalized size = 1.05

$$\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{x^3+1}}{\sqrt{3}(\sqrt[3]{2x+1})}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[1 + x^3]), x]

[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 + x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]

fricas [B] time = 2.05, size = 75, normalized size = 2.03

$$\frac{1}{3} \sqrt{6} 2^{1/6} \arctan\left(-\frac{\sqrt{6} 2^{1/6} (2x^5 + 2x^2 - 2^{2/3}(7x^4 + 4x) - 2^{1/3}(5x^3 + 2)) \sqrt{x^3 + 1}}{12(2x^6 + 3x^3 + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2), x, algorithm="fricas")

[Out] 1/3*sqrt(6)*2^(1/6)*arctan(-1/12*sqrt(6)*2^(1/6)*(2*x^5 + 2*x^2 - 2^(2/3)*(7*x^4 + 4*x) - 2^(1/3)*(5*x^3 + 2))*sqrt(x^3 + 1)/(2*x^6 + 3*x^3 + 1))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%{1, [1]%%} / %%{%%{[1,0,0]:[1,0,0,-2]%%}, [1]%%} Error: Bad Argument Value

maple [C] time = 0.07, size = 258, normalized size = 6.97

$$\frac{4\left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}\right) + 62^{\frac{2}{3}} \left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \text{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}, \frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{2^{\frac{2}{3}} - 1}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3 + 1} \left(2^{\frac{2}{3}} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2), x)

[Out] -4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+6*2^(2/3)*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)/(2

$$\frac{(-3/2+1/2\sqrt{3})^{1/2} \operatorname{EllipticPi}\left(\frac{(x+1)\sqrt{3}}{3/2-1/2\sqrt{3}}, \frac{(-3/2+1/2\sqrt{3})^{1/2}}{(-3/2-1/2\sqrt{3})^{1/2}}\right) - (-3/2-1/2\sqrt{3})^{1/2}}{(2^{2/3}-1)\sqrt{x^3+1}\left(x+2^{2/3}\right)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x - 2^{2/3}}{\sqrt{x^3+1}\left(x+2^{2/3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*x - 2^(2/3))/(sqrt(x^3 + 1)*(x + 2^(2/3))), x)

mupad [B] time = 3.59, size = 70, normalized size = 1.89

$$\frac{2^{2/3} \sqrt{3} \ln\left(\frac{\left(\sqrt{3} \operatorname{Im}\sqrt{x^3+1} + 2^{1/3} \sqrt{3} \operatorname{Re}\sqrt{x^3+1}\right)\left(\sqrt{3} \operatorname{Im}\sqrt{x^3+1} - 2^{1/3} \sqrt{3} \operatorname{Re}\sqrt{x^3+1}\right)^3}{(x+2^{2/3})^6}\right)}{3} \operatorname{Im}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x - 2^(2/3))/((x^3 + 1)^(1/2)*(x + 2^(2/3))),x)

[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2)*1i + (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i) * (3^(1/2)*1i - (x^3 + 1)^(1/2) + 2^(1/3)*3^(1/2)*x*1i)^3)/(x + 2^(2/3))^6)*1i)/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{2^{2/3}}{x\sqrt{x^3+1} + 2^{2/3}\sqrt{x^3+1}} \right) dx - \int \frac{2x}{x\sqrt{x^3+1} + 2^{2/3}\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(x**3+1)**(1/2),x)

[Out] -Integral(-2**(2/3)/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x) - Integral(2*x/(x*sqrt(x**3 + 1) + 2**(2/3)*sqrt(x**3 + 1)), x)

$$3.7 \quad \int \frac{2^{2/3} + 2x}{(2^{2/3} - x)\sqrt{1-x^3}} dx$$

Optimal. Leaf size=40

$$-\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.12, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2137, 203}

$$-\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]

[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[1 - x^3]])/Sqrt[3]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{2^{2/3} + 2x}{(2^{2/3} - x)\sqrt{1-x^3}} dx = -\left((2 \cdot 2^{2/3}) \text{Subst}\left(\int \frac{1}{1+3x^2} dx, x, \frac{1-\sqrt[3]{2}x}{\sqrt{1-x^3}} \right) \right) \\ = -\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{1-x^3}}\right)}{\sqrt{3}}$$

Mathematica [C] time = 0.37, size = 327, normalized size = 8.18

$$\frac{4\sqrt[4]{2}\sqrt{\frac{i(x-1)}{\sqrt{3+3i}}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i\sqrt{x^2+x+1}}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right)\right)\frac{2\sqrt{3}}{3i+\sqrt{3}}+\sqrt{-2ix+\sqrt{3}-i}\left((-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3}\right)x-\sqrt[3]{2}\sqrt{3}+2\sqrt{3}-3i\sqrt[3]{2}-6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right)\right)\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)}{\sqrt{3}\left(1+2\cdot 2^{2/3}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]),x]

[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] - (2*I)*x]*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3)

+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])]/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 - x^3])

IntegrateAlgebraic [A] time = 1.96, size = 41, normalized size = 1.02

$$\frac{2 \cdot 2^{2/3} \tan^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{3}(\sqrt[3]{2x-1})}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[1 - x^3]), x]

[Out] (-2*2^(2/3)*ArcTan[Sqrt[1 - x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]

fricas [B] time = 1.95, size = 76, normalized size = 1.90

$$-\frac{1}{3} \sqrt{6} 2^{1/6} \arctan\left(\frac{\sqrt{6} 2^{1/6} \left(2x^5 - 2x^2 + 2^{2/3}(7x^4 - 4x) - 2^{1/3}(5x^3 - 2)\right) \sqrt{-x^3 + 1}}{12(2x^6 - 3x^3 + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2), x, algorithm="fricas")

[Out] -1/3*sqrt(6)*2^(1/6)*arctan(1/12*sqrt(6)*2^(1/6)*(2*x^5 - 2*x^2 + 2^(2/3)*7*x^4 - 4*x) - 2^(1/3)*(5*x^3 - 2))*sqrt(-x^3 + 1)/(2*x^6 - 3*x^3 + 1))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error%%{1, [2]%%} / %%{%%{2, 0}:[1, 0, 0, -2]%%}, [2]%%} Error: Bad Argument Value

maple [C] time = 0.06, size = 253, normalized size = 6.32

$$\frac{4i\sqrt{3} \sqrt{i(x + \frac{1}{2} - \frac{i\sqrt{3}}{2})} \sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i(x + \frac{1}{2} + \frac{i\sqrt{3}}{2})} \sqrt{3} \operatorname{EllipticF}\left(\frac{\sqrt{\frac{i(x + \frac{1}{2} + \frac{i\sqrt{3}}{2})} \sqrt{3}}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right) + 2i2^{2/3} \sqrt{i(x + \frac{1}{2} - \frac{i\sqrt{3}}{2})} \sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i(x + \frac{1}{2} + \frac{i\sqrt{3}}{2})} \sqrt{3} \operatorname{EllipticPi}\left(\frac{\sqrt{\frac{i(x + \frac{1}{2} + \frac{i\sqrt{3}}{2})} \sqrt{3}}}}{3}, \frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3 + 1} \left(\frac{1}{2} + \frac{i\sqrt{3}}{2} - 2^{2/3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2), x)

[Out] 4/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2*I*2^(2/3)*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3))*EllipticPi(1/3*3^(1/2)*

$(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-2^(2/3)), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x + 2^{\frac{2}{3}}}{\sqrt{-x^3 + 1} \left(x - 2^{\frac{2}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*x + 2^(2/3))/(sqrt(-x^3 + 1)*(x - 2^(2/3))), x)

mupad [B] time = 3.63, size = 74, normalized size = 1.85

$$\frac{2^{2/3} \sqrt{3} \ln \left(\frac{(\sqrt{1-x^3} - \sqrt{3} i + 2^{1/3} \sqrt{3} x i)(\sqrt{3} i + \sqrt{1-x^3} - 2^{1/3} \sqrt{3} x i)^3}{(x - 2^{2/3})^6} \right) i}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + 2^(2/3))/((1 - x^3)^(1/2)*(x - 2^(2/3))),x)

[Out] (2^(2/3)*3^(1/2)*log((((1 - x^3)^(1/2) - 3^(1/2)*1i + 2^(1/3)*3^(1/2)*x*1i) * (3^(1/2)*1i + (1 - x^3)^(1/2) - 2^(1/3)*3^(1/2)*x*1i)^3)/(x - 2^(2/3))^6) * 1i)/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{1-x^3} - 2^{\frac{2}{3}}\sqrt{1-x^3}} dx - \int \frac{2x}{x\sqrt{1-x^3} - 2^{\frac{2}{3}}\sqrt{1-x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(-x**3+1)**(1/2),x)

[Out] -Integral(2**(2/3)/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x) - Integral(2*x/(x*sqrt(1 - x**3) - 2**(2/3)*sqrt(1 - x**3)), x)

$$3.8 \quad \int \frac{2^{2/3}+2x}{(2^{2/3}-x)\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=38

$$-\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2137, 206}

$$-\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]

[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 - 2^(1/3)*x))/Sqrt[-1 + x^3]])/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{2^{2/3} + 2x}{(2^{2/3} - x)\sqrt{-1 + x^3}} dx &= -\left((2 \cdot 2^{2/3}) \text{Subst}\left(\int \frac{1}{1 - 3x^2} dx, x, \frac{1 - \sqrt[3]{2}x}{\sqrt{-1 + x^3}} \right) \right) \\ &= -\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(1-\sqrt[3]{2}x)}{\sqrt{-1+x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.30, size = 325, normalized size = 8.55

$$\frac{4\sqrt[4]{2}\sqrt{\frac{i(x-1)}{\sqrt{3+3i}}}\left(6i\sqrt{3}\sqrt{2ix+\sqrt{3}+i\sqrt{x^2+x+1}}\Pi\left(\frac{2\sqrt{3}}{i+2\sqrt{2\sqrt{3}+\sqrt{3}}};\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}}\right)\right)\frac{2\sqrt{3}}{3+i\sqrt{3}}+\sqrt{-2ix+\sqrt{3}-i}\left((-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3})x-\sqrt[3]{2}\sqrt{3}+2\sqrt{3}-3i\sqrt[3]{2}-6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}}\right)\right)\frac{2\sqrt{3}}{3+i\sqrt{3}}\right)}{\sqrt{3}\left(1+2\sqrt[3]{2}-i\sqrt{3}\right)\sqrt{2ix+\sqrt{3}+i\sqrt{x^3-1}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]),x]

[Out] (-4*2^(1/6)*Sqrt[((-I)*(-1 + x))/(3*I + Sqrt[3])])*(Sqrt[-I + Sqrt[3]] - (2*I)*x)*(-6*I - (3*I)*2^(1/3) + 2*Sqrt[3] - 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3))

+ 4*Sqrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (6*I)*Sqrt[3]*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] + (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])]/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt[3] + (2*I)*x]*Sqrt[-1 + x^3])

IntegrateAlgebraic [A] time = 1.93, size = 39, normalized size = 1.03

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{x^3-1}}{\sqrt{3}\left(\sqrt[3]{2}x-1\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3) + 2*x)/((2^(2/3) - x)*Sqrt[-1 + x^3]), x]
 [Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 + x^3]/(Sqrt[3]*(-1 + 2^(1/3)*x))])/Sqrt[3]

fricas [B] time = 1.99, size = 238, normalized size = 6.26

$$\frac{1}{6} \sqrt{6} \log\left(\frac{x^{18} + 1440x^{15} + 17400x^{12} - 21056x^9 - 10368x^6 + 15360x^3 + 2\sqrt{6}(126x^{14} + 2664x^{11} - 4608x^5 + 2304x^2 + 2^{2/3}(310x^{13} + 2332x^{10} - 2656x^7 - 256x^4 + 512x) + 2^{1/3}(17x^{15} + 1058x^{12} + 2528x^9 - 5408x^6 + 2560x^3 - 512))\sqrt{x^3-1} + 24 \cdot 2^{2/3}(121x^{14} + 478x^{11} - 1144x^8 + 608x^5 - 64x^2) + 48 \cdot 2^{1/3}(5x^{16} + 176x^{13} + 83x^{10} - 680x^7 + 544x^4 - 128x) - 2048)}{x^{18} - 24x^{15} + 240x^{12} - 1280x^9 + 3840x^6 - 6144x^3 + 4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2), x, algorithm="fricas")
 [Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 + 1440*x^15 + 17400*x^12 - 21056*x^9 - 10368*x^6 + 15360*x^3 + 2*sqrt(6)*2^(1/6)*(126*x^14 + 2664*x^11 - 4608*x^5 + 2304*x^2 + 2^(2/3)*(x^16 + 310*x^13 + 2332*x^10 - 2656*x^7 - 256*x^4 + 512*x) + 2^(1/3)*(17*x^15 + 1058*x^12 + 2528*x^9 - 5408*x^6 + 2560*x^3 - 512))*sqrt(x^3 - 1) + 24*2^(2/3)*(x^17 + 121*x^14 + 478*x^11 - 1144*x^8 + 608*x^5 - 64*x^2) + 48*2^(1/3)*(5*x^16 + 176*x^13 + 83*x^10 - 680*x^7 + 544*x^4 - 128*x) - 2048)/(x^18 - 24*x^15 + 240*x^12 - 1280*x^9 + 3840*x^6 - 6144*x^3 + 4096))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2), x, algorithm="giac")
 [Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%{1, [2]} / %{{2,0}: [1,0,0,-2]}, [2]} Error: Bad Argument Value

maple [C] time = 0.06, size = 262, normalized size = 6.89

$$\frac{4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1-i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)+6\sqrt[3]{2}\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1-i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{-2^{\frac{2}{3}}+1},\sqrt{\frac{\frac{3}{2}+\frac{i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}\left(-2^{\frac{2}{3}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2), x)
 [Out] -4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), ((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

$$\frac{1}{2}I\sqrt{3} \left(\frac{1}{3/2 - 1/2 I\sqrt{3}} \right)^{1/2} - 6 \cdot 2^{2/3} \cdot \left(\frac{-3/2 - 1/2 I\sqrt{3}}{3/2 - 1/2 I\sqrt{3}} \right) \cdot \left(\frac{x-1}{-3/2 - 1/2 I\sqrt{3}} \right)^{1/2} \cdot \left(\frac{x+1/2 - 1/2 I\sqrt{3}}{3/2 - 1/2 I\sqrt{3}} \right)^{1/2} \cdot \left(\frac{x+1/2 + 1/2 I\sqrt{3}}{3/2 + 1/2 I\sqrt{3}} \right)^{1/2} / (x^3 - 1)^{1/2} / (-2^{2/3} + 1) \cdot \text{EllipticPi} \left(\left(\frac{x-1}{-3/2 - 1/2 I\sqrt{3}} \right)^{1/2}, \left(\frac{3/2 + 1/2 I\sqrt{3}}{-2^{2/3} + 1} \right) / \left(\frac{3/2 + 1/2 I\sqrt{3}}{3/2 - 1/2 I\sqrt{3}} \right)^{1/2} \right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x + 2^{\frac{2}{3}}}{\sqrt{x^3 - 1} \left(x - 2^{\frac{2}{3}} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)+2*x)/(2^(2/3)-x)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*x + 2^(2/3))/(sqrt(x^3 - 1)*(x - 2^(2/3))), x)

mupad [B] time = 2.86, size = 62, normalized size = 1.63

$$\frac{2^{2/3} \sqrt{3} \ln \left(\frac{\left(\sqrt{x^3 - 1} - \sqrt{3} + 2^{1/3} \sqrt{3} x \right)^3 \left(\sqrt{3} + \sqrt{x^3 - 1} - 2^{1/3} \sqrt{3} x \right)}{(x - 2^{2/3})^6} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + 2^(2/3))/(x^3 - 1)^(1/2)*(x - 2^(2/3))),x)

[Out] (2^(2/3)*3^(1/2)*log((((x^3 - 1)^(1/2) - 3^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^(1/2) + (x^3 - 1)^(1/2) - 2^(1/3)*3^(1/2)*x))/(x - 2^(2/3))^6))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{\frac{2}{3}}}{x\sqrt{x^3 - 1} - 2^{\frac{2}{3}}\sqrt{x^3 - 1}} dx - \int \frac{2x}{x\sqrt{x^3 - 1} - 2^{\frac{2}{3}}\sqrt{x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2**(2/3)+2*x)/(2**(2/3)-x)/(x**3-1)**(1/2),x)

[Out] -Integral(2**(2/3)/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x) - Integral(2*x/(x*sqrt(x**3 - 1) - 2**(2/3)*sqrt(x**3 - 1)), x)

$$3.9 \quad \int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=39

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(\sqrt[3]{2}x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.11, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2137, 206}

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(\sqrt[3]{2}x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]

[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*(1 + 2^(1/3)*x))/Sqrt[-1 - x^3]])/Sqrt[3]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{2^{2/3}-2x}{(2^{2/3}+x)\sqrt{-1-x^3}} dx &= (2 \cdot 2^{2/3}) \text{Subst}\left(\int \frac{1}{1-3x^2} dx, x, \frac{1+\sqrt[3]{2}x}{\sqrt{-1-x^3}}\right) \\ &= \frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{3}(1+\sqrt[3]{2}x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 0.29, size = 328, normalized size = 8.41

$$\frac{4\sqrt[4]{2}\sqrt{\frac{3(x+1)}{\sqrt{3}+3i}}\left(\sqrt{2ix+\sqrt{3}-i}\left((-3i\sqrt[3]{2}+4\sqrt{3}+\sqrt[3]{2}\sqrt{3})x+\sqrt[3]{2}\sqrt{3}-2\sqrt{3}+3i\sqrt[3]{2}+6i\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)-6i\sqrt{3}\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{i+2i2^{2/3}+\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[3]{3}}\right)\middle|\frac{2\sqrt{3}}{3i+\sqrt{3}}\right)\right)}{\sqrt{3}(1+2\cdot 2^{2/3}-i\sqrt{3})\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]),x]

[Out] (-4*2^(1/6)*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x] * (6*I + (3*I)*2^(1/3) - 2*Sqrt[3] + 2^(1/3)*Sqrt[3] + ((-3*I)*2^(1/3) + 4*I

```

qrt[3] + 2^(1/3)*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(
Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] - (6*I)*Sqrt[3]*Sqrt[I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(I + (2*I)*2^(2/3
) + Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqr
t[3])/(3*I + Sqrt[3])])]/(Sqrt[3]*(1 + 2*2^(2/3) - I*Sqrt[3])*Sqrt[I + Sqrt
[3] - (2*I)*x]*Sqrt[-1 - x^3])

```

IntegrateAlgebraic [A] time = 1.88, size = 41, normalized size = 1.05

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{-x^3-1}}{\sqrt{3}\left(\sqrt[3]{2x+1}\right)}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```

[In] IntegrateAlgebraic[(2^(2/3) - 2*x)/((2^(2/3) + x)*Sqrt[-1 - x^3]), x]
[Out] (2*2^(2/3)*ArcTanh[Sqrt[-1 - x^3]/(Sqrt[3]*(1 + 2^(1/3)*x))])/Sqrt[3]

```

fricas [B] time = 1.82, size = 241, normalized size = 6.18

$$\frac{1}{2} \sqrt{62} \log\left(\frac{x^{18} - 1440x^{15} + 17400x^{12} + 21056x^9 - 10368x^6 - 15360x^3 - 2\sqrt{62}(126x^{14} - 2664x^{11} + 4608x^8 + 2304x^5 + 2^5(1^4 - 310x^{13} + 2332x^{10} + 2656x^7 - 256x^4 - 512x) - 2^5(17x^{15} - 1058x^{12} + 2528x^9 + 5408x^6 + 2560x^3 + 512))\sqrt{-x^3-1} - 24 \cdot 2^{2/3}(x^{17} - 121x^{14} + 478x^{11} + 1144x^8 + 608x^5 + 64x^2) + 48 \cdot 2^{1/3}(5x^{16} - 176x^{13} + 83x^{10} + 680x^7 + 544x^4 + 128x) - 2048}{x^{18} + 24x^{15} + 240x^{12} + 1280x^9 + 3840x^6 + 6144x^3 + 4096}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2), x, algorithm="fricas")
[Out] 1/6*sqrt(6)*2^(1/6)*log((x^18 - 1440*x^15 + 17400*x^12 + 21056*x^9 - 10368*
x^6 - 15360*x^3 - 2*sqrt(6)*2^(1/6)*(126*x^14 - 2664*x^11 + 4608*x^8 + 2304
*x^5 + 2^(2/3)*(x^16 - 310*x^13 + 2332*x^10 + 2656*x^7 - 256*x^4 - 512*x) -
2^(1/3)*(17*x^15 - 1058*x^12 + 2528*x^9 + 5408*x^6 + 2560*x^3 + 512))*sqrt
(-x^3 - 1) - 24*2^(2/3)*(x^17 - 121*x^14 + 478*x^11 + 1144*x^8 + 608*x^5 +
64*x^2) + 48*2^(1/3)*(5*x^16 - 176*x^13 + 83*x^10 + 680*x^7 + 544*x^4 + 128
*x) - 2048)/(x^18 + 24*x^15 + 240*x^12 + 1280*x^9 + 3840*x^6 + 6144*x^3 + 4
096))

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2), x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%%{1, [1]%%}% / %%%{%%[1,0,0]:[1,0,0,-2]%%}, [1]%%}% Error: Bad Argument
Value

```

maple [C] time = 0.06, size = 249, normalized size = 6.38

$$\frac{4i\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i\left(x - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right) - 2i2^{2/3} \sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i\left(x - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticPi}\left(\frac{\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \frac{i\sqrt{3}}{2^{3/2} + \frac{1}{2} + \frac{i\sqrt{3}}{2}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3-1} \left(2^{2/3} + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2), x)
[Out] 4/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellipt
icF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2

```

$(I\sqrt{3})^{1/2}) - 2I\sqrt[3]{2}\sqrt{3}^{1/2} \cdot (I(x-1/2-1/2I\sqrt{3})\sqrt{3})^{1/2} \cdot ((x+1)/(3/2+1/2I\sqrt{3}))^{1/2} \cdot (-I(x-1/2+1/2I\sqrt{3})\sqrt{3})^{1/2} \cdot (-x^3-1)^{1/2} / (2^{2/3}+1/2+1/2I\sqrt{3}) \cdot \text{EllipticPi}(1/3\sqrt{3} \cdot (I(x-1/2-1/2I\sqrt{3})\sqrt{3})^{1/2}, I\sqrt{3}/(2^{2/3}+1/2+1/2I\sqrt{3}), (I\sqrt{3}/(3/2+1/2I\sqrt{3}))^{1/2})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x - 2^{\frac{2}{3}}}{\sqrt{-x^3 - 1} \left(x + 2^{\frac{2}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)-2*x)/(2^(2/3)+x)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*x - 2^(2/3))/(sqrt(-x^3 - 1)*(x + 2^(2/3))), x)

mupad [B] time = 2.84, size = 63, normalized size = 1.62

$$\frac{2^{2/3} \sqrt{3} \ln \left(\frac{(\sqrt{3} + \sqrt{-x^3 - 1} + 2^{1/3} \sqrt{3} x)^3 (\sqrt{3} - \sqrt{-x^3 - 1} + 2^{1/3} \sqrt{3} x)}{(x + 2^{2/3})^6} \right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x - 2^(2/3))/((- x^3 - 1)^(1/2)*(x + 2^(2/3))),x)

[Out] (2^(2/3)*3^(1/2)*log(((3^(1/2) + (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x)^3*(3^(1/2) - (- x^3 - 1)^(1/2) + 2^(1/3)*3^(1/2)*x))/(x + 2^(2/3))^6))/3

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2^{\frac{2}{3}}}{x\sqrt{-x^3 - 1} + 2^{\frac{2}{3}}\sqrt{-x^3 - 1}} \right) dx - \int \frac{2x}{x\sqrt{-x^3 - 1} + 2^{\frac{2}{3}}\sqrt{-x^3 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2**(2/3)-2*x)/(2**(2/3)+x)/(-x**3-1)**(1/2),x)

[Out] -Integral(-2**(2/3)/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x) - Integral(2*x/(x*sqrt(-x**3 - 1) + 2**(2/3)*sqrt(-x**3 - 1)), x)

$$3.10 \quad \int \frac{2^{2/3} \sqrt[3]{a} - 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{a + bx^3}} dx$$

Optimal. Leaf size=63

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a + bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.18, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 53, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2137, 203}

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a + bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[a + b*x^3]]/(Sqrt[3]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{a + bx^3}} dx = \frac{(2 \cdot 2^{2/3} \sqrt[3]{a}) \operatorname{Subst} \left(\int \frac{1}{1+3ax^2} dx, x, \frac{1 + \frac{\sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}}}{\sqrt{a + bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a + bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 1.12, size = 325, normalized size = 5.16

$$2 \sqrt{\frac{\sqrt[3]{a} + \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \left(\frac{2 \sqrt[3]{3} (\sqrt[3]{-1} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{\sqrt[3]{-1} - \frac{i \sqrt[3]{b} x}{\sqrt[3]{a}}} \operatorname{F}\left(\sin^{-1} \left(\sqrt{\frac{(-1)^{2/3} \sqrt[3]{b} x + \sqrt[3]{a}}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right)}{\sqrt{\frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}}} - \frac{3 \sqrt[3]{-1} 2^{2/3} (1 + \sqrt[3]{-1}) \sqrt[3]{a} \sqrt{\frac{1^{2/3} x^2 - \sqrt[3]{b} x}{a^{2/3} - \sqrt[3]{a}} + 1} \Pi \left(\frac{i \sqrt{3}}{\sqrt[3]{-1} + 2^{2/3}}, \sin^{-1} \left(\sqrt{\frac{(-1)^{2/3} \sqrt[3]{b} x + \sqrt[3]{a}}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right)}{\sqrt[3]{-1} + 2^{2/3}} \right) \sqrt{3} \sqrt[3]{b} \sqrt{a + bx^3}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((2*3^(1/4))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (3*(-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/((-1)^(1/3) + 2^(2/3)))/(Sqrt[3]*b^(1/3)*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 9.72, size = 65, normalized size = 1.03

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{3} \left(\sqrt[3]{2} \sqrt[6]{a} \sqrt[3]{bx+\sqrt{a}} \right)} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (-2*2^(2/3)*ArcTan[Sqrt[a + b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(1/3)*x))]/(Sqrt[3]*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3+a)^(1/2),x)

[Out] $\text{int}((2^{2/3}a^{1/3}-2b^{1/3}x)/(b^{1/3}x+2^{2/3}a^{1/3})/(bx^3+a)^{1/2},x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2b^{1/3}x - 2^{2/3}a^{1/3}}{\sqrt{bx^3 + a} \left(b^{1/3}x + 2^{2/3}a^{1/3} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2^{2/3}a^{1/3}-2b^{1/3}x)/(2^{2/3}a^{1/3}+b^{1/3}x)/(bx^3+a)^{1/2},x, \text{algorithm}=\text{"maxima"})$

[Out] $-\text{integrate}((2b^{1/3}x - 2^{2/3}a^{1/3})/(\text{sqrt}(bx^3 + a)*(b^{1/3}x + 2^{2/3}a^{1/3})), x)$

mupad [B] time = 5.81, size = 106, normalized size = 1.68

$$\frac{2^{2/3} \sqrt{3} \ln \left(\frac{\left(\sqrt{3} \sqrt{a} 1i - \sqrt{bx^3+a} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x 1i \right)^3 \left(\sqrt{3} \sqrt{a} 1i + \sqrt{bx^3+a} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x 1i \right)}{(2^{2/3} a^{1/3} + b^{1/3} x)^6} \right)}{3 a^{1/6} b^{1/3}} 1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2^{2/3}a^{1/3} - 2b^{1/3}x)/((a + bx^3)^{1/2}*(2^{2/3}a^{1/3} + b^{1/3}x)),x)$

[Out] $(2^{2/3}*3^{1/2}*\log(((3^{1/2}*a^{1/2}*1i - (a + bx^3)^{1/2} + 2^{1/3})*3^{1/2}*a^{1/6}*b^{1/3}*x*1i)^3*((3^{1/2}*a^{1/2}*1i + (a + bx^3)^{1/2} + 2^{1/3})*3^{1/2}*a^{1/6}*b^{1/3}*x*1i)))/(2^{2/3}*a^{1/3} + b^{1/3}*x)^6*1i)/(3*a^{1/6}*b^{1/3})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2^{2/3} \sqrt[3]{a}}{2^{2/3} \sqrt[3]{a} \sqrt{a + bx^3} + \sqrt[3]{b} x \sqrt{a + bx^3}} \right) dx - \int \frac{2 \sqrt[3]{b} x}{2^{2/3} \sqrt[3]{a} \sqrt{a + bx^3} + \sqrt[3]{b} x \sqrt{a + bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2^{2/3}a^{1/3}-2b^{1/3}x)/(2^{2/3}a^{1/3}+b^{1/3}x)/(bx^3+a)^{1/2},x)$

[Out] $-\text{Integral}(-2^{2/3}a^{1/3}/(2^{2/3}a^{1/3}*\text{sqrt}(a + bx^3) + b^{1/3}*x*\text{sqrt}(a + bx^3)), x) - \text{Integral}(2*b^{1/3}*x/(2^{2/3}a^{1/3}*\text{sqrt}(a + bx^3) + b^{1/3}*x*\text{sqrt}(a + bx^3)), x)$

$$3.11 \quad \int \frac{2^{2/3} \sqrt[3]{a} + 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{a - bx^3}} dx$$

Optimal. Leaf size=65

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.20, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2137, 203}

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (-2*2^(2/3)*ArcTan[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[a - b*x^3])/(Sqrt[3]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{2^{2/3} \sqrt[3]{a} + 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{a - bx^3}} dx = \frac{(2 \cdot 2^{2/3} \sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1+3ax^2} dx, x, \frac{1 - \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 1.13, size = 336, normalized size = 5.17

$$2 \sqrt{\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \left(\frac{2 (\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{\frac{\sqrt[3]{-1} (\sqrt[3]{a} + \sqrt[3]{-1} \sqrt[3]{b} x)}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}}}{\sqrt{\frac{\sqrt[3]{a} - (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}}} \right) F \left(\sin^{-1} \left(\sqrt{\frac{\sqrt[3]{a} - (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right) + \frac{\sqrt[3]{-1} 2^{2/3} (1 + \sqrt[3]{-1}) \sqrt[3]{a} \sqrt{\frac{3b^{2/3} x^2 + 3 \sqrt[3]{b} x}{a^{2/3} + \sqrt[3]{a}} + 3 \Pi \left(\frac{i \sqrt{3}}{\sqrt[3]{-1} + 2^{2/3}}; \sin^{-1} \left(\sqrt{\frac{\sqrt[3]{a} - (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right)}}{\sqrt[3]{-1} + 2^{2/3}}$$

$$\frac{\sqrt[3]{b} \sqrt{a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*((-1)^(1/3))*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] + ((-1)^(1/3)*2^(2/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[3 + (3*b^(1/3)*x)/a^(1/3) + (3*b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/((-1)^(1/3) + 2^(2/3)))/(b^(1/3)*Sqrt[a - b*x^3])

IntegrateAlgebraic [A] time = 9.73, size = 67, normalized size = 1.03

$$\frac{2 \cdot 2^{2/3} \tan^{-1} \left(\frac{\sqrt{a-bx^3}}{\sqrt{3} \left(\sqrt{a} - \sqrt[3]{2} \sqrt[6]{a} \sqrt[3]{bx} \right)} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (2*2^(2/3)*ArcTan[Sqrt[a - b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1/3)*x)))]/(Sqrt[3]*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.35, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3+a)^(1/2),x)

[Out] $\int \frac{(2^{2/3}a^{1/3} + 2b^{1/3}x)/(-b^{1/3}x + 2^{2/3}a^{1/3})}{(-bx^3 + a)^{1/2}} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2b^{1/3}x + 2^{2/3}a^{1/3}}{\sqrt{-bx^3 + a} \left(b^{1/3}x - 2^{2/3}a^{1/3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((2*b^(1/3)*x + 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - 2^(2/3)*a^(1/3))), x)`

mupad [B] time = 5.85, size = 107, normalized size = 1.65

$$\frac{2^{2/3} \sqrt{3} \ln \left(\frac{\left(\sqrt{a-bx^3} - \sqrt{3} \sqrt{a} \operatorname{li} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x \operatorname{li} \right) \left(\sqrt{3} \sqrt{a} \operatorname{li} + \sqrt{a-bx^3} - 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x \operatorname{li} \right)^3}{(2^{2/3} a^{1/3} - b^{1/3} x)^6} \right)}{3 a^{1/6} b^{1/3}} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) - b^(1/3)*x)), x)`

[Out] $(2^{2/3} * 3^{1/2} * \log(\frac{((a - b*x^3)^{1/2} - 3^{1/2} * a^{1/2} * \operatorname{li} + 2^{1/3} * 3^{1/2} * a^{1/6} * b^{1/3} * x * \operatorname{li}) * (3^{1/2} * a^{1/2} * \operatorname{li} + (a - b*x^3)^{1/2} - 2^{1/3} * 3^{1/2} * a^{1/6} * b^{1/3} * x * \operatorname{li})^3}{(2^{2/3} * a^{1/3} - b^{1/3} * x)^6} * \operatorname{li}) / (3 * a^{1/6} * b^{1/3}))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{2/3} \sqrt[3]{a}}{-2^{2/3} \sqrt[3]{a} \sqrt{a - bx^3} + \sqrt[3]{b} x \sqrt{a - bx^3}} dx - \int \frac{2 \sqrt[3]{b} x}{-2^{2/3} \sqrt[3]{a} \sqrt{a - bx^3} + \sqrt[3]{b} x \sqrt{a - bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(-b*x**3+a)**(1/2),x)`

[Out] `-Integral(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)`

$$3.12 \quad \int \frac{2^{2/3} \sqrt[3]{a} + 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{-a + bx^3}} dx$$

Optimal. Leaf size=66

$$\frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{bx^3 - a}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.20, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2137, 206}

$$\frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{bx^3 - a}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (-2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) - 2^(1/3)*b^(1/3)*x))/Sqrt[-a + b*x^3]]/(Sqrt[3]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{2^{2/3} \sqrt[3]{a} + 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{-a + bx^3}} dx = \frac{(2 \cdot 2^{2/3} \sqrt[3]{a}) \operatorname{Subst} \left(\int \frac{1}{1-3ax^2} dx, x, \frac{1 - \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} - \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{-a + bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.47, size = 390, normalized size = 5.91

$$\frac{2 \sqrt{\frac{\sqrt[3]{a} - \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \left(2 (\sqrt[3]{-1} + 2^{2/3}) (\sqrt[3]{-1} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{\frac{\sqrt[3]{-1} (\sqrt[3]{a} + \sqrt[3]{-1} \sqrt[3]{b} x)}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} F \left(\sin^{-1} \left(\sqrt{\frac{\sqrt[3]{a} - 1^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right) - \sqrt[3]{-1} 2^{2/3} \sqrt{3} (1 + \sqrt[3]{-1}) \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{a} - 1^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \sqrt{\frac{1^{2/3} a^2 + \sqrt[3]{b} x}{a^3}} + 1 \Pi \left(\frac{1 \sqrt{3}}{\sqrt[3]{-1} + 2^{2/3}}; \sin^{-1} \left(\sqrt{\frac{\sqrt[3]{a} - 1^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \middle| \sqrt[3]{-1} \right) \right)}{(\sqrt[3]{-1} + 2^{2/3}) \sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} - 1^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \sqrt{bx^3 - a}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (-2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(2*((-1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[((-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(((-1)^(1/3) + 2^(2/3))*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/(1 + (-1)^(1/3))*a^(1/3)])*Sqrt[-a + b*x^3])

IntegrateAlgebraic [A] time = 9.71, size = 68, normalized size = 1.03

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{bx^3-a}}{\sqrt{3}(\sqrt{a}-\sqrt[3]{2}\sqrt[3]{a}\sqrt[3]{bx})}\right)}{\sqrt{3}\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) + 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (-2*2^(2/3)*ArcTanh[Sqrt[-a + b*x^3]/(Sqrt[3]*(Sqrt[a] - 2^(1/3)*a^(1/6)*b^(1/3)*x))]/(Sqrt[3]*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)+2*b^(1/3)*x)/(2^(2/3)*a^(1/3)-b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.15, size = 0, normalized size = 0.00

$$\int \frac{2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{bx^3-a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*b^(1/3)*x+2^(2/3)*a^(1/3))/(-b^(1/3)*x+2^(2/3)*a^(1/3))/(b*x^3-a)^(1/2),x)

[Out] $\text{int}((2*b^{(1/3)}*x+2^{(2/3)}*a^{(1/3)})/(-b^{(1/3)}*x+2^{(2/3)}*a^{(1/3)})/(b*x^3-a)^{(1/2)},x)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2b^{1/3}x + 2^{2/3}a^{1/3}}{\sqrt{bx^3 - a} \left(b^{1/3}x - 2^{2/3}a^{1/3} \right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2^{(2/3)}*a^{(1/3)}+2*b^{(1/3)}*x)/(2^{(2/3)}*a^{(1/3)}-b^{(1/3)}*x)/(b*x^3-a)^{(1/2)},x, \text{algorithm}=\text{"maxima"})$

[Out] $-\text{integrate}((2*b^{(1/3)}*x + 2^{(2/3)}*a^{(1/3)})/(\text{sqrt}(b*x^3 - a)*(b^{(1/3)}*x - 2^{(2/3)}*a^{(1/3)})), x)$

mupad [B] time = 3.68, size = 102, normalized size = 1.55

$$\frac{\sqrt{3} 4^{1/3} \ln \left(\frac{(\sqrt{bx^3-a} + \sqrt{3} \sqrt{a} - 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x)(\sqrt{bx^3-a} - \sqrt{3} \sqrt{a} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x)^3}{(2^{2/3} a^{1/3} - b^{1/3} x)^6} \right)}{3 a^{1/6} b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((2^{(2/3)}*a^{(1/3)} + 2*b^{(1/3)}*x)/((b*x^3 - a)^{(1/2)}*(2^{(2/3)}*a^{(1/3)} - b^{(1/3)}*x)),x)$

[Out] $(3^{(1/2)}*4^{(1/3)}*\log((((b*x^3 - a)^{(1/2)} + 3^{(1/2)}*a^{(1/2)} - 2^{(1/3)}*3^{(1/2)})*a^{(1/6)}*b^{(1/3)}*x)*((b*x^3 - a)^{(1/2)} - 3^{(1/2)}*a^{(1/2)} + 2^{(1/3)}*3^{(1/2)})*a^{(1/6)}*b^{(1/3)}*x^3)/(2^{(2/3)}*a^{(1/3)} - b^{(1/3)}*x)^6)/(3*a^{(1/6)}*b^{(1/3)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2^{2/3} \sqrt[3]{a}}{-2^{2/3} \sqrt[3]{a} \sqrt{-a + bx^3} + \sqrt[3]{b} x \sqrt{-a + bx^3}} dx - \int \frac{2 \sqrt[3]{b} x}{-2^{2/3} \sqrt[3]{a} \sqrt{-a + bx^3} + \sqrt[3]{b} x \sqrt{-a + bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((2**(2/3)*a**(1/3)+2*b**(1/3)*x)/(2**(2/3)*a**(1/3)-b**(1/3)*x)/(b*x**3-a)**(1/2),x)$

[Out] $-\text{Integral}(2**(2/3)*a**(1/3)/(-2**(2/3)*a**(1/3)*\text{sqrt}(-a + b*x**3) + b**(1/3)*x*\text{sqrt}(-a + b*x**3)), x) - \text{Integral}(2*b**(1/3)*x/(-2**(2/3)*a**(1/3)*\text{sqrt}(-a + b*x**3) + b**(1/3)*x*\text{sqrt}(-a + b*x**3)), x)$

$$3.13 \quad \int \frac{2^{2/3} \sqrt[3]{a} - 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{-a - bx^3}} dx$$

Optimal. Leaf size=66

$$\frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{-a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.19, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2137, 206}

$$\frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{-a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (2*2^(2/3)*ArcTanh[(Sqrt[3]*a^(1/6)*(a^(1/3) + 2^(1/3)*b^(1/3)*x))/Sqrt[-a - b*x^3]]/(Sqrt[3]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{2^{2/3} \sqrt[3]{a} - 2 \sqrt[3]{b} x}{(2^{2/3} \sqrt[3]{a} + \sqrt[3]{b} x) \sqrt{-a - bx^3}} dx = \frac{(2 \cdot 2^{2/3} \sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1 - 3ax^2} dx, x, \frac{1 + \sqrt[3]{2} \sqrt[3]{b} x}{\sqrt[3]{a}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \cdot 2^{2/3} \tanh^{-1} \left(\frac{\sqrt{3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{2} \sqrt[3]{b} x)}{\sqrt{-a - bx^3}} \right)}{\sqrt{3} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.68, size = 375, normalized size = 5.68

$$\frac{2 \sqrt{\frac{\sqrt[3]{a} + \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \left(\sqrt[3]{-1} 2^{2/3} \sqrt{3} (1 + \sqrt[3]{-1}) \sqrt[3]{a} \sqrt{\frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \sqrt{\frac{b^{2/3} x^2 - \sqrt[3]{b} x}{a^{2/3}}} + 1 \right) \Pi \left(\frac{i \sqrt{3}}{\sqrt[3]{-1} + 2^{2/3}}; \sin^{-1} \left(\sqrt{\frac{(-1)^{2/3} \sqrt[3]{b} x + \sqrt[3]{a}}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \sqrt[3]{-1} \right) - \frac{2 (\sqrt[3]{-1} + 2^{2/3}) (\sqrt[3]{-1} \sqrt[3]{a} - \sqrt[3]{b} x) \sqrt{\sqrt[3]{-1} - \frac{i \sqrt[3]{b} x}{\sqrt[3]{a}}} \left(\sin^{-1} \left(\sqrt{\frac{(-1)^{2/3} \sqrt[3]{b} x + \sqrt[3]{a}}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \right) \sqrt[3]{-1} \right)}{\sqrt[3]{3}}}{(\sqrt[3]{-1} + 2^{2/3}) \sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} + (-1)^{2/3} \sqrt[3]{b} x}{(1 + \sqrt[3]{-1}) \sqrt[3]{a}}} \sqrt{-a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (-2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((-2*(-1)^(1/3) + 2^(2/3))*((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/3^(1/4) + (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(I*Sqrt[3])/((-1)^(1/3) + 2^(2/3)), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/(((1 + (-1)^(1/3))*a^(1/3)))*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a - b*x^3])

IntegrateAlgebraic [A] time = 9.68, size = 68, normalized size = 1.03

$$\frac{2 \cdot 2^{2/3} \tanh^{-1}\left(\frac{\sqrt{-a-bx^3}}{\sqrt{3}\left(\sqrt[3]{2}\sqrt[6]{a}\sqrt[3]{b}x+\sqrt{a}\right)}\right)}{\sqrt{3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((2^(2/3)*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (2*2^(2/3)*ArcTanh[Sqrt[-a - b*x^3]/(Sqrt[3]*(Sqrt[a] + 2^(1/3)*a^(1/6)*b^(1/3)*x))]/(Sqrt[3]*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.16, size = 0, normalized size = 0.00

$$\int \frac{-2b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2^{\frac{2}{3}}a^{\frac{1}{3}}\right)\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*b^(1/3)*x+2^(2/3)*a^(1/3))/(b^(1/3)*x+2^(2/3)*a^(1/3))/(-b*x^3-a)^(1/2),x)

[Out] $\int \frac{(-2b^{1/3}x + 2^{2/3}a^{1/3}) / (b^{1/3}x + 2^{2/3}a^{1/3})}{(-bx^3 - a)^{1/2}} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2b^{1/3}x - 2^{2/3}a^{1/3}}{\sqrt{-bx^3 - a} \left(b^{1/3}x + 2^{2/3}a^{1/3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2^(2/3)*a^(1/3)-2*b^(1/3)*x)/(2^(2/3)*a^(1/3)+b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((2*b^(1/3)*x - 2^(2/3)*a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x + 2^(2/3)*a^(1/3))), x)`

mupad [B] time = 3.65, size = 103, normalized size = 1.56

$$\frac{\sqrt{3} 4^{1/3} \ln \left(\frac{\left(\sqrt{-bx^3 - a} + \sqrt{3} \sqrt{a} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x \right)^3 \left(\sqrt{3} \sqrt{a} - \sqrt{-bx^3 - a} + 2^{1/3} \sqrt{3} a^{1/6} b^{1/3} x \right)}{\left(2^{2/3} a^{1/3} + b^{1/3} x \right)^6} \right)}{3 a^{1/6} b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2^(2/3)*a^(1/3) - 2*b^(1/3)*x)/((-a - b*x^3)^(1/2)*(2^(2/3)*a^(1/3) + b^(1/3)*x)),x)`

[Out] `(3^(1/2)*4^(1/3)*log((((-a - b*x^3)^(1/2) + 3^(1/2)*a^(1/2) + 2^(1/3)*3^(1/2)*a^(1/6)*b^(1/3)*x)^3*(3^(1/2)*a^(1/2) - (-a - b*x^3)^(1/2) + 2^(1/3)*3^(1/2)*a^(1/6)*b^(1/3)*x))/(2^(2/3)*a^(1/3) + b^(1/3)*x)^6)/(3*a^(1/6)*b^(1/3))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{2^{2/3} \sqrt[3]{a}}{2^{2/3} \sqrt[3]{a} \sqrt{-a - bx^3} + \sqrt[3]{b} x \sqrt{-a - bx^3}} \right) dx - \int \frac{2 \sqrt[3]{b} x}{2^{2/3} \sqrt[3]{a} \sqrt{-a - bx^3} + \sqrt[3]{b} x \sqrt{-a - bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2**(2/3)*a**(1/3)-2*b**(1/3)*x)/(2**(2/3)*a**(1/3)+b**(1/3)*x)/(-b*x**3-a)**(1/2),x)`

[Out] `-Integral(-2**(2/3)*a**(1/3)/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x) - Integral(2*b**(1/3)*x/(2**(2/3)*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)`

$$3.14 \quad \int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}} dx$$

Optimal. Leaf size=49

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3} \sqrt{c} (c+2dx)}{\sqrt{c^3+4d^3x^3}} \right)}{\sqrt{3} \sqrt{c} d}$$

Rubi [A] time = 0.12, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2137, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3} \sqrt{c} (c+2dx)}{\sqrt{c^3+4d^3x^3}} \right)}{\sqrt{3} \sqrt{c} d}$$

Antiderivative was successfully verified.

[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]), x]

[Out] (2*ArcTan[(Sqrt[3]*Sqrt[c]*(c + 2*d*x))/Sqrt[c^3 + 4*d^3*x^3]])/(Sqrt[3]*Sqrt[c]*d)

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2137

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(2*e)/d, Subst[Int[1/(1 + 3*a*x^2), x], x, (1 + (2*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 - 4*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{c-2dx}{(c+dx)\sqrt{c^3+4d^3x^3}} dx &= \frac{(2c) \text{Subst} \left(\int \frac{1}{1+3c^3x^2} dx, x, \frac{1+\frac{2dx}{c}}{\sqrt{c^3+4d^3x^3}} \right)}{d} \\ &= \frac{2 \tan^{-1} \left(\frac{\sqrt{3} \sqrt{c} (c+2dx)}{\sqrt{c^3+4d^3x^3}} \right)}{\sqrt{3} \sqrt{c} d} \end{aligned}$$

Mathematica [C] time = 1.10, size = 373, normalized size = 7.61

$$\frac{\sqrt{2} \sqrt{\frac{\sqrt{2}c+2dx}{(1+\sqrt{-1})^2}} \left(2 \sqrt{\frac{\sqrt{2}c-2(-1)^{2/3}dx}{(1+\sqrt{-1})^2}} (\sqrt{-1} (2+\sqrt{-2})c-2(\sqrt{-1}+2^{2/3})dx) F \left(\sin^{-1} \left(\frac{\sqrt{\frac{\sqrt{2}c+2(-1)^{2/3}dx}{(1+\sqrt{-1})^2}}}{\sqrt{2}} \right) \middle| \sqrt{-1} \right) - \sqrt{-1} 2^{2/3} \sqrt{3} (1+\sqrt{-1})c \sqrt{\frac{\sqrt{2}c+2(-1)^{2/3}dx}{(1+\sqrt{-1})^2}} \sqrt{\frac{4d^2x^2}{c^2} - \frac{2\sqrt{2}dx}{c} + 2^{2/3}} \Pi \left(\frac{i\sqrt{2}\sqrt{3}}{2+\sqrt{-2}}, \sin^{-1} \left(\frac{\sqrt{\frac{\sqrt{2}c+2(-1)^{2/3}dx}{(1+\sqrt{-1})^2}}}{\sqrt{2}} \right) \middle| \sqrt{-1} \right) \right)}{(2+\sqrt{-2})d \sqrt{\frac{\sqrt{2}c+2(-1)^{2/3}dx}{(1+\sqrt{-1})^2}} \sqrt{c^3+4d^3x^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]), x]

[Out] (2^(1/6)*Sqrt[(2^(1/3)*c + 2*d*x)/((1 + (-1)^(1/3))*c)])*(2*Sqrt[((-2)^(1/3)*c - 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)])*((-1)^(1/3)*(2 + (-2)^(1/3))*c

- 2*((-1)^(1/3) + 2^(2/3))*d*x)*EllipticF[ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]]/2^(1/6)], (-1)^(1/3)] - (-1)^(1/3)*2^(2/3)*Sqrt[3]*(1 + (-1)^(1/3))*c*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[2^(2/3) - (2*2^(1/3)*d*x)/c + (4*d^2*x^2)/c^2]*EllipticPi[(I*2^(1/3)*Sqrt[3])/(2 + (-2)^(1/3)), ArcSin[Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]]/2^(1/6)], (-1)^(1/3))]/((2 + (-2)^(1/3))*d*Sqrt[(2^(1/3)*c + 2*(-1)^(2/3)*d*x)/((1 + (-1)^(1/3))*c)]*Sqrt[c^3 + 4*d^3*x^3])

IntegrateAlgebraic [A] time = 1.25, size = 55, normalized size = 1.12

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{c^3 + 4d^3x^3}}{\sqrt{3}(c^{3/2} + 2\sqrt{c}dx)} \right)}{\sqrt{3} \sqrt{c} d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 + 4*d^3*x^3]), x]

[Out] (-2*ArcTan[Sqrt[c^3 + 4*d^3*x^3]/(Sqrt[3]*(c^(3/2) + 2*Sqrt[c]*d*x)))]/(Sqrt[3]*Sqrt[c]*d)

fricas [B] time = 2.12, size = 300, normalized size = 6.12

$$\left[\frac{\sqrt{3} \sqrt{\frac{1}{c}} \log \left(\frac{2d^6x^6 - 36cd^5x^5 - 18c^2d^4x^4 + 28c^3d^3x^3 + 18c^4d^2x^2 - c^6 - \sqrt{3}(4cd^4x^4 - 10c^2d^3x^3 - 18c^3d^2x^2 - 8c^4dx - c^5) \sqrt{4d^3x^3 + c^3} \sqrt{\frac{1}{c}}}{d^6x^6 + 6cd^5x^5 + 15c^2d^4x^4 + 20c^3d^3x^3 + 15c^4d^2x^2 + 6c^5dx + c^6} \right)}{6d}, \frac{\sqrt{3} \arctan \left(\frac{\sqrt{3} \sqrt{4d^3x^3 + c^3} (2d^3x^3 - 6cd^2x^2 - 6c^2dx - c^3)}{3(8d^4x^4 + 4cd^3x^3 + 2c^3dx + c^4) \sqrt{c}} \right)}{3\sqrt{c}d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2), x, algorithm="fricas")

[Out] [1/6*sqrt(3)*sqrt(-1/c)*log((2*d^6*x^6 - 36*c*d^5*x^5 - 18*c^2*d^4*x^4 + 28*c^3*d^3*x^3 + 18*c^4*d^2*x^2 - c^6 - sqrt(3)*(4*c*d^4*x^4 - 10*c^2*d^3*x^3 - 18*c^3*d^2*x^2 - 8*c^4*d*x - c^5))*sqrt(4*d^3*x^3 + c^3)*sqrt(-1/c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6))/d, -1/3*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(4*d^3*x^3 + c^3)*(2*d^3*x^3 - 6*c*d^2*x^2 - 6*c^2*d*x - c^3)/((8*d^4*x^4 + 4*c*d^3*x^3 + 2*c^3*d*x + c^4)*sqrt(c)))/(sqrt(c)*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2dx - c}{\sqrt{4d^3x^3 + c^3}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2), x, algorithm="giac")

[Out] integrate(-(2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)

maple [C] time = 0.07, size = 889, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2), x)

[Out] -4*((1/4*2^(1/3)-1/4*I*3^(1/2))*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2))*2^(1/3))*c/d*((x-(1/4*2^(1/3)+1/4*I*3^(1/2))*2^(1/3))*c/d)/((1/4*2^(1/3)-1/4*I*3^(1/2))*2^(1/3))*c/d-(1/4*2^(1/3)+1/4*I*3^(1/2))*2^(1/3))*c/d))^1/2*((x+1/2*2^(1/3))*c/d)/((1/4*2^(1/3)+1/4*I*3^(1/2))*2^(1/3))*c/d+1/2*2^(1/3))*c/d))^1/2

$$\frac{1}{2} * \left(\frac{x - (1/4 * 2^{1/3}) - 1/4 * I * 3^{1/2} * 2^{1/3}}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d} \right)^{1/2} / (4 * d^3 * x^3 + c^3)^{1/2} * \text{EllipticF} \left(\frac{x - (1/4 * 2^{1/3}) + 1/4 * I * 3^{1/2} * 2^{1/3}}{(1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d} \right)^{1/2}, \left(\frac{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d + 1/2 * 2^{1/3} * c/d} \right)^{1/2} + 6 * c/d * \left(\frac{(1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d}{(1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d} \right)^{1/2} * \left(\frac{x + 1/2 * 2^{1/3} * c/d}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d + 1/2 * 2^{1/3} * c/d} \right)^{1/2} * \left(\frac{x - (1/4 * 2^{1/3}) - 1/4 * I * 3^{1/2} * 2^{1/3}}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d} \right)^{1/2} / (4 * d^3 * x^3 + c^3)^{1/2} / \left(\frac{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d + c/d}{(1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d} \right)^{1/2}, \left(\frac{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d + c/d} \right), \left(\frac{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d - (1/4 * 2^{1/3} - 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d}{(1/4 * 2^{1/3} + 1/4 * I * 3^{1/2} * 2^{1/3}) * c/d + 1/2 * 2^{1/3} * c/d} \right)^{1/2} \right)$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{2 dx - c}{\sqrt{4 d^3 x^3 + c^3} (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(4*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*d*x - c)/(sqrt(4*d^3*x^3 + c^3)*(d*x + c)), x)

mupad [B] time = 4.73, size = 95, normalized size = 1.94

$$\frac{\sqrt{3} \ln \left(\frac{(-\sqrt{c^3+4d^3x^3} + \sqrt{3} c^{3/2} 1i + \sqrt{3} \sqrt{c} dx 2i)^3 (\sqrt{c^3+4d^3x^3} + \sqrt{3} c^{3/2} 1i + \sqrt{3} \sqrt{c} dx 2i)}{(c+dx)^6} \right) 1i}{3 \sqrt{c} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - 2*d*x)/((c^3 + 4*d^3*x^3)^(1/2)*(c + d*x)),x)

[Out] (3^(1/2)*log(((3^(1/2)*c^(3/2)*1i - (c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(1/2)*d*x*2i)^3*((c^3 + 4*d^3*x^3)^(1/2) + 3^(1/2)*c^(3/2)*1i + 3^(1/2)*c^(1/2)*d*x*2i)))/(c + d*x)^6)*1i)/(3*c^(1/2)*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \left(-\frac{c}{c\sqrt{c^3 + 4d^3x^3} + dx\sqrt{c^3 + 4d^3x^3}} \right) dx - \int \frac{2dx}{c\sqrt{c^3 + 4d^3x^3} + dx\sqrt{c^3 + 4d^3x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(4*d**3*x**3+c**3)**(1/2),x)

[Out] -Integral(-c/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x) - Integral(2*d*x/(c*sqrt(c**3 + 4*d**3*x**3) + d*x*sqrt(c**3 + 4*d**3*x**3)), x)

$$3.15 \quad \int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=23

$$\frac{2}{3} \tanh^{-1} \left(\frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

Rubi [A] time = 0.06, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2138, 206}

$$\frac{2}{3} \tanh^{-1} \left(\frac{(x+1)^2}{3\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((2 - x)*Sqrt[1 + x^3]), x]

[Out] (2*ArcTanh[(1 + x)^2/(3*Sqrt[1 + x^3])])/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(2-x)\sqrt{1+x^3}} dx &= 2 \text{Subst} \left(\int \frac{1}{9-x^2} dx, x, \frac{(1+x)^2}{\sqrt{1+x^3}} \right) \\ &= \frac{2}{3} \tanh^{-1} \left(\frac{(1+x)^2}{3\sqrt{1+x^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 46, normalized size = 2.00

$$\frac{1}{3} \log \left(\frac{(x+1)^2}{\sqrt{x^3+1}} + 3 \right) - \frac{1}{3} \log \left(3 - \frac{(x+1)^2}{\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((2 - x)*Sqrt[1 + x^3]), x]

[Out] -1/3*Log[3 - (1 + x)^2/Sqrt[1 + x^3]] + Log[3 + (1 + x)^2/Sqrt[1 + x^3]]/3

IntegrateAlgebraic [A] time = 0.88, size = 31, normalized size = 1.35

$$\frac{2}{3} \tanh^{-1} \left(\frac{\frac{x^2}{3} + \frac{2x}{3} + \frac{1}{3}}{\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)/((2 - x)*Sqrt[1 + x^3]), x]

[Out] (2*ArcTanh[(1/3 + (2*x)/3 + x^2/3)/Sqrt[1 + x^3]])/3

fricas [B] time = 2.02, size = 44, normalized size = 1.91

$$\frac{1}{3} \log\left(\frac{x^3 + 12x^2 + 6\sqrt{x^3 + 1}(x + 1) - 6x + 10}{x^3 - 6x^2 + 12x - 8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2), x, algorithm="fricas")

[Out] 1/3*log((x^3 + 12*x^2 + 6*sqrt(x^3 + 1)*(x + 1) - 6*x + 10)/(x^3 - 6*x^2 + 12*x - 8))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x + 1}{\sqrt{x^3 + 1}(x - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2), x, algorithm="giac")

[Out] integrate(-(x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)

maple [C] time = 0.04, size = 240, normalized size = 10.43

$$\frac{2\left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{\frac{\frac{x-1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{\frac{\frac{x-1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \operatorname{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right) + 2\left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{\frac{\frac{x-1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{\frac{\frac{x-1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \operatorname{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}, \frac{1}{2} - \frac{i\sqrt{3}}{6}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(2-x)/(x^3+1)^(1/2), x)

[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))+2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2), 1/2-1/6*I*3^(1/2), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x + 1}{\sqrt{x^3 + 1}(x - 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(x^3+1)^(1/2), x, algorithm="maxima")

[Out] -integrate((x + 1)/(sqrt(x^3 + 1)*(x - 2)), x)

mupad [B] time = 0.23, size = 205, normalized size = 8.91

$$\frac{(3 + \sqrt{3} \operatorname{li}) \sqrt{\frac{\frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}}{-\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \left(\operatorname{F}\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\right)\right) - \operatorname{Pi}\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}; \operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\right)\right) - \frac{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}{-\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}} \right) \sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}} \sqrt{\frac{\frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}}{\sqrt{x^3 + \left(-\left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) - 1\right) x - \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x + 1)/((x^3 + 1)^(1/2)*(x - 2)),x)`

[Out] $-\left(\sqrt[3]{3}i + 3\right)\left(x + \frac{\sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right)^{1/2} \cdot \left(\operatorname{ellipticF}\left(\operatorname{asin}\left(\frac{x + 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right), -\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right) - \operatorname{ellipticPi}\left(\frac{\sqrt[3]{3}i}{6} + \frac{1}{2}, \operatorname{asin}\left(\frac{x + 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right), -\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right)\right)\left(x + 1\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}\left(\frac{\sqrt[3]{3}i}{2} - x + \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}\right)\left(x^3 - x\left(\frac{\sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{1}{2}\right) + 1\right) - \left(\frac{\sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{1}{2}\right)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}} dx - \int \frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(2-x)/(x**3+1)**(1/2),x)`

[Out] $-\operatorname{Integral}\left(\frac{x}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}, x\right) - \operatorname{Integral}\left(\frac{1}{x\sqrt{x^3+1}-2\sqrt{x^3+1}}, x\right)$

$$3.16 \quad \int \frac{1-x}{(2+x)\sqrt{1-x^3}} dx$$

Optimal. Leaf size=27

$$-\frac{2}{3} \tanh^{-1} \left(\frac{(1-x)^2}{3\sqrt{1-x^3}} \right)$$

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2138, 206}

$$-\frac{2}{3} \tanh^{-1} \left(\frac{(1-x)^2}{3\sqrt{1-x^3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

[Out] (-2*ArcTanh[(1 - x)^2/(3*Sqrt[1 - x^3])])/3

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_)*(x_))/(((c_) + (d_)*(x_))*Sqrt[(a_) + (b_)*(x_)^3]), x_Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x}{(2+x)\sqrt{1-x^3}} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{9-x^2} dx, x, \frac{(1-x)^2}{\sqrt{1-x^3}} \right) \right) \\ &= -\frac{2}{3} \tanh^{-1} \left(\frac{(1-x)^2}{3\sqrt{1-x^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 54, normalized size = 2.00

$$\frac{1}{3} \log \left(3 - \frac{(1-x)^2}{\sqrt{1-x^3}} \right) - \frac{1}{3} \log \left(\frac{(1-x)^2}{\sqrt{1-x^3}} + 3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

[Out] Log[3 - (1 - x)^2/Sqrt[1 - x^3]]/3 - Log[3 + (1 - x)^2/Sqrt[1 - x^3]]/3

IntegrateAlgebraic [A] time = 0.88, size = 33, normalized size = 1.22

$$-\frac{2}{3} \tanh^{-1} \left(\frac{\frac{x^2}{3} - \frac{2x}{3} + \frac{1}{3}}{\sqrt{1-x^3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x)/((2 + x)*Sqrt[1 - x^3]),x]

[Out] (-2*ArcTanh[(1/3 - (2*x)/3 + x^2/3)/Sqrt[1 - x^3]])/3

fricas [B] time = 1.45, size = 47, normalized size = 1.74

$$\frac{1}{3} \log \left(-\frac{x^3 - 12x^2 - 6\sqrt{-x^3 + 1}(x - 1) - 6x - 10}{x^3 + 6x^2 + 12x + 8} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] 1/3*log(-(x^3 - 12*x^2 - 6*sqrt(-x^3 + 1)*(x - 1) - 6*x - 10)/(x^3 + 6*x^2 + 12*x + 8))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{-x^3+1}(x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)

maple [C] time = 0.04, size = 240, normalized size = 8.89

$$\frac{2i\sqrt{3} \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)} \sqrt{3} \sqrt{\frac{x-1}{\frac{3}{2}+i\sqrt{3}}} \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{\left(i+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}}}\right) - 2i\sqrt{3} \sqrt{i\left(x+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)} \sqrt{3} \sqrt{\frac{x-1}{\frac{3}{2}+i\sqrt{3}}} \sqrt{-i\left(x+\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticPi}\left(\frac{\sqrt{3} \sqrt{\left(i+\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}}}\right)}{3\sqrt{-x^3+1} \left(\frac{3}{2}+\frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(x+2)/(-x^3+1)^(1/2),x)

[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)/(3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{-x^3+1}(x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x - 1)/(sqrt(-x^3 + 1)*(x + 2)), x)

mupad [B] time = 0.18, size = 221, normalized size = 8.19

$$\frac{(3 + \sqrt{3} 1i) \sqrt{x^3 - 1} \sqrt{-\frac{x+\frac{1}{2}-\frac{\sqrt{3} 1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}} \sqrt{\frac{x+\frac{1}{2}+\frac{\sqrt{3} 1i}{2}}{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}} \left(F \left(\operatorname{asin} \left(\sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}} \right) \middle| -\frac{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3} 1i}{2}} \right) - \Pi \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{6}; \operatorname{asin} \left(\sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}} \right) \middle| -\frac{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}{-\frac{3}{2}+\frac{\sqrt{3} 1i}{2}} \right) \right) \sqrt{\frac{x-1}{\frac{3}{2}+\frac{\sqrt{3} 1i}{2}}}}{\sqrt{1-x^3} \sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) - 1} x + \left(-\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} 1i}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x - 1)/((1 - x^3)^(1/2)*(x + 2)),x)`

[Out] $((3^{1/2}i + 3)(x^3 - 1)^{1/2}(-(x - (3^{1/2}i)/2 + 1/2)/((3^{1/2}i)/2 - 3/2))^{1/2}((x + (3^{1/2}i)/2 + 1/2)/((3^{1/2}i)/2 + 3/2))^{1/2} * (\text{ellipticF}(\text{asin}((-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2}), -((3^{1/2}i)/2 + 3/2)/((3^{1/2}i)/2 - 3/2)) - \text{ellipticPi}((3^{1/2}i)/6 + 1/2, \text{asin}((-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2}), -((3^{1/2}i)/2 + 3/2)/((3^{1/2}i)/2 - 3/2)) * (-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2} / ((1 - x^3)^{1/2} * ((3^{1/2}i)/2 - 1/2) * ((3^{1/2}i)/2 + 1/2) - x * ((3^{1/2}i)/2 - 1/2) * ((3^{1/2}i)/2 + 1/2) + 1) + x^3)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{1-x^3} + 2\sqrt{1-x^3}} dx - \int \left(-\frac{1}{x\sqrt{1-x^3} + 2\sqrt{1-x^3}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/(2+x)/(-x**3+1)**(1/2),x)`

[Out] `-Integral(x/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-1/(x*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)`

$$3.17 \quad \int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=25

$$-\frac{2}{3} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right)$$

Rubi [A] time = 0.06, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2138, 203}

$$-\frac{2}{3} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/((2 + x)*Sqrt[-1 + x^3]), x]

[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-x}{(2+x)\sqrt{-1+x^3}} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{9+x^2} dx, x, \frac{(1-x)^2}{\sqrt{-1+x^3}} \right) \right) \\ &= -\frac{2}{3} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{-1+x^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$-\frac{2}{3} \tan^{-1} \left(\frac{(1-x)^2}{3\sqrt{x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/((2 + x)*Sqrt[-1 + x^3]), x]

[Out] (-2*ArcTan[(1 - x)^2/(3*Sqrt[-1 + x^3])])/3

IntegrateAlgebraic [A] time = 0.87, size = 21, normalized size = 0.84

$$\frac{2}{3} \tan^{-1} \left(\frac{3\sqrt{x^3-1}}{(x-1)^2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x)/((2 + x)*Sqrt[-1 + x^3]),x]

[Out] (2*ArcTan[(3*Sqrt[-1 + x^3])/(-1 + x)^2])/3

fricas [B] time = 1.66, size = 40, normalized size = 1.60

$$-\frac{1}{3} \arctan\left(\frac{(x^3 - 12x^2 - 6x - 10)\sqrt{x^3 - 1}}{6(x^4 - x^3 - x + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] -1/3*arctan(1/6*(x^3 - 12*x^2 - 6*x - 10)*sqrt(x^3 - 1)/(x^4 - x^3 - x + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x-1}{\sqrt{x^3-1}(x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)

maple [C] time = 0.04, size = 240, normalized size = 9.60

$$\frac{2\left(-\frac{3}{2} - \frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right) + 2\left(-\frac{3}{2} - \frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}, \frac{1}{2} + \frac{i\sqrt{3}}{6}, \sqrt{\frac{\frac{3}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x)/(x+2)/(x^3-1)^(1/2),x)

[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),1/2+1/6*I*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x-1}{\sqrt{x^3-1}(x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(2+x)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x - 1)/(sqrt(x^3 - 1)*(x + 2)), x)

mupad [B] time = 2.55, size = 205, normalized size = 8.20

$$\frac{(3 + \sqrt{3} \operatorname{li}) \sqrt{\frac{x+\frac{1}{2} - \frac{\sqrt{3} \operatorname{li}}{2}}{-\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\sqrt{\frac{x+\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\left(F\left(\operatorname{asin}\left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\right)\right) - \Pi\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{6}; \operatorname{asin}\left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}\right)\right) - \frac{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}{-\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}\right) \sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} \operatorname{li}}{2}}}}{\sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right) - 1} x + \left(-\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3} \operatorname{li}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x - 1)/((x^3 - 1)^(1/2)*(x + 2)),x)`

[Out] $((3^{1/2}i + 3)*(-x - (3^{1/2}i)/2 + 1/2)/((3^{1/2}i)/2 - 3/2))^{1/2} * ((x + (3^{1/2}i)/2 + 1/2)/((3^{1/2}i)/2 + 3/2))^{1/2} * (\text{ellipticF}(\text{asin}((-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2}), -((3^{1/2}i)/2 + 3/2)/((3^{1/2}i)/2 - 3/2)) - \text{ellipticPi}((3^{1/2}i)/6 + 1/2, \text{asin}((-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2}), -((3^{1/2}i)/2 + 3/2)/((3^{1/2}i)/2 - 3/2)) * (-x - 1)/((3^{1/2}i)/2 + 3/2))^{1/2} / (((3^{1/2}i)/2 - 1/2) * ((3^{1/2}i)/2 + 1/2) - x * ((3^{1/2}i)/2 - 1/2) * ((3^{1/2}i)/2 + 1/2) + 1) + x^3)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{x^3-1} + 2\sqrt{x^3-1}} dx - \int \left(-\frac{1}{x\sqrt{x^3-1} + 2\sqrt{x^3-1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)/(2+x)/(x**3-1)**(1/2),x)`

[Out] `-Integral(x/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-1/(x*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)`

$$3.18 \quad \int \frac{1+x}{(2-x)\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=25

$$\frac{2}{3} \tan^{-1} \left(\frac{(x+1)^2}{3\sqrt{-x^3-1}} \right)$$

Rubi [A] time = 0.07, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2138, 203}

$$\frac{2}{3} \tan^{-1} \left(\frac{(x+1)^2}{3\sqrt{-x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(2-x)\sqrt{-1-x^3}} dx &= 2 \text{Subst} \left(\int \frac{1}{9+x^2} dx, x, \frac{(1+x)^2}{\sqrt{-1-x^3}} \right) \\ &= \frac{2}{3} \tan^{-1} \left(\frac{(1+x)^2}{3\sqrt{-1-x^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2}{3} \tan^{-1} \left(\frac{(x+1)^2}{3\sqrt{-x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTan[(1 + x)^2/(3*Sqrt[-1 - x^3])])/3

IntegrateAlgebraic [A] time = 0.88, size = 23, normalized size = 0.92

$$-\frac{2}{3} \tan^{-1} \left(\frac{3\sqrt{-x^3-1}}{(x+1)^2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)/((2 - x)*Sqrt[-1 - x^3]),x]

[Out] (-2*ArcTan[(3*Sqrt[-1 - x^3])/(1 + x)^2])/3

fricas [A] time = 1.45, size = 38, normalized size = 1.52

$$-\frac{1}{3} \arctan\left(\frac{(x^3 + 12x^2 - 6x + 10)\sqrt{-x^3 - 1}}{6(x^4 + x^3 + x + 1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] -1/3*arctan(1/6*(x^3 + 12*x^2 - 6*x + 10)*sqrt(-x^3 - 1)/(x^4 + x^3 + x + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x+1}{\sqrt{-x^3-1}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)

maple [C] time = 0.04, size = 240, normalized size = 9.60

$$\frac{2i\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i\left(x - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right) + 2i\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}} \sqrt{-i\left(x - \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)} \sqrt{3} \operatorname{EllipticPi}\left(\frac{\sqrt{3} \sqrt{i\left(x - \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)} \sqrt{3}}{3}, \frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3-1} \left(-\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(2-x)/(-x^3-1)^(1/2),x)

[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(-3/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-3/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x+1}{\sqrt{-x^3-1}(x-2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(2-x)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x + 1)/(sqrt(-x^3 - 1)*(x - 2)), x)

mupad [B] time = 2.53, size = 221, normalized size = 8.84

$$\frac{(3 + \sqrt{3} i) \sqrt{x^3 + 1} \sqrt{\frac{x - \frac{1}{2} + \frac{\sqrt{3} i}{2}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \left(F\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}\right)\right) - \frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}} \right) - \Pi\left(\frac{1}{2} + \frac{\sqrt{3} i}{6}; \operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}\right)\right) - \frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}} \right) \sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \sqrt{\frac{1 - x + \frac{\sqrt{3} i}{2}}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}}{\sqrt{-x^3-1} \sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) - 1} x - \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x + 1)/((- x^3 - 1)^(1/2)*(x - 2)),x)`

[Out] $-\left(\sqrt[3]{3}i + 3\right)\left(x^3 + 1\right)^{1/2}\left(\frac{x + \sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right)^{1/2}\left(\operatorname{ellipticF}\left(\arcsin\left(\frac{x + 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right), -\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right)\right) - \operatorname{ellipticPi}\left(\frac{\sqrt[3]{3}i}{6} + \frac{1}{2}, \arcsin\left(\frac{x + 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right), -\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} - \frac{3}{2}\right)\right)\right)\left(\frac{x + 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right)\left(\frac{\sqrt[3]{3}i}{2} - x + \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}\left(\frac{-x^3 - 1}{\left(\frac{\sqrt[3]{3}i}{2} + \frac{3}{2}\right)^{1/2}}\right)\left(\frac{x^3 - x\left(\frac{\sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{1}{2}\right) + 1}{\left(\frac{\sqrt[3]{3}i}{2} - \frac{1}{2}\right)\left(\frac{\sqrt[3]{3}i}{2} + \frac{1}{2}\right)^{1/2}}\right)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}} dx - \int \frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)/(2-x)/(-x**3-1)**(1/2),x)`

[Out] $-\operatorname{Integral}\left(\frac{x}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}, x\right) - \operatorname{Integral}\left(\frac{1}{x\sqrt{-x^3-1}-2\sqrt{-x^3-1}}, x\right)$

$$3.19 \quad \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{(2\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=50

$$\frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.13, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2138, 206}

$$\frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*ArcTanh[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a + b*x^3])])/(3*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{(2\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \operatorname{Subst} \left(\int \frac{1}{9-ax^2} dx, x, \frac{\left(1 + \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a+bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 1.02

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a} \left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}} + 1 \right)^2}{3\sqrt{a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*ArcTanh[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2]/(3*Sqrt[a + b*x^3]))/(3*a^(1/6)*b^(1/3))

IntegrateAlgebraic [A] time = 6.62, size = 69, normalized size = 1.38

$$\frac{2 \tanh^{-1} \left(\frac{\frac{b^{2/3}x^2}{3\sqrt[6]{a}} + \frac{2}{3}\sqrt[6]{a}\sqrt[3]{b}x + \frac{\sqrt{a}}{3}}{\sqrt{a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*ArcTanh[(Sqrt[a]/3 + (2*a^(1/6)*b^(1/3)*x)/3 + (b^(2/3)*x^2)/(3*a^(1/6))]/Sqrt[a + b*x^3])/(3*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)

[Out] int((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x - 2a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(b*x^3 + a)*(b^(1/3)*x - 2*a^(1/3))), x)

mupad [B] time = 3.44, size = 65, normalized size = 1.30

$$\frac{\ln\left(\frac{(\sqrt{bx^3+a}+\sqrt{a})(\sqrt{bx^3+a}-\sqrt{a}+2a^{1/6}b^{1/3}x)^3}{x^3(b^{1/3}x-2a^{1/3})^3}\right)}{3a^{1/6}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(a + b*x^3)^(1/2)),x)

[Out] log((((a + b*x^3)^(1/2) + a^(1/2))*((a + b*x^3)^(1/2) - a^(1/2) + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{a+bx^3} + \sqrt[3]{b}x\sqrt{a+bx^3}} dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{a+bx^3} + \sqrt[3]{b}x\sqrt{a+bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(b*x**3+a)**(1/2),x)

[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(a + b*x**3) + b**(1/3)*x*sqrt(a + b*x**3)), x)

$$3.20 \quad \int \frac{\sqrt[3]{a} - \sqrt[3]{b}x}{(2\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=52

$$\frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.14, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2138, 206}

$$\frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (-2*ArcTanh[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[a - b*x^3])]/(3*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{\sqrt[3]{a} - \sqrt[3]{b}x}{(2\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a-bx^3}} dx = \frac{(2\sqrt[3]{a}) \operatorname{Subst} \left(\int \frac{1}{9-ax^2} dx, x, \frac{\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{a-bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tanh^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 1.02

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a} \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2]/(3*Sqrt[a - b*x^3]))/(3*a^(1/6)*b^(1/3))

IntegrateAlgebraic [A] time = 6.79, size = 70, normalized size = 1.35

$$-\frac{2 \tanh^{-1}\left(\frac{\frac{b^{2/3}x^2}{3\sqrt[6]{a}} - \frac{2}{3}\sqrt[6]{a}\sqrt[3]{b}x + \frac{\sqrt{a}}{3}}{\sqrt{a-bx^3}}\right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[a]/3 - (2*a^(1/6)*b^(1/3)*x)/3 + (b^(2/3)*x^2)/(3*a^(1/6))]/Sqrt[a - b*x^3]))/(3*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2), x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2), x)

[Out] int((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}}{\sqrt{-bx^3 + a}\left(b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(-b*x^3 + a)*(b^(1/3)*x + 2*a^(1/3))), x)

mupad [B] time = 3.59, size = 67, normalized size = 1.29

$$\frac{\ln\left(\frac{(\sqrt{a-bx^3}-\sqrt{a})(\sqrt{a-bx^3}+\sqrt{a}+2a^{1/6}b^{1/3}x)^3}{x^3(b^{1/3}x+2a^{1/3})^3}\right)}{3a^{1/6}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(a - b*x^3)^(1/2)),x)

[Out] log((((a - b*x^3)^(1/2) - a^(1/2))*((a - b*x^3)^(1/2) + a^(1/2) + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3))/(3*a^(1/6)*b^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}\right)dx-\int\frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{a-bx^3}+\sqrt[3]{b}x\sqrt{a-bx^3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(-b*x**3+a)**(1/2), x)

[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(a - b*x**3) + b**(1/3)*x*sqrt(a - b*x**3)), x)

$$3.21 \quad \int \frac{\sqrt[3]{a} - \sqrt[3]{b}x}{(2\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=53

$$\frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 45, $\frac{\text{number of rules}}{\text{integrand size}} = 0.044$, Rules used = {2138, 203}

$$\frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{bx^3-a}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (-2*ArcTan[(a^(1/3) - b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a + b*x^3])]/(3*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{\sqrt[3]{a} - \sqrt[3]{b}x}{(2\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{9+ax^2} dx, x, \frac{\left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a+bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} - \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{-a+bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] time = 0.03, size = 54, normalized size = 1.02

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \left(1 - \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{3\sqrt{bx^3-a}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (-2*ArcTan[(Sqrt[a]*(1 - (b^(1/3)*x)/a^(1/3))^2]/(3*Sqrt[-a + b*x^3]))/(3*a^(1/6)*b^(1/3))

IntegrateAlgebraic [A] time = 5.81, size = 65, normalized size = 1.23

$$\frac{2 \tan^{-1} \left(\frac{3 \sqrt[6]{a} \sqrt{bx^3 - a}}{a^{2/3} - 2 \sqrt[3]{a} \sqrt[3]{bx + b^{2/3}x^2}} \right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^(1/3) - b^(1/3)*x)/((2*a^(1/3) + b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (2*ArcTan[(3*a^(1/6)*Sqrt[-a + b*x^3])/(a^(2/3) - 2*a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)]/(3*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2), x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2), x)

[Out] int((-b^(1/3)*x+a^(1/3))/(b^(1/3)*x+2*a^(1/3))/(b*x^3-a)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}}{\sqrt{bx^3 - a} \left(b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)-b^(1/3)*x)/(2*a^(1/3)+b^(1/3)*x)/(b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x - a^(1/3))/(sqrt(b*x^3 - a)*(b^(1/3)*x + 2*a^(1/3))), x)

mupad [B] time = 5.45, size = 74, normalized size = 1.40

$$\frac{\ln\left(\frac{\left(\sqrt{bx^3-a}+\sqrt{a}\right)\left(\sqrt{a}+2a^{1/6}b^{1/3}x+\sqrt{bx^3-a}\right)^3}{x^3\left(b^{1/3}x+2a^{1/3}\right)^3}\right)}{3a^{1/6}b^{1/3}}1i$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^(1/3)*x - a^(1/3))/((b^(1/3)*x + 2*a^(1/3))*(b*x^3 - a)^(1/2)),x)

[Out] (log((((b*x^3 - a)^(1/2) + a^(1/2)*1i)*((b*x^3 - a)^(1/2)*1i + a^(1/2) + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x + 2*a^(1/3))^3)*1i)/(3*a^(1/6)*b^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{\sqrt[3]{a}}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}\right)dx-\int\frac{\sqrt[3]{b}x}{2\sqrt[3]{a}\sqrt{-a+bx^3}+\sqrt[3]{b}x\sqrt{-a+bx^3}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)-b**(1/3)*x)/(2*a**(1/3)+b**(1/3)*x)/(b*x**3-a)**(1/2),x)

[Out] -Integral(-a**(1/3)/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x) - Integral(b**(1/3)*x/(2*a**(1/3)*sqrt(-a + b*x**3) + b**(1/3)*x*sqrt(-a + b*x**3)), x)

$$3.22 \quad \int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{(2\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=53

$$\frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.14, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2138, 203}

$$\frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (2*ArcTan[(a^(1/3) + b^(1/3)*x)^2/(3*a^(1/6)*Sqrt[-a - b*x^3])]/(3*a^(1/6)*b^(1/3)))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\int \frac{\sqrt[3]{a} + \sqrt[3]{b}x}{(2\sqrt[3]{a} - \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{9+ax^2} dx, x, \frac{\left(1 + \frac{\sqrt[3]{b}x}{\sqrt[3]{a}}\right)^2}{\sqrt{-a-bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tan^{-1} \left(\frac{(\sqrt[3]{a} + \sqrt[3]{b}x)^2}{3\sqrt[6]{a}\sqrt{-a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.02

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{a} \left(\frac{\sqrt[3]{b}x}{\sqrt[3]{a}} + 1 \right)^2}{3\sqrt{-a-bx^3}} \right)}{3\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (2*ArcTan[(Sqrt[a]*(1 + (b^(1/3)*x)/a^(1/3))^2]/(3*Sqrt[-a - b*x^3]))]/(3*a^(1/6)*b^(1/3))

IntegrateAlgebraic [A] time = 5.62, size = 66, normalized size = 1.25

$$\frac{2 \tan^{-1} \left(\frac{3 \sqrt[6]{a} \sqrt{-a-bx^3}}{a^{2/3} + 2 \sqrt[3]{a} \sqrt[3]{b} x + b^{2/3} x^2} \right)}{3 \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a^(1/3) + b^(1/3)*x)/((2*a^(1/3) - b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (-2*ArcTan[(3*a^(1/6)*Sqrt[-a - b*x^3])/(a^(2/3) + 2*a^(1/3)*b^(1/3)*x + b^(2/3)*x^2)]/(3*a^(1/6)*b^(1/3))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2), x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + 2a^{\frac{1}{3}}\right) \sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2), x)

[Out] int((b^(1/3)*x+a^(1/3))/(-b^(1/3)*x+2*a^(1/3))/(-b*x^3-a)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}}{\sqrt{-bx^3 - a} \left(b^{\frac{1}{3}}x - 2a^{\frac{1}{3}}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a^(1/3)+b^(1/3)*x)/(2*a^(1/3)-b^(1/3)*x)/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] -integrate((b^(1/3)*x + a^(1/3))/(sqrt(-b*x^3 - a)*(b^(1/3)*x - 2*a^(1/3))), x)

mupad [B] time = 5.37, size = 78, normalized size = 1.47

$$\frac{\ln\left(\frac{\left(\sqrt{-bx^3-a}-\sqrt{a}\right) \operatorname{li}\left(2a^{1/6}b^{1/3}x-\sqrt{a}+\sqrt{-bx^3-a}\right)^3}{x^3\left(b^{1/3}x-2a^{1/3}\right)^3}\right) \operatorname{li}}{3a^{1/6}b^{1/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(b^(1/3)*x + a^(1/3))/((b^(1/3)*x - 2*a^(1/3))*(- a - b*x^3)^(1/2)),x)

[Out] (log(((((- a - b*x^3)^(1/2) - a^(1/2)*1i)*((- a - b*x^3)^(1/2)*1i - a^(1/2) + 2*a^(1/6)*b^(1/3)*x)^3)/(x^3*(b^(1/3)*x - 2*a^(1/3))^3))*1i)/(3*a^(1/6)*b^(1/3))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt[3]{a}}{-2\sqrt[3]{a}\sqrt{-a-bx^3} + \sqrt[3]{b}x\sqrt{-a-bx^3}} dx - \int \frac{\sqrt[3]{b}x}{-2\sqrt[3]{a}\sqrt{-a-bx^3} + \sqrt[3]{b}x\sqrt{-a-bx^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a**(1/3)+b**(1/3)*x)/(2*a**(1/3)-b**(1/3)*x)/(-b*x**3-a)**(1/2), x)

[Out] -Integral(a**(1/3)/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x) - Integral(b**(1/3)*x/(-2*a**(1/3)*sqrt(-a - b*x**3) + b**(1/3)*x*sqrt(-a - b*x**3)), x)

$$3.23 \quad \int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}} dx$$

Optimal. Leaf size=46

$$-\frac{2 \tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Rubi [A] time = 0.12, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2138, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Int[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]), x]

[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2138

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[(-2*e)/d, Subst[Int[1/(9 - a*x^2), x], x, (1 + (f*x)/e)^2/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b*c^3 + 8*a*d^3, 0] && EqQ[2*d*e + c*f, 0]

Rubi steps

$$\begin{aligned} \int \frac{c-2dx}{(c+dx)\sqrt{c^3-8d^3x^3}} dx &= \frac{(2c) \text{Subst}\left(\int \frac{1}{9-c^3x^2} dx, x, \frac{\left(1-\frac{2dx}{c}\right)^2}{\sqrt{c^3-8d^3x^3}}\right)}{d} \\ &= -\frac{2 \tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d} \end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{(c-2dx)^2}{3\sqrt{c}\sqrt{c^3-8d^3x^3}}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] Integrate[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]), x]

[Out] (-2*ArcTanh[(c - 2*d*x)^2/(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])])/(3*Sqrt[c]*d)

IntegrateAlgebraic [A] time = 1.29, size = 44, normalized size = 0.96

$$\frac{2 \tanh^{-1}\left(\frac{3\sqrt{c}\sqrt{c^3-8d^3x^3}}{(c-2dx)^2}\right)}{3\sqrt{c}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c - 2*d*x)/((c + d*x)*Sqrt[c^3 - 8*d^3*x^3]), x]
 [Out] (-2*ArcTanh[(3*Sqrt[c]*Sqrt[c^3 - 8*d^3*x^3])/(c - 2*d*x)^2])/(3*Sqrt[c]*d)

fricas [B] time = 1.76, size = 294, normalized size = 6.39

$$\left[\frac{\log\left(\frac{8d^6x^6-240cd^5x^5+408c^2d^4x^4+88c^3d^3x^3+156c^4d^2x^2+12c^5dx+17c^6-3(8d^4x^4-52cd^3x^3+12c^2d^2x^2-4c^3dx+5c^4)\sqrt{-8d^3x^3+c^3}\sqrt{c}}{d^6x^6+6cd^5x^5+15c^2d^4x^4+20c^3d^3x^3+15c^4d^2x^2+6c^5dx+c^6}\right)}{6\sqrt{c}d}, \frac{\sqrt{-c} \arctan\left(\frac{(4d^3x^3-24cd^2x^2-6c^2dx-5c^3)\sqrt{-8d^3x^3+c^3}\sqrt{c}}{3(16cd^4x^4-8c^2d^3x^3-2c^4dx+c^5)}\right)}{3cd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2), x, algorithm="fricas")
 [Out] [1/6*log((8*d^6*x^6 - 240*c*d^5*x^5 + 408*c^2*d^4*x^4 + 88*c^3*d^3*x^3 + 156*c^4*d^2*x^2 + 12*c^5*d*x + 17*c^6 - 3*(8*d^4*x^4 - 52*c*d^3*x^3 + 12*c^2*d^2*x^2 - 4*c^3*d*x + 5*c^4)*sqrt(-8*d^3*x^3 + c^3)*sqrt(c))/(d^6*x^6 + 6*c*d^5*x^5 + 15*c^2*d^4*x^4 + 20*c^3*d^3*x^3 + 15*c^4*d^2*x^2 + 6*c^5*d*x + c^6))/(sqrt(c)*d), -1/3*sqrt(-c)*arctan(1/3*(4*d^3*x^3 - 24*c*d^2*x^2 - 6*c^2*d*x - 5*c^3)*sqrt(-8*d^3*x^3 + c^3)*sqrt(-c)/(16*c*d^4*x^4 - 8*c^2*d^3*x^3 - 2*c^4*d*x + c^5))/(c*d)]

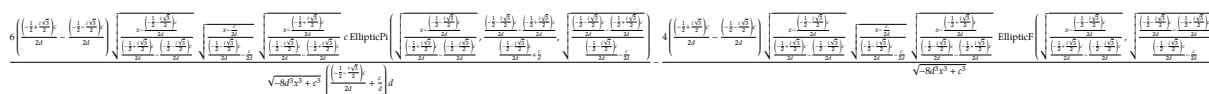
giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2dx - c}{\sqrt{-8d^3x^3 + c^3}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2), x, algorithm="giac")
 [Out] integrate(-(2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)

maple [C] time = 0.20, size = 650, normalized size = 14.13



Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2), x)
 [Out] -4*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^1/2*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^1/2*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^1/2/(-8*d^3*x^3+c^3)^(1/2)*EllipticF(((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^1/2, ((1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^1/2)+6*c/d*(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d)*((x-1/2*(-1/2-1/2*I*3^(1/2))*c/d)/(1/2*(-1/2+1/2*I*3^(1/2))*c/d-1/2*(-1/2-1/2*I*3^(1/2))*c/d))^1/2*((x-1/2*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*c/d))^1/2*((x-1/2*(-1/2+1/2*I*3^(1/2))*c/d)/(1/2*(-1/2-1/2*I*3^(1/2))*c/d-1/2*(-1/2+1/2*I*3^(1/2))*c/d))^1/2/(-8*d^3

$*x^3+c^3)^{(1/2)}/(1/2*(-1/2-1/2*I*3^{(1/2)})*c/d+c/d)*EllipticPi(((x-1/2*(-1/2-1/2*I*3^{(1/2)})*c/d)/(1/2*(-1/2+1/2*I*3^{(1/2)})*c/d-1/2*(-1/2-1/2*I*3^{(1/2)})*c/d))^{(1/2)},(1/2*(-1/2-1/2*I*3^{(1/2)})*c/d-1/2*(-1/2+1/2*I*3^{(1/2)})*c/d)/(1/2*(-1/2-1/2*I*3^{(1/2)})*c/d+c/d),((1/2*(-1/2-1/2*I*3^{(1/2)})*c/d-1/2*(-1/2+1/2*I*3^{(1/2)})*c/d)/(1/2*(-1/2-1/2*I*3^{(1/2)})*c/d-1/2*c/d))^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2dx - c}{\sqrt{-8d^3x^3 + c^3}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d^3*x^3+c^3)^(1/2),x, algorithm="maxima")

[Out] -integrate((2*d*x - c)/(sqrt(-8*d^3*x^3 + c^3)*(d*x + c)), x)

mupad [B] time = 3.13, size = 67, normalized size = 1.46

$$\frac{\ln\left(\frac{(\sqrt{c^3-8d^3x^3}-c^{3/2})(\sqrt{c^3-8d^3x^3}+c^{3/2}+4\sqrt{c}dx)^3}{x^3(c+dx)^3}\right)}{3\sqrt{c}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c - 2*d*x)/((c^3 - 8*d^3*x^3)^(1/2)*(c + d*x)),x)

[Out] log((((c^3 - 8*d^3*x^3)^(1/2) - c^(3/2))*((c^3 - 8*d^3*x^3)^(1/2) + c^(3/2) + 4*c^(1/2)*d*x)^3)/(x^3*(c + d*x)^3))/(3*c^(1/2)*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}}\right) dx - \int \frac{2dx}{c\sqrt{c^3-8d^3x^3}+dx\sqrt{c^3-8d^3x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*d*x+c)/(d*x+c)/(-8*d**3*x**3+c**3)**(1/2),x)

[Out] -Integral(-c/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x) - Integral(2*d*x/(c*sqrt(c**3 - 8*d**3*x**3) + d*x*sqrt(c**3 - 8*d**3*x**3)), x)

$$3.24 \quad \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{1 + x^3}} dx$$

Optimal. Leaf size=42

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Rubi [A] time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2140, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[-3 + 2*Sqrt[3]]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_)*(x_))/(((c_) + (d_)*(x_))*Sqrt[(a_) + (b_)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{1 + x^3}} dx = -\left(2 \text{Subst}\left(\int \frac{1}{1 + (3 - 2\sqrt{3})x^2} dx, x, \frac{1 + x}{\sqrt{1 + x^3}}\right)\right) = -\frac{2 \tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [C] time = 0.45, size = 267, normalized size = 6.36

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{5+3i}}}\left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}}\right)\right)+\sqrt{2ix+\sqrt{3}-i}\left((\sqrt{3}+(-2-i))x-i\sqrt{3}+(1+2i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}}\right)\right)\right)}{(-3+(2+i)\sqrt{3})\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^3+1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]

[Out] $(-2\sqrt{6}\sqrt{(I(1+x))/(3I+\sqrt{3})}(\sqrt{-I+\sqrt{3}+(2I)x}((1+2I)-I\sqrt{3}+((-2-I)+\sqrt{3})x)*\text{EllipticF}[\text{ArcSin}[\sqrt{I+\sqrt{3}-(2I)x}/(\sqrt{2}*3^{(1/4)})],(2\sqrt{3})/(3I+\sqrt{3})]+(4I)*\sqrt{I+\sqrt{3}-(2I)x}*\sqrt{1-x+x^2}*\text{EllipticPi}[\frac{(2I)\sqrt{3}}{-3+(2+I)\sqrt{3}},\text{ArcSin}[\sqrt{I+\sqrt{3}-(2I)x}/(\sqrt{2}*3^{(1/4)})}],(2\sqrt{3})/(3I+\sqrt{3})]))/((-3+(2+I)\sqrt{3})*\sqrt{I+\sqrt{3}-(2I)x}*\sqrt{1+x^3})$

IntegrateAlgebraic [A] time = 2.03, size = 53, normalized size = 1.26

$$-2\sqrt{\frac{1}{3}(3+2\sqrt{3})} \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3+1}}{x^2-x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[1 + x^3]),x]

[Out] $-2\sqrt{(3+2\sqrt{3})/3}*\text{ArcTanh}[(\sqrt{-3+2\sqrt{3}})*\sqrt{1+x^3}]/(1-x+x^2)$

fricas [B] time = 1.73, size = 205, normalized size = 4.88

$$\frac{1}{6}\sqrt{2\sqrt{3}+3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2-4(2x^6-18x^5+42x^4-8x^3-\sqrt{3}(x^6-12x^5+18x^4-16x^3-12x^2-8)+24x+8)\sqrt{x^3+1}\sqrt{2\sqrt{3}+3}+16\sqrt{3}(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4)+128x+112}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] $1/6*\sqrt{3}*\sqrt{2*\sqrt{3}+3}*\log((x^8-16*x^7+112*x^6-16*x^5+112*x^4+224*x^3+64*x^2-4*(2*x^6-18*x^5+42*x^4-8*x^3-\sqrt{3}*(x^6-12*x^5+18*x^4-16*x^3-12*x^2-8)+24*x+8)*\sqrt{x^3+1}*\sqrt{2*\sqrt{3}+3}+16*\sqrt{3}*(x^7-2*x^6+6*x^5+5*x^4+2*x^3+6*x^2+4*x+4)+128*x+112)/(x^8+8*x^7+16*x^6-16*x^5-56*x^4+32*x^3+64*x^2-64*x+16))$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x);OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%} / %%{%%{[-2,4]:[1,0,-3]%%},[2]%%} Error: Bad Argument Value

maple [C] time = 0.08, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3-i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3-i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3-i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\right)+4\left(\frac{3-i\sqrt{3}}{2}\right)\sqrt{\frac{x+1}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x-\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\text{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3-i\sqrt{3}}{2}}},\frac{\left(-\frac{3+i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3-i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\right)}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x)

[Out] $2*(3/2-1/2*I*3^{(1/2)})*((x+1)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))$

)^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(-3/2+1/2*I*3^(1/2))*3^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{x^3 + 1}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(x^3 + 1)*(x - sqrt(3) + 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x - 3^(1/2) + 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{\sqrt{(x + 1)(x^2 - x + 1)}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(x**3+1)**(1/2),x)

[Out] Integral((x + 1 + sqrt(3))/(sqrt((x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)

$$3.25 \quad \int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x)\sqrt{1 - x^3}} dx$$

Optimal. Leaf size=46

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Rubi [A] time = 0.11, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]), x]

[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[-3 + 2*Sqrt[3]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x)\sqrt{1 - x^3}} dx = 2 \text{Subst} \left(\int \frac{1}{1 + (3 - 2\sqrt{3})x^2} dx, x, \frac{1 - x}{\sqrt{1 - x^3}} \right) \\ = \frac{2 \tanh^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{-3 + 2\sqrt{3}}}$$

Mathematica [C] time = 0.49, size = 269, normalized size = 5.85

$$\frac{2\sqrt{6}\sqrt{\frac{i(x-1)}{\sqrt{3-3i}}}\left(4\sqrt{-2ix+\sqrt{3}-i}\sqrt{x^2+x+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt{3}}\right)\right)+\sqrt{2ix+\sqrt{3}+i}\left((1+2i)-i\sqrt{3}\right)x-\sqrt{3}+(2+i)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}-i}}{\sqrt{2}\sqrt{3}}\right)\right)\right)}{(1+2i)\sqrt{3}-3i}\sqrt{-2ix+\sqrt{3}-i}\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*S
qrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I
+ (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))
], (2*Sqrt[3])/(-3*I + Sqrt[3])])/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sq
rt[3] - (2*I)*x]*Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 2.15, size = 53, normalized size = 1.15

$$2\sqrt{\frac{1}{3}(3 + 2\sqrt{3})} \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3} - 3}\sqrt{1 - x^3}}{x^2 + x + 1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] 2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 +
x + x^2)]
```

fricas [B] time = 1.57, size = 207, normalized size = 4.50

$$\frac{1}{6}\sqrt{2\sqrt{3} + 3} \log\left(\frac{x^8 + 16x^7 + 112x^6 + 16x^5 + 112x^4 - 224x^3 + 64x^2 + 4(2x^6 + 18x^5 + 42x^4 + 8x^3 - \sqrt{3}(x^6 + 12x^5 + 18x^4 + 16x^3 - 12x^2 - 8) - 24x + 8)\sqrt{-x^3 + 1}\sqrt{2\sqrt{3} + 3} - 16\sqrt{3}(x^7 + 2x^6 + 6x^5 - 5x^4 + 2x^3 - 6x^2 + 4x - 4) - 128x + 112}{x^8 - 8x^7 + 16x^6 + 16x^5 - 56x^4 - 32x^3 + 64x^2 + 64x + 16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2)))/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) + 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*
x^4 - 224*x^3 + 64*x^2 + 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 - sqrt(3)*(x^6
+ 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(-x^3 + 1)*sqrt(2*
sqrt(3) + 3) - 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*
x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64
*x^2 + 64*x + 16))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2)))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding er
ror%%{%%{[1,1]:[1,0,-3]%%}, [2]%%}% / %%{%%{[-2,4]:[1,0,-3]%%}, [2]%%}% Err
or: Bad Argument Value
```

maple [C] time = 0.08, size = 243, normalized size = 5.28

$$\frac{2i\sqrt{3}\sqrt{i\left(x + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)}\sqrt{3}\sqrt{\frac{x-1}{\frac{3}{2} + i\sqrt{3}}}\sqrt{-i\left(x + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)}\sqrt{3}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{\left(x + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)}\sqrt{3}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + i\sqrt{3}}}\right) + 4i\sqrt{i\left(x + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)}\sqrt{3}\sqrt{\frac{x-1}{\frac{3}{2} + i\sqrt{3}}}\sqrt{-i\left(x + \frac{1}{2} + \frac{i\sqrt{3}}{2}\right)}\sqrt{3}\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\left(x + \frac{1}{2} - \frac{i\sqrt{3}}{2}\right)}\sqrt{3}}{3}, \frac{i\sqrt{3}}{-\frac{3}{2} + \sqrt{3} + \frac{i\sqrt{3}}{2}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2} + i\sqrt{3}}}\right)}{3\sqrt{-x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x+3^(1/2))/(1-x-3^(1/2)))/(-x^3+1)^(1/2),x)
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
```

```
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2)+4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-
3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1
)^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I
*3^(1/2))*3^(1/2))^(1/2), I*3^(1/2)/(-3/2+3^(1/2)+1/2*I*3^(1/2)), (I*3^(1/2)/
(-3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{-x^3 + 1} (x + \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x + sqrt(3) - 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(3^(1/2) - x + 1)/((1 - x^3)^(1/2)*(x + 3^(1/2) - 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{-(x-1)(x^2+x+1)} (x-1+\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2))/(-x**3+1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) - 1)/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))
), x)
```

$$3.26 \quad \int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x)\sqrt{-1 + x^3}} dx$$

Optimal. Leaf size=44

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Rubi [A] time = 0.10, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(1-x)}{\sqrt{x^3-1}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[-3 + 2*Sqrt[3]]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_)*(x_))/(((c_) + (d_)*(x_))*Sqrt[(a_) + (b_)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} - x}{(1 - \sqrt{3} - x)\sqrt{-1 + x^3}} dx = 2 \text{Subst} \left(\int \frac{1}{1 - (3 - 2\sqrt{3})x^2} dx, x, \frac{1 - x}{\sqrt{-1 + x^3}} \right) \\ = \frac{2 \tan^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}}(1-x)}{\sqrt{-1 + x^3}} \right)}{\sqrt{-3 + 2\sqrt{3}}}$$

Mathematica [C] time = 0.38, size = 267, normalized size = 6.07

$$\frac{2\sqrt{6} \sqrt{\frac{i(x-1)}{\sqrt{3}-3i}} \left(4\sqrt{-2ix + \sqrt{3} - i}\sqrt{x^2 + x + 1} \Pi \left(\frac{2\sqrt{3}}{-3i + (1+2i)\sqrt{3}}; \sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \Big|_{-3i + \sqrt{3}} \right) + \sqrt{2ix + \sqrt{3} + i} \left((1+2i) - i\sqrt{3} \right) x - \sqrt{3} + (2+i) F \left(\sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \Big|_{-3i + \sqrt{3}} \right) \right)}{(1+2i)\sqrt{3} - 3i} \sqrt{-2ix + \sqrt{3} - i}\sqrt{x^3 - 1}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x] * ((2 + I) - Sqrt[3] + ((1 + 2*I) - I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] + 4*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((-3*I + (1 + 2*I)*Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[-1 + x^3])

IntegrateAlgebraic [A] time = 2.10, size = 51, normalized size = 1.16

$$-2\sqrt{\frac{1}{3}(3+2\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] - x)/((1 - Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] -2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-3 + 2*Sqrt[3])*Sqrt[-1 + x^3))/(1 + x + x^2)]

fricas [A] time = 1.41, size = 50, normalized size = 1.14

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}+3} \arctan\left(\frac{(\sqrt{3}(x^2+4x-2)-6x+6)\sqrt{2\sqrt{3}+3}}{6\sqrt{x^3-1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*(sqrt(3)*(x^2 + 4*x - 2) - 6*x + 6)*sqrt(2*sqrt(3) + 3)/sqrt(x^3 - 1))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[1,1]:[1,0,-3]%%},[2]%%}% / %%{%%{[-2,4]:[1,0,-3]%%},[2]%%}% Error: Bad Argument Value

maple [C] time = 0.06, size = 245, normalized size = 5.57

$$\frac{2\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\sqrt{\frac{\frac{3+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\right)-4\left(-\frac{3}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}-\frac{i\sqrt{3}}{2}}}\sqrt{\frac{\frac{1+i\sqrt{3}}{2}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3}{2}-\frac{i\sqrt{3}}{2}}},\frac{\left(\frac{3+i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{\frac{3+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\right)}{\sqrt{x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x+3^(1/2))/(1-x-3^(1/2))/(x^3-1)^(1/2),x)

[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)))/(x^3-1)^(1/2)

$(2-1/2*I*3^{(1/2)})^{(1/2)}*((x+1/2-1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x+1/2+1/2*I*3^{(1/2)})/(3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3-1)^{(1/2)}*EllipticPi((x-1)/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}, 1/3*(3/2+1/2*I*3^{(1/2)})*3^{(1/2)}, ((3/2+1/2*I*3^{(1/2)})/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{x^3 - 1}(x + \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3^(1/2))/(1-x-3^(1/2)))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(3) - 1)/(sqrt(x^3 - 1)*(x + sqrt(3) - 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3^(1/2) - x + 1)/((x^3 - 1)^(1/2)*(x + 3^(1/2) - 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} - 1}{\sqrt{(x-1)(x^2+x+1)}(x-1+\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x+3**(1/2))/(1-x-3**(1/2)))/(x**3-1)**(1/2),x)

[Out] Integral((x - sqrt(3) - 1)/(sqrt((x - 1)*(x**2 + x + 1))*(x - 1 + sqrt(3))), x)

$$3.27 \quad \int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-1 - x^3}} dx$$

Optimal. Leaf size=44

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2140, 203}

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}(x+1)}{\sqrt{-x^3-1}} \right)}{\sqrt{2\sqrt{3}-3}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]), x]

[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[-3 + 2*Sqrt[3]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} + x}{(1 - \sqrt{3} + x)\sqrt{-1 - x^3}} dx = - \left(2 \text{Subst} \left(\int \frac{1}{1 - (3 - 2\sqrt{3})x^2} dx, x, \frac{1+x}{\sqrt{-1-x^3}} \right) \right) \\ = - \frac{2 \tan^{-1} \left(\frac{\sqrt{-3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}} \right)}{\sqrt{-3+2\sqrt{3}}}$$

Mathematica [C] time = 0.38, size = 269, normalized size = 6.11

$$\frac{2\sqrt{6} \sqrt{\frac{i(x+1)}{\sqrt{3}+3i}} \left(4i\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1} \Pi \left(\frac{2i\sqrt{3}}{-3+(2+i)\sqrt{3}}, \sin^{-1} \left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}} \right) \right) \frac{2\sqrt{3}}{3i+\sqrt{3}} + \sqrt{2ix+\sqrt{3}-i} \left((\sqrt{3}+(-2-i)x-i\sqrt{3}+(1+2i)) F \left(\sin^{-1} \left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt{3}} \right) \right) \frac{2\sqrt{3}}{3i+\sqrt{3}} \right) \right)}{(-3+(2+i)\sqrt{3})\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]

[Out] (-2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x] * ((1 + 2*I) - I*Sqrt[3] + ((-2 - I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + (4*I)*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(-3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])))/((-3 + (2 + I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])

IntegrateAlgebraic [A] time = 2.09, size = 55, normalized size = 1.25

$$2\sqrt{\frac{1}{3}(3+2\sqrt{3})} \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3}\sqrt{-x^3-1}}{x^2-x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] + x)/((1 - Sqrt[3] + x)*Sqrt[-1 - x^3]),x]

[Out] 2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x + x^2)]

fricas [A] time = 0.93, size = 59, normalized size = 1.34

$$\frac{1}{3} \sqrt{3} \sqrt{2\sqrt{3}+3} \arctan \left(\frac{\sqrt{-x^3-1} (\sqrt{3}(x^2-4x-2) + 6x+6) \sqrt{2\sqrt{3}+3}}{6(x^3+1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) + 3)*arctan(1/6*sqrt(-x^3 - 1)*(sqrt(3)*(x^2 - 4*x - 2) + 6*x + 6)*sqrt(2*sqrt(3) + 3)/(x^3 + 1))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[-1,-1]:[1,0,-3]%%},[2]%%} / %%{%%{[-2,4]:[1,0,-3]%%},[2]%%} E rror: Bad Argument Value

maple [C] time = 0.07, size = 247, normalized size = 5.61

$$\frac{2i\sqrt{3} \sqrt{i(x-\frac{1}{2}-\frac{i\sqrt{3}}{2})\sqrt{3}} \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}} \sqrt{-i(x-\frac{1}{2}+\frac{i\sqrt{3}}{2})\sqrt{3}} \operatorname{EllipticF}\left(\frac{\sqrt{3} \sqrt{(x-\frac{1}{2}-\frac{i\sqrt{3}}{2})\sqrt{3}}}{3}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right) - 4i \sqrt{i(x-\frac{1}{2}-\frac{i\sqrt{3}}{2})\sqrt{3}} \sqrt{\frac{x+1}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}} \sqrt{-i(x-\frac{1}{2}+\frac{i\sqrt{3}}{2})\sqrt{3}} \operatorname{EllipticPi}\left(\frac{\sqrt{3} \sqrt{(x-\frac{1}{2}-\frac{i\sqrt{3}}{2})\sqrt{3}}}{3}, \frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}}, \sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{\sqrt{-x^3-1} \left(\frac{3}{2} + \frac{i\sqrt{3}}{2} - \sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x)

[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)/(-x^3-1)^(1/2)*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2), I*3^(1/2)/(3/2+1/2*I*3^(1/2)-sqrt(3)), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

$2+1/2*I*3^{(1/2)})^{(1/2)}*(-I*(x-1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)/(-x^3-1)^{(1/2)/(3/2+1/2*I*3^{(1/2)}-3^{(1/2)})}*EllipticPi(1/3*3^{(1/2)}*(I*(x-1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},I*3^{(1/2)/(3/2+1/2*I*3^{(1/2)}-3^{(1/2)}), (I*3^{(1/2)/(3/2+1/2*I*3^{(1/2)}))^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} + 1}{\sqrt{-x^3 - 1}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3^(1/2))/(1+x-3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x - sqrt(3) + 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x - 3^(1/2) + 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 1 + \sqrt{3}}{\sqrt{-(x + 1)(x^2 - x + 1)}(x - \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+3**(1/2))/(1+x-3**(1/2))/(-x**3-1)**(1/2),x)

[Out] Integral((x + 1 + sqrt(3))/(sqrt(-(x + 1)*(x**2 - x + 1))*(x - sqrt(3) + 1)), x)

$$3.28 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=69

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.20, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst}\left(\int \frac{1}{1+(3-2\sqrt{3})ax^2} dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}(\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}}$$

Mathematica [C] time = 0.63, size = 322, normalized size = 4.67

$$2\sqrt{\frac{\sqrt[3]{a+\sqrt[3]{b}x}}{(1+\sqrt[3]{-1})\sqrt[3]{a}} \left(\frac{4(-1)^{5/6}(1+\sqrt[3]{-1})\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}+1}\Pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\sqrt[3]{-1}}\right)}{(1+2i)\sqrt{3-3i}\sqrt[3]{b}} - \frac{(\sqrt[3]{-1}\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{\sqrt[3]{-1}-\frac{i\sqrt[3]{b}x}{\sqrt[3]{a}}}\operatorname{F}\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\sqrt[3]{-1}}\right)}{\sqrt[3]{3}\sqrt[3]{b}\sqrt{\frac{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}} \right)}{\sqrt{a+bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)))/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)))/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)))/Sqrt[a + b*x^3]

IntegrateAlgebraic [A] time = 12.68, size = 115, normalized size = 1.67

$$\frac{2\sqrt{\frac{1}{3}(3+2\sqrt{3})} \tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}b^{2/3}x^2}}{\sqrt[6]{a}} - \sqrt{1+\frac{2}{\sqrt{3}}}\sqrt[6]{a}\sqrt[3]{b}x + \sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{a}}{\sqrt{a+bx^3}}\right)}{\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)])/Sqrt[a + b*x^3])/(a^(1/6)*b^(1/3))

fricas [A] time = 4.52, size = 1240, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a))*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x)))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 29

$20a^2b^6x^{18} + 24864a^3b^5x^{15} - 26576a^4b^4x^{12} - 56000a^5b^3x^9 - 115968a^6b^2x^6 - 56320a^7bx^3 - 1024a^8)$
 $\cdot b^{1/3} \cdot \sqrt{(2\sqrt{3} + 3)/(ab^{2/3})} + 32(9b^7x^{22} - 846a^2b^6x^{19} + 4617a^3b^5x^{16} + 5472a^4b^4x^{13} + 43776a^5b^3x^{10} + 98496a^6b^2x^7 + 59328a^7bx^4 + 4608a^8)$
 $\cdot \sqrt{3} \cdot (5b^7x^{22} - 505a^2b^6x^{19} + 2130a^3b^5x^{16} - 4928a^4b^4x^{13} - 28688a^5b^3x^{10} - 53760a^6b^2x^7 - 35200a^7bx^4 - 2560a^8)$
 $\cdot a^{2/3} \cdot b^{1/3} - 8(3b^7x^{23} - 1077a^2b^6x^{20} + 13320a^3b^5x^{17} - 19200a^4b^4x^{14} - 111360a^5b^3x^{11} - 345024a^6b^2x^8 - 328704a^7bx^5 - 61440a^8)$
 $\cdot 2\sqrt{3} \cdot (b^7x^{23} - 299a^2b^6x^{20} + 4260a^3b^5x^{17} + 1520a^4b^4x^{14} + 26720a^5b^3x^{11} + 105024a^6b^2x^8 + 93184a^7bx^5 + 17920a^8)$
 $\cdot a^{1/3} \cdot b^{2/3} + 32\sqrt{3} \cdot (35a^2b^7x^{21} - 1141a^3b^6x^{18} + 2544a^4b^5x^{15} + 6760a^5b^4x^{12} + 39520a^6b^3x^9 + 55680a^7b^2x^6 + 19712a^8)$
 $\cdot \sqrt{1/3} \cdot a^{1/3} \cdot \sqrt{-(2\sqrt{3} + 3)/(ab^{2/3})} \cdot \arctan(1/2 \cdot \sqrt{1/3} \cdot (a^{1/3} \cdot bx^2 + 2(\sqrt{3}x - 2x) \cdot a^{2/3} \cdot b^{2/3} + 2(\sqrt{3} \cdot a - a) \cdot b^{1/3})) \cdot \sqrt{-(2\sqrt{3} + 3)/(ab^{2/3})} / \sqrt{bx^3 + a}$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)

[Out] int((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 + a} \left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} - 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{a + bx^3} (-\sqrt{3} \sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

$$3.29 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=71

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Rubi [A] time = 0.19, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}}\right)}{\sqrt{2\sqrt{3}-3}\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\begin{aligned} \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{a-bx^3}} dx &= \frac{(2\sqrt[3]{a}) \text{Subst}\left(\int \frac{1}{1+(3-2\sqrt{3})ax^2} dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{b}} \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}(\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}}\right)}{\sqrt{-3+2\sqrt{3}}\sqrt[6]{a}\sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 1.44, size = 446, normalized size = 6.28

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{\frac{2i\sqrt[3]{a}+(\sqrt{3}+i)\sqrt[3]{bx}}{(\sqrt{3}-3i)\sqrt[3]{a}}}\sqrt{\frac{a^{2/3}x^2+\sqrt[3]{a}}{a^{2/3}}}\right)+11\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{i(1-i\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{a}}{(-3i+\sqrt{3})\sqrt[3]{a}}}\right)\right)^{\frac{1}{2}}(1+i\sqrt{3})+\sqrt{\frac{(\sqrt{3}-i)\sqrt[3]{a}+(\sqrt{3}+i)\sqrt[3]{bx}}{(\sqrt{3}-3i)\sqrt[3]{a}}}\left((-3+(2+i)\sqrt{3})\sqrt[3]{a}+(1+2i)\sqrt{3}-3i\right)\sqrt[3]{bx}F\left(\sin^{-1}\left(\sqrt{\frac{i(1-i\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{a}}{(-3i+\sqrt{3})\sqrt[3]{a}}}\right)\right)^{\frac{1}{2}}(1+i\sqrt{3})\right)}{(1+2i)\sqrt{3}-3i}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{a-bx^3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3]))*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3]))*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3],x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[(-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x]/((-3*I + Sqrt[3])*a^(1/3)))*((-3 + (2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-((2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2))/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[a - b*x^3])
```

IntegrateAlgebraic [A] time = 12.59, size = 115, normalized size = 1.62

$$\frac{2\sqrt{\frac{1}{3}(3 + 2\sqrt{3})} \tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}b^{2/3}x^2}}{\sqrt[3]{a}} + \sqrt{1+\frac{2}{\sqrt{3}}\sqrt[3]{a}\sqrt[3]{bx}} + \sqrt{1+\frac{2}{\sqrt{3}}\sqrt{a}}}{\sqrt{a-bx^3}}\right)}{\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3]))*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3]))*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3],x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)]/Sqrt[a - b*x^3])/(a^(1/6)*b^(1/3))
```

fricas [B] time = 4.32, size = 1294, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x)))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x
```

```
x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3
+ a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
+ 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18
+ 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 + a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3
+ a)*(sqrt(3)*a - a)*b^(1/3))*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 - a
))]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x
^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 + a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) - 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{-\sqrt{3} \sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{a - bx^3} (-\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

$$3.30 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=72

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{bx^3-a}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{bx^3-a}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]

[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{-a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1-(3-2\sqrt{3})ax^2} dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tan^{-1} \left(\frac{\sqrt{-3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{-a+bx^3}} \right)}{\sqrt{-3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.43, size = 447, normalized size = 6.21

$$\frac{2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\left(4\sqrt{3}\sqrt[3]{a}\sqrt{\frac{2i\sqrt[3]{a}+(\sqrt{3}+i)\sqrt[3]{bx}}{(\sqrt{3}-3i)\sqrt[3]{a}}}\sqrt{\frac{a^{2/3}x^2+\sqrt[3]{a}x}{a}}+11\pi\left(\frac{2\sqrt{3}}{-3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{i(1-i\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{a}}{(-3i+\sqrt{3})\sqrt[3]{a}}}\right)\right)^{\frac{1}{2}}(1+i\sqrt{3})\right)+\sqrt{\frac{(\sqrt{3}-i)\sqrt[3]{a}+(\sqrt{3}+i)\sqrt[3]{bx}}{(\sqrt{3}-3i)\sqrt[3]{a}}}\left((-3+(2+i)\sqrt{3})\sqrt[3]{a}+(1+2i)\sqrt{3}-3i\right)\sqrt[3]{bx}F\left(\sin^{-1}\left(\sqrt{\frac{i(1-i\sqrt{3})\sqrt[3]{a}+2\sqrt[3]{a}}{(-3i+\sqrt{3})\sqrt[3]{a}}}\right)\right)^{\frac{1}{2}}(1+i\sqrt{3})\right)}{(1+2i)\sqrt{3}-3i}\sqrt[3]{a}\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\sqrt{bx^3-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + Sqrt[3]))*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3]))*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(Sqrt[(-I + Sqrt[3])*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x]/((-3*I + Sqrt[3])*a^(1/3)))*((-3 + (2 + I)*Sqrt[3])*a^(1/3) + (-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*x)*EllipticF[ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2] + 4*Sqrt[3]*a^(1/3)*Sqrt[-((2*I)*a^(1/3) + (I + Sqrt[3])*b^(1/3)*x)/((-3*I + Sqrt[3])*a^(1/3))]*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[((-I)*(2*a^(1/3) + (1 - I*Sqrt[3])*b^(1/3)*x))/((-3*I + Sqrt[3])*a^(1/3))]], (1 + I*Sqrt[3])/2))/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)*Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*Sqrt[-a + b*x^3)]
```

IntegrateAlgebraic [A] time = 12.45, size = 116, normalized size = 1.61

$$\frac{2\sqrt{\frac{1}{3}(3 + 2\sqrt{3})} \tan^{-1}\left(\frac{\sqrt{1 + \frac{2}{\sqrt{3}}b^{2/3}x^2}}{\sqrt[3]{a}} + \sqrt{1 + \frac{2}{\sqrt{3}}\sqrt[3]{a}}\sqrt[3]{b}x + \sqrt{1 + \frac{2}{\sqrt{3}}\sqrt{a}}}{\sqrt{bx^3 - a}}\right)}{\sqrt[3]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[((1 + Sqrt[3]))*a^(1/3) - b^(1/3)*x)/(((1 - Sqrt[3]))*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]),x]
```

```
[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)]/Sqrt[-a + b*x^3])/(a^(1/6)*b^(1/3))
```

fricas [A] time = 4.25, size = 1245, normalized size = 17.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x - 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2
```

```

920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) + 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a
^6*b*x^4 - 4608*a^7*x - sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*
x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 + 2560*a^7*x))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 +
13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 + 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sq
rt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/
(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 +
4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*(sqrt
(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/sqrt(b*x^3 - a))]

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="giac")

```

[Out] Timed out

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x
^3-a)^(1/2),x)

```

```

[Out] int((-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b*x
^3-a)^(1/2),x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))
/(b*x^3-a)^(1/2),x, algorithm="maxima")

```

```

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x +
a^(1/3)*(sqrt(3) - 1))), x)

```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3} \sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a + bx^3} (-\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

$$3.31 \quad \int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=72

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx = -\frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1-(3-2\sqrt{3})ax^2} dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}} \right)}{\sqrt[3]{b}}$$

$$= -\frac{2 \tan^{-1} \left(\frac{\sqrt{-3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}} \right)}{\sqrt{-3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.53, size = 325, normalized size = 4.51

$$2\sqrt{\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \left(\frac{4(-1)^{5/6}(1 + \sqrt[3]{-1})\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}} - \frac{\sqrt[3]{bx}}{\sqrt[3]{a}}} + 1 \Pi\left(\frac{2\sqrt{3}}{-3i + (1+2i)\sqrt{3}}; \sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}\right)\sqrt[3]{-1}\right)}{(1+2i)\sqrt{3}-3i}\sqrt[3]{b} - \frac{(\sqrt[3]{-1}\sqrt[3]{a} - \sqrt[3]{bx})\sqrt{\sqrt[3]{-1} - \frac{i\sqrt[3]{bx}}{\sqrt[3]{a}}}}{\sqrt[3]{a}} F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}\right)\sqrt[3]{-1}\right)}{\sqrt[3]{a}} \right)}{\sqrt{-a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(-1)^(5/6)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[(2*Sqrt[3])/(-3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/((-3*I + (1 + 2*I)*Sqrt[3])*b^(1/3)))/Sqrt[-a - b*x^3]

IntegrateAlgebraic [A] time = 12.39, size = 118, normalized size = 1.64

$$2\sqrt{\frac{1}{3}}(3 + 2\sqrt{3}) \tan^{-1} \left(\frac{\frac{\sqrt{1 + \frac{2}{\sqrt{3}} b^{2/3} x^2}}{\sqrt[3]{a}} - \sqrt{1 + \frac{2}{\sqrt{3}} \sqrt[3]{a} \sqrt[3]{bx}} + \sqrt{1 + \frac{2}{\sqrt{3}} \sqrt{a}}}{\sqrt{-a - bx^3}}}{\sqrt[3]{a} \sqrt[3]{b}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)]/Sqrt[-a - b*x^3])/(a^(1/6)*b^(1/3))

fricas [B] time = 3.78, size = 1303, normalized size = 18.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) + 3)/(a*b^(2/3)))*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x - sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 - 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x - 2*sqrt(3)*(b^7

```

*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^
4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^
3 - a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b
^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 1382
4*a^7*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^1
4 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*
x^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 446
40*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x
^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^1
8 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a
^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt(-(2
*sqrt(3) + 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18
+ 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2
*x^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6
*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151
552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*s
qrt(3) + 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*x
^2 + 2*sqrt(-b*x^3 - a)*(sqrt(3)*x - 2*x)*a^(2/3)*b^(2/3) + 2*sqrt(-b*x^3 -
a)*(sqrt(3)*a - a)*b^(1/3))*sqrt((2*sqrt(3) + 3)/(a*b^(2/3)))/(b*x^3 + a)
]

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-
-b*x^3-a)^(1/2),x, algorithm="giac")

```

[Out] Timed out

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)

```

```

[Out] int((b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} - 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-
-b*x^3-a)^(1/2),x, algorithm="maxima")

```

```

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) + 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
- a^(1/3)*(sqrt(3) - 1))), x)

```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a - bx^3} (-\sqrt{3} \sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)), x)
```

$$3.32 \quad \int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a + bx^3}} dx$$

Optimal. Leaf size=73

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1 \right)}{\sqrt{a+bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1 \right)}{\sqrt{a+bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a + bx^3}} dx = -\frac{2 \text{Subst} \left(\int \frac{1}{1 + (3 - 2\sqrt{3})ax^2} dx, x, \frac{1 + \sqrt[3]{\frac{b}{a}} x}{\sqrt{a + bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = -\frac{2 \tanh^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(1 + \sqrt[3]{\frac{b}{a}} x \right)}{\sqrt{a + bx^3}} \right)}{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.28, size = 663, normalized size = 9.08

$$\frac{\left(\frac{3 \left(1080 \sqrt{a} \operatorname{F}_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) - 18176 \sqrt{a} \operatorname{F}_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) + 3 \sqrt{a} \operatorname{F}_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) \right) \sqrt{\frac{a}{a+b^2}} \operatorname{F}_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) + 12 (\sqrt{3}-3) \sqrt{\frac{a}{a+b^2}} \operatorname{F}_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) - 8 a^2 \left(\frac{1}{2}, \frac{1}{2}, 2, \frac{a^2}{4 \sqrt{a} (a+b^2)} \right) \right)}{24 (3\sqrt{3}-5) \sqrt{a+b^2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]
```

```
[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))])))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a - 6*Sqrt[3]*a))]))))/(24*(-5 + 3*Sqrt[3])*Sqrt[a + b*x^3])
```

IntegrateAlgebraic [A] time = 12.79, size = 90, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}(3 + 2\sqrt{3})} \sqrt{\frac{b}{a}} \tanh^{-1} \left(\frac{\sqrt{1 + \frac{2}{\sqrt{3}}} \sqrt{b} \sqrt{\frac{b}{a}} \sqrt{a + bx^3}}{a \left(\frac{b}{a} \right)^{2/3} + bx}} \right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]
```

```
[Out] (-2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[1 + 2/Sqrt[3]])*Sqrt[b]*(b/a)^(1/6)*Sqrt[a + b*x^3]/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]
```

fricas [A] time = 2.66, size = 1273, normalized size = 17.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2), x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) - 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^
```

```

5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 8601
6*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6
- 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt((2*sqrt(3)
) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x
^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*
b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a - a)
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.41index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.20, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}\right) \sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

```
[Out] int((1+(1/a*b)^(1/3)*x+3^(1/2))/(1+(1/a*b)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{bx^3 + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3) + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} + x \left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{bx^3 + a} \left(x \left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/2) + 1)),x)

[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((a + b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/2) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}}{\sqrt{a + bx^3} \left(x^3 \sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(b*x**3+a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) - sqrt(3) + 1)), x)

$$3.33 \quad \int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a - bx^3}} dx$$

Optimal. Leaf size=75

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.20, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*ArcTanh[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a - bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1 + (3 - 2\sqrt{3})ax^2} dx, x, \frac{1 - \sqrt[3]{\frac{b}{a}} x}{\sqrt{a - bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tanh^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(1 - \sqrt[3]{\frac{b}{a}} x\right)}{\sqrt{a - bx^3}} \right)}{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.18, size = 648, normalized size = 8.64

$$\frac{\left(\frac{3 \left(10080 \sqrt{3} F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) - 18176 \sqrt{3} F_1 \left(\frac{1}{2}, 1, \frac{b^2}{10a+b\sqrt{3}} \right) + 9216 \sqrt{3} F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) \right) \sqrt{1 - \frac{bx^3}{a}} \left(4 \sqrt{3} - 9 \right) F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) + 3 \sqrt{3} F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) + (9 - 3\sqrt{3}) F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) + (9 - 3\sqrt{3}) F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) \right)}{24(3\sqrt{3} - 9) \sqrt{a - bx^3}} - 12(\sqrt{3} - 3) \sqrt{b} \sqrt{1 - \frac{bx^3}{a}} F_1 \left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b\sqrt{3}} \right) - 8x^2 \left(\frac{1}{2} \right)^{2/3} \sqrt{3 - \frac{3bx^3}{a}} F_1 \left(\frac{1}{2}, 1, \frac{b^2}{10a+b\sqrt{3}} \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (x*(-12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - b*x^3*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))/(a*(2*(-5 + 3*Sqrt[3])*a + b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)] + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*Sqrt[3]*a)])))/(24*(-5 + 3*Sqrt[3])*Sqrt[a - b*x^3])

IntegrateAlgebraic [A] time = 12.62, size = 92, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}(3+2\sqrt{3})} \sqrt{\frac{6}{a}} \tanh^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b} \sqrt{\frac{6}{a}} \sqrt{a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[1 + 2/Sqrt[3]])*Sqrt[b]*(b/a)^(1/6)*Sqrt[a - b*x^3])/(a*(b/a)^(2/3) - b*x)]/Sqrt[b]

fricas [B] time = 2.85, size = 1330, normalized size = 17.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/((1-(b/a)^(1/3)*x-3^(1/2)))/(-b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 - 7168*a^8*x)))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14

```

4 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 - sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 + a))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(
a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
- 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 - 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*b*x^2 - 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 + a)*(sqrt
(3)*a - a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.42index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}\right) \sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((1-(1/a*b)^(1/3)*x+3^(1/2))/(1-(1/a*b)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{-bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3) - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} - x\left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{a - bx^3} \left(\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) - 1)),x)

[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{a - bx^3} \left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(-b*x**3+a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) - 1 + sqrt(3))), x)

$$3.34 \quad \int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a + bx^3}} dx$$

Optimal. Leaf size=76

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3 - a}} \right)}{\sqrt{2\sqrt{3} - 3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3 - a}} \right)}{\sqrt{2\sqrt{3} - 3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a + bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1 - (3 - 2\sqrt{3})ax^2} dx, x, \frac{1 - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tan^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(1 - \sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a + bx^3}} \right)}{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.59, size = 649, normalized size = 8.54

$$\frac{\left(\frac{3 \sqrt{10800} \sqrt{b} \operatorname{F}_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b^2}\right) - 18176 \sqrt{b} \operatorname{F}_1\left(\frac{1}{2}, 1, \frac{b^2}{10a+b^2}\right) + 8a^2 \sqrt{b} \operatorname{F}_1\left(\frac{1}{2}, 3, \frac{b^2}{10a+b^2}\right) + 3a \sqrt{b} \operatorname{F}_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{10a+b^2}\right) \sqrt{1 - \frac{b^2}{10a+b^2}} \right) \sqrt{1 - \frac{b^2}{10a+b^2}} + 12(\sqrt{3} - 3) \sqrt{b} \sqrt{1 - \frac{b^2}{10a+b^2}} \operatorname{F}_1\left(\frac{2}{3}, \frac{1}{2}, \frac{b^2}{10a+b^2}\right) - 8a^2 \left(\frac{1}{3}\right)^{2/3} \sqrt{3 - \frac{3b^2}{a}} \operatorname{F}_1\left(1, \frac{1}{2}, 1, 2, \frac{b^2}{10a+b^2}\right) \right)}{24(3\sqrt{3} - 5) \sqrt{bx^3 - a}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] $(x * (-12 * (-3 + \operatorname{Sqrt}[3]) * (b/a)^{(1/3)} * x * \operatorname{Sqrt}[1 - (b*x^3)/a] * \operatorname{AppellF1}[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] - 8 * (b/a)^{(2/3)} * x^2 * \operatorname{Sqrt}[3 - (3*b*x^3)/a] * \operatorname{AppellF1}[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] - (3 * (-18176 * a^3 * \operatorname{AppellF1}[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] + 10496 * \operatorname{Sqrt}[3] * a^3 * \operatorname{AppellF1}[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] - b*x^3 * (2 * (-5 + 3*\operatorname{Sqrt}[3]) * a + b*x^3) * \operatorname{Sqrt}[1 - (b*x^3)/a] * \operatorname{AppellF1}[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] * (8 * (-5 + 3*\operatorname{Sqrt}[3]) * a * \operatorname{AppellF1}[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] - 3 * b*x^3 * (\operatorname{AppellF1}[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] + (5 - 3*\operatorname{Sqrt}[3]) * \operatorname{AppellF1}[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)]))))) / (a * (2 * (-5 + 3*\operatorname{Sqrt}[3]) * a + b*x^3) * (8 * (-5 + 3*\operatorname{Sqrt}[3]) * a * \operatorname{AppellF1}[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] - 3 * b*x^3 * (\operatorname{AppellF1}[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)] + (5 - 3*\operatorname{Sqrt}[3]) * \operatorname{AppellF1}[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a - 6*\operatorname{Sqrt}[3]*a)]))) / (24 * (-5 + 3*\operatorname{Sqrt}[3]) * \operatorname{Sqrt}[-a + b*x^3])$

IntegrateAlgebraic [A] time = 12.58, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}(3+2\sqrt{3})}\sqrt{\frac{6}{a}}\tan^{-1}\left(\frac{\sqrt{1+\frac{2}{\sqrt{3}}}\sqrt{b}\sqrt{\frac{6}{a}}\sqrt{bx^3-a}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] - (b/a)^(1/3)*x)/((1 - Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] $(-2 * \operatorname{Sqrt}[(3 + 2 * \operatorname{Sqrt}[3])/3] * (b/a)^{(1/6)} * \operatorname{ArcTan}[\operatorname{Sqrt}[1 + 2/\operatorname{Sqrt}[3]] * \operatorname{Sqrt}[b] * (b/a)^{(1/6)} * \operatorname{Sqrt}[-a + b*x^3]/(a * (b/a)^{(2/3)} - b*x)]) / \operatorname{Sqrt}[b]$

fricas [A] time = 2.92, size = 1278, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/((1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2)), x, algorithm="fricas")

[Out] $[1/2 * \operatorname{sqrt}(1/3) * \operatorname{sqrt}(-(2 * \operatorname{sqrt}(3) + 3) * (b/a)^{(1/3)} / b) * \log((b^8 * x^{24} + 1840 * a * b^7 * x^{21} + 67264 * a^2 * b^6 * x^{18} + 58624 * a^3 * b^5 * x^{15} + 504064 * a^4 * b^4 * x^{12} - 2140160 * a^5 * b^3 * x^9 + 3100672 * a^6 * b^2 * x^6 - 1089536 * a^7 * b * x^3 + 28672 * a^8 - 4 * \operatorname{sqrt}(1/3) * (486 * a * b^7 * x^{20} + 28512 * a^2 * b^6 * x^{17} + 86832 * a^3 * b^5 * x^{14} + 14 * 5152 * a^4 * b^4 * x^{11} - 238464 * a^5 * b^3 * x^8 + 414720 * a^6 * b^2 * x^5 - 82944 * a^7 * b * x^2 + (3 * a * b^7 * x^{22} + 2688 * a^2 * b^6 * x^{19} + 56952 * a^3 * b^5 * x^{16} + 93504 * a^4 * b^4 * x^{13} - 63552 * a^5 * b^3 * x^{10} + 377856 * a^6 * b^2 * x^7 - 314880 * a^7 * b * x^4 + 24576 * a^8 * x - 2 * \operatorname{sqrt}(3) * (a * b^7 * x^{22} + 764 * a^2 * b^6 * x^{19} + 16860 * a^3 * b^5 * x^{16} + 197 * 92 * a^4 * b^4 * x^{13} + 42368 * a^5 * b^3 * x^{10} - 104448 * a^6 * b^2 * x^7 + 90880 * a^7 * b * x^4 - 7168 * a^8 * x)) * (b/a)^{(2/3)} - 6 * \operatorname{sqrt}(3) * (47 * a * b^7 * x^{20} + 2724 * a^2 * b^6 * x^{17} + 8976 * a^3 * b^5 * x^{14} + 4928 * a^4 * b^4 * x^{11} + 32448 * a^5 * b^3 * x^8 - 37632 * a^6 * b^2 * x^5 + 8192 * a^7 * b * x^2) + 2 * (30 * a * b^7 * x^{21} + 5010 * a^2 * b^6 * x^{18} + 44640 * a^3 * b^5 * x^{15} + 21360 * a^4 * b^4 * x^{12} + 79872 * a^5 * b^3 * x^9 - 233856 * a^6 * b^2 * x^6 + 860$

```

16*a^7*b*x^3 - 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt(-(2*sqrt
(3) + 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 + 61440*a^8*x^2 - 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7
*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x - sqrt(3)*(5*
a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(
1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 +
121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*
b*x^3 + 4096*a^8), -sqrt(1/3)*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*(sqrt(3)*a -
a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.44index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}\right) \sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/a*b)^(1/3)*x+1+3^(1/2))/(-1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((-1/a*b)^(1/3)*x+1+3^(1/2))/(-1/a*b)^(1/3)*x+1-3^(1/2))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1}{\sqrt{bx^3 - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x+3^(1/2))/(1-(b/a)^(1/3)*x-3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3) - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} - x\left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{bx^3 - a} \left(\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) - 1)), x)

[Out] int(-(3^(1/2) - x*(b/a)^(1/3) + 1)/((b*x^3 - a)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1}{\sqrt{-a + bx^3} \left(x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x+3**(1/2))/(1-(b/a)**(1/3)*x-3**(1/2))/(b*x**3-a)**(1/2), x)

[Out] Integral((x*(b/a)**(1/3) - sqrt(3) - 1)/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3) - 1 + sqrt(3))), x)

$$3.35 \quad \int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{-a - bx^3}} dx$$

Optimal. Leaf size=76

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{-a - bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.18, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{2\sqrt{3}-3} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{-a - bx^3}} \right)}{\sqrt{2\sqrt{3}-3} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (-2*ArcTan[(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[-3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{-a - bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1 - (3 - 2\sqrt{3})ax^2} dx, x, \frac{1 + \sqrt[3]{\frac{b}{a}} x}{\sqrt{-a - bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tan^{-1} \left(\frac{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \left(1 + \sqrt[3]{\frac{b}{a}} x\right)}{\sqrt{-a - bx^3}} \right)}{\sqrt{-3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.78, size = 666, normalized size = 8.76

$$\frac{\left(\frac{3 \sqrt{3} \sqrt{a} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3}\right) - 18176 a^{1/3} \sqrt{\frac{3}{a}} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3}\right)}{4 \sqrt{3} \sqrt{a} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3}\right)} + 12(\sqrt{3} - 3) \sqrt{\frac{3}{a}} \sqrt{-a - b x^3} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3}\right) - 8 a^{2/3} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{-a - b x^3}\right)} \right)}{24(3\sqrt{3} - 5) \sqrt{-a - b x^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (x*(12*(-3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -(b*x^3)/a, (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -(b*x^3)/a, (b*x^3)/(-10*a + 6*Sqrt[3]*a)] - (3*(-18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -(b*x^3)/a, (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -(b*x^3)/a, (b*x^3)/(-10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -(b*x^3)/a, (b*x^3)/(-10*a + 6*Sqrt[3]*a)]*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)]) + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)]) + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)])))/(a*(2*(-5 + 3*Sqrt[3])*a - b*x^3)*(8*(-5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)]) + (5 - 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -(b*x^3)/a, -(b*x^3)/(10*a - 6*Sqrt[3]*a)])))/(24*(-5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])

IntegrateAlgebraic [A] time = 12.61, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}(3 + 2\sqrt{3})} \sqrt{\frac{6}{a}} \tan^{-1}\left(\frac{\sqrt{1 + \frac{2}{\sqrt{3}} \sqrt{b} \sqrt{\frac{6}{a}} \sqrt{-a - b x^3}}}{a\left(\frac{b}{a}\right)^{2/3} + b x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[3] + (b/a)^(1/3)*x)/((1 - Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]),x]

[Out] (2*Sqrt[(3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[-a - b*x^3])/(a*(b/a)^(2/3) + b*x)])/Sqrt[b]

fricas [B] time = 3.17, size = 1339, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a^8*x - 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 + 7168*a^8*x))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 - sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3

```

- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^
14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 - sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 - a))*sqrt(-(2*sqrt(3) + 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 -
1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 - 2*sqrt(3)
*(a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 + 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 + 4608*a^8*x - sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 +
151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) + 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 + 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x - 2*a*x)*(b/a)^(2/3) + 2*sqrt(-b*x^3 - a)*(sqrt(
3)*a - a)*(b/a)^(1/3))*sqrt((2*sqrt(3) + 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.45index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}\right) \sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1+3^(1/2))/((1/a*b)^(1/3)*x+1-3^(1/2))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1}{\sqrt{-bx^3 - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x+3^(1/2))/(1+(b/a)^(1/3)*x-3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3) + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} + x \left(\frac{b}{a}\right)^{1/3} + 1}{\sqrt{-bx^3 - a} \left(x \left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/2) + 1)),x)

[Out] int((3^(1/2) + x*(b/a)^(1/3) + 1)/((- a - b*x^3)^(1/2)*(x*(b/a)^(1/3) - 3^(1/2) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}}{\sqrt{-a - bx^3} \left(x \sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x+3**(1/2))/(1+(b/a)**(1/3)*x-3**(1/2))/(-b*x**3-a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) + 1 + sqrt(3))/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3) - sqrt(3) + 1)), x)

$$3.36 \quad \int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{1 + x^3}} dx$$

Optimal. Leaf size=42

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Rubi [A] time = 0.09, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2140, 203}

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{x^3+1}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]), x]

[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{1 + x^3}} dx = - \left(2 \text{Subst} \left(\int \frac{1}{1 + (3 + 2\sqrt{3})x^2} dx, x, \frac{1 + x}{\sqrt{1 + x^3}} \right) \right) \\ = - \frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{1+x^3}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [C] time = 0.44, size = 269, normalized size = 6.40

$$\frac{2\sqrt{6} \sqrt{\frac{i(x+1)}{\sqrt{3+3i}}} \left(4\sqrt{-2ix + \sqrt{3} + i\sqrt{x^2 - x + 1}} \Pi \left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}}; \sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} + i}}{\sqrt{2} \sqrt{3}} \right) \Big|_{\frac{2\sqrt{3}}{3i+\sqrt{3}}} \right) + \sqrt{2ix + \sqrt{3} - i} \left((1+2i) + i\sqrt{3} \right) x - \sqrt{3} - (2+i) F \left(\sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} + i}}{\sqrt{2} \sqrt{3}} \right) \Big|_{\frac{2\sqrt{3}}{3i+\sqrt{3}}} \right) \right)}{(3i + (1 + 2i)\sqrt{3}) \sqrt{-2ix + \sqrt{3} + i\sqrt{x^3 + 1}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]

[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[t[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3])], ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])]))/((3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x^3])

IntegrateAlgebraic [A] time = 2.12, size = 53, normalized size = 1.26

$$-2\sqrt{\frac{1}{3}}(2\sqrt{3} - 3) \tan^{-1}\left(\frac{\sqrt{3 + 2\sqrt{3}} \sqrt{x^3 + 1}}{x^2 - x + 1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[1 + x^3]),x]

[Out] -2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 + x^3])/(1 - x + x^2)]

fricas [A] time = 0.91, size = 50, normalized size = 1.19

$$\frac{1}{3} \sqrt{3} \sqrt{2\sqrt{3} - 3} \arctan\left(\frac{(\sqrt{3}(x^2 - 4x - 2) - 6x - 6)\sqrt{2\sqrt{3} - 3}}{6\sqrt{x^3 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*(sqrt(3)*(x^2 - 4*x - 2) - 6*x - 6)*sqrt(2*sqrt(3) - 3)/sqrt(x^3 + 1))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[1,-1]:[1,0,-3]%%},[2]%%}% / %%{%%{[2,4]:[1,0,-3]%%},[2]%%}% Error: Bad Argument Value

maple [C] time = 0.03, size = 245, normalized size = 5.83

$$\frac{2\left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \text{EllipticF}\left(\sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}, \sqrt{\frac{\frac{3}{2} - \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}\right) - 4\left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right) \sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \sqrt{\frac{x - \frac{1}{2} + \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}} \text{EllipticPi}\left(\sqrt{\frac{x+1}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}, \frac{\left(-\frac{3}{2} + \frac{i\sqrt{3}}{2}\right)\sqrt{3}}{3}, \sqrt{\frac{\frac{3}{2} - \frac{i\sqrt{3}}{2}}{\frac{3}{2} - \frac{i\sqrt{3}}{2}}}\right)}{\sqrt{x^3 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x)

[Out] 2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-4*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)))/(x^3+1)^(1/2)

$$-1/2*I*3^{(1/2)})^{(1/2)}*((x-1/2-1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*((x-1/2+1/2*I*3^{(1/2)})/(-3/2+1/2*I*3^{(1/2)}))^{(1/2)}/(x^3+1)^{(1/2)}*EllipticPi(((x+1)/(3/2-1/2*I*3^{(1/2)}))^{(1/2)}, 1/3*(-3/2+1/2*I*3^{(1/2)})*3^{(1/2)}, ((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{x^3 + 1}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(3) + 1)/(sqrt(x^3 + 1)*(x + sqrt(3) + 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 3^(1/2) + 1)/((x^3 + 1)^(1/2)*(x + 3^(1/2) + 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{(x+1)(x^2-x+1)}(x+1+\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(x**3+1)**(1/2),x)

[Out] Integral((x - sqrt(3) + 1)/(sqrt((x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)

$$3.37 \quad \int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x)\sqrt{1 - x^3}} dx$$

Optimal. Leaf size=46

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Rubi [A] time = 0.10, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]

[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[(((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x]] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x)\sqrt{1 - x^3}} dx = 2 \text{Subst} \left(\int \frac{1}{1 + (3 + 2\sqrt{3})x^2} dx, x, \frac{1 - x}{\sqrt{1 - x^3}} \right) \\ = \frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{1-x^3}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [C] time = 0.49, size = 267, normalized size = 5.80

$$\frac{2\sqrt{6} \sqrt{\frac{i(x-1)}{\sqrt{3}-3i}} \left(\sqrt{2ix + \sqrt{3} + i} ((\sqrt{3} + (2+i)x + i\sqrt{3} + (1+2i)) F \left(\sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \Big|_{-3i+\sqrt{3}} \right) - 4i\sqrt{-2ix + \sqrt{3} - i} \sqrt{x^2 + x + 1} \Pi \left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}; \sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \Big|_{-3i+\sqrt{3}} \right) \right)}{(3 + (2+i)\sqrt{3}) \sqrt{-2ix + \sqrt{3} - i} \sqrt{1 - x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])]*(Sqrt[I + Sqrt[3] + (2*I)*x]
*((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I +
Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*
I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3]
)/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/
4))], (2*Sqrt[3])/(-3*I + Sqrt[3])]))/((3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt
[3] - (2*I)*x]*Sqrt[1 - x^3])
```

IntegrateAlgebraic [A] time = 2.06, size = 53, normalized size = 1.15

$$2\sqrt{\frac{1}{3}(2\sqrt{3}-3)} \tan^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{1-x^3}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[1 - x^3]),x]
[Out] 2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[1 - x^3])/(1 +
x + x^2)]
```

fricas [A] time = 1.46, size = 59, normalized size = 1.28

$$\frac{1}{3}\sqrt{3}\sqrt{2\sqrt{3}-3} \arctan\left(\frac{\sqrt{-x^3+1}(\sqrt{3}(x^2+4x-2)+6x-6)\sqrt{2\sqrt{3}-3}}{6(x^3-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2)))/(-x^3+1)^(1/2),x, algorithm="fricas")
[Out] 1/3*sqrt(3)*sqrt(2*sqrt(3) - 3)*arctan(1/6*sqrt(-x^3 + 1)*(sqrt(3)*(x^2 + 4
*x - 2) + 6*x - 6)*sqrt(2*sqrt(3) - 3)/(x^3 - 1))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2)))/(-x^3+1)^(1/2),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:Unable to divide, perhaps due to rounding er
ror%%{%%{[1,-1]:[1,0,-3]%%},[2]%%} / %%{%%{[2,4]:[1,0,-3]%%},[2]%%} Err
or: Bad Argument Value
```

maple [C] time = 0.04, size = 247, normalized size = 5.37

$$\frac{2i\sqrt{3}\sqrt{i(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i(x+\frac{1}{2}+\frac{i\sqrt{3}}{2})}\sqrt{3}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{i(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{3},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)-4i\sqrt{i(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\sqrt{-i(x+\frac{1}{2}+\frac{i\sqrt{3}}{2})}\sqrt{3}\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{i(x+\frac{1}{2}-\frac{i\sqrt{3}}{2})}\sqrt{\frac{x-1}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}}{3},\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{-\frac{3}{2}+\frac{i\sqrt{3}}{2}}}\right)}{3\sqrt{-x^3+1}\left(-\frac{3}{2}+\frac{i\sqrt{3}}{2}-\sqrt{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-x-3^(1/2))/(1-x+3^(1/2)))/(-x^3+1)^(1/2),x)
[Out] -2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3
^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*Elli
pticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+
1/2*I*3^(1/2)))^(1/2))-4*I*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-
```


$$-3/2+1/2*I*3^{(1/2)})^{(1/2)}*(-I*(x+1/2+1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)/(-x^3+1)^{(1/2)/(-3/2+1/2*I*3^{(1/2)}-3^{(1/2)})}*EllipticPi(1/3*3^{(1/2)}*(I*(x+1/2-1/2*I*3^{(1/2)})*3^{(1/2)})^{(1/2)},I*3^{(1/2)/(-3/2+1/2*I*3^{(1/2)}-3^{(1/2)}), (I*3^{(1/2)/(-3/2+1/2*I*3^{(1/2)})})^{(1/2)})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{-x^3 + 1}(x - \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) - 1)/(sqrt(-x^3 + 1)*(x - sqrt(3) - 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x + 3^(1/2) - 1)/((1 - x^3)^(1/2)*(3^(1/2) - x + 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - 1 + \sqrt{3}}{\sqrt{-(x - 1)(x^2 + x + 1)}(x - \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(-x**3+1)**(1/2),x)

[Out] Integral((x - 1 + sqrt(3))/(sqrt(-(x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)), x)

$$3.38 \quad \int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x)\sqrt{-1 + x^3}} dx$$

Optimal. Leaf size=44

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{x^3-1}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]), x]

[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_)*(x_))/(((c_) + (d_)*(x_))*Sqrt[(a_) + (b_)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} - x}{(1 + \sqrt{3} - x)\sqrt{-1 + x^3}} dx = 2 \text{Subst} \left(\int \frac{1}{1 - (3 + 2\sqrt{3})x^2} dx, x, \frac{1 - x}{\sqrt{-1 + x^3}} \right) \\ = \frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}}(1-x)}{\sqrt{-1+x^3}} \right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [C] time = 0.30, size = 265, normalized size = 6.02

$$\frac{2\sqrt{6} \sqrt{\frac{i(x-1)}{\sqrt{3}-3i}} \left(\sqrt{2ix + \sqrt{3} + i} ((\sqrt{3} + (2+i))x + i\sqrt{3} + (1+2i)) F \left(\sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \middle| \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right) - 4i\sqrt{-2ix + \sqrt{3} - i} \sqrt{x^2 + x + 1} \Pi \left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}; \sin^{-1} \left(\frac{\sqrt{-2ix + \sqrt{3} - i}}{\sqrt{2} \sqrt[4]{3}} \right) \middle| \frac{2\sqrt{3}}{-3i + \sqrt{3}} \right) \right)}{(3 + (2+i)\sqrt{3})\sqrt{-2ix + \sqrt{3} - i}\sqrt{x^3 - 1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] (2*Sqrt[6]*Sqrt[(I*(-1 + x))/(-3*I + Sqrt[3])])*(Sqrt[I + Sqrt[3] + (2*I)*x] * ((1 + 2*I) + I*Sqrt[3] + ((2 + I) + Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])] - (4*I)*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[1 + x + x^2]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[-I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(-3*I + Sqrt[3])])/(3 + (2 + I)*Sqrt[3])*Sqrt[-I + Sqrt[3] - (2*I)*x]*Sqrt[-1 + x^3])

IntegrateAlgebraic [A] time = 2.01, size = 51, normalized size = 1.16

$$-2\sqrt{\frac{1}{3}}(2\sqrt{3}-3)\tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}\sqrt{x^3-1}}{x^2+x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] - x)/((1 + Sqrt[3] - x)*Sqrt[-1 + x^3]),x]

[Out] -2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 + x^3])/(1 + x + x^2)]

fricas [B] time = 1.45, size = 204, normalized size = 4.64

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8+16x^7+112x^6+16x^5+112x^4-224x^3+64x^2-4(2x^6+18x^5+42x^4+8x^3+\sqrt{3}(x^6+12x^5+18x^4+16x^3-12x^2-8)-24x+8)\sqrt{x^3-1}\sqrt{2\sqrt{3}-3}+16\sqrt{3}(x^7+2x^6+6x^5-5x^4+2x^3-6x^2+4x-4)-128x+112}{x^8-8x^7+16x^6+16x^5-56x^4-32x^3+64x^2+64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 + 16*x^7 + 112*x^6 + 16*x^5 + 112*x^4 - 224*x^3 + 64*x^2 - 4*(2*x^6 + 18*x^5 + 42*x^4 + 8*x^3 + sqrt(3)*(x^6 + 12*x^5 + 18*x^4 + 16*x^3 - 12*x^2 - 8) - 24*x + 8)*sqrt(x^3 - 1)*sqrt(2*sqrt(3) - 3) + 16*sqrt(3)*(x^7 + 2*x^6 + 6*x^5 - 5*x^4 + 2*x^3 - 6*x^2 + 4*x - 4) - 128*x + 112)/(x^8 - 8*x^7 + 16*x^6 + 16*x^5 - 56*x^4 - 32*x^3 + 64*x^2 + 64*x + 16))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x)::OUTPUT:Unable to divide, perhaps due to rounding error: %%%{%%{[1,-1]:[1,0,-3]%%},[2]%%} / %%%{%%{[2,4]:[1,0,-3]%%},[2]%%} Error: Bad Argument Value

maple [C] time = 0.03, size = 245, normalized size = 5.57

$$2\left(-\frac{3-i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{-\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3+i\sqrt{3}}{2}}}\operatorname{EllipticF}\left(\sqrt{\frac{x-1}{-\frac{3-i\sqrt{3}}{2}}},\sqrt{\frac{3+i\sqrt{3}}{2}}\right)-4\left(-\frac{3-i\sqrt{3}}{2}\right)\sqrt{\frac{x-1}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3-i\sqrt{3}}{2}}}\sqrt{\frac{x+\frac{1+i\sqrt{3}}{2}}{\frac{3+i\sqrt{3}}{2}}}\operatorname{EllipticPi}\left(\sqrt{\frac{x-1}{-\frac{3-i\sqrt{3}}{2}}},-\frac{\left(\frac{3-i\sqrt{3}}{2}\right)\sqrt{3}}{3},\sqrt{\frac{3+i\sqrt{3}}{2}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x)

[Out] 2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)

)^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-4*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),-1/3*(3/2+1/2*I*3^(1/2))*3^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + \sqrt{3} - 1}{\sqrt{x^3 - 1}(x - \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3^(1/2))/(1-x+3^(1/2))/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(3) - 1)/(sqrt(x^3 - 1)*(x - sqrt(3) - 1)), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x + 3^(1/2) - 1)/((x^3 - 1)^(1/2)*(3^(1/2) - x + 1)),x)

[Out] \text{Hanged}

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - 1 + \sqrt{3}}{\sqrt{(x - 1)(x^2 + x + 1)}(x - \sqrt{3} - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-3**(1/2))/(1-x+3**(1/2))/(x**3-1)**(1/2),x)

[Out] Integral((x - 1 + sqrt(3))/(sqrt((x - 1)*(x**2 + x + 1))*(x - sqrt(3) - 1)), x)

$$3.39 \quad \int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-1 - x^3}} dx$$

Optimal. Leaf size=44

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Rubi [A] time = 0.09, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {2140, 206}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[3 + 2*Sqrt[3]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} + x}{(1 + \sqrt{3} + x)\sqrt{-1 - x^3}} dx = -\left(2 \text{Subst}\left(\int \frac{1}{1 - (3 + 2\sqrt{3})x^2} dx, x, \frac{1 + x}{\sqrt{-1 - x^3}}\right)\right) = -\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{3+2\sqrt{3}}}$$

Mathematica [C] time = 0.35, size = 271, normalized size = 6.16

$$\frac{2\sqrt{6}\sqrt{\frac{i(x+1)}{\sqrt{3+3i}}}\left(4\sqrt{-2ix+\sqrt{3}+i}\sqrt{x^2-x+1}\Pi\left(\frac{2\sqrt{3}}{3i+(1+2i)\sqrt{3}};\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\right)\frac{2\sqrt{3}}{3i+\sqrt{3}}+\sqrt{2ix+\sqrt{3}-i}\left((1+2i)+i\sqrt{3}\right)x-\sqrt{3}-(2+i)\right)F\left(\sin^{-1}\left(\frac{\sqrt{-2ix+\sqrt{3}+i}}{\sqrt{2}\sqrt[4]{3}}\right)\right)\frac{2\sqrt{3}}{3i+\sqrt{3}}}{(3i+(1+2i)\sqrt{3})\sqrt{-2ix+\sqrt{3}+i}\sqrt{-x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]

[Out] (2*Sqrt[6]*Sqrt[(I*(1 + x))/(3*I + Sqrt[3])]*(Sqrt[-I + Sqrt[3] + (2*I)*x]*((-2 - I) - Sqrt[3] + ((1 + 2*I) + I*Sqrt[3])*x)*EllipticF[ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])] + 4*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[1 - x + x^2]*EllipticPi[(2*Sqrt[3])/(3*I + (1 + 2*I)*Sqrt[3]), ArcSin[Sqrt[I + Sqrt[3] - (2*I)*x]/(Sqrt[2]*3^(1/4))], (2*Sqrt[3])/(3*I + Sqrt[3])])/(3*I + (1 + 2*I)*Sqrt[3])*Sqrt[I + Sqrt[3] - (2*I)*x]*Sqrt[-1 - x^3])

IntegrateAlgebraic [A] time = 2.14, size = 55, normalized size = 1.25

$$2\sqrt{\frac{1}{3}(2\sqrt{3} - 3)} \tanh^{-1}\left(\frac{\sqrt{3 + 2\sqrt{3}} \sqrt{-x^3 - 1}}{x^2 - x + 1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] + x)/((1 + Sqrt[3] + x)*Sqrt[-1 - x^3]),x]

[Out] 2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[-1 - x^3])/(1 - x + x^2)]

fricas [B] time = 0.93, size = 206, normalized size = 4.68

$$\frac{1}{6}\sqrt{3}\sqrt{2\sqrt{3}-3}\log\left(\frac{x^8-16x^7+112x^6-16x^5+112x^4+224x^3+64x^2+4(2x^6-18x^5+42x^4-8x^3+\sqrt{3}(x^6-12x^5+18x^4-16x^3-12x^2-8)+24x+8)\sqrt{-x^3-1}\sqrt{2\sqrt{3}-3}-16\sqrt{3}(x^7-2x^6+6x^5+5x^4+2x^3+6x^2+4x+4)+128x+112)}{x^8+8x^7+16x^6-16x^5-56x^4+32x^3+64x^2-64x+16}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="fricas")

[Out] 1/6*sqrt(3)*sqrt(2*sqrt(3) - 3)*log((x^8 - 16*x^7 + 112*x^6 - 16*x^5 + 112*x^4 + 224*x^3 + 64*x^2 + 4*(2*x^6 - 18*x^5 + 42*x^4 - 8*x^3 + sqrt(3)*(x^6 - 12*x^5 + 18*x^4 - 16*x^3 - 12*x^2 - 8) + 24*x + 8)*sqrt(-x^3 - 1)*sqrt(2*sqrt(3) - 3) - 16*sqrt(3)*(x^7 - 2*x^6 + 6*x^5 + 5*x^4 + 2*x^3 + 6*x^2 + 4*x + 4) + 128*x + 112)/(x^8 + 8*x^7 + 16*x^6 - 16*x^5 - 56*x^4 + 32*x^3 + 64*x^2 - 64*x + 16))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Unable to divide, perhaps due to rounding error%%{%%{[1,-1]:[1,0,-3]%%},[2]%%} / %%{%%{[2,4]:[1,0,-3]%%},[2]%%} Error: Bad Argument Value

maple [C] time = 0.03, size = 243, normalized size = 5.52

$$\frac{2i\sqrt{3}\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}\sqrt{\frac{x+1}{\frac{3}{2}+i\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}\sqrt{3}\operatorname{EllipticF}\left(\frac{\sqrt{3}\sqrt{\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}\sqrt{3}}{3},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}}}\right)+4i\sqrt{i\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}\sqrt{\frac{x+1}{\frac{3}{2}+i\sqrt{3}}}\sqrt{-i\left(x-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}\sqrt{3}\operatorname{EllipticPi}\left(\frac{\sqrt{3}\sqrt{\left(x-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}\sqrt{3}}{3},\frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}},\sqrt{\frac{i\sqrt{3}}{\frac{3}{2}+i\sqrt{3}}}\right)}{3\sqrt{-x^3-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x)

[Out] -2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*Ellip

```
ticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2)+4*I*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(3/2+3^(1/2)+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-x^3 - 1}(x + \sqrt{3} + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3^(1/2))/(1+x+3^(1/2))/(-x^3-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((x - sqrt(3) + 1)/(sqrt(-x^3 - 1)*(x + sqrt(3) + 1)), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.02

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - 3^(1/2) + 1)/((- x^3 - 1)^(1/2)*(x + 3^(1/2) + 1)),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x - \sqrt{3} + 1}{\sqrt{-(x+1)(x^2-x+1)}(x+1+\sqrt{3})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x-3**(1/2))/(1+x+3**(1/2))/(-x**3-1)**(1/2),x)
```

```
[Out] Integral((x - sqrt(3) + 1)/(sqrt(-(x + 1)*(x**2 - x + 1))*(x + 1 + sqrt(3))), x)
```

$$3.40 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=69

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.18, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]),x]

[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1+(3+2\sqrt{3})ax^2} dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a+bx^3}} \right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.59, size = 320, normalized size = 4.64

$$2\sqrt{\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \left(\frac{4\sqrt[3]{-1}(1 + \sqrt[3]{-1})\sqrt[3]{a} \sqrt{\frac{b^2/3x^2 - \sqrt[3]{bx}}{a^{2/3}} - \sqrt[3]{a}} + 11\pi \left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}} \sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \right)}{\sqrt[3]{-1}} \right) - \frac{(\sqrt[3]{-1}\sqrt[3]{a} - \sqrt[3]{bx})\sqrt{\sqrt[3]{-1} - i\sqrt[3]{bx}}}{\sqrt[3]{a}} F \left(\sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \right)}{\sqrt[3]{-1}} \right) \right)}{(3+(2+i)\sqrt{3})\sqrt[3]{b}} - \frac{(\sqrt[3]{-1}\sqrt[3]{a} - \sqrt[3]{bx})\sqrt{\sqrt[3]{-1} - i\sqrt[3]{bx}}}{\sqrt[3]{a}} F \left(\sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \right)}{\sqrt[3]{-1}} \right) \right)}{\sqrt[3]{3}\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{bx}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}}$$

$\sqrt{a + bx^3}$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-((((-1)^(1/3)*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)])/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/((3 + (2 + I)*Sqrt[3])*b^(1/3)))/Sqrt[a + b*x^3]

IntegrateAlgebraic [A] time = 12.51, size = 115, normalized size = 1.67

$$2\sqrt{\frac{1}{3}}(2\sqrt{3} - 3) \tan^{-1} \left(\frac{\sqrt{\frac{2}{\sqrt{3}} - 1} b^{2/3} x^2}{\sqrt[6]{a}} - \sqrt{\frac{2}{\sqrt{3}} - 1} \sqrt[6]{a} \sqrt[3]{b} x + \sqrt{\frac{2}{\sqrt{3}} - 1} \sqrt{a}}{\sqrt{a + bx^3}} \right)$$

$\sqrt[6]{a} \sqrt[3]{b}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[-1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)]/Sqrt[a + b*x^3])/(a^(1/6)*b^(1/3))

fricas [A] time = 4.78, size = 1236, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*sqrt(b*x^3 + a)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x)))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2

```

920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*
x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*b^(1/3))*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3))) + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x
^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^
6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x
^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a
^6*b*x^4 - 2560*a^7*x))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 +
13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5
*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b
^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 1050
24*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 32*sqr
t(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*
x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8))/
(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 + 121984
*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*b*x^3 +
4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(-1
/2*sqrt(1/3)*(a^(1/3)*b*x^2 - 2*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*(sqrt
(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/sqrt(b*x^3 + a))]

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="giac")

```

[Out] Timed out

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)

```

```

[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3+
a)^(1/2),x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{bx^3 + a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(
b*x^3+a)^(1/2),x, algorithm="maxima")

```

```

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 + a)*(b^(1/3)*x +
a^(1/3)*(sqrt(3) + 1))), x)

```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((a + b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt{3} \sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{a + bx^3} (\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(a + b*x**3)*(a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

$$3.41 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{a-bx^3}} dx$$

Optimal. Leaf size=71

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.18, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\begin{aligned} \int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{a-bx^3}} dx &= \frac{(2\sqrt[3]{a}) \text{Subst} \left(\int \frac{1}{1+(3+2\sqrt{3})ax^2} dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{a-bx^3}} \right)}{\sqrt[3]{b}} \\ &= \frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}} \end{aligned}$$

Mathematica [C] time = 0.82, size = 329, normalized size = 4.63

$$2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}\right)}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\operatorname{E}\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\right)\sqrt[3]{-1}}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}+1}\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\right)\sqrt[3]{-1}}{3+(2+i)\sqrt{3}}}\right)\sqrt[3]{b}\sqrt{a-bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[(-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x)]/((1 + (-1)^(1/3))*a^(1/3)))*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/(3 + (2 + I)*Sqrt[3]))/(b^(1/3)*Sqrt[a - b*x^3])

IntegrateAlgebraic [A] time = 12.46, size = 115, normalized size = 1.62

$$\frac{2\sqrt{\frac{1}{3}}(2\sqrt{3}-3)\tan^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt[6]{a}\sqrt[3]{bx}+\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{a}}\right)}{\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTan[(Sqrt[-1 + 2/Sqrt[3]])*Sqrt[a] + Sqrt[-1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)])/Sqrt[a - b*x^3]]/(a^(1/6)*b^(1/3))

fricas [B] time = 3.81, size = 1288, normalized size = 18.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 + 32*(9*b^7*x^22 + 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 - 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 - 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 - 4608*a^7*x + sqrt(3)*(5*b^7*x^22 + 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 + 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 + 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 + 2560*a^7*x)))*a^(2/3)*b^(1/3) + 8*(3*b^7*x^23 + 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 + 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 + 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 + 61440*a^7*x^2) + 2*sqrt(3)*(b^7*x^23 + 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 - 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 - 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 - 17920*a^7*x^2))*a^(1/3)*b^(2/3) - 4*sqrt(1/3)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7

```
*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x))*sqrt(-b*x^3 + a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*sqrt(-b*x^3 + a)*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3))) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a)*a^(1/3)*b*x^2 + 2*sqrt(-b*x^3 + a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(3)*a + a)*b^(1/3))*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 - a))
]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="giac")
```

[Out] Timed out

maple [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

```
[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3+a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{-bx^3 + a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 + a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))), x)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((a - b*x^3)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b}x}{\sqrt{a - bx^3} (-\sqrt{3} \sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b}x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3+a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(a - b*x**3)*(-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

$$3.42 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{-a+bx^3}} dx$$

Optimal. Leaf size=72

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.19, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{bx^3-a}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) - b^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a}-\sqrt[3]{b}x)\sqrt{-a+bx^3}} dx = \frac{(2\sqrt[3]{a}) \text{Subst}\left(\int \frac{1}{1-(3+2\sqrt{3})ax^2} dx, x, \frac{1-\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a+bx^3}}\right)}{\sqrt[3]{b}}$$

$$= \frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a}-\sqrt[3]{b}x)}{\sqrt{-a+bx^3}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.36, size = 330, normalized size = 4.58

$$2\sqrt{\frac{\sqrt[3]{a}-\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\left(\frac{\left(\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{bx}\right)\sqrt{\frac{\sqrt[3]{-1}\left(\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{bx}\right)}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\operatorname{F}\left(\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\right)\sqrt[3]{-1}}{\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}}-\frac{4\sqrt[3]{-1}\left(1+\sqrt[3]{-1}\right)\sqrt[3]{a}\sqrt{\frac{b^{2/3}x^2}{a^{2/3}}+\frac{\sqrt[3]{bx}}{\sqrt[3]{a}}+1}\Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{bx}}{(1+\sqrt[3]{-1})\sqrt[3]{a}}}\right)\right)\sqrt[3]{-1}}{3+(2+i)\sqrt{3}}}\right)$$

$$\frac{\sqrt[3]{b}\sqrt{bx^3-a}}{\sqrt[3]{b}\sqrt{bx^3-a}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (2*Sqrt[(a^(1/3) - b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*((((-1)^(1/3)*a^(1/3) + b^(1/3)*x)*Sqrt[(-1)^(1/3)*(a^(1/3) + (-1)^(1/3)*b^(1/3)*x)]/((1 + (-1)^(1/3))*a^(1/3)))*EllipticF[ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))] - (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 + (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) - (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)]/(3 + (2 + I)*Sqrt[3])))/(b^(1/3)*Sqrt[-a + b*x^3])

IntegrateAlgebraic [A] time = 12.48, size = 116, normalized size = 1.61

$$2\sqrt{\frac{1}{3}}(2\sqrt{3}-3)\tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}b^{2/3}x^2}{\sqrt[6]{a}}+\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt[6]{a}\sqrt[3]{b}x+\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{a}}{\sqrt{bx^3-a}}\right)$$

$$\frac{\sqrt[6]{a}\sqrt[3]{b}}{\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) - b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) - b^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] + Sqrt[-1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)]/Sqrt[-a + b*x^3])/(a^(1/6)*b^(1/3))

fricas [A] time = 4.06, size = 1239, normalized size = 17.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*sqrt(b*x^3 - a)*((3*b^7*x^22 + 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 + 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 + 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 + 24576*a^7*x + 2*sqrt(3)*(b^7*x^22 + 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 + 19792*a^3*b^4*x^13 + 42368*a^4*b^3*x^10 - 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 - 7168*a^7*x)))*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2)))*a^(1/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 29

$20a^2b^6x^{18} + 24864a^3b^5x^{15} + 26576a^4b^4x^{12} - 56000a^5b^3x^9 + 115968a^6b^2x^6 - 56320a^7b^1x^3 + 1024a^8)b^{1/3})\sqrt{(2\sqrt{3} - 3)/(ab^{2/3}))} + 32(9b^7x^{22} + 846ab^6x^{19} + 4617a^2b^5x^{16} - 5472a^3b^4x^{13} + 43776a^4b^3x^{10} - 98496a^5b^2x^7 + 59328a^6b^1x^4 - 4608a^7x + \sqrt{3}(5b^7x^{22} + 505ab^6x^{19} + 2130a^2b^5x^{16} + 4928a^3b^4x^{13} - 28688a^4b^3x^{10} + 53760a^5b^2x^7 - 35200a^6b^1x^4 + 2560a^7x))a^{2/3}b^{1/3} + 8(3b^7x^{23} + 1077ab^6x^{20} + 13320a^2b^5x^{17} + 19200a^3b^4x^{14} - 111360a^4b^3x^{11} + 345024a^5b^2x^8 - 328704a^6b^1x^5 + 61440a^7x^2) + 2\sqrt{3}(b^7x^{23} + 299ab^6x^{20} + 4260a^2b^5x^{17} - 1520a^3b^4x^{14} + 26720a^4b^3x^{11} - 105024a^5b^2x^8 + 93184a^6b^1x^5 - 17920a^7x^2))a^{1/3}b^{2/3} + 32\sqrt{3}(35ab^7x^{21} + 1141a^2b^6x^{18} + 2544a^3b^5x^{15} - 6760a^4b^4x^{12} + 39520a^5b^3x^9 - 55680a^6b^2x^6 + 19712a^7b^1x^3 - 512a^8))/(b^8x^{24} - 80ab^7x^{21} + 2368a^2b^6x^{18} - 30080a^3b^5x^{15} + 121984a^4b^4x^{12} + 240640a^5b^3x^9 + 151552a^6b^2x^6 + 40960a^7b^1x^3 + 4096a^8)), \sqrt{1/3}a^{1/3}\sqrt{-(2\sqrt{3} - 3)/(ab^{2/3}))}\arctan(1/2\sqrt{1/3}(a^{1/3}b^1x^2 + 2(\sqrt{3}x + 2x)a^{2/3}b^{2/3} - 2(\sqrt{3}a + a)b^{1/3}))\sqrt{-(2\sqrt{3} - 3)/(ab^{2/3}))}/\sqrt{bx^3 - a})]$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{-b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}}{\left(-b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)

[Out] int((-b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(-b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(b*x^3-a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{bx^3 - a}\left(b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} + 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(-b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((b^(1/3)*x + a^(1/3)*(sqrt(3) - 1))/(sqrt(b*x^3 - a)*(b^(1/3)*x - a^(1/3)*(sqrt(3) + 1))), x)

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x + a^(1/3)*(3^(1/2) - 1))/((b*x^3 - a)^(1/2)*(b^(1/3)*x - a^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a + bx^3} (-\sqrt{3} \sqrt[3]{a} - \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(-b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)/(sqrt(-a + b*x**3)*(-sqrt(3)*a**(1/3) - a**(1/3) + b**(1/3)*x)), x)
```

$$3.43 \quad \int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx$$

Optimal. Leaf size=72

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Rubi [A] time = 0.17, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 61, $\frac{\text{number of rules}}{\text{integrand size}} = 0.033$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*(a^(1/3) + b^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*a^(1/6)*b^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{(1-\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x}{((1+\sqrt{3})\sqrt[3]{a} + \sqrt[3]{b}x)\sqrt{-a-bx^3}} dx = -\frac{(2\sqrt[3]{a}) \text{Subst}\left(\int \frac{1}{1-(3+2\sqrt{3})ax^2} dx, x, \frac{1+\frac{\sqrt[3]{b}x}{\sqrt[3]{a}}}{\sqrt{-a-bx^3}}\right)}{\sqrt[3]{b}}$$

$$= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} (\sqrt[3]{a} + \sqrt[3]{b}x)}{\sqrt{-a-bx^3}}\right)}{\sqrt{3+2\sqrt{3}} \sqrt[6]{a} \sqrt[3]{b}}$$

Mathematica [C] time = 0.49, size = 323, normalized size = 4.49

$$2\sqrt{\frac{\sqrt[3]{a} + \sqrt[3]{bx}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}} \left(\frac{4\sqrt[3]{-1}(1 + \sqrt[3]{-1})\sqrt[3]{a} \sqrt{\frac{b^2/3x^2 - \sqrt[3]{bx}}{a^2/3} + 1} \Pi\left(\frac{2i\sqrt{3}}{3+(2+i)\sqrt{3}}, \sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}\right) \middle| \sqrt[3]{-1}\right) - \frac{(\sqrt[3]{-1}\sqrt[3]{a} - \sqrt[3]{bx})\sqrt{\sqrt[3]{-1} - i\sqrt[3]{bx}} F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}\sqrt[3]{bx} + \sqrt[3]{a}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}\right) \middle| \sqrt[3]{-1}\right)}{(3+(2+i)\sqrt{3})\sqrt[3]{b}}}{\sqrt[3]{3}\sqrt[3]{b} \sqrt{\frac{\sqrt[3]{a} + (-1)^{2/3}\sqrt[3]{bx}}{(1 + \sqrt[3]{-1})\sqrt[3]{a}}}} \right)}{\sqrt{-a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (2*Sqrt[(a^(1/3) + b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]*(-(((((-1)^(1/3))*a^(1/3) - b^(1/3)*x)*Sqrt[(-1)^(1/6) - (I*b^(1/3)*x)/a^(1/3)]*EllipticF[ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)))/(3^(1/4)*b^(1/3)*Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))])) + (4*(-1)^(1/3)*(1 + (-1)^(1/3))*a^(1/3)*Sqrt[1 - (b^(1/3)*x)/a^(1/3) + (b^(2/3)*x^2)/a^(2/3)]*EllipticPi[((2*I)*Sqrt[3])/(3 + (2 + I)*Sqrt[3]), ArcSin[Sqrt[(a^(1/3) + (-1)^(2/3)*b^(1/3)*x)/((1 + (-1)^(1/3))*a^(1/3))]], (-1)^(1/3)))/((3 + (2 + I)*Sqrt[3])*b^(1/3)))/Sqrt[-a - b*x^3]

IntegrateAlgebraic [A] time = 12.47, size = 118, normalized size = 1.64

$$\frac{2\sqrt{\frac{1}{3}}(2\sqrt{3} - 3) \tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}b^{2/3}x^2}{\sqrt[6]{a}} - \sqrt{\frac{2}{\sqrt{3}}-1}\sqrt[6]{a}\sqrt[3]{bx} + \sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{a}}\right)}{\sqrt[6]{a}\sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 - Sqrt[3])*a^(1/3) + b^(1/3)*x)/(((1 + Sqrt[3])*a^(1/3) + b^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[a] - Sqrt[-1 + 2/Sqrt[3]]*a^(1/6)*b^(1/3)*x + (Sqrt[-1 + 2/Sqrt[3]]*b^(2/3)*x^2)/a^(1/6)])/Sqrt[-a - b*x^3])/(a^(1/6)*b^(1/3))

fricas [B] time = 4.43, size = 1299, normalized size = 18.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-b*x^3-a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*a^(1/3)*sqrt((2*sqrt(3) - 3)/(a*b^(2/3)))*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 32*(9*b^7*x^22 - 846*a*b^6*x^19 + 4617*a^2*b^5*x^16 + 5472*a^3*b^4*x^13 + 43776*a^4*b^3*x^10 + 98496*a^5*b^2*x^7 + 59328*a^6*b*x^4 + 4608*a^7*x + sqrt(3)*(5*b^7*x^22 - 505*a*b^6*x^19 + 2130*a^2*b^5*x^16 - 4928*a^3*b^4*x^13 - 28688*a^4*b^3*x^10 - 53760*a^5*b^2*x^7 - 35200*a^6*b*x^4 - 2560*a^7*x)))*a^(2/3)*b^(1/3) - 8*(3*b^7*x^23 - 1077*a*b^6*x^20 + 13320*a^2*b^5*x^17 - 19200*a^3*b^4*x^14 - 111360*a^4*b^3*x^11 - 345024*a^5*b^2*x^8 - 328704*a^6*b*x^5 - 61440*a^7*x^2 + 2*sqrt(3)*(b^7*x^23 - 299*a*b^6*x^20 + 4260*a^2*b^5*x^17 + 1520*a^3*b^4*x^14 + 26720*a^4*b^3*x^11 + 105024*a^5*b^2*x^8 + 93184*a^6*b*x^5 + 17920*a^7*x^2))*a^(1/3)*b^(2/3) + 4*sqrt(1/3)*((3*b^7*x^22 - 2688*a*b^6*x^19 + 56952*a^2*b^5*x^16 - 93504*a^3*b^4*x^13 - 63552*a^4*b^3*x^10 - 377856*a^5*b^2*x^7 - 314880*a^6*b*x^4 - 24576*a^7*x + 2*sqrt(3)*(b^7*x

```

x^22 - 764*a*b^6*x^19 + 16860*a^2*b^5*x^16 - 19792*a^3*b^4*x^13 + 42368*a^4
*b^3*x^10 + 104448*a^5*b^2*x^7 + 90880*a^6*b*x^4 + 7168*a^7*x))*sqrt(-b*x^3
- a)*a^(2/3)*b^(2/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^
5*x^14 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824
*a^7*b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14
- 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x
^2))*sqrt(-b*x^3 - a)*a^(1/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 4464
0*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^
6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18
+ 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6
*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3 - a)*b^(1/3))*sqrt((2*s
qrt(3) - 3)/(a*b^(2/3))) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 +
2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x
^6 + 19712*a^7*b*x^3 + 512*a^8))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x
^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 15155
2*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*a^(1/3)*sqrt(-(2*s
qrt(3) - 3)/(a*b^(2/3)))*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*a^(1/3)*b*
x^2 - 2*sqrt(-b*x^3 - a)*(sqrt(3)*x + 2*x)*a^(2/3)*b^(2/3) - 2*sqrt(-b*x^3
- a)*(sqrt(3)*a + a)*b^(1/3))*sqrt(-(2*sqrt(3) - 3)/(a*b^(2/3)))/(b*x^3 + a
)))]

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
-b*x^3-a)^(1/2),x, algorithm="giac")

```

[Out] Timed out

maple [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x + (1 - \sqrt{3})a^{\frac{1}{3}}}{\left(b^{\frac{1}{3}}x + (1 + \sqrt{3})a^{\frac{1}{3}}\right)\sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)

```

```

[Out] int((b^(1/3)*x+(1-3^(1/2))*a^(1/3))/(b^(1/3)*x+(1+3^(1/2))*a^(1/3))/(-b*x^3
-a)^(1/2),x)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b^{\frac{1}{3}}x - a^{\frac{1}{3}}(\sqrt{3} - 1)}{\sqrt{-bx^3 - a}\left(b^{\frac{1}{3}}x + a^{\frac{1}{3}}(\sqrt{3} + 1)\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b^(1/3)*x+a^(1/3)*(1-3^(1/2)))/(b^(1/3)*x+a^(1/3)*(1+3^(1/2)))/(-
-b*x^3-a)^(1/2),x, algorithm="maxima")

```

```

[Out] integrate((b^(1/3)*x - a^(1/3)*(sqrt(3) - 1))/(sqrt(-b*x^3 - a)*(b^(1/3)*x
+ a^(1/3)*(sqrt(3) + 1))), x)

```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b^(1/3)*x - a^(1/3)*(3^(1/2) - 1))/((- a - b*x^3)^(1/2)*(b^(1/3)*x + a^(1/3)*(3^(1/2) + 1))),x)
```

```
[Out] \text{Hanged}
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{-\sqrt{3} \sqrt[3]{a} + \sqrt[3]{a} + \sqrt[3]{b} x}{\sqrt{-a - bx^3} (\sqrt[3]{a} + \sqrt{3} \sqrt[3]{a} + \sqrt[3]{b} x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b**(1/3)*x+a**(1/3)*(1-3**(1/2)))/(b**(1/3)*x+a**(1/3)*(1+3**(1/2)))/(-b*x**3-a)**(1/2),x)
```

```
[Out] Integral((-sqrt(3)*a**(1/3) + a**(1/3) + b**(1/3)*x)/(sqrt(-a - b*x**3)*(a**(1/3) + sqrt(3)*a**(1/3) + b**(1/3)*x)), x)
```

$$3.44 \quad \int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a + bx^3}} dx$$

Optimal. Leaf size=73

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.18, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$, Rules used = {2140, 203}

$$-\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (-2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{a + bx^3}} dx = -\frac{2 \text{Subst} \left(\int \frac{1}{1 + (3 + 2\sqrt{3})ax^2} dx, x, \frac{1 + \sqrt[3]{\frac{b}{a}} x}{\sqrt{a + bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = -\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 + \sqrt[3]{\frac{b}{a}} x\right)}{\sqrt{a+bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.22, size = 667, normalized size = 9.14

$$\frac{x \left(\frac{3^{1/3} \sqrt{a+b x^3} \left(\frac{1}{3} \sqrt{\frac{2}{3}} \sqrt{3} - 3 \right) + 18176 a^{3/2} \sqrt{a+b x^3} \left(\frac{1}{3} \sqrt{\frac{2}{3}} \sqrt{3} - 3 \right) \sqrt{\frac{a+b x^3}{a}} \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{(b x^3)}{a}, -\frac{(b x^3)}{(10 a+6 \sqrt{3} a)} \right]}{4 \sqrt{3} \sqrt{a+b x^3} \left(\frac{1}{3} \sqrt{\frac{2}{3}} \sqrt{3} - 3 \right) \sqrt{\frac{a+b x^3}{a}} \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{(b x^3)}{a}, -\frac{(b x^3)}{(10 a+6 \sqrt{3} a)} \right]} \right) + 12(3+\sqrt{3}) \sqrt{\frac{a+b x^3}{a}} \operatorname{AppellF1} \left[\frac{1}{3}, \frac{1}{2}, 1, \frac{4}{3}, -\frac{(b x^3)}{a}, -\frac{(b x^3)}{(10 a+6 \sqrt{3} a)} \right]}{24(5+3 \sqrt{3}) \sqrt{a+b x^3}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))))/(24*(5 + 3*Sqrt[3])*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 12.60, size = 90, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}(2\sqrt{3}-3)}\sqrt{\frac{6}{a}}\tan^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt{\frac{6}{a}}\sqrt{a+bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[a + b*x^3]), x]

[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]])*Sqrt[b]*(b/a)^(1/6)*Sqrt[a + b*x^3))/(a*(b/a)^(2/3) + b*x))]/Sqrt[b]

fricas [A] time = 2.98, size = 1270, normalized size = 17.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*(486*a*b^7*x^20 - 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 - 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 - 414720*a^6*b^2*x^5 - 82944*a^7*b*x^2 + (3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 + 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b

```

^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 860
16*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 2486
4*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^
6 - 56320*a^7*b*x^3 - 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 + a)*sqrt(-(2*sqrt
(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1077*a^2*b^6*x^20 + 13320*a^3*b
^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 - 345024*a^6*b^2*x^8 - 3
28704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 - 299*a^2*b^6*x^20
+ 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 + 105024*a^6*b
^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b/a)^(2/3) - 32*sqrt(3)*(35*a*b
^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 + 6760*a^4*b^4*x^12 + 39520
*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 + 512*a^8) + 32*(9*a*b^7
*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 + 5472*a^4*b^4*x^13 + 43776*a^
5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 + 4608*a^8*x + sqrt(3)*(5*
a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 - 4928*a^4*b^4*x^13 - 286
88*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 - 2560*a^8*x))*(b/a)^(
1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 + 30080*a^3*b^5*x^15 +
121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 - 40960*a^7*
b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(-
1/2*sqrt(1/3)*(b*x^2 - 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a +
a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 + a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2)))/(b*x^3+a)^(1/
2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}\right) \sqrt{bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2)))/(b*x^3+a)^(1/2)
,x)
```

```
[Out] int((((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2)))/(b*x^3+a)^(1/2)
,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{bx^3 + a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(b*x^3 + a)*(x*(b/a)^(1/3) + sqrt(3) + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1}{\sqrt{bx^3 + a} \left(\sqrt{3} + x \left(\frac{b}{a}\right)^{1/3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) + 1)),x)

[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((a + b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{a + bx^3} \left(x \sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(b*x**3+a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(a + b*x**3)*(x*(b/a)**(1/3) + 1 + sqrt(3))), x)

$$3.45 \quad \int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{a - bx^3}} dx$$

Optimal. Leaf size=75

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.19, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 203}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]), x]

[Out] (2*ArcTan[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{a - bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1+(3+2\sqrt{3})ax^2} dx, x, \frac{1 - \sqrt[3]{\frac{b}{a}}x}{\sqrt{a-bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tan^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - \sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 1.15, size = 649, normalized size = 8.65

$$\frac{\left(\frac{3 \sqrt{1089 \sqrt{3}} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} - 18176 \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} \sqrt{\frac{b^2}{a^2} (2(3 + \sqrt{3}) - b^2)} \sqrt{\frac{b^2}{a^2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}}{4 \sqrt{\frac{b^2}{a^2} (2(3 + \sqrt{3}) - b^2)} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}} \right) \left(\frac{10 \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} \sqrt{\frac{b^2}{a^2} (2(3 + \sqrt{3}) - b^2)} \sqrt{\frac{b^2}{a^2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}}}{4 \sqrt{\frac{b^2}{a^2} (2(3 + \sqrt{3}) - b^2)} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}} \right) - 12(3 + \sqrt{3}) \sqrt{\frac{b^2}{a^2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}} - 8 \sqrt{\frac{b^2}{a^2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}}} \sqrt{3 - \frac{3b^2}{a^2}} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} - \frac{b^2}{a^2} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} \sqrt{3 - \frac{3b^2}{a^2}} \sqrt{\frac{1}{2} + \frac{1}{2} \frac{b^2}{a^2}} \right)}{24(5 + 3\sqrt{3}) \sqrt{a - bx^3}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))/(24*(5 + 3*Sqrt[3])*Sqrt[a - b*x^3])

IntegrateAlgebraic [A] time = 12.66, size = 92, normalized size = 1.23

$$\frac{2\sqrt{\frac{1}{3}(2\sqrt{3}-3)}\sqrt{\frac{6}{a}}\tan^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt{\frac{6}{a}}\sqrt{a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}-bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[a - b*x^3]),x]

[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTan[(Sqrt[-1 + 2/Sqrt[3]])*Sqrt[b]*(b/a)^(1/6)*Sqrt[a - b*x^3)]/(a*(b/a)^(2/3) - b*x)]/Sqrt[b]

fricas [B] time = 3.23, size = 1324, normalized size = 17.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*((3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 - 7168*a^8*x))*sqrt(-b*x^3 + a)*(b/a)^(2/3) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 + 1024*a^8))*sqrt(-b*x^3 + a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 + 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14

```

14 + 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 + 69120*a^6*b^2*x^5 - 13824*a^7
*b*x^2 + sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4
928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))
*sqrt(-b*x^3 + a))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 +
1077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b
^3*x^11 + 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)
*(a*b^7*x^23 + 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 2
6720*a^5*b^3*x^11 - 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*
(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*
x^15 - 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^
7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^1
6 - 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*
b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^
5*x^16 + 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200
*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2
*b^6*x^18 - 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 +
151552*a^6*b^2*x^6 + 40960*a^7*b*x^3 + 4096*a^8)), sqrt(1/3)*sqrt((2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(1/2*sqrt(1/3)*(sqrt(-b*x^3 + a))*b*x^2 + 2*sq
rt(-b*x^3 + a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 + a)*(sqrt(
3)*a + a)*(b/a)^(1/3))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 - a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1
/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}}x + 1 - \sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}}x + 1 + \sqrt{3}\right)\sqrt{-bx^3 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/a*b)^(1/3)*x+1-3^(1/2))/(-1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

```
[Out] int((-1/a*b)^(1/3)*x+1-3^(1/2))/(-1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3+a)^(1
/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{-bx^3 + a}\left(x\left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(-b*x^3+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(-b*x^3 + a)*(x*(b/a)^(1/3) - sqrt(3) - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} + x \left(\frac{b}{a}\right)^{1/3} - 1}{\sqrt{a - bx^3} \left(\sqrt{3} - x \left(\frac{b}{a}\right)^{1/3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1/3) + 1)),x)

[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((a - b*x^3)^(1/2)*(3^(1/2) - x*(b/a)^(1/3) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3 \sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{a - bx^3} \left(x^3 \sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(-b*x**3+a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(a - b*x**3)*(x*(b/a)**(1/3) - sqrt(3) - 1)), x)

$$3.46 \quad \int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a + bx^3}} dx$$

Optimal. Leaf size=76

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3 - a}} \right)}{\sqrt{3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.19, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - x \sqrt[3]{\frac{b}{a}}\right)}{\sqrt{bx^3 - a}} \right)}{\sqrt{3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]

[Out] (2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 - (b/a)^(1/3)*x))/Sqrt[-a + b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} - \sqrt[3]{\frac{b}{a}}x}{\left(1 + \sqrt{3} - \sqrt[3]{\frac{b}{a}}x\right)\sqrt{-a + bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1 - (3 + 2\sqrt{3})ax^2} dx, x, \frac{1 - \sqrt[3]{\frac{b}{a}}x}{\sqrt{-a + bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 - \sqrt[3]{\frac{b}{a}}x\right)}{\sqrt{-a + bx^3}} \right)}{\sqrt{3 + 2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.64, size = 650, normalized size = 8.55

$$\frac{\left(\frac{\sqrt{10896} \sqrt{3} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) + 18176 \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) \operatorname{erf}\left(\frac{3 \sqrt{3} \sqrt{b} \sqrt{a x^3 - a}}{2 \sqrt{a^2}}\right) + \sqrt{\frac{3}{a}} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) \operatorname{erf}\left(\frac{3 \sqrt{3} \sqrt{b} \sqrt{a x^3 - a}}{2 \sqrt{a^2}}\right) + \sqrt{\frac{3}{a}} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) \operatorname{erf}\left(\frac{3 \sqrt{3} \sqrt{b} \sqrt{a x^3 - a}}{2 \sqrt{a^2}}\right)}{24(5 + 3\sqrt{3}) \sqrt{b x^3 - a}} \right) - 12(3 + \sqrt{3}) \sqrt{\frac{3}{a}} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) \operatorname{erf}\left(\frac{3 \sqrt{3} \sqrt{b} \sqrt{a x^3 - a}}{2 \sqrt{a^2}}\right) - 8 \sqrt{\frac{3}{a}} \operatorname{erf}\left(\frac{1}{2} \sqrt{\frac{3}{a}} \sqrt{\frac{b^2 x^2 - a}{a^2}}\right) \operatorname{erf}\left(\frac{3 \sqrt{3} \sqrt{b} \sqrt{a x^3 - a}}{2 \sqrt{a^2}}\right)}{24(5 + 3\sqrt{3}) \sqrt{b x^3 - a}} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]
```

```
[Out] (x*(-12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 - (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - 8*(b/a)^(2/3)*x^2*Sqrt[3 - (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + b*x^3*(2*(5 + 3*Sqrt[3])*a - b*x^3)*Sqrt[1 - (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))/(a*(2*(5 + 3*Sqrt[3])*a - b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, (b*x^3)/a, (b*x^3)/(10*a + 6*Sqrt[3]*a)])))/((24*(5 + 3*Sqrt[3])*Sqrt[-a + b*x^3]))
```

IntegrateAlgebraic [A] time = 12.76, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}}(2\sqrt{3} - 3)\sqrt{\frac{6}{a}} \tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}} - 1} \sqrt{b} \sqrt{\frac{6}{a}} \sqrt{b x^3 - a}}{a\left(\frac{b}{a}\right)^{2/3} - b x}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] - (b/a)^(1/3)*x)/((1 + Sqrt[3] - (b/a)^(1/3)*x)*Sqrt[-a + b*x^3]), x]
```

```
[Out] (2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]])*Sqrt[b]*(b/a)^(1/6)*Sqrt[-a + b*x^3])/(a*(b/a)^(2/3) - b*x)]/Sqrt[b]
```

fricas [A] time = 2.63, size = 1273, normalized size = 16.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2), x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 + 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 + 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 - 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 - 1089536*a^7*b*x^3 + 28672*a^8 - 4*sqrt(1/3)*(486*a*b^7*x^20 + 28512*a^2*b^6*x^17 + 86832*a^3*b^5*x^14 + 145152*a^4*b^4*x^11 - 238464*a^5*b^3*x^8 + 414720*a^6*b^2*x^5 - 82944*a^7*b*x^2 + (3*a*b^7*x^22 + 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 + 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 + 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 + 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 + 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 + 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 - 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 - 7168*a^8*x))*(b/a)^(2/3) + 6*sqrt(3)*(47*a*b^7*x^20 + 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 + 4928*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 - 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2) + 2*(30*a*b^7*x^21 + 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 + 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 - 233856*a^6*b^2*x^6 + 8601
```

```

6*a^7*b*x^3 - 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 + 2920*a^2*b^6*x^18 + 24864
*a^3*b^5*x^15 + 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 + 115968*a^6*b^2*x^6
- 56320*a^7*b*x^3 + 1024*a^8))*(b/a)^(1/3))*sqrt(b*x^3 - a)*sqrt((2*sqrt(3)
) - 3)*(b/a)^(1/3)/b) + 8*(3*a*b^7*x^23 + 1077*a^2*b^6*x^20 + 13320*a^3*b^5
*x^17 + 19200*a^4*b^4*x^14 - 111360*a^5*b^3*x^11 + 345024*a^6*b^2*x^8 - 328
704*a^7*b*x^5 + 61440*a^8*x^2 + 2*sqrt(3)*(a*b^7*x^23 + 299*a^2*b^6*x^20 +
4260*a^3*b^5*x^17 - 1520*a^4*b^4*x^14 + 26720*a^5*b^3*x^11 - 105024*a^6*b^2
*x^8 + 93184*a^7*b*x^5 - 17920*a^8*x^2))*(b/a)^(2/3) + 32*sqrt(3)*(35*a*b^7
*x^21 + 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^15 - 6760*a^4*b^4*x^12 + 39520*a
^5*b^3*x^9 - 55680*a^6*b^2*x^6 + 19712*a^7*b*x^3 - 512*a^8) + 32*(9*a*b^7*x
^22 + 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16 - 5472*a^4*b^4*x^13 + 43776*a^5*
b^3*x^10 - 98496*a^6*b^2*x^7 + 59328*a^7*b*x^4 - 4608*a^8*x + sqrt(3)*(5*a*
b^7*x^22 + 505*a^2*b^6*x^19 + 2130*a^3*b^5*x^16 + 4928*a^4*b^4*x^13 - 28688
*a^5*b^3*x^10 + 53760*a^6*b^2*x^7 - 35200*a^7*b*x^4 + 2560*a^8*x))*(b/a)^(1
/3))/(b^8*x^24 - 80*a*b^7*x^21 + 2368*a^2*b^6*x^18 - 30080*a^3*b^5*x^15 + 1
21984*a^4*b^4*x^12 + 240640*a^5*b^3*x^9 + 151552*a^6*b^2*x^6 + 40960*a^7*b*
x^3 + 4096*a^8)), sqrt(1/3)*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)*arctan(1/2
*sqrt(1/3)*(b*x^2 + 2*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*(sqrt(3)*a + a
*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/sqrt(b*x^3 - a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument Valueindex.cc ind
ex_m operator + Error: Bad Argument Value
```

maple [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}}{\left(-\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}\right) \sqrt{b x^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1/a*b)^(1/3)*x+1-3^(1/2))/(-1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/2),x)
```

```
[Out] int((-1/a*b)^(1/3)*x+1-3^(1/2))/(-1/a*b)^(1/3)*x+1+3^(1/2))/(b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} - 1}{\sqrt{b x^3 - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-(b/a)^(1/3)*x-3^(1/2))/(1-(b/a)^(1/3)*x+3^(1/2))/(b*x^3-a)^(1/2),x, algorithm="maxima")
```

[Out] integrate((x*(b/a)^(1/3) + sqrt(3) - 1)/(sqrt(b*x^3 - a)*(x*(b/a)^(1/3) - sqrt(3) - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{3} + x\left(\frac{b}{a}\right)^{1/3} - 1}{\sqrt{bx^3 - a} \left(\sqrt{3} - x\left(\frac{b}{a}\right)^{1/3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1/3) + 1)),x)

[Out] int(-(3^(1/2) + x*(b/a)^(1/3) - 1)/((b*x^3 - a)^(1/2)*(3^(1/2) - x*(b/a)^(1/3) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt[3]{\frac{b}{a}} - 1 + \sqrt{3}}{\sqrt{-a + bx^3} \left(x\sqrt[3]{\frac{b}{a}} - \sqrt{3} - 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(b/a)**(1/3)*x-3**(1/2))/(1-(b/a)**(1/3)*x+3**(1/2))/(b*x**3-a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) - 1 + sqrt(3))/(sqrt(-a + b*x**3)*(x*(b/a)**(1/3) - sqrt(3) - 1)), x)

$$3.47 \quad \int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{-a - bx^3}} dx$$

Optimal. Leaf size=76

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{-a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Rubi [A] time = 0.17, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 55, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2140, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(x \sqrt[3]{\frac{b}{a}} + 1\right)}{\sqrt{-a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(1 + (b/a)^(1/3)*x))/Sqrt[-a - b*x^3]])/(Sqrt[3 + 2*Sqrt[3]]*Sqrt[a]*(b/a)^(1/3))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2140

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := With[{k = Simplify[(d*e + 2*c*f)/(c*f)]}, Dist[((1 + k)*e)/d, Subst[Int[1/(1 + (3 + 2*k)*a*x^2), x], x, (1 + ((1 + k)*d*x)/c)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[d*e - c*f, 0] && EqQ[b^2*c^6 - 20*a*b*c^3*d^3 - 8*a^2*d^6, 0] && EqQ[6*a*d^4*e - c*f*(b*c^3 - 22*a*d^3), 0]

Rubi steps

$$\int \frac{1 - \sqrt{3} + \sqrt[3]{\frac{b}{a}} x}{\left(1 + \sqrt{3} + \sqrt[3]{\frac{b}{a}} x\right) \sqrt{-a - bx^3}} dx = \frac{2 \text{Subst} \left(\int \frac{1}{1 - (3 + 2\sqrt{3})ax^2} dx, x, \frac{1 + \sqrt[3]{\frac{b}{a}} x}{\sqrt{-a - bx^3}} \right)}{\sqrt[3]{\frac{b}{a}}} = \frac{2 \tanh^{-1} \left(\frac{\sqrt{3+2\sqrt{3}} \sqrt{a} \left(1 + \sqrt[3]{\frac{b}{a}} x\right)}{\sqrt{-a-bx^3}} \right)}{\sqrt{3+2\sqrt{3}} \sqrt{a} \sqrt[3]{\frac{b}{a}}}$$

Mathematica [C] time = 0.72, size = 670, normalized size = 8.82

$$\frac{\left(\frac{10896\sqrt{3}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) + 18176\sqrt{3}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) + 3\sqrt{3}\sqrt{a^2 - b^2}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) + 12(3 + \sqrt{3})\sqrt{\frac{a^2}{c^2} + 1}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) - 8\sqrt{3}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) + 12(3 + \sqrt{3})\sqrt{\frac{a^2}{c^2} + 1}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) - 8\sqrt{3}F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{b^2}{c^2}, \frac{b^2}{c^2}\right) \right)}{24(5 + 3\sqrt{3})\sqrt{-a - bx^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]), x]
```

```
[Out] (x*(12*(3 + Sqrt[3])*(b/a)^(1/3)*x*Sqrt[1 + (b*x^3)/a]*AppellF1[2/3, 1/2, 1, 5/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 8*(b/a)^(2/3)*x^2*Sqrt[3 + (3*b*x^3)/a]*AppellF1[1, 1/2, 1, 2, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - (3*(18176*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + 10496*Sqrt[3]*a^3*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - b*x^3*(2*(5 + 3*Sqrt[3])*a + b*x^3)*Sqrt[1 + (b*x^3)/a]*AppellF1[4/3, 1/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))])))/(a*(2*(5 + 3*Sqrt[3])*a + b*x^3)*(8*(5 + 3*Sqrt[3])*a*AppellF1[1/3, 1/2, 1, 4/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] - 3*b*x^3*(AppellF1[4/3, 1/2, 2, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))] + (5 + 3*Sqrt[3])*AppellF1[4/3, 3/2, 1, 7/3, -((b*x^3)/a), -((b*x^3)/(10*a + 6*Sqrt[3]*a))]))))/(24*(5 + 3*Sqrt[3])*Sqrt[-a - b*x^3])
```

IntegrateAlgebraic [A] time = 12.63, size = 93, normalized size = 1.22

$$\frac{2\sqrt{\frac{1}{3}(2\sqrt{3} - 3)}\sqrt{\frac{6}{a}}\tanh^{-1}\left(\frac{\sqrt{\frac{2}{\sqrt{3}}-1}\sqrt{b}\sqrt{\frac{6}{a}}\sqrt{-a-bx^3}}{a\left(\frac{b}{a}\right)^{2/3}+bx}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 - Sqrt[3] + (b/a)^(1/3)*x)/((1 + Sqrt[3] + (b/a)^(1/3)*x)*Sqrt[-a - b*x^3]), x]
```

```
[Out] (-2*Sqrt[(-3 + 2*Sqrt[3])/3]*(b/a)^(1/6)*ArcTanh[(Sqrt[-1 + 2/Sqrt[3]]*Sqrt[b]*(b/a)^(1/6)*Sqrt[-a - b*x^3])/(a*(b/a)^(2/3) + b*x)]/Sqrt[b]
```

fricas [B] time = 2.61, size = 1335, normalized size = 17.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2), x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(1/3)*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b)*log((b^8*x^24 - 1840*a*b^7*x^21 + 67264*a^2*b^6*x^18 - 58624*a^3*b^5*x^15 + 504064*a^4*b^4*x^12 + 2140160*a^5*b^3*x^9 + 3100672*a^6*b^2*x^6 + 1089536*a^7*b*x^3 + 28672*a^8 + 4*sqrt(1/3)*((3*a*b^7*x^22 - 2688*a^2*b^6*x^19 + 56952*a^3*b^5*x^16 - 93504*a^4*b^4*x^13 - 63552*a^5*b^3*x^10 - 377856*a^6*b^2*x^7 - 314880*a^7*b*x^4 - 24576*a^8*x + 2*sqrt(3)*(a*b^7*x^22 - 764*a^2*b^6*x^19 + 16860*a^3*b^5*x^16 - 19792*a^4*b^4*x^13 + 42368*a^5*b^3*x^10 + 104448*a^6*b^2*x^7 + 90880*a^7*b*x^4 + 7168*a^8*x)))*sqrt(-b*x^3 - a)*(b/a)^(2/3) - 2*(30*a*b^7*x^21 - 5010*a^2*b^6*x^18 + 44640*a^3*b^5*x^15 - 21360*a^4*b^4*x^12 + 79872*a^5*b^3*x^9 + 233856*a^6*b^2*x^6 + 86016*a^7*b*x^3 + 3072*a^8 + sqrt(3)*(17*a*b^7*x^21 - 2920*a^2*b^6*x^18 + 24864*a^3*b^5*x^15 - 26576*a^4*b^4*x^12 - 56000*a^5*b^3*x^9 - 115968*a^6*b^2*x^6 - 56320*a^7*b*x^3 - 1024*a^8))*sqrt(-b*x^3
```

```

- a)*(b/a)^(1/3) + 6*(81*a*b^7*x^20 - 4752*a^2*b^6*x^17 + 14472*a^3*b^5*x^14
4 - 24192*a^4*b^4*x^11 - 39744*a^5*b^3*x^8 - 69120*a^6*b^2*x^5 - 13824*a^7*
b*x^2 + sqrt(3)*(47*a*b^7*x^20 - 2724*a^2*b^6*x^17 + 8976*a^3*b^5*x^14 - 49
28*a^4*b^4*x^11 + 32448*a^5*b^3*x^8 + 37632*a^6*b^2*x^5 + 8192*a^7*b*x^2))*
sqrt(-b*x^3 - a))*sqrt((2*sqrt(3) - 3)*(b/a)^(1/3)/b) - 8*(3*a*b^7*x^23 - 1
077*a^2*b^6*x^20 + 13320*a^3*b^5*x^17 - 19200*a^4*b^4*x^14 - 111360*a^5*b^3
*x^11 - 345024*a^6*b^2*x^8 - 328704*a^7*b*x^5 - 61440*a^8*x^2 + 2*sqrt(3)*(
a*b^7*x^23 - 299*a^2*b^6*x^20 + 4260*a^3*b^5*x^17 + 1520*a^4*b^4*x^14 + 267
20*a^5*b^3*x^11 + 105024*a^6*b^2*x^8 + 93184*a^7*b*x^5 + 17920*a^8*x^2))*(b
/a)^(2/3) - 32*sqrt(3)*(35*a*b^7*x^21 - 1141*a^2*b^6*x^18 + 2544*a^3*b^5*x^
15 + 6760*a^4*b^4*x^12 + 39520*a^5*b^3*x^9 + 55680*a^6*b^2*x^6 + 19712*a^7*
b*x^3 + 512*a^8) + 32*(9*a*b^7*x^22 - 846*a^2*b^6*x^19 + 4617*a^3*b^5*x^16
+ 5472*a^4*b^4*x^13 + 43776*a^5*b^3*x^10 + 98496*a^6*b^2*x^7 + 59328*a^7*b*
x^4 + 4608*a^8*x + sqrt(3)*(5*a*b^7*x^22 - 505*a^2*b^6*x^19 + 2130*a^3*b^5*
x^16 - 4928*a^4*b^4*x^13 - 28688*a^5*b^3*x^10 - 53760*a^6*b^2*x^7 - 35200*a
^7*b*x^4 - 2560*a^8*x))*(b/a)^(1/3))/(b^8*x^24 + 80*a*b^7*x^21 + 2368*a^2*b
^6*x^18 + 30080*a^3*b^5*x^15 + 121984*a^4*b^4*x^12 - 240640*a^5*b^3*x^9 + 1
51552*a^6*b^2*x^6 - 40960*a^7*b*x^3 + 4096*a^8)), -sqrt(1/3)*sqrt(-(2*sqrt(
3) - 3)*(b/a)^(1/3)/b)*arctan(-1/2*sqrt(1/3)*(sqrt(-b*x^3 - a)*b*x^2 - 2*sq
rt(-b*x^3 - a)*(sqrt(3)*a*x + 2*a*x)*(b/a)^(2/3) - 2*sqrt(-b*x^3 - a)*(sqrt
(3)*a + a)*(b/a)^(1/3))*sqrt(-(2*sqrt(3) - 3)*(b/a)^(1/3)/b)/(b*x^3 + a))]

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueEvaluation t
ime: 0.43index.cc index_m operator + Error: Bad Argument Value
```

maple [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 - \sqrt{3}}{\left(\left(\frac{b}{a}\right)^{\frac{1}{3}} x + 1 + \sqrt{3}\right) \sqrt{-bx^3 - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

```
[Out] int(((1/a*b)^(1/3)*x+1-3^(1/2))/((1/a*b)^(1/3)*x+1+3^(1/2))/(-b*x^3-a)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \left(\frac{b}{a}\right)^{\frac{1}{3}} - \sqrt{3} + 1}{\sqrt{-bx^3 - a} \left(x \left(\frac{b}{a}\right)^{\frac{1}{3}} + \sqrt{3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)^(1/3)*x-3^(1/2))/(1+(b/a)^(1/3)*x+3^(1/2))/(-b*x^3-a)^(1/2),x, algorithm="maxima")

[Out] integrate((x*(b/a)^(1/3) - sqrt(3) + 1)/(sqrt(-b*x^3 - a)*(x*(b/a)^(1/3) + sqrt(3) + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x \left(\frac{b}{a}\right)^{1/3} - \sqrt{3} + 1}{\sqrt{-bx^3 - a} \left(\sqrt{3} + x \left(\frac{b}{a}\right)^{1/3} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) + 1)),x)

[Out] int((x*(b/a)^(1/3) - 3^(1/2) + 1)/((- a - b*x^3)^(1/2)*(3^(1/2) + x*(b/a)^(1/3) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt[3]{\frac{b}{a}} - \sqrt{3} + 1}{\sqrt{-a - bx^3} \left(x \sqrt[3]{\frac{b}{a}} + 1 + \sqrt{3}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(b/a)**(1/3)*x-3**(1/2))/(1+(b/a)**(1/3)*x+3**(1/2))/(-b*x**3-a)**(1/2),x)

[Out] Integral((x*(b/a)**(1/3) - sqrt(3) + 1)/(sqrt(-a - b*x**3)*(x*(b/a)**(1/3) + 1 + sqrt(3))), x)

$$3.48 \quad \int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx$$

Optimal. Leaf size=95

$$\frac{3 \log\left(d(2c+dx) - d\sqrt[3]{2c^3+d^3x^3}\right)}{2d} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d}$$

Rubi [A] time = 0.12, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = {2151}

$$\frac{3 \log\left(d(2c+dx) - d\sqrt[3]{2c^3+d^3x^3}\right)}{2d} - \frac{\sqrt{3} \tan^{-1}\left(\frac{\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}+1}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

[Out] -((Sqrt[3]*ArcTan[(1 + (2*(2*c + d*x))/(2*c^3 + d^3*x^3)^(1/3))/Sqrt[3]])/d) - Log[c + d*x]/d + (3*Log[d*(2*c + d*x) - d*(2*c^3 + d^3*x^3)^(1/3)])/(2*d)

Rule 2151

Int[((e_) + (f_.)*(x_))/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] := Simp[(Sqrt[3]*f*ArcTan[(1 + (2*Rt[b, 3]*(2*c + d*x))/(d*(a + b*x^3)^(1/3)))/Sqrt[3]])/(Rt[b, 3]*d), x] + (Simp[(f*Log[c + d*x])/(Rt[b, 3]*d), x] - Simp[(3*f*Log[Rt[b, 3]*(2*c + d*x) - d*(a + b*x^3)^(1/3)])/(2*Rt[b, 3]*d), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[d*e + c*f, 0] && EqQ[2*b*c^3 - a*d^3, 0]

Rubi steps

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx = -\frac{\sqrt{3} \tan^{-1}\left(\frac{1+\frac{2(2c+dx)}{\sqrt[3]{2c^3+d^3x^3}}}{\sqrt{3}}\right)}{d} - \frac{\log(c+dx)}{d} + \frac{3 \log\left(d(2c+dx) - d\sqrt[3]{2c^3+d^3x^3}\right)}{2d}$$

Mathematica [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{c-dx}{(c+dx)\sqrt[3]{2c^3+d^3x^3}} dx$$

Verification is not applicable to the result.

[In] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

[Out] Integrate[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)), x]

IntegrateAlgebraic [A] time = 1.42, size = 159, normalized size = 1.67

$$\frac{\log\left(\sqrt[3]{2c^3+d^3x^3} - 2c - dx\right)}{d} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{2c^3+d^3x^3}}{\sqrt[3]{2c^3+d^3x^3+4c+2dx}}\right)}{d} - \frac{\log\left(\left(2c^3+d^3x^3\right)^{2/3} + (2c+dx)\sqrt[3]{2c^3+d^3x^3} + 4c^2+4cdx+d^2x^2\right)}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c - d*x)/((c + d*x)*(2*c^3 + d^3*x^3)^(1/3)),x]

[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(2*c^3 + d^3*x^3)^(1/3))/(4*c + 2*d*x + (2*c^3 + d^3*x^3)^(1/3))])/d + Log[-2*c - d*x + (2*c^3 + d^3*x^3)^(1/3)]/d - Log[4*c^2 + 4*c*d*x + d^2*x^2 + (2*c + d*x)*(2*c^3 + d^3*x^3)^(1/3) + (2*c^3 + d^3*x^3)^(2/3)]/(2*d)

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (residue poly has multiple non-linear factors)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{dx - c}{(d^3x^3 + 2c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="giac")

[Out] integrate(-(d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{-dx + c}{(dx + c)(d^3x^3 + 2c^3)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)

[Out] int((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{dx - c}{(d^3x^3 + 2c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-d*x+c)/(d*x+c)/(d^3*x^3+2*c^3)^(1/3),x, algorithm="maxima")

[Out] -integrate((d*x - c)/((d^3*x^3 + 2*c^3)^(1/3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c - dx}{(2c^3 + d^3x^3)^{\frac{1}{3}}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)`

[Out] `int((c - d*x)/((2*c^3 + d^3*x^3)^(1/3)*(c + d*x)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{c}{c\sqrt[3]{2c^3 + d^3x^3} + dx\sqrt[3]{2c^3 + d^3x^3}} \right) dx - \int \frac{dx}{c\sqrt[3]{2c^3 + d^3x^3} + dx\sqrt[3]{2c^3 + d^3x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-d*x+c)/(d*x+c)/(d**3*x**3+2*c**3)**(1/3), x)`

[Out] `-Integral(-c/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/3)), x) - Integral(d*x/(c*(2*c**3 + d**3*x**3)**(1/3) + d*x*(2*c**3 + d**3*x**3)**(1/3)), x)`

$$3.49 \quad \int \frac{e+fx}{(c+dx)\sqrt[3]{-c^3+d^3x^3}} dx$$

Optimal. Leaf size=234

$$\frac{3(de - cf) \log\left(2^{2/3}d\sqrt[3]{d^3x^3 - c^3} + d(c - dx)\right)}{4\sqrt[3]{2}cd^2} + \frac{\sqrt{3}(de - cf) \tan^{-1}\left(\frac{1 - \sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3 - c^3}}\right)}{2\sqrt[3]{2}cd^2} - \frac{f \log\left(\sqrt[3]{d^3x^3 - c^3} - dx\right)}{2d^2} + \dots$$

Rubi [A] time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, number of rules / integrand size = 0.100, Rules used = {2152, 239, 2148}

$$\frac{3(de - cf) \log\left(2^{2/3}d\sqrt[3]{d^3x^3 - c^3} + d(c - dx)\right)}{4\sqrt[3]{2}cd^2} + \frac{\sqrt{3}(de - cf) \tan^{-1}\left(\frac{1 - \sqrt[3]{2}(c-dx)}{\sqrt[3]{d^3x^3 - c^3}}\right)}{2\sqrt[3]{2}cd^2} - \frac{f \log\left(\sqrt[3]{d^3x^3 - c^3} - dx\right)}{2d^2} + \frac{f \tan^{-1}\left(\frac{2dx}{\sqrt[3]{d^3x^3 - c^3}} + 1\right)}{\sqrt{3}d^2} + \frac{(de - cf) \log((c - dx)(c + dx)^2)}{4\sqrt[3]{2}cd^2}$$

Antiderivative was successfully verified.

[In] Int[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]

[Out] (f*ArcTan[(1 + (2*d*x)/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*d^2) + (Sqrt[3]*(d*e - c*f)*ArcTan[(1 - (2^(1/3)*(c - d*x))/(-c^3 + d^3*x^3)^(1/3))/Sqrt[3]]/(2*2^(1/3)*c*d^2) + ((d*e - c*f)*Log[(c - d*x)*(c + d*x)^2])/(4*2^(1/3)*c*d^2) - (f*Log[-(d*x) + (-c^3 + d^3*x^3)^(1/3)])/(2*d^2) - (3*(d*e - c*f)*Log[d*(c - d*x) + 2^(2/3)*d*(-c^3 + d^3*x^3)^(1/3)])/(4*2^(1/3)*c*d^2)

Rule 239

Int[((a_) + (b_.)*(x_)^3)^(-1/3), x_Symbol] := Simp[ArcTan[(1 + (2*Rt[b, 3]*x)/(a + b*x^3)^(1/3))/Sqrt[3]]/(Sqrt[3]*Rt[b, 3]), x] - Simp[Log[(a + b*x^3)^(1/3) - Rt[b, 3]*x]/(2*Rt[b, 3]), x] /; FreeQ[{a, b}, x]

Rule 2148

Int[1/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] := Simp[(Sqrt[3]*ArcTan[(1 - (2^(1/3)*Rt[b, 3]*(c - d*x))/(d*(a + b*x^3)^(1/3)))/Sqrt[3]]/(2^(4/3)*Rt[b, 3]*c), x] + (Simp[Log[(c + d*x)^2*(c - d*x)]/(2^(7/3)*Rt[b, 3]*c), x] - Simp[(3*Log[Rt[b, 3]*(c - d*x) + 2^(2/3)*d*(a + b*x^3)^(1/3)]/(2^(7/3)*Rt[b, 3]*c), x]) /; FreeQ[{a, b, c, d}, x] && EqQ[b*c^3 + a*d^3, 0]

Rule 2152

Int[((e_.) + (f_.)*(x_))/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*(x_)^3)^(1/3)), x_Symbol] := Dist[f/d, Int[1/(a + b*x^3)^(1/3), x], x] + Dist[(d*e - c*f)/d, Int[1/((c + d*x)*(a + b*x^3)^(1/3)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]

Rubi steps

$$\int \frac{e + fx}{(c + dx)\sqrt[3]{-c^3 + d^3x^3}} dx = \frac{f \int \frac{1}{\sqrt[3]{-c^3 + d^3x^3}} dx}{d} + \frac{(de - cf) \int \frac{1}{(c+dx)\sqrt[3]{-c^3 + d^3x^3}} dx}{d}$$

$$= \frac{f \tan^{-1} \left(\frac{1 + \frac{2dx}{\sqrt[3]{-c^3 + d^3x^3}}}{\sqrt{3}} \right)}{\sqrt{3} d^2} + \frac{\sqrt{3} (de - cf) \tan^{-1} \left(\frac{1 - \frac{\sqrt[3]{2}(c-dx)}{\sqrt[3]{-c^3 + d^3x^3}}}{\sqrt{3}} \right)}{2\sqrt[3]{2} cd^2} + \frac{(de - cf) \log((c - dx))}{4\sqrt[3]{2} cd^2}$$

Mathematica [F] time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{(c + dx)\sqrt[3]{-c^3 + d^3x^3}} dx$$

Verification is not applicable to the result.

[In] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]

[Out] Integrate[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]

IntegrateAlgebraic [F] time = 15.82, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{(c + dx)\sqrt[3]{-c^3 + d^3x^3}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e + f*x)/((c + d*x)*(-c^3 + d^3*x^3)^(1/3)), x]

[Out] $-1/2*(\text{Sqrt}[3]*e*\text{ArcTan}[(\text{Sqrt}[3]*(-c^3 + d^3*x^3)^{1/3})/(-2^{1/3}*c + 2^{1/3}*d*x + (-c^3 + d^3*x^3)^{1/3})])/(2^{1/3}*c*d) - (e*\text{Log}[2^{1/3}*c - 2^{1/3}*d*x + 2*(-c^3 + d^3*x^3)^{1/3}])/(2*2^{1/3}*c*d) + (e*\text{Log}[2^{2/3}*c^2 - 2*2^{2/3}*c*d*x + 2^{2/3}*d^2*x^2 + (-2*2^{1/3}*c + 2*2^{1/3}*d*x)*(-c^3 + d^3*x^3)^{1/3} + 4*(-c^3 + d^3*x^3)^{2/3}])/(4*2^{1/3}*c*d) + \text{Defer}[\text{IntegrateAlgebraic}[(f*x)/((c + d*x)*(-c^3 + d^3*x^3)^{1/3}), x]$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx + e}{(d^3x^3 - c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3), x, algorithm="giac")

[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{fx + e}{(dx + c)(d^3x^3 - c^3)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)

[Out] int((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{fx + e}{(d^3x^3 - c^3)^{\frac{1}{3}}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)/(d*x+c)/(d^3*x^3-c^3)^(1/3),x, algorithm="maxima")

[Out] integrate((f*x + e)/((d^3*x^3 - c^3)^(1/3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e + fx}{(d^3x^3 - c^3)^{1/3}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)),x)

[Out] int((e + f*x)/((d^3*x^3 - c^3)^(1/3)*(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e + fx}{\sqrt[3]{(-c + dx)(c^2 + cdx + d^2x^2)}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x+e)/(d*x+c)/(d**3*x**3-c**3)**(1/3),x)

[Out] Integral((e + f*x)/(((-c + d*x)*(c**2 + c*d*x + d**2*x**2))**(1/3)*(c + d*x)), x)

3.50 $\int x^2(a + bx)^n (c + dx^3) dx$

Optimal. Leaf size=160

$$-\frac{a(2b^3c - 5a^3d)(a + bx)^{n+2}}{b^6(n+2)} + \frac{(b^3c - 10a^3d)(a + bx)^{n+3}}{b^6(n+3)} + \frac{10a^2d(a + bx)^{n+4}}{b^6(n+4)} + \frac{a^2(b^3c - a^3d)(a + bx)^{n+1}}{b^6(n+1)} - \frac{5ad(a + bx)^{n+5}}{b^6(n+5)} + \frac{d(a + bx)^{n+6}}{b^6(n+6)}$$

Rubi [A] time = 0.11, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {1620}

$$\frac{a^2(b^3c - a^3d)(a + bx)^{n+1}}{b^6(n+1)} - \frac{a(2b^3c - 5a^3d)(a + bx)^{n+2}}{b^6(n+2)} + \frac{(b^3c - 10a^3d)(a + bx)^{n+3}}{b^6(n+3)} + \frac{10a^2d(a + bx)^{n+4}}{b^6(n+4)} - \frac{5ad(a + bx)^{n+5}}{b^6(n+5)} + \frac{d(a + bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x)^n*(c + d*x^3), x]

[Out] (a^2*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^6*(1 + n)) - (a*(2*b^3*c - 5*a^3*d)*(a + b*x)^(2 + n))/(b^6*(2 + n)) + ((b^3*c - 10*a^3*d)*(a + b*x)^(3 + n))/(b^6*(3 + n)) + (10*a^2*d*(a + b*x)^(4 + n))/(b^6*(4 + n)) - (5*a*d*(a + b*x)^(5 + n))/(b^6*(5 + n)) + (d*(a + b*x)^(6 + n))/(b^6*(6 + n))

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E xpon[Px, x], 2]]

Rubi steps

$$\begin{aligned} \int x^2(a + bx)^n (c + dx^3) dx &= \int \left(\frac{(a^2b^3c - a^5d)(a + bx)^n}{b^5} + \frac{a(-2b^3c + 5a^3d)(a + bx)^{1+n}}{b^5} + \frac{(b^3c - 10a^3d)(a + bx)^{2+n}}{b^5} \right) dx \\ &= \frac{a^2(b^3c - a^3d)(a + bx)^{1+n}}{b^6(1+n)} - \frac{a(2b^3c - 5a^3d)(a + bx)^{2+n}}{b^6(2+n)} + \frac{(b^3c - 10a^3d)(a + bx)^{3+n}}{b^6(3+n)} \end{aligned}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 0.83

$$\frac{(a + bx)^{n+1} \left(\frac{(a+bx)^2(b^3c-10a^3d)}{n+3} + \frac{a(a+bx)(5a^3d-2b^3c)}{n+2} + \frac{10a^2d(a+bx)^3}{n+4} + \frac{a^2b^3c-a^5d}{n+1} + \frac{d(a+bx)^5}{n+6} - \frac{5ad(a+bx)^4}{n+5} \right)}{b^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3), x]

[Out] ((a + b*x)^(1 + n)*((a^2*b^3*c - a^5*d)/(1 + n) + (a*(-2*b^3*c + 5*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - 10*a^3*d)*(a + b*x)^2)/(3 + n) + (10*a^2*d*(a + b*x)^3)/(4 + n) - (5*a*d*(a + b*x)^4)/(5 + n) + (d*(a + b*x)^5)/(6 + n))/b^6

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x^2(a + bx)^n (c + dx^3) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3),x]
[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3), x]
fricas [B]   time = 1.25, size = 490, normalized size = 3.06
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="fricas")
[Out] (2*a^3*b^3*c*n^3 + 30*a^3*b^3*c*n^2 + 148*a^3*b^3*c*n + 240*a^3*b^3*c - 120
*a^6*d + (b^6*d*n^5 + 15*b^6*d*n^4 + 85*b^6*d*n^3 + 225*b^6*d*n^2 + 274*b^6
*d*n + 120*b^6*d)*x^6 + (a*b^5*d*n^5 + 10*a*b^5*d*n^4 + 35*a*b^5*d*n^3 + 50
*a*b^5*d*n^2 + 24*a*b^5*d*n)*x^5 - 5*(a^2*b^4*d*n^4 + 6*a^2*b^4*d*n^3 + 11*
a^2*b^4*d*n^2 + 6*a^2*b^4*d*n)*x^4 + (b^6*c*n^5 + 18*b^6*c*n^4 + 240*b^6*c
+ (121*b^6*c + 20*a^3*b^3*d)*n^3 + 12*(31*b^6*c + 5*a^3*b^3*d)*n^2 + 4*(127
*b^6*c + 10*a^3*b^3*d)*n)*x^3 + (a*b^5*c*n^5 + 16*a*b^5*c*n^4 + 89*a*b^5*c*
n^3 + 2*(97*a*b^5*c - 30*a^4*b^2*d)*n^2 + 60*(2*a*b^5*c - a^4*b^2*d)*n)*x^2
- 2*(a^2*b^4*c*n^4 + 15*a^2*b^4*c*n^3 + 74*a^2*b^4*c*n^2 + 60*(2*a^2*b^4*c
- a^5*b*d)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6
*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

giac [B] time = 0.40, size = 835, normalized size = 5.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="giac")
[Out] ((b*x + a)^n*b^6*d*n^5*x^6 + (b*x + a)^n*a*b^5*d*n^5*x^5 + 15*(b*x + a)^n*b
^6*d*n^4*x^6 + 10*(b*x + a)^n*a*b^5*d*n^4*x^5 + 85*(b*x + a)^n*b^6*d*n^3*x^
6 + (b*x + a)^n*b^6*c*n^5*x^3 - 5*(b*x + a)^n*a^2*b^4*d*n^4*x^4 + 35*(b*x +
a)^n*a*b^5*d*n^3*x^5 + 225*(b*x + a)^n*b^6*d*n^2*x^6 + (b*x + a)^n*a*b^5*c
*n^5*x^2 + 18*(b*x + a)^n*b^6*c*n^4*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n^3*x^4
+ 50*(b*x + a)^n*a*b^5*d*n^2*x^5 + 274*(b*x + a)^n*b^6*d*n*x^6 + 16*(b*x +
a)^n*a*b^5*c*n^4*x^2 + 121*(b*x + a)^n*b^6*c*n^3*x^3 + 20*(b*x + a)^n*a^3*b
^3*d*n^3*x^3 - 55*(b*x + a)^n*a^2*b^4*d*n^2*x^4 + 24*(b*x + a)^n*a*b^5*d*n*
x^5 + 120*(b*x + a)^n*b^6*d*x^6 - 2*(b*x + a)^n*a^2*b^4*c*n^4*x + 89*(b*x +
a)^n*a*b^5*c*n^3*x^2 + 372*(b*x + a)^n*b^6*c*n^2*x^3 + 60*(b*x + a)^n*a^3*
b^3*d*n^2*x^3 - 30*(b*x + a)^n*a^2*b^4*d*n*x^4 - 30*(b*x + a)^n*a^2*b^4*c*n
^3*x + 194*(b*x + a)^n*a*b^5*c*n^2*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n^2*x^2 +
508*(b*x + a)^n*b^6*c*n*x^3 + 40*(b*x + a)^n*a^3*b^3*d*n*x^3 + 2*(b*x + a)
^n*a^3*b^3*c*n^3 - 148*(b*x + a)^n*a^2*b^4*c*n^2*x + 120*(b*x + a)^n*a*b^5*
c*n*x^2 - 60*(b*x + a)^n*a^4*b^2*d*n*x^2 + 240*(b*x + a)^n*b^6*c*x^3 + 30*(
b*x + a)^n*a^3*b^3*c*n^2 - 240*(b*x + a)^n*a^2*b^4*c*n*x + 120*(b*x + a)^n*
a^5*b*d*n*x + 148*(b*x + a)^n*a^3*b^3*c*n + 240*(b*x + a)^n*a^3*b^3*c - 120
*(b*x + a)^n*a^6*d)/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 162
4*b^6*n^2 + 1764*b^6*n + 720*b^6)
```

maple [B] time = 0.01, size = 451, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(b*x+a)^n*(d*x^3+c),x)
[Out] -(b*x+a)^(n+1)*(-b^5*d*n^5*x^5-15*b^5*d*n^4*x^5+5*a*b^4*d*n^4*x^4-85*b^5*d*
n^3*x^5+50*a*b^4*d*n^3*x^4-b^5*c*n^5*x^2-225*b^5*d*n^2*x^5-20*a^2*b^3*d*n^3
*x^3+175*a*b^4*d*n^2*x^4-18*b^5*c*n^4*x^2-274*b^5*d*n*x^5-120*a^2*b^3*d*n^2
```

```
*x^3+2*a*b^4*c*n^4*x+250*a*b^4*d*n*x^4-121*b^5*c*n^3*x^2-120*b^5*d*x^5+60*a^3*b^2*d*n^2*x^2-220*a^2*b^3*d*n*x^3+32*a*b^4*c*n^3*x+120*a*b^4*d*x^4-372*b^5*c*n^2*x^2+180*a^3*b^2*d*n*x^2-2*a^2*b^3*c*n^3-120*a^2*b^3*d*x^3+178*a*b^4*c*n^2*x-508*b^5*c*n*x^2-120*a^4*b*d*n*x+120*a^3*b^2*d*x^2-30*a^2*b^3*c*n^2+388*a*b^4*c*n*x-240*b^5*c*x^2-120*a^4*b*d*x-148*a^2*b^3*c*n+240*a*b^4*c*x+120*a^5*d-240*a^2*b^3*c)/b^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)
```

maxima [A] time = 0.98, size = 253, normalized size = 1.58

$$\frac{((n^2 + 3n + 2)b^3x^3 + (n^2 + n)ab^2x^2 - 2a^2bnx + 2a^3)(bx + a)^c}{(n^3 + 6n^2 + 11n + 6)b^3} + \frac{((n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^6x^6 + (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n)ab^5x^5 - 5(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + 20(n^3 + 3n^2 + 2n)a^3b^3x^3 - 60(n^2 + n)a^4b^2x^2 + 120a^5b^1x - 120a^6)(bx + a)^d}{(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c/((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
```

mupad [B] time = 3.20, size = 495, normalized size = 3.09

$$\frac{c \cdot x^{n+1} \left(\frac{d^2 (n^3 + 15n^2 + 85n + 225) + 274n + 120}{(n^3 + 6n^2 + 11n + 6)^2} \right) + \frac{2ab^2d^2 (n^3 + 15n^2 + 85n + 225) + 74n^3 + 120n^2}{(n^3 + 6n^2 + 11n + 6)^2} + \frac{d^2 (n^3 + 3n + 2) (20ab^2d^2 + 15n^3 + 74n^2 + 120n)}{P^2} + \frac{2d^2nx \left(-40bd^2 + 15n^3 + 74n^2 + 120n \right)}{P^2} + \frac{ad^2x^2 \left(n^3 + 10n^2 + 35n + 50 \right) + 24}{P^2} + \frac{a^2d^2x^3 \left(n^3 + 3n^2 + 2n \right) - 60(n^2 + n)a^4b^2x^2 + 120a^5b^1x - 120a^6}{P^2} + \frac{5d^2dx^4 \left(n^3 + 3n^2 + 2n \right)}{P^2}}{(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c + d*x^3)*(a + b*x)^n,x)

```
[Out] (a + b*x)^n*((d*x^6*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720) + (2*a^3*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^6*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (x^3*(3*n + n^2 + 2)*(120*b^3*c + 15*b^3*c*n^2 + b^3*c*n^3 + 20*a^3*d*n + 74*b^3*c*n))/(b^3*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (2*a^2*n*x*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^5*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*d*n*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) + (a*n*x^2*(n + 1)*(120*b^3*c - 60*a^3*d + 15*b^3*c*n^2 + b^3*c*n^3 + 74*b^3*c*n))/(b^4*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) - (5*a^2*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b^2*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)))
```

sympy [A] time = 7.63, size = 6397, normalized size = 39.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b*x+a)**n*(d*x**3+c),x)

```
[Out] Piecewise((a**n*(c*x**3/3 + d*x**6/6), Eq(b, 0)), (60*a**5*d*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d*x*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d*x**2*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d*x**2/(60*a
```


$$\begin{aligned}
& *5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a \\
& *b**10*x**4 + 60*b**11*x**5) - 2*a**2*b**3*c/(60*a**5*b**6 + 300*a**4*b**7*x \\
& x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x** \\
& **5) + 600*a**2*b**3*d*x**3*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + \\
& 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) \\
& + 900*a**2*b**3*d*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x** \\
& 2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 10*a*b**4*c*x/ \\
& (60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + \\
& 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4*log(a/b + x)/(60*a** \\
& 5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a* \\
& b**10*x**4 + 60*b**11*x**5) + 300*a*b**4*d*x**4/(60*a**5*b**6 + 300*a**4*b* \\
& *7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**1 \\
& 1*x**5) - 20*b**5*c*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x* \\
& *2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 60*b**5*d*x** \\
& 5*log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a \\
& **2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5), Eq(n, -6)), (-60*a**5*d* \\
& log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9 \\
& *x**3 + 12*b**10*x**4) - 125*a**5*d/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a** \\
& 2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**4*b*d*x*log(a/b + x) \\
& /(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b \\
& **10*x**4) - 440*a**4*b*d*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x \\
& **2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 360*a**3*b**2*d*x**2*log(a/b + x)/(\\
& 12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b** \\
& 10*x**4) - 540*a**3*b**2*d*x**2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b* \\
& *8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - a**2*b**3*c/(12*a**4*b**6 + 48* \\
& a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a** \\
& 2*b**3*d*x**3*log(a/b + x)/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x* \\
& *2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 240*a**2*b**3*d*x**3/(12*a**4*b**6 + \\
& 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 4*a \\
& *b**4*c*x/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x* \\
& *3 + 12*b**10*x**4) - 60*a*b**4*d*x**4*log(a/b + x)/(12*a**4*b**6 + 48*a**3 \\
& *b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12*b**10*x**4) - 6*b**5*c*x* \\
& *2/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8*x**2 + 48*a*b**9*x**3 + 12 \\
& *b**10*x**4) + 12*b**5*d*x**5/(12*a**4*b**6 + 48*a**3*b**7*x + 72*a**2*b**8 \\
& *x**2 + 48*a*b**9*x**3 + 12*b**10*x**4), Eq(n, -5)), (60*a**5*d*log(a/b + x) \\
&)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 110*a**5* \\
& d/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**4* \\
& b*d*x*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9* \\
& x**3) + 270*a**4*b*d*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b \\
& **9*x**3) + 180*a**3*b**2*d*x**2*log(a/b + x)/(6*a**3*b**6 + 18*a**2*b**7*x \\
& + 18*a*b**8*x**2 + 6*b**9*x**3) + 180*a**3*b**2*d*x**2/(6*a**3*b**6 + 18*a \\
& **2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 2*a**2*b**3*c/(6*a**3*b**6 + 1 \\
& 8*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) + 60*a**2*b**3*d*x**3*log(a/b \\
& + x)/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 6*a*b \\
& **4*c*x/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) - 15* \\
& a*b**4*d*x**4/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x**3) \\
& - 6*b**5*c*x**2/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9*x* \\
& *3) + 3*b**5*d*x**5/(6*a**3*b**6 + 18*a**2*b**7*x + 18*a*b**8*x**2 + 6*b**9 \\
& *x**3), Eq(n, -4)), (-60*a**5*d*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6 \\
& *b**8*x**2) - 90*a**5*d/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a** \\
& 4*b*d*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 120*a**4*b \\
& *d*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 60*a**3*b**2*d*x**2*log(a/ \\
& b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*a**2*b**3*c*log(a/b + \\
& x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 9*a**2*b**3*c/(6*a**2*b**6 + \\
& 12*a*b**7*x + 6*b**8*x**2) + 20*a**2*b**3*d*x**3/(6*a**2*b**6 + 12*a*b**7* \\
& x + 6*b**8*x**2) + 12*a*b**4*c*x*log(a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + \\
& 6*b**8*x**2) + 12*a*b**4*c*x/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) - 5* \\
& a*b**4*d*x**4/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 6*b**5*c*x**2*log \\
& (a/b + x)/(6*a**2*b**6 + 12*a*b**7*x + 6*b**8*x**2) + 2*b**5*d*x**5/(6*a**2
\end{aligned}$$

$$\begin{aligned}
& *b^{**6} + 12*a*b^{**7}*x + 6*b^{**8}*x^{**2}), \text{Eq}(n, -3)), (60*a^{**5}*d*\log(a/b + x)/(12 \\
& *a*b^{**6} + 12*b^{**7}*x) + 60*a^{**5}*d/(12*a*b^{**6} + 12*b^{**7}*x) + 60*a^{**4}*b*d*x*\log \\
& (a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) - 30*a^{**3}*b^{**2}*d*x^{**2}/(12*a*b^{**6} + 12*b* \\
& *7*x) - 24*a^{**2}*b^{**3}*c*\log(a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) - 24*a^{**2}*b^{**3}* \\
& c/(12*a*b^{**6} + 12*b^{**7}*x) + 10*a^{**2}*b^{**3}*d*x^{**3}/(12*a*b^{**6} + 12*b^{**7}*x) - 2 \\
& 4*a*b^{**4}*c*x*\log(a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) - 5*a*b^{**4}*d*x^{**4}/(12*a*b \\
& **6 + 12*b^{**7}*x) + 12*b^{**5}*c*x^{**2}/(12*a*b^{**6} + 12*b^{**7}*x) + 3*b^{**5}*d*x^{**5}/(\\
& 12*a*b^{**6} + 12*b^{**7}*x), \text{Eq}(n, -2)), (-a^{**5}*d*\log(a/b + x)/b^{**6} + a^{**4}*d*x/b \\
& **5 - a^{**3}*d*x^{**2}/(2*b^{**4}) + a^{**2}*c*\log(a/b + x)/b^{**3} + a^{**2}*d*x^{**3}/(3*b^{**3} \\
&) - a*c*x/b^{**2} - a*d*x^{**4}/(4*b^{**2}) + c*x^{**2}/(2*b) + d*x^{**5}/(5*b), \text{Eq}(n, -1) \\
&), (-120*a^{**6}*d*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 73 \\
& 5*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 120*a^{**5}*b*d*n*x*(\\
& a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 162 \\
& 4*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 60*a^{**4}*b^{**2}*d*n*x^{**2}*(a + b*x)* \\
& *n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n* \\
& *2 + 1764*b^{**6}*n + 720*b^{**6}) - 60*a^{**4}*b^{**2}*d*n*x^{**2}*(a + b*x)**n/(b^{**6}*n^{**} \\
& 6 + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b* \\
& *6*n + 720*b^{**6}) + 2*a^{**3}*b^{**3}*c*n^{**3}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**} \\
& 5 + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6} \\
&) + 30*a^{**3}*b^{**3}*c*n^{**2}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n \\
& **4 + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 148*a^{**3}*b \\
& **3*c*n*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n \\
& **3 + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 240*a^{**3}*b^{**3}*c*(a + b*x)* \\
& *n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n* \\
& *2 + 1764*b^{**6}*n + 720*b^{**6}) + 20*a^{**3}*b^{**3}*d*n^{**3}*x^{**3}*(a + b*x)**n/(b^{**6}* \\
& n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764 \\
& *b^{**6}*n + 720*b^{**6}) + 60*a^{**3}*b^{**3}*d*n^{**2}*x^{**3}*(a + b*x)**n/(b^{**6}*n^{**6} + 21 \\
& *b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + \\
& 720*b^{**6}) + 40*a^{**3}*b^{**3}*d*n*x^{**3}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + \\
& 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - \\
& 2*a^{**2}*b^{**4}*c*n^{**4}*x*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**} \\
& 4 + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 30*a^{**2}*b^{**4} \\
& *c*n^{**3}*x*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6} \\
& *n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 148*a^{**2}*b^{**4}*c*n^{**2}*x*(\\
& a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 162 \\
& 4*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 240*a^{**2}*b^{**4}*c*n*x*(a + b*x)**n/(b \\
& **6*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + \\
& 1764*b^{**6}*n + 720*b^{**6}) - 5*a^{**2}*b^{**4}*d*n^{**4}*x^{**4}*(a + b*x)**n/(b^{**6}*n^{**6} + \\
& 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}* \\
& n + 720*b^{**6}) - 30*a^{**2}*b^{**4}*d*n^{**3}*x^{**4}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}* \\
& n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b \\
& **6) - 55*a^{**2}*b^{**4}*d*n^{**2}*x^{**4}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 17 \\
& 5*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 30 \\
& *a^{**2}*b^{**4}*d*n*x^{**4}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} \\
& + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + a*b^{**5}*c*n^{**5}* \\
& x^{**2}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} \\
& + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 16*a*b^{**5}*c*n^{**4}*x^{**2}*(a + b* \\
& x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6} \\
& *n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 89*a*b^{**5}*c*n^{**3}*x^{**2}*(a + b*x)**n/(b^{**6}* \\
& n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764 \\
& *b^{**6}*n + 720*b^{**6}) + 194*a*b^{**5}*c*n^{**2}*x^{**2}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b \\
& **6*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 7 \\
& 20*b^{**6}) + 120*a*b^{**5}*c*n*x^{**2}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175 \\
& *b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + a*b \\
& **5*d*n^{**5}*x^{**5}*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 73 \\
& 5*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 10*a*b^{**5}*d*n^{**4}*x \\
& **5*(a + b*x)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} \\
& + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 35*a*b^{**5}*d*n^{**3}*x^{**5}*(a + b*x \\
&)**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*
\end{aligned}$$

```

n**2 + 1764*b**6*n + 720*b**6) + 50*a*b**5*d*n**2*x**5*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 24*a*b**5*d*n*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
*6) + b**6*c*n**5*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
*4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 18*b**6*c*n
**4*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 121*b**6*c*n**3*x**3*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b
**6*n**2 + 1764*b**6*n + 720*b**6) + 372*b**6*c*n**2*x**3*(a + b*x)**n/(b**
6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 508*b**6*c*n*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6
*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*
b**6) + 240*b**6*c*x**3*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + b**6*d*n**
5*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 15*b**6*d*n**4*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6
*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*d*n**3*x**6*(a + b*x)**n/(b**6*n*
*6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b
**6*n + 720*b**6) + 225*b**6*d*n**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*
n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b
**6) + 274*b**6*d*n*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*
n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*
d*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n*
*3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True))

```

3.51 $\int x(a + bx)^n (c + dx^3) dx$

Optimal. Leaf size=126

$$-\frac{a(b^3c - a^3d)(a + bx)^{n+1}}{b^5(n+1)} + \frac{(b^3c - 4a^3d)(a + bx)^{n+2}}{b^5(n+2)} + \frac{6a^2d(a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a + bx)^{n+4}}{b^5(n+4)} + \frac{d(a + bx)^{n+5}}{b^5(n+5)}$$

Rubi [A] time = 0.07, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {1620}

$$-\frac{a(b^3c - a^3d)(a + bx)^{n+1}}{b^5(n+1)} + \frac{(b^3c - 4a^3d)(a + bx)^{n+2}}{b^5(n+2)} + \frac{6a^2d(a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a + bx)^{n+4}}{b^5(n+4)} + \frac{d(a + bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)^n*(c + d*x^3), x]

[Out] -((a*(b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^5*(1 + n))) + ((b^3*c - 4*a^3*d)*(a + b*x)^(2 + n))/(b^5*(2 + n)) + (6*a^2*d*(a + b*x)^(3 + n))/(b^5*(3 + n)) - (4*a*d*(a + b*x)^(4 + n))/(b^5*(4 + n)) + (d*(a + b*x)^(5 + n))/(b^5*(5 + n))

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int x(a + bx)^n (c + dx^3) dx &= \int \left(\frac{a(-b^3c + a^3d)(a + bx)^n}{b^4} + \frac{(b^3c - 4a^3d)(a + bx)^{1+n}}{b^4} + \frac{6a^2d(a + bx)^{2+n}}{b^4} - \frac{4ad(a + bx)^{3+n}}{b^4} \right. \\ &\quad \left. + \frac{d(a + bx)^{4+n}}{b^4} \right) dx \\ &= -\frac{a(b^3c - a^3d)(a + bx)^{1+n}}{b^5(1+n)} + \frac{(b^3c - 4a^3d)(a + bx)^{2+n}}{b^5(2+n)} + \frac{6a^2d(a + bx)^{3+n}}{b^5(3+n)} - \frac{4ad(a + bx)^{4+n}}{b^5(4+n)} + \frac{d(a + bx)^{5+n}}{b^5(5+n)} \end{aligned}$$

Mathematica [A] time = 0.08, size = 104, normalized size = 0.83

$$\frac{(a + bx)^{n+1} \left(\frac{(a+bx)(b^3c-4a^3d)}{n+2} + \frac{a(a^3d-b^3c)}{n+1} + \frac{6a^2d(a+bx)^2}{n+3} + \frac{d(a+bx)^4}{n+5} - \frac{4ad(a+bx)^3}{n+4} \right)}{b^5}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)^n*(c + d*x^3), x]

[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d))/(1 + n) + ((b^3*c - 4*a^3*d)*(a + b*x))/(2 + n) + (6*a^2*d*(a + b*x)^2)/(3 + n) - (4*a*d*(a + b*x)^3)/(4 + n) + (d*(a + b*x)^4)/(5 + n)))/b^5

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int x(a + bx)^n (c + dx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3), x]

[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3), x]

fricas [B] time = 1.62, size = 348, normalized size = 2.76

$$\frac{(d^2 b^3 c^3 + 12 d^2 b^3 c^2 + 47 d^2 b^3 c - 24 d^2 - (b^5 d^4 + 10 b^5 d^3 + 35 b^5 d^2 + 50 b^5 d + 24 b^5) d^3 - (a b^4 d^4 + 6 a b^4 d^3 + 11 a b^4 d^2 + 6 a b^4 d) d^4 + 4 (d^2 b^3 d^2 + 3 d^2 b^3 d) d^3 - (b^5 c^4 + 13 b^5 c^3 + 60 b^5 c^2 + (59 b^5 c + 12 a^2 b^2 d) d^2 + (107 b^5 c + 12 a^2 b^2 d) d) d^2 - (a^2 c^4 + 12 a^2 c^3 + 47 a^2 c^2 + 12 (5 a^2 c - 2 a^2 b d) d) d c + a^2}{b^5 d^5 + 15 b^5 d^4 + 85 b^5 d^3 + 225 b^5 d^2 + 274 b^5 d + 120 b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c), x, algorithm="fricas")

[Out] $-(a^2 b^3 c n^3 + 12 a^2 b^3 c n^2 + 47 a^2 b^3 c n + 60 a^2 b^3 c - 24 a^5 * d - (b^5 d n^4 + 10 b^5 d n^3 + 35 b^5 d n^2 + 50 b^5 d n + 24 b^5 d) * x^5 - (a b^4 d n^4 + 6 a b^4 d n^3 + 11 a b^4 d n^2 + 6 a b^4 d n) * x^4 + 4 * (a^2 b^3 d n^3 + 3 a^2 b^3 d n^2 + 2 a^2 b^3 d n) * x^3 - (b^5 c n^4 + 13 b^5 c n^3 + 60 b^5 c + (59 b^5 c + 12 a^3 b^2 d) * n^2 + (107 b^5 c + 12 a^3 b^2 d) * n) * x^2 - (a b^4 c n^4 + 12 a b^4 c n^3 + 47 a b^4 c n^2 + 12 * (5 a b^4 c - 2 a^4 b d) * n) * x) * (b * x + a)^n / (b^5 n^5 + 15 b^5 n^4 + 85 b^5 n^3 + 225 b^5 n^2 + 274 b^5 n + 120 b^5)$

giac [B] time = 0.38, size = 577, normalized size = 4.58

$$\frac{(b^4 d n^4 + 10 b^4 d n^3 + 4 a b^3 d n^3 + 35 b^4 d n^2 + 50 b^4 d n + 24 b^4 d) n^4 - 4 a b^3 d n^3 + 11 a b^3 d n^2 + 6 a b^3 d n + 24 b^3 d}{(n^5 + 15 n^4 + 85 n^3 + 225 n^2 + 274 n + 120) b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c), x, algorithm="giac")

[Out] $((b * x + a)^n * b^5 * d * n^4 * x^5 + (b * x + a)^n * a * b^4 * d * n^4 * x^4 + 10 * (b * x + a)^n * b^5 * d * n^3 * x^5 + 6 * (b * x + a)^n * a * b^4 * d * n^3 * x^4 + 35 * (b * x + a)^n * b^5 * d * n^2 * x^5 + (b * x + a)^n * b^5 * c * n^4 * x^2 - 4 * (b * x + a)^n * a^2 * b^3 * d * n^3 * x^3 + 11 * (b * x + a)^n * a * b^4 * d * n^2 * x^4 + 50 * (b * x + a)^n * b^5 * d * n * x^5 + (b * x + a)^n * a * b^4 * c * n^4 * x + 13 * (b * x + a)^n * b^5 * c * n^3 * x^2 - 12 * (b * x + a)^n * a^2 * b^3 * d * n^2 * x^3 + 6 * (b * x + a)^n * a * b^4 * d * n * x^4 + 24 * (b * x + a)^n * b^5 * d * x^5 + 12 * (b * x + a)^n * a * b^4 * c * n^3 * x + 59 * (b * x + a)^n * b^5 * c * n^2 * x^2 + 12 * (b * x + a)^n * a^3 * b^2 * d * n^2 * x^2 - 8 * (b * x + a)^n * a^2 * b^3 * d * n * x^3 - (b * x + a)^n * a^2 * b^3 * c * n^3 + 47 * (b * x + a)^n * a * b^4 * c * n^2 * x + 107 * (b * x + a)^n * b^5 * c * n * x^2 + 12 * (b * x + a)^n * a^3 * b^2 * d * n * x^2 - 12 * (b * x + a)^n * a^2 * b^3 * c * n^2 + 60 * (b * x + a)^n * a * b^4 * c * n * x - 24 * (b * x + a)^n * a^4 * b * d * n * x + 60 * (b * x + a)^n * b^5 * c * x^2 - 47 * (b * x + a)^n * a^2 * b^3 * c * n - 60 * (b * x + a)^n * a^2 * b^3 * c + 24 * (b * x + a)^n * a^5 * d) / (b^5 n^5 + 15 b^5 n^4 + 85 b^5 n^3 + 225 b^5 n^2 + 274 b^5 n + 120 b^5)$

maple [B] time = 0.01, size = 283, normalized size = 2.25

$$\frac{(b^4 d n^4 + 10 b^4 d n^3 + 4 a b^3 d n^3 + 35 b^4 d n^2 + 50 b^4 d n + 24 b^4 d) n^4 - 4 a b^3 d n^3 + 11 a b^3 d n^2 + 6 a b^3 d n + 24 b^3 d}{(n^5 + 15 n^4 + 85 n^3 + 225 n^2 + 274 n + 120) b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x+a)^n*(d*x^3+c), x)

[Out] $(b * x + a)^{(n+1)} * (b^4 * d * n^4 * x^4 + 10 * b^4 * d * n^3 * x^4 - 4 * a * b^3 * d * n^3 * x^3 + 35 * b^4 * d * n^2 * x^4 - 24 * a * b^3 * d * n^2 * x^3 + b^4 * c * n^4 * x + 50 * b^4 * d * n * x^4 + 12 * a^2 * b^2 * d * n^2 * x^2 - 44 * a * b^3 * d * n * x^3 + 13 * b^4 * c * n^3 * x + 24 * b^4 * d * x^4 + 36 * a^2 * b^2 * d * n * x^2 - a * b^3 * c * n^3 - 2 * 4 * a * b^3 * d * x^3 + 59 * b^4 * c * n^2 * x - 24 * a^3 * b * d * n * x + 24 * a^2 * b^2 * d * x^2 - 12 * a * b^3 * c * n^2 + 107 * b^4 * c * n * x - 24 * a^3 * b * d * x - 47 * a * b^3 * c * n + 60 * b^4 * c * x + 24 * a^4 * d - 60 * a * b^3 * c) / b^5 / (n^5 + 15 n^4 + 85 n^3 + 225 n^2 + 274 n + 120)$

maxima [A] time = 1.02, size = 184, normalized size = 1.46

$$\frac{(b^2(n+1)x^2 + abnx - a^2)(bx + a)^n c}{(n^2 + 3n + 2)b^2} + \frac{((n^4 + 10n^3 + 35n^2 + 50n + 24)b^5 x^5 + (n^4 + 6n^3 + 11n^2 + 6n)ab^4 x^4 - 4(n^3 + 3n^2 + 2n)a^2 b^3 x^3 + 12(n^2 + n)a^3 b^2 x^2 - 24a^4 bnx + 24a^5)(bx + a)^n d}{(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c),x, algorithm="maxima")

[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c/((n^2 + 3*n + 2)*b^2) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)

mupad [B] time = 2.95, size = 363, normalized size = 2.88

$$(a + b*x)^n \left(\frac{d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)}{b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} - \frac{a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n)}{b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} + \frac{x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b^3*c*n)}{b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} + \frac{a*n*x*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n)}{b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} + \frac{a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6)}{b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} - \frac{4*a^2*d*n*x^3*(3*n + n^2 + 2)}{b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c + d*x^3)*(a + b*x)^n,x)

[Out] (a + b*x)^n*((d*x^5*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120) - (a^2*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^5*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (x^2*(n + 1)*(60*b^3*c + 12*b^3*c*n^2 + b^3*c*n^3 + 12*a^3*d*n + 47*b^3*c*n))/(b^3*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*n*x*(60*b^3*c - 24*a^3*d + 12*b^3*c*n^2 + b^3*c*n^3 + 47*b^3*c*n))/(b^4*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) + (a*d*n*x^4*(11*n + 6*n^2 + n^3 + 6))/(b*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)) - (4*a^2*d*n*x^3*(3*n + n^2 + 2))/(b^2*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)))

sympy [A] time = 4.86, size = 3704, normalized size = 29.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**n*(d*x**3+c),x)

[Out] Piecewise((a**n*(c*x**2/2 + d*x**5/5), Eq(b, 0)), (12*a**4*d*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 25*a**4*d/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a**3*b*d*x*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 88*a**3*b*d*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 72*a**2*b**2*d*x**2*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 108*a**2*b**2*d*x**2/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - a*b**3*c/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 48*a*b**3*d*x**3*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) - 4*b**4*c*x/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4) + 12*b**4*d*x**4*log(a/b + x)/(12*a**4*b**5 + 48*a**3*b**6*x + 72*a**2*b**7*x**2 + 48*a*b**8*x**3 + 12*b**9*x**4), Eq(n, -5)), (-24*a**4*d*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 44*a**4*d/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**3*b*d*x*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 108*a**3*b*d*x/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**2*d*x**2*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 72*a**2*b**2*d*x**2/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - a*b**3*c/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 24*a*b**3*d*x**3*log(a/b + x)/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) - 3*b**4*c*x/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3) + 6*b**4*d*x**4/(6*a**3*b**5 + 18*a**2*b**6*x + 18*a*b**7*x**2 + 6*b**8*x**3), Eq(n, -4)), (12*a**4*d*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**4*d/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 18*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 108*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 72*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 72*a**2*b**2*d*x**2/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 24*a*b**3*d*x**3*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 3*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 6*b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2))

$6*x + 2*b**7*x**2) + 24*a**3*b*d*x*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 24*a**3*b*d*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + 12*a**2*b**2*d*x**2*log(a/b + x)/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - a*b**3*c/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 4*a*b**3*d*x**3/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) - 2*b**4*c*x/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2) + b**4*d*x**4/(2*a**2*b**5 + 4*a*b**6*x + 2*b**7*x**2), Eq(n, -3), (-12*a**4*d*log(a/b + x)/(3*a*b**5 + 3*b**6*x) - 12*a**4*d/(3*a*b**5 + 3*b**6*x) - 12*a**3*b*d*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 6*a**2*b**2*d*x**2/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + 3*a*b**3*c/(3*a*b**5 + 3*b**6*x) - 2*a*b**3*d*x**3/(3*a*b**5 + 3*b**6*x) + 3*b**4*c*x*log(a/b + x)/(3*a*b**5 + 3*b**6*x) + b**4*d*x**4/(3*a*b**5 + 3*b**6*x), Eq(n, -2)), (a**4*d*log(a/b + x)/b**5 - a**3*d*x/b**4 + a**2*d*x**2/(2*b**3) - a*c*log(a/b + x)/b**2 - a*d*x**3/(3*b**2) + c*x/b + d*x**4/(4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a**4*b*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - a**2*b**3*c*n**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*c*n**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 47*a**2*b**3*c*n*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 60*a**2*b**3*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a*b**4*c*n**3*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 47*a*b**4*c*n**2*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*a*b**4*c*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*c*n**4*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 13*b**5*c*n**3*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 59*b**5*c*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 107*b**5*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 60*b**5*c*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*n**3*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 50*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5*d*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True)$

3.52 $\int (a + bx)^n (c + dx^3) dx$

Optimal. Leaf size=94

$$\frac{(b^3c - a^3d)(a + bx)^{n+1}}{b^4(n+1)} + \frac{3a^2d(a + bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a + bx)^{n+3}}{b^4(n+3)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)}$$

Rubi [A] time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1850}

$$\frac{(b^3c - a^3d)(a + bx)^{n+1}}{b^4(n+1)} + \frac{3a^2d(a + bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a + bx)^{n+3}}{b^4(n+3)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^n*(c + d*x^3), x]

[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx)^n (c + dx^3) dx &= \int \left(\frac{(b^3c - a^3d)(a + bx)^n}{b^3} + \frac{3a^2d(a + bx)^{1+n}}{b^3} - \frac{3ad(a + bx)^{2+n}}{b^3} + \frac{d(a + bx)^{3+n}}{b^3} \right) dx \\ &= \frac{(b^3c - a^3d)(a + bx)^{1+n}}{b^4(1+n)} + \frac{3a^2d(a + bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a + bx)^{3+n}}{b^4(3+n)} + \frac{d(a + bx)^{4+n}}{b^4(4+n)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 94, normalized size = 1.00

$$\frac{(b^3c - a^3d)(a + bx)^{n+1}}{b^4(n+1)} + \frac{3a^2d(a + bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a + bx)^{n+3}}{b^4(n+3)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^n*(c + d*x^3), x]

[Out] ((b^3*c - a^3*d)*(a + b*x)^(1 + n))/(b^4*(1 + n)) + (3*a^2*d*(a + b*x)^(2 + n))/(b^4*(2 + n)) - (3*a*d*(a + b*x)^(3 + n))/(b^4*(3 + n)) + (d*(a + b*x)^(4 + n))/(b^4*(4 + n))

IntegrateAlgebraic [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (a + bx)^n (c + dx^3) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3), x]

[Out] Defer[IntegrateAlgebraic] [(a + b*x)^n*(c + d*x^3), x]

fricas [B] time = 0.79, size = 222, normalized size = 2.36

$$\frac{(ab^3cn^3 + 9ab^3cn^2 + 26ab^3cn + 24ab^3c - 6a^4d + (b^4dn^3 + 6b^4dn^2 + 11b^4dn + 6b^4d)x^4 + (ab^3dn^3 + 3ab^3dn^2 + 2ab^3dn)x^3 - 3(a^2b^2dn^2 + a^2b^2dn)x^2 + (b^4cn^3 + 9b^4cn^2 + 24b^4c + 2(13b^4c + 3a^3bdn)x)(bx + a)^n}{b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c), x, algorithm="fricas")

[Out] (a*b^3*c*n^3 + 9*a*b^3*c*n^2 + 26*a*b^3*c*n + 24*a*b^3*c - 6*a^4*d + (b^4*d*n^3 + 6*b^4*d*n^2 + 11*b^4*d*n + 6*b^4*d)*x^4 + (a*b^3*d*n^3 + 3*a*b^3*d*n^2 + 2*a*b^3*d*n)*x^3 - 3*(a^2*b^2*d*n^2 + a^2*b^2*d*n)*x^2 + (b^4*c*n^3 + 9*b^4*c*n^2 + 24*b^4*c + 2*(13*b^4*c + 3*a^3*b*d)*n)*x)*(b*x + a)^n/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)

giac [B] time = 0.33, size = 361, normalized size = 3.84

$$\frac{(bx + a)^n(b^4dn^3 + 6b^4dn^2 + 11b^4dn + 6b^4d)x^4 + (ab^3dn^3 + 3ab^3dn^2 + 2ab^3dn)x^3 - 3(a^2b^2dn^2 + a^2b^2dn)x^2 + (b^4cn^3 + 9b^4cn^2 + 24b^4c + 2(13b^4c + 3a^3bdn)x)(bx + a)^n}{b^4n^4 + 10b^4n^3 + 35b^4n^2 + 50b^4n + 24b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c), x, algorithm="giac")

[Out] ((b*x + a)^n*b^4*d*n^3*x^4 + (b*x + a)^n*a*b^3*d*n^3*x^3 + 6*(b*x + a)^n*b^4*d*n^2*x^4 + 3*(b*x + a)^n*a*b^3*d*n^2*x^3 + 11*(b*x + a)^n*b^4*d*n*x^4 + (b*x + a)^n*b^4*c*n^3*x - 3*(b*x + a)^n*a^2*b^2*d*n^2*x^2 + 2*(b*x + a)^n*a*b^3*d*n*x^3 + 6*(b*x + a)^n*b^4*d*x^4 + (b*x + a)^n*a*b^3*c*n^3 + 9*(b*x + a)^n*b^4*c*n^2*x - 3*(b*x + a)^n*a^2*b^2*d*n*x^2 + 9*(b*x + a)^n*a*b^3*c*n^2 + 26*(b*x + a)^n*b^4*c*n*x + 6*(b*x + a)^n*a^3*b*d*n*x + 26*(b*x + a)^n*a*b^3*c*n + 24*(b*x + a)^n*b^4*c*x + 24*(b*x + a)^n*a*b^3*c - 6*(b*x + a)^n*a^4*d)/(b^4*n^4 + 10*b^4*n^3 + 35*b^4*n^2 + 50*b^4*n + 24*b^4)

maple [A] time = 0.00, size = 167, normalized size = 1.78

$$\frac{(-b^3dn^3x^3 - 6b^3dn^2x^3 + 3ab^2dn^2x^2 - 11b^3dnx^3 + 9ab^2dnx^2 - b^3cn^3 - 6dx^3b^3 - 6a^2bdnx + 6adx^2b^2 - 9b^3cn^2 - 6da^2xb - 26b^3cn + 6a^3d - 24b^3c)(bx + a)^{n+1}}{(n^4 + 10n^3 + 35n^2 + 50n + 24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^n*(d*x^3+c), x)

[Out] -(b*x+a)^(n+1)*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-b^3*c*n^3-6*b^3*d*x^3-6*a^2*b*d*n*x+6*a*b^2*d*x^2-9*b^3*c*n^2-6*a^2*b*d*x-26*b^3*c*n+6*a^3*d-24*b^3*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)

maxima [A] time = 0.96, size = 122, normalized size = 1.30

$$\frac{(bx + a)^{n+1}c}{b(n+1)} + \frac{((n^3 + 6n^2 + 11n + 6)b^4x^4 + (n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^nd}{(n^4 + 10n^3 + 35n^2 + 50n + 24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c), x, algorithm="maxima")

[Out] (b*x + a)^(n + 1)*c/(b*(n + 1)) + ((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4)

mupad [B] time = 2.95, size = 247, normalized size = 2.63

$$(a + bx)^n \left(\frac{x(6da^3bn + cb^4n^3 + 9cb^4n^2 + 26cb^4n + 24cb^4)}{b^4(n^4 + 10n^3 + 35n^2 + 50n + 24)} + \frac{a(-6da^3 + cb^3n^3 + 9cb^3n^2 + 26cb^3n + 24cb^3)}{b^4(n^4 + 10n^3 + 35n^2 + 50n + 24)} + \frac{dx^4(n^3 + 6n^2 + 11n + 6)}{n^4 + 10n^3 + 35n^2 + 50n + 24} - \frac{3a^2dnx^2(n+1)}{b^2(n^4 + 10n^3 + 35n^2 + 50n + 24)} + \frac{adnx^3(n^2 + 3n + 2)}{b(n^4 + 10n^3 + 35n^2 + 50n + 24)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)*(a + b*x)^n,x)`

[Out] $(a + b*x)^n * ((x*(24*b^4*c + 9*b^4*c*n^2 + b^4*c*n^3 + 26*b^4*c*n + 6*a^3*b*d*n)) / (b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*(24*b^3*c - 6*a^3*d + 9*b^3*c*n^2 + b^3*c*n^3 + 26*b^3*c*n)) / (b^4*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (d*x^4*(11*n + 6*n^2 + n^3 + 6)) / (50*n + 35*n^2 + 10*n^3 + n^4 + 24) - (3*a^2*d*n*x^2*(n + 1)) / (b^2*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)) + (a*d*n*x^3*(3*n + n^2 + 2)) / (b*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)))$

sympy [A] time = 2.79, size = 1906, normalized size = 20.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**n*(d*x**3+c),x)`

[Out] $\text{Piecewise}((a**n*(c*x + d*x**4/4), \text{Eq}(b, 0)), (6*a**3*d*\log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 11*a**3*d/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a**2*b*d*x*\log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 27*a**2*b*d*x/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2*\log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 18*a*b**2*d*x**2/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) - 2*b**3*c/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3) + 6*b**3*d*x**3*\log(a/b + x)/(6*a**3*b**4 + 18*a**2*b**5*x + 18*a*b**6*x**2 + 6*b**7*x**3), \text{Eq}(n, -4)), (-6*a**3*d*\log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 9*a**3*d/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x*\log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 12*a**2*b*d*x/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - 6*a*b**2*d*x**2*\log(a/b + x)/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) - b**3*c/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2) + 2*b**3*d*x**3/(2*a**2*b**4 + 4*a*b**5*x + 2*b**6*x**2), \text{Eq}(n, -3)), (6*a**3*d*\log(a/b + x)/(2*a*b**4 + 2*b**5*x) + 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*\log(a/b + x)/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 + 2*b**5*x) - 2*b**3*c/(2*a*b**4 + 2*b**5*x) + b**3*d*x**3/(2*a*b**4 + 2*b**5*x), \text{Eq}(n, -2)), (-a**3*d*\log(a/b + x)/b**4 + a**2*d*x/b**3 - a*d*x**2/(2*b**2) + c*\log(a/b + x)/b + d*x**3/(3*b), \text{Eq}(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a**3*b*d*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*c*n**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*a*b**3*c*n**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*a*b**3*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*a*b**3*c*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n**2*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 50*b**4*n + 24*b**4) + b**4*c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 9*b**4*c*n**2*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 26*b**4*c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 24*b**4*c*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d*n*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 50*b**4*n + 24*b**4))$

```
24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*  
n**2 + 50*b**4*n + 24*b**4), True))
```

$$3.53 \quad \int x^2(a + bx)^n (c + dx^3)^2 dx$$

Optimal. Leaf size=294

$$\frac{2a(b^3c - 4a^3d)(b^3c - a^3d)(a + bx)^{n+2}}{b^9(n+2)} - \frac{10ad(b^3c - 7a^3d)(a + bx)^{n+5}}{b^9(n+5)} + \frac{2d(b^3c - 28a^3d)(a + bx)^{n+6}}{b^9(n+6)} + \frac{28a^2d^2(a + bx)^{n+9}}{b^9(n+9)}$$

Rubi [A] time = 0.20, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1620}

$$\frac{(-20a^2b^3cd + 28a^6d^2 + b^6c^2)(a + bx)^{n+3}}{b^9(n+3)} + \frac{a^2(b^3c - a^3d)^2(a + bx)^{n+1}}{b^9(n+1)} - \frac{2a(b^3c - 4a^3d)(b^3c - a^3d)(a + bx)^{n+2}}{b^9(n+2)} + \frac{4a^2d(5b^3c - 14a^3d)(a + bx)^{n+4}}{b^9(n+4)} - \frac{10ad(b^3c - 7a^3d)(a + bx)^{n+5}}{b^9(n+5)} + \frac{2d(b^3c - 28a^3d)(a + bx)^{n+6}}{b^9(n+6)} + \frac{28a^2d^2(a + bx)^{n+7}}{b^9(n+7)} - \frac{8ad^2(a + bx)^{n+8}}{b^9(n+8)} + \frac{d^2(a + bx)^{n+9}}{b^9(n+9)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^2,x]

[Out] (a^2*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^9*(1 + n)) - (2*a*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^9*(2 + n)) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(3 + n))/(b^9*(3 + n)) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^(4 + n))/(b^9*(4 + n)) - (10*a*d*(b^3*c - 7*a^3*d)*(a + b*x)^(5 + n))/(b^9*(5 + n)) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^(6 + n))/(b^9*(6 + n)) + (28*a^2*d^2*(a + b*x)^(7 + n))/(b^9*(7 + n)) - (8*a*d^2*(a + b*x)^(8 + n))/(b^9*(8 + n)) + (d^2*(a + b*x)^(9 + n))/(b^9*(9 + n))

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol]
 := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E xpon[Px, x], 2]

Rubi steps

$$\int x^2(a + bx)^n (c + dx^3)^2 dx = \int \left(\frac{(ab^3c - a^4d)^2 (a + bx)^n}{b^8} - \frac{2(ab^6c^2 - 5a^4b^3cd + 4a^7d^2)(a + bx)^{1+n}}{b^8} + \frac{(b^6c^2 - 20a^3b^3cd + 28a^6d^2)(a + bx)^{2+n}}{b^8} \right) dx$$

$$= \frac{a^2(b^3c - a^3d)^2(a + bx)^{1+n}}{b^9(1+n)} - \frac{2a(b^3c - 4a^3d)(b^3c - a^3d)(a + bx)^{2+n}}{b^9(2+n)} + \frac{(b^6c^2 - 20a^3b^3cd + 28a^6d^2)(a + bx)^{3+n}}{b^9(3+n)}$$

Mathematica [A] time = 0.27, size = 252, normalized size = 0.86

$$\frac{(a + bx)^{n+1} \left(\frac{(ab^3c - a^4d)^2}{n+1} + \frac{2d(a+bx)^5(b^3c - 28a^3d)}{n+6} + \frac{10ad(a+bx)^4(7a^3d - b^3c)}{n+5} - \frac{2a(a+bx)(b^3c - 4a^3d)(b^3c - a^3d)}{n+2} + \frac{28a^2d^2(a+bx)^6}{n+7} + \frac{(a+bx)^2(28a^6d^2 - 20a^3b^3cd + b^6c^2)}{n+3} + \frac{4a^2d(a+bx)^3(5b^3c - 14a^3d)}{n+4} + \frac{d^2(a+bx)^8}{n+9} - \frac{8ad^2(a+bx)^7}{n+8} \right)}{b^9}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^2,x]

[Out] ((a + b*x)^(1 + n)*((a*b^3*c - a^4*d)^2/(1 + n) - (2*a*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x))/(2 + n) + ((b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^2)/(3 + n) + (4*a^2*d*(5*b^3*c - 14*a^3*d)*(a + b*x)^3)/(4 + n) + (10*a*d*(-(b^3*c) + 7*a^3*d)*(a + b*x)^4)/(5 + n) + (2*d*(b^3*c - 28*a^3*d)*(a + b*x)^5)/(6 + n) + (28*a^2*d^2*(a + b*x)^6)/(7 + n) - (8*a*d^2*(a + b*x)^7)/(8 + n) + (d^2*(a + b*x)^8)/(9 + n))/b^9

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x^2(a + bx)^n (c + dx^3)^2 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3)^2,x]

[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3)^2, x]

fricas [B] time = 1.00, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")

[Out] (2*a^3*b^6*c^2*n^6 + 78*a^3*b^6*c^2*n^5 + 1250*a^3*b^6*c^2*n^4 + 120960*a^3*b^6*c^2 - 120960*a^6*b^3*c*d + 40320*a^9*d^2 + (b^9*d^2*n^8 + 36*b^9*d^2*n^7 + 546*b^9*d^2*n^6 + 4536*b^9*d^2*n^5 + 22449*b^9*d^2*n^4 + 67284*b^9*d^2*n^3 + 118124*b^9*d^2*n^2 + 109584*b^9*d^2*n + 40320*b^9*d^2)*x^9 + (a*b^8*d^2*n^8 + 28*a*b^8*d^2*n^7 + 322*a*b^8*d^2*n^6 + 1960*a*b^8*d^2*n^5 + 6769*a*b^8*d^2*n^4 + 13132*a*b^8*d^2*n^3 + 13068*a*b^8*d^2*n^2 + 5040*a*b^8*d^2*n)*x^8 - 8*(a^2*b^7*d^2*n^7 + 21*a^2*b^7*d^2*n^6 + 175*a^2*b^7*d^2*n^5 + 735*a^2*b^7*d^2*n^4 + 1624*a^2*b^7*d^2*n^3 + 1764*a^2*b^7*d^2*n^2 + 720*a^2*b^7*d^2*n)*x^7 + 2*(b^9*c*d*n^8 + 39*b^9*c*d*n^7 + 60480*b^9*c*d + 4*(159*b^9*c*d + 7*a^3*b^6*d^2)*n^6 + 6*(939*b^9*c*d + 70*a^3*b^6*d^2)*n^5 + (29469*b^9*c*d + 2380*a^3*b^6*d^2)*n^4 + 9*(10279*b^9*c*d + 700*a^3*b^6*d^2)*n^3 + 2*(84307*b^9*c*d + 3836*a^3*b^6*d^2)*n^2 + 24*(6709*b^9*c*d + 140*a^3*b^6*d^2)*n)*x^6 + 2*(a*b^8*c*d*n^8 + 34*a*b^8*c*d*n^7 + 466*a*b^8*c*d*n^6 + 56*(59*a*b^8*c*d - 3*a^4*b^5*d^2)*n^5 + (12949*a*b^8*c*d - 1680*a^4*b^5*d^2)*n^4 + 2*(13883*a*b^8*c*d - 2940*a^4*b^5*d^2)*n^3 + 24*(1241*a*b^8*c*d - 350*a^4*b^5*d^2)*n^2 + 4032*(3*a*b^8*c*d - a^4*b^5*d^2)*n)*x^5 - 10*(a^2*b^7*c*d*n^7 + 30*a^2*b^7*c*d*n^6 + 346*a^2*b^7*c*d*n^5 + 24*(80*a^2*b^7*c*d - 7*a^5*b^4*d^2)*n^4 + (5269*a^2*b^7*c*d - 1008*a^5*b^4*d^2)*n^3 + 6*(1115*a^2*b^7*c*d - 308*a^5*b^4*d^2)*n^2 + 1008*(3*a^2*b^7*c*d - a^5*b^4*d^2)*n)*x^4 + 30*(351*a^3*b^6*c^2 - 8*a^6*b^3*c*d)*n^3 + (b^9*c^2*n^8 + 42*b^9*c^2*n^7 + 120960*b^9*c^2 + 8*(93*b^9*c^2 + 5*a^3*b^6*c*d)*n^6 + 18*(401*b^9*c^2 + 60*a^3*b^6*c*d)*n^5 + (41619*b^9*c^2 + 10600*a^3*b^6*c*d)*n^4 + 12*(12039*b^9*c^2 + 3750*a^3*b^6*c*d - 560*a^6*b^3*d^2)*n^3 + 4*(72569*b^9*c^2 + 18940*a^3*b^6*c*d - 5040*a^6*b^3*d^2)*n^2 + 48*(6289*b^9*c^2 + 840*a^3*b^6*c*d - 280*a^6*b^3*d^2)*n)*x^3 + 4*(12287*a^3*b^6*c^2 - 1440*a^6*b^3*c*d)*n^2 + (a*b^8*c^2*n^8 + 40*a*b^8*c^2*n^7 + 664*a*b^8*c^2*n^6 + 10*(589*a*b^8*c^2 - 12*a^4*b^5*c*d)*n^5 + (29839*a*b^8*c^2 - 3000*a^4*b^5*c*d)*n^4 + 10*(8479*a*b^8*c^2 - 2580*a^4*b^5*c*d)*n^3 + 24*(5029*a*b^8*c^2 - 3475*a^4*b^5*c*d + 840*a^7*b^2*d^2)*n^2 + 20160*(3*a*b^8*c^2 - 3*a^4*b^5*c*d + a^7*b^2*d^2)*n)*x^2 + 48*(2509*a^3*b^6*c^2 - 955*a^6*b^3*c*d)*n - 2*(a^2*b^7*c^2*n^7 + 39*a^2*b^7*c^2*n^6 + 625*a^2*b^7*c^2*n^5 + 15*(351*a^2*b^7*c^2 - 8*a^5*b^4*c*d)*n^4 + 2*(12287*a^2*b^7*c^2 - 1440*a^5*b^4*c*d)*n^3 + 24*(2509*a^2*b^7*c^2 - 955*a^5*b^4*c*d)*n^2 + 20160*(3*a^2*b^7*c^2 - 3*a^5*b^4*c*d + a^8*b*d^2)*n)*x*(b*x + a)^n/(b^9*n^9 + 45*b^9*n^8 + 870*b^9*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)

giac [B] time = 0.61, size = 2660, normalized size = 9.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")

[Out] $((b*x + a)^{n*b^9*d^2*n^8*x^9} + (b*x + a)^{n*a*b^8*d^2*n^8*x^8} + 36*(b*x + a)^{n*b^9*d^2*n^7*x^9} + 28*(b*x + a)^{n*a*b^8*d^2*n^7*x^8} + 546*(b*x + a)^{n*b^9*d^2*n^6*x^9} + 2*(b*x + a)^{n*b^9*c*d*n^8*x^6} - 8*(b*x + a)^{n*a^2*b^7*d^2*n^7*x^7} + 322*(b*x + a)^{n*a*b^8*d^2*n^6*x^8} + 4536*(b*x + a)^{n*b^9*d^2*n^5*x^9} + 2*(b*x + a)^{n*a*b^8*c*d*n^8*x^5} + 78*(b*x + a)^{n*b^9*c*d*n^7*x^6} - 168*(b*x + a)^{n*a^2*b^7*d^2*n^6*x^7} + 1960*(b*x + a)^{n*a*b^8*d^2*n^5*x^8} + 22449*(b*x + a)^{n*b^9*d^2*n^4*x^9} + 68*(b*x + a)^{n*a*b^8*c*d*n^7*x^5} + 1272*(b*x + a)^{n*b^9*c*d*n^6*x^6} + 56*(b*x + a)^{n*a^3*b^6*d^2*n^6*x^6} - 1400*(b*x + a)^{n*a^2*b^7*d^2*n^5*x^7} + 6769*(b*x + a)^{n*a*b^8*d^2*n^4*x^8} + 67284*(b*x + a)^{n*b^9*d^2*n^3*x^9} + (b*x + a)^{n*b^9*c^2*n^8*x^3} - 10*(b*x + a)^{n*a^2*b^7*c*d*n^7*x^4} + 932*(b*x + a)^{n*a*b^8*c*d*n^6*x^5} + 11268*(b*x + a)^{n*b^9*c*d*n^5*x^6} + 840*(b*x + a)^{n*a^3*b^6*d^2*n^5*x^6} - 5880*(b*x + a)^{n*a^2*b^7*d^2*n^4*x^7} + 13132*(b*x + a)^{n*a*b^8*d^2*n^3*x^8} + 118124*(b*x + a)^{n*b^9*d^2*n^2*x^9} + (b*x + a)^{n*a*b^8*c^2*n^8*x^2} + 42*(b*x + a)^{n*b^9*c^2*n^7*x^3} - 300*(b*x + a)^{n*a^2*b^7*c*d*n^6*x^4} + 6608*(b*x + a)^{n*a*b^8*c*d*n^5*x^5} - 336*(b*x + a)^{n*a^4*b^5*d^2*n^5*x^5} + 58938*(b*x + a)^{n*b^9*c*d*n^4*x^6} + 4760*(b*x + a)^{n*a^3*b^6*d^2*n^4*x^6} - 12992*(b*x + a)^{n*a^2*b^7*d^2*n^3*x^7} + 13068*(b*x + a)^{n*a*b^8*d^2*n^2*x^8} + 109584*(b*x + a)^{n*b^9*d^2*n*x^9} + 40*(b*x + a)^{n*a*b^8*c^2*n^7*x^2} + 744*(b*x + a)^{n*b^9*c^2*n^6*x^3} + 40*(b*x + a)^{n*a^3*b^6*c*d*n^6*x^3} - 3460*(b*x + a)^{n*a^2*b^7*c*d*n^5*x^4} + 25898*(b*x + a)^{n*a*b^8*c*d*n^4*x^5} - 3360*(b*x + a)^{n*a^4*b^5*d^2*n^4*x^5} + 185022*(b*x + a)^{n*b^9*c*d*n^3*x^6} + 12600*(b*x + a)^{n*a^3*b^6*d^2*n^3*x^6} - 14112*(b*x + a)^{n*a^2*b^7*d^2*n^2*x^7} + 5040*(b*x + a)^{n*a*b^8*d^2*n*x^8} + 40320*(b*x + a)^{n*b^9*d^2*x^9} - 2*(b*x + a)^{n*a^2*b^7*c^2*n^7*x} + 664*(b*x + a)^{n*a*b^8*c^2*n^6*x^2} + 7218*(b*x + a)^{n*b^9*c^2*n^5*x^3} + 1080*(b*x + a)^{n*a^3*b^6*c*d*n^5*x^3} - 19200*(b*x + a)^{n*a^2*b^7*c*d*n^4*x^4} + 1680*(b*x + a)^{n*a^5*b^4*d^2*n^4*x^4} + 55532*(b*x + a)^{n*a*b^8*c*d*n^3*x^5} - 11760*(b*x + a)^{n*a^4*b^5*d^2*n^3*x^5} + 337228*(b*x + a)^{n*b^9*c*d*n^2*x^6} + 15344*(b*x + a)^{n*a^3*b^6*d^2*n^2*x^6} - 5760*(b*x + a)^{n*a^2*b^7*d^2*n*x^7} - 78*(b*x + a)^{n*a^2*b^7*c^2*n^6*x} + 5890*(b*x + a)^{n*a*b^8*c^2*n^5*x^2} - 120*(b*x + a)^{n*a^4*b^5*c*d*n^5*x^2} + 41619*(b*x + a)^{n*b^9*c^2*n^4*x^3} + 10600*(b*x + a)^{n*a^3*b^6*c*d*n^4*x^3} - 52690*(b*x + a)^{n*a^2*b^7*c*d*n^3*x^4} + 10080*(b*x + a)^{n*a^5*b^4*d^2*n^3*x^4} + 59568*(b*x + a)^{n*a*b^8*c*d*n^2*x^5} - 16800*(b*x + a)^{n*a^4*b^5*d^2*n^2*x^5} + 322032*(b*x + a)^{n*b^9*c*d*n*x^6} + 6720*(b*x + a)^{n*a^3*b^6*d^2*n*x^6} + 2*(b*x + a)^{n*a^3*b^6*c^2*n^6} - 1250*(b*x + a)^{n*a^2*b^7*c^2*n^5*x} + 29839*(b*x + a)^{n*a*b^8*c^2*n^4*x^2} - 3000*(b*x + a)^{n*a^4*b^5*c*d*n^4*x^2} + 144468*(b*x + a)^{n*b^9*c^2*n^3*x^3} + 45000*(b*x + a)^{n*a^3*b^6*c*d*n^3*x^3} - 6720*(b*x + a)^{n*a^6*b^3*d^2*n^3*x^3} - 66900*(b*x + a)^{n*a^2*b^7*c*d*n^2*x^4} + 18480*(b*x + a)^{n*a^5*b^4*d^2*n^2*x^4} + 24192*(b*x + a)^{n*a*b^8*c*d*n*x^5} - 8064*(b*x + a)^{n*a^4*b^5*d^2*n*x^5} + 120960*(b*x + a)^{n*b^9*c*d*x^6} + 78*(b*x + a)^{n*a^3*b^6*c^2*n^5} - 10530*(b*x + a)^{n*a^2*b^7*c^2*n^4*x} + 240*(b*x + a)^{n*a^5*b^4*c*d*n^4*x} + 84790*(b*x + a)^{n*a*b^8*c^2*n^3*x^2} - 25800*(b*x + a)^{n*a^4*b^5*c*d*n^3*x^2} + 290276*(b*x + a)^{n*b^9*c^2*n^2*x^3} + 75760*(b*x + a)^{n*a^3*b^6*c*d*n^2*x^3} - 20160*(b*x + a)^{n*a^6*b^3*d^2*n^2*x^3} - 30240*(b*x + a)^{n*a^2*b^7*c*d*n*x^4} + 10080*(b*x + a)^{n*a^5*b^4*d^2*n*x^4} + 1250*(b*x + a)^{n*a^3*b^6*c^2*n^4} - 49148*(b*x + a)^{n*a^2*b^7*c^2*n^3*x} + 5760*(b*x + a)^{n*a^5*b^4*c*d*n^3*x} + 120696*(b*x + a)^{n*a*b^8*c^2*n^2*x^2} - 83400*(b*x + a)^{n*a^4*b^5*c*d*n^2*x^2} + 20160*(b*x + a)^{n*a^7*b^2*d^2*n^2*x^2} + 301872*(b*x + a)^{n*b^9*c^2*n*x^3} + 40320*(b*x + a)^{n*a^3*b^6*c*d*n*x^3} - 13440*(b*x + a)^{n*a^6*b^3*d^2*n*x^3} + 10530*(b*x + a)^{n*a^3*b^6*c^2*n^3} - 240*(b*x + a)^{n*a^6*b^3*c*d*n^3} - 120432*(b*x + a)^{n*a^2*b^7*c^2*n^2*x} + 45840*(b*x + a)^{n*a^5*b^4*c*d*n^2*x} + 60480*(b*x + a)^{n*a*b^8*c^2*n*x^2} - 60480*(b*x + a)^{n*a^4*b^5*c*d*n*x^2} + 20160*(b*x + a)^{n*a^7*b^2*d^2*n*x^2} + 120960*(b*x + a)^{n*b^9*c^2*x^3} + 49148*(b*x + a)^{n*a^3*b^6*c^2*n^2} - 5760*(b*x + a)^{n*a^6*b^3*c*d*n^2} - 120960*(b*x + a)^{n*a^2*b^7*c^2*n*x} + 120960*(b*x + a)^{n*a^5*b^4*c*d*n*x} - 40320*(b*x + a)^{n*a^8*b*d^2*n*x} + 120432*(b*x + a)^{n*a^3*b^6*c^2*n} - 45840*(b*x + a)^{n*a^6*b^3*c*d*n} + 120960*(b*x + a)^{n*a^3*b^6*c^2} - 120960*(b*x + a)^{n*a^6*b^3*c*d} + 40320*(b*x + a)^{n*a^9*d^2})/(b^9*n^9 + 45*b^9*n^8 + 870*b^9$

$*n^7 + 9450*b^9*n^6 + 63273*b^9*n^5 + 269325*b^9*n^4 + 723680*b^9*n^3 + 1172700*b^9*n^2 + 1026576*b^9*n + 362880*b^9)$

maple [B] time = 0.02, size = 1565, normalized size = 5.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(b*x+a)^n*(d*x^3+c)^2, x)$

[Out] $(b*x+a)^{(n+1)}*(b^8*d^2*n^8*x^8+36*b^8*d^2*n^7*x^8-8*a*b^7*d^2*n^7*x^7+546*b^8*d^2*n^6*x^8-224*a*b^7*d^2*n^6*x^7+2*b^8*c*d*n^8*x^5+4536*b^8*d^2*n^5*x^8+56*a^2*b^6*d^2*n^6*x^6-2576*a*b^7*d^2*n^5*x^7+78*b^8*c*d*n^7*x^5+22449*b^8*d^2*n^4*x^8+1176*a^2*b^6*d^2*n^5*x^6-10*a*b^7*c*d*n^7*x^4-15680*a*b^7*d^2*n^4*x^7+1272*b^8*c*d*n^6*x^5+67284*b^8*d^2*n^3*x^8-336*a^3*b^5*d^2*n^5*x^5+9800*a^2*b^6*d^2*n^4*x^6-340*a*b^7*c*d*n^6*x^4-54152*a*b^7*d^2*n^3*x^7+b^8*c^2*n^8*x^2+11268*b^8*c*d*n^5*x^5+118124*b^8*d^2*n^2*x^8-5040*a^3*b^5*d^2*n^4*x^5+40*a^2*b^6*c*d*n^6*x^3+41160*a^2*b^6*d^2*n^3*x^6-4660*a*b^7*c*d*n^5*x^4-105056*a*b^7*d^2*n^2*x^7+42*b^8*c^2*n^7*x^2+58938*b^8*c*d*n^4*x^5+109584*b^8*d^2*n*x^8+1680*a^4*b^4*d^2*n^4*x^4-28560*a^3*b^5*d^2*n^3*x^5+1200*a^2*b^6*c*d*n^5*x^3+90944*a^2*b^6*d^2*n^2*x^6-2*a*b^7*c^2*n^7*x-33040*a*b^7*c*d*n^4*x^4-104544*a*b^7*d^2*n*x^7+744*b^8*c^2*n^6*x^2+185022*b^8*c*d*n^3*x^5+40320*b^8*d^2*x^8+16800*a^4*b^4*d^2*n^3*x^4-120*a^3*b^5*c*d*n^5*x^2-75600*a^3*b^5*d^2*n^2*x^5+13840*a^2*b^6*c*d*n^4*x^3+98784*a^2*b^6*d^2*n*x^6-80*a*b^7*c^2*n^6*x-129490*a*b^7*c*d*n^3*x^4-40320*a*b^7*d^2*x^7+7218*b^8*c^2*n^5*x^2+337228*b^8*c*d*n^2*x^5-6720*a^5*b^3*d^2*n^3*x^3+58800*a^4*b^4*d^2*n^2*x^4-3240*a^3*b^5*c*d*n^4*x^2-92064*a^3*b^5*d^2*n*x^5+2*a^2*b^6*c^2*n^6+76800*a^2*b^6*c*d*n^3*x^3+40320*a^2*b^6*d^2*x^6-1328*a*b^7*c^2*n^5*x-277660*a*b^7*c*d*n^2*x^4+41619*b^8*c^2*n^4*x^2+322032*b^8*c*d*n*x^5-40320*a^5*b^3*d^2*n^2*x^3+240*a^4*b^4*c*d*n^4*x+84000*a^4*b^4*d^2*n*x^4-31800*a^3*b^5*c*d*n^3*x^2-40320*a^3*b^5*d^2*x^5+78*a^2*b^6*c^2*n^5+210760*a^2*b^6*c*d*n^2*x^3-11780*a*b^7*c^2*n^4*x-297840*a*b^7*c*d*n*x^4+144468*b^8*c^2*n^3*x^2+120960*b^8*c*d*x^5+20160*a^6*b^2*d^2*n^2*x^2-73920*a^5*b^3*d^2*n*x^3+6000*a^4*b^4*c*d*n^3*x+40320*a^4*b^4*d^2*x^4-135000*a^3*b^5*c*d*n^2*x^2+1250*a^2*b^6*c^2*n^4+267600*a^2*b^6*c*d*n*x^3-59678*a*b^7*c^2*n^3*x-120960*a*b^7*c*d*x^4+290276*b^8*c^2*n^2*x^2+60480*a^6*b^2*d^2*n*x^2-240*a^5*b^3*c*d*n^3-40320*a^5*b^3*d^2*x^3+51600*a^4*b^4*c*d*n^2*x-227280*a^3*b^5*c*d*n*x^2+10530*a^2*b^6*c^2*n^3+120960*a^2*b^6*c*d*x^3-169580*a*b^7*c^2*n^2*x+301872*b^8*c^2*n*x^2-40320*a^7*b*d^2*n*x+40320*a^6*b^2*d^2*x^2-5760*a^5*b^3*c*d*n^2+166800*a^4*b^4*c*d*n*x-120960*a^3*b^5*c*d*x^2+49148*a^2*b^6*c^2*n^2-241392*a*b^7*c^2*n*x+120960*b^8*c^2*x^2-40320*a^7*b*d^2*x-45840*a^5*b^3*c*d*n+120960*a^4*b^4*c*d*x+120432*a^2*b^6*c^2*n-120960*a*b^7*c^2*x+40320*a^8*d^2-120960*a^5*b^3*c*d+120960*a^2*b^6*c^2)/b^9/(n^9+45*n^8+870*n^7+9450*n^6+63273*n^5+269325*n^4+723680*n^3+1172700*n^2+1026576*n+362880)$

maxima [B] time = 0.81, size = 601, normalized size = 2.04

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^2*(b*x+a)^n*(d*x^3+c)^2, x, \text{algorithm}="maxima")$

[Out] $((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c^2/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 2*((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*c*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6) + ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 7$

$$35n^4 + 1624n^3 + 1764n^2 + 720n)a^2b^7x^7 + 56(n^6 + 15n^5 + 85n^4 + 225n^3 + 274n^2 + 120n)a^3b^6x^6 - 336(n^5 + 10n^4 + 35n^3 + 50n^2 + 24n)a^4b^5x^5 + 1680(n^4 + 6n^3 + 11n^2 + 6n)a^5b^4x^4 - 6720(n^3 + 3n^2 + 2n)a^6b^3x^3 + 20160(n^2 + n)a^7b^2x^2 - 40320a^8b^1x + 40320a^9)(b^9 + a^9)^{n+1} / ((n^9 + 45n^8 + 870n^7 + 9450n^6 + 63273n^5 + 269325n^4 + 723680n^3 + 1172700n^2 + 1026576n + 362880)b^9)$$

mupad [B] time = 3.73, size = 1410, normalized size = 4.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2(c + dx^3)^2(a + bx)^n, x)$

[Out]
$$\frac{(d^2x^9(a + bx)^n(109584n + 118124n^2 + 67284n^3 + 22449n^4 + 4536n^5 + 546n^6 + 36n^7 + n^8 + 40320))/(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880) + (2a^3(a + bx)^n(20160a^6d^2 + 60480b^6c^2 + 60216b^6c^2n + 24574b^6c^2n^2 + 5265b^6c^2n^3 + 625b^6c^2n^4 + 39b^6c^2n^5 + b^6c^2n^6 - 60480a^3b^3cd - 22920a^3b^3cdn - 2880a^3b^3cdn^2 - 120a^3b^3cdn^3))/(b^9(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) + (x^3(a + bx)^n(3n + n^2 + 2)(60480b^6c^2 - 6720a^6d^2n + 60216b^6c^2n + 24574b^6c^2n^2 + 5265b^6c^2n^3 + 625b^6c^2n^4 + 39b^6c^2n^5 + b^6c^2n^6 + 20160a^3b^3cdn + 7640a^3b^3cdn^2 + 960a^3b^3cdn^3 + 40a^3b^3cdn^4))/(b^6(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) + (2dx^6(a + bx)^n(504b^3c + 24b^3cn^2 + b^3cn^3 + 28a^3dn + 191b^3cn)(274n + 225n^2 + 85n^3 + 15n^4 + n^5 + 120))/(b^3(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) - (2a^2nx(a + bx)^n(20160a^6d^2 + 60480b^6c^2 + 60216b^6c^2n + 24574b^6c^2n^2 + 5265b^6c^2n^3 + 625b^6c^2n^4 + 39b^6c^2n^5 + b^6c^2n^6 - 60480a^3b^3cd - 22920a^3b^3cdn - 2880a^3b^3cdn^2 - 120a^3b^3cdn^3))/(b^8(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) + (ad^2nx^8(a + bx)^n(13068n + 13132n^2 + 6769n^3 + 1960n^4 + 322n^5 + 28n^6 + n^7 + 5040))/(b(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) - (8a^2d^2nx^7(a + bx)^n(1764n + 1624n^2 + 735n^3 + 175n^4 + 21n^5 + n^6 + 720))/(b^2(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) - (10a^2d^4nx^4(a + bx)^n(11n + 6n^2 + n^3 + 6)(504b^3c - 168a^3d + 24b^3cn^2 + b^3cn^3 + 191b^3cn))/(b^5(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) + (2adnx^5(a + bx)^n(50n + 35n^2 + 10n^3 + n^4 + 24)(504b^3c - 168a^3d + 24b^3cn^2 + b^3cn^3 + 191b^3cn))/(b^4(1026576n + 1172700n^2 + 723680n^3 + 269325n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880))$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**2,x)
```

```
[Out] Timed out
```

$$3.54 \quad \int x(a + bx)^n (c + dx^3)^2 dx$$

Optimal. Leaf size=248

$$-\frac{a(b^3c - a^3d)^2(a + bx)^{n+1}}{b^8(n+1)} + \frac{(b^3c - 7a^3d)(b^3c - a^3d)(a + bx)^{n+2}}{b^8(n+2)} - \frac{ad(8b^3c - 35a^3d)(a + bx)^{n+4}}{b^8(n+4)} + \frac{d(2b^3c - 35a^3d)(a + bx)^{n+8}}{b^8(n+8)}$$

Rubi [A] time = 0.15, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, number of rules / integrand size = 0.056, Rules used = {1620}

$$\frac{a(b^3c - a^3d)^2(a + bx)^{n+1}}{b^8(n+1)} + \frac{(b^3c - 7a^3d)(b^3c - a^3d)(a + bx)^{n+2}}{b^8(n+2)} + \frac{3a^2d(4b^3c - 7a^3d)(a + bx)^{n+3}}{b^8(n+3)} - \frac{ad(8b^3c - 35a^3d)(a + bx)^{n+4}}{b^8(n+4)} + \frac{d(2b^3c - 35a^3d)(a + bx)^{n+5}}{b^8(n+5)} + \frac{21a^2d^2(a + bx)^{n+6}}{b^8(n+6)} - \frac{7ad^2(a + bx)^{n+7}}{b^8(n+7)} + \frac{d^2(a + bx)^{n+8}}{b^8(n+8)}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*x)^n*(c + d*x^3)^2,x]

[Out] -((a*(b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^3*c - 7*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) - (a*d*(8*b^3*c - 35*a^3*d)*(a + b*x)^(4 + n))/(b^8*(4 + n)) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (21*a^2*d^2*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^2*(a + b*x)^(7 + n))/(b^8*(7 + n)) + (d^2*(a + b*x)^(8 + n))/(b^8*(8 + n))

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol]
 := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\int x(a + bx)^n (c + dx^3)^2 dx = \int \left(-\frac{a(-b^3c + a^3d)^2(a + bx)^n}{b^7} + \frac{(b^3c - 7a^3d)(b^3c - a^3d)(a + bx)^{1+n}}{b^7} - \frac{3a^2d(-4b^3c + 7a^3d)(a + bx)^{2+n}}{b^7} + \frac{ad(8b^3c - 35a^3d)(a + bx)^{3+n}}{b^7} - \frac{d(2b^3c - 35a^3d)(a + bx)^{4+n}}{b^7} + \frac{21a^2d^2(a + bx)^{5+n}}{b^7} - \frac{7ad^2(a + bx)^{6+n}}{b^7} + \frac{d^2(a + bx)^{7+n}}{b^7} \right) dx$$

$$= -\frac{a(b^3c - a^3d)^2(a + bx)^{1+n}}{b^8(1+n)} + \frac{(b^3c - 7a^3d)(b^3c - a^3d)(a + bx)^{2+n}}{b^8(2+n)} + \frac{3a^2d(4b^3c - 7a^3d)(a + bx)^{3+n}}{b^8(3+n)} - \frac{ad(8b^3c - 35a^3d)(a + bx)^{4+n}}{b^8(4+n)} + \frac{d(2b^3c - 35a^3d)(a + bx)^{5+n}}{b^8(5+n)} + \frac{21a^2d^2(a + bx)^{6+n}}{b^8(6+n)} - \frac{7ad^2(a + bx)^{7+n}}{b^8(7+n)} + \frac{d^2(a + bx)^{8+n}}{b^8(8+n)}$$

Mathematica [A] time = 0.21, size = 211, normalized size = 0.85

$$\frac{(a + bx)^{n+1} \left(\frac{d(a+bx)^4(2b^3c - 35a^3d)}{n+5} + \frac{ad(a+bx)^3(35a^3d - 8b^3c)}{n+4} + \frac{(a+bx)(b^3c - 7a^3d)(b^3c - a^3d)}{n+2} - \frac{a(b^3c - a^3d)^2}{n+1} + \frac{21a^2d^2(a+bx)^5}{n+6} + \frac{3a^2d(a+bx)^2(4b^3c - 7a^3d)}{n+3} + \frac{d^2(a+bx)^7}{n+8} - \frac{7ad^2(a+bx)^6}{n+7} \right)}{b^8}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^2,x]

[Out] ((a + b*x)^(1 + n)*(-(a*(b^3*c - a^3*d)^2)/(1 + n)) + ((b^3*c - 7*a^3*d)*(b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a^2*d*(4*b^3*c - 7*a^3*d)*(a + b*x)^2)/(3 + n) + (a*d*(-8*b^3*c + 35*a^3*d)*(a + b*x)^3)/(4 + n) + (d*(2*b^3*c - 35*a^3*d)*(a + b*x)^4)/(5 + n) + (21*a^2*d^2*(a + b*x)^5)/(6 + n) - (7*a*d^2*(a + b*x)^6)/(7 + n) + (d^2*(a + b*x)^7)/(8 + n))/b^8

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int x(a + bx)^n (c + dx^3)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3)^2, x]
```

fricas [B] time = 0.86, size = 1216, normalized size = 4.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] -(a^2*b^6*c^2*n^6 + 33*a^2*b^6*c^2*n^5 + 445*a^2*b^6*c^2*n^4 + 20160*a^2*b^6*c^2 - 16128*a^5*b^3*c*d + 5040*a^8*d^2 - (b^8*d^2*n^7 + 28*b^8*d^2*n^6 + 322*b^8*d^2*n^5 + 1960*b^8*d^2*n^4 + 6769*b^8*d^2*n^3 + 13132*b^8*d^2*n^2 + 13068*b^8*d^2*n + 5040*b^8*d^2)*x^8 - (a*b^7*d^2*n^7 + 21*a*b^7*d^2*n^6 + 175*a*b^7*d^2*n^5 + 735*a*b^7*d^2*n^4 + 1624*a*b^7*d^2*n^3 + 1764*a*b^7*d^2*n^2 + 720*a*b^7*d^2*n)*x^7 + 7*(a^2*b^6*d^2*n^6 + 15*a^2*b^6*d^2*n^5 + 85*a^2*b^6*d^2*n^4 + 225*a^2*b^6*d^2*n^3 + 274*a^2*b^6*d^2*n^2 + 120*a^2*b^6*d^2*n)*x^6 - 2*(b^8*c*d*n^7 + 31*b^8*c*d*n^6 + 8064*b^8*c*d + (391*b^8*c*d + 21*a^3*b^5*d^2)*n^5 + (2581*b^8*c*d + 210*a^3*b^5*d^2)*n^4 + (9544*b^8*c*d + 735*a^3*b^5*d^2)*n^3 + 2*(9782*b^8*c*d + 525*a^3*b^5*d^2)*n^2 + 72*(282*b^8*c*d + 7*a^3*b^5*d^2)*n)*x^5 - 2*(a*b^7*c*d*n^7 + 27*a*b^7*c*d*n^6 + 283*a*b^7*c*d*n^5 + 21*(69*a*b^7*c*d - 5*a^4*b^4*d^2)*n^4 + 2*(1874*a*b^7*c*d - 315*a^4*b^4*d^2)*n^3 + 3*(1524*a*b^7*c*d - 385*a^4*b^4*d^2)*n^2 + 126*(16*a*b^7*c*d - 5*a^4*b^4*d^2)*n)*x^4 + 3*(1045*a^2*b^6*c^2 - 16*a^5*b^3*c*d)*n^3 + 8*(a^2*b^6*c*d*n^6 + 24*a^2*b^6*c*d*n^5 + 211*a^2*b^6*c*d*n^4 + 3*(272*a^2*b^6*c*d - 35*a^5*b^3*d^2)*n^3 + 5*(260*a^2*b^6*c*d - 63*a^5*b^3*d^2)*n^2 + 42*(16*a^2*b^6*c*d - 5*a^5*b^3*d^2)*n)*x^3 + 2*(6077*a^2*b^6*c^2 - 504*a^5*b^3*c*d)*n^2 - (b^8*c^2*n^7 + 34*b^8*c^2*n^6 + 20160*b^8*c^2 + 2*(239*b^8*c^2 + 12*a^3*b^5*c*d)*n^5 + 4*(895*b^8*c^2 + 132*a^3*b^5*c*d)*n^4 + (15289*b^8*c^2 + 4008*a^3*b^5*c*d)*n^3 + 2*(18353*b^8*c^2 + 5784*a^3*b^5*c*d - 1260*a^6*b^2*d^2)*n^2 + 72*(621*b^8*c^2 + 112*a^3*b^5*c*d - 35*a^6*b^2*d^2)*n)*x^2 + 24*(1023*a^2*b^6*c^2 - 292*a^5*b^3*c*d)*n - (a*b^7*c^2*n^7 + 33*a*b^7*c^2*n^6 + 445*a*b^7*c^2*n^5 + 3*(1045*a*b^7*c^2 - 16*a^4*b^4*c*d)*n^4 + 2*(6077*a*b^7*c^2 - 504*a^4*b^4*c*d)*n^3 + 24*(1023*a*b^7*c^2 - 292*a^4*b^4*c*d)*n^2 + 1008*(20*a*b^7*c^2 - 16*a^4*b^4*c*d + 5*a^7*b*d^2)*n)*x*(b*x + a)^n/(b^8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8*n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
```

giac [B] time = 0.61, size = 2034, normalized size = 8.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^8*d^2*n^7*x^8 + (b*x + a)^n*a*b^7*d^2*n^7*x^7 + 28*(b*x + a)^n*b^8*d^2*n^6*x^8 + 21*(b*x + a)^n*a*b^7*d^2*n^6*x^7 + 322*(b*x + a)^n*b^8*d^2*n^5*x^8 + 2*(b*x + a)^n*b^8*c*d*n^7*x^5 - 7*(b*x + a)^n*a^2*b^6*d^2*n^6*x^6 + 175*(b*x + a)^n*a*b^7*d^2*n^5*x^7 + 1960*(b*x + a)^n*b^8*d^2*n^4*x^8 + 2*(b*x + a)^n*a*b^7*c*d*n^7*x^4 + 62*(b*x + a)^n*b^8*c*d*n^6*x^5 - 105*(b*x + a)^n*a^2*b^6*d^2*n^5*x^6 + 735*(b*x + a)^n*a*b^7*d^2*n^4*x^7 + 6769*(b*x + a)^n*b^8*d^2*n^3*x^8 + 54*(b*x + a)^n*a*b^7*c*d*n^6*x^4 + 782*(b*x + a)^n*b^8*c*d*n^5*x^5 + 42*(b*x + a)^n*a^3*b^5*d^2*n^5*x^5 - 595*(b*x + a)^n*a^2*b^6*d^2*n^4*x^6 + 1624*(b*x + a)^n*a*b^7*d^2*n^3*x^7 + 13132*(b*x + a)^n*b^8*d^2*n^2*x^8 + (b*x + a)^n*b^8*c^2*n^7*x^2 - 8*(b*x + a)^n*a^2*b^6*c*d*n^6*x^3 + 566*(b*x + a)^n*a*b^7*c*d*n^5*x^4 + 5162*(b*x + a)^n*b^8*c*d*n^4*x^5 + 420*(b*x + a)^n*a^3*b^5*d^2*n^4*x^5 - 1575*(b*x + a)^n*a^2*b^6*d^2*n^3*x^6 + 1764*(b*x + a)^n*a*b^7*d^2*n^2*x^7 + 13068*(b*x + a)^n*b^8*d^2*n*x^8 + (b*x + a)^n*a*b^7*c^2*n^7*x + 34*(b*x + a)^n*b^8*c^2*n^6*x^2 - 192*(
```

$$\begin{aligned}
& b^*x + a)^{\wedge}n*a^2*b^6*c*d*n^5*x^3 + 2898*(b^*x + a)^{\wedge}n*a*b^7*c*d*n^4*x^4 - 210*(\\
& b^*x + a)^{\wedge}n*a^4*b^4*d^2*n^4*x^4 + 19088*(b^*x + a)^{\wedge}n*b^8*c*d*n^3*x^5 + 1470*(\\
& b^*x + a)^{\wedge}n*a^3*b^5*d^2*n^3*x^5 - 1918*(b^*x + a)^{\wedge}n*a^2*b^6*d^2*n^2*x^6 + 720 \\
& *(b^*x + a)^{\wedge}n*a*b^7*d^2*n*x^7 + 5040*(b^*x + a)^{\wedge}n*b^8*d^2*x^8 + 33*(b^*x + a)^{\wedge} \\
& n*a*b^7*c^2*n^6*x + 478*(b^*x + a)^{\wedge}n*b^8*c^2*n^5*x^2 + 24*(b^*x + a)^{\wedge}n*a^3*b^ \\
& 5*c*d*n^5*x^2 - 1688*(b^*x + a)^{\wedge}n*a^2*b^6*c*d*n^4*x^3 + 7496*(b^*x + a)^{\wedge}n*a*b \\
& ^7*c*d*n^3*x^4 - 1260*(b^*x + a)^{\wedge}n*a^4*b^4*d^2*n^3*x^4 + 39128*(b^*x + a)^{\wedge}n*b \\
& ^8*c*d*n^2*x^5 + 2100*(b^*x + a)^{\wedge}n*a^3*b^5*d^2*n^2*x^5 - 840*(b^*x + a)^{\wedge}n*a^2 \\
& *b^6*d^2*n*x^6 - (b^*x + a)^{\wedge}n*a^2*b^6*c^2*n^6 + 445*(b^*x + a)^{\wedge}n*a*b^7*c^2*n^ \\
& 5*x + 3580*(b^*x + a)^{\wedge}n*b^8*c^2*n^4*x^2 + 528*(b^*x + a)^{\wedge}n*a^3*b^5*c*d*n^4*x^ \\
& 2 - 6528*(b^*x + a)^{\wedge}n*a^2*b^6*c*d*n^3*x^3 + 840*(b^*x + a)^{\wedge}n*a^5*b^3*d^2*n^3* \\
& x^3 + 9144*(b^*x + a)^{\wedge}n*a*b^7*c*d*n^2*x^4 - 2310*(b^*x + a)^{\wedge}n*a^4*b^4*d^2*n^2 \\
& *x^4 + 40608*(b^*x + a)^{\wedge}n*b^8*c*d*n*x^5 + 1008*(b^*x + a)^{\wedge}n*a^3*b^5*d^2*n*x^5 \\
& - 33*(b^*x + a)^{\wedge}n*a^2*b^6*c^2*n^5 + 3135*(b^*x + a)^{\wedge}n*a*b^7*c^2*n^4*x - 48*(\\
& b^*x + a)^{\wedge}n*a^4*b^4*c*d*n^4*x + 15289*(b^*x + a)^{\wedge}n*b^8*c^2*n^3*x^2 + 4008*(b^* \\
& x + a)^{\wedge}n*a^3*b^5*c*d*n^3*x^2 - 10400*(b^*x + a)^{\wedge}n*a^2*b^6*c*d*n^2*x^3 + 2520 \\
& *(b^*x + a)^{\wedge}n*a^5*b^3*d^2*n^2*x^3 + 4032*(b^*x + a)^{\wedge}n*a*b^7*c*d*n*x^4 - 1260* \\
& (b^*x + a)^{\wedge}n*a^4*b^4*d^2*n*x^4 + 16128*(b^*x + a)^{\wedge}n*b^8*c*d*x^5 - 445*(b^*x + \\
& a)^{\wedge}n*a^2*b^6*c^2*n^4 + 12154*(b^*x + a)^{\wedge}n*a*b^7*c^2*n^3*x - 1008*(b^*x + a)^{\wedge}n \\
& *a^4*b^4*c*d*n^3*x + 36706*(b^*x + a)^{\wedge}n*b^8*c^2*n^2*x^2 + 11568*(b^*x + a)^{\wedge}n* \\
& a^3*b^5*c*d*n^2*x^2 - 2520*(b^*x + a)^{\wedge}n*a^6*b^2*d^2*n^2*x^2 - 5376*(b^*x + a) \\
& ^{\wedge}n*a^2*b^6*c*d*n*x^3 + 1680*(b^*x + a)^{\wedge}n*a^5*b^3*d^2*n*x^3 - 3135*(b^*x + a)^{\wedge} \\
& n*a^2*b^6*c^2*n^3 + 48*(b^*x + a)^{\wedge}n*a^5*b^3*c*d*n^3 + 24552*(b^*x + a)^{\wedge}n*a*b^ \\
& 7*c^2*n^2*x - 7008*(b^*x + a)^{\wedge}n*a^4*b^4*c*d*n^2*x + 44712*(b^*x + a)^{\wedge}n*b^8*c^ \\
& 2*n*x^2 + 8064*(b^*x + a)^{\wedge}n*a^3*b^5*c*d*n*x^2 - 2520*(b^*x + a)^{\wedge}n*a^6*b^2*d^2 \\
& *n*x^2 - 12154*(b^*x + a)^{\wedge}n*a^2*b^6*c^2*n^2 + 1008*(b^*x + a)^{\wedge}n*a^5*b^3*c*d*n \\
& ^2 + 20160*(b^*x + a)^{\wedge}n*a*b^7*c^2*n*x - 16128*(b^*x + a)^{\wedge}n*a^4*b^4*c*d*n*x + \\
& 5040*(b^*x + a)^{\wedge}n*a^7*b*d^2*n*x + 20160*(b^*x + a)^{\wedge}n*b^8*c^2*x^2 - 24552*(b^*x \\
& + a)^{\wedge}n*a^2*b^6*c^2*n + 7008*(b^*x + a)^{\wedge}n*a^5*b^3*c*d*n - 20160*(b^*x + a)^{\wedge}n* \\
& a^2*b^6*c^2 + 16128*(b^*x + a)^{\wedge}n*a^5*b^3*c*d - 5040*(b^*x + a)^{\wedge}n*a^8*d^2)/(b^ \\
& 8*n^8 + 36*b^8*n^7 + 546*b^8*n^6 + 4536*b^8*n^5 + 22449*b^8*n^4 + 67284*b^8 \\
& *n^3 + 118124*b^8*n^2 + 109584*b^8*n + 40320*b^8)
\end{aligned}$$

maple [B] time = 0.02, size = 1142, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(b*x+a)^n*(d*x^3+c)^2,x)$

[Out] $-(b*x+a)^{(n+1)}*(-b^7*d^2*n^7*x^7-28*b^7*d^2*n^6*x^7+7*a*b^6*d^2*n^6*x^6-322$
 $*b^7*d^2*n^5*x^7+147*a*b^6*d^2*n^5*x^6-2*b^7*c*d*n^7*x^4-1960*b^7*d^2*n^4*x$
 $^7-42*a^2*b^5*d^2*n^5*x^5+1225*a*b^6*d^2*n^4*x^6-62*b^7*c*d*n^6*x^4-6769*b^$
 $7*d^2*n^3*x^7-630*a^2*b^5*d^2*n^4*x^5+8*a*b^6*c*d*n^6*x^3+5145*a*b^6*d^2*n^$
 $3*x^6-782*b^7*c*d*n^5*x^4-13132*b^7*d^2*n^2*x^7+210*a^3*b^4*d^2*n^4*x^4-357$
 $0*a^2*b^5*d^2*n^3*x^5+216*a*b^6*c*d*n^5*x^3+11368*a*b^6*d^2*n^2*x^6-b^7*c^2$
 $*n^7*x-5162*b^7*c*d*n^4*x^4-13068*b^7*d^2*n*x^7+2100*a^3*b^4*d^2*n^3*x^4-24$
 $*a^2*b^5*c*d*n^5*x^2-9450*a^2*b^5*d^2*n^2*x^5+2264*a*b^6*c*d*n^4*x^3+12348*$
 $a*b^6*d^2*n*x^6-34*b^7*c^2*n^6*x-19088*b^7*c*d*n^3*x^4-5040*b^7*d^2*x^7-840$
 $*a^4*b^3*d^2*n^3*x^3+7350*a^3*b^4*d^2*n^2*x^4-576*a^2*b^5*c*d*n^4*x^2-11508$
 $*a^2*b^5*d^2*n*x^5+a*b^6*c^2*n^6+11592*a*b^6*c*d*n^3*x^3+5040*a*b^6*d^2*x^6$
 $-478*b^7*c^2*n^5*x-39128*b^7*c*d*n^2*x^4-5040*a^4*b^3*d^2*n^2*x^3+48*a^3*b^$
 $4*c*d*n^4*x+10500*a^3*b^4*d^2*n*x^4-5064*a^2*b^5*c*d*n^3*x^2-5040*a^2*b^5*d$
 $^2*x^5+33*a*b^6*c^2*n^5+29984*a*b^6*c*d*n^2*x^3-3580*b^7*c^2*n^4*x-40608*b^$
 $7*c*d*n*x^4+2520*a^5*b^2*d^2*n^2*x^2-9240*a^4*b^3*d^2*n*x^3+1056*a^3*b^4*c*$
 $d*n^3*x+5040*a^3*b^4*d^2*x^4-19584*a^2*b^5*c*d*n^2*x^2+445*a*b^6*c^2*n^4+36$
 $576*a*b^6*c*d*n*x^3-15289*b^7*c^2*n^3*x-16128*b^7*c*d*x^4+7560*a^5*b^2*d^2*$
 $n*x^2-48*a^4*b^3*c*d*n^3-5040*a^4*b^3*d^2*x^3+8016*a^3*b^4*c*d*n^2*x-31200*$
 $a^2*b^5*c*d*n*x^2+3135*a*b^6*c^2*n^3+16128*a*b^6*c*d*x^3-36706*b^7*c^2*n^2*$
 $x-5040*a^6*b*d^2*n*x+5040*a^5*b^2*d^2*x^2-1008*a^4*b^3*c*d*n^2+23136*a^3*b^$

$4*c*d*n*x-16128*a^2*b^5*c*d*x^2+12154*a*b^6*c^2*n^2-44712*b^7*c^2*n*x-5040*a^6*b*d^2*x-7008*a^4*b^3*c*d*n+16128*a^3*b^4*c*d*x+24552*a*b^6*c^2*n-20160*b^7*c^2*x+5040*a^7*d^2-16128*a^4*b^3*c*d+20160*a*b^6*c^2)/b^8/(n^8+36*n^7+546*n^6+4536*n^5+22449*n^4+67284*n^3+118124*n^2+109584*n+40320)$

maxima [A] time = 0.98, size = 474, normalized size = 1.91

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")

[Out] $(b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + ((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040*a^8)*(b*x + a)^n*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8)$

mupad [B] time = 3.39, size = 1136, normalized size = 4.58

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c + d*x^3)^2*(a + b*x)^n,x)

[Out] $(d^2*x^8*(a + b*x)^n*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))/(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320) - (a^2*(a + b*x)^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^8*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (x^2*(n + 1)*(a + b*x)^n*(20160*b^6*c^2 - 2520*a^6*d^2*n + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 + 8064*a^3*b^3*c*d*n + 3504*a^3*b^3*c*d*n^2 + 504*a^3*b^3*c*d*n^3 + 24*a^3*b^3*c*d*n^4))/(b^6*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*n*x*(a + b*x)^n*(5040*a^6*d^2 + 20160*b^6*c^2 + 24552*b^6*c^2*n + 12154*b^6*c^2*n^2 + 3135*b^6*c^2*n^3 + 445*b^6*c^2*n^4 + 33*b^6*c^2*n^5 + b^6*c^2*n^6 - 16128*a^3*b^3*c*d - 7008*a^3*b^3*c*d*n - 1008*a^3*b^3*c*d*n^2 - 48*a^3*b^3*c*d*n^3))/(b^7*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (2*d*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(336*b^3*c + 21*b^3*c*n^2 + b^3*c*n^3 + 21*a^3*d*n + 146*b^3*c*n))/(b^3*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (a*d^2*n*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(b*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) - (7*a^2*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b^2*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) + (2*a*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(336*b^3*c - 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^4*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))$

) - (8*a^2*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(336*b^3*c - 105*a^3*d + 21*b^3*c*n^2 + b^3*c*n^3 + 146*b^3*c*n))/(b^5*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**n*(d*x**3+c)**2,x)

[Out] Timed out

$$3.55 \quad \int (a + bx)^n (c + dx^3)^2 dx$$

Optimal. Leaf size=203

$$\frac{(b^3c - a^3d)^2 (a + bx)^{n+1}}{b^7(n+1)} - \frac{3ad(2b^3c - 5a^3d)(a + bx)^{n+3}}{b^7(n+3)} + \frac{2d(b^3c - 10a^3d)(a + bx)^{n+4}}{b^7(n+4)} + \frac{15a^2d^2(a + bx)^{n+5}}{b^7(n+5)} + \frac{6ad^2(a + bx)^{n+6}}{b^7(n+6)} + \frac{d^2(a + bx)^{n+7}}{b^7(n+7)}$$

Rubi [A] time = 0.11, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1850}

$$\frac{(b^3c - a^3d)^2 (a + bx)^{n+1}}{b^7(n+1)} + \frac{6a^2d(b^3c - a^3d)(a + bx)^{n+2}}{b^7(n+2)} - \frac{3ad(2b^3c - 5a^3d)(a + bx)^{n+3}}{b^7(n+3)} + \frac{2d(b^3c - 10a^3d)(a + bx)^{n+4}}{b^7(n+4)} + \frac{15a^2d^2(a + bx)^{n+5}}{b^7(n+5)} - \frac{6ad^2(a + bx)^{n+6}}{b^7(n+6)} + \frac{d^2(a + bx)^{n+7}}{b^7(n+7)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^n*(c + d*x^3)^2,x]

[Out] ((b^3*c - a^3*d)^2*(a + b*x)^(1 + n))/(b^7*(1 + n)) + (6*a^2*d*(b^3*c - a^3*d)*(a + b*x)^(2 + n))/(b^7*(2 + n)) - (3*a*d*(2*b^3*c - 5*a^3*d)*(a + b*x)^(3 + n))/(b^7*(3 + n)) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^(4 + n))/(b^7*(4 + n)) + (15*a^2*d^2*(a + b*x)^(5 + n))/(b^7*(5 + n)) - (6*a*d^2*(a + b*x)^(6 + n))/(b^7*(6 + n)) + (d^2*(a + b*x)^(7 + n))/(b^7*(7 + n))

Rule 1850

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\begin{aligned} \int (a + bx)^n (c + dx^3)^2 dx &= \int \left(\frac{(b^3c - a^3d)^2 (a + bx)^n}{b^6} - \frac{6a^2d(-b^3c + a^3d)(a + bx)^{1+n}}{b^6} + \frac{3ad(-2b^3c + 5a^3d)(a + bx)^{2+n}}{b^6} \right. \\ &= \frac{(b^3c - a^3d)^2 (a + bx)^{1+n}}{b^7(1+n)} + \frac{6a^2d(b^3c - a^3d)(a + bx)^{2+n}}{b^7(2+n)} - \frac{3ad(2b^3c - 5a^3d)(a + bx)^{3+n}}{b^7(3+n)} \end{aligned}$$

Mathematica [A] time = 0.17, size = 172, normalized size = 0.85

$$\frac{(a + bx)^{n+1} \left(\frac{2d(a+bx)^3(b^3c-10a^3d)}{n+4} + \frac{3ad(a+bx)^2(5a^3d-2b^3c)}{n+3} + \frac{(b^3c-a^3d)^2}{n+1} + \frac{15a^2d^2(a+bx)^4}{n+5} + \frac{6a^2d(a+bx)(b^3c-a^3d)}{n+2} + \frac{d^2(a+bx)^6}{n+7} - \frac{6ad^2(a+bx)^5}{n+6} \right)}{b^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^n*(c + d*x^3)^2,x]

[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^2/(1 + n) + (6*a^2*d*(b^3*c - a^3*d)*(a + b*x))/(2 + n) + (3*a*d*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) + (2*d*(b^3*c - 10*a^3*d)*(a + b*x)^3)/(4 + n) + (15*a^2*d^2*(a + b*x)^4)/(5 + n) - (6*a*d^2*(a + b*x)^5)/(6 + n) + (d^2*(a + b*x)^6)/(7 + n))/b^7

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (a + bx)^n (c + dx^3)^2 dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3)^2,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + b*x)^n*(c + d*x^3)^2, x]
```

fricas [B] time = 0.65, size = 893, normalized size = 4.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="fricas")
```

```
[Out] (a*b^6*c^2*n^6 + 27*a*b^6*c^2*n^5 + 295*a*b^6*c^2*n^4 + 5040*a*b^6*c^2 - 25
20*a^4*b^3*c*d + 720*a^7*d^2 + (b^7*d^2*n^6 + 21*b^7*d^2*n^5 + 175*b^7*d^2*
n^4 + 735*b^7*d^2*n^3 + 1624*b^7*d^2*n^2 + 1764*b^7*d^2*n + 720*b^7*d^2)*x^
7 + (a*b^6*d^2*n^6 + 15*a*b^6*d^2*n^5 + 85*a*b^6*d^2*n^4 + 225*a*b^6*d^2*n^
3 + 274*a*b^6*d^2*n^2 + 120*a*b^6*d^2*n)*x^6 - 6*(a^2*b^5*d^2*n^5 + 10*a^2*
b^5*d^2*n^4 + 35*a^2*b^5*d^2*n^3 + 50*a^2*b^5*d^2*n^2 + 24*a^2*b^5*d^2*n)*x
^5 + 2*(b^7*c*d*n^6 + 24*b^7*c*d*n^5 + 1260*b^7*c*d + (226*b^7*c*d + 15*a^3
*b^4*d^2)*n^4 + 6*(176*b^7*c*d + 15*a^3*b^4*d^2)*n^3 + 5*(509*b^7*c*d + 33*
a^3*b^4*d^2)*n^2 + 18*(164*b^7*c*d + 5*a^3*b^4*d^2)*n)*x^4 + 3*(555*a*b^6*c
^2 - 4*a^4*b^3*c*d)*n^3 + 2*(a*b^6*c*d*n^6 + 21*a*b^6*c*d*n^5 + 163*a*b^6*c
*d*n^4 + 3*(189*a*b^6*c*d - 20*a^4*b^3*d^2)*n^3 + 4*(211*a*b^6*c*d - 45*a^4
*b^3*d^2)*n^2 + 60*(7*a*b^6*c*d - 2*a^4*b^3*d^2)*n)*x^3 + 8*(638*a*b^6*c^2
- 27*a^4*b^3*c*d)*n^2 - 6*(a^2*b^5*c*d*n^5 + 19*a^2*b^5*c*d*n^4 + 125*a^2*b
^5*c*d*n^3 + (317*a^2*b^5*c*d - 60*a^5*b^2*d^2)*n^2 + 30*(7*a^2*b^5*c*d - 2
*a^5*b^2*d^2)*n)*x^2 + 12*(669*a*b^6*c^2 - 107*a^4*b^3*c*d)*n + (b^7*c^2*n^
6 + 27*b^7*c^2*n^5 + 5040*b^7*c^2 + (295*b^7*c^2 + 12*a^3*b^4*c*d)*n^4 + 9*
(185*b^7*c^2 + 24*a^3*b^4*c*d)*n^3 + 4*(1276*b^7*c^2 + 321*a^3*b^4*c*d)*n^2
+ 36*(223*b^7*c^2 + 70*a^3*b^4*c*d - 20*a^6*b*d^2)*n)*x*(b*x + a)^n/(b^7*
n^7 + 28*b^7*n^6 + 322*b^7*n^5 + 1960*b^7*n^4 + 6769*b^7*n^3 + 13132*b^7*n^
2 + 13068*b^7*n + 5040*b^7)
```

giac [B] time = 0.50, size = 1477, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="giac")
```

```
[Out] ((b*x + a)^n*b^7*d^2*n^6*x^7 + (b*x + a)^n*a*b^6*d^2*n^6*x^6 + 21*(b*x + a)
^n*b^7*d^2*n^5*x^7 + 15*(b*x + a)^n*a*b^6*d^2*n^5*x^6 + 175*(b*x + a)^n*b^7
*d^2*n^4*x^7 + 2*(b*x + a)^n*b^7*c*d*n^6*x^4 - 6*(b*x + a)^n*a^2*b^5*d^2*n^
5*x^5 + 85*(b*x + a)^n*a*b^6*d^2*n^4*x^6 + 735*(b*x + a)^n*b^7*d^2*n^3*x^7
+ 2*(b*x + a)^n*a*b^6*c*d*n^6*x^3 + 48*(b*x + a)^n*b^7*c*d*n^5*x^4 - 60*(b*
x + a)^n*a^2*b^5*d^2*n^4*x^5 + 225*(b*x + a)^n*a*b^6*d^2*n^3*x^6 + 1624*(b*
x + a)^n*b^7*d^2*n^2*x^7 + 42*(b*x + a)^n*a*b^6*c*d*n^5*x^3 + 452*(b*x + a)
^n*b^7*c*d*n^4*x^4 + 30*(b*x + a)^n*a^3*b^4*d^2*n^4*x^4 - 210*(b*x + a)^n*a
^2*b^5*d^2*n^3*x^5 + 274*(b*x + a)^n*a*b^6*d^2*n^2*x^6 + 1764*(b*x + a)^n*b
^7*d^2*n*x^7 + (b*x + a)^n*b^7*c^2*n^6*x - 6*(b*x + a)^n*a^2*b^5*c*d*n^5*x^
2 + 326*(b*x + a)^n*a*b^6*c*d*n^4*x^3 + 2112*(b*x + a)^n*b^7*c*d*n^3*x^4 +
180*(b*x + a)^n*a^3*b^4*d^2*n^3*x^4 - 300*(b*x + a)^n*a^2*b^5*d^2*n^2*x^5 +
120*(b*x + a)^n*a*b^6*d^2*n*x^6 + 720*(b*x + a)^n*b^7*d^2*x^7 + (b*x + a)^
n*a*b^6*c^2*n^6 + 27*(b*x + a)^n*b^7*c^2*n^5*x - 114*(b*x + a)^n*a^2*b^5*c*
d*n^4*x^2 + 1134*(b*x + a)^n*a*b^6*c*d*n^3*x^3 - 120*(b*x + a)^n*a^4*b^3*d^
2*n^3*x^3 + 5090*(b*x + a)^n*b^7*c*d*n^2*x^4 + 330*(b*x + a)^n*a^3*b^4*d^2*
n^2*x^4 - 144*(b*x + a)^n*a^2*b^5*d^2*n*x^5 + 27*(b*x + a)^n*a*b^6*c^2*n^5
+ 295*(b*x + a)^n*b^7*c^2*n^4*x + 12*(b*x + a)^n*a^3*b^4*c*d*n^4*x - 750*(b
*x + a)^n*a^2*b^5*c*d*n^3*x^2 + 1688*(b*x + a)^n*a*b^6*c*d*n^2*x^3 - 360*(b
*x + a)^n*a^4*b^3*d^2*n^2*x^3 + 5904*(b*x + a)^n*b^7*c*d*n*x^4 + 180*(b*x +
```


$$a)^n * a^3 * b^4 * d^2 * n * x^4 + 295 * (b * x + a)^n * a * b^6 * c^2 * n^4 + 1665 * (b * x + a)^n * b^7 * c^2 * n^3 * x + 216 * (b * x + a)^n * a^3 * b^4 * c * d * n^3 * x - 1902 * (b * x + a)^n * a^2 * b^5 * c * d * n^2 * x^2 + 360 * (b * x + a)^n * a^5 * b^2 * d^2 * n^2 * x^2 + 840 * (b * x + a)^n * a * b^6 * c * d * n * x^3 - 240 * (b * x + a)^n * a^4 * b^3 * d^2 * n * x^3 + 2520 * (b * x + a)^n * b^7 * c * d * x^4 + 1665 * (b * x + a)^n * a * b^6 * c^2 * n^3 - 12 * (b * x + a)^n * a^4 * b^3 * c * d * n^3 + 5104 * (b * x + a)^n * b^7 * c^2 * n^2 * x + 1284 * (b * x + a)^n * a^3 * b^4 * c * d * n^2 * x - 1260 * (b * x + a)^n * a^2 * b^5 * c * d * n * x^2 + 360 * (b * x + a)^n * a^5 * b^2 * d^2 * n * x^2 + 5104 * (b * x + a)^n * a * b^6 * c^2 * n^2 - 216 * (b * x + a)^n * a^4 * b^3 * c * d * n^2 + 8028 * (b * x + a)^n * b^7 * c^2 * n * x + 2520 * (b * x + a)^n * a^3 * b^4 * c * d * n * x - 720 * (b * x + a)^n * a^6 * b * d^2 * n * x + 8028 * (b * x + a)^n * a * b^6 * c^2 * n - 1284 * (b * x + a)^n * a^4 * b^3 * c * d * n + 5040 * (b * x + a)^n * b^7 * c^2 * x + 5040 * (b * x + a)^n * a * b^6 * c^2 - 2520 * (b * x + a)^n * a^4 * b^3 * c * d + 720 * (b * x + a)^n * a^7 * d^2) / (b^7 * n^7 + 28 * b^7 * n^6 + 322 * b^7 * n^5 + 1960 * b^7 * n^4 + 6769 * b^7 * n^3 + 13132 * b^7 * n^2 + 13068 * b^7 * n + 5040 * b^7)$$

maple [B] time = 0.01, size = 793, normalized size = 3.91

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x+a)^n*(d*x^3+c)^2,x)

[Out] (b*x+a)^(n+1)*(b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+2*b^6*c*d*n^6*x^3+735*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-510*a*b^5*d^2*n^3*x^5+48*b^6*c*d*n^5*x^3+1624*b^6*d^2*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-6*a*b^5*c*d*n^5*x^2-1350*a*b^5*d^2*n^2*x^5+452*b^6*c*d*n^4*x^3+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+1050*a^2*b^4*d^2*n^2*x^4-126*a*b^5*c*d*n^4*x^2-1644*a*b^5*d^2*n*x^5+b^6*c^2*n^6+2112*b^6*c*d*n^3*x^3+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+12*a^2*b^4*c*d*n^4*x+1500*a^2*b^4*d^2*n*x^4-978*a*b^5*c*d*n^3*x^2-720*a*b^5*d^2*x^5+27*b^6*c^2*n^5+5090*b^6*c*d*n^2*x^3+360*a^4*b^2*d^2*n^2*x^2-1320*a^3*b^3*d^2*n*x^3+228*a^2*b^4*c*d*n^3*x+720*a^2*b^4*d^2*x^4-3402*a*b^5*c*d*n^2*x^2+295*b^6*c^2*n^4+5904*b^6*c*d*n*x^3+1080*a^4*b^2*d^2*n*x^2-12*a^3*b^3*c*d*n^3-720*a^3*b^3*d^2*x^3+1500*a^2*b^4*c*d*n^2*x-5064*a*b^5*c*d*n*x^2+1665*b^6*c^2*n^3+2520*b^6*c*d*x^3-720*a^5*b*d^2*n*x+720*a^4*b^2*d^2*x^2-216*a^3*b^3*c*d*n^2+3804*a^2*b^4*c*d*n*x-2520*a*b^5*c*d*x^2+5104*b^6*c^2*n^2-720*a^5*b*d^2*x-1284*a^3*b^3*c*d*n+2520*a^2*b^4*c*d*x+8028*b^6*c^2*n+720*a^6*d^2-2520*a^3*b^3*c*d+5040*b^6*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)

maxima [A] time = 0.66, size = 359, normalized size = 1.77

$$\frac{(b * x + a)^{n+1} * (b^6 * d^2 * n^6 * x^6 + 21 * b^6 * d^2 * n^5 * x^6 - 6 * a * b^5 * d^2 * n^5 * x^5 + 175 * b^6 * d^2 * n^4 * x^6 - 90 * a * b^5 * d^2 * n^4 * x^5 + 2 * b^6 * c * d * n^6 * x^3 + 735 * b^6 * d^2 * n^3 * x^6 + 30 * a^2 * b^4 * d^2 * n^4 * x^4 - 510 * a * b^5 * d^2 * n^3 * x^5 + 48 * b^6 * c * d * n^5 * x^3 + 1624 * b^6 * d^2 * n^2 * x^6 + 300 * a^2 * b^4 * d^2 * n^3 * x^4 - 6 * a * b^5 * c * d * n^5 * x^2 - 1350 * a * b^5 * d^2 * n^2 * x^5 + 452 * b^6 * c * d * n^4 * x^3 + 1764 * b^6 * d^2 * n * x^6 - 120 * a^3 * b^3 * d^2 * n^3 * x^3 + 1050 * a^2 * b^4 * d^2 * n^2 * x^4 - 126 * a * b^5 * c * d * n^4 * x^2 - 1644 * a * b^5 * d^2 * n * x^5 + b^6 * c^2 * n^6 + 2112 * b^6 * c * d * n^3 * x^3 + 720 * b^6 * d^2 * x^6 - 720 * a^3 * b^3 * d^2 * n^2 * x^3 + 12 * a^2 * b^4 * c * d * n^4 * x + 1500 * a^2 * b^4 * d^2 * n * x^4 - 978 * a * b^5 * c * d * n^3 * x^2 - 720 * a * b^5 * d^2 * x^5 + 27 * b^6 * c^2 * n^5 + 5090 * b^6 * c * d * n^2 * x^3 + 360 * a^4 * b^2 * d^2 * n^2 * x^2 - 1320 * a^3 * b^3 * d^2 * n * x^3 + 228 * a^2 * b^4 * c * d * n^3 * x + 720 * a^2 * b^4 * d^2 * x^4 - 3402 * a * b^5 * c * d * n^2 * x^2 + 295 * b^6 * c^2 * n^4 + 5904 * b^6 * c * d * n * x^3 + 1080 * a^4 * b^2 * d^2 * n * x^2 - 12 * a^3 * b^3 * c * d * n^3 - 720 * a^3 * b^3 * d^2 * x^3 + 1500 * a^2 * b^4 * c * d * n^2 * x - 5064 * a * b^5 * c * d * n * x^2 + 1665 * b^6 * c^2 * n^3 + 2520 * b^6 * c * d * x^3 - 720 * a^5 * b * d^2 * n * x + 720 * a^4 * b^2 * d^2 * x^2 - 216 * a^3 * b^3 * c * d * n^2 + 3804 * a^2 * b^4 * c * d * n * x - 2520 * a * b^5 * c * d * x^2 + 5104 * b^6 * c^2 * n^2 - 720 * a^5 * b * d^2 * x - 1284 * a^3 * b^3 * c * d * n + 2520 * a^2 * b^4 * c * d * x + 8028 * b^6 * c^2 * n + 720 * a^6 * d^2 - 2520 * a^3 * b^3 * c * d + 5040 * b^6 * c^2) / b^7 / (n^7 + 28 * n^6 + 322 * n^5 + 1960 * n^4 + 6769 * n^3 + 13132 * n^2 + 13068 * n + 5040)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c)^2,x, algorithm="maxima")

[Out] (b*x + a)^(n + 1)*c^2/(b*(n + 1)) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*d^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 13068*n + 5040)*b^7)

mupad [B] time = 3.19, size = 878, normalized size = 4.33

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c + d*x^3)^2*(a + b*x)^n,x)`

[Out] $(a*(a + b*x)^n*(720*a^6*d^2 + 5040*b^6*c^2 + 8028*b^6*c^2*n + 5104*b^6*c^2*n^2 + 1665*b^6*c^2*n^3 + 295*b^6*c^2*n^4 + 27*b^6*c^2*n^5 + b^6*c^2*n^6 - 2520*a^3*b^3*c*d - 1284*a^3*b^3*c*d*n - 216*a^3*b^3*c*d*n^2 - 12*a^3*b^3*c*d*n^3))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (d^2*x^7*(a + b*x)^n*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720))/(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040) + (x*(a + b*x)^n*(5040*b^7*c^2 + 8028*b^7*c^2*n + 5104*b^7*c^2*n^2 + 1665*b^7*c^2*n^3 + 295*b^7*c^2*n^4 + 27*b^7*c^2*n^5 + b^7*c^2*n^6 - 720*a^6*b*d^2*n + 2520*a^3*b^4*c*d*n + 1284*a^3*b^4*c*d*n^2 + 216*a^3*b^4*c*d*n^3 + 12*a^3*b^4*c*d*n^4))/(b^7*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*d*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(210*b^3*c + 18*b^3*c*n^2 + b^3*c*n^3 + 15*a^3*d*n + 107*b^3*c*n))/(b^3*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (a*d^2*n*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/(b*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d^2*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24))/(b^2*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) + (2*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^4*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) - (6*a^2*d*n*x^2*(n + 1)*(a + b*x)^n*(210*b^3*c - 60*a^3*d + 18*b^3*c*n^2 + b^3*c*n^3 + 107*b^3*c*n))/(b^5*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040))$

sympy [A] time = 14.44, size = 11851, normalized size = 58.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**n*(d*x**3+c)**2,x)`

[Out] `Piecewise((a**n*(c**2*x + c*d*x**4/2 + d**2*x**7/7), Eq(b, 0)), (60*a**6*d**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 147*a**6*d**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 360*a**5*b*d**2*x*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 822*a**5*b*d**2*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**4*b**2*d**2*x**2*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1875*a**4*b**2*d**2*x**2/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 2*a**3*b**3*c*d/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1200*a**3*b**3*d**2*x**3*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 2200*a**3*b**3*d**2*x**3/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) - 12*a**2*b**4*c*d*x/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 900*a**2*b**4*d**2*x**4*log(a/b + x)/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 + 900*a**2*b**11*x**4 + 360*a*b**12*x**5 + 60*b**13*x**6) + 1350*a**2*b**4*d**2*x**4/(60*a**6*b**7 + 360*a**5*b**8*x + 900*a**4*b**9*x**2 + 1200*a**3*b**10*x**3 +`

$$\begin{aligned}
& 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + 60b^{**13}x^{**6}) - 30a^{**}b^{**5}c^{**}d^{**}x^{**} \\
& 2/(60a^{**6}b^{**7} + 360a^{**5}b^{**8}x + 900a^{**4}b^{**9}x^{**2} + 1200a^{**3}b^{**10}x^{**} \\
& *3 + 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + 60b^{**13}x^{**6}) + 360a^{**}b^{**5}d^{**} \\
& **2x^{**5}\log(a/b + x)/(60a^{**6}b^{**7} + 360a^{**5}b^{**8}x + 900a^{**4}b^{**9}x^{**2} \\
& + 1200a^{**3}b^{**10}x^{**3} + 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + 60b^{**13}x^{**6}) \\
& + 360a^{**}b^{**5}d^{**2}x^{**5}/(60a^{**6}b^{**7} + 360a^{**5}b^{**8}x + 900a^{**4}b^{**9}x^{**2} \\
& + 1200a^{**3}b^{**10}x^{**3} + 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + 60 \\
& *b^{**13}x^{**6}) - 10b^{**6}c^{**2}/(60a^{**6}b^{**7} + 360a^{**5}b^{**8}x + 900a^{**4}b^{**9} \\
& *x^{**2} + 1200a^{**3}b^{**10}x^{**3} + 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + 60 \\
& *b^{**13}x^{**6}) - 40b^{**6}c^{**}d^{**}x^{**3}/(60a^{**6}b^{**7} + 360a^{**5}b^{**8}x + 900a^{**4}b^{**9} \\
& *x^{**2} + 1200a^{**3}b^{**10}x^{**3} + 900a^{**2}b^{**11}x^{**4} + 360a^{**}b^{**12}x^{**5} + \\
& 60b^{**13}x^{**6}) + 60b^{**6}d^{**2}x^{**6}\log(a/b + x)/(60a^{**6}b^{**7} + 360a^{**5}b^{**} \\
& *8x + 900a^{**4}b^{**9}x^{**2} + 1200a^{**3}b^{**10}x^{**3} + 900a^{**2}b^{**11}x^{**4} + 36 \\
& 0a^{**}b^{**12}x^{**5} + 60b^{**13}x^{**6}), \text{Eq}(n, -7)), (-60a^{**6}d^{**2}\log(a/b + x)/(1 \\
& 0a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 5 \\
& 0a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 137a^{**6}d^{**2}/(10a^{**5}b^{**7} + 50a^{**4}b^{**} \\
& 8x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12} \\
& *x^{**5}) - 300a^{**5}b^{**}d^{**2}x\log(a/b + x)/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 10 \\
& 0a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - \\
& 625a^{**5}b^{**}d^{**2}x/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 10 \\
& 0a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 600a^{**4}b^{**2}d^{**2}x \\
& **2\log(a/b + x)/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**} \\
& a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 1100a^{**4}b^{**2}d^{**2}x \\
& *2/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**} \\
& 3 + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - a^{**3}b^{**3}c^{**}d/(10a^{**5}b^{**7} + 50a^{**} \\
& 4b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10 \\
& b^{**12}x^{**5}) - 600a^{**3}b^{**3}d^{**2}x^{**3}\log(a/b + x)/(10a^{**5}b^{**7} + 50a^{**4} \\
& b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**} \\
& *12x^{**5}) - 900a^{**3}b^{**3}d^{**2}x^{**3}/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**} \\
& *3b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 5a^{**} \\
& **2b^{**4}c^{**}d^{**}x/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**} \\
& *2b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 300a^{**2}b^{**4}d^{**2}x^{**4} \\
& \log(a/b + x)/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2} \\
& *b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 300a^{**2}b^{**4}d^{**2}x^{**4}/(1 \\
& 0a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 5 \\
& 0a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 10a^{**}b^{**5}c^{**}d^{**}x^{**2}/(10a^{**5}b^{**7} + 50a^{**} \\
& 4b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10 \\
& b^{**12}x^{**5}) - 60a^{**}b^{**5}d^{**2}x^{**5}\log(a/b + x)/(10a^{**5}b^{**7} + 50a^{**4}b^{**8} \\
& *x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**} \\
& x^{**5}) - 2b^{**6}c^{**2}/(10a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 1 \\
& 00a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) - 10b^{**6}c^{**}d^{**}x^{**3}/(1 \\
& 0a^{**5}b^{**7} + 50a^{**4}b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 5 \\
& 0a^{**}b^{**11}x^{**4} + 10b^{**12}x^{**5}) + 10b^{**6}d^{**2}x^{**6}/(10a^{**5}b^{**7} + 50a^{**4} \\
& *b^{**8}x + 100a^{**3}b^{**9}x^{**2} + 100a^{**2}b^{**10}x^{**3} + 50a^{**}b^{**11}x^{**4} + 10b^{**} \\
& **12x^{**5}), \text{Eq}(n, -6)), (60a^{**6}d^{**2}\log(a/b + x)/(4a^{**4}b^{**7} + 16a^{**3}b^{**} \\
& **8x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**4}) + 125a^{**6}d^{**2} \\
& /(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**} \\
& *11x^{**4}) + 240a^{**5}b^{**}d^{**2}x\log(a/b + x)/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + \\
& 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**4}) + 440a^{**5}b^{**}d^{**2}x/(4a^{**} \\
& 4b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**} \\
& x^{**4}) + 360a^{**4}b^{**2}d^{**2}x^{**2}\log(a/b + x)/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x \\
& + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**4}) + 540a^{**4}b^{**2}d^{**2}x \\
& x^{**2}/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + \\
& 4b^{**11}x^{**4}) - 2a^{**3}b^{**3}c^{**}d/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**} \\
& 9x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**4}) + 240a^{**3}b^{**3}d^{**2}x^{**3}\log(a/b \\
& + x)/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + \\
& 4b^{**11}x^{**4}) + 240a^{**3}b^{**3}d^{**2}x^{**3}/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**} \\
& 2b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**4}) - 8a^{**2}b^{**4}c^{**}d^{**}x/(4a^{**} \\
& 4b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16a^{**}b^{**10}x^{**3} + 4b^{**11}x^{**}
\end{aligned}$$

$$\begin{aligned}
& 4) + 60a^{**2}b^{**4}d^{**2}x^{**4}\log(a/b + x)/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24 \\
& a^{**2}b^{**9}x^{**2} + 16ab^{**10}x^{**3} + 4b^{**11}x^{**4}) - 12ab^{**5}cd^{**2}x^{**2}/(4a \\
& a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16ab^{**10}x^{**3} + 4b^{**11}x \\
& **4) - 12ab^{**5}d^{**2}x^{**5}/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} \\
& + 16ab^{**10}x^{**3} + 4b^{**11}x^{**4}) - b^{**6}c^{**2}/(4a^{**4}b^{**7} + 16a^{**3}b^{**8} \\
& *x + 24a^{**2}b^{**9}x^{**2} + 16ab^{**10}x^{**3} + 4b^{**11}x^{**4}) - 8b^{**6}cd^{**2}x^{**3}/ \\
& (4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} + 16ab^{**10}x^{**3} + 4b^{**11}x \\
& **4) + 2b^{**6}d^{**2}x^{**6}/(4a^{**4}b^{**7} + 16a^{**3}b^{**8}x + 24a^{**2}b^{**9}x^{**2} \\
& + 16ab^{**10}x^{**3} + 4b^{**11}x^{**4}), \text{Eq}(n, -5)), (-60a^{**6}d^{**2}\log(a/b + \\
& x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) - 110a^{**6}d \\
& **2/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) - 180a^{**5} \\
& b^{**6}d^{**2}x\log(a/b + x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10} \\
& x^{**3}) - 270a^{**5}b^{**6}d^{**2}x/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} \\
& + 3b^{**10}x^{**3}) - 180a^{**4}b^{**2}d^{**2}x^{**2}\log(a/b + x)/(3a^{**3}b^{**7} + 9a^{**2} \\
& b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) - 180a^{**4}b^{**2}d^{**2}x^{**2}/(3a^{**3}b \\
& **7 + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + 6a^{**3}b^{**3}cd\log(\\
& a/b + x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + 11a \\
& **3b^{**3}cd/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) \\
& - 60a^{**3}b^{**3}d^{**2}x^{**3}\log(a/b + x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9} \\
& x^{**2} + 3b^{**10}x^{**3}) + 18a^{**2}b^{**4}cdx\log(a/b + x)/(3a^{**3}b^{**7} + 9a \\
& **2b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + 27a^{**2}b^{**4}cdx/(3a^{**3}b^{**7} \\
& + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + 15a^{**2}b^{**4}d^{**2}x^{**4} \\
& /(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + 18ab^{**5}c \\
& d^{**2}x^{**2}\log(a/b + x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + 3b^{**10} \\
& x^{**3}) + 18ab^{**5}cd^{**2}x^{**2}/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} + \\
& 3b^{**10}x^{**3}) - 3ab^{**5}d^{**2}x^{**5}/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x \\
& **2 + 3b^{**10}x^{**3}) - b^{**6}c^{**2}/(3a^{**3}b^{**7} + 9a^{**2}b^{**8}x + 9ab^{**9}x^{**2} \\
& + 3b^{**10}x^{**3}) + 6b^{**6}cd^{**2}x^{**3}\log(a/b + x)/(3a^{**3}b^{**7} + 9a^{**2}b^{**8} \\
& x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}) + b^{**6}d^{**2}x^{**6}/(3a^{**3}b^{**7} + 9a^{**2}b \\
& **8x + 9ab^{**9}x^{**2} + 3b^{**10}x^{**3}), \text{Eq}(n, -4)), (60a^{**6}d^{**2}\log(a/b + \\
& x)/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) + 90a^{**6}d^{**2}/(4a^{**2}b^{**7} + \\
& 8ab^{**8}x + 4b^{**9}x^{**2}) + 120a^{**5}b^{**6}d^{**2}x\log(a/b + x)/(4a^{**2}b^{**7} + 8 \\
& ab^{**8}x + 4b^{**9}x^{**2}) + 120a^{**5}b^{**6}d^{**2}x/(4a^{**2}b^{**7} + 8ab^{**8}x + 4 \\
& b^{**9}x^{**2}) + 60a^{**4}b^{**2}d^{**2}x^{**2}\log(a/b + x)/(4a^{**2}b^{**7} + 8ab^{**8}x \\
& + 4b^{**9}x^{**2}) - 24a^{**3}b^{**3}cd\log(a/b + x)/(4a^{**2}b^{**7} + 8ab^{**8}x + \\
& 4b^{**9}x^{**2}) - 36a^{**3}b^{**3}cd/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) - \\
& 20a^{**3}b^{**3}d^{**2}x^{**3}/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) - 48a^{**2}b \\
& **4cdx\log(a/b + x)/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) - 48a^{**2}b \\
& **4cdx/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) + 5a^{**2}b^{**4}d^{**2}x^{**4}/ \\
& (4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) - 24ab^{**5}cd^{**2}x^{**2}\log(a/b + x)/ \\
& (4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) - 2ab^{**5}d^{**2}x^{**5}/(4a^{**2}b^{**7} \\
& + 8ab^{**8}x + 4b^{**9}x^{**2}) - 2b^{**6}c^{**2}/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9} \\
& x^{**2}) + 8b^{**6}cd^{**2}x^{**3}/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}) + b^{**6}d \\
& **2x^{**6}/(4a^{**2}b^{**7} + 8ab^{**8}x + 4b^{**9}x^{**2}), \text{Eq}(n, -3)), (-60a^{**6}d \\
& **2\log(a/b + x)/(10ab^{**7} + 10b^{**8}x) - 60a^{**6}d^{**2}/(10ab^{**7} + 10b^{**8} \\
& *x) - 60a^{**5}b^{**6}d^{**2}x\log(a/b + x)/(10ab^{**7} + 10b^{**8}x) + 30a^{**4}b^{**2}d \\
& **2x^{**2}/(10ab^{**7} + 10b^{**8}x) + 60a^{**3}b^{**3}cd\log(a/b + x)/(10ab^{**7} \\
& + 10b^{**8}x) + 60a^{**3}b^{**3}cd/(10ab^{**7} + 10b^{**8}x) - 10a^{**3}b^{**3}d \\
& **2x^{**3}/(10ab^{**7} + 10b^{**8}x) + 60a^{**2}b^{**4}cdx\log(a/b + x)/(10ab^{**7} \\
& + 10b^{**8}x) + 5a^{**2}b^{**4}d^{**2}x^{**4}/(10ab^{**7} + 10b^{**8}x) - 30ab^{**5}c \\
& d^{**2}x^{**2}/(10ab^{**7} + 10b^{**8}x) - 3ab^{**5}d^{**2}x^{**5}/(10ab^{**7} + 10b^{**8} \\
& x) - 10b^{**6}c^{**2}/(10ab^{**7} + 10b^{**8}x) + 10b^{**6}cd^{**2}x^{**3}/(10ab^{**7} + 1 \\
& 0b^{**8}x) + 2b^{**6}d^{**2}x^{**6}/(10ab^{**7} + 10b^{**8}x), \text{Eq}(n, -2)), (a^{**6}d^{**2} \\
& \log(a/b + x)/b^{**7} - a^{**5}d^{**2}x/b^{**6} + a^{**4}d^{**2}x^{**2}/(2b^{**5}) - 2a^{**3}c \\
& d\log(a/b + x)/b^{**4} - a^{**3}d^{**2}x^{**3}/(3b^{**4}) + 2a^{**2}cdx/b^{**3} + a^{**2}d \\
& **2x^{**4}/(4b^{**3}) - acd^{**2}x^{**2}/b^{**2} - a^{**2}d^{**2}x^{**5}/(5b^{**2}) + c^{**2}\log(a/b + \\
& x)/b + 2cd^{**2}x^{**3}/(3b) + d^{**2}x^{**6}/(6b), \text{Eq}(n, -1)), (720a^{**7}d^{**2}(a + \\
& b*x)**n/(b^{**7}n^{**7} + 28b^{**7}n^{**6} + 322b^{**7}n^{**5} + 1960b^{**7}n^{**4} + 6769b \\
& **7n^{**3} + 13132b^{**7}n^{**2} + 13068b^{**7}n + 5040b^{**7}) - 720a^{**6}b^{**6}d^{**2}n
\end{aligned}$$

$$\begin{aligned}
& *x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 \\
& + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 360*a**5*b \\
& **2*d**2*n**2*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + \\
& 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b** \\
& *7) + 360*a**5*b**2*d**2*n*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32 \\
& 2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b** \\
& 7*n + 5040*b**7) - 12*a**4*b**3*c*d*n**3*(a + b*x)**n/(b**7*n**7 + 28*b**7* \\
& n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + \\
& 13068*b**7*n + 5040*b**7) - 216*a**4*b**3*c*d*n**2*(a + b*x)**n/(b**7*n**7 \\
& + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b** \\
& *7*n**2 + 13068*b**7*n + 5040*b**7) - 1284*a**4*b**3*c*d*n*(a + b*x)**n/(b** \\
& *7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + \\
& 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 2520*a**4*b**3*c*d*(a + b*x)* \\
& *n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n \\
& **3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 120*a**4*b**3*d**2*n**3 \\
& *x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n* \\
& *4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 360*a** \\
& 4*b**3*d**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n** \\
& 5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040 \\
& *b**7) - 240*a**4*b**3*d**2*n*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + \\
& 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068* \\
& b**7*n + 5040*b**7) + 12*a**3*b**4*c*d*n**4*x*(a + b*x)**n/(b**7*n**7 + 28* \\
& b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n* \\
& *2 + 13068*b**7*n + 5040*b**7) + 216*a**3*b**4*c*d*n**3*x*(a + b*x)**n/(b** \\
& 7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1 \\
& 3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1284*a**3*b**4*c*d*n**2*x*(a + \\
& b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769* \\
& b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*a**3*b**4*c* \\
& d*n*x*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n* \\
& *4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 30*a**3 \\
& *b**4*d**2*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 \\
& + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040* \\
& b**7) + 180*a**3*b**4*d**2*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 \\
& + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1306 \\
& 8*b**7*n + 5040*b**7) + 330*a**3*b**4*d**2*n**2*x**4*(a + b*x)**n/(b**7*n** \\
& 7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132* \\
& b**7*n**2 + 13068*b**7*n + 5040*b**7) + 180*a**3*b**4*d**2*n*x**4*(a + b*x) \\
& **n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7* \\
& n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*c*d*n**5*x \\
& **2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 \\
& + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 114*a**2* \\
& b**5*c*d*n**4*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + \\
& 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b** \\
& *7) - 750*a**2*b**5*c*d*n**3*x**2*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + \\
& 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b \\
& **7*n + 5040*b**7) - 1902*a**2*b**5*c*d*n**2*x**2*(a + b*x)**n/(b**7*n**7 + \\
& 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b** \\
& 7*n**2 + 13068*b**7*n + 5040*b**7) - 1260*a**2*b**5*c*d*n*x**2*(a + b*x)**n \\
& /(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n** \\
& 3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 6*a**2*b**5*d**2*n**5*x** \\
& 5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + \\
& 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) - 60*a**2*b** \\
& 5*d**2*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1 \\
& 960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7 \\
&) - 210*a**2*b**5*d**2*n**3*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 3 \\
& 22*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b* \\
& *7*n + 5040*b**7) - 300*a**2*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**7*n**7 + \\
& 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7 \\
& *n**2 + 13068*b**7*n + 5040*b**7) - 144*a**2*b**5*d**2*n*x**5*(a + b*x)**n/
\end{aligned}$$

$$\begin{aligned}
& (b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} \\
& + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + a*b^{**6}c^{**2}n^{**6}(a + b*x) \\
& **n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} \\
& + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 27*a*b^{**6}c^{**2}n^{**5}(a \\
& + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 676 \\
& 9*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 295*a*b^{**6}c^{**2} \\
& n^{**4}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} \\
& + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 1665*a* \\
& b^{**6}c^{**2}n^{**3}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 196 \\
& 0*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) \\
& + 5104*a*b^{**6}c^{**2}n^{**2}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} \\
& + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 50 \\
& 40*b^{**7}) + 8028*a*b^{**6}c^{**2}n(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322* \\
& b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n \\
& + 5040*b^{**7}) + 5040*a*b^{**6}c^{**2}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + \\
& 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b \\
& **7n + 5040*b^{**7}) + 2*a*b^{**6}c*d*n^{**6}x^{**3}(a + b*x)**n/(b^{**7}n^{**7} + 28*b* \\
& **7n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} \\
& + 13068*b^{**7}n + 5040*b^{**7}) + 42*a*b^{**6}c*d*n^{**5}x^{**3}(a + b*x)**n/(b^{**7}n \\
& **7 + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 1313 \\
& 2*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 326*a*b^{**6}c*d*n^{**4}x^{**3}(a + b*x) \\
&)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7} \\
& n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 1134*a*b^{**6}c*d*n^{**3}x \\
& x^{**3}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} \\
& + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 1688*a*b \\
& **6c*d*n^{**2}x^{**3}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + \\
& 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7} \\
&) + 840*a*b^{**6}c*d*n*x^{**3}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7} \\
& n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + \\
& 5040*b^{**7}) + a*b^{**6}d^{**2}n^{**6}x^{**6}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} \\
& + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068 \\
& *b^{**7}n + 5040*b^{**7}) + 15*a*b^{**6}d^{**2}n^{**5}x^{**6}(a + b*x)**n/(b^{**7}n^{**7} + 2 \\
& 8*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n \\
& **2 + 13068*b^{**7}n + 5040*b^{**7}) + 85*a*b^{**6}d^{**2}n^{**4}x^{**6}(a + b*x)**n/(b \\
& **7n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + \\
& 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 225*a*b^{**6}d^{**2}n^{**3}x^{**6}(a \\
& + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 676 \\
& 9*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 274*a*b^{**6}d^{**2} \\
& n^{**2}x^{**6}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b* \\
& **7n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 12 \\
& 0*a*b^{**6}d^{**2}n*x^{**6}(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} \\
& + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040* \\
& b^{**7}) + b^{**7}c^{**2}n^{**6}x(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n \\
& n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5 \\
& 040*b^{**7}) + 27*b^{**7}c^{**2}n^{**5}x(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 32 \\
& 2*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b** \\
& 7n + 5040*b^{**7}) + 295*b^{**7}c^{**2}n^{**4}x(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n \\
& **6 + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 1 \\
& 3068*b^{**7}n + 5040*b^{**7}) + 1665*b^{**7}c^{**2}n^{**3}x(a + b*x)**n/(b^{**7}n^{**7} + \\
& 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 13132*b^{**7} \\
& n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 5104*b^{**7}c^{**2}n^{**2}x(a + b*x)**n/(b** \\
& 7n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n^{**3} + 1 \\
& 3132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 8028*b^{**7}c^{**2}n*x(a + b*x)** \\
& n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b^{**7}n \\
& **3 + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 5040*b^{**7}c^{**2}x(a + b* \\
& x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + 6769*b** \\
& 7n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 2*b^{**7}c*d*n^{**6}x^{**4} \\
& *(a + b*x)**n/(b^{**7}n^{**7} + 28*b^{**7}n^{**6} + 322*b^{**7}n^{**5} + 1960*b^{**7}n^{**4} + \\
& 6769*b^{**7}n^{**3} + 13132*b^{**7}n^{**2} + 13068*b^{**7}n + 5040*b^{**7}) + 48*b^{**7}c*d*
\end{aligned}$$

```

n**5*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**
7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 452
*b**7*c*d*n**4*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 2112*b**7*c*d*n**3*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322
*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7
*n + 5040*b**7) + 5090*b**7*c*d*n**2*x**4*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 5904*b**7*c*d*n*x**4*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**
7*n**2 + 13068*b**7*n + 5040*b**7) + 2520*b**7*c*d*x**4*(a + b*x)**n/(b**7*
n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 131
32*b**7*n**2 + 13068*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n
/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**
3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*n**5*x**7*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 676
9*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 175*b**7*d**2*n
**4*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7
*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*
b**7*d**2*n**3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b
**7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 32
2*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**
7*n + 5040*b**7) + 1764*b**7*d**2*n*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*
n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 +
13068*b**7*n + 5040*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28
*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n
**2 + 13068*b**7*n + 5040*b**7), True))

```

3.56 $\int x^2(a + bx)^n (c + dx^3)^3 dx$

Optimal. Leaf size=459

$$-\frac{6ad^2(4b^3c - 55a^3d)(a + bx)^{n+8}}{b^{12}(n + 8)} + \frac{3d^2(b^3c - 55a^3d)(a + bx)^{n+9}}{b^{12}(n + 9)} - \frac{a(2b^3c - 11a^3d)(b^3c - a^3d)^2(a + bx)^{n+2}}{b^{12}(n + 2)} + \frac{55a^2d^2(b^3c - a^3d)(a + bx)^{n+10}}{b^{12}(n + 10)}$$

Rubi [A] time = 0.32, antiderivative size = 459, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1620}

$\frac{(b^3c - a^3d)(-2b^3c + 55a^3d + 9c^2)(a + bx)^{n+1}}{b^{12}(n + 3)} - \frac{3d^2(-56b^3c + 55a^3d + 10b^3c^2)(a + bx)^{n+2}}{b^{12}(n + 4)} - \frac{15ad(-14b^3c + 22a^3d + 9c^2)(a + bx)^{n+3}}{b^{12}(n + 5)} - \frac{3d(-56b^3c + 55a^3d + 9c^2)(a + bx)^{n+4}}{b^{12}(n + 6)} - \frac{42d^2(2b^3c - 11a^3d)(a + bx)^{n+5}}{b^{12}(n + 7)} - \frac{6ad^2(4b^3c - 55a^3d)(a + bx)^{n+6}}{b^{12}(n + 8)} - \frac{3d^2(b^3c - 55a^3d)(a + bx)^{n+7}}{b^{12}(n + 9)} - \frac{a^2(b^3c - a^3d)(a + bx)^{n+8}}{b^{12}(n + 10)} - \frac{a(2b^3c - 11a^3d)(b^3c - a^3d)^2(a + bx)^{n+2}}{b^{12}(n + 2)} - \frac{55a^2d^2(b^3c - a^3d)(a + bx)^{n+10}}{b^{12}(n + 10)} - \frac{11ad^2(b^3c - a^3d)(a + bx)^{n+11}}{b^{12}(n + 11)} - \frac{d^2(a + bx)^{n+12}}{b^{12}(n + 12)}$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
[Out] (a^2*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^12*(1 + n)) - (a*(2*b^3*c - 11*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^12*(2 + n)) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(3 + n))/(b^12*(3 + n)) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^(4 + n))/(b^12*(4 + n)) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^(5 + n))/(b^12*(5 + n)) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a + b*x)^(6 + n))/(b^12*(6 + n)) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x)^(7 + n))/(b^12*(7 + n)) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^(8 + n))/(b^12*(8 + n)) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^(9 + n))/(b^12*(9 + n)) + (55*a^2*d^3*(a + b*x)^(10 + n))/(b^12*(10 + n)) - (11*a*d^3*(a + b*x)^(11 + n))/(b^12*(11 + n)) + (d^3*(a + b*x)^(12 + n))/(b^12*(12 + n))
```

Rule 1620

```
Int[(Px_)*((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol]
:= Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\int x^2(a + bx)^n (c + dx^3)^3 dx = \int \left(-\frac{a^2(-b^3c + a^3d)^3(a + bx)^n}{b^{11}} + \frac{a(-b^3c + a^3d)^2(-2b^3c + 11a^3d)(a + bx)^{1+n}}{b^{11}} + \dots \right) dx$$

$$= \frac{a^2(b^3c - a^3d)^3(a + bx)^{1+n}}{b^{12}(1 + n)} - \frac{a(2b^3c - 11a^3d)(b^3c - a^3d)^2(a + bx)^{2+n}}{b^{12}(2 + n)} + \frac{(b^3c - a^3d)(b^6c^2 - 29a^3b^3cd + 55a^6d^2)(a + bx)^{3+n}}{b^{12}(3 + n)} + \dots$$

Mathematica [A] time = 0.47, size = 402, normalized size = 0.88

$(a + bx)^{n+1} \left(\frac{3d^2(a+bx)(b^3c - 55a^3d)}{b^{12}} + \frac{6ad^2(a+bx)^2(5b^3c - 4b^3c^2)}{b^{12}} + \frac{d(a+bx)(b^3c - a^3d)^2(11a^3d - 2b^3c)}{b^{12}} + \frac{55a^2d^2(a+bx)^3}{b^{12}} + \frac{3d(a+bx)^4(154a^3d^2 - 56a^3b^3cd + 9c^2)}{b^{12}} - \frac{15ad(a+bx)^5(22a^3d^2 - 14a^3b^3cd + 9c^2)}{b^{12}} + \frac{(a+bx)^6(5b^3c - 55a^3d)(55a^6d^2 - 29a^3b^3cd + 9c^2)}{b^{12}} + \frac{42a^2d^2(a+bx)^7(2b^3c - 11a^3d)}{b^{12}} + \frac{a^2(b^3c - a^3d)^2}{b^{12}} + \frac{3a^2d(a+bx)^9(55a^6d^2 - 56a^3b^3cd + 10b^3c^2)}{b^{12}} + \frac{d^2(a+bx)^{11}}{b^{12}} - \frac{11ad^2(a+bx)^{10}}{b^{12}} \right)$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + b*x)^n*(c + d*x^3)^3,x]
[Out] ((a + b*x)^(1 + n)*((a^2*(b^3*c - a^3*d)^3)/(1 + n) + (a*(b^3*c - a^3*d)^2*(-2*b^3*c + 11*a^3*d)*(a + b*x))/(2 + n) + ((b^3*c - a^3*d)*(b^6*c^2 - 29*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^2)/(3 + n) + (3*a^2*d*(10*b^6*c^2 - 56*a^3*b^3*c*d + 55*a^6*d^2)*(a + b*x)^3)/(4 + n) - (15*a*d*(b^6*c^2 - 14*a^3*b^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d + 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + b*x)^6)/(7 + n) - (6*a*d^2*(4*b^3*c - 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10 + n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n))
```


$$\begin{aligned} &^3*c*d + 22*a^6*d^2)*(a + b*x)^4)/(5 + n) + (3*d*(b^6*c^2 - 56*a^3*b^3*c*d \\ &+ 154*a^6*d^2)*(a + b*x)^5)/(6 + n) + (42*a^2*d^2*(2*b^3*c - 11*a^3*d)*(a + \\ &b*x)^6)/(7 + n) + (6*a*d^2*(-4*b^3*c + 55*a^3*d)*(a + b*x)^7)/(8 + n) + (3 \\ &*d^2*(b^3*c - 55*a^3*d)*(a + b*x)^8)/(9 + n) + (55*a^2*d^3*(a + b*x)^9)/(10 \\ &+ n) - (11*a*d^3*(a + b*x)^10)/(11 + n) + (d^3*(a + b*x)^11)/(12 + n))/b^ \\ &12 \end{aligned}$$

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int x^2(a + bx)^n (c + dx^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*x)^n*(c + d*x^3)^3,x]

[Out] Defer[IntegrateAlgebraic][x^2*(a + b*x)^n*(c + d*x^3)^3, x]

fricas [B] time = 0.94, size = 3564, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")

[Out] (2*a^3*b^9*c^3*n^9 + 144*a^3*b^9*c^3*n^8 + 4548*a^3*b^9*c^3*n^7 + 159667200
*a^3*b^9*c^3 - 239500800*a^6*b^6*c^2*d + 159667200*a^9*b^3*c*d^2 - 39916800
*a^12*d^3 + (b^12*d^3*n^11 + 66*b^12*d^3*n^10 + 1925*b^12*d^3*n^9 + 32670*b
^12*d^3*n^8 + 357423*b^12*d^3*n^7 + 2637558*b^12*d^3*n^6 + 13339535*b^12*d^3
*n^5 + 45995730*b^12*d^3*n^4 + 105258076*b^12*d^3*n^3 + 150917976*b^12*d^3
*n^2 + 120543840*b^12*d^3*n + 39916800*b^12*d^3)*x^12 + (a*b^11*d^3*n^11 +
55*a*b^11*d^3*n^10 + 1320*a*b^11*d^3*n^9 + 18150*a*b^11*d^3*n^8 + 157773*a*
b^11*d^3*n^7 + 902055*a*b^11*d^3*n^6 + 3416930*a*b^11*d^3*n^5 + 8409500*a*b
^11*d^3*n^4 + 12753576*a*b^11*d^3*n^3 + 10628640*a*b^11*d^3*n^2 + 3628800*a
*b^11*d^3*n)*x^11 - 11*(a^2*b^10*d^3*n^10 + 45*a^2*b^10*d^3*n^9 + 870*a^2*b
^10*d^3*n^8 + 9450*a^2*b^10*d^3*n^7 + 63273*a^2*b^10*d^3*n^6 + 269325*a^2*b
^10*d^3*n^5 + 723680*a^2*b^10*d^3*n^4 + 1172700*a^2*b^10*d^3*n^3 + 1026576*
a^2*b^10*d^3*n^2 + 362880*a^2*b^10*d^3*n)*x^10 + (3*b^12*c*d^2*n^11 + 207*b
^12*c*d^2*n^10 + 159667200*b^12*c*d^2 + 2*(3144*b^12*c*d^2 + 55*a^3*b^9*d^3
) *n^9 + 18*(6151*b^12*c*d^2 + 220*a^3*b^9*d^3)*n^8 + 3*(417309*b^12*c*d^2 +
20020*a^3*b^9*d^3)*n^7 + 567*(16777*b^12*c*d^2 + 880*a^3*b^9*d^3)*n^6 + 6*
(8226277*b^12*c*d^2 + 411565*a^3*b^9*d^3)*n^5 + 36*(4833097*b^12*c*d^2 + 20
5590*a^3*b^9*d^3)*n^4 + 40*(10142427*b^12*c*d^2 + 324841*a^3*b^9*d^3)*n^3 +
288*(2051288*b^12*c*d^2 + 41855*a^3*b^9*d^3)*n^2 + 5760*(82941*b^12*c*d^2
+ 770*a^3*b^9*d^3)*n)*x^9 + 3*(a*b^11*c*d^2*n^11 + 61*a*b^11*c*d^2*n^10 + 1
608*a*b^11*c*d^2*n^9 + 6*(4007*a*b^11*c*d^2 - 55*a^4*b^8*d^3)*n^8 + 21*(107
13*a*b^11*c*d^2 - 440*a^4*b^8*d^3)*n^7 + 21*(65289*a*b^11*c*d^2 - 5060*a^4*b
^8*d^3)*n^6 + 2*(2742001*a*b^11*c*d^2 - 323400*a^4*b^8*d^3)*n^5 + 2*(70625
74*a*b^11*c*d^2 - 1116885*a^4*b^8*d^3)*n^4 + 264*(84209*a*b^11*c*d^2 - 1641
5*a^4*b^8*d^3)*n^3 + 360*(52984*a*b^11*c*d^2 - 11979*a^4*b^8*d^3)*n^2 + 166
3200*(4*a*b^11*c*d^2 - a^4*b^8*d^3)*n)*x^8 - 24*(a^2*b^10*c*d^2*n^10 + 54*a
^2*b^10*c*d^2*n^9 + 1230*a^2*b^10*c*d^2*n^8 + 6*(2572*a^2*b^10*c*d^2 - 55*a
^5*b^7*d^3)*n^7 + 21*(5569*a^2*b^10*c*d^2 - 330*a^5*b^7*d^3)*n^6 + 42*(1315
3*a^2*b^10*c*d^2 - 1375*a^5*b^7*d^3)*n^5 + 10*(161702*a^2*b^10*c*d^2 - 2425
5*a^5*b^7*d^3)*n^4 + 24*(116917*a^2*b^10*c*d^2 - 22330*a^5*b^7*d^3)*n^3 + 3
60*(7192*a^2*b^10*c*d^2 - 1617*a^5*b^7*d^3)*n^2 + 237600*(4*a^2*b^10*c*d^2
- a^5*b^7*d^3)*n)*x^7 + 72*(1148*a^3*b^9*c^3 - 5*a^6*b^6*c^2*d)*n^6 + 3*(b^12
*c^2*d*n^11 + 72*b^12*c^2*d*n^10 + 79833600*b^12*c^2*d + (2285*b^12*c^2*d
+ 56*a^3*b^9*c*d^2)*n^9 + 12*(3505*b^12*c^2*d + 224*a^3*b^9*c*d^2)*n^8 + 3
*(165701*b^12*c^2*d + 17584*a^3*b^9*c*d^2)*n^7 + 48*(82167*b^12*c^2*d + 114
10*a^3*b^9*c*d^2 - 385*a^6*b^6*d^3)*n^6 + (21326135*b^12*c^2*d + 3263064*a^3
*b^9*c*d^2 - 277200*a^6*b^6*d^3)*n^5 + 12*(6509445*b^12*c^2*d + 946456*a^3

```

*b^9*c*d^2 - 130900*a^6*b^6*d^3)*n^4 + 4*(47131699*b^12*c^2*d + 5602072*a^3
*b^9*c*d^2 - 1039500*a^6*b^6*d^3)*n^3 + 96*(2946397*b^12*c^2*d + 236320*a^3
*b^9*c*d^2 - 52745*a^6*b^6*d^3)*n^2 + 2880*(81401*b^12*c^2*d + 3080*a^3*b^9
*c*d^2 - 770*a^6*b^6*d^3)*n)*x^6 + 6*(158683*a^3*b^9*c^3 - 3420*a^6*b^6*c^2
*d)*n^5 + 3*(a*b^11*c^2*d*n^11 + 67*a*b^11*c^2*d*n^10 + 1950*a*b^11*c^2*d*n
^9 + 6*(5385*a*b^11*c^2*d - 56*a^4*b^8*c*d^2)*n^8 + 3*(111851*a*b^11*c^2*d
- 4816*a^4*b^8*c*d^2)*n^7 + 3*(755417*a*b^11*c^2*d - 81424*a^4*b^8*c*d^2)*n
^6 + 560*(17848*a*b^11*c^2*d - 3687*a^4*b^8*c*d^2 + 198*a^7*b^5*d^3)*n^5 +
4*(7034735*a*b^11*c^2*d - 2313696*a^4*b^8*c*d^2 + 277200*a^7*b^5*d^3)*n^4 +
96*(498251*a*b^11*c^2*d - 227822*a^4*b^8*c*d^2 + 40425*a^7*b^5*d^3)*n^3 +
576*(75857*a*b^11*c^2*d - 43568*a^4*b^8*c*d^2 + 9625*a^7*b^5*d^3)*n^2 + 266
1120*(6*a*b^11*c^2*d - 4*a^4*b^8*c*d^2 + a^7*b^5*d^3)*n)*x^5 + 72*(100058*a
^3*b^9*c^3 - 6725*a^6*b^6*c^2*d)*n^4 - 15*(a^2*b^10*c^2*d*n^10 + 63*a^2*b^1
0*c^2*d*n^9 + 1698*a^2*b^10*c^2*d*n^8 + 6*(4253*a^2*b^10*c^2*d - 56*a^5*b^7
*c*d^2)*n^7 + 3*(77827*a^2*b^10*c^2*d - 4368*a^5*b^7*c*d^2)*n^6 + 3*(444109
*a^2*b^10*c^2*d - 63952*a^5*b^7*c*d^2)*n^5 + 4*(1166393*a^2*b^10*c^2*d - 32
4324*a^5*b^7*c*d^2 + 27720*a^8*b^4*d^3)*n^4 + 12*(789721*a^2*b^10*c^2*d - 3
38800*a^5*b^7*c*d^2 + 55440*a^8*b^4*d^3)*n^3 + 144*(68927*a^2*b^10*c^2*d -
38948*a^5*b^7*c*d^2 + 8470*a^8*b^4*d^3)*n^2 + 665280*(6*a^2*b^10*c^2*d - 4*
a^5*b^7*c*d^2 + a^8*b^4*d^3)*n)*x^4 + 8*(4473299*a^3*b^9*c^3 - 756675*a^6*b
^6*c^2*d + 15120*a^9*b^3*c*d^2)*n^3 + (b^12*c^3*n^11 + 75*b^12*c^3*n^10 + 1
59667200*b^12*c^3 + 4*(623*b^12*c^3 + 15*a^3*b^9*c^2*d)*n^9 + 18*(2683*b^12
*c^3 + 200*a^3*b^9*c^2*d)*n^8 + 3*(201527*b^12*c^3 + 30360*a^3*b^9*c^2*d)*n
^7 + 9*(568099*b^12*c^3 + 139760*a^3*b^9*c^2*d - 2240*a^6*b^6*c*d^2)*n^6 +
2*(14825779*b^12*c^3 + 5117670*a^3*b^9*c^2*d - 362880*a^6*b^6*c*d^2)*n^5 +
12*(9759623*b^12*c^3 + 4102800*a^3*b^9*c^2*d - 777840*a^6*b^6*c*d^2)*n^4 +
8*(38232551*b^12*c^3 + 16529190*a^3*b^9*c^2*d - 6229440*a^6*b^6*c*d^2 + 831
600*a^9*b^3*d^3)*n^3 + 576*(861864*b^12*c^3 + 298435*a^3*b^9*c^2*d - 163940
*a^6*b^6*c*d^2 + 34650*a^9*b^3*d^3)*n^2 + 5760*(76781*b^12*c^3 + 13860*a^3*
b^9*c^2*d - 9240*a^6*b^6*c*d^2 + 2310*a^9*b^3*d^3)*n)*x^3 + 144*(780996*a^3
*b^9*c^3 - 293635*a^6*b^6*c^2*d + 27720*a^9*b^3*c*d^2)*n^2 + (a*b^11*c^3*n^
11 + 73*a*b^11*c^3*n^10 + 2346*a*b^11*c^3*n^9 + 6*(7267*a*b^11*c^3 - 30*a^4
*b^8*c^2*d)*n^8 + 3*(172459*a*b^11*c^3 - 3480*a^4*b^8*c^2*d)*n^7 + 3*(13593
79*a*b^11*c^3 - 84120*a^4*b^8*c^2*d)*n^6 + 4*(5373821*a*b^11*c^3 - 817200*a
^4*b^8*c^2*d + 15120*a^7*b^5*c*d^2)*n^5 + 4*(18531227*a*b^11*c^3 - 6042105*
a^4*b^8*c^2*d + 514080*a^7*b^5*c*d^2)*n^4 + 72*(2189036*a*b^11*c^3 - 138005
5*a^4*b^8*c^2*d + 331800*a^7*b^5*c*d^2)*n^3 + 1440*(125842*a*b^11*c^3 - 137
481*a^4*b^8*c^2*d + 70644*a^7*b^5*c*d^2 - 13860*a^10*b^2*d^3)*n^2 + 1995840
0*(4*a*b^11*c^3 - 6*a^4*b^8*c^2*d + 4*a^7*b^5*c*d^2 - a^10*b^2*d^3)*n)*x^2
+ 2880*(70402*a^3*b^9*c^3 - 54321*a^6*b^6*c^2*d + 15204*a^9*b^3*c*d^2)*n -
2*(a^2*b^10*c^3*n^10 + 72*a^2*b^10*c^3*n^9 + 2274*a^2*b^10*c^3*n^8 + 36*(11
48*a^2*b^10*c^3 - 5*a^5*b^7*c^2*d)*n^7 + 3*(158683*a^2*b^10*c^3 - 3420*a^5*
b^7*c^2*d)*n^6 + 36*(100058*a^2*b^10*c^3 - 6725*a^5*b^7*c^2*d)*n^5 + 4*(447
3299*a^2*b^10*c^3 - 756675*a^5*b^7*c^2*d + 15120*a^8*b^4*c*d^2)*n^4 + 72*(7
80996*a^2*b^10*c^3 - 293635*a^5*b^7*c^2*d + 27720*a^8*b^4*c*d^2)*n^3 + 1440
*(70402*a^2*b^10*c^3 - 54321*a^5*b^7*c^2*d + 15204*a^8*b^4*c*d^2)*n^2 + 199
58400*(4*a^2*b^10*c^3 - 6*a^5*b^7*c^2*d + 4*a^8*b^4*c*d^2 - a^11*b*d^3)*n)*
x)*(b*x + a)^n/(b^12*n^12 + 78*b^12*n^11 + 2717*b^12*n^10 + 55770*b^12*n^9
+ 749463*b^12*n^8 + 6926634*b^12*n^7 + 44990231*b^12*n^6 + 206070150*b^12*n
^5 + 657206836*b^12*n^4 + 1414014888*b^12*n^3 + 1931559552*b^12*n^2 + 14864
42880*b^12*n + 479001600*b^12)

```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP

UT:sage2:=int(sage0,x):;OUTPUT:Polynomial exponent overflow. Error: Bad Argument Value

maple [B] time = 0.06, size = 3780, normalized size = 8.24

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b*x+a)^n*(d*x^3+c)^3,x)

[Out] $-(b*x+a)^{(n+1)}*(-b^{11}*d^3*n^{11}*x^{11}-66*b^{11}*d^3*n^{10}*x^{11}+11*a*b^{10}*d^3*n^{10}*x^{10}-1925*b^{11}*d^3*n^9*x^{11}+605*a*b^{10}*d^3*n^9*x^{10}-3*b^{11}*c*d^2*n^{11}*x^8-32670*b^{11}*d^3*n^8*x^{11}-110*a^2*b^9*d^3*n^9*x^9+14520*a*b^{10}*d^3*n^8*x^{10}-207*b^{11}*c*d^2*n^{10}*x^8-357423*b^{11}*d^3*n^7*x^{11}-4950*a^2*b^9*d^3*n^8*x^9+24*a*b^{10}*c*d^2*n^{10}*x^7+199650*a*b^{10}*d^3*n^7*x^{10}-6288*b^{11}*c*d^2*n^9*x^8-2637558*b^{11}*d^3*n^6*x^{11}+990*a^3*b^8*d^3*n^8*x^8-95700*a^2*b^9*d^3*n^7*x^9+1464*a*b^{10}*c*d^2*n^9*x^7+1735503*a*b^{10}*d^3*n^6*x^{10}-3*b^{11}*c^2*d*n^{11}*x^5-110718*b^{11}*c*d^2*n^8*x^8-13339535*b^{11}*d^3*n^5*x^{11}+35640*a^3*b^8*d^3*n^7*x^8-168*a^2*b^9*c*d^2*n^9*x^6-1039500*a^2*b^9*d^3*n^6*x^9+38592*a*b^{10}*c*d^2*n^8*x^7+9922605*a*b^{10}*d^3*n^5*x^{10}-216*b^{11}*c^2*d*n^{10}*x^5-1251927*b^{11}*c*d^2*n^7*x^8-45995730*b^{11}*d^3*n^4*x^{11}-7920*a^4*b^7*d^3*n^7*x^7+540540*a^3*b^8*d^3*n^6*x^8-9072*a^2*b^9*c*d^2*n^8*x^6-6960030*a^2*b^9*d^3*n^5*x^9+15*a*b^{10}*c^2*d*n^{10}*x^4+577008*a*b^{10}*c*d^2*n^7*x^7+37586230*a*b^{10}*d^3*n^4*x^{10}-6855*b^{11}*c^2*d*n^9*x^5-9512559*b^{11}*c*d^2*n^6*x^8-105258076*b^{11}*d^3*n^3*x^{11}-221760*a^4*b^7*d^3*n^6*x^7+1008*a^3*b^8*c*d^2*n^8*x^5+4490640*a^3*b^8*d^3*n^5*x^8-206640*a^2*b^9*c*d^2*n^7*x^6-29625750*a^2*b^9*d^3*n^4*x^9+1005*a*b^{10}*c^2*d*n^9*x^4+5399352*a*b^{10}*c*d^2*n^6*x^7+92504500*a*b^{10}*d^3*n^3*x^{10}-b^{11}*c^3*n^{11}*x^2-126180*b^{11}*c^2*d*n^8*x^5-49357662*b^{11}*c*d^2*n^5*x^8-150917976*b^{11}*d^3*n^2*x^{11}+55440*a^5*b^6*d^3*n^6*x^6-2550240*a^4*b^7*d^3*n^5*x^7+48384*a^3*b^8*c*d^2*n^7*x^5+22224510*a^3*b^8*d^3*n^4*x^8-60*a^2*b^9*c^2*d*n^9*x^3-2592576*a^2*b^9*c*d^2*n^6*x^6-79604800*a^2*b^9*d^3*n^3*x^9+29250*a*b^{10}*c^2*d*n^8*x^4+32905656*a*b^{10}*c*d^2*n^5*x^7+140289336*a*b^{10}*d^3*n^2*x^{10}-75*b^{11}*c^3*n^{10}*x^2-1491309*b^{11}*c^2*d*n^7*x^5-173991492*b^{11}*c*d^2*n^4*x^8-120543840*b^{11}*d^3*n*x^{11}+1164240*a^5*b^6*d^3*n^5*x^6-5040*a^4*b^7*c*d^2*n^7*x^4-15523200*a^4*b^7*d^3*n^4*x^7+949536*a^3*b^8*c*d^2*n^6*x^5+6661160*a^3*b^8*d^3*n^3*x^8-3780*a^2*b^9*c^2*d*n^8*x^3-19647432*a^2*b^9*c*d^2*n^5*x^6-128997000*a^2*b^9*d^3*n^2*x^9+2*a*b^{10}*c^3*n^{10}*x+484650*a*b^{10}*c^2*d*n^7*x^4+131616048*a*b^{10}*c*d^2*n^4*x^7+116915040*a*b^{10}*d^3*n*x^{10}-2492*b^{11}*c^3*n^9*x^2-11832048*b^{11}*c^2*d*n^6*x^5-405697080*b^{11}*c*d^2*n^3*x^8-39916800*b^{11}*d^3*x^{11}-332640*a^6*b^5*d^3*n^5*x^5+9702000*a^5*b^6*d^3*n^4*x^6-216720*a^4*b^7*c*d^2*n^6*x^4-53610480*a^4*b^7*d^3*n^3*x^7+180*a^3*b^8*c^2*d*n^8*x^2+9858240*a^3*b^8*c*d^2*n^5*x^5+116942760*a^3*b^8*d^3*n^2*x^8-101880*a^2*b^9*c^2*d*n^7*x^3-92807568*a^2*b^9*c*d^2*n^4*x^6-112923360*a^2*b^9*d^3*n*x^9+146*a*b^{10}*c^3*n^9*x+5033295*a*b^{10}*c^2*d*n^6*x^4+339003552*a*b^{10}*c*d^2*n^3*x^7+39916800*a*b^{10}*d^3*x^{10}-48294*b^{11}*c^3*n^8*x^2-63978405*b^{11}*c^2*d*n^5*x^5-590770944*b^{11}*c*d^2*n^2*x^8-4989600*a^6*b^5*d^3*n^4*x^5+20160*a^5*b^6*c*d^2*n^6*x^3+40748400*a^5*b^6*d^3*n^3*x^6-3664080*a^4*b^7*c*d^2*n^5*x^4-104005440*a^4*b^7*d^3*n^2*x^7+10800*a^3*b^8*c^2*d*n^7*x^2+58735152*a^3*b^8*c*d^2*n^4*x^5+108488160*a^3*b^8*d^3*n*x^8-2*a^2*b^9*c^3*n^9-1531080*a^2*b^9*c^2*d*n^6*x^3-271659360*a^2*b^9*c*d^2*n^3*x^6-39916800*a^2*b^9*d^3*x^9+4692*a*b^{10}*c^3*n^8*x+33993765*a*b^{10}*c^2*d*n^5*x^4+533548224*a*b^{10}*c*d^2*n^2*x^7-604581*b^{11}*c^3*n^7*x^2-234340020*b^{11}*c^2*d*n^4*x^5-477740160*b^{11}*c*d^2*n*x^8+1663200*a^7*b^4*d^3*n^4*x^4-28274400*a^6*b^5*d^3*n^3*x^5+786240*a^5*b^6*c*d^2*n^5*x^3+90034560*a^5*b^6*d^3*n^2*x^6-360*a^4*b^7*c^2*d*n^7*x-30970800*a^4*b^7*c*d^2*n^4*x^4-103498560*a^4*b^7*d^3*n*x^7+273240*a^3*b^8*c^2*d*n^6*x^2+204434496*a^3*b^8*c*d^2*n^3*x^5+39916800*a^3*b^8*d^3*x^8-144*a^2*b^9*c^3*n^8-14008860*a^2*b^9*c^2*d*n^5*x^3-471409344*a^2*b^9*c*d^2*n^2*x^6+87204*a*b^{10}*c^3*n^7*x+149923200*a*b^{10}*c^2*d*n^4*x^4+457781760*a*b^{10}*c*d^2*n*x^7-5112891*b^{11}*c^3*n^6*x^2-565580388*b^{11}*c^2*d*n^3*x^5-159667200*b^{11}*c*d^2*x^8+16632000*a^7*b^4*d^3*n^3*x^4-60480*a^6*$

```

b^5*c*d^2*n^5*x^2-74844000*a^6*b^5*d^3*n^2*x^5+11511360*a^5*b^6*c*d^2*n^4*x
^3+97796160*a^5*b^6*d^3*n*x^6-20880*a^4*b^7*c^2*d*n^6*x-138821760*a^4*b^7*c
*d^2*n^3*x^4-39916800*a^4*b^7*d^3*x^7+3773520*a^3*b^8*c^2*d*n^5*x^2+4033491
84*a^3*b^8*c*d^2*n^2*x^5-4548*a^2*b^9*c^3*n^7-79939620*a^2*b^9*c^2*d*n^4*x^
3-434972160*a^2*b^9*c*d^2*n*x^6+1034754*a*b^10*c^3*n^6*x+422084100*a*b^10*c
^2*d*n^3*x^4+159667200*a*b^10*c*d^2*x^7-29651558*b^11*c^3*n^5*x^2-848562336
*b^11*c^2*d*n^2*x^5-6652800*a^8*b^3*d^3*n^3*x^3+58212000*a^7*b^4*d^3*n^2*x^
4-2177280*a^6*b^5*c*d^2*n^4*x^2-91143360*a^6*b^5*d^3*n*x^5+360*a^5*b^6*c^2*
d*n^6+77837760*a^5*b^6*c*d^2*n^3*x^3+39916800*a^5*b^6*d^3*x^6-504720*a^4*b^
7*c^2*d*n^5*x-328063680*a^4*b^7*c*d^2*n^2*x^4+30706020*a^3*b^8*c^2*d*n^4*x^
2+408360960*a^3*b^8*c*d^2*n*x^5-82656*a^2*b^9*c^3*n^6-279934320*a^2*b^9*c^2
*d*n^3*x^3-159667200*a^2*b^9*c*d^2*x^6+8156274*a*b^10*c^3*n^5*x+717481440*a
*b^10*c^2*d*n^2*x^4-117115476*b^11*c^3*n^4*x^2-703304640*b^11*c^2*d*n*x^5-3
9916800*a^8*b^3*d^3*n^2*x^3+120960*a^7*b^4*c*d^2*n^4*x+83160000*a^7*b^4*d^3
*n*x^4-28002240*a^6*b^5*c*d^2*n^3*x^2-39916800*a^6*b^5*d^3*x^5+20520*a^5*b^
6*c^2*d*n^5+243936000*a^5*b^6*c*d^2*n^2*x^3-6537600*a^4*b^7*c^2*d*n^4*x-376
427520*a^4*b^7*c*d^2*n*x^4+147700800*a^3*b^8*c^2*d*n^3*x^2+159667200*a^3*b^
8*c*d^2*x^5-952098*a^2*b^9*c^3*n^5-568599120*a^2*b^9*c^2*d*n^2*x^3+42990568
*a*b^10*c^3*n^4*x+655404480*a*b^10*c^2*d*n*x^4-305860408*b^11*c^3*n^3*x^2-2
39500800*b^11*c^2*d*x^5+19958400*a^9*b^2*d^3*n^2*x^2-73180800*a^8*b^3*d^3*n
*x^3+4112640*a^7*b^4*c*d^2*n^3*x+39916800*a^7*b^4*d^3*x^4-149506560*a^6*b^5
*c*d^2*n^2*x^2+484200*a^5*b^6*c^2*d*n^4+336510720*a^5*b^6*c*d^2*n*x^3-48336
840*a^4*b^7*c^2*d*n^3*x-159667200*a^4*b^7*c*d^2*x^4+396700560*a^3*b^8*c^2*d
*n^2*x^2-7204176*a^2*b^9*c^3*n^4-595529280*a^2*b^9*c^2*d*n*x^3+148249816*a*
b^10*c^3*n^3*x+239500800*a*b^10*c^2*d*x^4-496433664*b^11*c^3*n^2*x^2+598752
00*a^9*b^2*d^3*n*x^2-120960*a^8*b^3*c*d^2*n^3-39916800*a^8*b^3*d^3*x^3+4777
9200*a^7*b^4*c*d^2*n^2*x-283288320*a^6*b^5*c*d^2*n*x^2+6053400*a^5*b^6*c^2*
d*n^3+159667200*a^5*b^6*c*d^2*x^3-198727920*a^4*b^7*c^2*d*n^2*x+515695680*a
^3*b^8*c^2*d*n*x^2-35786392*a^2*b^9*c^3*n^3-239500800*a^2*b^9*c^2*d*x^3+315
221184*a*b^10*c^3*n^2*x-442258560*b^11*c^3*n*x^2-39916800*a^10*b*d^3*n*x+39
916800*a^9*b^2*d^3*x^2-3991680*a^8*b^3*c*d^2*n^2+203454720*a^7*b^4*c*d^2*n*
x-159667200*a^6*b^5*c*d^2*x^2+42283440*a^5*b^6*c^2*d*n^2-395945280*a^4*b^7*
c^2*d*n*x+239500800*a^3*b^8*c^2*d*x^2-112463424*a^2*b^9*c^3*n^2+362424960*a
*b^10*c^3*n*x-159667200*b^11*c^3*x^2-39916800*a^10*b*d^3*x-43787520*a^8*b^3
*c*d^2*n+159667200*a^7*b^4*c*d^2*x+156444480*a^5*b^6*c^2*d*n-239500800*a^4*
b^7*c^2*d*x-202757760*a^2*b^9*c^3*n+159667200*a*b^10*c^3*x+39916800*a^11*d^
3-159667200*a^8*b^3*c*d^2+239500800*a^5*b^6*c^2*d-159667200*a^2*b^9*c^3)/b^
12/(n^12+78*n^11+2717*n^10+55770*n^9+749463*n^8+6926634*n^7+44990231*n^6+20
6070150*n^5+657206836*n^4+1414014888*n^3+1931559552*n^2+1486442880*n+479001
600)

```

maxima [B] time = 0.90, size = 1153, normalized size = 2.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x
+ a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^5 + 15*n^4 + 85*n^3 + 225
*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5
*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*
a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)
^n*c^2*d/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)
+ 3*((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 67284*n^3 + 118124*n
^2 + 109584*n + 40320)*b^9*x^9 + (n^8 + 28*n^7 + 322*n^6 + 1960*n^5 + 6769*
n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5
+ 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 +
85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*n^
3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^5*b^4*
```

$$\begin{aligned}
& x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n^2 + n)*a^7*b^2*x^2 - \\
& 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*c*d^2/((n^9 + 45*n^8 + 870*n^7 + 9 \\
& 450*n^6 + 63273*n^5 + 269325*n^4 + 723680*n^3 + 1172700*n^2 + 1026576*n + 3 \\
& 62880)*b^9) + ((n^{11} + 66*n^{10} + 1925*n^9 + 32670*n^8 + 357423*n^7 + 263755 \\
& 8*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 + 12054 \\
& 3840*n + 39916800)*b^{12}*x^{12} + (n^{11} + 55*n^{10} + 1320*n^9 + 18150*n^8 + 157 \\
& 773*n^7 + 902055*n^6 + 3416930*n^5 + 8409500*n^4 + 12753576*n^3 + 10628640* \\
& n^2 + 3628800*n)*a*b^{11}*x^{11} - 11*(n^{10} + 45*n^9 + 870*n^8 + 9450*n^7 + 632 \\
& 73*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362880*n)*a^ \\
& 2*b^{10}*x^{10} + 110*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67284*n^ \\
& 4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^3*b^9*x^9 - 990*(n^8 + 28*n^7 + 32 \\
& 2*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^4*b^8*x^8 + \\
& 7920*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*a^5* \\
& b^7*x^7 - 55440*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^6*b^6 \\
& *x^6 + 332640*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^7*b^5*x^5 - 1663200 \\
& *(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^8*b^4*x^4 + 6652800*(n^3 + 3*n^2 + 2*n)*a^9 \\
& *b^3*x^3 - 19958400*(n^2 + n)*a^{10}*b^2*x^2 + 39916800*a^{11}*b*n*x - 39916800 \\
& *a^{12})*(b*x + a)^n*d^3/((n^{12} + 78*n^{11} + 2717*n^{10} + 55770*n^9 + 749463*n^ \\
& 8 + 6926634*n^7 + 44990231*n^6 + 206070150*n^5 + 657206836*n^4 + 1414014888 \\
& *n^3 + 1931559552*n^2 + 1486442880*n + 479001600)*b^{12})
\end{aligned}$$

mupad [B] time = 7.14, size = 2896, normalized size = 6.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^2*(c + d*x^3)^3*(a + b*x)^n, x)$

[Out] $(2*a^3*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6)/(b^{12}*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^{10} + 78*n^{11} + n^{12} + 479001600)) + (d^3*x^{12}*(a + b*x)^n*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^{10} + n^{11} + 39916800))/(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^{10} + 78*n^{11} + n^{12} + 479001600) + (x^3*(a + b*x)^n*(3*n + n^2 + 2)*(79833600*b^9*c^3 + 6652800*a^9*d^3*n + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 + 39916800*a^3*b^6*c^2*d*n - 26611200*a^6*b^3*c*d^2*n + 26074080*a^3*b^6*c^2*d*n^2 - 7297920*a^6*b^3*c*d^2*n^2 + 7047240*a^3*b^6*c^2*d*n^3 - 665280*a^6*b^3*c*d^2*n^3 + 1008900*a^3*b^6*c^2*d*n^4 - 20160*a^6*b^3*c*d^2*n^4 + 80700*a^3*b^6*c^2*d*n^5 + 3420*a^3*b^6*c^2*d*n^6 + 60*a^3*b^6*c^2*d*n^7))/(b^9*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^{10} + 78*n^{11} + n^{12} + 479001600)) + (3*d*x^6*(a + b*x)^n*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120)*(665280*b^6*c^2 - 18480*a^6*d^2*n + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 + 73920*a^3*b^3*c*d*n + 20272*a^3*b^3*c*d*n^2 + 1848*a^3*b^3*c*d*n^3 + 56*a^3*b^3*c*d*n^4))/(b^6*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^{10} + 78*n^{11} + n^{12} + 479001600)) - (2*a^2*n*x*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 +$

$$\begin{aligned}
& 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6) / (b^{11} * (1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) + (d^2*x^9*(a + b*x)^n*(3960*b^3*c + 99*b^3*c*n^2 + 3*b^3*c*n^3 + 110*a^3*d*n + 1086*b^3*c*n)*(109584*n + 118124*n^2 + 67284*n^3 + 22449*n^4 + 4536*n^5 + 546*n^6 + 36*n^7 + n^8 + 40320)) / (b^3*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) + (a*d^3*n*x^{11}*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^{10} + 362880)) / (b*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) - (11*a^2*d^3*n*x^{10}*(a + b*x)^n*(1026576*n + 1172700*n^2 + 723680*n^3 + 269325*n^4 + 63273*n^5 + 9450*n^6 + 870*n^7 + 45*n^8 + n^9 + 362880)) / (b^2*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) + (a*n*x^2*(n + 1)*(a + b*x)^n*(79833600*b^9*c^3 - 19958400*a^9*d^3 + 101378880*b^9*c^3*n + 56231712*b^9*c^3*n^2 + 17893196*b^9*c^3*n^3 + 3602088*b^9*c^3*n^4 + 476049*b^9*c^3*n^5 + 41328*b^9*c^3*n^6 + 2274*b^9*c^3*n^7 + 72*b^9*c^3*n^8 + b^9*c^3*n^9 - 119750400*a^3*b^6*c^2*d + 79833600*a^6*b^3*c*d^2 - 78222240*a^3*b^6*c^2*d*n + 21893760*a^6*b^3*c*d^2*n - 21141720*a^3*b^6*c^2*d*n^2 + 1995840*a^6*b^3*c*d^2*n^2 - 3026700*a^3*b^6*c^2*d*n^3 + 60480*a^6*b^3*c*d^2*n^3 - 242100*a^3*b^6*c^2*d*n^4 - 10260*a^3*b^6*c^2*d*n^5 - 180*a^3*b^6*c^2*d*n^6) / (b^{10} * (1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) + (3*a*d*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(110880*a^6*d^2 + 665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 121632*a^3*b^3*c*d*n - 11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3)) / (b^7*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) - (15*a^2*d*n*x^4*(a + b*x)^n*(11*n + 6*n^2 + n^3 + 6)*(110880*a^6*d^2 + 665280*b^6*c^2 + 434568*b^6*c^2*n + 117454*b^6*c^2*n^2 + 16815*b^6*c^2*n^3 + 1345*b^6*c^2*n^4 + 57*b^6*c^2*n^5 + b^6*c^2*n^6 - 443520*a^3*b^3*c*d - 121632*a^3*b^3*c*d*n - 11088*a^3*b^3*c*d*n^2 - 336*a^3*b^3*c*d*n^3)) / (b^8*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) + (3*a*d^2*n*x^8*(a + b*x)^n*(1320*b^3*c - 330*a^3*d + 33*b^3*c*n^2 + b^3*c*n^3 + 362*b^3*c*n)*(13068*n + 13132*n^2 + 6769*n^3 + 1960*n^4 + 322*n^5 + 28*n^6 + n^7 + 5040)) / (b^4*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600)) - (24*a^2*d^2*n*x^7*(a + b*x)^n*(1320*b^3*c - 330*a^3*d + 33*b^3*c*n^2 + b^3*c*n^3 + 362*b^3*c*n)*(1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6 + 720)) / (b^5*(1486442880*n + 1931559552*n^2 + 1414014888*n^3 + 657206836*n^4 + 206070150*n^5 + 44990231*n^6 + 6926634*n^7 + 749463*n^8 + 55770*n^9 + 2717*n^10 + 78*n^11 + n^{12} + 479001600))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(b*x+a)**n*(d*x**3+c)**3,x)
```

```
[Out] Timed out
```

3.57 $\int x(a + bx)^n (c + dx^3)^3 dx$

Optimal. Leaf size=396

$$-\frac{21ad^2(b^3c - 10a^3d)(a + bx)^{n+7}}{b^{11}(n + 7)} + \frac{3d^2(b^3c - 40a^3d)(a + bx)^{n+8}}{b^{11}(n + 8)} - \frac{a(b^3c - a^3d)^3(a + bx)^{n+1}}{b^{11}(n + 1)} + \frac{(b^3c - 10a^3d)(b^3c - a^3d)^2(a + bx)^{n+2}}{b^{11}(n + 2)} - \frac{9a^2d(2b^3c - 5a^3d)(b^3c - a^3d)(a + bx)^{n+3}}{b^{11}(n + 3)} + \frac{45a^2d^2(a + bx)^{n+4}}{b^{11}(n + 4)} - \frac{10ad^3(a + bx)^{n+5}}{b^{11}(n + 5)} + \frac{d^3(a + bx)^{n+6}}{b^{11}(n + 6)} - \frac{63a^2d^2(b^3c - 4a^3d)(a + bx)^{n+7}}{b^{11}(n + 7)} - \frac{21ad^2(b^3c - 10a^3d)(a + bx)^{n+8}}{b^{11}(n + 8)} - \frac{3d^2(b^3c - 40a^3d)(a + bx)^{n+9}}{b^{11}(n + 9)} - \frac{a(b^3c - a^3d)^3(a + bx)^{n+10}}{b^{11}(n + 10)} - \frac{(b^3c - 10a^3d)(b^3c - a^3d)^2(a + bx)^{n+11}}{b^{11}(n + 11)} - \frac{9a^2d(2b^3c - 5a^3d)(b^3c - a^3d)(a + bx)^{n+12}}{b^{11}(n + 12)} - \frac{45a^2d^2(a + bx)^{n+13}}{b^{11}(n + 13)} - \frac{10ad^3(a + bx)^{n+14}}{b^{11}(n + 14)} + \frac{d^3(a + bx)^{n+15}}{b^{11}(n + 15)}$$

Rubi [A] time = 0.26, antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 18, number of rules / integrand size = 0.056, Rules used = {1620}

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x)^n*(c + d*x^3)^3,x]
[Out] -((a*(b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^11*(1 + n))) + ((b^3*c - 10*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^11*(2 + n)) + (9*a^2*d*(2*b^3*c - 5*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^11*(3 + n)) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^(4 + n))/(b^11*(4 + n)) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^(5 + n))/(b^11*(5 + n)) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^(6 + n))/(b^11*(6 + n)) - (21*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^(7 + n))/(b^11*(7 + n)) + (3*d^2*(b^3*c - 40*a^3*d)*(a + b*x)^(8 + n))/(b^11*(8 + n)) + (45*a^2*d^3*(a + b*x)^(9 + n))/(b^11*(9 + n)) - (10*a*d^3*(a + b*x)^(10 + n))/(b^11*(10 + n)) + (d^3*(a + b*x)^(11 + n))/(b^11*(11 + n))
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\int x(a + bx)^n (c + dx^3)^3 dx = \int \left(\frac{a(-b^3c + a^3d)^3(a + bx)^n}{b^{10}} + \frac{(b^3c - 10a^3d)(b^3c - a^3d)^2(a + bx)^{1+n}}{b^{10}} + \frac{9a^2d(2b^3c - 5a^3d)(b^3c - a^3d)(a + bx)^{2+n}}{b^{10}} - \frac{a(b^3c - a^3d)^3(a + bx)^{1+n}}{b^{11}(1 + n)} + \frac{(b^3c - 10a^3d)(b^3c - a^3d)^2(a + bx)^{2+n}}{b^{11}(2 + n)} + \frac{9a^2d(2b^3c - 5a^3d)(b^3c - a^3d)(a + bx)^{3+n}}{b^{11}(3 + n)} - \frac{3a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^4}{b^{11}(4 + n)} + \frac{3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^5}{b^{11}(5 + n)} + \frac{63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^6}{b^{11}(6 + n)} - \frac{21*a*d^2*(b^3*c - 10*a^3*d)*(a + b*x)^7}{b^{11}(7 + n)} + \frac{3*d^2*(b^3*c - 40*a^3*d)*(a + b*x)^8}{b^{11}(8 + n)} + \frac{45*a^2*d^3*(a + b*x)^9}{b^{11}(9 + n)} - \frac{10*a*d^3*(a + b*x)^{10}}{b^{11}(10 + n)} + \frac{d^3*(a + b*x)^{11}}{b^{11}(11 + n)} \right)$$

Mathematica [A] time = 0.38, size = 345, normalized size = 0.87

$$(a + bx)^{n+1} \left(\frac{3a^2(a+bx)^2(b^3c-40a^3d)}{b^{11}} + \frac{21ad^2(a+bx)^2(10a^3d-b^3c)}{b^{11}} + \frac{(a+bx)^2(b^3c-10a^3d)(b^3c-a^3d)^2}{b^{11}} + \frac{a^2(b^3c-a^3d)^3}{b^{11}} + \frac{45a^2d^2(a+bx)^2}{b^{11}} + \frac{3d(a+bx)^2(70a^6d^2-35a^3b^3cd+a^6d^2)}{b^{11}} - \frac{3ad(a+bx)^2(40a^6d^2-35a^3b^3cd+40a^6d^2)}{b^{11}} + \frac{63a^2d^2(a+bx)^2(b^3c-4a^3d)}{b^{11}} + \frac{9a^2d(a+bx)^2(b^3d-b^3c)(5a^3d-2b^3c)}{b^{11}} + \frac{d^3(a+bx)^3}{b^{11}} - \frac{10ad^3(a+bx)^3}{b^{11}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^3)^3,x]
[Out] ((a + b*x)^(1 + n)*((a*(-(b^3*c) + a^3*d)^3)/(1 + n) + ((b^3*c - 10*a^3*d)*(b^3*c - a^3*d)^2*(a + b*x))/(2 + n) + (9*a^2*d*(-(b^3*c) + a^3*d)*(-2*b^3*c + 5*a^3*d)*(a + b*x)^2)/(3 + n) - (3*a*d*(4*b^6*c^2 - 35*a^3*b^3*c*d + 40*a^6*d^2)*(a + b*x)^3)/(4 + n) + (3*d*(b^6*c^2 - 35*a^3*b^3*c*d + 70*a^6*d^2)*(a + b*x)^4)/(5 + n) + (63*a^2*d^2*(b^3*c - 4*a^3*d)*(a + b*x)^5)/(6 + n
```


) + (21*a*d^2*(-(b^3*c) + 10*a^3*d)*(a + b*x)^6)/(7 + n) + (3*d^2*(b^3*c - 40*a^3*d)*(a + b*x)^7)/(8 + n) + (45*a^2*d^3*(a + b*x)^8)/(9 + n) - (10*a*d^3*(a + b*x)^9)/(10 + n) + (d^3*(a + b*x)^10)/(11 + n))/b^11

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int x(a + bx)^n (c + dx^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*x)^n*(c + d*x^3)^3,x]

[Out] Defer[IntegrateAlgebraic][x*(a + b*x)^n*(c + d*x^3)^3, x]

fricas [B] time = 0.97, size = 2919, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")

[Out] $-(a^2*b^9*c^3*n^9 + 63*a^2*b^9*c^3*n^8 + 1734*a^2*b^9*c^3*n^7 + 19958400*a^2*b^9*c^3 - 23950080*a^5*b^6*c^2*d + 14968800*a^8*b^3*c*d^2 - 3628800*a^{11}*d^3 - (b^{11}*d^3*n^{10} + 55*b^{11}*d^3*n^9 + 1320*b^{11}*d^3*n^8 + 18150*b^{11}*d^3*n^7 + 157773*b^{11}*d^3*n^6 + 902055*b^{11}*d^3*n^5 + 3416930*b^{11}*d^3*n^4 + 8409500*b^{11}*d^3*n^3 + 12753576*b^{11}*d^3*n^2 + 10628640*b^{11}*d^3*n + 3628800*b^{11}*d^3)*x^{11} - (a*b^{10}*d^3*n^{10} + 45*a*b^{10}*d^3*n^9 + 870*a*b^{10}*d^3*n^8 + 9450*a*b^{10}*d^3*n^7 + 63273*a*b^{10}*d^3*n^6 + 269325*a*b^{10}*d^3*n^5 + 723680*a*b^{10}*d^3*n^4 + 1172700*a*b^{10}*d^3*n^3 + 1026576*a*b^{10}*d^3*n^2 + 362880*a*b^{10}*d^3*n)*x^{10} + 10*(a^2*b^9*d^3*n^9 + 36*a^2*b^9*d^3*n^8 + 546*a^2*b^9*d^3*n^7 + 4536*a^2*b^9*d^3*n^6 + 22449*a^2*b^9*d^3*n^5 + 67284*a^2*b^9*d^3*n^4 + 118124*a^2*b^9*d^3*n^3 + 109584*a^2*b^9*d^3*n^2 + 40320*a^2*b^9*d^3*n)*x^9 - 3*(b^{11}*c*d^2*n^{10} + 58*b^{11}*c*d^2*n^9 + 4989600*b^{11}*c*d^2 + 3*(487*b^{11}*c*d^2 + 10*a^3*b^8*d^3)*n^8 + 6*(3497*b^{11}*c*d^2 + 140*a^3*b^8*d^3)*n^7 + 21*(9027*b^{11}*c*d^2 + 460*a^3*b^8*d^3)*n^6 + 294*(3813*b^{11}*c*d^2 + 200*a^3*b^8*d^3)*n^5 + (4371359*b^{11}*c*d^2 + 203070*a^3*b^8*d^3)*n^4 + 2*(5512429*b^{11}*c*d^2 + 196980*a^3*b^8*d^3)*n^3 + 36*(473867*b^{11}*c*d^2 + 10890*a^3*b^8*d^3)*n^2 + 360*(40123*b^{11}*c*d^2 + 420*a^3*b^8*d^3)*n)*x^8 - 3*(a*b^{10}*c*d^2*n^{10} + 51*a*b^{10}*c*d^2*n^9 + 1104*a*b^{10}*c*d^2*n^8 + 6*(2209*a*b^{10}*c*d^2 - 40*a^4*b^7*d^3)*n^7 + 21*(4609*a*b^{10}*c*d^2 - 240*a^4*b^7*d^3)*n^6 + 21*(21119*a*b^{10}*c*d^2 - 2000*a^4*b^7*d^3)*n^5 + 2*(633433*a*b^{10}*c*d^2 - 88200*a^4*b^7*d^3)*n^4 + 12*(179733*a*b^{10}*c*d^2 - 32480*a^4*b^7*d^3)*n^3 + 360*(5449*a*b^{10}*c*d^2 - 1176*a^4*b^7*d^3)*n^2 + 21600*(33*a*b^{10}*c*d^2 - 8*a^4*b^7*d^3)*n)*x^7 + 18*(1519*a^2*b^9*c^3 - 4*a^5*b^6*c^2*d)*n^6 + 21*(a^2*b^9*c*d^2*n^9 + 45*a^2*b^9*c*d^2*n^8 + 834*a^2*b^9*c*d^2*n^7 + 30*(275*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n^6 + 3*(15763*a^2*b^9*c*d^2 - 1200*a^5*b^6*d^3)*n^5 + 15*(10651*a^2*b^9*c*d^2 - 1360*a^5*b^6*d^3)*n^4 + 4*(77069*a^2*b^9*c*d^2 - 13500*a^5*b^6*d^3)*n^3 + 60*(5119*a^2*b^9*c*d^2 - 1096*a^5*b^6*d^3)*n^2 + 3600*(33*a^2*b^9*c*d^2 - 8*a^5*b^6*d^3)*n)*x^6 + 3*(90643*a^2*b^9*c^3 - 1224*a^5*b^6*c^2*d)*n^5 - 3*(b^{11}*c^2*d*n^{10} + 61*b^{11}*c^2*d*n^9 + 7983360*b^{11}*c^2*d + 6*(270*b^{11}*c^2*d + 7*a^3*b^8*c*d^2)*n^8 + 210*(117*b^{11}*c^2*d + 8*a^3*b^8*c*d^2)*n^7 + 3*(78191*b^{11}*c^2*d + 8876*a^3*b^8*c*d^2)*n^6 + 3*(488231*b^{11}*c^2*d + 71120*a^3*b^8*c*d^2 - 3360*a^6*b^5*d^3)*n^5 + 2*(3008035*b^{11}*c^2*d + 459669*a^3*b^8*c*d^2 - 50400*a^6*b^5*d^3)*n^4 + 20*(795769*b^{11}*c^2*d + 105672*a^3*b^8*c*d^2 - 17640*a^6*b^5*d^3)*n^3 + 72*(356683*b^{11}*c^2*d + 33061*a^3*b^8*c*d^2 - 7000*a^6*b^5*d^3)*n^2 + 288*(78167*b^{11}*c^2*d + 3465*a^3*b^8*c*d^2 - 840*a^6*b^5*d^3)*n)*x^5 + 9*(196343*a^2*b^9*c^3 - 8600*a^5*b^6*c^2*d)*n^4 - 3*(a*b^{10}*c^2*d*n^{10} + 57*a*b^{10}*c^2*d*n^9 + 1392*a*b^{10}*c^2*d*n^8 + 6*(3167*a*b^{10}*c^2*d - 35*a^4*b^7*c*d^2)*n^7 + 15*(10571*a*b^{10}*c^2*d - 504*a^4*b^7*c*d^2)*n^6 + 3*(276811*a*b^{10}*c^2*d - 34300*a^4*b^7*c*d^2)*n^5 + 2*(1347169*a*b^{10}*c^2*d - 327600*a^4*b^7$

```

*c*d^2 + 25200*a^7*b^4*d^3)*n^4 + 42*(122334*a*b^10*c^2*d - 47045*a^4*b^7*c
*d^2 + 7200*a^7*b^4*d^3)*n^3 + 72*(71237*a*b^10*c^2*d - 36995*a^4*b^7*c*d^2
+ 7700*a^7*b^4*d^3)*n^2 + 7560*(264*a*b^10*c^2*d - 165*a^4*b^7*c*d^2 + 40*
a^7*b^4*d^3)*n)*x^4 + 8*(936802*a^2*b^9*c^3 - 107865*a^5*b^6*c^2*d + 1890*a
^8*b^3*c*d^2)*n^3 + 12*(a^2*b^9*c^2*d*n^9 + 54*a^2*b^9*c^2*d*n^8 + 1230*a^2
*b^9*c^2*d*n^7 + 6*(2552*a^2*b^9*c^2*d - 35*a^5*b^6*c*d^2)*n^6 + 33*(3413*a
^2*b^9*c^2*d - 210*a^5*b^6*c*d^2)*n^5 + 6*(82091*a^2*b^9*c^2*d - 13685*a^5*
b^6*c*d^2)*n^4 + 10*(121670*a^2*b^9*c^2*d - 40887*a^5*b^6*c*d^2 + 5040*a^8*
b^3*d^3)*n^3 + 24*(61997*a^2*b^9*c^2*d - 31220*a^5*b^6*c*d^2 + 6300*a^8*b^3
*d^3)*n^2 + 2520*(264*a^2*b^9*c^2*d - 165*a^5*b^6*c*d^2 + 40*a^8*b^3*d^3)*n
)*x^3 + 36*(554953*a^2*b^9*c^3 - 149048*a^5*b^6*c^2*d + 12600*a^8*b^3*c*d^2
)*n^2 - (b^11*c^3*n^10 + 64*b^11*c^3*n^9 + 19958400*b^11*c^3 + 3*(599*b^11*
c^3 + 12*a^3*b^8*c^2*d)*n^8 + 12*(2423*b^11*c^3 + 156*a^3*b^8*c^2*d)*n^7 +
3*(99757*b^11*c^3 + 13512*a^3*b^8*c^2*d)*n^6 + 24*(84959*b^11*c^3 + 19590*a
^3*b^8*c^2*d - 315*a^6*b^5*c*d^2)*n^5 + (9261503*b^11*c^3 + 3114324*a^3*b^8
*c^2*d - 234360*a^6*b^5*c*d^2)*n^4 + 4*(6868181*b^11*c^3 + 2875752*a^3*b^8*
c^2*d - 621810*a^6*b^5*c*d^2)*n^3 + 36*(1397573*b^11*c^3 + 577644*a^3*b^8*c
^2*d - 270690*a^6*b^5*c*d^2 + 50400*a^9*b^2*d^3)*n^2 + 720*(69851*b^11*c^3
+ 16632*a^3*b^8*c^2*d - 10395*a^6*b^5*c*d^2 + 2520*a^9*b^2*d^3)*n)*x^2 + 14
4*(210655*a^2*b^9*c^3 - 122502*a^5*b^6*c^2*d + 31395*a^8*b^3*c*d^2)*n - (a*
b^10*c^3*n^10 + 63*a*b^10*c^3*n^9 + 1734*a*b^10*c^3*n^8 + 18*(1519*a*b^10*c
^3 - 4*a^4*b^7*c^2*d)*n^7 + 3*(90643*a*b^10*c^3 - 1224*a^4*b^7*c^2*d)*n^6 +
9*(196343*a*b^10*c^3 - 8600*a^4*b^7*c^2*d)*n^5 + 8*(936802*a*b^10*c^3 - 10
7865*a^4*b^7*c^2*d + 1890*a^7*b^4*c*d^2)*n^4 + 36*(554953*a*b^10*c^3 - 1490
48*a^4*b^7*c^2*d + 12600*a^7*b^4*c*d^2)*n^3 + 144*(210655*a*b^10*c^3 - 1225
02*a^4*b^7*c^2*d + 31395*a^7*b^4*c*d^2)*n^2 + 90720*(220*a*b^10*c^3 - 264*a
^4*b^7*c^2*d + 165*a^7*b^4*c*d^2 - 40*a^10*b*d^3)*n)*x*(b*x + a)^n/(b^11*n
^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 263
7558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3
+ 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)

```

giac [B] time = 0.72, size = 4934, normalized size = 12.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")
```

```

[Out] ((b*x + a)^n*b^11*d^3*n^10*x^11 + (b*x + a)^n*a*b^10*d^3*n^10*x^10 + 55*(b*
x + a)^n*b^11*d^3*n^9*x^11 + 45*(b*x + a)^n*a*b^10*d^3*n^9*x^10 + 1320*(b*x
+ a)^n*b^11*d^3*n^8*x^11 + 3*(b*x + a)^n*b^11*c*d^2*n^10*x^8 - 10*(b*x + a
)^n*a^2*b^9*d^3*n^9*x^9 + 870*(b*x + a)^n*a*b^10*d^3*n^8*x^10 + 18150*(b*x
+ a)^n*b^11*d^3*n^7*x^11 + 3*(b*x + a)^n*a*b^10*c*d^2*n^10*x^7 + 174*(b*x +
a)^n*b^11*c*d^2*n^9*x^8 - 360*(b*x + a)^n*a^2*b^9*d^3*n^8*x^9 + 9450*(b*x
+ a)^n*a*b^10*d^3*n^7*x^10 + 157773*(b*x + a)^n*b^11*d^3*n^6*x^11 + 153*(b*
x + a)^n*a*b^10*c*d^2*n^9*x^7 + 4383*(b*x + a)^n*b^11*c*d^2*n^8*x^8 + 90*(b
*x + a)^n*a^3*b^8*d^3*n^8*x^8 - 5460*(b*x + a)^n*a^2*b^9*d^3*n^7*x^9 + 6327
3*(b*x + a)^n*a*b^10*d^3*n^6*x^10 + 902055*(b*x + a)^n*b^11*d^3*n^5*x^11 +
3*(b*x + a)^n*b^11*c^2*d*n^10*x^5 - 21*(b*x + a)^n*a^2*b^9*c*d^2*n^9*x^6 +
3312*(b*x + a)^n*a*b^10*c*d^2*n^8*x^7 + 62946*(b*x + a)^n*b^11*c*d^2*n^7*x^
8 + 2520*(b*x + a)^n*a^3*b^8*d^3*n^7*x^8 - 45360*(b*x + a)^n*a^2*b^9*d^3*n^
6*x^9 + 269325*(b*x + a)^n*a*b^10*d^3*n^5*x^10 + 3416930*(b*x + a)^n*b^11*d
^3*n^4*x^11 + 3*(b*x + a)^n*a*b^10*c^2*d*n^10*x^4 + 183*(b*x + a)^n*b^11*c^
2*d*n^9*x^5 - 945*(b*x + a)^n*a^2*b^9*c*d^2*n^8*x^6 + 39762*(b*x + a)^n*a*b
^10*c*d^2*n^7*x^7 - 720*(b*x + a)^n*a^4*b^7*d^3*n^7*x^7 + 568701*(b*x + a)^
n*b^11*c*d^2*n^6*x^8 + 28980*(b*x + a)^n*a^3*b^8*d^3*n^6*x^8 - 224490*(b*x
+ a)^n*a^2*b^9*d^3*n^5*x^9 + 723680*(b*x + a)^n*a*b^10*d^3*n^4*x^10 + 84095
00*(b*x + a)^n*b^11*d^3*n^3*x^11 + 171*(b*x + a)^n*a*b^10*c^2*d*n^9*x^4 + 4
860*(b*x + a)^n*b^11*c^2*d*n^8*x^5 + 126*(b*x + a)^n*a^3*b^8*c*d^2*n^8*x^5
- 17514*(b*x + a)^n*a^2*b^9*c*d^2*n^7*x^6 + 290367*(b*x + a)^n*a*b^10*c*d^2

```

$$\begin{aligned}
& n^6 x^7 - 15120(bx + a)^n a^4 b^7 d^3 n^6 x^7 + 3363066(bx + a)^n b^{11} \\
& * c^2 d^2 n^5 x^8 + 176400(bx + a)^n a^3 b^8 d^3 n^5 x^8 - 672840(bx + a)^n \\
& a^2 b^9 d^3 n^4 x^9 + 1172700(bx + a)^n a b^{10} d^3 n^3 x^{10} + 12753576 \\
& (bx + a)^n b^{11} d^3 n^2 x^{11} + (bx + a)^n b^{11} c^3 n^{10} x^2 - 12(bx + a) \\
&)^n a^2 b^9 c^2 d n^9 x^3 + 4176(bx + a)^n a b^{10} c^2 d n^8 x^4 + 73710(bx + a) \\
&)^n b^{11} c^2 d n^7 x^5 + 5040(bx + a)^n a^3 b^8 c^2 d n^7 x^5 - 17 \\
& 3250(bx + a)^n a^2 b^9 c^2 d n^6 x^6 + 5040(bx + a)^n a^5 b^6 d^3 n^6 x^6 \\
& + 1330497(bx + a)^n a b^{10} c^2 d n^5 x^7 - 126000(bx + a)^n a^4 b^7 d^3 n^5 x^7 \\
& + 13114077(bx + a)^n b^{11} c^2 d n^4 x^8 + 609210(bx + a)^n a^3 b^8 d^3 n^4 x^8 \\
& - 1181240(bx + a)^n a^2 b^9 d^3 n^3 x^9 + 1026576(bx + a)^n a b^{10} d^3 n^2 x^{10} \\
& + 10628640(bx + a)^n b^{11} d^3 n x^{11} + (bx + a)^n a b^{10} c^3 n^{10} x + 64(bx + a)^n b^{11} c^3 n^9 x^2 \\
& - 648(bx + a)^n a^2 b^9 c^2 d n^8 x^3 + 57006(bx + a)^n a b^{10} c^2 d n^7 x^4 - 630(bx + a) \\
&)^n a^4 b^7 c^2 d n^7 x^4 + 703719(bx + a)^n b^{11} c^2 d n^6 x^5 + 798 \\
& 84(bx + a)^n a^3 b^8 c^2 d n^6 x^5 - 993069(bx + a)^n a^2 b^9 c^2 d n^5 x^6 \\
& + 75600(bx + a)^n a^5 b^6 d^3 n^5 x^6 + 3800598(bx + a)^n a b^{10} c^2 d n^4 x^7 \\
& - 529200(bx + a)^n a^4 b^7 d^3 n^4 x^7 + 33074574(bx + a)^n b^{11} c^2 d n^3 x^8 \\
& + 1181880(bx + a)^n a^3 b^8 d^3 n^3 x^8 - 1095840(bx + a)^n a^2 b^9 d^3 n^2 x^9 \\
& + 362880(bx + a)^n a b^{10} d^3 n x^{10} + 3628 \\
& 800(bx + a)^n b^{11} d^3 x^{11} + 63(bx + a)^n a b^{10} c^3 n^9 x + 1797(bx + a) \\
&)^n b^{11} c^3 n^8 x^2 + 36(bx + a)^n a^3 b^8 c^2 d n^8 x^2 - 14760(bx + a)^n a^2 b^9 c^2 d n^7 x^3 \\
& + 475695(bx + a)^n a b^{10} c^2 d n^6 x^4 - 22680(bx + a)^n a^4 b^7 c^2 d n^6 x^4 \\
& + 4394079(bx + a)^n b^{11} c^2 d n^5 x^5 + 640080(bx + a)^n a^3 b^8 c^2 d n^5 x^5 \\
& - 30240(bx + a)^n a^6 b^5 d^3 n^5 x^5 - 3355065(bx + a)^n a^2 b^9 c^2 d n^4 x^6 \\
& + 428400(bx + a)^n a^5 b^6 d^3 n^4 x^6 + 6470388(bx + a)^n a b^{10} c^2 d n^3 x^7 \\
& - 116928 \\
& 0(bx + a)^n a^4 b^7 d^3 n^3 x^7 + 51177636(bx + a)^n b^{11} c^2 d n^2 x^8 \\
& + 1176120(bx + a)^n a^3 b^8 d^3 n^2 x^8 - 403200(bx + a)^n a^2 b^9 d^3 n x^9 \\
& - (bx + a)^n a^2 b^9 c^3 n^9 + 1734(bx + a)^n a b^{10} c^3 n^8 x + 29076(bx + a)^n b^{11} c^3 n^7 x^2 \\
& + 1872(bx + a)^n a^3 b^8 c^2 d n^7 x^2 - 183744(bx + a)^n a^2 b^9 c^2 d n^6 x^3 \\
& + 2520(bx + a)^n a^5 b^6 c^2 d n^6 x^3 + 2491299(bx + a)^n a b^{10} c^2 d n^5 x^4 \\
& - 308700(bx + a)^n a^4 b^7 c^2 d n^5 x^4 + 18048210(bx + a)^n b^{11} c^2 d n^4 x^5 \\
& + 2758014(bx + a)^n a^3 b^8 c^2 d n^4 x^5 - 302400(bx + a)^n a^6 b^5 d^3 n^4 x^5 \\
& - 6473796(bx + a)^n a^2 b^9 c^2 d n^3 x^6 + 1134000(bx + a)^n a^5 b^6 d^3 n^3 x^6 \\
& + 5884920(bx + a)^n a b^{10} c^2 d n^2 x^7 - 1270080(bx + a)^n a^4 b^7 d^3 n^2 x^7 \\
& + 43332840(bx + a)^n b^{11} c^2 d n x^8 + 453600(bx + a)^n a^3 b^8 d^3 n x^8 \\
& - 63(bx + a)^n a^2 b^9 c^3 n^8 + 27342(bx + a)^n a b^{10} c^3 n^7 x - 72(bx + a)^n a^4 b^7 c^2 d n^7 x \\
& + 299271(bx + a)^n b^{11} c^3 n^6 x^2 + 40536(bx + a)^n a^3 b^8 c^2 d n^6 x^2 - 1351548(bx + a) \\
&)^n a^2 b^9 c^2 d n^5 x^3 + 83160(bx + a)^n a^5 b^6 c^2 d n^5 x^3 + 80 \\
& 83014(bx + a)^n a b^{10} c^2 d n^4 x^4 - 1965600(bx + a)^n a^4 b^7 c^2 d n^4 x^4 \\
& + 151200(bx + a)^n a^7 b^4 d^3 n^4 x^4 + 47746140(bx + a)^n b^{11} c^2 d n^3 x^5 \\
& + 6340320(bx + a)^n a^3 b^8 c^2 d n^3 x^5 - 1058400(bx + a)^n a^6 b^5 d^3 n^3 x^5 \\
& - 6449940(bx + a)^n a^2 b^9 c^2 d n^2 x^6 + 13 \\
& 80960(bx + a)^n a^5 b^6 d^3 n^2 x^6 + 2138400(bx + a)^n a b^{10} c^2 d n x^7 \\
& - 518400(bx + a)^n a^4 b^7 d^3 n x^7 + 14968800(bx + a)^n b^{11} c^2 d n^2 x^8 \\
& - 1734(bx + a)^n a^2 b^9 c^3 n^7 + 271929(bx + a)^n a b^{10} c^3 n^6 x - 3672(bx + a)^n a^4 b^7 c^2 d n^6 x \\
& + 2039016(bx + a)^n b^{11} c^3 n^5 x^2 + 470160(bx + a)^n a^3 b^8 c^2 d n^5 x^2 - 7560(bx + a)^n a^6 b^5 c^2 d n^5 x^2 \\
& - 5910552(bx + a)^n a^2 b^9 c^2 d n^4 x^3 + 985320(bx + a)^n a^5 b^6 c^2 d n^4 x^3 \\
& + 15414084(bx + a)^n a b^{10} c^2 d n^3 x^4 - 5 \\
& 927670(bx + a)^n a^4 b^7 c^2 d n^3 x^4 + 907200(bx + a)^n a^7 b^4 d^3 n^3 x^4 \\
& + 77043528(bx + a)^n b^{11} c^2 d n^2 x^5 + 7141176(bx + a)^n a^3 b^8 c^2 d n^2 x^5 \\
& - 1512000(bx + a)^n a^6 b^5 d^3 n^2 x^5 - 2494800(bx + a)^n a^2 b^9 c^2 d n x^6 \\
& + 604800(bx + a)^n a^5 b^6 d^3 n x^6 - 27342(bx + a)^n a^2 b^9 c^3 n^6 \\
& + 72(bx + a)^n a^5 b^6 c^2 d n^6 + 1767087(bx + a)^n a b^{10} c^3 n^5 x - 77400(bx + a)^n a^4 b^7 c^2 d n^5 x \\
& + 9261503(bx + a)^n b^{11} c^3 n^4 x^2 + 3114324(bx + a)^n a^3 b^8 c^2 d n^4 x^2 -
\end{aligned}$$

$$\begin{aligned}
& 234360*(b*x + a)^n*a^6*b^5*c*d^2*n^4*x^2 - 14600400*(b*x + a)^n*a^2*b^9*c^2*d*n^3*x^3 + 4906440*(b*x + a)^n*a^5*b^6*c*d^2*n^3*x^3 - 604800*(b*x + a)^n*a^8*b^3*d^3*n^3*x^3 + 15387192*(b*x + a)^n*a*b^10*c^2*d*n^2*x^4 - 7990920*(b*x + a)^n*a^4*b^7*c*d^2*n^2*x^4 + 1663200*(b*x + a)^n*a^7*b^4*d^3*n^2*x^4 + 67536288*(b*x + a)^n*b^11*c^2*d*n*x^5 + 2993760*(b*x + a)^n*a^3*b^8*c*d^2*n*x^5 - 725760*(b*x + a)^n*a^6*b^5*d^3*n*x^5 - 271929*(b*x + a)^n*a^2*b^9*c^3*n^5 + 3672*(b*x + a)^n*a^5*b^6*c^2*d*n^5 + 7494416*(b*x + a)^n*a*b^10*c^3*n^4*x - 862920*(b*x + a)^n*a^4*b^7*c^2*d*n^4*x + 15120*(b*x + a)^n*a^7*b^4*c*d^2*n^4*x + 27472724*(b*x + a)^n*b^11*c^3*n^3*x^2 + 11503008*(b*x + a)^n*a^3*b^8*c^2*d*n^3*x^2 - 2487240*(b*x + a)^n*a^6*b^5*c*d^2*n^3*x^2 - 17855136*(b*x + a)^n*a^2*b^9*c^2*d*n^2*x^3 + 8991360*(b*x + a)^n*a^5*b^6*c*d^2*n^2*x^3 - 1814400*(b*x + a)^n*a^8*b^3*d^3*n^2*x^3 + 5987520*(b*x + a)^n*a*b^10*c^2*d*n*x^4 - 3742200*(b*x + a)^n*a^4*b^7*c*d^2*n*x^4 + 907200*(b*x + a)^n*a^7*b^4*d^3*n*x^4 + 23950080*(b*x + a)^n*b^11*c^2*d*x^5 - 1767087*(b*x + a)^n*a^2*b^9*c^3*n^4 + 77400*(b*x + a)^n*a^5*b^6*c^2*d*n^4 + 19978308*(b*x + a)^n*a*b^10*c^3*n^3*x - 5365728*(b*x + a)^n*a^4*b^7*c^2*d*n^3*x + 453600*(b*x + a)^n*a^7*b^4*c*d^2*n^3*x + 50312628*(b*x + a)^n*b^11*c^3*n^2*x^2 + 20795184*(b*x + a)^n*a^3*b^8*c^2*d*n^2*x^2 - 9744840*(b*x + a)^n*a^6*b^5*c*d^2*n^2*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n^2*x^2 - 7983360*(b*x + a)^n*a^2*b^9*c^2*d*n*x^3 + 4989600*(b*x + a)^n*a^5*b^6*c*d^2*n*x^3 - 1209600*(b*x + a)^n*a^8*b^3*d^3*n*x^3 - 7494416*(b*x + a)^n*a^2*b^9*c^3*n^3 + 862920*(b*x + a)^n*a^5*b^6*c^2*d*n^3 - 15120*(b*x + a)^n*a^8*b^3*c*d^2*n^3 + 30334320*(b*x + a)^n*a*b^10*c^3*n^2*x - 17640288*(b*x + a)^n*a^4*b^7*c^2*d*n^2*x + 4520880*(b*x + a)^n*a^7*b^4*c*d^2*n^2*x + 50292720*(b*x + a)^n*b^11*c^3*n*x^2 + 11975040*(b*x + a)^n*a^3*b^8*c^2*d*n*x^2 - 7484400*(b*x + a)^n*a^6*b^5*c*d^2*n*x^2 + 1814400*(b*x + a)^n*a^9*b^2*d^3*n*x^2 - 19978308*(b*x + a)^n*a^2*b^9*c^3*n^2 + 5365728*(b*x + a)^n*a^5*b^6*c^2*d*n^2 - 453600*(b*x + a)^n*a^8*b^3*c*d^2*n^2 + 19958400*(b*x + a)^n*a*b^10*c^3*n*x - 23950080*(b*x + a)^n*a^4*b^7*c^2*d*n*x + 14968800*(b*x + a)^n*a^7*b^4*c*d^2*n*x - 3628800*(b*x + a)^n*a^10*b*d^3*n*x + 19958400*(b*x + a)^n*b^11*c^3*x^2 - 30334320*(b*x + a)^n*a^2*b^9*c^3*n + 17640288*(b*x + a)^n*a^5*b^6*c^2*d*n - 4520880*(b*x + a)^n*a^8*b^3*c*d^2*n - 19958400*(b*x + a)^n*a^2*b^9*c^3 + 23950080*(b*x + a)^n*a^5*b^6*c^2*d - 14968800*(b*x + a)^n*a^8*b^3*c*d^2 + 3628800*(b*x + a)^n*a^11*d^3)/(b^11*n^11 + 66*b^11*n^10 + 1925*b^11*n^9 + 32670*b^11*n^8 + 357423*b^11*n^7 + 2637558*b^11*n^6 + 13339535*b^11*n^5 + 45995730*b^11*n^4 + 105258076*b^11*n^3 + 150917976*b^11*n^2 + 120543840*b^11*n + 39916800*b^11)
\end{aligned}$$

maple [B] time = 0.04, size = 2972, normalized size = 7.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x*(b*x+a)^n*(d*x^3+c)^3,x)$

[Out] $(b*x+a)^{(n+1)}*(b^{10}*d^3*n^{10}*x^{10}+55*b^{10}*d^3*n^9*x^{10}-10*a*b^9*d^3*n^9*x^{10}+1320*b^{10}*d^3*n^8*x^{10}-450*a*b^9*d^3*n^8*x^9+3*b^{10}*c*d^2*n^{10}*x^7+18150*b^{10}*d^3*n^7*x^{10}+90*a^2*b^8*d^3*n^8*x^8-8700*a*b^9*d^3*n^7*x^9+174*b^{10}*c*d^2*n^9*x^7+157773*b^{10}*d^3*n^6*x^{10}+3240*a^2*b^8*d^3*n^7*x^8-21*a*b^9*c*d^2*n^9*x^6-94500*a*b^9*d^3*n^6*x^9+4383*b^{10}*c*d^2*n^8*x^7+902055*b^{10}*d^3*n^5*x^{10}-720*a^3*b^7*d^3*n^7*x^7+49140*a^2*b^8*d^3*n^6*x^8-1071*a*b^9*c*d^2*n^8*x^6-632730*a*b^9*d^3*n^5*x^9+3*b^{10}*c^2*d*n^{10}*x^4+62946*b^{10}*c*d^2*n^7*x^7+3416930*b^{10}*d^3*n^4*x^{10}-20160*a^3*b^7*d^3*n^6*x^7+126*a^2*b^8*c*d^2*n^8*x^5+408240*a^2*b^8*d^3*n^5*x^8-23184*a*b^9*c*d^2*n^7*x^6-2693250*a*b^9*d^3*n^4*x^9+183*b^{10}*c^2*d*n^9*x^4+568701*b^{10}*c*d^2*n^6*x^7+8409500*b^{10}*d^3*n^3*x^{10}+5040*a^4*b^6*d^3*n^6*x^6-231840*a^3*b^7*d^3*n^5*x^7+5670*a^2*b^8*c*d^2*n^7*x^5+2020410*a^2*b^8*d^3*n^4*x^8-12*a*b^9*c^2*d*n^9*x^3-278334*a*b^9*c*d^2*n^6*x^6-7236800*a*b^9*d^3*n^3*x^9+4860*b^{10}*c^2*d*n^8*x^4+3363066*b^{10}*c*d^2*n^5*x^7+12753576*b^{10}*d^3*n^2*x^{10}+105840*a^4*b^6*d^3*n^5*x^6-630*a^3*b^7*c*d^2*n^7*x^4-1411200*a^3*b^7*d^3*n^4*x^7+105084*a^2*b^8*c*d^2*n$

$$\begin{aligned} &^6*x^5+6055560*a^2*b^8*d^3*n^3*x^8-684*a*b^9*c^2*d*n^8*x^3-2032569*a*b^9*c* \\ &d^2*n^5*x^6-11727000*a*b^9*d^3*n^2*x^9+b^10*c^3*n^10*x+73710*b^10*c^2*d*n^7 \\ &*x^4+13114077*b^10*c*d^2*n^4*x^7+10628640*b^10*d^3*n*x^10-30240*a^5*b^5*d^3 \\ &*n^5*x^5+882000*a^4*b^6*d^3*n^4*x^6-25200*a^3*b^7*c*d^2*n^6*x^4-4873680*a^3 \\ &*b^7*d^3*n^3*x^7+36*a^2*b^8*c^2*d*n^8*x^2+1039500*a^2*b^8*c*d^2*n^5*x^5+106 \\ &31160*a^2*b^8*d^3*n^2*x^8-16704*a*b^9*c^2*d*n^7*x^3-9313479*a*b^9*c*d^2*n^4 \\ &*x^6-10265760*a*b^9*d^3*n*x^9+64*b^10*c^3*n^9*x+703719*b^10*c^2*d*n^6*x^4+3 \\ &3074574*b^10*c*d^2*n^3*x^7+3628800*b^10*d^3*x^10-453600*a^5*b^5*d^3*n^4*x^5 \\ &+2520*a^4*b^6*c*d^2*n^6*x^3+3704400*a^4*b^6*d^3*n^3*x^6-399420*a^3*b^7*c*d^ \\ &2*n^5*x^4-9455040*a^3*b^7*d^3*n^2*x^7+1944*a^2*b^8*c^2*d*n^7*x^2+5958414*a^ \\ &2*b^8*c*d^2*n^4*x^5+9862560*a^2*b^8*d^3*n*x^8-a*b^9*c^3*n^9-228024*a*b^9*c^ \\ &2*d*n^6*x^3-26604186*a*b^9*c*d^2*n^3*x^6-3628800*a*b^9*d^3*x^9+1797*b^10*c^ \\ &3*n^8*x+4394079*b^10*c^2*d*n^5*x^4+51177636*b^10*c*d^2*n^2*x^7+151200*a^6*b \\ &^4*d^3*n^4*x^4-2570400*a^5*b^5*d^3*n^3*x^5+90720*a^4*b^6*c*d^2*n^5*x^3+8184 \\ &960*a^4*b^6*d^3*n^2*x^6-72*a^3*b^7*c^2*d*n^7*x-3200400*a^3*b^7*c*d^2*n^4*x^ \\ &4-9408960*a^3*b^7*d^3*n*x^7+44280*a^2*b^8*c^2*d*n^6*x^2+20130390*a^2*b^8*c* \\ &d^2*n^3*x^5+3628800*a^2*b^8*d^3*x^8-63*a*b^9*c^3*n^8-1902780*a*b^9*c^2*d*n^ \\ &5*x^3-45292716*a*b^9*c*d^2*n^2*x^6+29076*b^10*c^3*n^7*x+18048210*b^10*c^2*d \\ &*n^4*x^4+43332840*b^10*c*d^2*n*x^7+1512000*a^6*b^4*d^3*n^3*x^4-7560*a^5*b^5 \\ &*c*d^2*n^5*x^2-6804000*a^5*b^5*d^3*n^2*x^5+1234800*a^4*b^6*c*d^2*n^4*x^3+88 \\ &90560*a^4*b^6*d^3*n*x^6-3744*a^3*b^7*c^2*d*n^6*x-13790070*a^3*b^7*c*d^2*n^3 \\ &*x^4-3628800*a^3*b^7*d^3*x^7+551232*a^2*b^8*c^2*d*n^5*x^2+38842776*a^2*b^8* \\ &c*d^2*n^2*x^5-1734*a*b^9*c^3*n^7-9965196*a*b^9*c^2*d*n^4*x^3-41194440*a*b^9 \\ &*c*d^2*n*x^6+299271*b^10*c^3*n^6*x+47746140*b^10*c^2*d*n^3*x^4+14968800*b^1 \\ &0*c*d^2*x^7-604800*a^7*b^3*d^3*n^3*x^3+5292000*a^6*b^4*d^3*n^2*x^4-249480*a \\ &^5*b^5*c*d^2*n^4*x^2-8285760*a^5*b^5*d^3*n*x^5+72*a^4*b^6*c^2*d*n^6+7862400 \\ &*a^4*b^6*c*d^2*n^3*x^3+3628800*a^4*b^6*d^3*x^6-81072*a^3*b^7*c^2*d*n^5*x-31 \\ &701600*a^3*b^7*c*d^2*n^2*x^4+4054644*a^2*b^8*c^2*d*n^4*x^2+38699640*a^2*b^8 \\ &*c*d^2*n*x^5-27342*a*b^9*c^3*n^6-32332056*a*b^9*c^2*d*n^3*x^3-14968800*a*b^ \\ &9*c*d^2*x^6+2039016*b^10*c^3*n^5*x+77043528*b^10*c^2*d*n^2*x^4-3628800*a^7* \\ &b^3*d^3*n^2*x^3+15120*a^6*b^4*c*d^2*n^4*x+7560000*a^6*b^4*d^3*n*x^4-2955960 \\ &*a^5*b^5*c*d^2*n^3*x^2-3628800*a^5*b^5*d^3*x^5+3672*a^4*b^6*c^2*d*n^5+23710 \\ &680*a^4*b^6*c*d^2*n^2*x^3-940320*a^3*b^7*c^2*d*n^4*x-35705880*a^3*b^7*c*d^2 \\ &*n*x^4+17731656*a^2*b^8*c^2*d*n^3*x^2+14968800*a^2*b^8*c*d^2*x^5-271929*a*b \\ &^9*c^3*n^5-61656336*a*b^9*c^2*d*n^2*x^3+9261503*b^10*c^3*n^4*x+67536288*b^1 \\ &0*c^2*d*n*x^4+1814400*a^8*b^2*d^3*n^2*x^2-6652800*a^7*b^3*d^3*n*x^3+468720* \\ &a^6*b^4*c*d^2*n^3*x+3628800*a^6*b^4*d^3*x^4-14719320*a^5*b^5*c*d^2*n^2*x^2+ \\ &77400*a^4*b^6*c^2*d*n^4+31963680*a^4*b^6*c*d^2*n*x^3-6228648*a^3*b^7*c^2*d* \\ &n^3*x-14968800*a^3*b^7*c*d^2*x^4+43801200*a^2*b^8*c^2*d*n^2*x^2-1767087*a*b \\ &^9*c^3*n^4-61548768*a*b^9*c^2*d*n*x^3+27472724*b^10*c^3*n^3*x+23950080*b^10 \\ &*c^2*d*x^4+5443200*a^8*b^2*d^3*n*x^2-15120*a^7*b^3*c*d^2*n^3-3628800*a^7*b^ \\ &3*d^3*x^3+4974480*a^6*b^4*c*d^2*n^2*x-26974080*a^5*b^5*c*d^2*n*x^2+862920*a \\ &^4*b^6*c^2*d*n^3+14968800*a^4*b^6*c*d^2*x^3-23006016*a^3*b^7*c^2*d*n^2*x+53 \\ &565408*a^2*b^8*c^2*d*n*x^2-7494416*a*b^9*c^3*n^3-23950080*a*b^9*c^2*d*x^3+5 \\ &0312628*b^10*c^3*n^2*x-3628800*a^9*b*d^3*n*x+3628800*a^8*b^2*d^3*x^2-453600 \\ &*a^7*b^3*c*d^2*n^2+19489680*a^6*b^4*c*d^2*n*x-14968800*a^5*b^5*c*d^2*x^2+53 \\ &65728*a^4*b^6*c^2*d*n^2-41590368*a^3*b^7*c^2*d*n*x+23950080*a^2*b^8*c^2*d*x \\ &^2-19978308*a*b^9*c^3*n^2+50292720*b^10*c^3*n*x-3628800*a^9*b*d^3*x-4520880 \\ &*a^7*b^3*c*d^2*n+14968800*a^6*b^4*c*d^2*x+17640288*a^4*b^6*c^2*d*n-23950080 \\ &*a^3*b^7*c^2*d*x-30334320*a*b^9*c^3*n+19958400*b^10*c^3*x+3628800*a^10*d^3- \\ &14968800*a^7*b^3*c*d^2+23950080*a^4*b^6*c^2*d-19958400*a*b^9*c^3)/b^11/(n^1 \\ &1+66*n^10+1925*n^9+32670*n^8+357423*n^7+2637558*n^6+13339535*n^5+45995730*n \\ &^4+105258076*n^3+150917976*n^2+120543840*n+39916800) \end{aligned}$$

maxima [B] time = 0.85, size = 953, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")

```
[Out] (b^2*(n + 1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^3/((n^2 + 3*n + 2)*b^2) + 3
*((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2
- 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n^4 + 85*n^3 + 225*n^
2 + 274*n + 120)*b^5) + 3*((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 +
13132*n^2 + 13068*n + 5040)*b^8*x^8 + (n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1
624*n^3 + 1764*n^2 + 720*n)*a*b^7*x^7 - 7*(n^6 + 15*n^5 + 85*n^4 + 225*n^3
+ 274*n^2 + 120*n)*a^2*b^6*x^6 + 42*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)
*a^3*b^5*x^5 - 210*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^4*b^4*x^4 + 840*(n^3 + 3*
n^2 + 2*n)*a^5*b^3*x^3 - 2520*(n^2 + n)*a^6*b^2*x^2 + 5040*a^7*b*n*x - 5040
*a^8)*(b*x + a)^n*c*d^2/((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449*n^4 + 6
7284*n^3 + 118124*n^2 + 109584*n + 40320)*b^8) + ((n^10 + 55*n^9 + 1320*n^8
+ 18150*n^7 + 157773*n^6 + 902055*n^5 + 3416930*n^4 + 8409500*n^3 + 127535
76*n^2 + 10628640*n + 3628800)*b^11*x^11 + (n^10 + 45*n^9 + 870*n^8 + 9450*
n^7 + 63273*n^6 + 269325*n^5 + 723680*n^4 + 1172700*n^3 + 1026576*n^2 + 362
880*n)*a*b^10*x^10 - 10*(n^9 + 36*n^8 + 546*n^7 + 4536*n^6 + 22449*n^5 + 67
284*n^4 + 118124*n^3 + 109584*n^2 + 40320*n)*a^2*b^9*x^9 + 90*(n^8 + 28*n^7
+ 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2 + 5040*n)*a^3*b^8*
x^8 - 720*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*n^3 + 1764*n^2 + 720*n)*
a^4*b^7*x^7 + 5040*(n^6 + 15*n^5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a^5*
b^6*x^6 - 30240*(n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a^6*b^5*x^5 + 15120
0*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^7*b^4*x^4 - 604800*(n^3 + 3*n^2 + 2*n)*a^8
*b^3*x^3 + 1814400*(n^2 + n)*a^9*b^2*x^2 - 3628800*a^10*b*n*x + 3628800*a^1
1)*(b*x + a)^n*d^3/((n^11 + 66*n^10 + 1925*n^9 + 32670*n^8 + 357423*n^7 + 2
637558*n^6 + 13339535*n^5 + 45995730*n^4 + 105258076*n^3 + 150917976*n^2 +
120543840*n + 39916800)*b^11)
```

mupad [B] time = 5.60, size = 2436, normalized size = 6.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^3)^3*(a + b*x)^n,x)
```

```
[Out] (d^3*x^11*(a + b*x)^n*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^
4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362880
0))/(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 13339535*
n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^11 + 39
916800) - (a^2*(a + b*x)^n*(19958400*b^9*c^3 - 3628800*a^9*d^3 + 30334320*b
^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 1767087*b^9*c^3*n^4
+ 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n^7 + 63*b^9*c^3*n
^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 14968800*a^6*b^3*c*d^2 - 176402
88*a^3*b^6*c^2*d*n + 4520880*a^6*b^3*c*d^2*n - 5365728*a^3*b^6*c^2*d*n^2 +
453600*a^6*b^3*c*d^2*n^2 - 862920*a^3*b^6*c^2*d*n^3 + 15120*a^6*b^3*c*d^2*n
^3 - 77400*a^3*b^6*c^2*d*n^4 - 3672*a^3*b^6*c^2*d*n^5 - 72*a^3*b^6*c^2*d*n^
6))/(b^11*(120543840*n + 150917976*n^2 + 105258076*n^3 + 45995730*n^4 + 133
39535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 1925*n^9 + 66*n^10 + n^1
1 + 39916800)) + (x^2*(n + 1)*(a + b*x)^n*(19958400*b^9*c^3 + 1814400*a^9*d
^3*n + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9*c^3*n^3 + 17
67087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1734*b^9*c^3*n
^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 + 11975040*a^3*b^6*c^2*d*n - 7484400*a^6*
b^3*c*d^2*n + 8820144*a^3*b^6*c^2*d*n^2 - 2260440*a^6*b^3*c*d^2*n^2 + 26828
64*a^3*b^6*c^2*d*n^3 - 226800*a^6*b^3*c*d^2*n^3 + 431460*a^3*b^6*c^2*d*n^4
- 7560*a^6*b^3*c*d^2*n^4 + 38700*a^3*b^6*c^2*d*n^5 + 1836*a^3*b^6*c^2*d*n^6
+ 36*a^3*b^6*c^2*d*n^7))/(b^9*(120543840*n + 150917976*n^2 + 105258076*n^3
+ 45995730*n^4 + 13339535*n^5 + 2637558*n^6 + 357423*n^7 + 32670*n^8 + 192
5*n^9 + 66*n^10 + n^11 + 39916800)) + (a*n*x*(a + b*x)^n*(19958400*b^9*c^3
- 3628800*a^9*d^3 + 30334320*b^9*c^3*n + 19978308*b^9*c^3*n^2 + 7494416*b^9
*c^3*n^3 + 1767087*b^9*c^3*n^4 + 271929*b^9*c^3*n^5 + 27342*b^9*c^3*n^6 + 1
734*b^9*c^3*n^7 + 63*b^9*c^3*n^8 + b^9*c^3*n^9 - 23950080*a^3*b^6*c^2*d + 1
```

$$\begin{aligned}
& 4968800a^6b^3cd^2 - 17640288a^3b^6c^2dn + 4520880a^6b^3cd^2n \\
& - 5365728a^3b^6c^2dn^2 + 453600a^6b^3cd^2n^2 - 862920a^3b^6c^2 \\
& *dn^3 + 15120a^6b^3cd^2n^3 - 77400a^3b^6c^2dn^4 - 3672a^3b^6c^2 \\
& ^2dn^5 - 72a^3b^6c^2dn^6) / (b^{10}(120543840n + 150917976n^2 + 1052 \\
& 58076n^3 + 45995730n^4 + 13339535n^5 + 2637558n^6 + 357423n^7 + 32670n \\
& n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) + (3d^5x^5(a + b^6x)^n(50n + \\
& 35n^2 + 10n^3 + n^4 + 24)(332640b^6c^2 - 10080a^6d^2n + 245004b^6 \\
& *c^2n + 74524b^6c^2n^2 + 11985b^6c^2n^3 + 1075b^6c^2n^4 + 51b^6c^2 \\
& c^2n^5 + b^6c^2n^6 + 41580a^3b^3cdn + 12558a^3b^3cdn^2 + 1260a^3 \\
& a^3b^3cdn^3 + 42a^3b^3cdn^4) / (b^6(120543840n + 150917976n^2 + \\
& 105258076n^3 + 45995730n^4 + 13339535n^5 + 2637558n^6 + 357423n^7 + 32 \\
& 670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) + (3d^2x^8(a + b^6x)^n(\\
& 990b^3c + 30b^3cn^2 + b^3cn^3 + 30a^3dn + 299b^3cn) * (13068n + \\
& 13132n^2 + 6769n^3 + 1960n^4 + 322n^5 + 28n^6 + n^7 + 5040)) / (b^3(12 \\
& 0543840n + 150917976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 2 \\
& 637558n^6 + 357423n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800) \\
&) + (a^3d^3n^10x^{10}(a + b^6x)^n(1026576n + 1172700n^2 + 723680n^3 + 26932 \\
& 5n^4 + 63273n^5 + 9450n^6 + 870n^7 + 45n^8 + n^9 + 362880)) / (b^6(120543 \\
& 840n + 150917976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 26375 \\
& 58n^6 + 357423n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) - \\
& (10a^2d^3n^9x^9(a + b^6x)^n(109584n + 118124n^2 + 67284n^3 + 22449n^4 \\
& + 4536n^5 + 546n^6 + 36n^7 + n^8 + 40320)) / (b^2(120543840n + 1509179 \\
& 76n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 2637558n^6 + 357423 \\
& *n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) + (3a^2d^2n^7x^7 * \\
& (a + b^6x)^n(990b^3c - 240a^3d + 30b^3cn^2 + b^3cn^3 + 299b^3cn) * \\
& (1764n + 1624n^2 + 735n^3 + 175n^4 + 21n^5 + n^6 + 720)) / (b^4(12054 \\
& 3840n + 150917976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 2637 \\
& 558n^6 + 357423n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) - \\
& (21a^2d^2n^6x^6(a + b^6x)^n(990b^3c - 240a^3d + 30b^3cn^2 + b^3c \\
& cn^3 + 299b^3cn) * (274n + 225n^2 + 85n^3 + 15n^4 + n^5 + 120)) / (b^5 * \\
& (120543840n + 150917976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 \\
& + 2637558n^6 + 357423n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 399168 \\
& 00)) + (3a^2d^2n^4x^4(a + b^6x)^n(11n + 6n^2 + n^3 + 6) * (50400a^6d^2 + 3 \\
& 32640b^6c^2 + 245004b^6c^2n + 74524b^6c^2n^2 + 11985b^6c^2n^3 + \\
& 1075b^6c^2n^4 + 51b^6c^2n^5 + b^6c^2n^6 - 207900a^3b^3cd - 6279 \\
& 0a^3b^3cdn - 6300a^3b^3cdn^2 - 210a^3b^3cdn^3)) / (b^7(120543 \\
& 840n + 150917976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 26375 \\
& 58n^6 + 357423n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800)) - \\
& (12a^2d^2n^3x^3(a + b^6x)^n(3n + n^2 + 2) * (50400a^6d^2 + 332640b^6c^2 \\
& + 245004b^6c^2n + 74524b^6c^2n^2 + 11985b^6c^2n^3 + 1075b^6c^2n^4 \\
& + 51b^6c^2n^5 + b^6c^2n^6 - 207900a^3b^3cd - 62790a^3b^3cd \\
& *n - 6300a^3b^3cdn^2 - 210a^3b^3cdn^3)) / (b^8(120543840n + 15091 \\
& 7976n^2 + 105258076n^3 + 45995730n^4 + 13339535n^5 + 2637558n^6 + 3574 \\
& 23n^7 + 32670n^8 + 1925n^9 + 66n^{10} + n^{11} + 39916800))
\end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x+a)**n*(d*x**3+c)**3,x)

[Out] Timed out

$$3.58 \quad \int (a + bx)^n (c + dx^3)^3 dx$$

Optimal. Leaf size=337

$$\frac{18ad^2 (b^3c - 7a^3d) (a + bx)^{n+6}}{b^{10}(n+6)} + \frac{3d^2 (b^3c - 28a^3d) (a + bx)^{n+7}}{b^{10}(n+7)} + \frac{(b^3c - a^3d)^3 (a + bx)^{n+1}}{b^{10}(n+1)} - \frac{9ad (b^3c - 4a^3d) (b^3c - a^3d)^2 (a + bx)^{n+2}}{b^{10}(n+2)}$$

Rubi [A] time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, number of rules / integrand size = 0.059, Rules used = {1850}

$$\frac{3d(-20a^2b^3cd + 28a^6d^2 + b^6c^2)(a + bx)^{n+4}}{b^{10}(n+4)} + \frac{9a^2d^2(5b^3c - 14a^3d)(a + bx)^{n+5}}{b^{10}(n+5)} - \frac{18ad^2(b^3c - 7a^3d)(a + bx)^{n+6}}{b^{10}(n+6)} + \frac{3d^2(b^3c - 28a^3d)(a + bx)^{n+7}}{b^{10}(n+7)} + \frac{(b^3c - a^3d)^3(a + bx)^{n+1}}{b^{10}(n+1)} + \frac{9a^2d(b^3c - a^3d)^2(a + bx)^{n+2}}{b^{10}(n+2)} - \frac{9ad(b^3c - 4a^3d)(b^3c - a^3d)^2(a + bx)^{n+3}}{b^{10}(n+3)} + \frac{36a^2d^2(a + bx)^{n+4}}{b^{10}(n+4)} - \frac{9ad^2(a + bx)^{n+5}}{b^{10}(n+5)} + \frac{d^3(a + bx)^{n+6}}{b^{10}(n+6)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x)^n*(c + d*x^3)^3, x]

[Out] ((b^3*c - a^3*d)^3*(a + b*x)^(1 + n))/(b^10*(1 + n)) + (9*a^2*d*(b^3*c - a^3*d)^2*(a + b*x)^(2 + n))/(b^10*(2 + n)) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^(3 + n))/(b^10*(3 + n)) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^(4 + n))/(b^10*(4 + n)) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^(5 + n))/(b^10*(5 + n)) - (18*a*d^2*(b^3*c - 7*a^3*d)*(a + b*x)^(6 + n))/(b^10*(6 + n)) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^(7 + n))/(b^10*(7 + n)) + (36*a^2*d^3*(a + b*x)^(8 + n))/(b^10*(8 + n)) - (9*a*d^3*(a + b*x)^(9 + n))/(b^10*(9 + n)) + (d^3*(a + b*x)^(10 + n))/(b^10*(10 + n))

Rule 1850

Int[(Pq_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n}, x] && PolyQ[Pq, x] && (IGtQ[p, 0] || EqQ[n, 1])

Rubi steps

$$\int (a + bx)^n (c + dx^3)^3 dx = \int \left(\frac{(b^3c - a^3d)^3 (a + bx)^n}{b^9} + \frac{9d (ab^3c - a^4d)^2 (a + bx)^{1+n}}{b^9} + \frac{9ad (b^3c - 4a^3d) (-b^3c - a^3d)^2 (a + bx)^{2+n}}{b^9} \right) dx$$

$$= \frac{(b^3c - a^3d)^3 (a + bx)^{1+n}}{b^{10}(1+n)} + \frac{9a^2d (b^3c - a^3d)^2 (a + bx)^{2+n}}{b^{10}(2+n)} - \frac{9ad (b^3c - 4a^3d) (b^3c - a^3d)^2 (a + bx)^{3+n}}{b^{10}(3+n)}$$

Mathematica [A] time = 0.36, size = 290, normalized size = 0.86

$$\frac{(a + bx)^{n+1} \left(\frac{9d(a+bx)(ab^3c - a^4d)^2}{n+2} + \frac{3d^2(a+bx)^2(b^3c - 28a^3d)}{n+7} + \frac{18ad^2(a+bx)^3(7a^3d - b^3c)}{n+6} - \frac{9ad(a+bx)^2(b^3c - 4a^3d)(b^3c - a^3d)}{n+3} + \frac{(b^3c - a^3d)^3}{n+1} + \frac{36a^2d^2(a+bx)^7}{n+8} + \frac{3d(a+bx)^2(28a^6d^2 - 20a^3b^3cd + b^6c^2)}{n+4} + \frac{9a^2d^2(a+bx)^4(5b^3c - 14a^3d)}{n+5} + \frac{d^3(a+bx)^9}{n+10} - \frac{9ad^3(a+bx)^5}{n+9} \right)}{b^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x)^n*(c + d*x^3)^3, x]

[Out] ((a + b*x)^(1 + n)*((b^3*c - a^3*d)^3/(1 + n) + (9*d*(a*b^3*c - a^4*d)^2*(a + b*x))/(2 + n) - (9*a*d*(b^3*c - 4*a^3*d)*(b^3*c - a^3*d)*(a + b*x)^2)/(3 + n) + (3*d*(b^6*c^2 - 20*a^3*b^3*c*d + 28*a^6*d^2)*(a + b*x)^3)/(4 + n) + (9*a^2*d^2*(5*b^3*c - 14*a^3*d)*(a + b*x)^4)/(5 + n) + (18*a*d^2*(-(b^3*c) + 7*a^3*d)*(a + b*x)^5)/(6 + n) + (3*d^2*(b^3*c - 28*a^3*d)*(a + b*x)^6)/(7 + n) + (36*a^2*d^3*(a + b*x)^7)/(8 + n) - (9*a*d^3*(a + b*x)^8)/(9 + n) + (d^3*(a + b*x)^9)/(10 + n))/b^10

IntegrateAlgebraic [F] time = 0.10, size = 0, normalized size = 0.00

$$\int (a + bx)^n (c + dx^3)^3 dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x)^n*(c + d*x^3)^3,x]

[Out] Defer[IntegrateAlgebraic] [(a + b*x)^n*(c + d*x^3)^3, x]

fricas [B] time = 0.84, size = 2313, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="fricas")

[Out] (a*b^9*c^3*n^9 + 54*a*b^9*c^3*n^8 + 1266*a*b^9*c^3*n^7 + 362880*a*b^9*c^3 - 2721600*a^4*b^6*c^2*d + 1555200*a^7*b^3*c*d^2 - 362880*a^10*d^3 + (b^10*d^3*n^9 + 45*b^10*d^3*n^8 + 870*b^10*d^3*n^7 + 9450*b^10*d^3*n^6 + 63273*b^10*d^3*n^5 + 269325*b^10*d^3*n^4 + 723680*b^10*d^3*n^3 + 1172700*b^10*d^3*n^2 + 1026576*b^10*d^3*n + 362880*b^10*d^3)*x^10 + (a*b^9*d^3*n^9 + 36*a*b^9*d^3*n^8 + 546*a*b^9*d^3*n^7 + 4536*a*b^9*d^3*n^6 + 22449*a*b^9*d^3*n^5 + 67284*a*b^9*d^3*n^4 + 118124*a*b^9*d^3*n^3 + 109584*a*b^9*d^3*n^2 + 40320*a*b^9*d^3*n)*x^9 - 9*(a^2*b^8*d^3*n^8 + 28*a^2*b^8*d^3*n^7 + 322*a^2*b^8*d^3*n^6 + 1960*a^2*b^8*d^3*n^5 + 6769*a^2*b^8*d^3*n^4 + 13132*a^2*b^8*d^3*n^3 + 13068*a^2*b^8*d^3*n^2 + 5040*a^2*b^8*d^3*n)*x^8 + 3*(b^10*c*d^2*n^9 + 48*b^10*c*d^2*n^8 + 518400*b^10*c*d^2 + 24*(41*b^10*c*d^2 + a^3*b^7*d^3)*n^7 + 6*(1877*b^10*c*d^2 + 84*a^3*b^7*d^3)*n^6 + 21*(3759*b^10*c*d^2 + 200*a^3*b^7*d^3)*n^5 + 42*(8321*b^10*c*d^2 + 420*a^3*b^7*d^3)*n^4 + 4*(242639*b^10*c*d^2 + 9744*a^3*b^7*d^3)*n^3 + 72*(22439*b^10*c*d^2 + 588*a^3*b^7*d^3)*n^2 + 1440*(1003*b^10*c*d^2 + 12*a^3*b^7*d^3)*n)*x^7 + 18*(938*a*b^9*c^3 - a^4*b^6*c^2*d)*n^6 + 3*(a*b^9*c*d^2*n^9 + 42*a*b^9*c*d^2*n^8 + 732*a*b^9*c*d^2*n^7 + 6*(1145*a*b^9*c*d^2 - 28*a^4*b^6*d^3)*n^6 + 9*(4191*a*b^9*c*d^2 - 280*a^4*b^6*d^3)*n^5 + 24*(5132*a*b^9*c*d^2 - 595*a^4*b^6*d^3)*n^4 + 4*(57887*a*b^9*c*d^2 - 9450*a^4*b^6*d^3)*n^3 + 48*(4715*a*b^9*c*d^2 - 959*a^4*b^6*d^3)*n^2 + 2880*(30*a*b^9*c*d^2 - 7*a^4*b^6*d^3)*n)*x^6 + 3*(46963*a*b^9*c^3 - 270*a^4*b^6*c^2*d)*n^5 - 18*(a^2*b^8*c*d^2*n^8 + 37*a^2*b^8*c*d^2*n^7 + 547*a^2*b^8*c*d^2*n^6 + (4135*a^2*b^8*c*d^2 - 168*a^5*b^5*d^3)*n^5 + 4*(4261*a^2*b^8*c*d^2 - 420*a^5*b^5*d^3)*n^4 + 4*(9487*a^2*b^8*c*d^2 - 1470*a^5*b^5*d^3)*n^3 + 48*(871*a^2*b^8*c*d^2 - 175*a^5*b^5*d^3)*n^2 + 576*(30*a^2*b^8*c*d^2 - 7*a^5*b^5*d^3)*n)*x^5 + 18*(42287*a*b^9*c^3 - 835*a^4*b^6*c^2*d)*n^4 + 3*(b^10*c^2*d*n^9 + 51*b^10*c^2*d*n^8 + 907200*b^10*c^2*d + 6*(186*b^10*c^2*d + 5*a^3*b^7*c*d^2)*n^7 + 6*(2281*b^10*c^2*d + 165*a^3*b^7*c*d^2)*n^6 + 3*(34343*b^10*c^2*d + 4150*a^3*b^7*c*d^2)*n^5 + 3*(163313*b^10*c^2*d + 24750*a^3*b^7*c*d^2 - 1680*a^6*b^4*d^3)*n^4 + 2*(728587*b^10*c^2*d + 107160*a^3*b^7*c*d^2 - 15120*a^6*b^4*d^3)*n^3 + 36*(71689*b^10*c^2*d + 7810*a^3*b^7*c*d^2 - 1540*a^6*b^4*d^3)*n^2 + 360*(6751*b^10*c^2*d + 360*a^3*b^7*c*d^2 - 84*a^6*b^4*d^3)*n)*x^4 + 2*(1327882*a*b^9*c^3 - 73575*a^4*b^6*c^2*d + 1080*a^7*b^3*c*d^2)*n^3 + 3*(a*b^9*c^2*d*n^9 + 48*a*b^9*c^2*d*n^8 + 972*a*b^9*c^2*d*n^7 + 30*(359*a*b^9*c^2*d - 4*a^4*b^6*c*d^2)*n^6 + 3*(23573*a*b^9*c^2*d - 1200*a^4*b^6*c*d^2)*n^5 + 6*(46297*a*b^9*c^2*d - 6500*a^4*b^6*c*d^2)*n^4 + 4*(155957*a*b^9*c^2*d - 45000*a^4*b^6*c*d^2 + 5040*a^7*b^3*d^3)*n^3 + 120*(5911*a*b^9*c^2*d - 2644*a^4*b^6*c*d^2 + 504*a^7*b^3*d^3)*n^2 + 2880*(105*a*b^9*c^2*d - 60*a^4*b^6*c*d^2 + 14*a^7*b^3*d^3)*n)*x^3 + 72*(79913*a*b^9*c^3 - 11131*a^4*b^6*c^2*d + 810*a^7*b^3*c*d^2)*n^2 - 9*(a^2*b^8*c^2*d*n^8 + 46*a^2*b^8*c^2*d*n^7 + 880*a^2*b^8*c^2*d*n^6 + 10*(901*a^2*b^8*c^2*d - 12*a^5*b^5*c*d^2)*n^5 + (52699*a^2*b^8*c^2*d - 3360*a^5*b^5*c*d^2)*n^4 + 8*(21548*a^2*b^8*c^2*d - 4035*a^5*b^5*c*d^2)*n^3 + 60*(4651*a^2*b^8*c^2*d - 1924*a^5*b^5*c*d^2 + 336*a^8*b^2*d^3)*n^2 + 1440*(105*a^2*b^8*c^2*d - 60*a^5*b

$$\begin{aligned} &^5*c*d^2 + 14*a^8*b^2*d^3)*n)*x^2 + 360*(19444*a*b^9*c^3 - 6393*a^4*b^6*c^2 \\ &*d + 1452*a^7*b^3*c*d^2)*n + (b^{10}*c^3*n^9 + 54*b^{10}*c^3*n^8 + 3628800*b^{10} \\ &*c^3 + 6*(211*b^{10}*c^3 + 3*a^3*b^7*c^2*d)*n^7 + 18*(938*b^{10}*c^3 + 45*a^3*b \\ &^7*c^2*d)*n^6 + 3*(46963*b^{10}*c^3 + 5010*a^3*b^7*c^2*d)*n^5 + 18*(42287*b^{10} \\ &0*c^3 + 8175*a^3*b^7*c^2*d - 120*a^6*b^4*c*d^2)*n^4 + 4*(663941*b^{10}*c^3 + \\ &200358*a^3*b^7*c^2*d - 14580*a^6*b^4*c*d^2)*n^3 + 72*(79913*b^{10}*c^3 + 3196 \\ &5*a^3*b^7*c^2*d - 7260*a^6*b^4*c*d^2)*n^2 + 1440*(4861*b^{10}*c^3 + 1890*a^3* \\ &b^7*c^2*d - 1080*a^6*b^4*c*d^2 + 252*a^9*b*d^3)*n)*x*(b*x + a)^n/(b^{10}*n^{10} \\ &+ 55*b^{10}*n^9 + 1320*b^{10}*n^8 + 18150*b^{10}*n^7 + 157773*b^{10}*n^6 + 902055 \\ &*b^{10}*n^5 + 3416930*b^{10}*n^4 + 8409500*b^{10}*n^3 + 12753576*b^{10}*n^2 + 10628 \\ &640*b^{10}*n + 3628800*b^{10}) \end{aligned}$$

giac [B] time = 0.67, size = 3874, normalized size = 11.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="giac")

[Out] ((b*x + a)^n*b^{10}*d^3*n^9*x^{10} + (b*x + a)^n*a*b^9*d^3*n^9*x^9 + 45*(b*x + a)^n*b^{10}*d^3*n^8*x^{10} + 36*(b*x + a)^n*a*b^9*d^3*n^8*x^9 + 870*(b*x + a)^n*b^{10}*d^3*n^7*x^{10} + 3*(b*x + a)^n*b^{10}*c*d^2*n^9*x^7 - 9*(b*x + a)^n*a^2*b^8*d^3*n^8*x^8 + 546*(b*x + a)^n*a*b^9*d^3*n^7*x^9 + 9450*(b*x + a)^n*b^{10}*d^3*n^6*x^{10} + 3*(b*x + a)^n*a*b^9*c*d^2*n^9*x^6 + 144*(b*x + a)^n*b^{10}*c*d^2*n^8*x^7 - 252*(b*x + a)^n*a^2*b^8*d^3*n^7*x^8 + 4536*(b*x + a)^n*a*b^9*d^3*n^6*x^9 + 63273*(b*x + a)^n*b^{10}*d^3*n^5*x^{10} + 126*(b*x + a)^n*a*b^9*c*d^2*n^8*x^6 + 2952*(b*x + a)^n*b^{10}*c*d^2*n^7*x^7 + 72*(b*x + a)^n*a^3*b^7*d^3*n^7*x^7 - 2898*(b*x + a)^n*a^2*b^8*d^3*n^6*x^8 + 22449*(b*x + a)^n*a*b^9*d^3*n^5*x^9 + 269325*(b*x + a)^n*b^{10}*d^3*n^4*x^{10} + 3*(b*x + a)^n*b^{10}*c^2*d*n^9*x^4 - 18*(b*x + a)^n*a^2*b^8*c*d^2*n^8*x^5 + 2196*(b*x + a)^n*a*b^9*c*d^2*n^7*x^6 + 33786*(b*x + a)^n*b^{10}*c*d^2*n^6*x^7 + 1512*(b*x + a)^n*a^3*b^7*d^3*n^6*x^7 - 17640*(b*x + a)^n*a^2*b^8*d^3*n^5*x^8 + 67284*(b*x + a)^n*a*b^9*d^3*n^4*x^9 + 723680*(b*x + a)^n*b^{10}*d^3*n^3*x^{10} + 3*(b*x + a)^n*a*b^9*c^2*d*n^9*x^3 + 153*(b*x + a)^n*b^{10}*c^2*d*n^8*x^4 - 666*(b*x + a)^n*a^2*b^8*c*d^2*n^7*x^5 + 20610*(b*x + a)^n*a*b^9*c*d^2*n^6*x^6 - 504*(b*x + a)^n*a^4*b^6*d^3*n^6*x^6 + 236817*(b*x + a)^n*b^{10}*c*d^2*n^5*x^7 + 12600*(b*x + a)^n*a^3*b^7*d^3*n^5*x^7 - 60921*(b*x + a)^n*a^2*b^8*d^3*n^4*x^8 + 118124*(b*x + a)^n*a*b^9*d^3*n^3*x^9 + 1172700*(b*x + a)^n*b^{10}*d^3*n^2*x^{10} + 144*(b*x + a)^n*a*b^9*c^2*d*n^8*x^3 + 3348*(b*x + a)^n*b^{10}*c^2*d*n^7*x^4 + 90*(b*x + a)^n*a^3*b^7*c*d^2*n^7*x^4 - 9846*(b*x + a)^n*a^2*b^8*c*d^2*n^6*x^5 + 113157*(b*x + a)^n*a*b^9*c*d^2*n^5*x^6 - 7560*(b*x + a)^n*a^4*b^6*d^3*n^5*x^6 + 1048446*(b*x + a)^n*b^{10}*c*d^2*n^4*x^7 + 52920*(b*x + a)^n*a^3*b^7*d^3*n^4*x^7 - 118188*(b*x + a)^n*a^2*b^8*d^3*n^3*x^8 + 109584*(b*x + a)^n*a*b^9*d^3*n^2*x^9 + 1026576*(b*x + a)^n*b^{10}*d^3*n*x^{10} + (b*x + a)^n*b^{10}*c^3*n^9*x - 9*(b*x + a)^n*a^2*b^8*c^2*d*n^8*x^2 + 2916*(b*x + a)^n*a*b^9*c^2*d*n^7*x^3 + 41058*(b*x + a)^n*b^{10}*c^2*d*n^6*x^4 + 2970*(b*x + a)^n*a^3*b^7*c*d^2*n^6*x^4 - 74430*(b*x + a)^n*a^2*b^8*c*d^2*n^5*x^5 + 3024*(b*x + a)^n*a^5*b^5*d^3*n^5*x^5 + 369504*(b*x + a)^n*a*b^9*c*d^2*n^4*x^6 - 42840*(b*x + a)^n*a^4*b^6*d^3*n^4*x^6 + 2911668*(b*x + a)^n*b^{10}*c*d^2*n^3*x^7 + 116928*(b*x + a)^n*a^3*b^7*d^3*n^3*x^7 - 117612*(b*x + a)^n*a^2*b^8*d^3*n^2*x^8 + 40320*(b*x + a)^n*a*b^9*d^3*n*x^9 + 362880*(b*x + a)^n*b^{10}*d^3*x^{10} + (b*x + a)^n*a*b^9*c^3*n^9 + 54*(b*x + a)^n*b^{10}*c^3*n^8*x - 414*(b*x + a)^n*a^2*b^8*c^2*d*n^7*x^2 + 32310*(b*x + a)^n*a*b^9*c^2*d*n^6*x^3 - 360*(b*x + a)^n*a^4*b^6*c*d^2*n^6*x^3 + 309087*(b*x + a)^n*b^{10}*c^2*d*n^5*x^4 + 37350*(b*x + a)^n*a^3*b^7*c*d^2*n^5*x^4 - 306792*(b*x + a)^n*a^2*b^8*c*d^2*n^4*x^5 + 30240*(b*x + a)^n*a^5*b^5*d^3*n^4*x^5 + 694644*(b*x + a)^n*a*b^9*c*d^2*n^3*x^6 - 113400*(b*x + a)^n*a^4*b^6*d^3*n^3*x^6 + 4846824*(b*x + a)^n*b^{10}*c*d^2*n^2*x^7 + 127008*(b*x + a)^n*a^3*b^7*d^3*n^2*x^7 - 45360*(b*x + a)^n*a^2*b^8*d^3*n*x^8 + 54*(b*x + a)^n*a*b^9*c^3*n^8 + 1266*(b*x + a)^n*b^{10}*c^3*n^7*x + 18*(b*x + a)^n*a^3*b^7*c^2*d*n^7*x - 7920*(b*x + a)^n*a^2*

$$\begin{aligned}
& b^8 c^2 d^n n^6 x^2 + 212157 (b x + a)^n a b^9 c^2 d^n n^5 x^3 - 10800 (b x + a)^n a^4 b^6 c d^2 n^5 x^3 + 1469817 (b x + a)^n b^{10} c^2 d^n n^4 x^4 + 222750 \\
& (b x + a)^n a^3 b^7 c d^2 n^4 x^4 - 15120 (b x + a)^n a^6 b^4 d^3 n^4 x^4 - 683064 (b x + a)^n a^2 b^8 c d^2 n^3 x^5 + 105840 (b x + a)^n a^5 b^5 d^3 \\
& n^3 x^5 + 678960 (b x + a)^n a b^9 c d^2 n^2 x^6 - 138096 (b x + a)^n a^4 b^6 d^3 n^2 x^6 + 4332960 (b x + a)^n b^{10} c d^2 n x^7 + 51840 (b x + a)^n a^3 b^7 d^3 n x^7 \\
& + 1266 (b x + a)^n a b^9 c^3 n^7 + 16884 (b x + a)^n b^{10} c^3 n^6 x + 810 (b x + a)^n a^3 b^7 c^2 d^n n^6 x - 81090 (b x + a)^n a^2 b^8 c^2 d^n n^5 x^2 \\
& + 1080 (b x + a)^n a^5 b^5 c d^2 n^5 x^2 + 833346 (b x + a)^n a b^9 c^2 d^n n^4 x^3 - 117000 (b x + a)^n a^4 b^6 c d^2 n^4 x^3 + 4371522 \\
& (b x + a)^n b^{10} c^2 d^n n^3 x^4 + 642960 (b x + a)^n a^3 b^7 c d^2 n^3 x^4 - 90720 (b x + a)^n a^6 b^4 d^3 n^3 x^4 - 752544 (b x + a)^n a^2 b^8 c d^2 n^2 x^5 \\
& + 151200 (b x + a)^n a^5 b^5 d^3 n^2 x^5 + 259200 (b x + a)^n a b^9 c d^2 n x^6 - 60480 (b x + a)^n a^4 b^6 d^3 n x^6 + 1555200 (b x + a)^n b^{10} c d^2 n x^7 \\
& + 16884 (b x + a)^n a b^9 c^3 n^6 - 18 (b x + a)^n a^4 b^6 c^2 d^n n^6 + 140889 (b x + a)^n b^{10} c^3 n^5 x + 15030 (b x + a)^n a^3 b^7 c^2 d^n n^5 x \\
& - 474291 (b x + a)^n a^2 b^8 c^2 d^n n^4 x^2 + 30240 (b x + a)^n a^5 b^5 c d^2 n^4 x^2 + 1871484 (b x + a)^n a b^9 c^2 d^n n^3 x^3 - 540000 (b x + a)^n a^4 b^6 c d^2 n^3 x^3 \\
& + 60480 (b x + a)^n a^7 b^3 d^3 n^3 x^3 + 7742412 (b x + a)^n b^{10} c^2 d^n n^2 x^4 + 843480 (b x + a)^n a^3 b^7 c d^2 n^2 x^4 - 166320 (b x + a)^n a^6 b^4 d^3 n^2 x^4 - 311040 (b x + a)^n a^2 b^8 c d^2 n x^5 \\
& + 72576 (b x + a)^n a^5 b^5 d^3 n x^5 + 140889 (b x + a)^n a b^9 c^3 n^5 - 810 (b x + a)^n a^4 b^6 c^2 d^n n^5 + 761166 (b x + a)^n b^{10} c^3 n^4 x \\
& + 147150 (b x + a)^n a^3 b^7 c^2 d^n n^4 x - 2160 (b x + a)^n a^6 b^4 c d^2 n^4 x - 1551456 (b x + a)^n a^2 b^8 c^2 d^n n^3 x^2 + 290520 (b x + a)^n a^5 b^5 c d^2 n^3 x^2 \\
& + 2127960 (b x + a)^n a b^9 c^2 d^n n^2 x^3 - 951840 (b x + a)^n a^4 b^6 c d^2 n^2 x^3 + 181440 (b x + a)^n a^7 b^3 d^3 n^2 x^3 + 7291080 (b x + a)^n b^{10} c^2 d^n n x^4 \\
& + 388800 (b x + a)^n a^3 b^7 c d^2 n x^4 - 90720 (b x + a)^n a^6 b^4 d^3 n x^4 + 761166 (b x + a)^n a b^9 c^3 n^4 - 15030 (b x + a)^n a^4 b^6 c^2 d^n n^4 \\
& + 2655764 (b x + a)^n b^{10} c^3 n^3 x + 801432 (b x + a)^n a^3 b^7 c^2 d^n n^3 x - 58320 (b x + a)^n a^6 b^4 c d^2 n^3 x - 2511540 (b x + a)^n a^2 b^8 c^2 d^n n^2 x^2 \\
& + 1038960 (b x + a)^n a^5 b^5 c d^2 n^2 x^2 - 181440 (b x + a)^n a^8 b^2 d^3 n^2 x^2 + 907200 (b x + a)^n a b^9 c^2 d^n n x^3 - 518400 (b x + a)^n a^4 b^6 c d^2 n x^3 \\
& + 120960 (b x + a)^n a^7 b^3 d^3 n x^3 + 2721600 (b x + a)^n b^{10} c^2 d^n x^4 + 2655764 (b x + a)^n a b^9 c^3 n^3 - 147150 (b x + a)^n a^4 b^6 c^2 d^n n^3 \\
& + 2160 (b x + a)^n a^7 b^3 c d^2 n^3 + 5753736 (b x + a)^n b^{10} c^3 n^2 x + 2301480 (b x + a)^n a^3 b^7 c^2 d^n n^2 x - 522720 (b x + a)^n a^6 b^4 c d^2 n^2 x \\
& - 1360800 (b x + a)^n a^2 b^8 c^2 d^n n x^2 + 777600 (b x + a)^n a^5 b^5 c d^2 n x^2 - 181440 (b x + a)^n a^8 b^2 d^3 n x^2 + 5753736 (b x + a)^n a b^9 c^3 n^2 \\
& - 801432 (b x + a)^n a^4 b^6 c^2 d^n n^2 + 58320 (b x + a)^n a^7 b^3 c d^2 n^2 + 6999840 (b x + a)^n b^{10} c^3 n x + 2721600 (b x + a)^n a^3 b^7 c^2 d^n n x \\
& - 1555200 (b x + a)^n a^6 b^4 c d^2 n x + 362880 (b x + a)^n a^9 b^3 d^3 n x + 6999840 (b x + a)^n a b^9 c^3 n - 2301480 (b x + a)^n a^4 b^6 c^2 d^n n \\
& + 522720 (b x + a)^n a^7 b^3 c d^2 n + 3628800 (b x + a)^n b^{10} c^3 x + 3628800 (b x + a)^n a b^9 c^3 - 2721600 (b x + a)^n a^4 b^6 c^2 d^n + 1555200 \\
& (b x + a)^n a^7 b^3 c d^2 - 362880 (b x + a)^n a^{10} d^3) / (b^{10} n^{10} + 55 b^{10} n^9 + 1320 b^{10} n^8 + 18150 b^{10} n^7 + 157773 b^{10} n^6 + 902055 b^{10} n^5 \\
& + 3416930 b^{10} n^4 + 8409500 b^{10} n^3 + 12753576 b^{10} n^2 + 10628640 b^{10} n + 3628800 b^{10})
\end{aligned}$$

maple [B] time = 0.03, size = 2280, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b*x+a)^n*(d*x^3+c)^3,x)$

[Out] $-(b*x+a)^{(n+1)}*(-b^9*d^3*n^9*x^9-45*b^9*d^3*n^8*x^9+9*a*b^8*d^3*n^8*x^8-870*b^9*d^3*n^7*x^9+324*a*b^8*d^3*n^7*x^8-3*b^9*c*d^2*n^9*x^6-9450*b^9*d^3*n^6$

```

*x^9-72*a^2*b^7*d^3*n^7*x^7+4914*a*b^8*d^3*n^6*x^8-144*b^9*c*d^2*n^8*x^6-63
273*b^9*d^3*n^5*x^9-2016*a^2*b^7*d^3*n^6*x^7+18*a*b^8*c*d^2*n^8*x^5+40824*a
*b^8*d^3*n^5*x^8-2952*b^9*c*d^2*n^7*x^6-269325*b^9*d^3*n^4*x^9+504*a^3*b^6*
d^3*n^6*x^6-23184*a^2*b^7*d^3*n^5*x^7+756*a*b^8*c*d^2*n^7*x^5+202041*a*b^8*
d^3*n^4*x^8-3*b^9*c^2*d*n^9*x^3-33786*b^9*c*d^2*n^6*x^6-723680*b^9*d^3*n^3*
x^9+10584*a^3*b^6*d^3*n^5*x^6-90*a^2*b^7*c*d^2*n^7*x^4-141120*a^2*b^7*d^3*n
^4*x^7+13176*a*b^8*c*d^2*n^6*x^5+605556*a*b^8*d^3*n^3*x^8-153*b^9*c^2*d*n^8
*x^3-236817*b^9*c*d^2*n^5*x^6-1172700*b^9*d^3*n^2*x^9-3024*a^4*b^5*d^3*n^5*
x^5+88200*a^3*b^6*d^3*n^4*x^6-3330*a^2*b^7*c*d^2*n^6*x^4-487368*a^2*b^7*d^3
*n^3*x^7+9*a*b^8*c^2*d*n^8*x^2+123660*a*b^8*c*d^2*n^5*x^5+1063116*a*b^8*d^3
*n^2*x^8-3348*b^9*c^2*d*n^7*x^3-1048446*b^9*c*d^2*n^4*x^6-1026576*b^9*d^3*n
*x^9-45360*a^4*b^5*d^3*n^4*x^5+360*a^3*b^6*c*d^2*n^6*x^3+370440*a^3*b^6*d^3
*n^3*x^6-49230*a^2*b^7*c*d^2*n^5*x^4-945504*a^2*b^7*d^3*n^2*x^7+432*a*b^8*c
^2*d*n^7*x^2+678942*a*b^8*c*d^2*n^4*x^5+986256*a*b^8*d^3*n*x^8-b^9*c^3*n^9-
41058*b^9*c^2*d*n^6*x^3-2911668*b^9*c*d^2*n^3*x^6-362880*b^9*d^3*x^9+15120*
a^5*b^4*d^3*n^4*x^4-257040*a^4*b^5*d^3*n^3*x^5+11880*a^3*b^6*c*d^2*n^5*x^3+
818496*a^3*b^6*d^3*n^2*x^6-18*a^2*b^7*c^2*d*n^7*x-372150*a^2*b^7*c*d^2*n^4*
x^4-940896*a^2*b^7*d^3*n*x^7+8748*a*b^8*c^2*d*n^6*x^2+2217024*a*b^8*c*d^2*n
^3*x^5+362880*a*b^8*d^3*x^8-54*b^9*c^3*n^8-309087*b^9*c^2*d*n^5*x^3-4846824
*b^9*c*d^2*n^2*x^6+151200*a^5*b^4*d^3*n^3*x^4-1080*a^4*b^5*c*d^2*n^5*x^2-68
0400*a^4*b^5*d^3*n^2*x^5+149400*a^3*b^6*c*d^2*n^4*x^3+889056*a^3*b^6*d^3*n*
x^6-828*a^2*b^7*c^2*d*n^6*x-1533960*a^2*b^7*c*d^2*n^3*x^4-362880*a^2*b^7*d^
3*x^7+96930*a*b^8*c^2*d*n^5*x^2+4167864*a*b^8*c*d^2*n^2*x^5-1266*b^9*c^3*n^
7-1469817*b^9*c^2*d*n^4*x^3-4332960*b^9*c*d^2*n*x^6-60480*a^6*b^3*d^3*n^3*x
^3+529200*a^5*b^4*d^3*n^2*x^4-32400*a^4*b^5*c*d^2*n^4*x^2-828576*a^4*b^5*d^
3*n*x^5+18*a^3*b^6*c^2*d*n^6+891000*a^3*b^6*c*d^2*n^3*x^3+362880*a^3*b^6*d^
3*x^6-15840*a^2*b^7*c^2*d*n^5*x-3415320*a^2*b^7*c*d^2*n^2*x^4+636471*a*b^8*
c^2*d*n^4*x^2+4073760*a*b^8*c*d^2*n*x^5-16884*b^9*c^3*n^6-4371522*b^9*c^2*d
*n^3*x^3-1555200*b^9*c*d^2*x^6-362880*a^6*b^3*d^3*n^2*x^3+2160*a^5*b^4*c*d^
2*n^4*x+756000*a^5*b^4*d^3*n*x^4-351000*a^4*b^5*c*d^2*n^3*x^2-362880*a^4*b^
5*d^3*x^5+810*a^3*b^6*c^2*d*n^5+2571840*a^3*b^6*c*d^2*n^2*x^3-162180*a^2*b^
7*c^2*d*n^4*x-3762720*a^2*b^7*c*d^2*n*x^4+2500038*a*b^8*c^2*d*n^3*x^2+15552
00*a*b^8*c*d^2*x^5-140889*b^9*c^3*n^5-7742412*b^9*c^2*d*n^2*x^3+181440*a^7*
b^2*d^3*n^2*x^2-665280*a^6*b^3*d^3*n*x^3+60480*a^5*b^4*c*d^2*n^3*x+362880*a
^5*b^4*d^3*x^4-1620000*a^4*b^5*c*d^2*n^2*x^2+15030*a^3*b^6*c^2*d*n^4+337392
0*a^3*b^6*c*d^2*n*x^3-948582*a^2*b^7*c^2*d*n^3*x-1555200*a^2*b^7*c*d^2*x^4+
5614452*a*b^8*c^2*d*n^2*x^2-761166*b^9*c^3*n^4-7291080*b^9*c^2*d*n*x^3+5443
20*a^7*b^2*d^3*n*x^2-2160*a^6*b^3*c*d^2*n^3-362880*a^6*b^3*d^3*x^3+581040*a
^5*b^4*c*d^2*n^2*x-2855520*a^4*b^5*c*d^2*n*x^2+147150*a^3*b^6*c^2*d*n^3+155
5200*a^3*b^6*c*d^2*x^3-3102912*a^2*b^7*c^2*d*n^2*x+6383880*a*b^8*c^2*d*n*x^
2-2655764*b^9*c^3*n^3-2721600*b^9*c^2*d*x^3-362880*a^8*b*d^3*n*x+362880*a^7
*b^2*d^3*x^2-58320*a^6*b^3*c*d^2*n^2+2077920*a^5*b^4*c*d^2*n*x-1555200*a^4*
b^5*c*d^2*x^2+801432*a^3*b^6*c^2*d*n^2-5023080*a^2*b^7*c^2*d*n*x+2721600*a*
b^8*c^2*d*x^2-5753736*b^9*c^3*n^2-362880*a^8*b*d^3*x-522720*a^6*b^3*c*d^2*n
+1555200*a^5*b^4*c*d^2*x+2301480*a^3*b^6*c^2*d*n-2721600*a^2*b^7*c^2*d*x-69
99840*b^9*c^3*n+362880*a^9*d^3-1555200*a^6*b^3*c*d^2+2721600*a^3*b^6*c^2*d-
3628800*b^9*c^3)/b^10/(n^10+55*n^9+1320*n^8+18150*n^7+157773*n^6+902055*n^5
+3416930*n^4+8409500*n^3+12753576*n^2+10628640*n+3628800)

```

maxima [B] time = 0.78, size = 770, normalized size = 2.28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x+a)^n*(d*x^3+c)^3,x, algorithm="maxima")
```

```
[Out] (b*x + a)^(n + 1)*c^3/(b*(n + 1)) + 3*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (
n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^
4)*(b*x + a)^n*c^2*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + 3*((n^6 +
21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n
```

$$\begin{aligned} &^5 + 85n^4 + 225n^3 + 274n^2 + 120n) * a * b^6 * x^6 - 6 * (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n) * a^2 * b^5 * x^5 + 30 * (n^4 + 6n^3 + 11n^2 + 6n) * a^3 * b^4 * x^4 \\ &- 120 * (n^3 + 3n^2 + 2n) * a^4 * b^3 * x^3 + 360 * (n^2 + n) * a^5 * b^2 * x^2 - 720 * a^6 * b * n * x + 720 * a^7) * (b * x + a)^n * c * d^2 / ((n^7 + 28n^6 + 322n^5 + 1960n^4 + 6769n^3 + 13132n^2 + 13068n + 5040) * b^7) + ((n^9 + 45n^8 + 870n^7 + 9450n^6 + 63273n^5 + 269325n^4 + 723680n^3 + 1172700n^2 + 1026576n + 362880) * b^{10} * x^{10} + (n^9 + 36n^8 + 546n^7 + 4536n^6 + 22449n^5 + 67284n^4 + 118124n^3 + 109584n^2 + 40320n) * a * b^9 * x^9 - 9 * (n^8 + 28n^7 + 322n^6 + 1960n^5 + 6769n^4 + 13132n^3 + 13068n^2 + 5040n) * a^2 * b^8 * x^8 + 72 * (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n) * a^3 * b^7 * x^7 - 504 * (n^6 + 15n^5 + 85n^4 + 225n^3 + 274n^2 + 120n) * a^4 * b^6 * x^6 + 3024 * (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n) * a^5 * b^5 * x^5 - 15120 * (n^4 + 6n^3 + 11n^2 + 6n) * a^6 * b^4 * x^4 + 60480 * (n^3 + 3n^2 + 2n) * a^7 * b^3 * x^3 - 181440 * (n^2 + n) * a^8 * b^2 * x^2 + 362880 * a^9 * b * n * x - 362880 * a^{10}) * (b * x + a)^n * d^3 / ((n^{10} + 55n^9 + 1320n^8 + 18150n^7 + 157773n^6 + 902055n^5 + 3416930n^4 + 8409500n^3 + 12753576n^2 + 10628640n + 3628800) * b^{10}) \end{aligned}$$

mupad [B] time = 4.34, size = 2001, normalized size = 5.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((c + d * x^3)^3 * (a + b * x)^n, x)$

[Out]
$$\begin{aligned} &((a + b * x)^n * (3628800 * a * b^9 * c^3 - 362880 * a^{10} * d^3 - 2721600 * a^4 * b^6 * c^2 * d + 1555200 * a^7 * b^3 * c * d^2 + 5753736 * a * b^9 * c^3 * n^2 + 2655764 * a * b^9 * c^3 * n^3 + 761166 * a * b^9 * c^3 * n^4 + 140889 * a * b^9 * c^3 * n^5 + 16884 * a * b^9 * c^3 * n^6 + 1266 * a * b^9 * c^3 * n^7 + 54 * a * b^9 * c^3 * n^8 + a * b^9 * c^3 * n^9 + 6999840 * a * b^9 * c^3 * n - 2301480 * a^4 * b^6 * c^2 * d * n + 522720 * a^7 * b^3 * c * d^2 * n - 801432 * a^4 * b^6 * c^2 * d * n^2 + 58320 * a^7 * b^3 * c * d^2 * n^2 - 147150 * a^4 * b^6 * c^2 * d * n^3 + 2160 * a^7 * b^3 * c * d^2 * n^3 - 15030 * a^4 * b^6 * c^2 * d * n^4 - 810 * a^4 * b^6 * c^2 * d * n^5 - 18 * a^4 * b^6 * c^2 * d * n^6)) / (b^{10} * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) + (x * (a + b * x))^n * (3628800 * b^{10} * c^3 + 6999840 * b^{10} * c^3 * n + 5753736 * b^{10} * c^3 * n^2 + 2655764 * b^{10} * c^3 * n^3 + 761166 * b^{10} * c^3 * n^4 + 140889 * b^{10} * c^3 * n^5 + 16884 * b^{10} * c^3 * n^6 + 1266 * b^{10} * c^3 * n^7 + 54 * b^{10} * c^3 * n^8 + b^{10} * c^3 * n^9 + 362880 * a^9 * b * d^3 * n + 2721600 * a^3 * b^7 * c^2 * d * n - 1555200 * a^6 * b^4 * c * d^2 * n + 2301480 * a^3 * b^7 * c^2 * d * n^2 - 522720 * a^6 * b^4 * c * d^2 * n^2 + 801432 * a^3 * b^7 * c^2 * d * n^3 - 58320 * a^6 * b^4 * c * d^2 * n^3 + 147150 * a^3 * b^7 * c^2 * d * n^4 - 2160 * a^6 * b^4 * c * d^2 * n^4 + 15030 * a^3 * b^7 * c^2 * d * n^5 + 810 * a^3 * b^7 * c^2 * d * n^6 + 18 * a^3 * b^7 * c^2 * d * n^7)) / (b^{10} * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) + (d^3 * x^{10} * (a + b * x))^n * (1026576 * n + 1172700 * n^2 + 723680 * n^3 + 269325 * n^4 + 63273 * n^5 + 9450 * n^6 + 870 * n^7 + 45 * n^8 + n^9 + 362880)) / (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800) + (3 * d^2 * x^7 * (a + b * x))^n * (720 * b^3 * c + 27 * b^3 * c * n^2 + b^3 * c * n^3 + 24 * a^3 * d * n + 242 * b^3 * c * n) * (1764 * n + 1624 * n^2 + 735 * n^3 + 175 * n^4 + 21 * n^5 + n^6 + 720)) / (b^3 * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) + (3 * d * x^4 * (a + b * x))^n * (11 * n + 6 * n^2 + n^3 + 6) * (151200 * b^6 * c^2 - 5040 * a^6 * d^2 * n + 127860 * b^6 * c^2 * n + 44524 * b^6 * c^2 * n^2 + 8175 * b^6 * c^2 * n^3 + 835 * b^6 * c^2 * n^4 + 45 * b^6 * c^2 * n^5 + b^6 * c^2 * n^6 + 21600 * a^3 * b^3 * c * d * n + 7260 * a^3 * b^3 * c * d * n^2 + 810 * a^3 * b^3 * c * d * n^3 + 30 * a^3 * b^3 * c * d * n^4)) / (b^6 * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) + (a * d^3 * n * x^9 * (a + b * x))^n * (109584 * n + 118124 * n^2 + 67284 * n^3 + 22449 * n^4 + 4536 * n^5 + 546 * n^6 + 36 * n^7 + n^8 + 40320)) / (b * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) - (9 * a^2 * d^3 * n * x^8 * (a + b * x))^n * (13068 * n + 13132 * n^2 + 6769 * n^3 + 1960 * n^4 + 322 * n^5 + 28 * n^6 + n^7 + 5040)) / (b^2 * (10628640 * n + 12753576 * n^2 + 8409500 * n^3 + 3416930 * n^4 + 902055 * n^5 + 157773 * n^6 + 18150 * n^7 + 1320 * n^8 + 55 * n^9 + n^{10} + 3628800)) \end{aligned}$$

```

0*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n
^9 + n^10 + 3628800)) + (3*a*d*n*x^3*(a + b*x)^n*(3*n + n^2 + 2)*(20160*a^6
*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524*b^6*c^2*n^2 + 8175*b^6*c^2
*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^2*n^6 - 86400*a^3*b^3*c*d -
29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 120*a^3*b^3*c*d*n^3))/(b^7*(1
0628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*
n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (9*a^2*d*n*x^2*(n
+ 1)*(a + b*x)^n*(20160*a^6*d^2 + 151200*b^6*c^2 + 127860*b^6*c^2*n + 44524
*b^6*c^2*n^2 + 8175*b^6*c^2*n^3 + 835*b^6*c^2*n^4 + 45*b^6*c^2*n^5 + b^6*c^
2*n^6 - 86400*a^3*b^3*c*d - 29040*a^3*b^3*c*d*n - 3240*a^3*b^3*c*d*n^2 - 12
0*a^3*b^3*c*d*n^3))/(b^8*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930
*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 362
8800)) + (3*a*d^2*n*x^6*(a + b*x)^n*(720*b^3*c - 168*a^3*d + 27*b^3*c*n^2 +
b^3*c*n^3 + 242*b^3*c*n)*(274*n + 225*n^2 + 85*n^3 + 15*n^4 + n^5 + 120))/
(b^4*(10628640*n + 12753576*n^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 +
157773*n^6 + 18150*n^7 + 1320*n^8 + 55*n^9 + n^10 + 3628800)) - (18*a^2*d^2
*n*x^5*(a + b*x)^n*(50*n + 35*n^2 + 10*n^3 + n^4 + 24)*(720*b^3*c - 168*a^3
*d + 27*b^3*c*n^2 + b^3*c*n^3 + 242*b^3*c*n))/(b^5*(10628640*n + 12753576*n
^2 + 8409500*n^3 + 3416930*n^4 + 902055*n^5 + 157773*n^6 + 18150*n^7 + 1320
*n^8 + 55*n^9 + n^10 + 3628800))

```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x+a)**n*(d*x**3+c)**3,x)

[Out] Timed out

$$3.59 \quad \int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=16

$$2 \tan^{-1} \left(\frac{x+1}{\sqrt{x^3+1}} \right)$$

Rubi [A] time = 0.08, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2146, 203}

$$2 \tan^{-1} \left(\frac{x+1}{\sqrt{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]

[Out] 2*ArcTan[(1 + x)/Sqrt[1 + x^3]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2146

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]

Rubi steps

$$\begin{aligned} \int \frac{2-2x-x^2}{(2+x^2)\sqrt{1+x^3}} dx &= 2 \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \frac{1+x}{\sqrt{1+x^3}} \right) \\ &= 2 \tan^{-1} \left(\frac{1+x}{\sqrt{1+x^3}} \right) \end{aligned}$$

Mathematica [C] time = 0.87, size = 296, normalized size = 18.50

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1} \left(\frac{\sqrt{3}(1+\sqrt[3]{-1})\left(\sqrt[3]{-1}-x\right)F\left(\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\right)}{(-1)^{2/3}x+1} - \frac{3i(\sqrt{2}-i)\Gamma\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\right)}{(-1)^{5/6}+\sqrt{2}} + \frac{3(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6})\Gamma\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right)\right)}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}} \right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]

[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2]))*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*Sqrt[3] + Sqrt[6]))*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi

n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))], (-1)^(1/3)]/(5*I + 2*Sqrt[2 + Sqrt[3] + (2*I)*Sqrt[6]])/(3*Sqrt[1 + x^3])

IntegrateAlgebraic [A] time = 1.03, size = 23, normalized size = 1.44

$$2 \tan^{-1} \left(\frac{\sqrt{x^3 + 1}}{x^2 - x + 1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[1 + x^3]),x]

[Out] 2*ArcTan[Sqrt[1 + x^3]/(1 - x + x^2)]

fricas [A] time = 0.81, size = 19, normalized size = 1.19

$$- \arctan \left(\frac{x^2 - 2x}{2\sqrt{x^3 + 1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] -arctan(1/2*(x^2 - 2*x)/sqrt(x^3 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)

maple [C] time = 0.07, size = 1640, normalized size = 102.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x)

[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3*I*2^(1/2)*(1/(3/2-1/2*I*3^(1/2)))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2)))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I*3^(1/2))/(-1-I*2^(1/2))),((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(3/2-1/2*I*3^(1/2)))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2)))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(-3/2+1/2*I*3^(1/2)))*x-1/2/(-3/2+1/2*I*3^(1/2))+1/2*I/(-3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3+1)^(1/2)/(-1-I*2^(1/2))*EllipticPi(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2),(-3/2+1/2*I

$$\begin{aligned} & *3^{(1/2)} / (-1 - I * 2^{(1/2)}), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} \\ & + I * (1 / (3/2 - 1/2 * I * 3^{(1/2)}) * x + 1 / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * (1 / (-3/2 - 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 - 1/2 * I * 3^{(1/2)}) - 1/2 * I / (-3/2 - 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} * \\ & (1 / (-3/2 + 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 + 1/2 * I * 3^{(1/2)}) + 1/2 * I / (-3/2 + 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} / (x^3 + 1)^{(1/2)} / (-1 - I * 2^{(1/2)}) * \text{EllipticPi}((x + 1) / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)}, \\ & (-3/2 + 1/2 * I * 3^{(1/2)}) / (-1 - I * 2^{(1/2)}), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * 3^{(1/2)} + 3 * I * 2^{(1/2)} * (1 / (3/2 - 1/2 * I * 3^{(1/2)}) * x + 1 / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * (1 / (-3/2 - 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 - 1/2 * I * 3^{(1/2)}) - 1/2 * I / (-3/2 - 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} * (1 / (-3/2 + 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 + 1/2 * I * 3^{(1/2)}) + 1/2 * I / (-3/2 + 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} / (x^3 + 1)^{(1/2)} / (I * 2^{(1/2)} - 1) * \text{EllipticPi}((x + 1) / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)}, \\ & (-3/2 + 1/2 * I * 3^{(1/2)}) / (I * 2^{(1/2)} - 1), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} + 2^{(1/2)} * (1 / (3/2 - 1/2 * I * 3^{(1/2)}) * x + 1 / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * (1 / (-3/2 - 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 - 1/2 * I * 3^{(1/2)}) - 1/2 * I / (-3/2 - 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} * (1 / (-3/2 + 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 + 1/2 * I * 3^{(1/2)}) + 1/2 * I / (-3/2 + 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} / (x^3 + 1)^{(1/2)} / (I * 2^{(1/2)} - 1) * \text{EllipticPi}((x + 1) / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)}, \\ & (-3/2 + 1/2 * I * 3^{(1/2)}) / (I * 2^{(1/2)} - 1), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * 3^{(1/2)} - 3 * (1 / (3/2 - 1/2 * I * 3^{(1/2)}) * x + 1 / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * (1 / (-3/2 - 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 - 1/2 * I * 3^{(1/2)}) - 1/2 * I / (-3/2 - 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} * (1 / (-3/2 + 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 + 1/2 * I * 3^{(1/2)}) + 1/2 * I / (-3/2 + 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} / (x^3 + 1)^{(1/2)} / (I * 2^{(1/2)} - 1) * \text{EllipticPi}((x + 1) / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)}, \\ & (-3/2 + 1/2 * I * 3^{(1/2)}) / (I * 2^{(1/2)} - 1), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} + I * (1 / (3/2 - 1/2 * I * 3^{(1/2)}) * x + 1 / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * (1 / (-3/2 - 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 - 1/2 * I * 3^{(1/2)}) - 1/2 * I / (-3/2 - 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} * (1 / (-3/2 + 1/2 * I * 3^{(1/2)}) * x - 1/2 / (-3/2 + 1/2 * I * 3^{(1/2)}) + 1/2 * I / (-3/2 + 1/2 * I * 3^{(1/2)}) * 3^{(1/2)})^{(1/2)} / (x^3 + 1)^{(1/2)} / (I * 2^{(1/2)} - 1) * \text{EllipticPi}((x + 1) / (3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)}, \\ & (-3/2 + 1/2 * I * 3^{(1/2)}) / (I * 2^{(1/2)} - 1), ((-3/2 + 1/2 * I * 3^{(1/2)}) / (-3/2 - 1/2 * I * 3^{(1/2)}))^{(1/2)} * 3^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(x^2+2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(x^2 + 2)), x)

mupad [B] time = 0.20, size = 273, normalized size = 17.06

$$\frac{(3 + \sqrt{3}i) \sqrt{\frac{x - \frac{1}{2} + \frac{\sqrt{3}i}{2}}{-\frac{3}{2} + \frac{\sqrt{3}i}{2}}} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3}i}{2}}} \sqrt{\frac{\frac{1}{2} - x + \frac{\sqrt{3}i}{2}}{\frac{3}{2} + \frac{\sqrt{3}i}{2}}} \left(-F\left(\text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3}i}{2}}}\right), \frac{\frac{3}{2} + \frac{\sqrt{3}i}{2}}{-\frac{3}{2} + \frac{\sqrt{3}i}{2}}\right) + \Pi\left(\frac{\frac{3}{2} + \frac{\sqrt{3}i}{2}}{1 + \sqrt{2}i}; \text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3}i}{2}}}\right), \frac{\frac{3}{2} + \frac{\sqrt{3}i}{2}}{-\frac{3}{2} + \frac{\sqrt{3}i}{2}}\right) + \Pi\left(-\frac{\frac{3}{2} + \frac{\sqrt{3}i}{2}}{-1 + \sqrt{2}i}; \text{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3}i}{2}}}\right), \frac{\frac{3}{2} + \frac{\sqrt{3}i}{2}}{-\frac{3}{2} + \frac{\sqrt{3}i}{2}}\right) \right)}{\sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) - 1} x - \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)\left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(x^3 + 1)^(1/2)),x)

[Out] ((3^(1/2)*1i + 3)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2) * ((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2) * (((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2) * (ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2) + ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))))/(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} dx - \int \frac{x^2}{x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} dx - \int \left(-\frac{2}{x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-2*x+2)/(x**2+2)/(x**3+1)**(1/2), x)

[Out] -Integral(2*x/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(x**2/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x) - Integral(-2/(x**2*sqrt(x**3 + 1) + 2*sqrt(x**3 + 1)), x)

$$3.60 \quad \int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}} dx$$

Optimal. Leaf size=20

$$-2 \tan^{-1} \left(\frac{1-x}{\sqrt{1-x^3}} \right)$$

Rubi [A] time = 0.09, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2146, 203}

$$-2 \tan^{-1} \left(\frac{1-x}{\sqrt{1-x^3}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]

[Out] -2*ArcTan[(1 - x)/Sqrt[1 - x^3]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2146

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]

Rubi steps

$$\begin{aligned} \int \frac{2+2x-x^2}{(2+x^2)\sqrt{1-x^3}} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \frac{1-x}{\sqrt{1-x^3}} \right) \right) \\ &= -2 \tan^{-1} \left(\frac{1-x}{\sqrt{1-x^3}} \right) \end{aligned}$$

Mathematica [C] time = 0.73, size = 280, normalized size = 14.00

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}\sqrt{x^2+x+1} \left(\frac{\sqrt{3}(1+\sqrt[3]{-1})(x+\sqrt[3]{-1})\text{F}\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{2/3}x-1} + \frac{6(1+i\sqrt{2})\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{i+2\sqrt{2}-\sqrt{3}} + \frac{3(1-i\sqrt{2})\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\middle|\sqrt[3]{-1}\right)}{(-1)^{5/6}-\sqrt{2}} \right)}{3\sqrt{1-x^3}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]

[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))])*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(-I + 2*Sqrt[2] + Sqrt[3]))

)x)/(1 + (-1)^(1/3))]], (-1)^(1/3)))/((-1)^(5/6) - Sqrt[2]))/(3*Sqrt[1 - x^3])

IntegrateAlgebraic [A] time = 1.03, size = 23, normalized size = 1.15

$$-2 \tan^{-1} \left(\frac{\sqrt{1-x^3}}{x^2+x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[1 - x^3]),x]

[Out] -2*ArcTan[Sqrt[1 - x^3]/(1 + x + x^2)]

fricas [A] time = 0.83, size = 28, normalized size = 1.40

$$-\arctan \left(\frac{\sqrt{-x^3+1}(x^2+2x)}{2(x^3-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] -arctan(1/2*sqrt(-x^3 + 1)*(x^2 + 2*x)/(x^3 - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

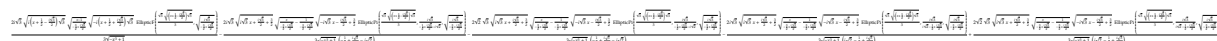
$$\int -\frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)

maple [C] time = 0.06, size = 732, normalized size = 36.60



Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x)

[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(-1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))-2/3*I*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)-1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+2/3*2^(1/2)*3

$$\begin{aligned} & \sqrt[3]{1/2} * (I * 3^{1/2} * x + 1/2 * I * 3^{1/2} + 3/2)^{1/2} * (1 / (-3/2 + 1/2 * I * 3^{1/2})) * x - 1 / (-3/2 + 1/2 * I * 3^{1/2}) \\ & \sqrt[3]{1/2} * (-I * 3^{1/2} * x - 1/2 * I * 3^{1/2} + 3/2)^{1/2} / (-x^3 + 1)^{1/2} \\ & / (I * 2^{1/2} - 1/2 + 1/2 * I * 3^{1/2}) * \text{EllipticPi}(1/3 * 3^{1/2} * (I * (x + 1/2 - 1/2 * I * 3^{1/2}) * 3^{1/2}))^{1/2}, \\ & I * 3^{1/2} / (I * 2^{1/2} - 1/2 + 1/2 * I * 3^{1/2}), (I * 3^{1/2} / (-3/2 + 1/2 * I * 3^{1/2}))^{1/2} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(x^2+2)/(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(x^2 + 2)), x)

mupad [B] time = 2.83, size = 292, normalized size = 14.60

$$\frac{(3 + \sqrt{3}i) \sqrt{x^3 - 1} \sqrt{\frac{1 - \sqrt{3}i}{2 + \sqrt{3}i}} \sqrt{\frac{1 + \sqrt{3}i}{2 + \sqrt{3}i}} \sqrt{\frac{x-1}{2 + \sqrt{3}i}} \left(-F\left(\arcsin\left(\sqrt{\frac{x-1}{2 + \sqrt{3}i}}\right) \middle| \frac{3 + \sqrt{3}i}{-3 + \sqrt{3}i}\right) + \Pi\left(\frac{3 + \sqrt{3}i}{2 + \sqrt{3}i}; \arcsin\left(\sqrt{\frac{x-1}{2 + \sqrt{3}i}}\right) \middle| \frac{3 + \sqrt{3}i}{-3 + \sqrt{3}i}\right) + \Pi\left(\frac{3 + \sqrt{3}i}{-1 + \sqrt{2}i}; \arcsin\left(\sqrt{\frac{x-1}{2 + \sqrt{3}i}}\right) \middle| \frac{3 + \sqrt{3}i}{-3 + \sqrt{3}i}\right) \right)}{\sqrt{1 - x^3} \sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) - 1} x + \left(-\frac{1}{2} + \frac{\sqrt{3}i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3}i}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - x^2 + 2)/((x^2 + 2)*(1 - x^3)^(1/2)),x)

[Out]
$$\begin{aligned} & -((3^{1/2} * 1i + 3) * (x^3 - 1)^{1/2} * (-x - (3^{1/2} * 1i) / 2 + 1/2) / ((3^{1/2} * 1i) / 2 - 3/2))^{1/2} * ((x + (3^{1/2} * 1i) / 2 + 1/2) / ((3^{1/2} * 1i) / 2 + 3/2))^{1/2} \\ & * (-x - 1) / ((3^{1/2} * 1i) / 2 + 3/2)^{1/2} * \text{ellipticPi}(((3^{1/2} * 1i) / 2 + 3/2) / (2^{1/2} * 1i + 1), \arcsin((-x - 1) / ((3^{1/2} * 1i) / 2 + 3/2))^{1/2}), \\ & -((3^{1/2} * 1i) / 2 + 3/2) / ((3^{1/2} * 1i) / 2 - 3/2)) - \text{ellipticF}(\arcsin((-x - 1) / ((3^{1/2} * 1i) / 2 + 3/2))^{1/2}), \\ & -((3^{1/2} * 1i) / 2 + 3/2) / ((3^{1/2} * 1i) / 2 - 3/2)) + \text{ellipticPi}(-((3^{1/2} * 1i) / 2 + 3/2) / (2^{1/2} * 1i - 1), \arcsin((-x - 1) / ((3^{1/2} * 1i) / 2 + 3/2))^{1/2}), \\ & -((3^{1/2} * 1i) / 2 + 3/2) / ((3^{1/2} * 1i) / 2 - 3/2))) / ((1 - x^3)^{1/2} * ((3^{1/2} * 1i) / 2 - 1/2) * ((3^{1/2} * 1i) / 2 + 1/2) - x * ((3^{1/2} * 1i) / 2 - 1/2) * ((3^{1/2} * 1i) / 2 + 1/2) + 1) + x^3)^{1/2} \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \left(-\frac{2x}{x^2 \sqrt{1 - x^3} + 2 \sqrt{1 - x^3}} \right) dx - \int \frac{x^2}{x^2 \sqrt{1 - x^3} + 2 \sqrt{1 - x^3}} dx - \int \left(-\frac{2}{x^2 \sqrt{1 - x^3} + 2 \sqrt{1 - x^3}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2*x+2)/(x**2+2)/(-x**3+1)**(1/2),x)

[Out]
$$\begin{aligned} & -\text{Integral}(-2*x/(x**2*\text{sqrt}(1 - x**3) + 2*\text{sqrt}(1 - x**3)), x) - \text{Integral}(x**2 / \\ & / (x**2*\text{sqrt}(1 - x**3) + 2*\text{sqrt}(1 - x**3)), x) - \text{Integral}(-2/(x**2*\text{sqrt}(1 - \\ & x**3) + 2*\text{sqrt}(1 - x**3)), x) \end{aligned}$$

$$3.61 \quad \int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=18

$$-2 \tanh^{-1} \left(\frac{1-x}{\sqrt{x^3-1}} \right)$$

Rubi [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2146, 206}

$$-2 \tanh^{-1} \left(\frac{1-x}{\sqrt{x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]

[Out] -2*ArcTanh[(1 - x)/Sqrt[-1 + x^3]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2146

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]

Rubi steps

$$\begin{aligned} \int \frac{2+2x-x^2}{(2+x^2)\sqrt{-1+x^3}} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{1-x}{\sqrt{-1+x^3}} \right) \right) \\ &= -2 \tanh^{-1} \left(\frac{1-x}{\sqrt{-1+x^3}} \right) \end{aligned}$$

Mathematica [C] time = 0.23, size = 278, normalized size = 15.44

$$\frac{2\sqrt{\frac{1-x}{1+\sqrt[3]{-1}}}}{\sqrt{x^2+x+1}} \left(\frac{\sqrt{3}(1+\sqrt[3]{-1})(x+\sqrt[3]{-1}) \text{F}\left(\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\right)\sqrt[3]{-1}}{(-1)^{2/3}x-1} + \frac{6(1+i\sqrt{2})\Pi\left(\frac{2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\right)\sqrt[3]{-1}}{i+2\sqrt{2}-\sqrt{3}} + \frac{3(1-i\sqrt{2})\Pi\left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}};\sin^{-1}\left(\sqrt{\frac{1-(-1)^{2/3}x}{1+\sqrt[3]{-1}}}\right)\right)\sqrt[3]{-1}}{(-1)^{5/6}-\sqrt{2}} \right) \frac{1}{3\sqrt{x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 2*x - x^2)/((2 + x^2)*Sqrt[-1 + x^3]),x]

[Out] (2*Sqrt[(1 - x)/(1 + (-1)^(1/3))])*Sqrt[1 + x + x^2]*((Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(-1 + (-1)^(2/3)*x) + (6*(1 + I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(I + 2*Sqrt[2] - Sqrt[3]) + (3*(1 - I*Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(-I + 2*Sqrt[2] + Sqrt[3]))/3


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pticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))
-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(-I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(-I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))
-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))-I*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))
-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)-3*I*2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))
)*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+2^(1/2)*(1/(-3/2-1/2*I*3^(1/2))
*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))
-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3^(1/2))
)*x+1/2/(3/2+1/2*I*3^(1/2))+1/2*I/(3/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(x^3-1)^(1/2)
/(I*2^(1/2)+1)*EllipticPi(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2), (3/2+1/2*I*3^(1/2))/(I*2^(1/2)+1),
((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))*3^(1/2)

```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(x^2+2)/(x^3-1)^(1/2), x, algorithm="maxima")

[Out] -integrate((x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(x^2 + 2)), x)

mupad [B] time = 2.77, size = 276, normalized size = 15.33

$$\frac{(3 + \sqrt{3} i) \sqrt{\frac{x + \frac{1}{2} \sqrt{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \sqrt{\frac{x + \frac{1}{2} \sqrt{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \left(-F \left(\operatorname{asin} \left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \right) \middle| -\frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{\frac{3}{2} + \frac{\sqrt{3} i}{2}} \right) + \Pi \left(\frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{1 + \sqrt{2} i}; \operatorname{asin} \left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \right) \middle| -\frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}} \right) + \Pi \left(-\frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{-1 + \sqrt{2} i}; \operatorname{asin} \left(\sqrt{\frac{x-1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \right) \middle| -\frac{\frac{3}{2} + \frac{\sqrt{3} i}{2}}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}} \right) \right)}{\sqrt{x^3 + \left(-\left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) - 1 \right) x + \left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2} \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - x^2 + 2)/((x^2 + 2)*(x^3 - 1)^(1/2)), x)

[Out] -((3^(1/2)*1i + 3)*(-(x - (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + (3^(1/2)*1i)/2 + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(-(x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*((3^(1/2)*1i)/2 + 3/2)^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1), asin((-x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin((-x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin((-x - 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2), -(3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/(((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + x^3)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(-\frac{2x}{x^2\sqrt{x^3-1} + 2\sqrt{x^3-1}} \right) dx - \int \frac{x^2}{x^2\sqrt{x^3-1} + 2\sqrt{x^3-1}} dx - \int \left(-\frac{2}{x^2\sqrt{x^3-1} + 2\sqrt{x^3-1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2*x+2)/(x**2+2)/(x**3-1)**(1/2), x)

[Out] -Integral(-2*x/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(x**2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x) - Integral(-2/(x**2*sqrt(x**3 - 1) + 2*sqrt(x**3 - 1)), x)

$$3.62 \quad \int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=18

$$2 \tanh^{-1} \left(\frac{x+1}{\sqrt{-x^3-1}} \right)$$

Rubi [A] time = 0.08, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2146, 206}

$$2 \tanh^{-1} \left(\frac{x+1}{\sqrt{-x^3-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]

[Out] 2*ArcTanh[(1 + x)/Sqrt[-1 - x^3]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2146

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] := -Dist[g/e, Subst[Int[1/(1 + a*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, e, f, g, h}, x] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*c*g - 4*a*e*h, 0]

Rubi steps

$$\begin{aligned} \int \frac{2-2x-x^2}{(2+x^2)\sqrt{-1-x^3}} dx &= 2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{1+x}{\sqrt{-1-x^3}} \right) \\ &= 2 \tanh^{-1} \left(\frac{1+x}{\sqrt{-1-x^3}} \right) \end{aligned}$$

Mathematica [C] time = 0.56, size = 298, normalized size = 16.56

$$\frac{2\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}\sqrt{x^2-x+1} \left(\frac{\sqrt{3}(1+\sqrt[3]{-1})(\sqrt[3]{-1-x}) \left(\sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}} \right) \right) \sqrt[3]{-1}}{(-1)^{2/3}x+1} - \frac{3i(\sqrt{2}-i)\Gamma \left(\frac{-2\sqrt{3}}{-i-2\sqrt{2}+\sqrt{3}} \right) \sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}} \right) \sqrt[3]{-1}}{(-1)^{5/6}+\sqrt{2}} + \frac{3(5+i\sqrt{2}+i\sqrt{3}+\sqrt{6})\Gamma \left(\frac{2\sqrt{3}}{-i+2\sqrt{2}+\sqrt{3}} \right) \sin^{-1} \left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}} \right) \sqrt[3]{-1}}{5i+2\sqrt{2}+\sqrt{3}+2i\sqrt{6}} \right)}{3\sqrt{-x^3-1}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]

[Out] (2*Sqrt[(1 + x)/(1 + (-1)^(1/3))])*Sqrt[1 - x + x^2]*((Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(1 + (-1)^(2/3)*x) - ((3*I)*(-I + Sqrt[2])*EllipticPi[(2*Sqrt[3])/(-I - 2*Sqrt[2] + Sqrt[3]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/((-1)^(5/6) + Sqrt[2]) + (3*(5 + I*Sqrt[2] + I*Sqrt[3] + Sqrt[6])*EllipticPi[(2*Sqrt[3])/(-I + 2*Sqrt[2] + Sqrt[3]), ArcSi

```
n[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))], (-1)^(1/3)]/(5*I + 2*Sqrt[2]
+ Sqrt[3] + (2*I)*Sqrt[6]))/(3*Sqrt[-1 - x^3])
```

IntegrateAlgebraic [A] time = 1.04, size = 25, normalized size = 1.39

$$-2 \tanh^{-1} \left(\frac{\sqrt{-x^3 - 1}}{x^2 - x + 1} \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + x^2)*Sqrt[-1 - x^3]),x]
```

```
[Out] -2*ArcTanh[Sqrt[-1 - x^3]/(1 - x + x^2)]
```

fricas [A] time = 0.82, size = 28, normalized size = 1.56

$$\log \left(-\frac{x^2 - 2x - 2\sqrt{-x^3 - 1}}{x^2 + 2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="fricas")
```

```
[Out] log(-(x^2 - 2*x - 2*sqrt(-x^3 - 1))/(x^2 + 2))
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)
```

maple [C] time = 0.06, size = 724, normalized size = 40.22



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x)
```

```
[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^1/2*((x+1)/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^1/2/(-x^3-1)^(1/2)*Elliptic
F(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^1/2,(I*3^(1/2)/(3/2+1/2
*I*3^(1/2)))^(1/2))-2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/
2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*
I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2))*EllipticP
i(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^1/2,I*3^(1/2)/(1/2+1/2*I*
3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*
3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(
1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/
2*I*3^(1/2)-I*2^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1
/2))^1/2,I*3^(1/2)/(1/2+1/2*I*3^(1/2)-I*2^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(
1/2)))^(1/2))+2/3*2^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1
/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(
1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3
*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^1/2,I*3^(1/2)/(I*2^(1/2)+1/2+1
/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+2/3*I*3^(1/2)*(I*3^(1/
```

2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2),I*3^(1/2)/(I*2^(1/2)+1/2+1/2*I*3^(1/2)),(I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(x^2+2)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] -integrate((x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(x^2 + 2)), x)

mupad [B] time = 0.11, size = 289, normalized size = 16.06

$$\frac{(3 + \sqrt{3} i) \sqrt{x^3 + 1} \sqrt{\frac{1 - \sqrt{3} i}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}} \sqrt{\frac{1-x+\frac{\sqrt{3} i}{2}}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}}{\sqrt{-x^3 - 1} \sqrt{x^3 + \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) - 1}} x - \left(-\frac{1}{2} + \frac{\sqrt{3} i}{2}\right) \left(\frac{1}{2} + \frac{\sqrt{3} i}{2}\right)} \left(-F\left(\operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}\right)\right), \frac{3 + \sqrt{3} i}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}\right) + \Pi\left(\frac{3 + \sqrt{3} i}{1 + \sqrt{2} i}, \operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}\right)\right), \frac{3 + \sqrt{3} i}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}\right) + \Pi\left(\frac{3 + \sqrt{3} i}{-1 + \sqrt{2} i}, \operatorname{asin}\left(\sqrt{\frac{x+1}{\frac{3}{2} + \frac{\sqrt{3} i}{2}}}\right)\right), \frac{3 + \sqrt{3} i}{-\frac{3}{2} + \frac{\sqrt{3} i}{2}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + x^2 - 2)/((x^2 + 2)*(- x^3 - 1)^(1/2)),x)

[Out] ((3^(1/2)*1i + 3)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) - ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)) + ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(2^(1/2)*1i - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}} dx - \int \frac{x^2}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}} dx - \int \left(-\frac{2}{x^2\sqrt{-x^3 - 1} + 2\sqrt{-x^3 - 1}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-2*x+2)/(x**2+2)/(-x**3-1)**(1/2),x)

[Out] -Integral(2*x/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)

$$3.63 \quad \int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}} dx$$

Optimal. Leaf size=30

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Rubi [A] time = 0.09, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2145, 204}

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{x^3+1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[1 + d]*(1 + x))/Sqrt[1 + x^3]])/Sqrt[1 + d]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 2145

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rubi steps

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{1+x^3}} dx = -\left(4 \text{Subst}\left(\int \frac{1}{-2-(2+2d)x^2} dx, x, \frac{1+x}{\sqrt{1+x^3}}\right)\right) = \frac{2 \tan^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{1+x^3}}\right)}{\sqrt{1+d}}$$

Mathematica [C] time = 1.36, size = 424, normalized size = 14.13

$$\frac{\sqrt{\frac{x+1}{1+\sqrt{1+x^3}}}\left(\frac{2\sqrt{5}(1+\sqrt{1+x^3})\sqrt{1+x^3}\sqrt{\frac{1+2\sqrt{1+x^3}}{1+\sqrt{1+x^3}}}}{(1+\sqrt{1+x^3})^2} - \frac{3\left(\left(1+\sqrt{1+x^3}\right)\sqrt{1+x^3}\sqrt{\frac{1+2\sqrt{1+x^3}}{1+\sqrt{1+x^3}}}\right)^2\sqrt{1+x^3}\sqrt{\frac{1+2\sqrt{1+x^3}}{1+\sqrt{1+x^3}}}}{2\sqrt{1+x^3}\sqrt{\frac{1+2\sqrt{1+x^3}}{1+\sqrt{1+x^3}}}}\right)}{\left(\sqrt{1+d}\sqrt{1+x^3}\right)^2\sqrt{1+d}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]

[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2])

+ (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]]], (-1)^(1/3)))/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d + d^2]))/(3*Sqrt[1 + x^3])

IntegrateAlgebraic [A] time = 2.31, size = 37, normalized size = 1.23

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{d+1} \sqrt{x^3+1}}{x^2-x+1} \right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[1 + d]*Sqrt[1 + x^3])/(1 - x + x^2)]/Sqrt[1 + d]

fricas [A] time = 0.96, size = 181, normalized size = 6.03

$$\left[\frac{\sqrt{-d-1} \log \left(\frac{2(3d+4)x^3 - x^4 - (d^2+2d+4)x^2 - d^2 + 4\sqrt{x^3+1}((d+2)x - x^2 + d)\sqrt{-d-1} - 2(d^2+2d)x + 4d + 4}{2dx^3 + x^4 + (d^2+2d+4)x^2 + d^2 + 2(d^2+2d)x + 4d + 4} \right)}{2(d+1)}, \frac{\arctan \left(\frac{\sqrt{x^3+1}((d+2)x - x^2 + d)\sqrt{d+1}}{2((d+1)x^3 + d + 1)} \right)}{\sqrt{d+1}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-d - 1)*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - d^2 + 4*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1) - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/(d + 1), -arctan(-1/2*sqrt(x^3 + 1)*((d + 2)*x - x^2 + d)*sqrt(d + 1)/((d + 1)*x^3 + d + 1))/sqrt(d + 1)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + 2x - 2}{\sqrt{x^3 + 1}(dx + x^2 + d + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(x^3 + 1)*(d*x + x^2 + d + 2)), x)

maple [C] time = 0.07, size = 4397, normalized size = 146.57

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x)

[Out] -2*(3/2-1/2*I*3^(1/2))*((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2-1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x-1/2+1/2*I*3^(1/2))/(-3/2+1/2*I*3^(1/2)))^(1/2)/(x^3+1)^(1/2)*EllipticF(((x+1)/(3/2-1/2*I*3^(1/2)))^(1/2), ((-3/2+1/2*I*3^(1/2))/(-3/2-1/2*I*3^(1/2)))^(1/2))-3/2/(d^2-4*d-8)^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/(3/2-1/2*I*3^(1/2)))^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/2/(-3/2-1/2*I*3^(1/2))-1/2*I/(-3/2-1/2*I*3^(1/2))*3^(1/2)))^(1/2)*(1/(-3/2+1/2*

$/2*I*3^{(1/2)})/(-1+1/2*d+1/2*(d^2-4*d-8)^{(1/2)}), ((-3/2+1/2*I*3^{(1/2)})/(-3/2-1/2*I*3^{(1/2)}))^{(1/2)}*d^2*3^{(1/2)}$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for more details)Is d^2-4*(d+2) positive, negative or zero?

mupad [B] time = 2.82, size = 632, normalized size = 21.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2*x + x^2 - 2)/((x^3 + 1)^(1/2)*(d + d*x + x^2 + 2)),x)

[Out]
$$-(2*((3^{(1/2)}*1i)/2 + 3/2)*((x + (3^{(1/2)}*1i)/2 - 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*((3^{(1/2)}*1i)/2 - x + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticF(asin(((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}), -(3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2)))/(x^3 - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) - ((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2))^{(1/2)} - (2*((3^{(1/2)}*1i)/2 + 3/2)*((x + (3^{(1/2)}*1i)/2 - 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*((3^{(1/2)}*1i)/2 - x + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticPi(((3^{(1/2)}*1i)/2 + 3/2)/((d^2 - 4*d - 8)^{(1/2)}/2 - d/2 + 1), asin(((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}), -(3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2))*(d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^{(1/2)}/2) + 4))/((x^3 - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) - ((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2))^{(1/2)}*(d^2 - 4*d - 8)^{(1/2)}*((d^2 - 4*d - 8)^{(1/2)}/2 - d/2 + 1)) - (2*((3^{(1/2)}*1i)/2 + 3/2)*((x + (3^{(1/2)}*1i)/2 - 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*((3^{(1/2)}*1i)/2 - x + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticPi(-((3^{(1/2)}*1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^{(1/2)}/2 - 1), asin(((x + 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}), -(3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2))*(d - (d - 2)*(d/2 + (d^2 - 4*d - 8)^{(1/2)}/2) + 4))/((x^3 - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) - ((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2))^{(1/2)}*(d^2 - 4*d - 8)^{(1/2)}*(d/2 + (d^2 - 4*d - 8)^{(1/2)}/2 - 1))$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{x^3+1} + d\sqrt{x^3+1} + x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} dx - \int \frac{x^2}{dx\sqrt{x^3+1} + d\sqrt{x^3+1} + x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} dx - \int \left(-\frac{2}{dx\sqrt{x^3+1} + d\sqrt{x^3+1} + x^2\sqrt{x^3+1} + 2\sqrt{x^3+1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(x**3+1)**(1/2),x)

[Out]
$$-\text{Integral}(2*x/(d*x*\text{sqrt}(x**3 + 1) + d*\text{sqrt}(x**3 + 1) + x**2*\text{sqrt}(x**3 + 1) + 2*\text{sqrt}(x**3 + 1)), x) - \text{Integral}(x**2/(d*x*\text{sqrt}(x**3 + 1) + d*\text{sqrt}(x**3 + 1) + x**2*\text{sqrt}(x**3 + 1) + 2*\text{sqrt}(x**3 + 1)), x) - \text{Integral}(-2/(d*x*\text{sqrt}(x**3 + 1) + d*\text{sqrt}(x**3 + 1) + x**2*\text{sqrt}(x**3 + 1) + 2*\text{sqrt}(x**3 + 1)), x)$$

3.64 $\int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{1-x^3}} dx$

Optimal. Leaf size=38

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {2145, 204}

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (-2*ArcTan[(Sqrt[1 - d]*(1 - x))/Sqrt[1 - x^3]])/Sqrt[1 - d]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 2145

```
Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]
```

Rubi steps

$$\int \frac{2 + 2x - x^2}{(2 - d + dx + x^2)\sqrt{1 - x^3}} dx = 4 \text{Subst} \left(\int \frac{1}{-2 - (2 - 2d)x^2} dx, x, \frac{1 - x}{\sqrt{1 - x^3}} \right) = -\frac{2 \tan^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{1-x^3}}\right)}{\sqrt{1-d}}$$

Mathematica [C] time = 1.49, size = 427, normalized size = 11.24

$$\frac{\sqrt{\frac{1-x}{1+\sqrt{-1}}}\sqrt{x^2+x+1} \left(\frac{2\sqrt{1+\sqrt{-1}}\sqrt{\frac{1-x}{1+\sqrt{-1}}}\sqrt{\frac{1-x}{1+\sqrt{-1}}}}{(1+\sqrt{-1})^{3/2}} + \frac{2\sqrt{1+\sqrt{-1}}\sqrt{\frac{1-x}{1+\sqrt{-1}}}\sqrt{\frac{1-x}{1+\sqrt{-1}}}}{(1+\sqrt{-1})^{3/2}} \right)}{3\sqrt{1-x^3}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]),x]
```

```
[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2
```

$$\left[\frac{2 \operatorname{tanh}^{-1}\left(\frac{\sqrt{d-1}\sqrt{1-x^3}}{x^2+x+1}\right)}{\sqrt{d-1}} \right]$$

IntegrateAlgebraic [A] time = 2.45, size = 37, normalized size = 0.97

$$\frac{2 \operatorname{tanh}^{-1}\left(\frac{\sqrt{d-1}\sqrt{1-x^3}}{x^2+x+1}\right)}{\sqrt{d-1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[1 - x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[-1 + d]*Sqrt[1 - x^3])/(1 + x + x^2)]/Sqrt[-1 + d]

fricas [A] time = 0.98, size = 191, normalized size = 5.03

$$\left[\frac{\log\left(\frac{2(3d-4)x^3-x^4-(d^2-2d+4)x^2-4\sqrt{-x^3+1}((d-2)x-x^2-d)\sqrt{d-1}-d^2+2(d^2-2d)x-4d+4}{2dx^3+x^4+(d^2-2d+4)x^2+d^2-2(d^2-2d)x-4d+4}\right)}{2\sqrt{d-1}}, \frac{\sqrt{-d+1} \arctan\left(\frac{\sqrt{-x^3+1}((d-2)x-x^2-d)\sqrt{d+1}}{2((d-1)x^3-d+1)}\right)}{d-1} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2), x, algorithm="fricas")

[Out] [1/2*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - 4*sqrt(-x^3 + 1)*(d - 2)*x - x^2 - d)*sqrt(d - 1) - d^2 + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4)/sqrt(d - 1), -sqrt(-d + 1)*arctan(-1/2*sqrt(-x^3 + 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1)/((d - 1)*x^3 - d + 1))/(d - 1)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 2x - 2}{\sqrt{-x^3 + 1}(dx + x^2 - d + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2), x, algorithm="giac")

[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(-x^3 + 1)*(d*x + x^2 - d + 2)), x)

maple [C] time = 0.07, size = 1908, normalized size = 50.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(-x^3+1)^(1/2), x)

[Out] 2/3*I*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x-1)/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x+1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3+1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(-3/2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2+4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)*(1/(-3/2+1/2*I*3^(1/2))*x-1/(-3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3+1)^(1/2)/(-1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2+4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2-1/2*I*3^(1/2))*3

mupad [B] time = 0.14, size = 677, normalized size = 17.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - x^2 + 2)/((1 - x^3)^(1/2)*(d*x - d + x^2 + 2)),x)`

[Out]
$$\begin{aligned} & (2*((3^{(1/2)}*1i)/2 + 3/2)*(x^3 - 1)^{(1/2)}*(-(x - (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*(-(x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticF(\text{asin}((-x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}, -((3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2)))/((1 - x^3)^{(1/2)}*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) + x^3)^{(1/2)} + (2*((3^{(1/2)}*1i)/2 + 3/2)*(x^3 - 1)^{(1/2)}*(-(x - (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*(-(x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticPi(((3^{(1/2)}*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^{(1/2)}/2 + 1), \text{asin}((-x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}, -((3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2))*d + (d + 2)*(d/2 - (4*d + d^2 - 8)^{(1/2)}/2) - 4))/((1 - x^3)^{(1/2)}*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) + x^3)^{(1/2)}*(d/2 - (4*d + d^2 - 8)^{(1/2)}/2 + 1)*(4*d + d^2 - 8)^{(1/2)}) - (2*((3^{(1/2)}*1i)/2 + 3/2)*(x^3 - 1)^{(1/2)}*(-(x - (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 - 3/2))^{(1/2)}*((x + (3^{(1/2)}*1i)/2 + 1/2)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*(-(x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}*ellipticPi(((3^{(1/2)}*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^{(1/2)}/2 + 1), \text{asin}((-x - 1)/((3^{(1/2)}*1i)/2 + 3/2))^{(1/2)}, -((3^{(1/2)}*1i)/2 + 3/2)/((3^{(1/2)}*1i)/2 - 3/2))*d + (d + 2)*(d/2 + (4*d + d^2 - 8)^{(1/2)}/2) - 4))/((1 - x^3)^{(1/2)}*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) - x*((3^{(1/2)}*1i)/2 - 1/2)*((3^{(1/2)}*1i)/2 + 1/2) + 1) + x^3)^{(1/2)}*(d/2 + (4*d + d^2 - 8)^{(1/2)}/2 + 1)*(4*d + d^2 - 8)^{(1/2)}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{2x}{dx\sqrt{1-x^3} - d\sqrt{1-x^3} + x^2\sqrt{1-x^3} + 2\sqrt{1-x^3}} \right) dx - \int \frac{x^2}{dx\sqrt{1-x^3} - d\sqrt{1-x^3} + x^2\sqrt{1-x^3} + 2\sqrt{1-x^3}} dx - \int \left(\frac{2}{dx\sqrt{1-x^3} - d\sqrt{1-x^3} + x^2\sqrt{1-x^3} + 2\sqrt{1-x^3}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(-x**3+1)**(1/2),x)`

[Out] `-Integral(-2*x/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(x**2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x) - Integral(-2/(d*x*sqrt(1 - x**3) - d*sqrt(1 - x**3) + x**2*sqrt(1 - x**3) + 2*sqrt(1 - x**3)), x)`

$$3.65 \quad \int \frac{2+2x-x^2}{(2-d+dx+x^2)\sqrt{-1+x^3}} dx$$

Optimal. Leaf size=36

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Rubi [A] time = 0.09, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2145, 207}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{x^3-1}}\right)}{\sqrt{1-d}}$$

Antiderivative was successfully verified.

[In] Int[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]

[Out] (-2*ArcTanh[(Sqrt[1 - d]*(1 - x))/Sqrt[-1 + x^3]])/Sqrt[1 - d]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2145

Int[((f_) + (g_.)*(x_) + (h_.)*(x_)^2)/(((c_) + (d_.)*(x_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rubi steps

$$\int \frac{2 + 2x - x^2}{(2 - d + dx + x^2)\sqrt{-1 + x^3}} dx = 4 \text{Subst}\left(\int \frac{1}{-2 - (-2 + 2d)x^2} dx, x, \frac{1 - x}{\sqrt{-1 + x^3}}\right) = -\frac{2 \tanh^{-1}\left(\frac{\sqrt{1-d}(1-x)}{\sqrt{-1+x^3}}\right)}{\sqrt{1-d}}$$

Mathematica [C] time = 0.46, size = 425, normalized size = 11.81

$$\frac{\sqrt{\frac{1-x}{1+\sqrt{-1+x^3}}}\sqrt{x^2+x+1} \left(\frac{2\sqrt{1+\sqrt{-1}}\sqrt{1+\sqrt{-1}} \left(\sqrt{\frac{1-x}{1+\sqrt{-1+x^3}}}\sqrt{-1} \right) + \dots}{(-1)^{2/3}-1} \right)}{3\sqrt{x^3-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]

[Out] (Sqrt[(1 - x)/(1 + (-1)^(1/3))]*Sqrt[1 + x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) + x)*EllipticF[ArcSin[Sqrt[(1 - (-1)^(2/3)*x)/(1 + (-1)^(1/3))]]], (-1)^(1/3)])/(-1 + (-1)^(2/3)*x) + ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 - 4*Sqrt[-8 + 4*d + d^2] + 2*(-1)^(1/3)*Sqrt[-8 + 4*d + d^2

$$\left[\frac{(1 + (-1)^{1/3})d(-4 + \sqrt{-8 + 4d + d^2}) \operatorname{EllipticPi}\left[\frac{(2I)\sqrt{3}}{2(-1)^{1/3} - d + \sqrt{-8 + 4d + d^2}}\right], \operatorname{ArcSin}\left[\frac{\sqrt{(1 - (-1)^{2/3})x}}{1 + (-1)^{1/3}}\right]}{(1 + (-1)^{1/3})d^2 - 4\sqrt{-8 + 4d + d^2} + 2(-1)^{1/3}\sqrt{-8 + 4d + d^2} + (1 + (-1)^{1/3})d(4 + \sqrt{-8 + 4d + d^2})} \right. \\ \left. \operatorname{EllipticPi}\left[\frac{(-2I)\sqrt{3}}{-2(-1)^{1/3} + d + \sqrt{-8 + 4d + d^2}}\right], \operatorname{ArcSin}\left[\frac{\sqrt{(1 - (-1)^{2/3})x}}{1 + (-1)^{1/3}}\right], (-1)^{1/3}\right] \Big/ \left((-2 - (-1)^{2/3} + d + (-1)^{1/3}d)\sqrt{-8 + 4d + d^2} \right) \Big/ (3\sqrt{-1 + x^3})$$

IntegrateAlgebraic [A] time = 2.32, size = 35, normalized size = 0.97

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{d-1}\sqrt{x^3-1}}{x^2+x+1}\right)}{\sqrt{d-1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 2*x - x^2)/((2 - d + d*x + x^2)*Sqrt[-1 + x^3]),x]

[Out] (2*ArcTan[(Sqrt[-1 + d]*Sqrt[-1 + x^3])/(1 + x + x^2)]/Sqrt[-1 + d]

fricas [A] time = 0.94, size = 187, normalized size = 5.19

$$\left[\frac{\sqrt{-d+1} \log\left(\frac{2(3d-4)x^3-x^4-(d^2-2d+4)x^2-d^2+4\sqrt{x^3-1}((d-2)x-x^2-d)\sqrt{-d+1}+2(d^2-2d)x-4d+4}{2dx^3+x^4+(d^2-2d+4)x^2+d^2-2(d^2-2d)x-4d+4}\right)}{2(d-1)}, \frac{\arctan\left(\frac{-\sqrt{x^3-1}((d-2)x-x^2-d)\sqrt{-d+1}}{2((d-1)x^3-d+1)}\right)}{\sqrt{d-1}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-d + 1)*log(-(2*(3*d - 4)*x^3 - x^4 - (d^2 - 2*d + 4)*x^2 - d^2 + 4*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(-d + 1) + 2*(d^2 - 2*d)*x - 4*d + 4)/(2*d*x^3 + x^4 + (d^2 - 2*d + 4)*x^2 + d^2 - 2*(d^2 - 2*d)*x - 4*d + 4))/(d - 1), -arctan(-1/2*sqrt(x^3 - 1)*((d - 2)*x - x^2 - d)*sqrt(d - 1)/((d - 1)*x^3 - d + 1))/sqrt(d - 1)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 - 2x - 2}{\sqrt{x^3 - 1}(dx + x^2 - d + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="giac")

[Out] integrate(-(x^2 - 2*x - 2)/(sqrt(x^3 - 1)*(d*x + x^2 - d + 2)), x)

maple [C] time = 0.05, size = 4437, normalized size = 123.25

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x)

[Out] -2*(-3/2-1/2*I*3^(1/2))*((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2-1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2)*((x+1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)))^(1/2)/(x^3-1)^(1/2)*EllipticF(((x-1)/(-3/2-1/2*I*3^(1/2)))^(1/2),((3/2+1/2*I*3^(1/2))/(3/2-1/2*I*3^(1/2)))^(1/2))+3/2/(d^2+4*d-8)^(1/2)*(1/(-3/2-1/2*I*3^(1/2))*x-1/(-3/2-1/2*I*3^(1/2)))^(1/2)*(1/(3/2-1/2*I*3^(1/2))*x+1/2/(3/2-1/2*I*3^(1/2))-1/2*I/(3/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*(1/(3/2+1/2*I*3

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2*x+2)/(d*x+x^2-d+2)/(x^3-1)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d^2-4*(2-d)>0)', see `assume?` for more details)Is d^2-4*(2-d) positive, negative or zero?

mupad [B] time = 2.80, size = 629, normalized size = 17.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x - x^2 + 2)/((x^3 - 1)^(1/2)*(d*x - d + x^2 + 2)),x)

[Out]
$$\begin{aligned} & (2*((3^{1/2}*1i)/2 + 3/2)*(-(x - (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 - 3/2))^{1/2} * ((x + (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * (-(x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * \text{ellipticF}(\text{asin}((-x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2}), \\ & -((3^{1/2}*1i)/2 + 3/2)/((3^{1/2}*1i)/2 - 3/2))/(((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) - x*((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) + 1) + x^3)^{1/2} + (2*((3^{1/2}*1i)/2 + 3/2)*(-(x - (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 - 3/2))^{1/2} * ((x + (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * (-(x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * \text{ellipticPi} \\ & (((3^{1/2}*1i)/2 + 3/2)/(d/2 - (4*d + d^2 - 8)^{1/2}/2 + 1), \text{asin}((-x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2}), \\ & -((3^{1/2}*1i)/2 + 3/2)/((3^{1/2}*1i)/2 - 3/2))* (d + (d + 2)*(d/2 - (4*d + d^2 - 8)^{1/2}/2) - 4))/(((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) - x*((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) + 1) + x^3)^{1/2} * (d/2 - (4*d + d^2 - 8)^{1/2}/2 + 1) * (4*d + d^2 - 8)^{1/2}) \\ & - (2*((3^{1/2}*1i)/2 + 3/2)*(-(x - (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 - 3/2))^{1/2} * ((x + (3^{1/2}*1i)/2 + 1/2)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * (-(x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2} * \text{ellipticPi}(((3^{1/2}*1i)/2 + 3/2)/(d/2 + (4*d + d^2 - 8)^{1/2}/2 + 1), \text{asin}((-x - 1)/((3^{1/2}*1i)/2 + 3/2))^{1/2}), \\ & -((3^{1/2}*1i)/2 + 3/2)/((3^{1/2}*1i)/2 - 3/2))* (d + (d + 2)*(d/2 + (4*d + d^2 - 8)^{1/2}/2) - 4))/(((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) - x*((3^{1/2}*1i)/2 - 1/2)*((3^{1/2}*1i)/2 + 1/2) + 1) + x^3)^{1/2} * (d/2 + (4*d + d^2 - 8)^{1/2}/2 + 1) * (4*d + d^2 - 8)^{1/2}) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\left(-\frac{2x}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx-\int\frac{x^2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}dx-\int\left(-\frac{2}{dx\sqrt{x^3-1}-d\sqrt{x^3-1}+x^2\sqrt{x^3-1}+2\sqrt{x^3-1}}\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2*x+2)/(d*x+x**2-d+2)/(x**3-1)**(1/2),x)

[Out]
$$-\text{Integral}(-2*x/(d*x*\text{sqrt}(x**3 - 1) - d*\text{sqrt}(x**3 - 1) + x**2*\text{sqrt}(x**3 - 1) + 2*\text{sqrt}(x**3 - 1)), x) - \text{Integral}(x**2/(d*x*\text{sqrt}(x**3 - 1) - d*\text{sqrt}(x**3 - 1) + x**2*\text{sqrt}(x**3 - 1) + 2*\text{sqrt}(x**3 - 1)), x) - \text{Integral}(-2/(d*x*\text{sqrt}(x**3 - 1) - d*\text{sqrt}(x**3 - 1) + x**2*\text{sqrt}(x**3 - 1) + 2*\text{sqrt}(x**3 - 1)), x)$$

$$3.66 \quad \int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2145, 207}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+1}(x+1)}{\sqrt{-x^3-1}}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

[In] Int[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]

[Out] (2*ArcTanh[(Sqrt[1 + d]*(1 + x))/Sqrt[-1 - x^3]])/Sqrt[1 + d]

Rule 207

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 2145

Int[((f_) + (g_)*(x_) + (h_)*(x_)^2)/(((c_) + (d_)*(x_) + (e_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_)^3]), x_Symbol] :> Dist[-2*g*h, Subst[Int[1/(2*e*h - (b*d*f - 2*a*e*h)*x^2), x], x, (1 + (2*h*x)/g)/Sqrt[a + b*x^3]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b*d*f - 2*a*e*h, 0] && EqQ[b*g^3 - 8*a*h^3, 0] && EqQ[g^2 + 2*f*h, 0] && EqQ[b*d*f + b*c*g - 4*a*e*h, 0]

Rubi steps

$$\int \frac{2-2x-x^2}{(2+d+dx+x^2)\sqrt{-1-x^3}} dx = -\left(4 \text{Subst}\left(\int \frac{1}{-2-(-2-2d)x^2} dx, x, \frac{1+x}{\sqrt{-1-x^3}}\right)\right) = \frac{2 \tanh^{-1}\left(\frac{\sqrt{1+d}(1+x)}{\sqrt{-1-x^3}}\right)}{\sqrt{1+d}}$$

Mathematica [C] time = 0.55, size = 426, normalized size = 13.31

$$\frac{\sqrt{\frac{x+1}{1+\sqrt{-1-x^3}}}\left(\frac{2\sqrt{5}(1+\sqrt{-1})\sqrt{\frac{1+2\sqrt{2}x+1}{1+\sqrt{-1}}}}{(1+\sqrt{-1})^{3/2}}\right)^{3/2} - 3\left(\frac{((1+\sqrt{-1})^2)(\sqrt{2-4d-8+d^2-2\sqrt{-1}\sqrt{2-4d-8+d^2-4d-8+d^2-8}}{\sqrt{-1-x^3}})\sqrt{\frac{1+2\sqrt{2}x+1}{1+\sqrt{-1}}}}{(1+\sqrt{-1})^{3/2}}\right)^{3/2}}{(1+\sqrt{-1})^{3/2}\sqrt{2-4d-8}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]),x]

[Out] (Sqrt[(1 + x)/(1 + (-1)^(1/3))]*Sqrt[1 - x + x^2]*((2*Sqrt[3]*(1 + (-1)^(1/3)))*((-1)^(1/3) - x)*EllipticF[ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)])/(1 + (-1)^(2/3)*x) - ((3*I)*((8 + 8*(-1)^(1/3) - (1 + (-1)^(1/3))*d^2 + 4*Sqrt[-8 - 4*d + d^2] - 2*(-1)^(1/3)*Sqrt[-8 - 4*d + d^2]

+ (1 + (-1)^(1/3))*d*(4 + Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d - Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)] + ((1 + (-1)^(1/3))*d^2 + (1 + (-1)^(1/3))*d*(-4 + Sqrt[-8 - 4*d + d^2]) - 2*(4 + 4*(-1)^(1/3) - 2*Sqrt[-8 - 4*d + d^2] + (-1)^(1/3)*Sqrt[-8 - 4*d + d^2]))*EllipticPi[((2*I)*Sqrt[3])/(2*(-1)^(1/3) + d + Sqrt[-8 - 4*d + d^2]), ArcSin[Sqrt[(1 + (-1)^(2/3)*x)/(1 + (-1)^(1/3))]], (-1)^(1/3))]/((2 + (-1)^(2/3) + d + (-1)^(1/3)*d)*Sqrt[-8 - 4*d + d^2]))/(3*Sqrt[-1 - x^3])

IntegrateAlgebraic [A] time = 2.36, size = 39, normalized size = 1.22

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+1} \sqrt{-x^3-1}}{x^2-x+1}\right)}{\sqrt{d+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 - 2*x - x^2)/((2 + d + d*x + x^2)*Sqrt[-1 - x^3]), x]

[Out] (-2*ArcTanh[(Sqrt[1 + d]*Sqrt[-1 - x^3])/(1 - x + x^2)]/Sqrt[1 + d]

fricas [B] time = 1.06, size = 185, normalized size = 5.78

$$\left[\frac{\log\left(\frac{2(3d+4)x^3-x^4-(d^2+2d+4)x^2-4\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{d+1}-d^2-2(d^2+2d)x+4d+4}{2dx^3+x^4+(d^2+2d+4)x^2+d^2+2(d^2+2d)x+4d+4}\right)}{2\sqrt{d+1}}, -\frac{\sqrt{-d-1} \arctan\left(\frac{\sqrt{-x^3-1}((d+2)x-x^2+d)\sqrt{-d-1}}{2((d+1)x^3+d+1)}\right)}{d+1} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2), x, algorithm="fricas")

[Out] [1/2*log(-(2*(3*d + 4)*x^3 - x^4 - (d^2 + 2*d + 4)*x^2 - 4*sqrt(-x^3 - 1)*(d + 2)*x - x^2 + d)*sqrt(d + 1) - d^2 - 2*(d^2 + 2*d)*x + 4*d + 4)/(2*d*x^3 + x^4 + (d^2 + 2*d + 4)*x^2 + d^2 + 2*(d^2 + 2*d)*x + 4*d + 4))/sqrt(d + 1), -sqrt(-d - 1)*arctan(-1/2*sqrt(-x^3 - 1)*((d + 2)*x - x^2 + d)*sqrt(-d - 1)/((d + 1)*x^3 + d + 1))/(d + 1)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 + 2x - 2}{\sqrt{-x^3 - 1}(dx + x^2 + d + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2), x, algorithm="giac")

[Out] integrate(-(x^2 + 2*x - 2)/(sqrt(-x^3 - 1)*(d*x + x^2 + d + 2)), x)

maple [C] time = 0.06, size = 1888, normalized size = 59.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2), x)

[Out] 2/3*I*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2)*((x+1)/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*(x-1/2+1/2*I*3^(1/2))*3^(1/2))^(1/2)/(-x^3-1)^(1/2)*EllipticF(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))+1/3*I/(d^2-4*d-8)^(1/2)*3^(1/2)*(I*3^(1/2)*x-1/2*I*3^(1/2)+3/2)^(1/2)*(1/(3/2+1/2*I*3^(1/2))*x+1/(3/2+1/2*I*3^(1/2)))^(1/2)*(-I*3^(1/2)*x+1/2*I*3^(1/2)+3/2)^(1/2)/(-x^3-1)^(1/2)/(1/2+1/2*I*3^(1/2)+1/2*d-1/2*(d^2-4*d-8)^(1/2))*EllipticPi(1/3*3^(1/2)*(I*(x-1/2-1/2*I*3^(1/2))*3^(1/2))^(1/2), (I*3^(1/2)/(3/2+1/2*I*3^(1/2)))^(1/2))

$$\begin{aligned} &))^{(1/2)}, I^3^{(1/2)} / ((1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / \\ &(3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d^{-2-1/3} * I^3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} \\ &+ 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} \\ &* x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d - 1/2 * \\ &d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / \\ &(3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d - 4/3 * I / (d^2 - 4 * d - 8)^{(1/2)} * 3^{(1/2)} * (I^3^{(1/2)} * x - \\ &1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * \\ &(-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 \\ &+ 1/2 * d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d + 2/3 * I^3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} \\ &+ 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 \\ &* d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} - 8/3 * I / (d^2 - 4 * d - 8)^{(1/2)} * 3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * \\ &(-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d - 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) - 1/3 * I / (d^2 - 4 * d - 8)^{(1/2)} * 3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d^{-2-1/3} * I^3^{(1/2)} \\ &* (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * \\ &I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d + 4/3 * I / (d^2 - 4 * d - 8)^{(1/2)} * 3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)} \\ &I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) + 8/3 * I / (d^2 - 4 * \\ &d - 8)^{(1/2)} * 3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(\\ &1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)}, I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d \\ &+ 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) * d + 2/3 * I \\ &* 3^{(1/2)} * (I^3^{(1/2)} * x - 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} * (1 / (3/2 + 1/2 * I^3^{(1/2)})) * x + 1 / (\\ &3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)} * (-I^3^{(1/2)} * x + 1/2 * I^3^{(1/2)} + 3/2)^{(1/2)} / (-x^3 - 1)^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)} * \text{EllipticPi}(1/3 * I^3^{(1/2)} * (I * (x - 1/2 - 1/2 * I^3^{(1/2)}) * 3^{(1/2)})^{(1/2)}, I^3^{(1/2)} / (1/2 + 1/2 * I^3^{(1/2)} + 1/2 * d + 1/2 * (d^2 - 4 * d - 8)^{(1/2)}), (I^3^{(1/2)} / (3/2 + 1/2 * I^3^{(1/2)}))^{(1/2)}) \end{aligned}$$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2-2*x+2)/(d*x+x^2+d+2)/(-x^3-1)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(d^2-4*(d+2)>0)', see `assume?` for more details)Is d^2-4*(d+2) positive, negative or zero?

mupad [B] time = 0.12, size = 680, normalized size = 21.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x + x^2 - 2)/((- x^3 - 1)^(1/2)*(d + d*x + x^2 + 2)),x)
[Out] - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticF(asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2)))/((- x^3 - 1)^(1/2)*(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(((3^(1/2)*1i)/2 + 3/2)/((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*((d - (d - 2)*(d/2 - (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*((d^2 - 4*d - 8)^(1/2)/2 - d/2 + 1)) - (2*((3^(1/2)*1i)/2 + 3/2)*(x^3 + 1)^(1/2)*((x + (3^(1/2)*1i)/2 - 1/2)/((3^(1/2)*1i)/2 - 3/2))^(1/2)*((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*(((3^(1/2)*1i)/2 - x + 1/2)/((3^(1/2)*1i)/2 + 3/2))^(1/2)*ellipticPi(-((3^(1/2)*1i)/2 + 3/2)/(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1), asin(((x + 1)/((3^(1/2)*1i)/2 + 3/2))^(1/2)), -((3^(1/2)*1i)/2 + 3/2)/((3^(1/2)*1i)/2 - 3/2))*((d - (d - 2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2) + 4))/((- x^3 - 1)^(1/2)*(x^3 - x*((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2) + 1) - ((3^(1/2)*1i)/2 - 1/2)*((3^(1/2)*1i)/2 + 1/2))^(1/2)*(d^2 - 4*d - 8)^(1/2)*(d/2 + (d^2 - 4*d - 8)^(1/2)/2 - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2x}{dx\sqrt{-x^3-1} + d\sqrt{-x^3-1} + x^2\sqrt{-x^3-1} + 2\sqrt{-x^3-1}} dx - \int \frac{x^2}{dx\sqrt{-x^3-1} + d\sqrt{-x^3-1} + x^2\sqrt{-x^3-1} + 2\sqrt{-x^3-1}} dx - \int \left(\frac{2}{dx\sqrt{-x^3-1} + d\sqrt{-x^3-1} + x^2\sqrt{-x^3-1} + 2\sqrt{-x^3-1}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2-2*x+2)/(d*x+x**2+d+2)/(-x**3-1)**(1/2),x)
[Out] -Integral(2*x/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(x**2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x) - Integral(-2/(d*x*sqrt(-x**3 - 1) + d*sqrt(-x**3 - 1) + x**2*sqrt(-x**3 - 1) + 2*sqrt(-x**3 - 1)), x)
```

$$3.67 \quad \int x^m \left(c (a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=161

$$\frac{a^3 cx^{m+1} \sqrt{c(a+bx^2)^2}}{(m+1)(a+bx^2)} + \frac{3a^2 bcx^{m+3} \sqrt{c(a+bx^2)^2}}{(m+3)(a+bx^2)} + \frac{b^3 cx^{m+7} \sqrt{c(a+bx^2)^2}}{(m+7)(a+bx^2)} + \frac{3ab^2 cx^{m+5} \sqrt{c(a+bx^2)^2}}{(m+5)(a+bx^2)}$$

Rubi [A] time = 0.13, antiderivative size = 205, normalized size of antiderivative = 1.27, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1989, 1112, 270}

$$\frac{a^3 cx^{m+1} \sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(m+1)(a+bx^2)} + \frac{3a^2 bcx^{m+3} \sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(m+3)(a+bx^2)} + \frac{3ab^2 cx^{m+5} \sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(m+5)(a+bx^2)} + \frac{b^3 cx^{m+7} \sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(m+7)(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^m*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (a^3*c*x^(1+m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((1+m)*(a + b*x^2)) + (3*a^2*b*c*x^(3+m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((3+m)*(a + b*x^2)) + (3*a*b^2*c*x^(5+m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((5+m)*(a + b*x^2)) + (b^3*c*x^(7+m)*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/((7+m)*(a + b*x^2))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] :> Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int x^m \left(c (a + bx^2)^2 \right)^{3/2} dx &= \int x^m (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int x^m (abc + b^2cx^2)^3 dx}{b^2c (abc + b^2cx^2)} \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int (a^3b^3c^3x^m + 3a^2b^4c^3x^{2+m} + 3ab^5c^3x^{4+m} + b^6c^3x^{6+m}) dx}{b^2c (abc + b^2cx^2)} \\ &= \frac{a^3cx^{1+m}\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(1+m)(a+bx^2)} + \frac{3a^2bcx^{3+m}\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(3+m)(a+bx^2)} + \frac{3ab^2cx^{5+m}\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(5+m)(a+bx^2)} + \frac{b^3cx^{7+m}\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{(7+m)(a+bx^2)} \end{aligned}$$

Mathematica [A] time = 0.10, size = 132, normalized size = 0.82

$$\frac{x^{m+1} \left(c(a + bx^2)^2 \right)^{3/2} \left(a^3(m^3 + 15m^2 + 71m + 105) + 3a^2b(m^3 + 13m^2 + 47m + 35)x^2 + 3ab^2(m^3 + 11m^2 + 31m + 21)x^4 + b^3(m^3 + 9m^2 + 23m + 15)x^6 \right)}{(m+1)(m+3)(m+5)(m+7)(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^m*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^(1 + m)*(c*(a + b*x^2)^2)^(3/2)*(a^3*(105 + 71*m + 15*m^2 + m^3) + 3*a^2*b*(35 + 47*m + 13*m^2 + m^3)*x^2 + 3*a*b^2*(21 + 31*m + 11*m^2 + m^3)*x^4 + b^3*(15 + 23*m + 9*m^2 + m^3)*x^6))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*(a + b*x^2)^3)

IntegrateAlgebraic [F] time = 0.83, size = 0, normalized size = 0.00

$$\int x^m \left(c(a + bx^2)^2 \right)^{3/2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^m*(c*(a + b*x^2)^2)^(3/2), x]

[Out] Defer[IntegrateAlgebraic][x^m*(c*(a + b*x^2)^2)^(3/2), x]

fricas [A] time = 0.71, size = 233, normalized size = 1.45

$$\frac{\left((b^3cm^3 + 9b^3cm^2 + 23b^3cm + 15b^3c)x^7 + 3(ab^2cm^3 + 11ab^2cm^2 + 31ab^2cm + 21ab^2c)x^5 + 3(a^2bcm^3 + 13a^2bcm^2 + 47a^2bcm + 35a^2bc)x^3 + (a^3cm^3 + 15a^3cm^2 + 71a^3cm + 105a^3c)x \right) \sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{am^4 + 16am^3 + 86am^2 + (bm^4 + 16bm^3 + 86bm^2 + 176bm + 105b)x^2 + 176am + 105a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2), x, algorithm="fricas")

[Out] ((b^3*c*m^3 + 9*b^3*c*m^2 + 23*b^3*c*m + 15*b^3*c)*x^7 + 3*(a*b^2*c*m^3 + 11*a*b^2*c*m^2 + 31*a*b^2*c*m + 21*a*b^2*c)*x^5 + 3*(a^2*b*c*m^3 + 13*a^2*b*c*m^2 + 47*a^2*b*c*m + 35*a^2*b*c)*x^3 + (a^3*c*m^3 + 15*a^3*c*m^2 + 71*a^3*c*m + 105*a^3*c)*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*x^m/(a*m^4 + 16*a*m^3 + 86*a*m^2 + (b*m^4 + 16*b*m^3 + 86*b*m^2 + 176*b*m + 105*b)*x^2 + 176*a*m + 105*a)

giac [B] time = 0.38, size = 355, normalized size = 2.20

$$\frac{(b^3m^3x^7 + 9b^3m^2x^7 + 23b^3mx^7 + 15b^3x^7) \operatorname{sgn}(bx^2 + a) + 3(a^2b^2m^3x^5 + 11a^2b^2m^2x^5 + 31a^2b^2mx^5 + 21a^2b^2x^5) \operatorname{sgn}(bx^2 + a) + 3(a^3m^3x^3 + 15a^3m^2x^3 + 71a^3mx^3 + 105a^3x^3) \operatorname{sgn}(bx^2 + a) \sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{(m+7)(m+5)(m+3)(m+1)(bx^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(c*(b*x^2+a)^2)^(3/2), x, algorithm="giac")

[Out] (b^3*m^3*x^7*x^m*sgn(b*x^2 + a) + 9*b^3*m^2*x^7*x^m*sgn(b*x^2 + a) + 3*a*b^2*m^3*x^5*x^m*sgn(b*x^2 + a) + 23*b^3*m*x^7*x^m*sgn(b*x^2 + a) + 33*a*b^2*m^2*x^5*x^m*sgn(b*x^2 + a) + 15*b^3*x^7*x^m*sgn(b*x^2 + a) + 3*a^2*b*m^3*x^3*x^m*sgn(b*x^2 + a) + 93*a*b^2*m*x^5*x^m*sgn(b*x^2 + a) + 39*a^2*b*m^2*x^3*x^m*sgn(b*x^2 + a) + 63*a*b^2*x^5*x^m*sgn(b*x^2 + a) + a^3*m^3*x*x^m*sgn(b*x^2 + a) + 141*a^2*b*m*x^3*x^m*sgn(b*x^2 + a) + 15*a^3*m^2*x*x^m*sgn(b*x^2 + a) + 105*a^2*b*x^3*x^m*sgn(b*x^2 + a) + 71*a^3*m*x*x^m*sgn(b*x^2 + a) + 105*a^3*x*x^m*sgn(b*x^2 + a))*c^(3/2)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)

maple [A] time = 0.01, size = 200, normalized size = 1.24

$$\frac{(b^3m^3x^6 + 9b^3m^2x^6 + 3a^2b^2m^3x^4 + 23b^3mx^6 + 33a^2b^2m^2x^4 + 15b^3x^6 + 3a^2bm^3x^2 + 93a^2bm^2x^4 + 39a^2bm^2x^2 + 63ab^2x^4 + a^3m^3 + 141a^2bm^2x^2 + 15a^3m^2 + 105a^2bx^2 + 71a^3m + 105a^3) \left((bx^2 + a)^2 \right)^{3/2} x^{m+1}}{(m+7)(m+5)(m+3)(m+1)(bx^2+a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(c*(b*x^2+a)^2)^(3/2),x)`

[Out] $x^{(m+1)}*(b^3*m^3*x^6+9*b^3*m^2*x^6+3*a*b^2*m^3*x^4+23*b^3*m*x^6+33*a*b^2*m^2*x^4+15*b^3*x^6+3*a^2*b*m^3*x^2+93*a*b^2*m*x^4+39*a^2*b*m^2*x^2+63*a*b^2*x^4+a^3*m^3+141*a^2*b*m*x^2+15*a^3*m^2+105*a^2*b*x^2+71*a^3*m+105*a^3)*(c*(b*x^2+a)^2)^(3/2)/(7+m)/(5+m)/(3+m)/(m+1)/(b*x^2+a)^3$

maxima [A] time = 0.99, size = 119, normalized size = 0.74

$$\frac{\left((m^3+9m^2+23m+15)b^3c^{\frac{3}{2}}x^7+3(m^3+11m^2+31m+21)ab^2c^{\frac{3}{2}}x^5+3(m^3+13m^2+47m+35)a^2bc^{\frac{3}{2}}x^3+(m^3+15m^2+71m+105)a^3c^{\frac{3}{2}}x\right)x^m}{m^4+16m^3+86m^2+176m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^m*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")`

[Out] $((m^3+9m^2+23m+15)*b^3*c^{(3/2)}*x^7+3*(m^3+11m^2+31m+21)*a*b^2*c^{(3/2)}*x^5+3*(m^3+13m^2+47m+35)*a^2*b*c^{(3/2)}*x^3+(m^3+15m^2+71m+105)*a^3*c^{(3/2)}*x)*x^m/(m^4+16m^3+86m^2+176m+105)$

mupad [B] time = 2.97, size = 234, normalized size = 1.45

$$x^m \left(\frac{3a^2cx^3\sqrt{c(bx^2+a)^2}(m^3+13m^2+47m+35)}{m^4+16m^3+86m^2+176m+105} + \frac{b^2cx^7\sqrt{c(bx^2+a)^2}(m^3+9m^2+23m+15)}{m^4+16m^3+86m^2+176m+105} + \frac{3abcx^5\sqrt{c(bx^2+a)^2}(m^3+11m^2+31m+21)}{m^4+16m^3+86m^2+176m+105} + \frac{a^3cx\sqrt{c(bx^2+a)^2}(m^3+15m^2+71m+105)}{b(m^4+16m^3+86m^2+176m+105)} \right) \frac{a}{b} + x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^m*(c*(a+b*x^2)^2)^(3/2),x)`

[Out] $(x^m*((3*a^2*c*x^3*(c*(a+b*x^2)^2)^(1/2)*(47*m+13*m^2+m^3+35))/(176*m+86*m^2+16*m^3+m^4+105)+(b^2*c*x^7*(c*(a+b*x^2)^2)^(1/2)*(23*m+9*m^2+m^3+15))/(176*m+86*m^2+16*m^3+m^4+105)+(3*a*b*c*x^5*(c*(a+b*x^2)^2)^(1/2)*(31*m+11*m^2+m^3+21))/(176*m+86*m^2+16*m^3+m^4+105)+(a^3*c*x*(c*(a+b*x^2)^2)^(1/2)*(71*m+15*m^2+m^3+105))/(b*(176*m+86*m^2+16*m^3+m^4+105))))/(a/b+x^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(c*(b*x**2+a)**2)**(3/2),x)`

[Out] Timed out

$$3.68 \quad \int x^5 \left(c(a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=143

$$\frac{a^3cx^6\sqrt{c(a+bx^2)^2}}{6(a+bx^2)} + \frac{3a^2bcx^8\sqrt{c(a+bx^2)^2}}{8(a+bx^2)} + \frac{b^3cx^{12}\sqrt{c(a+bx^2)^2}}{12(a+bx^2)} + \frac{3ab^2cx^{10}\sqrt{c(a+bx^2)^2}}{10(a+bx^2)}$$

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1989, 1111, 645}

$$\frac{c(a+bx^2)^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{12b^3} - \frac{ac(a+bx^2)^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{5b^3} + \frac{a^2c(a+bx^2)^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{8b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (a^2*c*(a + b*x^2)^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(8*b^3) - (a*c*(a + b*x^2)^4*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(5*b^3) + (c*(a + b*x^2)^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(12*b^3))

Rule 645

Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*FracPart[p]))], Int[ExpandLinearProduct[(b/2 + c*x)^(2*p), (d + e*x)^m, b/2, c, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 0] && EqQ[m - 2*p + 1, 0]

Rule 1111

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int x^5 \left(c(a + bx^2)^2 \right)^{3/2} dx &= \int x^5 (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx \\ &= \frac{1}{2} \text{Subst} \left(\int x^2 (a^2c + 2abcx + b^2cx^2)^{3/2} dx, x, x^2 \right) \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \text{Subst} \left(\int \left(\frac{a^2(abc+b^2cx)^3}{b^2} - \frac{2a(abc+b^2cx)^4}{b^3c} + \frac{(abc+b^2cx)^5}{b^4c^2} \right) dx, x, x^2 \right)}{2b^2c(abc + b^2cx^2)} \\ &= \frac{a^2c(a + bx^2)^3\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{8b^3} - \frac{ac(a + bx^2)^4\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{5b^3} + \frac{c(a + bx^2)^5\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{12b^3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^6 (20a^3 + 45a^2bx^2 + 36ab^2x^4 + 10b^3x^6) (c(a + bx^2)^2)^{3/2}}{120(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^6*(c*(a + b*x^2)^2)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6))/(120*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.82, size = 74, normalized size = 0.52

$$\frac{x^6 (20a^3 + 45a^2bx^2 + 36ab^2x^4 + 10b^3x^6) (a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{120(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^6*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(20*a^3 + 45*a^2*b*x^2 + 36*a*b^2*x^4 + 10*b^3*x^6))/(120*(a + b*x^2)^3)

fricas [A] time = 0.67, size = 74, normalized size = 0.52

$$\frac{(10b^3cx^{12} + 36ab^2cx^{10} + 45a^2bcx^8 + 20a^3cx^6)\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{120(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2), x, algorithm="fricas")

[Out] 1/120*(10*b^3*c*x^12 + 36*a*b^2*c*x^10 + 45*a^2*b*c*x^8 + 20*a^3*c*x^6)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.30, size = 72, normalized size = 0.50

$$\frac{1}{120} (10b^3x^{12}\operatorname{sgn}(bx^2 + a) + 36ab^2x^{10}\operatorname{sgn}(bx^2 + a) + 45a^2bx^8\operatorname{sgn}(bx^2 + a) + 20a^3x^6\operatorname{sgn}(bx^2 + a))c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2), x, algorithm="giac")

[Out] 1/120*(10*b^3*x^12*sgn(b*x^2 + a) + 36*a*b^2*x^10*sgn(b*x^2 + a) + 45*a^2*b*x^8*sgn(b*x^2 + a) + 20*a^3*x^6*sgn(b*x^2 + a))*c^(3/2)

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{(10b^3x^6 + 36ab^2x^4 + 45a^2bx^2 + 20a^3) \left((bx^2 + a)^2 c \right)^{\frac{3}{2}} x^6}{120(bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((b*x^2+a)^2*c)^(3/2), x)

[Out] 1/120*x^6*(10*b^3*x^6+36*a*b^2*x^4+45*a^2*b*x^2+20*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^2+a)^3

maxima [A] time = 1.03, size = 136, normalized size = 0.95

$$\frac{(b^2cx^4 + 2abcx^2 + a^2c)^{\frac{3}{2}}a^2x^2}{8b^2} + \frac{(b^2cx^4 + 2abcx^2 + a^2c)^{\frac{3}{2}}a^3}{8b^3} + \frac{(b^2cx^4 + 2abcx^2 + a^2c)^{\frac{5}{2}}x^2}{12b^2c} - \frac{7(b^2cx^4 + 2abcx^2 + a^2c)^{\frac{5}{2}}a}{60b^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")

[Out] 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^2*x^2/b^2 + 1/8*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)*a^3/b^3 + 1/12*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*x^2/(b^2*c) - 7/60*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(5/2)*a/(b^3*c)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^5 \left(c (b x^2 + a)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(c*(a + b*x^2)^2)^(3/2),x)

[Out] int(x^5*(c*(a + b*x^2)^2)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(c*(b*x**2+a)**2)**(3/2),x)

[Out] Timed out

$$3.69 \quad \int x^4 \left(c(a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=143

$$\frac{a^3 cx^5 \sqrt{c(a + bx^2)^2}}{5(a + bx^2)} + \frac{3a^2 bcx^7 \sqrt{c(a + bx^2)^2}}{7(a + bx^2)} + \frac{b^3 cx^{11} \sqrt{c(a + bx^2)^2}}{11(a + bx^2)} + \frac{ab^2 cx^9 \sqrt{c(a + bx^2)^2}}{3(a + bx^2)}$$

Rubi [A] time = 0.11, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1989, 1112, 270}

$$\frac{b^3 cx^{11} \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{11(a + bx^2)} + \frac{ab^2 cx^9 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{3(a + bx^2)} + \frac{3a^2 bcx^7 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{7(a + bx^2)} + \frac{a^3 cx^5 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{5(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^4*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (a^3*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a^2*b*c*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (a*b^2*c*x^9*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (b^3*c*x^11*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(11*(a + b*x^2))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int x^4 \left(c(a + bx^2)^2 \right)^{3/2} dx &= \int x^4 (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int x^4 (abc + b^2cx^2)^3 dx}{b^2c(abc + b^2cx^2)} \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int (a^3b^3c^3x^4 + 3a^2b^4c^3x^6 + 3ab^5c^3x^8 + b^6c^3x^{10}) dx}{b^2c(abc + b^2cx^2)} \\ &= \frac{a^3cx^5\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{5(a + bx^2)} + \frac{3a^2bcx^7\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{7(a + bx^2)} + \frac{ab^2cx^9\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{3(a + bx^2)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{x^5 (231a^3 + 495a^2bx^2 + 385ab^2x^4 + 105b^3x^6) \left(c(a + bx^2)^2 \right)^{3/2}}{1155 (a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4*(c*(a + b*x^2)^2)^(3/2),x]

[Out] (x^5*(c*(a + b*x^2)^2)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6))/(1155*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.07, size = 74, normalized size = 0.52

$$\frac{x^5 (231a^3 + 495a^2bx^2 + 385ab^2x^4 + 105b^3x^6) (a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{1155 (a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4*(c*(a + b*x^2)^2)^(3/2),x]

[Out] (x^5*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(231*a^3 + 495*a^2*b*x^2 + 385*a*b^2*x^4 + 105*b^3*x^6))/(1155*(a + b*x^2)^3)

fricas [A] time = 0.87, size = 74, normalized size = 0.52

$$\frac{(105 b^3 c x^{11} + 385 a b^2 c x^9 + 495 a^2 b c x^7 + 231 a^3 c x^5) \sqrt{b^2 c x^4 + 2 a b c x^2 + a^2 c}}{1155 (b x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")

[Out] 1/1155*(105*b^3*c*x^11 + 385*a*b^2*c*x^9 + 495*a^2*b*c*x^7 + 231*a^3*c*x^5)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.29, size = 72, normalized size = 0.50

$$\frac{1}{1155} (105 b^3 x^{11} \operatorname{sgn}(b x^2 + a) + 385 a b^2 x^9 \operatorname{sgn}(b x^2 + a) + 495 a^2 b x^7 \operatorname{sgn}(b x^2 + a) + 231 a^3 x^5 \operatorname{sgn}(b x^2 + a)) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")

[Out] 1/1155*(105*b^3*x^11*sgn(b*x^2 + a) + 385*a*b^2*x^9*sgn(b*x^2 + a) + 495*a^2*b*x^7*sgn(b*x^2 + a) + 231*a^3*x^5*sgn(b*x^2 + a))*c^(3/2)

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{(105b^3x^6 + 385ab^2x^4 + 495a^2bx^2 + 231a^3) \left((bx^2 + a)^2 c \right)^{\frac{3}{2}} x^5}{1155 (bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*((b*x^2+a)^2*c)^(3/2),x)

[Out] 1/1155*x^5*(105*b^3*x^6+385*a*b^2*x^4+495*a^2*b*x^2+231*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^2+a)^3

maxima [A] time = 1.22, size = 47, normalized size = 0.33

$$\frac{1}{11} b^3 c^{\frac{3}{2}} x^{11} + \frac{1}{3} a b^2 c^{\frac{3}{2}} x^9 + \frac{3}{7} a^2 b c^{\frac{3}{2}} x^7 + \frac{1}{5} a^3 c^{\frac{3}{2}} x^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")

[Out] 1/11*b^3*c^(3/2)*x^11 + 1/3*a*b^2*c^(3/2)*x^9 + 3/7*a^2*b*c^(3/2)*x^7 + 1/5*a^3*c^(3/2)*x^5

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^4 \left(c (b x^2 + a)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(c*(a + b*x^2)^2)^(3/2),x)

[Out] int(x^4*(c*(a + b*x^2)^2)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4*(c*(b*x**2+a)**2)**(3/2),x)

[Out] Timed out

$$3.70 \quad \int x^3 \left(c(a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=66

$$\frac{c(a + bx^2)^4 \sqrt{c(a + bx^2)^2}}{10b^2} - \frac{ac(a + bx^2)^3 \sqrt{c(a + bx^2)^2}}{8b^2}$$

Rubi [A] time = 0.11, antiderivative size = 78, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1989, 1111, 640, 609}

$$\frac{(a^2c + 2abcx^2 + b^2cx^4)^{5/2}}{10b^2c} - \frac{a(a + bx^2)(a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{8b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c*(a + b*x^2)^2)^(3/2), x]

[Out] -(a*(a + b*x^2)*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(8*b^2) + (a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(5/2)/(10*b^2*c)

Rule 609

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && NeQ[p, -2^(-1)]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1111

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2] && IntegerQ[(m - 1)/2] && (GtQ[m, 0] || LtQ[0, 4*p, -m - 1])

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int x^3 \left(c(a + bx^2)^2 \right)^{3/2} dx &= \int x^3 (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx \\
&= \frac{1}{2} \text{Subst} \left(\int x (a^2c + 2abcx + b^2cx^2)^{3/2} dx, x, x^2 \right) \\
&= \frac{(a^2c + 2abcx^2 + b^2cx^4)^{5/2}}{10b^2c} - \frac{a \text{Subst} \left(\int (a^2c + 2abcx + b^2cx^2)^{3/2} dx, x, x^2 \right)}{2b} \\
&= -\frac{a(a + bx^2)(a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{8b^2} + \frac{(a^2c + 2abcx^2 + b^2cx^4)^{5/2}}{10b^2c}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.95

$$\frac{x^4 (10a^3 + 20a^2bx^2 + 15ab^2x^4 + 4b^3x^6) \left(c(a + bx^2)^2 \right)^{3/2}}{40(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^4*(c*(a + b*x^2)^2)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6))/(40*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.76, size = 74, normalized size = 1.12

$$\frac{x^4 (10a^3 + 20a^2bx^2 + 15ab^2x^4 + 4b^3x^6) (a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{40(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^4*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(10*a^3 + 20*a^2*b*x^2 + 15*a*b^2*x^4 + 4*b^3*x^6))/(40*(a + b*x^2)^3)

fricas [A] time = 0.92, size = 74, normalized size = 1.12

$$\frac{(4b^3cx^{10} + 15ab^2cx^8 + 20a^2bcx^6 + 10a^3cx^4) \sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{40(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2), x, algorithm="fricas")

[Out] 1/40*(4*b^3*c*x^10 + 15*a*b^2*c*x^8 + 20*a^2*b*c*x^6 + 10*a^3*c*x^4)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.30, size = 48, normalized size = 0.73

$$\frac{1}{40} (4b^3x^{10} + 15ab^2x^8 + 20a^2bx^6 + 10a^3x^4) c^{\frac{3}{2}} \text{sgn}(bx^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*(b*x^2+a)^2)^(3/2), x, algorithm="giac")

[Out] $\frac{1}{40} \cdot (4b^3x^{10} + 15a^2b^2x^8 + 20a^2bx^6 + 10a^3x^4) \cdot c^{3/2} \cdot \text{sgn}(bx^2 + a)$

maple [A] time = 0.01, size = 60, normalized size = 0.91

$$\frac{(4b^3x^6 + 15a^2b^2x^4 + 20a^2bx^2 + 10a^3) \left((bx^2 + a)^2 c \right)^{3/2} x^4}{40 (bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*((b*x^2+a)^2*c)^(3/2),x)`

[Out] $\frac{1}{40} x^4 (4b^3x^6 + 15a^2b^2x^4 + 20a^2bx^2 + 10a^3) \cdot ((bx^2+a)^2c)^{3/2} / (bx^2+a)^3$

maxima [A] time = 1.04, size = 98, normalized size = 1.48

$$-\frac{(b^2cx^4 + 2abcx^2 + a^2c)^{3/2} ax^2}{8b} - \frac{(b^2cx^4 + 2abcx^2 + a^2c)^{3/2} a^2}{8b^2} + \frac{(b^2cx^4 + 2abcx^2 + a^2c)^{5/2}}{10b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")`

[Out] $-\frac{1}{8} (b^2cx^4 + 2abcx^2 + a^2c)^{3/2} \cdot ax^2/b - \frac{1}{8} (b^2cx^4 + 2abcx^2 + a^2c)^{5/2} \cdot a^2/b^2 + \frac{1}{10} (b^2cx^4 + 2abcx^2 + a^2c)^{5/2} / (b^2c)$

mupad [B] time = 2.83, size = 50, normalized size = 0.76

$$\frac{(-a^2 + 3abx^2 + 4b^2x^4) (ca^2 + 2cabx^2 + cb^2x^4)^{3/2}}{40b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(c*(a+b*x^2)^2)^(3/2),x)`

[Out] $((4b^2x^4 - a^2 + 3abx^2) \cdot (a^2c + b^2cx^4 + 2abcx^2)^{3/2}) / (40b^2)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(c*(b*x**2+a)**2)**(3/2),x)`

[Out] Timed out

$$3.71 \quad \int x^2 \left(c(a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=143

$$\frac{a^3 cx^3 \sqrt{c(a + bx^2)^2}}{3(a + bx^2)} + \frac{3a^2 bcx^5 \sqrt{c(a + bx^2)^2}}{5(a + bx^2)} + \frac{b^3 cx^9 \sqrt{c(a + bx^2)^2}}{9(a + bx^2)} + \frac{3ab^2 cx^7 \sqrt{c(a + bx^2)^2}}{7(a + bx^2)}$$

Rubi [A] time = 0.10, antiderivative size = 187, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1989, 1112, 270}

$$\frac{b^3 cx^9 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{9(a + bx^2)} + \frac{3ab^2 cx^7 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{7(a + bx^2)} + \frac{3a^2 bcx^5 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{5(a + bx^2)} + \frac{a^3 cx^3 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{3(a + bx^2)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (a^3*c*x^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(3*(a + b*x^2)) + (3*a^2*b*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2)) + (3*a*b^2*c*x^7*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(7*(a + b*x^2)) + (b^3*c*x^9*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(9*(a + b*x^2))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int x^2 \left(c(a + bx^2)^2 \right)^{3/2} dx &= \int x^2 (a^2c + 2abcx^2 + b^2cx^4)^{3/2} dx \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int x^2 (abc + b^2cx^2)^3 dx}{b^2c(abc + b^2cx^2)} \\ &= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int (a^3b^3c^3x^2 + 3a^2b^4c^3x^4 + 3ab^5c^3x^6 + b^6c^3x^8) dx}{b^2c(abc + b^2cx^2)} \\ &= \frac{a^3cx^3\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{3(a + bx^2)} + \frac{3a^2bcx^5\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{5(a + bx^2)} + \frac{3ab^2cx^7\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{7(a + bx^2)} + \frac{b^3cx^9\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{9(a + bx^2)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 63, normalized size = 0.44

$$\frac{(105a^3x^3 + 189a^2bx^5 + 135ab^2x^7 + 35b^3x^9) \left(c(a + bx^2)^2 \right)^{3/2}}{315(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c*(a + b*x^2)^2)^(3/2),x]

[Out] ((c*(a + b*x^2)^2)^(3/2)*(105*a^3*x^3 + 189*a^2*b*x^5 + 135*a*b^2*x^7 + 35*b^3*x^9))/(315*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.07, size = 74, normalized size = 0.52

$$\frac{x^3(105a^3 + 189a^2bx^2 + 135ab^2x^4 + 35b^3x^6)(a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{315(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(c*(a + b*x^2)^2)^(3/2),x]

[Out] (x^3*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(105*a^3 + 189*a^2*b*x^2 + 135*a*b^2*x^4 + 35*b^3*x^6))/(315*(a + b*x^2)^3)

fricas [A] time = 0.95, size = 74, normalized size = 0.52

$$\frac{(35b^3cx^9 + 135ab^2cx^7 + 189a^2bcx^5 + 105a^3cx^3)\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{315(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="fricas")

[Out] 1/315*(35*b^3*c*x^9 + 135*a*b^2*c*x^7 + 189*a^2*b*c*x^5 + 105*a^3*c*x^3)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.27, size = 72, normalized size = 0.50

$$\frac{1}{315} (35b^3x^9 \operatorname{sgn}(bx^2 + a) + 135ab^2x^7 \operatorname{sgn}(bx^2 + a) + 189a^2bx^5 \operatorname{sgn}(bx^2 + a) + 105a^3x^3 \operatorname{sgn}(bx^2 + a))c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="giac")

[Out] 1/315*(35*b^3*x^9*sgn(b*x^2 + a) + 135*a*b^2*x^7*sgn(b*x^2 + a) + 189*a^2*b*x^5*sgn(b*x^2 + a) + 105*a^3*x^3*sgn(b*x^2 + a))*c^(3/2)

maple [A] time = 0.01, size = 60, normalized size = 0.42

$$\frac{(35b^3x^6 + 135ab^2x^4 + 189a^2bx^2 + 105a^3) \left((bx^2 + a)^2 c \right)^{\frac{3}{2}} x^3}{315(bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((b*x^2+a)^2*c)^(3/2),x)

[Out] 1/315*x^3*(35*b^3*x^6+135*a*b^2*x^4+189*a^2*b*x^2+105*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^2+a)^3

maxima [A] time = 1.20, size = 47, normalized size = 0.33

$$\frac{1}{9} b^3 c^{\frac{3}{2}} x^9 + \frac{3}{7} a b^2 c^{\frac{3}{2}} x^7 + \frac{3}{5} a^2 b c^{\frac{3}{2}} x^5 + \frac{1}{3} a^3 c^{\frac{3}{2}} x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")

[Out] 1/9*b^3*c^(3/2)*x^9 + 3/7*a*b^2*c^(3/2)*x^7 + 3/5*a^2*b*c^(3/2)*x^5 + 1/3*a^3*c^(3/2)*x^3

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(c (b x^2 + a)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*(a + b*x^2)^2)^(3/2),x)

[Out] int(x^2*(c*(a + b*x^2)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(c (a + b x^2)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*(b*x**2+a)**2)**(3/2),x)

[Out] Integral(x**2*(c*(a + b*x**2)**2)**(3/2), x)

$$3.72 \quad \int x \left(c (a + bx^2)^2 \right)^{3/2} dx$$

Optimal. Leaf size=32

$$\frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^2}}{8b}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1591, 15, 30}

$$\frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^2}}{8b}$$

Antiderivative was successfully verified.

[In] Int[x*(c*(a + b*x^2)^2)^(3/2),x]

[Out] (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^2])/(8*b)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int x \left(c (a + bx^2)^2 \right)^{3/2} dx &= \frac{\text{Subst} \left(\int (cx^2)^{3/2} dx, x, a + bx^2 \right)}{2b} \\ &= \frac{\left(c \sqrt{c(a+bx^2)^2} \right) \text{Subst} \left(\int x^3 dx, x, a + bx^2 \right)}{2b(a+bx^2)} \\ &= \frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^2}}{8b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.91

$$\frac{(a+bx^2) \left(c(a+bx^2)^2 \right)^{3/2}}{8b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c*(a + b*x^2)^2)^(3/2), x]

[Out] ((a + b*x^2)*(c*(a + b*x^2)^2)^(3/2))/(8*b)

IntegrateAlgebraic [B] time = 0.70, size = 73, normalized size = 2.28

$$\frac{x^2 (4a^3 + 6a^2bx^2 + 4ab^2x^4 + b^3x^6) (a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{8(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x^2*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(4*a^3 + 6*a^2*b*x^2 + 4*a*b^2*x^4 + b^3*x^6))/(8*(a + b*x^2)^3)

fricas [B] time = 0.80, size = 73, normalized size = 2.28

$$\frac{(b^3cx^8 + 4ab^2cx^6 + 6a^2bcx^4 + 4a^3cx^2)\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{8(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^2)^(3/2), x, algorithm="fricas")

[Out] 1/8*(b^3*c*x^8 + 4*a*b^2*c*x^6 + 6*a^2*b*c*x^4 + 4*a^3*c*x^2)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.25, size = 25, normalized size = 0.78

$$\frac{(bx^2 + a)^4 c^{\frac{3}{2}} \operatorname{sgn}(bx^2 + a)}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^2)^(3/2), x, algorithm="giac")

[Out] 1/8*(b*x^2 + a)^4*c^(3/2)*sgn(b*x^2 + a)/b

maple [B] time = 0.01, size = 59, normalized size = 1.84

$$\frac{(b^3x^6 + 4ab^2x^4 + 6a^2bx^2 + 4a^3)\left((bx^2 + a)^2c\right)^{\frac{3}{2}}x^2}{8(bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^2+a)^2*c)^(3/2), x)

[Out] 1/8*x^2*(b^3*x^6+4*a*b^2*x^4+6*a^2*b*x^2+4*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^2+a)^3

maxima [B] time = 0.93, size = 60, normalized size = 1.88

$$\frac{1}{8} (b^2cx^4 + 2abcx^2 + a^2c)^{\frac{3}{2}} x^2 + \frac{(b^2cx^4 + 2abcx^2 + a^2c)^{\frac{3}{2}} a}{8b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{8}(b^2cx^4 + 2abcx^2 + a^2c)^{3/2}x^2 + \frac{1}{8}(b^2cx^4 + 2abcx^2 + a^2c)^{3/2}a/b$

mupad [B] time = 2.84, size = 40, normalized size = 1.25

$$\frac{(b^2x^2 + ab)(ca^2 + 2cabx^2 + cb^2x^4)^{3/2}}{8b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(c*(a + b*x^2)^2)^(3/2),x)

[Out] $((a*b + b^2*x^2)*(a^2*c + b^2*c*x^4 + 2*a*b*c*x^2)^{3/2})/(8*b^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(c(a + bx^2)^2 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x**2+a)**2)**(3/2),x)

[Out] Integral(x*(c*(a + b*x**2)**2)**(3/2), x)

$$3.73 \quad \int \left(c \left(a + bx^2 \right)^2 \right)^{3/2} dx$$

Optimal. Leaf size=135

$$\frac{a^3 cx \sqrt{c(a+bx^2)^2}}{a+bx^2} + \frac{a^2 bcx^3 \sqrt{c(a+bx^2)^2}}{a+bx^2} + \frac{b^3 cx^7 \sqrt{c(a+bx^2)^2}}{7(a+bx^2)} + \frac{3ab^2 cx^5 \sqrt{c(a+bx^2)^2}}{5(a+bx^2)}$$

Rubi [A] time = 0.05, antiderivative size = 175, normalized size of antiderivative = 1.30, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1988, 1088, 194}

$$\frac{b^3 x^7 (a^2 c + 2abcx^2 + b^2 cx^4)^{3/2}}{7(a+bx^2)^3} + \frac{3ab^2 x^5 (a^2 c + 2abcx^2 + b^2 cx^4)^{3/2}}{5(a+bx^2)^3} + \frac{a^2 bx^3 (a^2 c + 2abcx^2 + b^2 cx^4)^{3/2}}{(a+bx^2)^3} + \frac{a^3 x (a^2 c + 2abcx^2 + b^2 cx^4)^{3/2}}{(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^2)^(3/2), x]

[Out] (a^3*x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (a^2*b*x^3*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(a + b*x^2)^3 + (3*a*b^2*x^5*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(5*(a + b*x^2)^3) + (b^3*x^7*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2))/(7*(a + b*x^2)^3)

Rule 194

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]

Rule 1088

Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^p/(b + 2*c*x^2)^(2*p), Int[(b + 2*c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1988

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \left(c \left(a + bx^2 \right)^2 \right)^{3/2} dx &= \int \left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2} dx \\ &= \frac{\left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2} \int \left(2abc + 2b^2 cx^2 \right)^3 dx}{\left(2abc + 2b^2 cx^2 \right)^3} \\ &= \frac{\left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2} \int \left(8a^3 b^3 c^3 + 24a^2 b^4 c^3 x^2 + 24ab^5 c^3 x^4 + 8b^6 c^3 x^6 \right) dx}{\left(2abc + 2b^2 cx^2 \right)^3} \\ &= \frac{a^3 x \left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2}}{\left(a + bx^2 \right)^3} + \frac{a^2 bx^3 \left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2}}{\left(a + bx^2 \right)^3} + \frac{3ab^2 x^5 \left(a^2 c + 2abcx^2 + b^2 cx^4 \right)^{3/2}}{5 \left(a + bx^2 \right)^3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 61, normalized size = 0.45

$$\frac{(35a^3x + 35a^2bx^3 + 21ab^2x^5 + 5b^3x^7) \left(c(a + bx^2)^2 \right)^{3/2}}{35(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^2)^(3/2), x]

[Out] ((c*(a + b*x^2)^2)^(3/2)*(35*a^3*x + 35*a^2*b*x^3 + 21*a*b^2*x^5 + 5*b^3*x^7))/(35*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 0.53

$$\frac{x(35a^3 + 35a^2bx^2 + 21ab^2x^4 + 5b^3x^6) \left(a^2c + 2abcx^2 + b^2cx^4 \right)^{3/2}}{35(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2), x]

[Out] (x*(a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(35*a^3 + 35*a^2*b*x^2 + 21*a*b^2*x^4 + 5*b^3*x^6))/(35*(a + b*x^2)^3)

fricas [A] time = 0.67, size = 72, normalized size = 0.53

$$\frac{(5b^3cx^7 + 21ab^2cx^5 + 35a^2bcx^3 + 35a^3cx) \sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{35(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2), x, algorithm="fricas")

[Out] 1/35*(5*b^3*c*x^7 + 21*a*b^2*c*x^5 + 35*a^2*b*c*x^3 + 35*a^3*c*x)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.33, size = 46, normalized size = 0.34

$$\frac{1}{35} (5b^3x^7 + 21ab^2x^5 + 35a^2bx^3 + 35a^3x) c^{\frac{3}{2}} \operatorname{sgn}(bx^2 + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2), x, algorithm="giac")

[Out] 1/35*(5*b^3*x^7 + 21*a*b^2*x^5 + 35*a^2*b*x^3 + 35*a^3*x)*c^(3/2)*sgn(b*x^2 + a)

maple [A] time = 0.00, size = 58, normalized size = 0.43

$$\frac{(5b^3x^6 + 21ab^2x^4 + 35a^2bx^2 + 35a^3) \left((bx^2 + a)^2 c \right)^{\frac{3}{2}} x}{35(bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^2*c)^(3/2), x)

[Out] 1/35*x*(5*b^3*x^6+21*a*b^2*x^4+35*a^2*b*x^2+35*a^3)*((b*x^2+a)^2*c)^(3/2)/(b*x^2+a)^3

maxima [A] time = 0.96, size = 43, normalized size = 0.32

$$\frac{1}{7} b^3 c^{\frac{3}{2}} x^7 + \frac{3}{5} a b^2 c^{\frac{3}{2}} x^5 + a^2 b c^{\frac{3}{2}} x^3 + a^3 c^{\frac{3}{2}} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2),x, algorithm="maxima")

[Out] 1/7*b^3*c^(3/2)*x^7 + 3/5*a*b^2*c^(3/2)*x^5 + a^2*b*c^(3/2)*x^3 + a^3*c^(3/2)*x

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(c (b x^2 + a)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^2)^(3/2),x)

[Out] int((c*(a + b*x^2)^2)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c (a + b x^2)^2 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**2)**(3/2),x)

[Out] Integral((c*(a + b*x**2)**2)**(3/2), x)

$$3.74 \quad \int \frac{(c(ax^2+b)^2)^{3/2}}{x} dx$$

Optimal. Leaf size=139

$$\frac{a^3 c \log(x) \sqrt{c(a+bx^2)^2}}{a+bx^2} + \frac{3a^2 bcx^2 \sqrt{c(a+bx^2)^2}}{2(a+bx^2)} + \frac{b^3 cx^6 \sqrt{c(a+bx^2)^2}}{6(a+bx^2)} + \frac{3ab^2 cx^4 \sqrt{c(a+bx^2)^2}}{4(a+bx^2)}$$

Rubi [A] time = 0.10, antiderivative size = 183, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3 cx^6 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{6(a+bx^2)} + \frac{3ab^2 cx^4 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{4(a+bx^2)} + \frac{3a^2 bcx^2 \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{2(a+bx^2)} + \frac{a^3 c \log(x) \sqrt{a^2 c + 2abcx^2 + b^2 cx^4}}{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^2)^(3/2)/x,x]

[Out] (3*a^2*b*c*x^2*sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(2*(a + b*x^2)) + (3*a*b^2*c*x^4*sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(4*(a + b*x^2)) + (b^3*c*x^6*sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(6*(a + b*x^2)) + (a^3*c*sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*Log[x])/(a + b*x^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{(c(a+bx^2)^2)^{3/2}}{x} dx &= \int \frac{(a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{x} dx \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int \frac{(abc+b^2cx^2)^3}{x} dx}{b^2c(abc + b^2cx^2)} \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \operatorname{Subst}\left(\int \frac{(abc+b^2cx)^3}{x} dx, x, x^2\right)}{2b^2c(abc + b^2cx^2)} \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \operatorname{Subst}\left(\int \left(3a^2b^4c^3 + \frac{a^3b^3c^3}{x} + 3ab^5c^3x + b^6c^3x^2\right) dx, x, x^2\right)}{2b^2c(abc + b^2cx^2)} \\
&= \frac{3a^2bcx^2\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{2(a+bx^2)} + \frac{3ab^2cx^4\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{4(a+bx^2)} + \frac{b^3cx^6\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{6(a+bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 62, normalized size = 0.45

$$\frac{(c(a+bx^2)^2)^{3/2} (12a^3 \log(x) + bx^2(18a^2 + 9abx^2 + 2b^2x^4))}{12(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x, x]

[Out] ((c*(a + b*x^2)^2)^(3/2)*(b*x^2*(18*a^2 + 9*a*b*x^2 + 2*b^2*x^4) + 12*a^3*Log[x]))/(12*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.57, size = 263, normalized size = 1.89

$$\frac{1}{24}\sqrt{a^2c + 2abcx^2 + b^2cx^4} (11a^2c + 7abcx^2 + 2b^2cx^4) + \frac{1}{24}(-18a^2cx^2\sqrt{b^2c} - 9abcx^4\sqrt{b^2c} - 2b^2cx^6\sqrt{b^2c}) + \frac{1}{2}a^3c^{3/2} \operatorname{tanh}^{-1}\left(\frac{x^2\sqrt{b^2c}}{a\sqrt{c}} - \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{a\sqrt{c}}\right) - \frac{a^3c\sqrt{b^2c} \log(x^2(abc + b^2cx^2) - x^2\sqrt{b^2c}\sqrt{a^2c + 2abcx^2 + b^2cx^4})}{4b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x, x]

[Out] (Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*(11*a^2*c + 7*a*b*c*x^2 + 2*b^2*c*x^4))/24 + (-18*a^2*c*Sqrt[b^2*c]*x^2 - 9*a*b*c*Sqrt[b^2*c]*x^4 - 2*b^2*c*Sqrt[b^2*c]*x^6)/24 + (a^3*c^(3/2)*ArcTanh[(Sqrt[b^2*c]*x^2)/(a*Sqrt[c]) - Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(a*Sqrt[c])])/2 - (a^3*c*Sqrt[b^2*c]*Log[x^2*(a*b*c + b^2*c*x^2) - Sqrt[b^2*c]*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]])/(4*b)

fricas [A] time = 0.72, size = 73, normalized size = 0.53

$$\frac{(2b^3cx^6 + 9ab^2cx^4 + 18a^2bcx^2 + 12a^3c \log(x))\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{12(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x, x, algorithm="fricas")

[Out] 1/12*(2*b^3*c*x^6 + 9*a*b^2*c*x^4 + 18*a^2*b*c*x^2 + 12*a^3*c*log(x))*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^2 + a)

giac [A] time = 0.35, size = 73, normalized size = 0.53

$$\frac{1}{12} \left(2b^3x^6 \operatorname{sgn}(bx^2 + a) + 9ab^2x^4 \operatorname{sgn}(bx^2 + a) + 18a^2bx^2 \operatorname{sgn}(bx^2 + a) + 6a^3 \log(x^2) \operatorname{sgn}(bx^2 + a) \right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="giac")

[Out] 1/12*(2*b^3*x^6*sgn(b*x^2 + a) + 9*a*b^2*x^4*sgn(b*x^2 + a) + 18*a^2*b*x^2*sgn(b*x^2 + a) + 6*a^3*log(x^2)*sgn(b*x^2 + a))*c^(3/2)

maple [A] time = 0.02, size = 59, normalized size = 0.42

$$\frac{\left((bx^2 + a)^2 c \right)^{\frac{3}{2}} \left(2b^3x^6 + 9ab^2x^4 + 18a^2bx^2 + 12a^3 \ln(x) \right)}{12 (bx^2 + a)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^2*c)^(3/2)/x,x)

[Out] 1/12*((b*x^2+a)^2*c)^(3/2)*(2*b^3*x^6+9*a*b^2*x^4+18*a^2*b*x^2+12*a^3*ln(x))/((b*x^2+a)^3)

maxima [A] time = 0.98, size = 171, normalized size = 1.23

$$\frac{1}{2} (-1)^{2b^2cx^2+2abc} a^3 c^{\frac{3}{2}} \log(2b^2cx^2 + 2abc) - \frac{1}{2} (-1)^{2abx^2+2a^2c} a^3 c^{\frac{3}{2}} \log\left(2abc + \frac{2a^2c}{x^2}\right) + \frac{1}{4} \sqrt{b^2cx^4 + 2abx^2 + a^2c} abx^2 + \frac{3}{4} \sqrt{b^2cx^4 + 2abx^2 + a^2c} a^2c + \frac{1}{6} (b^2cx^4 + 2abx^2 + a^2c)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x,x, algorithm="maxima")

[Out] 1/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^3*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 1/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^3*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 1/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c*x^2 + 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a^2*c + 1/6*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c (bx^2 + a)^2 \right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^2)^(3/2)/x,x)

[Out] int((c*(a + b*x^2)^2)^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c (a + bx^2)^2 \right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**2)**(3/2)/x,x)

[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x, x)

$$3.75 \quad \int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2} dx$$

Optimal. Leaf size=134

$$-\frac{a^3c\sqrt{c(a+bx^2)^2}}{x(a+bx^2)} + \frac{3a^2bcx\sqrt{c(a+bx^2)^2}}{a+bx^2} + \frac{b^3cx^5\sqrt{c(a+bx^2)^2}}{5(a+bx^2)} + \frac{ab^2cx^3\sqrt{c(a+bx^2)^2}}{a+bx^2}$$

Rubi [A] time = 0.09, antiderivative size = 178, normalized size of antiderivative = 1.33, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1989, 1112, 270}

$$\frac{b^3cx^5\sqrt{a^2c+2abcx^2+b^2cx^4}}{5(a+bx^2)} + \frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2} + \frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2} - \frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x(a+bx^2)}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^2,x]

[Out] -((a^3*c*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(x*(a + b*x^2))) + (3*a^2*b*c*x*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (a*b^2*c*x^3*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(a + b*x^2) + (b^3*c*x^5*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4])/(5*(a + b*x^2))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{\left(c(a+bx^2)^2\right)^{3/2}}{x^2} dx &= \int \frac{\left(a^2c+2abcx^2+b^2cx^4\right)^{3/2}}{x^2} dx \\ &= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \frac{(abc+b^2cx^2)^3}{x^2} dx}{b^2c(abc+b^2cx^2)} \\ &= \frac{\sqrt{a^2c+2abcx^2+b^2cx^4} \int \left(3a^2b^4c^3 + \frac{a^3b^3c^3}{x^2} + 3ab^5c^3x^2 + b^6c^3x^4\right) dx}{b^2c(abc+b^2cx^2)} \\ &= -\frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{x(a+bx^2)} + \frac{3a^2bcx\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2} + \frac{ab^2cx^3\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.46

$$\frac{(-5a^3 + 15a^2bx^2 + 5ab^2x^4 + b^3x^6) \left(c(a + bx^2)^2 \right)^{3/2}}{5x(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^2,x]

[Out] ((c*(a + b*x^2)^2)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*(a + b*x^2)^3)

IntegrateAlgebraic [A] time = 0.10, size = 73, normalized size = 0.54

$$\frac{(-5a^3 + 15a^2bx^2 + 5ab^2x^4 + b^3x^6) (a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{5x(a + bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x^2,x]

[Out] ((a^2*c + 2*a*b*c*x^2 + b^2*c*x^4)^(3/2)*(-5*a^3 + 15*a^2*b*x^2 + 5*a*b^2*x^4 + b^3*x^6))/(5*x*(a + b*x^2)^3)

fricas [A] time = 0.48, size = 72, normalized size = 0.54

$$\frac{(b^3cx^6 + 5ab^2cx^4 + 15a^2bcx^2 - 5a^3c)\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{5(bx^3 + ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="fricas")

[Out] 1/5*(b^3*c*x^6 + 5*a*b^2*c*x^4 + 15*a^2*b*c*x^2 - 5*a^3*c)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^3 + a*x)

giac [A] time = 0.24, size = 69, normalized size = 0.51

$$\frac{1}{5} \left(b^3x^5 \operatorname{sgn}(bx^2 + a) + 5ab^2x^3 \operatorname{sgn}(bx^2 + a) + 15a^2bx \operatorname{sgn}(bx^2 + a) - \frac{5a^3 \operatorname{sgn}(bx^2 + a)}{x} \right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="giac")

[Out] 1/5*(b^3*x^5*sgn(b*x^2 + a) + 5*a*b^2*x^3*sgn(b*x^2 + a) + 15*a^2*b*x*sgn(b*x^2 + a) - 5*a^3*sgn(b*x^2 + a)/x)*c^(3/2)

maple [A] time = 0.01, size = 60, normalized size = 0.45

$$\frac{(-b^3x^6 - 5ab^2x^4 - 15a^2bx^2 + 5a^3) \left((bx^2 + a)^2 c \right)^{\frac{3}{2}}}{5(bx^2 + a)^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^2*c)^(3/2)/x^2,x)

[Out] $-1/5*(-b^3*x^6-5*a*b^2*x^4-15*a^2*b*x^2+5*a^3)*((b*x^2+a)^2*c)^{(3/2)}/x/(b*x^2+a)^3$

maxima [A] time = 0.97, size = 48, normalized size = 0.36

$$\frac{b^3 c^{\frac{3}{2}} x^6 + 5 a b^2 c^{\frac{3}{2}} x^4 + 15 a^2 b c^{\frac{3}{2}} x^2 - 5 a^3 c^{\frac{3}{2}}}{5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*(b*x^2+a)^2)^(3/2)/x^2,x, algorithm="maxima")`

[Out] $1/5*(b^3*c^{(3/2)}*x^6 + 5*a*b^2*c^{(3/2)}*x^4 + 15*a^2*b*c^{(3/2)}*x^2 - 5*a^3*c^{(3/2)})/x$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c(bx^2 + a)^2\right)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*(a + b*x^2)^2)^(3/2)/x^2,x)`

[Out] `int((c*(a + b*x^2)^2)^(3/2)/x^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a + bx^2)^2\right)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*(b*x**2+a)**2)**(3/2)/x**2,x)`

[Out] `Integral((c*(a + b*x**2)**2)**(3/2)/x**2, x)`

$$3.76 \quad \int \frac{(c(a+bx^2)^2)^{3/2}}{x^3} dx$$

Optimal. Leaf size=140

$$-\frac{a^3c\sqrt{c(a+bx^2)^2}}{2x^2(a+bx^2)} + \frac{3a^2bc\log(x)\sqrt{c(a+bx^2)^2}}{a+bx^2} + \frac{b^3cx^4\sqrt{c(a+bx^2)^2}}{4(a+bx^2)} + \frac{3ab^2cx^2\sqrt{c(a+bx^2)^2}}{2(a+bx^2)}$$

Rubi [A] time = 0.10, antiderivative size = 184, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1989, 1112, 266, 43}

$$\frac{b^3cx^4\sqrt{a^2c+2abcx^2+b^2cx^4}}{4(a+bx^2)} + \frac{3ab^2cx^2\sqrt{a^2c+2abcx^2+b^2cx^4}}{2(a+bx^2)} - \frac{a^3c\sqrt{a^2c+2abcx^2+b^2cx^4}}{2x^2(a+bx^2)} + \frac{3a^2bc\log(x)\sqrt{a^2c+2abcx^2+b^2cx^4}}{a+bx^2}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^2)^(3/2)/x^3,x]

[Out] $-(a^3c\sqrt{a^2c+2abx^2c+b^2cx^4})/(2x^2(a+bx^2)) + (3a^2bc\log(x)\sqrt{a^2c+2abx^2c+b^2cx^4})/(2(a+bx^2)) + (b^3cx^4\sqrt{a^2c+2abx^2c+b^2cx^4})/(4(a+bx^2)) + (3a^2bc\sqrt{a^2c+2abx^2c+b^2cx^4})\log(x)/(a+bx^2)$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1112

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(d*x)^m*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, m, p}, x] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p - 1/2]

Rule 1989

Int[(u_)^(p_.)*((d_.)*(x_))^(m_.), x_Symbol] := Int[(d*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{d, m, p}, x] && TrinomialQ[u, x] && !TrinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{(c(a+bx^2)^2)^{3/2}}{x^3} dx &= \int \frac{(a^2c + 2abcx^2 + b^2cx^4)^{3/2}}{x^3} dx \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \int \frac{(abc+b^2cx^2)^3}{x^3} dx}{b^2c(abc + b^2cx^2)} \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \operatorname{Subst}\left(\int \frac{(abc+b^2cx)^3}{x^2} dx, x, x^2\right)}{2b^2c(abc + b^2cx^2)} \\
&= \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4} \operatorname{Subst}\left(\int \left(3ab^5c^3 + \frac{a^3b^3c^3}{x^2} + \frac{3a^2b^4c^3}{x} + b^6c^3x\right) dx, x, x^2\right)}{2b^2c(abc + b^2cx^2)} \\
&= -\frac{a^3c\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{2x^2(a + bx^2)} + \frac{3ab^2cx^2\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{2(a + bx^2)} + \frac{b^3cx^4\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{4(a + bx^2)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 65, normalized size = 0.46

$$\frac{(c(a+bx^2)^2)^{3/2} (2a^3 - 12a^2bx^2 \log(x) - 6ab^2x^4 - b^3x^6)}{4x^2(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^2)^(3/2)/x^3, x]

[Out] -1/4*((c*(a + b*x^2)^2)^(3/2)*(2*a^3 - 6*a*b^2*x^4 - b^3*x^6 - 12*a^2*b*x^2*Log[x]))/(x^2*(a + b*x^2)^3)

IntegrateAlgebraic [B] time = 1.02, size = 324, normalized size = 2.31

$$\frac{\frac{3}{2}a^2bc^{3/2} \operatorname{tanh}^{-1}\left(\frac{x^2\sqrt{b^2c}}{a\sqrt{c}} - \frac{\sqrt{a^2c + 2abcx^2 + b^2cx^4}}{a\sqrt{c}}\right) - \frac{3}{4}a^2c\sqrt{b^2c} \log\left(x^2(abc + b^2cx^2) - x^2\sqrt{b^2c}\sqrt{a^2c + 2abcx^2 + b^2cx^4}\right) + \frac{c\sqrt{b^2c}(-8a^3 - 21a^2bx^2 + 24ab^2x^4 + 4b^3x^6)\sqrt{a^2c + 2abcx^2 + b^2cx^4} - c(-8a^3 - 21a^2bx^2 + 24ab^2x^4 + 4b^3x^6)(abc + b^2cx^2)}{16x^2\sqrt{b^2c}(a + bx^2) - 16bx^2\sqrt{a^2c + 2abcx^2 + b^2cx^4}}}{4x^2(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^2)^(3/2)/x^3, x]

[Out] (-c*(a*b*c + b^2*c*x^2)*(-8*a^3 - 21*a^2*b*x^2 + 24*a*b^2*x^4 + 4*b^3*x^6) + c*Sqrt[b^2*c]*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]*(-8*a^3 - 21*a^2*b*x^2 + 24*a*b^2*x^4 + 4*b^3*x^6))/(16*Sqrt[b^2*c]*x^2*(a + b*x^2) - 16*b*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]) + (3*a^2*b*c^(3/2)*ArcTanh[(Sqrt[b^2*c]*x^2)/(a*Sqrt[c]) - Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]/(a*Sqrt[c])])/2 - (3*a^2*c*Sqrt[b^2*c]*Log[x^2*(a*b*c + b^2*c*x^2) - Sqrt[b^2*c]*x^2*Sqrt[a^2*c + 2*a*b*c*x^2 + b^2*c*x^4]])/4

fricas [A] time = 0.69, size = 76, normalized size = 0.54

$$\frac{(b^3cx^6 + 6ab^2cx^4 + 12a^2bcx^2 \log(x) - 2a^3c)\sqrt{b^2cx^4 + 2abcx^2 + a^2c}}{4(bx^4 + ax^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3, x, algorithm="fricas")

[Out] 1/4*(b^3*c*x^6 + 6*a*b^2*c*x^4 + 12*a^2*b*c*x^2*log(x) - 2*a^3*c)*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)/(b*x^4 + a*x^2)

giac [A] time = 0.29, size = 91, normalized size = 0.65

$$\frac{1}{4} \left(b^3 x^4 \operatorname{sgn}(bx^2 + a) + 6 ab^2 x^2 \operatorname{sgn}(bx^2 + a) + 6 a^2 b \log(x^2) \operatorname{sgn}(bx^2 + a) - \frac{2(3 a^2 b x^2 \operatorname{sgn}(bx^2 + a) + a^3 \operatorname{sgn}(bx^2 + a))}{x^2} \right) c^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="giac")

[Out] 1/4*(b^3*x^4*sgn(b*x^2 + a) + 6*a*b^2*x^2*sgn(b*x^2 + a) + 6*a^2*b*log(x^2)*sgn(b*x^2 + a) - 2*(3*a^2*b*x^2*sgn(b*x^2 + a) + a^3*sgn(b*x^2 + a))/x^2)*c^(3/2)

maple [A] time = 0.02, size = 61, normalized size = 0.44

$$\frac{\left((bx^2 + a)^2 c \right)^{\frac{3}{2}} \left(b^3 x^6 + 6a b^2 x^4 + 12a^2 b x^2 \ln(x) - 2a^3 \right)}{4 (bx^2 + a)^3 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^2*c)^(3/2)/x^3,x)

[Out] 1/4*((b*x^2+a)^2*c)^(3/2)*(b^3*x^6+6*a*b^2*x^4+12*b*a^2*ln(x)*x^2-2*a^3)/(b*x^2+a)^3/x^2

maxima [A] time = 0.99, size = 176, normalized size = 1.26

$$\frac{3}{2} (-1)^{2b^2cx^2+2abc} a^2bc^{\frac{3}{2}} \log(2b^2cx^2+2abc) - \frac{3}{2} (-1)^{2abcx^2+2a^2c} a^2bc^{\frac{3}{2}} \log\left(2abc + \frac{2a^2c}{x^2}\right) + \frac{3}{4} \sqrt{b^2cx^4+2abcx^2+a^2c} b^2cx^2 + \frac{9}{4} \sqrt{b^2cx^4+2abcx^2+a^2c} abc - \frac{(b^2cx^4+2abcx^2+a^2c)^{\frac{3}{2}}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^2)^(3/2)/x^3,x, algorithm="maxima")

[Out] 3/2*(-1)^(2*b^2*c*x^2 + 2*a*b*c)*a^2*b*c^(3/2)*log(2*b^2*c*x^2 + 2*a*b*c) - 3/2*(-1)^(2*a*b*c*x^2 + 2*a^2*c)*a^2*b*c^(3/2)*log(2*a*b*c + 2*a^2*c/x^2) + 3/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*b^2*c*x^2 + 9/4*sqrt(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)*a*b*c - 1/2*(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)^(3/2)/x^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c (bx^2 + a)^2 \right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^2)^(3/2)/x^3,x)

[Out] int((c*(a + b*x^2)^2)^(3/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c (a + bx^2)^2 \right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**2)**(3/2)/x**3,x)

[Out] Integral((c*(a + b*x**2)**2)**(3/2)/x**3, x)

$$3.77 \quad \int x^2 \left(c(a + bx^2)^3 \right)^{3/2} dx$$

Optimal. Leaf size=253

$$\frac{21a^{9/2}c\sqrt{c(a+bx^2)^3}\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{1024b^{3/2}\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{21a^5cx\sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4cx^3\sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{7}{128}a^3cx^3\sqrt{c(a+bx^2)^3}$$

Rubi [A] time = 0.25, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6720, 279, 321, 217, 206}

$$\frac{21a^6c\sqrt{c(a+bx^2)^3}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{1024b^{3/2}(a+bx^2)^{3/2}} + \frac{21a^5cx\sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4cx^3\sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{7}{128}a^3cx^3\sqrt{c(a+bx^2)^3} + \frac{21}{320}a^2cx^3(a+bx^2)\sqrt{c(a+bx^2)^3} + \frac{3}{40}acx^3(a+bx^2)^2\sqrt{c(a+bx^2)^3} + \frac{1}{12}cx^3(a+bx^2)^3\sqrt{c(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c*(a + b*x^2)^3)^(3/2), x]

[Out] (7*a^3*c*x^3*Sqrt[c*(a + b*x^2)^3])/128 + (21*a^5*c*x*Sqrt[c*(a + b*x^2)^3])/(1024*b*(a + b*x^2)) + (21*a^4*c*x^3*Sqrt[c*(a + b*x^2)^3])/(512*(a + b*x^2)) + (21*a^2*c*x^3*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/320 + (3*a*c*x^3*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/40 + (c*x^3*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/12 - (21*a^6*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(1024*b^(3/2)*(a + b*x^2)^(3/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 279

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*(a + b*x^n)^p/(c*(m + n*p + 1)), x] + Dist[(a*n*p)/(m + n*p + 1), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ

[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
 \int x^2 \left(c(a+bx^2)^3 \right)^{3/2} dx &= \frac{\left(c\sqrt{c(a+bx^2)^3} \right) \int x^2 (a+bx^2)^{9/2} dx}{(a+bx^2)^{3/2}} \\
 &= \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(3ac\sqrt{c(a+bx^2)^3} \right) \int x^2 (a+bx^2)^{7/2} dx}{4(a+bx^2)^{3/2}} \\
 &= \frac{3}{40} acx^3 (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(21a^2c\sqrt{c(a+bx^2)^3} \right) \int x^2 (a+bx^2)^{5/2} dx}{8(a+bx^2)^{3/2}} \\
 &= \frac{21}{320} a^2 cx^3 (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{3}{40} acx^3 (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(7a^3c\sqrt{c(a+bx^2)^3} \right) \int x^2 (a+bx^2)^{3/2} dx}{128(a+bx^2)^{3/2}} \\
 &= \frac{7}{128} a^3 cx^3 \sqrt{c(a+bx^2)^3} + \frac{21}{320} a^2 cx^3 (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{3}{40} acx^3 (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} \\
 &= \frac{7}{128} a^3 cx^3 \sqrt{c(a+bx^2)^3} + \frac{21a^4 cx^3 \sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{21}{320} a^2 cx^3 (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} \\
 &= \frac{7}{128} a^3 cx^3 \sqrt{c(a+bx^2)^3} + \frac{21a^5 cx \sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4 cx^3 \sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{21}{320} a^2 cx^3 (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3} \\
 &= \frac{7}{128} a^3 cx^3 \sqrt{c(a+bx^2)^3} + \frac{21a^5 cx \sqrt{c(a+bx^2)^3}}{1024b(a+bx^2)} + \frac{21a^4 cx^3 \sqrt{c(a+bx^2)^3}}{512(a+bx^2)} + \frac{21}{320} a^2 cx^3 (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{12} cx^3 (a+bx^2)^3 \sqrt{c(a+bx^2)^3}
 \end{aligned}$$

Mathematica [A] time = 0.15, size = 143, normalized size = 0.57

$$\frac{\left(c(a+bx^2)^3 \right)^{3/2} \left(\sqrt{bx} \sqrt{\frac{bx^2}{a} + 1} \left(315a^5 + 4910a^4bx^2 + 11432a^3b^2x^4 + 12144a^2b^3x^6 + 6272ab^4x^8 + 1280b^5x^{10} \right) - 315a^{11/2} \sinh^{-1} \left(\frac{\sqrt{bx}}{\sqrt{a}} \right) \right)}{15360b^{3/2} (a+bx^2)^4 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c*(a + b*x^2)^3)^(3/2), x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(315*a^5 + 4910*a^4*b*x^2 + 11432*a^3*b^2*x^4 + 12144*a^2*b^3*x^6 + 6272*a*b^4*x^8 + 1280*b^5*x^10) - 315*a^(11/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(15360*b^(3/2)*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])

IntegrateAlgebraic [A] time = 10.96, size = 174, normalized size = 0.69

$$\frac{\left(c(a+bx^2)^3\right)^{3/2} \left(\frac{21a^6c^{3/2} \log(\sqrt{a+bx^2}-\sqrt{bx})}{1024b^{3/2}} + \frac{\sqrt{a+bx^2}(315a^5c^{3/2}x+4910a^4bc^{3/2}x^3+11432a^3b^2c^{3/2}x^5+12144a^2b^3c^{3/2}x^7+6272ab^4c^{3/2}x^9+1280b^5c^{3/2}x^{11})}{15360b}\right)}{c^{3/2}(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(c*(a + b*x^2)^3)^(3/2), x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(315*a^5*c^(3/2)*x + 4910*a^4*b*c^(3/2)*x^3 + 11432*a^3*b^2*c^(3/2)*x^5 + 12144*a^2*b^3*c^(3/2)*x^7 + 6272*a*b^4*c^(3/2)*x^9 + 1280*b^5*c^(3/2)*x^11))/(15360*b) + (21*a^6*c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]]/(1024*b^(3/2))))/(c^(3/2)*(a + b*x^2)^(9/2))

fricas [A] time = 0.84, size = 433, normalized size = 1.71

$$\frac{315(a^6c^2 + a^7c) \sqrt{c} \log\left(\frac{2a^6c^2 + a^7c + \sqrt{bcx^2 + ac} \sqrt{bcx^2 + ac}}{2a^6c^2 + a^7c}\right) + 2(1280b^5c^{11} + 6272a^6c^9 + 12144a^5b^2c^7 + 11432a^4b^3c^5 + 4910a^3b^4c^3 + 315a^2b^5c) \sqrt{bcx^2 + ac} + 315(a^6c^2 + a^7c) \sqrt{c} \arctan\left(\frac{\sqrt{bcx^2 + ac} \sqrt{bcx^2 + ac}}{\sqrt{bcx^2 + ac}}\right) + (1280b^5c^{11} + 6272a^6c^9 + 12144a^5b^2c^7 + 11432a^4b^3c^5 + 4910a^3b^4c^3 + 315a^2b^5c) \sqrt{bcx^2 + ac}}{15360(b^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2), x, algorithm="fricas")

[Out] [1/30720*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^2 + a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*b*x*sqrt(c/b))/(b*x^2 + a) + 2*(1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b), 1/15360*(315*(a^6*b*c*x^2 + a^7*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*b*x*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (1280*b^5*c*x^11 + 6272*a*b^4*c*x^9 + 12144*a^2*b^3*c*x^7 + 11432*a^3*b^2*c*x^5 + 4910*a^4*b*c*x^3 + 315*a^5*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b^2*x^2 + a*b)]

giac [A] time = 0.51, size = 177, normalized size = 0.70

$$\frac{1}{15360} \left(\frac{315 a^6 c \log\left(\frac{-\sqrt{bc} x + \sqrt{bcx^2 + ac}}{\sqrt{bc} b}\right) \operatorname{sgn}(bx^2 + a)}{\sqrt{bc} b} + \left(\frac{315 a^5 \operatorname{sgn}(bx^2 + a)}{b} + 2(2455 a^4 \operatorname{sgn}(bx^2 + a) + 4(1429 a^3 b \operatorname{sgn}(bx^2 + a) + 2(759 a^2 b^2 \operatorname{sgn}(bx^2 + a) + 8(10 b^4 x^2 \operatorname{sgn}(bx^2 + a) + 49 a^3 b^3 \operatorname{sgn}(bx^2 + a)) x^2) x^2) \sqrt{bcx^2 + ac} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2), x, algorithm="giac")

[Out] 1/15360*(315*a^6*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/(sqrt(b*c)*b) + (315*a^5*sgn(b*x^2 + a)/b + 2*(2455*a^4*sgn(b*x^2 + a) + 4*(1429*a^3*b*sgn(b*x^2 + a) + 2*(759*a^2*b^2*sgn(b*x^2 + a) + 8*(10*b^4*x^2*sgn(b*x^2 + a) + 49*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c

maple [A] time = 0.04, size = 236, normalized size = 0.93

$$\frac{\left((bx^2 + a)^3 c\right)^{3/2} \left(315 a^6 c^3 \ln\left(\frac{bcx + \sqrt{bcx^2 + ac}}{\sqrt{bc}}\right) - 1280 (bcx^2 + ac)^{5/2} \sqrt{bc} b^3 x^7 + 315 \sqrt{bc} \sqrt{bcx^2 + ac} a^2 c^2 x - 3712 \sqrt{bc} (bcx^2 + ac)^{5/2} a b^2 x^5 + 210 \sqrt{bc} (bcx^2 + ac)^{5/2} a^4 c x - 3440 \sqrt{bc} (bcx^2 + ac)^{5/2} a^2 b x^3 - 840 \sqrt{bc} (bcx^2 + ac)^{5/2} a^2 x\right)}{15360 (bx^2 + a)^3 ((bx^2 + a)c)^{3/2} \sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*(b*x^2+a)^3)^(3/2), x)

[Out] -1/15360*(c*(b*x^2+a)^3)^(3/2)/b*(-1280*x^7*(b*c*x^2+a*c)^(5/2)*b^3*(b*c)^(1/2)-3712*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^5*a*b^2-3440*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^3*a^2*b+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^6*c^3-840*(b*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x*a^3+210*(b*c)^(1/2)*

$(b*c*x^2+a*c)^{(3/2)}*x*a^4*c+315*(b*c)^{(1/2)}*(b*c*x^2+a*c)^{(1/2)}*x*a^5*c^2/(b*x^2+a)^3/(c*(b*x^2+a))^{(3/2)}/c/(b*c)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((bx^2 + a)^3 c \right)^{\frac{3}{2}} x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2)*x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(c (bx^2 + a)^3 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c*(a + b*x^2)^3)^(3/2),x)

[Out] int(x^2*(c*(a + b*x^2)^3)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c*(b*x**2+a)**3)**(3/2),x)

[Out] Timed out

$$3.78 \quad \int x \left(c \left(a + bx^2 \right)^3 \right)^{3/2} dx$$

Optimal. Leaf size=32

$$\frac{c \left(a + bx^2 \right)^4 \sqrt{c \left(a + bx^2 \right)^3}}{11b}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1591, 15, 30}

$$\frac{c \left(a + bx^2 \right)^4 \sqrt{c \left(a + bx^2 \right)^3}}{11b}$$

Antiderivative was successfully verified.

[In] Int[x*(c*(a + b*x^2)^3)^(3/2),x]

[Out] (c*(a + b*x^2)^4*Sqrt[c*(a + b*x^2)^3])/(11*b)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int x \left(c \left(a + bx^2 \right)^3 \right)^{3/2} dx &= \frac{\text{Subst} \left(\int \left(cx^3 \right)^{3/2} dx, x, a + bx^2 \right)}{2b} \\ &= \frac{\left(c \sqrt{c \left(a + bx^2 \right)^3} \right) \text{Subst} \left(\int x^{9/2} dx, x, a + bx^2 \right)}{2b \left(a + bx^2 \right)^{3/2}} \\ &= \frac{c \left(a + bx^2 \right)^4 \sqrt{c \left(a + bx^2 \right)^3}}{11b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.91

$$\frac{\left(a + bx^2 \right) \left(c \left(a + bx^2 \right)^3 \right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c*(a + b*x^2)^3)^(3/2),x]

[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)

IntegrateAlgebraic [A] time = 13.18, size = 29, normalized size = 0.91

$$\frac{(a + bx^2) \left(c(a + bx^2)^3 \right)^{3/2}}{11b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(c*(a + b*x^2)^3)^(3/2),x]

[Out] ((a + b*x^2)*(c*(a + b*x^2)^3)^(3/2))/(11*b)

fricas [B] time = 0.64, size = 87, normalized size = 2.72

$$\frac{(b^4cx^8 + 4ab^3cx^6 + 6a^2b^2cx^4 + 4a^3bcx^2 + a^4c)\sqrt{b^3cx^6 + 3ab^2cx^4 + 3a^2bcx^2 + a^3c}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")

[Out] 1/11*(b^4*c*x^8 + 4*a*b^3*c*x^6 + 6*a^2*b^2*c*x^4 + 4*a^3*b*c*x^2 + a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)/b

giac [A] time = 0.31, size = 28, normalized size = 0.88

$$\frac{(bcx^2 + ac)^{\frac{11}{2}} \operatorname{sgn}(bx^2 + a)}{11bc^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")

[Out] 1/11*(b*c*x^2 + a*c)^(11/2)*sgn(b*x^2 + a)/(b*c^4)

maple [A] time = 0.01, size = 26, normalized size = 0.81

$$\frac{(bx^2 + a) \left((bx^2 + a)^3 c \right)^{\frac{3}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^2+a)^3*c)^(3/2),x)

[Out] 1/11*(b*x^2+a)/b*((b*x^2+a)^3*c)^(3/2)

maxima [B] time = 1.08, size = 70, normalized size = 2.19

$$\frac{\left(b^4c^{\frac{3}{2}}x^8 + 4ab^3c^{\frac{3}{2}}x^6 + 6a^2b^2c^{\frac{3}{2}}x^4 + 4a^3bc^{\frac{3}{2}}x^2 + a^4c^{\frac{3}{2}} \right) (bx^2 + a)^{\frac{3}{2}}}{11b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")

[Out] $1/11*(b^4*c^{(3/2)}*x^8 + 4*a*b^3*c^{(3/2)}*x^6 + 6*a^2*b^2*c^{(3/2)}*x^4 + 4*a^3*b*c^{(3/2)}*x^2 + a^4*c^{(3/2)})*(b*x^2 + a)^{(3/2)}/b$

mupad [B] time = 2.71, size = 62, normalized size = 1.94

$$\sqrt{c(bx^2 + a)^3} \left(\frac{a^4 c}{11b} + \frac{4a^3 c x^2}{11} + \frac{b^3 c x^8}{11} + \frac{6a^2 b c x^4}{11} + \frac{4ab^2 c x^6}{11} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*(a + b*x^2)^3)^(3/2), x)`

[Out] $(c*(a + b*x^2)^3)^{(1/2)}*((a^4*c)/(11*b) + (4*a^3*c*x^2)/11 + (b^3*c*x^8)/11 + (6*a^2*b*c*x^4)/11 + (4*a*b^2*c*x^6)/11)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*(b*x**2+a)**3)**(3/2), x)`

[Out] Timed out

$$3.79 \quad \int \left(c (a + bx^2)^3 \right)^{3/2} dx$$

Optimal. Leaf size=207

$$\frac{63a^{7/2}c\sqrt{c(a+bx^2)^3} \sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)}{256\sqrt{b}\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{63a^4cx\sqrt{c(a+bx^2)^3}}{256(a+bx^2)} + \frac{21}{128}a^3cx\sqrt{c(a+bx^2)^3} + \frac{21}{160}a^2cx(a+bx^2)\sqrt{c(a+bx^2)^3}$$

Rubi [A] time = 0.07, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {6720, 195, 217, 206}

$$\frac{63a^4cx\sqrt{c(a+bx^2)^3}}{256(a+bx^2)} + \frac{21}{128}a^3cx\sqrt{c(a+bx^2)^3} + \frac{21}{160}a^2cx(a+bx^2)\sqrt{c(a+bx^2)^3} + \frac{63a^5c\sqrt{c(a+bx^2)^3} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{256\sqrt{b}(a+bx^2)^{3/2}} + \frac{9}{80}acx(a+bx^2)^2\sqrt{c(a+bx^2)^3} + \frac{1}{10}cx(a+bx^2)^3\sqrt{c(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^3)^(3/2), x]

[Out] (21*a^3*c*x*Sqrt[c*(a + b*x^2)^3])/128 + (63*a^4*c*x*Sqrt[c*(a + b*x^2)^3])/(256*(a + b*x^2)) + (21*a^2*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/160 + (9*a*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/80 + (c*x*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/10 + (63*a^5*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(256*Sqrt[b]*(a + b*x^2)^(3/2))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
\int \left(c(a+bx^2)^3 \right)^{3/2} dx &= \frac{\left(c\sqrt{c(a+bx^2)^3} \right) \int (a+bx^2)^{9/2} dx}{(a+bx^2)^{3/2}} \\
&= \frac{1}{10} cx (a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(9ac\sqrt{c(a+bx^2)^3} \right) \int (a+bx^2)^{7/2} dx}{10(a+bx^2)^{3/2}} \\
&= \frac{9}{80} acx (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{10} cx (a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(63a^2c\sqrt{c(a+bx^2)^3} \right) \int (a+bx^2)^{5/2} dx}{80(a+bx^2)^{3/2}} \\
&= \frac{21}{160} a^2 cx (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{80} acx (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{10} cx (a+bx^2)^3 \sqrt{c(a+bx^2)^3} \\
&= \frac{21}{128} a^3 cx \sqrt{c(a+bx^2)^3} + \frac{21}{160} a^2 cx (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{80} acx (a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
&= \frac{21}{128} a^3 cx \sqrt{c(a+bx^2)^3} + \frac{63a^4 cx \sqrt{c(a+bx^2)^3}}{256(a+bx^2)} + \frac{21}{160} a^2 cx (a+bx^2) \sqrt{c(a+bx^2)^3} \\
&= \frac{21}{128} a^3 cx \sqrt{c(a+bx^2)^3} + \frac{63a^4 cx \sqrt{c(a+bx^2)^3}}{256(a+bx^2)} + \frac{21}{160} a^2 cx (a+bx^2) \sqrt{c(a+bx^2)^3} \\
&= \frac{21}{128} a^3 cx \sqrt{c(a+bx^2)^3} + \frac{63a^4 cx \sqrt{c(a+bx^2)^3}}{256(a+bx^2)} + \frac{21}{160} a^2 cx (a+bx^2) \sqrt{c(a+bx^2)^3}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 132, normalized size = 0.64

$$\frac{\left(c(a+bx^2)^3 \right)^{3/2} \left(315a^{9/2} \sinh^{-1} \left(\frac{\sqrt{bx^2}}{\sqrt{a}} \right) + \sqrt{bx^2} \sqrt{\frac{bx^2}{a} + 1} (965a^4 + 1490a^3bx^2 + 1368a^2b^2x^4 + 656ab^3x^6 + 128b^4x^8) \right)}{1280\sqrt{b} (a+bx^2)^4 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^3)^(3/2), x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]*(965*a^4 + 1490*a^3*b*x^2 + 1368*a^2*b^2*x^4 + 656*a*b^3*x^6 + 128*b^4*x^8) + 315*a^(9/2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/(1280*Sqrt[b]*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])

IntegrateAlgebraic [A] time = 14.69, size = 155, normalized size = 0.75

$$\frac{\left(c(a+bx^2)^3 \right)^{3/2} \left(\frac{\sqrt{a+bx^2} (965a^4c^{3/2}x + 1490a^3bc^{3/2}x^3 + 1368a^2b^2c^{3/2}x^5 + 656ab^3c^{3/2}x^7 + 128b^4c^{3/2}x^9)}{1280} - \frac{63a^5c^{3/2} \log(\sqrt{a+bx^2} - \sqrt{bx^2})}{256\sqrt{b}} \right)}{c^{3/2} (a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2), x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(965*a^4*c^(3/2)*x + 1490*a^3*b*c^(3/2)*x^3 + 1368*a^2*b^2*c^(3/2)*x^5 + 656*a*b^3*c^(3/2)*x^7 + 128*b^4*c^(3/2)*x^9))/1280 - (63*a^5*c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(256*Sqrt[b]))/(c^(3/2)*(a + b*x^2)^(9/2))

fricas [A] time = 0.62, size = 402, normalized size = 1.94

$$\frac{315 (a^5 b c^2 + a^6 c) \sqrt{c} \log \left(\frac{2 (128 b^4 c^3 + 656 a b^3 c^2 + 1368 a^2 b^2 c + 1490 a^3 b c + 965 a^4 c) \sqrt{b c^2 + a^2} + 2 (128 b^4 c^3 + 656 a b^3 c^2 + 1368 a^2 b^2 c + 1490 a^3 b c + 965 a^4 c) \sqrt{b c^2 + a^2}}{2560 (b x^2 + a)} \right) - 128 b^4 c^3 + 656 a b^3 c^2 + 1368 a^2 b^2 c + 1490 a^3 b c + 965 a^4 c}{1280 (b x^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="fricas")

[Out] [1/2560*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(c/b)*log(-(2*b^2*c*x^4 + 3*a*b*c*x^2 + a^2*c + 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*sqrt(c/b))/(b*x^2 + a) + 2*(128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), -1/1280*(315*(a^5*b*c*x^2 + a^6*c)*sqrt(-c/b)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*sqrt(-c/b)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (128*b^4*c*x^9 + 656*a*b^3*c*x^7 + 1368*a^2*b^2*c*x^5 + 1490*a^3*b*c*x^3 + 965*a^4*c*x)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]

giac [A] time = 0.33, size = 153, normalized size = 0.74

$$\frac{1}{1280} \left(\frac{315 a^5 c \log \left(\frac{-\sqrt{b c} x + \sqrt{b c x^2 + a c}}{\sqrt{b c}} \right) \operatorname{sgn}(b x^2 + a)}{\sqrt{b c}} - (965 a^4 \operatorname{sgn}(b x^2 + a) + 2 (745 a^3 b \operatorname{sgn}(b x^2 + a) + 4 (171 a^2 b^2 \operatorname{sgn}(b x^2 + a) + 2 (8 b^4 x^2 \operatorname{sgn}(b x^2 + a) + 41 a b^3 \operatorname{sgn}(b x^2 + a)) x^2) x^2) \sqrt{b c x^2 + a c} x \right) c \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="giac")

[Out] -1/1280*(315*a^5*c*log(abs(-sqrt(b*c)*x + sqrt(b*c*x^2 + a*c)))*sgn(b*x^2 + a)/sqrt(b*c) - (965*a^4*sgn(b*x^2 + a) + 2*(745*a^3*b*sgn(b*x^2 + a) + 4*(171*a^2*b^2*sgn(b*x^2 + a) + 2*(8*b^4*x^2*sgn(b*x^2 + a) + 41*a*b^3*sgn(b*x^2 + a))*x^2)*x^2)*sqrt(b*c*x^2 + a*c)*x)*c

maple [A] time = 0.02, size = 205, normalized size = 0.99

$$\frac{\left((b x^2 + a)^3 c \right)^{\frac{3}{2}} \left(315 a^5 c^3 \ln \left(\frac{b c x + \sqrt{b c x^2 + a c} \sqrt{b c}}{\sqrt{b c}} \right) + 315 \sqrt{b c x^2 + a c} \sqrt{b c} a^4 c^2 x + 128 (b c x^2 + a c)^{\frac{5}{2}} \sqrt{b c} b^2 x^5 + 210 (b c x^2 + a c)^{\frac{3}{2}} \sqrt{b c} a^3 c x + 400 (b c x^2 + a c)^{\frac{5}{2}} \sqrt{b c} a b x^3 + 440 (b c x^2 + a c)^{\frac{5}{2}} \sqrt{b c} a^2 x \right)}{1280 (b x^2 + a)^3 \left((b x^2 + a) c \right)^{\frac{3}{2}} \sqrt{b c} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^3*c)^(3/2),x)

[Out] 1/1280*((b*x^2+a)^3*c)^(3/2)*(128*x^5*(b*c*x^2+a*c)^(5/2)*b^2*(b*c)^(1/2)+400*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^3*a*b+440*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x*a^2+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*x*a^3*c+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*x*a^4*c^2+315*ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*a^5*c^3)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((b x^2 + a)^3 c \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2),x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(c (b x^2 + a)^3 \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^3)^(3/2), x)

[Out] int((c*(a + b*x^2)^3)^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**3)**(3/2), x)

[Out] Timed out

$$3.80 \quad \int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x} dx$$

Optimal. Leaf size=192

$$\frac{a^4 c \sqrt{c(a+bx^2)^3}}{a+bx^2} + \frac{1}{3} a^3 c \sqrt{c(a+bx^2)^3} - \frac{a^3 c \sqrt{c(a+bx^2)^3} \tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{1}{5} a^2 c (a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7} a c (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{9} c (a+bx^2)^3 \sqrt{c(a+bx^2)^3}$$

Rubi [A] time = 0.22, antiderivative size = 194, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6720, 266, 50, 63, 208}

$$\frac{a^4 c \sqrt{c(a+bx^2)^3}}{a+bx^2} + \frac{1}{3} a^3 c \sqrt{c(a+bx^2)^3} + \frac{1}{5} a^2 c (a+bx^2) \sqrt{c(a+bx^2)^3} - \frac{a^{9/2} c \sqrt{c(a+bx^2)^3} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{(a+bx^2)^{3/2}} + \frac{1}{7} a c (a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{9} c (a+bx^2)^3 \sqrt{c(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^3)^(3/2)/x,x]

[Out] (a^3*c*Sqrt[c*(a + b*x^2)^3])/3 + (a^4*c*Sqrt[c*(a + b*x^2)^3])/(a + b*x^2) + (a^2*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/5 + (a*c*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/7 + (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/9 - (a^(9/2)*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(a + b*x^2)^(3/2)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
```


[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
 \int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x} dx &= \frac{\left(c\sqrt{c(a+bx^2)^3}\right) \int \frac{(a+bx^2)^{9/2}}{x} dx}{(a+bx^2)^{3/2}} \\
 &= \frac{\left(c\sqrt{c(a+bx^2)^3}\right) \text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x} dx, x, x^2\right)}{2(a+bx^2)^{3/2}} \\
 &= \frac{1}{9}c(a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(ac\sqrt{c(a+bx^2)^3}\right) \text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x} dx, x, x^2\right)}{2(a+bx^2)^{3/2}} \\
 &= \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{9}c(a+bx^2)^3 \sqrt{c(a+bx^2)^3} + \frac{\left(a^2c\sqrt{c(a+bx^2)^3}\right)}{2(a+bx^2)^{3/2}} \\
 &= \frac{1}{5}a^2c(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3} + \frac{1}{9}c(a+bx^2)^3 \sqrt{c(a+bx^2)^3} \\
 &= \frac{1}{3}a^3c \sqrt{c(a+bx^2)^3} + \frac{1}{5}a^2c(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
 &= \frac{1}{3}a^3c \sqrt{c(a+bx^2)^3} + \frac{a^4c \sqrt{c(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
 &= \frac{1}{3}a^3c \sqrt{c(a+bx^2)^3} + \frac{a^4c \sqrt{c(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
 &= \frac{1}{3}a^3c \sqrt{c(a+bx^2)^3} + \frac{a^4c \sqrt{c(a+bx^2)^3}}{a+bx^2} + \frac{1}{5}a^2c(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{1}{7}ac(a+bx^2)^2 \sqrt{c(a+bx^2)^3}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 111, normalized size = 0.58

$$\frac{\left(c(a+bx^2)^3\right)^{3/2} \left(\sqrt{a+bx^2} (563a^4 + 506a^3bx^2 + 408a^2b^2x^4 + 185ab^3x^6 + 35b^4x^8) - 315a^{9/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{315(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x,x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*(Sqrt[a + b*x^2]*(563*a^4 + 506*a^3*b*x^2 + 408*a^2*b^2*x^4 + 185*a*b^3*x^6 + 35*b^4*x^8) - 315*a^(9/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]))/(315*(a + b*x^2)^(9/2))

IntegrateAlgebraic [A] time = 16.44, size = 146, normalized size = 0.76

$$\frac{\left(c(a+bx^2)^3\right)^{3/2} \left(\frac{1}{315} \sqrt{a+bx^2} \left(563a^4c^{3/2} + 506a^3bc^{3/2}x^2 + 408a^2b^2c^{3/2}x^4 + 185ab^3c^{3/2}x^6 + 35b^4c^{3/2}x^8\right) - a^{9/2}c^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{c^{3/2}(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x,x]

[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(563*a^4*c^(3/2) + 506*a^3*b*c^(3/2)*x^2 + 408*a^2*b^2*c^(3/2)*x^4 + 185*a*b^3*c^(3/2)*x^6 + 35*b^4*c^(3/2)*x^8))/315 - a^(9/2)*c^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(c^(3/2)*(a + b*x^2)^(9/2))

fricas [A] time = 0.71, size = 391, normalized size = 2.04

$$\frac{315(a^4bc^2 + a^5c)\sqrt{ac} \log\left(\frac{b^2x^2 + a^2 - \sqrt{bcx^2 + ac} \sqrt{bcx^2 + ac}}{b^2x^2 + a^2}\right) + 2(35b^4c^4 + 185ab^3c^4 + 408a^2b^2c^4 + 506a^3bc^4 + 563a^4c^4)\sqrt{bcx^2 + ac} + 315(a^4bc^2 + a^5c)\sqrt{-ac} \arctan\left(\frac{b^2x^2 + a^2 - \sqrt{bcx^2 + ac} \sqrt{bcx^2 + ac}}{b^2x^2 + a^2}\right) + (35b^4c^4 + 185ab^3c^4 + 408a^2b^2c^4 + 506a^3bc^4 + 563a^4c^4)\sqrt{bcx^2 + ac} + 315(a^4bc^2 + a^5c)\sqrt{-ac}}{630(b^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="fricas")

[Out] [1/630*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a), 1/315*(315*(a^4*b*c*x^2 + a^5*c)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (35*b^4*c*x^8 + 185*a*b^3*c*x^6 + 408*a^2*b^2*c*x^4 + 506*a^3*b*c*x^2 + 563*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^2 + a)]

giac [A] time = 0.36, size = 185, normalized size = 0.96

$$\frac{1}{315} \left(\frac{315a^5 \arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right) \operatorname{sgn}(bx^2+a)}{\sqrt{-ac}} + \frac{315\sqrt{bcx^2+ac} a^4 c^4 \operatorname{sgn}(bx^2+a) + 105(bc^2+ac)^{\frac{3}{2}} a^3 c^4 \operatorname{sgn}(bx^2+a) + 63(bc^2+ac)^{\frac{5}{2}} a^2 c^4 \operatorname{sgn}(bx^2+a) + 45(bc^2+ac)^{\frac{7}{2}} a c^4 \operatorname{sgn}(bx^2+a) + 35(bc^2+ac)^{\frac{9}{2}} c^4 \operatorname{sgn}(bx^2+a)}{c^{45}} \right) c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="giac")

[Out] 1/315*(315*a^5*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-a*c) + (315*sqrt(b*c*x^2 + a*c)*a^4*c^4*sgn(b*x^2 + a) + 105*(b*c*x^2 + a*c)^(3/2)*a^3*c^4*sgn(b*x^2 + a) + 63*(b*c*x^2 + a*c)^(5/2)*a^2*c^4*sgn(b*x^2 + a) + 45*(b*c*x^2 + a*c)^(7/2)*a*c^4*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(9/2)*c^4*sgn(b*x^2 + a))/c^45*c^2

maple [A] time = 0.02, size = 221, normalized size = 1.15

$$\frac{\left((bx^2+a)^3 c\right)^{\frac{3}{2}} \left(315a^5 c^3 \ln\left(\frac{2ac+2\sqrt{ac}\sqrt{bcx^2+ac}}{x}\right) - 315\sqrt{ac}\sqrt{bcx^2+ac} a^4 c^2 - 35\sqrt{ac}(bcx^2+ac)^{\frac{5}{2}} b^2 x^4 - 105\sqrt{ac}(bcx^2+ac)^{\frac{3}{2}} a^2 c - 115\sqrt{ac}(bcx^2+ac)^{\frac{5}{2}} abx^2 - 189((bx^2+a)c)^{\frac{5}{2}} \sqrt{ac} a^2 + 46\sqrt{ac}(bcx^2+ac)^{\frac{5}{2}} a^2\right)}{315(bx^2+a)^3((bx^2+a)c)^{\frac{3}{2}} \sqrt{ac} c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^3*c)^(3/2)/x,x)

[Out] -1/315*((b*x^2+a)^3*c)^(3/2)*(-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^4*b^2-15*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*x^2*a*b+315*ln(2*((a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)+a*c)/x)*a^5*c^3-189*a^2*((b*x^2+a)*c)^(5/2)*(a*c)^(1/2)+46*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2-105*(a*c)^(1/2)*(b*c*x^2+a*c)^(3/2)*a^3*c-315*(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2)*a^4*c^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left((bx^2 + a)^3 c\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x,x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c(bx^2 + a)^3\right)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^3)^(3/2)/x,x)

[Out] int((c*(a + b*x^2)^3)^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c(a + bx^2)^3\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**3)**(3/2)/x,x)

[Out] Integral((c*(a + b*x**2)**3)**(3/2)/x, x)

$$3.81 \quad \int \frac{(c(ax^2+b)^3)^{3/2}}{x^2} dx$$

Optimal. Leaf size=208

$$\frac{315a^{5/2}\sqrt{b}c\sqrt{c(a+bx^2)^3}\sinh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{128\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{315a^3bcx\sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{105}{64}a^2bcx\sqrt{c(a+bx^2)^3} + \frac{21}{16}abcx(a+bx^2)$$

Rubi [A] time = 0.19, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 19, number of rules / integrand size = 0.263, Rules used = {6720, 277, 195, 217, 206}

$$\frac{315a^3bcx\sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{105}{64}a^2bcx\sqrt{c(a+bx^2)^3} + \frac{315a^4\sqrt{b}c\sqrt{c(a+bx^2)^3}\tanh^{-1}\left(\frac{\sqrt{bx}}{\sqrt{a+bx^2}}\right)}{128(a+bx^2)^{3/2}} + \frac{21}{16}abcx(a+bx^2)\sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c(a+bx^2)^3}}{x} + \frac{9}{8}bcx(a+bx^2)^2\sqrt{c(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^2,x]

[Out] (105*a^2*b*c*x*Sqrt[c*(a + b*x^2)^3])/64 + (315*a^3*b*c*x*Sqrt[c*(a + b*x^2)^3])/(128*(a + b*x^2)) + (21*a*b*c*x*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/16 + (9*b*c*x*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/8 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/x + (315*a^4*Sqrt[b]*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(128*(a + b*x^2)^(3/2))

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^2} dx &= \frac{\left(c\sqrt{c(a+bx^2)^3}\right) \int \frac{(a+bx^2)^{9/2}}{x^2} dx}{(a+bx^2)^{3/2}} \\
&= -\frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{x} + \frac{\left(9bc\sqrt{c(a+bx^2)^3}\right) \int (a+bx^2)^{7/2} dx}{(a+bx^2)^{3/2}} \\
&= \frac{9}{8}bcx(a+bx^2)^2 \sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{x} + \frac{\left(63abc\sqrt{c(a+bx^2)^3}\right)}{8(a+bx^2)} \\
&= \frac{21}{16}abcx(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{8}bcx(a+bx^2)^2 \sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{x} \\
&= \frac{105}{64}a^2bcx \sqrt{c(a+bx^2)^3} + \frac{21}{16}abcx(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{8}bcx(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
&= \frac{105}{64}a^2bcx \sqrt{c(a+bx^2)^3} + \frac{315a^3bcx \sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{21}{16}abcx(a+bx^2) \sqrt{c(a+bx^2)^3} \\
&= \frac{105}{64}a^2bcx \sqrt{c(a+bx^2)^3} + \frac{315a^3bcx \sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{21}{16}abcx(a+bx^2) \sqrt{c(a+bx^2)^3} \\
&= \frac{105}{64}a^2bcx \sqrt{c(a+bx^2)^3} + \frac{315a^3bcx \sqrt{c(a+bx^2)^3}}{128(a+bx^2)} + \frac{21}{16}abcx(a+bx^2) \sqrt{c(a+bx^2)^3}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 65, normalized size = 0.31

$$\frac{a^4 \left(c(a+bx^2)^3\right)^{3/2} {}_2F_1\left(-\frac{9}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x(a+bx^2)^4 \sqrt{\frac{bx^2}{a} + 1}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^2, x]

[Out] -((a^4*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[-9/2, -1/2, 1/2, -(b*x^2)/a]))/(x*(a + b*x^2)^4*Sqrt[1 + (b*x^2)/a])

IntegrateAlgebraic [A] time = 19.20, size = 157, normalized size = 0.75

$$\frac{\left(c(a+bx^2)^3\right)^{3/2} \left(\frac{\sqrt{a+bx^2}(-128a^4c^{3/2}+325a^3bc^{3/2}x^2+210a^2b^2c^{3/2}x^4+88ab^3c^{3/2}x^6+16b^4c^{3/2}x^8)}{128x} - \frac{315}{128}a^4\sqrt{b}c^{3/2} \log\left(\sqrt{a+bx^2} - \sqrt{b}x\right)\right)}{c^{3/2}(a+bx^2)^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x^2, x]

[Out] $((c*(a + b*x^2)^3)^{(3/2)}*((\text{Sqrt}[a + b*x^2]*(-128*a^4*c^{(3/2)} + 325*a^3*b*c^{(3/2)}*x^2 + 210*a^2*b^2*c^{(3/2)}*x^4 + 88*a*b^3*c^{(3/2)}*x^6 + 16*b^4*c^{(3/2)}*x^8)))/(128*x - (315*a^4*\text{Sqrt}[b]*c^{(3/2)}*\text{Log}[-(\text{Sqrt}[b]*x) + \text{Sqrt}[a + b*x^2]])/128))/(c^{(3/2)}*(a + b*x^2)^{(9/2)})$

fricas [A] time = 0.73, size = 396, normalized size = 1.90

$$\frac{315(a^4bx^3 + a^5c) \log\left(\frac{-16b^4cx^8 + 88ab^3cx^6 + 210a^2b^2cx^4 + 325a^3b^2cx^2 - 128a^4c}{256(b^3 + ax)}\right) + 2(16b^4cx^8 + 88ab^3cx^6 + 210a^2b^2cx^4 + 325a^3b^2cx^2 - 128a^4c)\sqrt{bcx^2 + ac}}{128(b^3 + ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="fricas")

[Out] $[1/256*(315*(a^4*b*c*x^3 + a^5*c*x)*\text{sqrt}(b*c)*\log(-(2*b^2*c*x^4 + 3*a*b*c*x^2 + a^2*c + 2*\text{sqrt}(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*\text{sqrt}(b*c)*x)/(b*x^2 + a)) + 2*(16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*\text{sqrt}(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x), -1/128*(315*(a^4*b*c*x^3 + a^5*c*x)*\text{sqrt}(-b*c)*\text{arctan}(\text{sqrt}(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))*\text{sqrt}(-b*c)*x/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) - (16*b^4*c*x^8 + 88*a*b^3*c*x^6 + 210*a^2*b^2*c*x^4 + 325*a^3*b*c*x^2 - 128*a^4*c)*\text{sqrt}(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^3 + a*x)]$

giac [A] time = 0.54, size = 185, normalized size = 0.89

$$\frac{1}{256} \left(\frac{512 \sqrt{bc} a^5 \text{sgn}(bx^2 + a)}{(\sqrt{bc}x - \sqrt{bcx^2 + ac})^2 - ac} - 315 \sqrt{bc} a^4 \log\left(\frac{\sqrt{bc}x - \sqrt{bcx^2 + ac}}{\sqrt{bc}x + \sqrt{bcx^2 + ac}}\right) \text{sgn}(bx^2 + a) + 2(325 a^3 b \text{sgn}(bx^2 + a) + 2(105 a^2 b^2 \text{sgn}(bx^2 + a) + 4(2 b^4 x^2 \text{sgn}(bx^2 + a) + 11 a b^3 \text{sgn}(bx^2 + a))x^2) \sqrt{bcx^2 + ac} x \right) \right) c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="giac")

[Out] $1/256*(512*\text{sqrt}(b*c)*a^5*c*\text{sgn}(b*x^2 + a)/((\text{sqrt}(b*c)*x - \text{sqrt}(b*c*x^2 + a*c))^2 - a*c) - 315*\text{sqrt}(b*c)*a^4*\log((\text{sqrt}(b*c)*x - \text{sqrt}(b*c*x^2 + a*c))^2) * \text{sgn}(b*x^2 + a) + 2*(325*a^3*b*\text{sgn}(b*x^2 + a) + 2*(105*a^2*b^2*\text{sgn}(b*x^2 + a) + 4*(2*b^4*x^2*\text{sgn}(b*x^2 + a) + 11*a*b^3*\text{sgn}(b*x^2 + a))*x^2)*\text{sqrt}(b*c*x^2 + a*c)*x)*c$

maple [A] time = 0.02, size = 215, normalized size = 1.03

$$\frac{((bx^2 + a)^3 c)^{\frac{3}{2}} \left(315 a^4 b c^3 x \ln\left(\frac{bcx + \sqrt{bcx^2 + ac} \sqrt{bc}}{\sqrt{bc}}\right) + 315 \sqrt{bcx^2 + ac} \sqrt{bc} a^3 b c^2 x^2 + 210 (bcx^2 + ac)^{\frac{3}{2}} \sqrt{bc} a^2 b c x^2 + 16 (bcx^2 + ac)^{\frac{5}{2}} \sqrt{bc} b^2 x^4 + 56 (bcx^2 + ac)^{\frac{5}{2}} \sqrt{bc} a b x^2 - 128 (bcx^2 + ac)^{\frac{5}{2}} \sqrt{bc} a^2 \right)}{128 (bx^2 + a)^3 ((bx^2 + a)c)^{\frac{3}{2}} \sqrt{bc} cx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^3*c)^(3/2)/x^2,x)

[Out] $1/128*((b*x^2+a)^3*c)^(3/2)*(16*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^4*b^2+56*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*x^2*a*b+210*(b*c*x^2+a*c)^(3/2)*(b*c)^(1/2)*x^2*a^2*b*c+315*(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2)*x^2*a^3*b*c^2+315*\ln((b*c*x+(b*c*x^2+a*c)^(1/2)*(b*c)^(1/2))/(b*c)^(1/2))*x*a^4*b*c^3-128*(b*c*x^2+a*c)^(5/2)*(b*c)^(1/2)*a^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/(b*c)^(1/2)/x$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((bx^2 + a)^3 c)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^2,x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c(bx^2 + a)^3\right)^{3/2}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^3)^(3/2)/x^2,x)

[Out] int((c*(a + b*x^2)^3)^(3/2)/x^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**2,x)

[Out] Timed out

$$3.82 \quad \int \frac{(c(ax^2+b)^3)^{3/2}}{x^3} dx$$

Optimal. Leaf size=202

$$\frac{9a^3bc\sqrt{c(a+bx^2)^3}}{2(a+bx^2)} + \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} - \frac{9a^2bc\sqrt{c(a+bx^2)^3} \tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2\left(\frac{bx^2}{a}+1\right)^{3/2}} + \frac{9}{10}abc(a+bx^2)\sqrt{c(a+bx^2)^3}$$

Rubi [A] time = 0.22, antiderivative size = 204, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {6720, 266, 47, 50, 63, 208}

$$\frac{9a^3bc\sqrt{c(a+bx^2)^3}}{2(a+bx^2)} + \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} - \frac{9a^{7/2}bc\sqrt{c(a+bx^2)^3} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2(a+bx^2)^{3/2}} + \frac{9}{10}abc(a+bx^2)\sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3\sqrt{c(a+bx^2)^3}}{2x^2} + \frac{9}{14}bc(a+bx^2)^2\sqrt{c(a+bx^2)^3}$$

Antiderivative was successfully verified.

[In] Int[(c*(a + b*x^2)^3)^(3/2)/x^3, x]

[Out] (3*a^2*b*c*Sqrt[c*(a + b*x^2)^3])/2 + (9*a^3*b*c*Sqrt[c*(a + b*x^2)^3])/(2*(a + b*x^2)) + (9*a*b*c*(a + b*x^2)*Sqrt[c*(a + b*x^2)^3])/10 + (9*b*c*(a + b*x^2)^2*Sqrt[c*(a + b*x^2)^3])/14 - (c*(a + b*x^2)^3*Sqrt[c*(a + b*x^2)^3])/((2*x^2) - (9*a^(7/2)*b*c*Sqrt[c*(a + b*x^2)^3]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*(a + b*x^2)^(3/2))

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266


```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x
] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ
[v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(c(a+bx^2)^3\right)^{3/2}}{x^3} dx &= \frac{\left(c\sqrt{c(a+bx^2)^3}\right) \int \frac{(a+bx^2)^{9/2}}{x^3} dx}{(a+bx^2)^{3/2}} \\
&= \frac{\left(c\sqrt{c(a+bx^2)^3}\right) \text{Subst}\left(\int \frac{(a+bx)^{9/2}}{x^2} dx, x, x^2\right)}{2(a+bx^2)^{3/2}} \\
&= -\frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{2x^2} + \frac{\left(9bc\sqrt{c(a+bx^2)^3}\right) \text{Subst}\left(\int \frac{(a+bx)^{7/2}}{x} dx, x, x^2\right)}{4(a+bx^2)^{3/2}} \\
&= \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{2x^2} + \frac{\left(9abc\sqrt{c(a+bx^2)^3}\right)}{4} \\
&= \frac{9}{10}abc(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3} - \frac{c(a+bx^2)^3 \sqrt{c(a+bx^2)^3}}{2x^2} \\
&= \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} + \frac{9}{10}abc(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
&= \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} + \frac{9a^3bc\sqrt{c(a+bx^2)^3}}{2(a+bx^2)} + \frac{9}{10}abc(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
&= \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} + \frac{9a^3bc\sqrt{c(a+bx^2)^3}}{2(a+bx^2)} + \frac{9}{10}abc(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3} \\
&= \frac{3}{2}a^2bc\sqrt{c(a+bx^2)^3} + \frac{9a^3bc\sqrt{c(a+bx^2)^3}}{2(a+bx^2)} + \frac{9}{10}abc(a+bx^2) \sqrt{c(a+bx^2)^3} + \frac{9}{14}bc(a+bx^2)^2 \sqrt{c(a+bx^2)^3}
\end{aligned}$$

Mathematica [C] time = 0.02, size = 48, normalized size = 0.24

$$\frac{b(a+bx^2)\left(c(a+bx^2)^3\right)^{3/2} {}_2F_1\left(2, \frac{11}{2}; \frac{13}{2}; \frac{bx^2}{a} + 1\right)}{11a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] (b*(a + b*x^2)*(c*(a + b*x^2)^3)^(3/2)*Hypergeometric2F1[2, 11/2, 13/2, 1 + (b*x^2)/a])/(11*a^2)
```

IntegrateAlgebraic [A] time = 20.09, size = 152, normalized size = 0.75

$$\frac{\left(c(a + bx^2)^3\right)^{3/2} \left(\frac{\sqrt{a+bx^2} \left(-35a^4c^{3/2} + 388a^3bc^{3/2}x^2 + 156a^2b^2c^{3/2}x^4 + 58ab^3c^{3/2}x^6 + 10b^4c^{3/2}x^8\right)}{70x^2} - \frac{9}{2}a^{7/2}bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)\right)}{c^{3/2}(a + bx^2)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*(a + b*x^2)^3)^(3/2)/x^3,x]
```

```
[Out] ((c*(a + b*x^2)^3)^(3/2)*((Sqrt[a + b*x^2]*(-35*a^4*c^(3/2) + 388*a^3*b*c^(3/2)*x^2 + 156*a^2*b^2*c^(3/2)*x^4 + 58*a*b^3*c^(3/2)*x^6 + 10*b^4*c^(3/2)*x^8))/(70*x^2) - (9*a^(7/2)*b*c^(3/2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/2)/(c^(3/2)*(a + b*x^2)^(9/2))
```

fricas [A] time = 0.81, size = 411, normalized size = 2.03

$$\frac{315(a^4b^2cx^4 + a^4bcx^2)\sqrt{ac} \log\left(\frac{2a^2 + 2abx^2 + 2a^2x^4 - 2\sqrt{bcx^2+ac}\sqrt{bx^2+a}}{2a^2}\right) + 2(10b^4cx^8 + 58ab^3cx^6 + 156a^2b^2cx^4 + 388a^3bcx^2 - 35a^4c)\sqrt{bcx^2+ac} + a^4c}{140(bx^4 + ax^2)} - \frac{315(a^4b^2cx^4 + a^4bcx^2)\sqrt{-ac} \arctan\left(\frac{2\sqrt{bcx^2+ac}\sqrt{bx^2+a}}{2a^2}\right) + (10b^4cx^8 + 58ab^3cx^6 + 156a^2b^2cx^4 + 388a^3bcx^2 - 35a^4c)\sqrt{bcx^2+ac} + a^4c}{70(bx^4 + ax^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/140*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(a*c)*log(-(b^2*c*x^4 + 3*a*b*c*x^2 + 2*a^2*c - 2*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(a*c))/(b*x^4 + a*x^2)) + 2*(10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2), 1/70*(315*(a^3*b^2*c*x^4 + a^4*b*c*x^2)*sqrt(-a*c)*arctan(sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c)*sqrt(-a*c)/(b^2*c*x^4 + 2*a*b*c*x^2 + a^2*c)) + (10*b^4*c*x^8 + 58*a*b^3*c*x^6 + 156*a^2*b^2*c*x^4 + 388*a^3*b*c*x^2 - 35*a^4*c)*sqrt(b^3*c*x^6 + 3*a*b^2*c*x^4 + 3*a^2*b*c*x^2 + a^3*c))/(b*x^4 + a*x^2)]
```

giac [A] time = 0.39, size = 204, normalized size = 1.01

$$\frac{\left(\frac{315a^4b^2c \arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right) \operatorname{sgn}(bx^2+a)}{\sqrt{-ac}} - \frac{35\sqrt{bcx^2+ac}a^4b \operatorname{sgn}(bx^2+a)}{x^2} + \frac{2\left(140\sqrt{bcx^2+ac}a^3b^2c^2 \operatorname{sgn}(bx^2+a) + 35(bc^2+ac)^{\frac{3}{2}}a^2b^2c^2 \operatorname{sgn}(bx^2+a) + 14(bc^2+ac)^{\frac{5}{2}}ab^2c^{19} \operatorname{sgn}(bx^2+a) + 5(bc^2+ac)^{\frac{7}{2}}b^2c^{18} \operatorname{sgn}(bx^2+a)\right)}{c^{21}}\right)}{70b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/70*(315*a^4*b^2*c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))*sgn(b*x^2 + a)/sqrt(-a*c) - 35*sqrt(b*c*x^2 + a*c)*a^4*b*sgn(b*x^2 + a)/x^2 + 2*(140*sqrt(b*c*x^2 + a*c)*a^3*b^2*c^21*sgn(b*x^2 + a) + 35*(b*c*x^2 + a*c)^(3/2)*a^2*b^2*c^20*sgn(b*x^2 + a) + 14*(b*c*x^2 + a*c)^(5/2)*a*b^2*c^19*sgn(b*x^2 + a) + 5*(b*c*x^2 + a*c)^(7/2)*b^2*c^18*sgn(b*x^2 + a))/c^21*c/b
```

maple [A] time = 0.02, size = 238, normalized size = 1.18

$$\frac{\left((bx^2 + a)^3 c\right)^{\frac{3}{2}} \left(-315a^4 b c^3 x^2 \ln\left(\frac{2ax + 2\sqrt{ac}\sqrt{bcx^2+ac}}{c}\right) + 315\sqrt{bcx^2+ac}\sqrt{ac}a^3 b c^2 x^2 + 105(bc^2 + ac)^{\frac{3}{2}}\sqrt{ac}a^2 b c x^2 + 10\sqrt{ac}(bc^2 + ac)^{\frac{5}{2}}b^2 x^4 - 4\sqrt{ac}(bc^2 + ac)^{\frac{5}{2}}abx^2 + 42((bx^2 + a)c)^{\frac{5}{2}}\sqrt{ac}abx^2 - 35\sqrt{ac}(bc^2 + ac)^{\frac{5}{2}}a^2\right)}{70(bx^2 + a)^3((bx^2 + a)c)^{\frac{3}{2}}\sqrt{ac}cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^3*c)^(3/2)/x^3,x)

[Out] 1/70*((b*x^2+a)^3*c)^(3/2)*(10*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*b^2*x^4-315*ln(2*(a*c+(a*c)^(1/2)*(b*c*x^2+a*c)^(1/2))/x)*x^2*a^4*b*c^3-4*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a*b*x^2+105*(b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*b*c+315*(b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a^3*b*c^2+42*a*b*((b*x^2+a)*c)^(5/2)*x^2*(a*c)^(1/2)-35*(a*c)^(1/2)*(b*c*x^2+a*c)^(5/2)*a^2)/(b*x^2+a)^3/((b*x^2+a)*c)^(3/2)/c/x^2/(a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\left(bx^2 + a\right)^3 c\right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^3)^(3/2)/x^3,x, algorithm="maxima")

[Out] integrate(((b*x^2 + a)^3*c)^(3/2)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\left(bx^2 + a\right)^3\right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^3)^(3/2)/x^3,x)

[Out] int((c*(a + b*x^2)^3)^(3/2)/x^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**3)**(3/2)/x**3,x)

[Out] Timed out

$$3.83 \quad \int x^2 \left(\frac{c}{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=77

$$\frac{\sqrt{a} c \sqrt{\frac{bx^2}{a} + 1} \sqrt{\frac{c}{a+bx^2}} \sinh^{-1} \left(\frac{\sqrt{b} x}{\sqrt{a}} \right) - \frac{cx \sqrt{\frac{c}{a+bx^2}}}{b}}{b^{3/2}}$$

Rubi [A] time = 0.14, antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {6720, 288, 217, 206}

$$\frac{c \sqrt{a+bx^2} \sqrt{\frac{c}{a+bx^2}} \tanh^{-1} \left(\frac{\sqrt{b} x}{\sqrt{a+bx^2}} \right) - \frac{cx \sqrt{\frac{c}{a+bx^2}}}{b}}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*x*Sqrt[c/(a + b*x^2)])/b) + (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/b^(3/2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
\int x^2 \left(\frac{c}{a+bx^2}\right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{x^2}{(a+bx^2)^{3/2}} dx \\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{1}{\sqrt{a+bx^2}} dx}{b} \\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{b} \\
&= -\frac{cx\sqrt{\frac{c}{a+bx^2}}}{b} + \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 89, normalized size = 1.16

$$\frac{\sqrt{a}\sqrt{\frac{bx^2}{a}+1}\left(\frac{c}{a+bx^2}\right)^{3/2}\left((a+bx^2)\sinh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)-\sqrt{a}\sqrt{b}x\sqrt{\frac{bx^2}{a}+1}\right)}{b^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(c/(a + b*x^2))^(3/2), x]

[Out] (Sqrt[a]*(c/(a + b*x^2))^(3/2)*Sqrt[1 + (b*x^2)/a]*(-(Sqrt[a]*Sqrt[b]*x*Sqrt[1 + (b*x^2)/a]) + (a + b*x^2)*ArcSinh[(Sqrt[b]*x)/Sqrt[a]]))/b^(3/2)

IntegrateAlgebraic [A] time = 5.72, size = 88, normalized size = 1.14

$$\frac{(a+bx^2)^{3/2}\left(\frac{c}{a+bx^2}\right)^{3/2}\left(-\frac{c^{3/2}\log(\sqrt{a+bx^2}-\sqrt{b}x)}{b^{3/2}}-\frac{c^{3/2}x}{b\sqrt{a+bx^2}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(c/(a + b*x^2))^(3/2), x]

[Out] ((c/(a + b*x^2))^(3/2)*(a + b*x^2)^(3/2)*(-(c^(3/2)*x)/(b*Sqrt[a + b*x^2])) - (c^(3/2)*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/b^(3/2))/c^(3/2)

fricas [A] time = 0.64, size = 141, normalized size = 1.83

$$\left[\frac{2cx\sqrt{\frac{c}{bx^2+a}} - c\sqrt{\frac{c}{b}} \log\left(-2bcx^2 - ac - 2(b^2x^3 + abx)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{b}}\right)}{2b}, \frac{cx\sqrt{\frac{c}{bx^2+a}} + c\sqrt{\frac{-c}{b}} \arctan\left(\frac{bx\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{-c}{b}}}{c}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c/(b*x^2+a))^(3/2), x, algorithm="fricas")

[Out] [-1/2*(2*c*x*sqrt(c/(b*x^2 + a)) - c*sqrt(c/b)*log(-2*b*c*x^2 - a*c - 2*(b^2*x^3 + a*b*x)*sqrt(c/(b*x^2 + a))*sqrt(c/b)))/b, -(c*x*sqrt(c/(b*x^2 + a)) + c*sqrt(-c/b)*arctan(b*x*sqrt(c/(b*x^2 + a))*sqrt(-c/b)/c))/b]

giac [A] time = 0.46, size = 71, normalized size = 0.92

$$-\left(\frac{cx\text{sgn}(bx^2+a)}{\sqrt{bcx^2+ac}b} + \frac{c\log\left(\left|-\sqrt{bc}x + \sqrt{bcx^2+ac}\right|\right)\text{sgn}(bx^2+a)}{\sqrt{bc}b}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="giac")

[Out] $-(c*x*\text{sgn}(b*x^2 + a)/(\text{sqrt}(b*c*x^2 + a*c)*b) + c*\log(\text{abs}(-\text{sqrt}(b*c)*x + \text{sqrt}(b*c*x^2 + a*c))))*\text{sgn}(b*x^2 + a)/(\text{sqrt}(b*c)*b))*c$

maple [A] time = 0.01, size = 60, normalized size = 0.78

$$\frac{\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}(bx^2+a)\left(b^{\frac{3}{2}}x - \sqrt{bx^2+a} b \ln\left(\sqrt{b}x + \sqrt{bx^2+a}\right)\right)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c/(b*x^2+a))^(3/2),x)

[Out] $-(c/(b*x^2+a))^{3/2}*(b*x^2+a)*(x*b^{3/2}-\ln(b^{1/2}*x+(b*x^2+a)^{1/2}))*b*(b*x^2+a)^{1/2}/b^{5/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")

[Out] integrate(x^2*(c/(b*x^2+a))^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 \left(\frac{c}{bx^2+a}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(c/(a+b*x^2))^(3/2),x)

[Out] int(x^2*(c/(a+b*x^2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(c/(b*x**2+a))**(3/2),x)

[Out] Integral(x**2*(c/(a+b*x**2))**(3/2), x)

$$3.84 \quad \int x \left(\frac{c}{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=21

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1591, 15, 30}

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] Int[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int x \left(\frac{c}{a+bx^2} \right)^{3/2} dx &= \frac{\text{Subst} \left(\int \left(\frac{c}{x} \right)^{3/2} dx, x, a+bx^2 \right)}{2b} \\ &= \frac{\left(c\sqrt{\frac{c}{a+bx^2}} \sqrt{a+bx^2} \right) \text{Subst} \left(\int \frac{1}{x^{3/2}} dx, x, a+bx^2 \right)}{2b} \\ &= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{b} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

IntegrateAlgebraic [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{c\sqrt{\frac{c}{a+bx^2}}}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(c/(a + b*x^2))^(3/2),x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/b)

fricas [A] time = 0.90, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="fricas")

[Out] -c*sqrt(c/(b*x^2 + a))/b

giac [A] time = 0.39, size = 28, normalized size = 1.33

$$-\frac{c^2\operatorname{sgn}(bx^2 + a)}{\sqrt{bcx^2 + ac}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="giac")

[Out] -c^2*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*b)

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$-\frac{(bx^2 + a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1/(b*x^2+a)*c)^(3/2),x)

[Out] -(b*x^2+a)/b*(1/(b*x^2+a)*c)^(3/2)

maxima [A] time = 1.06, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c/(b*x^2+a))^(3/2),x, algorithm="maxima")

[Out] -c*sqrt(c/(b*x^2 + a))/b

mupad [B] time = 2.64, size = 19, normalized size = 0.90

$$-\frac{c\sqrt{\frac{c}{bx^2+a}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c/(a + b*x^2))^(3/2),x)`

[Out] `-(c*(c/(a + b*x^2))^(1/2))/b`

sympy [A] time = 1.51, size = 53, normalized size = 2.52

$$\begin{cases} -\frac{ac^{\frac{3}{2}}\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{b} - c^{\frac{3}{2}}x^2\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} & \text{for } b \neq 0 \\ \frac{x^2\left(\frac{c}{a}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c/(b*x**2+a))**(3/2),x)`

[Out] `Piecewise((-a*c**(3/2)*(1/(a + b*x**2))**(3/2)/b - c**(3/2)*x**2*(1/(a + b*x**2))**(3/2), Ne(b, 0)), (x**2*(c/a)**(3/2)/2, True))`

$$3.85 \quad \int \left(\frac{c}{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=21

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6720, 191}

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2), x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \left(\frac{c}{a+bx^2} \right)^{3/2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}} \sqrt{a+bx^2} \right) \int \frac{1}{(a+bx^2)^{3/2}} dx \\ &= \frac{cx\sqrt{\frac{c}{a+bx^2}}}{a} \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 1.00

$$\frac{cx\sqrt{\frac{c}{a+bx^2}}}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c/(a + b*x^2))^(3/2), x]

[Out] (c*x*Sqrt[c/(a + b*x^2)])/a

IntegrateAlgebraic [A] time = 5.62, size = 27, normalized size = 1.29

$$\frac{x(a+bx^2)\left(\frac{c}{a+bx^2}\right)^{3/2}}{a}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2),x]

[Out] (x*(c/(a + b*x^2))^(3/2)*(a + b*x^2))/a

fricas [A] time = 0.50, size = 19, normalized size = 0.90

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="fricas")

[Out] c*x*sqrt(c/(b*x^2 + a))/a

giac [A] time = 0.48, size = 28, normalized size = 1.33

$$\frac{c^2x\operatorname{sgn}(bx^2 + a)}{\sqrt{bcx^2 + ac}a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="giac")

[Out] c^2*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a)

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$\frac{(bx^2 + a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(b*x^2+a)*c)^(3/2),x)

[Out] (b*x^2+a)/a*x*(1/(b*x^2+a)*c)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{c}{bx^2 + a}\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2),x, algorithm="maxima")

[Out] integrate((c/(b*x^2 + a))^(3/2), x)

mupad [B] time = 2.74, size = 19, normalized size = 0.90

$$\frac{cx\sqrt{\frac{c}{bx^2+a}}}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c/(a + b*x^2))^(3/2),x)

[Out] (c*x*(c/(a + b*x^2))^(1/2))/a

sympy [A] time = 1.46, size = 66, normalized size = 3.14

$$\begin{cases} c^{\frac{3}{2}}x\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}} + \frac{bc^{\frac{3}{2}}x^3\left(\frac{1}{a+bx^2}\right)^{\frac{3}{2}}}{a} & \text{for } a \neq 0 \\ -\frac{c^{\frac{3}{2}}x\left(\frac{1}{b}\right)^{\frac{3}{2}}\left(\frac{1}{x^2}\right)^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x**2+a))**(3/2),x)

[Out] Piecewise((c**(3/2)*x*(1/(a + b*x**2))**(3/2) + b*c**(3/2)*x**3*(1/(a + b*x**2))**(3/2)/a, Ne(a, 0)), (-c**(3/2)*x*(1/b)**(3/2)*(x**(-2))**(3/2)/2, True))

$$3.86 \quad \int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x} dx$$

Optimal. Leaf size=71

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{bx^2}{a} + 1}\sqrt{\frac{c}{a+bx^2}} \tanh^{-1}\left(\sqrt{\frac{bx^2}{a} + 1}\right)}{a}$$

Rubi [A] time = 0.14, antiderivative size = 73, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6720, 266, 51, 63, 208}

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2)/x,x]

[Out] (c*Sqrt[c/(a + b*x^2)]/a - (c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/a^(3/2)

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6720

```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^
FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x]
&& !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[
v, x] && EqQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{1}{x(a+bx^2)^{3/2}} dx \\
&= \frac{1}{2} \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, x^2\right) \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^2\right)}{2a} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} + \frac{\left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx^2}\right)}{ab} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{a^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 38, normalized size = 0.54

$$\frac{c\sqrt{\frac{c}{a+bx^2}} {}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{bx^2}{a} + 1\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[(c/(a + b*x^2))^(3/2)/x,x]

[Out] (c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*x^2)/a])/a

IntegrateAlgebraic [A] time = 0.08, size = 60, normalized size = 0.85

$$\frac{c\sqrt{\frac{c}{a+bx^2}}}{a} - \frac{c^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\frac{c}{a+bx^2}}}{\sqrt{c}}\right)}{a^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x,x]

[Out] (c*Sqrt[c/(a + b*x^2)]/a - (c^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c/(a + b*x^2)])]/Sqrt[c]))/a^(3/2)

fricas [A] time = 0.68, size = 138, normalized size = 1.94

$$\left[\frac{c\sqrt{\frac{c}{a}} \log\left(-\frac{bcx^2+2ac-2(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right) + 2c\sqrt{\frac{c}{bx^2+a}}}{2a}, \frac{c\sqrt{\frac{-c}{a}} \arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{-c}{a}}}{c}\right) + c\sqrt{\frac{c}{bx^2+a}}}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="fricas")

[Out] [1/2*(c*sqrt(c/a)*log(-(b*c*x^2 + 2*a*c - 2*(a*b*x^2 + a^2)*sqrt(c/(b*x^2 + a))*sqrt(c/a))/x^2) + 2*c*sqrt(c/(b*x^2 + a)))/a, (c*sqrt(-c/a)*arctan(a*sqrt(c/(b*x^2 + a))*sqrt(-c/a)/c) + c*sqrt(c/(b*x^2 + a)))/a]

giac [A] time = 0.25, size = 59, normalized size = 0.83

$$c \left(\frac{c \arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac} a} + \frac{c}{\sqrt{bcx^2+ac} a} \right) \operatorname{sgn}(bx^2+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="giac")

[Out] c*(c*arctan(sqrt(b*c*x^2 + a*c)/sqrt(-a*c))/(sqrt(-a*c)*a) + c/(sqrt(b*c*x^2 + a*c)*a))*sgn(b*x^2 + a)

maple [A] time = 0.01, size = 64, normalized size = 0.90

$$\frac{\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}} (bx^2+a) \left(\sqrt{bx^2+a} a \ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right) - a^{\frac{3}{2}}\right)}{a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(b*x^2+a)*c)^(3/2)/x,x)

[Out] -(1/(b*x^2+a)*c)^(3/2)*(b*x^2+a)*(ln(2*(a^(1/2)*(b*x^2+a)^(1/2)+a)/x)*a*(b*x^2+a)^(1/2)-a^(3/2))/a^(5/2)

maxima [A] time = 1.95, size = 80, normalized size = 1.13

$$\frac{1}{2} c \left(\frac{c \log\left(\frac{a\sqrt{\frac{c}{bx^2+a}} - \sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}} + \sqrt{ac}}\right)}{\sqrt{ac} a} + \frac{2\sqrt{\frac{c}{bx^2+a}}}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x,x, algorithm="maxima")

[Out] 1/2*c*(c*log((a*sqrt(c/(b*x^2 + a)) - sqrt(a*c))/(a*sqrt(c/(b*x^2 + a)) + sqrt(a*c)))/(sqrt(a*c)*a) + 2*sqrt(c/(b*x^2 + a))/a)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{bx^2+a}\right)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c/(a + b*x^2))^(3/2)/x,x)

[Out] int((c/(a + b*x^2))^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x**2+a))**(3/2)/x,x)

[Out] Integral((c/(a + b*x**2))**(3/2)/x, x)

$$3.87 \quad \int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx$$

Optimal. Leaf size=48

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6720, 271, 191}

$$-\frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2)/x^2,x]

[Out] -((c*Sqrt[c/(a + b*x^2)])/(a*x)) - (2*b*c*x*Sqrt[c/(a + b*x^2)]/a^2

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 271

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m + 1)*(a + b*x^n)^(p + 1))/(a*(m + 1)), x] - Dist[(b*(m + n*(p + 1) + 1))/(a*(m + 1)), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^2} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{1}{x^2(a+bx^2)^{3/2}} dx \\ &= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{\left(2bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{1}{(a+bx^2)^{3/2}} dx}{a} \\ &= -\frac{c\sqrt{\frac{c}{a+bx^2}}}{ax} - \frac{2bcx\sqrt{\frac{c}{a+bx^2}}}{a^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.67

$$\frac{c(a+2bx^2)\sqrt{\frac{c}{a+bx^2}}}{a^2x}$$

Antiderivative was successfully verified.

[In] Integrate[(c/(a + b*x^2))^(3/2)/x^2,x]

[Out] -((c*Sqrt[c/(a + b*x^2)]*(a + 2*b*x^2))/(a^2*x))

IntegrateAlgebraic [A] time = 5.65, size = 38, normalized size = 0.79

$$-\frac{(a + bx^2)(a + 2bx^2)\left(\frac{c}{a+bx^2}\right)^{3/2}}{a^2x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x^2,x]

[Out] -(((c/(a + b*x^2))^(3/2)*(a + b*x^2)*(a + 2*b*x^2))/(a^2*x))

fricas [A] time = 0.43, size = 32, normalized size = 0.67

$$-\frac{(2bcx^2 + ac)\sqrt{\frac{c}{bx^2+a}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="fricas")

[Out] -(2*b*c*x^2 + a*c)*sqrt(c/(b*x^2 + a))/(a^2*x)

giac [A] time = 0.34, size = 81, normalized size = 1.69

$$\left[\frac{bcx \operatorname{sgn}(bx^2 + a)}{\sqrt{bcx^2 + ac} a^2} - \frac{2\sqrt{bc} \operatorname{csgn}(bx^2 + a)}{\left(\left(\sqrt{bc}x - \sqrt{bcx^2 + ac}\right)^2 - ac\right)a} \right] c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="giac")

[Out] -(b*c*x*sgn(b*x^2 + a)/(sqrt(b*c*x^2 + a*c)*a^2) - 2*sqrt(b*c)*c*sgn(b*x^2 + a)/(((sqrt(b*c)*x - sqrt(b*c*x^2 + a*c))^2 - a*c)*a)*c

maple [A] time = 0.01, size = 37, normalized size = 0.77

$$-\frac{(bx^2 + a)(2bx^2 + a)\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}}{a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(b*x^2+a)*c)^(3/2)/x^2,x)

[Out] -(b*x^2+a)*(2*b*x^2+a)*(1/(b*x^2+a)*c)^(3/2)/a^2/x

maxima [A] time = 0.96, size = 46, normalized size = 0.96

$$-\frac{2b^2c^{\frac{3}{2}}x^4 + 3abc^{\frac{3}{2}}x^2 + a^2c^{\frac{3}{2}}}{(bx^2 + a)^{\frac{3}{2}}a^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^2,x, algorithm="maxima")

[Out] $-(2*b^2*c^{(3/2)}*x^4 + 3*a*b*c^{(3/2)}*x^2 + a^2*c^{(3/2)})/((b*x^2 + a)^{(3/2)}*a^{2*x})$

mupad [B] time = 2.83, size = 54, normalized size = 1.12

$$\frac{\left(\frac{bc}{a} + \frac{2b^2cx^2}{a^2}\right) \sqrt{\frac{c}{bx^2+a}} \left(\frac{a}{b} + x^2\right)}{bx^3 + ax}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c/(a + b*x^2))^(3/2)/x^2,x)`

[Out] $-(((b*c)/a + (2*b^2*c*x^2)/a^2)*(c/(a + b*x^2))^{(1/2)}*(a/b + x^2))/(a*x + b*x^3)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c/(b*x**2+a))**(3/2)/x**2,x)`

[Out] `Integral((c/(a + b*x**2))**(3/2)/x**2, x)`

$$3.88 \quad \int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3} dx$$

Optimal. Leaf size=104

$$-\frac{3bc\sqrt{\frac{c}{a+bx^2}}}{2a^2} + \frac{3bc\sqrt{\frac{bx^2}{a}+1}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\sqrt{\frac{bx^2}{a}+1}\right)}{2a^2} - \frac{c\sqrt{\frac{c}{a+bx^2}}}{2ax^2}$$

Rubi [A] time = 0.15, antiderivative size = 112, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {6720, 266, 51, 63, 208}

$$-\frac{3c(a+bx^2)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2} + \frac{3bc\sqrt{a+bx^2}\sqrt{\frac{c}{a+bx^2}}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}} + \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2}$$

Antiderivative was successfully verified.

[In] Int[(c/(a + b*x^2))^(3/2)/x^3,x]

[Out] (c*Sqrt[c/(a + b*x^2)]/(a*x^2) - (3*c*Sqrt[c/(a + b*x^2)]*(a + b*x^2))/(2*a^2*x^2) + (3*b*c*Sqrt[c/(a + b*x^2)]*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(5/2)))

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{c}{a+bx^2}\right)^{3/2}}{x^3} dx &= \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \int \frac{1}{x^3(a+bx^2)^{3/2}} dx \\
&= \frac{1}{2} \left(c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}} dx, x, x^2\right) \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} + \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, x^2\right)}{2a} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}(a+bx^2)}{2a^2x^2} - \frac{\left(3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, x^2\right)}{4a^2} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}(a+bx^2)}{2a^2x^2} - \frac{\left(3c\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2}\right) \text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx^2}\right)}{2a^2} \\
&= \frac{c\sqrt{\frac{c}{a+bx^2}}}{ax^2} - \frac{3c\sqrt{\frac{c}{a+bx^2}}(a+bx^2)}{2a^2x^2} + \frac{3bc\sqrt{\frac{c}{a+bx^2}}\sqrt{a+bx^2} \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 40, normalized size = 0.38

$$\frac{bc\sqrt{\frac{c}{a+bx^2}} {}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{bx^2}{a} + 1\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(c/(a + b*x^2))^(3/2)/x^3,x]

[Out] -((b*c*Sqrt[c/(a + b*x^2)]*Hypergeometric2F1[-1/2, 2, 1/2, 1 + (b*x^2)/a])/a^2)

IntegrateAlgebraic [A] time = 0.13, size = 77, normalized size = 0.74

$$\frac{3bc^{3/2} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\frac{c}{a+bx^2}}}{\sqrt{c}}\right)}{2a^{5/2}} - \frac{c(a + 3bx^2)\sqrt{\frac{c}{a+bx^2}}}{2a^2x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(c/(a + b*x^2))^(3/2)/x^3,x]

[Out] -1/2*(c*Sqrt[c/(a + b*x^2)]*(a + 3*b*x^2))/(a^2*x^2) + (3*b*c^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[c/(a + b*x^2)])/Sqrt[c]])/(2*a^(5/2))

fricas [A] time = 0.70, size = 175, normalized size = 1.68

$$\left[\frac{3bcx^2\sqrt{\frac{c}{a}} \log\left(-\frac{bcx^2+2ac+2(abx^2+a^2)\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{x^2}\right) - 2(3bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{4a^2x^2}, -\frac{3bcx^2\sqrt{\frac{c}{a}} \arctan\left(\frac{a\sqrt{\frac{c}{bx^2+a}}\sqrt{\frac{c}{a}}}{c}\right) + (3bcx^2+ac)\sqrt{\frac{c}{bx^2+a}}}{2a^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="fricas")

[Out] $[1/4*(3*b*c*x^2*\sqrt{c/a})*\log(-(b*c*x^2 + 2*a*c + 2*(a*b*x^2 + a^2)*\sqrt{c/(b*x^2 + a)})*\sqrt{c/a))/x^2) - 2*(3*b*c*x^2 + a*c)*\sqrt{c/(b*x^2 + a)))/(a^2*x^2), -1/2*(3*b*c*x^2*\sqrt{-c/a})*\arctan(a*\sqrt{c/(b*x^2 + a)})*\sqrt{-c/a}/c) + (3*b*c*x^2 + a*c)*\sqrt{c/(b*x^2 + a)))/(a^2*x^2)]$

giac [A] time = 0.33, size = 103, normalized size = 0.99

$$-\frac{1}{2}c \left(\frac{3bc \arctan\left(\frac{\sqrt{bcx^2+ac}}{\sqrt{-ac}}\right)}{\sqrt{-ac}a^2} + \frac{2abc^2 - 3(bc x^2 + ac)bc}{\left(\sqrt{bcx^2+ac}ac - (bcx^2+ac)^{\frac{3}{2}}\right)a^2} \right) \operatorname{sgn}(bx^2+a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="giac")

[Out] $-1/2*c*(3*b*c*\arctan(\sqrt{b*c*x^2 + a*c})/\sqrt{-a*c})/(\sqrt{-a*c}*a^2) + (2*a*b*c^2 - 3*(b*c*x^2 + a*c)*b*c)/((\sqrt{b*c*x^2 + a*c}*a*c - (b*c*x^2 + a*c)^{(3/2}))*a^2))*\operatorname{sgn}(b*x^2 + a)$

maple [A] time = 0.01, size = 81, normalized size = 0.78

$$\frac{\left(\frac{c}{bx^2+a}\right)^{\frac{3}{2}}(bx^2+a) \left(3\sqrt{bx^2+a} abx^2 \ln\left(\frac{2a+2\sqrt{bx^2+a}\sqrt{a}}{x}\right) - 3a^{\frac{3}{2}}bx^2 - a^{\frac{5}{2}}\right)}{2a^{\frac{7}{2}}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(b*x^2+a)*c)^(3/2)/x^3,x)

[Out] $1/2*(1/(b*x^2+a)*c)^{(3/2)}*(b*x^2+a)*(3*(b*x^2+a)^{(1/2)}*\ln(2*(a+(b*x^2+a)^{(1/2)})*a^{(1/2)}))/x)*x^2*a*b-3*a^{(3/2)}*x^2*b-a^{(5/2)})/a^{(7/2)}/x^2$

maxima [A] time = 1.99, size = 121, normalized size = 1.16

$$-\frac{1}{4}bc \left(\frac{2c\sqrt{\frac{c}{bx^2+a}}}{a^2c - \frac{a^3c}{bx^2+a}} + \frac{3c \log\left(\frac{a\sqrt{\frac{c}{bx^2+a}} - \sqrt{ac}}{a\sqrt{\frac{c}{bx^2+a}} + \sqrt{ac}}\right)}{\sqrt{ac}a^2} + \frac{4\sqrt{\frac{c}{bx^2+a}}}{a^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c/(b*x^2+a))^(3/2)/x^3,x, algorithm="maxima")

[Out] $-1/4*b*c*(2*c*\sqrt{c/(b*x^2 + a)})/(a^2*c - a^3*c/(b*x^2 + a)) + 3*c*\log((a*\sqrt{c/(b*x^2 + a)} - \sqrt{a*c})/(a*\sqrt{c/(b*x^2 + a)} + \sqrt{a*c}))/(\sqrt{a*c}*a^2) + 4*\sqrt{c/(b*x^2 + a)}/a^2$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{c}{bx^2+a}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c/(a + b*x^2))^(3/2)/x^3,x)

[Out] int((c/(a + b*x^2))^(3/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{c}{a+bx^2}\right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c/(b*x**2+a))**(3/2)/x**3,x)
```

```
[Out] Integral((c/(a + b*x**2))**(3/2)/x**3, x)
```

$$3.89 \quad \int x^7 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=138

$$-\frac{2a^3(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^4} + \frac{6a^2(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^4} + \frac{2(a+bx^2)^4\left(c\sqrt{a+bx^2}\right)^{3/2}}{19b^4} - \frac{2a(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{5b^4}$$

Rubi [A] time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, integrand size = 21, number of rules / integrand size = 0.143, Rules used = {6720, 266, 43}

$$\frac{6a^2c(a+bx^2)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^4} - \frac{2a^3c(a+bx^2)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^4} + \frac{2c(a+bx^2)^{9/2}\sqrt{c\sqrt{a+bx^2}}}{19b^4} - \frac{2ac(a+bx^2)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{5b^4}$$

Antiderivative was successfully verified.

[In] Int[x^7*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (-2*a^3*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^4) + (6*a^2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^4) - (2*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(7/2))/(5*b^4) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(9/2))/(19*b^4)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int x^7 \left(c\sqrt{a+bx^2} \right)^{3/2} dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \int x^7 (a+bx^2)^{3/4} dx}{\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst} \left(\int x^3 (a+bx)^{3/4} dx, x, x^2 \right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst} \left(\int \left(-\frac{a^3(a+bx)^{3/4}}{b^3} + \frac{3a^2(a+bx)^{7/4}}{b^3} - \frac{3a(a+bx)^{11/4}}{b^3} + \frac{(a+bx)^{15/4}}{b^3} \right) dx, x, x^2 \right)}{2\sqrt[4]{a+bx^2}} \\ &= -\frac{2a^3c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{3/2}}{7b^4} + \frac{6a^2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{5/2}}{11b^4} - \frac{2ac\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{7/2}}{5b^4} + \frac{2c(a+bx^2)^{9/2}\sqrt{c\sqrt{a+bx^2}}}{19b^4} \end{aligned}$$

Mathematica [A] time = 0.04, size = 63, normalized size = 0.46

$$\frac{2(a + bx^2)(-128a^3 + 224a^2bx^2 - 308ab^2x^4 + 385b^3x^6)(c\sqrt{a + bx^2})^{3/2}}{7315b^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-128*a^3 + 224*a^2*b*x^2 - 308*a*b^2*x^4 + 385*b^3*x^6))/(7315*b^4)

IntegrateAlgebraic [A] time = 19.49, size = 83, normalized size = 0.60

$$\frac{2\sqrt{a + bx^2}\sqrt{c\sqrt{a + bx^2}}(128a^4c - 96a^3bcx^2 + 84a^2b^2cx^4 - 77ab^3cx^6 - 385b^4cx^8)}{7315b^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^7*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (-2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(128*a^4*c - 96*a^3*b*c*x^2 + 84*a^2*b^2*c*x^4 - 77*a*b^3*c*x^6 - 385*b^4*c*x^8))/(7315*b^4)

fricas [A] time = 0.92, size = 75, normalized size = 0.54

$$\frac{2(385b^4cx^8 + 77ab^3cx^6 - 84a^2b^2cx^4 + 96a^3bcx^2 - 128a^4c)\sqrt{bx^2 + a}\sqrt{\sqrt{bx^2 + a}c}}{7315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 2/7315*(385*b^4*c*x^8 + 77*a*b^3*c*x^6 - 84*a^2*b^2*c*x^4 + 96*a^3*b*c*x^2 - 128*a^4*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^4

giac [A] time = 0.34, size = 137, normalized size = 0.99

$$\frac{2c^{\frac{3}{2}}\left(\frac{19\left(77(bx^2+a)^{\frac{15}{4}}-315(bx^2+a)^{\frac{11}{4}}+495(bx^2+a)^{\frac{7}{4}}-385(bx^2+a)^{\frac{3}{4}}\right)a}{b^3}+\frac{1155(bx^2+a)^{\frac{19}{4}}-5852(bx^2+a)^{\frac{15}{4}}+11970(bx^2+a)^{\frac{11}{4}}-12540(bx^2+a)^{\frac{7}{4}}+7315(bx^2+a)^{\frac{3}{4}}a^4}{b^3}\right)}{21945b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")

[Out] 2/21945*c^(3/2)*(19*(77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)*a/b^3 + (1155*(b*x^2 + a)^(19/4) - 5852*(b*x^2 + a)^(15/4)*a + 11970*(b*x^2 + a)^(11/4)*a^2 - 12540*(b*x^2 + a)^(7/4)*a^3 + 7315*(b*x^2 + a)^(3/4)*a^4)/b^3/b

maple [A] time = 0.01, size = 58, normalized size = 0.42

$$\frac{2(bx^2 + a)(-385b^3x^6 + 308ab^2x^4 - 224a^2bx^2 + 128a^3)\left(\sqrt{bx^2 + a}c\right)^{\frac{3}{2}}}{7315b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x)

[Out] $-2/7315*(b*x^2+a)*(-385*b^3*x^6+308*a*b^2*x^4-224*a^2*b*x^2+128*a^3)*(c*(b*x^2+a)^{(1/2)})^{(3/2)}/b^4$

maxima [A] time = 0.98, size = 85, normalized size = 0.62

$$\frac{2 \left(1045 \left(\sqrt{bx^2 + ac} \right)^{\frac{7}{2}} a^3 c^6 - 1995 \left(\sqrt{bx^2 + ac} \right)^{\frac{11}{2}} a^2 c^4 + 1463 \left(\sqrt{bx^2 + ac} \right)^{\frac{15}{2}} ac^2 - 385 \left(\sqrt{bx^2 + ac} \right)^{\frac{19}{2}} \right)}{7315 b^4 c^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")

[Out] $-2/7315*(1045*(\text{sqrt}(b*x^2 + a)*c)^{(7/2)}*a^3*c^6 - 1995*(\text{sqrt}(b*x^2 + a)*c)^{(11/2)}*a^2*c^4 + 1463*(\text{sqrt}(b*x^2 + a)*c)^{(15/2)}*a*c^2 - 385*(\text{sqrt}(b*x^2 + a)*c)^{(19/2)})/(b^4*c^8)$

mupad [B] time = 2.96, size = 109, normalized size = 0.79

$$\sqrt{c \sqrt{bx^2 + a}} \left(\frac{2cx^8 \sqrt{bx^2 + a}}{19} - \frac{256a^4 c \sqrt{bx^2 + a}}{7315b^4} + \frac{2acx^6 \sqrt{bx^2 + a}}{95b} - \frac{24a^2 cx^4 \sqrt{bx^2 + a}}{1045b^2} + \frac{192a^3 cx^2 \sqrt{bx^2 + a}}{7315b^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(c*(a + b*x^2)^(1/2))^(3/2),x)

[Out] $(c*(a + b*x^2)^{(1/2)})^{(1/2)}*((2*c*x^8*(a + b*x^2)^{(1/2)})/19 - (256*a^4*c*(a + b*x^2)^{(1/2)})/(7315*b^4) + (2*a*c*x^6*(a + b*x^2)^{(1/2)})/(95*b) - (24*a^2*c*x^4*(a + b*x^2)^{(1/2)})/(1045*b^2) + (192*a^3*c*x^2*(a + b*x^2)^{(1/2)})/(7315*b^3))$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**7*(c*(b*x**2+a)**(1/2))**(3/2),x)

[Out] Timed out

$$3.90 \quad \int x^5 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=102

$$\frac{2a^2(a+bx^2)\left(c\sqrt{a+bx^2}\right)^{3/2}}{7b^3} + \frac{2(a+bx^2)^3\left(c\sqrt{a+bx^2}\right)^{3/2}}{15b^3} - \frac{4a(a+bx^2)^2\left(c\sqrt{a+bx^2}\right)^{3/2}}{11b^3}$$

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6720, 266, 43}

$$\frac{2a^2c(a+bx^2)^{3/2}\sqrt{c\sqrt{a+bx^2}}}{7b^3} + \frac{2c(a+bx^2)^{7/2}\sqrt{c\sqrt{a+bx^2}}}{15b^3} - \frac{4ac(a+bx^2)^{5/2}\sqrt{c\sqrt{a+bx^2}}}{11b^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (2*a^2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^3) - (4*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^3) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(7/2))/(15*b^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int x^5 \left(c\sqrt{a+bx^2} \right)^{3/2} dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \int x^5 (a+bx^2)^{3/4} dx}{\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst}\left(\int x^2(a+bx)^{3/4} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst}\left(\int \left(\frac{a^2(a+bx)^{3/4}}{b^2} - \frac{2a(a+bx)^{7/4}}{b^2} + \frac{(a+bx)^{11/4}}{b^2}\right) dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{2a^2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{3/2}}{7b^3} - \frac{4ac\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{5/2}}{11b^3} + \frac{2c\sqrt{c\sqrt{a+bx^2}}(a+bx^2)^{7/2}}{15b^3} \end{aligned}$$

Mathematica [A] time = 0.03, size = 52, normalized size = 0.51

$$\frac{2(a + bx^2)(32a^2 - 56abx^2 + 77b^2x^4)(c\sqrt{a + bx^2})^{3/2}}{1155b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(32*a^2 - 56*a*b*x^2 + 77*b^2*x^4))/(1155*b^3)

IntegrateAlgebraic [A] time = 25.56, size = 71, normalized size = 0.70

$$\frac{2\sqrt{a + bx^2}\sqrt{c\sqrt{a + bx^2}}(32a^3c - 24a^2bcx^2 + 21ab^2cx^4 + 77b^3cx^6)}{1155b^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(32*a^3*c - 24*a^2*b*c*x^2 + 21*a*b^2*c*x^4 + 77*b^3*c*x^6))/(1155*b^3)

fricas [A] time = 0.72, size = 63, normalized size = 0.62

$$\frac{2(77b^3cx^6 + 21ab^2cx^4 - 24a^2bcx^2 + 32a^3c)\sqrt{bx^2 + a}\sqrt{\sqrt{bx^2 + a}c}}{1155b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2), x, algorithm="fricas")

[Out] 2/1155*(77*b^3*c*x^6 + 21*a*b^2*c*x^4 - 24*a^2*b*c*x^2 + 32*a^3*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^3

giac [A] time = 0.24, size = 109, normalized size = 1.07

$$\frac{2c^{\frac{3}{2}}\left(\frac{5\left(21(bx^2+a)^{\frac{11}{4}}-66(bx^2+a)^{\frac{7}{4}}a+77(bx^2+a)^{\frac{3}{4}}a^2\right)a}{b^2}+\frac{77(bx^2+a)^{\frac{15}{4}}-315(bx^2+a)^{\frac{11}{4}}a+495(bx^2+a)^{\frac{7}{4}}a^2-385(bx^2+a)^{\frac{3}{4}}a^3}{b^2}\right)}{1155b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2), x, algorithm="giac")

[Out] 2/1155*c^(3/2)*(5*(21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)*a/b^2 + (77*(b*x^2 + a)^(15/4) - 315*(b*x^2 + a)^(11/4)*a + 495*(b*x^2 + a)^(7/4)*a^2 - 385*(b*x^2 + a)^(3/4)*a^3)/b^2)/b

maple [A] time = 0.01, size = 47, normalized size = 0.46

$$\frac{2(bx^2 + a)(77x^4b^2 - 56abx^2 + 32a^2)\left(\sqrt{bx^2 + a}c\right)^{\frac{3}{2}}}{1155b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*((b*x^2+a)^(1/2)*c)^(3/2), x)

[Out] $2/1155*(b*x^2+a)*(77*b^2*x^4-56*a*b*x^2+32*a^2)*((b*x^2+a)^{(1/2)}*c)^{(3/2)}/b^3$

maxima [A] time = 0.93, size = 64, normalized size = 0.63

$$\frac{2 \left(165 \left(\sqrt{bx^2 + ac} \right)^{\frac{7}{2}} a^2 c^4 - 210 \left(\sqrt{bx^2 + ac} \right)^{\frac{11}{2}} ac^2 + 77 \left(\sqrt{bx^2 + ac} \right)^{\frac{15}{2}} \right)}{1155 b^3 c^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")`

[Out] $2/1155*(165*(\text{sqrt}(b*x^2 + a)*c)^{(7/2)}*a^2*c^4 - 210*(\text{sqrt}(b*x^2 + a)*c)^{(11/2)}*a*c^2 + 77*(\text{sqrt}(b*x^2 + a)*c)^{(15/2)})/(b^3*c^6)$

mupad [B] time = 2.90, size = 88, normalized size = 0.86

$$\sqrt{c \sqrt{bx^2 + a}} \left(\frac{2cx^6 \sqrt{bx^2 + a}}{15} + \frac{64a^3 c \sqrt{bx^2 + a}}{1155b^3} + \frac{2acx^4 \sqrt{bx^2 + a}}{55b} - \frac{16a^2 c x^2 \sqrt{bx^2 + a}}{385b^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(c*(a + b*x^2)^(1/2))^(3/2),x)`

[Out] $(c*(a + b*x^2)^{(1/2)})^{(1/2)}*((2*c*x^6*(a + b*x^2)^{(1/2)})/15 + (64*a^3*c*(a + b*x^2)^{(1/2)})/(1155*b^3) + (2*a*c*x^4*(a + b*x^2)^{(1/2)})/(55*b) - (16*a^2*c*x^2*(a + b*x^2)^{(1/2)})/(385*b^2))$

sympy [A] time = 88.61, size = 116, normalized size = 1.14

$$\begin{cases} \frac{64a^3c^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{1155b^3} - \frac{16a^2c^{\frac{3}{2}}x^2(a+bx^2)^{\frac{3}{4}}}{385b^2} + \frac{2ac^{\frac{3}{2}}x^4(a+bx^2)^{\frac{3}{4}}}{55b} + \frac{2c^{\frac{3}{2}}x^6(a+bx^2)^{\frac{3}{4}}}{15} & \text{for } b \neq 0 \\ \frac{x^6(\sqrt{ac})^{\frac{3}{2}}}{6} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(c*(b*x**2+a)**(1/2))**(3/2),x)`

[Out] `Piecewise((64*a**3*c**(3/2)*(a + b*x**2)**(3/4)/(1155*b**3) - 16*a**2*c**(3/2)*x**2*(a + b*x**2)**(3/4)/(385*b**2) + 2*a*c**(3/2)*x**4*(a + b*x**2)**(3/4)/(55*b) + 2*c**(3/2)*x**6*(a + b*x**2)**(3/4)/15, Ne(b, 0)), (x**6*(sqrt(a)*c)**(3/2)/6, True))`

$$3.91 \quad \int x^3 \left(c\sqrt{a+bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=66

$$\frac{2(a+bx^2)^2 \left(c\sqrt{a+bx^2} \right)^{3/2}}{11b^2} - \frac{2a(a+bx^2) \left(c\sqrt{a+bx^2} \right)^{3/2}}{7b^2}$$

Rubi [A] time = 0.14, antiderivative size = 74, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6720, 266, 43}

$$\frac{2c(a+bx^2)^{5/2} \sqrt{c\sqrt{a+bx^2}}}{11b^2} - \frac{2ac(a+bx^2)^{3/2} \sqrt{c\sqrt{a+bx^2}}}{7b^2}$$

Antiderivative was successfully verified.

[In] Int[x^3*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (-2*a*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b^2) + (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(5/2))/(11*b^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int x^3 \left(c\sqrt{a+bx^2} \right)^{3/2} dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \int x^3 (a+bx^2)^{3/4} dx}{\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst} \left(\int x(a+bx)^{3/4} dx, x, x^2 \right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}} \right) \text{Subst} \left(\int \left(-\frac{a+(bx)^{3/4}}{b} + \frac{(a+bx)^{7/4}}{b} \right) dx, x, x^2 \right)}{2\sqrt[4]{a+bx^2}} \\ &= -\frac{2ac\sqrt{c\sqrt{a+bx^2}} (a+bx^2)^{3/2}}{7b^2} + \frac{2c\sqrt{c\sqrt{a+bx^2}} (a+bx^2)^{5/2}}{11b^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 41, normalized size = 0.62

$$\frac{2(a+bx^2)(7bx^2-4a)(c\sqrt{a+bx^2})^{3/2}}{77b^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*(-4*a + 7*b*x^2))/(77*b^2)

IntegrateAlgebraic [A] time = 12.36, size = 59, normalized size = 0.89

$$\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}(4a^2c-3abcx^2-7b^2cx^4)}{77b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (-2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(4*a^2*c - 3*a*b*c*x^2 - 7*b^2*c*x^4))/(77*b^2)

fricas [A] time = 0.65, size = 51, normalized size = 0.77

$$\frac{2(7b^2cx^4 + 3abcx^2 - 4a^2c)\sqrt{bx^2 + a}\sqrt{\sqrt{bx^2 + a}c}}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 2/77*(7*b^2*c*x^4 + 3*a*b*c*x^2 - 4*a^2*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b^2

giac [A] time = 0.30, size = 81, normalized size = 1.23

$$\frac{2\left(\frac{11\left(3(bx^2+a)^{\frac{7}{4}}-7(bx^2+a)^{\frac{3}{4}}a\right)a}{b} + \frac{21(bx^2+a)^{\frac{11}{4}}-66(bx^2+a)^{\frac{7}{4}}a+77(bx^2+a)^{\frac{3}{4}}a^2}{b}\right)c^{\frac{3}{2}}}{231b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")

[Out] 2/231*(11*(3*(b*x^2 + a)^(7/4) - 7*(b*x^2 + a)^(3/4)*a)*a/b + (21*(b*x^2 + a)^(11/4) - 66*(b*x^2 + a)^(7/4)*a + 77*(b*x^2 + a)^(3/4)*a^2)/b)*c^(3/2)/b

maple [A] time = 0.01, size = 36, normalized size = 0.55

$$\frac{2(bx^2+a)(-7bx^2+4a)(\sqrt{bx^2+a}c)^{\frac{3}{2}}}{77b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((b*x^2+a)^(1/2)*c)^(3/2),x)

[Out] -2/77*(b*x^2+a)*(-7*b*x^2+4*a)*((b*x^2+a)^(1/2)*c)^(3/2)/b^2

maxima [A] time = 0.93, size = 43, normalized size = 0.65

$$\frac{2 \left(11 \left(\sqrt{bx^2 + a} \right)^{\frac{7}{2}} ac^2 - 7 \left(\sqrt{bx^2 + a} \right)^{\frac{11}{2}} \right)}{77 b^2 c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")

[Out] -2/77*(11*(sqrt(b*x^2 + a)*c)^(7/2)*a*c^2 - 7*(sqrt(b*x^2 + a)*c)^(11/2))/(b^2*c^4)

mupad [B] time = 2.89, size = 67, normalized size = 1.02

$$\sqrt{c} \sqrt{bx^2 + a} \left(\frac{2cx^4 \sqrt{bx^2 + a}}{11} - \frac{8a^2c \sqrt{bx^2 + a}}{77b^2} + \frac{6acx^2 \sqrt{bx^2 + a}}{77b} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(c*(a + b*x^2)^(1/2))^(3/2),x)

[Out] (c*(a + b*x^2)^(1/2))^(1/2)*((2*c*x^4*(a + b*x^2)^(1/2))/11 - (8*a^2*c*(a + b*x^2)^(1/2))/(77*b^2) + (6*a*c*x^2*(a + b*x^2)^(1/2))/(77*b))

sympy [A] time = 38.30, size = 87, normalized size = 1.32

$$\begin{cases} -\frac{8a^2c^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{77b^2} + \frac{6ac^{\frac{3}{2}}x^2(a+bx^2)^{\frac{3}{4}}}{77b} + \frac{2c^{\frac{3}{2}}x^4(a+bx^2)^{\frac{3}{4}}}{11} & \text{for } b \neq 0 \\ \frac{x^4(\sqrt{a}c)^{\frac{3}{2}}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(c*(b*x**2+a)**(1/2))**(3/2),x)

[Out] Piecewise((-8*a**2*c**(3/2)*(a + b*x**2)**(3/4)/(77*b**2) + 6*a*c**(3/2)*x**2*(a + b*x**2)**(3/4)/(77*b) + 2*c**(3/2)*x**4*(a + b*x**2)**(3/4)/11, Ne(b, 0)), (x**4*(sqrt(a)*c)**(3/2)/4, True))

$$3.92 \quad \int x \left(c\sqrt{a + bx^2} \right)^{3/2} dx$$

Optimal. Leaf size=36

$$\frac{2c(a + bx^2)^{3/2} \sqrt{c\sqrt{a + bx^2}}}{7b}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1591, 15, 30}

$$\frac{2c(a + bx^2)^{3/2} \sqrt{c\sqrt{a + bx^2}}}{7b}$$

Antiderivative was successfully verified.

[In] Int[x*(c*Sqrt[a + b*x^2])^(3/2), x]

[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*(a + b*x^2)^(3/2))/(7*b)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :=> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :=> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] :=> With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned} \int x \left(c\sqrt{a + bx^2} \right)^{3/2} dx &= \frac{\text{Subst} \left(\int (c\sqrt{x})^{3/2} dx, x, a + bx^2 \right)}{2b} \\ &= \frac{\left(c\sqrt{c\sqrt{a + bx^2}} \right) \text{Subst} \left(\int x^{3/4} dx, x, a + bx^2 \right)}{2b\sqrt[4]{a + bx^2}} \\ &= \frac{2c\sqrt{c\sqrt{a + bx^2}} (a + bx^2)^{3/2}}{7b} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 0.86

$$\frac{2(a + bx^2) \left(c\sqrt{a + bx^2} \right)^{3/2}}{7b}$$

Antiderivative was successfully verified.

[In] Integrate[x*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (2*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2))/(7*b)

IntegrateAlgebraic [A] time = 9.75, size = 45, normalized size = 1.25

$$\frac{2\sqrt{a+bx^2}\sqrt{c\sqrt{a+bx^2}}(ac+bcx^2)}{7b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(c*Sqrt[a + b*x^2])^(3/2),x]

[Out] (2*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*(a*c + b*c*x^2))/(7*b)

fricas [A] time = 1.08, size = 37, normalized size = 1.03

$$\frac{2(bc x^2 + ac)\sqrt{bx^2 + a}\sqrt{\sqrt{bx^2 + a}c}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="fricas")

[Out] 2/7*(b*c*x^2 + a*c)*sqrt(b*x^2 + a)*sqrt(sqrt(b*x^2 + a)*c)/b

giac [A] time = 0.27, size = 17, normalized size = 0.47

$$\frac{2(bx^2 + a)^{\frac{7}{4}}c^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="giac")

[Out] 2/7*(b*x^2 + a)^(7/4)*c^(3/2)/b

maple [A] time = 0.00, size = 26, normalized size = 0.72

$$\frac{2(bx^2 + a)\left(\sqrt{bx^2 + a}c\right)^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^2+a)^(1/2)*c)^(3/2),x)

[Out] 2/7*(b*x^2+a)*((b*x^2+a)^(1/2)*c)^(3/2)/b

maxima [A] time = 0.91, size = 25, normalized size = 0.69

$$\frac{2(bx^2 + a)\left(\sqrt{bx^2 + a}c\right)^{\frac{3}{2}}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(c*(b*x^2+a)^(1/2))^(3/2),x, algorithm="maxima")

[Out] 2/7*(b*x^2 + a)*(sqrt(b*x^2 + a)*c)^(3/2)/b

mupad [B] time = 2.77, size = 28, normalized size = 0.78

$$\frac{2c(bx^2 + a)^{3/2} \sqrt{c} \sqrt{bx^2 + a}}{7b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*(a + b*x^2)^(1/2))^(3/2),x)`

[Out] `(2*c*(a + b*x^2)^(3/2)*(c*(a + b*x^2)^(1/2))^(1/2))/(7*b)`

sympy [A] time = 14.89, size = 58, normalized size = 1.61

$$\begin{cases} \frac{2ac^{\frac{3}{2}}(a+bx^2)^{\frac{3}{4}}}{7b} + \frac{2c^{\frac{3}{2}}x^2(a+bx^2)^{\frac{3}{4}}}{7} & \text{for } b \neq 0 \\ \frac{x^2(\sqrt{ac})^{\frac{3}{2}}}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(c*(b*x**2+a)**(1/2))**(3/2),x)`

[Out] `Piecewise((2*a*c**(3/2)*(a + b*x**2)**(3/4)/(7*b) + 2*c**(3/2)*x**2*(a + b*x**2)**(3/4)/7, Ne(b, 0)), (x**2*(sqrt(a)*c)**(3/2)/2, True))`

$$3.93 \quad \int \frac{(c\sqrt{a+bx^2})^{3/2}}{x} dx$$

Optimal. Leaf size=117

$$\frac{2}{3} (c\sqrt{a+bx^2})^{3/2} + \frac{(c\sqrt{a+bx^2})^{3/2} \tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}} - \frac{(c\sqrt{a+bx^2})^{3/2} \tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Rubi [A] time = 0.16, antiderivative size = 141, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6720, 266, 50, 63, 298, 203, 206}

$$\frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}} \tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} - \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}} \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} + \frac{2}{3}c\sqrt{a+bx^2} \sqrt{c\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x,x]

[Out] (2*c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/3 + (a^(3/4)*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)]/(a + b*x^2)^(1/4) - (a^(3/4)*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]/(a + b*x^2)^(1/4))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !IntegerQ[a/b, 0]

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \frac{(c\sqrt{a+bx^2})^{3/2}}{x} dx &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right) \int \frac{(a+bx^2)^{3/4}}{x} dx}{\sqrt[4]{a+bx^2}} \\ &= \frac{\left(c\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \frac{(a+bx)^{3/4}}{x} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2} + \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}} \\ &= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2} + \frac{\left(2ac\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}} dx, x, \sqrt[4]{a+bx^2}\right)}{b\sqrt[4]{a+bx^2}} \\ &= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2} - \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a-x^2}} dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt[4]{a+bx^2}} + \frac{\left(ac\sqrt{c\sqrt{a+bx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a+x^2}} dx, x, \sqrt[4]{a+bx^2}\right)}{\sqrt[4]{a+bx^2}} \\ &= \frac{2}{3}c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2} + \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}} \tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} - \frac{a^{3/4}c\sqrt{c\sqrt{a+bx^2}} \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{\sqrt[4]{a+bx^2}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 96, normalized size = 0.82

$$\frac{\left(c\sqrt{a+bx^2}\right)^{3/2} \left(3a^{3/4} \tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) - 3a^{3/4} \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right) + 2(a+bx^2)^{3/4}\right)}{3(a+bx^2)^{3/4}}$$

Antiderivative was successfully verified.

[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x,x]

[Out] ((c*Sqrt[a + b*x^2])^(3/2)*(2*(a + b*x^2)^(3/4) + 3*a^(3/4)*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)] - 3*a^(3/4)*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)]))/(3*(a + b*x^2)^(3/4))

IntegrateAlgebraic [B] time = 72.03, size = 593, normalized size = 5.07

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(c*Sqrt[a + b*x^2])^(3/2)/x,x]
```

```
[Out] Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[(a + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*((Sqrt[a]*c*ArcTan[((2*a)/b + x^2 + (2*Sqrt[a]*Sqrt[a + b*x^2])/b - (2*x^2*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]))/x^2])Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2])/b + (Sqrt[a]*c*Log[Sqrt[2]*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]))/x^2] - Sqrt[2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2])/b) + Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2]*((2*c)/3 + Sqrt[(a + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]*(-((a*c*ArcTan[((2*a)/b + x^2 + (2*Sqrt[a]*Sqrt[a + b*x^2])/b - (2*x^2*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]))/x^2])Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2])/b)/x^2)))/(b*(a + b*x^2))) - (a*c*Log[Sqrt[2]*Sqrt[(Sqrt[a]*(Sqrt[a] + Sqrt[a + b*x^2]))/x^2] - Sqrt[2]*Sqrt[(a + b*x^2 + Sqrt[a]*Sqrt[a + b*x^2])/x^2]])/(b*(a + b*x^2)))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.41, size = 190, normalized size = 1.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="giac")
```

```
[Out] -1/12*(6*sqrt(2)*(-a)^(3/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) + 6*sqrt(2)*(-a)^(3/4)*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4)) - 3*sqrt(2)*(-a)^(3/4)*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) + 3*sqrt(2)*(-a)^(3/4)*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a)) - 8*(b*x^2 + a)^(3/4)*c^(3/2)
```

maple [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(\sqrt{bx^2 + ac})^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)
```

```
[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x,x)
```

maxima [A] time = 1.97, size = 118, normalized size = 1.01

$$\frac{3ac^4 \left(\frac{2 \arctan\left(\frac{\sqrt{\sqrt{bx^2+ac}}}{(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}} + \frac{\log\left(\frac{\sqrt{\sqrt{bx^2+ac}} - (ac^2)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+ac}} + (ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}}\right) + 4 \left(\sqrt{bx^2+ac}\right)^{\frac{3}{2}} c^2}{6c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x,x, algorithm="maxima")

[Out] 1/6*(3*a*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a))*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) + log((sqrt(sqrt(b*x^2 + a))*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a))*c) + (a*c^2)^(1/4))/(a*c^2)^(1/4)) + 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^2)/c^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c\sqrt{bx^2+a})^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x,x)

[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c\sqrt{a+bx^2})^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x,x)

[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x, x)

$$3.94 \quad \int \frac{(c\sqrt{a+bx^2})^{3/2}}{x^3} dx$$

Optimal. Leaf size=133

$$\frac{(c\sqrt{a+bx^2})^{3/2}}{2x^2} + \frac{3b(c\sqrt{a+bx^2})^{3/2} \tan^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}} - \frac{3b(c\sqrt{a+bx^2})^{3/2} \tanh^{-1}\left(\sqrt[4]{\frac{bx^2}{a}+1}\right)}{4a\left(\frac{bx^2}{a}+1\right)^{3/4}}$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6720, 266, 47, 63, 298, 203, 206}

$$\frac{c\sqrt{a+bx^2} \sqrt{c\sqrt{a+bx^2}}}{2x^2} + \frac{3bc\sqrt{c\sqrt{a+bx^2}} \tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a} \sqrt[4]{a+bx^2}} - \frac{3bc\sqrt{c\sqrt{a+bx^2}} \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a} \sqrt[4]{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]

[Out] -(c*Sqrt[c*Sqrt[a + b*x^2]]*Sqrt[a + b*x^2])/(2*x^2) + (3*b*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTan[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^(1/4)*(a + b*x^2)^(1/4)) - (3*b*c*Sqrt[c*Sqrt[a + b*x^2]]*ArcTanh[(a + b*x^2)^(1/4)/a^(1/4)])/(4*a^(1/4)*(a + b*x^2)^(1/4))

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b}

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !IntegerQ[a/b, 0]

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \frac{(c\sqrt{a+bx^2})^{3/2}}{x^3} dx &= \frac{(c\sqrt{c\sqrt{a+bx^2}}) \int \frac{(a+bx^2)^{3/4}}{x^3} dx}{\sqrt[4]{a+bx^2}} \\ &= \frac{(c\sqrt{c\sqrt{a+bx^2}}) \operatorname{Subst}\left(\int \frac{(a+bx)^{3/4}}{x^2} dx, x, x^2\right)}{2\sqrt[4]{a+bx^2}} \\ &= -\frac{c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2}}{2x^2} + \frac{(3bc\sqrt{c\sqrt{a+bx^2}}) \operatorname{Subst}\left(\int \frac{1}{x\sqrt[4]{a+bx}} dx, x, x^2\right)}{8\sqrt[4]{a+bx^2}} \\ &= -\frac{c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2}}{2x^2} + \frac{(3c\sqrt{c\sqrt{a+bx^2}}) \operatorname{Subst}\left(\int \frac{x^2}{-\frac{a}{b}+\frac{x^4}{b}} dx, x, \sqrt[4]{a+bx^2}\right)}{2\sqrt[4]{a+bx^2}} \\ &= -\frac{c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2}}{2x^2} - \frac{(3bc\sqrt{c\sqrt{a+bx^2}}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a-x^2}} dx, x, \sqrt[4]{a+bx^2}\right)}{4\sqrt[4]{a+bx^2}} + \frac{(3bc\sqrt{c\sqrt{a+bx^2}}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+x^2}} dx, x, \sqrt[4]{a+bx^2}\right)}{4\sqrt[4]{a+bx^2}} \\ &= -\frac{c\sqrt{c\sqrt{a+bx^2}} \sqrt{a+bx^2}}{2x^2} + \frac{3bc\sqrt{c\sqrt{a+bx^2}} \tan^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}} - \frac{3bc\sqrt{c\sqrt{a+bx^2}} \tanh^{-1}\left(\frac{\sqrt[4]{a+bx^2}}{\sqrt[4]{a}}\right)}{4\sqrt[4]{a}\sqrt[4]{a+bx^2}} \end{aligned}$$

Mathematica [C] time = 0.01, size = 50, normalized size = 0.38

$$\frac{2b(a+bx^2)(c\sqrt{a+bx^2})^{3/2}}{7a^2} {}_2F_1\left(\frac{7}{4}, 2; \frac{11}{4}; \frac{bx^2}{a} + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[(c*Sqrt[a + b*x^2])^(3/2)/x^3, x]

[Out] (2*b*(c*Sqrt[a + b*x^2])^(3/2)*(a + b*x^2)*Hypergeometric2F1[7/4, 2, 11/4, 1 + (b*x^2)/a])/(7*a^2)

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(c*Sqrt[a + b*x^2])^(3/2)/x^3,x]

[Out] \$Aborted

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="fricas")

[Out] Timed out

giac [A] time = 0.40, size = 212, normalized size = 1.59

$$\frac{\left(\frac{6\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{2(-a)^{\frac{1}{4}}+2(bx^2+a)^{\frac{1}{4}}}}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{6\sqrt{2}b^2 \arctan\left(\frac{\sqrt{2}\sqrt{2(-a)^{\frac{1}{4}}-2(bx^2+a)^{\frac{1}{4}}}}{2(-a)^{\frac{1}{4}}}\right)}{(-a)^{\frac{1}{4}}} + \frac{3\sqrt{2}(-a)^{\frac{3}{4}}b^2 \log\left(\sqrt{2}\sqrt{(bx^2+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}}\right)}{a} + \frac{3\sqrt{2}b^2 \log\left(-\sqrt{2}\sqrt{(bx^2+a)^{\frac{1}{4}}(-a)^{\frac{1}{4}}+\sqrt{bx^2+a}+\sqrt{-a}}\right)}{(-a)^{\frac{1}{4}}} - \frac{8(bx^2+a)^{\frac{3}{4}}b}{x^2} \right) c^{\frac{3}{2}}}{16b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="giac")

[Out] 1/16*(6*sqrt(2)*b^2*arctan(1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) + 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 6*sqrt(2)*b^2*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a)^(1/4) - 2*(b*x^2 + a)^(1/4))/(-a)^(1/4))/(-a)^(1/4) + 3*sqrt(2)*(-a)^(3/4)*b^2*log(sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a))/a + 3*sqrt(2)*b^2*log(-sqrt(2)*(b*x^2 + a)^(1/4)*(-a)^(1/4) + sqrt(b*x^2 + a) + sqrt(-a))/(-a)^(1/4) - 8*(b*x^2 + a)^(3/4)*b/x^2)*c^(3/2)/b

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{\left(\sqrt{bx^2 + ac}\right)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)

[Out] int(((b*x^2+a)^(1/2)*c)^(3/2)/x^3,x)

maxima [A] time = 2.40, size = 138, normalized size = 1.04

$$\frac{\left(3c^4 \left(\frac{2 \arctan\left(\frac{\sqrt{\sqrt{bx^2+ac}}}{(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}} + \frac{\log\left(\frac{\sqrt{\sqrt{bx^2+ac}}-(ac^2)^{\frac{1}{4}}}{\sqrt{\sqrt{bx^2+ac}}+(ac^2)^{\frac{1}{4}}}\right)}{(ac^2)^{\frac{1}{4}}}\right) - \frac{4\left(\sqrt{bx^2+ac}\right)^{\frac{3}{2}}c^4}{(bx^2+a)c^2-ac^2} \right) b}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x^2+a)^(1/2))^(3/2)/x^3,x, algorithm="maxima")

[Out] 1/8*(3*c^4*(2*arctan(sqrt(sqrt(b*x^2 + a)*c)/(a*c^2)^(1/4))/(a*c^2)^(1/4) + log((sqrt(sqrt(b*x^2 + a)*c) - (a*c^2)^(1/4))/(sqrt(sqrt(b*x^2 + a)*c) + (a*c^2)^(1/4)))/(a*c^2)^(1/4)) - 4*(sqrt(b*x^2 + a)*c)^(3/2)*c^4/((b*x^2 + a)*c^2 - a*c^2))*b/c^2

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(c\sqrt{bx^2+a}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)

[Out] int((c*(a + b*x^2)^(1/2))^(3/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(c\sqrt{a+bx^2}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*(b*x**2+a)**(1/2))**(3/2)/x**3, x)

[Out] Integral((c*sqrt(a + b*x**2))**(3/2)/x**3, x)

3.95 $\int \sqrt{(b-x)(-a+x)} dx$

Optimal. Leaf size=71

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1981, 612, 621, 204}

$$-\frac{1}{4}(a+b-2x)\sqrt{x(a+b)-ab-x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(b - x)*(-a + x)], x]

[Out] -((a + b - 2*x)*Sqrt[-(a*b) + (a + b)*x - x^2])/4 - ((a - b)^2*ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2]])/8

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QuadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \sqrt{(b-x)(-a+x)} dx &= \int \sqrt{-ab + (a+b)x - x^2} dx \\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab + (a+b)x - x^2} + \frac{1}{8}(a-b)^2 \int \frac{1}{\sqrt{-ab + (a+b)x - x^2}} dx \\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab + (a+b)x - x^2} + \frac{1}{4}(a-b)^2 \text{Subst}\left(\int \frac{1}{-4-x^2} dx, x, \frac{a+b-2x}{\sqrt{-ab + (a+b)x - x^2}}\right) \\ &= -\frac{1}{4}(a+b-2x)\sqrt{-ab + (a+b)x - x^2} - \frac{1}{8}(a-b)^2 \tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab + (a+b)x - x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.16, size = 106, normalized size = 1.49

$$\frac{(a-x)\left((a-b)^{5/2}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)-(a-x)(b-x)(a+b-2x)\right)}{4(x-a)\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(b-x)*(-a+x)],x]

[Out] ((a-x)*(-((a+b-2*x)*(a-x)*(b-x))+(a-b)^(5/2)*Sqrt[(a-x)/(a-b)]*Sqrt[b-x]*ArcSinh[Sqrt[b-x]/Sqrt[a-b]])/(4*(-a+x)*Sqrt[(a-x)*(-b+x)])

IntegrateAlgebraic [C] time = 0.29, size = 82, normalized size = 1.15

$$\frac{1}{4}\sqrt{-ab+ax+bx-x^2}(-a-b+2x)+\frac{1}{8}i\left(a^2-2ab+b^2\right)\log\left(-2i\sqrt{x(a+b)-ab-x^2}+a+b-2x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(b-x)*(-a+x)],x]

[Out] ((-a-b+2*x)*Sqrt[-(a*b)+a*x+b*x-x^2])/4+(I/8)*(a^2-2*a*b+b^2)*Log[a+b-2*x-(2*I)*Sqrt[-(a*b)+(a+b)*x-x^2]]

fricas [A] time = 0.73, size = 80, normalized size = 1.13

$$-\frac{1}{8}\left(a^2-2ab+b^2\right)\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2x)}{2(ab-(a+b)x+x^2)}\right)-\frac{1}{4}\sqrt{-ab+(a+b)x-x^2}(a+b-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="fricas")

[Out] -1/8*(a^2-2*a*b+b^2)*arctan(-1/2*sqrt(-a*b+(a+b)*x-x^2)*(a+b-2*x)/(a*b-(a+b)*x+x^2))-1/4*sqrt(-a*b+(a+b)*x-x^2)*(a+b-2*x)

giac [A] time = 0.29, size = 61, normalized size = 0.86

$$\frac{1}{8}\left(a^2-2ab+b^2\right)\arcsin\left(\frac{a+b-2x}{a-b}\right)\operatorname{sgn}(-a+b)-\frac{1}{4}\sqrt{-ab+ax+bx-x^2}(a+b-2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="giac")

[Out] 1/8*(a^2-2*a*b+b^2)*arcsin((a+b-2*x)/(a-b))*sgn(-a+b)-1/4*sqrt(-a*b+a*x+b*x-x^2)*(a+b-2*x)

maple [A] time = 0.02, size = 122, normalized size = 1.72

$$\frac{a^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{ab\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{4}+\frac{b^2\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)}{8}-\frac{(a+b-2x)\sqrt{-ab-x^2+(a+b)x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b-x)*(-a+x))^(1/2),x)

[Out] -1/4*(a+b-2*x)*(-a*b+(a+b)*x-x^2)^(1/2)-1/4*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*a*b+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*a^2+1/8*arctan((x-1/2*b-1/2*a)/(-a*b+(a+b)*x-x^2)^(1/2))*b^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b-x)*(-a+x))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more details)Is b-a zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-(a-x)(b-x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a-x)*(b-x))^(1/2),x)

[Out] int((-a-x)*(b-x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{(-a+x)(b-x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b-x)*(-a+x))**(1/2),x)

[Out] Integral(sqrt((-a+x)*(b-x)), x)

$$3.96 \quad \int \frac{1}{\sqrt{(b-x)(-a+x)}} dx$$

Optimal. Leaf size=32

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1981, 621, 204}

$$-\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{x(a+b)-ab-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(b - x)*(-a + x)],x]

[Out] -ArcTan[(a + b - 2*x)/(2*Sqrt[-(a*b) + (a + b)*x - x^2])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(b-x)(-a+x)}} dx &= \int \frac{1}{\sqrt{-ab + (a+b)x - x^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{-4 - x^2} dx, x, \frac{a+b-2x}{\sqrt{-ab + (a+b)x - x^2}}\right) \\ &= -\tan^{-1}\left(\frac{a+b-2x}{2\sqrt{-ab + (a+b)x - x^2}}\right) \end{aligned}$$

Mathematica [B] time = 0.03, size = 72, normalized size = 2.25

$$\frac{2\sqrt{a-b}\sqrt{b-x}\sqrt{\frac{a-x}{a-b}}\sinh^{-1}\left(\frac{\sqrt{b-x}}{\sqrt{a-b}}\right)}{\sqrt{(a-x)(x-b)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(b - x)*(-a + x)],x]

[Out] $(-2\sqrt{a-b}\sqrt{(a-x)/(a-b)}\sqrt{b-x}\operatorname{ArcSinh}[\sqrt{b-x}/\sqrt{a-b}])/\sqrt{(a-x)(-b+x)}$

IntegrateAlgebraic [C] time = 0.28, size = 34, normalized size = 1.06

$$i \log\left(-2i\sqrt{x(a+b)-ab-x^2} + a + b - 2x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(b-x)*(-a+x)],x]

[Out] I*Log[a + b - 2*x - (2*I)*Sqrt[-(a*b) + (a + b)*x - x^2]]

fricas [A] time = 0.89, size = 43, normalized size = 1.34

$$-\arctan\left(-\frac{\sqrt{-ab+(a+b)x-x^2}(a+b-2x)}{2(ab-(a+b)x+x^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="fricas")

[Out] -arctan(-1/2*sqrt(-a*b + (a + b)*x - x^2)*(a + b - 2*x)/(a*b - (a + b)*x + x^2))

giac [A] time = 0.43, size = 22, normalized size = 0.69

$$\arcsin\left(\frac{a+b-2x}{a-b}\right)\operatorname{sgn}(-a+b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="giac")

[Out] arcsin((a + b - 2*x)/(a - b))*sgn(-a + b)

maple [A] time = 0.01, size = 28, normalized size = 0.88

$$\arctan\left(\frac{-\frac{a}{2}-\frac{b}{2}+x}{\sqrt{-ab-x^2+(a+b)x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b-x)*(-a+x))^(1/2),x)

[Out] arctan((-1/2*a-1/2*b+x)/(-a*b-x^2+(a+b)*x)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b-x)*(-a+x))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b-a>0)', see `assume?` for more details)Is b-a zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{\sqrt{-(a-x)(b-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-(a - x)*(b - x))^(1/2), x)`

[Out] `int(1/(-(a - x)*(b - x))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-a+x)(b-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b-x)*(-a+x))**(1/2), x)`

[Out] `Integral(1/sqrt((-a + x)*(b - x)), x)`

$$3.97 \quad \int x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx$$

Optimal. Leaf size=244

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3} - \frac{\sqrt{e}(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}} + \frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6bd^2e} - \frac{(c+dx^2)^2(ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^3}$$

Rubi [A] time = 0.33, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 463, 455, 385, 208}

$$\frac{(c+dx^2)(-a^2d^2-2abcd+11b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^2d^3} - \frac{\sqrt{e}(bc-ad)(a^2d^2+2abcd+5b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}} + \frac{(c+dx^2)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6bd^2e} - \frac{(c+dx^2)^2(ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8bd^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] ((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)/(16*b^2*d^3) - ((3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(8*b*d^3) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2)^3)/(6*b*d^2*e) - ((b*c - a*d)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(5/2)*d^(7/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.))
)^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n)
^(1/q)], x)] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx = ((bc-ad)e) \operatorname{Subst} \left(\int \frac{x^2(-ae+cx^2)^2}{(be-dx^2)^4} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} (c+dx^2)^3}{6bd^2e} - \frac{(bc-ad) \operatorname{Subst} \left(\int \frac{x^2(-3(2a^2d^2e^2-(bce-ade)^2)+6bc^2dex^2)}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6bd^2}$$

$$= -\frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} (c+dx^2)^3}{6bd^2e} + \frac{(bc-ad) \operatorname{Subst} \left(\int \frac{3d(bc-ade)}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6bd^2e}$$

$$= \frac{(11b^2c^2 - 2abcd - a^2d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} (c+dx^2)^3}{6bd^2e}$$

$$= \frac{(11b^2c^2 - 2abcd - a^2d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{16b^2d^3} - \frac{(3bc+ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{8bd^3} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} (c+dx^2)^3}{6bd^2e}$$

Mathematica [A] time = 0.52, size = 198, normalized size = 0.81

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(-\frac{3(a^2d^2+2abcd+5b^2c^2)(bc-ad)^{3/2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) - b\sqrt{d} (c+dx^2) (3a^2d^2 - 2abd(dx^2 - 2c) + b^2(-15c^2 + 10cdx^2 - 8d^2x^4)) \right)}{48b^3d^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2*a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c - a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]/Sqrt[a + b*x^2]))/(48*b^3*d^(7/2))

IntegrateAlgebraic [A] time = 0.48, size = 225, normalized size = 0.92

$$\frac{(-3a^2cd^2 - 3a^2d^3x^2 - 4abc^2d - 2abcd^2x^2 + 2abd^3x^4 + 15b^2c^3 + 5b^2c^2dx^2 - 2b^2cd^2x^4 + 8b^2d^3x^6) \sqrt{\frac{ae+bcx^2}{c+dx^2}} - \sqrt{e}(-a^3d^3 - a^2bcd^2 - 3ab^2c^2d + 5b^3c^3) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{ae+bcx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right)}{48b^2d^3 - 16b^{5/2}d^{7/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 + 5*b^2*c^2*d*x^2 - 2*a*b*c*d^2*x^2 - 3*a^2*d^3*x^2 - 2*b^2*c*d^2*x^4 + 2*a

$$*b*d^3*x^4 + 8*b^2*d^3*x^6))/(48*b^2*d^3) - ((5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*\text{Sqrt}[e]*\text{ArcTanh}[(\text{Sqrt}[d]*\text{Sqrt}[(a*e + b*e*x^2)/(c + d*x^2)])]/(\text{Sqrt}[b]*\text{Sqrt}[e]))/(16*b^(5/2)*d^(7/2))$$

fricas [A] time = 0.96, size = 541, normalized size = 2.22

$$\frac{3(5b^3c^3 - 3ab^2c^2d - a^2bcd^2 - a^3d^3)\sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{ae + bex^2}}{\sqrt{(c + dx^2)b}}\right) + 4(2b^2d^3x^4 + b^2c^2d + abcd^2 + (3b^2cd - ab^2d^2)\sqrt{e})\sqrt{e} + 15b^2c^2e - 4ab^2cd - 3a^2d^2e}{96b^2d^3} + \frac{3(5b^2c^2d - 2ab^2cd^2 - 3a^2d^3)x^2\sqrt{e}}{b^2d^3} \operatorname{arctan}\left(\frac{1}{2}\sqrt{\frac{2b^2d^3x^2 + b^2cd + a^2d^2}{d^2}}\sqrt{\frac{e}{b^2d^3}}\right) + 2(8b^2d^3x^6 + 15b^2c^3 - 4ab^2cd^2 - 3a^2cd^2 - 2(b^2cd^2 - ab^2d^3)x^4 + (5b^2c^2d - 2ab^2cd^2 - 3a^2d^3)x^2)\sqrt{e}}{96b^2d^3} \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2*d^3*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d)))/(b*e*x^2 + a*e) + 2*(8*b^2*d^3*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3)]

giac [A] time = 0.73, size = 243, normalized size = 1.00

$$\frac{1}{96} \left(2\sqrt{bdx^4e + bcx^2e + adx^2e + ace} \left(2x^2 \left(\frac{4x^2}{d} - \frac{5b^2cd - abd^2}{b^2d^3} \right) + \frac{15b^2c^2 - 4abcd - 3a^2d^2}{b^2d^3} \right) + \frac{3(5b^2c^2d - 3ab^2cd^2 - a^2bd^3)e^{1/2} \log\left(-bce - ade - 2(\sqrt{bd}x^2e^{1/2} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})\sqrt{bd}e^{1/2}\right)}{\sqrt{bd}b^2d^3} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] 1/96*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2*(4*x^2/d - (5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b^2*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*e^(-1/2)*log(abs(-b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b^2*d^3))*sgn(d*x^2 + c)

maple [B] time = 0.07, size = 527, normalized size = 2.16

$$\frac{\sqrt{\frac{bdx^4e + bcx^2e + adx^2e + ace}{bd}} \left(\frac{2x^2 \left(\frac{4x^2}{d} - \frac{5b^2cd - abd^2}{b^2d^3} \right) + \frac{15b^2c^2 - 4abcd - 3a^2d^2}{b^2d^3}}{\sqrt{bd}} + \frac{3(5b^2c^2d - 3ab^2cd^2 - a^2bd^3)e^{1/2} \log\left(-bce - ade - 2(\sqrt{bd}x^2e^{1/2} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})\sqrt{bd}e^{1/2}\right)}{\sqrt{bd}b^2d^3} \right) \operatorname{sgn}(dx^2 + c)}{96\sqrt{bd}b^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)

[Out] 1/96*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*a*b*d^2*(b*d)^(1/2)-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^2*c*b^2*d*(b*d)^(1/2)+3*d^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^3+3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a^2*c*b*d^2+9*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*c^2*b^2*d-15*b^3*ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*d*(b*d)^(1/2)-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a^2*d^2*(b*d)^(1/2)-24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*a*c*b*d*(b*d)^(1/2)+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*c^2*b^2*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/b^2/(b*d)^(1/2)

maxima [A] time = 2.28, size = 414, normalized size = 1.70

$$\frac{1}{96} e \left(\frac{2 \left(3 \left(11 b^3 c^3 d^2 - 13 a b^2 c^2 d^3 + a^2 b c d^4 + a^3 d^5 \right) \left(\frac{(b x^2 + a)^{5/2}}{d x^2 + c} \right) - 8 \left(5 b^4 c^3 d - 3 a b^3 c^2 d^2 - 3 a^2 b^2 c d^3 + a^3 b d^4 \right) \left(\frac{(b x^2 + a)^{3/2}}{d x^2 + c} \right) e + 3 \left(5 b^5 c^3 - 3 a b^4 c^2 d - a^2 b^3 c d^2 - a^3 b^2 d^3 \right) \sqrt{\frac{(b x^2 + a)^2}{d x^2 + c}} \right)}{b^5 d^3 e^3 - \frac{3 (b x^2 + a)^4 e^4}{d^2 + c} + \frac{3 (b x^2 + a)^3 e^5}{(d x^2 + c)^2} - \frac{(b x^2 + a)^2 e^6}{(d x^2 + c)^3}} + \frac{3 \left(5 b^3 c^3 - 3 a b^2 c^2 d - a^2 b c d^2 - a^3 d^3 \right) \log \left(\frac{d \sqrt{\frac{(b x^2 + a)^2}{d x^2 + c} - \sqrt{b d e}}}{d \sqrt{\frac{(b x^2 + a)^2}{d x^2 + c} + \sqrt{b d e}}} \right)}{\sqrt{b d e} b^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/96*e*(2*(3*(11*b^3*c^3*d^2 - 13*a*b^2*c^2*d^3 + a^2*b*c*d^4 + a^3*d^5))*((b*x^2 + a)*e/(d*x^2 + c))^(5/2) - 8*(5*b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e + 3*(5*b^5*c^3 - 3*a*b^4*c^2*d - a^2*b^3*c*d^2 - a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^2/(b^5*d^3*e^3 - 3*(b*x^2 + a)*b^4*d^4*e^3/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^3*d^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^2*d^6*e^3/(d*x^2 + c)^3) + 3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^2*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{\frac{e (b x^2 + a)}{d x^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.98 \quad \int x^3 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}} dx$$

Optimal. Leaf size=161

$$\frac{\sqrt{e}(bc-ad)(ad+3bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}} + \frac{(c+dx^2)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{4d^2} - \frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{8bd^2}$$

Rubi [A] time = 0.16, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}(bc-ad)(ad+3bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}} + \frac{(c+dx^2)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{4d^2} - \frac{(c+dx^2)(5bc-ad)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{8bd^2}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] -((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b*d^2) + (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*d^2) + ((b*c - a*d)*(3*b*c + a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])]/(Sqrt[b]*Sqrt[e]))/(8*b^(3/2)*d^(5/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx = ((bc-ad)e) \text{Subst} \left(\int \frac{x^2(-ae+cx^2)}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{4d^2} - \frac{((bc-ad)e) \text{Subst} \left(\int \frac{(bc-ad)e+4cdx^2}{(be-dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4d^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{4d^2} + \frac{((bc-ad)(3bc+ad)e) \text{Subst} \left(\int \frac{1}{be-dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{8bd^2}$$

$$= -\frac{(5bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{8bd^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)^2}{4d^2} + \frac{(bc-ad)(3bc+ad)\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{8b^{3/2}d^{5/2}}$$

Mathematica [A] time = 0.38, size = 149, normalized size = 0.93

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(b\sqrt{d} (c+dx^2) (ad-3bc+2bdx^2) + \frac{(ad+3bc)(bc-ad)^{3/2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{\sqrt{a+bx^2}} \right)}{8b^2d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]
[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(c + d*x^2)*(-3*b*c + a*d + 2*b*d*x^2) + ((b*c - a*d)^(3/2)*(3*b*c + a*d)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(8*b^2*d^(5/2))
```

IntegrateAlgebraic [A] time = 0.28, size = 154, normalized size = 0.96

$$\frac{\sqrt{e} (-a^2d^2 - 2abcd + 3b^2c^2) \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}} \right) + \frac{(acd + ad^2x^2 - 3bc^2 - bcdx^2 + 2bd^2x^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8bd^2}}{8b^{3/2}d^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-3*b*c^2 + a*c*d - b*c*d*x^2 + a*d^2*x^2 + 2*b*d^2*x^4))/(8*b*d^2) + ((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2))]/(Sqrt[b]*Sqrt[e])])/(8*b^(3/2)*d^(5/2))
```

fricas [A] time = 0.90, size = 407, normalized size = 2.53

$$\frac{(3b^2d^2 - 2abcd - a^2d^2)\sqrt{\frac{ae+bx^2}{c+dx^2}} \log\left(\frac{8b^2d^2c^2 + 8(b^2cd + abd^2)cx^2 + (b^2d^2 + 6abcd + a^2d^2)x^4 - 4(2b^2d^2x^4 + b^2c^2d + abcd^2 + (3b^2cd^2 + abd^2)x^2)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{32bd^2}\right) - 4(2bd^2x^4 - 3bc^2 + acd - (bcd - ad^2)x^2)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{16bd^2} + \frac{(acd + ad^2x^2 - 3bc^2 - bcdx^2 + 2bd^2x^4)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{8bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2), x, algorithm="fricas")
```

```
[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4
+ 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b
^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d
- (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/16*((
3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) -
2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a
e)/(d*x^2 + c)))/(b*d^2)]
```

giac [A] time = 0.82, size = 185, normalized size = 1.15

$$\frac{1}{16} \left(\frac{2\sqrt{bdx^4e + bcx^2e + adx^2e + ace} \left(\frac{2x^2}{d} - \frac{3bc - ad}{bd^2} \right) - \frac{(3b^2c^2e - 2abcde - a^2d^2e)^{\frac{1}{2}} \log \left(\frac{-bce - ade - 2(\sqrt{bd}x^2e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})\sqrt{bd}e^{\frac{1}{2}}}{\sqrt{bd}bd^2} \right)}{\sqrt{bd}bd^2}}{\sqrt{bd}bd^2} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)*(2*x^2/d - (3*b*c -
a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*e^(-1/2)*log(abs(-
b*c*e - a*d*e - 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d
*x^2*e + a*c*e))*sqrt(b*d)*e^(1/2)))/(sqrt(b*d)*b*d^2))*sgn(d*x^2 + c)
```

maple [B] time = 0.04, size = 342, normalized size = 2.12

$$\frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}(dx^2+c) \left(-d^2d^2 \ln \left(\frac{2bd^2+abd+e+2\sqrt{bd^4+ad^2+bd^2+ac}\sqrt{bd}}{2\sqrt{bd}} \right) - 2abcd \ln \left(\frac{2bd^2+abd+e+2\sqrt{bd^4+ad^2+bd^2+ac}\sqrt{bd}}{2\sqrt{bd}} \right) + 3d^2c^2 \ln \left(\frac{2bd^2+abd+e+2\sqrt{bd^4+ad^2+bd^2+ac}\sqrt{bd}}{2\sqrt{bd}} \right) + 4\sqrt{bd}x^4 + adx^2 + bcx^2 + ac \sqrt{bd}bd^2 + 2\sqrt{bd}x^4 + adx^2 + bcx^2 + ac \sqrt{bd}ad - 6\sqrt{bd}x^4 + adx^2 + bcx^2 + ac \sqrt{bd}bc \right)}{16\sqrt{(dx^2+c)(bx^2+a)}\sqrt{bd}bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(b*d)^(1/2)*x^2*b*d-d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x
^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*a^2-2*ln(1/2*(2*b*d*x^2+a*d
+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*a*c*b*
d+3*b^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2)))/(b*d)^(1/2))*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2
)*a*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((d*x^2+c)*(b*
x^2+a))^(1/2)/d^2/b/(b*d)^(1/2)
```

maxima [A] time = 2.25, size = 269, normalized size = 1.67

$$\frac{1}{16} e \left(\frac{2 \left((5b^2c^2d - 6abcd^2 + a^2d^3) \left(\frac{(bx^2+a)e}{dx^2+c} \right)^{\frac{3}{2}} - (3b^3c^2 - 2ab^2cd - a^2bd^2) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e \right)}{b^3d^2e^2 - \frac{2(bx^2+a)b^2d^3e^2}{dx^2+c} + \frac{(bx^2+a)^2bd^4e^2}{(dx^2+c)^2}} - \frac{(3b^2c^2 - 2abcd - a^2d^2) \log \left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde}bd^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*((5*b^2*c^2*d - 6*a*b*c*d^2 + a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c)
)^(3/2) - (3*b^3*c^2 - 2*a*b^2*c*d - a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 +
c))*e)/(b^3*d^2*e^2 - 2*(b*x^2 + a)*b^2*d^3*e^2/(d*x^2 + c) + (b*x^2 + a)^
2*b*d^4*e^2/(d*x^2 + c)^2) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((d*sqrt(
(b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)
) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*d^2))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2), x)

[Out] Timed out

$$3.99 \quad \int x \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}} dx$$

Optimal. Leaf size=103

$$\frac{(c+dx^2)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{2d} - \frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1960, 288, 208}

$$\frac{(c+dx^2)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{2d} - \frac{\sqrt{e}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)/(2*d) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*Sqrt[b]*d^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1)/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1)/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/(c_. + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m+1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned} \int x \sqrt{\frac{e(a+bx^2)}{c+dx^2}} dx &= ((bc-ad)e) \operatorname{Subst} \left(\int \frac{x^2}{(be-dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\ &= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{2d} - \frac{((bc-ad)e) \operatorname{Subst} \left(\int \frac{1}{be-dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2d} \\ &= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c+dx^2)}{2d} - \frac{(bc-ad)\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2\sqrt{b} d^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.28, size = 143, normalized size = 1.39

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(b\sqrt{d} (a+bx^2) (c+dx^2) - \sqrt{a+bx^2} (bc-ad)^{3/2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) \right)}{2bd^{3/2} (a+bx^2)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(a + b*x^2)*(c + d*x^2) - (b*c - a*d)^(3/2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b*d^(3/2)*(a + b*x^2))

IntegrateAlgebraic [A] time = 0.16, size = 107, normalized size = 1.04

$$\frac{(c+dx^2) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2d} - \frac{\sqrt{e} (bc-ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2\sqrt{b} d^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

[Out] ((c + d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(2*d) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(Sqrt[b]*Sqrt[e]))]/(2*Sqrt[b]*d^(3/2)))

fricas [A] time = 0.68, size = 313, normalized size = 3.04

$$\frac{(bc-ad)\sqrt{\frac{e}{bd}} \log \left(8b^2d^2ex^4 + 8(b^2cd + abd^2)ex^2 + (b^2c^2 + 6abcd + a^2d^2)e + 4(2b^2d^3x^4 + b^2c^2d + a*b*c*d^2 + (3b^2cd^2 + abd^3)x^2) \sqrt{\frac{bc+ae}{d^2+e}} \sqrt{\frac{e}{bd}} \right) - 4(dx^2+c) \sqrt{\frac{bc+ae}{d^2+e}} (bc-ad) \sqrt{\frac{e}{bd}} \arctan \left(\frac{(2bd^2+bc+ad) \sqrt{\frac{bc+ae}{d^2+e}} \sqrt{\frac{e}{bd}}}{2(bc+ae)} \right) + 2(dx^2+c) \sqrt{\frac{bc+ae}{d^2+e}}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2), x, algorithm="fricas")

[Out] [-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))]/d, 1/4*(b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))]

$2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d]$

giac [A] time = 0.82, size = 149, normalized size = 1.45

$$\frac{1}{4} \left(\frac{(bce - ade)\sqrt{bd} e^{\left(\frac{1}{2}\right)} \log\left(\left| -\sqrt{bd} bce^{\frac{1}{2}} - \sqrt{bd} ade^{\frac{1}{2}} - 2\left(\sqrt{bd} x^2 e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace}\right)bd \right|\right)}{bd^2} + \frac{2\sqrt{bdx^4e + bcx^2e + adx^2e + ace}}{d} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] $1/4*((b*c*e - a*d*e)*sqrt(b*d)*e^{(-1/2)}*\log(\operatorname{abs}(-sqrt(b*d)*b*c*e^{(1/2)} - sqrt(b*d)*a*d*e^{(1/2)} - 2*(sqrt(b*d)*x^2*e^{(1/2)} - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*b*d))/(b*d^2) + 2*sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e)/d)*\operatorname{sgn}(d*x^2 + c)$

maple [B] time = 0.02, size = 200, normalized size = 1.94

$$\frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}} (dx^2+c) \left(ad \ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right) - bc \ln\left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}}\right) + 2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd} \right)}{4\sqrt{(dx^2+c)(bx^2+a)}\sqrt{bd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)

[Out] $1/4*((b*x^2+a)/(d*x^2+c)*e)^{(1/2)}*(d*x^2+c)*(a*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^{(1/2)}*(b*d)^{(1/2)})/(b*d)^{(1/2)})*d-b*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^{(1/2)}*(b*d)^{(1/2)})/(b*d)^{(1/2)})*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)})/((d*x^2+c)*(b*x^2+a))^{(1/2)}/d/(b*d)^{(1/2)}$

maxima [A] time = 2.25, size = 145, normalized size = 1.41

$$\frac{1}{4} e \left(\frac{2(bc - ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{bde - \frac{(bx^2+a)d^2e}{dx^2+c}} + \frac{(bc - ad) \log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}}\right)}{\sqrt{bde}d} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] $1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b*d*e - (b*x^2 + a)*d^2*e/(d*x^2 + c)) + (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{\frac{e(bx^2 + a)}{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)

[Out] int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

$$3.100 \quad \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx$$

Optimal. Leaf size=112

$$\frac{\sqrt{b} \sqrt{e} \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right)}{\sqrt{d}} - \frac{\sqrt{a} \sqrt{e} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{c}}$$

Rubi [A] time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1960, 481, 208}

$$\frac{\sqrt{b} \sqrt{e} \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right)}{\sqrt{d}} - \frac{\sqrt{a} \sqrt{e} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]

[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2))]/(Sqrt[a]*Sqrt[e]))/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2))]/(Sqrt[b]*Sqrt[e]))/Sqrt[d]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 481

Int[((e_.)*(x_)^(m_.))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> -Dist[(a*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(a + b*x^n), x], x] + Dist[(c*e^n)/(b*c - a*d), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m, 2*n - 1]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{x^2}{(-ae + cx^2)(be - dx^2)} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= (ae) \operatorname{Subst} \left(\int \frac{1}{-ae + cx^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) + (be) \operatorname{Subst} \left(\int \frac{1}{be - dx^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= \frac{\sqrt{a} \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{c}} + \frac{\sqrt{b} \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.18, size = 173, normalized size = 1.54

$$\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{c} \sqrt{bc - ad} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) - \sqrt{a} \sqrt{d} \sqrt{c + dx^2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) \right)}{\sqrt{c} \sqrt{d} \sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]] - Sqrt[a]*Sqrt[d]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/Sqrt[a]*Sqrt[c + d*x^2]]))/(Sqrt[c]*Sqrt[d]*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.15, size = 116, normalized size = 1.04

$$\frac{\sqrt{b} \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{d}} - \frac{\sqrt{a} \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]

[Out] -((Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2))]/Sqrt[a]*Sqrt[e]])/Sqrt[c]) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2))]/Sqrt[b]*Sqrt[e]])/Sqrt[d]

fricas [A] time = 1.34, size = 865, normalized size = 7.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")

[Out] [1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))] + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*sqrt(-b

```
*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 + 6*a*b*c
*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2
*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x
^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b*e*x^2 + a^2
*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3
*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))), 1/2*
sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c)))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e/d)*arctan(1/2*(2
*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e
*x^2 + a*b*e))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Error: Bad Argument Type
```

maple [B] time = 0.04, size = 179, normalized size = 1.60

$$\frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}} (dx^2+c) \left(-\sqrt{bd} a \ln \left(\frac{adx^2+bcx^2+2ac+2\sqrt{ac}\sqrt{bdx^4+adx^2+bcx^2+ac}}{x^2} \right) + \sqrt{ac} b \ln \left(\frac{2bdx^2+ad+bc+2\sqrt{bdx^4+adx^2+bcx^2+ac}\sqrt{bd}}{2\sqrt{bd}} \right) \right)}{2\sqrt{(dx^2+c)(bx^2+a)}\sqrt{bd}\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x,x)
```

```
[Out] 1/2*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*b*(a*c)^(1/2)-a
*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*
c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)
```

maxima [A] time = 2.24, size = 149, normalized size = 1.33

$$\frac{1}{2} \left(\frac{a \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace}} - \frac{b \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde}} \right) e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*(a*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2
+ a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/sqrt(a*c*e) - b*log((d*sqrt((b*x^2 + a
)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b
*d*e)))/sqrt(b*d*e))*e
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)

[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)

[Out] Timed out

$$3.101 \quad \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} dx$$

Optimal. Leaf size=127

$$\frac{(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{\sqrt{e}(bc - ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

Rubi [A] time = 0.09, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1960, 288, 208}

$$\frac{(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{\sqrt{e}(bc - ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]

[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{x^2}{(-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)} + \frac{((bc - ad)e) \text{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2c}$$

$$= \frac{(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)} - \frac{(bc - ad)\sqrt{e} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2\sqrt{a} c^{3/2}}$$

Mathematica [A] time = 0.10, size = 133, normalized size = 1.05

$$\frac{\sqrt{c + dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(-\frac{(bc-ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{\sqrt{a} c^{3/2}} - \frac{\sqrt{a+bx^2} \sqrt{c+dx^2}}{cx^2} \right)}{2\sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(c*x^2)) - ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*c^(3/2)))/(2*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.16, size = 113, normalized size = 0.89

$$\frac{\sqrt{e} (ad - bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2\sqrt{a} c^{3/2}} + \frac{(-c - dx^2) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]

[Out] ((-c - d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(2*c*x^2) + (((-b*c) + a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*Sqrt[a]*c^(3/2)))

fricas [A] time = 1.20, size = 333, normalized size = 2.62

$$\left| \frac{(bc - ad)x^2 \sqrt{\frac{e}{ac}} \log \left(\frac{(b^2z^2 + 6abcd + a^2d^2)cx^4 + 8a^2c^2c + 8(ab^2c + a^2cd)cx^2 + 4(2a^2c^3 + (abc^2d + a^2cd^2)cx^4 + (abc^3 + 3a^2c^2d)x^2) \sqrt{\frac{bc^2+ae}{d^2+c}} \sqrt{\frac{e}{ac}}}{x^4} \right) + 4(dx^2 + c) \sqrt{\frac{bc^2+ae}{d^2+c}} (bc - ad)x^2 \sqrt{\frac{e}{ac}} \arctan \left(\frac{(bc+ad)x^2 + 2ac}{2(bc^2+ae)} \sqrt{\frac{bc^2+ae}{d^2+c}} \sqrt{\frac{e}{ac}} \right) - 2(dx^2 + c) \sqrt{\frac{bc^2+ae}{d^2+c}}}{8cx^2} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")

[Out] [-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c)))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*

$$d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)}*\sqrt{e/(a*c)})/x^4 + 4*(d*x^2 + c)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*\sqrt{-e/(a*c)})*\arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)}*\sqrt{-e/(a*c)})/(b*e*x^2 + a*e)) - 2*(d*x^2 + c)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]$$

giac [B] time = 0.68, size = 269, normalized size = 2.12

$$\frac{1}{2} \left(\frac{(bce - ade) \arctan\left(\frac{\sqrt{bd}x^2e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}c} - \frac{(\sqrt{bd}x^2e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})bce + (\sqrt{bd}x^2e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})ade + 2\sqrt{bd}ace^{\frac{3}{2}}}{(ace - (\sqrt{bd}x^2e^{\frac{1}{2}} - \sqrt{bdx^4e + bcx^2e + adx^2e + ace})^2)c} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")

[Out] 1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*b*c*e + (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a*d*e + 2*sqrt(b*d)*a*c*e^(3/2))/(a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)*c))*sgn(d*x^2 + c)

maple [B] time = 0.05, size = 326, normalized size = 2.57

$$\frac{\sqrt{\frac{bd+a}{2d^2+c}}(dx^2+c)\left(-a^2cd^2\ln\left(\frac{ad^2+bc^2+2ac\sqrt{\frac{bd+a}{2d^2+c}}\sqrt{bdx^4+bcx^2+adx^2+ac}}{d^2}\right)+ab^2c^2\ln\left(\frac{ad^2+bx^2+2ac\sqrt{\frac{bd+a}{2d^2+c}}\sqrt{bdx^4+bcx^2+ac}}{d^2}\right)-2\sqrt{bd}x^4+ad^2+bcx^2+ac\sqrt{ac}bdx^4-2\sqrt{bd}x^4+ad^2+bcx^2+ac\sqrt{ac}adx^2-2\sqrt{bd}x^4+ad^2+bcx^2+ac\sqrt{ac}bcx^2+2(bdx^4+ad^2+bcx^2+ac)^{\frac{3}{2}}\sqrt{ac}\right)}{4\sqrt{(d^2+c)(bx^2+a)}\sqrt{ac}a^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^3,x)

[Out] -1/4*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)-a^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*c*x^2+c^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b*a*x^2-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b*c*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^2/a/x^2/(a*c)^(1/2)

maxima [A] time = 2.23, size = 145, normalized size = 1.14

$$\frac{1}{4} e \left(\frac{2(bc - ad)\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{ace - \frac{(bx^2+a)c^2e}{dx^2+c}} + \frac{(bc - ad) \log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}}\right)}{\sqrt{ace}c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")

[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a*c*e - (b*x^2 + a)*c^2*e/(d*x^2 + c)) + (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

```
[Out] Timed out
```

$$3.102 \quad \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5} dx$$

Optimal. Leaf size=208

$$\frac{\sqrt{e}(3ad+bc)(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Rubi [A] time = 0.17, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 455, 385, 208}

$$\frac{\sqrt{e}(3ad+bc)(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}} - \frac{(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc-5ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2\left(a-\frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5, x]

[Out] -((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(4*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))^2) + ((b*c - 5*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(8*a*c^2*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b*c + 3*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/((Sqrt[a]*Sqrt[e]))]/(8*a^(3/2)*c^(5/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,

Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b * e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5} dx &= ((bc - ad)e) \text{Subst} \left(\int \frac{x^2 (be - dx^2)}{(-ae + cx^2)^3} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2} - \frac{((bc - ad)e) \text{Subst} \left(\int \frac{-((bc-ad)e)+4cdx^2}{(-ae+cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4c^2} \\ &= \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc - 5ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{((bc - ad)(bc + 3ad)e) \text{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{8ac^2} \\ &= \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2} + \frac{(bc - 5ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^2 \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(bc - ad)(bc + 3ad) \sqrt{e} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{8a^{3/2} c^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 174, normalized size = 0.84

$$\frac{\sqrt{c + dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(x^4 (-3a^2 d^2 + 2abcd + b^2 c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) + \sqrt{a} \sqrt{c} \sqrt{a + bx^2} \sqrt{c + dx^2} (-2ac + 3adx^2 - bcx^2) \right)}{8a^{3/2} c^{5/2} x^4 \sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5, x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.27, size = 161, normalized size = 0.77

$$\frac{(-2ac^2 + acdx^2 + 3ad^2x^4 - bc^2x^2 - bcdx^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8ac^2x^4} - \frac{\sqrt{e} (3a^2d^2 - 2abcd - b^2c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{3/2} c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5, x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*a*c^2 - b*c^2*x^2 + a*c*d*x^2 - b*c*d*x^4 + 3*a*d^2*x^4))/(8*a*c^2*x^4) - (((-b^2*c^2) - 2*a*b*c*d + 3*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]))/(8*a^(3/2)*c^(5/2))

fricas [A] time = 2.32, size = 427, normalized size = 2.05

$$\frac{\left((b^2c^2 + 2abcd - 3a^2d^2)x^4 \sqrt{\frac{e}{ac}} \log\left(\frac{(b^2c^2 + 2abcd + 3a^2d^2)x^4 + 8(abcd + a^2d^2)\sqrt{\frac{e}{ac}}}{32a^2x^4} \right) + 4((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \arctan\left(\frac{(bc + ad)\sqrt{\frac{e}{ac}}}{2(bcd + ad)} \right) + 2((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \right)}{16a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")

[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c)))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]

giac [B] time = 0.82, size = 598, normalized size = 2.88

$$\frac{\left(\frac{(b^2c^2 + 2abcd - 3a^2d^2)x^4 \sqrt{\frac{e}{ac}} \log\left(\frac{(b^2c^2 + 2abcd + 3a^2d^2)x^4 + 8(abcd + a^2d^2)\sqrt{\frac{e}{ac}}}{32a^2x^4} \right) + 4((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \arctan\left(\frac{(bc + ad)\sqrt{\frac{e}{ac}}}{2(bcd + ad)} \right) + 2((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \right)}{16a^2x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")

[Out] -1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/sqrt(-a*c*e)*a*c^2) - ((sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a*b^2*c^3*e^2 + 10*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*c*d^2*e^2 + (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*b^2*c^2*e + 2*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a*b*c*d*e - 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d)*a^3*c^2*d*e^(5/2) + 8*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2*sqrt(b*d)*a*b*c^2*e^(3/2))/((a*c*e - (sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^2)^2*a*c^2))*sgn(d*x^2 + c)

maple [B] time = 0.07, size = 558, normalized size = 2.68

$$\frac{\left(\frac{(b^2c^2 + 2abcd - 3a^2d^2)x^4 \sqrt{\frac{e}{ac}} \log\left(\frac{(b^2c^2 + 2abcd + 3a^2d^2)x^4 + 8(abcd + a^2d^2)\sqrt{\frac{e}{ac}}}{32a^2x^4} \right) + 4((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \arctan\left(\frac{(bc + ad)\sqrt{\frac{e}{ac}}}{2(bcd + ad)} \right) + 2((bcd - 3ad^2)x^4 + 2ac^2 + (bc^2 - ad)x^2) \sqrt{\frac{e}{ac}} \right)}{16a^2x^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^5,x)

[Out] 1/16*((b*x^2+a)/(d*x^2+c)*e)^(1/2)*(d*x^2+c)*(-10*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)-2*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)-3*a^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4+c^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^2*a*x^4-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*d*a*x^2*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*b*c*x^2*(a*c)^(1/2)-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*a*c*(a*c)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/c^3/a^2/x^4/(a*c)^(1/2)

maxima [A] time = 2.17, size = 265, normalized size = 1.27

$$-\frac{1}{16}e^{\left(\frac{2 \left((b^2c^3 - 6abc^2d + 5a^2cd^2) \left(\frac{(bx^2+a)e}{dx^2+c} \right)^{\frac{3}{2}} + (ab^2c^2 + 2a^2bcd - 3a^3d^2) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e \right)}{a^3c^2e^2 - \frac{2(bx^2+a)a^2c^3e^2}{dx^2+c} + \frac{(bx^2+a)^2ac^4e^2}{(dx^2+c)^2}} \right) + \frac{(b^2c^2 + 2abcd - 3a^2d^2) \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace} ac^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")

[Out] -1/16*e*(2*((b^2*c^3 - 6*a*b*c^2*d + 5*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2) + (a*b^2*c^2 + 2*a^2*b*c*d - 3*a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 + c)))/((a^3*c^2*e^2 - 2*(b*x^2 + a)*a^2*c^3*e^2/(d*x^2 + c) + (b*x^2 + a)^2*a*c^4*e^2/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*c^2))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)

[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)

[Out] Timed out

$$3.103 \quad \int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx$$

Optimal. Leaf size=318

$$\frac{(-11a^2d^2 + 2abcd + b^2c^2)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \sqrt{e} (5a^2d^2 + 2abcd + b^2c^2)(bc - ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^2c^3\left(a - \frac{c(a+bx^2)}{c+dx^2}\right) 16a^{5/2}c^{7/2}}$$

Rubi [A] time = 0.31, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 463, 455, 385, 208}

$$\frac{(-11a^2d^2 + 2abcd + b^2c^2)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \sqrt{e} (5a^2d^2 + 2abcd + b^2c^2)(bc - ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{16a^2c^3\left(a - \frac{c(a+bx^2)}{c+dx^2}\right) 16a^{5/2}c^{7/2}} + \frac{e^2(bc - ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{6ac^2\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(3ad + bc)(bc - ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]

[Out] ((b*c - a*d)^2*(b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(8*a*c^3*(a - (c*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d - 11*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(16*a^2*c^3*(a - (c*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) - ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/((Sqrt[a]*Sqrt[e]))]/(16*a^(5/2)*c^(7/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] := -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)

```
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1960

```
Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_))
)^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{x^2 (be - dx^2)^2}{(-ae + cx^2)^4} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^3} + \frac{(bc - ad) \text{Subst} \left(\int \frac{x^2 (-3(2b^2c^2e^2 - (bce - ade)^2) + 6acd^2ex^2)}{(-ae+cx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6ac^2}$$

$$= \frac{(bc - ad)^2 (bc + 3ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)^2} + \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^3} - \frac{(bc - ad) \text{Subst} \left(\int \frac{3c(bc-ad)(bc+3ad)}{(-ae+cx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{24a^2}$$

$$= \frac{(bc - ad)^2 (bc + 3ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)^2} - \frac{(bc - ad) (b^2c^2 + 2abcd - 11a^2d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)} + \frac{(bc - ad)^3 e^2}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

$$= \frac{(bc - ad)^2 (bc + 3ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8ac^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)^2} - \frac{(bc - ad) (b^2c^2 + 2abcd - 11a^2d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16a^2c^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)} + \frac{(bc - ad)^3 e^2}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Mathematica [A] time = 0.18, size = 222, normalized size = 0.70

$$\frac{\sqrt{c + dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{a} \sqrt{c} \sqrt{a + bx^2} \sqrt{c + dx^2} (a^2 (-8c^2 + 10cdx^2 - 15d^2x^4) - 2abcx^2 (c - 2dx^2) + 3b^2c^2x^4) - 3x^6 (-5a^3d^3 + 3a^2bcd^2 + ab^2c^2d + b^3c^3) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) \right)}{48a^5c^7/2x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7, x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2*(-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.47, size = 234, normalized size = 0.74

$$\frac{(-8a^2c^3 + 2a^2c^2dx^2 - 5a^2cd^2x^4 - 15a^2d^3x^6 - 2abc^3x^2 + 2abc^2dx^4 + 4abcd^2x^6 + 3b^2c^3x^4 + 3b^2c^2dx^6) \sqrt{\frac{ae+bx^2}{c+dx^2}} + \frac{\sqrt{e} (5a^3d^3 - 3a^2bcd^2 - ab^2c^2d - b^3c^3) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{16a^{5/2}c^{7/2}}}{48a^2c^3x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-8*a^2*c^3 - 2*a*b*c^3*x^2 + 2*a^2*c^2*d*x^2 + 3*b^2*c^3*x^4 + 2*a*b*c^2*d*x^4 - 5*a^2*c*d^2*x^4 + 3*b^2*c^2*d*x^6 + 4*a*b*c*d^2*x^6 - 15*a^2*d^3*x^6))/(48*a^2*c^3*x^6) + (((-b^3*c^3) - a*b^2*c^2*d - 3*a^2*b*c*d^2 + 5*a^3*d^3)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(16*a^(5/2)*c^(7/2))

fricas [A] time = 6.17, size = 561, normalized size = 1.76

$$\frac{3 \sqrt{e} \sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right) - 4 \left((3b^2c^3 + 4abcd^2 - 15a^2d^3)x^6 - 2abc^3x^2 + 2abc^2dx^4 - 5a^2cd^2x^4 + 3b^2c^3x^4 + 3b^2c^2dx^6 \right) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{48a^2c^3x^6} + \frac{((3b^2c^3 + 4abcd^2 - 15a^2d^3)x^6 - 2abc^3x^2 + 2abc^2dx^4 - 5a^2cd^2x^4 + 3b^2c^3x^4 + 3b^2c^2dx^6) \sqrt{e} \operatorname{arctanh}\left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}}\right) - 2 \left((3b^2c^3 + 4abcd^2 - 15a^2d^3)x^6 - 2abc^3x^2 + 2abc^2dx^4 - 5a^2cd^2x^4 + 3b^2c^3x^4 + 3b^2c^2dx^6 \right) \sqrt{e}}{16a^{5/2}c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")

[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c)))/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]

giac [B] time = 0.98, size = 1076, normalized size = 3.38

$$\frac{1}{48} \frac{(3(b^3c^3e + ab^2c^2de + 3a^2b^2cd^2e - 5a^3d^3e) \operatorname{arctan}(-(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace}))/\sqrt{-ace})/(\sqrt{-ace}a^2c^3) - (3(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})a^2b^3c^5e^3 + 51(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})a^3b^2c^4de^3 + 105(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})a^4bc^3d^2e^3 + 33(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})a^5c^2d^3e^3 + 8(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^3ab^3c^4e^2 + 72(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^3a^2b^2c^3d^2e^2 + 24(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^3a^3bc^2d^2e^2 - 40(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^3a^4cd^3e^2 - 3(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^5ab^3c^3e - 3(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^5a^2b^2c^2de - 9(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^5a^2b^2cd^2e + 15(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^5a^2b^2cd^2e + 15(\sqrt{bd}x^2e^{1/2} - \sqrt{bd^2x^4e + bcx^2e + adx^2e + ace})^5a^2b^2cd^2e)}{16a^{5/2}c^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")

[Out] 1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan(-(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^3*b^2*c^4*d*e^3 + 105*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^4*b*c^3*d^2*e^3 + 33*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))*a^5*c^2*d^3*e^3 + 8*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a*b^3*c^4*e^2 + 72*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^2*b^2*c^3*d^2*e^2 + 24*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^3*a^4*c*d^3*e^2 - 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*b^3*c^3*e - 3*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^2*b^2*c^2*d*e - 9*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^2*b^2*c*d^2*e + 15*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^2*b^2*c*d^2*e + 15*(sqrt(b*d)*x^2*e^(1/2) - sqrt(b*d*x^4*e + b*c*x^2*e + a*d*x^2*e + a*c*e))^5*a^2*b^2*c*d^2*e)

$(x^4e + b*x^2e + a*d*x^2e + a*c*e)^{5*a^3*d^3e + 16*\sqrt{b*d}*a^4*b*c^4*d*e^{(7/2)} + 48*\sqrt{b*d}*a^5*c^3*d^2*e^{(7/2)} + 48*(\sqrt{b*d}*x^2*e^{(1/2)} - \sqrt{b*d*x^4e + b*c*x^2e + a*d*x^2e + a*c*e})^2*\sqrt{b*d}*a^2*b^2*c^4*e^{(5/2)} + 144*(\sqrt{b*d}*x^2*e^{(1/2)} - \sqrt{b*d*x^4e + b*c*x^2e + a*d*x^2e + a*c*e})^2*\sqrt{b*d}*a^3*b*c^3*d*e^{(5/2)})/((a*c*e - (\sqrt{b*d}*x^2*e^{(1/2)} - \sqrt{b*d*x^4e + b*c*x^2e + a*d*x^2e + a*c*e})^2)^3*a^2*c^3))*\text{sgn}(d*x^2 + c)$

maple [B] time = 0.08, size = 849, normalized size = 2.67

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((b*x^2+a)/(d*x^2+c)*e)^(1/2)/x^7,x)`

[Out] $-1/96*((b*x^2+a)/(d*x^2+c)*e)^{(1/2)}*(d*x^2+c)*(-66*b*d^3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*x^8*a^2*(a*c)^{(1/2)}-24*b^2*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*x^8*a*c*(a*c)^{(1/2)}-6*b^3*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*x^8*c^2*(a*c)^{(1/2)}-15*a^4*\ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^{(1/2)}*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)})/x^2)*d^3*c*x^6+9*\ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^{(1/2)}*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)})/x^2)*d^2*b*a^3*c^2*x^6+3*\ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^{(1/2)}*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)})/x^2)*d*b^2*a^2*c^3*x^6+3*c^4*\ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^{(1/2)}*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)})/x^2)*b^3*a*x^6-66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*d^3*a^3*x^6*(a*c)^{(1/2)}-54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*d^2*b*a^2*c*x^6*(a*c)^{(1/2)}-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*b^2*d*a*c^2*x^6*(a*c)^{(1/2)}-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*b^3*c^3*x^6*(a*c)^{(1/2)}+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*d^2*a^2*x^4*(a*c)^{(1/2)}+24*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*d*b*a*c*x^4*(a*c)^{(1/2)}+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*b^2*c^2*x^4*(a*c)^{(1/2)}-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*d*a^2*c*x^2*(a*c)^{(1/2)}-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*b*a*c^2*x^2*(a*c)^{(1/2)}+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*a^2*c^2*(a*c)^{(1/2)})/((d*x^2+c)*(b*x^2+a))^{(1/2)}/c^4/a^3/x^6/(a*c)^{(1/2)}$

maxima [A] time = 2.29, size = 410, normalized size = 1.29

$$\frac{1}{96} \left(\frac{2 \left(3(b^3c^3 + ab^2cd - 13a^2bc^2d^2 + 11a^3c^2d^3) \left(\frac{(bx^2+a)^{5/2}}{dx^2+c} \right) - 8(ab^3c^4 - 3a^2b^2c^3d - 3a^3bc^2d^2 + 5a^4cd^3) \left(\frac{(bx^2+a)^{3/2}}{dx^2+c} \right) e - 3(a^2b^3c^3 + a^3b^2c^2d + 3a^4bcd^2 - 5a^5d^3) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} \right)}{a^5c^3e^3 - \frac{3(bx^2+a)^4e^3}{dx^2+c} + \frac{3(bx^2+a)^2a^2c^2e^3}{(dx^2+c)^2} - \frac{(bx^2+a)^3a^2c^2e^3}{(dx^2+c)^3}} - \frac{3(b^3c^3 + ab^2cd + 3a^2bc^2d - 5a^3d^3) \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace}a^3c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")`

[Out] $-1/96*e*(2*(3*(b^3*c^5 + a*b^2*c^4*d - 13*a^2*b*c^3*d^2 + 11*a^3*c^2*d^3)*(b*x^2 + a)*e/(d*x^2 + c))^{(5/2)} - 8*(a*b^3*c^4 - 3*a^2*b^2*c^3*d - 3*a^3*b*c^2*d^2 + 5*a^4*c*d^3)*(b*x^2 + a)*e/(d*x^2 + c))^{(3/2)}*e - 3*(a^2*b^3*c^3 + a^3*b^2*c^2*d + 3*a^4*b*c*d^2 - 5*a^5*d^3)*\sqrt{((b*x^2 + a)*e/(d*x^2 + c))*e^2}/(a^5*c^3*e^3 - 3*(b*x^2 + a)*a^4*c^4*e^3/(d*x^2 + c) + 3*(b*x^2 + a)^2*a^3*c^5*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a^2*c^6*e^3/(d*x^2 + c)^3) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*\log((c*\sqrt{((b*x^2 + a)*e/(d*x^2 + c))} - \sqrt{a*c*e})/(c*\sqrt{((b*x^2 + a)*e/(d*x^2 + c))} + \sqrt{a*c*e}))/(\sqrt{a*c*e})*a^2*c^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

```
[Out] Timed out
```

$$3.104 \quad \int x^5 \left(\frac{e^{(a+bx^2)}}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=282

$$\frac{e(c+dx^2)(-5a^2d^2-50abcd+79b^2c^2)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{48bd^4} - \frac{e^{3/2}(bc-ad)(-a^2d^2-10abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

Rubi [A] time = 0.38, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1960, 463, 455, 1157, 388, 208}

$$\frac{e^{3/2}(bc-ad)(-a^2d^2-10abcd+35b^2c^2)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}} + \frac{e(c+dx^2)(-5a^2d^2-50abcd+79b^2c^2)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{48bd^4} + \frac{c^2e(bc-ad)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{d^4} + \frac{(c+dx^2)^3\left(\frac{e^{(a+bx^2)}}{c+dx^2}\right)^{5/2}}{6bd^2e} - \frac{e(c+dx^2)^2(ad+11bc)\sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{24d^4}$$

Antiderivative was successfully verified.

[In] Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (c^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/d^4 + ((79*b^2*c^2 - 50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(4*8*b*d^4) - (((11*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*d^4) + (((e*(a + b*x^2))/(c + d*x^2))^(5/2)*(c + d*x^2)^3)/(6*b*d^2*e) - ((b*c - a*d)*(35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e]))]/(16*b^(3/2)*d^(9/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(2), x_Symbol] := -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1157

```
Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_),
x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_))
)^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx = ((bc-ad)e) \operatorname{Subst} \left(\int \frac{x^4 (-ae+cx^2)^2}{(be-dx^2)^4} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{5/2} (c+dx^2)^3}{6bd^2e} - \frac{(bc-ad) \operatorname{Subst} \left(\int \frac{x^4 (-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2)}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6bd^2}$$

$$= -\frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)^2}{24d^4} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{5/2} (c+dx^2)^3}{6bd^2e} - \frac{(bc-ad) \operatorname{Subst} \left(\int \frac{x^4 (-6a^2d^2e^2+5(bce-ade)^2+6bc^2dex^2)}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6bd^2}$$

$$= \frac{(79b^2c^2 - 50abcd - 5a^2d^2)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{24d^4}$$

$$= \frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2 - 50abcd - 5a^2d^2)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{24d^4}$$

$$= \frac{c^2(bc-ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^4} + \frac{(79b^2c^2 - 50abcd - 5a^2d^2)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{48bd^4} - \frac{(11bc+ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}{24d^4}$$

Mathematica [A] time = 0.54, size = 294, normalized size = 1.04

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(b\sqrt{d}\sqrt{bc-ad} (3a^2d^2(c+dx^2) + a^2bd(-100c^2 - 35cdx^2 + 17d^2x^4)) + ad^2(105c^3 - 65c^2dx^2 - 52cd^2x^4 + 22d^3x^6) + b^3x^2(105c^3 + 35c^2dx^2 - 14cd^2x^4 + 8d^3x^6) - 3\sqrt{a+bx^2}(bc-ad)^2(-a^2d^2 - 10abcd + 35b^2c^2) \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{bc-ad}}\right) \right)}{48b^2d^2(a+bx^2)\sqrt{bc-ad}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[b*c - a*d]*(3*a^3*d^2*(c + d*x^2) + a^2*b*d*(-100*c^2 - 35*c*d*x^2 + 17*d^2*x^4) + b^3*x^2*(105*c^3 + 35*c^2*d*x^2 - 14*c*d^2*x^4 + 8*d^3*x^6) + a*b^2*(105*c^3 - 65*c^2*d*x^2 - 52*c*d^2*x^4 + 22*d^3*x^6)) - 3*(b*c - a*d)^2*(35*b^2*c^2 - 10*a*b*c*d

- a^2*d^2)*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^2*d^(9/2)*Sqrt[b*c - a*d]*(a + b*x^2))

IntegrateAlgebraic [A] time = 0.48, size = 233, normalized size = 0.83

$$\frac{\sqrt{\frac{ae+bx^2}{c+dx^2}} (3a^2cd^2e + 3a^2d^3ex^2 - 100abc^2de - 38abcd^2ex^2 + 14abd^3ex^4 + 105b^2c^3e + 35b^2c^2dex^2 - 14b^2cd^2ex^4 + 8b^2d^3ex^6)}{48bd^4} - \frac{e^{3/2} (a^3d^3 + 9a^2bcd^2 - 45ab^2c^2d + 35b^3c^3) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{c}} \right)}{16b^{3/2}d^{9/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(105*b^2*c^3*e - 100*a*b*c^2*d*e + 3*a^2*c*d^2*e + 35*b^2*c^2*d*e*x^2 - 38*a*b*c*d^2*e*x^2 + 3*a^2*d^3*e*x^2 - 14*b^2*c*d^2*e*x^4 + 14*a*b*d^3*e*x^4 + 8*b^2*d^3*e*x^6))/(48*b*d^4) - ((35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]]/(Sqrt[b]*Sqrt[e]))/(16*b^(3/2)*d^(9/2))

fricas [A] time = 3.46, size = 553, normalized size = 1.96

$$\frac{\sqrt{\frac{ae+bx^2}{c+dx^2}} (3a^2cd^2e + 3a^2d^3ex^2 - 100abc^2de - 38abcd^2ex^2 + 14abd^3ex^4 + 105b^2c^3e + 35b^2c^2dex^2 - 14b^2cd^2ex^4 + 8b^2d^3ex^6)}{48bd^4} - \frac{e^{3/2} (a^3d^3 + 9a^2bcd^2 - 45ab^2c^2d + 35b^3c^3) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{c}} \right)}{16b^{3/2}d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d)))/(b*e*x^2 + a*e) + 2*(8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x^2*d+c)]Evaluation time: 0.84Unable to divide, perhaps due to rounding error% %%%{2, [0, 5, 0]%%}, [2, 0, 0, 0]%%}+%%{%%{[-4, [0, 4, 0]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]}%%, [1, 0, 0, 1]%%}+%%{%%{2, [1, 4, 1]%%}, [0, 0, 0, 2]%%} / %%{%%{1, [0, 2, 2]%%}, [2, 0, 0, 0]%%}+%%{%%{[-2, [0, 1, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]}%%, [1, 0, 0, 1]%%}+%%{%%{1, [1, 1, 3]%%}, [0, 0, 0, 2]%%} Error: Bad Argument Value

maple [B] time = 0.08, size = 1027, normalized size = 3.64

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*((b*x^2+a)/(d*x^2+c)*e)^{(3/2)}, x)$

[Out] $\frac{1}{96}*(12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*x^4*a*b*d^3-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*x^4*b^2*c*d^2-3*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*x^2*a^3*d^4-27*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*x^2*a^2*b*c*d^3+135*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*x^2*a*b^2*c^2*d^2-105*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*x^2*b^3*c^3*d+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*(b*d)^{(1/2)}*x^2*b*d^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*x^2*a^2*d^3-108*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*x^2*a*b*c*d^2+54*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*x^2*b^2*c^2*d-3*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*a^3*c*d^3-27*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*a^2*b*c^2*d^2+135*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*a*b^2*c^3*d-105*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)))/(b*d)^{(1/2)})*b^3*c^4+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*(b*d)^{(1/2)}*b*c*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*a^2*c*d^2-120*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*a*b*c^2*d+114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*b^2*c^3-96*(b*d)^{(1/2)}*((d*x^2+c)*(b*x^2+a))^{(1/2)}*a*b*c^2*d+96*(b*d)^{(1/2)}*((d*x^2+c)*(b*x^2+a))^{(1/2)}*b^2*c^3)/d^4/b*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^{(3/2)}/(b*d)^{(1/2)}/((d*x^2+c)*(b*x^2+a))^{(1/2)}/(b*x^2+a)$

maxima [A] time = 2.12, size = 454, normalized size = 1.61

$$\frac{1}{96} \left(\frac{2 \left(3 \left(29 b^3 c^3 d^2 - 51 a b^2 c^2 d + 23 a^2 b c d^2 - a^3 d^3 \right) \left(\frac{(b^2 + a)^2}{d^2 + c} \right)^{\frac{5}{2}} - 8 \left(17 b^4 c^3 d - 27 a b^3 c^2 d + 9 a^2 b^2 c d^2 + a^3 b d^3 \right) \left(\frac{(b^2 + a)^2}{d^2 + c} \right)^{\frac{3}{2}} + 3 \left(19 b^5 c^3 - 29 a b^4 c^2 d + 9 a^2 b^3 c d^2 + a^3 b^2 d^3 \right) \sqrt{\frac{(b^2 + a)^2}{d^2 + c}} \right) \sqrt{\frac{(b^2 + a)^2}{d^2 + c}}}{b^4 d^4 c^3 - \frac{3(b^2 + a)b^3 c^2 d^2}{d^2 + c} + \frac{3(b^2 + a)^2 b^2 c d^2}{(a^2 + c)^2} - \frac{(b^2 + a)^3 b d^2}{(a^2 + c)^3}} + \frac{96(bc^3 - ac^2d)\sqrt{\frac{(b^2 + a)^2}{d^2 + c}}}{d^4} + \frac{3(35b^3c^3 - 45ab^2c^2d + 9a^2bcd^2 + a^3d^3)\log\left(\frac{\sqrt{\frac{(b^2 + a)^2}{d^2 + c}} - \sqrt{bc}}{\sqrt{\frac{(b^2 + a)^2}{d^2 + c}} + \sqrt{bc}}\right)}{\sqrt{bde}bd^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^5*(e*(b*x^2+a)/(d*x^2+c))^{(3/2)}, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{96}*e*(2*(3*(29*b^3*c^3*d^2 - 51*a*b^2*c^2*d^3 + 23*a^2*b*c*d^4 - a^3*d^5))*((b*x^2 + a)*e/(d*x^2 + c))^{(5/2)}*e - 8*(17*b^4*c^3*d - 27*a*b^3*c^2*d^2 + 9*a^2*b^2*c*d^3 + a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^{(3/2)}*e^2 + 3*(19*b^5*c^3 - 29*a*b^4*c^2*d + 9*a^2*b^3*c*d^2 + a^3*b^2*d^3)*\text{sqrt}((b*x^2 + a)*e/(d*x^2 + c))*e^3)/(b^4*d^4*e^3 - 3*(b*x^2 + a)*b^3*d^5*e^3/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^2*d^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*b*d^7*e^3/(d*x^2 + c)^3) + 96*(b*c^3 - a*c^2*d)*\text{sqrt}((b*x^2 + a)*e/(d*x^2 + c))/d^4 + 3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*\log((d*\text{sqrt}((b*x^2 + a)*e/(d*x^2 + c)) - \text{sqrt}(b*d*e))/(d*\text{sqrt}((b*x^2 + a)*e/(d*x^2 + c)) + \text{sqrt}(b*d*e)))/(\text{sqrt}(b*d*e)*b*d^4)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(\frac{e(bx^2 + a)}{dx^2 + c} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^5*((e*(a + b*x^2))/(c + d*x^2))^{(3/2)}, x)$

[Out] $\text{int}(x^5*((e*(a + b*x^2))/(c + d*x^2))^{(3/2)}, x)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

$$3.105 \quad \int x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=199

$$\frac{3e^{3/2}(bc-ad)(5bc-ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{8\sqrt{b} d^{7/2}} + \frac{be(c+dx^2)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3} - \frac{e(c+dx^2)(9bc-5ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3} - \frac{ce(bc-ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}$$

Rubi [A] time = 0.22, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 455, 1157, 388, 208}

$$\frac{3e^{3/2}(bc-ad)(5bc-ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{8\sqrt{b} d^{7/2}} + \frac{be(c+dx^2)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3} - \frac{e(c+dx^2)(9bc-5ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8d^3} - \frac{ce(bc-ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3}$$

Antiderivative was successfully verified.

[In] Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] -((c*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/d^3) - ((9*b*c - 5*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)/(8*d^3) + (b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2/(4*d^3) + (3*(b*c - a*d)*(5*b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/((Sqrt[b]*Sqrt[e])))/(8*Sqrt[b]*d^(7/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p+1))/(2*b^(m/2 + 1)*(p+1)), x] + Dist[1/(2*b^(m/2 + 1)*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*x^2*Together[(b^(m/2)*x^(m-2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q+1))/(2*d*(q+1)), x] + Dist[1/(2*d*(q+1)), Int[(d + e*x^2)^(q+1)*ExpandToSum[2*d*(q+1)*Qx + R*(2*q+3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -

$b*d*e + a*e^2, 0]$ && IGtQ[p, 0] && LtQ[q, -1]

Rule 1960

Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^3 \left(\frac{e(a + bx^2)}{c + dx^2} \right)^{3/2} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{x^4 (-ae + cx^2)}{(be - dx^2)^3} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= \frac{be \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4d^3} + \frac{((bc - ad)e) \operatorname{Subst} \left(\int \frac{-b(bc-ad)e^2 - 4d(bc-ad)ex^2 - 4cd^2x^4}{(be-dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4d^3} \\ &= -\frac{(9bc - 5ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8d^3} + \frac{be \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4d^3} - \frac{(bc - ad) \operatorname{Subst} \left(\int \frac{-b(bc-ad)e^2 - 4d(bc-ad)ex^2 - 4cd^2x^4}{(be-dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4d^3} \\ &= -\frac{c(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc - 5ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8d^3} + \frac{be \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4d^3} \\ &= -\frac{c(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{d^3} - \frac{(9bc - 5ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8d^3} + \frac{be \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4d^3} \end{aligned}$$

Mathematica [A] time = 0.62, size = 191, normalized size = 0.96

$$\frac{e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(3\sqrt{bc-ad} (a^2d^2 - 6abcd + 5b^2c^2) \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) + b\sqrt{d} \sqrt{a+bx^2} (ad(13c + 5dx^2) + b(-15c^2 - 5cdx^2 + 2d^2x^4)) \right)}{8bd^{7/2} \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*Sqrt[a + b*x^2]*(a*d*(13*c + 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*Sqrt[b*c - a*d]*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]))/(8*b*d^(7/2)*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.37, size = 157, normalized size = 0.79

$$\frac{3e^{3/2} (a^2d^2 - 6abcd + 5b^2c^2) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{8\sqrt{b} d^{7/2}} + \frac{\sqrt{\frac{ae+bx^2}{c+dx^2}} (13acde + 5ad^2ex^2 - 15bc^2e - 5bcdex^2 + 2bd^2ex^4)}{8d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-15*b*c^2*e + 13*a*c*d*e - 5*b*c*d*e*x^2 + 5*a*d^2*e*x^2 + 2*b*d^2*e*x^4))/(8*d^3) + (3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*Sqrt[b]*d^(7/2))
```

fricas [A] time = 1.93, size = 417, normalized size = 2.10

$$\frac{3(5b^2c^2 - 6abcd + d^2e^2)\sqrt{e}\log\left(\frac{8b^2d^2e^2 + 8(b^2cd + abd^2)e^2 + (b^2c^2 + 6abcd + d^2e^2)e + 4(2b^2d^2e^2 + abcd^2 + (3b^2d^2 + abcd^2)\sqrt{\frac{cd}{2d^2}}\sqrt{e}}{32d^3}\right) + 4(2bd^2e^2 - 5(bcd - ad^2)e^2 - (15b^2 - 13acd)\sqrt{\frac{cd}{2d^2}}\sqrt{e})}{16d^3} - \frac{3(5b^2c^2 - 6abcd + d^2e^2)\sqrt{e}\arctan\left(\frac{2(b^2cd - ad^2)\sqrt{\frac{cd}{2d^2}}\sqrt{e}}{2(b^2cd + abcd^2)}\right) - 2(2b^2d^2e^2 - 5(bcd - ad^2)e^2 - (15b^2 - 13acd)\sqrt{\frac{cd}{2d^2}}\sqrt{e})}{16d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d)))/(b*e*x^2 + a*e) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.76Unable to divide, perhaps due to rounding error%
%%{%%{2, [0, 4, 0]%%}, [2, 0, 0, 0]%%}+%%{%%{[%%{-4, [0, 3, 0]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}%%}, [1, 0, 0, 1]%%}+%%{%%{2, [1, 3, 1]%%}, [0, 0, 0, 2]%%} / %%{%%{1, [0, 2, 2]%%}, [2, 0, 0, 0]%%}+%%{%%{[%%{-2, [0, 1, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}%%}, [1, 0, 0, 1]%%}+%%{%%{1, [1, 1, 3]%%}, [0, 0, 0, 2]%%} Error: Bad
Argument Value
```

maple [B] time = 0.06, size = 679, normalized size = 3.41

$$\frac{1}{16} \left(4(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2} x^4 bd^2 + 3 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) x^2 a^2 d^3 - 18 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) x^2 a b c d^2 + 15 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) x^2 a b c d^2 + 10 (bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2} x^2 a d^2 - 10 (bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2} x^2 b c d + 3 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) a^2 c d^2 - 18 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) a b c^2 d + 15 \ln\left(\frac{1}{2} (2bdx^2 + ad + b^2c + 2(bdx^4 + adx^2 + b^2c^2 + a^2c)^{1/2} (bd)^{1/2}) / (bd)^{1/2}\right) b^2 c^3 + 16 ((dx^2 + c) (bx^2 + a))^{1/2} (bd)^{1/2} a c d - 16 ((dx^2 + c) (bx^2 + a))^{1/2} (bd)^{1/2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] 1/16*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b*d^2+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*x^2*a^2*d^3-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*x^2*a*b*c*d^2+15*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*x^2*b^2*c^2*d+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*a*d^2-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^2*b*c*d+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*a^2*c*d^2-18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*a*b*c^2*d+15*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*b^2*c^3+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)
```

$/2)*b*c^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*a*c*d-14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*b*c^2)/d^3*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^{(3/2)}/(b*d)^{(1/2)}/((d*x^2+c)*(b*x^2+a))^{(1/2)}/(b*x^2+a)$

maxima [A] time = 2.29, size = 303, normalized size = 1.52

$$\frac{1}{16} e^{\left(\frac{2 \left((9 b^2 c^2 d - 14 a b c d^2 + 5 a^2 d^3) \left(\frac{(b x^2 + a) e}{d x^2 + c} \right)^{\frac{3}{2}} - (7 b^3 c^2 - 10 a b^2 c d + 3 a^2 b d^2) \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} c^2 \right)}{b^2 d^3 e^2 - \frac{2 (b x^2 + a) b d^4 e^2}{d x^2 + c} + \frac{(b x^2 + a)^2 d^5 e^2}{(d x^2 + c)^2}} - \frac{3 (5 b^2 c^2 - 6 a b c d + a^2 d^2) e \log \left(\frac{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} + \sqrt{b d e}} \right)}{\sqrt{b d e} d^3} - \frac{16 (b c^2 - a c d) \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}}}{d^3} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{16} e^{\left(2 \left((9 b^2 c^2 d - 14 a b c d^2 + 5 a^2 d^3) \left(\frac{(b x^2 + a) e}{d x^2 + c} \right)^{\frac{3}{2}} - (7 b^3 c^2 - 10 a b^2 c d + 3 a^2 b d^2) \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} c^2 \right) \right) e - (5 b^2 c^2 - 6 a b c d + a^2 d^2) e \log \left(\frac{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} + \sqrt{b d e}} \right) - 16 (b c^2 - a c d) \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}}}{b^2 d^3 e^2 - \frac{2 (b x^2 + a) b d^4 e^2}{d x^2 + c} + \frac{(b x^2 + a)^2 d^5 e^2}{(d x^2 + c)^2}} - \frac{3 (5 b^2 c^2 - 6 a b c d + a^2 d^2) e \log \left(\frac{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} + \sqrt{b d e}} \right)}{\sqrt{b d e} d^3} - \frac{16 (b c^2 - a c d) \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}}}{d^3}}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(\frac{e (b x^2 + a)}{d x^2 + c} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)

[Out] int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

$$3.106 \quad \int x \left(\frac{e^{(a+bx^2)}}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=141

$$\frac{3\sqrt{b} e^{3/2} (bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2d^{5/2}} + \frac{3e(bc - ad) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{2d^2} + \frac{(c + dx^2) \left(\frac{e^{(a+bx^2)}}{c+dx^2} \right)^{3/2}}{2d}$$

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1960, 288, 321, 208}

$$\frac{3\sqrt{b} e^{3/2} (bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2d^{5/2}} + \frac{3e(bc - ad) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{2d^2} + \frac{(c + dx^2) \left(\frac{e^{(a+bx^2)}}{c+dx^2} \right)^{3/2}}{2d}$$

Antiderivative was successfully verified.

[In] Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*d^2) + (((e*(a + b*x^2))/(c + d*x^2))^(3/2)*(c + d*x^2))/(2*d) - (3*Sqrt[b]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e]))]/(2*d^(5/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[m + n*(p + 1) + 1]/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} dx &= ((bc-ad)e) \operatorname{Subst} \left(\int \frac{x^4}{(be-dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} (c+dx^2)}{2d} - \frac{(3(bc-ad)e) \operatorname{Subst} \left(\int \frac{x^2}{be-dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2d} \\
&= \frac{3(bc-ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} (c+dx^2)}{2d} - \frac{(3b(bc-ad)e^2) \operatorname{Subst} \left(\int \frac{1}{be-dx^2} dx, x \right)}{2d^2} \\
&= \frac{3(bc-ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d^2} + \frac{\left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2} (c+dx^2)}{2d} - \frac{3\sqrt{b}(bc-ad)e^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2d^{5/2}}
\end{aligned}$$

Mathematica [C] time = 0.06, size = 96, normalized size = 0.68

$$\frac{e(a+bx^2)^2 \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}} {}_2F_1 \left(\frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \frac{d(bx^2+a)}{ad-bc} \right)}{5bc-5ad}$$

Antiderivative was successfully verified.

[In] Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (e*(a + b*x^2)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*Hypergeometric2F1[3/2, 5/2, 7/2, (d*(a + b*x^2))/(-b*c + a*d)]/(5*b*c - 5*a*d)

IntegrateAlgebraic [A] time = 0.20, size = 120, normalized size = 0.85

$$\frac{e(-2ad + 3bc + bdx^2) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2d^2} - \frac{3e^{3/2} (b^{3/2}c - a\sqrt{b}d) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2d^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (e*(3*b*c - 2*a*d + b*d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(2*d^2) - (3*(b^(3/2)*c - a*Sqrt[b]*d)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*d^(5/2))

fricas [A] time = 1.33, size = 328, normalized size = 2.33

$$\frac{3(bc-ad)\sqrt{\frac{bc}{d}} \log \left(8b^2d^2ex^4 + 8(b^2cd + abd^2)ex^2 + (b^2c^2 + 6abcd + a^2d^2)c + 4(2bd^2x^4 + bc^2d + acd^2 + (3bd^2 + ad^2)x^2) \sqrt{\frac{bc+ax}{d}} \sqrt{\frac{bx^2+cx}{d}} \right) - 4(bdx^2 + (3bc-2ad)b) \sqrt{\frac{bc+ax}{d}} \sqrt{\frac{bx^2+cx}{d}}}{8d^2} - \frac{3(bc-ad)\sqrt{\frac{bc}{d}} e \arctan \left(\frac{(2bd^2+bc+ad)\sqrt{\frac{bc}{d}} \sqrt{\frac{bx^2+cx}{d}}}{2(b^2c^2+ab^2)} \right) + 2(bdx^2 + (3bc-2ad)b) \sqrt{\frac{bc+ax}{d}} \sqrt{\frac{bx^2+cx}{d}}}{4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2

+ c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 2*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/d^2]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x^2*d+c)]Evaluation time: 0.67Unable to divide, perhaps due to rounding error% %%%{2, [0, 3, 0]%%}, [2, 0, 0, 0]%%}+%%%{%%%{4, [0, 2, 0]%%}, 0]: [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 0, 0, 1]%%}+%%%{%%%{2, [1, 2, 1]%%}, [0, 0, 0, 2]%%} / %%%{%%%{1, [0, 2, 2]%%}, [2, 0, 0, 0]%%}+%%%{%%%{2, [0, 1, 2]%%}, 0]: [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 0, 0, 1]%%}+%%%{%%%{1, [1, 1, 3]%%}, [0, 0, 0, 2]%%} Error: Bad Argument Value

maple [B] time = 0.05, size = 432, normalized size = 3.06

$$\frac{-3abd^2 \ln\left(\frac{2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} - \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c}}{2\sqrt{bd}}\right) + 3bd^2 \ln\left(\frac{2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c}}{2\sqrt{bd}}\right) - 3abd \ln\left(\frac{2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} - \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c}}{2\sqrt{bd}}\right) + 3bd^2 \ln\left(\frac{2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c}}{2\sqrt{bd}}\right) - 2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + 4\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} - 4\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} - 2\sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c} + \sqrt{bd} \sqrt{bx^2+a} \sqrt{dx^2+c}}{4\sqrt{bd} \sqrt{dx^2+c} \sqrt{bx^2+a} \sqrt{dx^2+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)

[Out] -1/4*(-3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*b^2*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*b^2*c^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/d^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*x^2+a)

maxima [A] time = 2.21, size = 189, normalized size = 1.34

$$\frac{1}{4} \left(\frac{2(b^2c - abd) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e}{bd^2e - \frac{(bx^2+a)d^3e}{dx^2+c}} + \frac{3(b^2c - abd)e \log\left(\frac{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d\sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}}\right)}{\sqrt{bde} d^2} + \frac{4(bc - ad) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{d^2} \right) e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/4*(2*(b^2*c - a*b*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(b*d^2*e - (b*x^2 + a)*d^3*e/(d*x^2 + c)) + 3*(b^2*c - a*b*d)*e*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/d^2)*e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(\frac{e(bx^2 + a)}{dx^2 + c} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)`

[Out] `int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2), x)`

[Out] Timed out

$$3.107 \quad \int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x} dx$$

Optimal. Leaf size=151

$$-\frac{a^{3/2}e^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Rubi [A] time = 0.19, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 479, 522, 208}

$$-\frac{a^{3/2}e^{3/2} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2} \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}} - \frac{e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]

[Out] -(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(c*d)) - (a^(3/2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[a]*Sqrt[e])])/c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e])])/d^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 479

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_.))/(((a_) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{x^4}{(-ae + cx^2)(be - dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= -\frac{(bc - ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{((bc - ad)e) \operatorname{Subst} \left(\int \frac{-abe^2 + (bc+ad)ex^2}{(-ae+cx^2)(be-dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{cd} \\
&= -\frac{(bc - ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} + \frac{(a^2e^2) \operatorname{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{c} + \frac{(b^2e^2) \operatorname{Subst} \left(\int \frac{1}{be-dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{d} \\
&= -\frac{(bc - ad)e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd} - \frac{a^{3/2}e^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{d^{3/2}}
\end{aligned}$$

Mathematica [A] time = 1.28, size = 193, normalized size = 1.28

$$\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{d} \left(-\frac{a^{3/2}d\sqrt{c+dx^2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) + \frac{ad}{c} - b \right) + \frac{b\sqrt{bc-ad} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{\sqrt{a+bx^2}} \right)}{d^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((b*Sqrt[b*c - a*d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2] + Sqrt[d]*(-b + (a*d)/c - (a^(3/2)*d*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/Sqrt[a]*Sqrt[c + d*x^2]])/(c^(3/2)*Sqrt[a + b*x^2]))/d^(3/2)

IntegrateAlgebraic [A] time = 0.26, size = 156, normalized size = 1.03

$$-\frac{a^{3/2}e^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{d^{3/2}} + \frac{e(ad - bc)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{cd}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]

[Out] ((-(b*c) + a*d)*e*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(c*d) - (a^(3/2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/Sqrt[a]*Sqrt[e]])/c^(3/2) + (b^(3/2)*e^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/Sqrt[b]*Sqrt[e]])/d^(3/2)

fricas [A] time = 2.46, size = 1049, normalized size = 6.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 +
(b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (
3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + a*
d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e +
8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3
+ 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) - 4*
(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*sqrt(-b
*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c)))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b
*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt
((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2
+ 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b*e*x^2 + a^2*e
) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (
b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*
b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(b
*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2*(a*d*sqrt(-a*e/c)
*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*e*x^2 + a
*b*e)) - 2*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.52Error: Bad Argument Type
```

```
maple [B] time = 0.06, size = 401, normalized size = 2.66
```

$$\frac{\left(\sqrt{d} \, x^2 \sqrt{e} \ln \left(\frac{d^2 x^2 + 2 a d x + a^2}{d^2 x^2 + 2 a d x + a^2} \sqrt{\frac{d^2 x^2 + 2 a d x + a^2}{d^2 x^2 + 2 a d x + a^2}} \right) + \sqrt{d} \, x^2 \sqrt{e} \ln \left(\frac{2 d^2 x^2 + 2 a d x + a^2}{2 \sqrt{d}} \sqrt{\frac{d^2 x^2 + 2 a d x + a^2}{d^2 x^2 + 2 a d x + a^2}} \right) - \sqrt{d} \, x^2 \sqrt{e} \ln \left(\frac{d^2 x^2 + 2 a d x + a^2}{2 \sqrt{d}} \sqrt{\frac{d^2 x^2 + 2 a d x + a^2}{d^2 x^2 + 2 a d x + a^2}} \right) + \sqrt{d} \, x^2 \ln \left(\frac{2 d^2 x^2 + 2 a d x + a^2}{2 \sqrt{d}} \sqrt{\frac{d^2 x^2 + 2 a d x + a^2}{d^2 x^2 + 2 a d x + a^2}} \right) + 2 \sqrt{(d^2 + c)(b x^2 + a)} \sqrt{d} \sqrt{e} \operatorname{arctan} \left(\frac{d x^2 + c}{\sqrt{d} \sqrt{e}} \right) \left(\frac{d^2 + c}{d^2 + c} \right)^{\frac{3}{2}}}{2 \sqrt{d} \sqrt{e} (b x^2 + a) \sqrt{(d^2 + c)(b x^2 + a)} \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*b^2*c*d-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*d^2+ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a^2*c*d+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*b*c)/c/d*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 2.27, size = 197, normalized size = 1.30

$$\frac{1}{2} \left(\frac{a^2 e \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace} c} - \frac{b^2 e \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde} d} - \frac{2(bc - ad) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{cd} \right) e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")

[Out] 1/2*(a^2*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c) - b^2*e*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*d) - 2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(c*d))*e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c} \right)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)

[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)

[Out] Timed out

$$3.108 \quad \int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3} dx$$

Optimal. Leaf size=165

$$\frac{3\sqrt{a} e^{3/2}(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}} + \frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)}$$

Rubi [A] time = 0.10, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 288, 321, 208}

$$\frac{3\sqrt{a} e^{3/2}(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}} + \frac{3e(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc-ad)\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)}$$

Antiderivative was successfully verified.

[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]

[Out] (3*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*c^2) + ((b*c - a*d)*((e*(a + b*x^2))/(c + d*x^2))^(3/2))/(2*c*(a - (c*(a + b*x^2))/(c + d*x^2))) - (3*Sqrt[a]*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !ILtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-a*e) + c*x^q)^(Simplify[(m+1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ

[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{x^4}{(-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= \frac{(bc - ad) \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3(bc - ad)e) \operatorname{Subst} \left(\int \frac{x^2}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2c} \\
&= \frac{3(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc - ad) \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} + \frac{(3a(bc - ad)e^2) \operatorname{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2c^2} \\
&= \frac{3(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c^2} + \frac{(bc - ad) \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{2c \left(a - \frac{c(a+bx^2)}{c+dx^2}\right)} - \frac{3\sqrt{a} (bc - ad)e^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2c^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 146, normalized size = 0.88

$$\frac{e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(\sqrt{c} \sqrt{a+bx^2} (2bcx^2 - a(c+3dx^2)) - 3\sqrt{a} x^2 \sqrt{c+dx^2} (bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) \right)}{2c^{5/2} x^2 \sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 - a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.21, size = 127, normalized size = 0.77

$$\frac{3e^{3/2} (a^{3/2}d - \sqrt{a}bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2c^{5/2}} + \frac{e(-ac - 3adx^2 + 2bcx^2) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2c^2 x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x]

[Out] (e*(-(a*c) + 2*b*c*x^2 - 3*a*d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(2*c^2*x^2) + (3*(-(Sqrt[a]*b*c) + a^(3/2)*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*c^(5/2))

fricas [A] time = 2.57, size = 350, normalized size = 2.12

$$\frac{3(bc - ad) \sqrt{\frac{ae}{c}} x^2 \log \left(\frac{(b^2 x^2 + 6abcd + a^2 d^2) e^{x^4} + 8a^2 c^2 x + 8(ab^2 c + a^2 d) e^{x^2} + 4((bc^2 d + ad^2) x^4 + 2ac^3 + (bc^2 + 3ac^2 d) x^2) \sqrt{\frac{ae}{c}} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{s^4} \right) - 4((2bc - 3ad) e^{x^2} - ace) \sqrt{\frac{ae+bx^2}{c+dx^2}} - 3(bc - ad) \sqrt{\frac{ae}{c}} e^{x^2} \arctan \left(\frac{(bc+ad)x^2 + 2ac}{2(abx^2 + a^2 d)} \sqrt{\frac{ae}{c}} \sqrt{\frac{ae+bx^2}{c+dx^2}} \right) + 2((2bc - 3ad) e^{x^2} - ace) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8c^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] [-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^2)
*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan(1/2*
((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a
*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c)))/(c^2*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x^2
*d+c)]Evaluation time: 0.55Unable to divide, perhaps due to rounding error%
%%{%%{2, [0, 1, 0]%%}, [6, 0, 0]%%}+%%{%%{[-4, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%},
[5, 1, 0]%%}+%%{%%{2, [1, 0, 1]%%}, [4, 2, 0]%%}+%%{%%{[-4, [0, 1, 1]%%}, [4, 1, 1
]%%}+%%{%%{[%%{8, [0, 0, 1]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [3, 2, 1]%%}+
%%{%%{[-4, [1, 0, 2]%%}, [2, 3, 1]%%}+%%{%%{2, [0, 1, 2]%%}, [2, 2, 2]%%}+%%{%%
{%%{[-4, [0, 0, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [1, 3, 2]%%}+%%{%%{2, [
1, 0, 3]%%}, [0, 4, 2]%%} / %%{%%{1, [0, 2, 0]%%}, [6, 0, 0]%%}+%%{%%{[%%{-2, [
0, 1, 0]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [5, 1, 0]%%}+%%{%%{1, [1, 1, 1]%%}
, [4, 2, 0]%%}+%%{%%{[-2, [0, 2, 1]%%}, [4, 1, 1]%%}+%%{%%{[%%{4, [0, 1, 1]%%}, 0
] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [3, 2, 1]%%}+%%{%%{[-2, [1, 1, 2]%%}, [2, 3, 1]%%}
}+%%{%%{1, [0, 2, 2]%%}, [2, 2, 2]%%}+%%{%%{[%%{-2, [0, 1, 2]%%}, 0] : [1, 0, %%{-
1, [1, 1, 1]%%}}]%%}, [1, 3, 2]%%}+%%{%%{1, [1, 1, 3]%%}, [0, 4, 2]%%} Error: Bad
Argument Value
```

maple [B] time = 0.08, size = 641, normalized size = 3.88

```
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```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^3,x)
```

```
[Out] -1/4*(-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b*d^2-3*ln((a*
d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)
*x^4*a^2*c*d^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b
*c*x^2+a*c)^(1/2))/x^2)*x^4*a*b*c^2*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
*(a*c)^(1/2)*x^4*a*d^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^
4*b*c*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2))/x^2)*x^2*a^2*c^2*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a*b*c^3+4*((d*x^2+c)*(b*x^2+a))
^(1/2)*(a*c)^(1/2)*x^2*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*
b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*d-2*(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^2*a*c*d-2*(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(a*c)^(1/2)*x^2*b*c^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)
^(1/2)*c*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^2/c^3/(b*x^
2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 2.26, size = 189, normalized size = 1.15

$$\frac{1}{4} \left(\frac{2(abc - a^2d) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{ac^2e - \frac{(bx^2+a)c^3e}{dx^2+c}} + \frac{3(abc - a^2d)e \log\left(\frac{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c\sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}}\right)}{\sqrt{ace}c^2} + \frac{4(bc - ad) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{c^2} \right) e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")

[Out] 1/4*(2*(a*b*c - a^2*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e/(a*c^2*e - (b*x^2 + a)*c^3*e/(d*x^2 + c)) + 3*(a*b*c - a^2*d)*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c^2) + 4*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^2)*e

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)

[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)

[Out] Timed out

$$3.109 \quad \int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5} dx$$

Optimal. Leaf size=256

$$\frac{3e^{3/2}(bc-5ad)(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}} - \frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)}$$

Rubi [A] time = 0.22, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 455, 1157, 388, 208}

$$\frac{ae^3(bc-ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2} + \frac{e^2(5bc-9ad)(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)} - \frac{3e^{3/2}(bc-5ad)(bc-ad) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}} - \frac{de(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3}$$

Antiderivative was successfully verified.

[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]

[Out] -((d*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/c^3) - (a*(b*c - a*d)^2*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(4*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) + ((5*b*c - 9*a*d)*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(8*c^3*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - 5*a*d)*(b*c - a*d)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[a]*Sqrt[e]))]/(8*Sqrt[a]*c^(7/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2-1)*(b*c - a*d)*x*(a + b*x^2)^(p+1)/(2*b^(m/2+1)*(p+1)), x] + Dist[1/(2*b^(m/2+1)*(p+1)), Int[(a + b*x^2)^(p+1)*ExpandToSum[2*b*(p+1)*x^2*Together[(b^(m/2)*x^(m-2)*(c + d*x^2) - (-a)^(m/2-1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2-1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x

```
, 0}}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1960

```
Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{x^4 (be - dx^2)}{(-ae + cx^2)^3} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= -\frac{a(bc - ad)^2 e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{((bc - ad)e) \text{Subst} \left(\int \frac{-a(bc-ad)e^2 - 4c(bc-ad)ex^2 + 4c^2dx^4}{(-ae+cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4c^3}$$

$$= -\frac{a(bc - ad)^2 e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} + \frac{(5bc - 9ad)(bc - ad)e^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(bc - ad) \text{Subst} \left(\int \frac{-a(3bc-7ad)}{-ae} \right)}{8a}$$

$$= -\frac{d(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc - ad)^2 e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} + \frac{(5bc - 9ad)(bc - ad)e^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} + \dots$$

$$= -\frac{d(bc - ad)e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^3} - \frac{a(bc - ad)^2 e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} + \frac{(5bc - 9ad)(bc - ad)e^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8c^3 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \dots$$

Mathematica [A] time = 0.13, size = 186, normalized size = 0.73

$$\frac{e \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(3x^4 \sqrt{c + dx^2} (5a^2d^2 - 6abcd + b^2c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) + \sqrt{a} \sqrt{c} \sqrt{a + bx^2} (a(2c^2 - 5cdx^2 - 15d^2x^4) + bcx^2(5c + 13dx^2)) \right)}{8\sqrt{a} c^{7/2} x^4 \sqrt{a + bx^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x]
[Out] -1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2])
```

IntegrateAlgebraic [A] time = 0.29, size = 159, normalized size = 0.62

$$\frac{3e^{3/2} (5a^2d^2 - 6abcd + b^2c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8\sqrt{a} c^{7/2}} - \frac{e (2ac^2 - 5acdx^2 - 15ad^2x^4 + 5bc^2x^2 + 13bcdx^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8c^3x^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)

[Out] $-1/8*(e*\text{Sqrt}[(a*e + b*e*x^2)/(c + d*x^2)]*(2*a*c^2 + 5*b*c^2*x^2 - 5*a*c*d*x^2 + 13*b*c*d*x^4 - 15*a*d^2*x^4))/(c^3*x^4) - (3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e^{(3/2)}*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[(a*e + b*e*x^2)/(c + d*x^2)])]/(\text{Sqrt}[a]*\text{Sqrt}[e]))/(8*\text{Sqrt}[a]*c^{(7/2)})$

fricas [A] time = 7.44, size = 435, normalized size = 1.70

$$\frac{3(b^2c^2 - 6abcd + 5a^2d^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}} \log\left(\frac{(b^2c^2 - 6abcd + 5a^2d^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}} + (13bcd - 15ad^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{32c^3x^4}\right) - 4((13bcd - 15ad^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}} + 2ac^2e + 5(bc^2 - acd)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}}) \arctan\left(\frac{(b^2c^2 - 6abcd + 5a^2d^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2(bc^2 + ad)}\right) - 2((13bcd - 15ad^2)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}} + 2ac^2e + 5(bc^2 - acd)e^{3/2} \sqrt{\frac{ae+bx^2}{c+dx^2}})}{16c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")

[Out] $[1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*\text{sqrt}(e/(a*c))*\log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*\text{sqrt}((b*e*x^2 + a*e)/(d*x^2 + c))*\text{sqrt}(e/(a*c)))/x^4 - 4*((13*b*c*d - 15*a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*\text{sqrt}((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*\text{sqrt}(-e/(a*c))*\arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*\text{sqrt}((b*e*x^2 + a*e)/(d*x^2 + c))*\text{sqrt}(-e/(a*c)))/(b*e*x^2 + a*e) - 2*((13*b*c*d - 15*a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*\text{sqrt}((b*e*x^2 + a*e)/(d*x^2 + c)))/(c^3*x^4)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x^2*d+c)]Evaluation time: 0.56Unable to divide, perhaps due to rounding error%%{%%}{2, [4, 1, 4]%%}, [2, 7, 0]%%}+%%{%%}{-8, [3, 2, 4]%%}, [2, 6, 1]%%}+%%{%%}{12, [2, 3, 4]%%}, [2, 5, 2]%%}+%%{%%}{-8, [1, 4, 4]%%}, [2, 4, 3]%%}+%%{%%}{2, [0, 5, 4]%%}, [2, 3, 4]%%}+%%{%%}{[%%]{-4, [4, 0, 4]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 8, 0]%%}+%%{%%}{[%%]{16, [3, 1, 4]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 7, 1]%%}+%%{%%}{[%%]{-24, [2, 2, 4]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 6, 2]%%}+%%{%%}{[%%]{16, [1, 3, 4]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 5, 3]%%}+%%{%%}{[%%]{-4, [0, 4, 4]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 4, 4]%%}+%%{%%}{2, [5, 0, 5]%%}, [0, 9, 0]%%}+%%{%%}{-8, [4, 1, 5]%%}, [0, 8, 1]%%}+%%{%%}{12, [3, 2, 5]%%}, [0, 7, 2]%%}+%%{%%}{-8, [2, 3, 5]%%}, [0, 6, 3]%%}+%%{%%}{2, [1, 4, 5]%%}, [0, 5, 4]%%} / %%{%%}{1, [0, 2, 0]%%}, [2, 0, 0]%%}+%%{%%}{[%%]{-2, [0, 1, 0]%%}, 0} : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 1, 0]%%}+%%{%%}{1, [1, 1, 1]%%}, [0, 2, 0]%%} Error: Bad Argument Value

maple [B] time = 0.07, size = 1042, normalized size = 4.07

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^5,x)
```

```
[Out] 1/16*(-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b*d^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^2*c*d^2-15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^3*c*d^3+18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^2*b*c^2*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^2*c^3*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*d^3-26*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b*c*d^2+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^2*c^2*d-15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*c^2*d^2+18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b*c^3*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a*b^2*c^4+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*d^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b*c*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c^2*d+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*b^2*c^3+14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*c*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*b*c^2-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*c^2)/a*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^4/c^4/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

maxima [A] time = 2.48, size = 303, normalized size = 1.18

$$-\frac{1}{16} e \left(\frac{2 \left((5b^2c^3 - 14abcd + 9a^2cd^2) \left(\frac{bx^2+a}{dx^2+c} \right)^{\frac{3}{2}} e - (3ab^2c^2 - 10a^2bcd + 7a^3d^2) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e^2 \right)}{a^2c^3e^2 - \frac{2(bx^2+a)ac^4e^2}{dx^2+c} + \frac{(bx^2+a)^2c^5e^2}{(dx^2+c)^2}} - \frac{3(b^2c^2 - 6abcd + 5a^2d^2)e \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace}c^3} + \frac{16(bcd - ad^2) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{c^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] -1/16*e*(2*((5*b^2*c^3 - 14*a*b*c^2*d + 9*a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - (3*a*b^2*c^2 - 10*a^2*b*c*d + 7*a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^2)/(a^2*c^3*e^2 - 2*(b*x^2 + a)*a*c^4*e^2/(d*x^2 + c) + (b*x^2 + a)^2*c^5*e^2/(d*x^2 + c)^2) - 3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*c^3) + 16*(b*c*d - a*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c} \right)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

```
[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

```
[Out] Timed out
```

$$3.110 \quad \int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7} dx$$

Optimal. Leaf size=366

$$\frac{e^2(-79a^2d^2 + 50abcd + 5b^2c^2)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} + \frac{e^{3/2}(-35a^2d^2 + 10abcd + b^2c^2)(bc - ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{48ac^4\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{16a^{3/2}c^{9/2}}{16a^{3/2}c^{9/2}}$$

Rubi [A] time = 0.37, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1960, 463, 455, 1157, 388, 208}

$$\frac{e^2(-79a^2d^2 + 50abcd + 5b^2c^2)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} + \frac{e^{3/2}(-35a^2d^2 + 10abcd + b^2c^2)(bc - ad)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{48ac^4\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)} + \frac{d^2e(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{e^2(bc - ad)^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{e^3(11ad + bc)(bc - ad)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4\left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Antiderivative was successfully verified.

[In] Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x]

[Out] (d^2*(b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/c^4 + ((b*c - a*d)^3*e^2*((e*(a + b*x^2))/(c + d*x^2))^(5/2))/(6*a*c^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^3) + ((b*c - a*d)^2*(b*c + 11*a*d)*e^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(24*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((b*c - a*d)*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(48*a*c^4*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^2)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/((Sqrt[a]*Sqrt[e]))]/(16*a^(3/2)*c^(9/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(2), x_Symbol] := -Simp[(b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)

$(p + 1) \text{Simp}[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1) * x^n, x], x] /;$ FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1157

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 1960

Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{x^4 (be - dx^2)^2}{(-ae + cx^2)^4} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc - ad) \text{Subst} \left(\int \frac{x^4 (-6b^2c^2e^2 + 5(bce - ade)^2 + 6acd^2ex^2)}{(-ae + cx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6ac^2}$$

$$= \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc - ad)^2 (bc + 11ad) e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc - ad) \text{Subst} \left(\int \frac{ac(bc - ad)}{(-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{48ac^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

$$= \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc - ad)^2 (bc + 11ad) e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2} - \frac{(bc - ad) (5b^2c^2 + 50abc)}{48ac^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

$$= \frac{d^2 (bc - ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc - ad)^2 (bc + 11ad) e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

$$= \frac{d^2 (bc - ad) e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{c^4} + \frac{(bc - ad)^3 e^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{5/2}}{6ac^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^3} + \frac{(bc - ad)^2 (bc + 11ad) e^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24c^4 \left(ae - \frac{ce(a+bx^2)}{c+dx^2}\right)^2}$$

Mathematica [A] time = 0.19, size = 245, normalized size = 0.67

$$e^{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(3x^6\sqrt{c+dx^2} (35a^3d^3 - 45a^2bcd^2 + 9ab^2c^2d + b^3c^3) \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}} \right) - \sqrt{a}\sqrt{c}\sqrt{a+bx^2} (a^2(8c^3 - 14c^2dx^2 + 35cd^2x^4 + 105d^3x^6) + 2abcx^2(7c^2 - 19cdx^2 - 50d^2x^4) + 3b^2c^2x^4(c+dx^2)) \right)} \frac{1}{48a^{3/2}c^{9/2}x^6\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]

[Out] (e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2])*(3*b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) + a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(48*a^(3/2)*c^(9/2)*x^6*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.49, size = 234, normalized size = 0.64

$$\frac{e^{3/2} (35a^3d^3 - 45a^2bcd^2 + 9ab^2c^2d + b^3c^3) \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}} \right)}{16a^{3/2}c^{9/2}} - \frac{e (8a^2c^3 - 14a^2c^2dx^2 + 35a^2cd^2x^4 + 105a^2d^3x^6 + 14abc^3x^2 - 38abc^2dx^4 - 100abcd^2x^6 + 3b^2c^3x^4 + 3b^2c^2dx^6) \sqrt{\frac{a+bx^2}{c+dx^2}}}{48ac^4x^6}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]

[Out] -1/48*(e*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(8*a^2*c^3 + 14*a*b*c^3*x^2 - 14*a^2*c^2*d*x^2 + 3*b^2*c^3*x^4 - 38*a*b*c^2*d*x^4 + 35*a^2*c*d^2*x^4 + 3*b^2*c^2*d*x^6 - 100*a*b*c*d^2*x^6 + 105*a^2*d^3*x^6))/(a*c^4*x^6) + ((b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2))]/(Sqrt[a]*Sqrt[e])])/(16*a^(3/2)*c^(9/2))

fricas [A] time = 20.91, size = 573, normalized size = 1.57

$$\frac{3 \left(3 \sqrt{c^2} + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3 \right) \sqrt{\frac{a+bx^2}{c+dx^2}} \operatorname{arctan} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) - \left(14 a^2 c^3 - 100 a b c^2 d x^2 + 105 a^2 c d^2 x^4 + 14 a b c^3 x^2 - 38 a b c^2 d x^4 - 100 a b c d^2 x^6 + 3 b^2 c^3 x^4 + 3 b^2 c^2 d x^6 \right) \sqrt{\frac{a+bx^2}{c+dx^2}}}{16 a^{3/2} c^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")

[Out] [1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqr t(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a* b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b* c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^ 4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e *x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*c^3 + 9 *a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*arctan(1/2 *(b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c)) / (b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes

constant sign by intervals (correct if the argument is real): Check [abs(x^2 *d+c)] Evaluation time: 0.57 Unable to divide, perhaps due to rounding error%
 %%{2, [5, 1, 5]%%}, [2, 9, 0]%%}+%%{-10, [4, 2, 5]%%}, [2, 8, 1]%%}+%%{-20, [3, 3, 5]%%}, [2, 7, 2]%%}+%%{-20, [2, 4, 5]%%}, [2, 6, 3]%%}+%%{-10, [1, 5, 5]%%}, [2, 5, 4]%%}+%%{-2, [0, 6, 5]%%}, [2, 4, 5]%%}+%%{-4, [5, 0, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 10, 0]%%}+%%{-20, [4, 1, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 9, 1]%%}+%%{-40, [3, 2, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 8, 2]%%}+%%{-40, [2, 3, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 7, 3]%%}+%%{-20, [1, 4, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 6, 4]%%}+%%{-4, [0, 5, 5]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 5, 5]%%}+%%{-2, [6, 0, 6]%%}, [0, 11, 0]%%}+%%{-10, [5, 1, 6]%%}, [0, 10, 1]%%}+%%{-20, [4, 2, 6]%%}, [0, 9, 2]%%}+%%{-20, [3, 3, 6]%%}, [0, 8, 3]%%}+%%{-10, [2, 4, 6]%%}, [0, 7, 4]%%}+%%{-2, [1, 5, 6]%%}, [0, 6, 5]%%} / %%{-1, [0, 2, 0]%%}, [2, 0, 0]%%}+%%{-2, [0, 1, 0]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}]%%}, [1, 1, 0]%%}+%%{-1, [1, 1, 1]%%}, [0, 2, 0]%%} Error: Bad Argument Value

maple [B] time = 0.10, size = 1498, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x^2+a)/(d*x^2+c)*e)^(3/2)/x^7, x)

[Out] -1/96*(-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a^2*b*d^4+72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*a*b^2*c*d^3-216*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a^2*b*c*d^3+138*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*c^2*d^2-72*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a*b*c*d^2-42*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*c^2*d^2+66*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c^3*d-96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a^2*b*c^2*d^2-60*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*b*c^2*d-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^4*c^2*d^3-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^5+174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*a^2*d^3+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^3*c^4-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c^3-105*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^4*c*d^4-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a^3*d^4+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^10*b^3*c^2*d^2+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^3*b*c^2*d^3-27*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a^2*b^2*c^3*d^2-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^8*a*b^3*c^4*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^3*c^3*d+135*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^3*b*c^3*d^2-27*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^2*b^2*c^4*d-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^6*b^2*c^2*d-174*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^3*c*d^3+96*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*x^6*a^3*c*d^3+114*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a^2*c*d^2-44*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*c^2*d+12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*b*c^3+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c^3/a^2*(d*x^2+c)*((b*x^2+a)/(d*x^2+c)*e)^(3/2)/(a*c)^(1/2)/x^6/c^5/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)

maxima [A] time = 3.04, size = 453, normalized size = 1.24

$$\frac{1}{96} \left(2 \left(3 \left(b^2 c^3 - 23 a b^2 c^2 d + 51 a^2 b c^2 d^2 - 29 a^3 c^2 d^3 \right) \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{5}{2}} + 8 \left(a b^3 c^4 + 9 a^2 b^2 c^3 d - 27 a^3 b c^2 d^2 + 17 a^4 c d^3 \right) \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{3}{2}} + 3 \left(a^2 b^3 c^3 + 9 a^3 b^2 c^2 d - 29 a^4 b c d^2 + 19 a^5 d^3 \right) \sqrt{\frac{b x^2 + a}{d x^2 + c}} + c^3 \right) + \frac{96 (b c d^2 - a d^3) \sqrt{\frac{b x^2 + a}{d x^2 + c}}}{c^4} - \frac{3 (b^2 c^3 + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3) \log \left(\frac{\sqrt{\frac{b x^2 + a}{d x^2 + c}} - \sqrt{a c}}{c \sqrt{\frac{b x^2 + a}{d x^2 + c}} + \sqrt{a c}} \right)}{\sqrt{a c} c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")

[Out] 1/96*e*(2*(3*(b^3*c^5 - 23*a*b^2*c^4*d + 51*a^2*b*c^3*d^2 - 29*a^3*c^2*d^3) * ((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e + 8*(a*b^3*c^4 + 9*a^2*b^2*c^3*d - 27*a^3*b*c^2*d^2 + 17*a^4*c*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - 3*(a^2*b^3*c^3 + 9*a^3*b^2*c^2*d - 29*a^4*b*c*d^2 + 19*a^5*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^3)/(a^4*c^4*e^3 - 3*(b*x^2 + a)*a^3*c^5*e^3/(d*x^2 + c) + 3*(b*x^2 + a)^2*a^2*c^6*e^3/(d*x^2 + c)^2 - (b*x^2 + a)^3*a*c^7*e^3/(d*x^2 + c)^3) + 96*(b*c*d^2 - a*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/c^4 - 3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*c^4)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)

[Out] int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)

[Out] Timed out

$$3.111 \quad \int x \sqrt{\frac{1-x^2}{1+x^2}} dx$$

Optimal. Leaf size=51

$$\frac{1}{2} \sqrt{\frac{1-x^2}{x^2+1}} (x^2+1) - \tan^{-1} \left(\sqrt{\frac{1-x^2}{x^2+1}} \right)$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1960, 288, 204}

$$\frac{1}{2} \sqrt{\frac{1-x^2}{x^2+1}} (x^2+1) - \tan^{-1} \left(\sqrt{\frac{1-x^2}{x^2+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]

[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m+1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned} \int x \sqrt{\frac{1-x^2}{1+x^2}} dx &= - \left(2 \text{Subst} \left(\int \frac{x^2}{(-1-x^2)^2} dx, x, \sqrt{\frac{1-x^2}{1+x^2}} \right) \right) \\ &= \frac{1}{2} \sqrt{\frac{1-x^2}{1+x^2}} (1+x^2) + \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{\frac{1-x^2}{1+x^2}} \right) \\ &= \frac{1}{2} \sqrt{\frac{1-x^2}{1+x^2}} (1+x^2) - \tan^{-1} \left(\sqrt{\frac{1-x^2}{1+x^2}} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.69

$$\frac{\sqrt{\frac{1-x^2}{x^2+1}} \sqrt{x^2+1} \left(\sqrt{x^2+1} (x^2-1) + 2\sqrt{1-x^2} \sin^{-1} \left(\frac{\sqrt{1-x^2}}{\sqrt{2}} \right) \right)}{2(x^2-1)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]

[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*Sqrt[1 + x^2]*((-1 + x^2)*Sqrt[1 + x^2] + 2*Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x^2]/Sqrt[2]]))/(2*(-1 + x^2))

IntegrateAlgebraic [A] time = 0.05, size = 51, normalized size = 1.00

$$\frac{1}{2} \sqrt{\frac{1-x^2}{x^2+1}} (x^2+1) - \tan^{-1} \left(\sqrt{\frac{1-x^2}{x^2+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[(1 - x^2)/(1 + x^2)],x]

[Out] (Sqrt[(1 - x^2)/(1 + x^2)]*(1 + x^2))/2 - ArcTan[Sqrt[(1 - x^2)/(1 + x^2)]]

fricas [A] time = 0.72, size = 55, normalized size = 1.08

$$\frac{1}{2} (x^2+1) \sqrt{\frac{x^2-1}{x^2+1}} - \arctan \left(\frac{(x^2+1) \sqrt{\frac{x^2-1}{x^2+1}} - 1}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")

[Out] 1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - 1)/x^2)

giac [A] time = 0.28, size = 18, normalized size = 0.35

$$\frac{1}{2} \sqrt{-x^4+1} + \frac{1}{2} \arcsin(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")

[Out] 1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)

maple [A] time = 0.02, size = 52, normalized size = 1.02

$$\frac{\sqrt{-\frac{x^2-1}{x^2+1}} (x^2+1) \left(\arcsin(x^2) + \sqrt{-x^4+1} \right)}{2\sqrt{-(x^2-1)(x^2+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((-x^2+1)/(x^2+1))^(1/2),x)

[Out] 1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))/(-(x^2-1)*(x^2+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{-\frac{x^2-1}{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)

mupad [B] time = 2.67, size = 55, normalized size = 1.08

$$-\operatorname{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right) - \frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)

[Out] - atan((-x^2 - 1)/(x^2 + 1))^(1/2) - (-x^2 - 1)/(x^2 + 1))^(1/2)/((x^2 - 1)/(x^2 + 1) - 1)

sympy [A] time = 21.54, size = 39, normalized size = 0.76

$$\left\{ \frac{\sqrt{1-x^2} \sqrt{x^2+1}}{2} - \operatorname{asin}\left(\frac{\sqrt{2} \sqrt{1-x^2}}{2}\right) \text{ for } x > -1 \wedge x < 1 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)

[Out] Piecewise((sqrt(1 - x**2)*sqrt(x**2 + 1)/2 - asin(sqrt(2)*sqrt(1 - x**2)/2), (x > -1) & (x < 1))

$$3.112 \quad \int x \sqrt{\frac{5-7x^2}{7+5x^2}} dx$$

Optimal. Leaf size=72

$$\frac{1}{10} \sqrt{\frac{5-7x^2}{5x^2+7}} (5x^2+7) - \frac{37 \tan^{-1} \left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^2}{5x^2+7}} \right)}{5\sqrt{35}}$$

Rubi [A] time = 0.03, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1960, 288, 204}

$$\frac{1}{10} \sqrt{\frac{5-7x^2}{5x^2+7}} (5x^2+7) - \frac{37 \tan^{-1} \left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^2}{5x^2+7}} \right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]

[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !I LtQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m+1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned} \int x \sqrt{\frac{5-7x^2}{7+5x^2}} dx &= - \left(74 \text{Subst} \left(\int \frac{x^2}{(-7-5x^2)^2} dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}} \right) \right) \\ &= \frac{1}{10} \sqrt{\frac{5-7x^2}{7+5x^2}} (7+5x^2) + \frac{37}{5} \text{Subst} \left(\int \frac{1}{-7-5x^2} dx, x, \sqrt{\frac{5-7x^2}{7+5x^2}} \right) \\ &= \frac{1}{10} \sqrt{\frac{5-7x^2}{7+5x^2}} (7+5x^2) - \frac{37 \tan^{-1} \left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^2}{7+5x^2}} \right)}{5\sqrt{35}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 104, normalized size = 1.44

$$\frac{\sqrt{\frac{5-7x^2}{5x^2+7}} \sqrt{5x^2+7} \left(35\sqrt{5x^2+7} (7x^2-5) - 74\sqrt{35} \sqrt{7x^2-5} \sinh^{-1} \left(\sqrt{\frac{5}{74}} \sqrt{7x^2-5} \right) \right)}{350(7x^2-5)}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]

[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*Sqrt[7 + 5*x^2]*(35*Sqrt[7 + 5*x^2]*(-5 + 7*x^2) - 74*Sqrt[35]*Sqrt[-5 + 7*x^2]*ArcSinh[Sqrt[5/74]*Sqrt[-5 + 7*x^2]]))/(350*(-5 + 7*x^2))

IntegrateAlgebraic [A] time = 0.07, size = 72, normalized size = 1.00

$$\frac{1}{10} \sqrt{\frac{5-7x^2}{5x^2+7}} (5x^2+7) - \frac{37 \tan^{-1} \left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^2}{5x^2+7}} \right)}{5\sqrt{35}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]

[Out] (Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(7 + 5*x^2))/10 - (37*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]])/(5*Sqrt[35])

fricas [A] time = 0.66, size = 77, normalized size = 1.07

$$\frac{1}{10} (5x^2+7) \sqrt{-\frac{7x^2-5}{5x^2+7}} - \frac{37}{350} \sqrt{35} \arctan \left(\frac{\sqrt{35} (35x^2+12) \sqrt{-\frac{7x^2-5}{5x^2+7}}}{35(7x^2-5)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")

[Out] 1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^2 + 12)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(7*x^2 - 5))

giac [A] time = 0.35, size = 30, normalized size = 0.42

$$\frac{37}{350} \sqrt{35} \arcsin \left(\frac{35}{37} x^2 + \frac{12}{37} \right) + \frac{1}{10} \sqrt{-35x^4 - 24x^2 + 35}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")

[Out] 37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 35)

maple [A] time = 0.03, size = 78, normalized size = 1.08

$$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}} (5x^2+7) \left(37\sqrt{35} \arcsin \left(\frac{35x^2}{37} + \frac{12}{37} \right) + 35\sqrt{-35x^4 - 24x^2 + 35} \right)}{350\sqrt{-(7x^2-5)}(5x^2+7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)

[Out] $\frac{1}{350} * (-7x^2 - 5) / (5x^2 + 7)^{(1/2)} * (5x^2 + 7) * (37 * 35^{(1/2)} * \arcsin(35 / (37x^2 + 12/37)) + 35 * (-35x^4 - 24x^2 + 35)^{(1/2)}) / (-7x^2 - 5) * (5x^2 + 7)^{(1/2)}$

maxima [A] time = 2.05, size = 76, normalized size = 1.06

$$-\frac{37}{175} \sqrt{35} \arctan\left(\frac{1}{7} \sqrt{35} \sqrt{\frac{7x^2 - 5}{5x^2 + 7}}\right) - \frac{37 \sqrt{\frac{7x^2 - 5}{5x^2 + 7}}}{5 \left(\frac{5(7x^2 - 5)}{5x^2 + 7} - 7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")`

[Out] $-37/175 * \sqrt{35} * \arctan(1/7 * \sqrt{35} * \sqrt{-(7x^2 - 5)/(5x^2 + 7)}) - 37/5 * \sqrt{-(7x^2 - 5)/(5x^2 + 7)} / (5 * (7x^2 - 5)/(5x^2 + 7) - 7)$

mupad [B] time = 0.21, size = 88, normalized size = 1.22

$$\frac{37 \sqrt{35} \operatorname{atan}\left(\frac{\sqrt{5} \sqrt{7} \sqrt{\frac{7x^2 - 5}{5x^2 + 7}}}{7}\right)}{175} - \frac{37 \sqrt{5} \sqrt{7} \sqrt{35} \sqrt{\frac{7x^2 - 5}{5x^2 + 7}}}{1225 \left(\frac{5x^2 - \frac{25}{7}}{5x^2 + 7} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)`

[Out] $-(37 * 35^{(1/2)} * \operatorname{atan}((5^{(1/2)} * 7^{(1/2)} * (-7x^2 - 5) / (5x^2 + 7))^{(1/2)} / 7)) / 175 - (37 * 5^{(1/2)} * 7^{(1/2)} * 35^{(1/2)} * (-7x^2 - 5) / (5x^2 + 7))^{(1/2)} / (1225 * ((5x^2 - 25/7) / (5x^2 + 7) - 1))$

sympy [A] time = 66.86, size = 66, normalized size = 0.92

$$\left\{ \frac{5 \sqrt{35} \left(\frac{\sqrt{25 - 35x^2} \sqrt{35x^2 + 49}}{125} - \frac{74 \operatorname{asin}\left(\frac{\sqrt{74} \sqrt{25 - 35x^2}}{74}\right)}{125} \right)}{14} \right. \quad \left. \text{for } x > -\frac{\sqrt{35}}{7} \wedge x < \frac{\sqrt{35}}{7} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)`

[Out] `Piecewise((5*sqrt(35)*(sqrt(25 - 35*x**2)*sqrt(35*x**2 + 49)/125 - 74*asin(sqrt(74)*sqrt(25 - 35*x**2)/74)/125)/14, (x > -sqrt(35)/7) & (x < sqrt(35)/7))`

$$3.113 \quad \int x^2 \sqrt{\frac{1-x^3}{1+x^3}} dx$$

Optimal. Leaf size=53

$$\frac{1}{3} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1) - \frac{2}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{x^3+1}} \right)$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1960, 288, 204}

$$\frac{1}{3} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1) - \frac{2}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]

[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3)]])/3

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^n*(m-n+1))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[m+n*(p+1)+1/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-a*e) + c*x^q)^(Simplify[(m+1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m+1)/n]]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{\frac{1-x^3}{1+x^3}} dx &= - \left(\frac{4}{3} \text{Subst} \left(\int \frac{x^2}{(-1-x^2)^2} dx, x, \sqrt{\frac{1-x^3}{1+x^3}} \right) \right) \\ &= \frac{1}{3} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3) + \frac{2}{3} \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{\frac{1-x^3}{1+x^3}} \right) \\ &= \frac{1}{3} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3) - \frac{2}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{1+x^3}} \right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 86, normalized size = 1.62

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}} \sqrt{x^3+1} \left(\sqrt{x^3+1} (x^3-1) + 2\sqrt{1-x^3} \sin^{-1} \left(\frac{\sqrt{1-x^3}}{\sqrt{2}} \right) \right)}{3(x^3-1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]

[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*((-1 + x^3)*Sqrt[1 + x^3] + 2*Sqrt[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(3*(-1 + x^3))

IntegrateAlgebraic [A] time = 0.05, size = 53, normalized size = 1.00

$$\frac{1}{3} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1) - \frac{2}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]

[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/3 - (2*ArcTan[Sqrt[(1 - x^3)/(1 + x^3)]])/3

fricas [A] time = 0.75, size = 55, normalized size = 1.04

$$\frac{1}{3} (x^3+1) \sqrt{-\frac{x^3-1}{x^3+1}} - \frac{2}{3} \arctan \left(\frac{(x^3+1) \sqrt{-\frac{x^3-1}{x^3+1}} - 1}{x^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")

[Out] 1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)

giac [A] time = 0.29, size = 22, normalized size = 0.42

$$\frac{1}{3} \left(\sqrt{-x^6+1} + \arcsin(x^3) \right) \operatorname{sgn}(x^3+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")

[Out] 1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)

maple [A] time = 0.10, size = 68, normalized size = 1.28

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}} \sqrt{-(x^3+1)(x^3-1)} \arcsin(x^3)}{3(x^3-1)} + \frac{(x^3+1) \sqrt{-\frac{x^3-1}{x^3+1}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)

[Out] 1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{-\frac{x^3-1}{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")

[Out] integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)

mupad [B] time = 2.67, size = 56, normalized size = 1.06

$$-\frac{2 \operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3} - \frac{2 \sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3(x^3-1)}{x^3+1} - 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)

[Out] - (2*atan((-x^3 - 1)/(x^3 + 1))^(1/2))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/((3*(x^3 - 1))/(x^3 + 1) - 3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)

[Out] Timed out

$$3.114 \quad \int x^8 \sqrt{\frac{1-x^3}{1+x^3}} dx$$

Optimal. Leaf size=113

$$-\frac{1}{9} \left(\frac{1-x^3}{x^3+1} \right)^{3/2} (x^3+1)^3 - \frac{1}{6} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1)^2 + \frac{1}{2} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1) - \frac{1}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{x^3+1}} \right)$$

Rubi [A] time = 0.06, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1960, 463, 455, 385, 204}

$$-\frac{1}{9} \left(\frac{1-x^3}{x^3+1} \right)^{3/2} (x^3+1)^3 - \frac{1}{6} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1)^2 + \frac{1}{2} \sqrt{\frac{1-x^3}{x^3+1}} (x^3+1) - \frac{1}{3} \tan^{-1} \left(\sqrt{\frac{1-x^3}{x^3+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]

[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3))/2 - (Sqrt[(1 - x^3)/(1 + x^3)]*(1 + x^3)^2)/6 - (((1 - x^3)/(1 + x^3))^(3/2)*(1 + x^3)^3)/9 - ArcTan[Sqrt[(1 - x^3)/(1 + x^3)]]/3

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 463

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,

```
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int x^8 \sqrt{\frac{1-x^3}{1+x^3}} dx &= -\left(\frac{4}{3} \text{Subst}\left(\int \frac{x^2(-1+x^2)^2}{(-1-x^2)^4} dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right)\right) \\
 &= -\frac{1}{9} \left(\frac{1-x^3}{1+x^3}\right)^{3/2} (1+x^3)^3 - \frac{2}{9} \text{Subst}\left(\int \frac{x^2(6-6x^2)}{(-1-x^2)^3} dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right) \\
 &= -\frac{1}{6} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3)^2 - \frac{1}{9} \left(\frac{1-x^3}{1+x^3}\right)^{3/2} (1+x^3)^3 + \frac{1}{18} \text{Subst}\left(\int \frac{12-24x^2}{(-1-x^2)^2} dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right) \\
 &= \frac{1}{2} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3) - \frac{1}{6} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3)^2 - \frac{1}{9} \left(\frac{1-x^3}{1+x^3}\right)^{3/2} (1+x^3)^3 + \frac{1}{3} \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{\frac{1-x^3}{1+x^3}}\right) \\
 &= \frac{1}{2} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3) - \frac{1}{6} \sqrt{\frac{1-x^3}{1+x^3}} (1+x^3)^2 - \frac{1}{9} \left(\frac{1-x^3}{1+x^3}\right)^{3/2} (1+x^3)^3 - \frac{1}{3} \tan^{-1}\left(\sqrt{\frac{1-x^3}{1+x^3}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 98, normalized size = 0.87

$$\frac{\sqrt{\frac{1-x^3}{x^3+1}} \sqrt{x^3+1} \left(6\sqrt{1-x^3} \sin^{-1}\left(\frac{\sqrt{1-x^3}}{\sqrt{2}}\right) + \sqrt{x^3+1} (2x^9 - 5x^6 + 7x^3 - 4)\right)}{18(x^3-1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)], x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*Sqrt[1 + x^3]*(Sqrt[1 + x^3]*(-4 + 7*x^3 - 5*x^6
+ 2*x^9) + 6*Sqrt[1 - x^3]*ArcSin[Sqrt[1 - x^3]/Sqrt[2]]))/(18*(-1 + x^3))
```

IntegrateAlgebraic [A] time = 0.07, size = 63, normalized size = 0.56

$$\frac{1}{18} \sqrt{\frac{1-x^3}{x^3+1}} (2x^9 - x^6 + x^3 + 4) - \frac{1}{3} \tan^{-1}\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^8*Sqrt[(1 - x^3)/(1 + x^3)], x]
```

```
[Out] (Sqrt[(1 - x^3)/(1 + x^3)]*(4 + x^3 - x^6 + 2*x^9))/18 - ArcTan[Sqrt[(1 - x
^3)/(1 + x^3)]]/3
```

fricas [A] time = 0.78, size = 65, normalized size = 0.58

$$\frac{1}{18} (2x^9 - x^6 + x^3 + 4) \sqrt{\frac{x^3-1}{x^3+1}} - \frac{1}{3} \arctan\left(\frac{(x^3+1)\sqrt{\frac{x^3-1}{x^3+1}} - 1}{x^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*((-x^3+1)/(x^3+1))^(1/2), x, algorithm="fricas")
```

[Out] $1/18*(2*x^9 - x^6 + x^3 + 4)*\sqrt{-(x^3 - 1)/(x^3 + 1)} - 1/3*\arctan(((x^3 + 1)*\sqrt{-(x^3 - 1)/(x^3 + 1)} - 1)/x^3)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sqrt{-\frac{x^3 - 1}{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")`

[Out] `integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)`

maple [A] time = 0.07, size = 80, normalized size = 0.71

$$-\frac{\sqrt{-\frac{x^3-1}{x^3+1}} \sqrt{-(x^3+1)(x^3-1)} \arcsin(x^3)}{6(x^3-1)} + \frac{(2x^6-3x^3+4)(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)`

[Out] $1/18*(2*x^6-3*x^3+4)*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*(-(x^3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)*\arcsin(x^3)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^8 \sqrt{-\frac{x^3 - 1}{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")`

[Out] `integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)`

mupad [B] time = 2.66, size = 101, normalized size = 0.89

$$\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9} - \frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3} + \frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18} - \frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18} + \frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)`

[Out] $(2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - \operatorname{atan}((- (x^3 - 1)/(x^3 + 1))^(1/2))/3 + (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)`

[Out] Timed out

$$3.115 \quad \int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}} dx$$

Optimal. Leaf size=106

$$\frac{1}{250} \sqrt{\frac{5-7x^5}{5x^5+7}} (5x^5+7)^2 - \frac{27}{350} \sqrt{\frac{5-7x^5}{5x^5+7}} (5x^5+7) + \frac{2257 \tan^{-1}\left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Rubi [A] time = 0.06, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1960, 455, 385, 204}

$$\frac{1}{250} \sqrt{\frac{5-7x^5}{5x^5+7}} (5x^5+7)^2 - \frac{27}{350} \sqrt{\frac{5-7x^5}{5x^5+7}} (5x^5+7) + \frac{2257 \tan^{-1}\left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}}$$

Antiderivative was successfully verified.

[In] Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]

[Out] (-27*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5))/350 + (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(7 + 5*x^5)^2)/250 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]])/(875*Sqrt[35])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 455

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)]/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int x^9 \sqrt{\frac{5-7x^5}{7+5x^5}} dx &= -\left(\frac{148}{5} \operatorname{Subst}\left(\int \frac{x^2(-5+7x^2)}{(-7-5x^2)^3} dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right)\right) \\
&= \frac{1}{250} \sqrt{\frac{5-7x^5}{7+5x^5}} (7+5x^5)^2 + \frac{37}{125} \operatorname{Subst}\left(\int \frac{-74+140x^2}{(-7-5x^2)^2} dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right) \\
&= -\frac{27}{350} \sqrt{\frac{5-7x^5}{7+5x^5}} (7+5x^5) + \frac{1}{250} \sqrt{\frac{5-7x^5}{7+5x^5}} (7+5x^5)^2 - \frac{2257}{875} \operatorname{Subst}\left(\int \frac{1}{-7-5x^2} dx, x, \sqrt{\frac{5-7x^5}{7+5x^5}}\right) \\
&= -\frac{27}{350} \sqrt{\frac{5-7x^5}{7+5x^5}} (7+5x^5) + \frac{1}{250} \sqrt{\frac{5-7x^5}{7+5x^5}} (7+5x^5)^2 + \frac{2257 \tan^{-1}\left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^5}{7+5x^5}}\right)}{875\sqrt{35}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 109, normalized size = 1.03

$$\frac{\sqrt{\frac{5-7x^5}{5x^5+7}} \sqrt{5x^5+7} \left(4514\sqrt{35} \sqrt{7x^5-5} \sinh^{-1}\left(\sqrt{\frac{5}{74}} \sqrt{7x^5-5}\right) + 35\sqrt{5x^5+7} (245x^{10}-777x^5+430)\right)}{61250(7x^5-5)}$$

Antiderivative was successfully verified.

[In] Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)], x]

[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*Sqrt[7 + 5*x^5]*(35*Sqrt[7 + 5*x^5]*(430 - 77*x^5 + 245*x^10) + 4514*Sqrt[35]*Sqrt[-5 + 7*x^5]*ArcSinh[Sqrt[5/74]*Sqrt[-5 + 7*x^5]]))/(61250*(-5 + 7*x^5))

IntegrateAlgebraic [A] time = 0.09, size = 77, normalized size = 0.73

$$\frac{2257 \tan^{-1}\left(\sqrt{\frac{5}{7}} \sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{875\sqrt{35}} + \frac{\sqrt{\frac{5-7x^5}{5x^5+7}} (175x^{10} - 185x^5 - 602)}{1750}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)], x]

[Out] (Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(-602 - 185*x^5 + 175*x^10))/1750 + (2257*ArcTan[Sqrt[5/7]*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]])/(875*Sqrt[35])

fricas [A] time = 0.81, size = 82, normalized size = 0.77

$$\frac{1}{1750} (175x^{10} - 185x^5 - 602) \sqrt{-\frac{7x^5-5}{5x^5+7}} + \frac{2257}{61250} \sqrt{35} \arctan\left(\frac{\sqrt{35}(35x^5+12)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{35(7x^5-5)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2), x, algorithm="fricas")

[Out] 1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/61250*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)))/(7*x^5 - 5))

giac [A] time = 0.37, size = 47, normalized size = 0.44

$$\frac{1}{61250} \left(35 \sqrt{-35x^{10} - 24x^5 + 35(35x^5 - 86)} - 2257 \sqrt{35} \arcsin\left(\frac{35}{37} x^5 + \frac{12}{37}\right)\right) \operatorname{sgn}(5x^5 + 7)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")

[Out] 1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arcsin(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)

maple [C] time = 0.28, size = 130, normalized size = 1.23

$$\frac{2257\sqrt{\frac{7x^5-5}{5x^5+7}}\sqrt{-(5x^5+7)(7x^5-5)}\operatorname{RootOf}(_Z^2+35)\ln(35x^5\operatorname{RootOf}(_Z^2+35)+12\operatorname{RootOf}(_Z^2+35)+35\sqrt{-35x^{10}-24x^5+35})}{61250(7x^5-5)} + \frac{(35x^5-86)(5x^5+7)\sqrt{\frac{7x^5-5}{5x^5+7}}}{1750}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)

[Out] 1/1750*(35*x^5-86)*(5*x^5+7)*((-7*x^5-5)/(5*x^5+7))^(1/2)-2257/61250*RootOf(_Z^2+35)*ln(35*RootOf(_Z^2+35)*x^5+12*RootOf(_Z^2+35)+35*(-35*x^10-24*x^5+35)^(1/2))*((-7*x^5-5)/(5*x^5+7))^(1/2)*(-(5*x^5+7)*(7*x^5-5))^(1/2)/(7*x^5-5)

maxima [A] time = 1.99, size = 121, normalized size = 1.14

$$\frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{\frac{7x^5-5}{5x^5+7}}\right) - \frac{37\left(675\left(-\frac{7x^5-5}{5x^5+7}\right)^{\frac{3}{2}} + 427\sqrt{\frac{7x^5-5}{5x^5+7}}\right)}{875\left(\frac{25(7x^5-5)^2}{(5x^5+7)^2} - \frac{70(7x^5-5)}{5x^5+7} + 49\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")

[Out] 2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt((-7*x^5 - 5)/(5*x^5 + 7))) - 37/875*(675*(-7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt((-7*x^5 - 5)/(5*x^5 + 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49)

mupad [B] time = 2.99, size = 134, normalized size = 1.26

$$\frac{2257\sqrt{35}\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{\frac{7x^5-5}{5x^5+7}}}{7}\right)}{30625} - \frac{43\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{\frac{7x^5-5}{5x^5+7}}}{4375} - \frac{37\sqrt{5}\sqrt{7}\sqrt{35}x^5\sqrt{\frac{7x^5-5}{5x^5+7}}}{12250} + \frac{\sqrt{5}\sqrt{7}\sqrt{35}x^{10}\sqrt{\frac{7x^5-5}{5x^5+7}}}{350}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)

[Out] (2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))/30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/4375 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12250 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)

[Out] Timed out

$$3.116 \quad \int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx$$

Optimal. Leaf size=52

$$\frac{\sqrt{-\frac{x^2}{1-x^2}} \sqrt{x^2-1} \tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6719, 444, 63, 203}

$$\frac{\sqrt{-\frac{x^2}{1-x^2}} \sqrt{x^2-1} \tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]

[Out] (Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[2]*x)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 6719

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx &= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}} \sqrt{-1+x^2}\right) \int \frac{x}{\sqrt{-1+x^2}(1+x^2)} dx}{x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}} \sqrt{-1+x^2}\right) \text{Subst}\left(\int \frac{1}{\sqrt{-1+x}(1+x)} dx, x, x^2\right)}{2x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+x^2}} \sqrt{-1+x^2}\right) \text{Subst}\left(\int \frac{1}{2+x^2} dx, x, \sqrt{-1+x^2}\right)}{x} \\
&= \frac{\sqrt{-\frac{x^2}{1-x^2}} \sqrt{-1+x^2} \tan^{-1}\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.94

$$\frac{\sqrt{\frac{x^2}{x^2-1}} \sqrt{x^2-1} \tan^{-1}\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2), x]

[Out] (Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt[2]*x)

IntegrateAlgebraic [A] time = 0.18, size = 29, normalized size = 0.56

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\frac{x^2}{x^2-1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^2/(-1 + x^2)]/(1 + x^2), x]

[Out] ArcTan[x/(Sqrt[2]*Sqrt[x^2/(-1 + x^2)])]/Sqrt[2]

fricas [A] time = 0.55, size = 32, normalized size = 0.62

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1)))/x

giac [C] time = 0.42, size = 40, normalized size = 0.77

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{x^2-1}\right) \text{sgn}(x^2-1) \text{sgn}(x) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} i \sqrt{2}\right) \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sqrt(2)*arctan(1/2*I*sqrt(2))*sgn(x)

maple [A] time = 0.02, size = 42, normalized size = 0.81

$$\frac{\sqrt{\frac{x^2}{x^2-1}} \sqrt{x^2-1} \sqrt{2} \arctan\left(\frac{\sqrt{x^2-1} \sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2/(x^2-1))^(1/2)/(x^2+1),x)

[Out] 1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)

[Out] int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)

[Out] Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)

$$3.117 \quad \int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{-\frac{x^2}{(a+1)x^2-a+1}} \sqrt{(a+1)x^2+a-1} \tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Rubi [A] time = 0.19, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6719, 444, 63, 205}

$$\frac{\sqrt{-\frac{x^2}{(a+1)x^2-a+1}} \sqrt{(a+1)x^2+a-1} \tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2), x]

[Out] (Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt[-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 6719

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2} dx &= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}} \sqrt{-1+a+(1+a)x^2}\right) \int \frac{x}{(1+x^2)\sqrt{-1+a+(1+a)x^2}} dx}{x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}} \sqrt{-1+a+(1+a)x^2}\right) \text{Subst}\left(\int \frac{1}{(1+x)\sqrt{-1+a+(1+a)x}} dx, x, x^2\right)}{2x} \\
&= \frac{\left(\sqrt{\frac{x^2}{-1+a+(1+a)x^2}} \sqrt{-1+a+(1+a)x^2}\right) \text{Subst}\left(\int \frac{1}{1-\frac{-1+a}{1+a}+\frac{x^2}{1+a}} dx, x, \sqrt{-1+a+(1+a)x^2}\right)}{(1+a)x} \\
&= \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}} \sqrt{-1+a+(1+a)x^2} \tan^{-1}\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 65, normalized size = 0.96

$$\frac{\sqrt{ax^2+a+x^2-1} \sqrt{\frac{x^2}{(a+1)x^2+a-1}} \tan^{-1}\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2/(-1+a+(1+a)*x^2)]/(1+x^2),x]

[Out] (Sqrt[-1+a+x^2+a*x^2]*Sqrt[x^2/(-1+a+(1+a)*x^2)]*ArcTan[Sqrt[-1+a+(1+a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)

IntegrateAlgebraic [A] time = 0.34, size = 35, normalized size = 0.51

$$\frac{\tan^{-1}\left(\frac{x}{\sqrt{2}\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^2/(-1+a+(1+a)*x^2)]/(1+x^2),x]

[Out] ArcTan[x/(Sqrt[2]*Sqrt[x^2/(-1+a+(1+a)*x^2+a*x^2)])]/Sqrt[2]

fricas [A] time = 0.64, size = 42, normalized size = 0.62

$$\frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{2}((a+1)x^2+a-3)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a+1)*x^2+a-3)*sqrt(x^2/((a+1)*x^2+a-1)))/x

giac [A] time = 0.45, size = 61, normalized size = 0.90

$$\frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{ax^2+x^2+a-1}\right) \operatorname{sgn}(ax^2+x^2+a-1) \operatorname{sgn}(x) - \frac{1}{2} \sqrt{2} \arctan\left(\frac{1}{2} \sqrt{2} \sqrt{a-1}\right) \operatorname{sgn}(a-1) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2 + a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*sgn(x)

maple [A] time = 0.04, size = 60, normalized size = 0.88

$$\frac{\sqrt{\frac{x^2}{ax^2+x^2+a-1}} \sqrt{ax^2+x^2+a-1} \sqrt{2} \arctan\left(\frac{\sqrt{ax^2+x^2+a-1} \sqrt{2}}{2}\right)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)

[Out] 1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2*(a*x^2+x^2+a-1)^(1/2)*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)

[Out] int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)

[Out] Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)

$$3.118 \quad \int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=281

$$\frac{(bc - ad)(5a^2d^2 + 2abcd + b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}} + \frac{(c + dx^2)(5a^2d^2 + 2abcd + b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e} - \frac{(c + dx^2)^2}{16b^3d^2e}$$

Rubi [A] time = 0.29, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 413, 385, 199, 208}

$$\frac{(c + dx^2)(5a^2d^2 + 2abcd + b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{16b^3d^2e} + \frac{(bc - ad)(5a^2d^2 + 2abcd + b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}} - \frac{(c + dx^2)^2(5ad + 3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{24b^2d^2e} - \frac{(c + dx^2)^3\left(a - \frac{c(a+bx^2)}{c+dx^2}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bde(bc - ad)}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(16*b^3*d^2*e) - ((3*b*c + 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b^2*d^2*e) - (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3*(a - (c*(a + b*x^2))/(c + d*x^2)))/(6*b*d*(b*c - a*d)*e) + ((b*c - a*d)*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(16*b^(7/2)*d^(5/2)*Sqrt[e])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 413

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p + 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p + q) + 1))*x^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.))
)^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx = ((bc - ad)e) \operatorname{Subst} \left(\int \frac{(-ae + cx^2)^2}{(be - dx^2)^4} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)}{6bd(bc - ad)e} - \frac{(bc - ad) \operatorname{Subst} \left(\int \frac{-a(bc+5ad)e^2+3c(bc+ad)ex^2}{(be-dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{6bd}$$

$$= -\frac{(3bc + 5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)}{6bd(bc - ad)e} + \frac{((bc - ad)(b^2c^2 + 2abcd + 5a^2d^2))\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{16b^3d^2e}$$

$$= \frac{(b^2c^2 + 2abcd + 5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{16b^3d^2e} - \frac{(3bc + 5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)}{6bd(bc - ad)e}$$

$$= \frac{(b^2c^2 + 2abcd + 5a^2d^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{16b^3d^2e} - \frac{(3bc + 5ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^2d^2e} - \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3 \left(a - \frac{c(a+bx^2)}{c+dx^2} \right)}{6bd(bc - ad)e}$$

Mathematica [A] time = 0.39, size = 224, normalized size = 0.80

$$\frac{\sqrt{a + bx^2} \left(3\sqrt{bc - ad} (5a^2d^2 + 2abcd + b^2c^2) \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) + \sqrt{d} \sqrt{a + bx^2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} (15a^2d^2 - 2abd(2c + 5dx^2) + b^2(-3c^2 + 2cdx^2 + 8d^2x^4)) \right)}{48b^3d^{5/2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])

IntegrateAlgebraic [A] time = 0.43, size = 226, normalized size = 0.80

$$\frac{(15a^2cd^2 + 15a^2d^3x^2 - 4abcd^2 - 14abcd^2x^2 - 10abd^3x^4 - 3b^2c^3 - b^2c^2dx^2 + 10b^2cd^2x^4 + 8b^2d^3x^6) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{48b^3d^2e} + \frac{(-5a^3d^3 + 3a^2bcd^2 + ab^2c^2d + b^3c^3) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-3*b^2*c^3 - 4*a*b*c^2*d + 15*a^2*c*d^2 - b^2*c^2*d*x^2 - 14*a*b*c*d^2*x^2 + 15*a^2*d^3*x^2 + 10*b^2*c*d^2*x^4 - 10*a*b*d^3*x^4 + 8*b^2*d^3*x^6))/(48*b^3*d^2*e) + ((b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/Sqrt[b]*Sqrt[e])/(16*b^(7/2)*d^(5/2)*Sqrt[e])
```

fricas [A] time = 0.64, size = 545, normalized size = 1.94

$$\frac{3 \sqrt{d} \sqrt{a e + b e x^2} \sqrt{c + d x^2} \sqrt{48 b^3 d^2 e} \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a e + b e x^2}}{\sqrt{b} \sqrt{e}}\right) + (b^3 c^3 + a b^2 c^2 d + 3 a^2 b c d^2 - 5 a^3 d^3) \sqrt{d} \sqrt{a e + b e x^2} \sqrt{c + d x^2}}{16 b^{7/2} d^{5/2} \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^4*d^3*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{1, [0,1,0]
%%}, [2,0]%%}+%%{%%}{-2,0]: [1,0,%%{-1, [1,1,1]%%}]}%%}, [1,1]%%}+%%{%%}{
1, [1,0,1]%%}, [0,2]%%} / %%{%%}{1, [0,2,0]%%}, [2,0]%%}+%%{%%}{[%%{-2, [0
,1,0]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]}%%}, [1,1]%%}+%%{%%}{1, [1,1,1]%%}, [0
,2]%%} Error: Bad Argument Value
```

maple [B] time = 0.05, size = 527, normalized size = 1.88

$$\frac{(b^2 + a) \sqrt{48 b^3 d^2 e} \operatorname{arctanh}\left(\frac{\sqrt{d} \sqrt{a e + b e x^2}}{\sqrt{b} \sqrt{e}}\right) + (b^3 c^3 + a b^2 c^2 d + 3 a^2 b c d^2 - 5 a^3 d^3) \sqrt{d} \sqrt{a e + b e x^2} \sqrt{c + d x^2}}{16 b^{7/2} d^{5/2} \sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/96*(b*x^2+a)/b^3*(-36*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b
*d^2*x^2-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b^2*c*d*x^2-15*
a^3*d^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*
d)^(1/2))/(b*d)^(1/2))+9*a^2*b*c*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a*b^2*c^2*d*ln(1/2*(2
*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(
1/2))+3*b^3*c^3*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(b*d)^(1/2))/(b*d)^(1/2))+16*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(b*d)
^(1/2)*b*d+30*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a^2*d^2-24*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*b*c*d-6*(b*d*x^4+a*d*x^2+b
```

$$c*x^2+a*c)^{(1/2)}*(b*d)^{(1/2)}*b^2*c^2)/((b*x^2+a)/(d*x^2+c)*e)^{(1/2)}/((d*x^2+c)*(b*x^2+a))^{(1/2)}/d^2/(b*d)^{(1/2)}$$

maxima [A] time = 2.27, size = 413, normalized size = 1.47

$$\frac{1}{96} e^{\left(\frac{2 \left(3 \left(b^3 c^3 d^2 + a b^2 c^2 d^3 + 3 a^2 b c d^4 - 5 a^3 d^5 \right) \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{3}{2}} + 8 \left(b^4 c^3 d - 3 a b^3 c^2 d^2 - 3 a^2 b^2 c d^3 + 5 a^3 b d^4 \right) \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{3}{2}} e - 3 \left(b^3 c^3 + a b^2 c^2 d - 13 a^2 b^2 c d^2 + 11 a^3 b^2 d^3 \right) \sqrt{\frac{b x^2 + a}{d x^2 + c}} e^2 \right)}{3 \left(b^3 c^3 + a b^2 c^2 d + 3 a^2 b c d^2 - 5 a^3 d^3 \right) \log \left(\frac{d \sqrt{\frac{b x^2 + a}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{b x^2 + a}{d x^2 + c}} + \sqrt{b d e}} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/96*e*(2*(3*(b^3*c^3*d^2 + a*b^2*c^2*d^3 + 3*a^2*b*c*d^4 - 5*a^3*d^5)*(b*x^2 + a)*e/(d*x^2 + c))^(5/2) + 8*(b^4*c^3*d - 3*a*b^3*c^2*d^2 - 3*a^2*b^2*c*d^3 + 5*a^3*b*d^4)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e - 3*(b^5*c^3 + a*b^4*c^2*d - 13*a^2*b^3*c*d^2 + 11*a^3*b^2*d^3)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^2)/(b^6*d^2*e^4 - 3*(b*x^2 + a)*b^5*d^3*e^4/(d*x^2 + c) + 3*(b*x^2 + a)^2*b^4*d^4*e^4/(d*x^2 + c)^2 - (b*x^2 + a)^3*b^3*d^5*e^4/(d*x^2 + c)^3) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/sqrt(b*d*e))

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)

[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

$$3.119 \quad \int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=169

$$\frac{(bc-ad)(3ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}} - \frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de} + \frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Rubi [A] time = 0.13, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 385, 199, 208}

$$\frac{(bc-ad)(3ad+bc) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}} - \frac{(c+dx^2)(3ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8b^2de} + \frac{(c+dx^2)^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bde}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] -((b*c + 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^2*d*e) + (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b*d*e) - ((b*c - a*d)*(b*c + 3*a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{-ae + cx^2}{(be - dx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4bde} - \frac{((bc - ad)(bc + 3ad)e) \operatorname{Subst} \left(\int \frac{1}{(be - dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4bd} \\
&= -\frac{(bc + 3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4bde} - \frac{((bc - ad)(bc + 3ad)) \operatorname{Subst} \left(\int \frac{\sqrt{d}}{(be - dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{8b^2d} \\
&= -\frac{(bc + 3ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8b^2de} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4bde} - \frac{(bc - ad)(bc + 3ad) \tanh^{-1} \left(\frac{\sqrt{d}}{\sqrt{bc - ad}} \right)}{8b^{5/2}d^{3/2}\sqrt{e}}
\end{aligned}$$

Mathematica [A] time = 0.34, size = 172, normalized size = 1.02

$$\frac{\sqrt{d} (a + bx^2) \sqrt{\frac{b(c+dx^2)}{bc-ad}} (b(c + 2dx^2) - 3ad) - \sqrt{a + bx^2} \sqrt{bc - ad} (3ad + bc) \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{8b^2d^{3/2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

[Out] (Sqrt[d]*(a + b*x^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-3*a*d + b*(c + 2*d*x^2)) - Sqrt[b*c - a*d]*(b*c + 3*a*d)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^2*d^(3/2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])

IntegrateAlgebraic [A] time = 0.27, size = 158, normalized size = 0.93

$$\frac{(3a^2d^2 - 2abcd - b^2c^2) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{8b^{5/2}d^{3/2}\sqrt{e}} + \frac{(-3acd - 3ad^2x^2 + bc^2 + 3bcdx^2 + 2bd^2x^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8b^2de}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)], x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(b*c^2 - 3*a*c*d + 3*b*c*d*x^2 - 3*a*d^2*x^2 + 2*b*d^2*x^4))/(8*b^2*d*e) + (((-b^2*c^2) - 2*a*b*c*d + 3*a^2*d^2)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^(5/2)*d^(3/2)*Sqrt[e])

fricas [A] time = 0.78, size = 413, normalized size = 2.44

$$\frac{(b^2d^2 + 2abcd - 3a^2d^2)\sqrt{de} \log\left(8b^2d^2cx^4 + 8(b^2cd + abd^2)c^2 + (b^2d^2 + 6abcd + a^2d^2)c + 4(2bd^2c^2 + bc^2 + acd + 3bcd + ad^2)c^2\sqrt{de} \sqrt{\frac{ae+bx^2}{c+dx^2}}\right) - 4(2b^2d^2c^2 + b^2c^2d - 3abcd + 3(b^2cd^2 - abd^2)c^2)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{32b^2d^2e} + \frac{(b^2d^2 + 2abcd - 3a^2d^2)\sqrt{de} \operatorname{arctanh}\left(\frac{(b^2d^2 + 2abcd - 3a^2d^2)\sqrt{de} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2(b^2cd + abd^2)c^2}\right) + 2(2b^2d^2c^2 + b^2c^2d - 3abcd + 3(b^2cd^2 - abd^2)c^2)\sqrt{\frac{ae+bx^2}{c+dx^2}}}{16b^2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x^2 + a*b*d*e) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error{1, [0,1,0] %%, [2,0]%%}+%%{-2,0]: [1,0,%%{-1, [1,1,1]%%}]%%, [1,1]%%}+%%{-1, [1,1,0,1]%%}, [0,2]%%} / %%{-1, [0,2,0]%%}, [2,0]%%}+%%{-2, [0,1,0]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%, [1,1]%%}+%%{-1, [1,1,1]%%}, [0,2]%%} Error: Bad Argument Value

maple [B] time = 0.04, size = 342, normalized size = 2.02

$$\frac{(bx^2+a) \left(3a^2 d^2 \ln \left(\frac{2bd^2+ad+bc+2\sqrt{bd^2+ad+bc} \sqrt{bx^2+ac}}{2\sqrt{bd}} \right) - 2abcd \ln \left(\frac{2bd^2+ad+bc+2\sqrt{bd^2+ad+bc} \sqrt{bx^2+ac}}{2\sqrt{bd}} \right) - d^2 \ln \left(\frac{2bd^2+ad+bc+2\sqrt{bd^2+ad+bc} \sqrt{bx^2+ac}}{2\sqrt{bd}} \right) + 4\sqrt{bd} x^4 + adx^2 + bcx^2 + ac \sqrt{bd} bx^2 - 6\sqrt{bd} x^4 + adx^2 + bcx^2 + ac \sqrt{bd} ad + 2\sqrt{bd} x^4 + adx^2 + bcx^2 + ac \sqrt{bd} bc \right)}{16 \sqrt{\frac{bx^2+a}{d^2+c}} \sqrt{(dx^2+c)} (bx^2+a) \sqrt{bd} b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)

[Out] 1/16*(b*x^2+a)*(4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2+3*d^2*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*a^2-2*a*b*c*d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*b^2*c^2-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*c)/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/b^2/d/(b*d)^(1/2)

maxima [A] time = 2.23, size = 268, normalized size = 1.59

$$\frac{1}{16} e^{\left(\frac{2 \left((b^2 c^2 d + 2 a b c d^2 - 3 a^2 d^3) \left(\frac{bx^2+a}{dx^2+c} \right)^{\frac{3}{2}} + (b^3 c^2 - 6 a b^2 c d + 5 a^2 b d^2) \sqrt{\frac{bx^2+a}{dx^2+c}} e \right)}{b^4 d e^3 - \frac{2 (bx^2+a) b^3 d^2 e^3}{dx^2+c} + \frac{(bx^2+a)^2 b^2 d^3 e^3}{(dx^2+c)^2}} + \frac{(b^2 c^2 + 2 a b c d - 3 a^2 d^2) \log \left(\frac{d \sqrt{\frac{bx^2+a}{dx^2+c}} - \sqrt{b d e}}{d \sqrt{\frac{bx^2+a}{dx^2+c}} + \sqrt{b d e}} \right)}{\sqrt{b d e} b^2 d e} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/16*e*(2*((b^2*c^2*d + 2*a*b*c*d^2 - 3*a^2*d^3)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2) + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e)/(b^4*d*e^3 - 2*(b*x^2 + a)*b^3*d^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*b^2*d^3*e^3/(d*x^2 + c)^2) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log((d*sqrt((

$(b*x^2 + a)*e/(d*x^2 + c) - \text{sqrt}(b*d*e)/(d*\text{sqrt}((b*x^2 + a)*e/(d*x^2 + c) + \text{sqrt}(b*d*e)))/(\text{sqrt}(b*d*e)*b^2*d*e)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2), x)

[Out] Timed out

$$3.120 \quad \int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=106

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2b^{3/2} \sqrt{d} \sqrt{e}} + \frac{(c + dx^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Rubi [A] time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1960, 199, 208}

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2b^{3/2} \sqrt{d} \sqrt{e}} + \frac{(c + dx^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(2*b*e) + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2))]/(Sqrt[b]*Sqrt[e])]/(2*b^(3/2)*Sqrt[d]*Sqrt[e])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*(a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{1}{(be - dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{2be} + \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{be - dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2b}$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{2be} + \frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2b^{3/2} \sqrt{d} \sqrt{e}}$$

Mathematica [A] time = 0.13, size = 152, normalized size = 1.43

$$\frac{\sqrt{a + bx^2} \left(\sqrt{d} \sqrt{a + bx^2} \sqrt{\frac{b(c+dx^2)}{bc-ad}} + \sqrt{bc - ad} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) \right)}{2b\sqrt{d} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] (Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)] + Sqrt[b*c - a*d]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(2*b*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])

IntegrateAlgebraic [A] time = 0.16, size = 110, normalized size = 1.04

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{2b^{3/2} \sqrt{d} \sqrt{e}} + \frac{(c + dx^2) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2be}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]

[Out] ((c + d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(2*b*e) + ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3/2)*Sqrt[d]*Sqrt[e])

fricas [A] time = 0.66, size = 313, normalized size = 2.95

$$\frac{\sqrt{bde}(bc - ad) \log \left(8b^2d^2ex^4 + 8(b^2cd + abdf)ex^2 + (b^2c^2 + 6abcd + a^2d^2)e - 4(2bf^2x^4 + bc^2 + acd + (3bcd + ad^2)x^2) \sqrt{bde} \sqrt{\frac{bc^2+ae}{d^2+c}} \right) - 4(bf^2x^2 + bcd) \sqrt{\frac{bc^2+ae}{d^2+c}} \sqrt{-bde}(bc - ad) \arctan \left(\frac{2bf^2x^2 + bc + ad}{2(b^2d^2 + abde)} \sqrt{\frac{bc^2+ae}{d^2+c}} \right) - 2(bf^2x^2 + bcd) \sqrt{\frac{bc^2+ae}{d^2+c}}}{8b^2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))] - 4

```
*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4*(sqrt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{1,[0,1,0]
%%}, [2,0]%%}+%%{%%}{-2,0]:[1,0,%%{-1,[1,1,1]%%}]%%}, [1,1]%%}+%%{%%}{
1,[1,0,1]%%}, [0,2]%%} / %%{%%}{1,[0,2,0]%%}, [2,0]%%}+%%{%%}{%%{-2,[0
,1,0]%%},0]:[1,0,%%{-1,[1,1,1]%%}]%%}, [1,1]%%}+%%{%%}{1,[1,1,1]%%}, [0
,2]%%} Error: Bad Argument Value
```

maple [B] time = 0.03, size = 200, normalized size = 1.89

$$\frac{(bx^2 + a) \left(-ad \ln \left(\frac{2bdx^2 + ad + bc + 2\sqrt{bd}x^4 + adx^2 + bcx^2 + ac}{2\sqrt{bd}} \right) + bc \ln \left(\frac{2bdx^2 + ad + bc + 2\sqrt{bd}x^4 + adx^2 + bcx^2 + ac}{2\sqrt{bd}} \right) + 2\sqrt{bd}x^4 + adx^2 + bcx^2 + ac \right) \sqrt{bd}}{4\sqrt{\frac{(bx^2+a)e}{dx^2+c}} \sqrt{(dx^2+c)(bx^2+a)} \sqrt{bd} b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] 1/4*(b*x^2+a)*(-d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)
^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*a+b*c*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2))/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x
^2+a))^(1/2)/b/(b*d)^(1/2)
```

maxima [A] time = 2.18, size = 153, normalized size = 1.44

$$\frac{1}{4} e \left(\frac{2(bc - ad) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{b^2 e^2 - \frac{(bx^2+a)bde^2}{dx^2+c}} - \frac{(bc - ad) \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde} be} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(b^2*e^2 - (b*x^2 + a)
*b*d*e^2/(d*x^2 + c)) - (b*c - a*d)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d
*e)*b*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

```
[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.121 \quad \int \frac{1}{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=112

$$\frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{b} \sqrt{e}} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a} \sqrt{e}}$$

Rubi [A] time = 0.11, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1960, 391, 208}

$$\frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{b} \sqrt{e}} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a} \sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

```
[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[e])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 1960

```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{1}{x \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx = ((bc - ad)e) \operatorname{Subst} \left(\int \frac{1}{(-ae + cx^2)(be - dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= c \operatorname{Subst} \left(\int \frac{1}{-ae + cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) + d \operatorname{Subst} \left(\int \frac{1}{be - dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= -\frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a} \sqrt{e}} + \frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{b} \sqrt{e}}$$

Mathematica [A] time = 0.24, size = 190, normalized size = 1.70

$$\frac{\sqrt{a+bx^2} \left(\sqrt{a} \sqrt{d} \sqrt{c+dx^2} \sqrt{bc-ad} \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) - b\sqrt{c} (c+dx^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) \right)}{\sqrt{a} b (c+dx^2)^{3/2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] (Sqrt[a + b*x^2]*(Sqrt[a]*Sqrt[d]*Sqrt[b*c - a*d]*Sqrt[c + d*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]] - b*Sqrt[c]*(c + d*x^2)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*b*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^(3/2))

IntegrateAlgebraic [A] time = 0.19, size = 116, normalized size = 1.04

$$\frac{\sqrt{d} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{b} \sqrt{e}} - \frac{\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{\sqrt{a} \sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] -((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2))/(c + d*x^2])]/(Sqrt[a]*Sqrt[e]))/(Sqrt[a]*Sqrt[e])) + (Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2))/(c + d*x^2])]/(Sqrt[b]*Sqrt[e]))/(Sqrt[b]*Sqrt[e])

fricas [A] time = 0.99, size = 881, normalized size = 7.87



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))] + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2

```
+ a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*
c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e))/x^4), -1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d
+ a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c +
a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2
+ a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*
d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3
*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))),
1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(-d/(b*e))*arctan(
1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e)
)/(b*d*x^2 + a*d))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{1, [2,1,2]
%%}, [2,3,0]%%}+%%{%%}{-2, [1,2,2]%%}, [2,2,1]%%}+%%{%%}{1, [0,3,2]%%}, [2,
1,2]%%}+%%{%%}{-2, [2,0,2]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,4,0
]%%}+%%{%%}{4, [1,1,2]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,3,1]%%}+
%%{%%}{-2, [0,2,2]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,2,2]%%}+%%{%%}
%%{1, [3,0,3]%%}, [0,5,0]%%}+%%{%%}{-2, [2,1,3]%%}, [0,4,1]%%}+%%{%%}{1, [
1,2,3]%%}, [0,3,2]%%} / %%{%%}{1, [0,2,0]%%}, [2,0,0]%%}+%%{%%}{-2, [
0,1,0]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,1,0]%%}+%%{%%}{1, [1,1,1]%%}
, [0,2,0]%%} Error: Bad Argument Value

maple [B] time = 0.04, size = 179, normalized size = 1.60

$$\frac{(bx^2 + a) \left(-\sqrt{bd} \operatorname{c} \ln \left(\frac{adx^2 + bcx^2 + 2ac + 2\sqrt{ac} \sqrt{bdx^4 + adx^2 + bcx^2 + ac}}{x^2} \right) + \sqrt{ac} \operatorname{d} \ln \left(\frac{2bdx^2 + ad + bc + 2\sqrt{bd} \sqrt{bdx^4 + adx^2 + bcx^2 + ac} \sqrt{bd}}{2\sqrt{bd}} \right) \right)}{2\sqrt{\frac{(bx^2+a)e}{dx^2+c}} \sqrt{(dx^2+c)(bx^2+a)} \sqrt{bd} \sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)

[Out] 1/2*(b*x^2+a)*(d*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2))/(b*d)^(1/2)*(a*c)^(1/2)-c*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*(b*d)^(1/2))/(b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2)

maxima [A] time = 1.96, size = 155, normalized size = 1.38

$$\frac{1}{2} e \left(\frac{c \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace} e} - \frac{d \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde} e} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*e*(c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*e) - d*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*e))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)

[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

$$3.122 \quad \int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=130

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{3/2} \sqrt{c} \sqrt{e}} + \frac{(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Rubi [A] time = 0.08, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1960, 199, 208}

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{3/2} \sqrt{c} \sqrt{e}} + \frac{(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] ((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*a*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/ (Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e])

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\int \frac{1}{x^3 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx = ((bc - ad)e) \text{Subst} \left(\int \frac{1}{(-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)$$

$$= \frac{(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(bc - ad) \text{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2a}$$

$$= \frac{(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2a \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} + \frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{3/2} \sqrt{c} \sqrt{e}}$$

Mathematica [A] time = 0.12, size = 133, normalized size = 1.02

$$\frac{\sqrt{a+bx^2} \left(-\frac{(ad-bc) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{a^{3/2} \sqrt{c}} - \frac{\sqrt{a+bx^2} \sqrt{c+dx^2}}{ax^2} \right)}{2\sqrt{c+dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] (Sqrt[a + b*x^2]*(-(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^2)) - ((-(b*c) + a*d)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(a^(3/2)*Sqrt[c]))/(2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])

IntegrateAlgebraic [A] time = 0.17, size = 116, normalized size = 0.89

$$\frac{(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bex^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{3/2} \sqrt{c} \sqrt{e}} + \frac{(-c - dx^2) \sqrt{\frac{ae+bex^2}{c+dx^2}}}{2aex^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] ((-c - d*x^2)*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(2*a*e*x^2) + ((b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e]))

fricas [A] time = 1.03, size = 333, normalized size = 2.56

$$\frac{\sqrt{ace}(bc - ad)x^2 \log \left(\frac{(b^2c^2 + 6abcd + a^2d^2)e^{ax^2} + 8a^2c^2e + 8(ab^2c + a^2cd)e^{ax^2} - 4((bcd + ad^2)x^4 + 2ac^2 + (bc^2 + 3acd))^2 \sqrt{ace} \sqrt{\frac{bc^2 + ac}{d^2 + c}}}{x^4} \right) + 4(acdx^2 + ac^2) \sqrt{\frac{bc^2 + ac}{d^2 + c}} - \sqrt{ace}(bc - ad)x^2 \arctan \left(\frac{\sqrt{-ace}((bc + ad)x^2 + 2ac) \sqrt{\frac{bc^2 + ac}{d^2 + c}}}{2(abcdx^2 + a^2ce)} \right) + 2(acdx^2 + ac^2) \sqrt{\frac{bc^2 + ac}{d^2 + c}}}{8a^2cex^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

```
[Out] [-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b*c*e*x^2 + a^2*c*e) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*e*x^2)]
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error{1, [0,1,0]
}, [6,0,0]}+{[-2,0]: [1,0,{-1, [1,1,1]}]}}, [5,1,0]}+{1, [1,0,1]}}, [4,2,0]}+{[-2, [0,1,1]}}, [4,1,1]}+{4, [0,0,1]}}, 0]: [1,0,{-1, [1,1,1]}]}}, [3,2,1]}+{[-2, [1,0,2]}}, [2,3,1]}+{1, [0,1,2]}}, [2,2,2]}+{[-2, [0,0,2]}}, 0]: [1,0,{-1, [1,1,1]}]}}, [1,3,2]}+{1, [1,0,3]}}, [0,4,2]} / {1, [0,2,0]}}, [6,0,0]}+{[-2, [0,1,0]}}, 0]: [1,0,{-1, [1,1,1]}]}}, [5,1,0]}+{1, [1,1,1]}}, [4,2,0]}+{[-2, [0,2,1]}}, [4,1,1]}+{4, [0,1,1]}}, 0]: [1,0,{-1, [1,1,1]}]}}, [3,2,1]}+{[-2, [1,1,2]}}, [2,3,1]}+{1, [0,2,2]}}, [2,2,2]}+{[-2, [0,1,2]}}, 0]: [1,0,{-1, [1,1,1]}]}}, [1,3,2]}+{1, [1,1,3]}}, [0,4,2]} Error: Bad Argument Value
```

```
maple [B] time = 0.05, size = 326, normalized size = 2.51
```

$$\frac{(bx^2 + a) \left(a^2 cd x^2 \ln \left(\frac{ad^2 + bcx^2 + 2ac\sqrt{bdx^2 + ad^2 + bcx^2 + ac}}{2c} \right) - ab^2 x^2 \ln \left(\frac{ad^2 + bcx^2 + 2ac\sqrt{bdx^2 + ad^2 + bcx^2 + ac}}{2c} \right) - 2\sqrt{bdx^2 + ad^2 + bcx^2 + ac} \sqrt{ac} bdx^4 - 2\sqrt{bdx^2 + ad^2 + bcx^2 + ac} \sqrt{ac} adx^2 - 2\sqrt{bdx^2 + ad^2 + bcx^2 + ac} \sqrt{ac} bcx^2 + 2(bdx^4 + adx^2 + bcx^2 + ac)^{\frac{3}{2}} \sqrt{ac} \right)}{4\sqrt{\frac{bx^2+a}{x^2+c}} \sqrt{(dx^2+c)(bx^2+a)} \sqrt{ac} a^2 c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)
```

```
[Out] -1/4*(b*x^2+a)*(-2*b*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^4*(a*c)^(1/2)+a^2*c*d*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-a*b*c^2*x^2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d*a*x^2*(a*c)^(1/2)-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*b*c*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2))/((b*x^2+a)/(d*x^2+c)*e)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a^2/c/x^2/(a*c)^(1/2)
```

```
maxima [A] time = 1.52, size = 153, normalized size = 1.18
```

$$\frac{1}{4} e \left(\frac{2(bc - ad) \sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{a^2 e^2 - \frac{(bx^2+a)ace^2}{dx^2+c}} - \frac{(bc - ad) \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace} ae} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*e*(2*(b*c - a*d)*sqrt((b*x^2 + a)*e/(d*x^2 + c))/(a^2*e^2 - (b*x^2 + a)
*a*c*e^2/(d*x^2 + c)) - (b*c - a*d)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c))
- sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c
*e)*a*e))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

```
[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

```
[Out] Timed out
```

$$3.123 \quad \int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx$$

Optimal. Leaf size=218

$$\frac{(ad + 3bc)(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{5/2}c^{3/2}\sqrt{e}} - \frac{(ad + 3bc)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{e(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2}$$

Rubi [A] time = 0.14, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 385, 199, 208}

$$\frac{(ad + 3bc)(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{5/2}c^{3/2}\sqrt{e}} - \frac{(ad + 3bc)(bc - ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2c \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{e(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] -((b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(4*a*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2 - ((b*c - a*d)*(3*b*c + a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(8*a^2*c*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) - ((b*c - a*d)*(3*b*c + a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/((Sqrt[a]*Sqrt[e]))]/(8*a^(5/2)*c^(3/2)*Sqrt[e]))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))

$\wedge(1/q)], x]] /; \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \ \&\& \ \text{FractionQ}[p] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} dx &= ((bc - ad)e) \text{Subst} \left(\int \frac{be - dx^2}{(-ae + cx^2)^3} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\ &= -\frac{(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{((bc - ad)(3bc + ad)e) \text{Subst} \left(\int \frac{1}{(-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{4ac} \\ &= -\frac{(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(bc - ad)(3bc + ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2 c \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} + \frac{((bc - ad)(3bc + ad)) \text{Subst} \left(\int \frac{1}{(-ae + cx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{8a^2 c} \\ &= -\frac{(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(bc - ad)(3bc + ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^2 c \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(bc - ad)(3bc + ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{8a^{5/2} c^{3/2} \sqrt{e}} \end{aligned}$$

Mathematica [A] time = 0.16, size = 173, normalized size = 0.79

$$\frac{\sqrt{a} \sqrt{c} (a + bx^2) \sqrt{c + dx^2} (3bcx^2 - a(2c + dx^2)) - x^4 \sqrt{a + bx^2} (-a^2 d^2 - 2abcd + 3b^2 c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{8a^{5/2} c^{3/2} x^4 \sqrt{c + dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] (Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2)) - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])

IntegrateAlgebraic [A] time = 0.28, size = 164, normalized size = 0.75

$$\frac{(-2ac^2 - 3acdx^2 - ad^2x^4 + 3bc^2x^2 + 3bcdx^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8a^2 cex^4} + \frac{(a^2 d^2 + 2abcd - 3b^2 c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{5/2} c^{3/2} \sqrt{e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*a*c^2 + 3*b*c^2*x^2 - 3*a*c*d*x^2 + 3*b*c*d*x^4 - a*d^2*x^4))/(8*a^2*c*e*x^4) + ((-3*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt[c]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(8*a^(5/2)*c^(3/2)*Sqrt[e])

fricas [A] time = 2.24, size = 443, normalized size = 2.03

$$\frac{(3b^2c^2 - 2abcd - a^2d^2)\sqrt{ac} \log\left(\frac{(b^2c^2 + ad^2 + 2abdc)\sqrt{ac} + (b^2c^2 + ad^2 + 2abdc)\sqrt{ac}}{2}\right) + 4(2a^2c^3 - (3abc^2d - a^2cd^2)^2 - 3(abc^3 - a^2c^2d)^2)\sqrt{\frac{ac}{2c^2}}}{32a^2c^2d^4} + \frac{(3b^2c^2 - 2abcd - a^2d^2)\sqrt{-ac} \arctan\left(\frac{\sqrt{-ac}(b^2c^2 + ad^2 + 2abdc)\sqrt{ac}}{2(abcd^2 + a^2c^2)}\right) - 2(2a^2c^3 - (3abc^2d - a^2cd^2)^2 - 3(abc^3 - a^2c^2d)^2)\sqrt{\frac{ac}{2c^2}}}{16a^2c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^3*c^2*e*x^4)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{1, [4, 1, 4] %%}, [2, 7, 0]%%}+%%{%%}{-4, [3, 2, 4] %%}, [2, 6, 1]%%}+%%{%%}{6, [2, 3, 4] %%}, [2, 5, 2]%%}+%%{%%}{-4, [1, 4, 4] %%}, [2, 4, 3]%%}+%%{%%}{1, [0, 5, 4] %%}, [2, 3, 4] %%}+%%{%%}{[-2, [4, 0, 4] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 8, 0]%%}+ %%{%%}{%%{8, [3, 1, 4] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 7, 1]%%}+%%{%%}{ %%{-12, [2, 2, 4] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 6, 2]%%}+%%{%%}{[%% {8, [1, 3, 4] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 5, 3]%%}+%%{%%}{[%%{-2, [0, 4, 4] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 4, 4]%%}+%%{%%}{1, [5, 0, 5] %%}, [0, 9, 0]%%}+%%{%%}{-4, [4, 1, 5] %%}, [0, 8, 1]%%}+%%{%%}{6, [3, 2, 5] %%}, [0, 7, 2]%%}+%%{%%}{-4, [2, 3, 5] %%}, [0, 6, 3]%%}+%%{%%}{1, [1, 4, 5] %%}, [0, 5, 4]%%} / %%{%%}{1, [0, 2, 0] %%}, [2, 0, 0]%%}+%%{%%}{[%%{-2, [0, 1, 0] %%}, 0] : [1, 0, %%{-1, [1, 1, 1] %%}] %%}, [1, 1, 0]%%}+%%{%%}{1, [1, 1, 1] %%}, [0, 2, 0]%%} Error: Bad Argument Value

maple [B] time = 0.06, size = 558, normalized size = 2.56

$$\frac{(b^2c^2 + ad^2 + 2abdc)\sqrt{ac} \log\left(\frac{(b^2c^2 + ad^2 + 2abdc)\sqrt{ac} + (b^2c^2 + ad^2 + 2abdc)\sqrt{ac}}{2}\right) + 4(2a^2c^3 - (3abc^2d - a^2cd^2)^2 - 3(abc^3 - a^2c^2d)^2)\sqrt{\frac{ac}{2c^2}}}{32a^2c^2d^4} + \frac{(3b^2c^2 - 2abcd - a^2d^2)\sqrt{-ac} \arctan\left(\frac{\sqrt{-ac}(b^2c^2 + ad^2 + 2abdc)\sqrt{ac}}{2(abcd^2 + a^2c^2)}\right) - 2(2a^2c^3 - (3abc^2d - a^2cd^2)^2 - 3(abc^3 - a^2c^2d)^2)\sqrt{\frac{ac}{2c^2}}}{16a^2c^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(1/2),x)

[Out] 1/16*(b*x^2+a)*(-2*b*d^2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*a*(a*c)^(1/2)-10*b^2*d*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*x^6*c*(a*c)^(1/2)+a^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d^2*c*x^4+2*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*d*b*a^2*c^2*x^4-3*c^3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*b^2*a*x^4-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*d^2*a^2*x^4*(a*c)^(1/2)-8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*c*d-10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*b^2*c^2*x^4*(a*c)^(1/2)+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a*

$$d*x^2+10*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*b*c*x^2*(a*c)^{(1/2)}-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*(a*c)^{(1/2)}*a*c/((b*x^2+a)/(d*x^2+c)*e)^{(1/2)}/((d*x^2+c)*(b*x^2+a))^{(1/2)}/a^3/c^2/x^4/(a*c)^{(1/2)}$$

maxima [A] time = 1.76, size = 271, normalized size = 1.24

$$\frac{1}{16} e \left(\frac{2 \left((3b^2c^3 - 2abcd - a^2cd^2) \left(\frac{(bx^2+a)e}{dx^2+c} \right)^{\frac{3}{2}} - (5ab^2c^2 - 6a^2bcd + a^3d^2) \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e \right)}{a^4ce^3 - \frac{2(bx^2+a)a^3c^2e^3}{dx^2+c} + \frac{(bx^2+a)^2a^2c^3e^3}{(dx^2+c)^2}} + \frac{(3b^2c^2 - 2abcd - a^2d^2) \log \left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}} \right)}{\sqrt{ace} a^2ce} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] 1/16*e*(2*((3*b^2*c^3 - 2*a*b*c^2*d - a^2*c*d^2)*((b*x^2 + a)*e/(d*x^2 + c))^(3/2) - (5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e)/(a^4*c*e^3 - 2*(b*x^2 + a)*a^3*c^2*e^3/(d*x^2 + c) + (b*x^2 + a)^2*a^2*c^3*e^3/(d*x^2 + c)^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a^2*c*e)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \sqrt{\frac{e(bx^2+a)}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)

[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)

[Out] Timed out

$$3.124 \quad \int \frac{x^5}{\left(\frac{e^{(a+bx^2)}}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=354

$$\frac{(c+dx^2)^3 (7a^2d^2 - 2abcd + b^2c^2) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{6b^2de^2(bc-ad)^2} - \frac{a^2(c+dx^2)^3}{be(bc-ad)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}} - \frac{(bc-ad)(5ad(2bc-7ad) + b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{6} \sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}$$

Rubi [A] time = 0.38, antiderivative size = 348, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1960, 462, 385, 199, 208}

$$\frac{a^2(c+dx^2)^3}{be(bc-ad)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}} - \frac{(bc-ad)(5ad(2bc-7ad) + b^2c^2) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{6} \sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}} + \frac{(c+dx^2)^3 \left(\frac{c^2}{d} - \frac{a(2bc-7ad)}{b^2}\right) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{6e^2(bc-ad)^2} - \frac{(c+dx^2)^2 \left(\frac{5a(2bc-7ad)}{b^2} + \frac{c^2}{d}\right) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{24be^2(bc-ad)} - \frac{(c+dx^2)(5ad(2bc-7ad) + b^2c^2) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{16b^4d^2}$$

Antiderivative was successfully verified.

[In] Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] -((b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)/(16*b^4*d*e^2) - ((c^2/d + (5*a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(24*b*(b*c - a*d)*e^2) - (a^2*(c + d*x^2)^3)/(b*(b*c - a*d)^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((c^2/d - (a*(2*b*c - 7*a*d))/b^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^3)/(6*(b*c - a*d)^2*e^2) - ((b*c - a*d)*(b^2*c^2 + 5*a*d*(2*b*c - 7*a*d))*ArcTanh[Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e])]/(16*b^(9/2)*d^(3/2)*e^(3/2))

Rule 199

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[(x*(a + b*x^n)^(p + 1))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (IntegerQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denominator[p + 1/n] < Denominator[p])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 385

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := -Simp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n + p, 0])

Rule 462

Int[((e_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^2, x_Symbol] := Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x] /; Free

$Q[\{a, b, c, d, e, p\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LtQ}[m, -1] \ \&\& \ \text{GtQ}[n, 0]$

Rule 1960

$\text{Int}[(x_)^{(m_.)} * (((e_.) * ((a_.) + (b_.) * (x_)^{(n_.)}))) / ((c_) + (d_.) * (x_)^{(n_.)})]^{(p_.)}, x_Symbol] \ :> \ \text{With}[\{q = \text{Denominator}[p]\}, \text{Dist}[(q * e * (b * c - a * d)) / n, \text{Subst}[\text{Int}[(x^{(q * (p + 1) - 1) * (-a * e) + c * x^q)^{(\text{Simplify}[(m + 1) / n] - 1)} / (b * e - d * x^q)^{(\text{Simplify}[(m + 1) / n] + 1)}, x], x, ((e * (a + b * x^n)) / (c + d * x^n))^{(1 / q)}], x]] \ /; \ \text{FreeQ}[\{a, b, c, d, e, m, n\}, x] \ \&\& \ \text{FractionQ}[p] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1) / n]]$

Rubi steps

$$\begin{aligned} \int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx &= ((bc - ad)e) \text{Subst} \left(\int \frac{(-ae + cx^2)^2}{x^2 (be - dx^2)^4} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= -\frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(bc - ad) \text{Subst} \left(\int \frac{-a(2bc - 7ad)e^2 + bc^2 ex^2}{(be - dx^2)^4} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{b} \\ &= -\frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3}{6b^2 d (bc - ad)^2 e^2} - \frac{(bc - ad) (b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^3 d (bc - ad) e^2} \\ &= -\frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3}{6b^2 d (bc - ad)^2 e^2} - \frac{(bc - ad) (b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^3 d (bc - ad) e^2} + \frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \\ &= -\frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3}{6b^2 d (bc - ad)^2 e^2} - \frac{(bc - ad) (b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^3 d (bc - ad) e^2} + \frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \\ &= -\frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^3}{6b^2 d (bc - ad)^2 e^2} - \frac{(bc - ad) (b^2 c^2 - 2abcd + 7a^2 d^2) \sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{24b^3 d (bc - ad) e^2} + \frac{a^2 (c + dx^2)^3}{b(bc - ad)^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \end{aligned}$$

Mathematica [A] time = 0.48, size = 247, normalized size = 0.70

$$\frac{\sqrt{d} \sqrt{\frac{b(c+dx^2)}{bc-ad}} (105a^3 d^2 + 5a^2 b d (7dx^2 - 20c) + ab^2 (3c^2 - 38cdx^2 - 14d^2 x^4) + b^3 x^2 (3c^2 + 14cdx^2 + 8d^2 x^4)) - 3\sqrt{a + bx^2} \sqrt{bc - ad} (-35a^2 d^2 + 10abcd + b^2 c^2) \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{48b^4 d^{3/2} e \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c + 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 14*

$$c*d*x^2 + 8*d^2*x^4)) - 3*sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2*d^2)*sqrt[a + b*x^2]*ArcSinh[(sqrt[d]*sqrt[a + b*x^2])/sqrt[b*c - a*d]]/(48*b^4*d^(3/2)*e*sqrt[(e*(a + b*x^2))/(c + d*x^2)]*sqrt[(b*(c + d*x^2))/(b*c - a*d]))$$

IntegrateAlgebraic [A] time = 0.59, size = 293, normalized size = 0.83

$$\frac{(-35a^3d^3 + 45a^2bcd^2 - 9ab^2c^2d - b^3c^3) \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{ac+bx^2}{c+dx^2}}}{\sqrt{b}\sqrt{c}}\right) + (105a^3cd^2 + 105a^3d^3x^2 - 100a^2b^2cd - 65a^2bcd^2x^2 + 35a^2bd^3x^4 + 3ab^2c^3 - 35ab^2c^2dx^2 - 52ab^2cd^2x^4 - 14ab^2d^3x^6 + 3b^3c^3x^2 + 17b^3c^2dx^4 + 22b^3cd^2x^6 + 8b^3d^3x^8)\sqrt{\frac{ac+bx^2}{c+dx^2}}}{16b^{9/2}d^{3/2}e^{3/2} + 48b^4d^2(a + bx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(3*a*b^2*c^3 - 100*a^2*b*c^2*d + 105*a^3*c*d^2 + 3*b^3*c^3*x^2 - 35*a*b^2*c^2*d*x^2 - 65*a^2*b*c*d^2*x^2 + 105*a^3*d^3*x^2 + 17*b^3*c^2*d*x^4 - 52*a*b^2*c*d^2*x^4 + 35*a^2*b*d^3*x^4 + 22*b^3*c*d^2*x^6 - 14*a*b^2*d^3*x^6 + 8*b^3*d^3*x^8))/(48*b^4*d*e^2*(a + b*x^2)) + ((- (b^3*c^3) - 9*a*b^2*c^2*d + 45*a^2*b*c*d^2 - 35*a^3*d^3)*ArcTanh[(sqrt[d]*sqrt[(a*e + b*e*x^2)/(c + d*x^2)]]/(sqrt[b]*sqrt[e]))/(16*b^(9/2)*d^(3/2)*e^(3/2))

fricas [A] time = 2.85, size = 781, normalized size = 2.21

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x^2 + a*b*d*e) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x); OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{2, [1,2,2] %%}, [2,1,2,0] %%}+%%{%%}{-4, [2,1,2] %%}, [2,1,1,1] %%}+%%{%%}{2, [3,0,2] %%}, [2,1,0,2] %%}+%%{%%}{[%%]{-4, [0,2,2] %%}, 0]: [1,0, %%]{-1, [1,1,1] %%} %%},

```
[1, 1, 3, 0] + %%%{%%{[%%{8, [1, 1, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [1, 1, 2, 1]%%} + %%%{%%{[%%{-4, [2, 0, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [1, 1, 1, 2]%%} + %%%{%%{2, [0, 3, 3]%%}, [0, 1, 4, 0]%%} + %%%{%%{-4, [1, 2, 3]%%}, [0, 1, 3, 1]%%} + %%%{%%{2, [2, 1, 3]%%}, [0, 1, 2, 2]%%} / %%%{%%{1, [2, 0, 2]%%}, [2, 0, 0, 0]%%} + %%%{%%{-2, [1, 0, 2]%%}, 0] : [1, 0, %%{-1, [1, 1, 1]%%}}]%%}, [1, 0, 1, 0]%%} + %%%{%%{1, [1, 1, 3]%%}, [0, 0, 2, 0]%%} Error: Bad Argument Value
```

maple [B] time = 0.07, size = 1027, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2), x)

[Out]
$$\frac{1}{96} * (-60 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * x^4 * a * b^2 * d^2 + 12 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * x^4 * b^3 * c * d - 105 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * x^2 * a^3 * b * d^3 + 135 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * x^2 * a^2 * b^2 * c * d^2 - 27 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * x^2 * a * b^3 * c^2 * d - 3 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * x^2 * b^4 * c^3 + 16 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(3/2)} * (b * d)^{(1/2)} * x^2 * b^2 * d + 54 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * x^2 * a^2 * b * d^2 - 108 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * x^2 * a * b^2 * c * d + 6 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * x^2 * b^3 * c^2 - 105 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * a^4 * d^3 + 135 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * a^3 * b * c * d^2 - 27 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * a^2 * b^2 * c^2 * d - 3 * \ln(1/2 * (2 * b * d * x^2 + a * d + b * c + 2 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2))} / (b * d)^{(1/2)}) * a * b^3 * c^3 + 16 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(3/2)} * (b * d)^{(1/2)} * a * b * d + 114 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * a^3 * d^2 - 120 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * a^2 * b * c * d + 6 * (b * d * x^4 + a * d * x^2 + b * c * x^2 + a * c)^{(1/2)} * (b * d)^{(1/2)} * a * b^2 * c^2 + 96 * (b * d)^{(1/2)} * ((d * x^2 + c) * (b * x^2 + a))^{(1/2)} * a^3 * d^2 - 96 * (b * d)^{(1/2)} * ((d * x^2 + c) * (b * x^2 + a))^{(1/2)} * a^2 * b * c * d / d / b^4 * (b * x^2 + a) / (b * d)^{(1/2)} / (((d * x^2 + c) * (b * x^2 + a))^{(1/2)} / (d * x^2 + c) / (b * x^2 + a) / (d * x^2 + c) * e)^{(3/2)}$$

maxima [A] time = 1.83, size = 465, normalized size = 1.31

$$\frac{1}{96} e^{\left(2 \left(48 (a^2 b^4 c d - a^3 b^3 d^2) e^3 + \frac{3 (b^3 d^2 + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3) (b x^2 + a)^3}{(d x^2 + c)^3} - \frac{8 (b^4 c^3 d + 9 a b^3 c^2 d^2 - 45 a^2 b^2 c d^3 + 35 a^3 b d^4) (b x^2 + a)^2}{(d x^2 + c)^2} - \frac{3 (b^5 c^3 - 23 a b^4 c^2 d + 99 a^2 b^3 c d^2 - 77 a^3 b^2 d^3) (b x^2 + a)}{d x^2 + c} \right) \log \left(\frac{d \sqrt{\frac{b x^2 + a}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{b x^2 + a}{d x^2 + c}} + \sqrt{b d e}} \right) + \frac{3 (b^3 c^3 + 9 a b^2 c^2 d - 45 a^2 b c d^2 + 35 a^3 d^3) e^3}{\sqrt{b d e} b^4 d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="maxima")

[Out]
$$\frac{1}{96} * e * (2 * (48 * (a^2 * b^4 * c * d - a^3 * b^3 * d^2) * e^3 + 3 * (b^3 * c^3 * d^2 + 9 * a * b^2 * c^2 * d^3 - 45 * a^2 * b * c * d^4 + 35 * a^3 * d^5) * (b * x^2 + a)^3 * e^3 / (d * x^2 + c)^3 - 8 * (b^4 * c^3 * d + 9 * a * b^3 * c^2 * d^2 - 45 * a^2 * b^2 * c * d^3 + 35 * a^3 * b * d^4) * (b * x^2 + a)^2 * e^3 / (d * x^2 + c)^2 - 3 * (b^5 * c^3 - 23 * a * b^4 * c^2 * d + 99 * a^2 * b^3 * c * d^2 - 77 * a^3 * b^2 * d^3) * (b * x^2 + a) * e^3 / (d * x^2 + c)) / (b^4 * d^4 * ((b * x^2 + a) * e / (d * x^2 + c))^{(7/2)} * e^2 - 3 * b^5 * d^3 * ((b * x^2 + a) * e / (d * x^2 + c))^{(5/2)} * e^3 + 3 * b^6 * d^2 * ((b * x^2 + a) * e / (d * x^2 + c))^{(3/2)} * e^4 - b^7 * d * \sqrt{(b * x^2 + a) * e / (d * x^2 + c)} * e^5) + 3 * (b^3 * c^3 + 9 * a * b^2 * c^2 * d - 45 * a^2 * b * c * d^2 + 35 * a^3 * d^3) * \log((d * \sqrt{e} * \sqrt{(b * x^2 + a) * e / (d * x^2 + c)} - \sqrt{b * d * e}) / (d * \sqrt{e} * \sqrt{(b * x^2 + a) * e / (d * x^2 + c)} + \sqrt{b * d * e}))) / (\sqrt{b * d * e} * b^4 * d * e^2)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(\frac{e^{(bx^2+a)}}{dx^2+c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)

[Out] int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2), x)

[Out] Timed out

$$3.125 \quad \int \frac{x^3}{\left(\frac{e^{(a+bx^2)}}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=202

$$\frac{3(bc - 5ad)(bc - ad) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right)}{8b^{7/2} \sqrt{d} e^{3/2}} + \frac{(c + dx^2)(3bc - 7ad) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{8b^3 e^2} + \frac{a(bc - ad)}{b^3 e \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}} + \frac{(c + dx^2)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{4b^2 e^2}$$

Rubi [A] time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 456, 453, 208}

$$\frac{(c + dx^2)^2 \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{4b^2 e^2} + \frac{(c + dx^2)(3bc - 7ad) \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{8b^3 e^2} + \frac{3(bc - 5ad)(bc - ad) \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}}\right)}{8b^{7/2} \sqrt{d} e^{3/2}} + \frac{a(bc - ad)}{b^3 e \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] (a*(b*c - a*d))/(b^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + ((3*b*c - 7*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))/(8*b^3*e^2) + (Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)^2)/(4*b^2*e^2) + (3*(b*c - 5*a*d)*(b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(8*b^(7/2)*Sqrt[d]*e^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1)))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] := Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,

```
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = ((bc - ad)e) \operatorname{Subst} \left(\int \frac{-ae + cx^2}{x^2 (be - dx^2)^3} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4b^2e^2} - \frac{1}{4}((bc - ad)e) \operatorname{Subst} \left(\int \frac{\frac{4a}{b} - \frac{3(bc-ad)x^2}{b^2e}}{x^2 (be - dx^2)^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{(3bc - 7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4b^2e^2} + \frac{1}{8}((bc - ad)e) \operatorname{Subst} \left(\int \frac{-\frac{8a}{b^2e} + \frac{3}{x^2} (be - dx^2)}{x^2 (be - dx^2)^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right)$$

$$= \frac{a(bc - ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc - 7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4b^2e^2} + \frac{(3bc - 5ad)(bc - ad)}{8b^3e^2}$$

$$= \frac{a(bc - ad)}{b^3e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3bc - 7ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)}{8b^3e^2} + \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}} (c + dx^2)^2}{4b^2e^2} + \frac{3(bc - 5ad)(bc - ad)}{8b^3e^2}$$

Mathematica [A] time = 0.32, size = 190, normalized size = 0.94

$$\frac{\sqrt{d} \sqrt{\frac{b(c+dx^2)}{bc-ad}} (-15a^2d + ab(13c - 5dx^2) + b^2x^2(5c + 2dx^2)) + 3\sqrt{a + bx^2} (bc - 5ad)\sqrt{bc - ad} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right)}{8b^3\sqrt{d}e\sqrt{\frac{b(c+dx^2)}{bc-ad}} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-15*a^2*d + a*b*(13*c - 5*d*x^2) + b^2*x^2*(5*c + 2*d*x^2)) + 3*(b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]/(8*b^3*Sqrt[d]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])

IntegrateAlgebraic [A] time = 0.34, size = 203, normalized size = 1.00

$$\frac{3(5a^2d^2 - 6abcd + b^2c^2) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ac+bx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right) + (-15a^2cd - 15a^2d^2x^2 + 13abc^2 + 8abcdx^2 - 5abd^2x^4 + 5b^2c^2x^2 + 7b^2cdx^4 + 2b^2d^2x^6) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{8b^{7/2}\sqrt{d}e^{3/2} + 8b^3e^2(a + bx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]


```
[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(13*a*b*c^2 - 15*a^2*c*d + 5*b^2*c^2*x^2
+ 8*a*b*c*d*x^2 - 15*a^2*d^2*x^2 + 7*b^2*c*d*x^4 - 5*a*b*d^2*x^4 + 2*b^2*d
^2*x^6))/(8*b^3*e^2*(a + b*x^2)) + (3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*Arc
Tanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2))]/(Sqrt[b]*Sqrt[e]))/(8*b^(
7/2)*Sqrt[d]*e^(3/2))
```

fricas [A] time = 1.94, size = 585, normalized size = 2.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*
a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x
^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (
3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(2
*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3
)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a
*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2 - 6*a^
2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sqrt(-b*d*
e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c)))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15
*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d
^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 +
a*b^4*d*e^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{2,[1,2,2]
%%}, [2,1,2,0]%%}+%%{%%}{-4,[2,1,2]%%}, [2,1,1,1]%%}+%%{%%}{2,[3,0,2]%%
}, [2,1,0,2]%%}+%%{%%}{[%%]{-4,[0,2,2]%%},0]:[1,0,%%{-1,[1,1,1]%%}]%%},
[1,1,3,0]%%}+%%{%%}{[%%]{8,[1,1,2]%%},0]:[1,0,%%{-1,[1,1,1]%%}]%%}, [1,1
,2,1]%%}+%%{%%}{[%%]{-4,[2,0,2]%%},0]:[1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,1,
2]%%}+%%{%%}{2,[0,3,3]%%}, [0,1,4,0]%%}+%%{%%}{-4,[1,2,3]%%}, [0,1,3,1]
%%}+%%{%%}{2,[2,1,3]%%}, [0,1,2,2]%%} / %%{%%}{1,[2,0,2]%%}, [2,0,0,0]%%
}+%%{%%}{[%%]{-2,[1,0,2]%%},0]:[1,0,%%{-1,[1,1,1]%%}]%%}, [1,0,1,0]%%}
+%%{%%}{1,[1,1,3]%%}, [0,0,2,0]%%} Error: Bad Argument Value
```

maple [B] time = 0.06, size = 679, normalized size = 3.36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)
```

```
[Out] -1/16*(-4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*x^4*b^2*d-15*ln(1
/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b
*d)^(1/2))*x^2*a^2*b*d^2+18*ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*
c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*x^2*a*b^2*c*d-3*ln(1/2*(2*b*d*x^
2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*x
```

$$\begin{aligned} & \cdot 2 \cdot b^3 \cdot c^2 + 10 \cdot (b \cdot d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)} \cdot x^2 \cdot a \cdot b \cdot d - 10 \cdot \\ & (b \cdot d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)} \cdot x^2 \cdot b^2 \cdot c - 15 \cdot \ln(1/2 \cdot (2 \cdot b \cdot d \cdot \\ & x^2 + a \cdot d + b \cdot c + 2 \cdot (b \cdot d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)}) / (b \cdot d)^{(1/2)}) \\ & \cdot a^3 \cdot d^2 + 18 \cdot \ln(1/2 \cdot (2 \cdot b \cdot d \cdot x^2 + a \cdot d + b \cdot c + 2 \cdot (b \cdot d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \\ & \cdot (b \cdot d)^{(1/2)}) / (b \cdot d)^{(1/2)}) \cdot a^2 \cdot b \cdot c \cdot d - 3 \cdot \ln(1/2 \cdot (2 \cdot b \cdot d \cdot x^2 + a \cdot d + b \cdot c + 2 \cdot (b \cdot d \cdot x^4 \\ & + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)}) / (b \cdot d)^{(1/2)}) \cdot a \cdot b^2 \cdot c^2 + 16 \cdot ((d \cdot x^2 + \\ & c) \cdot (b \cdot x^2 + a))^{(1/2)} \cdot (b \cdot d)^{(1/2)} \cdot a^2 \cdot d - 16 \cdot ((d \cdot x^2 + c) \cdot (b \cdot x^2 + a))^{(1/2)} \cdot (b \cdot d)^{(1/2)} \\ & \cdot a \cdot b \cdot c + 14 \cdot (b \cdot d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)} \cdot a^2 \cdot d - 10 \cdot (b \cdot \\ & d \cdot x^4 + a \cdot d \cdot x^2 + b \cdot c \cdot x^2 + a \cdot c)^{(1/2)} \cdot (b \cdot d)^{(1/2)} \cdot a \cdot b \cdot c) / b^3 \cdot (b \cdot x^2 + a) / (b \cdot d)^{(1/2)} \\ & / ((d \cdot x^2 + c) \cdot (b \cdot x^2 + a))^{(1/2)} / (d \cdot x^2 + c) / ((b \cdot x^2 + a) / (d \cdot x^2 + c) \cdot e)^{(3/2)} \end{aligned}$$

maxima [A] time = 1.81, size = 311, normalized size = 1.54

$$\frac{1}{16} e \left(\frac{2 \left(8 (ab^3c - a^2b^2d) e^2 - \frac{3(b^2c^2d - 6abcd^2 + 5a^2d^3)(bx^2+a)^2 e^2}{(dx^2+c)^2} + \frac{5(b^3c^2 - 6ab^2cd + 5a^2bd^2)(bx^2+a)e^2}{dx^2+c} \right) - 3(b^2c^2 - 6abcd + 5a^2d^2) \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{b^3d^2 \left(\frac{(bx^2+a)e}{dx^2+c} \right)^{\frac{5}{2}} e^2 - 2b^4d \left(\frac{(bx^2+a)e}{dx^2+c} \right)^{\frac{3}{2}} e^3 + b^5 \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e^4} - \frac{3(b^2c^2 - 6abcd + 5a^2d^2) \log \left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}} \right)}{\sqrt{bde} b^3 e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out]
$$\frac{1}{16} e \cdot (2 \cdot (8 \cdot (a \cdot b^3 \cdot c - a^2 \cdot b^2 \cdot d) \cdot e^2 - 3 \cdot (b^2 \cdot c^2 \cdot d - 6 \cdot a \cdot b \cdot c \cdot d^2 + 5 \cdot a^2 \cdot b \cdot d^3) \cdot (b \cdot x^2 + a)^2 \cdot e^2 / (d \cdot x^2 + c)^2 + 5 \cdot (b^3 \cdot c^2 - 6 \cdot a \cdot b^2 \cdot c \cdot d + 5 \cdot a^2 \cdot b \cdot d^2) \cdot (b \cdot x^2 + a) \cdot e^2 / (d \cdot x^2 + c)) / (b^3 \cdot d^2 \cdot ((b \cdot x^2 + a) \cdot e / (d \cdot x^2 + c))^{(5/2)} \cdot e^2 - 2 \cdot b^4 \cdot d \cdot ((b \cdot x^2 + a) \cdot e / (d \cdot x^2 + c))^{(3/2)} \cdot e^3 + b^5 \cdot \sqrt{((b \cdot x^2 + a) \cdot e / (d \cdot x^2 + c))} \cdot e^4 - 3 \cdot (b^2 \cdot c^2 - 6 \cdot a \cdot b \cdot c \cdot d + 5 \cdot a^2 \cdot d^2) \cdot \log((d \cdot \sqrt{((b \cdot x^2 + a) \cdot e / (d \cdot x^2 + c))} - \sqrt{b \cdot d \cdot e}) / (d \cdot \sqrt{((b \cdot x^2 + a) \cdot e / (d \cdot x^2 + c))} + \sqrt{b \cdot d \cdot e}))) / (\sqrt{b \cdot d \cdot e} \cdot b^3 \cdot e^2))$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\left(\frac{e(bx^2+a)}{dx^2+c} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)

[Out] int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

$$3.126 \quad \int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=146

$$\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}} - \frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Rubi [A] time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{d}(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}} - \frac{3(bc-ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c+dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]

[Out] (-3*(b*c - a*d))/(2*b^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (c + d*x^2)/(2*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (3*Sqrt[d]*(b*c - a*d)*ArcTanh[Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e]))/(2*b^(5/2)*e^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-a*e) + c*x^q)^(Simplify[(m+1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ

[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx &= ((bc - ad)e) \text{Subst} \left(\int \frac{1}{x^2 (be - dx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= \frac{c + dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3(bc - ad)) \text{Subst} \left(\int \frac{1}{x^2 (be - dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2b} \\
&= -\frac{3(bc - ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c + dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{(3d(bc - ad)) \text{Subst} \left(\int \frac{1}{be - dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2b^2e} \\
&= -\frac{3(bc - ad)}{2b^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c + dx^2}{2be\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{3\sqrt{d}(bc - ad) \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}} \right)}{2b^{5/2}e^{3/2}}
\end{aligned}$$

Mathematica [C] time = 0.07, size = 86, normalized size = 0.59

$$-\frac{(a + bx^2) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}, \frac{1}{2}, \frac{d(bx^2+a)}{ad-bc}\right)}{b\left(\frac{b(c+dx^2)}{bc-ad}\right)^{3/2}\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] -(((a + b*x^2)*Hypergeometric2F1[-3/2, -1/2, 1/2, (d*(a + b*x^2))/(-(b*c) + a*d)])/(b*((e*(a + b*x^2))/(c + d*x^2))^(3/2)*((b*(c + d*x^2))/(b*c - a*d))^(3/2)))

IntegrateAlgebraic [A] time = 0.25, size = 153, normalized size = 1.05

$$\frac{3(bc\sqrt{d} - ad^{3/2}) \tanh^{-1} \left(\frac{\sqrt{d}\sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{b}\sqrt{e}} \right)}{2b^{5/2}e^{3/2}} + \frac{(3acd + 3ad^2x^2 - 2bc^2 - bcdx^2 + bd^2x^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2b^2e^2(a + bx^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(a*e + b*e*x^2)/(c + d*x^2)]*(-2*b*c^2 + 3*a*c*d - b*c*d*x^2 + 3*a*d^2*x^2 + b*d^2*x^4))/(2*b^2*e^2*(a + b*x^2)) + (3*(b*c*Sqrt[d] - a*d^(3/2))*ArcTanh[(Sqrt[d]*Sqrt[(a*e + b*e*x^2)/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(5/2)*e^(3/2))

fricas [A] time = 1.01, size = 443, normalized size = 3.03

$$\frac{3((b^2 - abd)cx^2 + (abc - a^2d))\sqrt{c} \log\left(\frac{8b^2d^2x^4 + b^2c^2 + 6abcd + a^2d^2 + 8(b^2cd + abd^2)x^2 - 4(2b^2d^2x^4 + b^2c^2 + abcd + (3b^2cd + abd^2)x^2)\sqrt{\frac{d}{b^2c^2}}}{8(b^2d^2 + abd^2)}\right) - 4(b^2d^2x^4 + b^2c^2 + 3abd - (bcd - 3abd^2)x^2)\sqrt{\frac{d}{b^2c^2}} - 3((b^2 - abd)cx^2 + (abc - a^2d))\sqrt{c} \operatorname{arctan}\left(\frac{(b^2d^2 + abd^2)\sqrt{\frac{d}{b^2c^2}}}{2(b^2d^2 + abd^2)}\right) - 2(b^2d^2x^4 - 2b^2c^2 + 3abd - (bcd - 3abd^2)x^2)\sqrt{\frac{d}{b^2c^2}}}{8(b^2d^2 + abd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] $[-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*\sqrt{d/(b*e)}*\log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)}*\sqrt{d/(b*e)}) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*a*d^2)*x^2)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*\sqrt{-d/(b*e)}*\operatorname{arctan}(1/2*(2*b*d*x^2 + b*c + a*d)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)}*\sqrt{-d/(b*e)})/(b*d*x^2 + a*d) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*c*d - 3*a*d^2)*x^2)*\sqrt{(b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^2*e^2)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{2, [1,2,2] %%}, [2,1,2,0]%%}+%%{%%}{-4, [2,1,2]%%}, [2,1,1,1]%%}+%%{%%}{2, [3,0,2]%%}, [2,1,0,2]%%}+%%{%%}{[%%]{-4, [0,2,2]%%}, 0] : [1,0,%%{-1, [1,1,1]%%}}%%}, [1,1,3,0]%%}+%%{%%}{[%%]{8, [1,1,2]%%}, 0] : [1,0,%%{-1, [1,1,1]%%}}%%}, [1,1,2,1]%%}+%%{%%}{[%%]{-4, [2,0,2]%%}, 0] : [1,0,%%{-1, [1,1,1]%%}}%%}, [1,1,1,2]%%}+%%{%%}{2, [0,3,3]%%}, [0,1,4,0]%%}+%%{%%}{-4, [1,2,3]%%}, [0,1,3,1]%%}+%%{%%}{2, [2,1,3]%%}, [0,1,2,2]%%} / %%{%%}{1, [2,0,2]%%}, [2,0,0,0]%%}+%%{%%}{[%%]{-2, [1,0,2]%%}, 0] : [1,0,%%{-1, [1,1,1]%%}}%%}, [1,0,1,0]%%}+%%{%%}{1, [1,1,3]%%}, [0,0,2,0]%%} Error: Bad Argument Value

maple [B] time = 0.05, size = 432, normalized size = 2.96

$$\frac{(-3abd^2x^2 \ln\left(\frac{2bd^2x^2 + 2\sqrt{bd^2x^2 + a^2d^2} + b^2c^2 + a^2d^2}{2bd^2}\right) + 3bd^2x^2 \ln\left(\frac{2bd^2x^2 + 2\sqrt{bd^2x^2 + a^2d^2} + b^2c^2 + a^2d^2}{2bd^2}\right) - 3a^2d^2 \ln\left(\frac{2bd^2x^2 + 2\sqrt{bd^2x^2 + a^2d^2} + b^2c^2 + a^2d^2}{2bd^2}\right) + 3abd \ln\left(\frac{2bd^2x^2 + 2\sqrt{bd^2x^2 + a^2d^2} + b^2c^2 + a^2d^2}{2bd^2}\right) + 2\sqrt{bd^2x^2 + a^2d^2} + bcx^2 + ac\sqrt{bd^2x^2 + a^2d^2} + 2\sqrt{bd^2x^2 + a^2d^2} + bcx^2 + ac\sqrt{bd^2x^2 + a^2d^2} + 4\sqrt{(d^2+c)(b^2+a)}\sqrt{bd^2x^2 + a^2d^2} - 4\sqrt{(d^2+c)(b^2+a)}\sqrt{bd^2x^2 + a^2d^2})}{4\sqrt{d}\sqrt{(d^2+c)(b^2+a)}(d^2+c)\sqrt{\frac{d}{b^2c^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)

[Out] $1/4*(-3*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))*x^2*a*b*d^2+3*b^2*c*d*x^2*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*b*d*x^2-3*a^2*d^2*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+3*a*b*c*d*\ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c))^(1/2)*(b*d)^(1/2))/(b*d)^(1/2))+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)*a*d+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c)/b^2*(b*x^2+a)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)$

maxima [A] time = 1.74, size = 199, normalized size = 1.36

$$\frac{1}{4} e \left(\frac{2 \left(2 (b^2 c - a b d) e - \frac{3 (b c d - a d^2) (b x^2 + a) e}{d x^2 + c} \right) - 3 (b c d - a d^2) \log \left(\frac{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} - \sqrt{b d e}}{d \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} + \sqrt{b d e}} \right)}{b^2 d \left(\frac{(b x^2 + a) e}{d x^2 + c} \right)^{\frac{3}{2}} e^2 - b^3 \sqrt{\frac{(b x^2 + a) e}{d x^2 + c}} e^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/4*e*(2*(2*(b^2*c - a*b*d)*e - 3*(b*c*d - a*d^2)*(b*x^2 + a)*e/(d*x^2 + c))/(b^2*d*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - b^3*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^3) - 3*(b*c*d - a*d^2)*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b^2*e^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(\frac{e(bx^2+a)}{dx^2+c} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)

[Out] int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

$$3.127 \quad \int \frac{1}{x \left(\frac{e^{(a+bx^2)}}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=152

$$-\frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2} e^{3/2}} + \frac{d^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{b^{3/2} e^{3/2}} + \frac{bc - ad}{abe \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}$$

Rubi [A] time = 0.20, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 480, 522, 208}

$$-\frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2} e^{3/2}} + \frac{d^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{b^{3/2} e^{3/2}} + \frac{bc - ad}{abe \sqrt{\frac{e^{(a+bx^2)}}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]

[Out] (b*c - a*d)/(a*b*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[a]*Sqrt[e]))]/(a^(3/2)*e^(3/2)) + (d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[b]*Sqrt[e]))]/(b^(3/2)*e^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q) + b*d*(m + n*(p + q + 2) + 1)*x^n, x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 522

Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] := Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))

$\wedge(1/q)], x]] /; \text{FreeQ}\{a, b, c, d, e, m, n\}, x\} \&\& \text{FractionQ}[p] \&\& \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx &= ((bc - ad)e) \text{Subst} \left(\int \frac{1}{x^2 (-ae + cx^2) (be - dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\ &= \frac{bc - ad}{abe \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc - ad) \text{Subst} \left(\int \frac{-((bc+ad)e)+cdx^2}{(-ae+cx^2)(be-dx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{abe} \\ &= \frac{bc - ad}{abe \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{c^2 \text{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{ae} + \frac{d^2 \text{Subst} \left(\int \frac{1}{be-dx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{be} \\ &= \frac{bc - ad}{abe \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{a^{3/2} e^{3/2}} + \frac{d^{3/2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{b^{3/2} e^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.35, size = 253, normalized size = 1.66

$$\frac{-a^{3/2} d^{3/2} \sqrt{a+bx^2} \sqrt{c+dx^2} (ad-bc) \sqrt{\frac{b(c+dx^2)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{a+bx^2}}{\sqrt{bc-ad}} \right) - b(c+dx^2) \sqrt{bc-ad} \left(bc^{3/2} \sqrt{a+bx^2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{e}} \right) + \sqrt{a} \sqrt{c+dx^2} (ad-bc) \right)}{a^{3/2} b^2 e (c+dx^2)^{3/2} \sqrt{bc-ad} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x]

[Out] $(-a^{3/2} d^{3/2} (-b*c + a*d) \sqrt{a + b*x^2} \sqrt{c + d*x^2} \sqrt{(b*(c + d*x^2))/(b*c - a*d)} \text{ArcSinh}[(\sqrt{d} \sqrt{a + b*x^2})/\sqrt{b*c - a*d}] - b \sqrt{b*c - a*d} (c + d*x^2) (\sqrt{a} (-b*c + a*d) \sqrt{c + d*x^2} + b*c^{3/2} \sqrt{a + b*x^2} \text{ArcTanh}[(\sqrt{c} \sqrt{a + b*x^2})/(\sqrt{a} \sqrt{c + d*x^2})]))/(a^{3/2} b^2 \sqrt{b*c - a*d} e \sqrt{(e*(a + b*x^2))/(c + d*x^2)}) * (c + d*x^2)^{3/2}$

IntegrateAlgebraic [B] time = 0.73, size = 419, normalized size = 2.76

$$\frac{bc^{5/2} \sqrt{\frac{ae+bcx^2}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bcx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right) - ad^{5/2} \sqrt{\frac{ae+bcx^2}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bcx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right) + c^{3/2} d \sqrt{\frac{ae+bcx^2}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bcx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right) + \frac{bc^2}{ae} + \frac{cd^{3/2} \sqrt{\frac{ae+bcx^2}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{ae+bcx^2}{c+dx^2}}}{\sqrt{b} \sqrt{e}} \right)}{\sqrt{b} e^{3/2}} + \frac{ad^2}{be} - \frac{2cd}{e}}{bc \sqrt{\frac{ae+bcx^2}{c+dx^2}} - ad \sqrt{\frac{ae+bcx^2}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x]

[Out] $((b*c^2)/(a*e) - (2*c*d)/e + (a*d^2)/(b*e) - (b*c^{5/2} \sqrt{(a*e + b*e*x^2)/(c + d*x^2)} \text{ArcTanh}[(\sqrt{c} \sqrt{(a*e + b*e*x^2)/(c + d*x^2)})/(\sqrt{a} \sqrt{e})]))/(a^{3/2} * e^{3/2}) + (c^{3/2} * d \sqrt{(a*e + b*e*x^2)/(c + d*x^2)})$

$$\frac{\text{ArcTanh}\left[\frac{\sqrt{c}\sqrt{(aex^2)/(c+dx^2)}}{\sqrt{a}\sqrt{e}}\right]}{\sqrt{a}e^{3/2}} + \frac{(cd^{3/2})\sqrt{(aex^2)/(c+dx^2)}\text{ArcTanh}\left[\frac{\sqrt{d}\sqrt{(aex^2)/(c+dx^2)}}{\sqrt{b}\sqrt{e}}\right]}{\sqrt{b}e^{3/2}} - \frac{(ad^{5/2})\sqrt{(aex^2)/(c+dx^2)}\text{ArcTanh}\left[\frac{\sqrt{d}\sqrt{(aex^2)/(c+dx^2)}}{\sqrt{b}\sqrt{e}}\right]}{\sqrt{b}e^{3/2}} \frac{1}{(b^{3/2}e^{3/2})} \frac{1}{(b^2c\sqrt{(aex^2)/(c+dx^2)} - a^2d\sqrt{(aex^2)/(c+dx^2)})}$$

fricas [B] time = 2.15, size = 1293, normalized size = 8.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out]
$$\frac{1}{4} \left((abde^2x^2 + a^2de) \sqrt{d/(be)} \log(8b^2d^2x^4 + b^2c^2 + 6ab^2cd + a^2d^2 + 8(b^2cd + abd^2)x^2 + 4(2b^2d^2x^4 + b^2c^2 + ab^2cd + (3b^2cd + abd^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{d/(be)} \right) + (b^2cex^2 + abc^2e) \sqrt{c/(ae)} \log((b^2c^2 + 6ab^2cd + a^2d^2)x^4 + 8a^2c^2 + 8(ab^2c^2 + a^2cd)x^2 - 4((ab^2cd + a^2d^2)x^4 + 2a^2c^2 + (ab^2c^2 + 3a^2cd)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{c/(ae)}) / x^4 + 4(b^2c^2 - acd + (bcd - ad^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) / (ab^2e^2x^2 + a^2be^2) , -1/4(2(abde^2x^2 + a^2de) \sqrt{-d/(be)} \arctan(1/2(2bdx^2 + bc + ad) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{-d/(be)}) / (bdx^2 + ad) - (b^2cex^2 + abc^2e) \sqrt{c/(ae)} \log((b^2c^2 + 6ab^2cd + a^2d^2)x^4 + 8a^2c^2 + 8(ab^2c^2 + a^2cd)x^2 - 4((ab^2cd + a^2d^2)x^4 + 2a^2c^2 + (ab^2c^2 + 3a^2cd)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{c/(ae)}) / x^4 - 4(b^2c^2 - acd + (bcd - ad^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) / (ab^2e^2x^2 + a^2be^2) , 1/4(2(b^2cex^2 + abc^2e) \sqrt{-c/(ae)} \arctan(1/2((bc + ad)x^2 + 2ac) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{-c/(ae)}) / (bcx^2 + ac) + (abdex^2 + a^2de) \sqrt{d/(be)} \log(8b^2d^2x^4 + b^2c^2 + 6ab^2cd + a^2d^2 + 8(b^2cd + abd^2)x^2 + 4(2b^2d^2x^4 + b^2c^2 + ab^2cd + (3b^2cd + abd^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{d/(be)}) + 4(b^2c^2 - acd + (bcd - ad^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) / (ab^2e^2x^2 + a^2be^2) , 1/2((b^2cex^2 + abc^2e) \sqrt{-c/(ae)} \arctan(1/2((bc + ad)x^2 + 2ac) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{-c/(ae)}) / (bcx^2 + ac) - (abdex^2 + a^2de) \sqrt{-d/(be)} \arctan(1/2(2bdx^2 + bc + ad) \sqrt{(bex^2 + a)/(dx^2 + c)}) \sqrt{-d/(be)}) / (bdx^2 + ad) + 2(b^2c^2 - acd + (bcd - ad^2)x^2) \sqrt{(bex^2 + a)/(dx^2 + c)}) / (ab^2e^2x^2 + a^2be^2)]$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%{2,[1,2,2]%%}, [2,1,3,0]%%}+%%{%%{-4,[2,1,2]%%}, [2,1,2,1]%%}+%%{%%{2,[3,0,2]%%}, [2,1,1,2]%%}+%%{%%{%%{-4,[0,2,2]%%}, 0]: [1,0,%%{-1,[1,1,1]%%}}%%}, [1,1,4,0]%%}+%%{%%{%%{8,[1,1,2]%%}, 0]: [1,0,%%{-1,[1,1,1]%%}}%%}, [1,1,3,1]%%}+%%{%%{%%{-4,[2,0,2]%%}, 0]: [1,0,%%{-1,[1,1,1]%%}}%%}, [1,1,2,2]%%}+%%{%%{2,[0,3,3]%%}, [0,1,5,0]%%}+%%{%%{-4,[1,2,3]%%}, [0,1,4,1]%%}+%%{%%{2,[2,1,3]%%}, [0,1,3,2]%%} / %%{%%{1,[2,0,0]%%}, [2,0,0,0]%%}+%%{%%{%%{-2,[1,0,0]%%}, 0]: [1,0,%%{-1,[1,1,1]%%}}%%}, [1,0,1,0]%%}+%%{%%{1,[1,1,1]%%}, [0,0,2,0]%%} Error: Bad Argument Value

maple [B] time = 0.06, size = 401, normalized size = 2.64

$$\frac{\left(-\sqrt{ac} ab d^2 \ln\left(\frac{2bd^2 + ad^2 + 2\sqrt{bd^2 + ad^2 + bc^2 + ac} \sqrt{bd}}{2\sqrt{bd}}\right) + \sqrt{bd} b^2 c^2 \ln\left(\frac{ad^2 + bc^2 + 2bc + 2\sqrt{bd^2 + ad^2 + bc^2 + ac}}{d^2}\right) - \sqrt{ac} a^2 d^2 \ln\left(\frac{2bd^2 + ad^2 + 2\sqrt{bd^2 + ad^2 + bc^2 + ac} \sqrt{bd}}{2\sqrt{bd}}\right) + \sqrt{bd} ab c^2 \ln\left(\frac{ad^2 + bc^2 + 2bc + 2\sqrt{bd^2 + ad^2 + bc^2 + ac}}{d^2}\right) + 2\sqrt{(d^2 + c)(b^2 + a)} \sqrt{bd} \sqrt{ac} ad - 2\sqrt{(d^2 + c)(b^2 + a)} \sqrt{bd} \sqrt{ac} bc\right) (b^2 + a)}{2\sqrt{ac} \sqrt{bd} \sqrt{(d^2 + c)(b^2 + a)} (d^2 + c) \left(\frac{bd^2 + ac}{d^2 + c}\right)^{\frac{3}{2}} ab}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x^2+a)/(d*x^2+c)*e)^(3/2), x)

[Out] -1/2*(-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*(a*c)^(1/2)*x^2*a*b*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*b^2*c^2-ln(1/2*(2*b*d*x^2+a*d+b*c+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)))/(b*d)^(1/2))*(a*c)^(1/2)*a^2*d^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*a*b*c^2+2*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*a*d-2*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*(a*c)^(1/2)*b*c)/a/b*(b*x^2+a)/(a*c)^(1/2)/(b*d)^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)

maxima [A] time = 1.92, size = 204, normalized size = 1.34

$$\frac{1}{2} e \left(\frac{c^2 \log\left(\frac{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{ace}}\right)}{\sqrt{ace} ae^2} - \frac{d^2 \log\left(\frac{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} - \sqrt{bde}}{d \sqrt{\frac{(bx^2+a)e}{dx^2+c}} + \sqrt{bde}}\right)}{\sqrt{bde} be^2} + \frac{2(bc - ad)}{ab \sqrt{\frac{(bx^2+a)e}{dx^2+c}} e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2), x, algorithm="maxima")

[Out] 1/2*e*(c^2*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a*e^2) - d^2*log((d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(b*d*e))/(d*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(b*d*e)))/(sqrt(b*d*e)*b*e^2) + 2*(b*c - a*d)/(a*b*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^2))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)

[Out] int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2), x)

[Out] Timed out

$$3.128 \quad \int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=170

$$\frac{3\sqrt{c}(bc-ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{5/2}e^{3/2}} - \frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Rubi [A] time = 0.11, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 290, 325, 208}

$$\frac{3\sqrt{c}(bc-ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{5/2}e^{3/2}} - \frac{3(bc-ad)}{2a^2e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc-ad}{2a\sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]

[Out] (-3*(b*c - a*d))/(2*a^2*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) + (b*c - a*d)/(2*a*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))) + (3*Sqrt[c]*(b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/ (Sqrt[a]*Sqrt[e])])/(2*a^(5/2)*e^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 290

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*n*(p+1)), x] + Dist[(m+n*(p+1)+1)/(a*n*(p+1)), Int[(c*x)^m*(a + b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && LtQ[p, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 325

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a + b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1960

Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-a*e) + c*x^q)^(Simplify[(m+1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m+1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ

[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{1}{x^2 (-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right) \\
&= \frac{bc - ad}{2a \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(3(bc - ad)) \operatorname{Subst} \left(\int \frac{1}{x^2 (-ae+cx^2)} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2a} \\
&= -\frac{3(bc - ad)}{2a^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc - ad}{2a \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} - \frac{(3c(bc - ad)) \operatorname{Subst} \left(\int \frac{1}{-ae+cx^2} dx, x, \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \right)}{2a^2 e} \\
&= -\frac{3(bc - ad)}{2a^2 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} + \frac{bc - ad}{2a \sqrt{\frac{e(a+bx^2)}{c+dx^2}} \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)} + \frac{3\sqrt{c}(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{5/2} e^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 148, normalized size = 0.87

$$\frac{3\sqrt{c} x^2 \sqrt{a+bx^2} (bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right) - \sqrt{a} \sqrt{c+dx^2} (a(c - 2dx^2) + 3bcx^2)}{2a^{5/2} e x^2 \sqrt{c+dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x]

[Out] $(-\sqrt{a} \sqrt{c+dx^2} (3bcx^2 + a(c - 2dx^2)) + 3\sqrt{c} (bcx^2 - ad) \sqrt{a+bx^2} \operatorname{ArcTanh}[\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}}]) / (2a^{5/2} e x^2 \sqrt{c+dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}})$

IntegrateAlgebraic [A] time = 0.27, size = 159, normalized size = 0.94

$$\frac{(-ac^2 + acdx^2 + 2ad^2x^4 - 3bc^2x^2 - 3bcdx^4) \sqrt{\frac{ae+bx^2}{c+dx^2}}}{2a^2 e^2 x^2 (a+bx^2)} - \frac{3(a\sqrt{c}d - bc^{3/2}) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{2a^{5/2} e^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x]

[Out] $(\sqrt{\frac{ae+bx^2}{c+dx^2}} (-(ac^2 - 3bc^2x^2 + a^2cdx^2 - 3b^2cdx^4 + 2ad^2x^4)) / (2a^{5/2} e^2 x^2 \sqrt{a+bx^2}) - (3(-bc^{3/2}) + a\sqrt{c}d) \operatorname{ArcTanh}[\frac{\sqrt{c} \sqrt{\frac{ae+bx^2}{c+dx^2}}}{\sqrt{a} \sqrt{e}}]) / (2a^{5/2} e^{3/2}))$

fricas [A] time = 3.11, size = 469, normalized size = 2.76

$$\frac{3((b^2c - abd)cx^4 + (abc - a^2d)cx^2)\sqrt{\frac{c}{a}} \log\left(\frac{(b^2c - abd)cx^4 + (abc - a^2d)cx^2 + 4((3bd - 2ad^2)c^2 + (a^2c + b^2d^2))\sqrt{\frac{c}{a}}}{8(b^2c^2 + a^2d^2)}\right) + 4((3bd - 2ad^2)c^2 + a^2 + (3b^2 - ad^2)c)\sqrt{\frac{c}{a}}}{4(b^2c^2 + a^2d^2)} - 3((b^2c - abd)cx^4 + (abc - a^2d)cx^2)\sqrt{\frac{c}{a}} \arctan\left(\frac{(b^2c - abd)cx^4 + (abc - a^2d)cx^2}{2(b^2c^2 + a^2d^2)}\right) + 2((3bd - 2ad^2)c^2 + a^2 + (3b^2 - ad^2)c)\sqrt{\frac{c}{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4 + 4*((3*b*c*d - 2*a*d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e)))/(b*c*x^2 + a*c) + 2*((3*b*c*d - 2*a*d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*b*e^2*x^4 + a^3*e^2*x^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{2, [1,0,0] %%}, [6,1,0,0]%%}+%%{%%}{[-4,0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [5,1,1,0]%%}+ %%{%%}{2, [0,1,1]%%}, [4,1,2,0]%%}+%%{%%}{-4, [1,0,1]%%}, [4,1,1,1]%%}+%% {%%}{[%%]{8, [0,0,1]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [3,1,2,1]%%}+%%{%% {-4, [0,1,2]%%}, [2,1,3,1]%%}+%%{%%}{2, [1,0,2]%%}, [2,1,2,2]%%}+%%{%%}{[%%{-4, [0,0,2]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,1,3,2]%%}+%%{%%}{2, [0,1,3]%%}, [0,1,4,2]%%} / %%{%%}{1, [2,0,0]%%}, [6,0,0,0]%%}+%%{%%}{[%%]{ -2, [1,0,0]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [5,0,1,0]%%}+%%{%%}{1, [1,1, 1]%%}, [4,0,2,0]%%}+%%{%%}{-2, [2,0,1]%%}, [4,0,1,1]%%}+%%{%%}{[%%]{4, [1, 0,1]%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [3,0,2,1]%%}+%%{%%}{-2, [1,1,2]%% } , [2,0,3,1]%%}+%%{%%}{1, [2,0,2]%%}, [2,0,2,2]%%}+%%{%%}{[%%]{-2, [1,0,2] %%%}, 0]: [1,0,%%{-1, [1,1,1]%%}]%%}, [1,0,3,2]%%}+%%{%%}{1, [1,1,3]%%}, [0,0 ,4,2]%%} Error: Bad Argument Value

maple [B] time = 0.07, size = 641, normalized size = 3.77

$$\frac{1}{4} \sqrt{\frac{c}{a}} \left((b^2 c^2 + 6 a b c d + a^2 d^2) x^4 + 8 a^2 c^2 + 8 (a b c^2 + a^2 c d) x^2 - 4 ((a b c d + a^2 d^2) x^4 + 2 a^2 c^2 + (a b c^2 + 3 a^2 c d) x^2) \sqrt{\frac{c}{a}} \right) \log\left(\frac{(b^2 c^2 + 6 a b c d + a^2 d^2) x^4 + 8 a^2 c^2 + 8 (a b c^2 + a^2 c d) x^2 - 4 ((a b c d + a^2 d^2) x^4 + 2 a^2 c^2 + (a b c^2 + 3 a^2 c d) x^2) \sqrt{\frac{c}{a}}}{8 (b^2 c^2 + a^2 d^2)}\right) + \frac{4 ((3 b c d - 2 a d^2) x^4 + a c^2 + (3 b c^2 - a c d) x^2) \sqrt{\frac{c}{a}} \arctan\left(\frac{(b^2 c^2 + 6 a b c d + a^2 d^2) x^4 + 8 a^2 c^2 + 8 (a b c^2 + a^2 c d) x^2 - 4 ((a b c d + a^2 d^2) x^4 + 2 a^2 c^2 + (a b c^2 + 3 a^2 c d) x^2) \sqrt{\frac{c}{a}}}{2 (b^2 c^2 + a^2 d^2)}\right) + 2 ((3 b d - 2 a d^2) c^2 + a^2 + (3 b^2 - a d^2) c) \sqrt{\frac{c}{a}}}{4 (b^2 c^2 + a^2 d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)

[Out] 1/4*(2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^2*d-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a*b^2*c^2+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*b^2*c-3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^3*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^2*a^2*b*c^2+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^2*a*

$$b*c-2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*(a*c)^{(1/2)}*x^2*b+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(1/2)}*(a*c)^{(1/2)}*x^2*a^2*d+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^{(3/2)}*(a*c)^{(1/2)}*a*(b*x^2+a)/(a*c)^{(1/2)}/x^2/a^3/((d*x^2+c)*(b*x^2+a))^{(1/2)}/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^{(3/2)}$$

maxima [A] time = 1.97, size = 197, normalized size = 1.16

$$\frac{1}{4} e \left(\frac{2 \left(2 (abc - a^2 d) e - \frac{3 (bc^2 - acd) (bx^2 + a) e}{dx^2 + c} \right)}{a^2 c \left(\frac{(bx^2 + a) e}{dx^2 + c} \right)^{\frac{3}{2}} e^2 - a^3 \sqrt{\frac{(bx^2 + a) e}{dx^2 + c}} e^3} - \frac{3 (bc - ad) c \log \left(\frac{c \sqrt{\frac{(bx^2 + a) e}{dx^2 + c}} - \sqrt{ace}}{c \sqrt{\frac{(bx^2 + a) e}{dx^2 + c}} + \sqrt{ace}} \right)}{\sqrt{ace} a^2 e^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/4*e*(2*(2*(a*b*c - a^2*d)*e - 3*(b*c^2 - a*c*d)*(b*x^2 + a)*e/(d*x^2 + c))/(a^2*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^2 - a^3*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^3) - 3*(b*c - a*d)*c*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a^2*e^2))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(\frac{e^{(bx^2+a)}}{dx^2+c} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)

[Out] int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)

[Out] Timed out

$$3.129 \quad \int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=255

$$\frac{3(5bc - ad)(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{7/2} \sqrt{c} e^{3/2}} - \frac{(7bc - 3ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3 \left(ae^2 - \frac{ce^2(a+bx^2)}{c+dx^2} \right)} + \frac{b(bc - ad)}{a^3 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)}$$

Rubi [A] time = 0.23, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1960, 456, 453, 208}

$$\frac{(7bc - 3ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3 \left(ae^2 - \frac{ce^2(a+bx^2)}{c+dx^2} \right)} - \frac{3(5bc - ad)(bc - ad) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a} \sqrt{e}} \right)}{8a^{7/2} \sqrt{c} e^{3/2}} - \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} + \frac{b(bc - ad)}{a^3 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]

[Out] (b*(b*c - a*d))/(a^3*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]) - ((b*c - a*d)^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(4*a^2*(a*e - (c*e*(a + b*x^2))/(c + d*x^2))^2) - ((7*b*c - 3*a*d)*(b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(8*a^3*(a*e^2 - (c*e^2*(a + b*x^2))/(c + d*x^2))) - (3*(b*c - a*d)*(5*b*c - a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(Sqrt[a]*Sqrt[e]))]/(8*a^(7/2)*Sqrt[c]*e^(3/2)))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 453

Int[((e_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]

Rule 456

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*ExpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c - a*d)*x^(-m + 2)]/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])

Rule 1960

```
Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_)))/((c_) + (d_)*(x_)^(n_))
)^(p_), x_Symbol] :=> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n,
Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1))/(b
*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ
[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2} \right)^{3/2}} dx &= ((bc - ad)e) \operatorname{Subst} \left(\int \frac{be - dx^2}{x^2 (-ae + cx^2)^3} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= -\frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{1}{4} ((bc - ad)e) \operatorname{Subst} \left(\int \frac{\frac{4b}{a} + \frac{3(bc-ad)x^2}{a^2e}}{x^2 (-ae + cx^2)^2} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= -\frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(7bc - 3ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3 \left(ae^2 - \frac{ce^2(a+bx^2)}{c+dx^2} \right)} + \frac{1}{8} ((bc - ad)e) \operatorname{Subst} \left(\int \frac{\frac{8b}{a^2e} + \dots}{x^2 (-ae + cx^2)} dx, x, \sqrt{\frac{e(a + bx^2)}{c + dx^2}} \right) \\ &= \frac{b(bc - ad)}{a^3 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(7bc - 3ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3 \left(ae^2 - \frac{ce^2(a+bx^2)}{c+dx^2} \right)} + \frac{3(bc - ad)(5bc - \dots)}{\dots} \\ &= \frac{b(bc - ad)}{a^3 e \sqrt{\frac{e(a+bx^2)}{c+dx^2}}} - \frac{(bc - ad)^2 \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2 \left(ae - \frac{ce(a+bx^2)}{c+dx^2} \right)^2} - \frac{(7bc - 3ad)(bc - ad) \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{8a^3 \left(ae^2 - \frac{ce^2(a+bx^2)}{c+dx^2} \right)} - \frac{3(bc - ad)(5bc - \dots)}{\dots} \end{aligned}$$

Mathematica [A] time = 0.12, size = 189, normalized size = 0.74

$$\frac{\sqrt{a} \sqrt{c} \sqrt{c + dx^2} (-a^2 (2c + 5dx^2) + abx^2 (5c - 13dx^2) + 15b^2 cx^4) - 3x^4 \sqrt{a + bx^2} (a^2 d^2 - 6abcd + 5b^2 c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a+bx^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{8a^{7/2} \sqrt{c} e x^4 \sqrt{c + dx^2} \sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]

[Out] (Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2) - a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]/(8*a^(7/2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])

IntegrateAlgebraic [A] time = 0.37, size = 209, normalized size = 0.82

$$\frac{(-2a^2c^2 - 7a^2cdx^2 - 5a^2d^2x^4 + 5abc^2x^2 - 8abcdx^4 - 13abd^2x^6 + 15b^2c^2x^4 + 15b^2cdx^6) \sqrt{\frac{ae+bx^2}{c+dx^2}} - 3(a^2d^2 - 6abcd + 5b^2c^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{ae+bx^2}}{\sqrt{a} \sqrt{e}} \right)}{8a^3e^2x^4(a + bx^2) 8a^{7/2} \sqrt{c} e^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]

[Out] $(\sqrt{(a e + b e x^2)/(c + d x^2)}(-2 a^2 c^2 + 5 a b c^2 x^2 - 7 a^2 c d x^2 + 15 b^2 c^2 x^4 - 8 a b c d x^4 - 5 a^2 d^2 x^4 + 15 b^2 c d x^6 - 13 a b d^2 x^6))/(8 a^3 e^2 x^4 (a + b x^2)) - (3(5 b^2 c^2 - 6 a b c d + a^2 d^2) \operatorname{ArcTanh}(\sqrt{c} \sqrt{(a e + b e x^2)/(c + d x^2)}))/(\sqrt{a} \sqrt{e})]/(8 a^{7/2} \sqrt{c} e^{3/2})$

fricas [A] time = 10.11, size = 613, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] $[1/32(3((5b^3c^2 - 6ab^2cd + a^2bd^2)x^6 + (5ab^2c^2 - 6a^2b^2cd + a^3d^2)x^4)\sqrt{ace})\log(((b^2c^2 + 6ab^2cd + a^2d^2)ex^4 + 8a^2c^2e + 8(ab^2c^2 + a^2cd)ex^2 - 4((b^2cd + ad^2)x^4 + 2ac^2 + (b^2c^2 + 3ac^2d)x^2)\sqrt{ace})\sqrt{(bex^2 + a)/(dx^2 + c)})/x^4 + 4((15ab^2c^2d - 13a^2b^2cd^2)x^6 - 2a^3c^3 + (15ab^2c^3 - 8a^2b^2cd - 5a^3cd^2)x^4 + (5a^2b^2c^3 - 7a^3c^2d)x^2)\sqrt{(bex^2 + a)/(dx^2 + c)})/(a^4b^2ce^2x^6 + a^5ce^2x^4), 1/16(3((5b^3c^2 - 6ab^2cd + a^2bd^2)x^6 + (5ab^2c^2 - 6a^2b^2cd + a^3d^2)x^4)\sqrt{-ace})\arctan(1/2\sqrt{-ace}((b^2c^2 + a^2d^2)ex^2 + 2ac^2)\sqrt{(bex^2 + a)/(dx^2 + c)})/(ab^2ce^2x^2 + a^2ce)) + 2((15ab^2c^2d - 13a^2b^2cd^2)x^6 - 2a^3c^3 + (15ab^2c^3 - 8a^2b^2cd - 5a^3cd^2)x^4 + (5a^2b^2c^3 - 7a^3c^2d)x^2)\sqrt{(bex^2 + a)/(dx^2 + c)})/(a^4b^2ce^2x^6 + a^5ce^2x^4)]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep*d+c)]Unable to divide, perhaps due to rounding error%%{%%}{2,[1,4,4]%%}, [2,1,7,0]%%}+%%{%%}{-8,[2,3,4]%%}, [2,1,6,1]%%}+%%{%%}{12,[3,2,4]%%}, [2,1,5,2]%%}+%%{%%}{-8,[4,1,4]%%}, [2,1,4,3]%%}+%%{%%}{2,[5,0,4]%%}, [2,1,3,4]%%}+%%{%%}{[%%]{-4,[0,4,4]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,8,0]%%}+%%{%%}{[%%]{16,[1,3,4]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,7,1]%%}+%%{%%}{[%%]{-24,[2,2,4]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,6,2]%%}+%%{%%}{[%%]{16,[3,1,4]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,5,3]%%}+%%{%%}{[%%]{-4,[4,0,4]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,1,4,4]%%}+%%{%%}{2,[0,5,5]%%}, [0,1,9,0]%%}+%%{%%}{-8,[1,4,5]%%}, [0,1,8,1]%%}+%%{%%}{12,[2,3,5]%%}, [0,1,7,2]%%}+%%{%%}{-8,[3,2,5]%%}, [0,1,6,3]%%}+%%{%%}{2,[4,1,5]%%}, [0,1,5,4]%%} / %%{%%}{1,[2,0,0]%%}, [2,0,0,0]%%}+%%{%%}{-2,[1,0,0]%%},0}: [1,0,%%{-1,[1,1,1]%%}]%%}, [1,0,1,0]%%}+%%{%%}{1,[1,1,1]%%}, [0,0,2,0]%%} Error: Bad Argument Value

maple [B] time = 0.08, size = 1042, normalized size = 4.09

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/((b*x^2+a)/(d*x^2+c)*e)^(3/2),x)

```
[Out] -1/16*(-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*a*b^2*d^2+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^8*b^3*c*d+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^3*b*c*d^2-18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a^2*b^2*c^2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^6*a*b^3*c^3-12*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a^2*b*d^2+26*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*a*b^2*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^6*b^3*c^2+3*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^4*c*d^2-18*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^3*b*c^2*d+15*ln((a*d*x^2+b*c*x^2+2*a*c+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2))/x^2)*x^4*a^2*b^2*c^3+16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a^2*b*c*d-16*((d*x^2+c)*(b*x^2+a))^(1/2)*(a*c)^(1/2)*x^4*a*b^2*c^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*a*b*d-18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^4*b^2*c-6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^3*d^2+8*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a^2*b*c*d+18*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(a*c)^(1/2)*x^4*a*b^2*c^2+6*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a^2*d-14*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*x^2*a*b*c+4*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(3/2)*(a*c)^(1/2)*a^2*c)/c*(b*x^2+a)/(a*c)^(1/2)/x^4/a^4/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/((b*x^2+a)/(d*x^2+c)*e)^(3/2)
```

maxima [A] time = 1.96, size = 311, normalized size = 1.22

$$\frac{1}{16} e^{\left(\frac{2 \left(8(a^2 b^2 c - a^3 b d) e^2 + \frac{3(5 b^2 c^3 - 6 a b c^2 d + a^2 c d^2)(b x^2 + a)^2 e^2}{(d x^2 + c)^2} - \frac{5(5 a b^2 c^2 - 6 a^2 b c d + a^3 d^2)(b x^2 + a)^2}{d x^2 + c} \right)}{a^3 c^2 \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{5}{2}} e^2 - 2 a^4 c \left(\frac{b x^2 + a}{d x^2 + c} \right)^{\frac{3}{2}} e^3 + a^5 \sqrt{\frac{b x^2 + a}{d x^2 + c}} e^4} + \frac{3(5 b^2 c^2 - 6 a b c d + a^2 d^2) \log \left(\frac{c \sqrt{\frac{b x^2 + a}{d x^2 + c}} - \sqrt{a c e}}{c \sqrt{\frac{b x^2 + a}{d x^2 + c}} + \sqrt{a c e}} \right)}{\sqrt{a c e} a^3 e^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/16*e*(2*(8*(a^2*b^2*c - a^3*b*d)*e^2 + 3*(5*b^2*c^3 - 6*a*b*c^2*d + a^2*c*d^2)*(b*x^2 + a)^2*e^2/(d*x^2 + c)^2 - 5*(5*a*b^2*c^2 - 6*a^2*b*c*d + a^3*d^2)*(b*x^2 + a)*e^2/(d*x^2 + c))/(a^3*c^2*((b*x^2 + a)*e/(d*x^2 + c))^(5/2)*e^2 - 2*a^4*c*((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*e^3 + a^5*sqrt((b*x^2 + a)*e/(d*x^2 + c))*e^4) + 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*log((c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) - sqrt(a*c*e))/(c*sqrt((b*x^2 + a)*e/(d*x^2 + c)) + sqrt(a*c*e)))/(sqrt(a*c*e)*a^3*e^2)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(\frac{e(bx^2+a)}{dx^2+c} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

```
[Out] int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

```
[Out] Timed out
```

$$3.130 \quad \int x^5 \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal. Leaf size=216

$$\frac{(-8a^2c^2 + 4abc + b^2)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16a^2d^3} + \frac{b(8a^2c^2 + 4abc + b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3} + \frac{(c + dx^2)^3 \left(\frac{ac+adx^2+b}{c+dx^2}\right)}{6ad^3}$$

Rubi [A] time = 0.62, antiderivative size = 259, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 9, integrand size = 21, number of rules / integrand size = 0.429, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{(8a^2c^2 + 4abc + b^2)(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{16a^2d^3} + \frac{b(8a^2c^2 + 4abc + b^2) \sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{16a^{5/2}d^3 \sqrt{a(c+dx^2)+b}} - \frac{(8ac + 3b)(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}} (a(c+dx^2)+b)}{24a^2d^3} + \frac{x^2(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}} (a(c+dx^2)+b)}{6ad^2}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[a + b/(c + d*x^2)],x]

[Out] ((b^2 + 4*a*b*c + 8*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(16*a^2*d^3) - ((3*b + 8*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(24*a^2*d^3) + (x^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(6*a*d^2) + (b*(b^2 + 4*a*b*c + 8*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(16*a^(5/2)*d^3*Sqrt[b + a*(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 90

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; FreeQ

[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int x^5 \sqrt{a + \frac{b}{c + dx^2}} dx &= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \int \frac{x^5 \sqrt{b + a(c + dx^2)}}{\sqrt{c + dx^2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \int \frac{x^5 \sqrt{b + ac + adx^2}}{\sqrt{c + dx^2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \text{Subst}\left(\int \frac{x^2 \sqrt{b + ac + adx}}{\sqrt{c + dx}} dx, x, x^2\right)}{2\sqrt{b + a(c + dx^2)}} \\
&= \frac{x^2(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{6ad^2} + \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b + ac}}{\sqrt{c + dx}} dx, x, x^2\right)}{6ad^2 \sqrt{b + a(c + dx^2)}} \\
&= -\frac{(3b + 8ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24a^2d^3} + \frac{x^2(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{6ad^2} \\
&= \frac{(b^2 + 4abc + 8a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16a^2d^3} - \frac{(3b + 8ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24a^2d^3} \\
&= \frac{(b^2 + 4abc + 8a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16a^2d^3} - \frac{(3b + 8ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24a^2d^3} \\
&= \frac{(b^2 + 4abc + 8a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16a^2d^3} - \frac{(3b + 8ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24a^2d^3} \\
&= \frac{(b^2 + 4abc + 8a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16a^2d^3} - \frac{(3b + 8ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24a^2d^3}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 137, normalized size = 0.63

$$\frac{\sqrt{a} (c + dx^2) \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (8a^2(c^2 - cdx^2 + d^2x^4) + 2ab(dx^2 - 5c) - 3b^2) + 3b(8a^2c^2 + 4abc + b^2) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}}\right)}{48a^{5/2}d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[a + b/(c + d*x^2)], x]

[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2 + 2*a*b*(-5*c + d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) + 3*b*(b^2 + 4*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(5/2)*d^3)

IntegrateAlgebraic [A] time = 0.27, size = 161, normalized size = 0.75

$$\frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (8a^2c^3 + 8a^2d^3x^6 - 10abc^2 - 8abcdx^2 + 2abd^2x^4 - 3b^2c - 3b^2dx^2)}{48a^2d^3} + \frac{(8a^2bc^2 + 4ab^2c + b^3) \tanh^{-1}\left(\frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}}{\sqrt{a}}\right)}{16a^{5/2}d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5*Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b^2*c - 10*a*b*c^2 + 8*a^2*c^3 - 3*b^2*d*x^2 - 8*a*b*c*d*x^2 + 2*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a^2*d^3) + ((b^3 + 4*a*b^2*c + 8*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/((16*a^(5/2)*d^3)
```

fricas [A] time = 0.97, size = 423, normalized size = 1.96

$$\frac{3(8a^2bc^2 + 4abd^2 + b^3)\sqrt{a} \log\left(\frac{8a^2d^2x^4 + 8a^2c^3 + 8(2a^2c + ab)d^2 + 8abc + 4(2ad^2 + 4ac + 3bd^2 + 2ac^2 + b)\sqrt{\frac{a^2d^2x^2 + a^2c + b}{d^2d^2}}}{192a^2d^3}\right) + 4(8a^2d^2x^4 + 4a^2c^3 + 2a^2bd^2x^4 + 8a^2c^3 - 10a^2b^2c - 3abd^2 - (8a^2c + 3ab^2)d^2)\sqrt{\frac{a^2d^2x^2 + a^2c + b}{d^2d^2}}}{96a^2d^3} - \frac{3(8a^2bc^2 + 4abd^2 + b^3)\sqrt{a} \operatorname{arctan}\left(\frac{2ad^2x^2 + 2a^2c + b}{2d^2x^2 + 2a^2c + b}\right) - 2(8a^2d^2x^4 + 2a^2bd^2x^4 + 8a^2c^3 - 10a^2b^2c - 3abd^2 - (8a^2c + 3ab^2)d^2)\sqrt{\frac{a^2d^2x^2 + a^2c + b}{d^2d^2}}}{96a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^3), -1/96*(3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(8*a^3*d^3*x^6 + 2*a^2*b*d^2*x^4 + 8*a^3*c^3 - 10*a^2*b*c^2 - 3*a*b^2*c - (8*a^2*b*c + 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^3)]
```

giac [A] time = 0.49, size = 219, normalized size = 1.01

$$\frac{1}{96} \left(2\sqrt{ad^2x^4 + 2acd^2 + bd^2x^2 + ac^2 + bc} \left(2x^2 \left(\frac{4x^2}{d} - \frac{4a^2cd^2 - abd^3}{a^2d^5} \right) + \frac{8a^2c^2d^2 - 10abcd^2 - 3b^2d^4}{a^2d^5} \right) - \frac{3(8a^2bc^2 + 4ab^2c + b^3) \log\left(\frac{-2acd - 2(\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acd^2 + bd^2x^2 + ac^2 + bc}}\sqrt{a}|d| - bd)}{a^2d^2|d|} \right)}{a^2d^2|d|} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/96*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/d - (4*a^2*c*d^3 - a*b*d^3)/(a^2*d^5)) + (8*a^2*c^2*d^2 - 10*a*b*c*d^2 - 3*b^2*d^2)/(a^2*d^5)) - 3*(8*a^2*b*c^2 + 4*a*b^2*c + b^3)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d^2*abs(d))*sgn(d*x^2 + c)
```

maple [B] time = 0.06, size = 533, normalized size = 2.47

$$\frac{\sqrt{\frac{a^2d^2x^2 + a^2c + b}{d^2d^2}} \left(\frac{3(8a^2bc^2 + 4ab^2c + b^3) \log\left(\frac{-2acd - 2(\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acd^2 + bd^2x^2 + ac^2 + bc}}\sqrt{a}|d| - bd)}{a^2d^2|d|} \right)}{a^2d^2|d|} \right) \operatorname{sgn}(dx^2 + c) - 2(8a^3d^3x^6 + 2a^2bd^2x^4 + 8a^3c^3 - 10a^2b^2c - 3abd^2 - (8a^2c + 3ab^2)d^2)\sqrt{\frac{a^2d^2x^2 + a^2c + b}{d^2d^2}}}{96a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^3*(-48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*c*a^2*d*(a*d^2)^(1/2)+24*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^2*b*a*d*(a*d^2)^(1/2)+12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^2*c*a*d+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*a*(a*d^2)^(1/2)-36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*c*b*a*(a*d^2)^(1/2)-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*b^2*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(a*d^2)^(1/2)
```

maxima [A] time = 1.72, size = 328, normalized size = 1.52

$$\frac{3(8a^2bc^2 - 4ab^2c - b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8(6a^3bc^2 - ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + 3(8a^4bc^2 + 4a^3b^2c + a^2b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^5d^3 - \frac{3(adx^2+ac+b)a^4d^3}{dx^2+c} + \frac{3(adx^2+ac+b)^2a^3d^3}{(dx^2+c)^2} - \frac{(adx^2+ac+b)^3a^2d^3}{(dx^2+c)^3}\right)} (8a^2c^2 + 4abc + b^2)b \log\left(\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] $-1/48*(3*(8*a^2*b*c^2 - 4*a*b^2*c - b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{5/2} - 8*(6*a^3*b*c^2 - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{3/2} + 3*(8*a^4*b*c^2 + 4*a^3*b^2*c + a^2*b^3)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3 - 3*(a*d*x^2 + a*c + b)*a^4*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a^3*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^2*d^3/(d*x^2 + c)^3 - 1/32*(8*a^2*c^2 + 4*a*b*c + b^2)*b*\text{log}(-(\text{sqrt}(a) - \text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(\text{sqrt}(a) + \text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^{(5/2)*d^3})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \sqrt{a + \frac{b}{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a + b/(c + d*x^2))^(1/2),x)

[Out] int(x^5*(a + b/(c + d*x^2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5*(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

$$3.131 \quad \int x^3 \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal. Leaf size=141

$$-\frac{b(4ac + b) \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{8a^{3/2}d^2} + \frac{(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2} + \frac{(b - 4ac)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8ad^2}$$

Rubi [A] time = 0.47, antiderivative size = 181, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{b(4ac + b)\sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}} \right)}{8a^{3/2}d^2 \sqrt{a(c + dx^2) + b}} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}} (a(c + dx^2) + b)}{4ad^2} - \frac{(4ac + b)(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{8ad^2}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[a + b/(c + d*x^2)],x]

[Out] -((b + 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(8*a*d^2) + ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*a*d^2) - (b*(b + 4*a*c)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(3/2)*d^2*Sqrt[b + a*(c + d*x^2)])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 80

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p
+ 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int x^3 \sqrt{a + \frac{b}{c + dx^2}} dx &= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \int \frac{x^3 \sqrt{b + a(c + dx^2)}}{\sqrt{c + dx^2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \int \frac{x^3 \sqrt{b + ac + adx^2}}{\sqrt{c + dx^2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \text{Subst}\left(\int \frac{x \sqrt{b + ac + adx}}{\sqrt{c + dx}} dx, x, x^2\right)}{2\sqrt{b + a(c + dx^2)}} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4ad^2} - \frac{\left((b + 4ac)\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b}}{\sqrt{c + dx}} dx, x, x^2\right)}{8ad\sqrt{b + a(c + dx^2)}} \\
&= -\frac{(b + 4ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{8ad^2} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4ad^2} - \frac{b(b + 4ac)}{8ad\sqrt{b + a(c + dx^2)}} \\
&= -\frac{(b + 4ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{8ad^2} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4ad^2} - \frac{b(b + 4ac)}{8ad\sqrt{b + a(c + dx^2)}} \\
&= -\frac{(b + 4ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{8ad^2} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4ad^2} - \frac{b(b + 4ac)}{8ad\sqrt{b + a(c + dx^2)}} \\
&= -\frac{(b + 4ac)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{8ad^2} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4ad^2} - \frac{b(b + 4ac)}{8ad\sqrt{b + a(c + dx^2)}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 97, normalized size = 0.69

$$\frac{\sqrt{a} (c + dx^2) \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (-2ac + 2adx^2 + b) - b(4ac + b) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[a + b/(c + d*x^2)],x]

[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b - 2*a*c + 2*a*d*x^2) - b*(b + 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a^(3/2)*d^2)

IntegrateAlgebraic [A] time = 0.19, size = 116, normalized size = 0.82

$$\frac{(-4abc - b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}}}{\sqrt{a}}\right)}{8a^{3/2}d^2} + \frac{\sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (-2ac^2 + 2ad^2x^4 + bc + bdx^2)}{8ad^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*Sqrt[a + b/(c + d*x^2)], x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(b*c - 2*a*c^2 + b*d*x^2 + 2*a*d^2*x^4)/(8*a*d^2) + ((-b^2 - 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a]))/(8*a^(3/2)*d^2)

fricas [A] time = 0.73, size = 325, normalized size = 2.30

$$\frac{(4abc + b^2)\sqrt{a} \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2c^2 + ab)dx^2 + 8abc + b^2 - 4(2ad^2x^4 + (4ac + b)dx^2 + 2ac^2 + bc)\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{32a^2d^2}\right) + 4(2a^2d^2x^4 + abdx^2 - 2a^2c^2 + abc)\sqrt{\frac{ad^2+ac+b}{d^2+c}} + (4abc + b^2)\sqrt{-a} \arctan\left(\frac{(2ab^2+2ac^2)\sqrt{-a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{2(d^2x^2+a^2+ab)}\right) + 2(2a^2d^2x^4 + abdx^2 - 2a^2c^2 + abc)\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{16a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2), x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2), 1/16*((4*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(2*a^2*d^2*x^4 + a*b*d*x^2 - 2*a^2*c^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2)]

giac [A] time = 0.44, size = 159, normalized size = 1.13

$$\frac{1}{16} \left(\frac{2\sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc} \left(\frac{2x^2}{d} - \frac{2acd - bd}{ad^3} \right) + \frac{(4abc + b^2) \log\left(\left| -2acd - 2\left(\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}}\right)\sqrt{a}|d| - bd \right| \right)}{a^2d|d|} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2), x, algorithm="giac")

[Out] 1/16*(2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/d - (2*a*c*d - b*d)/(a*d^3)) + (4*a*b*c + b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(3/2)*d*abs(d)))*sgn(d*x^2 + c)

maple [B] time = 0.04, size = 354, normalized size = 2.51

$$\frac{\sqrt{\frac{ad^2+ac+b}{d^2+c}} (dx^2+c) \left(-4abcd \ln\left(\frac{2a^2d^2+2ad^2bd+2\sqrt{a}b^2+2ad^2bd^2+2a^2bc\sqrt{a}}{2\sqrt{a}d^2}\right) + 4\sqrt{a}d^2x^4 + 2acd^2 + bdx^2 + a^2 + bc\sqrt{a}d^2 - b^2d \ln\left(\frac{2a^2d^2+2ad^2bd+2\sqrt{a}b^2+2ad^2bd^2+2a^2bc\sqrt{a}}{2\sqrt{a}d^2}\right) - 4\sqrt{a}d^2x^4 + 2acd^2 + bdx^2 + a^2 + bc\sqrt{a}d^2 - ac + 2\sqrt{a}d^2x^4 + 2acd^2 + bdx^2 + a^2 + bc\sqrt{a}d^2 \right)}{16\sqrt{(dx^2+c)(ad^2+ac+b)}\sqrt{a}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b/(d*x^2+c))^(1/2), x)

[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*x^2*a*d-4*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2)*a*b*c*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*a*c-ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2)*b^2*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/(a*d^2)^(1/2)

maxima [A] time = 1.75, size = 218, normalized size = 1.55

$$\frac{(4abc - b^2) \left(\frac{adx^2+ac+b}{dx^2+c} \right)^{\frac{3}{2}} - (4a^2bc + ab^2) \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8 \left(a^3d^2 - \frac{2(adx^2+ac+b)a^2d^2}{dx^2+c} + \frac{(adx^2+ac+b)^2ad^2}{(dx^2+c)^2} \right)} + \frac{(4ac + b)b \log\left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}} \right)}{16a^{\frac{3}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out]
$$-1/8*((4*a*b*c - b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{3/2} - (4*a^2*b*c + a*b^2)*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(a^3*d^2 - 2*(a*d*x^2 + a*c + b)*a^2*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a*d^2/(d*x^2 + c)^2) + 1/16*(4*a*c + b)*b*\log(-(\sqrt{a} - \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(\sqrt{a} + \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/(a^{3/2}*d^2)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \sqrt{a + \frac{b}{d x^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b/(c + d*x^2))^(1/2),x)

[Out] int(x^3*(a + b/(c + d*x^2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

$$3.132 \quad \int x \sqrt{a + \frac{b}{c+dx^2}} dx$$

Optimal. Leaf size=69

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1591, 242, 47, 63, 208}

$$\frac{(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{2d} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a + b/(c + d*x^2)],x]

[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(2*d) + (b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
```

PolyQ[Qr, x]

Rubi steps

$$\begin{aligned}
\int x \sqrt{a + \frac{b}{c + dx^2}} dx &= \frac{\text{Subst}\left(\int \sqrt{a + \frac{b}{x}} dx, x, c + dx^2\right)}{2d} \\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x^2} dx, x, \frac{1}{c+dx^2}\right)}{2d} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2d} - \frac{b \text{Subst}\left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \frac{1}{c+dx^2}\right)}{4d} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2d} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{c+dx^2}}\right)}{2d} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a} d}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.12

$$\frac{\sqrt{a} (c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}} + b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a} d}$$

Antiderivative was successfully verified.

`[In] Integrate[x*Sqrt[a + b/(c + d*x^2)], x]``[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)`**IntegrateAlgebraic [A]** time = 0.12, size = 85, normalized size = 1.23

$$\frac{(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2d} + \frac{b \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2\sqrt{a} d}$$

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[x*Sqrt[a + b/(c + d*x^2)], x]``[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(2*d) + (b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*Sqrt[a]*d)`**fricas [A]** time = 0.76, size = 267, normalized size = 3.87

$$\frac{\sqrt{a} b \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 + 4(2ad^2x^4 + (4ac + b)dx^2 + 2ac^2 + bc)\sqrt{a}\sqrt{\frac{ad^2x^2+b}{dx^2+c}}}{8ad}\right) + 4(adx^2 + ac)\sqrt{\frac{ad^2x^2+b}{dx^2+c}} - \sqrt{-a} b \arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{ad^2x^2+b}{dx^2+c}}}{2(a^2dx^2+a^2c+ab)}\right) - 2(adx^2 + ac)\sqrt{\frac{ad^2x^2+b}{dx^2+c}}}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c))*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d), -1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b) - 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d)]

giac [B] time = 0.51, size = 127, normalized size = 1.84

$$-\frac{1}{4} \left(\frac{b \log \left(\left| -8 a^3 c d - 8 \left(\sqrt{a d^2} x^2 - \sqrt{a d^2} x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c \right) a | d | - 4 \sqrt{a} b d \right| \right)}{\sqrt{a} |d|} - \frac{2 \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c}}{d} \right) \operatorname{sgn}(d x^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] -1/4*(b*log(abs(-8*a^(3/2)*c*d - 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*abs(d) - 4*sqrt(a)*b*d))/(sqrt(a)*abs(d)) - 2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/d)*sgn(d*x^2 + c)

maple [B] time = 0.02, size = 180, normalized size = 2.61

$$\frac{\sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} (d x^2 + c) \left(b d \ln \left(\frac{2 a d^2 x^2 + 2 a c d + b d + 2 \sqrt{a d^2} x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c \sqrt{a d^2}}{2 \sqrt{a d^2}} \right) + 2 \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c} \sqrt{a d^2} \right)}{4 \sqrt{(d x^2 + c) (a d x^2 + a c + b)} \sqrt{a d^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b/(d*x^2+c))^(1/2),x)

[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(b*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2)*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/d/(a*d^2)^(1/2)

maxima [B] time = 1.64, size = 126, normalized size = 1.83

$$-\frac{b \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{2 \left(a d - \frac{(a d x^2 + a c + b) d}{d x^2 + c} \right)} - \frac{b \log \left(-\frac{\sqrt{a} - \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{\sqrt{a} + \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}} \right)}{4 \sqrt{a} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*d - (a*d*x^2 + a*c + b)*d/(d*x^2 + c)) - 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d)

mupad [B] time = 3.01, size = 120, normalized size = 1.74

$$\frac{\sqrt{\frac{b(d x^2 + c) + a(d x^2 + c)^2}{(d x^2 + c)^2}} (d x^2 + c) \left(\frac{b \ln \left(\frac{\frac{b}{2} + a(d x^2 + c) + \sqrt{a} \sqrt{b(d x^2 + c) + a(d x^2 + c)^2}}{\sqrt{a}} \right)}{\sqrt{a} \sqrt{b(d x^2 + c) + a(d x^2 + c)^2}} + 2 \right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b/(c + d*x^2))^(1/2),x)`

[Out] `((b*(c + d*x^2) + a*(c + d*x^2)^2)/(c + d*x^2)^2)^(1/2)*(c + d*x^2)*((b*log((b/2 + a*(c + d*x^2) + a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2))/a^(1/2)))/(a^(1/2)*(b*(c + d*x^2) + a*(c + d*x^2)^2)^(1/2)) + 2))/(4*d)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b/(d*x**2+c))**(1/2),x)`

[Out] `Integral(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)`

$$3.133 \quad \int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x} dx$$

Optimal. Leaf size=96

$$\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right) - \frac{\sqrt{ac+b} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{\sqrt{c}}$$

Rubi [A] time = 0.43, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}} \right)}{\sqrt{a(c+dx^2)+b}} - \frac{\sqrt{ac+b} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{\sqrt{c} \sqrt{a(c+dx^2)+b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/(c + d*x^2)]/x,x]

[Out] (Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]/Sqrt[b + a*(c + d*x^2)] - (Sqrt[b + a*c]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/Sqrt[c]*Sqrt[b + a*(c + d*x^2)]])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 105

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] := Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\text{Sqrt}[a + b \cdot x^2]] \text{ /; FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_ \cdot)} \cdot ((c_ + (d_ \cdot)(x_)^{(n_)})^{(q_ \cdot)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q, x], x, x^n], x] \text{ /; FreeQ}\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1975

$\text{Int}[(u_)^{(p_ \cdot)} \cdot (v_)^{(q_ \cdot)} \cdot ((e_ \cdot)(x_)^{(m_ \cdot)}), x_Symbol] \rightarrow \text{Int}[(e \cdot x)^m \cdot \text{ExpandToSum}[u, x]^p \cdot \text{ExpandToSum}[v, x]^q, x] \text{ /; FreeQ}\{e, m, p, q\}, x] \ \&\& \ \text{BinomialQ}\{u, v\}, x] \ \&\& \ \text{EqQ}[\text{BinomialDegree}[u, x] - \text{BinomialDegree}[v, x], 0] \ \&\& \ !\text{BinomialMatchQ}\{u, v\}, x]$

Rule 6722

$\text{Int}[(u_ \cdot) \cdot ((a_ \cdot) + (b_ \cdot)(v_)^{(n_)})^{(p_ \cdot)}, x_Symbol] \rightarrow \text{Dist}[(a + b \cdot v^n)^{\text{FracPart}[p]} / (v^{(n \cdot \text{FracPart}[p])} \cdot (b + a/v^n)^{\text{FracPart}[p]}), \text{Int}[u \cdot v^{(n \cdot p)} \cdot (b + a/v^n)^p, x], x] \text{ /; FreeQ}\{a, b, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ \text{ILtQ}[n, 0] \ \&\& \ \text{BinomialQ}[v, x] \ \&\& \ !\text{LinearQ}[v, x]$

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+a(c+dx^2)}}{x\sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+ac+adx^2}}{x\sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x\sqrt{c+dx}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
&= -\frac{\left((-b-ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2\right) + \left(ad\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(a\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2}\right) - \left((-b-ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{b+a(c+dx^2)}} \\
&= -\frac{\sqrt{b+ac} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+a(c+dx^2)}}\right) + \left(a\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{c} \sqrt{b+a(c+dx^2)}} \\
&= \frac{\sqrt{a} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right) - \sqrt{b+ac} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+a(c+dx^2)}}
\end{aligned}$$

Mathematica [A] time = 0.14, size = 80, normalized size = 0.83

$$\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{\sqrt{ac+b} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x,x]

[Out] Sqrt[a]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (Sqrt[b + a*c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2))]/Sqrt[b + a*c]])/Sqrt[c]

IntegrateAlgebraic [A] time = 0.18, size = 108, normalized size = 1.12

$$\frac{\sqrt{-ac-b} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{\sqrt{c}} + \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x,x]

```
[Out] (Sqrt[-b - a*c]*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2))]/(b + a*c)]/Sqrt[c] + Sqrt[a]*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]
```

fricas [B] time = 0.92, size = 927, normalized size = 9.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 1/4*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4), 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c))/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 1/4*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))), -1/2*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 1/2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c))/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2))]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(d*x^2+c)]Error: Bad Argument Type
```

maple [B] time = 0.04, size = 235, normalized size = 2.45

$$\frac{\sqrt{\frac{ad^2x^2+ac+b}{dx^2+c}}(dx^2+c)\left(-acd\ln\left(\frac{2ad^2x^2+2acd+bd+2\sqrt{ad^2x^2+2acd+bdx^2+ac^2+bc}\sqrt{ad^2}}{2\sqrt{ad^2}}\right)+\sqrt{ac^2+bc}\sqrt{ad^2}\ln\left(\frac{2acd^2x^2+bdx^2+2ac^2+2bc+2\sqrt{ac^2+bc}\sqrt{ad^2x^2+2acd+bdx^2+ac^2+bc}}{x^2}\right)\right)}{2\sqrt{(dx^2+c)(ad^2x^2+ac+b)}\sqrt{ad^2}c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(1/2)/x,x)
```

```
[Out] -1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*a*c*d+(a*c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)+2*b*c)/x^2)*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/c/(a*d^2)^(1/2)
```

maxima [A] time = 1.46, size = 159, normalized size = 1.66

$$\frac{(ac + b) \log \left(\frac{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}} \right)}{2 \sqrt{(ac + b)c}} - \frac{1}{2} \sqrt{a} \log \left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")

[Out] 1/2*(a*c + b)*log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/sqrt((a*c + b)*c) - 1/2*sqrt(a)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(1/2)/x,x)

[Out] int((a + b/(c + d*x^2))^(1/2)/x, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))**(1/2)/x,x)

[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x, x)

$$3.134 \quad \int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3} dx$$

Optimal. Leaf size=104

$$\frac{bd \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{2c^{3/2} \sqrt{ac+b}} - \frac{(c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

Rubi [A] time = 0.39, antiderivative size = 140, normalized size of antiderivative = 1.35, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{bd \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{2c^{3/2} \sqrt{ac+b} \sqrt{a(c+dx^2)+b}} - \frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/(c + d*x^2)]/x^3,x]

[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(2*c*x^2) + (b*d*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2)]/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])))/(2*c^(3/2)*Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)))/((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[(n*(d*e - c*f))/(m + 1)*(b*e - a*f), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^3} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+a(c+dx^2)}}{x^3 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\ &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+ac+adx^2}}{x^3 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\ &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^2 \sqrt{c+dx}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\ &= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2cx^2} - \frac{(bd\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2\right)}{4c\sqrt{b+a(c+dx^2)}} \\ &= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2cx^2} - \frac{(bd\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}) \text{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2} dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{2c\sqrt{b+a(c+dx^2)}} \\ &= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2cx^2} + \frac{bd\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+a(c+dx^2)}}\right)}{2c^{3/2} \sqrt{b+ac} \sqrt{b+a(c+dx^2)}} \end{aligned}$$

Mathematica [B] time = 0.47, size = 212, normalized size = 2.04

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(-2\sqrt{c(ac+b)}(c+dx^2)(ac+adx^2+b) - 2bdx^2 \log(x)\sqrt{(c+dx^2)(a(c+dx^2)+b)} + bdx^2\sqrt{(c+dx^2)(ac+adx^2+b)} \log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)} + 2ac(c+dx^2) + b(2c+dx^2)\right)\right)}{4cx^2\sqrt{c(ac+b)}(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^3, x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]]) / (4*c*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))

IntegrateAlgebraic [A] time = 0.19, size = 120, normalized size = 1.15

$$\frac{bd \tan^{-1} \left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b} \right)}{2c^{3/2} \sqrt{-ac-b}} + \frac{(-c-dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2cx^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^3,x]

[Out] ((-c - d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*c*x^2) + (b*d*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(2*c^(3/2)*Sqrt[-b - a*c])

fricas [B] time = 0.64, size = 433, normalized size = 4.16

$$\frac{\sqrt{ac^2+bc} b dx^2 \log \left(\frac{(8d^2x^2+8abc+8b^2)^2 d^4+8d^2c^3+16abc^3+8b^2c^2+8(2d^2c^2+3abd^2+b^2)d^2+4((2ac+b)d^2+2ac^2+(4ac^2+3bc)d^2+2b^2)\sqrt{ac^2+bc}\sqrt{\frac{ac^2+bc}{d^2+cx}}}{x^4} \right) - 4(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{ac^2+bc}{d^2+cx}} - \sqrt{-ac^2-bc} b dx^2 \arctan \left(\frac{(2ac+b)d^2+2ac^2+2b^2)\sqrt{-ac^2-bc}\sqrt{\frac{ac^2+bc}{d^2+cx}}}{2(d^2c^2+3abd^2+b^2)d^2} \right) + 2(ac^3+(ac^2+bc)dx^2+bc^2)\sqrt{\frac{ac^2+bc}{d^2+cx}}}{8(ac^3+bc^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")

[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3 + b*c^2)*x^2), -1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^3 + b*c^2)*x^2)]

giac [B] time = 0.46, size = 281, normalized size = 2.70

$$\frac{1}{2} \left(\frac{bd \arctan \left(-\frac{\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}}}{\sqrt{-ac^2 - bc}} \right)}{\sqrt{-ac^2 - bc}} + \frac{2a^{\frac{3}{2}}c^2|d| + 2(\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}})acd + 2\sqrt{a}bc|d| + (\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}})bd}{(ac^2 - (\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}})^2 + bc)c} \right) \operatorname{sgn}(dx^2 + c)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")

[Out] -1/2*(b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*c) + (2*a^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*c))*sgn(dx^2 + c)

maple [B] time = 0.05, size = 454, normalized size = 4.37

$$\frac{\sqrt{\frac{ad^2x^2}{d^2+c}}(d^2+c) \left(-bd^2d^2x^2 \ln \left(\frac{bd^2d^2x^2+2bd^2c+2bd^2c\sqrt{\frac{ad^2x^2}{d^2+c}}}{2\sqrt{bd^2+2acd^2+bd^2x^2+ac^2+bc}} \sqrt{\frac{ad^2x^2}{d^2+c}} \right) - 2\sqrt{bd^2+2acd^2+bd^2x^2+ac^2+bc} \sqrt{\frac{ad^2x^2}{d^2+c}} \ln \left(\frac{bd^2d^2x^2+2bd^2c+2bd^2c\sqrt{\frac{ad^2x^2}{d^2+c}}}{4\sqrt{(d^2+c)(ad^2+ac+b)}} \right) - 4\sqrt{bd^2+2acd^2+bd^2x^2+ac^2+bc} \sqrt{\frac{ad^2x^2}{d^2+c}} \right) - 4\sqrt{bd^2+2acd^2+bd^2x^2+ac^2+bc} \sqrt{\frac{ad^2x^2}{d^2+c}} \ln \left(\frac{bd^2d^2x^2+2bd^2c+2bd^2c\sqrt{\frac{ad^2x^2}{d^2+c}}}{4\sqrt{(d^2+c)(ad^2+ac+b)}} \right) - 4\sqrt{bd^2+2acd^2+bd^2x^2+ac^2+bc} \sqrt{\frac{ad^2x^2}{d^2+c}}}{4\sqrt{(d^2+c)(ad^2+ac+b)}}(ac+b)\sqrt{\frac{ad^2x^2}{d^2+c}} \sqrt{\frac{ad^2x^2}{d^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(1/2)/x^3,x)

[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)-ln((2*a*c*d*x^2+b*d*x

$$\begin{aligned} & \int \frac{(2ax^2 + 2bx + 2c)\sqrt{ax^2 + bx + c}}{x^3} dx \\ &= \frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2 + bc - \frac{(adx^2+ac+b)c^2}{dx^2+c}\right)} - \frac{bd \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c} \end{aligned}$$

maxima [A] time = 1.48, size = 156, normalized size = 1.50

$$\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^2 + bc - \frac{(adx^2+ac+b)c^2}{dx^2+c}\right)} - \frac{bd \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")

[Out]
$$-1/2*b*d*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}/(a*c^2 + b*c - (a*d*x^2 + a*c + b)*c^2/(d*x^2 + c)) - 1/4*b*d*\log((c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} - \sqrt{(a*c + b)*c})/(c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} + \sqrt{(a*c + b)*c}))/(\sqrt{(a*c + b)*c})*c$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(1/2)/x^3,x)

[Out] int((a + b/(c + d*x^2))^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))**(1/2)/x**3,x)

[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**3, x)

$$3.135 \quad \int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^5} dx$$

Optimal. Leaf size=174

$$\frac{bd^2(4ac + 3b) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{8c^{5/2}(ac + b)^{3/2}} + \frac{d(4ac + 5b)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^2x^2(ac + b)} - \frac{(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4c^2x^4}$$

Rubi [A] time = 0.51, antiderivative size = 218, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{bd^2(4ac + 3b)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{8c^{5/2}(ac + b)^{3/2} \sqrt{a(c + dx^2) + b}} + \frac{d(4ac + 3b)(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{8c^2x^2(ac + b)} - \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}} (a(c + dx^2) + b)}{4cx^4(ac + b)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/(c + d*x^2)]/x^5,x]

[Out] ((3*b + 4*a*c)*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(8*c^2*(b + a*c)*x^2) - ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*c*(b + a*c)*x^4) - (b*(3*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTan h[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(5/2)*(b + a*c)^(3/2)*Sqrt[b + a*(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^5} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+a(c+dx^2)}}{x^5 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
 &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+ac+adx^2}}{x^5 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
 &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3 \sqrt{c+dx}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
 &= \frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{4c(b+ac)x^4} - \frac{\left((3b+4ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^3 \sqrt{c+dx}} dx, x, x^2\right)}{8c(b+ac)\sqrt{b+a(c+dx^2)}} \\
 &= \frac{(3b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{4c(b+ac)x^4} + \frac{b(3b+4ac)}{8c^2(b+ac)x^2} \\
 &= \frac{(3b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{4c(b+ac)x^4} + \frac{b(3b+4ac)}{8c^2(b+ac)x^2} \\
 &= \frac{(3b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{8c^2(b+ac)x^2} - \frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{4c(b+ac)x^4} - \frac{b(3b+4ac)}{8c^2(b+ac)x^2}
 \end{aligned}$$

Mathematica [A] time = 0.48, size = 278, normalized size = 1.60

$$\frac{\sqrt{\frac{ac+abd^2}{c+dx^2}} \left(2\sqrt{c(ac+b)}(c+dx^2)(2d^2c(c-d^2x^2)+ab(4c^2-3cdx^2-3d^2x^4)+b^2(2c-3d^2)) - 2bd^2x^4 \log(4ac+3b)\sqrt{c+dx^2}(a(c+dx^2)+b) + bd^2x^4(4ac+3b)\sqrt{c+dx^2}(a(c+dx^2)+b) \log(2\sqrt{c(ac+b)}\sqrt{c+dx^2}(ac+adx^2+b) + 2ac(c+dx^2)+b(2c+dx^2))\right)}{16cx^4(c+ac+b)^{3/2}(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b^2*(2*c - 3*d*x^2) + a*b*(4*c^2 - 3*c*d*x^2 - 3*d^2*x^4) + 2*a^2*c*(c^2 - d^2*x^4)) - 2*b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + b*(3*b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/(c*(c*(b + a*c))^(3/2)*x^4*(b + a*(c + d*x^2)))
```

IntegrateAlgebraic [A] time = 0.33, size = 168, normalized size = 0.97

$$\frac{d^2(4abc + 3b^2) \tan^{-1}\left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right) + \sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-2ac^3 + 2acd^2x^4 - 2bc^2 + bcdx^2 + 3bd^2x^4)}{8c^{5/2}(-ac - b)^{3/2} + 8c^2x^4(ac + b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + b*c*d*x^2 + 3*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^2*(b + a*c)*x^4) + (((3*b^2 + 4*a*b*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[-b - a*c])*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/(b + a*c)))/(8*c^(5/2)*(-b - a*c)^(3/2))
```

fricas [A] time = 0.93, size = 577, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4 - 4*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4), 1/16*((4*a*b*c + 3*b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 5*a*b*c^2 + 3*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - (a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*x^4)]
```

giac [B] time = 0.58, size = 713, normalized size = 4.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*((4*a*b*c*d^2 + 3*b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a*c^3 + b*c^2)*sqrt(-a*c^2 - b*c)) + (8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*
```

$$c^3 d \operatorname{abs}(d) + 24 a^{5/2} b^4 d \operatorname{abs}(d) + 36 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c}) a^2 b^3 c^3 d^2 + 8 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c})^2 a^{3/2} b^2 c^3 d \operatorname{abs}(d) + 24 a^{3/2} b^2 c^3 d \operatorname{abs}(d) - 4 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c})^3 a b^3 c^3 d^2 + 25 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c}) a b^2 c^2 d^2 + 8 \sqrt{a} b^3 c^2 d \operatorname{abs}(d) - 3 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c})^3 b^2 d^2 + 5 (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c}) b^3 c^3 d^2 / ((a c^3 + b c^2) (a c^2 - (\sqrt{a d^2} x^2 - \sqrt{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c})^2 + b c)^2) \operatorname{sgn}(d x^2 + c)$$

maple [B] time = 0.06, size = 923, normalized size = 5.30

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b/(d*x^2+c))^(1/2)/x^5,x)`

[Out] $\frac{1}{16} \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{1/2} (d x^2 + c) (-12 a^2 d^3 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} x^6 c (a c^2 + b c)^{3/2} - 4 \ln((2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2} (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}) / x^2) x^4 a^3 b^3 c^5 d^2 - 10 a d^3 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} x^6 b (a c^2 + b c)^{3/2} - 11 \ln((2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2} (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}) / x^2) x^4 a^2 b^2 c^4 d^2 - 20 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} a^2 c^2 d^2 x^4 (a c^2 + b c)^{3/2} - 10 \ln((2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2} (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}) / x^2) x^4 a b^3 c^3 d^2 - 28 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} a c d^2 b x^4 (a c^2 + b c)^{3/2} - 3 \ln((2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2} (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}) / x^2) x^4 b^4 c^2 d^2 - 10 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} b^2 d^2 x^4 (a c^2 + b c)^{3/2} + 12 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} a c d x^2 (a c^2 + b c)^{3/2} + 10 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} b d x^2 (a c^2 + b c)^{3/2} - 4 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} a c^2 - 4 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} b c) / ((d x^2 + c) (a d x^2 + a c + b))^{1/2} / c^3 / (a c + b)^2 / x^4 / (a c^2 + b c)^{3/2}$

maxima [B] time = 1.53, size = 322, normalized size = 1.85

$$\frac{(4 a b c + 3 b^2) d^2 \log\left(\frac{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} - \sqrt{(a c + b) c}}{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} + \sqrt{(a c + b) c}}\right)}{16 (a c^3 + b c^2) \sqrt{(a c + b) c}} - \frac{(4 a b c^2 + 5 b^2 c) d^2 \left(\frac{a d x^2 + a c + b}{d x^2 + c}\right)^{3/2} - (4 a^2 b c^2 + 7 a b^2 c + 3 b^3) d^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{8 \left(a^3 c^5 + 3 a^2 b c^4 + 3 a b^2 c^3 + b^3 c^2 + \frac{(a c^5 + b c^4) (a d x^2 + a c + b)^2}{(d x^2 + c)^2} - \frac{2 (a^2 c^5 + 2 a b c^4 + b^2 c^3) (a d x^2 + a c + b)}{d x^2 + c} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")`

[Out] $\frac{1}{16} (4 a b^3 c + 3 b^2) d^2 \log\left(\frac{c \sqrt{(a d x^2 + a c + b) / (d x^2 + c)}}{c \sqrt{(a d x^2 + a c + b) / (d x^2 + c)} + \sqrt{(a c + b) c}}\right) - \sqrt{(a c + b) c} / (c \sqrt{(a d x^2 + a c + b) / (d x^2 + c)} + \sqrt{(a c + b) c}) - 1/8 ((4 a b^3 c^2 + 5 b^2 c) d^2 ((a d x^2 + a c + b) / (d x^2 + c))^{3/2} - (4 a^2 b^3 c^2 + 7 a b^2 c^3 + 3 b^3) d^2 \sqrt{(a d x^2 + a c + b) / (d x^2 + c)}) / (a^3 c^5 + 3 a^2 b c^4 + 3 a b^2 c^3 + b^3 c^2 + (a c^5 + b c^4) (a d x^2 + a c + b)^2 / (d x^2 + c)^2 - 2 (a^2 c^5 + 2 a b c^4 + b^2 c^3) (a d x^2 + a c + b) / (d x^2 + c))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(1/2)/x^5,x)

[Out] int((a + b/(c + d*x^2))^(1/2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))*(1/2)/x**5,x)

[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**5, x)

$$3.136 \quad \int \frac{\sqrt{a + \frac{b}{c + dx^2}}}{x^7} dx$$

Optimal. Leaf size=265

$$\frac{bd^3 (8a^2c^2 + 12abc + 5b^2) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right) d^2 (8a^2c^2 + 20abc + 11b^2) (c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}} + d(4ac + 3b^2)}{16c^{7/2}(ac + b)^{5/2} - 16c^3x^2(ac + b)^2}$$

Rubi [A] time = 0.61, antiderivative size = 271, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, integrand size = 21, number of rules / integrand size = 0.381, Rules used = {6722, 1975, 446, 99, 151, 12, 93, 208}

$$\frac{bd^3 (8a^2c^2 + 12abc + 5b^2) \sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right) - d^2(2ac + 5b)(4ac + 3b) (c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} + \frac{d(4ac + 5b) (c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{24c^2x^4(ac + b)} - \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{6cx^6}}{16c^{7/2}(ac + b)^{5/2} \sqrt{a(c + dx^2) + b} - 48c^3x^2(ac + b)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b/(c + d*x^2)]/x^7, x]

[Out] -((c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(6*c*x^6) + ((5*b + 4*a*c)*d*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(24*c^2*(b + a*c)*x^4) - ((5*b + 2*a*c)*(3*b + 4*a*c)*d^2*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(48*c^3*(b + a*c)^2*x^2) + (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2)]/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(16*c^(7/2)*(b + a*c)^(5/2)*Sqrt[b + a*(c + d*x^2)])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 99

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)))/((m + 1)*(b*e - a*f)), x] - Dist[1/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n + p + 2)*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h]*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]

, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && !LtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + \frac{b}{c+dx^2}}}{x^7} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+a(c+dx^2)}}{x^7 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{\sqrt{b+ac+adx^2}}{x^7 \sqrt{c+dx^2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x^4 \sqrt{c+dx}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{-\frac{1}{2}(5b+4ac)d-2ad^2x}{x^3 \sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2\right)}{6c\sqrt{b+a(c+dx^2)}} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{d}{x^3 \sqrt{c+dx}} dx, x, x^2\right)}{12c^2} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{48c^3(b+ac)^2} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{48c^3(b+ac)^2} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{48c^3(b+ac)^2} \\
&= -\frac{(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{(5b+4ac)d(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{24c^2(b+ac)x^4} - \frac{(5b+2ac)(3b+4ac)d^2(c+dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{48c^3(b+ac)^2}
\end{aligned}$$

Mathematica [A] time = 0.50, size = 245, normalized size = 0.92

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(\frac{3bd^3(8a^2+12abc+5b^2)(c+dx^2) \left(2\log(x) - \log\left(2\sqrt{c(ac+b)} \sqrt{c+dx^2} \sqrt{ac+adx^2+b} + 2ac(c+dx^2) + b(2c+dx^2) \right) \right)}{\sqrt{c(ac+b)} \sqrt{(c+dx^2)(a(c+dx^2)+b)}} + 2d^3(8a^2c^2 + 26abc + 15b^2) + \frac{16c^3(ac+b)^2}{x^6} - \frac{4bc^2d(ac+b)}{x^4} + \frac{2bcd^2(8ac+5b)}{x^2} \right)}{96c^3(ac+b)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b/(c + d*x^2)]/x^7, x]

[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(15*b^2 + 26*a*b*c + 8*a^2*c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (4*b*c^2*(b + a*c)*d)/x^4 + (2*b*c*(5*b + 8*a*c)*d^2)/x^2 + (3*b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*d^3*(c + d*x^2)*(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/((Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)])))/(c^3*(b + a*c)^2)

IntegrateAlgebraic [A] time = 0.52, size = 248, normalized size = 0.94

$$\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}(-8a^2c^5-8a^2c^2d^3x^6-16abc^4+2abc^3dx^2-8abc^2d^2x^4-26abcd^3x^6-8b^2c^3+2b^2c^2dx^2-5b^2cd^2x^4-15b^2d^3x^6)}{48c^3x^6(ac+b)^2} + \frac{d^3(8a^2bc^2+12ab^2c+5b^3)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{ac+b}\right)}{16c^{7/2}(-ac-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b/(c + d*x^2)]/x^7,x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-8*b^2*c^3 - 16*a*b*c^4 - 8*a^2*c^5 + 2*b^2*c^2*d*x^2 + 2*a*b*c^3*d*x^2 - 5*b^2*c*d^2*x^4 - 8*a*b*c^2*d^2*x^4 - 15*b^2*d^3*x^6 - 26*a*b*c*d^3*x^6 - 8*a^2*c^2*d^3*x^6))/(48*c^3*(b + a*c)^2*x^6) + ((5*b^3 + 12*a*b^2*c + 8*a^2*b*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[-b - a*c])*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/(b + a*c))/(16*c^(7/2)*(-b - a*c)^(5/2))

fricas [A] time = 1.53, size = 755, normalized size = 2.85

$$\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}(-8a^2c^5-8a^2c^2d^3x^6-16abc^4+2abc^3dx^2-8abc^2d^2x^4-26abcd^3x^6-8b^2c^3+2b^2c^2dx^2-5b^2cd^2x^4-15b^2d^3x^6)}{48c^3x^6(ac+b)^2} + \frac{d^3(8a^2bc^2+12ab^2c+5b^3)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{ac+b}\right)}{16c^{7/2}(-ac-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")

[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6), -1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 34*a^2*b*c^3 + 41*a*b^2*c^2 + 15*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (8*a^2*b*c^4 + 13*a*b^2*c^3 + 5*b^3*c^2)*d^2*x^4 - 2*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^7 + 3*a^2*b*c^6 + 3*a*b^2*c^5 + b^3*c^4)*x^6)]

giac [B] time = 0.77, size = 1414, normalized size = 5.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")

[Out] -1/48*(3*(8*a^2*b*c^2*d^3 + 12*a*b^2*c*d^3 + 5*b^3*d^3)*arctan(-(sqrt(a*d^2*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c)))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*sqrt(-a*c^2 - b*c)) + (64*a^(11/2)*c^8*d^2*abs(d) + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^5*c^7*d^3 + 192*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(9/2)*c^6*d^2*abs(d) + 304*a^(9/2)*b*c^7*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^4*c^5*d^3 + 744*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^4*b*c^6*d^3 + 528*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(7/2)*b*c^5*d^2*abs(d) + 576*a^(7/2)*b^2*c^6*d^2*abs(d) + 64*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^3*b*c^4*d^3 + 1116*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3

$$\begin{aligned}
& *b^2*c^5*d^3 + 480*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 \\
& + a*c^2 + b*c))^2*a^{(5/2)}*b^2*c^4*d^2*\text{abs}(d) + 544*a^{(5/2)}*b^3*c^5*d^2*\text{abs}(d) \\
& + 24*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 \\
& + b*c))^5*a^2*b*c^2*d^3 - 96*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x \\
& ^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*b^2*c^3*d^3 + 801*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 \\
& + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b^3*c^4*d^3 + 144*(\text{sqrt}(a*d^2)*x^2 \\
& - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^{(3/2)}*b^3*c^3*d^2*\text{abs}(d) \\
& + 256*a^{(3/2)}*b^4*c^4*d^2*\text{abs}(d) + 36*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 \\
& + b*d*x^2 + a*c^2 + b*c))^5*a*b^2*c*d^3 - 136*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 \\
& + b*d*x^2 + a*c^2 + b*c))^3*a*b^3*c^2*d^3 + 270*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x \\
& ^2 + b*d*x^2 + a*c^2 + b*c))*a*b^4*c^3*d^3 + 48*\text{sqrt}(a)*b^5*c^3*d^2*\text{abs}(d) \\
& + 15*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^5*b^3*d^3 \\
& - 40*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*b^4*c*d^3 \\
& + 33*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b^5*c^2*d^3 \\
& /((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*c^2 - (\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a \\
& *c^2 + b*c))^2 + b*c)^3))*\text{sgn}(d*x^2 + c)
\end{aligned}$$

maple [B] time = 0.08, size = 1518, normalized size = 5.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a+b/(d*x^2+c))^{(1/2)}/x^7, x)$

[Out]
$$\begin{aligned}
& -1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^{(1/2)}*(d*x^2+c)*(-24*\ln((2*a*c*d*x^2+b*d*x \\
& x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+ \\
& b*c)^{(1/2}))/x^2)*x^6*a^5*b*c^8*d^3-96*a^3*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+ \\
& 2+a*c^2+b*c)^{(1/2)}*x^8*c^2*(a*c^2+b*c)^{(5/2)}-108*\ln((2*a*c*d*x^2+b*d*x^2+2* \\
& a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2}))/x^2)*x^6*a^4*b^2*c^7*d^3 \\
& -156*a^2*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*x^8*c*b*(a*c^2+b*c)^{(5/2)}-195*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c \\
& ^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2}))/x^2)*x^6*a^3*b^3*c^6*d^3 \\
& -66*a*d^4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*x^8*b^2*(a*c^2+b*c)^{(5/2)}-144*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2 \\
& +b*c)^{(1/2)}*a^3*c^3*d^3*(a*c^2+b*c)^{(5/2)}*x^6-177*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2}))/x^2)*x^6*a^2*b^4*c^5*d^3 \\
& -324*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*a^2*c^2*d^3*b*(a*c^2+b*c)^{(5/2)}*x^6-81*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2}))/x^2)*x^6*a*b^5*c^4*d^3 \\
& -252*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*a*c*d^3*b^2*(a*c^2+b*c)^{(5/2)}*x^6-15*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2}))/x^2)*x^6*b^6*c^3*d^3 \\
& +96*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*a^2*c^2*d^2*(a*c^2+b*c)^{(5/2)}*x^4-66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*b^3*d^3*(a*c^2+b*c)^{(5/2)}*x^6 \\
& +156*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*a*c*d^2*b*(a*c^2+b*c)^{(5/2)}*x^4+66*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*b^2*d^2*(a*c^2+b*c)^{(5/2)}*x^4 \\
& -48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*x^2*a^2*c^3*d-84*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*x^2*a*b*c^2*d \\
& -36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*x^2*b^2*c*d+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*a^2*c^4 \\
& +32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*a*b*c^3+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(5/2)}*b^2*c^2 \\
& /((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/c^4/(a*c+b)^3/x^6/(a*c^2+b*c)^{(5/2)}
\end{aligned}$$

maxima [B] time = 1.91, size = 557, normalized size = 2.10

$$\frac{(8a^2bc^2 + 12ab^2c + 5b^3)d^3 \log\left(\frac{\sqrt{\frac{a^2c^2+ac+b}{d^2+c}} - \sqrt{ac+b}}{\sqrt{\frac{a^2c^2+ac+b}{d^2+c}} + \sqrt{ac+b}}\right)}{32(a^2c^5 + 2abc^4 + b^2c^3)\sqrt{ac+b}} - \frac{3(8a^2bc^4 + 20ab^2c^3 + 11b^3c^2)d^3\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{5}{2}} - 8(6a^2bc^4 + 18a^2b^2c^3 + 17ab^3c^2 + 5b^4c)d^3\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{3}{2}} + 3(8a^4bc^4 + 28a^3b^2c^3 + 37a^2b^3c^2 + 22ab^4c + 5b^5)d^3\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{48\left(a^5c^8 + 5a^4bc^7 + 10a^3b^2c^6 + 10a^2b^3c^5 + 5ab^4c^4 + b^5c^3 - \frac{(a^2c^2+2ab^2c^2+b^2c^2)(ad^2+ac+b)}{(d^2+c)^3} + \frac{3(a^2c^2+3a^2b^2c^2+3ab^2c^2+b^3c^2)(ad^2+ac+b)}{(d^2+c)^2} - \frac{3(a^2c^2+4a^2b^2c^2+6a^2b^2c^2+4ab^3c^2+b^4c^2)(ad^2+ac+b)}{d^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")

[Out]
$$-1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*d^3*\log((c*\sqrt{(a*d*x^2 + a*c + b)}/(d*x^2 + c)) - \sqrt{(a*c + b)*c})/(c*\sqrt{(a*d*x^2 + a*c + b)}/(d*x^2 + c) + \sqrt{(a*c + b)*c}))/((a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*\sqrt{(a*c + b)*c}) - 1/48*(3*(8*a^2*b*c^4 + 20*a*b^2*c^3 + 11*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 18*a^2*b^2*c^3 + 17*a*b^3*c^2 + 5*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 28*a^3*b^2*c^3 + 37*a^2*b^3*c^2 + 22*a*b^4*c + 5*b^5)*d^3*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*c^8 + 5*a^4*b*c^7 + 10*a^3*b^2*c^6 + 10*a^2*b^3*c^5 + 5*a*b^4*c^4 + b^5*c^3 - (a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4)*(a*d*x^2 + a*c + b)/(d*x^2 + c))$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a + \frac{b}{dx^2+c}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(1/2)/x^7,x)

[Out] int((a + b/(c + d*x^2))^(1/2)/x^7, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))**(1/2)/x**7,x)

[Out] Integral(sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))/x**7, x)

$$3.137 \quad \int x^5 \left(a + \frac{b}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=249

$$\frac{(-24a^2c^2 + 60abc + 5b^2)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3} - \frac{b(-24a^2c^2 + 12abc + b^2) \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{16a^{3/2}d^3} - \frac{bc^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^3}$$

Rubi [A] time = 0.73, antiderivative size = 311, normalized size of antiderivative = 1.25, number of steps used = 10, number of rules used = 9, integrand size = 21, number of rules / integrand size = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$\frac{(-24a^2c^2 + 12abc + b^2)(c + dx^2) \sqrt{\frac{b}{c+dx^2}} \sqrt{a + \frac{b}{c+dx^2}} (a(c + dx^2) + b)}{24abd^3} - \frac{(-24a^2c^2 + 12abc + b^2)(c + dx^2) \sqrt{\frac{b}{c+dx^2}}}{16ad^3} - \frac{b(-24a^2c^2 + 12abc + b^2) \sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}} \right)}{16a^{3/2}d^3 \sqrt{a(c+dx^2)+b}} - \frac{c^2 \sqrt{\frac{b}{c+dx^2}} (a(c + dx^2) + b)^2}{bd^3} + \frac{(c + dx^2) \sqrt{\frac{b}{c+dx^2}} (a(c + dx^2) + b)^2}{6ad^3}$$

Antiderivative was successfully verified.

[In] Int[x^5*(a + b/(c + d*x^2))^(3/2), x]

[Out] -((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(16*a*d^3) - ((b^2 + 12*a*b*c - 24*a^2*c^2)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(24*a*b*d^3) - (c^2*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(b*d^3) + ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(6*a*d^3) - (b*(b^2 + 12*a*b*c - 24*a^2*c^2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(16*a^(3/2)*d^3*Sqrt[b + a*(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c + d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1) + c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n

```
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int x^5 \left(a + \frac{b}{c + dx^2} \right)^{3/2} dx &= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \int \frac{x^5 (b + a(c + dx^2))^{3/2}}{(c + dx^2)^{3/2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \int \frac{x^5 (b + ac + adx^2)^{3/2}}{(c + dx^2)^{3/2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \text{Subst} \left(\int \frac{x^2 (b + ac + adx)^{3/2}}{(c + dx)^{3/2}} dx, x, x^2 \right)}{2\sqrt{b + a(c + dx^2)}} \\
&= -\frac{c^2 \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{bd^3} + \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \text{Subst} \left(\int \frac{(b + ac + adx)^{3/2}}{\sqrt{b + a(c + dx^2)}} dx \right)}{bd^3 \sqrt{b + a(c + dx^2)}} \\
&= -\frac{c^2 \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{bd^3} + \frac{(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{6ad^3} \\
&= -\frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{24abd^3} - \frac{c^2 \sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{bd^3} \\
&= -\frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16ad^3} - \frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2)}{24abd^3} \\
&= -\frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16ad^3} - \frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2)}{24abd^3} \\
&= -\frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16ad^3} - \frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2)}{24abd^3} \\
&= -\frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2) \sqrt{a + \frac{b}{c + dx^2}}}{16ad^3} - \frac{(b^2 + 12abc - 24a^2c^2)(c + dx^2)}{24abd^3}
\end{aligned}$$

Mathematica [A] time = 0.41, size = 142, normalized size = 0.57

$$\frac{\sqrt{a} \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (8a^2(c^3 + d^3x^6) - 2ab(47c^2 + 16cdx^2 - 7d^2x^4) + 3b^2(c + dx^2)) - 3b(-24a^2c^2 + 12abc + b^2) \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}} \right)}{48a^{3/2}d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*(c + d*x^2) - 2*a*b*(47*c^2 + 16*c*d*x^2 - 7*d^2*x^4) + 8*a^2*(c^3 + d^3*x^6)) - 3*b*(b^2 + 12*a*b*c - 24*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(3/2)*d^3)

IntegrateAlgebraic [A] time = 0.28, size = 163, normalized size = 0.65

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (8a^2c^3 + 8a^2d^3x^6 - 94abc^2 - 32abcdx^2 + 14abd^2x^4 + 3b^2c + 3b^2dx^2)}{48ad^3} + \frac{(24a^2bc^2 - 12ab^2c - b^3) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{3/2}d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b^2*c - 94*a*b*c^2 + 8*a^2*c^3 + 3*b^2*d*x^2 - 32*a*b*c*d*x^2 + 14*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a*d^3) + ((-b^3 - 12*a*b^2*c + 24*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/(16*a^(3/2)*d^3)

fricas [A] time = 0.85, size = 427, normalized size = 1.71

$$\frac{3(24a^2bc^2 - 12ab^2c - b^3)\sqrt{a}\log\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + 4(8a^2d^3 + 14a^2bd^2 + 8a^2c^3 - 94a^2b^2c - 32abcdx^2 - 32a^2b^2c - 3a^2b^2d^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48ad^3} + \frac{3(24a^2bc^2 - 12ab^2c - b^3)\sqrt{a}\operatorname{arctan}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) - 2(8a^2d^3 + 14a^2bd^2 + 8a^2c^3 - 94a^2b^2c - 32abcdx^2 - 32a^2b^2c - 3a^2b^2d^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{96a^{3/2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/192*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^3), -1/96*(3*(24*a^2*b*c^2 - 12*a*b^2*c - b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b) - 2*(8*a^3*d^3*x^6 + 14*a^2*b*d^2*x^4 + 8*a^3*c^3 - 94*a^2*b*c^2 + 3*a*b^2*c - (32*a^2*b*c - 3*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^3)]

giac [B] time = 2.10, size = 527, normalized size = 2.12

$$\frac{1}{48} \sqrt{a} \sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(\frac{2(4ax^2 \operatorname{sgn}(d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc))}{d} - \frac{4a^3c^2d^6 \operatorname{sgn}(d^2x^2 + c) - 7a^2bd^6 \operatorname{sgn}(d^2x^2 + c)}{a^2d^8} \right) x^2 + \frac{8a^3c^2d^5 \operatorname{sgn}(d^2x^2 + c) - 46a^2b^2c^2d^5 \operatorname{sgn}(d^2x^2 + c) + 3a^2b^2d^5 \operatorname{sgn}(d^2x^2 + c)}{a^2d^8} - \frac{1}{96} (24a^{5/2}b^2c^2 \operatorname{sgn}(d^2x^2 + c) - 12a^{3/2}b^2c^2 \operatorname{sgn}(d^2x^2 + c) - \sqrt{a}b^3 \operatorname{sgn}(d^2x^2 + c)) \log\left(\frac{\sqrt{(-2a^{5/2}c^3d - 6(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc))} \cdot a^2c^2 \operatorname{abs}(d) - 6(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)}{\sqrt{a}d^2x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}\right) - \frac{2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}{\sqrt{a}d^2x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc} \sqrt{a}b^2c^2d - a^{3/2}b^2c^2d - 2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)^3 a \operatorname{abs}(d) - 2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc) a b^2 c^2 \operatorname{abs}(d) - (\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)^2 \sqrt{a} b^2 d)}{a^2 d^2 \operatorname{abs}(d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*(4*a*x^2*sgn(d*x^2 + c)/d - (4*a^3*c*d^6*sgn(d*x^2 + c) - 7*a^2*b*d^6*sgn(d*x^2 + c))/(a^2*d^8)))*x^2 + (8*a^3*c^2*d^5*sgn(d*x^2 + c) - 46*a^2*b^2*c^2*d^5*sgn(d*x^2 + c) + 3*a^2*b^2*d^5*sgn(d*x^2 + c))/(a^2*d^8) - 1/96*(24*a^(5/2)*b^2*c^2*sgn(d*x^2 + c) - 12*a^(3/2)*b^2*c^2*sgn(d*x^2 + c) - sqrt(a)*b^3*sgn(d*x^2 + c))*log(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b^2*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*abs(d) - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*abs(d) - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d))/(a^2*d^2*abs(d))

maple [B] time = 0.07, size = 1018, normalized size = 4.09

$$\frac{1}{48} \sqrt{a} \sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(\frac{2(4ax^2 \operatorname{sgn}(d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc))}{d} - \frac{4a^3c^2d^6 \operatorname{sgn}(d^2x^2 + c) - 7a^2bd^6 \operatorname{sgn}(d^2x^2 + c)}{a^2d^8} \right) x^2 + \frac{8a^3c^2d^5 \operatorname{sgn}(d^2x^2 + c) - 46a^2b^2c^2d^5 \operatorname{sgn}(d^2x^2 + c) + 3a^2b^2d^5 \operatorname{sgn}(d^2x^2 + c)}{a^2d^8} - \frac{1}{96} (24a^{5/2}b^2c^2 \operatorname{sgn}(d^2x^2 + c) - 12a^{3/2}b^2c^2 \operatorname{sgn}(d^2x^2 + c) - \sqrt{a}b^3 \operatorname{sgn}(d^2x^2 + c)) \log\left(\frac{\sqrt{(-2a^{5/2}c^3d - 6(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc))} \cdot a^2c^2 \operatorname{abs}(d) - 6(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)}{\sqrt{a}d^2x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}\right) - \frac{2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}{\sqrt{a}d^2x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc} \sqrt{a}b^2c^2d - a^{3/2}b^2c^2d - 2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)^3 a \operatorname{abs}(d) - 2(\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc) a b^2 c^2 \operatorname{abs}(d) - (\sqrt{a}d^2)x^2 - \sqrt{a}d^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc)^2 \sqrt{a} b^2 d)}{a^2 d^2 \operatorname{abs}(d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(a+b/(d*x^2+c))^(3/2), x)


```
[Out] -1/96*(48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4
*a^2*c*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)
*x^4*a*b*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2))*x^2*a^2*b*c^2*d^2+48*(
a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a^2*c^2*d
+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b
*c)^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2))*x^2*a*b^2*c*d^2-16*(a*d^2*x^4+2*a*c
*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*x^2*a*d+96*(a*d^2*x^4+2*a*c*d
*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^2*a*b*c*d+3*ln(1/2*(2*a*d^2*x
^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1
/2)))/(a*d^2)^(1/2))*x^2*b^3*d^2-72*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2
*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2))*a^2
*b*c^3*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^
2*b^2*d+36*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2
+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2))*a*b^2*c^2*d+96*((d*x^2+c)*(
a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*a*b*c^2-16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x
^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*
c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c^2+3*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)))/(a*d^2)^(1/2))
*b^3*c*d-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*b^
2*c)/d^3/a*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/((d*x^2+c)*(a*d*
x^2+a*c+b))^(1/2)
```

maxima [A] time = 1.71, size = 368, normalized size = 1.48

$$\frac{bc^2 \sqrt{\frac{adx^2+ac+b}{d^2+c}}}{d^3} - \frac{3(8a^2bc^2 - 20ab^2c + b^3) \left(\frac{adx^2+ac+b}{d^2+c}\right)^{\frac{5}{2}} - 8(6a^3bc^2 - 12a^2b^2c - ab^3) \left(\frac{adx^2+ac+b}{d^2+c}\right)^{\frac{3}{2}} + 3(8a^4bc^2 - 12a^3b^2c - a^2b^3) \sqrt{\frac{adx^2+ac+b}{d^2+c}}}{48 \left(a^4d^3 - \frac{3(adx^2+ac+b)a^3d^3}{d^2+c} + \frac{3(adx^2+ac+b)^2 a^2d^3}{(d^2+c)^2} - \frac{(adx^2+ac+b)^3 ad^3}{(d^2+c)^3} \right)} - \frac{(24a^2c^2 - 12abc - b^2)b \log\left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{d^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{d^2+c}}}\right)}{32a^{\frac{3}{2}}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

```
[Out] -b*c^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^3 - 1/48*(3*(8*a^2*b*c^2 - 2
0*a*b^2*c + b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^2 -
12*a^2*b^2*c - a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b
*c^2 - 12*a^3*b^2*c - a^2*b^3)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*
d^3 - 3*(a*d*x^2 + a*c + b)*a^3*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a
^2*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a*d^3/(d*x^2 + c)^3) - 1/32*(2
4*a^2*c^2 - 12*a*b*c - b^2)*b*log(-sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x
^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^(3/2)*d^3)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^5 \left(a + \frac{b}{d x^2 + c} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b/(c + d*x^2))^(3/2),x)
```

```
[Out] int(x^5*(a + b/(c + d*x^2))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^5 \left(\frac{ac + adx^2 + b}{c + dx^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(a+b/(d*x**2+c))**(3/2),x)
```

```
[Out] Integral(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)
```

$$3.138 \quad \int x^3 \left(a + \frac{b}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=172

$$\frac{a(c+dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4d^2} + \frac{(5b-4ac)(c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8d^2} + \frac{bc \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{d^2} + \frac{3b(b-4ac) \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{8\sqrt{a}d^2}$$

Rubi [A] time = 0.54, antiderivative size = 222, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{c\sqrt{a+\frac{b}{c+dx^2}}(a(c+dx^2)+b)^2}{bd^2} + \frac{(b-4ac)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}(a(c+dx^2)+b)}{4bd^2} + \frac{3(b-4ac)(c+dx^2)\sqrt{a+\frac{b}{c+dx^2}}}{8d^2} + \frac{3b(b-4ac)\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{8\sqrt{a}d^2\sqrt{a(c+dx^2)+b}}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b/(c + d*x^2))^(3/2),x]

[Out] (3*(b - 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(8*d^2) + ((b - 4*a*c)*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(4*b*d^2) + (c*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(b*d^2) + (3*b*(b - 4*a*c)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*Sqrt[a]*d^2*Sqrt[b + a*(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.),
x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a + \frac{b}{c + dx^2} \right)^{3/2} dx &= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \int \frac{x^3 (b + a(c + dx^2))^{3/2}}{(c + dx^2)^{3/2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \int \frac{x^3 (b + ac + adx^2)^{3/2}}{(c + dx^2)^{3/2}} dx}{\sqrt{b + a(c + dx^2)}} \\
&= \frac{\left(\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \text{Subst} \left(\int \frac{x (b + ac + adx)^{3/2}}{(c + dx)^{3/2}} dx, x, x^2 \right)}{2\sqrt{b + a(c + dx^2)}} \\
&= \frac{c\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{bd^2} + \frac{\left((b - 4ac)\sqrt{c + dx^2} \sqrt{a + \frac{b}{c + dx^2}} \right) \text{Subst} \left(\int \frac{(b + ac + adx)^{3/2}}{\sqrt{c + dx}} dx, x, x^2 \right)}{2bd\sqrt{b + a(c + dx^2)}} \\
&= \frac{(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4bd^2} + \frac{c\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))^2}{bd^2} + \dots \\
&= \frac{3(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{8d^2} + \frac{(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4bd^2} \\
&= \frac{3(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{8d^2} + \frac{(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4bd^2} \\
&= \frac{3(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{8d^2} + \frac{(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4bd^2} \\
&= \frac{3(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{8d^2} + \frac{(b - 4ac)(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}} (b + a(c + dx^2))}{4bd^2}
\end{aligned}$$

Mathematica [A] time = 0.24, size = 104, normalized size = 0.60

$$\frac{\sqrt{a} \sqrt{\frac{ac + adx^2 + b}{c + dx^2}} (-2ac^2 + 2ad^2x^4 + 13bc + 5bdx^2) + 3b(b - 4ac) \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}} \right)}{8\sqrt{a}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b/(c + d*x^2))^(3/2),x]

[Out] (Sqrt[a]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a*d^2*x^4) + 3*b*(b - 4*a*c)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*Sqrt[a]*d^2)

IntegrateAlgebraic [A] time = 0.18, size = 115, normalized size = 0.67

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(-2ac^2 + 2ad^2x^4 + 13bc + 5bdx^2\right)}{8d^2} - \frac{3(4abc - b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8\sqrt{a}d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(13*b*c - 2*a*c^2 + 5*b*d*x^2 + 2*a*d^2*x^4))/(8*d^2) - (3*(-b^2 + 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/(8*Sqrt[a]*d^2)

fricas [A] time = 0.79, size = 335, normalized size = 1.95

$$\frac{3(4abc - b^2)\sqrt{a} \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 - 4(2ad^2x^4 + (4ac + b)dx^2 + 2ac^2 + bc)\sqrt{\frac{ad^2x^2 + a}{d^2 + c}}}{32ad^2}\right) + 4(2a^2d^2x^4 + 5abd^2 - 2a^2c^2 + 13abc)\sqrt{\frac{ad^2x^2 + a}{d^2 + c}}}{16ad^2} - \frac{3(4abc - b^2)\sqrt{-a} \arctan\left(\frac{(2ad^2 + 2abx)\sqrt{\frac{ad^2x^2 + a}{d^2 + c}}}{2(d^2x^2 + c + ab)}\right) + 2(2a^2d^2x^4 + 5abd^2 - 2a^2c^2 + 13abc)\sqrt{\frac{ad^2x^2 + a}{d^2 + c}}}{16ad^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/32*(3*(4*a*b*c - b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2), 1/16*(3*(4*a*b*c - b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(2*a^2*d^2*x^4 + 5*a*b*d*x^2 - 2*a^2*c^2 + 13*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a*d^2)]

giac [B] time = 1.99, size = 438, normalized size = 2.55

$$\frac{1}{8}\sqrt{a}\sqrt{d^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc} \left(\frac{2ax^2 \operatorname{sgn}(dx^2 + c)}{d} - \frac{(2a^2cd^2 \operatorname{sgn}(dx^2 + c) - 5abd^2 \operatorname{sgn}(dx^2 + c))}{(ad^4)} \right) + \frac{1}{16}(4a^{3/2}bc \operatorname{sgn}(dx^2 + c) - \sqrt{a}b^2 \operatorname{sgn}(dx^2 + c)) \log\left(\frac{abs(-2a^{5/2}c^3d - 6(\sqrt{ad^2})x^2 - \sqrt{ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc})a^2c^2 \operatorname{abs}(d) - 6(\sqrt{ad^2})x^2 - \sqrt{ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc})^2 a^{3/2}cd - a^{3/2}b^2c^2d - 2(\sqrt{ad^2})x^2 - \sqrt{ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc})^3}{a \operatorname{abs}(d) - 2(\sqrt{ad^2})x^2 - \sqrt{ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc})a^2c^2 \operatorname{abs}(d) - (\sqrt{ad^2})x^2 - \sqrt{ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc})^2 \sqrt{a}bd}\right) / (ad \operatorname{abs}(d))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*a*x^2*sgn(d*x^2 + c)/d - (2*a^2*c*d^2*sgn(d*x^2 + c) - 5*a*b*d^2*sgn(d*x^2 + c))/(a*d^4)) + 1/16*(4*a^(3/2)*b*c*sgn(d*x^2 + c) - sqrt(a)*b^2*sgn(d*x^2 + c))*log(abs(-2*a^(5/2)*c^3*d - 6*(sqrt(a*d^2))*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*c^2*abs(d) - 6*(sqrt(a*d^2))*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*c*d - a^(3/2)*b^2*c^2*d - 2*(sqrt(a*d^2))*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*abs(d) - 2*(sqrt(a*d^2))*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b*c*abs(d) - (sqrt(a*d^2))*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b*d)/(a*d*abs(d))

maple [B] time = 0.06, size = 593, normalized size = 3.45

$$\frac{1}{16}(4(a^2d^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc)^{1/2}(ad^2)^{1/2}x^4 + ad^2 - 12 \ln\left(\frac{1}{2}(2ad^2x^2 + 2acd + bdx^2 + 2(ad^2x^4 + 2acd^2x^2 + bdx^2 + ac^2 + bc))\right) \sqrt{a}bd) / (ad \operatorname{abs}(d))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b/(d*x^2+c))^(3/2), x)

[Out] 1/16*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*x^4+a*d^2-12*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)))*sqrt(a)*b*d)/(a*d*abs(d))

$$\frac{c^2+bc}{(a^2d^2)^{1/2}} \cdot \frac{(a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot x^2 \cdot a^2 b^2 c^2 d^2 + 3 \ln\left(\frac{1}{2} \cdot \frac{(2a^2d^2x^2 + 2a^2cd + b^2d + 2(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2}) \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot x^2 \cdot b^2 d^2 + 10 \cdot \frac{(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2} \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot x^2 \cdot b^2 d - 12 \cdot \ln\left(\frac{1}{2} \cdot \frac{(2a^2d^2x^2 + 2a^2cd + b^2d + 2(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2}) \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot a^2 b^2 c^2 d - 4 \cdot \frac{(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2} \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot a^2 c^2 + 3 \cdot \ln\left(\frac{1}{2} \cdot \frac{(2a^2d^2x^2 + 2a^2cd + b^2d + 2(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2}) \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot b^2 c^2 d + 16 \cdot \left(\frac{dx^2+c}{(a^2d^2x^2 + a^2c + b)}\right)^{1/2} \cdot \frac{(a^2d^2)^{1/2} \cdot b^2 c + 10 \cdot \frac{(a^2d^2x^4 + 2a^2cdx^2 + b^2dx^2 + a^2c^2 + bc)^{1/2} \cdot (a^2d^2)^{1/2}}{(a^2d^2)^{1/2}} \cdot b^2 c}{d^2} \cdot \left(\frac{a^2d^2x^2 + a^2c + b}{(dx^2+c)}\right)^{1/2} \cdot \frac{(a^2d^2)^{1/2}}{\left(\frac{dx^2+c}{(a^2d^2x^2 + a^2c + b)}\right)^{1/2}}\right)^{1/2}\right)$$

maxima [A] time = 1.71, size = 247, normalized size = 1.44

$$\frac{bc \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d^2} + \frac{3(4ac-b)b \log\left(\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16\sqrt{a}d^2} - \frac{(4abc-5b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4a^2bc-3ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^2d^2 - \frac{2(adx^2+ac+b)ad^2}{dx^2+c} + \frac{(adx^2+ac+b)^2d^2}{(dx^2+c)^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] b*c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/d^2 + 3/16*(4*a*c - b)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(sqrt(a)*d^2) - 1/8*((4*a*b*c - 5*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c - 3*a*b^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d^2 - 2*(a*d*x^2 + a*c + b)*a*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*d^2/(d*x^2 + c)^2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x^3 \left(a + \frac{b}{dx^2 + c} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b/(c + d*x^2))^(3/2),x)

[Out] int(x^3*(a + b/(c + d*x^2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(\frac{ac + adx^2 + b}{c + dx^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)

$$3.139 \quad \int x \left(a + \frac{b}{c+dx^2} \right)^{3/2} dx$$

Optimal. Leaf size=94

$$\frac{(c+dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} - \frac{3b\sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{3\sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{2d}$$

Rubi [A] time = 0.07, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {1591, 242, 47, 50, 63, 208}

$$\frac{(c+dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} - \frac{3b\sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{3\sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b/(c + d*x^2))^(3/2), x]

[Out] (-3*b*Sqrt[a + b/(c + d*x^2)]/(2*d) + ((c + d*x^2)*(a + b/(c + d*x^2))^(3/2))/(2*d) + (3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 242

`Int[((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]`

Rule 1591

`Int[((a_) + (b_)*(Pq_)^(n_))^(p_)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]`

Rubi steps

$$\begin{aligned}
 \int x \left(a + \frac{b}{c + dx^2} \right)^{3/2} dx &= \frac{\text{Subst} \left(\int \left(a + \frac{b}{x} \right)^{3/2} dx, x, c + dx^2 \right)}{2d} \\
 &= -\frac{\text{Subst} \left(\int \frac{(a+bx)^{3/2}}{x^2} dx, x, \frac{1}{c+dx^2} \right)}{2d} \\
 &= \frac{(c + dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} - \frac{(3b) \text{Subst} \left(\int \frac{\sqrt{a+bx}}{x} dx, x, \frac{1}{c+dx^2} \right)}{4d} \\
 &= -\frac{3b\sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{(c + dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} - \frac{(3ab) \text{Subst} \left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{c+dx^2} \right)}{4d} \\
 &= -\frac{3b\sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{(c + dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} - \frac{(3a) \text{Subst} \left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{c+dx^2}} \right)}{2d} \\
 &= -\frac{3b\sqrt{a + \frac{b}{c+dx^2}}}{2d} + \frac{(c + dx^2) \left(a + \frac{b}{c+dx^2} \right)^{3/2}}{2d} + \frac{3\sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{2d}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 79, normalized size = 0.84

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(a(c + dx^2) - 2b \right) + 3\sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{2d}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b + a*(c + d*x^2)) + 3*Sqrt[a]*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*d)

IntegrateAlgebraic [A] time = 0.13, size = 91, normalized size = 0.97

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (ac + adx^2 - 2b)}{2d} + \frac{3\sqrt{a} b \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(a + b/(c + d*x^2))^(3/2), x]

[Out] $((-2*b + a*c + a*d*x^2)*\sqrt{(b + a*c + a*d*x^2)/(c + d*x^2)})/(2*d) + (3*\sqrt{a}*b*\text{ArcTanh}[\sqrt{(b + a*c + a*d*x^2)/(c + d*x^2)}/\sqrt{a}])/(2*d)$

fricas [A] time = 0.61, size = 269, normalized size = 2.86

$$\frac{3\sqrt{a}b \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 + 4(2ad^2x^4 + (4ac + b)dx^2 + 2ac^2 + bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8d}\right) + 4(adx^2 + ac - 2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - 3\sqrt{-a}b \arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(d^2dx^2+2c+ab)}\right) - 2(adx^2 + ac - 2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] $[1/8*(3*\sqrt{a}*b*\log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c))*\sqrt{a}*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}) + 4*(a*d*x^2 + a*c - 2*b)*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}/d, -1/4*(3*\sqrt{-a}*b*\arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*\sqrt{-a}*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(a^2*d*x^2 + a^2*c + a*b)) - 2*(a*d*x^2 + a*c - 2*b)*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}/d]$

giac [B] time = 1.82, size = 387, normalized size = 4.12

$$\frac{\sqrt{a}b \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 + 4(2ad^2x^4 + (4ac + b)dx^2 + 2ac^2 + bc)\sqrt{a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8d}\right) + 4(adx^2 + ac - 2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - 3\sqrt{-a}b \arctan\left(\frac{(2adx^2+2ac+b)\sqrt{-a}\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2(d^2dx^2+2c+ab)}\right) - 2(adx^2 + ac - 2b)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] $-1/4*\sqrt{a}*b*\log(\text{abs}(-2*a^{(5/2)}*c^3*d - 6*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})*a^2*c^2*\text{abs}(d) - 6*(\sqrt{a*d^2})*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^2*a^{(3/2)}*c*d - a^{(3/2)}*b*c^2*d - 2*(\sqrt{a*d^2})*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^3*a*\text{abs}(d) - 2*(\sqrt{a*d^2})*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})*a*b*c*\text{abs}(d) - (\sqrt{a*d^2})*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^2*\sqrt{a}*b*d))*\text{sgn}(d*x^2 + c)/\text{abs}(d) - 1/4*\sqrt{a}*b*\text{abs}(d)*\log(\text{abs}(a))*\text{sgn}(d*x^2 + c)/d^2 + 1/2*\sqrt{a}*b*\sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c}*a*\text{sgn}(d*x^2 + c)/d$

maple [B] time = 0.05, size = 336, normalized size = 3.57

$$\frac{(-3ab d^2 \ln\left(\frac{2a d^2 x^4 + 2acd + b d^2 + \sqrt{a d^2 x^4 + 2acd + b d^2 + a c^2 + b c} \sqrt{a d^2}}{2 \sqrt{a d^2}}\right) - 3abcd \ln\left(\frac{2a d^2 x^4 + 2acd + b d^2 + \sqrt{a d^2 x^4 + 2acd + b d^2 + a c^2 + b c} \sqrt{a d^2}}{2 \sqrt{a d^2}}\right) - 2 \sqrt{a d^2 x^4 + 2acd x^2 + b d x^2 + a c^2 + b c} \sqrt{a d^2} - 2 \sqrt{a d^2 x^4 + 2acd x^2 + b d x^2 + a c^2 + b c} \sqrt{a d^2} ac + 4 \sqrt{(d x^2 + c)(a d x^2 + a c + b)} \sqrt{a d^2} b) \sqrt{\frac{a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c}{d^2}}}{4 \sqrt{(d x^2 + c)(a d x^2 + a c + b)} \sqrt{a d^2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b/(d*x^2+c))^(3/2), x)

[Out] $-1/4*(-3*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2)*x^2*a*b*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*d*x^2-3*a*b*c*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^2)^(1/2)*b)/d*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*d^2)^(1/2)$

maxima [A] time = 2.24, size = 156, normalized size = 1.66

$$\frac{ab\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ad - \frac{(adx^2+ac+b)d}{dx^2+c}\right)} - \frac{3\sqrt{a}b \log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4d} - \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] $-1/2*a*b*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}/(a*d - (a*d*x^2 + a*c + b)*d/(d*x^2 + c)) - 3/4*\sqrt{a}*b*\log(-(\sqrt{a} - \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(\sqrt{a} + \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/d - b*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}/d$

mupad [B] time = 3.74, size = 61, normalized size = 0.65

$$\frac{\left(a + \frac{b}{dx^2+c}\right)^{3/2} (dx^2 + c) {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; -\frac{a(dx^2+c)}{b}\right)}{d\left(\frac{a(dx^2+c)}{b} + 1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b/(c + d*x^2))^(3/2),x)

[Out] $-\left((a + b/(c + d*x^2))^{3/2}*(c + d*x^2)*\text{hypergeom}([-3/2, -1/2], 1/2, -(a*(c + d*x^2))/b)\right)/(d*((a*(c + d*x^2))/b + 1)^{3/2})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(\frac{ac + adx^2 + b}{c + dx^2} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)

$$3.140 \quad \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x} dx$$

Optimal. Leaf size=126

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(ac+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

Rubi [A] time = 0.49, antiderivative size = 206, normalized size of antiderivative = 1.63, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{a^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a(c+dx^2)+b}} - \frac{(ac+b)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}}\right)}{c^{3/2} \sqrt{a(c+dx^2)+b}} + \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

[In] Int[(a + b/(c + d*x^2))^(3/2)/x,x]

[Out] (b*Sqrt[a + b/(c + d*x^2)]/c + (a^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]/Sqrt[b + a*(c + d*x^2)] - ((b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(c^(3/2)*Sqrt[b + a*(c + d*x^2)])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 157

Int[((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p

$p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 206

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_) + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_) + (b_)*(x_)^2], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n_)})^{(p_)}*((c_) + (d_)*(x_)^{(n_)})^{(q_)}], x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

$\text{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_))^{(m_)}], x_Symbol] := \text{Int}[(e*x)^m*\text{ExpandToSum}[u, x]^p*\text{ExpandToSum}[v, x]^q, x] /;$ FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

$\text{Int}[(u_)*((a_) + (b_)*(v_)^{(n_)})^{(p_)}], x_Symbol] := \text{Dist}[(a + b*v^n)^{\text{FracPart}[p]}/(v^{(n*\text{FracPart}[p])*(b + a/v^n)^{\text{FracPart}[p]})}, \text{Int}[u*v^{(n*p)}*(b + a/v^n)^p, x], x] /;$ FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+a(c+dx^2))^{3/2}}{x(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+ac+adx^2)^{3/2}}{x(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x(c+dx)^{3/2}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
&= \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c} + \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\frac{1}{2}(b+ac)^2 d + \frac{1}{2}a^2 cd^2 x}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2\right)}{cd\sqrt{b+a(c+dx^2)}} \\
&= \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c} + \frac{\left((b+ac)^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2\right)}{2c\sqrt{b+a(c+dx^2)}} + \dots \\
&= \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c} + \frac{\left(a^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{b+a(c+dx^2)}} + \dots \\
&= \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c} - \frac{(b+ac)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+a(c+dx^2)}}\right)}{c^{3/2} \sqrt{b+a(c+dx^2)}} + \dots \\
&= \frac{b\sqrt{a + \frac{b}{c+dx^2}}}{c} + \frac{a^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+a(c+dx^2)}} - \frac{(b+ac)^{3/2} \sqrt{c+dx^2}}{c^{3/2}} \dots
\end{aligned}$$

Mathematica [A] time = 0.23, size = 118, normalized size = 0.94

$$\frac{a^{3/2} c^{3/2} \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right) + b\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}} - (ac+b)^{3/2} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x, x]

[Out] (b*Sqrt[c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)] + a^(3/2)*c^(3/2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]] - (b + a*c)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2))]/Sqrt[b + a*c]])/c^(3/2)

IntegrateAlgebraic [A] time = 0.21, size = 139, normalized size = 1.10

$$a^{3/2} \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) - \frac{(-ac-b)^{3/2} \tan^{-1}\left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{c^{3/2}} + \frac{b\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x,x]
```

```
[Out] (b*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/c - ((-b - a*c)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(b + a*c))]/c^(3/2) + a^(3/2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])
```

fricas [A] time = 1.02, size = 1073, normalized size = 8.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] [1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a*c + b)*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2))*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4 + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c, -1/4*(2*sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2))*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4 - 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c, 1/4*(a^(3/2)*c*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 2*(a*c + b)*sqrt(-a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 4*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c, -1/2*(sqrt(-a)*a*c*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - (a*c + b)*sqrt(-a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) - 2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/c]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,x);OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(d*x^2+c)]Evaluation time: 0.71Error: Bad Argument Type
```

maple [B] time = 0.06, size = 652, normalized size = 5.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b/(d*x^2+c))^(3/2)/x,x)
```

```
[Out] 1/2*(ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2
+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))*x^2*a^2*c^2*d^2-(a*d^2)^(1/2)*(a*
c^2+b*c)^(1/2)*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a
*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*c*d+ln(1/2*(2*a*d
^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2
)^(1/2))/(a*d^2)^(1/2))*a^2*c^3*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a*c
*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d
*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*b*d-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*a*c^2-(a*d^2)^(1/2)*(a*c^2+b*c)^(1/2)*ln((2*a
*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b
*d*x^2+a*c^2+b*c)^(1/2))/x^2)*b*c+2*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*d^
2)^(1/2)*b*c*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)/(a*d^2)^(1/2)/c^2/((d*x^2+c
)*(a*d*x^2+a*c+b))^(1/2)
```

maxima [A] time = 2.11, size = 201, normalized size = 1.60

$$-\frac{1}{2}a^3 \log\left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right) + \frac{b\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c} + \frac{(a^2c^2 + 2abc + b^2) \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] -1/2*a^(3/2)*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a)
+ sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))) + b*sqrt((a*d*x^2 + a*c + b)/(d
*x^2 + c))/c + 1/2*(a^2*c^2 + 2*a*b*c + b^2)*log((c*sqrt((a*d*x^2 + a*c + b
)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)
) + sqrt((a*c + b)*c)))/(sqrt((a*c + b)*c)*c)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/(c + d*x^2))^(3/2)/x,x)
```

```
[Out] int((a + b/(c + d*x^2))^(3/2)/x, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x**2+c))**(3/2)/x,x)
```

```
[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x, x)
```

$$3.141 \quad \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx$$

Optimal. Leaf size=138

$$\frac{3bd\sqrt{ac+b} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{2c^{5/2}} - \frac{3bd\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2c^2} - \frac{(c+dx^2)\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{2cx^2}$$

Rubi [A] time = 0.53, antiderivative size = 170, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 1975, 446, 94, 93, 208}

$$-\frac{3bd\sqrt{a+\frac{b}{c+dx^2}}}{2c^2} + \frac{3bd\sqrt{ac+b}\sqrt{c+dx^2}\sqrt{a+\frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{2c^{5/2}\sqrt{a(c+dx^2)+b}} - \frac{\sqrt{a+\frac{b}{c+dx^2}}(a(c+dx^2)+b)}{2cx^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b/(c + d*x^2))^(3/2)/x^3,x]

[Out] (-3*b*d*Sqrt[a + b/(c + d*x^2)]/(2*c^2) - (Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(2*c*x^2) + (3*b*Sqrt[b + a*c]*d*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2)]/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])))/(2*c^(5/2)*Sqrt[b + a*(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !SumSimplerQ[p, 1] && !SumSimplerQ[m, 1]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_.)*(x_))^(m_), x_Symbol] :> Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi

alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && !LtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^3} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+a(c+dx^2))^{3/2}}{x^3(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\ &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+ac+adx^2)^{3/2}}{x^3(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\ &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^2(c+dx)^{3/2}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\ &= -\frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{2cx^2} - \frac{\left(3bd\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\sqrt{b+ac+adx}}{x(c+dx)^{3/2}} dx, x, x^2\right)}{4c\sqrt{b+a(c+dx^2)}} \\ &= -\frac{3bd\sqrt{a + \frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right)}{4c^2\sqrt{b+a(c+dx^2)}} \\ &= -\frac{3bd\sqrt{a + \frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{2cx^2} - \frac{\left(3b(b+ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right)}{2c^2\sqrt{b+a(c+dx^2)}} \\ &= -\frac{3bd\sqrt{a + \frac{b}{c+dx^2}}}{2c^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{2cx^2} + \frac{3b\sqrt{b+ac}d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}{2c^{5/2}\sqrt{b+a(c+dx^2)}} \end{aligned}$$

Mathematica [A] time = 0.58, size = 256, normalized size = 1.86

$$\frac{\sqrt{\frac{ac+bdx^2}{c+dx^2}} \left(-2\sqrt{c(ac+b)} \left(a^2c(c+dx^2)^2 + ab(2c^2+5cdx^2+3d^2x^4) + b^2(c+3dx^2) \right) - 6bdx^2 \log(x)(ac+b) \sqrt{(c+dx^2)(a(c+dx^2)+b)} + 3bdx^2(ac+b) \sqrt{(c+dx^2)(a(c+dx^2)+b)} \log(2\sqrt{c(ac+b)} \sqrt{(c+dx^2)(ac+adx^2+b)} + 2ac(c+dx^2) + b(2c+dx^2)) \right)}{4c^2x^2\sqrt{c(ac+b)} (a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^3,x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(a^2*c*(c + d*x^2)^2 + b^2*(c + 3*d*x^2) + a*b*(2*c^2 + 5*c*d*x^2 + 3*d^2*x^4)) - 6*b*(b + a*c)*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + 3*b*(b + a*c)*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/(4*c^2*Sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))

IntegrateAlgebraic [A] time = 0.20, size = 136, normalized size = 0.99

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-ac^2 - acdx^2 - bc - 3bdx^2)}{2c^2x^2} - \frac{3bd\sqrt{-ac-b} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2c^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^3,x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c) - a*c^2 - 3*b*d*x^2 - a*c*d*x^2))/(2*c^2*x^2) - (3*b*Sqrt[-b - a*c]*d*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/ (b + a*c)])/ (2*c^(5/2))

fricas [A] time = 0.88, size = 404, normalized size = 2.93

$$\left[\frac{3bdx^2\sqrt{\frac{ac+b}{c}} \log\left(\frac{(8a^2d^2+8abc+3d^2)c^2+8a^2b^2+16abd^2+8(2a^2c^2+3ad^2+3d^2)c^2+4((2a^2+bc)d^2+2ac^2+2bc^2+4ac^2+3bc^2)d^2)\sqrt{\frac{ac+b}{c}}\sqrt{\frac{ac+b}{c}}}{8c^2x^2}\right) - 4((ac+3b)dx^2+ac^2+bc)\sqrt{\frac{ac+b}{c}}}{3bdx^2\sqrt{\frac{ac+b}{c}} \arctan\left(\frac{(2ac+b)d^2+2a^2c^2+2bc^2)\sqrt{\frac{ac+b}{c}}\sqrt{\frac{ac+b}{c}}}{2(a^2c^2+(ac+ab)d^2+2abc+3b^2)}\right)} + 2((ac+3b)dx^2+ac^2+bc)\sqrt{\frac{ac+b}{c}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")

[Out] [1/8*(3*b*d*x^2*sqrt((a*c + b)/c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c^2 + b*c)*d^2*x^4 + 2*a*c^4 + 2*b*c^3 + (4*a*c^3 + 3*b*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt((a*c + b)/c))/x^4) - 4*((a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2), -1/4*(3*b*d*x^2*sqrt(-(a*c + b)/c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-(a*c + b)/c)/(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)) + 2*((a*c + 3*b)*d*x^2 + a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(c^2*x^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")

[Out] undef

maple [B] time = 0.07, size = 820, normalized size = 5.94



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(3/2)/x^3,x)

[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a*d^3-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)))/x^2)*x^4*a*b*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^4*a*c*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*b^2*c*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^4*b*d^2-3*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^2*a*b*c^3*d-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^2*a*c^2*d-3*ln((

$*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}/x^2)*x^2*b^2*c^2*d+4*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*c^2+b*c)^{(1/2)}*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(1/2)}*x^2*d-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(1/2)}*x^2*b*c*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(1/2)}*c)/(a*c^2+b*c)^{(1/2)}/x^2/c^3/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}$

maxima [A] time = 2.41, size = 202, normalized size = 1.46

$$\frac{(abc + b^2)d\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(ac^3 + bc^2 - \frac{(adx^2+ac+b)c^3}{dx^2+c}\right)} - \frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{c^2} - \frac{3(abc + b^2)d \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")

[Out] $-1/2*(a*b*c + b^2)*d*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a*c^3 + b*c^2 - (a*d*x^2 + a*c + b)*c^3/(d*x^2 + c)) - b*d*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^2 - 3/4*(a*b*c + b^2)*d*\text{log}((c*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))) - \text{sqrt}((a*c + b)*c))/(c*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)) + \text{sqrt}((a*c + b)*c)))/(\text{sqrt}((a*c + b)*c)*c^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(3/2)/x^3,x)

[Out] int((a + b/(c + d*x^2))^(3/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))**(3/2)/x**3,x)

[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**3, x)

$$3.142 \quad \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^5} dx$$

Optimal. Leaf size=205

$$\frac{3bd^2(4ac + 5b) \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{7/2}\sqrt{ac+b}} + \frac{bd^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{c^3} + \frac{d(4ac + 9b)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8c^3x^2} - \frac{(ac + b)(c + dx^2)^2}{4c^3x^4}$$

Rubi [A] time = 0.59, antiderivative size = 260, normalized size of antiderivative = 1.27, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(4ac + 5b)\sqrt{a + \frac{b}{c+dx^2}}}{8c^3(ac + b)} - \frac{3bd^2(4ac + 5b)\sqrt{c + dx^2}\sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1}\left(\frac{\sqrt{ac+b}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{a(c+dx^2)+b}}\right)}{8c^{7/2}\sqrt{ac+b}\sqrt{a(c+dx^2)+b}} + \frac{d(4ac + 5b)\sqrt{a + \frac{b}{c+dx^2}}(a(c + dx^2) + b)}{8c^2x^2(ac + b)} - \frac{\sqrt{a + \frac{b}{c+dx^2}}(a(c + dx^2) + b)^2}{4cx^4(ac + b)}$$

Antiderivative was successfully verified.

[In] Int[(a + b/(c + d*x^2))^(3/2)/x^5,x]

[Out] (3*b*(5*b + 4*a*c)*d^2*Sqrt[a + b/(c + d*x^2)]/(8*c^3*(b + a*c)) + ((5*b + 4*a*c)*d*Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2)))/(8*c^2*(b + a*c)*x^2) - (Sqrt[a + b/(c + d*x^2)]*(b + a*(c + d*x^2))^2)/(4*c*(b + a*c)*x^4) - (3*b*(5*b + 4*a*c)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(7/2)*Sqrt[b + a*c]*Sqrt[b + a*(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !SumSimplerQ[p, 1] && !SumSimplerQ[m, 1]

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^5} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+a(c+dx^2))^{3/2}}{x^5(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
 &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+ac+adx^2)^{3/2}}{x^5(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
 &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^3(c+dx)^{3/2}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
 &= -\frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))^2}{4c(b+ac)x^4} - \frac{\left((5b+4ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{(b+ac+ax)}{x^2(c+dx)} dx, x, x^2\right)}{8c(b+ac)\sqrt{b+a(c+dx^2)}} \\
 &= \frac{(5b+4ac)d\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))^2}{4c(b+ac)x^4} + \frac{(3b(5b+4ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}})}{4c(b+ac)} \\
 &= \frac{3b(5b+4ac)d^2\sqrt{a + \frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))^2}{4c(b+ac)x^4} + \frac{(3b(5b+4ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}})}{4c(b+ac)} \\
 &= \frac{3b(5b+4ac)d^2\sqrt{a + \frac{b}{c+dx^2}}}{8c^3(b+ac)} + \frac{(5b+4ac)d\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))}{8c^2(b+ac)x^2} - \frac{\sqrt{a + \frac{b}{c+dx^2}} (b+a(c+dx^2))^2}{4c(b+ac)x^4} + \frac{(3b(5b+4ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}})}{4c(b+ac)}
 \end{aligned}$$

Mathematica [A] time = 0.33, size = 190, normalized size = 0.93

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(-\frac{2c^2(ac+b)}{x^4} + \frac{3bd^2(4ac+5b)(c+dx^2)(2\log(x)-\log(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)}+2ac(c+dx^2)+b(2c+dx^2)))}{2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(a(c+dx^2)+b)}} \right) + d^2(2ac+15b) + \frac{5bcd}{x^2}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*((15*b + 2*a*c)*d^2 - (2*c^2*(b + a*c))/x^4 + (5*b*c*d)/x^2 + (3*b*(5*b + 4*a*c)*d^2*(c + d*x^2)*(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)])))/(2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))])))/(8*c^3)
```

IntegrateAlgebraic [A] time = 0.27, size = 162, normalized size = 0.79

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-2ac^3 + 2acd^2x^4 - 2bc^2 + 5bcdx^2 + 15bd^2x^4)}{8c^3x^4} - \frac{3d^2(4abc + 5b^2) \tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8c^{7/2}\sqrt{-ac-b}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^5,x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 + 5*b*c*d*x^2 + 15*b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c^3*x^4) - (3*(5*b^2 + 4*a*b*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)])/(b + a*c)])/(8*c^(7/2)*Sqrt[-b - a*c])
```

fricas [A] time = 1.70, size = 557, normalized size = 2.72

$$\frac{\left(\frac{3(abc + 5b^2)\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{32(a^2 + bc^2)x^4} + \frac{d(2c^2 - (2c^2 + 17ad^2 + 15b^2)d^2 + 4ad^4 + 2b^2 - 5(ab^2 + b^2)bc^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16(a^2 + bc^2)x^4} - \frac{3(abc + 5b^2)\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{16(a^2 + bc^2)x^4} \right)}{8c^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] [1/32*(3*(4*a*b*c + 5*b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4), 1/16*(3*(4*a*b*c + 5*b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(2*a^2*c^5 - (2*a^2*c^3 + 17*a*b*c^2 + 15*b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 - 5*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a*c^5 + b*c^4)*x^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

undef

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] undef
```

maple [B] time = 0.08, size = 1653, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(3/2)/x^5,x)

[Out]
$$\frac{1}{16} \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{1/2} \left(-12 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^8 a^2 c d^4 - 12 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^6 a^3 b c^5 d^3 - 18 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^8 a b d^4 - 39 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^6 a^2 b^2 c^4 d^3 - 32 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^6 a^2 c^2 d^3 - 42 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^6 a b^3 c^3 d^3 - 12 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^4 a^3 b c^6 d^2 - 62 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^6 a b c d^3 - 15 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^6 b^4 c^2 d^3 - 39 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^4 a^2 b^2 c^5 d^2 - 18 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^6 b^2 d^3 - 20 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^4 a^2 c^3 d^2 - 42 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^4 a b^3 c^4 d^2 + 16 \left(\frac{d x^2 + c}{(d x^2 + c)} \right)^{1/2} (a c^2 + b c)^{3/2} x^4 a b c^2 d^2 + 12 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} x^4 a c d^2 - 44 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^4 a b c^2 d^2 - 15 \ln \left(\frac{(2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2}) (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2}}{x^2} \right) x^4 b^4 c^3 d^2 + 16 \left(\frac{d x^2 + c}{(d x^2 + c)} \right)^{1/2} (a c^2 + b c)^{3/2} x^4 b^2 c d^2 + 18 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} x^4 b d^2 - 18 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a c^2 + b c)^{3/2} x^4 b^2 c d^2 + 8 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} x^2 a c^2 d + 14 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} x^2 b c d - 4 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} a c^3 - 4 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{3/2} (a c^2 + b c)^{3/2} b c^2 \right) / (a c^2 + b c)^{3/2} / x^4 / (a c + b) / c^4 / \left(\frac{d x^2 + c}{(d x^2 + c)} \right)^{1/2}$$

maxima [A] time = 2.42, size = 313, normalized size = 1.53

$$\frac{b d^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{c^3} + \frac{3 (4 a b c + 5 b^2) d^2 \log \left(\frac{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} - \sqrt{(a c + b) c}}{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} + \sqrt{(a c + b) c}} \right)}{16 \sqrt{(a c + b) c} c^3} - \frac{(4 a b c^2 + 9 b^2 c) d^2 \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{\frac{3}{2}} - (4 a^2 b c^2 + 11 a b^2 c + 7 b^3) d^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{8 \left(a^2 c^5 + 2 a b c^4 + b^2 c^3 + \frac{(a d x^2 + a c + b)^2 c^5}{(d x^2 + c)^2} - \frac{2 (a c^5 + b c^4) (a d x^2 + a c + b)}{d x^2 + c} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")

[Out]
$$b d^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} / c^3 + \frac{3}{16} (4 a b c + 5 b^2) d^2 \log \left(\frac{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} - \sqrt{(a c + b) c}}{c \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} + \sqrt{(a c + b) c}} \right) / \left(\sqrt{(a c + b) c} \right) c^3 - \frac{1}{8} (4 a b c^2 + 9 b^2 c) d^2 \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{3/2} - \frac{(4 a^2 b c^2 + 11 a b^2 c + 7 b^3) d^2 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{(a^2 c^5 + 2 a b c^4 + b^2 c^3 + (a d x^2 + a c + b)^2 c^5 / (d x^2 + c))^2} - \frac{2 (a c^5 + b c^4) (a d x^2 + a c + b)}{(d x^2 + c)}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/(c + d*x^2))^(3/2)/x^5, x)

[Out] int((a + b/(c + d*x^2))^(3/2)/x^5, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{3/2}}{x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x**2+c))**(3/2)/x**5, x)

[Out] Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**5, x)

$$3.143 \quad \int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx$$

Optimal. Leaf size=292

$$\frac{bd^3 \left(24a^2c^2 + 60abc + 35b^2\right) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{16c^{9/2}(ac+b)^{3/2}} - \frac{d^2 \left(24a^2c^2 + 108abc + 79b^2\right) (c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{48c^4x^2(ac+b)} - \frac{bd^3}{48c^4x^2(ac+b)}$$

Rubi [A] time = 0.73, antiderivative size = 287, normalized size of antiderivative = 0.98, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 98, 151, 152, 12, 93, 208}

$$\frac{d^3(8a^2c^2 + 110abc + 105b^2) \sqrt{a + \frac{b}{c+dx^2}}}{48c^4(ac+b)} + \frac{bd^3(24a^2c^2 + 60abc + 35b^2) \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{16c^{9/2}(ac+b)^{3/2} \sqrt{a(c+dx^2)+b}} - \frac{bd^2(32ac + 35b) \sqrt{a + \frac{b}{c+dx^2}}}{48c^3x^2(ac+b)} + \frac{7bd \sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{(ac+b) \sqrt{a + \frac{b}{c+dx^2}}}{6cx^6}$$

Antiderivative was successfully verified.

[In] Int[(a + b/(c + d*x^2))^(3/2)/x^7, x]

[Out] -((105*b^2 + 110*a*b*c + 8*a^2*c^2)*d^3*Sqrt[a + b/(c + d*x^2)]/(48*c^4*(b + a*c)) - ((b + a*c)*Sqrt[a + b/(c + d*x^2)]/(6*c*x^6) + (7*b*d*Sqrt[a + b/(c + d*x^2)]/(24*c^2*x^4) - (b*(35*b + 32*a*c)*d^2*Sqrt[a + b/(c + d*x^2)]/(48*c^3*(b + a*c)*x^2) + (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2)]/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])))/(16*c^(9/2)*(b + a*c)^(3/2)*Sqrt[b + a*(c + d*x^2)])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 93

Int[(((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 98

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 151

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_)*((g_) + (h_)*(x_)), x_Symbol] := Simp[((b*g - a*h)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d

$x)^n(e + f*x)^p \text{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[m]

Rule 152

$\text{Int}[(a + b*x)^m * (c + d*x)^n * (e + f*x)^p, x_Symbol] := \text{Simp}[(b*g - a*h)*(a + b*x)^{m+1} * (c + d*x)^{n+1} * (e + f*x)^{p+1} / ((m+1)*(b*c - a*d)*(b*e - a*f)), x] + \text{Dist}[1/((m+1)*(b*c - a*d)*(b*e - a*f)), \text{Int}[(a + b*x)^{m+1} * (c + d*x)^n * (e + f*x)^p \text{Simp}[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegerQ[2*m, 2*n, 2*p]

Rule 208

$\text{Int}[(a + b*x^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

$\text{Int}(x)^m * (a + b*x^n)^p * (c + d*x^n)^q, x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m+1)/n) - 1} * (a + b*x)^p * (c + d*x)^q, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m+1)/n]]

Rule 1975

$\text{Int}(u)^p * (v)^q * (e*x)^m, x_Symbol] := \text{Int}[(e*x)^m * \text{ExpandToSum}[u, x]^p * \text{ExpandToSum}[v, x]^q, x] /;$ FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

$\text{Int}(u)^p * (a + b*v^n)^p, x_Symbol] := \text{Dist}[(a + b*v^n)^{\text{FracPart}[p]} / (v^{(n*\text{FracPart}[p])} * (b + a/v^n)^{\text{FracPart}[p]}), \text{Int}[u*v^{(n*p)} * (b + a/v^n)^p, x], x] /;$ FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{\left(a + \frac{b}{c+dx^2}\right)^{3/2}}{x^7} dx &= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+a(c+dx^2))^{3/2}}{x^7(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \int \frac{(b+ac+adx^2)^{3/2}}{x^7(c+dx^2)^{3/2}} dx}{\sqrt{b+a(c+dx^2)}} \\
&= \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{(b+ac+adx)^{3/2}}{x^4(c+dx)^{3/2}} dx, x, x^2\right)}{2\sqrt{b+a(c+dx^2)}} \\
&= -\frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\frac{7}{2}b(b+ac)d+3abd^2x}{x^3(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{6c\sqrt{b+a(c+dx^2)}} \\
&= -\frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} + \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{\frac{1}{4}b(b+ac)(35b}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{12c^2(b+ac)\sqrt{b+a(c+dx^2)}} \\
&= -\frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{b(35b+32ac)d^2\sqrt{a + \frac{b}{c+dx^2}}}{48c^3(b+ac)x^2} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{b(35b+32ac)d^2}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{48c^3(b+ac)x^2} \\
&= -\frac{(105b^2+110abc+8a^2c^2)d^3\sqrt{a + \frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{b(35b+32ac)d^2}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{48c^3(b+ac)x^2} \\
&= -\frac{(105b^2+110abc+8a^2c^2)d^3\sqrt{a + \frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{b(35b+32ac)d^2}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{48c^3(b+ac)x^2} \\
&= -\frac{(105b^2+110abc+8a^2c^2)d^3\sqrt{a + \frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{b(35b+32ac)d^2}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{48c^3(b+ac)x^2} \\
&= -\frac{(105b^2+110abc+8a^2c^2)d^3\sqrt{a + \frac{b}{c+dx^2}}}{48c^4(b+ac)} - \frac{(b+ac)\sqrt{a + \frac{b}{c+dx^2}}}{6cx^6} + \frac{7bd\sqrt{a + \frac{b}{c+dx^2}}}{24c^2x^4} - \frac{\left(\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}\right) \text{Subst}\left(\int \frac{b(35b+32ac)d^2}{x^2(c+dx)^{3/2}\sqrt{b+ac+adx}} dx, x, x^2\right)}{48c^3(b+ac)x^2}
\end{aligned}$$

Mathematica [A] time = 0.52, size = 245, normalized size = 0.84

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(\frac{3bd^3(24a^2c^2+60abc+35b^2)(c+dx^2) \left(2 \log(x) - \log\left(2\sqrt{c(ac+b)} \sqrt{(c+dx^2)(ac+adx^2+b)} + 2ac(c+dx^2)+b(2c+dx^2) \right) \right)}{\sqrt{c(ac+b)} \sqrt{(c+dx^2)(ac+adx^2+b)}} \right) + 2d^3(8a^2c^2+110abc+105b^2) + \frac{16c^3(ac+b)^2}{x^6} - \frac{28bd^2d(ac+b)}{x^4} + \frac{2bcd^2(32ac+35b)}{x^2} \right)}{96c^4(ac+b)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b/(c + d*x^2))^(3/2)/x^7, x]

[Out] -1/96*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*(105*b^2 + 110*a*b*c + 8*a^2*c^2)*d^3 + (16*c^3*(b + a*c)^2)/x^6 - (28*b*c^2*(b + a*c)*d)/x^4 + (2*b*c*(35*b + 32*a*c)*d^2)/x^2 + (3*b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*d^3*(c + d*x^2)*(2*Log[x] - Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b +

$a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)])))/(Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]))/(c^4*(b + a*c))$

IntegrateAlgebraic [A] time = 0.45, size = 248, normalized size = 0.85

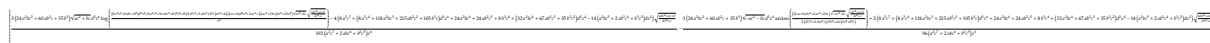
$$\frac{\sqrt{\frac{ac+ad^2+b}{c+dx^2}} (-8a^2c^5 - 8a^2c^2d^3x^6 - 16abc^4 + 14abc^3dx^2 - 32abc^2d^2x^4 - 110abcd^3x^6 - 8b^2c^3 + 14b^2c^2dx^2 - 35b^2cd^2x^4 - 105b^2d^3x^6)}{48c^4x^6(ac+b)} - \frac{d^3(24a^2bc^2 + 60ab^2c + 35b^3) \tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+ad^2+b}{c+dx^2}}}{ac+b}\right)}{16c^{9/2}(-ac-b)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b/(c + d*x^2))^(3/2)/x^7, x]

[Out] $(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-8*b^2*c^3 - 16*a*b*c^4 - 8*a^2*c^5 + 14*b^2*c^2*d*x^2 + 14*a*b*c^3*d*x^2 - 35*b^2*c*d^2*x^4 - 32*a*b*c^2*d^2*x^4 - 105*b^2*d^3*x^6 - 110*a*b*c*d^3*x^6 - 8*a^2*c^2*d^3*x^6))/(48*c^4*(b + a*c)*x^6) - ((35*b^3 + 60*a*b^2*c + 24*a^2*b*c^2)*d^3*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/(b + a*c)))/(16*c^(9/2)*(-b - a*c)^(3/2))$

fricas [A] time = 2.31, size = 733, normalized size = 2.51



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7, x, algorithm="fricas")

[Out] $[1/192*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(a*c^2 + b*c)*d^3*x^6*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6), -1/96*(3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*sqrt(-a*c^2 - b*c)*d^3*x^6*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(8*a^3*c^7 + (8*a^3*c^4 + 118*a^2*b*c^3 + 215*a*b^2*c^2 + 105*b^3*c)*d^3*x^6 + 24*a^2*b*c^6 + 24*a*b^2*c^5 + 8*b^3*c^4 + (32*a^2*b*c^4 + 67*a*b^2*c^3 + 35*b^3*c^2)*d^2*x^4 - 14*(a^2*b*c^5 + 2*a*b^2*c^4 + b^3*c^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^7 + 2*a*b*c^6 + b^2*c^5)*x^6)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7, x, algorithm="giac")

[Out] undef

maple [B] time = 0.08, size = 2605, normalized size = 8.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b/(d*x^2+c))^(3/2)/x^7, x)

[Out] $-1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(-738*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+2*a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(5/2)*x^8*a*b^2*c*d^4-540*(a*d^2*x^4+2*a*c*d$

$$\begin{aligned}
& *x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^6 * a^2 * b * c^3 * d^3 + 96 * (a * c^2 \\
& + b * c)^{(5/2)} * ((d * x^2 + c) * (a * d * x^2 + a * c + b))^{(1/2)} * x^6 * a^2 * b * c^3 * d^3 + 276 * (a * d^2 * \\
& x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * x^6 * a * b * c * d^3 - 56 \\
& 4 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^6 * a * b \\
& ^2 * c^2 * d^3 + 192 * (a * c^2 + b * c)^{(5/2)} * ((d * x^2 + c) * (a * d * x^2 + a * c + b))^{(1/2)} * x^6 * a * b^2 \\
& * c^2 * d^3 + 168 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} \\
& * x^4 * a * b * c^2 * d^2 - 76 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * \\
& c^2 + b * c)^{(5/2)} * x^2 * a * b * c^3 * d - 276 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} \\
& * (a * c^2 + b * c)^{(5/2)} * x^{10} * a^2 * b * c * d^5 - 816 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + \\
& a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^8 * a^2 * b * c^2 * d^4 - 144 * (a * d^2 * x^4 + 2 * a * c * \\
& d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^6 * a^3 * c^4 * d^3 - 105 * \ln((2 * \\
& a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + \\
& b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * b^6 * c^3 * d^4 - 174 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b \\
& * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^8 * b^3 * d^4 - 105 * \ln((2 * a * c * d * x^2 + b \\
& * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c \\
& ^2 + b * c)^{(1/2)}) / x^2) * x^6 * b^6 * c^4 * d^3 + 174 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} \\
& * (a * c^2 + b * c)^{(5/2)} * x^6 * b^2 * d^3 + 32 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} \\
& * (a * c^2 + b * c)^{(5/2)} * a * b * c^4 - 927 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} \\
& * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^6 * a^2 * b^4 * c^6 * d^3 + 96 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} \\
& * (a * c^2 + b * c)^{(5/2)} * x^6 * a^2 * c^2 * d^3 - 495 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} \\
& * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^6 * a * b^5 * c^5 * d^3 + 48 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} \\
& * (a * c^2 + b * c)^{(5/2)} * x^4 * a^2 * c^3 * d^2 - 174 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^6 * b^3 * c * d^3 + 96 * (a * c^2 + b * c)^{(5/2)} * ((d * x^2 + c) \\
& * (a * d * x^2 + a * c + b))^{(1/2)} * x^6 * b^3 * c * d^3 + 114 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * \\
& c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * x^4 * b^2 * c * d^2 - 32 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * \\
& d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * x^2 * a^2 * c^4 * d - 44 * (a * d^2 * x^4 + 2 * a * c * \\
& d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * x^2 * b^2 * c^2 * d - 72 * \ln((2 * a * c \\
& * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d \\
& * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * a^5 * b * c^8 * d^4 - 96 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d \\
& * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^{10} * a^3 * c^2 * d^5 - 396 * \ln((2 * a * c * d * x^2 + \\
& b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + \\
& a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * a^4 * b^2 * c^7 * d^4 - 861 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * \\
& c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * a^3 * b^3 * c^6 * d^4 - 72 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^6 * a^5 * b * c^9 * d^3 - 174 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^{10} * a * b^2 * d^5 - 240 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)} * (a * c^2 + b * c)^{(5/2)} * x^8 * a^3 * c^3 * d^4 - 927 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * a^2 * b^4 * c^5 * d^4 - 396 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^6 * a^4 * b^2 * c^8 * d^3 - 495 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^8 * a * b^5 * c^4 * d^4 - 861 * \ln((2 * a * c * d * x^2 + b * d * x^2 + 2 * a * c^2 + 2 * b * c + 2 * (a * c^2 + b * c)^{(1/2)} * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(1/2)}) / x^2) * x^6 * a^3 * b^3 * c^7 * d^3 + 16 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * a^2 * c^5 + 16 * (a * d^2 * x^4 + 2 * a * c * d * x^2 + b * d * x^2 + a * c^2 + b * c)^{(3/2)} * (a * c^2 + b * c)^{(5/2)} * b^2 * c^3) / (a * c^2 + b * c)^{(5/2)} / x^6 / (a * c + b)^2 / c^5 / ((d * x^2 + c) * (a * d * x^2 + a * c + b))^{(1/2)}
\end{aligned}$$

maxima [B] time = 2.49, size = 534, normalized size = 1.83

$$\frac{(24a^2b^2c^2 + 60ab^2c + 35b^3)d^3 \log\left(\frac{c\sqrt{\frac{ad^2+ac+b}{d^2+c}} + \sqrt{(ac+b)c}}{c\sqrt{\frac{ad^2+ac+b}{d^2+c}} + \sqrt{(ac+b)c}}\right)}{32(ac^5 + bc^4)\sqrt{(ac+b)c}} - \frac{bd^3\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{c^4} - \frac{3(8a^2bc^4 + 36ab^2c^3 + 29b^3c^2)d^3\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{5}{2}} - 8(6a^3bc^4 + 30a^2b^2c^3 + 41ab^3c^2 + 17b^4c)d^3\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{3}{2}} + 3(8a^4bc^4 + 44a^3b^2c^3 + 83a^2b^3c^2 + 66ab^4c + 19b^5)d^3\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{48\left(a^6c^5 + 4a^5bc^7 + 6a^4b^2c^6 + 4ab^2c^5 + b^4c^4 - \frac{(a^3+bc^3)(ad^2+ac+b)}{(d^2+c)^3} + \frac{3(a^2b+2ab^2c^2+bc^3)(ad^2+ac+b)}{(d^2+c)^2} - \frac{3(a^2b^3+3a^2bc^2+3a^2b^2c^2+bc^3)(ad^2+ac+b)}{d^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")

[Out] -1/32*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*d^3*log((c*sqrt((a*d*x^2 + a*c +

$b)/(d*x^2 + c)) - \text{sqrt}((a*c + b)*c))/(c*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)) + \text{sqrt}((a*c + b)*c)))/((a*c^5 + b*c^4)*\text{sqrt}((a*c + b)*c)) - b*d^3*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))/c^4 - 1/48*(3*(8*a^2*b*c^4 + 36*a*b^2*c^3 + 29*b^3*c^2)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 8*(6*a^3*b*c^4 + 30*a^2*b^2*c^3 + 41*a*b^3*c^2 + 17*b^4*c)*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*(8*a^4*b*c^4 + 44*a^3*b^2*c^3 + 83*a^2*b^3*c^2 + 66*a*b^4*c + 19*b^5)*d^3*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^8 + 4*a^3*b*c^7 + 6*a^2*b^2*c^6 + 4*a*b^3*c^5 + b^4*c^4 - (a*c^8 + b*c^7)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 3*(a^2*c^8 + 2*a*b*c^7 + b^2*c^6)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(a^3*c^8 + 3*a^2*b*c^7 + 3*a*b^2*c^6 + b^3*c^5)*(a*d*x^2 + a*c + b)/(d*x^2 + c))$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a + \frac{b}{dx^2+c}\right)^{3/2}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b/(c + d*x^2))^(3/2)/x^7, x)`

[Out] `int((a + b/(c + d*x^2))^(3/2)/x^7, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}}{x^7} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b/(d*x**2+c))**(3/2)/x**7, x)`

[Out] `Integral(((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)/x**7, x)`

$$3.144 \quad \int \frac{x^5}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal. Leaf size=225

$$\frac{(8ac + 5b)(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{24a^2d^3} - \frac{b(8a^2c^2 + 12abc + 5b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{7/2}d^3} + \frac{(8a^2c^2 + 12abc + 5b^2)(c + dx^2)}{16a^3d^3}$$

Rubi [A] time = 0.62, antiderivative size = 267, normalized size of antiderivative = 1.19, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 90, 80, 50, 63, 217, 206}

$$\frac{(8a^2c^2 + 12abc + 5b^2)(a(c + dx^2) + b)}{16a^3d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{b(8a^2c^2 + 12abc + 5b^2)\sqrt{a(c + dx^2) + b} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{16a^{7/2}d^3\sqrt{c + dx^2}\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(8ac + 5b)(c + dx^2)(a(c + dx^2) + b)}{24a^2d^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c + dx^2)(a(c + dx^2) + b)}{6ad^2\sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^5/Sqrt[a + b/(c + d*x^2)], x]

[Out] ((5*b^2 + 12*a*b*c + 8*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^3*d^3*Sqrt[a + b/(c + d*x^2)]) - ((5*b + 8*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(24*a^2*d^3*Sqrt[a + b/(c + d*x^2)]) + (x^2*(c + d*x^2)*(b + a*(c + d*x^2)))/(6*a*d^2*Sqrt[a + b/(c + d*x^2)]) - (b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)])/(16*a^(7/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 90

Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))], x], x]

$(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2)))*x, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n, p\}, x] \ \&\& \ \text{NeQ}[n + p + 3, 0]$

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1*\text{ArcTanh}[\text{Rt}[-b, 2]*x]/\text{Rt}[a, 2])]/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)}*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; \text{FreeQ}[\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1975

$\text{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_)^{(m_)}), x_Symbol] \rightarrow \text{Int}[(e*x)^m*\text{ExpandToSum}[u, x]^p*\text{ExpandToSum}[v, x]^q, x] /; \text{FreeQ}[\{e, m, p, q\}, x] \ \&\& \ \text{BinomialQ}[\{u, v\}, x] \ \&\& \ \text{EqQ}[\text{BinomialDegree}[u, x] - \text{BinomialDegree}[v, x], 0] \ \&\& \ !\text{BinomialMatchQ}[\{u, v\}, x]$

Rule 6722

$\text{Int}[(u_)*((a_ + (b_)*(v_)^{(n_)})^{(p_)}), x_Symbol] \rightarrow \text{Dist}[(a + b*v^n)^{\text{FracPart}[p]}/(v^{(n*\text{FracPart}[p])})*(b + a/v^n)^{\text{FracPart}[p]}, \text{Int}[u*v^{(n*p)}*(b + a/v^n)^p, x], x] /; \text{FreeQ}[\{a, b, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{LtQ}[n, 0] \ \&\& \ \text{BinomialQ}[v, x] \ \&\& \ !\text{LinearQ}[v, x]$

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^5 \sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^5 \sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \text{Subst} \left(\int \frac{x^2 \sqrt{c+dx}}{\sqrt{b+ac+adx}} dx, x, x^2 \right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)} \text{Subst} \left(\int \frac{\sqrt{c+dx} \left(-c(b+ac) - \frac{1}{2}(5b+8ac)dx \right)}{\sqrt{b+ac+adx}} dx, x, x^2 \right)}{6ad^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2 d^3 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(-2ac(b+ac) - \frac{1}{2}(5b+8ac)dx \right)}{6ad^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(5b^2+12abc+8a^2c^2)(b+a(c+dx^2))}{16a^3 d^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2 d^3 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(5b^2+12abc+8a^2c^2)(b+a(c+dx^2))}{16a^3 d^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2 d^3 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(5b^2+12abc+8a^2c^2)(b+a(c+dx^2))}{16a^3 d^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2 d^3 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(5b^2+12abc+8a^2c^2)(b+a(c+dx^2))}{16a^3 d^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(5b+8ac)(c+dx^2)(b+a(c+dx^2))}{24a^2 d^3 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{x^2(c+dx^2)(b+a(c+dx^2))}{6ad^2 \sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 140, normalized size = 0.62

$$\frac{\sqrt{a}(c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}} (8a^2(c^2-cdx^2+d^2x^4) + 2ab(13c-5dx^2) + 15b^2) - 3b(8a^2c^2 + 12abc + 5b^2) \tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{48a^{7/2}d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/Sqrt[a + b/(c + d*x^2)], x]

[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2 + 2*a*b*(13*c - 5*d*x^2) + 8*a^2*(c^2 - c*d*x^2 + d^2*x^4)) - 3*b*(5*b^2 + 12*a*b*c + 8*a^2*c^2)*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(48*a^(7/2)*d^3)

IntegrateAlgebraic [A] time = 0.27, size = 163, normalized size = 0.72

$$\frac{(-8a^2bc^2 - 12ab^2c - 5b^3) \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right) + \sqrt{\frac{ac+adx^2+b}{c+dx^2}} (8a^2c^3 + 8a^2d^3x^6 + 26abc^2 + 16abcdx^2 - 10abd^2x^4 + 15b^2c + 15b^2dx^2)}{16a^{7/2}d^3 + 48a^3d^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^5/Sqrt[a + b/(c + d*x^2)],x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(15*b^2*c + 26*a*b*c^2 + 8*a^2*c^3 + 15*b^2*d*x^2 + 16*a*b*c*d*x^2 - 10*a*b*d^2*x^4 + 8*a^2*d^3*x^6))/(48*a^3*d^3) + ((-5*b^3 - 12*a*b^2*c - 8*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/(16*a^(7/2)*d^3)
```

fricas [A] time = 0.77, size = 425, normalized size = 1.89

$$\frac{3(8a^2bc^2 + 12abd^2 + 5b^3)\sqrt{a} \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)d^2 + 8abc + b^2 - 4(2a^2d^2 + (4ac + 3bd^2 + 2ac^2 + bc)\sqrt{\frac{a(d^2x^2 + c)}{c+d^2x^2}})}{192d^3}\right) + 4(8a^3d^3 - 10a^2bd^2 + 8a^2c^2 + 26a^2bc^2 + 15abd^2 + (16a^2c + 15ab^2)d^2)\sqrt{\frac{a(d^2x^2 + c)}{c+d^2x^2}}}{96a^3d^3} + \frac{3(8a^2bc^2 + 12abd^2 + 5b^3)\sqrt{a} \arctan\left(\frac{d^2x^2 + a + b\sqrt{\frac{a(d^2x^2 + c)}{c+d^2x^2}}}{2(d^2x^2 + c)}\right) + 2(8a^3d^3 - 10a^2bd^2 + 8a^2c^2 + 26a^2bc^2 + 15abd^2 + (16a^2c + 15ab^2)d^2)\sqrt{\frac{a(d^2x^2 + c)}{c+d^2x^2}}}{96a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3), 1/96*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b) + 2*(8*a^3*d^3*x^6 - 10*a^2*b*d^2*x^4 + 8*a^3*c^3 + 26*a^2*b*c^2 + 15*a*b^2*c + (16*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^3)]
```

giac [A] time = 0.57, size = 274, normalized size = 1.22

$$\frac{1}{48} \frac{\sqrt{a^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}}{a^3d^3} \left(2x^2 \frac{4x^2}{a^3d^3 \operatorname{sgn}(d^2x^2 + c)} - \frac{4a^2cd \operatorname{sgn}(d^2x^2 + c) + 5abd \operatorname{sgn}(d^2x^2 + c)}{a^3d^3} \right) + \frac{8a^2c^2d^2 \operatorname{sgn}(d^2x^2 + c) + 26abcd \operatorname{sgn}(d^2x^2 + c) + 15b^2d^2 \operatorname{sgn}(d^2x^2 + c)}{a^3d^3} \left(\frac{(8a^2bc^2 + 12abd^2 + 5b^3) \log\left(\left| \frac{-2acd - 2\left(\sqrt{a^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}\right)\sqrt{d|d-bd}}{32a^2d^3d \operatorname{sgn}(d^2x^2 + c)} \right| \right)}{32a^2d^3d \operatorname{sgn}(d^2x^2 + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2*(4*x^2/(a*d*sgn(d*x^2 + c)) - (4*a^2*c*d^3*sgn(d*x^2 + c) + 5*a*b*d^3*sgn(d*x^2 + c))/(a^3*d^5)) + (8*a^2*c^2*d^2*sgn(d*x^2 + c) + 26*a*b*c*d^2*sgn(d*x^2 + c) + 15*b^2*d^2*sgn(d*x^2 + c))/(a^3*d^5) + 1/32*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(7/2)*d^2*abs(d)*sgn(d*x^2 + c))
```

maple [B] time = 0.05, size = 533, normalized size = 2.37

$$\frac{\frac{1}{48} \sqrt{a^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc} \left(2x^2 \frac{4x^2}{a^3d^3 \operatorname{sgn}(d^2x^2 + c)} - \frac{4a^2cd \operatorname{sgn}(d^2x^2 + c) + 5abd \operatorname{sgn}(d^2x^2 + c)}{a^3d^3} \right) + \frac{8a^2c^2d^2 \operatorname{sgn}(d^2x^2 + c) + 26abcd \operatorname{sgn}(d^2x^2 + c) + 15b^2d^2 \operatorname{sgn}(d^2x^2 + c)}{a^3d^3} \left(\frac{(8a^2bc^2 + 12abd^2 + 5b^3) \log\left(\left| \frac{-2acd - 2\left(\sqrt{a^2x^4 + 2acdx^2 + bd^2x^2 + ac^2 + bc}\right)\sqrt{d|d-bd}}{32a^2d^3d \operatorname{sgn}(d^2x^2 + c)} \right| \right)}{32a^2d^3d \operatorname{sgn}(d^2x^2 + c)} \right)}{32a^2d^3d \operatorname{sgn}(d^2x^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(a+b/(d*x^2+c))^(1/2),x)
```

```
[Out] 1/96*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/a^3/d^3*(-48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a^2*c*d*x^2-36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*d*x^2-24*a^2*b*c^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-36*a*b^2*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(3/2)*(a*d^2)^(1/2)*a+36*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2)*a*b*c-15*b^3*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+30*(a*d^2*x^4+2*a*c*d*x^2+b
```

$$d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/(a*d^2)^{(1/2)}$$

maxima [A] time = 2.28, size = 340, normalized size = 1.51

$$\frac{3(8a^2bc^2 + 12ab^2c + 5b^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}} - 8(6a^3bc^2 + 12a^2b^2c + 5ab^3)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + 3(8a^4bc^2 + 20a^3b^2c + 11a^2b^3)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{48\left(a^6d^3 - \frac{3(adx^2+ac+b)a^5d^3}{dx^2+c} + \frac{3(adx^2+ac+b)^2a^4d^3}{(dx^2+c)^2} - \frac{(adx^2+ac+b)^3a^3d^3}{(dx^2+c)^3}\right)} + \frac{(8a^2c^2 + 12abc + 5b^2)b \log\left(\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{32a^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] $-1/48*(3*(8*a^2*b*c^2 + 12*a*b^2*c + 5*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{(5/2)} - 8*(6*a^3*b*c^2 + 12*a^2*b^2*c + 5*a*b^3)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{(3/2)} + 3*(8*a^4*b*c^2 + 20*a^3*b^2*c + 11*a^2*b^3)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^3 - 3*(a*d*x^2 + a*c + b)*a^5*d^3/(d*x^2 + c) + 3*(a*d*x^2 + a*c + b)^2*a^4*d^3/(d*x^2 + c)^2 - (a*d*x^2 + a*c + b)^3*a^3*d^3/(d*x^2 + c)^3) + 1/32*(8*a^2*c^2 + 12*a*b*c + 5*b^2)*b*\text{log}(-(\text{sqrt}(a) - \text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(\text{sqrt}(a) + \text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^{(7/2)}*d^3)$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\sqrt{a + \frac{b}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b/(c + d*x^2))^(1/2),x)

[Out] int(x^5/(a + b/(c + d*x^2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x**5/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

$$3.145 \quad \int \frac{x^3}{\sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal. Leaf size=148

$$\frac{b(4ac + 3b) \tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{8a^{5/2}d^2} - \frac{(4ac + 3b)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^2d^2} + \frac{(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4ad^2}$$

Rubi [A] time = 0.46, antiderivative size = 189, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6722, 1975, 446, 80, 50, 63, 217, 206}

$$-\frac{(4ac + 3b)(a(c + dx^2) + b)}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{b(4ac + 3b) \sqrt{a(c + dx^2) + b} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}} \right)}{8a^{5/2}d^2 \sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c + dx^2)(a(c + dx^2) + b)}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^3/Sqrt[a + b/(c + d*x^2)],x]

[Out] -((3*b + 4*a*c)*(b + a*(c + d*x^2)))/(8*a^2*d^2*Sqrt[a + b/(c + d*x^2)]) + ((c + d*x^2)*(b + a*(c + d*x^2)))/(4*a*d^2*Sqrt[a + b/(c + d*x^2)]) + (b*(3*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(5/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_.)*(v_)^(q_.)*((e_.)*(x_)^(m_.), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^3 \sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^3 \sqrt{c+dx^2}}{\sqrt{b+ac+adx^2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{x \sqrt{c+dx}}{\sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(c+dx^2)(b+a(c+dx^2))}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{\left((3b+4ac)\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{\sqrt{b+ac+adx}} dx, x, x^2\right)}{8ad\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(3b+4ac)(b+a(c+dx^2))}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)(b+a(c+dx^2))}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a(c+dx^2)}\right)}{16a^2d\sqrt{c+dx^2}} \\
&= -\frac{(3b+4ac)(b+a(c+dx^2))}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)(b+a(c+dx^2))}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a(c+dx^2)}\right)}{8a^2d^2\sqrt{c+dx^2}} \\
&= -\frac{(3b+4ac)(b+a(c+dx^2))}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)(b+a(c+dx^2))}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(b(3b+4ac)\sqrt{b+a(c+dx^2)}\right)}{8a^2d^2\sqrt{c+dx^2}} \\
&= -\frac{(3b+4ac)(b+a(c+dx^2))}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)(b+a(c+dx^2))}{4ad^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{b(3b+4ac)\sqrt{b+a(c+dx^2)}}{8a^{5/2}d^2\sqrt{c+dx^2}}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 101, normalized size = 0.68

$$\frac{b(4ac + 3b) \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a} (c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}} (2a(c - dx^2) + 3b)}{8a^{5/2}d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/Sqrt[a + b/(c + d*x^2)], x]

[Out] $(-\sqrt{a}(c + dx^2)\sqrt{\frac{b + ac + a dx^2}{c + dx^2}}(3b + 2a(c - dx^2))) + b(3b + 4ac)\operatorname{ArcTanh}\left[\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}}\right]/(8a^{5/2}d^2)$

IntegrateAlgebraic [A] time = 0.19, size = 118, normalized size = 0.80

$$\frac{(4abc + 3b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + \sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-2ac^2 + 2ad^2x^4 - 3bc - 3bdx^2)}{8a^{5/2}d^2} + \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-2ac^2 + 2ad^2x^4 - 3bc - 3bdx^2)}{8a^2d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/Sqrt[a + b/(c + d*x^2)], x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-3*b*c - 2*a*c^2 - 3*b*d*x^2 + 2*a*d^2*x^4))/(8*a^2*d^2) + ((3*b^2 + 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/(8*a^(5/2)*d^2)

fricas [A] time = 0.54, size = 333, normalized size = 2.25

$$\frac{(4abc + 3b^2)\sqrt{a} \log\left(\frac{8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 + 4(2a^2d^2x^4 + (4ac + b)dx^2 + 2a^2c + bc)\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{32a^3d^2}\right) + 4(2a^2d^2x^4 - 3abd^2 - 2a^2c^2 - 3abc)\sqrt{\frac{ad^2+ac+b}{d^2+c}} - 2(2a^2d^2x^4 - 3abd^2 - 2a^2c^2 - 3abc)\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{16a^3d^2} - \frac{(4abc + 3b^2)\sqrt{-a} \arctan\left(\frac{(2abd^2 + 2ac + b)\sqrt{-a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{2(a^2d^2 + ac + ab)}\right)}{16a^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2), x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + 3*b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2), -1/16*((4*a*b*c + 3*b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - 2*(2*a^2*d^2*x^4 - 3*a*b*d*x^2 - 2*a^2*c^2 - 3*a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^3*d^2)]

giac [A] time = 0.52, size = 191, normalized size = 1.29

$$\frac{1}{8}\sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}\left(\frac{2x^2}{ad\operatorname{sgn}(dx^2 + c)} - \frac{2ac\operatorname{sgn}(dx^2 + c) + 3bd\operatorname{sgn}(dx^2 + c)}{a^2d^3}\right) - \frac{(4abc + 3b^2)\log\left(\left|-2acd - 2\left(\sqrt{ad^2}x^2 - \sqrt{ad^2}x^4 + 2acdx^2 + bdx^2 + ac^2 + bc\right)\sqrt{a}|d| - bd\right|\right)}{16a^2d|\operatorname{sgn}(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2), x, algorithm="giac")

[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a*d*sgn(d*x^2 + c)) - (2*a*c*d*sgn(d*x^2 + c) + 3*b*d*sgn(d*x^2 + c))/(a^2*d^3)) - 1/16*(4*a*b*c + 3*b^2)*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(5/2)*d*abs(d)*sgn(d*x^2 + c))

maple [B] time = 0.04, size = 354, normalized size = 2.39

$$\frac{\sqrt{\frac{ad^2+ac+b}{d^2+c}}\left(4abcd\ln\left(\frac{2a^2d^2+2ad+bd^2\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{2a^2d^2}\right) + 4\sqrt{a}b^2x^3 + 2acd^2x^2 + bdx^2 + ac^2 + bc\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}} + 3b^2d\ln\left(\frac{2a^2d^2+2ad+bd^2\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}{2a^2d^2}\right) - 4\sqrt{a}b^2x^3 + 2acd^2x^2 + bdx^2 + ac^2 + bc\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}} - 6\sqrt{a}b^2x^3 + 2acd^2x^2 + bdx^2 + ac^2 + bc\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}\right)}{16\sqrt{(dx^2+c)(ad^2+ac+b)}\sqrt{a}\sqrt{\frac{ad^2+ac+b}{d^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+b/(d*x^2+c))^(1/2), x)

[Out] 1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)/d^2*(4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*a*d*x^2+4*a*b*c*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*a*c+3*b^2*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2)*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a^2/(a*d^2)^(1/2)

maxima [A] time = 2.49, size = 223, normalized size = 1.51

$$\frac{(4abc + 3b^2)\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4a^2bc + 5ab^2)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4d^2 - \frac{2(adx^2+ac+b)a^3d^2}{dx^2+c} + \frac{(adx^2+ac+b)^2a^2d^2}{(dx^2+c)^2}\right)} - \frac{(4ac + 3b)b \log\left(\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{16a^{\frac{5}{2}}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out]
$$-1/8*((4*a*b*c + 3*b^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) - (4*a^2*b*c + 5*a*b^2)*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(a^4*d^2 - 2*(a*d*x^2 + a*c + b)*a^3*d^2/(d*x^2 + c) + (a*d*x^2 + a*c + b)^2*a^2*d^2/(d*x^2 + c)^2) - 1/16*(4*a*c + 3*b)*b*\log(-(\sqrt{a} - \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(\sqrt{a} + \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/(a^{5/2}*d^2)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\sqrt{a + \frac{b}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a + b/(c + d*x^2))^(1/2),x)

[Out] int(x^3/(a + b/(c + d*x^2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(x**3/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)

$$3.146 \quad \int \frac{x}{\sqrt{a + \frac{b}{c + dx^2}}} dx$$

Optimal. Leaf size=72

$$\frac{(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{2ad} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1591, 242, 51, 63, 208}

$$\frac{(c + dx^2)\sqrt{a + \frac{b}{c + dx^2}}}{2ad} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c + dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b/(c + d*x^2)], x]

[Out] ((c + d*x^2)*Sqrt[a + b/(c + d*x^2)]/(2*a*d) - (b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[
Pq, x], q*Coeff[Pq, x, q]*Qr] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] &&
```

PolyQ[Qr, x]

Rubi steps

$$\begin{aligned}
\int \frac{x}{\sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a + \frac{b}{x}}} dx, x, c + dx^2\right)}{2d} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x^2 \sqrt{a+bx}} dx, x, \frac{1}{c+dx^2}\right)}{2d} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2ad} + \frac{b \text{Subst}\left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \frac{1}{c+dx^2}\right)}{4ad} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2ad} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{c+dx^2}}\right)}{2ad} \\
&= \frac{(c + dx^2) \sqrt{a + \frac{b}{c+dx^2}}}{2ad} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 70, normalized size = 0.97

$$\frac{\sqrt{a} (c + dx^2) \sqrt{a + \frac{b}{c+dx^2}} - b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b/(c + d*x^2)],x]

[Out] (Sqrt[a]*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)] - b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)

IntegrateAlgebraic [A] time = 0.11, size = 88, normalized size = 1.22

$$\frac{(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2ad} - \frac{b \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[a + b/(c + d*x^2)],x]

[Out] ((c + d*x^2)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*a*d) - (b*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]])/(2*a^(3/2)*d)

fricas [A] time = 0.74, size = 267, normalized size = 3.71

$$\frac{\sqrt{a} b \log\left(\frac{8 a^2 d^2 x^4 + 8 a^2 c^2 + 8 (2 a^2 c + a b) d x^2 + 8 a b c + b^2 - 4 (2 a d^2 x^4 + (4 a c + b) d x^2 + 2 a c^2 + b c) \sqrt{a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{8 a^2 d}\right) + 4 (a d x^2 + a c) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} - \sqrt{-a} b \arctan\left(\frac{(2 a d x^2 + 2 a c + b) \sqrt{-a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{2 (a^2 d x^2 + a^2 c + a b)}\right) + 2 (a d x^2 + a c) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{4 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/8*(sqrt(a)*b*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c))*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d), 1/4*(sqrt(-a)*b*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a*d*x^2 + a*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d)]

giac [B] time = 0.51, size = 138, normalized size = 1.92

$$\frac{b \log \left(\left| -2acd - 2 \left(\sqrt{ad^2 x^2 - \sqrt{ad^2 x^4 + 2acd x^2 + bdx^2 + ac^2 + bc}} \right) \sqrt{a} |d| - bd \right| \right)}{4 a^{\frac{3}{2}} |d| \operatorname{sgn}(dx^2 + c)} + \frac{\sqrt{ad^2 x^4 + 2acd x^2 + bdx^2 + ac^2 + bc}}{2 ad \operatorname{sgn}(dx^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] 1/4*b*log(abs(-2*a*c*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*sqrt(a)*abs(d) - b*d))/(a^(3/2)*abs(d)*sgn(d*x^2 + c)) + 1/2*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a*d*sgn(d*x^2 + c))

maple [B] time = 0.03, size = 184, normalized size = 2.56

$$\frac{\sqrt{\frac{ad^2 x^2 + ac + b}{dx^2 + c}} (dx^2 + c) \left(-bd \ln \left(\frac{2ad^2 x^2 + 2acd + bd + 2\sqrt{ad^2 x^4 + 2acd x^2 + bdx^2 + ac^2 + bc} \sqrt{ad^2}}{2\sqrt{ad^2}} \right) + 2\sqrt{ad^2 x^4 + 2acd x^2 + bdx^2 + ac^2 + bc} \sqrt{ad^2} \right)}{4\sqrt{(dx^2 + c)(ad^2 x^2 + ac + b)} \sqrt{ad^2} ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b/(d*x^2+c))^(1/2),x)

[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-b*d*ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^(1/2)*(a*d^2)^(1/2))/(a*d^2)^(1/2))+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*d^2)^(1/2))/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/a/d/(a*d^2)^(1/2)

maxima [B] time = 2.81, size = 129, normalized size = 1.79

$$-\frac{b \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2 \left(a^2 d - \frac{(adx^2+ac+b)ad}{dx^2+c} \right)} + \frac{b \log \left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}} \right)}{4 a^{\frac{3}{2}} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] -1/2*b*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^2*d - (a*d*x^2 + a*c + b)*a*d/(d*x^2 + c)) + 1/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(3/2)*d)

mupad [B] time = 3.34, size = 111, normalized size = 1.54

$$\frac{\sqrt{\frac{a(dx^2+c)}{b}} + 1 (dx^2 + c) \left(\frac{3\sqrt{b}\sqrt{b+a(dx^2+c)}}{2a(dx^2+c)} + \frac{b^{3/2} \operatorname{asin}\left(\frac{\sqrt{a}\sqrt{dx^2+c} + 1}{\sqrt{b}}\right)}{2a^{3/2}(dx^2+c)^{3/2}} \right)}{3d \sqrt{a + \frac{b}{dx^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b/(c + d*x^2))^(1/2),x)`

[Out] $((a*(c + d*x^2))/b + 1)^{(1/2)}*(c + d*x^2)*((b^{(3/2)}*asin((a^{(1/2)}*(c + d*x^2)^{(1/2)}*1i)/b^{(1/2)})*3i)/(2*a^{(3/2)}*(c + d*x^2)^{(3/2)}) + (3*b^{(1/2)}*(b + a*(c + d*x^2))^{(1/2)})/(2*a*(c + d*x^2)))/(3*d*(a + b/(c + d*x^2))^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b/(d*x**2+c))**(1/2),x)`

[Out] `Integral(x/sqrt((a*c + a*d*x**2 + b)/(c + d*x**2)), x)`

$$3.147 \quad \int \frac{1}{x \sqrt{a + \frac{b}{c + dx^2}}} dx$$

Optimal. Leaf size=96

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Rubi [A] time = 0.41, antiderivative size = 184, normalized size of antiderivative = 1.92, number of steps used = 9, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {6722, 1975, 446, 105, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a(c+dx^2)+b} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{\sqrt{a} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{\sqrt{c} \sqrt{a(c+dx^2)+b} \tanh^{-1}\left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}}\right)}{\sqrt{ac+b} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b/(c + d*x^2)]), x]

[Out] (Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]/(Sqrt[a]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]) - (Sqrt[c]*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/Sqrt[c]*Sqrt[b + a*(c + d*x^2)]]/(Sqrt[b + a*c]*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]))

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int((((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 105

Int((((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

Rule 206

Int(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] \cdot \text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] \text{ ; FreeQ}\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Subst}[\text{Int}[1/(1 - b \cdot x^2), x], x, x/\text{Sqrt}[a + b \cdot x^2]] \text{ ; FreeQ}\{a, b\}, x] \ \&\& \ !\text{GtQ}[a, 0]$

Rule 446

$\text{Int}[(x_)^{(m_ \cdot)} \cdot ((a_ + (b_ \cdot)(x_)^{(n_)})^{(p_ \cdot)} \cdot ((c_ + (d_ \cdot)(x_)^{(n_)})^{(q_ \cdot)}), x_Symbol] \rightarrow \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1) \cdot (a + b \cdot x)^p \cdot (c + d \cdot x)^q, x], x, x^n], x] \text{ ; FreeQ}\{a, b, c, d, m, n, p, q\}, x] \ \&\& \ \text{NeQ}[b \cdot c - a \cdot d, 0] \ \&\& \ \text{IntegerQ}[\text{Simplify}[(m + 1)/n]]$

Rule 1975

$\text{Int}[(u_)^{(p_ \cdot)} \cdot (v_)^{(q_ \cdot)} \cdot ((e_ \cdot)(x_)^{(m_ \cdot)}), x_Symbol] \rightarrow \text{Int}[(e \cdot x)^m \cdot \text{ExpandToSum}[u, x]^p \cdot \text{ExpandToSum}[v, x]^q, x] \text{ ; FreeQ}\{e, m, p, q\}, x] \ \&\& \ \text{BinomialQ}\{u, v\}, x] \ \&\& \ \text{EqQ}[\text{BinomialDegree}[u, x] - \text{BinomialDegree}[v, x], 0] \ \&\& \ !\text{BinomialMatchQ}\{u, v\}, x]$

Rule 6722

$\text{Int}[(u_ \cdot) \cdot ((a_ \cdot) + (b_ \cdot)(v_)^{(n_)})^{(p_ \cdot)}, x_Symbol] \rightarrow \text{Dist}[(a + b \cdot v^n)^{\text{FracPart}[p]} / (v^{(n \cdot \text{FracPart}[p])} \cdot (b + a/v^n)^{\text{FracPart}[p]}), \text{Int}[u \cdot v^{(n \cdot p)} \cdot (b + a/v^n)^p, x], x] \text{ ; FreeQ}\{a, b, p\}, x] \ \&\& \ !\text{IntegerQ}[p] \ \&\& \ !\text{LtQ}[n, 0] \ \&\& \ \text{BinomialQ}[v, x] \ \&\& \ !\text{LinearQ}[v, x]$

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x\sqrt{b+a(c+dx^2)}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x\sqrt{b+ac+adx^2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x\sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\left(c\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(d\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(c\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{\sqrt{c}\sqrt{b+a(c+dx^2)} \tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \sqrt{c+dx^2}\right)}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{a}\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{\sqrt{c}\sqrt{b+a(c+dx^2)} \tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{\sqrt{b+ac}\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 80, normalized size = 0.83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{\sqrt{ac+b}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b/(c + d*x^2)]), x]

[Out] ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]/Sqrt[a] - (Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/Sqrt[b + a*c]

IntegrateAlgebraic [A] time = 0.15, size = 109, normalized size = 1.14

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{\sqrt{a}} - \frac{\sqrt{c} \tan^{-1}\left(\frac{\sqrt{c}\sqrt{-ac-b}\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{\sqrt{-ac-b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b/(c + d*x^2)]), x]

[Out] $-\left(\frac{\sqrt{c} \operatorname{ArcTan}\left[\frac{\sqrt{c} \sqrt{-b - a c}}{\sqrt{(b + a c + a d x^2)}}\right]}{(c + d x^2)}\right) / \sqrt{-b - a c} + \operatorname{ArcTanh}\left[\frac{\sqrt{c} \sqrt{(b + a c + a d x^2)}}{(c + d x^2)}\right] / \sqrt{a} / \sqrt{a}$

fricas [B] time = 0.77, size = 972, normalized size = 10.12

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{4} \left(a \sqrt{\frac{c}{a c + b}} \log\left(\frac{\left((8 a^2 c^2 + 8 a b c + b^2) d^2 x^4 + 8 a^2 c^4 + 16 a^2 b c^3 + 8 b^2 c^2 + 8 (2 a^2 c^3 + 3 a b c^2 + b^2 c) d x^2 - 4 \left((2 a^2 c^2 + 3 a b c + b^2) d^2 x^4 + 2 a^2 c^4 + 4 a b c^3 + 2 b^2 c^2 + (4 a^2 c^3 + 7 a b c^2 + 3 b^2 c) d x^2 \right) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} \sqrt{\frac{c}{a c + b}} \right)}{x^4} + \sqrt{a} \log\left(\frac{8 a^2 d^2 x^4 + 8 a^2 c^2 + 8 (2 a^2 c + a b) d x^2 + 8 a b c + b^2 + 4 (2 a d^2 x^4 + (4 a c + b) d x^2 + 2 a c^2 + b c) \sqrt{a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} \right)}{a}, \frac{1}{4} \left(a \sqrt{\frac{c}{a c + b}} \log\left(\frac{\left((8 a^2 c^2 + 8 a b c + b^2) d^2 x^4 + 8 a^2 c^4 + 16 a^2 b c^3 + 8 b^2 c^2 + 8 (2 a^2 c^3 + 3 a b c^2 + b^2 c) d x^2 - 4 \left((2 a^2 c^2 + 3 a b c + b^2) d^2 x^4 + 2 a^2 c^4 + 4 a b c^3 + 2 b^2 c^2 + (4 a^2 c^3 + 7 a b c^2 + 3 b^2 c) d x^2 \right) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} \sqrt{\frac{c}{a c + b}} \right)}{x^4} - 2 \sqrt{-a} \arctan\left(\frac{1}{2} \frac{(2 a d x^2 + 2 a c + b) \sqrt{-a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{a^2 d x^2 + a^2 c + a b} \right) \right) / a, \frac{1}{4} \left(2 a \sqrt{-c} \sqrt{\frac{c}{a c + b}} \arctan\left(\frac{1}{2} \frac{(2 a c + b) d x^2 + 2 a c^2 + 2 b c}{(a d x^2 + a c + b) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}} \sqrt{\frac{-c}{a c + b}} \right) \sqrt{\frac{c}{a c + b}} \right) / (a c d x^2 + a c^2 + b c) + \sqrt{a} \log\left(\frac{8 a^2 d^2 x^4 + 8 a^2 c^2 + 8 (2 a^2 c + a b) d x^2 + 8 a b c + b^2 + 4 (2 a d^2 x^4 + (4 a c + b) d x^2 + 2 a c^2 + b c) \sqrt{a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} \right)}{a}, \frac{1}{2} \left(a \sqrt{-c} \sqrt{\frac{c}{a c + b}} \arctan\left(\frac{1}{2} \frac{(2 a c + b) d x^2 + 2 a c^2 + 2 b c}{(a d x^2 + a c + b) \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}} \sqrt{\frac{-c}{a c + b}} \right) \sqrt{\frac{c}{a c + b}} \right) / (a c d x^2 + a c^2 + b c) - \sqrt{-a} \arctan\left(\frac{1}{2} \frac{(2 a d x^2 + 2 a c + b) \sqrt{-a} \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{a^2 d x^2 + a^2 c + a b} \right) \right) / a \right]$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")`

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(c+d*t_nostep)] Error: Bad Argument Type

maple [B] time = 0.04, size = 312, normalized size = 3.25

$$\frac{\sqrt{\frac{a d^2 + a c + b}{d x^2 + c}} (d x^2 + c) \left(a c d \ln \left(\frac{2 a d^2 x^2 + 2 a c d + b d + 2 \sqrt{a d^2 x^4 + 2 a c d x^2 + a^2 c^2 + b c} \sqrt{a d^2}}{2 \sqrt{a d^2}} \right) + b d \ln \left(\frac{2 a d^2 x^2 + 2 a c d + b d + 2 \sqrt{a d^2 x^4 + 2 a c d x^2 + a^2 c^2 + b c} \sqrt{a d^2}}{2 \sqrt{a d^2}} \right) - \sqrt{a c^2 + b c} \sqrt{a d^2} \ln \left(\frac{2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 \sqrt{a c^2 + b c} \sqrt{a d^2 x^2 + 2 a c d x^2 + a^2 c^2 + b c}}{x^2} \right) \right)}{2 \sqrt{(d x^2 + c) (a d x^2 + a c + b)} (a c + b) \sqrt{a d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a+b/(d*x^2+c))^(1/2),x)`

[Out] $\frac{1}{2} \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{1/2} (d x^2 + c) (a c d \ln(1/2 (2 a d^2 x^2 + 2 a c d + b d + 2 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a d^2)^{1/2}) / (a d^2)^{1/2}) + b d \ln(1/2 (2 a d^2 x^2 + 2 a c d + b d + 2 (a d^2 x^4 + 2 a c d x^2 + b d x^2 + a c^2 + b c)^{1/2} (a d^2)^{1/2}) / (a d^2)^{1/2}) - (a c^2 + b c)^{1/2} (a d^2)^{1/2} \ln((2 a c d x^2 + b d x^2 + 2 a c^2 + 2 b c + 2 (a c^2 + b c)^{1/2} (a$

$*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}/x^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/(a*c+b)/(a*d^2)^{(1/2)}$

maxima [A] time = 3.34, size = 155, normalized size = 1.61

$$\frac{c \log \left(\frac{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}} \right)}{2 \sqrt{(ac+b)c}} - \frac{\log \left(-\frac{\sqrt{a} - \sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a} + \sqrt{\frac{adx^2+ac+b}{dx^2+c}}} \right)}{2 \sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] $1/2*c*\log((c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} - \sqrt{(a*c + b)*c})/(c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} + \sqrt{(a*c + b)*c}))/\sqrt{(a*c + b)*c} - 1/2*\log(-(\sqrt{a} - \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(\sqrt{a} + \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/\sqrt{a}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{a + \frac{b}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b/(c + d*x^2))^(1/2)),x)

[Out] int(1/(x*(a + b/(c + d*x^2))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(1/(x*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)

$$3.148 \quad \int \frac{1}{x^3 \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal. Leaf size=108

$$\frac{(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac + b)} - \frac{bd \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{2\sqrt{c} (ac + b)^{3/2}}$$

Rubi [A] time = 0.39, antiderivative size = 148, normalized size of antiderivative = 1.37, number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{a(c + dx^2) + b}{2x^2(ac + b) \sqrt{a + \frac{b}{c+dx^2}}} - \frac{bd \sqrt{a(c + dx^2) + b} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{2\sqrt{c} (ac + b)^{3/2} \sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*Sqrt[a + b/(c + d*x^2)]),x]

[Out] -(b + a*(c + d*x^2))/(2*(b + a*c)*x^2*Sqrt[a + b/(c + d*x^2)]) - (b*d*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(2*Sqrt[c]*(b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x^3 \sqrt{b+a(c+dx^2)}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x^3 \sqrt{b+ac+adx^2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2 \sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= -\frac{b+a(c+dx^2)}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(bd\sqrt{b+a(c+dx^2)}) \operatorname{Subst}\left(\int \frac{1}{x\sqrt{c+dx}\sqrt{b+ac+adx}} dx, x, x^2\right)}{4(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= -\frac{b+a(c+dx^2)}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(bd\sqrt{b+a(c+dx^2)}) \operatorname{Subst}\left(\int \frac{1}{-c-(-b-ac)x^2} dx, x, \frac{\sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}}\right)}{2(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= -\frac{b+a(c+dx^2)}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{bd\sqrt{b+a(c+dx^2)} \tanh^{-1}\left(\frac{\sqrt{b+ac}\sqrt{c+dx^2}}{\sqrt{c}\sqrt{b+a(c+dx^2)}}\right)}{2\sqrt{c}(b+ac)^{3/2}\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \end{aligned}$$

Mathematica [A] time = 0.30, size = 210, normalized size = 1.94

$$\frac{c\sqrt{\frac{ac+ad^2+b}{c+dx^2}} \left(2\sqrt{c(ac+b)}(c+dx^2)(ac+adx^2+b) - 2bdx^2 \log(x)\sqrt{(c+dx^2)(a(c+dx^2)+b)} + bdx^2\sqrt{(c+dx^2)(ac+adx^2+b)} \log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)} + 2ac(c+dx^2) + b(2c+dx^2)\right) \right)}{4x^2(c(ac+b))^{3/2}(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*Sqrt[a + b/(c + d*x^2)]), x]
```

```
[Out] -1/4*(c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b + a*c + a*d*x^2) - 2*b*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + b*d*x^2*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/((c*(b + a*c))^(3/2)*x^2*(b + a*(c + d*x^2)))
```

IntegrateAlgebraic [A] time = 0.20, size = 131, normalized size = 1.21

$$\frac{(-c - dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2x^2(ac + b)} - \frac{bd \tan^{-1} \left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b} \right)}{2\sqrt{c} \sqrt{-ac - b} (ac + b)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*sqrt[a + b/(c + d*x^2)]), x]

[Out] ((-c - d*x^2)*sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(2*(b + a*c)*x^2) - (b*d*ArcTan[(sqrt[c]*sqrt[-b - a*c]*sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/(b + a*c)])/(2*sqrt[c]*sqrt[-b - a*c]*(b + a*c))

fricas [B] time = 0.89, size = 451, normalized size = 4.18

$$\frac{\sqrt{ac^2 + bc} b dx^2 \log \left(\frac{(8a^2d^2 + 8abc + b^2)d^2x^4 + 8a^2d^2 + 16abd^2 + 8b^2d^2 + 8(2ac + b^2)d^2x^2 + 4(2ac + b^2)d^2 + 2a^2(4ac^2 + 3bc)d^2 + 2b^2) \sqrt{\frac{ac+bx^2}{c+dx^2}}}{8(a^2c^3 + 2abc^2 + b^2c)x^2} - 4(ac^3 + (ac^2 + bc)dx^2 + bc^2) \sqrt{\frac{ac+bx^2}{c+dx^2}} \sqrt{-ac^2 - bc} b dx^2 \arctan \left(\frac{(2ac + b^2)d^2 + 2a^2(4ac^2 + 3bc) \sqrt{\frac{ac+bx^2}{c+dx^2}}}{2(d^2 + 2abx^2 + (d^2 + ab)b^2x^2)} \right)}{4(a^2c^3 + 2abc^2 + b^2c)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2), x, algorithm="fricas")

[Out] [1/8*(sqrt(a*c^2 + b*c)*b*d*x^2*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) - 4*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2), 1/4*(sqrt(-a*c^2 - b*c)*b*d*x^2*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*(a*c^3 + (a*c^2 + b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*x^2)]

giac [B] time = 0.70, size = 315, normalized size = 2.92

$$\frac{bd \arctan \left(\frac{\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}}}{\sqrt{-ac^2 - bc}} \right)}{2\sqrt{-ac^2 - bc} (\text{acsgn}(dx^2 + c) + \text{bsgn}(dx^2 + c))} - \frac{2a^{\frac{3}{2}}c^2|d| + 2(\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}})acd + 2\sqrt{a}bc|d| + (\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}})bd}{2(ac^2 - (\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}})^2 + bc)(\text{acsgn}(dx^2 + c) + \text{bsgn}(dx^2 + c))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2), x, algorithm="giac")

[Out] 1/2*b*d*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/(sqrt(-a*c^2 - b*c)*(a*c*sgn(d*x^2 + c) + b*sgn(d*x^2 + c))) - 1/2*(2*a^(3/2)*c^2*abs(d) + 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*c*d + 2*sqrt(a)*b*c*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*b*d)/((a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)*(a*c*sgn(d*x^2 + c) + b*sgn(d*x^2 + c)))

maple [B] time = 0.04, size = 452, normalized size = 4.19

$$\frac{\sqrt{\frac{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}}{-ac^2 - bc}} (1/2 + \frac{1}{2} \sqrt{\frac{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}}{-ac^2 - bc}}) \ln \left(\frac{(8a^2d^2 + 8abc + b^2)d^2x^4 + 8a^2d^2 + 16abd^2 + 8b^2d^2 + 8(2ac + b^2)d^2x^2 + 4(2ac + b^2)d^2 + 2a^2(4ac^2 + 3bc)d^2 + 2b^2) \sqrt{\frac{ac+bx^2}{c+dx^2}}}{8(a^2c^3 + 2abc^2 + b^2c)x^2} \right) - 2\sqrt{ad^2x^2 - \sqrt{ad^2x^4 + 2acdx^2 + bdx^2 + ac^2 + bc}} \sqrt{-ac^2 - bc} b dx^2 \arctan \left(\frac{(2ac + b^2)d^2 + 2a^2(4ac^2 + 3bc) \sqrt{\frac{ac+bx^2}{c+dx^2}}}{2(d^2 + 2abx^2 + (d^2 + ab)b^2x^2)} \right)}{4\sqrt{(d^2 + c)(ad^2 + ac + b)} (ac + b) \sqrt{-ac^2 - bc} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b/(d*x^2+c))^(1/2), x)

[Out] -1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-2*a*d^2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*x^4*(a*c^2+b*c)^(1/2)+ln((2*a*c*d*x^2+b*d*x

$$\frac{d^2 + 2ac^2 + 2bc + 2(a^2c^2 + b^2c)^{1/2} (ad^2x^4 + 2acd^2x^2 + bd^2x^2 + a^2c^2 + b^2c)^{1/2}}{x^2} \cdot x^2 \cdot a^2 b^2 c^2 d - 4(a^2d^2x^4 + 2acd^2x^2 + bd^2x^2 + a^2c^2 + b^2c)^{1/2} \cdot a^2 c^2 d^2 x^2 (a^2c^2 + b^2c)^{1/2} + \ln\left(\frac{(2acd^2x^2 + bd^2x^2 + 2a^2c^2 + 2b^2c + 2(a^2c^2 + b^2c)^{1/2})(ad^2x^4 + 2acd^2x^2 + bd^2x^2 + a^2c^2 + b^2c)^{1/2}}{x^2}\right) \cdot x^2 \cdot b^2 c^2 d - 2(a^2d^2x^4 + 2acd^2x^2 + bd^2x^2 + a^2c^2 + b^2c)^{1/2} \cdot bd^2x^2 (a^2c^2 + b^2c)^{1/2} + 2(a^2d^2x^4 + 2acd^2x^2 + bd^2x^2 + a^2c^2 + b^2c)^{3/2} (a^2c^2 + b^2c)^{1/2}\right) / ((d^2x^2 + c)(ad^2x^2 + a^2c + b))^{1/2} / (a^2c + b)^2 / c / x^2 / (a^2c^2 + b^2c)^{1/2}$$

maxima [A] time = 2.78, size = 173, normalized size = 1.60

$$-\frac{bd\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{2\left(a^2c^2 + 2abc + b^2 - \frac{(adx^2+ac+b)(ac^2+bc)}{dx^2+c}\right)} + \frac{bd \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{4\sqrt{(ac+b)c}(ac+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(1/2),x, algorithm="maxima")

[Out] $-\frac{1}{2}bd\sqrt{\frac{(ad^2x^2 + a^2c + b)(d^2x^2 + c)}{(a^2c^2 + 2abc + b^2 - (ad^2x^2 + a^2c + b)(a^2c^2 + b^2c)/(d^2x^2 + c))}} + \frac{1}{4}bd\log\left(\frac{c\sqrt{\frac{(ad^2x^2 + a^2c + b)(d^2x^2 + c)}{(a^2c^2 + 2abc + b^2 - (ad^2x^2 + a^2c + b)(a^2c^2 + b^2c)/(d^2x^2 + c))}} - \sqrt{(a^2c + b)c}}{c\sqrt{\frac{(ad^2x^2 + a^2c + b)(d^2x^2 + c)}{(a^2c^2 + 2abc + b^2 - (ad^2x^2 + a^2c + b)(a^2c^2 + b^2c)/(d^2x^2 + c))}} + \sqrt{(a^2c + b)c}}\right) / (\sqrt{(a^2c + b)c}(a^2c + b))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \sqrt{a + \frac{b}{dx^2+c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)),x)

[Out] int(1/(x^3*(a + b/(c + d*x^2))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b/(d*x**2+c))**(1/2),x)

[Out] Integral(1/(x**3*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)

$$3.149 \quad \int \frac{1}{x^5 \sqrt{a + \frac{b}{c+dx^2}}} dx$$

Optimal. Leaf size=177

$$\frac{bd^2(4ac+b) \tanh^{-1}\left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}}\right)}{8c^{3/2}(ac+b)^{5/2}} + \frac{d(4ac+b)(c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8cx^2(ac+b)^2} - \frac{(c+dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4cx^4(ac+b)}$$

Rubi [A] time = 0.47, antiderivative size = 218, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{bd^2(4ac+b) \sqrt{a(c+dx^2)+b} \tanh^{-1}\left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}}\right)}{8c^{3/2}(ac+b)^{5/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{d(4ac+b)(a(c+dx^2)+b)}{8cx^2(ac+b)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)(a(c+dx^2)+b)}{4cx^4(ac+b) \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]

[Out] ((b + 4*a*c)*d*(b + a*(c + d*x^2)))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*x^2)]) - ((c + d*x^2)*(b + a*(c + d*x^2)))/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c + d*x^2)]) + (b*(b + 4*a*c)*d^2*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*c^(3/2)*(b + a*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/(m + 1)*(b*e - a*f), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^5 \sqrt{a + \frac{b}{c+dx^2}}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x^5 \sqrt{b+a(c+dx^2)}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{\sqrt{c+dx^2}}{x^5 \sqrt{b+ac+adx^2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^3 \sqrt{b+ac+adx}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= -\frac{(c+dx^2)(b+a(c+dx^2))}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{\left((b+4ac)d\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x^2 \sqrt{b+ac+adx}} dx, x, x^2\right)}{8c(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)(b+a(c+dx^2))}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(b(b+4ac)d^2 \sqrt{b+a(c+dx^2)})}{16c(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)(b+a(c+dx^2))}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(b(b+4ac)d^2 \sqrt{b+a(c+dx^2)})}{8c(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
 &= \frac{(b+4ac)d(b+a(c+dx^2))}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)(b+a(c+dx^2))}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{b(b+4ac)d^2 \sqrt{b+a(c+dx^2)}}{8c^{3/2}(b+ac)^{5/2} \sqrt{a + \frac{b}{c+dx^2}}}
 \end{aligned}$$

Mathematica [A] time = 0.48, size = 269, normalized size = 1.52

$$\frac{\sqrt{\frac{c^2+bd^2+4b^2}{c+dx^2}} \left(-2\sqrt{c(ac+b)}(c+dx^2)(2a^2c(c^2-d^2x^4)+ab(4c^2+cdx^2+d^2x^4)+b^2(2c+dx^2))-2bd^2x^4 \log(x)(4ac+b)\sqrt{(c+dx^2)(a(c+dx^2)+b)}+bd^2x^4(4ac+b)\sqrt{(c+dx^2)(a(c+dx^2)+b)} \log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)}+2ac(c+dx^2)+b(2c+dx^2)\right) \right)}{16a^2(c(ac+b))^2(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]

[Out] (c*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b^2*(2*c + d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + c*d*x^2 + d^2*x^4)) - 2*b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x + b*(b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/(16*(c*(b + a*c))^(5/2)*x^4*(b + a*(c + d*x^2)))

IntegrateAlgebraic [A] time = 0.33, size = 174, normalized size = 0.98

$$\frac{d^2(4abc + b^2) \tan^{-1}\left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{8c^{3/2} \sqrt{-ac-b} (ac+b)^2} + \frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-2ac^3 + 2acd^2x^4 - 2bc^2 - 3bcdx^2 - bd^2x^4)}{8cx^4(ac+b)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*Sqrt[a + b/(c + d*x^2)]),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b*c^2 - 2*a*c^3 - 3*b*c*d*x^2 - b*d^2*x^4 + 2*a*c*d^2*x^4))/(8*c*(b + a*c)^2*x^4) + ((b^2 + 4*a*b*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/(b + a*c)))/(8*c^(3/2)*Sqrt[-b - a*c]*(b + a*c)^2)

fricas [A] time = 1.02, size = 593, normalized size = 3.35

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="fricas")

[Out] [1/32*((4*a*b*c + b^2)*sqrt(a*c^2 + b*c)*d^2*x^4*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4 - 4*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4), -1/16*((4*a*b*c + b^2)*sqrt(-a*c^2 - b*c)*d^2*x^4*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) + 2*(2*a^2*c^5 - (2*a^2*c^3 + a*b*c^2 - b^2*c)*d^2*x^4 + 4*a*b*c^4 + 2*b^2*c^3 + 3*(a*b*c^3 + b^2*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*x^4)]

giac [B] time = 10.39, size = 815, normalized size = 4.60

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2),x, algorithm="giac")

[Out] -1/8*(4*a*b*c*d^2 + b^2*d^2)*arctan(-(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))/sqrt(-a*c^2 - b*c))/((a^2*c^3*sgn(d*x^2 + c) + 2*a*b*c^2*sgn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*sqrt(-a*c^2 - b*c)) + 1/8*(8*a^(7/2)*c^5*d*abs(d) + 16*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^4*d^2 + 8*(sqrt(a*d^2)*x^2 - sqrt

$$\begin{aligned} & (a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)^2*a^{(5/2)}*c^3*d*abs(d) + \\ & 16*a^{(5/2)}*b*c^4*d*abs(d) + 28*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d \\ & *x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c^3*d^2 + 16*(sqrt(a*d^2)*x^2 - sqrt(a \\ & *d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^{(3/2)}*b*c^2*d*abs(d) + \\ & 8*a^{(3/2)}*b^2*c^3*d*abs(d) + 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d \\ & *x^2 + b*d*x^2 + a*c^2 + b*c))^3*a*b*c*d^2 + 13*(sqrt(a*d^2)*x^2 - sqrt(a*d \\ & ^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*c^2*d^2 + 8*(sqrt(a*d^ \\ & 2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*sqrt(a)*b \\ & ^2*c*d*abs(d) + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + \\ & a*c^2 + b*c))^3*b^2*d^2 + (sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 \\ & + b*d*x^2 + a*c^2 + b*c))*b^3*c*d^2)/((a^2*c^3*sgn(d*x^2 + c) + 2*a*b*c^2*s \\ & gn(d*x^2 + c) + b^2*c*sgn(d*x^2 + c))*(a*c^2 - (sqrt(a*d^2)*x^2 - sqrt(a*d^ \\ & 2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2 + b*c)^2) \end{aligned}$$

maple [B] time = 0.05, size = 922, normalized size = 5.21

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(a+b/(d*x^2+c))^(1/2), x)

[Out] $\frac{1}{16} \left(\frac{a*d*x^2+a*c+b}{(d*x^2+c)} \right)^{(1/2)} (d*x^2+c) (-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)} (a*c^2+b*c)^{(3/2)} a^2*c*d^3*x^6+4*a^3*b*c^5*d^2*x^4 \ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)} (a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2) - 2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)} (a*c^2+b*c)^{(3/2)} a*b*d^3*x^6+9*a^2*b^2*c^4*d^2*x^4 \ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)} (a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2) - 20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)} (a*c^2+b*c)^{(3/2)} a^2*c^2*d^2*x^4+6*a*b^3*c^3*d^2*x^4 \ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)} (a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2) - 12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)} (a*c^2+b*c)^{(3/2)} a*b*c*d^2*x^4+b^4*c^2*d^2*x^4 \ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)} (a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2) - 2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)} (a*c^2+b*c)^{(3/2)} *b^2*d^2*x^4+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)} (a*c^2+b*c)^{(3/2)} a*c*d*x^2+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)} (a*c^2+b*c)^{(3/2)} *b*d*x^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)} (a*c^2+b*c)^{(3/2)} *a*c^2-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)} (a*c^2+b*c)^{(3/2)} *b*c) / ((d*x^2+c) (a*d*x^2+a*c+b))^{(1/2)} / (a*c+b)^3/c^2/x^4 / (a*c^2+b*c)^{(3/2)}$

maxima [B] time = 2.37, size = 359, normalized size = 2.03

$$\frac{(4abc + b^2)d^2 \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{16(a^2c^3 + 2abc^2 + b^2c)\sqrt{(ac+b)c}} - \frac{(4abc^2 + b^2c)d^2\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} - (4a^2bc^2 + 3ab^2c - b^3)d^2\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{8\left(a^4c^5 + 4a^3bc^4 + 6a^2b^2c^3 + 4ab^3c^2 + b^4c + \frac{(a^2c^5 + 2abc^4 + b^2c^3)(adx^2+ac+b)^2}{(dx^2+c)^2} - \frac{2(a^3c^5 + 3a^2bc^4 + 3ab^2c^3 + b^3c^2)(adx^2+ac+b)}{dx^2+c}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(1/2), x, algorithm="maxima")

[Out] $-1/16*(4*a*b*c + b^2)*d^2*\log((c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - sqrt((a*c + b)*c))/(c*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) + sqrt((a*c + b)*c)))/((a^2*c^3 + 2*a*b*c^2 + b^2*c)*sqrt((a*c + b)*c)) - 1/8*((4*a*b*c^2 + b^2*c)*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{(3/2)} - (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c + (a^2*c^5 + 2*a*b*c^4 + b^2*c^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 2*(a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c))$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^5 \sqrt{a + \frac{b}{dx^2 + c}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^5*(a + b/(c + d*x^2))^(1/2)),x)`

[Out] `int(1/(x^5*(a + b/(c + d*x^2))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(a+b/(d*x**2+c))**(1/2),x)`

[Out] `Integral(1/(x**5*sqrt((a*c + a*d*x**2 + b)/(c + d*x**2))), x)`

$$3.150 \quad \int \frac{x^5}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=310

$$\frac{(6a^2c^2 + 12abc + 7b^2)(c + dx^2)^3 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{6a^2b^2d^3} - \frac{b(24a^2c^2 + 60abc + 35b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{16a^{9/2}d^3} + \frac{(24a^2c^2 + 60abc + 35b^2)(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{a^2bd^3} + \frac{(ac+b)^2(c+dx^2)^2}{a^2bd^3} + \frac{(c+dx^2)^2(a(c+dx^2)+b)}{6a^2d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(24a^2c^2 + 60abc + 35b^2)(a(c+dx^2)+b)}{16a^4d^3\sqrt{a+\frac{b}{c+dx^2}}} + \frac{(24a^2c^2 + 60abc + 35b^2)(c+dx^2)(a(c+dx^2)+b)}{24a^3bd^3\sqrt{a+\frac{b}{c+dx^2}}}$$

Rubi [A] time = 0.78, antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, integrand size = 21, number of rules / integrand size = 0.429, Rules used = {6722, 1975, 446, 89, 80, 50, 63, 217, 206}

$$\frac{(24a^2c^2 + 60abc + 35b^2)(c + dx^2)(a(c + dx^2) + b)}{24a^3bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(24a^2c^2 + 60abc + 35b^2)(a(c + dx^2) + b)}{16a^4d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{b(24a^2c^2 + 60abc + 35b^2)\sqrt{a(c + dx^2) + b} \tanh^{-1}\left(\frac{\sqrt{a}\sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{16a^{9/2}d^3\sqrt{c+dx^2}\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c + dx^2)^2(a(c + dx^2) + b)}{6a^2d^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(ac + b)^2(c + dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a + b/(c + d*x^2))^(3/2), x]

[Out] ((b + a*c)^2*(c + d*x^2)^2)/(a^2*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(b + a*(c + d*x^2)))/(16*a^4*d^3*Sqrt[a + b/(c + d*x^2)]) - ((35*b^2 + 60*a*b*c + 24*a^2*c^2)*(c + d*x^2)*(b + a*(c + d*x^2)))/(24*a^3*b*d^3*Sqrt[a + b/(c + d*x^2)]) + ((c + d*x^2)^2*(b + a*(c + d*x^2)))/(6*a^2*d^3*Sqrt[a + b/(c + d*x^2)]) - (b*(35*b^2 + 60*a*b*c + 24*a^2*c^2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)])/(16*a^(9/2)*d^3*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 89

Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c

```
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
+ 1)*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
(EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] || !SumSimplerQ[p,
1])))
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 446

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975

```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_)^(m_)), x_Symbol] := Int[(e*x)^m*Expa
ndToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && Binomi
alQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !
BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/
v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && Bin
omialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^5(c+dx^2)^{3/2}}{(b+a(c+dx^2))^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^5(c+dx^2)^{3/2}}{(b+ac+adx^2)^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{x^2(c+dx)^{3/2}}{(b+ac+adx)^{3/2}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}\left(\frac{1}{2}(b+ac)(5b+4ac)d - \frac{1}{2}abd^2x\right)}{\sqrt{b+ac+adx}} dx, x, x^2\right)}{a^2bd^3\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)^2(b+a(c+dx^2))}{6a^2d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{\left((35b^2+60abc+24a^2c^2)\sqrt{b+a(c+dx^2)}\right)}{12a^2bd^2\sqrt{c+dx^2}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)(b+a(c+dx^2))}{24a^3bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(c+dx^2)^2}{6a^2d^3\sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(b+a(c+dx^2))}{16a^4d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)}{24a^2bd^2\sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(b+a(c+dx^2))}{16a^4d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)}{24a^2bd^2\sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{(b+ac)^2(c+dx^2)^2}{a^2bd^3\sqrt{a + \frac{b}{c+dx^2}}} + \frac{(35b^2+60abc+24a^2c^2)(b+a(c+dx^2))}{16a^4d^3\sqrt{a + \frac{b}{c+dx^2}}} - \frac{(35b^2+60abc+24a^2c^2)(c+dx^2)}{24a^2bd^2\sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [C] time = 11.65, size = 1215, normalized size = 3.92

Warning: Unable to verify antiderivative.

[In] Integrate[x^5/(a + b/(c + d*x^2))^(3/2), x]

[Out] (b*(2835*a^2*(b + a*c)^2*(a + b/(c + d*x^2)) - 3240*a^2*c*(b + a*c)*(a + b/(c + d*x^2))^2 + 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2 + 765*a^2*c^2*(a + b/(c + d*x^2))^3 + 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^3 - 105*a*c^2*(a + b/(c + d*x^2))^4 + (300*(b + a*c)^2*(a + b/(c + d*x^2))^3*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2)]])/(1 + b/(a*c + a*d*x^2))^(3/2) + (60*c*(b + a*c)*(a + b/(c + d*x^2))^2)/((1 + b/(a*c + a*d*x^2))^(3/2))

+ b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))]/(1 + b/(a*c + a*d*x^2))^(3/2) + (120*a*c^2*(a + b/(c + d*x^2))^4*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))]/Sqrt[1 + b/(a*c + a*d*x^2)] - 2835*a^3*(b + a*c)^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] - 765*a^3*c^2*(a + b/(c + d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] - 1365*a*(b + a*c)^2*(a + b/(c + d*x^2))^2*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] - 300*a*c*(b + a*c)*(a + b/(c + d*x^2))^3*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] + 105*a*c^2*(a + b/(c + d*x^2))^4*Sqrt[1 + b/(a*c + a*d*x^2)]*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] + 3240*a^4*c*(b + a*c)*(1 + b/(a*c + a*d*x^2))^(3/2)*ArcTanh[Sqrt[1 + b/(a*c + a*d*x^2))] - 760*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 1040*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 344*c^2*(a + b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2}, {1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 256*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 448*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 192*c^2*(a + b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2, 2}, {1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 32*(b + a*c)^2*(a + b/(c + d*x^2))^3*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] + 64*c*(b + a*c)*(a + b/(c + d*x^2))^4*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)] - 32*c^2*(a + b/(c + d*x^2))^5*HypergeometricPFQ[{1/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 7/2}, 1 + b/(a*c + a*d*x^2)))/(720*a^5*d^3*(a + b/(c + d*x^2))^(5/2))

IntegrateAlgebraic [A] time = 0.38, size = 253, normalized size = 0.82

$$\frac{(-24a^2bc^2 - 60ab^2c - 35b^3) \tanh^{-1}\left(\frac{\sqrt{ac+ad^2+b}}{\sqrt{a}}\right) + \sqrt{\frac{ac+ad^2+b}{c+dx^2}} (8a^3c^4 + 8a^3c^2dx^2 + 8a^3cd^2x^6 + 8a^3d^4x^8 + 118a^2bc^3 + 150a^2bc^2dx^2 + 18a^2bcd^2x^4 - 14a^2bd^3x^6 + 215ab^2c^2 + 250ab^2cdx^2 + 35ab^2d^2x^4 + 105b^3c + 105b^3dx^2)}{16a^{9/2}d^3} + \frac{48a^4d^3(ac + adx^2 + b)}{48a^4d^3(ac + adx^2 + b)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(105*b^3*c + 215*a*b^2*c^2 + 118*a^2*b*c^3 + 8*a^3*c^4 + 105*b^3*d*x^2 + 250*a*b^2*c*d*x^2 + 150*a^2*b*c^2*d*x^2 + 8*a^3*c^3*d*x^2 + 35*a*b^2*d^2*x^4 + 18*a^2*b*c*d^2*x^4 - 14*a^2*b*d^3*x^6 + 8*a^3*c*d^3*x^6 + 8*a^3*d^4*x^8))/(48*a^4*d^3*(b + a*c + a*d*x^2)) + ((-35*b^3 - 60*a*b^2*c - 24*a^2*b*c^2)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/((16*a^(9/2)*d^3))

fricas [A] time = 0.84, size = 675, normalized size = 2.18



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/192*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(8*a^4*d^4*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3), 1/96*(3*(24*a^3*b*c^3 + 84*a^2*b^2*c^2 + 95*a*b^3*c + 35*b^4 + (24*a^3*b*c^2 + 60*a^2*b^2*c + 35*a*b^3)*d*x^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(8*a^4*d^4

$$*x^8 + 2*(4*a^4*c - 7*a^3*b)*d^3*x^6 + 8*a^4*c^4 + 118*a^3*b*c^3 + (18*a^3*b*c + 35*a^2*b^2)*d^2*x^4 + 215*a^2*b^2*c^2 + 105*a*b^3*c + (8*a^4*c^3 + 150*a^3*b*c^2 + 250*a^2*b^2*c + 105*a*b^3)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^6*d^4*x^2 + (a^6*c + a^5*b)*d^3)]$$

giac [B] time = 1.67, size = 597, normalized size = 1.93

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] $\frac{1}{48}\sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c}*(2*x^2*(4*x^2/(a^2*d*\text{sgn}(d*x^2 + c)) - (4*a^{11}*c*d^6*\text{sgn}(d*x^2 + c) + 11*a^{10}*b*d^6*\text{sgn}(d*x^2 + c))/(a^{13}*d^8)) + (8*a^{11}*c^2*d^5*\text{sgn}(d*x^2 + c) + 62*a^{10}*b*c*d^5*\text{sgn}(d*x^2 + c) + 57*a^9*b^2*d^5*\text{sgn}(d*x^2 + c))/(a^{13}*d^8)) + \frac{1}{96}*(24*a^{(5/2)}*b*c^2 + 60*a^{(3/2)}*b^2*c + 35*\sqrt{a}*b^3)*\log(\text{abs}(-2*a^{(7/2)}*c^3*d - 6*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c}))*a^3*c^2*\text{abs}(d) - 6*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^2*a^{(5/2)}*c*d - 5*a^{(5/2)}*b*c^2*d - 2*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^3*a^2*\text{abs}(d) - 10*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c}))*a^2*b*c*\text{abs}(d) - 5*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c})^2*a^{(3/2)}*b*d - 4*a^{(3/2)}*b^2*c*d - 4*(\sqrt{a*d^2}*x^2 - \sqrt{a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c}))*a*b^2*\text{abs}(d) - \sqrt{a}*b^3*d)/(a^5*d^2*\text{abs}(d)*\text{sgn}(d*x^2 + c))$

maple [B] time = 0.07, size = 1240, normalized size = 4.00

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a+b/(d*x^2+c))^(3/2),x)

[Out] $\frac{1}{96}*((a*d*x^2+a*c+b)/(d*x^2+c))^{(1/2)}*(d*x^2+c)/a^4/d^3*(-48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*x^4*a^3*c*d^2-60*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*x^4*a^2*b*d^2-72*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*x^2*a^3*b*c^2*d^2-48*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*x^2*a^3*c^2*d-180*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*x^2*a^2*b^2*c*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*d^2)^{(1/2)}*x^2*a^2*d-105*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*x^2*a*b^3*d^2-72*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*a^3*b*c^3*d+54*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*x^2*a*b^2*d-252*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*a^2*b^2*c^2*d+96*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*a^2*b*c^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*d^2)^{(1/2)}*a^2*c+108*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*a^2*b*c^2-285*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*a*b^3*c*d+192*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*a*b^2*c+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*d^2)^{(1/2)}*a*b+222*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*a*b^2*c-105*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*b^4*d+96*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*b^3+114*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*b^3)/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/(a*d^2)^{(1/2)}/(a*d*x^2+a*c+b)$

maxima [A] time = 2.36, size = 389, normalized size = 1.25

$$\frac{48 a^5 b c^2 + 96 a^4 b^2 c + 48 a^3 b^3 - \frac{3(24 a^2 b^2 c + 60 a b^2 c + 35 b^3)(a d x^2 + a c + b)^3}{(d x^2 + c)^3} + \frac{8(24 a^3 b c^2 + 60 a^2 b^2 c + 35 a b^3)(a d x^2 + a c + b)^2}{(d x^2 + c)^2} - \frac{3(56 a^4 b c^2 + 132 a^3 b^2 c + 77 a^2 b^3)(a d x^2 + a c + b)}{d x^2 + c}}{48 \left(a^7 d^3 \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}} - 3 a^6 d^3 \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{\frac{3}{2}} + 3 a^5 d^3 \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{\frac{5}{2}} - a^4 d^3 \left(\frac{a d x^2 + a c + b}{d x^2 + c} \right)^{\frac{7}{2}} \right)} + \frac{(24 a^2 c^2 + 60 a b c + 35 b^2) b \log \left(-\frac{\sqrt{a} - \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}}{\sqrt{a} + \sqrt{\frac{a d x^2 + a c + b}{d x^2 + c}}} \right)}{32 a^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/48*(48*a^5*b*c^2 + 96*a^4*b^2*c + 48*a^3*b^3 - 3*(24*a^2*b*c^2 + 60*a*b^2*c + 35*b^3)*(a*d*x^2 + a*c + b)^3/(d*x^2 + c)^3 + 8*(24*a^3*b*c^2 + 60*a^2*b^2*c + 35*a*b^3)*(a*d*x^2 + a*c + b)^2/(d*x^2 + c)^2 - 3*(56*a^4*b*c^2 + 132*a^3*b^2*c + 77*a^2*b^3)*(a*d*x^2 + a*c + b)/(d*x^2 + c))/(a^7*d^3*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - 3*a^6*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + 3*a^5*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - a^4*d^3*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(7/2)) + 1/32*(24*a^2*c^2 + 60*a*b*c + 35*b^2)*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(9/2)*d^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5}{\left(a + \frac{b}{d x^2 + c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a + b/(c + d*x^2))^(3/2),x)

[Out] int(x^5/(a + b/(c + d*x^2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{\left(\frac{a c + a d x^2 + b}{c + d x^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(x**5/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)

$$3.151 \quad \int \frac{x^3}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=187

$$\frac{3b(4ac + 5b) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{8a^{7/2}d^2} - \frac{(4ac + 7b)(c + dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8a^3d^2} - \frac{b(ac + b)}{a^3d^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} + \frac{(c + dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4a^2d^2}$$

Rubi [A] time = 0.57, antiderivative size = 242, normalized size of antiderivative = 1.29, number of steps used = 9, number of rules used = 8, integrand size = 21, number of rules / integrand size = 0.381, Rules used = {6722, 1975, 446, 78, 50, 63, 217, 206}

$$\frac{(4ac + 5b)(c + dx^2)(a(c + dx^2) + b)}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(4ac + 5b)(a(c + dx^2) + b)}{8a^3d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{3b(4ac + 5b) \sqrt{a(c + dx^2) + b} \tanh^{-1}\left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}}\right)}{8a^{7/2}d^2 \sqrt{c + dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(ac + b)(c + dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b/(c + d*x^2))^(3/2), x]

[Out] -(((b + a*c)*(c + d*x^2)^2)/(a*b*d^2*Sqrt[a + b/(c + d*x^2)])) - (3*(5*b + 4*a*c)*(b + a*(c + d*x^2)))/(8*a^3*d^2*Sqrt[a + b/(c + d*x^2)]) + ((5*b + 4*a*c)*(c + d*x^2)*(b + a*(c + d*x^2)))/(4*a^2*b*d^2*Sqrt[a + b/(c + d*x^2)]) + (3*b*(5*b + 4*a*c)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]])/(8*a^(7/2)*d^2*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt

$Q[a, 0] \parallel LtQ[b, 0]$)

Rule 217

$Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] \rightarrow Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[\{a, b\}, x] \&\& !GtQ[a, 0]$

Rule 446

$Int[(x_)^{(m_)}*((a_) + (b_)*(x_)^{(n)})^{(p_)}*((c_) + (d_)*(x_)^{(n)})^{(q_)}], x_Symbol] \rightarrow Dist[1/n, Subst[Int[x^{(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /; FreeQ[\{a, b, c, d, m, n, p, q\}, x] \&\& NeQ[b*c - a*d, 0] \&\& IntegerQ[Simplify[(m + 1)/n]]$

Rule 1975

$Int[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_))^{(m_)}], x_Symbol] \rightarrow Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[\{e, m, p, q\}, x] \&\& BinomialQ[\{u, v\}, x] \&\& EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] \&\& !BinomialMatchQ[\{u, v\}, x]$

Rule 6722

$Int[(u_)*((a_) + (b_)*(v_)^{(n)})^{(p_)}], x_Symbol] \rightarrow Dist[(a + b*v^n)^{FracPart[p]}/(v^{(n*FracPart[p])*(b + a/v^n)^{FracPart[p]})], Int[u*v^{(n*p)}*(b + a/v^n)^p, x], x] /; FreeQ[\{a, b, p\}, x] \&\& !IntegerQ[p] \&\& ILtQ[n, 0] \&\& BinomialQ[v, x] \&\& !LinearQ[v, x]$

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^3(c+dx^2)^{3/2}}{(b+a(c+dx^2))^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{x^3(c+dx^2)^{3/2}}{(b+ac+adx^2)^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{x(c+dx)^{3/2}}{(b+ac+adx)^{3/2}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left((5b+4ac)\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{\sqrt{b+ac+adx}} dx, x, x^2\right)}{2abd\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(5b+4ac)\sqrt{b+a(c+dx^2)}}{8a^2d^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(5b+4ac)(b+a(c+dx^2))}{8a^3d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(5b+4ac)(b+a(c+dx^2))}{8a^3d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(5b+4ac)(b+a(c+dx^2))}{8a^3d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{(b+ac)(c+dx^2)^2}{abd^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3(5b+4ac)(b+a(c+dx^2))}{8a^3d^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{(5b+4ac)(c+dx^2)(b+a(c+dx^2))}{4a^2bd^2 \sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 133, normalized size = 0.71

$$\frac{3b(4ac+5b)\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \operatorname{tanh}^{-1}\left(\frac{\sqrt{a+\frac{b}{c+dx^2}}}{\sqrt{a}}\right) - \sqrt{a}\left(2a^2(c^2-d^2x^4) + ab(17c+5dx^2) + 15b^2\right)}{8a^{7/2}d^2\sqrt{a+\frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b/(c + d*x^2))^(3/2), x]

[Out] (- (Sqrt[a]*(15*b^2 + a*b*(17*c + 5*d*x^2) + 2*a^2*(c^2 - d^2*x^4))) + 3*b*(5*b + 4*a*c)*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(8*a^(7/2)*d^2*Sqrt[a + b/(c + d*x^2)])

IntegrateAlgebraic [A] time = 0.24, size = 189, normalized size = 1.01

$$\frac{3(4abc + 5b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + \sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^{7/2}d^2} + \frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^3d^2(ac + adx^2 + b)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a + b/(c + d*x^2))^(3/2), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-15*b^2*c - 17*a*b*c^2 - 2*a^2*c^3 - 15*b^2*d*x^2 - 22*a*b*c*d*x^2 - 2*a^2*c^2*d*x^2 - 5*a*b*d^2*x^4 + 2*a^2*c*d^2*x^4 + 2*a^2*d^3*x^6))/(8*a^3*d^2*(b + a*c + a*d*x^2)) + (3*(5*b^2 + 4*a*b*c)*ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/Sqrt[a])/((8*a^(7/2)*d^2)

fricas [A] time = 0.84, size = 541, normalized size = 2.89

$$\frac{3(4abc + 5b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + \sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^{7/2}d^2} + \frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^3d^2(ac + adx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2), x, algorithm="fricas")

[Out] [1/32*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2), -1/16*(3*(4*a^2*b*c^2 + 9*a*b^2*c + (4*a^2*b*c + 5*a*b^2)*d*x^2 + 5*b^3)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b) - 2*(2*a^3*d^3*x^6 + (2*a^3*c - 5*a^2*b)*d^2*x^4 - 2*a^3*c^3 - 17*a^2*b*c^2 - 15*a*b^2*c - (2*a^3*c^2 + 22*a^2*b*c + 15*a*b^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^5*d^3*x^2 + (a^5*c + a^4*b)*d^2)]

giac [B] time = 1.57, size = 510, normalized size = 2.73

$$\frac{3(4abc + 5b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + \sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^{7/2}d^2} + \frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^3d^2(ac + adx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b/(d*x^2+c))^(3/2), x, algorithm="giac")

[Out] 1/8*sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)*(2*x^2/(a^2*d*sgn(d*x^2 + c)) - (2*a^6*c*d^2 + 7*a^5*b*d^2)/(a^8*d^4*sgn(d*x^2 + c))) - 1/16*(4*a^(3/2)*b*c + 5*sqrt(a)*b^2)*log(abs(-2*a^(7/2)*c^3*d - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*abs(d) - 6*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(5/2)*c*d - 5*a^(5/2)*b*c^2*d - 2*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*abs(d) - 10*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*abs(d) - 5*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^(3/2)*b*d - 4*a^(3/2)*b^2*c*d - 4*(sqrt(a*d^2)*x^2 - sqrt(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*abs(d) - sqrt(a)*b^3*d))/(a^4*d*abs(d)*sgn(d*x^2 + c))

maple [B] time = 0.06, size = 783, normalized size = 4.19

$$\frac{3(4abc + 5b^2) \tanh^{-1}\left(\frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}}}{\sqrt{a}}\right) + \sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^{7/2}d^2} + \frac{\sqrt{\frac{ac+dx^2+b}{c+dx^2}} (-2a^2c^3 - 2a^2c^2dx^2 + 2a^2cd^2x^4 + 2a^2d^3x^6 - 17abc^2 - 22abcdx^2 - 5abd^2x^4 - 15b^2c - 15b^2dx^2)}{8a^3d^2(ac + adx^2 + b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(a+b/(d*x^2+c))^{3/2}, x)$

[Out]
$$-1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^{1/2}*(d*x^2+c)/a^3/d^2*(-4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}*x^4*a^2*d^2-12*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}))/((a*d^2)^{1/2})*x^2*a^2*b*c*d^2-15*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}))/((a*d^2)^{1/2})*x^2*a*b^2*d^2+10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}*a*b*d*x^2-12*a^2*b*c^2*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}))/((a*d^2)^{1/2}))+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}*a^2*c^2-27*a*b^2*c*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}))/((a*d^2)^{1/2}))+16*((d*x^2+c)*(a*d*x^2+a*c+b))^{1/2}*(a*d^2)^{1/2}*a*b*c+18*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}*a*b*c-15*b^3*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}))/((a*d^2)^{1/2}))+16*((d*x^2+c)*(a*d*x^2+a*c+b))^{1/2}*(a*d^2)^{1/2}*b^2+14*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{1/2}*(a*d^2)^{1/2}*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^{1/2}/(a*d^2)^{1/2}/(a*d*x^2+a*c+b)$$

maxima [A] time = 2.31, size = 262, normalized size = 1.40

$$\frac{8a^3bc + 8a^2b^2 + \frac{3(ax^2+ac+b)^2(4abc+5b^2)}{(dx^2+c)^2} - \frac{5(4a^2bc+5ab^2)(ax^2+ac+b)}{dx^2+c} - \frac{3(4ac+5b)b \log\left(\frac{\sqrt{a}-\sqrt{\frac{ax^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{ax^2+ac+b}{dx^2+c}}}\right)}{16a^{\frac{7}{2}}d^2}}{8\left(a^5d^2\sqrt{\frac{ax^2+ac+b}{dx^2+c}} - 2a^4d^2\left(\frac{ax^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}} + a^3d^2\left(\frac{ax^2+ac+b}{dx^2+c}\right)^{\frac{5}{2}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x^3/(a+b/(d*x^2+c))^{3/2}, x, \text{algorithm}="maxima")$

[Out]
$$-1/8*(8*a^3*b*c + 8*a^2*b^2 + 3*(a*d*x^2 + a*c + b)^2*(4*a*b*c + 5*b^2))/(d*x^2 + c)^2 - 5*(4*a^2*b*c + 5*a*b^2)*(a*d*x^2 + a*c + b)/(d*x^2 + c)/(a^5*d^2*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} - 2*a^4*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{3/2} + a^3*d^2*((a*d*x^2 + a*c + b)/(d*x^2 + c))^{5/2}) - 3/16*(4*a*c + 5*b)*b*\log(-(\sqrt{a} - \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/(\sqrt{a} + \sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)}))/((a^{7/2}*d^2)$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{\left(a + \frac{b}{dx^2+c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(x^3/(a + b/(c + d*x^2))^{3/2}, x)$

[Out] $\text{int}(x^3/(a + b/(c + d*x^2))^{3/2}, x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(x**3/(a+b/(d*x**2+c))**(3/2), x)$

[Out] $\text{Integral}(x**3/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)$

$$3.152 \quad \int \frac{x}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx$$

Optimal. Leaf size=100

$$-\frac{3b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d} + \frac{3b}{2a^2d\sqrt{a + \frac{b}{c+dx^2}}} + \frac{c + dx^2}{2ad\sqrt{a + \frac{b}{c+dx^2}}}$$

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1591, 242, 51, 63, 208}

$$\frac{3(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}}{2a^2d} - \frac{3b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{c + dx^2}{ad\sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b/(c + d*x^2))^(3/2),x]

[Out] -((c + d*x^2)/(a*d*Sqrt[a + b/(c + d*x^2)])) + (3*(c + d*x^2)*Sqrt[a + b/(c + d*x^2)])/(2*a^2*d) - (3*b*ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]])/(2*a^(5/2)*d)

Rule 51

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[
n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1591

```
Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[P
q, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[I
```

nt[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rubi steps

$$\begin{aligned}
 \int \frac{x}{\left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a + \frac{b}{x}\right)^{3/2}} dx, x, c + dx^2\right)}{2d} \\
 &= -\frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx)^{3/2}} dx, x, \frac{1}{c+dx^2}\right)}{2d} \\
 &= -\frac{c + dx^2}{ad\sqrt{a + \frac{b}{c+dx^2}}} - \frac{3 \text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx}} dx, x, \frac{1}{c+dx^2}\right)}{2ad} \\
 &= -\frac{c + dx^2}{ad\sqrt{a + \frac{b}{c+dx^2}}} + \frac{3(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}}{2a^2d} + \frac{(3b) \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, \frac{1}{c+dx^2}\right)}{4a^2d} \\
 &= -\frac{c + dx^2}{ad\sqrt{a + \frac{b}{c+dx^2}}} + \frac{3(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}}{2a^2d} + \frac{3 \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + \frac{b}{c+dx^2}}\right)}{2a^2d} \\
 &= -\frac{c + dx^2}{ad\sqrt{a + \frac{b}{c+dx^2}}} + \frac{3(c + dx^2)\sqrt{a + \frac{b}{c+dx^2}}}{2a^2d} - \frac{3b \tanh^{-1}\left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}
 \end{aligned}$$

Mathematica [C] time = 0.06, size = 50, normalized size = 0.50

$$\frac{{}_2F_1\left(-\frac{1}{2}, 2; \frac{1}{2}; \frac{a + \frac{b}{dx^2+c}}{a}\right)}{a^2d\sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b/(c + d*x^2))^(3/2), x]

[Out] (b*Hypergeometric2F1[-1/2, 2, 1/2, (a + b/(c + d*x^2))/a])/(a^2*d*Sqrt[a + b/(c + d*x^2)])

IntegrateAlgebraic [A] time = 0.19, size = 127, normalized size = 1.27

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (ac^2 + 2acdx^2 + ad^2x^4 + 3bc + 3bdx^2)}{2a^2d(ac + adx^2 + b)} - \frac{3b \tanh^{-1}\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}}\right)}{2a^{5/2}d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b/(c + d*x^2))^(3/2), x]

[Out] $(\text{Sqrt}[(b + a*c + a*d*x^2)/(c + d*x^2)]*(3*b*c + a*c^2 + 3*b*d*x^2 + 2*a*c*d*x^2 + a*d^2*x^4))/(2*a^2*d*(b + a*c + a*d*x^2)) - (3*b*\text{ArcTanh}[\text{Sqrt}[(b + a*c + a*d*x^2)/(c + d*x^2)]]/\text{Sqrt}[a])/(2*a^{(5/2)*d})$

fricas [B] time = 0.89, size = 395, normalized size = 3.95

$$\frac{3(abd^2 + abc + b^2)\sqrt{a} \log\left(8a^2d^2x^4 + 8a^2c^2 + 8(2a^2c + ab)dx^2 + 8abc + b^2 - 4(2ad^2x^4 + (4ac + b)dx^2 + 2a^2 + bc)\sqrt{\frac{ad^2x^2 + a^2c + b}{d^2x^2 + c}}\right) + 4(a^2d^2x^4 + a^2c^2 + (2a^2c + 3ab)dx^2 + 3abc)\sqrt{\frac{ad^2x^2 + a^2c + b}{d^2x^2 + c}} - 3(abd^2 + abc + b^2)\sqrt{-a} \arctan\left(\frac{(2ad^2x^2 + a^2c + b)\sqrt{\frac{ad^2x^2 + a^2c + b}{d^2x^2 + c}}}{2(d^2x^2 + c)\sqrt{a}}\right) + 2(a^2d^2x^4 + a^2c^2 + (2a^2c + 3ab)dx^2 + 3abc)\sqrt{\frac{ad^2x^2 + a^2c + b}{d^2x^2 + c}}}{8(a^2d^2x^2 + (a^2c + a^2b)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] $[1/8*(3*(a*b*d*x^2 + a*b*c + b^2)*\text{sqrt}(a)*\log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 - 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*\text{sqrt}(a)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c))) + 4*(a^2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d), 1/4*(3*(a*b*d*x^2 + a*b*c + b^2)*\text{sqrt}(-a)*\arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*\text{sqrt}(-a)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) + 2*(a^2*d^2*x^4 + a^2*c^2 + (2*a^2*c + 3*a*b)*d*x^2 + 3*a*b*c)*\text{sqrt}((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*d^2*x^2 + (a^4*c + a^3*b)*d)]$

giac [B] time = 1.55, size = 449, normalized size = 4.49

$$\frac{\log\left(\frac{2a^2d^2x^4 + 4(\sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}) - \sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}))}{4d^2\text{sgn}(d^2x^2 + c)}\right) + 2(\sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}) - \sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}))}{4d^2\text{sgn}(d^2x^2 + c)}\right) + \frac{\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}}{2d\text{sgn}(d^2x^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] $1/4*b*\log(\text{abs}(-2*a^{(7/2)*c^3*d} - 6*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^3*c^2*\text{abs}(d) - 6*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^{(5/2)*c*d} - 5*a^{(5/2)*b*c^2*d} - 2*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^3*a^2*\text{abs}(d) - 10*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a^2*b*c*\text{abs}(d) - 5*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))^2*a^{(3/2)*b*d} - 4*a^{(3/2)*b^2*c*d} - 4*(\text{sqrt}(a*d^2)*x^2 - \text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c))*a*b^2*\text{abs}(d) - \text{sqrt}(a)*b^3*d))/(a^{(5/2)*\text{abs}(d)*\text{sgn}(d*x^2 + c)} + 1/2*\text{sqrt}(a*d^2*x^4 + 2*a*c*d*x^2 + b*d*x^2 + a*c^2 + b*c)/(a^2*d*\text{sgn}(d*x^2 + c))$

maple [B] time = 0.05, size = 478, normalized size = 4.78

$$\frac{\frac{\sqrt{d^2x^2 + c}}{2d} \ln\left(\frac{2a^2d^2x^4 + 4(\sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}) - \sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}))}{4d^2\text{sgn}(d^2x^2 + c)}\right) - 2ab\sqrt{a} \ln\left(\frac{2a^2d^2x^4 + 4(\sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}) - \sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}))}{4d^2\text{sgn}(d^2x^2 + c)}\right) + 2(\sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}) - \sqrt{d^2x^2 + c}(\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}))}{4d^2\text{sgn}(d^2x^2 + c)}\right) + \frac{\sqrt{d^2x^2 + 2ad^2x^2 + bd^2x^2 + a^2c + b}}{2d\text{sgn}(d^2x^2 + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+b/(d*x^2+c))^(3/2),x)

[Out] $1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^{(1/2)}*(d*x^2+c)/a^2/d*(-3*a*b*d^2*x^2*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)}+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*a*d*x^2-3*a*b*c*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)}-3*b^2*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)}+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*a*c+4*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*b+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*d^2)^{(1/2)}*(a*d^2)^{(1/2)}*b)/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/(a*d^2)^{(1/2)}/(a*d*x^2+a*c+b)$

maxima [A] time = 2.22, size = 161, normalized size = 1.61

$$\frac{2ab - \frac{3(adx^2+ac+b)b}{dx^2+c}}{2\left(a^3d\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - a^2d\left(\frac{adx^2+ac+b}{dx^2+c}\right)^{\frac{3}{2}}\right)} + \frac{3b \log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{4a^{\frac{5}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] 1/2*(2*a*b - 3*(a*d*x^2 + a*c + b)*b/(d*x^2 + c))/(a^3*d*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)) - a^2*d*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2)) + 3/4*b*log(-(sqrt(a) - sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(sqrt(a) + sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))))/(a^(5/2)*d)

mupad [B] time = 3.93, size = 61, normalized size = 0.61

$$\frac{\left(\frac{a(dx^2+c)}{b} + 1\right)^{3/2} (dx^2 + c) {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{7}{2}; -\frac{a(dx^2+c)}{b}\right)}{5d\left(a + \frac{b}{dx^2+c}\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b/(c + d*x^2))^(3/2),x)

[Out] (((a*(c + d*x^2))/b + 1)^(3/2)*(c + d*x^2)*hypergeom([3/2, 5/2], 7/2, -(a*(c + d*x^2))/b))/(5*d*(a + b/(c + d*x^2))^(3/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\left(\frac{ac+adx^2+b}{c+dx^2}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(x/((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2), x)

$$3.153 \quad \int \frac{1}{x \left(a + \frac{b}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=134

$$\frac{\tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{(ac+b)^{3/2}} - \frac{b}{a(ac+b) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}$$

Rubi [A] time = 0.51, antiderivative size = 214, normalized size of antiderivative = 1.60, number of steps used = 10, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {6722, 1975, 446, 98, 157, 63, 217, 206, 93, 208}

$$\frac{\sqrt{a(c+dx^2)+b} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{a(c+dx^2)+b}} \right)}{a^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c^{3/2} \sqrt{a(c+dx^2)+b} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{(ac+b)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{b}{a(ac+b) \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b/(c + d*x^2))^(3/2)),x]

[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + (Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/Sqrt[b + a*(c + d*x^2)]]/(a^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)]) - (c^(3/2)*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/Sqrt[c]*Sqrt[b + a*(c + d*x^2)]]/((b + a*c)^(3/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 157

Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p

$p, x], x] + \text{Dist}[(b*g - a*h)/b, \text{Int}[(c + d*x)^n*(e + f*x)^p/(a + b*x), x], x] /;$ FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 206

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(1*\text{ArcTanh}[(\text{Rt}[-b, 2]*x)/\text{Rt}[a, 2]])/(\text{Rt}[a, 2]*\text{Rt}[-b, 2]), x] /;$ FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

$\text{Int}[(a_ + (b_)*(x_)^2)^{-1}, x_Symbol] := \text{Simp}[(\text{Rt}[-(a/b), 2]*\text{ArcTanh}[x/\text{Rt}[-(a/b), 2]])/a, x] /;$ FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

$\text{Int}[1/\text{Sqrt}[(a_ + (b_)*(x_)^2)], x_Symbol] := \text{Subst}[\text{Int}[1/(1 - b*x^2), x], x, x/\text{Sqrt}[a + b*x^2]] /;$ FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 446

$\text{Int}[(x_)^{(m_)}*((a_ + (b_)*(x_)^{(n_)})^{(p_)}*((c_ + (d_)*(x_)^{(n_)})^{(q_)}), x_Symbol] := \text{Dist}[1/n, \text{Subst}[\text{Int}[x^{(\text{Simplify}[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q}, x], x, x^n], x] /;$ FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

$\text{Int}[(u_)^{(p_)}*(v_)^{(q_)}*((e_)*(x_)^{(m_)}), x_Symbol] := \text{Int}[(e*x)^m*\text{ExpandToSum}[u, x]^p*\text{ExpandToSum}[v, x]^q, x] /;$ FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && !BinomialMatchQ[{u, v}, x]

Rule 6722

$\text{Int}[(u_)*((a_ + (b_)*(v_)^{(n_)})^{(p_)}), x_Symbol] := \text{Dist}[(a + b*v^n)^{\text{FracPart}[p]}/(v^{(n*\text{FracPart}[p])})*(b + a/v^n)^{\text{FracPart}[p]}, \text{Int}[u*v^{(n*p)}*(b + a/v^n)^p, x], x] /;$ FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \left(a + \frac{b}{c+dx^2} \right)^{3/2}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x(b+a(c+dx^2))^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x(b+ac+adx^2)^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst} \left(\int \frac{(c+dx)^{3/2}}{x(b+ac+adx)^{3/2}} dx, x, x^2 \right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{b}{a(b+ac)\sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst} \left(\int \frac{\frac{1}{2}ac^2d + \frac{1}{2}(b+ac)d^2x}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2 \right)}{a(b+ac)d\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{b}{a(b+ac)\sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(c^2 \sqrt{b+a(c+dx^2)} \right) \operatorname{Subst} \left(\int \frac{1}{x\sqrt{c+dx} \sqrt{b+ac+adx}} dx, x, x^2 \right)}{2(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{c^2 \sqrt{b+a(c+dx^2)}}{2(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{b}{a(b+ac)\sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax^2}} dx, x, \sqrt{c+dx^2} \right)}{a\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{c^2 \sqrt{b+a(c+dx^2)}}{2(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{b}{a(b+ac)\sqrt{a + \frac{b}{c+dx^2}}} - \frac{c^{3/2} \sqrt{b+a(c+dx^2)} \tanh^{-1} \left(\frac{\sqrt{b+ac} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{b+a(c+dx^2)}} \right)}{(b+ac)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)}}{(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\
&= -\frac{b}{a(b+ac)\sqrt{a + \frac{b}{c+dx^2}}} + \frac{\sqrt{b+a(c+dx^2)} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{b+a(c+dx^2)}} \right)}{a^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c^{3/2} \sqrt{b+a(c+dx^2)}}{(b+ac)^{3/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}
\end{aligned}$$

Mathematica [A] time = 0.46, size = 110, normalized size = 0.82

$$\frac{\tanh^{-1} \left(\frac{\sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{a}} \right)}{a^{3/2}} - \frac{c^{3/2} \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{a + \frac{b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{(ac+b)^{3/2}} - \frac{b}{a(ac+b)\sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b/(c + d*x^2))^(3/2)), x]

[Out] -(b/(a*(b + a*c)*Sqrt[a + b/(c + d*x^2)])) + ArcTanh[Sqrt[a + b/(c + d*x^2)]/Sqrt[a]]/a^(3/2) - (c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[a + b/(c + d*x^2)])/Sqrt[b + a*c]])/(b + a*c)^(3/2)

IntegrateAlgebraic [A] time = 0.30, size = 169, normalized size = 1.26

$$\frac{\tanh^{-1} \left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{a}} \right)}{a^{3/2}} + \frac{c^{3/2} \tan^{-1} \left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b} \right)}{(-ac-b)^{3/2}} + \frac{(-bc - bdx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{a(ac+b)(ac+adx^2+b)}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c) - b*d*x^2))/(a*(b + a*c)*(b + a*c + a*d*x^2)) + (c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2))]/(b + a*c)]/(-b - a*c)^(3/2) + ArcTanh[Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]/Sqrt[a]]/a^(3/2)
```

fricas [B] time = 1.02, size = 1477, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) + (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) - 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/4*(2*(a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4) + 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2), 1/4*(2*(a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b)))/(a*c*d*x^2 + a*c^2 + b*c)) + (a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(a)*log(8*a^2*d^2*x^4 + 8*a^2*c^2 + 8*(2*a^2*c + a*b)*d*x^2 + 8*a*b*c + b^2 + 4*(2*a*d^2*x^4 + (4*a*c + b)*d*x^2 + 2*a*c^2 + b*c)*sqrt(a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))) - 4*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2), -1/2*((a^2*c^2 + (a^2*c + a*b)*d*x^2 + 2*a*b*c + b^2)*sqrt(-a)*arctan(1/2*(2*a*d*x^2 + 2*a*c + b)*sqrt(-a)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*d*x^2 + a^2*c + a*b)) - (a^3*c*d*x^2 + a^3*c^2 + a^2*b*c)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b)))/(a*c*d*x^2 + a*c^2 + b*c)) + 2*(a*b*d*x^2 + a*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^4*c^2 + 2*a^3*b*c + a^2*b^2 + (a^4*c + a^3*b)*d*x^2)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(c+d
*t_nostep)]Error: Bad Argument Type
```

maple [B] time = 0.06, size = 1015, normalized size = 7.57

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(a+b/(d*x^2+c))^(3/2),x)`

[Out]
$$-1/2*((a*d*x^2+a*c+b)/(d*x^2+c))^{(1/2)}*(d*x^2+c)/a*(-\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*x^2*a^3*c^2*d^2-2*a^2*b*c*d^2*x^2*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})+(a*d^2)^{(1/2)}*(a*c^2+b*c)^{(1/2)}*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c))^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)})/x^2)*x^2*a^2*c*d-a*b^2*d^2*x^2*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})- \ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})*a^3*c^3*d-3*a^2*b*c^2*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})+(a*d^2)^{(1/2)}*(a*c^2+b*c)^{(1/2)}*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c))^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)})/x^2)*a^2*c^2-3*a*b^2*c*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})+(a*d^2)^{(1/2)}*(a*c^2+b*c)^{(1/2)}*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c))^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)})/x^2)*a*b*c+2*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*a*b*c-b^3*d*\ln(1/2*(2*a*d^2*x^2+2*a*c*d+b*d+2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c))^{(1/2)}*(a*d^2)^{(1/2)})/(a*d^2)^{(1/2)})+2*((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}*(a*d^2)^{(1/2)}*b^2)/((d*x^2+c)*(a*d*x^2+a*c+b))^{(1/2)}/(a*c+b)^2/(a*d^2)^{(1/2)}/(a*d*x^2+a*c+b)$$

maxima [A] time = 2.23, size = 201, normalized size = 1.50

$$\frac{c^2 \log\left(\frac{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c\sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}}\right)}{2\sqrt{(ac+b)c}(ac+b)} - \frac{b}{(a^2c+ab)\sqrt{\frac{adx^2+ac+b}{dx^2+c}}} - \frac{\log\left(-\frac{\sqrt{a}-\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}{\sqrt{a}+\sqrt{\frac{adx^2+ac+b}{dx^2+c}}}\right)}{2a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")`

[Out]
$$1/2*c^2*\log((c*\sqrt{(a*d*x^2+a*c+b)/(d*x^2+c)} - \sqrt{(a*c+b)*c}))/ (c*\sqrt{(a*d*x^2+a*c+b)/(d*x^2+c)} + \sqrt{(a*c+b)*c}))/(\sqrt{(a*c+b)*c}*(a*c+b)) - b/((a^2*c+a*b)*\sqrt{(a*d*x^2+a*c+b)/(d*x^2+c)}) - 1/2*\log(-(\sqrt{a} - \sqrt{(a*d*x^2+a*c+b)/(d*x^2+c)}))/(\sqrt{a} + \sqrt{(a*d*x^2+a*c+b)/(d*x^2+c)}))/a^{(3/2)}$$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\left(a + \frac{b}{dx^2+c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a+b/(c+d*x^2))^(3/2)),x)`

[Out] `int(1/(x*(a+b/(c+d*x^2))^(3/2)),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \left(\frac{ac+adx^2+b}{c+dx^2} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(1/(x*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)

$$3.154 \quad \int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=146

$$\frac{3bd}{2(ac+b)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{c+dx^2}{2x^2(ac+b) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{3b\sqrt{c}d \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{2(ac+b)^{5/2}}$$

Rubi [A] time = 0.45, antiderivative size = 174, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6722, 1975, 446, 94, 93, 208}

$$\frac{3bd}{2(ac+b)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c+dx^2}{2x^2(ac+b) \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}d \sqrt{a(c+dx^2)+b} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{2(ac+b)^{5/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]
```

```
[Out] (3*b*d)/(2*(b + a*c)^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x^2)/(2*(b + a*c)*x^2*Sqrt[a + b/(c + d*x^2)]) - (3*b*Sqrt[c]*d*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(2*(b + a*c)^(5/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1975


```
Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]
```

Rule 6722

```
Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x^3(b+a(c+dx^2))^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x^3(b+ac+adx^2)^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{\sqrt{b+a(c+dx^2)} \operatorname{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= -\frac{c+dx^2}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3bd\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}} dx, x, x^2\right)}{4(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{3bd}{2(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}} dx, x, x^2\right)}{4(b+ac)^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{3bd}{2(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3bcd\sqrt{b+a(c+dx^2)}\right) \operatorname{Subst}\left(\int \frac{\sqrt{c+dx}}{x(b+ac+adx)^{3/2}} dx, x, x^2\right)}{2(b+ac)^2 \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \\ &= \frac{3bd}{2(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{c+dx^2}{2(b+ac)x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3b\sqrt{c}d\sqrt{b+a(c+dx^2)} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c+dx^2}}\right)}{2(b+ac)^{5/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} \end{aligned}$$

Mathematica [A] time = 0.42, size = 229, normalized size = 1.57

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \left(-2\sqrt{c(ac+b)}(c+dx^2)(ac(c+dx^2)+b(c-2dx^2))+6bcdx^2 \log(x)\sqrt{(c+dx^2)(a(c+dx^2)+b)}-3bcdx^2\sqrt{(c+dx^2)(a(c+dx^2)+b)} \log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)}+2ac(c+dx^2)+b(2c+dx^2)\right)\right)}{4x^2(ac+b)^2\sqrt{c(ac+b)}(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b/(c + d*x^2))^(3/2)), x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b*(c - 2*d*x^2) + a*c*(c + d*x^2)) + 6*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] - 3*b*c*d*x^2*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2

$a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*sqrt[c*(b + a*c)]*sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)])))/(4*(b + a*c)^2*sqrt[c*(b + a*c)]*x^2*(b + a*(c + d*x^2)))$

IntegrateAlgebraic [A] time = 0.32, size = 183, normalized size = 1.25

$$\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} (-ac^3 - 2ac^2dx^2 - acd^2x^4 - bc^2 + bcdx^2 + 2bd^2x^4)}{2x^2(ac + b)^2 (ac + adx^2 + b)} - \frac{3b\sqrt{c} d \tan^{-1}\left(\frac{\sqrt{c} \sqrt{-ac-b} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{ac+b}\right)}{2\sqrt{-ac-b}(ac + b)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b/(c + d*x^2))^(3/2)),x]

[Out] (sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-(b*c^2) - a*c^3 + b*c*d*x^2 - 2*a*c^2*d*x^2 + 2*b*d^2*x^4 - a*c*d^2*x^4))/(2*(b + a*c)^2*x^2*(b + a*c + a*d*x^2)) - (3*b*sqrt[c]*d*ArcTan[(sqrt[c]*sqrt[-b - a*c])*sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]]/(b + a*c)]/(2*sqrt[-b - a*c]*(b + a*c)^2)

fricas [A] time = 0.77, size = 599, normalized size = 4.10

$$\frac{3\left(\frac{bd^2x^4 + (abc + b^2)dx^2}{2c^2}\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \log\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\right) - \frac{4((ac-2b)d^2x^4 + ac^3 + (2ac^2-bc)dx^2 + bc^2)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{2\sqrt{ac-b}} - \frac{3\left(\frac{bd^2x^4 + (abc + b^2)dx^2}{2c^2}\right) \arctan\left(\frac{\sqrt{\frac{ac+adx^2+b}{c+dx^2}} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}\right)}{2\sqrt{ac-b}} - 2\left((ac-2b)d^2x^4 + ac^3 + (2ac^2-bc)dx^2 + bc^2\right)\sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8\left(\frac{bd^2x^4 + (abc + b^2)dx^2}{2c^2}\right)^2 + (ac^3 + 3ab^2c + 3ab^2c + b^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/8*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(c/(a*c + b))*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 - 4*((2*a^2*c^2 + 3*a*b*c + b^2)*d^2*x^4 + 2*a^2*c^4 + 4*a*b*c^3 + 2*b^2*c^2 + (4*a^2*c^3 + 7*a*b*c^2 + 3*b^2*c)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(c/(a*c + b)))/x^4 - 4*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2), 1/4*(3*(a*b*d^2*x^4 + (a*b*c + b^2)*d*x^2)*sqrt(-c/(a*c + b))*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c))*sqrt(-c/(a*c + b)))/(a*c*d*x^2 + a*c^2 + b*c)) - 2*((a*c - 2*b)*d^2*x^4 + a*c^3 + (2*a*c^2 - b*c)*d*x^2 + b*c^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^3*c^2 + 2*a^2*b*c + a*b^2)*d*x^4 + (a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*x^2)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] undef

maple [B] time = 0.07, size = 1088, normalized size = 7.45

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+b/(d*x^2+c))^(3/2),x)

[Out] 1/4*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(1/2)*x^6*a^2*d^3-3*ln((2*a*c*d*x^2+b*d

$$\begin{aligned} & x^2 + 2ac^2 + 2b^2c + 2(ac^2 + b^2c)^{1/2} (ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} / x^2 \\ & x^4 a^2 b^2 c^2 d^2 + 6(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} (ac^2 + b^2c)^{1/2} x^4 a^2 c^2 d^2 - 3 \ln((2acd^2x^2 + b^2d^2x^2 + 2ac^2 + 2b^2c + 2(ac^2 + b^2c)^{1/2}) / x^2) \\ & x^4 a^2 b^2 c^2 d^2 + 4(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} (ac^2 + b^2c)^{1/2} x^4 a^2 b^2 d^2 - 3 \ln((2acd^2x^2 + b^2d^2x^2 + 2ac^2 + 2b^2c + 2(ac^2 + b^2c)^{1/2}) / x^2) \\ & x^2 a^2 b^2 c^3 d + 4(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} (ac^2 + b^2c)^{1/2} x^2 a^2 c^2 d - 6 \ln((2acd^2x^2 + b^2d^2x^2 + 2ac^2 + 2b^2c + 2(ac^2 + b^2c)^{1/2}) / x^2) \\ & x^2 a^2 b^2 c^2 d + 4((dx^2 + c)(ad^2x^2 + ac + b))^{1/2} (ac^2 + b^2c)^{1/2} x^2 a^2 b^2 c^2 d - 2(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{3/2} (ac^2 + b^2c)^{1/2} \\ & x^2 a^2 d + 6(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} (ac^2 + b^2c)^{1/2} x^2 a^2 b^2 c^2 d - 3 \ln((2acd^2x^2 + b^2d^2x^2 + 2ac^2 + 2b^2c + 2(ac^2 + b^2c)^{1/2}) / x^2) \\ & x^2 b^3 c^2 d + 4((dx^2 + c)(ad^2x^2 + ac + b))^{1/2} (ac^2 + b^2c)^{1/2} x^2 b^2 d + 2(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{1/2} (ac^2 + b^2c)^{1/2} \\ & x^2 b^2 d - 2(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{3/2} (ac^2 + b^2c)^{1/2} a^2 c - 2(ad^2x^4 + 2acd^2x^2 + b^2d^2x^2 + ac^2 + b^2c)^{3/2} \\ & (ac^2 + b^2c)^{1/2} b / ((dx^2 + c)(ad^2x^2 + ac + b))^{1/2} / (ac + b)^3 / (ad^2x^2 + ac + b) / x^2 / (ac^2 + b^2c)^{1/2} \end{aligned}$$

maxima [A] time = 2.34, size = 247, normalized size = 1.69

$$\frac{3bcd \log \left(\frac{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} - \sqrt{(ac+b)c}}{c \sqrt{\frac{adx^2+ac+b}{dx^2+c}} + \sqrt{(ac+b)c}} \right)}{4(a^2c^2 + 2abc + b^2)\sqrt{(ac+b)c}} + \frac{\frac{3(adx^2+ac+b)bcd}{dx^2+c} - 2(abc + b^2)d}{2 \left((a^2c^3 + 2abc^2 + b^2c) \left(\frac{adx^2+ac+b}{dx^2+c} \right)^{\frac{3}{2}} - (a^3c^3 + 3a^2bc^2 + 3ab^2c + b^3) \sqrt{\frac{adx^2+ac+b}{dx^2+c}} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] $\frac{3}{4} b^2 c^2 d \log \left(\frac{c \sqrt{(a d x^2 + a c + b) / (d x^2 + c)} - \sqrt{(a c + b) c}}{c \sqrt{(a d x^2 + a c + b) / (d x^2 + c)} + \sqrt{(a c + b) c}} \right) / \left((a^2 c^2 + 2 a^2 b c + b^2) \sqrt{(a c + b) c} \right) + \frac{1}{2} * (3 (a d x^2 + a c + b) b^2 c^2 d / (d x^2 + c) - 2 (a b^2 c + b^2) d) / \left((a^2 c^3 + 2 a^2 b c^2 + b^2 c) * ((a d x^2 + a c + b) / (d x^2 + c))^{3/2} - (a^3 c^3 + 3 a^2 b c^2 + 3 a b^2 c + b^3) \sqrt{(a d x^2 + a c + b) / (d x^2 + c)} \right)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^3 \left(a + \frac{b}{dx^2+c} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)),x)

[Out] int(1/(x^3*(a + b/(c + d*x^2))^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \left(\frac{ac+adx^2+b}{c+dx^2} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b/(d*x**2+c))**(3/2),x)

[Out] Integral(1/(x**3*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)

$$3.155 \quad \int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2} \right)^{3/2}} dx$$

Optimal. Leaf size=212

$$\frac{abd^2}{(ac+b)^3 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}} - \frac{3bd^2(b-4ac) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{\sqrt{ac+b}} \right)}{8\sqrt{c}(ac+b)^{7/2}} - \frac{d(3b-4ac)(c+dx^2) \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{8x^2(ac+b)^3} - \frac{(c+dx^2)^2 \sqrt{\frac{ac+adx^2+b}{c+dx^2}}}{4x^4(ac+b)^2}$$

Rubi [A] time = 0.58, antiderivative size = 246, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6722, 1975, 446, 96, 94, 93, 208}

$$\frac{3bd^2(b-4ac)}{8c(ac+b)^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{3bd^2(b-4ac) \sqrt{a(c+dx^2)+b} \tanh^{-1} \left(\frac{\sqrt{ac+b} \sqrt{c+dx^2}}{\sqrt{c} \sqrt{a(c+dx^2)+b}} \right)}{8\sqrt{c}(ac+b)^{7/2} \sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}} - \frac{d(b-4ac)(c+dx^2)}{8cx^2(ac+b)^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)^2}{4cx^4(ac+b) \sqrt{a + \frac{b}{c+dx^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]

[Out] (3*b*(b - 4*a*c)*d^2)/(8*c*(b + a*c)^3*Sqrt[a + b/(c + d*x^2)]) - ((b - 4*a*c)*d*(c + d*x^2))/(8*c*(b + a*c)^2*x^2*Sqrt[a + b/(c + d*x^2)]) - (c + d*x^2)^2/(4*c*(b + a*c)*x^4*Sqrt[a + b/(c + d*x^2)]) - (3*b*(b - 4*a*c)*d^2*Sqrt[b + a*(c + d*x^2)]*ArcTanh[(Sqrt[b + a*c]*Sqrt[c + d*x^2])/(Sqrt[c]*Sqrt[b + a*(c + d*x^2)])])/(8*Sqrt[c]*(b + a*c)^(7/2)*Sqrt[c + d*x^2]*Sqrt[a + b/(c + d*x^2)])

Rule 93

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !SumSimplerQ[p, 1] && !SumSimplerQ[m, 1]

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1975

Int[(u_)^(p_)*(v_)^(q_)*((e_)*(x_))^(m_), x_Symbol] := Int[(e*x)^m*ExpandToSum[u, x]^p*ExpandToSum[v, x]^q, x] /; FreeQ[{e, m, p, q}, x] && BinomialQ[{u, v}, x] && EqQ[BinomialDegree[u, x] - BinomialDegree[v, x], 0] && ! BinomialMatchQ[{u, v}, x]

Rule 6722

Int[(u_)*((a_) + (b_)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rubi steps

$$\int \frac{1}{x^5 \left(a + \frac{b}{c+dx^2}\right)^{3/2}} dx = \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x^5(b+a(c+dx^2))^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)} \int \frac{(c+dx^2)^{3/2}}{x^5(b+ac+adx^2)^{3/2}} dx}{\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= \frac{\sqrt{b+a(c+dx^2)} \text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^3(b+ac+adx)^{3/2}} dx, x, x^2\right)}{2\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= -\frac{(c+dx^2)^2}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left((b-4ac)d\sqrt{b+a(c+dx^2)}\right) \text{Subst}\left(\int \frac{(c+dx)^{3/2}}{x^2(b+ac+adx)^{3/2}} dx, x, x^2\right)}{8c(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= -\frac{(b-4ac)d(c+dx^2)}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)^2}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d^2\sqrt{b+a(c+dx^2)}\right)}{16c(b+ac)^2 \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(b-4ac)d(c+dx^2)}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)^2}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d\sqrt{b+a(c+dx^2)}\right)}{8c(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

$$= \frac{3b(b-4ac)d^2}{8c(b+ac)^3 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(b-4ac)d(c+dx^2)}{8c(b+ac)^2 x^2 \sqrt{a + \frac{b}{c+dx^2}}} - \frac{(c+dx^2)^2}{4c(b+ac)x^4 \sqrt{a + \frac{b}{c+dx^2}}} + \frac{\left(3b(b-4ac)d\sqrt{b+a(c+dx^2)}\right)}{8c(b+ac)\sqrt{c+dx^2} \sqrt{a + \frac{b}{c+dx^2}}}$$

Mathematica [A] time = 0.57, size = 281, normalized size = 1.33

$$\frac{\sqrt{\frac{ac+bd^2+b}{c+dx^2}} \left(2\sqrt{c(ac+b)}(c+dx^2)(2a^2c(c-d^2x^4)+ab(4c^2+5cdx^2)+b^2(2c+5dx^2))+6bd^2x^4\log(x)(4ac-b)\sqrt{(c+dx^2)(a(c+dx^2)+b)}+3bd^2x^4(b-4ac)\sqrt{(c+dx^2)(a(c+dx^2)+b)}\log\left(2\sqrt{c(ac+b)}\sqrt{(c+dx^2)(ac+adx^2+b)}+2ac(c+dx^2)+b(2c+dx^2)\right)\right)}{16a^4(ac+b)^3\sqrt{c(ac+b)}(a(c+dx^2)+b)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^5*(a + b/(c + d*x^2))^(3/2)), x]

[Out] -1/16*(Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(2*Sqrt[c*(b + a*c)]*(c + d*x^2)*(b^2*(2*c + 5*d*x^2) + 2*a^2*c*(c^2 - d^2*x^4) + a*b*(4*c^2 + 5*c*d*x^2 + 13*d^2*x^4)) + 6*b*(-b + 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[x] + 3*b*(b - 4*a*c)*d^2*x^4*Sqrt[(c + d*x^2)*(b + a*(c + d*x^2))]*Log[2*a*c*(c + d*x^2) + b*(2*c + d*x^2) + 2*Sqrt[c*(b + a*c)]*Sqrt[(c + d*x^2)*(b + a*c + a*d*x^2)]])/((b + a*c)^3*Sqrt[c*(b + a*c)]*x^4*(b + a*(c + d*x^2)))

IntegrateAlgebraic [A] time = 0.41, size = 259, normalized size = 1.22

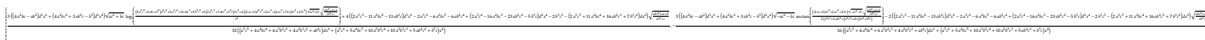
$$\frac{\sqrt{\frac{ac+bd^2+b}{c+dx^2}} \left(-2a^2c^4 - 2a^2c^3dx^2 + 2a^2c^2d^2x^4 + 2a^2cd^3x^6 - 4abc^3 - 9abc^2dx^2 - 18abcd^2x^4 - 13abd^3x^6 - 2b^2c^2 - 7b^2cdx^2 - 5b^2d^2x^4\right)}{8x^4(ac+b)^3(ac+adx^2+b)} - \frac{3d^2(b^2-4abc)\tan^{-1}\left(\frac{\sqrt{c}\sqrt{ac-b}\sqrt{\frac{ac+bd^2+b}{c+dx^2}}}{ac+b}\right)}{8\sqrt{c}\sqrt{ac-b}(ac+b)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^5*(a + b/(c + d*x^2))^(3/2)),x]

[Out] (Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2)]*(-2*b^2*c^2 - 4*a*b*c^3 - 2*a^2*c^4 - 7*b^2*c*d*x^2 - 9*a*b*c^2*d*x^2 - 2*a^2*c^3*d*x^2 - 5*b^2*d^2*x^4 - 18*a*b*c*d^2*x^4 + 2*a^2*c^2*d^2*x^4 - 13*a*b*d^3*x^6 + 2*a^2*c*d^3*x^6))/(8*(b + a*c)^3*x^4*(b + a*c + a*d*x^2)) - (3*(b^2 - 4*a*b*c)*d^2*ArcTan[(Sqrt[c]*Sqrt[-b - a*c]*Sqrt[(b + a*c + a*d*x^2)/(c + d*x^2])]/(b + a*c)))/(8*Sqrt[c]*Sqrt[-b - a*c]*(b + a*c)^3)

fricas [B] time = 1.43, size = 961, normalized size = 4.53



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="fricas")

[Out] [1/32*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(a*c^2 + b*c)*log(((8*a^2*c^2 + 8*a*b*c + b^2)*d^2*x^4 + 8*a^2*c^4 + 16*a*b*c^3 + 8*b^2*c^2 + 8*(2*a^2*c^3 + 3*a*b*c^2 + b^2*c)*d*x^2 + 4*((2*a*c + b)*d^2*x^4 + 2*a*c^3 + (4*a*c^2 + 3*b*c)*d*x^2 + 2*b*c^2)*sqrt(a*c^2 + b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/x^4) + 4*((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4), -1/16*(3*((4*a^2*b*c - a*b^2)*d^3*x^6 + (4*a^2*b*c^2 + 3*a*b^2*c - b^3)*d^2*x^4)*sqrt(-a*c^2 - b*c)*arctan(1/2*((2*a*c + b)*d*x^2 + 2*a*c^2 + 2*b*c)*sqrt(-a*c^2 - b*c)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/(a^2*c^3 + 2*a*b*c^2 + (a^2*c^2 + a*b*c)*d*x^2 + b^2*c)) - 2*((2*a^3*c^3 - 11*a^2*b*c^2 - 13*a*b^2*c)*d^3*x^6 - 2*a^3*c^6 - 6*a^2*b*c^5 - 6*a*b^2*c^4 + (2*a^3*c^4 - 16*a^2*b*c^3 - 23*a*b^2*c^2 - 5*b^3*c)*d^2*x^4 - 2*b^3*c^3 - (2*a^3*c^5 + 11*a^2*b*c^4 + 16*a*b^2*c^3 + 7*b^3*c^2)*d*x^2)*sqrt((a*d*x^2 + a*c + b)/(d*x^2 + c)))/((a^5*c^5 + 4*a^4*b*c^4 + 6*a^3*b^2*c^3 + 4*a^2*b^3*c^2 + a*b^4*c)*d*x^6 + (a^5*c^6 + 5*a^4*b*c^5 + 10*a^3*b^2*c^4 + 10*a^2*b^3*c^3 + 5*a*b^4*c^2 + b^5*c)*x^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

undef

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="giac")

[Out] undef

maple [B] time = 0.08, size = 1947, normalized size = 9.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(a+b/(d*x^2+c))^(3/2),x)

[Out] -1/16*((a*d*x^2+a*c+b)/(d*x^2+c))^(1/2)*(d*x^2+c)*(-33*ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^(1/2)*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2))/x^2)*x^4*a^3*b^2*c^5*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a^2*b*c^2*d^2+16*((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-10*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^4*a*b^2*c*d^2-2*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^(1/2)*(a*c^2+b*c)^(3/2)*x^2*a*b*c*d+2*(a*d^2*x^4+2*a*

$c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^6*a^2*b*c*d^3+3*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^4*b^5*c^2*d^2-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^4*b^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*x^2*b^2*d+8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*a*b*c^2-3*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^4*a*b^4*c^3*d^2+6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*x^4*a*b*d^2-8*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*x^2*a^2*c^2*d+12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^8*a^3*c*d^4-12*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^6*a^4*b*c^5*d^3-6*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^8*a^2*b*d^4-21*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^6*a^3*b^2*c^4*d^3+32*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^6*a^3*c^2*d^3-6*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^6*a^2*b^3*c^3*d^3-12*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^4*a^4*b*c^6*d^2+3*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^6*a*b^4*c^2*d^3+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*a^2*c^3+4*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*b^2*c-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^6*a*b^2*d^3+20*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)}*(a*c^2+b*c)^{(3/2)}*x^4*a^3*c^3*d^2-27*\ln((2*a*c*d*x^2+b*d*x^2+2*a*c^2+2*b*c+2*(a*c^2+b*c)^{(1/2)}*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(1/2)})/x^2)*x^4*a^2*b^3*c^4*d^2-12*(a*d^2*x^4+2*a*c*d*x^2+b*d*x^2+a*c^2+b*c)^{(3/2)}*(a*c^2+b*c)^{(3/2)}*x^4*a^2*c*d^2)/c/((d*x^2+c)*(a*d*x^2+a*c+b))^(1/2)/(a*c+b)^4/x^4/(a*c^2+b*c)^(3/2)/(a*d*x^2+a*c+b)$

maxima [B] time = 2.35, size = 450, normalized size = 2.12

$$\frac{3(4abc - b^2)d^2 \log\left(\frac{c\sqrt{\frac{ad^2+ac+b}{d^2+c}} - \sqrt{ac+bc}}{c\sqrt{\frac{ad^2+ac+b}{d^2+c}} + \sqrt{ac+bc}}\right)}{16(a^3c^3 + 3a^2bc^2 + 3ab^2c + b^3)\sqrt{ac+b}c} \frac{8(a^3bc^2 + 2a^2b^2c + ab^3)d^2 + \frac{3(4ab^2c^2 - b^2)(ad^2+ac+b)^2d^2}{(d^2+c)^2} - \frac{5(4a^2b^2+3ab^2c-b^3)(ad^2+ac+b)^2}{d^2+c}}{8\left(a^3c^5 + 3a^2bc^4 + 3ab^2c^3 + b^3c^2\right)\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{5}{2}} - 2(a^4c^5 + 4a^3bc^4 + 6a^2b^2c^3 + 4ab^3c^2 + b^4c)\left(\frac{ad^2+ac+b}{d^2+c}\right)^{\frac{3}{2}} + (a^5c^5 + 5a^4bc^4 + 10a^3b^2c^3 + 10a^2b^3c^2 + 5ab^4c + b^5)\sqrt{\frac{ad^2+ac+b}{d^2+c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(a+b/(d*x^2+c))^(3/2),x, algorithm="maxima")

[Out] $-3/16*(4*a*b*c - b^2)*d^2*\log((c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})/(c*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)} + \sqrt{(a*c + b)*c}))/((a^3*c^3 + 3*a^2*b*c^2 + 3*a*b^2*c + b^3)*\sqrt{(a*c + b)*c}) - 1/8*(8*(a^3*b*c^2 + 2*a^2*b^2*c + a*b^3)*d^2 + 3*(4*a*b*c^2 - b^2*c)*(a*d*x^2 + a*c + b)^2*d^2/(d*x^2 + c)^2 - 5*(4*a^2*b*c^2 + 3*a*b^2*c - b^3)*(a*d*x^2 + a*c + b)*d^2/(d*x^2 + c))/((a^3*c^5 + 3*a^2*b*c^4 + 3*a*b^2*c^3 + b^3*c^2)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(5/2) - 2*(a^4*c^5 + 4*a^3*b*c^4 + 6*a^2*b^2*c^3 + 4*a*b^3*c^2 + b^4*c)*((a*d*x^2 + a*c + b)/(d*x^2 + c))^(3/2) + (a^5*c^5 + 5*a^4*b*c^4 + 10*a^3*b^2*c^3 + 10*a^2*b^3*c^2 + 5*a*b^4*c + b^5)*\sqrt{(a*d*x^2 + a*c + b)/(d*x^2 + c)})$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^5 \left(a + \frac{b}{d x^2 + c}\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + b/(c + d*x^2))^(3/2)),x)

[Out] `int(1/(x^5*(a + b/(c + d*x^2))^(3/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^5 \left(\frac{ac+adx^2+b}{c+dx^2} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**5/(a+b/(d*x**2+c))**(3/2),x)`

[Out] `Integral(1/(x**5*((a*c + a*d*x**2 + b)/(c + d*x**2))**(3/2)), x)`

$$3.156 \quad \int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx$$

Optimal. Leaf size=75

$$\frac{3\sqrt{ax^{23}} \sinh^{-1}(x^{5/2})}{20x^{23/2}} - \frac{3\sqrt{x^5+1} \sqrt{ax^{23}}}{20x^9} + \frac{\sqrt{x^5+1} \sqrt{ax^{23}}}{10x^4}$$

Rubi [A] time = 0.02, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1} \sqrt{ax^{23}}}{10x^4} - \frac{3\sqrt{x^5+1} \sqrt{ax^{23}}}{20x^9} + \frac{3\sqrt{ax^{23}} \sinh^{-1}(x^{5/2})}{20x^{23/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^23]/Sqrt[1 + x^5], x]

[Out] (-3*Sqrt[a*x^23]*Sqrt[1 + x^5])/(20*x^9) + (Sqrt[a*x^23]*Sqrt[1 + x^5])/(10*x^4) + (3*Sqrt[a*x^23]*ArcSinh[x^(5/2)])/(20*x^(23/2))

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n*(m - n + 1)))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{23}}}{\sqrt{1+x^5}} dx &= \frac{\sqrt{ax^{23}} \int \frac{x^{23/2}}{\sqrt{1+x^5}} dx}{x^{23/2}} \\
&= \frac{\sqrt{ax^{23}} \sqrt{1+x^5}}{10x^4} - \frac{(3\sqrt{ax^{23}}) \int \frac{x^{13/2}}{\sqrt{1+x^5}} dx}{4x^{23/2}} \\
&= -\frac{3\sqrt{ax^{23}} \sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}} \sqrt{1+x^5}}{10x^4} + \frac{(3\sqrt{ax^{23}}) \int \frac{x^{3/2}}{\sqrt{1+x^5}} dx}{8x^{23/2}} \\
&= -\frac{3\sqrt{ax^{23}} \sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}} \sqrt{1+x^5}}{10x^4} + \frac{(3\sqrt{ax^{23}}) \text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}} dx, x, \sqrt{x}\right)}{4x^{23/2}} \\
&= -\frac{3\sqrt{ax^{23}} \sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}} \sqrt{1+x^5}}{10x^4} + \frac{(3\sqrt{ax^{23}}) \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, x^{5/2}\right)}{20x^{23/2}} \\
&= -\frac{3\sqrt{ax^{23}} \sqrt{1+x^5}}{20x^9} + \frac{\sqrt{ax^{23}} \sqrt{1+x^5}}{10x^4} + \frac{3\sqrt{ax^{23}} \sinh^{-1}(x^{5/2})}{20x^{23/2}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.65

$$\frac{\sqrt{ax^{23}} \left(3 \sinh^{-1}(x^{5/2}) + \sqrt{x^5 + 1} (2x^5 - 3) x^{5/2} \right)}{20x^{23/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^23]/Sqrt[1 + x^5], x]

[Out] (Sqrt[a*x^23]*(x^(5/2)*Sqrt[1 + x^5]*(-3 + 2*x^5) + 3*ArcSinh[x^(5/2)]))/(20*x^(23/2))

IntegrateAlgebraic [A] time = 1.43, size = 84, normalized size = 1.12

$$\frac{\sqrt{a} x^{23/2} \left(\frac{1}{20} \sqrt{x^5 + 1} (2\sqrt{a} x^{15/2} - 3\sqrt{a} x^{5/2}) + \frac{3}{20} \sqrt{a} \log(x^{5/2} + \sqrt{x^5 + 1}) \right)}{\sqrt{ax^{23}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^23]/Sqrt[1 + x^5], x]

[Out] (Sqrt[a]*x^(23/2)*((Sqrt[1 + x^5]*(-3*Sqrt[a]*x^(5/2) + 2*Sqrt[a]*x^(15/2)))/20 + (3*Sqrt[a]*Log[x^(5/2) + Sqrt[1 + x^5]])/20))/Sqrt[a*x^23]

fricas [A] time = 0.80, size = 169, normalized size = 2.25

$$\left[\frac{3\sqrt{a}x^9 \log\left(\frac{-8ax^{19}+8ax^{14}+ax^9+4\sqrt{ax^{23}}(2x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^9}\right) + 4\sqrt{ax^{23}}(2x^5-3)\sqrt{x^5+1}}{80x^9}, -\frac{3\sqrt{-a}x^9 \arctan\left(\frac{\sqrt{ax^{23}}(2x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2(ax^{19}+ax^{14})}\right) - 2\sqrt{ax^{23}}(2x^5-3)\sqrt{x^5+1}}{40x^9} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2), x, algorithm="fricas")

[Out] [1/80*(3*sqrt(a)*x^9*log(-(8*a*x^19 + 8*a*x^14 + a*x^9 + 4*sqrt(a*x^23))*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^9) + 4*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 + 1))/x^9, -1/40*(3*sqrt(-a)*x^9*arctan(1/2*sqrt(a*x^23)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(a*x^19 + a*x^14)) - 2*sqrt(a*x^23)*(2*x^5 - 3)*sqrt(x^5 + 1))/x^9]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)

maple [A] time = 0.07, size = 64, normalized size = 0.85

$$\frac{(2x^5 - 3) \sqrt{x^5 + 1} \sqrt{ax^{23}}}{20x^9} + \frac{3\sqrt{ax^{23}} \sqrt{(x^5 + 1)ax} \operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{20\sqrt{x^5 + 1} \sqrt{a} x^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^23)^(1/2)/(x^5+1)^(1/2),x)

[Out] 1/20/x^9*(2*x^5-3)*(x^5+1)^(1/2)*(a*x^23)^(1/2)+3/20/a^(1/2)*arcsinh(x^(5/2))*(a*x^23)^(1/2)/x^12*(a*x*(x^5+1))^(1/2)/(x^5+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^23)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^23)/sqrt(x^5 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2),x)

[Out] int((a*x^23)^(1/2)/(x^5 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{23}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**23)**(1/2)/(x**5+1)**(1/2),x)

[Out] Integral(sqrt(a*x**23)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)

$$3.157 \quad \int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx$$

Optimal. Leaf size=50

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}(x^{5/2})}{5x^{13/2}}$$

Rubi [A] time = 0.01, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 321, 329, 275, 215}

$$\frac{\sqrt{x^5+1}\sqrt{ax^{13}}}{5x^4} - \frac{\sqrt{ax^{13}}\sinh^{-1}(x^{5/2})}{5x^{13/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^13]/Sqrt[1 + x^5], x]

[Out] (Sqrt[a*x^13]*Sqrt[1 + x^5])/(5*x^4) - (Sqrt[a*x^13]*ArcSinh[x^(5/2)])/(5*x^(13/2))

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 321

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^{13}}}{\sqrt{1+x^5}} dx &= \frac{\sqrt{ax^{13}} \int \frac{x^{13/2}}{\sqrt{1+x^5}} dx}{x^{13/2}} \\
&= \frac{\sqrt{ax^{13}} \sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}} \int \frac{x^{3/2}}{\sqrt{1+x^5}} dx}{2x^{13/2}} \\
&= \frac{\sqrt{ax^{13}} \sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}} \operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}} dx, x, \sqrt{x}\right)}{x^{13/2}} \\
&= \frac{\sqrt{ax^{13}} \sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}} \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, x^{5/2}\right)}{5x^{13/2}} \\
&= \frac{\sqrt{ax^{13}} \sqrt{1+x^5}}{5x^4} - \frac{\sqrt{ax^{13}} \sinh^{-1}\left(x^{5/2}\right)}{5x^{13/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 0.84

$$\frac{\sqrt{ax^{13}} \left(x^{5/2} \sqrt{x^5 + 1} - \sinh^{-1}\left(x^{5/2}\right) \right)}{5x^{13/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^13]/Sqrt[1 + x^5], x]

[Out] (Sqrt[a*x^13]*(x^(5/2)*Sqrt[1 + x^5] - ArcSinh[x^(5/2)])/(5*x^(13/2))

IntegrateAlgebraic [A] time = 0.32, size = 69, normalized size = 1.38

$$\frac{\sqrt{a} x^{13/2} \left(\frac{1}{5} \sqrt{a} x^{5/2} \sqrt{x^5 + 1} - \frac{1}{5} \sqrt{a} \log\left(x^{5/2} + \sqrt{x^5 + 1}\right) \right)}{\sqrt{ax^{13}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^13]/Sqrt[1 + x^5], x]

[Out] (Sqrt[a]*x^(13/2)*((Sqrt[a]*x^(5/2)*Sqrt[1 + x^5])/5 - (Sqrt[a]*Log[x^(5/2) + Sqrt[1 + x^5]]/5))/Sqrt[a*x^13]

fricas [B] time = 1.09, size = 153, normalized size = 3.06

$$\left[\frac{\sqrt{a} x^4 \log\left(-\frac{8ax^{14}+8ax^9+ax^4-4\sqrt{ax^{13}}(2x^5+1)\sqrt{x^5+1}\sqrt{a}}{x^4}\right) + 4\sqrt{ax^{13}}\sqrt{x^5+1} - \sqrt{-a}x^4 \arctan\left(\frac{\sqrt{ax^{13}}(2x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2(ax^{14}+ax^9)}\right) + 2\sqrt{ax^{13}}\sqrt{x^5+1}}{20x^4}, \frac{\sqrt{-a}x^4 \arctan\left(\frac{\sqrt{ax^{13}}(2x^5+1)\sqrt{x^5+1}\sqrt{-a}}{2(ax^{14}+ax^9)}\right) + 2\sqrt{ax^{13}}\sqrt{x^5+1}}{10x^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2), x, algorithm="fricas")

[Out] [1/20*(sqrt(a)*x^4*log(-(8*a*x^14 + 8*a*x^9 + a*x^4 - 4*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a))/x^4) + 4*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4, 1/10*(sqrt(-a)*x^4*arctan(1/2*sqrt(a*x^13)*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(-a)/(a*x^14 + a*x^9)) + 2*sqrt(a*x^13)*sqrt(x^5 + 1))/x^4]

giac [A] time = 0.32, size = 68, normalized size = 1.36

$$\frac{a^{\frac{11}{2}} \log\left(-\sqrt{ax} a^{\frac{5}{2}} x^2 + \sqrt{a^6 x^5 + a^6}\right)}{5|a|^5} + \frac{\sqrt{a^6 x^5 + a^6} \sqrt{ax} x^2}{5a^2|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{5}a^{11/2}\log(-\sqrt{ax}a^{5/2}x^2 + \sqrt{a^6x^5 + a^6})/\text{abs}(a)^5 + 1/5\sqrt{a^6x^5 + a^6}\sqrt{ax}x^2/(a^2\text{abs}(a))$

maple [A] time = 0.06, size = 57, normalized size = 1.14

$$\frac{\sqrt{ax^{13}}\sqrt{x^5+1}}{5x^4} - \frac{\sqrt{ax^{13}}\sqrt{(x^5+1)ax}\operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5\sqrt{x^5+1}\sqrt{a}x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^13)^(1/2)/(x^5+1)^(1/2),x)

[Out] $\frac{1}{5}(ax^{13})^{1/2}(x^5+1)^{1/2}/x^4 - 1/5/a^{1/2}\operatorname{arcsinh}(x^{5/2})(ax^{13})^{1/2}/x^7((x^5+1)ax)^{1/2}/(x^5+1)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^13)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(ax^13)/sqrt(x^5 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2),x)

[Out] int((a*x^13)^(1/2)/(x^5 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{13}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**13)**(1/2)/(x**5+1)**(1/2),x)

[Out] Integral(sqrt(a*x**13)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)

$$3.158 \quad \int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx$$

Optimal. Leaf size=24

$$\frac{2\sqrt{ax^3} \sinh^{-1}(x^{5/2})}{5x^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 329, 275, 215}

$$\frac{2\sqrt{ax^3} \sinh^{-1}(x^{5/2})}{5x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^3]/Sqrt[1 + x^5], x]

[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 275

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_)*(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^3}}{\sqrt{1+x^5}} dx &= \frac{\sqrt{ax^3} \int \frac{x^{3/2}}{\sqrt{1+x^5}} dx}{x^{3/2}} \\ &= \frac{(2\sqrt{ax^3}) \text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^{10}}} dx, x, \sqrt{x}\right)}{x^{3/2}} \\ &= \frac{(2\sqrt{ax^3}) \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, x^{5/2}\right)}{5x^{3/2}} \\ &= \frac{2\sqrt{ax^3} \sinh^{-1}(x^{5/2})}{5x^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\frac{2\sqrt{ax^3} \sinh^{-1}\left(x^{5/2}\right)}{5x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^3]/Sqrt[1 + x^5], x]

[Out] (2*Sqrt[a*x^3]*ArcSinh[x^(5/2)])/(5*x^(3/2))

IntegrateAlgebraic [A] time = 0.33, size = 35, normalized size = 1.46

$$\frac{2ax^{3/2} \log\left(x^{5/2} + \sqrt{x^5 + 1}\right)}{5\sqrt{ax^3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^3]/Sqrt[1 + x^5], x]

[Out] (2*a*x^(3/2)*Log[x^(5/2) + Sqrt[1 + x^5]])/(5*Sqrt[a*x^3])

fricas [B] time = 0.83, size = 98, normalized size = 4.08

$$\left[\frac{1}{10} \sqrt{a} \log\left(-8ax^{10} - 8ax^5 - 4(2x^6 + x)\sqrt{x^5 + 1}\sqrt{ax^3}\sqrt{a} - a\right), -\frac{1}{5} \sqrt{-a} \arctan\left(\frac{(2x^5 + 1)\sqrt{x^5 + 1}\sqrt{ax^3}\sqrt{-a}}{2(ax^9 + ax^4)}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2), x, algorithm="fricas")

[Out] [1/10*sqrt(a)*log(-8*a*x^10 - 8*a*x^5 - 4*(2*x^6 + x)*sqrt(x^5 + 1)*sqrt(a*x^3)*sqrt(a) - a), -1/5*sqrt(-a)*arctan(1/2*(2*x^5 + 1)*sqrt(x^5 + 1)*sqrt(a*x^3)*sqrt(-a)/(a*x^9 + a*x^4)]]

giac [B] time = 0.19, size = 58, normalized size = 2.42

$$-\frac{2a^{\frac{3}{2}} \log\left(-\sqrt{ax} a^{\frac{5}{2}} x^2 + \sqrt{a^6 x^5 + a^6}\right) \operatorname{sgn}(x)}{5|a|} + \frac{2a^{\frac{3}{2}} \log\left(a^2|a|\right) \operatorname{sgn}(x)}{5|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2), x, algorithm="giac")

[Out] -2/5*a^(3/2)*log(-sqrt(a*x)*a^(5/2)*x^2 + sqrt(a^6*x^5 + a^6))*sgn(x)/abs(a) + 2/5*a^(3/2)*log(a^2*abs(a))*sgn(x)/abs(a)

maple [A] time = 0.05, size = 17, normalized size = 0.71

$$\frac{2\sqrt{a} x^3 \operatorname{arcsinh}\left(x^{\frac{5}{2}}\right)}{5x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3)^(1/2)/(x^5+1)^(1/2), x)

[Out] 2/5*arcsinh(x^(5/2))*(a*x^3)^(1/2)/x^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^3)/sqrt(x^5 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ax^3}}{\sqrt{x^5+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2),x)

[Out] int((a*x^3)^(1/2)/(x^5 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^3}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**3)**(1/2)/(x**5+1)**(1/2),x)

[Out] Integral(sqrt(a*x**3)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)

$$3.159 \quad \int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} dx$$

Optimal. Leaf size=23

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 264}

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^7]/Sqrt[1 + x^5], x]

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{1+x^5}} dx &= \left(\sqrt{\frac{a}{x^7}} x^{7/2} \right) \int \frac{1}{x^{7/2} \sqrt{1+x^5}} dx \\ &= -\frac{2}{5} \sqrt{\frac{a}{x^7}} x \sqrt{1+x^5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2}{5}x\sqrt{x^5+1}\sqrt{\frac{a}{x^7}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^7]/Sqrt[1 + x^5], x]

[Out] (-2*Sqrt[a/x^7]*x*Sqrt[1 + x^5])/5

IntegrateAlgebraic [A] time = 14.88, size = 28, normalized size = 1.22

$$\frac{2x^{15}\sqrt{x^5+1}\left(\frac{a}{x^7}\right)^{5/2}}{5a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a/x^7]/Sqrt[1 + x^5],x]

[Out] $(-2*(a/x^7)^{(5/2)}*x^{15}*Sqrt[1 + x^5])/(5*a^2)$

fricas [A] time = 0.60, size = 17, normalized size = 0.74

$$-\frac{2}{5} \sqrt{x^5 + 1} x \sqrt{\frac{a}{x^7}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="fricas")

[Out] $-2/5*\text{sqrt}(x^5 + 1)*x*\text{sqrt}(a/x^7)$

giac [A] time = 0.24, size = 28, normalized size = 1.22

$$-\frac{2 a^4 \left(\frac{\sqrt{a + \frac{a}{x^5}}}{a^3} - \frac{1}{a^2} \right)}{5 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="giac")

[Out] $-2/5*a^4*(\text{sqrt}(a + a/x^5)/a^3 - 1/a^{(5/2)})/\text{abs}(a)$

maple [B] time = 0.00, size = 37, normalized size = 1.61

$$\frac{2(x+1)(x^4 - x^3 + x^2 - x + 1) \sqrt{\frac{a}{x^7}} x}{5 \sqrt{x^5 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^7)^(1/2)/(x^5+1)^(1/2),x)

[Out] $-2/5*x*(x+1)*(x^4-x^3+x^2-x+1)*(a/x^7)^{(1/2)}/(x^5+1)^{(1/2)}$

maxima [B] time = 2.22, size = 41, normalized size = 1.78

$$\frac{2(\sqrt{a} x^6 + \sqrt{a} x)}{5 \sqrt{x^4 - x^3 + x^2 - x + 1} \sqrt{x + 1} x^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^7)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")

[Out] $-2/5*(\text{sqrt}(a)*x^6 + \text{sqrt}(a)*x)/(\text{sqrt}(x^4 - x^3 + x^2 - x + 1)*\text{sqrt}(x + 1)*x^{(7/2)})$

mupad [B] time = 2.69, size = 17, normalized size = 0.74

$$\frac{2 x \sqrt{x^5 + 1} \sqrt{\frac{a}{x^7}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^7)^(1/2)/(x^5 + 1)^(1/2),x)

[Out] $-(2*x*(x^5 + 1)^{(1/2)}*(a/x^7)^{(1/2)})/5$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^7}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x**7)**(1/2)/(x**5+1)**(1/2), x)

[Out] Integral(sqrt(a/x**7)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)

$$3.160 \quad \int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} dx$$

Optimal. Leaf size=49

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}} - \frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 271, 264}

$$\frac{4}{15}x^6\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}} - \frac{2}{15}x\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^17]/Sqrt[1 + x^5], x]

[Out] (-2*Sqrt[a/x^17]*x*Sqrt[1 + x^5])/15 + (4*Sqrt[a/x^17]*x^6*Sqrt[1 + x^5])/15

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 264

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rule 271

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x^(m+1)*(a+b*x^n)^(p+1))/(a*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*(m+1)), Int[x^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[(m+1)/n+p+1] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{1+x^5}} dx &= \left(\sqrt{\frac{a}{x^{17}}} x^{17/2} \right) \int \frac{1}{x^{17/2} \sqrt{1+x^5}} dx \\ &= -\frac{2}{15} \sqrt{\frac{a}{x^{17}}} x \sqrt{1+x^5} - \frac{1}{3} \left(2 \sqrt{\frac{a}{x^{17}}} x^{17/2} \right) \int \frac{1}{x^{7/2} \sqrt{1+x^5}} dx \\ &= -\frac{2}{15} \sqrt{\frac{a}{x^{17}}} x \sqrt{1+x^5} + \frac{4}{15} \sqrt{\frac{a}{x^{17}}} x^6 \sqrt{1+x^5} \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.61

$$-\frac{2}{15}x(1-2x^5)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^17]/Sqrt[1 + x^5], x]

[Out] (-2*Sqrt[a/x^17]*x*(1 - 2*x^5)*Sqrt[1 + x^5])/15

IntegrateAlgebraic [A] time = 9.88, size = 35, normalized size = 0.71

$$\frac{2x^{35}\sqrt{x^5+1}(2x^5-1)\left(\frac{a}{x^{17}}\right)^{5/2}}{15a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a/x^17]/Sqrt[1 + x^5], x]

[Out] (2*(a/x^17)^(5/2)*x^35*Sqrt[1 + x^5]*(-1 + 2*x^5))/(15*a^2)

fricas [A] time = 0.52, size = 25, normalized size = 0.51

$$\frac{2}{15}(2x^6-x)\sqrt{x^5+1}\sqrt{\frac{a}{x^{17}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2), x, algorithm="fricas")

[Out] 2/15*(2*x^6 - x)*sqrt(x^5 + 1)*sqrt(a/x^17)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(x)]
 Warning, integration of abs or sign assumes constant sign by intervals (cor
 rect if the argument is real):Check [abs(t_nostep)]Warning, integration of
 abs or sign assumes constant sign by intervals (correct if the argument is
 real):Check [abs(t_nostep)]sym2poly/r2sym(const gen & e,const index_m & i,c
 onst vecteur & l) Error: Bad Argument Value

maple [A] time = 0.00, size = 44, normalized size = 0.90

$$\frac{2(x+1)(x^4-x^3+x^2-x+1)(2x^5-1)\sqrt{\frac{a}{x^{17}}}}{15\sqrt{x^5+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^17)^(1/2)/(x^5+1)^(1/2), x)

[Out] 2/15*x*(x+1)*(x^4-x^3+x^2-x+1)*(2*x^5-1)*(a/x^17)^(1/2)/(x^5+1)^(1/2)

maxima [A] time = 2.87, size = 50, normalized size = 1.02

$$\frac{2(2\sqrt{a}x^{11} + \sqrt{a}x^6 - \sqrt{a}x)}{15\sqrt{x^4-x^3+x^2-x+1}\sqrt{x+1}x^{17/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^17)^(1/2)/(x^5+1)^(1/2),x, algorithm="maxima")

[Out] 2/15*(2*sqrt(a)*x^11 + sqrt(a)*x^6 - sqrt(a)*x)/(sqrt(x^4 - x^3 + x^2 - x + 1)*sqrt(x + 1)*x^(17/2))

mupad [B] time = 2.67, size = 29, normalized size = 0.59

$$\frac{\sqrt{\frac{a}{x^{17}}} \left(\frac{4x^{11}}{15} + \frac{2x^6}{15} - \frac{2x}{15} \right)}{\sqrt{x^5 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^17)^(1/2)/(x^5 + 1)^(1/2),x)

[Out] ((a/x^17)^(1/2)*((2*x^6)/15 - (2*x)/15 + (4*x^11)/15))/(x^5 + 1)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^{17}}}}{\sqrt{(x+1)(x^4-x^3+x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x**17)**(1/2)/(x**5+1)**(1/2),x)

[Out] Integral(sqrt(a/x**17)/sqrt((x + 1)*(x**4 - x**3 + x**2 - x + 1)), x)

$$3.161 \quad \int \frac{\sqrt{ax^6}}{x(1-x^4)} dx$$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {15, 298, 203, 206}

$$\frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^6]/(x*(1 - x^4)),x]

[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^6}}{x(1-x^4)} dx &= \frac{\sqrt{ax^6} \int \frac{x^2}{1-x^4} dx}{x^3} \\ &= \frac{\sqrt{ax^6} \int \frac{1}{1-x^2} dx}{2x^3} - \frac{\sqrt{ax^6} \int \frac{1}{1+x^2} dx}{2x^3} \\ &= -\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$\frac{\sqrt{ax^6} (\log(1-x) - \log(x+1) + 2 \tan^{-1}(x))}{4x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^6]/(x*(1 - x^4)), x]

[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3

IntegrateAlgebraic [A] time = 0.06, size = 29, normalized size = 0.78

$$\sqrt{ax^6} \left(\frac{\tanh^{-1}(x)}{2x^3} - \frac{\tan^{-1}(x)}{2x^3} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^6]/(x*(1 - x^4)), x]

[Out] Sqrt[a*x^6]*(-1/2*ArcTan[x]/x^3 + ArcTanh[x]/(2*x^3))

fricas [A] time = 0.73, size = 29, normalized size = 0.78

$$\frac{\sqrt{ax^6} \left(2 \arctan(x) - \log\left(\frac{x+1}{x-1}\right) \right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(1/2)/x/(-x^4+1), x, algorithm="fricas")

[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3

giac [A] time = 0.18, size = 29, normalized size = 0.78

$$-\frac{1}{4} \left(2 \arctan(x) \operatorname{sgn}(x) - \log(|x+1| \operatorname{sgn}(x)) + \log(|x-1| \operatorname{sgn}(x)) \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(1/2)/x/(-x^4+1), x, algorithm="giac")

[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*sqrt(a)

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$\frac{\sqrt{ax^6} (2 \arctan(x) + \ln(x-1) - \ln(x+1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^6)^(1/2)/x/(-x^4+1), x)

[Out] -1/4*(a*x^6)^(1/2)*(ln(x-1)-ln(x+1)+2*arctan(x))/x^3

maxima [A] time = 2.53, size = 26, normalized size = 0.70

$$-\frac{1}{2} \sqrt{a} \arctan(x) + \frac{1}{4} \sqrt{a} \log(x+1) - \frac{1}{4} \sqrt{a} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")

[Out] -1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$-\int \frac{\sqrt{ax^6}}{x(x^4-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(a*x^6)^(1/2)/(x*(x^4 - 1)),x)

[Out] -int((a*x^6)^(1/2)/(x*(x^4 - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5-x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**6)**(1/2)/x/(-x**4+1),x)

[Out] -Integral(sqrt(a*x**6)/(x**5 - x), x)

$$3.162 \quad \int \frac{\sqrt{ax^6}}{x-x^5} dx$$

Optimal. Leaf size=37

$$\frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {15, 1584, 298, 203, 206}

$$\frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^6]/(x - x^5), x]

[Out] -(Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^6}}{x-x^5} dx &= \frac{\sqrt{ax^6} \int \frac{x^3}{x-x^5} dx}{x^3} \\
&= \frac{\sqrt{ax^6} \int \frac{x^2}{1-x^4} dx}{x^3} \\
&= \frac{\sqrt{ax^6} \int \frac{1}{1-x^2} dx}{2x^3} - \frac{\sqrt{ax^6} \int \frac{1}{1+x^2} dx}{2x^3} \\
&= -\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 0.89

$$-\frac{\sqrt{ax^6} (\log(1-x) - \log(x+1) + 2 \tan^{-1}(x))}{4x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^6]/(x - x^5), x]

[Out] -1/4*(Sqrt[a*x^6]*(2*ArcTan[x] + Log[1 - x] - Log[1 + x]))/x^3

IntegrateAlgebraic [A] time = 0.05, size = 29, normalized size = 0.78

$$\sqrt{ax^6} \left(\frac{\tanh^{-1}(x)}{2x^3} - \frac{\tan^{-1}(x)}{2x^3} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^6]/(x - x^5), x]

[Out] Sqrt[a*x^6]*(-1/2*ArcTan[x]/x^3 + ArcTanh[x]/(2*x^3))

fricas [A] time = 0.55, size = 29, normalized size = 0.78

$$-\frac{\sqrt{ax^6} \left(2 \arctan(x) - \log\left(\frac{x+1}{x-1}\right) \right)}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(1/2)/(-x^5+x), x, algorithm="fricas")

[Out] -1/4*sqrt(a*x^6)*(2*arctan(x) - log((x + 1)/(x - 1)))/x^3

giac [A] time = 0.20, size = 29, normalized size = 0.78

$$-\frac{1}{4} \left(2 \arctan(x) \operatorname{sgn}(x) - \log(|x+1|) \operatorname{sgn}(x) + \log(|x-1|) \operatorname{sgn}(x) \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(1/2)/(-x^5+x), x, algorithm="giac")

[Out] -1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*sqrt(a)

maple [A] time = 0.01, size = 28, normalized size = 0.76

$$-\frac{\sqrt{ax^6} (2 \arctan(x) + \ln(x-1) - \ln(x+1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^6)^(1/2)/(-x^5+x),x)`

[Out] `-1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3`

maxima [A] time = 2.08, size = 26, normalized size = 0.70

$$-\frac{1}{2}\sqrt{a}\arctan(x) + \frac{1}{4}\sqrt{a}\log(x+1) - \frac{1}{4}\sqrt{a}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")`

[Out] `-1/2*sqrt(a)*arctan(x) + 1/4*sqrt(a)*log(x + 1) - 1/4*sqrt(a)*log(x - 1)`

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{ax^6}}{x-x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^6)^(1/2)/(x - x^5),x)`

[Out] `int((a*x^6)^(1/2)/(x - x^5), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^6}}{x^5-x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x**6)**(1/2)/(-x**5+x),x)`

[Out] `-Integral(sqrt(a*x**6)/(x**5 - x), x)`

$$3.163 \quad \int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx$$

Optimal. Leaf size=71

$$\frac{a\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{a\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} - \frac{1}{5}ax^2\sqrt{ax^6} - \frac{a\sqrt{ax^6}}{x^2}$$

Rubi [A] time = 0.01, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {15, 302, 212, 206, 203}

$$-\frac{1}{5}ax^2\sqrt{ax^6} - \frac{a\sqrt{ax^6}}{x^2} + \frac{a\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{a\sqrt{ax^6} \tanh^{-1}(x)}{2x^3}$$

Antiderivative was successfully verified.

[In] Int[(a*x^6)^(3/2)/(x*(1 - x^4)), x]

[Out] -((a*Sqrt[a*x^6])/x^2) - (a*x^2*Sqrt[a*x^6])/5 + (a*Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + (a*Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rubi steps

$$\begin{aligned}
\int \frac{(ax^6)^{3/2}}{x(1-x^4)} dx &= \frac{(a\sqrt{ax^6}) \int \frac{x^8}{1-x^4} dx}{x^3} \\
&= \frac{(a\sqrt{ax^6}) \int \left(-1 - x^4 + \frac{1}{1-x^4}\right) dx}{x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{(a\sqrt{ax^6}) \int \frac{1}{1-x^4} dx}{x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{(a\sqrt{ax^6}) \int \frac{1}{1-x^2} dx}{2x^3} + \frac{(a\sqrt{ax^6}) \int \frac{1}{1+x^2} dx}{2x^3} \\
&= -\frac{a\sqrt{ax^6}}{x^2} - \frac{1}{5}ax^2\sqrt{ax^6} + \frac{a\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{a\sqrt{ax^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 0.62

$$\frac{a\sqrt{ax^6} (4x^5 + 20x + 5 \log(1-x) - 5 \log(x+1) - 10 \tan^{-1}(x))}{20x^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^6)^(3/2)/(x*(1 - x^4)),x]

[Out] -1/20*(a*Sqrt[a*x^6]*(20*x + 4*x^5 - 10*ArcTan[x] + 5*Log[1 - x] - 5*Log[1 + x]))/x^3

IntegrateAlgebraic [A] time = 0.07, size = 43, normalized size = 0.61

$$(ax^6)^{3/2} \left(\frac{\tan^{-1}(x)}{2x^9} + \frac{\tanh^{-1}(x)}{2x^9} + \frac{-x^4 - 5}{5x^8} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a*x^6)^(3/2)/(x*(1 - x^4)),x]

[Out] (a*x^6)^(3/2)*((-5 - x^4)/(5*x^8) + ArcTan[x]/(2*x^9) + ArcTanh[x]/(2*x^9))

fricas [A] time = 0.66, size = 41, normalized size = 0.58

$$\frac{\sqrt{ax^6} \left(4ax^5 + 20ax - 10a \arctan(x) - 5a \log\left(\frac{x+1}{x-1}\right) \right)}{20x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="fricas")

[Out] -1/20*sqrt(a*x^6)*(4*a*x^5 + 20*a*x - 10*a*arctan(x) - 5*a*log((x + 1)/(x - 1)))/x^3

giac [A] time = 0.18, size = 42, normalized size = 0.59

$$-\frac{1}{20} \left(4x^5 \operatorname{sgn}(x) + 20x \operatorname{sgn}(x) - 10 \arctan(x) \operatorname{sgn}(x) - 5 \log(|x+1|) \operatorname{sgn}(x) + 5 \log(|x-1|) \operatorname{sgn}(x) \right) a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^6)^(3/2)/x/(-x^4+1),x, algorithm="giac")

[Out] $-1/20*(4*x^5*\text{sgn}(x) + 20*x*\text{sgn}(x) - 10*\arctan(x)*\text{sgn}(x) - 5*\log(\text{abs}(x + 1))*\text{sgn}(x) + 5*\log(\text{abs}(x - 1))*\text{sgn}(x))*a^{(3/2)}$

maple [A] time = 0.01, size = 38, normalized size = 0.54

$$\frac{(ax^6)^{\frac{3}{2}}(4x^5 + 20x - 10\arctan(x) + 5\ln(x-1) - 5\ln(x+1))}{20x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x^6)^(3/2)/x/(-x^4+1), x)`

[Out] $-1/20*(a*x^6)^{(3/2)}*(4*x^5+5*\ln(x-1)-5*\ln(x+1)-10*\arctan(x)+20*x)/x^9$

maxima [A] time = 2.09, size = 40, normalized size = 0.56

$$-\frac{1}{5}a^{\frac{3}{2}}x^5 - a^{\frac{3}{2}}x + \frac{1}{2}a^{\frac{3}{2}}\arctan(x) + \frac{1}{4}a^{\frac{3}{2}}\log(x+1) - \frac{1}{4}a^{\frac{3}{2}}\log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x^6)^(3/2)/x/(-x^4+1), x, algorithm="maxima")`

[Out] $-1/5*a^{(3/2)}*x^5 - a^{(3/2)}*x + 1/2*a^{(3/2)}*\arctan(x) + 1/4*a^{(3/2)}*\log(x + 1) - 1/4*a^{(3/2)}*\log(x - 1)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{(ax^6)^{3/2}}{x(x^4-1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(a*x^6)^(3/2)/(x*(x^4 - 1)), x)`

[Out] $-\text{int}((a*x^6)^{(3/2)}/(x*(x^4 - 1)), x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(ax^6)^{\frac{3}{2}}}{x^5 - x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x**6)**(3/2)/x/(-x**4+1), x)`

[Out] $-\text{Integral}((a*x**6)**(3/2)/(x**5 - x), x)$

$$3.164 \quad \int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx$$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {212, 206, 203, 15, 298}

$$\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)), x]

[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x(1-x^4)} \right) dx &= \int \frac{1}{1-x^4} dx - \int \frac{\sqrt{ax^6}}{x(1-x^4)} dx \\
&= \frac{1}{2} \int \frac{1}{1-x^2} dx + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{\sqrt{ax^6} \int \frac{x^2}{1-x^4} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{ax^6} \int \frac{1}{1-x^2} dx}{2x^3} + \frac{\sqrt{ax^6} \int \frac{1}{1+x^2} dx}{2x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 29, normalized size = 0.59

$$\frac{1}{2} \left(\frac{\sqrt{ax^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)), x]

[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2

IntegrateAlgebraic [A] time = 0.07, size = 48, normalized size = 0.98

$$\left(\frac{\sqrt{ax^6}}{2x^3} + \frac{1}{2} \right) \tan^{-1}(x) - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x*(1 - x^4)), x]

[Out] (1/2 + Sqrt[a*x^6]/(2*x^3))*ArcTan[x] + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

fricas [B] time = 0.65, size = 256, normalized size = 5.22

$$\frac{x^3 \sqrt{\frac{(a+1)x^4+2\sqrt{a^2}}{x^3}} \log\left(\frac{(a-1)x^4-(a-1)x^2-2\left(x^3-\sqrt{a^2}\right)\sqrt{\frac{(a+1)x^4+2\sqrt{a^2}}{x^3}}}{x^4+x^2}\right) + x^3 \log(x+1) - x^3 \log(x-1) - \sqrt{a^2}(\log(x+1) - \log(x-1))}{4x^3} - \frac{2x^3 \sqrt{\frac{(a+1)x^4+2\sqrt{a^2}}{x^3}} \arctan\left(\frac{(x^3-\sqrt{a^2})\sqrt{\frac{(a+1)x^4+2\sqrt{a^2}}{x^3}}}{(a-1)x^2}\right) + x^3 \log(x+1) - x^3 \log(x-1) - \sqrt{a^2}(\log(x+1) - \log(x-1))}{4x^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1), x, algorithm="fricas")

[Out] [1/4*(x^3*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)*log(((a - 1)*x^4 - (a - 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3))/((x^4 + x^2)) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)*arctan(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)/((a - 1)*x^2) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1)))/x^3]

giac [A] time = 0.18, size = 48, normalized size = 0.98

$$\frac{1}{4} \left(2 \arctan(x) \operatorname{sgn}(x) - \log(|x+1| \operatorname{sgn}(x) + \log(|x-1| \operatorname{sgn}(x))) \sqrt{a} + \frac{1}{2} \arctan(x) + \frac{1}{4} \log(|x+1|) - \frac{1}{4} \log(|x-1|) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="giac")

[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\arctan(x)}{2} + \frac{\sqrt{ax^6} (2 \arctan(x) + \ln(x-1) - \ln(x+1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x)

[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3

maxima [A] time = 2.70, size = 42, normalized size = 0.86

$$\frac{1}{2} \sqrt{a} \arctan(x) - \frac{1}{4} \sqrt{a} \log(x+1) + \frac{1}{4} \sqrt{a} \log(x-1) + \frac{1}{2} \arctan(x) + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/x/(-x^4+1),x, algorithm="maxima")

[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{ax^6}}{x(x^4-1)} - \frac{1}{x^4-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1),x)

[Out] int((a*x^6)^(1/2)/(x*(x^4 - 1)) - 1/(x^4 - 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x} dx - \int \left(-\frac{\sqrt{ax^6}}{x^5-x} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/x/(-x**4+1),x)

[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)

$$3.165 \quad \int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx$$

Optimal. Leaf size=49

$$\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {212, 206, 203, 15, 1584, 298}

$$\frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5), x]

[Out] ArcTan[x]/2 + (Sqrt[a*x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \left(\frac{1}{1-x^4} - \frac{\sqrt{ax^6}}{x-x^5} \right) dx &= \int \frac{1}{1-x^4} dx - \int \frac{\sqrt{ax^6}}{x-x^5} dx \\
&= \frac{1}{2} \int \frac{1}{1-x^2} dx + \frac{1}{2} \int \frac{1}{1+x^2} dx - \frac{\sqrt{ax^6} \int \frac{x^3}{x-x^5} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{ax^6} \int \frac{x^2}{1-x^4} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{ax^6} \int \frac{1}{1-x^2} dx}{2x^3} + \frac{\sqrt{ax^6} \int \frac{1}{1+x^2} dx}{2x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{ax^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.59

$$\frac{1}{2} \left(\frac{\sqrt{ax^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5), x]

[Out] (ArcTan[x] + (Sqrt[a*x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2

IntegrateAlgebraic [A] time = 0.05, size = 48, normalized size = 0.98

$$\left(\frac{\sqrt{ax^6}}{2x^3} + \frac{1}{2} \right) \tan^{-1}(x) - \frac{\sqrt{ax^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x^4)^(-1) - Sqrt[a*x^6]/(x - x^5), x]

[Out] (1/2 + Sqrt[a*x^6]/(2*x^3))*ArcTan[x] + ArcTanh[x]/2 - (Sqrt[a*x^6]*ArcTanh[x])/(2*x^3)

fricas [B] time = 0.67, size = 256, normalized size = 5.22

$$\left| \frac{x^3 \sqrt{-\frac{(a+1)x^2 + 2\sqrt{ax^6}}{3}} \log\left(\frac{(a-1)x^4 - (a-1)x^2 - 2\sqrt{ax^6}}{x^4 + x^2}\right) + x^3 \log(x+1) - x^3 \log(x-1) - \sqrt{ax^6} (\log(x+1) - \log(x-1))}{4x^3} - \frac{2x^3 \sqrt{\frac{(a+1)x^2 + 2\sqrt{ax^6}}{3}} \arctan\left(\frac{(x^2 - \sqrt{ax^6}) \sqrt{\frac{(a+1)x^2 + 2\sqrt{ax^6}}{3}}}{(a-1)x^2}\right) + x^3 \log(x+1) - x^3 \log(x-1) - \sqrt{ax^6} (\log(x+1) - \log(x-1))}{4x^3} \right|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x), x, algorithm="fricas")

[Out] [1/4*(x^3*sqrt(-(a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3]*log(((a - 1)*x^4 - (a - 1)*x^2 - 2*(x^3 - sqrt(a*x^6))*sqrt(-(a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)/(x^4 + x^2) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1))/x^3, 1/4*(2*x^3*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)*arctan(-(x^3 - sqrt(a*x^6))*sqrt(((a + 1)*x^3 + 2*sqrt(a*x^6)))/x^3)/((a - 1)*x^2) + x^3*log(x + 1) - x^3*log(x - 1) - sqrt(a*x^6)*(log(x + 1) - log(x - 1))/x^3]

giac [A] time = 0.19, size = 48, normalized size = 0.98

$$\frac{1}{4} (2 \arctan(x) \operatorname{sgn}(x) - \log(|x + 1|) \operatorname{sgn}(x) + \log(|x - 1|) \operatorname{sgn}(x)) \sqrt{a} + \frac{1}{2} \arctan(x) + \frac{1}{4} \log(|x + 1|) - \frac{1}{4} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="giac")

[Out] 1/4*(2*arctan(x)*sgn(x) - log(abs(x + 1))*sgn(x) + log(abs(x - 1))*sgn(x))*
sqrt(a) + 1/2*arctan(x) + 1/4*log(abs(x + 1)) - 1/4*log(abs(x - 1))

maple [A] time = 0.01, size = 37, normalized size = 0.76

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\operatorname{arctan}(x)}{2} + \frac{\sqrt{ax^6} (2 \operatorname{arctan}(x) + \ln(x-1) - \ln(x+1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x)

[Out] 1/2*arctanh(x)+1/2*arctan(x)+1/4*(a*x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))
)/x^3

maxima [A] time = 2.04, size = 42, normalized size = 0.86

$$\frac{1}{2} \sqrt{a} \operatorname{arctan}(x) - \frac{1}{4} \sqrt{a} \log(x+1) + \frac{1}{4} \sqrt{a} \log(x-1) + \frac{1}{2} \operatorname{arctan}(x) + \frac{1}{4} \log(x+1) - \frac{1}{4} \log(x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^4+1)-(a*x^6)^(1/2)/(-x^5+x),x, algorithm="maxima")

[Out] 1/2*sqrt(a)*arctan(x) - 1/4*sqrt(a)*log(x + 1) + 1/4*sqrt(a)*log(x - 1) + 1
/2*arctan(x) + 1/4*log(x + 1) - 1/4*log(x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{1}{x^4-1} - \frac{\sqrt{ax^6}}{x-x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5),x)

[Out] int(- 1/(x^4 - 1) - (a*x^6)^(1/2)/(x - x^5), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x^5-x} dx - \int \left(-\frac{\sqrt{ax^6}}{x^5-x} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**4+1)-(a*x**6)**(1/2)/(-x**5+x),x)

[Out] -Integral(x/(x**5 - x), x) - Integral(-sqrt(a*x**6)/(x**5 - x), x)

$$3.166 \quad \int \frac{\sqrt{ax^3}}{x-x^3} dx$$

Optimal. Leaf size=44

$$\frac{\sqrt{ax^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{ax^3} \tan^{-1}(\sqrt{x})}{x^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {15, 1584, 329, 298, 203, 206}

$$\frac{\sqrt{ax^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{ax^3} \tan^{-1}(\sqrt{x})}{x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^3]/(x - x^3), x]

[Out] -((Sqrt[a*x^3]*ArcTan[Sqrt[x]])/x^(3/2)) + (Sqrt[a*x^3]*ArcTanh[Sqrt[x]])/x^(3/2)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{ax^3}}{x-x^3} dx &= \frac{\sqrt{ax^3} \int \frac{x^{3/2}}{x-x^3} dx}{x^{3/2}} \\
&= \frac{\sqrt{ax^3} \int \frac{\sqrt{x}}{1-x^2} dx}{x^{3/2}} \\
&= \frac{(2\sqrt{ax^3}) \text{Subst}\left(\int \frac{x^2}{1-x^4} dx, x, \sqrt{x}\right)}{x^{3/2}} \\
&= \frac{\sqrt{ax^3} \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{x}\right)}{x^{3/2}} - \frac{\sqrt{ax^3} \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right)}{x^{3/2}} \\
&= -\frac{\sqrt{ax^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} + \frac{\sqrt{ax^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{\sqrt{ax^3} \left(\tanh^{-1}(\sqrt{x}) - \tan^{-1}(\sqrt{x}) \right)}{x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^3]/(x - x^3), x]

[Out] (Sqrt[a*x^3]*(-ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]))/x^(3/2)

IntegrateAlgebraic [A] time = 0.08, size = 51, normalized size = 1.16

$$\sqrt{a} \tan^{-1}\left(\frac{\sqrt{ax^3}}{\sqrt{a}x^2}\right) + \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{ax^3}}{\sqrt{a}x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^3]/(x - x^3), x]

[Out] Sqrt[a]*ArcTan[Sqrt[a*x^3]/(Sqrt[a]*x^2)] + Sqrt[a]*ArcTanh[Sqrt[a*x^3]/(Sqrt[a]*x^2)]

fricas [A] time = 0.70, size = 127, normalized size = 2.89

$$\left[-\sqrt{a} \arctan\left(\frac{\sqrt{ax^3}}{\sqrt{a}x}\right) + \frac{1}{2} \sqrt{a} \log\left(\frac{ax^2 + ax + 2\sqrt{ax^3}\sqrt{a}}{x^2 - x}\right), -\sqrt{-a} \arctan\left(\frac{\sqrt{ax^3}\sqrt{-a}}{ax}\right) + \frac{1}{2} \sqrt{-a} \log\left(\frac{ax^2 - ax - 2\sqrt{ax^3}\sqrt{-a}}{x^2 + x}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(-x^3+x), x, algorithm="fricas")

[Out] [-sqrt(a)*arctan(sqrt(a*x^3)/(sqrt(a)*x)) + 1/2*sqrt(a)*log((a*x^2 + a*x + 2*sqrt(a*x^3)*sqrt(a))/(x^2 - x)), -sqrt(-a)*arctan(sqrt(a*x^3)*sqrt(-a)/(a*x)) + 1/2*sqrt(-a)*log((a*x^2 - a*x - 2*sqrt(a*x^3)*sqrt(-a))/(x^2 + x))]

giac [A] time = 0.21, size = 43, normalized size = 0.98

$$\frac{\left(\frac{a^2 \arctan\left(\frac{\sqrt{ax}}{\sqrt{-a}}\right)}{\sqrt{-a}} + a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{ax}}{\sqrt{a}}\right) \right) \text{sgn}(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="giac")

[Out] -(a^2*arctan(sqrt(a*x)/sqrt(-a))/sqrt(-a) + a^(3/2)*arctan(sqrt(a*x)/sqrt(a))) *sgn(x)/a

maple [A] time = 0.02, size = 43, normalized size = 0.98

$$\frac{\sqrt{ax^3} \left(\operatorname{arctanh}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right) - \operatorname{arctan}\left(\frac{\sqrt{ax}}{\sqrt{a}}\right) \right) \sqrt{a}}{\sqrt{ax} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3)^(1/2)/(-x^3+x),x)

[Out] (a*x^3)^(1/2)*a^(1/2)*(arctanh((a*x)^(1/2)/a^(1/2))-arctan((a*x)^(1/2)/a^(1/2)))/x/(a*x)^(1/2)

maxima [A] time = 2.18, size = 32, normalized size = 0.73

$$-\sqrt{a} \arctan(\sqrt{x}) + \frac{1}{2} \sqrt{a} \log(\sqrt{x} + 1) - \frac{1}{2} \sqrt{a} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^3)^(1/2)/(-x^3+x),x, algorithm="maxima")

[Out] -sqrt(a)*arctan(sqrt(x)) + 1/2*sqrt(a)*log(sqrt(x) + 1) - 1/2*sqrt(a)*log(sqrt(x) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{ax^3}}{x-x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^3)^(1/2)/(x - x^3),x)

[Out] int((a*x^3)^(1/2)/(x - x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{ax^3}}{x^3-x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**3)**(1/2)/(-x**3+x),x)

[Out] -Integral(sqrt(a*x**3)/(x**3 - x), x)

$$3.167 \quad \int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=44

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {15, 321, 215}

$$\frac{\sqrt{x^2+1}\sqrt{ax^4}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^2], x]

[Out] (Sqrt[a*x^4]*Sqrt[1 + x^2])/(2*x) - (Sqrt[a*x^4]*ArcSinh[x])/(2*x^2)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^4}}{\sqrt{1+x^2}} dx &= \frac{\sqrt{ax^4}}{x^2} \int \frac{x^2}{\sqrt{1+x^2}} dx \\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}}{2x^2} \int \frac{1}{\sqrt{1+x^2}} dx \\ &= \frac{\sqrt{ax^4}\sqrt{1+x^2}}{2x} - \frac{\sqrt{ax^4}\sinh^{-1}(x)}{2x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.73

$$\frac{\sqrt{ax^4} \left(x\sqrt{x^2+1} - \sinh^{-1}(x) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^2],x]

[Out] (Sqrt[a*x^4]*(x*Sqrt[1 + x^2] - ArcSinh[x]))/(2*x^2)

IntegrateAlgebraic [A] time = 0.09, size = 61, normalized size = 1.39

$$\frac{\sqrt{a} x^2 \left(\frac{1}{2} \sqrt{a} \sqrt{x^2 + 1} x + \frac{1}{2} \sqrt{a} \log \left(\sqrt{x^2 + 1} - x \right) \right)}{\sqrt{a x^4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^4]/Sqrt[1 + x^2],x]

[Out] (Sqrt[a]*x^2*((Sqrt[a]*x*Sqrt[1 + x^2])/2 + (Sqrt[a]*Log[-x + Sqrt[1 + x^2]]/2))/Sqrt[a*x^4]

fricas [A] time = 0.55, size = 42, normalized size = 0.95

$$\frac{\sqrt{a x^4} \sqrt{x^2 + 1} x + \sqrt{a x^4} \log \left(-x + \sqrt{x^2 + 1} \right)}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/2*(sqrt(a*x^4)*sqrt(x^2 + 1)*x + sqrt(a*x^4)*log(-x + sqrt(x^2 + 1)))/x^2

giac [A] time = 0.19, size = 27, normalized size = 0.61

$$\frac{1}{2} \left(\sqrt{x^2 + 1} x + \log \left(-x + \sqrt{x^2 + 1} \right) \right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")

[Out] 1/2*(sqrt(x^2 + 1)*x + log(-x + sqrt(x^2 + 1)))*sqrt(a)

maple [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{\sqrt{a x^4} \left(\sqrt{x^2 + 1} x - \operatorname{arcsinh}(x) \right)}{2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^4)^(1/2)/(x^2+1)^(1/2),x)

[Out] 1/2*(a*x^4)^(1/2)*(x*(x^2+1)^(1/2)-arcsinh(x))/x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a x^4}}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^4)/sqrt(x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a x^4}}{\sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

```
[Out] int((a*x^4)^(1/2)/(x^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**2+1)**(1/2), x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt(x**2 + 1), x)
```

$$3.168 \quad \int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=22

$$\frac{\sqrt{x^2+1} \sqrt{ax^2}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 261}

$$\frac{\sqrt{x^2+1} \sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^2}}{\sqrt{1+x^2}} dx &= \frac{\sqrt{ax^2} \int \frac{x}{\sqrt{1+x^2}} dx}{x} \\ &= \frac{\sqrt{ax^2} \sqrt{1+x^2}}{x} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$\frac{\sqrt{x^2+1} \sqrt{ax^2}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2]/Sqrt[1 + x^2], x]

[Out] (Sqrt[a*x^2]*Sqrt[1 + x^2])/x

IntegrateAlgebraic [A] time = 0.03, size = 21, normalized size = 0.95

$$\frac{ax\sqrt{x^2+1}}{\sqrt{ax^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2]/Sqrt[1 + x^2],x]

[Out] (a*x*Sqrt[1 + x^2])/Sqrt[a*x^2]

fricas [A] time = 0.52, size = 18, normalized size = 0.82

$$\frac{\sqrt{ax^2} \sqrt{x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] sqrt(a*x^2)*sqrt(x^2 + 1)/x

giac [A] time = 0.16, size = 19, normalized size = 0.86

$$\left(\sqrt{x^2 + 1} \operatorname{sgn}(x) - \operatorname{sgn}(x)\right) \sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")

[Out] (sqrt(x^2 + 1)*sgn(x) - sgn(x))*sqrt(a)

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$\frac{\sqrt{ax^2} \sqrt{x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2)^(1/2)/(x^2+1)^(1/2),x)

[Out] (a*x^2)^(1/2)*(x^2+1)^(1/2)/x

maxima [A] time = 1.90, size = 19, normalized size = 0.86

$$\frac{\sqrt{a} x^2 + \sqrt{a}}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] (sqrt(a)*x^2 + sqrt(a))/sqrt(x^2 + 1)

mupad [B] time = 2.67, size = 19, normalized size = 0.86

$$\frac{\sqrt{a} \sqrt{x^2 + 1} \sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2)^(1/2)/(x^2 + 1)^(1/2),x)

[Out] (a^(1/2)*(x^2 + 1)^(1/2)*(x^2)^(1/2))/x

sympy [A] time = 0.48, size = 20, normalized size = 0.91

$$\frac{\sqrt{a} \sqrt{x^2 + 1} \sqrt{x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2)**(1/2)/(x**2+1)**(1/2),x)

[Out] sqrt(a)*sqrt(x**2 + 1)*sqrt(x**2)/x

$$3.169 \quad \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=22

$$x \left(-\sqrt{\frac{a}{x^2}} \right) \tanh^{-1} \left(\sqrt{x^2 + 1} \right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 266, 63, 207}

$$x \left(-\sqrt{\frac{a}{x^2}} \right) \tanh^{-1} \left(\sqrt{x^2 + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^2], x]

[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :=> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :=> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^2}} dx &= \left(\sqrt{\frac{a}{x^2}} x \right) \int \frac{1}{x\sqrt{1+x^2}} dx \\
&= \frac{1}{2} \left(\sqrt{\frac{a}{x^2}} x \right) \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^2 \right) \\
&= \left(\sqrt{\frac{a}{x^2}} x \right) \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^2} \right) \\
&= -\sqrt{\frac{a}{x^2}} x \tanh^{-1} \left(\sqrt{1+x^2} \right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$x \left(-\sqrt{\frac{a}{x^2}} \right) \tanh^{-1} \left(\sqrt{x^2 + 1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^2], x]

[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])

IntegrateAlgebraic [A] time = 0.05, size = 22, normalized size = 1.00

$$x \left(-\sqrt{\frac{a}{x^2}} \right) \tanh^{-1} \left(\sqrt{x^2 + 1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a/x^2]/Sqrt[1 + x^2], x]

[Out] -(Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^2]])

fricas [A] time = 0.72, size = 76, normalized size = 3.45

$$\left[x \sqrt{\frac{a}{x^2}} \log \left(\frac{\sqrt{x^2 + 1} - 1}{x} \right), 2 \sqrt{-a} \arctan \left(-\frac{\sqrt{-a} x^2 \sqrt{\frac{a}{x^2}} - \sqrt{x^2 + 1} \sqrt{-a} x \sqrt{\frac{a}{x^2}}}{a} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2), x, algorithm="fricas")

[Out] [x*sqrt(a/x^2)*log((sqrt(x^2 + 1) - 1)/x), 2*sqrt(-a)*arctan(-(sqrt(-a)*x^2*sqrt(a/x^2) - sqrt(x^2 + 1)*sqrt(-a)*x*sqrt(a/x^2))/a)]

giac [A] time = 0.21, size = 30, normalized size = 1.36

$$-\frac{1}{2} \sqrt{a} \left(\log \left(\sqrt{x^2 + 1} + 1 \right) - \log \left(\sqrt{x^2 + 1} - 1 \right) \right) \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^2)^(1/2)/(x^2+1)^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(a)*(log(sqrt(x^2 + 1) + 1) - log(sqrt(x^2 + 1) - 1))*sgn(x)

maple [A] time = 0.01, size = 19, normalized size = 0.86

$$-\sqrt{\frac{a}{x^2}} x \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)`

[Out] `-(a/x^2)^(1/2)*x*arctanh(1/(x^2+1)^(1/2))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x^2)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a/x^2)/sqrt(x^2+1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a/x^2)^(1/2)/(x^2+1)^(1/2),x)`

[Out] `int((a/x^2)^(1/2)/(x^2+1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a/x**2)**(1/2)/(x**2+1)**(1/2),x)`

[Out] `Integral(sqrt(a/x**2)/sqrt(x**2+1), x)`

$$3.170 \quad \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=21

$$x\sqrt{x^2+1} \left(-\sqrt{\frac{a}{x^4}} \right)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 264}

$$x\sqrt{x^2+1} \left(-\sqrt{\frac{a}{x^4}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^4]/Sqrt[1 + x^2], x]

[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 264

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[(m+1)/n+p+1, 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{1+x^2}} dx &= \left(\sqrt{\frac{a}{x^4}} x^2 \right) \int \frac{1}{x^2 \sqrt{1+x^2}} dx \\ &= -\sqrt{\frac{a}{x^4}} x \sqrt{1+x^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.00

$$x\sqrt{x^2+1} \left(-\sqrt{\frac{a}{x^4}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^4]/Sqrt[1 + x^2], x]

[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])

IntegrateAlgebraic [A] time = 0.07, size = 21, normalized size = 1.00

$$x\sqrt{x^2+1} \left(-\sqrt{\frac{a}{x^4}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a/x^4]/Sqrt[1 + x^2],x]

[Out] -(Sqrt[a/x^4]*x*Sqrt[1 + x^2])

fricas [A] time = 0.49, size = 30, normalized size = 1.43

$$-x^2\sqrt{\frac{a}{x^4}} - \sqrt{x^2+1}x\sqrt{\frac{a}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] -x^2*sqrt(a/x^4) - sqrt(x^2 + 1)*x*sqrt(a/x^4)

giac [A] time = 0.18, size = 22, normalized size = 1.05

$$\frac{2\sqrt{a}}{(x - \sqrt{x^2+1})^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(a)/((x - sqrt(x^2 + 1))^2 - 1)

maple [A] time = 0.00, size = 18, normalized size = 0.86

$$-\sqrt{\frac{a}{x^4}}\sqrt{x^2+1}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^4)^(1/2)/(x^2+1)^(1/2),x)

[Out] -x*(a/x^4)^(1/2)*(x^2+1)^(1/2)

maxima [A] time = 1.97, size = 23, normalized size = 1.10

$$\frac{\sqrt{a}x^2 + \sqrt{a}}{\sqrt{x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^4)^(1/2)/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] -(sqrt(a)*x^2 + sqrt(a))/(sqrt(x^2 + 1)*x)

mupad [B] time = 2.87, size = 18, normalized size = 0.86

$$-\sqrt{a}x\sqrt{x^2+1}\sqrt{\frac{1}{x^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^4)^(1/2)/(x^2 + 1)^(1/2),x)

[Out] -a^(1/2)*x*(x^2 + 1)^(1/2)*(1/x^4)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^4}}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a/x**4)**(1/2)/(x**2+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a/x**4)/sqrt(x**2 + 1), x)
```

$$3.171 \quad \int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=25

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Rubi [A] time = 0.00, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {15, 261}

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^4]/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax^4}}{\sqrt{1+x^3}} dx &= \frac{\sqrt{ax^4} \int \frac{x^2}{\sqrt{1+x^3}} dx}{x^2} \\ &= \frac{2\sqrt{ax^4} \sqrt{1+x^3}}{3x^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{x^3+1}\sqrt{ax^4}}{3x^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^4]/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[a*x^4]*Sqrt[1 + x^3])/(3*x^2)

IntegrateAlgebraic [A] time = 0.03, size = 26, normalized size = 1.04

$$\frac{2ax^2\sqrt{x^3+1}}{3\sqrt{ax^4}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^4]/Sqrt[1 + x^3],x]

[Out] (2*a*x^2*Sqrt[1 + x^3])/(3*Sqrt[a*x^4])

fricas [A] time = 0.58, size = 19, normalized size = 0.76

$$\frac{2\sqrt{ax^4}\sqrt{x^3+1}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(a*x^4)*sqrt(x^3 + 1)/x^2

giac [A] time = 0.17, size = 12, normalized size = 0.48

$$\frac{2}{3}\sqrt{x^3+1}\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="giac")

[Out] 2/3*sqrt(x^3 + 1)*sqrt(a)

maple [A] time = 0.01, size = 31, normalized size = 1.24

$$\frac{2(x+1)(x^2-x+1)\sqrt{ax^4}}{3\sqrt{x^3+1}x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^4)^(1/2)/(x^3+1)^(1/2),x)

[Out] 2/3*(x+1)*(x^2-x+1)/x^2*(a*x^4)^(1/2)/(x^3+1)^(1/2)

maxima [A] time = 2.26, size = 28, normalized size = 1.12

$$\frac{2(\sqrt{a}x^3 + \sqrt{a})}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^4)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] 2/3*(sqrt(a)*x^3 + sqrt(a))/(sqrt(x^2 - x + 1)*sqrt(x + 1))

mupad [B] time = 2.91, size = 20, normalized size = 0.80

$$\frac{2\sqrt{a}\sqrt{x^3+1}\sqrt{x^4}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^4)^(1/2)/(x^3 + 1)^(1/2),x)

[Out] (2*a^(1/2)*(x^3 + 1)^(1/2)*(x^4)^(1/2))/(3*x^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^4}}{\sqrt{(x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x**4)**(1/2)/(x**3+1)**(1/2),x)
```

```
[Out] Integral(sqrt(a*x**4)/sqrt((x + 1)*(x**2 - x + 1)), x)
```


$$3.172 \quad \int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=23

$$\frac{2}{3}\sqrt{a} \sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {329, 275, 215}

$$\frac{2}{3}\sqrt{a} \sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x]/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[a]*ArcSinh[(a*x)^(3/2)/a^(3/2)])/3

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 329

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^n)^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{ax}}{\sqrt{1+x^3}} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{x^2}{\sqrt{1+\frac{x^6}{a^3}}} dx, x, \sqrt{ax}\right)}{a} \\ &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a^3}}} dx, x, (ax)^{3/2}\right)}{3a} \\ &= \frac{2}{3}\sqrt{a} \sinh^{-1}\left(\frac{(ax)^{3/2}}{a^{3/2}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 0.96

$$\frac{2\sqrt{ax} \sinh^{-1}(x^{3/2})}{3\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x]/Sqrt[1 + x^3], x]

[Out] (2*Sqrt[a*x]*ArcSinh[x^(3/2)])/(3*Sqrt[x])

IntegrateAlgebraic [A] time = 7.39, size = 44, normalized size = 1.91

$$-\frac{2}{3}\sqrt{a} \log\left((ax)^{3/2} - a^{3/2}\sqrt{\frac{a^3x^3 + a^3}{a^3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x]/Sqrt[1 + x^3], x]

[Out] (-2*Sqrt[a]*Log[(a*x)^(3/2) - a^(3/2)*Sqrt[(a^3 + a^3*x^3)/a^3]])/3

fricas [B] time = 0.77, size = 85, normalized size = 3.70

$$\left[\frac{1}{6}\sqrt{a} \log\left(-8ax^6 - 8ax^3 - 4(2x^4 + x)\sqrt{x^3 + 1}\sqrt{ax}\sqrt{a} - a\right), -\frac{1}{3}\sqrt{-a} \arctan\left(\frac{2\sqrt{x^3 + 1}\sqrt{ax}\sqrt{-a}x}{2ax^3 + a}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2), x, algorithm="fricas")

[Out] [1/6*sqrt(a)*log(-8*a*x^6 - 8*a*x^3 - 4*(2*x^4 + x)*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(a) - a), -1/3*sqrt(-a)*arctan(2*sqrt(x^3 + 1)*sqrt(a*x)*sqrt(-a)*x/(2*a*x^3 + a))]

giac [B] time = 0.19, size = 35, normalized size = 1.52

$$\frac{2a^{\frac{5}{2}} \log\left(-\sqrt{ax}a^{\frac{3}{2}}x + \sqrt{a^4x^3 + a^4}\right)}{3|a|^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2), x, algorithm="giac")

[Out] -2/3*a^(5/2)*log(-sqrt(a*x)*a^(3/2)*x + sqrt(a^4*x^3 + a^4))/abs(a)^2

maple [C] time = 0.14, size = 321, normalized size = 13.96

$$\frac{4\sqrt{ax}\sqrt{x^3+1}(1+i\sqrt{3})\sqrt{\frac{(3+i\sqrt{3})x}{(1+i\sqrt{3})(x+1)}}(x+1)^2\sqrt{\frac{2x+i\sqrt{3}-1}{(i\sqrt{3}-1)(x+1)}}\sqrt{\frac{-2x+i\sqrt{3}+1}{(1+i\sqrt{3})(x+1)}}\left(\operatorname{EllipticF}\left(\sqrt{\frac{(3+i\sqrt{3})x}{(1+i\sqrt{3})(x+1)}}, \sqrt{\frac{(-3+i\sqrt{3})(1+i\sqrt{3})}{(i\sqrt{3}-1)(3+i\sqrt{3})}}\right) - \operatorname{EllipticPi}\left(\sqrt{\frac{(3+i\sqrt{3})x}{(1+i\sqrt{3})(x+1)}}, \frac{1+i\sqrt{3}}{3+i\sqrt{3}}, \sqrt{\frac{(-3+i\sqrt{3})(1+i\sqrt{3})}{(i\sqrt{3}-1)(3+i\sqrt{3})}}\right)\right)}{\sqrt{(x^3+1)ax}(3+i\sqrt{3})\sqrt{-(x+1)(2x+i\sqrt{3}-1)(-2x+i\sqrt{3}+1)ax}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x)^(1/2)/(x^3+1)^(1/2), x)

[Out] -4*(a*x)^(1/2)*(x^3+1)^(1/2)*a*(1+I*3^(1/2))*((3+I*3^(1/2))/(1+I*3^(1/2)))/(x+1)*x)^(1/2)*(x+1)^2*((2*x+I*3^(1/2)-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((-2*x+I*3^(1/2)+1)/(1+I*3^(1/2))/(x+1))^(1/2)*(EllipticF(((3+I*3^(1/2))/(1+I*3^(1/2)))/(x+1)*x)^(1/2), ((-3+I*3^(1/2))*(1+I*3^(1/2)))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))-EllipticPi(((3+I*3^(1/2))/(1+I*3^(1/2)))/(x+1)*x)^(1/2), (1+I*3^(1/2))/(3+I*3^(1/2)), ((-3+I*3^(1/2))*(1+I*3^(1/2)))/(I*3^(1/2)-1)/(3+I*3^(1/2)))^(1/2))/((x^3+1)*a*x)^(1/2)/(3+I*3^(1/2))/(-(x+1)*(2*x+I*3^(1/2)-1)*(-2*x+I*3^(1/2)+1)*a*x)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax}}{\sqrt{x^3 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x)/sqrt(x^3 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{ax}}{\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)

[Out] int((a*x)^(1/2)/(x^3 + 1)^(1/2), x)

sympy [A] time = 1.15, size = 14, normalized size = 0.61

$$\frac{2\sqrt{a} \operatorname{asinh}\left(x^{\frac{3}{2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x)**(1/2)/(x**3+1)**(1/2),x)

[Out] 2*sqrt(a)*asinh(x**(3/2))/3

$$3.173 \quad \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx$$

Optimal. Leaf size=24

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {15, 266, 63, 207}

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a/x^2]/Sqrt[1 + x^3], x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{1+x^3}} dx &= \left(\sqrt{\frac{a}{x^2}} x\right) \int \frac{1}{x\sqrt{1+x^3}} dx \\ &= \frac{1}{3} \left(\sqrt{\frac{a}{x^2}} x\right) \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^3\right) \\ &= \frac{1}{3} \left(2\sqrt{\frac{a}{x^2}} x\right) \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^3}\right) \\ &= -\frac{2}{3} \sqrt{\frac{a}{x^2}} x \tanh^{-1}\left(\sqrt{1+x^3}\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a/x^2]/Sqrt[1 + x^3], x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3

IntegrateAlgebraic [A] time = 0.05, size = 24, normalized size = 1.00

$$-\frac{2}{3}x\sqrt{\frac{a}{x^2}} \tanh^{-1}\left(\sqrt{x^3+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a/x^2]/Sqrt[1 + x^3], x]

[Out] (-2*Sqrt[a/x^2]*x*ArcTanh[Sqrt[1 + x^3]])/3

fricas [A] time = 0.79, size = 68, normalized size = 2.83

$$\left[\frac{1}{3}x\sqrt{\frac{a}{x^2}} \log\left(\frac{x^3 - 2\sqrt{x^3+1} + 2}{x^3}\right), \frac{2}{3}\sqrt{-a} \arctan\left(\frac{\sqrt{x^3+1}\sqrt{-a}x\sqrt{\frac{a}{x^2}}}{ax^3+a}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2), x, algorithm="fricas")

[Out] [1/3*x*sqrt(a/x^2)*log((x^3 - 2*sqrt(x^3 + 1) + 2)/x^3), 2/3*sqrt(-a)*arctan(sqrt(x^3 + 1)*sqrt(-a)*x*sqrt(a/x^2)/(a*x^3 + a))]

giac [A] time = 0.20, size = 31, normalized size = 1.29

$$-\frac{1}{3}\sqrt{a}\left(\log\left(\sqrt{x^3+1}+1\right)-\log\left(\left|\sqrt{x^3+1}-1\right|\right)\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2), x, algorithm="giac")

[Out] -1/3*sqrt(a)*(log(sqrt(x^3 + 1) + 1) - log(abs(sqrt(x^3 + 1) - 1)))*sgn(x)

maple [A] time = 0.01, size = 19, normalized size = 0.79

$$\frac{2\sqrt{\frac{a}{x^2}}x \operatorname{arctanh}\left(\sqrt{x^3+1}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^2)^(1/2)/(x^3+1)^(1/2), x)

[Out] -2/3*x*arctanh((x^3+1)^(1/2))*(a/x^2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x^2)^(1/2)/(x^3+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a/x^2)/sqrt(x^3 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{x^3+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2),x)

[Out] int((a/x^2)^(1/2)/(x^3 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a}{x^2}}}{\sqrt{(x+1)(x^2-x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a/x**2)**(1/2)/(x**3+1)**(1/2),x)

[Out] Integral(sqrt(a/x**2)/sqrt((x + 1)*(x**2 - x + 1)), x)

$$3.174 \quad \int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}} \right) dx$$

Optimal. Leaf size=34

$$\frac{2x^{1-n}\sqrt{x^n+1}\sqrt{ax^{2n}}}{n+2}$$

Rubi [C] time = 0.03, antiderivative size = 80, normalized size of antiderivative = 2.35, number of steps used = 5, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {15, 364, 245}

$$\frac{2x^{1-n}\sqrt{ax^{2n}} {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; 1 + \frac{1}{n}; -x^n\right)}{n+2} + \frac{x\sqrt{ax^{2n}} {}_2F_1\left(\frac{1}{2}, 1 + \frac{1}{n}; 2 + \frac{1}{n}; -x^n\right)}{n+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1 + x^n]), x]

[Out] (x*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, 1 + n^(-1), 2 + n^(-1), -x^n])/(1 + n) + (2*x^(1 - n)*Sqrt[a*x^(2*n)]*Hypergeometric2F1[1/2, n^(-1), 1 + n^(-1), -x^n])/(2 + n)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}} \right) dx &= \frac{2 \int \frac{x^{-n}\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx}{2+n} + \int \frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} dx \\ &= \left(x^{-n}\sqrt{ax^{2n}} \right) \int \frac{x^n}{\sqrt{1+x^n}} dx + \frac{\left(2x^{-n}\sqrt{ax^{2n}} \right) \int \frac{1}{\sqrt{1+x^n}} dx}{2+n} \\ &= \frac{x\sqrt{ax^{2n}} {}_2F_1\left(\frac{1}{2}, 1 + \frac{1}{n}; 2 + \frac{1}{n}; -x^n\right)}{1+n} + \frac{2x^{1-n}\sqrt{ax^{2n}} {}_2F_1\left(\frac{1}{2}, \frac{1}{n}; 1 + \frac{1}{n}; -x^n\right)}{2+n} \end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 0.97

$$\frac{2ax^{n+1}\sqrt{x^n+1}}{(n+2)\sqrt{ax^{2n}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1 + x^n]), x]

[Out] (2*a*x^(1 + n)*Sqrt[1 + x^n])/((2 + n)*Sqrt[a*x^(2*n)])

IntegrateAlgebraic [F] time = 1.67, size = 0, normalized size = 0.00

$$\int \left(\frac{\sqrt{ax^{2n}}}{\sqrt{1+x^n}} + \frac{2x^{-n}\sqrt{ax^{2n}}}{(2+n)\sqrt{1+x^n}} \right) dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1 + x^n]), x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a*x^(2*n)]/Sqrt[1 + x^n] + (2*Sqrt[a*x^(2*n)])/((2 + n)*x^n*Sqrt[1 + x^n]), x]

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)^(1/2), x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (has polynomial part)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^{2n}}}{\sqrt{x^n+1}} + \frac{2\sqrt{ax^{2n}}}{(n+2)\sqrt{x^n+1}x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(a*x^(2*n))/sqrt(x^n + 1) + 2*sqrt(a*x^(2*n))/((n + 2)*sqrt(x^n + 1)*x^n), x)

maple [A] time = 0.05, size = 30, normalized size = 0.88

$$\frac{2\sqrt{x^n+1}\sqrt{ax^{2n}}xx^{-n}}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^(2*n))^(1/2)/(x^n+1)^(1/2)+2*(a*x^(2*n))^(1/2)/(n+2)/(x^n)/(x^n+1)^(1/2), x)

[Out] 2*x*(x^n+1)^(1/2)/(n+2)*(a*(x^n)^2)^(1/2)/(x^n)

maxima [A] time = 1.51, size = 18, normalized size = 0.53

$$\frac{2\sqrt{a}\sqrt{x^n+1}x}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^(2*n))^(1/2)/(1+x^n)^(1/2)+2*(a*x^(2*n))^(1/2)/(2+n)/(x^n)/(1+x^n)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(a)*sqrt(x^n + 1)*x/(n + 2)

mupad [B] time = 2.89, size = 43, normalized size = 1.26

$$\frac{\sqrt{a}x^{2n}\left(\frac{2x}{n+2} + \frac{2x^{n+1}}{n+2}\right)}{x^n\sqrt{x^n+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^(2*n))^(1/2)/(x^n + 1)^(1/2) + (2*(a*x^(2*n))^(1/2))/(x^n*(x^n + 1)^(1/2)*(n + 2)),x)

[Out] ((a*x^(2*n))^(1/2)*((2*x)/(n + 2) + (2*x^(n + 1))/(n + 2)))/(x^n*(x^n + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{2\sqrt{ax^{2n}}}{\sqrt{x^n+1}} dx + \int \frac{n\sqrt{ax^{2n}}}{\sqrt{x^n+1}} dx + \int \frac{2x^{-n}\sqrt{ax^{2n}}}{\sqrt{x^n+1}} dx}{n+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**(2*n))**(1/2)/(1+x**n)**(1/2)+2*(a*x**(2*n))**(1/2)/(2+n)/(x**n)/(1+x**n)**(1/2),x)

[Out] (Integral(2*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(n*sqrt(a*x**(2*n))/sqrt(x**n + 1), x) + Integral(2*x**(-n)*sqrt(a*x**(2*n))/sqrt(x**n + 1), x))/(n + 2)

3.175 $\int (ax^m)^r dx$

Optimal. Leaf size=16

$$\frac{x(ax^m)^r}{mr+1}$$

Rubi [A] time = 0.00, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {15, 30}

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m)^r, x]

[Out] (x*(a*x^m)^r)/(1 + m*r)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] := Simp[x^(m+1)/(m+1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (ax^m)^r dx &= (x^{-mr} (ax^m)^r) \int x^{mr} dx \\ &= \frac{x(ax^m)^r}{1+mr} \end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$\frac{x(ax^m)^r}{mr+1}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m)^r, x]

[Out] (x*(a*x^m)^r)/(1 + m*r)

IntegrateAlgebraic [F] time = 0.02, size = 0, normalized size = 0.00

$$\int (ax^m)^r dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^m)^r, x]

[Out] Defer[IntegrateAlgebraic] [(a*x^m)^r, x]

fricas [A] time = 0.66, size = 20, normalized size = 1.25

$$\frac{x e^{(mr \log(x) + r \log(a))}}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r,x, algorithm="fricas")

[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)

giac [A] time = 0.21, size = 20, normalized size = 1.25

$$\frac{x e^{(mr \log(x) + r \log(a))}}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r,x, algorithm="giac")

[Out] x*e^(m*r*log(x) + r*log(a))/(m*r + 1)

maple [A] time = 0.00, size = 17, normalized size = 1.06

$$\frac{x (a x^m)^r}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r,x)

[Out] x*(a*x^m)^r/(m*r+1)

maxima [A] time = 1.48, size = 17, normalized size = 1.06

$$\frac{a^r x (x^m)^r}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r,x, algorithm="maxima")

[Out] a^r*x*(x^m)^r/(m*r + 1)

mupad [B] time = 3.40, size = 16, normalized size = 1.00

$$\frac{x (a x^m)^r}{mr + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r,x)

[Out] (x*(a*x^m)^r)/(m*r + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^r x (x^m)^r}{mr+1} & \text{for } m \neq -\frac{1}{r} \\ \int \left(a x^{-\frac{1}{r}} \right)^r dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**m)**r,x)

[Out] Piecewise((a**r*x*(x**m)**r/(m*r + 1), Ne(m, -1/r)), (Integral((a*x**(-1/r))**r, x), True))

3.176 $\int (ax^m)^r (bx^n)^s dx$

Optimal. Leaf size=26

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m)^r*(b*x^n)^s,x]

[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (ax^m)^r (bx^n)^s dx &= (x^{-mr} (ax^m)^r) \int x^{mr} (bx^n)^s dx \\ &= (x^{-mr-ns} (ax^m)^r (bx^n)^s) \int x^{mr+ns} dx \\ &= \frac{x (ax^m)^r (bx^n)^s}{1 + mr + ns} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s}{mr + ns + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m)^r*(b*x^n)^s,x]

[Out] (x*(a*x^m)^r*(b*x^n)^s)/(1 + m*r + n*s)

IntegrateAlgebraic [F] time = 0.04, size = 0, normalized size = 0.00

$$\int (ax^m)^r (bx^n)^s dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^m)^r*(b*x^n)^s,x]

[Out] Defer[IntegrateAlgebraic] [(a*x^m)^r*(b*x^n)^s, x]

fricas [A] time = 0.65, size = 32, normalized size = 1.23

$$\frac{x e^{(mr \log(x) + ns \log(x) + r \log(a) + s \log(b))}}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="fricas")

[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)

giac [A] time = 0.24, size = 32, normalized size = 1.23

$$\frac{x e^{(mr \log(x) + ns \log(x) + r \log(a) + s \log(b))}}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="giac")

[Out] x*e^(m*r*log(x) + n*s*log(x) + r*log(a) + s*log(b))/(m*r + n*s + 1)

maple [A] time = 0.00, size = 27, normalized size = 1.04

$$\frac{x (a x^m)^r (b x^n)^s}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r*(b*x^n)^s,x)

[Out] x*(a*x^m)^r*(b*x^n)^s/(m*r+n*s+1)

maxima [A] time = 1.08, size = 32, normalized size = 1.23

$$\frac{a^r b^s x e^{(r \log(x^m) + s \log(x^n))}}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s,x, algorithm="maxima")

[Out] a^r*b^s*x*e^(r*log(x^m) + s*log(x^n))/(m*r + n*s + 1)

mupad [B] time = 2.92, size = 26, normalized size = 1.00

$$\frac{x (a x^m)^r (b x^n)^s}{mr + ns + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r*(b*x^n)^s,x)

[Out] (x*(a*x^m)^r*(b*x^n)^s)/(m*r + n*s + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{a^r b^s x (x^m)^r (x^n)^s}{mr + ns + 1} & \text{for } m \neq -\frac{ns+1}{r} \\ \int (bx^n)^s \left(ax^{-\frac{1}{r}} x^{-\frac{ns}{r}}\right)^r dx & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**m)**r*(b*x**n)**s,x)

[Out] Piecewise((a**r*b**s*x*(x**m)**r*(x**n)**s/(m*r + n*s + 1), Ne(m, -(n*s + 1)/r)), (Integral((b*x**n)**s*(a*x**(-1/r)*x**(-n*s/r))**r, x), True))

3.177 $\int (ax^m)^r (bx^n)^s (cx^p)^t dx$

Optimal. Leaf size=36

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {15, 30}

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

[In] Int[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]

[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int (ax^m)^r (bx^n)^s (cx^p)^t dx &= (x^{-mr} (ax^m)^r) \int x^{mr} (bx^n)^s (cx^p)^t dx \\ &= (x^{-mr-ns} (ax^m)^r (bx^n)^s) \int x^{mr+ns} (cx^p)^t dx \\ &= (x^{-mr-ns-pt} (ax^m)^r (bx^n)^s (cx^p)^t) \int x^{mr+ns+pt} dx \\ &= \frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{1 + mr + ns + pt} \end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{x (ax^m)^r (bx^n)^s (cx^p)^t}{mr + ns + pt + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]

[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(1 + m*r + n*s + p*t)

IntegrateAlgebraic [F] time = 0.07, size = 0, normalized size = 0.00

$$\int (ax^m)^r (bx^n)^s (cx^p)^t dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x]

[Out] Defer[IntegrateAlgebraic] [(a*x^m)^r*(b*x^n)^s*(c*x^p)^t, x]

fricas [A] time = 0.52, size = 44, normalized size = 1.22

$$\frac{x e^{(mr \log(x) + ns \log(x) + pt \log(x) + r \log(a) + s \log(b) + t \log(c))}}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="fricas")

[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c)) / (m*r + n*s + p*t + 1)

giac [A] time = 0.24, size = 44, normalized size = 1.22

$$\frac{x e^{(mr \log(x) + ns \log(x) + pt \log(x) + r \log(a) + s \log(b) + t \log(c))}}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="giac")

[Out] x*e^(m*r*log(x) + n*s*log(x) + p*t*log(x) + r*log(a) + s*log(b) + t*log(c)) / (m*r + n*s + p*t + 1)

maple [A] time = 0.00, size = 37, normalized size = 1.03

$$\frac{x (a x^m)^r (b x^n)^s (c x^p)^t}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)

[Out] x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t/(m*r+n*s+p*t+1)

maxima [A] time = 1.27, size = 44, normalized size = 1.22

$$\frac{a^r b^s c^t x e^{(r \log(x^m) + s \log(x^n) + t \log(x^p))}}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x, algorithm="maxima")

[Out] a^r*b^s*c^t*x*e^(r*log(x^m) + s*log(x^n) + t*log(x^p))/(m*r + n*s + p*t + 1)

mupad [B] time = 3.10, size = 36, normalized size = 1.00

$$\frac{x (a x^m)^r (b x^n)^s (c x^p)^t}{mr + ns + pt + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^m)^r*(b*x^n)^s*(c*x^p)^t,x)

[Out] (x*(a*x^m)^r*(b*x^n)^s*(c*x^p)^t)/(m*r + n*s + p*t + 1)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**m)**r*(b*x**n)**s*(c*x**p)**t,x)

[Out] Timed out

$$3.178 \quad \int \frac{x^2}{\sqrt{a+bx} + \sqrt{c+bx}} dx$$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} - \frac{2(bx+c)^{7/2}}{7b^3(a-c)} + \frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Rubi [A] time = 0.13, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2104, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{2c^2(bx+c)^{3/2}}{3b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} - \frac{2(bx+c)^{7/2}}{7b^3(a-c)} + \frac{4c(bx+c)^{5/2}}{5b^3(a-c)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(a - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(a - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(a - c)) - (2*c^2*(c + b*x)^(3/2))/(3*b^3*(a - c)) + (4*c*(c + b*x)^(5/2))/(5*b^3*(a - c)) - (2*(c + b*x)^(7/2))/(7*b^3*(a - c))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2104

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx} + \sqrt{c+bx}} dx &= -\frac{b \int x^2 \sqrt{a+bx} dx}{-ab+bc} + \frac{b \int x^2 \sqrt{c+bx} dx}{-ab+bc} \\ &= -\frac{b \int \left(\frac{a^2 \sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2} \right) dx}{-ab+bc} + \frac{b \int \left(\frac{c^2 \sqrt{c+bx}}{b^2} - \frac{2c(c+bx)^{3/2}}{b^2} + \frac{(c+bx)^{5/2}}{b^2} \right) dx}{-ab+bc} \\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(a-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(a-c)} + \frac{2(a+bx)^{7/2}}{7b^3(a-c)} - \frac{2c^2(c+bx)^{3/2}}{3b^3(a-c)} + \frac{4c(c+bx)^{5/2}}{5b^3(a-c)} - \end{aligned}$$

Mathematica [A] time = 0.17, size = 140, normalized size = 0.95

$$\frac{2(8a^3\sqrt{a+bx} - 4a^2bx\sqrt{a+bx} + 15b^3x^3(\sqrt{a+bx} - \sqrt{bx+c}) + 3ab^2x^2\sqrt{a+bx} - 3b^2cx^2\sqrt{bx+c} - 8c^3\sqrt{bx+c} + 4bc^2x\sqrt{bx+c})}{105b^3(a-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] $(2*(8*a^3*\text{Sqrt}[a + b*x] - 4*a^2*b*x*\text{Sqrt}[a + b*x] + 3*a*b^2*x^2*\text{Sqrt}[a + b*x] - 8*c^3*\text{Sqrt}[c + b*x] + 4*b*c^2*x*\text{Sqrt}[c + b*x] - 3*b^2*c*x^2*\text{Sqrt}[c + b*x] + 15*b^3*x^3*(\text{Sqrt}[a + b*x] - \text{Sqrt}[c + b*x])))/(105*b^3*(a - c))$

IntegrateAlgebraic [A] time = 0.65, size = 157, normalized size = 1.07

$$\frac{2\sqrt{a+bx}(8a^3-4a^2(bx+c)+4a^2c+3a(bx+c)^2-6ac(bx+c)+3ac^2+45c^2(bx+c)+15(bx+c)^3-45c(bx+c)^2-15c^3)}{105b^3(a-c)} - \frac{2(35c^2(bx+c)^{3/2}+15(bx+c)^{7/2}-42c(bx+c)^{5/2})}{105b^3(a-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] $(2*\text{Sqrt}[a + b*x]*(8*a^3 + 4*a^2*c + 3*a*c^2 - 15*c^3 - 4*a^2*(c + b*x) - 6*a*c*(c + b*x) + 45*c^2*(c + b*x) + 3*a*(c + b*x)^2 - 45*c*(c + b*x)^2 + 15*(c + b*x)^3)/(105*b^3*(a - c)) - (2*(35*c^2*(c + b*x)^{(3/2)} - 42*c*(c + b*x)^{(5/2)} + 15*(c + b*x)^{(7/2)}))/(105*b^3*(a - c))$

fricas [A] time = 0.58, size = 94, normalized size = 0.64

$$\frac{2\left(\left(15b^3x^3 + 3ab^2x^2 - 4a^2bx + 8a^3\right)\sqrt{bx+a} - \left(15b^3x^3 + 3b^2cx^2 - 4bc^2x + 8c^3\right)\sqrt{bx+c}\right)}{105\left(ab^3 - b^3c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")

[Out] $2/105*((15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*\text{sqrt}(b*x + a) - (15*b^3*x^3 + 3*b^2*c*x^2 - 4*b*c^2*x + 8*c^3)*\text{sqrt}(b*x + c))/(a*b^3 - b^3*c)$

giac [B] time = 0.27, size = 390, normalized size = 2.65

$$\frac{2}{105} \left(\left(\frac{5(a^2b^3 - 2ab^2c + b^3c)(bx+a)}{a^3b^3 - 3a^2b^2c + 3ab^2c^2 - b^3c^3} - \frac{15a^3b^3 - 31a^2b^2c + 17ab^2c^2 - b^3c^3}{a^3b^3 - 3a^2b^2c + 3ab^2c^2 - b^3c^3} + \frac{45a^4b^3 - 96a^3b^2c + 53a^2b^2c^2 + 2ab^2c^3 - 4b^3c^4}{a^3b^3 - 3a^2b^2c + 3ab^2c^2 - b^3c^3} \right) (bx+a) - \frac{15a^4b^3 - 33a^3b^2c + 17a^2b^2c^2 - 3a^2b^2c^3 + 12ab^2c^4 - 8b^3c^5}{a^3b^3 - 3a^2b^2c + 3ab^2c^2 - b^3c^3} \right) \sqrt{bx+c} + \frac{2(15(bx+a)^2 - 42(bx+a)^{3/2}a + 35(bx+a)^{5/2}a^2)}{105(ab^3 - b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")

[Out] $-2/105*((3*(b*x + a)*(5*(a^2*b^9 - 2*a*b^9*c + b^9*c^2)*(b*x + a)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3) - (15*a^3*b^9 - 31*a^2*b^9*c + 17*a*b^9*c^2 - b^9*c^3)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3)) + (45*a^4*b^9 - 96*a^3*b^9*c + 53*a^2*b^9*c^2 + 2*a*b^9*c^3 - 4*b^9*c^4)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*(b*x + a) - (15*a^5*b^9 - 33*a^4*b^9*c + 17*a^3*b^9*c^2 - 3*a^2*b^9*c^3 + 12*a*b^9*c^4 - 8*b^9*c^5)/(a^3*b^12 - 3*a^2*b^12*c + 3*a*b^12*c^2 - b^12*c^3))*\text{sqrt}(b*x + c) + 2/105*(15*(b*x + a)^{(7/2)} - 42*(b*x + a)^{(5/2)}*a + 35*(b*x + a)^{(3/2)}*a^2)/(a*b^3 - b^3*c)$

maple [A] time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^{\frac{3}{2}}a^2}{3} - \frac{4(bx+a)^{\frac{5}{2}}a}{5} + \frac{2(bx+a)^{\frac{7}{2}}}{7}}{(a-c)b^3} - \frac{2\left(\frac{(bx+c)^{\frac{3}{2}}c^2}{3} - \frac{2(bx+c)^{\frac{5}{2}}c}{5} + \frac{(bx+c)^{\frac{7}{2}}}{7}\right)}{(a-c)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)

[Out] $2/(a-c)/b^3*(1/7*(b*x+a)^{(7/2)}-2/5*(b*x+a)^{(5/2)}*a+1/3*a^2*(b*x+a)^{(3/2)})-2/(a-c)/b^3*(1/7*(b*x+c)^{(7/2)}-2/5*(b*x+c)^{(5/2)}*c+1/3*c^2*(b*x+c)^{(3/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx+a} + \sqrt{bx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c)), x)

mupad [B] time = 2.95, size = 179, normalized size = 1.22

$$\frac{2x^3\sqrt{a+bx}}{7(a-c)} - \frac{2x^3\sqrt{c+bx}}{7(a-c)} + \frac{16a^3\sqrt{a+bx}}{105b^3(a-c)} - \frac{16c^3\sqrt{c+bx}}{105b^3(a-c)} + \frac{2ax^2\sqrt{a+bx}}{35b(a-c)} - \frac{8a^2x\sqrt{a+bx}}{105b^2(a-c)} - \frac{2cx^2\sqrt{c+bx}}{35b(a-c)} + \frac{8c^2x\sqrt{c+bx}}{105b^2(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)

[Out] (2*x^3*(a + b*x)^(1/2))/(7*(a - c)) - (2*x^3*(c + b*x)^(1/2))/(7*(a - c)) + (16*a^3*(a + b*x)^(1/2))/(105*b^3*(a - c)) - (16*c^3*(c + b*x)^(1/2))/(105*b^3*(a - c)) + (2*a*x^2*(a + b*x)^(1/2))/(35*b*(a - c)) - (8*a^2*x*(a + b*x)^(1/2))/(105*b^2*(a - c)) - (2*c*x^2*(c + b*x)^(1/2))/(35*b*(a - c)) + (8*c^2*x*(c + b*x)^(1/2))/(105*b^2*(a - c))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx} + \sqrt{bx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)

[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c)), x)

$$3.179 \quad \int \frac{x}{\sqrt{a+bx} + \sqrt{c+bx}} dx$$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(a-c)} - \frac{2(bx+c)^{5/2}}{5b^2(a-c)} + \frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2104, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(a-c)} - \frac{2(bx+c)^{5/2}}{5b^2(a-c)} + \frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a - c))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2104

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx} + \sqrt{c+bx}} dx &= -\frac{b \int x\sqrt{a+bx} dx}{-ab+bc} + \frac{b \int x\sqrt{c+bx} dx}{-ab+bc} \\ &= -\frac{b \int \left(-\frac{a\sqrt{a+bx}}{b} + \frac{(a+bx)^{3/2}}{b}\right) dx}{-ab+bc} + \frac{b \int \left(-\frac{c\sqrt{c+bx}}{b} + \frac{(c+bx)^{3/2}}{b}\right) dx}{-ab+bc} \\ &= -\frac{2a(a+bx)^{3/2}}{3b^2(a-c)} + \frac{2(a+bx)^{5/2}}{5b^2(a-c)} + \frac{2c(c+bx)^{3/2}}{3b^2(a-c)} - \frac{2(c+bx)^{5/2}}{5b^2(a-c)} \end{aligned}$$

Mathematica [A] time = 0.10, size = 95, normalized size = 1.00

$$\frac{2(a+bx)^{5/2}}{5b^2(a-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(a-c)} - \frac{2(bx+c)^{5/2}}{5b^2(a-c)} + \frac{2c(bx+c)^{3/2}}{3b^2(a-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(a - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(a - c)) + (2*c*(c + b*x)^(3/2))/(3*b^2*(a - c)) - (2*(c + b*x)^(5/2))/(5*b^2*(a - c))

IntegrateAlgebraic [A] time = 0.57, size = 101, normalized size = 1.06

$$\frac{2(5c(bx+c)^{3/2} - 3(bx+c)^{5/2})}{15b^2(a-c)} - \frac{2\sqrt{a+bx}(2a^2 - a(bx+c) + ac - 3(bx+c)^2 + 6c(bx+c) - 3c^2)}{15b^2(a-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x]),x]

[Out] (-2*Sqrt[a + b*x]*(2*a^2 + a*c - 3*c^2 - a*(c + b*x) + 6*c*(c + b*x) - 3*(c + b*x)^2))/(15*b^2*(a - c)) + (2*(5*c*(c + b*x)^(3/2) - 3*(c + b*x)^(5/2)))/(15*b^2*(a - c))

fricas [A] time = 0.54, size = 70, normalized size = 0.74

$$\frac{2\left(\left(3b^2x^2 + abx - 2a^2\right)\sqrt{bx+a} - \left(3b^2x^2 + bcx - 2c^2\right)\sqrt{bx+c}\right)}{15\left(ab^2 - b^2c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")

[Out] 2/15*((3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a) - (3*b^2*x^2 + b*c*x - 2*c^2)*sqrt(b*x + c))/(a*b^2 - b^2*c)

giac [B] time = 0.21, size = 206, normalized size = 2.17

$$\frac{2\left(\left(bx+a\right)\left(\frac{3(ab^2-b^2c)(bx+a)}{a^2b^3-2ab^3c+b^3c^2} - \frac{6a^2b^2-7ab^2c+b^2c^2}{a^2b^3-2ab^3c+b^3c^2}\right) + \frac{3a^3b^2-4a^2b^2c-ab^2c^2+2b^2c^3}{a^2b^3-2ab^3c+b^3c^2}\right)\sqrt{bx+c} - \frac{3(bx+a)^{\frac{5}{2}}-5(bx+a)^{\frac{3}{2}}a}{ab-bc}}{15b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")

[Out] -2/15*(((b*x + a)*(3*(a*b^2 - b^2*c)*(b*x + a)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2) - (6*a^2*b^2 - 7*a*b^2*c + b^2*c^2)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)) + (3*a^3*b^2 - 4*a^2*b^2*c - a*b^2*c^2 + 2*b^2*c^3)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2))*sqrt(b*x + c) - (3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(a*b - b*c))/b

maple [A] time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3} + \frac{2(bx+a)^{\frac{5}{2}}}{5}}{(a-c)b^2} - \frac{2\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3} + \frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a-c)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)

[Out] 2/(a-c)/b^2*(1/5*(b*x+a)^(5/2)-1/3*a*(b*x+a)^(3/2))-2/(a-c)/b^2*(1/5*(b*x+c)^(5/2)-1/3*(b*x+c)^(3/2)*c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx+a} + \sqrt{bx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c)), x)

mupad [B] time = 2.70, size = 129, normalized size = 1.36

$$\frac{2x^2\sqrt{a+bx}}{5(a-c)} - \frac{2x^2\sqrt{c+bx}}{5(a-c)} - \frac{4a^2\sqrt{a+bx}}{15b^2(a-c)} + \frac{4c^2\sqrt{c+bx}}{15b^2(a-c)} + \frac{2ax\sqrt{a+bx}}{15b(a-c)} - \frac{2cx\sqrt{c+bx}}{15b(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)

[Out] (2*x^2*(a + b*x)^(1/2))/(5*(a - c)) - (2*x^2*(c + b*x)^(1/2))/(5*(a - c)) - (4*a^2*(a + b*x)^(1/2))/(15*b^2*(a - c)) + (4*c^2*(c + b*x)^(1/2))/(15*b^2*(a - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(a - c)) - (2*c*x*(c + b*x)^(1/2))/(15*b*(a - c))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+bx} + \sqrt{bx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c)), x)

$$3.180 \quad \int \frac{1}{\sqrt{a+bx} + \sqrt{c+bx}} dx$$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Rubi [A] time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {6689}

$$\frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1), x]

[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))

Rule 6689

Int[(u_)*((e_)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+bx} + \sqrt{c+bx}} dx &= \frac{\int (\sqrt{a+bx} - \sqrt{c+bx}) dx}{a-c} \\ &= \frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(c+bx)^{3/2}}{3b(a-c)} \end{aligned}$$

Mathematica [A] time = 0.05, size = 35, normalized size = 0.74

$$\frac{2 \left((a+bx)^{3/2} - (bx+c)^{3/2} \right)}{3b(a-c)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1), x]

[Out] (2*((a + b*x)^(3/2) - (c + b*x)^(3/2)))/(3*b*(a - c))

IntegrateAlgebraic [A] time = 0.35, size = 47, normalized size = 1.00

$$\frac{2(a+bx)^{3/2}}{3b(a-c)} - \frac{2(bx+c)^{3/2}}{3b(a-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-1), x]

[Out] (2*(a + b*x)^(3/2))/(3*b*(a - c)) - (2*(c + b*x)^(3/2))/(3*b*(a - c))

fricas [A] time = 0.71, size = 29, normalized size = 0.62

$$\frac{2\left((bx+a)^{\frac{3}{2}} - (bx+c)^{\frac{3}{2}}\right)}{3(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")

[Out] 2/3*((b*x + a)^(3/2) - (b*x + c)^(3/2))/(a*b - b*c)

giac [A] time = 0.21, size = 75, normalized size = 1.60

$$-\frac{2}{3}\sqrt{bx+c}\left(\frac{(bx+a)b}{ab^2-b^2c} - \frac{ab-bc}{ab^2-b^2c}\right) + \frac{2(bx+a)^{\frac{3}{2}}}{3(ab-bc)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")

[Out] -2/3*sqrt(b*x + c)*((b*x + a)*b/(a*b^2 - b^2*c) - (a*b - b*c)/(a*b^2 - b^2*c)) + 2/3*(b*x + a)^(3/2)/(a*b - b*c)

maple [A] time = 0.00, size = 40, normalized size = 0.85

$$\frac{2(bx+a)^{\frac{3}{2}}}{3(a-c)b} - \frac{2(bx+c)^{\frac{3}{2}}}{3(a-c)b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)

[Out] 2/3*(b*x+a)^(3/2)/b/(a-c)-2/3*(b*x+c)^(3/2)/b/(a-c)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a} + \sqrt{bx+c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x + a) + sqrt(b*x + c)), x)

mupad [B] time = 2.71, size = 79, normalized size = 1.68

$$\frac{2x\sqrt{a+bx}}{3(a-c)} - \frac{2x\sqrt{c+bx}}{3(a-c)} + \frac{2a\sqrt{a+bx}}{3b(a-c)} - \frac{2c\sqrt{c+bx}}{3b(a-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2)),x)

[Out] (2*x*(a + b*x)^(1/2))/(3*(a - c)) - (2*x*(c + b*x)^(1/2))/(3*(a - c)) + (2*a*(a + b*x)^(1/2))/(3*b*(a - c)) - (2*c*(c + b*x)^(1/2))/(3*b*(a - c))

sympy [A] time = 0.71, size = 136, normalized size = 2.89

$$\begin{cases} \frac{2a}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{4bx}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2c}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} + \frac{2\sqrt{a+bx}\sqrt{bx+c}}{3b\sqrt{a+bx}+3b\sqrt{bx+c}} & \text{for } b \neq 0 \\ \frac{x}{\sqrt{a}+\sqrt{c}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)
```

```
[Out] Piecewise((2*a/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 4*b*x/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 2*c/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)) + 2*sqrt(a + b*x)*sqrt(b*x + c)/(3*b*sqrt(a + b*x) + 3*b*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c)), True))
```

$$3.181 \quad \int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})} dx$$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{bx+c}}{a-c} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c} + \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2104, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{bx+c}}{a-c} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c} + \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{a-c}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]

[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c) + (2*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2104

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})} dx &= -\frac{b \int \frac{\sqrt{a+bx}}{x} dx}{-ab+bc} + \frac{b \int \frac{\sqrt{c+bx}}{x} dx}{-ab+bc} \\
&= \frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{c+bx}}{a-c} + \frac{a \int \frac{1}{x\sqrt{a+bx}} dx}{a-c} - \frac{c \int \frac{1}{x\sqrt{c+bx}} dx}{a-c} \\
&= \frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{c+bx}}{a-c} + \frac{(2a) \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b(a-c)} - \frac{(2c) \operatorname{Subst}\left(\int \frac{1}{-\frac{c}{b} + \frac{x^2}{b}} dx, x, \sqrt{c+bx}\right)}{b(a-c)} \\
&= \frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{c+bx}}{a-c} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a-c} + \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{a-c}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 75, normalized size = 0.77

$$\frac{2\left(\sqrt{a+bx} - \sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \sqrt{bx+c} + \sqrt{c} \tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)\right)}{a-c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]

[Out] (2*(Sqrt[a + b*x] - Sqrt[c + b*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]))/(a - c)

IntegrateAlgebraic [B] time = 0.89, size = 275, normalized size = 2.84

$$\frac{2\sqrt{a+bx}}{a-c} - \frac{2\sqrt{bx+c}}{a-c} - \frac{2\sqrt{-(\sqrt{a} + \sqrt{c})^2} \tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}} - \frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a} - \sqrt{c})(\sqrt{a} + \sqrt{c})} - \frac{2\sqrt{-(\sqrt{a} - \sqrt{c})^2} \tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}} - \frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a} - \sqrt{c})(\sqrt{a} + \sqrt{c})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]

[Out] (2*Sqrt[a + b*x])/(a - c) - (2*Sqrt[c + b*x])/(a - c) - (2*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c])/(Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c]) - (2*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c])/(Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c])

fricas [A] time = 0.61, size = 318, normalized size = 3.28

$$\left[\frac{\sqrt{a} \log\left(\frac{(b+2\sqrt{a}\sqrt{c}+2)}{a-c}\right) + \sqrt{c} \log\left(\frac{(b+2\sqrt{a}\sqrt{c}-2)}{a-c}\right) - 2\sqrt{bx+a} + 2\sqrt{bx+c}}{a-c} - 2\sqrt{c} \arctan\left(\frac{\sqrt{bx+c}}{a-c}\right) + \sqrt{a} \log\left(\frac{(b+2\sqrt{a}\sqrt{c}+2)}{a-c}\right) - 2\sqrt{bx+a} + 2\sqrt{bx+c}}{a-c} - 2\sqrt{a} \arctan\left(\frac{\sqrt{bx+a}}{a-c}\right) - \sqrt{c} \log\left(\frac{(b+2\sqrt{a}\sqrt{c}-2)}{a-c}\right) + 2\sqrt{bx+a} - 2\sqrt{bx+c}}{a-c} - 2\sqrt{a} \arctan\left(\frac{\sqrt{bx+a}}{a-c}\right) - \sqrt{c} \log\left(\frac{(b+2\sqrt{a}\sqrt{c}-2)}{a-c}\right) + \sqrt{bx+a} - \sqrt{bx+c}}{a-c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="fricas")

[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a - c), -(2*sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(b*x + c))/(a - c), (2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a) - 2*sqrt(b*x + c))/(a - c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-c)*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(b*x + a) - sqrt(b*x + c))/(a - c)]

giac [B] time = 0.86, size = 1016, normalized size = 10.47

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")

[Out] $2*a*\arctan(\sqrt{b*x+a}/\sqrt{-a})/(\sqrt{-a}*(a-c)) - 2*(a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + 2*(a*c^2 + \sqrt{a*c})*c^2)*(a-c)^2*\operatorname{sgn}(-a+c) - 2*(a*c^2 + \sqrt{a*c})*a*c*(a-c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2 + c^3)*\sqrt{a*c})*\operatorname{abs}(-a+c)*\operatorname{sgn}(-a+c) - (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*\sqrt{a*c})*\operatorname{abs}(-a+c) - (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 + (a^3*c - a^2*c^2 - a*c^3 + c^4)*\sqrt{a*c})*\operatorname{sgn}(-a+c) + (a^4 - a^3*c - a^2*c^2 + a*c^3)*\sqrt{a*c}*\arctan(-(\sqrt{b*x+a} - \sqrt{b*x+c}))/\sqrt{-(a^2 - c^2 + \sqrt{(a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a-c)})}/(a-c))/((\sqrt{-a}*a^4 - a^4*\sqrt{-c} - 4*\sqrt{-a}*a^3*c + 4*a^3*\sqrt{-c}*c + 6*\sqrt{-a}*a^2*c^2 - 6*a^2*\sqrt{-c}*c^2 - 4*\sqrt{-a}*a*c^3 + 4*a*\sqrt{-c}*c^3 + \sqrt{-a}*c^4 - \sqrt{-c}*c^4)*\operatorname{abs}(-a+c)) + 2*(a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - 2*(a*c^2 + \sqrt{a*c})*c^2)*(a-c)^2*\operatorname{sgn}(-a+c) - 2*(a*c^2 - \sqrt{a*c})*a*c*(a-c)^2 + (a^2*c^2 - 2*a*c^3 + c^4 - (a^2*c - 2*a*c^2 + c^3)*\sqrt{a*c})*\operatorname{abs}(-a+c)*\operatorname{sgn}(-a+c) + (a^3*c - 2*a^2*c^2 + a*c^3 + (a^2*c - 2*a*c^2 + c^3)*\sqrt{a*c})*\operatorname{abs}(-a+c) + (a^4*c - a^3*c^2 - a^2*c^3 + a*c^4 - (a^3*c - a^2*c^2 - a*c^3 + c^4)*\sqrt{a*c})*\operatorname{sgn}(-a+c) + (a^4 - a^3*c - a^2*c^2 + a*c^3)*\sqrt{a*c}*\arctan(-(\sqrt{b*x+a} - \sqrt{b*x+c}))/\sqrt{-(a^2 - c^2 - \sqrt{(a^2 - c^2)^2 - (a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(a-c)})}/(a-c))/((\sqrt{-a}*a^4 - a^4*\sqrt{-c} - 4*\sqrt{-a}*a^3*c + 4*a^3*\sqrt{-c}*c + 6*\sqrt{-a}*a^2*c^2 - 6*a^2*\sqrt{-c}*c^2 - 4*\sqrt{-a}*a*c^3 + 4*a*\sqrt{-c}*c^3 + \sqrt{-a}*c^4 - \sqrt{-c}*c^4)*\operatorname{abs}(-a+c)) + 2*\sqrt{b*x+a}/(a-c) - 2*\sqrt{b*x+c}/(a-c)$

maple [A] time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a}}{a-c} - \frac{-2\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) + 2\sqrt{bx+c}}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)

[Out] $1/(a-c)*(2*(b*x+a)^(1/2)-2*a^(1/2)*\operatorname{arctanh}((b*x+a)^(1/2)/a^(1/2)))-1/(a-c)*(2*(b*x+c)^(1/2)-2*c^(1/2)*\operatorname{arctanh}((b*x+c)^(1/2)/c^(1/2)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{bx+a} + \sqrt{bx+c})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x+a) + sqrt(b*x+c))), x)

mupad [B] time = 18.08, size = 2983, normalized size = 30.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a+b*x)^(1/2) + (c+b*x)^(1/2))),x)

$$\begin{aligned}
& (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i + a^{5/2} \cdot c^2 \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i \\
& + a^3c^3 \cdot (a + bx)^{1/2} \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i + a^{3/2} \cdot c^{5/2} \cdot (a + bx)^{1/2} \\
& \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i + a^{5/2} \cdot c^{3/2} \cdot (a + bx)^{1/2} \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i \\
& - a^2c^2 \cdot (c + bx)^{1/2} \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 4i - a^{3/2} \cdot c^{5/2} \cdot (c + bx)^{1/2} \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i \\
& - a^{5/2} \cdot c^{3/2} \cdot (c + bx)^{1/2} \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i) / (2a^5c^{3/2} - 4a^4c^{5/2} + 2a^{5/2} \cdot c^4 + 2a^3c^{7/2} - 4a^{7/2} \cdot c^3 + 2a^{9/2} \cdot c^2 - 2a^2c^4 \cdot (a + bx)^{1/2} + 4a^3c^3 \cdot (a + bx)^{1/2} - 2a^4c^2 \cdot (a + bx)^{1/2} - 2a^{3/2} \cdot c^{9/2} \cdot (a + bx)^{1/2} + 2a^{5/2} \cdot c^{7/2} \cdot (a + bx)^{1/2} + 2a^{7/2} \cdot c^{5/2} \cdot (a + bx)^{1/2} - 2a^{9/2} \cdot c^{3/2} \cdot (a + bx)^{1/2} + 2a^2c^4 \cdot (c + bx)^{1/2} - 4a^3c^3 \cdot (c + bx)^{1/2} + 2a^4c^2 \cdot (c + bx)^{1/2})) \cdot (a^3c^3 + a^3c - 2a^2c^2)^{1/2} \cdot 2i - c^{3/2} \cdot \log(((a + bx)^{1/2} - a^{1/2}) / ((c + bx)^{1/2} - c^{1/2})) \cdot (a + bx)^{1/2} - c^{3/2} \cdot \log(((a + bx)^{1/2} - a^{1/2}) / ((c + bx)^{1/2} - c^{1/2})) \cdot (c + bx)^{1/2} / (c^{1/2} \cdot (a^{1/2} - c^{1/2})) \cdot (a^{1/2} + c^{1/2})^2 \cdot ((a + bx)^{1/2} + (c + bx)^{1/2} - a^{1/2} - c^{1/2}))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{bx+c})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)

[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))), x)

$$3.182 \quad \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})} dx$$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(a-c)} + \frac{\sqrt{bx+c}}{x(a-c)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)} + \frac{b \tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2104, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(a-c)} + \frac{\sqrt{bx+c}}{x(a-c)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)} + \frac{b \tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])),x]

[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]/(Sqrt[a]*(a - c))) + (b*ArcTanh[Sqrt[c + b*x]/Sqrt[c]]/(a - c)*Sqrt[c])

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2104

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})} dx &= -\frac{b \int \frac{\sqrt{a+bx}}{x^2} dx}{-ab+bc} + \frac{b \int \frac{\sqrt{c+bx}}{x^2} dx}{-ab+bc} \\
&= -\frac{\sqrt{a+bx}}{(a-c)x} + \frac{\sqrt{c+bx}}{(a-c)x} + \frac{b \int \frac{1}{x\sqrt{a+bx}} dx}{2(a-c)} - \frac{b \int \frac{1}{x\sqrt{c+bx}} dx}{2(a-c)} \\
&= -\frac{\sqrt{a+bx}}{(a-c)x} + \frac{\sqrt{c+bx}}{(a-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{a-c} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{c}{b} + \frac{x^2}{b}} dx, x, \sqrt{c+bx}\right)}{a-c} \\
&= -\frac{\sqrt{a+bx}}{(a-c)x} + \frac{\sqrt{c+bx}}{(a-c)x} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)} + \frac{b \tanh^{-1}\left(\frac{\sqrt{c+bx}}{\sqrt{c}}\right)}{(a-c)\sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 99, normalized size = 0.96

$$\frac{bx\sqrt{\frac{bx}{c}+1} \tanh^{-1}\left(\sqrt{\frac{bx}{c}+1}\right)+bx+c}{\sqrt{bx+c}} - \frac{bx\sqrt{\frac{bx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)+a+bx}{\sqrt{a+bx}}}{x(a-c)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])), x]

[Out] (-((a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])/Sqrt[a + b*x]) + (c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]])/Sqrt[c + b*x])/((a - c)*x)

IntegrateAlgebraic [B] time = 1.37, size = 301, normalized size = 2.92

$$-\frac{\sqrt{a+bx}}{x(a-c)} + \frac{\sqrt{bx+c}}{x(a-c)} - \frac{b\sqrt{-(\sqrt{a}+\sqrt{c})^2} \tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}} - \frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{\sqrt{a}\sqrt{c}(\sqrt{a}-\sqrt{c})(\sqrt{a}+\sqrt{c})} + \frac{b\sqrt{-(\sqrt{a}-\sqrt{c})^2} \tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}} - \frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{\sqrt{a}\sqrt{c}(\sqrt{a}-\sqrt{c})(\sqrt{a}+\sqrt{c})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])), x]

[Out] -(Sqrt[a + b*x]/((a - c)*x)) + Sqrt[c + b*x]/((a - c)*x) - (b*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c])/(Sqrt[a]*(Sqrt[a] - Sqrt[c])* (Sqrt[a] + Sqrt[c])*Sqrt[c]) + (b*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c])/(Sqrt[a]*(Sqrt[a] - Sqrt[c])* (Sqrt[a] + Sqrt[c])*Sqrt[c])

fricas [A] time = 0.59, size = 399, normalized size = 3.87

$$\left[\frac{\sqrt{a} b c x \log\left(\frac{(a+2\sqrt{a}\sqrt{c}+bx)}{x}\right) + ab\sqrt{c} x \log\left(\frac{(a-2\sqrt{a}\sqrt{c}+bx)}{x}\right) + 2\sqrt{bx+a} ac - 2\sqrt{bx+c} ac}{2(a^2c-ac^2)x} - \frac{2ab\sqrt{c} x \arctan\left(\frac{\sqrt{bx+c}}{\sqrt{a}}\right) + \sqrt{a} b c x \log\left(\frac{(a+2\sqrt{a}\sqrt{c}+bx)}{x}\right) + 2\sqrt{bx+a} ac - 2\sqrt{bx+c} ac}{2(a^2c-ac^2)x} - \frac{ab\sqrt{c} x \log\left(\frac{(a+2\sqrt{a}\sqrt{c}+bx)}{x}\right) - 2\sqrt{bx+a} ac + 2\sqrt{bx+c} ac}{2(a^2c-ac^2)x} - \frac{\sqrt{a} b c x \arctan\left(\frac{\sqrt{bx+c}}{\sqrt{a}}\right) - ab\sqrt{c} x \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) - \sqrt{bx+a} ac + \sqrt{bx+c} ac}{(a^2c-ac^2)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)), x, algorithm="fricas")

[Out] [-1/2*(sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + a*b*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x), -1/2*(2*a*b*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + sqrt(a)*b*c*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a*c - 2*sqrt(b*x + c)*a*c)/((a^2*c - a*c^2)*x),

$$\frac{1}{2} \cdot (2 \cdot \sqrt{-a} \cdot b \cdot c \cdot x \cdot \arctan(\sqrt{bx+a} \cdot \sqrt{-a}/a) - a \cdot b \cdot \sqrt{c} \cdot x \cdot \log((bx - 2 \cdot \sqrt{bx+c} \cdot \sqrt{c} + 2 \cdot c)/x) - 2 \cdot \sqrt{bx+a} \cdot a \cdot c + 2 \cdot \sqrt{bx+c} \cdot a \cdot c) / ((a^2 \cdot c - a \cdot c^2) \cdot x), (\sqrt{-a} \cdot b \cdot c \cdot x \cdot \arctan(\sqrt{bx+a} \cdot \sqrt{-a}/a) - a \cdot b \cdot \sqrt{-c} \cdot x \cdot \arctan(\sqrt{bx+c} \cdot \sqrt{-c}/c) - \sqrt{bx+a} \cdot a \cdot c + \sqrt{bx+c} \cdot a \cdot c) / ((a^2 \cdot c - a \cdot c^2) \cdot x)]$$

giac [B] time = 10.21, size = 1190, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="giac")

[Out] $b \cdot \arctan(\sqrt{bx+a}/\sqrt{-a}) / (\sqrt{-a} \cdot (a-c)) + (2 \cdot (a \cdot c^2 + \sqrt{a \cdot c} \cdot c^2) \cdot (a-c)^2 \cdot b \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) + 2 \cdot (a \cdot c^2 + \sqrt{a \cdot c} \cdot a \cdot c) \cdot (a-c)^2 \cdot b + (a^2 \cdot c^2 - 2 \cdot a \cdot c^3 + c^4 + (a^2 \cdot c - 2 \cdot a \cdot c^2 + c^3) \cdot \sqrt{a \cdot c}) \cdot b \cdot \operatorname{abs}(a-c) \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) + (a^3 \cdot c - 2 \cdot a^2 \cdot c^2 + a \cdot c^3 + (a^2 \cdot c - 2 \cdot a \cdot c^2 + c^3) \cdot \sqrt{a \cdot c}) \cdot b \cdot \operatorname{abs}(a-c) - (a^4 \cdot c - a^3 \cdot c^2 - a^2 \cdot c^3 + a \cdot c^4 + (a^3 \cdot c - a^2 \cdot c^2 - a \cdot c^3 + c^4) \cdot \sqrt{a \cdot c})) \cdot b \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) - (a^4 \cdot c - a^3 \cdot c^2 - a^2 \cdot c^3 + a \cdot c^4 + (a^4 - a^3 \cdot c - a^2 \cdot c^2 + a \cdot c^3) \cdot \sqrt{a \cdot c}) \cdot b) \cdot \arctan(-(\sqrt{bx+a} - \sqrt{bx+c}) / \sqrt{-(a^2 - c^2 + \sqrt{(a^2 - c^2)^2 - (a^3 - 3 \cdot a^2 \cdot c + 3 \cdot a \cdot c^2 - c^3) \cdot (a-c)}) / (a-c)}) / ((\sqrt{-a} \cdot a^4 \cdot c - a^4 \cdot \sqrt{-c} \cdot c - 4 \cdot \sqrt{-a} \cdot a^3 \cdot c^2 + 4 \cdot a^3 \cdot \sqrt{-c} \cdot c^2 + 6 \cdot \sqrt{-a} \cdot a^2 \cdot c^3 - 6 \cdot a^2 \cdot \sqrt{-c} \cdot c^3 - 4 \cdot \sqrt{-a} \cdot a \cdot c^4 + 4 \cdot a \cdot \sqrt{-c} \cdot c^4 + \sqrt{-a} \cdot c^5 - \sqrt{-c} \cdot c^5) \cdot \operatorname{abs}(a-c) - (2 \cdot (a \cdot c^2 + \sqrt{a \cdot c} \cdot c^2) \cdot (a-c)^2 \cdot b \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) - 2 \cdot (a \cdot c^2 + \sqrt{a \cdot c} \cdot a \cdot c) \cdot (a-c)^2 \cdot b + (a^2 \cdot c^2 - 2 \cdot a \cdot c^3 + c^4 - (a^2 \cdot c - 2 \cdot a \cdot c^2 + c^3) \cdot \sqrt{a \cdot c}) \cdot b \cdot \operatorname{abs}(a-c) \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) - (a^3 \cdot c - 2 \cdot a^2 \cdot c^2 + a \cdot c^3 + (a^2 \cdot c - 2 \cdot a \cdot c^2 + c^3) \cdot \sqrt{a \cdot c}) \cdot b \cdot \operatorname{abs}(a-c) - (a^4 \cdot c - a^3 \cdot c^2 - a^2 \cdot c^3 + a \cdot c^4 - (a^3 \cdot c - a^2 \cdot c^2 - a \cdot c^3 + c^4) \cdot \sqrt{a \cdot c})) \cdot b \cdot \operatorname{sgn}(2 \cdot a - 2 \cdot c) + (a^4 \cdot c - a^3 \cdot c^2 - a^2 \cdot c^3 + a \cdot c^4 - (a^4 - a^3 \cdot c - a^2 \cdot c^2 + a \cdot c^3) \cdot \sqrt{a \cdot c}) \cdot b) \cdot \arctan(-(\sqrt{bx+a} - \sqrt{bx+c}) / \sqrt{-(a^2 - c^2 - \sqrt{(a^2 - c^2)^2 - (a^3 - 3 \cdot a^2 \cdot c + 3 \cdot a \cdot c^2 - c^3) \cdot (a-c)}) / (a-c)}) / ((\sqrt{-a} \cdot a^4 \cdot c - a^4 \cdot \sqrt{-c} \cdot c - 4 \cdot \sqrt{-a} \cdot a^3 \cdot c^2 + 4 \cdot a^3 \cdot \sqrt{-c} \cdot c^2 + 6 \cdot \sqrt{-a} \cdot a^2 \cdot c^3 - 6 \cdot a^2 \cdot \sqrt{-c} \cdot c^3 - 4 \cdot \sqrt{-a} \cdot a \cdot c^4 + 4 \cdot a \cdot \sqrt{-c} \cdot c^4 + \sqrt{-a} \cdot c^5 - \sqrt{-c} \cdot c^5) \cdot \operatorname{abs}(a-c)) - 2 \cdot (b \cdot (\sqrt{bx+a} - \sqrt{bx+c})^3 - a \cdot b \cdot (\sqrt{bx+a} - \sqrt{bx+c}) + b \cdot c \cdot (\sqrt{bx+a} - \sqrt{bx+c})) / (((\sqrt{bx+a} - \sqrt{bx+c})^4 - 2 \cdot a \cdot (\sqrt{bx+a} - \sqrt{bx+c})^2 - 2 \cdot c \cdot (\sqrt{bx+a} - \sqrt{bx+c})^2 + a^2 - 2 \cdot a \cdot c + c^2) \cdot (a-c) - \sqrt{bx+a} / ((a-c) \cdot x))$

maple [A] time = 0.02, size = 88, normalized size = 0.85

$$\frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx} \right) b}{a-c} - \frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{2\sqrt{c}} - \frac{\sqrt{bx+c}}{2bx} \right) b}{a-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x)

[Out] $2/(a-c) \cdot b \cdot (-1/2 \cdot (b \cdot x + a)^{1/2} / b / x - 1/2 \cdot \operatorname{arctanh}((b \cdot x + a)^{1/2} / a^{1/2}) / a^{1/2}) - 2/(a-c) \cdot b \cdot (-1/2 \cdot (b \cdot x + c)^{1/2} / b / x - 1/2 \cdot c^{1/2} \cdot \operatorname{arctanh}((b \cdot x + c)^{1/2} / c^{1/2}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(\sqrt{bx+a} + \sqrt{bx+c})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))), x)

mupad [B] time = 18.88, size = 2642, normalized size = 25.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))),x)

[Out] (b*atan(((b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*((2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*((2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) - (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*((4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2)))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*((2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*((4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2)))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*((2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*((4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2)))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2)*((a^3*b*c^(7/2) - a^(7/2)*b*c^3 - a^2*b*c^(9/2) + a^(9/2)*b*c^2)/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) + (((a + b*x)^(1/2) - a^(1/2))*((2*a^(3/2)*b*c^5 - 2*a^5*b*c^(3/2) + 2*a^4*b*c^(5/2) - 2*a^(5/2)*b*c^4))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)) + (b*(a*c^(1/2) + a^(1/2)*c)*((a^(5/2)*c^(11/2) - a^(7/2)*c^(9/2) - a^(9/2)*c^(7/2) + a^(11/2)*c^(5/2))/(a^3*c^5 - 2*a^4*c^4 + a^5*c^3) - (((a + b*x)^(1/2) - a^(1/2))*((4*a^2*c^6 - 12*a^3*c^5 + 16*a^4*c^4 - 12*a^5*c^3 + 4*a^6*c^2)))/(2*((c + b*x)^(1/2) - c^(1/2))*(a^3*c^5 - 2*a^4*c^4 + a^5*c^3)))*((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3/2)*c^(1/2)))^(1/2))/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2))))*1i)/(2*(2*a^2*c^3 - 2*a^3*c^2 + a^(3/2)*c^(7/2) - a^(7/2)*c^(3/2)))

$$\frac{\left(2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}\right)^{1/2}}{\left(2\left(2a^2c^3 - 2a^3c^2 + a^{3/2}c^{7/2} - a^{7/2}c^{3/2}\right)\right)^{1/2}} \cdot \frac{\left(a^{1/2}c^{1/2} + a^{1/2}c\right) \cdot \left(a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}\right)}{\left(2a^2c^3 - 2a^3c^2 + a^{3/2}c^{7/2} - a^{7/2}c^{3/2}\right)}$$

$$- \left(\frac{a^{1/2}b}{4(ac - a^2)} - \frac{bc^{1/2}}{4(ac - c^2)} - \left(\frac{a^{1/2}(a^2b/4 - bc^2/4 + abc/4)}{a^3c - a^2c^2} - \frac{c^{1/2}(bc^2/4 - a^2b/4 + abc/4)}{ac^3 - a^2c^2}\right) \cdot \frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}}\right)^2$$

$$+ \left(\frac{a^{1/2}(ab/4 - 3bc/4)}{ac^2 - a^2c} - \frac{c^{1/2}(3ab/4 - bc/4)}{ac^2 - a^2c}\right) \cdot \frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}}$$

$$+ \frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}} \cdot \frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}} \cdot \frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}}$$

$$- \frac{(a + c) \cdot \left((a + bx)^{1/2} - a^{1/2}\right)^2}{a^{1/2}c^{1/2} \cdot \left((c + bx)^{1/2} - c^{1/2}\right)^2} - \log\left(\frac{(a + bx)^{1/2} - a^{1/2}}{(c + bx)^{1/2} - c^{1/2}}\right) \cdot \frac{b \cdot \left(2a^{1/2}c - (b(a^{1/2} + c^{1/2}))\right)}{2c(a - c)} - \frac{b \cdot \left((a + bx)^{1/2} - a^{1/2}\right)}{4a^{1/2}c^{1/2} \cdot (a^{1/2} - c^{1/2})} \cdot \left((c + bx)^{1/2} - c^{1/2}\right)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(\sqrt{a + bx} + \sqrt{bx + c}\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2)),x)

[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))), x)

$$3.183 \quad \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$$

Optimal. Leaf size=228

$$\frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2} - \frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)} - \frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)}$$

Rubi [A] time = 0.37, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6689, 90, 80, 50, 63, 217, 206}

$$\frac{x(a+bx)^{3/2}(bx+c)^{3/2}}{2b^2(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(bx+c)^{3/2}}{12b^3(a-c)^2} + \frac{(4ac-5(a+c)^2)(a+bx)^{3/2}\sqrt{bx+c}}{16b^3(a-c)^2} - \frac{(4ac-5(a+c)^2)\sqrt{a+bx}\sqrt{bx+c}}{32b^3(a-c)} - \frac{(4ac-5(a+c)^2)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{32b^3} + \frac{bx^4}{2(a-c)^2} + \frac{x^3(a+c)}{3(a-c)^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] ((a + c)*x^3)/(3*(a - c)^2) + (b*x^4)/(2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*Sqrt[a + b*x]*Sqrt[c + b*x])/(32*b^3*(a - c)) + ((4*a*c - 5*(a + c)^2)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(16*b^3*(a - c)^2) + (5*(a + c)*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(12*b^3*(a - c)^2) - (x*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(2*b^2*(a - c)^2) - ((4*a*c - 5*(a + c)^2)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(32*b^3)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 90

Int[((a_.) + (b_.)*(x_))^(2)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 3)), x] + Dist[1/(d*f*(n + p + 3)), Int[(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(n + p + 3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 3, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx &= \frac{\int \left(a \left(1 + \frac{c}{a} \right) x^2 + 2bx^3 - 2x^2 \sqrt{a+bx} \sqrt{c+bx} \right) dx}{(a-c)^2} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} - \frac{2 \int x^2 \sqrt{a+bx} \sqrt{c+bx} dx}{(a-c)^2} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} - \frac{\int \sqrt{a+bx} \sqrt{c+bx} (-ac - \frac{5}{2}x^2)}{2b^2(a-c)^2} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}(c+bx)^{3/2}}{12b^3(a-c)^2} - \frac{x(a+bx)^{3/2}(c+bx)^{3/2}}{2b^2(a-c)^2} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(4ac - 5(a+c)^2)(a+bx)^{3/2}\sqrt{c+bx}}{16b^3(a-c)^2} + \frac{5(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{12b^3(a-c)^2} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2 + 6ac + 5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac - 5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{16b^3(a-c)} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2 + 6ac + 5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac - 5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{16b^3(a-c)} \\ &= \frac{(a+c)x^3}{3(a-c)^2} + \frac{bx^4}{2(a-c)^2} + \frac{(5a^2 + 6ac + 5c^2)\sqrt{a+bx}\sqrt{c+bx}}{32b^3(a-c)} + \frac{(4ac - 5(a+c)^2)\sqrt{a+bx}\sqrt{c+bx}}{16b^3(a-c)} \end{aligned}$$

Mathematica [A] time = 1.29, size = 361, normalized size = 1.58

$$\frac{-15a^2\sqrt{a+bx}\sqrt{c+bx} + \frac{3\sqrt{b(a-c)}\sqrt{a^2+6ac+5c^2}\sqrt{\frac{a-c}{a+bx}} \operatorname{arctanh}\left(\frac{\sqrt{a-c}\sqrt{a+bx}}{\sqrt{a+bx}}\right)}{\sqrt{b(a-c)}\sqrt{a+bx}} + a^2\sqrt{a+bx}\sqrt{c+bx} + c(10bx+7c) - 16b^3c^2(3\sqrt{a+bx}\sqrt{c+bx} - 2c) - 8b^2c^2\sqrt{a+bx}\sqrt{c+bx} - a(8b^2\sqrt{a+bx}\sqrt{c+bx} - 7c^2\sqrt{a+bx}\sqrt{c+bx} + 4bc\sqrt{a+bx}\sqrt{c+bx} - 32b^3c^2) - 15c^2\sqrt{a+bx}\sqrt{c+bx} + 10bc^2\sqrt{a+bx}\sqrt{c+bx} + 48b^4c^4}{96b^3(a-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] (48*b^4*x^4 - 15*a^3*Sqrt[a + b*x]*Sqrt[c + b*x] - 15*c^3*Sqrt[a + b*x]*Sqrt[c + b*x] + 10*b*c^2*x*Sqrt[a + b*x]*Sqrt[c + b*x] - 8*b^2*c*x^2*Sqrt[a +

$b*x]*\text{Sqrt}[c + b*x] + a^2*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + b*x]*(7*c + 10*b*x) - 16*b^3*x^3*(-2*c + 3*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + b*x]) - a*(-32*b^3*x^3 - 7*c^2*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + b*x] + 4*b*c*x*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + b*x] + 8*b^2*x^2*\text{Sqrt}[a + b*x]*\text{Sqrt}[c + b*x]) + (3*\text{Sqrt}[b]*(-a + c)^3*(5*a^2 + 6*a*c + 5*c^2)*\text{Sqrt}[-((c + b*x)/(a - c))]*\text{ArcSinh}[(\text{Sqrt}[b]*\text{Sqrt}[a + b*x])/(\text{Sqrt}[b*(-a + c)])]/(\text{Sqrt}[b*(-a + c)]*\text{Sqrt}[c + b*x]))/(96*b^3*(a - c)^2)$

IntegrateAlgebraic [A] time = 0.60, size = 291, normalized size = 1.28

$$\frac{(-5c^2 - 6ac - 5a^2) \log(\sqrt{a+bx} - \sqrt{bx+c})}{33b^3} + \frac{\sqrt{a+bx}(-15a^2\sqrt{bx+c} + 10a^2(bx+c)^{3/2} - 3a^2c\sqrt{bx+c} + 3ac^2\sqrt{bx+c} - 8a(bx+c)^{3/2} + 12ac(bx+c)^{3/2} + 15c^2\sqrt{bx+c} - 118c^2(bx+c)^{3/2} - 48(bx+c)^{3/2} + 136c(bx+c)^{3/2})}{96b^3(a-c)^2} + \frac{6ac^2(bx+c) + 2a(bx+c)^3 - 6a(bx+c)^2 - 6c^3(bx+c) + 12c^2(bx+c)^2 + 3(bx+c)^3 - 10c(bx+c)^2}{ab^3(a-c)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] $(\text{Sqrt}[a + b*x]*(-15*a^3*\text{Sqrt}[c + b*x] - 3*a^2*c*\text{Sqrt}[c + b*x] + 3*a*c^2*\text{Sqrt}[c + b*x] + 15*c^3*\text{Sqrt}[c + b*x] + 10*a^2*(c + b*x)^{(3/2)} + 12*a*c*(c + b*x)^{(3/2)} - 118*c^2*(c + b*x)^{(3/2)} - 8*a*(c + b*x)^{(5/2)} + 136*c*(c + b*x)^{(5/2)} - 48*(c + b*x)^{(7/2)}))/ (96*b^3*(a - c)^2) + (6*a*c^2*(c + b*x) - 6*c^3*(c + b*x) - 6*a*c*(c + b*x)^2 + 12*c^2*(c + b*x)^2 + 2*a*(c + b*x)^3 - 10*c*(c + b*x)^3 + 3*(c + b*x)^4)/(6*b^3*(a - c)^2) + ((-5*a^2 - 6*a*c - 5*c^2)*\text{Log}[\text{Sqrt}[a + b*x] - \text{Sqrt}[c + b*x]])/(32*b^3)$

fricas [A] time = 0.63, size = 196, normalized size = 0.86

$$\frac{96b^4x^4 + 64(ab^3 + b^3c)x^3 - 2(48b^3x^3 + 15a^3 - 7a^2c - 7ac^2 + 15c^3 + 8(ab^2 + b^2c)x^2 - 2(5a^2b - 2abc + 5bc^2)x)\sqrt{bx+a}\sqrt{bx+c} - 3(5a^4 - 4a^3c - 2a^2c^2 - 4ac^3 + 5c^4)\log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c)}{192(a^2b^3 - 2ab^3c + b^3c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] $1/192*(96*b^4*x^4 + 64*(a*b^3 + b^3*c)*x^3 - 2*(48*b^3*x^3 + 15*a^3 - 7*a^2*c - 7*a*c^2 + 15*c^3 + 8*(a*b^2 + b^2*c)*x^2 - 2*(5*a^2*b - 2*a*b*c + 5*b*c^2)*x)*\text{sqrt}(b*x + a)*\text{sqrt}(b*x + c) - 3*(5*a^4 - 4*a^3*c - 2*a^2*c^2 - 4*a*c^3 + 5*c^4)*\log(-2*b*x + 2*\text{sqrt}(b*x + a)*\text{sqrt}(b*x + c) - a - c))/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)$

giac [B] time = 0.39, size = 797, normalized size = 3.50

$$\frac{1}{192} \left(\frac{96b^4x^4 + 64(ab^3 + b^3c)x^3 - 2(48b^3x^3 + 15a^3 - 7a^2c - 7ac^2 + 15c^3 + 8(ab^2 + b^2c)x^2 - 2(5a^2b - 2abc + 5bc^2)x)\sqrt{bx+a}\sqrt{bx+c} - 3(5a^4 - 4a^3c - 2a^2c^2 - 4ac^3 + 5c^4)\log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c)}{a^2b^3 - 2ab^3c + b^3c^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")

[Out] $-1/96*(2*(4*(b*x + a)*(6*(a^5*b^9 - 5*a^4*b^9*c + 10*a^3*b^9*c^2 - 10*a^2*b^9*c^3 + 5*a*b^9*c^4 - b^9*c^5)*(b*x + a)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7) - (17*a^6*b^9 - 86*a^5*b^9*c + 175*a^4*b^9*c^2 - 180*a^3*b^9*c^3 + 95*a^2*b^9*c^4 - 22*a*b^9*c^5 + b^9*c^6)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7)) + (59*a^7*b^9 - 301*a^6*b^9*c + 615*a^5*b^9*c^2 - 625*a^4*b^9*c^3 + 305*a^3*b^9*c^4 - 39*a^2*b^9*c^5 - 19*a*b^9*c^6 + 5*b^9*c^7)/(a^7*b^12 - 7*a^6*b^12*c + 21*a^5*b^12*c^2 - 35*a^4*b^12*c^3 + 35*a^3*b^12*c^4 - 21*a^2*b^12*c^5 + 7*a*b^12*c^6 - b^12*c^7))*\text{sqrt}(b*x + a)*\text{sqrt}(b*x + c) + 1/6*(3*(b*x + a)^4 - 10*(b*x + a)^3*a + 12*(b*x + a)^2*a^2 - 6*(b*x + a)*a^3 + 2*(b*x + a)^3*c - 6*(b*x + a)^2*a*c + 6*(b*x + a)*a^2*c)/(a^2*b^3 - 2*a*b^3*c + b^3*c^2)$

$\int \frac{x^2}{(\sqrt{bx+a} + \sqrt{bx+c})^2} dx = \frac{1}{32}(5a^2 + 6ac + 5c^2) \log(\sqrt{bx+a} + \sqrt{bx+c}) + \frac{1}{b^3}$

maple [C] time = 0.02, size = 604, normalized size = 2.65

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)

[Out] $\frac{1}{3}x^3/(a-c)^2a + \frac{1}{3}x^3/(a-c)^2c + \frac{1}{2}bx^4/(a-c)^2 - \frac{1}{192}(a-c)^2(bx+a)^{1/2}(bx+c)^{1/2} \cdot (96 \operatorname{csgn}(b)x^3b^3(b^2x^2+abx+bcx+ac)^{1/2} + 16 \operatorname{csgn}(b)x^2ab^2(b^2x^2+abx+bcx+ac)^{1/2} + 16 \operatorname{csgn}(b)x^2b^2c(b^2x^2+abx+bcx+ac)^{1/2} - 20(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)xa^2b + 8(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)xa^2b^2 - 20(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)xa^2bc - 20(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)xb^2c + 30(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)a^3 - 14(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)a^2c - 14(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)a^2c + 30(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b)c^3 - 15 \ln(1/2(2(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b) + 2bx+a+c) \operatorname{csgn}(b))a^4 + 12 \ln(1/2(2(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b) + 2bx+a+c) \operatorname{csgn}(b))a^3c + 6 \ln(1/2(2(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b) + 2bx+a+c) \operatorname{csgn}(b))a^2c^2 + 12 \ln(1/2(2(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b) + 2bx+a+c) \operatorname{csgn}(b))a^2c^2 - 15 \ln(1/2(2(b^2x^2+abx+bcx+ac)^{1/2} \operatorname{csgn}(b) + 2bx+a+c) \operatorname{csgn}(b))c^4) \operatorname{csgn}(b) / b^3 (b^2x^2+abx+bcx+ac)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{bx+a} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)

mupad [B] time = 81.17, size = 1358, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)

[Out] $\frac{x^3(a+c)}{3(a-c)^2} - \frac{((a+b*x)^{1/2} - a^{1/2})^{15}((3ac)/8 + (5a^2)/16 + (5c^2)/16)}{b^3((c+b*x)^{1/2} - c^{1/2})^{15}} + \frac{((a+b*x)^{1/2} - a^{1/2})^3((23ac^3)/12 + (23a^3c)/12 - (115a^4)/48 - (115c^4)/48 + (349a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^3(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})^{13}((23ac^3)/12 + (23a^3c)/12 - (115a^4)/48 - (115c^4)/48 + (349a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^{13}(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})^5((3917ac^3)/12 + (3917a^3c)/12 + (383a^4)/48 + (383c^4)/48 + (7279a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^5(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})^{11}((3917ac^3)/12 + (3917a^3c)/12 + (383a^4)/48 + (383c^4)/48 + (7279a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^{11}(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})^7((17567ac^3)/12 + (17567a^3c)/12 + (2789a^4)/48 + (2789c^4)/48 + (28213a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^7(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})^9((17567ac^3)/12 + (17567a^3c)/12 + (2789a^4)/48 + (2789c^4)/48 + (28213a^2c^2)/8)}{((c+b*x)^{1/2} - c^{1/2})^9(a^2b^3 + b^3c^2 - 2ab^3c)} + \frac{((a+b*x)^{1/2} - a^{1/2})((3ac)/8 + (5a^2)/16 + (5c^2)/16)}{b^3((c+b*x)^{1/2} - c^{1/2})} - (a^2b^3 + b^3c^2 - 2ab^3c)$

```
(1/2)*c^(1/2)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^4)/(((c +
b*x)^(1/2) - c^(1/2))^4*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)
)*(192*a*c^2 + 192*a^2*c)*((a + b*x)^(1/2) - a^(1/2))^12)/(((c + b*x)^(1/2)
- c^(1/2))^12*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*
x)^(1/2) - a^(1/2))^6*((5120*a*c^2)/3 + (5120*a^2*c)/3 + 256*a^3 + 256*c^3)
)/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^3 + b^3*c^2 - 2*a*b^3*c)) - (a^(1/2)
)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^10*((5120*a*c^2)/3 + (5120*a^2*c)/3 +
256*a^3 + 256*c^3))/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^3 + b^3*c^2 - 2
*a*b^3*c)) - (a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^8*((10112*a*c^2)/
3 + (10112*a^2*c)/3 + 512*a^3 + 512*c^3))/(((c + b*x)^(1/2) - c^(1/2))^8*(a
^2*b^3 + b^3*c^2 - 2*a*b^3*c)))/((28*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b
*x)^(1/2) - c^(1/2))^4 - (8*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2)
- c^(1/2))^2 - (56*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/
2))^6 + (70*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 -
(56*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + (28*((
a + b*x)^(1/2) - a^(1/2))^12)/((c + b*x)^(1/2) - c^(1/2))^12 - (8*((a + b*x)
)^(1/2) - a^(1/2))^14)/((c + b*x)^(1/2) - c^(1/2))^14 + ((a + b*x)^(1/2) -
a^(1/2))^16)/((c + b*x)^(1/2) - c^(1/2))^16 + 1) + (b*x^4)/(2*(a - c)^2) + (
atanh(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(6*a*c + 5*a
^2 + 5*c^2))/(16*b^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)

[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)

$$3.184 \quad \int \frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$$

Optimal. Leaf size=165

$$\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2} + \frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)} - \frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2} + \frac{2bx}{3(a-c)}$$

Rubi [A] time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {6689, 80, 50, 63, 217, 206}

$$\frac{2(a+bx)^{3/2}(bx+c)^{3/2}}{3b^2(a-c)^2} + \frac{(a+c)(a+bx)^{3/2}\sqrt{bx+c}}{2b^2(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{bx+c}}{4b^2(a-c)} - \frac{(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{4b^2} + \frac{2bx^3}{3(a-c)^2} + \frac{x^2(a+c)}{2(a-c)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] ((a + c)*x^2)/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2) - ((a + c)*Sqrt[a + b*x]*Sqrt[c + b*x])/(4*b^2*(a - c)) + ((a + c)*(a + b*x)^(3/2)*Sqrt[c + b*x])/(2*b^2*(a - c)^2) - (2*(a + b*x)^(3/2)*(c + b*x)^(3/2))/(3*b^2*(a - c)^2) - ((a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(4*b^2)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx &= \frac{\int \left(a \left(1 + \frac{c}{a} \right) x + 2bx^2 - 2x\sqrt{a+bx}\sqrt{c+bx} \right) dx}{(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{2 \int x\sqrt{a+bx}\sqrt{c+bx} dx}{(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2} + \frac{(a+c) \int \sqrt{a+bx}\sqrt{c+bx} dx}{b(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} - \frac{2(a+bx)^{3/2}(c+bx)^{3/2}}{3b^2(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} \\ &= \frac{(a+c)x^2}{2(a-c)^2} + \frac{2bx^3}{3(a-c)^2} - \frac{(a+c)\sqrt{a+bx}\sqrt{c+bx}}{4b^2(a-c)} + \frac{(a+c)(a+bx)^{3/2}\sqrt{c+bx}}{2b^2(a-c)^2} \end{aligned}$$

Mathematica [A] time = 0.88, size = 229, normalized size = 1.39

$$\frac{3a^2\sqrt{a+bx}\sqrt{bx+c} - 2a(bx\sqrt{a+bx}\sqrt{bx+c} + c\sqrt{a+bx}\sqrt{bx+c} - 3b^2x^2) + (4bx+3c)(-2bx\sqrt{a+bx}\sqrt{bx+c} + c\sqrt{a+bx}\sqrt{bx+c} + 2b^2x^2)}{12b^2(a-c)^2} - \frac{(a+c)\sqrt{b(c-a)}\sqrt{\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{4b^{5/2}\sqrt{bx+c}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] (3*a^2*Sqrt[a + b*x]*Sqrt[c + b*x] + (3*c + 4*b*x)*(2*b^2*x^2 + c*Sqrt[a + b*x]*Sqrt[c + b*x] - 2*b*x*Sqrt[a + b*x]*Sqrt[c + b*x]) - 2*a*(-3*b^2*x^2 + c*Sqrt[a + b*x]*Sqrt[c + b*x] + b*x*Sqrt[a + b*x]*Sqrt[c + b*x]))/(12*b^2*(a - c)^2) - (Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]])/(4*b^(5/2)*Sqrt[c + b*x])

IntegrateAlgebraic [A] time = 0.41, size = 183, normalized size = 1.11

$$\frac{\sqrt{a+bx}(3a^2\sqrt{bx+c} - 2a(bx+c)^{3/2} - 3c^2\sqrt{bx+c} - 8(bx+c)^{5/2} + 14c(bx+c)^{3/2})}{12b^2(a-c)^2} + \frac{3a(bx+c)^2 - 6ac(bx+c) + 6c^2(bx+c) + 4(bx+c)^3 - 9c(bx+c)^2}{6b^2(a-c)^2} + \frac{(a+c)\log(\sqrt{a+bx} - \sqrt{bx+c})}{4b^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^2,x]

[Out] (Sqrt[a + b*x]*(3*a^2*Sqrt[c + b*x] - 3*c^2*Sqrt[c + b*x] - 2*a*(c + b*x)^(3/2) + 14*c*(c + b*x)^(3/2) - 8*(c + b*x)^(5/2)))/(12*b^2*(a - c)^2) + (-6*

$$a*c*(c + b*x) + 6*c^2*(c + b*x) + 3*a*(c + b*x)^2 - 9*c*(c + b*x)^2 + 4*(c + b*x)^3 / (6*b^2*(a - c)^2) + ((a + c)*\text{Log}[\text{Sqrt}[a + b*x] - \text{Sqrt}[c + b*x]]) / (4*b^2)$$

fricas [A] time = 0.76, size = 149, normalized size = 0.90

$$\frac{16b^3x^3 + 12(ab^2 + b^2c)x^2 - 2(8b^2x^2 - 3a^2 + 2ac - 3c^2 + 2(ab + bc)x)\sqrt{bx + a}\sqrt{bx + c} + 3(a^3 - a^2c - ac^2 + c^3)\log(-2bx + 2\sqrt{bx + a}\sqrt{bx + c} - a - c)}{24(a^2b^2 - 2ab^2c + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/24*(16*b^3*x^3 + 12*(a*b^2 + b^2*c)*x^2 - 2*(8*b^2*x^2 - 3*a^2 + 2*a*c - 3*c^2 + 2*(a*b + b*c)*x)*sqrt(b*x + a)*sqrt(b*x + c) + 3*(a^3 - a^2*c - a*c^2 + c^3)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c)/(a^2*b^2 - 2*a*b^2*c + b^2*c^2)

giac [B] time = 0.27, size = 445, normalized size = 2.70

$$\frac{(2(bx + a) \left(\frac{4(a^2b^2 - 3a^2b^2c + 3ab^2c^2 - b^2c^3)(bx + a)}{a^2b^3 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5} - \frac{7a^4b^2 - 22a^2b^2c + 24a^2b^2c^2 - 10ab^2c^3 + a^2c^4}{a^2b^3 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5} + \frac{3(a^2b^2 - 3a^2b^2c + 2a^2b^2c^2 + 2a^2b^2c^3 - 3a^2c^4 + b^2c^5)}{a^2b^3 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5} \right) \sqrt{bx + a} \sqrt{bx + c} - \frac{3(a+c) \log\left(\frac{-\sqrt{bx+a} + \sqrt{bx+c}}{b}\right) - \frac{2(4(bx+a)^3 - 9(bx+a)^2 + 6(bx+a) - 6)}{b^2 - 2abc + bc^2}}{12b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")

[Out] -1/12*((2*(b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5) - (7*a^4*b^2 - 22*a^3*b^2*c + 24*a^2*b^2*c^2 - 10*a*b^2*c^3 + b^2*c^4)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5)) + 3*(a^5*b^2 - 3*a^4*b^2*c + 2*a^3*b^2*c^2 + 2*a^2*b^2*c^3 - 3*a*b^2*c^4 + b^2*c^5)/(a^5*b^3 - 5*a^4*b^3*c + 10*a^3*b^3*c^2 - 10*a^2*b^3*c^3 + 5*a*b^3*c^4 - b^3*c^5))*sqrt(b*x + a)*sqrt(b*x + c) - 3*(a + c)*log(abs(-sqrt(b*x + a) + sqrt(b*x + c)))/b - 2*(4*(b*x + a)^3 - 9*(b*x + a)^2*a + 6*(b*x + a)*a^2 + 3*(b*x + a)^2*c - 6*(b*x + a)*a*c)/(a^2*b - 2*a*b*c + b*c^2))/b

maple [C] time = 0.02, size = 431, normalized size = 2.61

$$\frac{\frac{2b^3x^3}{3a^2b^3 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5} + \frac{12(ab^2 + b^2c)x^2}{a^2b^3 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^2c^4 - b^2c^5} - \frac{2(8b^2x^2 - 3a^2 + 2ac - 3c^2 + 2(ab + bc)x)\sqrt{bx + a}\sqrt{bx + c}}{24(a^2b^2 - 2ab^2c + b^2c^2)} + \frac{3(a^3 - a^2c - ac^2 + c^3)\log\left(\frac{-\sqrt{bx+a} + \sqrt{bx+c}}{b}\right) - \frac{2(4(bx+a)^3 - 9(bx+a)^2 + 6(bx+a) - 6)}{b^2 - 2abc + bc^2}}{12b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)

[Out] 1/2*x^2/(a-c)^2*a+1/2*x^2/(a-c)^2*c+2/3*b*x^3/(a-c)^2-1/24/(a-c)^2*(b*x+a)^(1/2)*(b*x+c)^(1/2)*(16*csgn(b)*x^2*b^2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+4*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*a*b+4*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*x*b*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a^2+4*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*a*c-6*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)))*csgn(b))*a^3-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)))*csgn(b))*a^2*c-3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)))*csgn(b))*a*c^2+3*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)))*csgn(b))*c^3)*csgn(b)/b^2/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{bx + a} + \sqrt{bx + c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^2, x)

mupad [B] time = 37.52, size = 1012, normalized size = 6.13

(\frac{1}{2} \sqrt{bx+a} - \frac{1}{2} \sqrt{bx+c}) / (b^2 (\sqrt{bx+c} - \sqrt{bx+a})) + ((\sqrt{bx+a} - \sqrt{bx+c})^{11} (a/2 + c/2)) / (b^2 (\sqrt{bx+c} - \sqrt{bx+a}))^{11} - ((\sqrt{bx+a} - \sqrt{bx+c})^3 ((101ac^2)/2 + (101a^2c)/2 + (17a^3)/6 + (17c^3)/6)) / ((\sqrt{bx+c} - \sqrt{bx+a})^3 (a^2b^2 + b^2c^2 - 2ab^2c)) - ((\sqrt{bx+a} - \sqrt{bx+c})^9 ((101ac^2)/2 + (101a^2c)/2 + (17a^3)/6 + (17c^3)/6)) / ((\sqrt{bx+c} - \sqrt{bx+a})^9 (a^2b^2 + b^2c^2 - 2ab^2c)) - ((\sqrt{bx+a} - \sqrt{bx+c})^5 (269ac^2 + 269a^2c + 19a^3 + 19c^3)) / ((\sqrt{bx+c} - \sqrt{bx+a})^5 (a^2b^2 + b^2c^2 - 2ab^2c)) - ((\sqrt{bx+a} - \sqrt{bx+c})^7 (269ac^2 + 269a^2c + 19a^3 + 19c^3)) / ((\sqrt{bx+c} - \sqrt{bx+a})^7 (a^2b^2 + b^2c^2 - 2ab^2c)) + (16a^{3/2}c^{3/2}(\sqrt{bx+a} - \sqrt{bx+c})^2) / ((\sqrt{bx+c} - \sqrt{bx+a})^2 (a^2b^2 + b^2c^2 - 2ab^2c)) + (16a^{3/2}c^{3/2}(\sqrt{bx+a} - \sqrt{bx+c})^{10}) / ((\sqrt{bx+c} - \sqrt{bx+a})^{10} (a^2b^2 + b^2c^2 - 2ab^2c)) + (a^{1/2}c^{1/2}(\sqrt{bx+a} - \sqrt{bx+c})^4 (192ac + 64a^2 + 64c^2)) / ((\sqrt{bx+c} - \sqrt{bx+a})^4 (a^2b^2 + b^2c^2 - 2ab^2c)) + (a^{1/2}c^{1/2}(\sqrt{bx+a} - \sqrt{bx+c})^8 (192ac + 64a^2 + 64c^2)) / ((\sqrt{bx+c} - \sqrt{bx+a})^8 (a^2b^2 + b^2c^2 - 2ab^2c)) + (a^{1/2}c^{1/2}(\sqrt{bx+a} - \sqrt{bx+c})^6 ((1312ac)/3 + 128a^2 + 128c^2)) / ((\sqrt{bx+c} - \sqrt{bx+a})^6 (a^2b^2 + b^2c^2 - 2ab^2c)) / ((15(\sqrt{bx+a} - \sqrt{bx+c})^4) / ((\sqrt{bx+c} - \sqrt{bx+a})^4 - (6(\sqrt{bx+a} - \sqrt{bx+c})^2) / ((\sqrt{bx+c} - \sqrt{bx+a})^2 - (20(\sqrt{bx+a} - \sqrt{bx+c})^6) / ((\sqrt{bx+c} - \sqrt{bx+a})^6 + (15(\sqrt{bx+a} - \sqrt{bx+c})^8) / ((\sqrt{bx+c} - \sqrt{bx+a})^8 - (6(\sqrt{bx+a} - \sqrt{bx+c})^{10}) / ((\sqrt{bx+c} - \sqrt{bx+a})^{10} + ((\sqrt{bx+a} - \sqrt{bx+c})^{12}) / ((\sqrt{bx+c} - \sqrt{bx+a})^{12} + 1) - (atanh((\sqrt{bx+a} - \sqrt{bx+c}) / ((\sqrt{bx+c} - \sqrt{bx+a}) * (a + c))) / (2b^2) + (x^2(a + c)) / (2(a - c)^2) + (2bx^3) / (3(a - c)^2))

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^2,x)

[Out] (((a + b*x)^(1/2) - a^(1/2))*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))) + (((a + b*x)^(1/2) - a^(1/2))^11*(a/2 + c/2))/(b^2*((c + b*x)^(1/2) - c^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*((101*a*c^2)/2 + (101*a^2*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^9*((101*a*c^2)/2 + (101*a^2*c)/2 + (17*a^3)/6 + (17*c^3)/6))/(((c + b*x)^(1/2) - c^(1/2))^9*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^5*(269*a*c^2 + 269*a^2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^5*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) - (((a + b*x)^(1/2) - a^(1/2))^7*(269*a*c^2 + 269*a^2*c + 19*a^3 + 19*c^3))/(((c + b*x)^(1/2) - c^(1/2))^7*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*(a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) - c^(1/2))^2*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (16*a^(3/2)*c^(3/2)*(a + b*x)^(1/2) - a^(1/2))^10)/(((c + b*x)^(1/2) - c^(1/2))^10*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*(a + b*x)^(1/2) - a^(1/2))^4*(192*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^4*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*(a + b*x)^(1/2) - a^(1/2))^8*(192*a*c + 64*a^2 + 64*c^2))/(((c + b*x)^(1/2) - c^(1/2))^8*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) + (a^(1/2)*c^(1/2)*(a + b*x)^(1/2) - a^(1/2))^6*((1312*a*c)/3 + 128*a^2 + 128*c^2))/(((c + b*x)^(1/2) - c^(1/2))^6*(a^2*b^2 + b^2*c^2 - 2*a*b^2*c)) / ((15*((a + b*x)^(1/2) - a^(1/2))^4)/((c + b*x)^(1/2) - c^(1/2))^4 - (6*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 - (20*((a + b*x)^(1/2) - a^(1/2))^6)/((c + b*x)^(1/2) - c^(1/2))^6 + (15*((a + b*x)^(1/2) - a^(1/2))^8)/((c + b*x)^(1/2) - c^(1/2))^8 - (6*((a + b*x)^(1/2) - a^(1/2))^10)/((c + b*x)^(1/2) - c^(1/2))^10 + ((a + b*x)^(1/2) - a^(1/2))^12/((c + b*x)^(1/2) - c^(1/2))^12 + 1) - (atanh((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)))*(a + c))/(2*b^2) + (x^2*(a + c))/(2*(a - c)^2) + (2*b*x^3)/(3*(a - c)^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{a+bx} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(b*x + c))**2, x)

$$3.185 \quad \int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$$

Optimal. Leaf size=63

$$\frac{(a-c)^2}{8b(\sqrt{a+bx} + \sqrt{bx+c})^4} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b}$$

Rubi [A] time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.81, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6689, 50, 63, 217, 206}

$$\frac{bx^2}{(a-c)^2} - \frac{(a+bx)^{3/2}\sqrt{bx+c}}{b(a-c)^2} + \frac{\sqrt{a+bx}\sqrt{bx+c}}{2b(a-c)} + \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{2b} + \frac{x(a+c)}{(a-c)^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2), x]

[Out] ((a + c)*x)/(a - c)^2 + (b*x^2)/(a - c)^2 + (Sqrt[a + b*x]*Sqrt[c + b*x])/(2*b*(a - c)) - ((a + b*x)^(3/2)*Sqrt[c + b*x])/(b*(a - c)^2) + ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]]/(2*b)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^2} dx &= \frac{\int \left(a \left(1 + \frac{c}{a} \right) + 2bx - 2\sqrt{a+bx} \sqrt{c+bx} \right) dx}{(a-c)^2} \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} - \frac{2 \int \sqrt{a+bx} \sqrt{c+bx} dx}{(a-c)^2} \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} - \frac{(a+bx)^{3/2} \sqrt{c+bx}}{b(a-c)^2} + \frac{\int \frac{\sqrt{a+bx}}{\sqrt{c+bx}} dx}{2(a-c)} \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} + \frac{\sqrt{a+bx} \sqrt{c+bx}}{2b(a-c)} - \frac{(a+bx)^{3/2} \sqrt{c+bx}}{b(a-c)^2} + \frac{1}{4} \int \frac{1}{\sqrt{a+bx} \sqrt{c+bx}} dx \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} + \frac{\sqrt{a+bx} \sqrt{c+bx}}{2b(a-c)} - \frac{(a+bx)^{3/2} \sqrt{c+bx}}{b(a-c)^2} + \frac{\text{Subst} \left(\int \frac{1}{\sqrt{-a+c}} dx \right)}{2b} \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} + \frac{\sqrt{a+bx} \sqrt{c+bx}}{2b(a-c)} - \frac{(a+bx)^{3/2} \sqrt{c+bx}}{b(a-c)^2} + \frac{\text{Subst} \left(\int \frac{1}{1-x^2} dx \right)}{2b} \\
&= \frac{(a+c)x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} + \frac{\sqrt{a+bx} \sqrt{c+bx}}{2b(a-c)} - \frac{(a+bx)^{3/2} \sqrt{c+bx}}{b(a-c)^2} + \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}} \right)}{2b}
\end{aligned}$$

Mathematica [B] time = 0.56, size = 179, normalized size = 2.84

$$\frac{2bx(bx - \sqrt{a+bx} \sqrt{bx+c}) + a(2bx - \sqrt{a+bx} \sqrt{bx+c}) + c(2bx - \sqrt{a+bx} \sqrt{bx+c}) + \frac{\sqrt{b(c-a)}^3 \sqrt{\frac{bx+c}{c-a}} \sinh^{-1} \left(\frac{\sqrt{b} \sqrt{a+bx}}{\sqrt{b(c-a)}} \right)}{\sqrt{b(c-a)} \sqrt{bx+c}} + 2c^2}{2b(a-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2), x]

[Out] (2*c^2 + 2*b*x*(b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + a*(2*b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + c*(2*b*x - Sqrt[a + b*x]*Sqrt[c + b*x]) + (Sqrt[b]*(-a + c)^3*Sqrt[(c + b*x)/(-a + c)]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]])/(Sqrt[b*(-a + c)]*Sqrt[c + b*x]))/(2*b*(a - c)^2)

IntegrateAlgebraic [A] time = 0.27, size = 122, normalized size = 1.94

$$\frac{\sqrt{a+bx} (-a\sqrt{bx+c} - 2(bx+c)^{3/2} + c\sqrt{bx+c})}{2b(a-c)^2} + \frac{a(bx+c) + (bx+c)^2 - c(bx+c)}{b(a-c)^2} - \frac{\log(\sqrt{a+bx} - \sqrt{bx+c})}{2b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-2), x]

[Out] (Sqrt[a + b*x]*(-a*Sqrt[c + b*x]) + c*Sqrt[c + b*x] - 2*(c + b*x)^(3/2))/(2*b*(a - c)^2) + (a*(c + b*x) - c*(c + b*x) + (c + b*x)^2)/(b*(a - c)^2) - Log[Sqrt[a + b*x] - Sqrt[c + b*x]]/(2*b)

fricas [B] time = 0.71, size = 103, normalized size = 1.63

$$\frac{4b^2x^2 - 2(2bx + a + c)\sqrt{bx+a}\sqrt{bx+c} + 4(ab + bc)x - (a^2 - 2ac + c^2) \log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c)}{4(a^2b - 2abc + bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] $\frac{1}{4}(4b^2x^2 - 2(2bx + a + c)\sqrt{bx+a}\sqrt{bx+c} + 4(ab + bc)x - (a^2 - 2ac + c^2)\log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c))/(a^2b - 2abc + b^2c^2)$

giac [B] time = 0.24, size = 189, normalized size = 3.00

$$-\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}\left(\frac{2(ab-bc)(bx+a)}{a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3}-\frac{a^2b-2abc+bc^2}{a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3}\right)+\frac{(bx+a)^2-(bx+a)a+(bx+a)c}{a^2b-2abc+bc^2}-\frac{\log(|-\sqrt{bx+a}+\sqrt{bx+c}|)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")

[Out] $-\frac{1}{2}\sqrt{bx+a}\sqrt{bx+c}(2(ab-bc)(bx+a)/(a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3)-(a^2b-2abc+bc^2)/(a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3))+((bx+a)^2-(bx+a)a+(bx+a)c)/(a^2b-2abc+bc^2)-\frac{1}{2}\log(\text{abs}(-\sqrt{bx+a}+\sqrt{bx+c}))/b$

maple [B] time = 0.01, size = 377, normalized size = 5.98

$$\frac{\sqrt{bx+a}\sqrt{bx+c} \ln\left(\frac{c^2+2ab+bc}{\sqrt{bx+c}} + \sqrt{bx+a}\sqrt{bx+c}\right)}{4(a-c)^2\sqrt{bx+c}\sqrt{bx+a}\sqrt{b^2}} - \frac{\sqrt{bx+a}\sqrt{bx+c} \ln\left(\frac{c^2+2ab+bc}{\sqrt{bx+c}} + \sqrt{bx+a}\sqrt{bx+c}\right)}{2(a-c)^2\sqrt{bx+c}\sqrt{bx+a}\sqrt{b^2}} + \frac{bx^2}{(a-c)^2} + \frac{\sqrt{bx+a}\sqrt{bx+c} \ln\left(\frac{c^2+2ab+bc}{\sqrt{bx+c}} + \sqrt{bx+a}\sqrt{bx+c}\right)}{4(a-c)^2\sqrt{bx+c}\sqrt{bx+a}\sqrt{b^2}} + \frac{ax}{(a-c)^2} - \frac{cx}{(a-c)^2} - \frac{\sqrt{bx+c}\sqrt{bx+a}}{2(a-c)^2b} + \frac{\sqrt{bx+a}\sqrt{bx+c}}{2(a-c)^2b} - \frac{\sqrt{bx+a}\sqrt{bx+c}}{(a-c)^2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)

[Out] $\frac{x}{(a-c)^2} + \frac{x}{(a-c)^2} + \frac{bx^2}{(a-c)^2} - \frac{1}{(a-c)^2} - \frac{1}{(a-c)^2} \frac{1}{b} (bx+a)^{(1/2)} (bx+c)^{(3/2)} - \frac{1}{2} \frac{1}{(a-c)^2} \frac{1}{b} (bx+c)^{(1/2)} (bx+a)^{(1/2)} a + \frac{1}{2} \frac{1}{(a-c)^2} \frac{1}{b} (bx+c)^{(1/2)} (bx+a)^{(1/2)} c + \frac{1}{4} \frac{1}{(a-c)^2} ((bx+a)(bx+c))^{(1/2)} / ((bx+c)^{(1/2)} / (bx+a)^{(1/2)}) * \ln((1/2*ab+1/2*bc+xb^2)/(b^2)^{(1/2)} + (b^2*x^2+(a+b*bc)*x+ac)^{(1/2)}) / (b^2)^{(1/2)} * a^2 - \frac{1}{2} \frac{1}{(a-c)^2} ((bx+a)(bx+c))^{(1/2)} / ((bx+c)^{(1/2)} / (bx+a)^{(1/2)}) * \ln((1/2*ab+1/2*bc+xb^2)/(b^2)^{(1/2)} + (b^2*x^2+(a+b*bc)*x+ac)^{(1/2)}) / (b^2)^{(1/2)} * ac + \frac{1}{4} \frac{1}{(a-c)^2} ((bx+a)(bx+c))^{(1/2)} / ((bx+c)^{(1/2)} / (bx+a)^{(1/2)}) * \ln((1/2*ab+1/2*bc+xb^2)/(b^2)^{(1/2)} + (b^2*x^2+(a+b*bc)*x+ac)^{(1/2)}) / (b^2)^{(1/2)} * c^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sqrt{bx+a} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((sqrt(b*x+a)+sqrt(b*x+c))^(-2),x)

mupad [B] time = 0.24, size = 110, normalized size = 1.75

$$\frac{bx^2}{(a-c)^2} + \frac{x(a+c)}{(a-c)^2} + \frac{\ln(a+c+2\sqrt{a+bx}\sqrt{c+bx}+2bx)(ab-bc)^2}{4b^3(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}\left(\frac{x}{2} + \frac{ab+bc}{4b^2}\right)}{(a-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a+b*x)^(1/2)+(c+b*x)^(1/2))^2,x)

[Out] $\frac{(bx^2)/(a-c)^2 + (x*(a+c))/(a-c)^2 + (\log(a+c+2*(a+b*x)^(1/2)*(c+b*x)^(1/2)+2*b*x)*(a*b-b*c)^2)/(4*b^3*(a-c)^2) - (2*(a+b*x)^(1/2)*(c+b*x)^(1/2)*(x/2+(a*b+b*c)/(4*b^2)))/(a-c)^2}$

sympy [A] time = 1.04, size = 388, normalized size = 6.16

$$\left(\frac{2a \log(\sqrt{a+bx} + \sqrt{bx+c})}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{a}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4bx \log(\sqrt{a+bx} + \sqrt{bx+c})}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2bx}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{2c \log(\sqrt{a+bx} + \sqrt{bx+c})}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{c}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} + \frac{4\sqrt{a+bx}\sqrt{bx+c} \log(\sqrt{a+bx} + \sqrt{bx+c})}{4ab+8b^2x+4bc+8b\sqrt{a+bx}\sqrt{bx+c}} \right) \text{ for } b \neq 0$$

$$\left(\frac{x}{(\sqrt{a} + \sqrt{c})^2} \right) \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)

[Out] Piecewise((2*a*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + a/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*b*x*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*b*x/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 2*c*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + c/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)) + 4*sqrt(a + b*x)*sqrt(b*x + c)*log(sqrt(a + b*x) + sqrt(b*x + c))/(4*a*b + 8*b**2*x + 4*b*c + 8*b*sqrt(a + b*x)*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**2, True))

$$3.186 \quad \int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$$

Optimal. Leaf size=133

$$\frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2} - \frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2}$$

Rubi [A] time = 0.23, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6689, 101, 157, 63, 217, 206, 93, 208}

$$\frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2} - \frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]
```

```
[Out] (2*b*x)/(a - c)^2 - (2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 - (2*(a + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]])/(a - c)^2 + (4*Sqrt[a]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(a - c)^2 + ((a + c)*Log[x])/(a - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 101

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a*f) + b*n*(d*e - c*f))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))
```

Rule 157

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6689

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^2} dx = \frac{\int \left(2b + \frac{a(1+\frac{c}{a})}{x} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{x} \right) dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x} dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} + \frac{2\int \frac{-ac-\frac{1}{2}b(a+c)x}{x\sqrt{a+bx}\sqrt{c+bx}} dx}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{(2ac)\int \frac{1}{x\sqrt{a+bx}\sqrt{c+bx}} dx}{(a-c)^2} - \frac{b(a+c)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{(4ac)\text{Subst}\left(\int \frac{1}{-a+cx^2} dx, x, \frac{\sqrt{c+bx}}{\sqrt{a+bx}}\right)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{(a+c)\log(x)}{(a-c)^2} - \frac{b(a+c)}{(a-c)^2}$$

$$= \frac{2bx}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2} - \frac{2(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}}\right)}{(a-c)^2} + \frac{4\sqrt{a}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}}\right)}{(a-c)^2} - \frac{b(a+c)}{(a-c)^2}$$

Mathematica [A] time = 1.07, size = 195, normalized size = 1.47

$$\frac{\sqrt{b}\left(-2(c\sqrt{a+bx} + bx(\sqrt{a+bx} - \sqrt{bx+c})) + (a+c)\log(x)\sqrt{bx+c} + 4\sqrt{a}\sqrt{c}\sqrt{bx+c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)\right) - 2(a+c)\sqrt{b(c-a)}\sqrt{\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right)}{\sqrt{b}(a-c)^2\sqrt{bx+c}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2), x]

[Out] (-2*Sqrt[b*(-a + c)]*(a + c)*Sqrt[-((c + b*x)/(a - c))]*ArcSinh[(Sqrt[b]*Sqrt[a + b*x])/Sqrt[b*(-a + c)]] + Sqrt[b]*(-2*(c*Sqrt[a + b*x] + b*x*(Sqrt[a

+ b*x] - Sqrt[c + b*x])) + 4*Sqrt[a]*Sqrt[c]*Sqrt[c + b*x]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])] + (a + c)*Sqrt[c + b*x]*Log[x])/ (Sqrt[b]*(a - c)^2*Sqrt[c + b*x])

IntegrateAlgebraic [A] time = 0.38, size = 144, normalized size = 1.08

$$\frac{2(bx+c)}{(a-c)^2} - \frac{2\sqrt{a+bx}\sqrt{bx+c}}{(a-c)^2} + \frac{\log(-\sqrt{a+bx}\sqrt{bx+c} + \sqrt{a}\sqrt{c} + bx)}{(\sqrt{a} + \sqrt{c})^2} + \frac{\log(\sqrt{a+bx}\sqrt{bx+c} + \sqrt{a}\sqrt{c} - bx)}{(\sqrt{a} - \sqrt{c})^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^2), x]

[Out] (-2*Sqrt[a + b*x]*Sqrt[c + b*x])/(a - c)^2 + (2*(c + b*x))/(a - c)^2 + Log[Sqrt[a]*Sqrt[c] + b*x - Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] + Sqrt[c])^2 + Log[Sqrt[a]*Sqrt[c] - b*x + Sqrt[a + b*x]*Sqrt[c + b*x]]/(Sqrt[a] - Sqrt[c])^2

fricas [A] time = 0.83, size = 290, normalized size = 2.18

$$\frac{2bx + (a+c)\log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c) + (a+c)\log(x) + 2\sqrt{ac}\log\left(\frac{2a^2+2ac^2+2\sqrt{ac}\sqrt{bx+a}\sqrt{bx+c}}{a^2-2ac+c^2}\right) - 2\sqrt{bx+a}\sqrt{bx+c}}{a^2-2ac+c^2} - \frac{2bx + (a+c)\log(-2bx + 2\sqrt{bx+a}\sqrt{bx+c} - a - c) + (a+c)\log(x) - 4\sqrt{-ac}\arctan\left(\frac{-\sqrt{-ac}\sqrt{bx+a}\sqrt{bx+c}}{a}\right) - 2\sqrt{bx+a}\sqrt{bx+c}}{a^2-2ac+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] [(2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a + c)*log(x) + 2*sqrt(a*c)*log((2*a^2*c + 2*a*c^2 + 2*(2*a*c + sqrt(a*c))*(a + c))*sqrt(b*x + a)*sqrt(b*x + c) + (a^2*b + 2*a*b*c + b*c^2)*x + 2*(2*a*c + (a*b + b*c)*x)*sqrt(a*c)))/x - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*c + c^2), (2*b*x + (a + c)*log(-2*b*x + 2*sqrt(b*x + a)*sqrt(b*x + c) - a - c) + (a + c)*log(x) - 4*sqrt(-a*c)*arctan(-(sqrt(-a*c)*b*x - sqrt(-a*c)*sqrt(b*x + a)*sqrt(b*x + c))/(a*c)) - 2*sqrt(b*x + a)*sqrt(b*x + c))/(a^2 - 2*a*c + c^2)]

giac [A] time = 0.67, size = 194, normalized size = 1.46

$$\frac{4ac\arctan\left(\frac{(\sqrt{bx+a}-\sqrt{bx+c})^2-a-c}{2\sqrt{-ac}}\right)}{(a^2-2ac+c^2)\sqrt{-ac}} - \frac{2(a^2-2ac+c^2)\sqrt{bx+a}\sqrt{bx+c}}{a^4-4a^3c+6a^2c^2-4ac^3+c^4} + \frac{(a+c)\log\left(\frac{(\sqrt{bx+a}-\sqrt{bx+c})^2}{a^2-2ac+c^2}\right)}{a^2-2ac+c^2} + \frac{(a+c)\log(|bx|)}{a^2-2ac+c^2} + \frac{2(bx+a)}{a^2-2ac+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 4*a*c*arctan(1/2*((sqrt(b*x + a) - sqrt(b*x + c))^2 - a - c)/sqrt(-a*c))/((a^2 - 2*a*c + c^2)*sqrt(-a*c)) - 2*(a^2 - 2*a*c + c^2)*sqrt(b*x + a)*sqrt(b*x + c)/(a^4 - 4*a^3*c + 6*a^2*c^2 - 4*a*c^3 + c^4) + (a + c)*log((sqrt(b*x + a) - sqrt(b*x + c))^2)/(a^2 - 2*a*c + c^2) + (a + c)*log(abs(b*x))/(a^2 - 2*a*c + c^2) + 2*(b*x + a)/(a^2 - 2*a*c + c^2)

maple [C] time = 0.02, size = 258, normalized size = 1.94

$$\frac{\frac{a\ln(x)}{(a-c)^2} + \frac{2bx}{(a-c)^2} + \frac{c\ln(x)}{(a-c)^2} + \frac{\sqrt{bx+a}\sqrt{bx+c}\left(2ac\operatorname{csign}(b)\ln\left(\frac{abx+bx+2a+2\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}}{x}\right) - \sqrt{ac}\ln\left(\frac{2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\operatorname{csign}(b)}{2}\right)\right) - \sqrt{ac}\ln\left(\frac{2bx+a+c+2\sqrt{b^2x^2+abx+bcx+ac}\operatorname{csign}(b)}{2}\right) - 2\sqrt{b^2x^2+abx+bcx+ac}\sqrt{ac}\operatorname{csign}(b)}{(a-c)^2\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}}}{(a-c)^2\sqrt{ac}\sqrt{b^2x^2+abx+bcx+ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)

[Out] 1/(a-c)^2*a*ln(x)+1/(a-c)^2*c*ln(x)+2*b*x/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x+c)^(1/2)*(2*csign(b)*ln((a*b*x+b*c*x+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)+2*a*c)/x)*a*c-2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1/2)*csign

(b)-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)
)*csgn(b))*a-(a*c)^(1/2)*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)
)*csgn(b))*csgn(b))*c)*csgn(b)/(a*c)^(1/2)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{bx+a} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)

mupad [B] time = 11.14, size = 524, normalized size = 3.94

$$\frac{2bx}{(a-c)^2} - \ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{c+bx}-\sqrt{c}}+1\right)\left(\frac{4c}{(a-c)^2} + \frac{2}{a-c}\right) - \frac{(\sqrt{a+bz}-\sqrt{a})(a+c)}{(\sqrt{a+bz}-\sqrt{c})(a^2-2ac+c^2)} + \frac{(\sqrt{a+bz}-\sqrt{a})(a+c)}{(\sqrt{a+bz}-\sqrt{c})(a^2-2ac+c^2)} - \frac{2b\sqrt{c}(\sqrt{a+bz}-\sqrt{a})^2}{(\sqrt{a+bz}-\sqrt{c})(a^2-2ac+c^2)} - \frac{2\ln\left(\frac{\sqrt{a+bz}-\sqrt{a}}{\sqrt{a+bz}-\sqrt{c}}-1\right)(a+c)}{(a-c)^2} + \frac{\ln(x)(a+c)}{a^2-2ac+c^2} + \frac{2\sqrt{a}\sqrt{c}\ln\left(\frac{\sqrt{a+bz}-\sqrt{a}}{\sqrt{a+bz}-\sqrt{c}}\right)}{(a-c)^2} - \frac{2\sqrt{a}\sqrt{c}\ln\left(\frac{a+(\sqrt{a+bz}-\sqrt{a})}{\sqrt{a+bz}-\sqrt{c}}-\sqrt{a}\sqrt{c} + \frac{(\sqrt{a+bz}-\sqrt{a})}{\sqrt{a+bz}-\sqrt{c}} - \frac{\sqrt{a}\sqrt{c}(\sqrt{a+bz}-\sqrt{a})^2}{(\sqrt{a+bz}-\sqrt{c})^2}\right)}{a^2-2ac+c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)

[Out] (2*b*x)/(a - c)^2 - log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)) + 1)*((4*c)/(a - c)^2 + 2/(a - c)) - (((a + b*x)^(1/2) - a^(1/2))^3*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))^3*(a^2 - 2*a*c + c^2)) + (((a + b*x)^(1/2) - a^(1/2))*(4*a + 4*c))/(((c + b*x)^(1/2) - c^(1/2))*(a^2 - 2*a*c + c^2)) - (16*a^(1/2)*c^(1/2)*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) - c^(1/2))^2*(a^2 - 2*a*c + c^2))/(((a + b*x)^(1/2) - a^(1/2))^4/((c + b*x)^(1/2) - c^(1/2))^4 - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((c + b*x)^(1/2) - c^(1/2))^2 + 1) + (2*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2)) - 1)*(a + c))/(a - c)^2 + (log(x)*(a + c))/(a^2 - 2*a*c + c^2) + (2*a^(1/2)*c^(1/2)*log(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))))/(a - c)^2 - (2*a^(1/2)*c^(1/2)*log((a*((a + b*x)^(1/2) - a^(1/2)))/((c + b*x)^(1/2) - c^(1/2)) - a^(1/2)*c^(1/2) + (c*((a + b*x)^(1/2) - a^(1/2))))/(((c + b*x)^(1/2) - c^(1/2)) - (a^(1/2)*c^(1/2))*((a + b*x)^(1/2) - a^(1/2))^2)/(((c + b*x)^(1/2) - c^(1/2))^2)/(a^2 - 2*a*c + c^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)

[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)

$$3.187 \quad \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})^2} dx$$

Optimal. Leaf size=141

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2} + \frac{2b\log(x)}{(a-c)^2} + \frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2} - \frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2} - \frac{a+c}{x(a-c)^2}$$

Rubi [A] time = 0.22, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {6689, 97, 157, 63, 217, 206, 93, 208}

$$\frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2} + \frac{2b\log(x)}{(a-c)^2} + \frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}(a-c)^2} - \frac{4b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{bx+c}}\right)}{(a-c)^2} - \frac{a+c}{x(a-c)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2), x]

[Out] -((a + c)/((a - c)^2*x)) + (2*Sqrt[a + b*x]*Sqrt[c + b*x])/((a - c)^2*x) - (4*b*ArcTanh[Sqrt[a + b*x]/Sqrt[c + b*x]]/(a - c)^2 + (2*b*(a + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[a]*Sqrt[c + b*x])])/(Sqrt[a]*(a - c)^2*Sqrt[c]) + (2*b*Log[x])/(a - c)^2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int((((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 97

Int(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 157

Int((((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6689

```
Int[(u_)*((e_)*Sqrt[(a_) + (b_)*(x_)^(n_.)] + (f_)*Sqrt[(c_) + (d_)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{c+bx})^2} dx = \frac{\int \left(\frac{a(1+\frac{c}{a})}{x^2} + \frac{2b}{x} - \frac{2\sqrt{a+bx}\sqrt{c+bx}}{x^2} \right) dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2b \log(x)}{(a-c)^2} - \frac{2 \int \frac{\sqrt{a+bx}\sqrt{c+bx}}{x^2} dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b \log(x)}{(a-c)^2} - \frac{2 \int \frac{\frac{1}{2}b(a+c)+b^2x}{x\sqrt{a+bx}\sqrt{c+bx}} dx}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b \log(x)}{(a-c)^2} - \frac{(2b^2) \int \frac{1}{\sqrt{a+bx}\sqrt{c+bx}} dx}{(a-c)^2} - \frac{b(a+c)}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b \log(x)}{(a-c)^2} - \frac{(4b) \text{Subst} \left(\int \frac{1}{\sqrt{-a+c+x^2}} dx, x, \frac{\sqrt{a+bx}}{\sqrt{a-c}} \right)}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} + \frac{2b(a+c) \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}} \right)}{\sqrt{a}(a-c)^2 \sqrt{c}} + \frac{2b \log(x)}{(a-c)^2}$$

$$= -\frac{a+c}{(a-c)^2 x} + \frac{2\sqrt{a+bx}\sqrt{c+bx}}{(a-c)^2 x} - \frac{4b \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{c+bx}} \right)}{(a-c)^2} + \frac{2b(a+c) \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{c+bx}} \right)}{\sqrt{a}(a-c)^2 \sqrt{c}}$$

Mathematica [A] time = 0.96, size = 205, normalized size = 1.45

$$\frac{a(-\sqrt{bx+c})+2c\sqrt{a+bx}+2bx\sqrt{a+bx}-c\sqrt{bx+c}+2bx\log(x)\sqrt{bx+c}}{x} - 4\sqrt{b}\sqrt{b(c-a)}\sqrt{\frac{bx+c}{a-c}}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{a+bx}}{\sqrt{b(c-a)}}\right) + \frac{2b(a+c)\sqrt{bx+c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a}\sqrt{bx+c}}\right)}{\sqrt{a}\sqrt{c}}$$

$$(a-c)^2\sqrt{bx+c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2), x]

[Out] $(-4\sqrt{b}\sqrt{b(-a+c)}\sqrt{-(c+bx)/(a-c)}\operatorname{ArcSinh}(\sqrt{b}\sqrt{a+bx}/\sqrt{b(-a+c)}) + (2b(a+c)\sqrt{c+bx}\operatorname{ArcTanh}(\sqrt{c}\sqrt{a+bx}/(\sqrt{a}\sqrt{c+bx}))/(\sqrt{a}\sqrt{c}) + (2c\sqrt{a+bx} + 2b\sqrt{bx+c} - a\sqrt{c+bx} - c\sqrt{c+bx} + 2b\sqrt{bx+c}\sqrt{c+bx}\operatorname{Log}(x))/x)/((a-c)^2\sqrt{c+bx})$

IntegrateAlgebraic [A] time = 0.75, size = 221, normalized size = 1.57

$$-\frac{ab-2b(bx+c)+3bc}{bx(a-c)^2} + \frac{2\sqrt{a+bx}\sqrt{bx+c}}{x(a-c)^2} + \frac{2b\log(\sqrt{a+bx}\sqrt{bx+c}(2c-2(bx+c))-bx(-a-2(bx+c)+c))}{(\sqrt{a}-\sqrt{c})^2(\sqrt{a}+\sqrt{c})^2} + \frac{2b(a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}\sqrt{bx+c}-bx}{\sqrt{a}\sqrt{c}}\right)}{\sqrt{a}\sqrt{c}(\sqrt{a}-\sqrt{c})^2(\sqrt{a}+\sqrt{c})^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^2),x]

[Out] $(2\sqrt{a+bx}\sqrt{c+bx})/((a-c)^2x) - (ab+3b^2c-2b^2(c+bx))/(b(a-c)^2x) + (2b(a+c)\operatorname{ArcTanh}(-bx+\sqrt{a+bx}\sqrt{c+bx})/(\sqrt{a}\sqrt{c+bx}))/(\sqrt{a}(\sqrt{a}-\sqrt{c})^2(\sqrt{a}+\sqrt{c})^2) + (2b\sqrt{c}\operatorname{Log}(-bx+(a+c-2(c+bx)))) + \sqrt{a+bx}\sqrt{c+bx}(2c-2(c+bx))/(\sqrt{a}-\sqrt{c})^2(\sqrt{a}+\sqrt{c})^2)$

fricas [A] time = 0.66, size = 367, normalized size = 2.60

$$\frac{2abx\log(-2bx+2\sqrt{bx+a}\sqrt{bx+c}-a-c)+2abx\log(bx)+2abx+(ab+bc)\sqrt{bx+c}\log\left(\frac{2bx+2a^2+2(a+c)\sqrt{bx+c}\sqrt{bx+c}-a^2-2ac}{2\sqrt{-ac}}\right)+2\sqrt{bx+a}\sqrt{bx+c}-a^2-2ac}{(a^2-2ac+c^2)^2} + \frac{2abx\log(-2bx+2\sqrt{bx+a}\sqrt{bx+c}-a-c)+2abx\log(bx)+2abx+(ab+bc)\sqrt{bx+c}\log\left(\frac{2bx+2a^2+2(a+c)\sqrt{bx+c}\sqrt{bx+c}-a^2-2ac}{2\sqrt{-ac}}\right)+2\sqrt{bx+a}\sqrt{bx+c}-a^2-2ac}{(a^2-2ac+c^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] $[(2ab^2c^2x\log(-2bx+2\sqrt{bx+a}\sqrt{bx+c})-a-c)+2ab^2c^2x\log(x)+2ab^2c^2x+(ab+bc)\sqrt{ac}x\log((2a^2c+2ac^2+2(2ac+\sqrt{ac})(a+c))\sqrt{bx+a}\sqrt{bx+c})+(a^2b+2ab^2c+b^2c^2)x+2(2ac+(ab+bc)x)\sqrt{ac})/x+2\sqrt{bx+a}\sqrt{bx+c}(a^2c-a^2c-a^2c^2)/((a^3c-2a^2c^2+ac^3)x), (2ab^2c^2x\log(-2bx+2\sqrt{bx+a}\sqrt{bx+c})-a-c)+2ab^2c^2x\log(x)+2ab^2c^2x-2(ab+bc)\sqrt{-ac}x\arctan(-\sqrt{-ac}bx-\sqrt{-ac}\sqrt{bx+a}\sqrt{bx+c})/(ac)+2\sqrt{bx+a}\sqrt{bx+c}(a^2c-a^2c-a^2c^2)/((a^3c-2a^2c^2+ac^3)x)]$

giac [B] time = 1.89, size = 311, normalized size = 2.21

$$\frac{2b\log\left(\frac{\sqrt{bx+a}-\sqrt{bx+c}}{a^2-2ac+c^2}\right)+\frac{2b\log(bx)}{a^2-2ac+c^2}+\frac{2(ab+bc)\arctan\left(\frac{\sqrt{bx+a}-\sqrt{bx+c}}{2\sqrt{-ac}}\right)}{(a^2-2ac+c^2)\sqrt{-ac}}-\frac{4(ab(\sqrt{bx+a}-\sqrt{bx+c})^2+bc(\sqrt{bx+a}-\sqrt{bx+c})^2-a^2b+2abc-bc^2)}{((\sqrt{bx+a}-\sqrt{bx+c})^4-2a(\sqrt{bx+a}-\sqrt{bx+c})^2-2c(\sqrt{bx+a}-\sqrt{bx+c})^2+a^2-2ac+c^2)(a^2-2ac+c^2)}-\frac{2(bx+a)b-ab+bc}{(a^2-2ac+c^2)bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="giac")

[Out] $2b\log((\sqrt{bx+a}-\sqrt{bx+c})^2)/(a^2-2ac+c^2)+2b\log(\operatorname{abs}(bx))/(a^2-2ac+c^2)+2(ab+bc)\arctan(1/2((\sqrt{bx+a}-\sqrt{bx+c})^2-a-c)/\sqrt{-ac})/((a^2-2ac+c^2)\sqrt{-ac})-4(ab(\sqrt{bx+a}-\sqrt{bx+c})^2+b^2c(\sqrt{bx+a}-\sqrt{bx+c})^2-a^2b+2ab^2c-b^2c^2)/(((\sqrt{bx+a}-\sqrt{bx+c})^4-2a(\sqrt{bx+a}-\sqrt{bx+c})^2-2c(\sqrt{bx+a}-\sqrt{bx+c})^2+a^2-2ac+c^2)(a^2-2ac+c^2))-(2(bx+a)b-ab+bc)/((a^2-2ac+c^2)bx)$

maple [C] time = 0.02, size = 274, normalized size = 1.94

$$\frac{2b\ln(x)}{(a-c)^2}-\frac{a}{(a-c)^2x}-\frac{c}{(a-c)^2x}+\frac{\sqrt{bx+a}\sqrt{bx+c}\left(abx\operatorname{csign}(b)\ln\left(\frac{abx+cx+2a+2\sqrt{ac}\sqrt{bx+a}\sqrt{bx+c}}{x}\right)+bcx\operatorname{csign}(b)\ln\left(\frac{abx+cx+2a+2\sqrt{ac}\sqrt{bx+a}\sqrt{bx+c}}{x}\right)-2\sqrt{ac}bx\ln\left(\frac{2bx+a+2\sqrt{bx+a}\sqrt{bx+c}\operatorname{csign}(b)}{2}\right)\operatorname{sgn}(b)\right)}{(a-c)^2\sqrt{bx+a}\sqrt{bx+c}+abx+bcx+ac\sqrt{ac}x}\operatorname{csign}(b)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x)

[Out] -1/x/(a-c)^2*a-1/x/(a-c)^2*c+2*b*ln(x)/(a-c)^2+1/(a-c)^2*(b*x+a)^(1/2)*(b*x+c)^(1/2)*(csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2))/x)*x*a+b+csgn(b)*ln((a*b*x+b*c*x+2*a*c+2*(a*c)^(1/2)*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2))/x)*x*b*c-2*ln(1/2*(2*b*x+a+c+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*csgn(b)))*csgn(b))*x*b*(a*c)^(1/2)+2*(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)*(a*c)^(1/2)*csgn(b))*csgn(b)/(b^2*x^2+a*b*x+b*c*x+a*c)^(1/2)/x/(a*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(\sqrt{bx+a} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^2), x)

mupad [B] time = 28.82, size = 7637, normalized size = 54.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^2),x)

[Out] (2*b*log(x))/(a^2 - 2*a*c + c^2) - (((a + b*x)^(1/2) - a^(1/2))^2*((a^2*b)/2 + (b*c^2)/2 - (3*a*b*c)/2))/(((c + b*x)^(1/2) - c^(1/2))^2*(a*c^3 + a^3*c - 2*a^2*c^2)) - b/(2*(a^2 - 2*a*c + c^2)) + (a^(1/2)*c^(1/2)*((a*b)/2 + (b*c)/2)*((a + b*x)^(1/2) - a^(1/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a*c^3 + a^3*c - 2*a^2*c^2)))/(((a + b*x)^(1/2) - a^(1/2))/((c + b*x)^(1/2) - c^(1/2))) + ((a + b*x)^(1/2) - a^(1/2))^3/((c + b*x)^(1/2) - c^(1/2))^3 - ((a + c)*((a + b*x)^(1/2) - a^(1/2))^2)/(a^(1/2)*c^(1/2)*((c + b*x)^(1/2) - c^(1/2))^2)) + (b*atan(((b*((4*(4*a^4*b^3*c^12 + 8*a^5*b^3*c^11 - 32*a^6*b^3*c^10 - 8*a^7*b^3*c^9 + 56*a^8*b^3*c^8 - 8*a^9*b^3*c^7 - 32*a^10*b^3*c^6 + 8*a^11*b^3*c^5 + 4*a^12*b^3*c^4)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*b*((4*(16*a^6*b*c^14 - 4*a^5*b*c^15 + 12*a^7*b*c^13 - 192*a^8*b*c^12 + 504*a^9*b*c^11 - 672*a^10*b*c^10 + 504*a^11*b*c^9 - 192*a^12*b*c^8 + 12*a^13*b*c^7 + 16*a^14*b*c^6 - 4*a^15*b*c^5)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) + (4*b*((4*(a^(9/2)*c^(35/2) - 8*a^(11/2)*c^(33/2) + 27*a^(13/2)*c^(31/2) - 49*a^(15/2)*c^(29/2) + 50*a^(17/2)*c^(27/2) - 27*a^(19/2)*c^(25/2) + 6*a^(21/2)*c^(23/2) + 6*a^(23/2)*c^(21/2) - 27*a^(25/2)*c^(19/2) + 50*a^(27/2)*c^(17/2) - 49*a^(29/2)*c^(15/2) + 27*a^(31/2)*c^(13/2) - 8*a^(33/2)*c^(11/2) + a^(35/2)*c^(9/2)))/(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7) - (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^4*c^18 - 47*a^5*c^17 + 268*a^6*c^16 - 982*a^7*c^15 + 2564*a^8*c^14 - 4993*a^9*c^13 + 7404*a^10*c^12 - 8436*a^11*c^11 + 7404*a^12*c^10 - 4993*a^13*c^9 + 2564*a^14*c^8 - 982*a^15*c^7) + 268*a^16*c^6 - 47*a^17*c^5 + 4*a^18*c^4)))/(((c + b*x)^(1/2) - c^(1/2))*(a^7*c^15 - 8*a^8*c^14 + 28*a^9*c^13 - 56*a^10*c^12 + 70*a^11*c^11 - 56*a^12*c^10 + 28*a^13*c^9 - 8*a^14*c^8 + a^15*c^7)))/((a - c)^2 + (2*((a + b*x)^(1/2) - a^(1/2))*(4*a^(7/2)*b*c^(33/2) - 43*a^(9/2)*b*c^(31/2) + 231*a^(11/2)*b*c^(29/2) - 749*a^(13/2)*b*c^(27/2) + 1505*a^(15/2)*b*c^(25/2) - 1770*a^(17/2)*b*c^(23/2) + 822*a^(19/2)*b*c^(21/2) + 822*a^(21/2)*b*c^(19/2) - 1770*a^(23/2)*b*c^(17/2) + 1505*a^(25/2)*b*c^(15/2) - 749*a^(27/2)*b*c^(13

$$\begin{aligned}
& /2) + 231a^{(29/2)}b^2c^{(11/2)} - 43a^{(31/2)}b^2c^{(9/2)} + 4a^{(33/2)}b^2c^{(7/2)} \\
&))/(((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} \\
& + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) / (a - c)^2 - (4*(a^{(7/2)}b^2c^{(29/2)} + 12a^{(9/2)}b^2c^{(27/2)} - 100 \\
& *a^{(11/2)}b^2c^{(25/2)} + 285a^{(13/2)}b^2c^{(23/2)} - 390a^{(15/2)}b^2c^{(21/2)} + 192a^{(17/2)}b^2c^{(19/2)} + 192a^{(19/2)}b^2c^{(17/2)} - 390a^{(21/2)} \\
& *b^2c^{(15/2)} + 285a^{(23/2)}b^2c^{(13/2)} - 100a^{(25/2)}b^2c^{(11/2)} + 12a^{(27/2)}b^2c^{(9/2)} + a^{(29/2)}b^2c^{(7/2)})) / (a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} \\
& + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7) + (2*((a + b^2x)^{(1/2)} - a^{(1/2)})*(73a^4b^2c^{14} - 570a^5b^2c^{13} + 2053a^6b^2c^{12} - 4568a^7b^2c^{11} \\
& + 7090a^8b^2c^{10} - 8156a^9b^2c^9 + 7090a^{10}b^2c^8 - 4568a^{11}b^2c^7 + 2053a^{12}b^2c^6 - 570a^{13}b^2c^5 + 73a^{14}b^2c^4)) / (((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} \\
& - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) / (a - c)^2 - (2*((a + b^2x)^{(1/2)} - a^{(1/2)}) \\
& *(65a^{(7/2)}b^3c^{(25/2)} - 427a^{(9/2)}b^3c^{(23/2)} + 1256a^{(11/2)}b^3c^{(21/2)} - 1856a^{(13/2)}b^3c^{(19/2)} + 962a^{(15/2)}b^3c^{(17/2)} + 962a^{(17/2)}b^3c^{(15/2)} \\
& - 1856a^{(19/2)}b^3c^{(13/2)} + 1256a^{(21/2)}b^3c^{(11/2)} - 427a^{(23/2)}b^3c^{(9/2)} + 65a^{(25/2)}b^3c^{(7/2)})) / (((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} \\
& - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) * 4i) / (a - c)^2 + (b*((4*(4a^4b^3c^{12} + 8a^5b^3c^{11} - 32a^6b^3c^{10} - 8a^7b^3c^9 \\
& + 56a^8b^3c^8 - 8a^9b^3c^7 - 32a^{10}b^3c^6 + 8a^{11}b^3c^5 + 4a^{12}b^3c^4)) / (a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} \\
& + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7) + (4*b*((4*(a^{(7/2)}b^2c^{(29/2)} + 12a^{(9/2)}b^2c^{(27/2)} - 100a^{(11/2)}b^2c^{(25/2)} + 285a^{(13/2)}b^2c^{(23/2)} - 390a^{(15/2)}b^2c^{(21/2)} \\
& + 192a^{(17/2)}b^2c^{(19/2)} + 192a^{(19/2)}b^2c^{(17/2)} - 390a^{(21/2)}b^2c^{(15/2)} + 285a^{(23/2)}b^2c^{(13/2)} - 100a^{(25/2)}b^2c^{(11/2)} + 12a^{(27/2)}b^2c^{(9/2)} + a^{(29/2)}b^2c^{(7/2)})) \\
& / (a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7) + (4*b*((4*(16a^6b^2c^{14} - 4a^5b^2c^{15} + 12a^7b^2c^{13} \\
& - 192a^8b^2c^{12} + 504a^9b^2c^{11} - 672a^{10}b^2c^{10} + 504a^{11}b^2c^9 - 192a^{12}b^2c^8 + 12a^{13}b^2c^7 + 16a^{14}b^2c^6 - 4a^{15}b^2c^5)) / (a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} \\
& - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7) - (4*b*((4*(a^{(9/2)}c^{(35/2)} - 8a^{(11/2)}c^{(33/2)} + 27a^{(13/2)}c^{(31/2)} - 49a^{(15/2)}c^{(29/2)} \\
& + 50a^{(17/2)}c^{(27/2)} - 27a^{(19/2)}c^{(25/2)} + 6a^{(21/2)}c^{(23/2)} + 6a^{(23/2)}c^{(21/2)} - 27a^{(25/2)}c^{(19/2)} + 50a^{(27/2)}c^{(17/2)} - 49a^{(29/2)}c^{(15/2)} \\
& + 27a^{(31/2)}c^{(13/2)} - 8a^{(33/2)}c^{(11/2)} + a^{(35/2)}c^{(9/2)})) / (a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 \\
& + a^{15}c^7) - (2*((a + b^2x)^{(1/2)} - a^{(1/2)})*(4a^4c^{18} - 47a^5c^{17} + 268a^6c^{16} - 982a^7c^{15} + 2564a^8c^{14} - 4993a^9c^{13} + 7404a^{10}c^{12} - 8436a^{11}c^{11} \\
& + 7404a^{12}c^{10} - 4993a^{13}c^9 + 2564a^{14}c^8 - 982a^{15}c^7 + 268a^{16}c^6 - 47a^{17}c^5 + 4a^{18}c^4)) / (((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} - 8a^8c^{14} + 28a^9c^{13} \\
& - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) / (a - c)^2 + (2*((a + b^2x)^{(1/2)} - a^{(1/2)})*(4a^{(7/2)}b^2c^{(33/2)} - 43a^{(9/2)}b^2c^{(31/2)} \\
& + 231a^{(11/2)}b^2c^{(29/2)} - 749a^{(13/2)}b^2c^{(27/2)} + 1505a^{(15/2)}b^2c^{(25/2)} - 1770a^{(17/2)}b^2c^{(23/2)} + 822a^{(19/2)}b^2c^{(21/2)} + 822a^{(21/2)}b^2c^{(19/2)} - 1770a^{(23/2)}b^2c^{(17/2)} \\
& + 1505a^{(25/2)}b^2c^{(15/2)} - 749a^{(27/2)}b^2c^{(13/2)} + 231a^{(29/2)}b^2c^{(11/2)} - 43a^{(31/2)}b^2c^{(9/2)} + 4a^{(33/2)}b^2c^{(7/2)})) / (((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} - 8a^8c^{14} \\
& + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) / (a - c)^2 - (2*((a + b^2x)^{(1/2)} - a^{(1/2)})*(73a^4b^2c^{14} - 570a^5b^2c^{13} + 2053a^6b^2c^{12} \\
& - 4568a^7b^2c^{11} + 7090a^8b^2c^{10} - 8156a^9b^2c^9 + 7090a^{10}b^2c^8 - 4568a^{11}b^2c^7 + 2053a^{12}b^2c^6 - 570a^{13}b^2c^5 + 73a^{14}b^2c^4)) / (((c + b^2x)^{(1/2)} - c^{(1/2)})*(a^7c^{15} - 8a^8c^{14} \\
& + 28a^9c^{13} - 56a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7))) / (a - c)^2
\end{aligned}$$

$$\begin{aligned}
& 6a^{10}c^{12} + 70a^{11}c^{11} - 56a^{12}c^{10} + 28a^{13}c^9 - 8a^{14}c^8 + a^{15}c^7) + (4b*((4*(a^{(7/2)}*b^2*c^{(29/2)} + 12*a^{(9/2)}*b^2*c^{(27/2)} - 100*a^{(11/2)}*b^2*c^{(25/2)} + 285*a^{(13/2)}*b^2*c^{(23/2)} - 390*a^{(15/2)}*b^2*c^{(21/2)} + 192*a^{(17/2)}*b^2*c^{(19/2)} + 192*a^{(19/2)}*b^2*c^{(17/2)} - 390*a^{(21/2)}*b^2*c^{(15/2)} + 285*a^{(23/2)}*b^2*c^{(13/2)} - 100*a^{(25/2)}*b^2*c^{(11/2)} + 12*a^{(27/2)}*b^2*c^{(9/2)} + a^{(29/2)}*b^2*c^{(7/2)})))/(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7) + (4*b*((4*(16*a^6*b*c^{14} - 4*a^5*b*c^{15} + 12*a^7*b*c^{13} - 192*a^8*b*c^{12} + 504*a^9*b*c^{11} - 672*a^{10}*b*c^{10} + 504*a^{11}*b*c^9 - 192*a^{12}*b*c^8 + 12*a^{13}*b*c^7 + 16*a^{14}*b*c^6 - 4*a^{15}*b*c^5)))/(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7) - (4*b*((4*(a^{(9/2)}*c^{(35/2)} - 8*a^{(11/2)}*c^{(33/2)} + 27*a^{(13/2)}*c^{(31/2)} - 49*a^{(15/2)}*c^{(29/2)} + 50*a^{(17/2)}*c^{(27/2)} - 27*a^{(19/2)}*c^{(25/2)} + 6*a^{(21/2)}*c^{(23/2)} + 6*a^{(23/2)}*c^{(21/2)} - 27*a^{(25/2)}*c^{(19/2)} + 50*a^{(27/2)}*c^{(17/2)} - 49*a^{(29/2)}*c^{(15/2)} + 27*a^{(31/2)}*c^{(13/2)} - 8*a^{(33/2)}*c^{(11/2)} + a^{(35/2)}*c^{(9/2)})))/(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7) - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(4*a^4*c^{18} - 47*a^5*c^{17} + 268*a^6*c^{16} - 982*a^7*c^{15} + 2564*a^8*c^{14} - 4993*a^9*c^{13} + 7404*a^{10}*c^{12} - 8436*a^{11}*c^{11} + 7404*a^{12}*c^{10} - 4993*a^{13}*c^9 + 2564*a^{14}*c^8 - 982*a^{15}*c^7 + 268*a^{16}*c^6 - 47*a^{17}*c^5 + 4*a^{18}*c^4)))/(((c + b*x)^{(1/2)} - c^{(1/2)})*(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7)))/((a + b*x)^{(1/2)} - a^{(1/2)})*(4*a^{(7/2)}*b*c^{(33/2)} - 43*a^{(9/2)}*b*c^{(31/2)} + 231*a^{(11/2)}*b*c^{(29/2)} - 749*a^{(13/2)}*b*c^{(27/2)} + 1505*a^{(15/2)}*b*c^{(25/2)} - 1770*a^{(17/2)}*b*c^{(23/2)} + 822*a^{(19/2)}*b*c^{(21/2)} + 822*a^{(21/2)}*b*c^{(19/2)} - 1770*a^{(23/2)}*b*c^{(17/2)} + 1505*a^{(25/2)}*b*c^{(15/2)} - 749*a^{(27/2)}*b*c^{(13/2)} + 231*a^{(29/2)}*b*c^{(11/2)} - 43*a^{(31/2)}*b*c^{(9/2)} + 4*a^{(33/2)}*b*c^{(7/2)})))/(((c + b*x)^{(1/2)} - c^{(1/2)})*(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7)))/((a - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(73*a^4*b^2*c^{14} - 570*a^5*b^2*c^{13} + 2053*a^6*b^2*c^{12} - 4568*a^7*b^2*c^{11} + 7090*a^8*b^2*c^{10} - 8156*a^9*b^2*c^9 + 7090*a^{10}*b^2*c^8 - 4568*a^{11}*b^2*c^7 + 2053*a^{12}*b^2*c^6 - 570*a^{13}*b^2*c^5 + 73*a^{14}*b^2*c^4)))/(((c + b*x)^{(1/2)} - c^{(1/2)})*(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7)))/((a - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(65*a^{(7/2)}*b^3*c^{(25/2)} - 427*a^{(9/2)}*b^3*c^{(23/2)} + 1256*a^{(11/2)}*b^3*c^{(21/2)} - 1856*a^{(13/2)}*b^3*c^{(19/2)} + 962*a^{(15/2)}*b^3*c^{(17/2)} + 962*a^{(17/2)}*b^3*c^{(15/2)} - 1856*a^{(19/2)}*b^3*c^{(13/2)} + 1256*a^{(21/2)}*b^3*c^{(11/2)} - 427*a^{(23/2)}*b^3*c^{(9/2)} + 65*a^{(25/2)}*b^3*c^{(7/2)})))/(((c + b*x)^{(1/2)} - c^{(1/2)})*(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7)))/((a - c)^2 + (4*((a + b*x)^{(1/2)} - a^{(1/2)})*(224*a^5*b^4*c^9 - 112*a^4*b^4*c^8 - 112*a^6*b^4*c^8 - 448*a^7*b^4*c^7 + 112*a^8*b^4*c^6 + 224*a^9*b^4*c^5 - 112*a^{10}*b^4*c^4)))/(((c + b*x)^{(1/2)} - c^{(1/2)})*(a^7*c^{15} - 8*a^8*c^{14} + 28*a^9*c^{13} - 56*a^{10}*c^{12} + 70*a^{11}*c^{11} - 56*a^{12}*c^{10} + 28*a^{13}*c^9 - 8*a^{14}*c^8 + a^{15}*c^7)))*8i)/(a - c)^2 - (log((a*((a + b*x)^{(1/2)} - a^{(1/2)})))/((c + b*x)^{(1/2)} - c^{(1/2)}) - a^{(1/2)}*c^{(1/2)} + (c*((a + b*x)^{(1/2)} - a^{(1/2)})))/((c + b*x)^{(1/2)} - c^{(1/2)}) - (a^{(1/2)}*c^{(1/2)}*((a + b*x)^{(1/2)} - a^{(1/2)})^2)/((c + b*x)^{(1/2)} - c^{(1/2)})^2)*(a^{(1/2)}*b*c^{(3/2)} + a^{(3/2)}*b*c^{(1/2)}))/(a*c^3 + a^3*c - 2*a^2*c^2) - (a + c)/(x*(a^2 - 2*a*c + c^2)) + (log(((a + b*x)^{(1/2)} - a^{(1/2)})/((c + b*x)^{(1/2)} - c^{(1/2)}))*(a^{(1/2)}*b*c^{(3/2)} + a^{(3/2)}*b*c^{(1/2)}))/(a*c^3 + a^3*c - 2*a^2*c^2) + (b*((a + b*x)^{(1/2)} - a^{(1/2)}))/(2*(a - c)^2*((c + b*x)^{(1/2)} - c^{(1/2)}))
\end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{bx+c})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**2,x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**2), x)
```

$$3.188 \quad \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$$

Optimal. Leaf size=375

$$-\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3} - \frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3} - \frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3}$$

Rubi [A] time = 0.37, antiderivative size = 375, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, number of rules / integrand size = 0.080, Rules used = {6689, 43}

$$\frac{24a^2(a+bx)^{3/2}}{5b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} - \frac{8a^3(a+bx)^{5/2}}{3b^3(a-c)^3} - \frac{24c^2(bx+c)^{5/2}}{5b^3(a-c)^3} + \frac{8c^3(bx+c)^{3/2}}{3b^3(a-c)^3} - \frac{2c^2(3a+c)(bx+c)^{3/2}}{3b^3(a-c)^3} + \frac{8(a+bx)^{9/2}}{9b^3(a-c)^3} + \frac{2(a+3c)(a+bx)^{7/2}}{7b^3(a-c)^3} - \frac{24a(a+bx)^{7/2}}{7b^3(a-c)^3} - \frac{4a(a+3c)(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{8(bx+c)^{9/2}}{9b^3(a-c)^3} + \frac{24c(bx+c)^{7/2}}{7b^3(a-c)^3} - \frac{2(3a+c)(bx+c)^{7/2}}{7b^3(a-c)^3} + \frac{4c(3a+c)(bx+c)^{5/2}}{5b^3(a-c)^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]

[Out]
$$\frac{-8a^3(a+bx)^{3/2}}{(3b^3(a-c)^3)} + \frac{(2a^2(a+3c)(a+bx)^{3/2})}{(3b^3(a-c)^3)} + \frac{(24a^2(a+bx)^{5/2})}{(5b^3(a-c)^3)} - \frac{(4a^3(a+bx)^{9/2})}{(5b^3(a-c)^3)} + \frac{(24a^2(a+bx)^{7/2})}{(7b^3(a-c)^3)} - \frac{(24a^2(a+bx)^{5/2})}{(7b^3(a-c)^3)} + \frac{(8c^3(bx+c)^{3/2})}{(7b^3(a-c)^3)} + \frac{(8c^3(bx+c)^{9/2})}{(9b^3(a-c)^3)} + \frac{(8c^2(3a+c)(bx+c)^{3/2})}{(3b^3(a-c)^3)} - \frac{(24c^2(bx+c)^{5/2})}{(3b^3(a-c)^3)} - \frac{(24c^2(bx+c)^{7/2})}{(5b^3(a-c)^3)} + \frac{(4c^3(bx+c)^{5/2})}{(5b^3(a-c)^3)} + \frac{(4c^3(bx+c)^{7/2})}{(7b^3(a-c)^3)} - \frac{(2c^2(3a+c)(bx+c)^{7/2})}{(7b^3(a-c)^3)} - \frac{(2c^2(3a+c)(bx+c)^{5/2})}{(7b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{3/2})}{(7b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{9/2})}{(9b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{7/2})}{(9b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{5/2})}{(9b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{3/2})}{(9b^3(a-c)^3)} - \frac{(8c^2(3a+c)(bx+c)^{1/2})}{(9b^3(a-c)^3)}$$

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_.), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx &= \frac{\int \left(a \left(1 + \frac{3c}{a} \right) x^2 \sqrt{a+bx} + 4bx^3 \sqrt{a+bx} - 3a \left(1 + \frac{c}{3a} \right) x^2 \sqrt{c+bx} - 4bx^3 \sqrt{c+bx} \right)}{(a-c)^3} \\ &= \frac{(4b) \int x^3 \sqrt{a+bx} dx}{(a-c)^3} - \frac{(4b) \int x^3 \sqrt{c+bx} dx}{(a-c)^3} - \frac{(3a+c) \int x^2 \sqrt{c+bx} dx}{(a-c)^3} + \frac{(a-c) \int x^2 \sqrt{a+bx} dx}{(a-c)^3} \\ &= \frac{(4b) \int \left(-\frac{a^3 \sqrt{a+bx}}{b^3} + \frac{3a^2(a+bx)^{3/2}}{b^3} - \frac{3a(a+bx)^{5/2}}{b^3} + \frac{(a+bx)^{7/2}}{b^3} \right) dx}{(a-c)^3} - \frac{(4b) \int \left(-\frac{c^3 \sqrt{c+bx}}{b^3} \right) dx}{(a-c)^3} \\ &= -\frac{8a^3(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{2a^2(a+3c)(a+bx)^{3/2}}{3b^3(a-c)^3} + \frac{24a^2(a+bx)^{5/2}}{5b^3(a-c)^3} - \frac{4a(a+3c)(a+bx)^{3/2}}{5b^3(a-c)^3} \end{aligned}$$

Mathematica [A] time = 0.40, size = 282, normalized size = 0.75

$$\frac{2(-40a^4\sqrt{a+bx} + 4a^3\sqrt{a+bx}(5bx+18c) - 3a^2bx\sqrt{a+bx}(5bx+12c) + a(5b^3x^3(13\sqrt{a+bx} - 27\sqrt{bx+c}) + 27b^2cx^2(\sqrt{a+bx} - \sqrt{bx+c}) - 72c^2\sqrt{bx+c} + 36bc^2\sqrt{bx+c}) + 5(28b^4x^4(\sqrt{a+bx} - \sqrt{bx+c}) + b^3cx^3(27\sqrt{a+bx} - 13\sqrt{bx+c}) + 3b^2c^2x^2\sqrt{bx+c} + 8c^4\sqrt{bx+c} - 4b^2cx\sqrt{bx+c}))}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]

[Out] (2*(-40*a^4*Sqrt[a + b*x] - 3*a^2*b*x*Sqrt[a + b*x]*(12*c + 5*b*x) + 4*a^3*Sqrt[a + b*x]*(18*c + 5*b*x) + a*(-72*c^3*Sqrt[c + b*x] + 36*b*c^2*x*Sqrt[c + b*x] + 5*b^3*x^3*(13*Sqrt[a + b*x] - 27*Sqrt[c + b*x])) + 27*b^2*c*x^2*(Sqrt[a + b*x] - Sqrt[c + b*x])) + 5*(8*c^4*Sqrt[c + b*x] - 4*b*c^3*x*Sqrt[c + b*x] + 3*b^2*c^2*x^2*Sqrt[c + b*x] + b^3*c*x^3*(27*Sqrt[a + b*x] - 13*Sqrt[c + b*x])) + 28*b^4*x^4*(Sqrt[a + b*x] - Sqrt[c + b*x]))/(315*b^3*(a - c)^3)

IntegrateAlgebraic [A] time = 0.82, size = 267, normalized size = 0.71

$$\frac{2\sqrt{a+bx}(40a^4 - 20a^3(bx+c) - 52a^2c + 15a^2(bx+c)^2 + 6a^2c(bx+c) - 21a^2c^2 - 141ac^2(bx+c) - 65a^2c^2 + 168ac(bx+c)^2 + 38ac^2 + 155c^2(bx+c) - 435c^2(bx+c)^2 - 140(bx+c)^3 + 425c^3(bx+c)^2 - 5c^4)}{315b^3(a-c)^3} - \frac{2(315a^2c^2(bx+c)^{3/2} + 135a(bx+c)^{5/2} - 378ac(bx+c)^{5/2} - 315c^2(bx+c)^{3/2} + 630c^2(bx+c)^{5/2} + 140(bx+c)^{7/2} - 495c(bx+c)^{7/2})}{315b^3(a-c)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]

[Out] (-2*Sqrt[a + b*x]*(40*a^4 - 52*a^3*c - 21*a^2*c^2 + 38*a*c^3 - 5*c^4 - 20*a^3*(c + b*x) + 6*a^2*c*(c + b*x) - 141*a*c^2*(c + b*x) + 155*c^3*(c + b*x) + 15*a^2*(c + b*x)^2 + 168*a*c*(c + b*x)^2 - 435*c^2*(c + b*x)^2 - 65*a*(c + b*x)^3 + 425*c*(c + b*x)^3 - 140*(c + b*x)^4)/(315*b^3*(a - c)^3) - (2*(315*a*c^2*(c + b*x)^(3/2) - 315*c^3*(c + b*x)^(3/2) - 378*a*c*(c + b*x)^(5/2) + 630*c^2*(c + b*x)^(5/2) + 135*a*(c + b*x)^(7/2) - 495*c*(c + b*x)^(7/2) + 140*(c + b*x)^(9/2)))/(315*b^3*(a - c)^3)

fricas [A] time = 0.55, size = 208, normalized size = 0.55

$$\frac{2((140b^4x^4 - 40a^4 + 72a^3c + 5(13ab^3 + 27b^3c)x^3 - 3(5a^2b^2 - 9ab^2c)x^2 + 4(5a^3b - 9a^2bc)x)\sqrt{bx+a} - (140b^4x^4 + 72ac^3 - 40c^4 + 5(27ab^3 + 13b^3c)x^3 + 3(9ab^2c - 5b^2c^2)x^2 - 4(9abc^2 - 5bc^3)x)\sqrt{bx+c})}{315(a^3b^3 - 3a^2b^3c + 3ab^3c^2 - b^3c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")

[Out] 2/315*((140*b^4*x^4 - 40*a^4 + 72*a^3*c + 5*(13*a*b^3 + 27*b^3*c)*x^3 - 3*(5*a^2*b^2 - 9*a*b^2*c)*x^2 + 4*(5*a^3*b - 9*a^2*b*c)*x)*sqrt(b*x + a) - (140*b^4*x^4 + 72*a*c^3 - 40*c^4 + 5*(27*a*b^3 + 13*b^3*c)*x^3 + 3*(9*a*b^2*c - 5*b^2*c^2)*x^2 - 4*(9*a*b*c^2 - 5*b*c^3)*x)*sqrt(b*x + c))/(a^3*b^3 - 3*a^2*b^3*c + 3*a*b^3*c^2 - b^3*c^3)

giac [B] time = 0.81, size = 1447, normalized size = 3.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")

[Out] -2/315*((5*(b*x + a)*(28*(a^9*b^12 - 9*a^8*b^12*c + 36*a^7*b^12*c^2 - 84*a^6*b^12*c^3 + 126*a^5*b^12*c^4 - 126*a^4*b^12*c^5 + 84*a^3*b^12*c^6 - 36*a^2*b^12*c^7 + 9*a*b^12*c^8 - b^12*c^9)*(b*x + a)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12) - (85*a^10*b^12 - 778*a^9*b^12*c + 3177*a^8*b^12*c^2 - 7608*a^7*b^12*c^3 + 11802*a^6*b^12*c^4 - 12348*a^5*b^12*c^5 + 8778*a^4*b^12*c^6 - 4152*a^3*b^12*c^7 + 1233*a^2*b^12*c^8 - 202*a*b^12*c^9 + 13*b^12*c^10)/(a^12*b^15 - 12*a^11*b^15*c + 66*a^10*b^15*c^2 - 220*a^9*b^15*c^3 + 495*a^8*b^15*c^4 - 792*a^7*b^15*c^5 + 924*a^6*b^15*c^6 - 792*a^5*b^15*c^7 + 495*a^4*b^15*c^8 - 220*a^3*b^15*c^9 + 66*a^2*b^15*c^10 - 12*a*b^15*c^11 + b^15*c^12))

$\wedge{15}c^6 - 792a^5b^15c^7 + 495a^4b^15c^8 - 220a^3b^15c^9 + 66a^2b^15c^{10} - 12ab^15c^{11} + b^15c^{12})) + 3*(145a^{11}b^{12} - 1361a^{10}b^{12} * c + 5719a^9b^{12}c^2 - 14151a^8b^{12}c^3 + 22794a^7b^{12}c^4 - 24906a^6b^{12}c^5 + 18606a^5b^{12}c^6 - 9294a^4b^{12}c^7 + 2901a^3b^{12}c^8 - 4 69a^2b^{12}c^9 + 11ab^{12}c^{10} + 5b^{12}c^{11})/(a^{12}b^{15} - 12a^{11}b^{15}c + 66a^{10}b^{15}c^2 - 220a^9b^{15}c^3 + 495a^8b^{15}c^4 - 792a^7b^{15}c^5 + 924a^6b^{15}c^6 - 792a^5b^{15}c^7 + 495a^4b^{15}c^8 - 220a^3b^{15}c^9 + 66a^2b^{15}c^{10} - 12ab^{15}c^{11} + b^{15}c^{12}))* (bx + a) - (155a^{12}b^{12} - 1536a^{11}b^{12}c + 6855a^{10}b^{12}c^2 - 18170a^9b^{12}c^3 + 31770a^8b^{12}c^4 - 38520a^7b^{12}c^5 + 33222a^6b^{12}c^6 - 20700a^5b^{12}c^7 + 9495a^4b^{12}c^8 - 3320a^3b^{12}c^9 + 915a^2b^{12}c^{10} - 186ab^{12}c^{11} + 20b^{12}c^{12})/(a^{12}b^{15} - 12a^{11}b^{15}c + 66a^{10}b^{15}c^2 - 220a^9b^{15}c^3 + 495a^8b^{15}c^4 - 792a^7b^{15}c^5 + 924a^6b^{15}c^6 - 792a^5b^{15}c^7 + 495a^4b^{15}c^8 - 220a^3b^{15}c^9 + 66a^2b^{15}c^{10} - 12ab^{15}c^{11} + b^{15}c^{12}))* (bx + a) + (5a^{13}b^{12} - 83a^{12}b^{12}c + 543a^{11}b^{12}c^2 - 1925a^{10}b^{12}c^3 + 4070a^9b^{12}c^4 - 4950a^8b^{12}c^5 + 2 046a^7b^{12}c^6 + 3894a^6b^{12}c^7 - 8415a^5b^{12}c^8 + 8305a^4b^{12}c^9 - 5005a^3b^{12}c^{10} + 1887a^2b^{12}c^{11} - 412ab^{12}c^{12} + 40b^{12}c^{13})/(a^{12}b^{15} - 12a^{11}b^{15}c + 66a^{10}b^{15}c^2 - 220a^9b^{15}c^3 + 495a^8b^{15}c^4 - 792a^7b^{15}c^5 + 924a^6b^{15}c^6 - 792a^5b^{15}c^7 + 495a^4b^{15}c^8 - 220a^3b^{15}c^9 + 66a^2b^{15}c^{10} - 12ab^{15}c^{11} + b^{15}c^{12}))*sqrt(bx + c) + 2/315*(140*(bx + a)^(9/2) - 495*(bx + a)^(7/2)*a + 630*(bx + a)^(5/2)*a^2 - 315*(bx + a)^(3/2)*a^3 + 135*(bx + a)^(7/2)*c - 378*(bx + a)^(5/2)*ac + 315*(bx + a)^(3/2)*a^2*c)/(a^3b^3 - 3a^2b^3c + 3ab^3c^2 - b^3c^3)$

maple [A] time = 0.01, size = 294, normalized size = 0.78

$$\frac{2 \left(\frac{(bx+a)^3 a^2}{3} - \frac{2(bx+a)^2 a^2}{5} + \frac{(bx+a)^2}{7} \right) a}{(a-c)^3 b^3} - \frac{6 \left(\frac{(bx+c)^3 c^2}{3} - \frac{2(bx+c)^2 c^2}{5} + \frac{(bx+c)^2}{7} \right) a}{(a-c)^3 b^3} + \frac{6 \left(\frac{(bx+a)^3 a^2}{3} - \frac{2(bx+a)^2 a^2}{5} + \frac{(bx+a)^2}{7} \right) c}{(a-c)^3 b^3} - \frac{2 \left(\frac{(bx+c)^3 c^2}{3} - \frac{2(bx+c)^2 c^2}{5} + \frac{(bx+c)^2}{7} \right) c}{(a-c)^3 b^3} + \frac{8(bx+a)^3 a^2}{3} + \frac{24(bx+c)^2 a^2}{5} - \frac{24(bx+a)^2 a}{7} + \frac{8(bx+c)^2}{9} - \frac{8 \left(\frac{(bx+c)^3 c^3}{3} + \frac{3(bx+c)^2 c^2}{5} - \frac{3(bx+c)^2 c}{7} + \frac{(bx+c)^2}{9} \right)}{(a-c)^3 b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)

[Out] 2/(a-c)^3*a/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))+6/(a-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-6/(a-c)^3*a/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c)^(7/2))-2/(a-c)^3*c/b^3*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c)^(7/2))+8/(a-c)^3/b^3*(1/9*(b*x+a)^(9/2)-3/7*a*(b*x+a)^(7/2)+3/5*a^2*(b*x+a)^(5/2)-1/3*a^3*(b*x+a)^(3/2))-8/(a-c)^3/b^3*(1/9*(b*x+c)^(9/2)-3/7*c*(b*x+c)^(7/2)+3/5*c^2*(b*x+c)^(5/2)-1/3*c^3*(b*x+c)^(3/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{bx+a} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)

mupad [B] time = 3.30, size = 529, normalized size = 1.41

$$\frac{x^2 \left(\frac{4bx}{9(a-c)} - \frac{24(bx+c)}{9(a-c)^2} \right) \sqrt{bx+c}}{7a} + \frac{x^2 \left(\frac{4bx}{9(a-c)} - \frac{24(bx+c)}{9(a-c)^2} \right) \sqrt{bx+a}}{7a} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+a}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+a}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+a}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+a}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+a}}{15b^2} + \frac{8x^2 \left(\frac{24(bx+c)}{9(a-c)^2} + \frac{4(bx+c)}{9(a-c)} \right) \sqrt{bx+c}}{15b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)

```
[Out] (x^3*((64*b*c)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2)
)/(7*b) - (x^3*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a +
b*x)^(1/2))/(7*b) - (8*c^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(
a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15*b^3)
- (x^2*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c)/(9*(a - c)^3) - (2*b*(3*
a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(5*b) + (8*a^2*((2*(3*a*c + a
^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3
)))/(7*b))*(a + b*x)^(1/2))/(15*b^3) + (x^2*((2*(3*a*c + a^2))/(a - c)^3 + (
6*a*((64*a*b)/(9*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x
)^(1/2))/(5*b) + (8*b*x^4*(a + b*x)^(1/2))/(9*(a - c)^3) - (8*b*x^4*(c + b*
x)^(1/2))/(9*(a - c)^3) + (4*c*x*((2*c*(3*a + c))/(a - c)^3 + (6*c*((64*b*c
)/(9*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(7*b))*(c + b*x)^(1/2))/(15
*b^2) - (4*a*x*((2*(3*a*c + a^2))/(a - c)^3 + (6*a*((64*a*b)/(9*(a - c)^3)
- (2*b*(5*a + 3*c))/(a - c)^3))/(7*b))*(a + b*x)^(1/2))/(15*b^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(b*x + c))**3, x)
```


$$3.189 \quad \int \frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$$

Optimal. Leaf size=261

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3} + \frac{8(a+bx)^{7/2}}{7b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8(bx+c)^{3/2}}{7b^2(a-c)^3}$$

Rubi [A] time = 0.24, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6689, 43}

$$\frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8c^2(bx+c)^{3/2}}{3b^2(a-c)^3} + \frac{8(a+bx)^{7/2}}{7b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{8(bx+c)^{3/2}}{7b^2(a-c)^3} + \frac{16c(bx+c)^{5/2}}{5b^2(a-c)^3} - \frac{2(3a+c)(bx+c)^{5/2}}{5b^2(a-c)^3} + \frac{2c(3a+c)(bx+c)^{3/2}}{3b^2(a-c)^3}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]

[Out] (8*a^2*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (2*a*(a + 3*c)*(a + b*x)^(3/2))/(3*b^2*(a - c)^3) - (16*a*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(5/2))/(5*b^2*(a - c)^3) + (8*(a + b*x)^(7/2))/(7*b^2*(a - c)^3) - (8*c^2*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (2*c*(3*a + c)*(c + b*x)^(3/2))/(3*b^2*(a - c)^3) + (16*c*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(5/2))/(5*b^2*(a - c)^3) - (8*(c + b*x)^(7/2))/(7*b^2*(a - c)^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_.), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx &= \frac{\int \left(a \left(1 + \frac{3c}{a} \right) x \sqrt{a+bx} + 4bx^2 \sqrt{a+bx} - 3a \left(1 + \frac{c}{3a} \right) x \sqrt{c+bx} - 4bx^2 \sqrt{c+bx} \right) dx}{(a-c)^3} \\ &= \frac{(4b) \int x^2 \sqrt{a+bx} dx}{(a-c)^3} - \frac{(4b) \int x^2 \sqrt{c+bx} dx}{(a-c)^3} - \frac{(3a+c) \int x \sqrt{c+bx} dx}{(a-c)^3} + \frac{(a+c) \int x \sqrt{a+bx} dx}{(a-c)^3} \\ &= \frac{(4b) \int \left(\frac{a^2 \sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2} \right) dx}{(a-c)^3} - \frac{(4b) \int \left(\frac{c^2 \sqrt{c+bx}}{b^2} - \frac{2c(c+bx)^{3/2}}{b^2} + \frac{(c+bx)^{5/2}}{b^2} \right) dx}{(a-c)^3} \\ &= \frac{8a^2(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{2a(a+3c)(a+bx)^{3/2}}{3b^2(a-c)^3} - \frac{16a(a+bx)^{5/2}}{5b^2(a-c)^3} + \frac{2(a+3c)(a+bx)^{5/2}}{5b^2(a-c)^3} \end{aligned}$$

Mathematica [A] time = 0.26, size = 214, normalized size = 0.82

$$\frac{2(6a^3\sqrt{a+bx} - a^2\sqrt{a+bx}(3bx+14c) + 20b^3x^3(\sqrt{a+bx} - \sqrt{bx+c}) + a(b^2x^2(11\sqrt{a+bx} - 21\sqrt{bx+c}) + 7bcx(\sqrt{a+bx} - \sqrt{bx+c}) + 14c^2\sqrt{bx+c}) + b^2cx^2(21\sqrt{a+bx} - 11\sqrt{bx+c}) - 6c^3\sqrt{bx+c} + 3bc^2x\sqrt{bx+c})}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*(6*a^3*Sqrt[a + b*x] - 6*c^3*Sqrt[c + b*x] + 3*b*c^2*x*Sqrt[c + b*x] - a^2*Sqrt[a + b*x]*(14*c + 3*b*x) + b^2*c*x^2*(21*Sqrt[a + b*x] - 11*Sqrt[c + b*x])) + 20*b^3*x^3*(Sqrt[a + b*x] - Sqrt[c + b*x]) + a*(14*c^2*Sqrt[c + b*x] + b^2*x^2*(11*Sqrt[a + b*x] - 21*Sqrt[c + b*x])) + 7*b*c*x*(Sqrt[a + b*x] - Sqrt[c + b*x]))/(35*b^2*(a - c)^3)
```

IntegrateAlgebraic [A] time = 0.64, size = 180, normalized size = 0.69

$$\frac{2\sqrt{a+bx}(6a^3-3a^2(bx+c)-11a^2c+11a(bx+c)^2-15ac(bx+c)+4a^2+18c^2(bx+c)+20(bx+c)^3-39c(bx+c)^2+c^3)+2(-21a(bx+c)^{5/2}+35ac(bx+c)^{3/2}-35c^2(bx+c)^{3/2}-20(bx+c)^{7/2}+49c(bx+c)^{5/2})}{35b^2(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[c + b*x])^3,x]
```

```
[Out] (2*Sqrt[a + b*x]*(6*a^3 - 11*a^2*c + 4*a*c^2 + c^3 - 3*a^2*(c + b*x) - 15*a*c*(c + b*x) + 18*c^2*(c + b*x) + 11*a*(c + b*x)^2 - 39*c*(c + b*x)^2 + 20*(c + b*x)^3))/(35*b^2*(a - c)^3) + (2*(35*a*c*(c + b*x)^(3/2) - 35*c^2*(c + b*x)^(3/2) - 21*a*(c + b*x)^(5/2) + 49*c*(c + b*x)^(5/2) - 20*(c + b*x)^(7/2)))/(35*b^2*(a - c)^3)
```

fricas [A] time = 0.71, size = 159, normalized size = 0.61

$$\frac{2((20b^3x^3+6a^3-14a^2c+(11ab^2+21b^2c)x^2-(3a^2b-7abc)x)\sqrt{bx+a}-(20b^3x^3-14ac^2+6c^3+(21ab^2+11b^2c)x^2+(7abc-3bc^2)x)\sqrt{bx+c})}{35(a^3b^2-3a^2b^2c+3ab^2c^2-b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/35*((20*b^3*x^3 + 6*a^3 - 14*a^2*c + (11*a*b^2 + 21*b^2*c)*x^2 - (3*a^2*b - 7*a*b*c)*x)*sqrt(b*x + a) - (20*b^3*x^3 - 14*a*c^2 + 6*c^3 + (21*a*b^2 + 11*b^2*c)*x^2 + (7*a*b*c - 3*b*c^2)*x)*sqrt(b*x + c))/(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)
```

giac [B] time = 0.38, size = 866, normalized size = 3.32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/35((((b*x + a)*(20*(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)*(b*x + a)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9) - (39*a^7*b^3 - 245*a^6*b^3*c + 651*a^5*b^3*c^2 - 945*a^4*b^3*c^3 + 805*a^3*b^3*c^4 - 399*a^2*b^3*c^5 + 105*a*b^3*c^6 - 11*b^3*c^7)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9)) + 3*(6*a^8*b^3 - 41*a^7*b^3*c + 119*a^6*b^3*c^2 - 189*a^5*b^3*c^3 + 175*a^4*b^3*c^4 - 91*a^3*b^3*c^5 + 21*a^2*b^3*c^6 + a*b^3*c^7 - b^3*c^8)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*(b*x + a) + (a^9*b^3 - 2*a^8*b^3*c - 20*a^7*b^3*c^2 + 112*a^6*b^3*c^3 - 266*a^5*b^3*c^4 + 364*a^4*b^3*c^5 - 308*a^3*b^3*c^6 + 160*a^2*b^3*c^7 - 47*a*b^3*c^8 + 6*b^3*c^9)/(a^9*b^4 - 9*a^8*b^4*c + 36*a^7*b^4*c^2 - 84*a^6*b^4*c^3 + 126*a^5*b^4*c^4 - 126*a^4*b^4*c^5 + 84*a^3*b^4*c^6 - 36*a^2*b^4*c^7 + 9*a*b^4*c^8 - b^4*c^9))*sqrt(b*x + c) - (20*(b*x + a)^(7/2) - 49*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 + 21*
```

$$(b*x + a)^{(5/2)}*c - 35*(b*x + a)^{(3/2)}*a*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)/b$$

maple [A] time = 0.00, size = 222, normalized size = 0.85

$$\frac{2\left(-\frac{(bx+a)^2a}{3} + \frac{(bx+a)^5}{5}\right)a}{(a-c)^3b^2} - \frac{6\left(-\frac{(bx+c)^2c}{3} + \frac{(bx+c)^5}{5}\right)a}{(a-c)^3b^2} + \frac{6\left(-\frac{(bx+a)^2a}{3} + \frac{(bx+a)^5}{5}\right)c}{(a-c)^3b^2} - \frac{2\left(-\frac{(bx+c)^2c}{3} + \frac{(bx+c)^5}{5}\right)c}{(a-c)^3b^2} + \frac{8(bx+a)^2a^2}{3} - \frac{16(bx+a)^2a}{5} + \frac{8(bx+a)^2}{7} - \frac{8\left(\frac{(bx+c)^2c^2}{3} - \frac{2(bx+c)^2c}{5} + \frac{(bx+c)^2}{7}\right)}{(a-c)^3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)

[Out] 2/(a-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+6/(a-c)^3*c/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-6/(a-c)^3*a/b^2*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2))-2/(a-c)^3*c/b^2*(-1/3*(b*x+c)^(3/2)*c+1/5*(b*x+c)^(5/2))+8/(a-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-8/(a-c)^3/b^2*(1/3*(b*x+c)^(3/2)*c^2-2/5*(b*x+c)^(5/2)*c+1/7*(b*x+c)^(7/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{bx+a} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(b*x + c))^3, x)

mupad [B] time = 3.21, size = 385, normalized size = 1.48

$$\frac{x^2 \left(\frac{48bc}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right) \sqrt{c+bx}}{5b} - \frac{x^2 \left(\frac{48ab}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right) \sqrt{a+bx}}{5b} - \frac{2a \left(\frac{24(4b+3c)}{(a-c)^3} + \frac{4 \left(\frac{48bc}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right)}{(a-c)^3} \right) \sqrt{a+bx}}{3b^2} + \frac{8bx^3 \sqrt{a+bx}}{7(a-c)^3} + \frac{2c \left(\frac{24(4b+3c)}{(a-c)^3} + \frac{4 \left(\frac{48bc}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right)}{(a-c)^3} \right) \sqrt{c+bx}}{3b^2} - \frac{8bx^3 \sqrt{c+bx}}{7(a-c)^3} + \frac{x \left(\frac{24(4b+3c)}{(a-c)^3} + \frac{4 \left(\frac{48bc}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right)}{(a-c)^3} \right) \sqrt{a+bx}}{3b} - \frac{x \left(\frac{24(4b+3c)}{(a-c)^3} + \frac{4 \left(\frac{48bc}{7(a-c)^3} - \frac{24(5a+3c)}{5b} \right)}{(a-c)^3} \right) \sqrt{c+bx}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)

[Out] (x^2*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^(1/2))/(5*b) - (x^2*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a + b*x)^(1/2))/(5*b) - (2*a*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (8*b*x^3*(a + b*x)^(1/2))/(7*(a - c)^3) + (2*c*((2*c*(3*a + c))/(a - c)^3 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/2))/(3*b^2) - (8*b*x^3*(c + b*x)^(1/2))/(7*(a - c)^3) + (x*((2*a*(a + 3*c))/(a - c)^3 + (4*a*((48*a*b)/(7*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(5*b))*(a + b*x)^(1/2))/(3*b) - (x*((2*c*(3*a + c))/(a - c)^3 + (4*c*((48*b*c)/(7*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(5*b))*(c + b*x)^(1/2))/(3*b)

sympy [A] time = 2.70, size = 942, normalized size = 3.61

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)

[Out] Piecewise((12*a**2/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*a*b*x/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 10

```

5*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 44*a*c/(35*a*b**2*sqrt(
a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x
*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 36*a
*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x
+ c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sq
rt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 40*b**2*x**2/(35*a*b**2*sqrt(a + b
*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt
(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 54*b*c*x/
(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b
*x) + 140*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(
b*x + c)) + 30*b*x*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 1
05*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x +
c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)) + 12*c**2/(35*a*b
**2*sqrt(a + b*x) + 105*a*b**2*sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 14
0*b**3*x*sqrt(b*x + c) + 105*b**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)
) + 36*c*sqrt(a + b*x)*sqrt(b*x + c)/(35*a*b**2*sqrt(a + b*x) + 105*a*b**2*
sqrt(b*x + c) + 140*b**3*x*sqrt(a + b*x) + 140*b**3*x*sqrt(b*x + c) + 105*b
**2*c*sqrt(a + b*x) + 35*b**2*c*sqrt(b*x + c)), Ne(b, 0)), (x**2/(2*(sqrt(a
) + sqrt(c))**3), True))

```

$$3.190 \quad \int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$$

Optimal. Leaf size=64

$$\frac{(a-c)^2}{10b(\sqrt{a+bx} + \sqrt{bx+c})^5} - \frac{1}{2b(\sqrt{a+bx} + \sqrt{bx+c})}$$

Rubi [B] time = 0.09, antiderivative size = 151, normalized size of antiderivative = 2.36, number of steps used = 6, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6689, 43}

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8a(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8(bx+c)^{5/2}}{5b(a-c)^3} + \frac{8c(bx+c)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3), x]

[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*x)^(5/2))/(5*b*(a - c)^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_.), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(\sqrt{a+bx} + \sqrt{c+bx})^3} dx &= \frac{\int \left(a \left(1 + \frac{3c}{a} \right) \sqrt{a+bx} + 4bx\sqrt{a+bx} - 3a \left(1 + \frac{c}{3a} \right) \sqrt{c+bx} - 4bx\sqrt{c+bx} \right) dx}{(a-c)^3} \\ &= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b) \int x\sqrt{a+bx} dx}{(a-c)^3} - \frac{(4b) \int x\sqrt{c+bx} dx}{(a-c)^3} \\ &= \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} + \frac{(4b) \int \left(-\frac{a\sqrt{a+bx}}{b} + \frac{(a+bx)^{3/2}}{b} \right) dx}{(a-c)^3} \\ &= -\frac{8a(a+bx)^{3/2}}{3b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} + \frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{8c(c+bx)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(c+bx)^{3/2}}{3b(a-c)^3} \end{aligned}$$

Mathematica [B] time = 0.15, size = 151, normalized size = 2.36

$$\frac{8(a+bx)^{5/2}}{5b(a-c)^3} + \frac{2(a+3c)(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8a(a+bx)^{3/2}}{3b(a-c)^3} - \frac{8(bx+c)^{5/2}}{5b(a-c)^3} + \frac{8c(bx+c)^{3/2}}{3b(a-c)^3} - \frac{2(3a+c)(bx+c)^{3/2}}{3b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3), x]
```

```
[Out] (-8*a*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (2*(a + 3*c)*(a + b*x)^(3/2))/(3*b*(a - c)^3) + (8*(a + b*x)^(5/2))/(5*b*(a - c)^3) + (8*c*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (2*(3*a + c)*(c + b*x)^(3/2))/(3*b*(a - c)^3) - (8*(c + b*x)^(5/2))/(5*b*(a - c)^3)
```

IntegrateAlgebraic [A] time = 0.53, size = 110, normalized size = 1.72

$$\frac{2\sqrt{a+bx}(a^2-3a(bx+c)-2ac-4(bx+c)^2+3c(bx+c)+c^2)}{5b(a-c)^3} - \frac{2(5a(bx+c)^{3/2}+4(bx+c)^{5/2}-5c(bx+c)^{3/2})}{5b(a-c)^3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[c + b*x])^(-3), x]
```

```
[Out] (-2*Sqrt[a + b*x]*(a^2 - 2*a*c + c^2 - 3*a*(c + b*x) + 3*c*(c + b*x) - 4*(c + b*x)^2))/(5*b*(a - c)^3) - (2*(5*a*(c + b*x)^(3/2) - 5*c*(c + b*x)^(3/2) + 4*(c + b*x)^(5/2)))/(5*b*(a - c)^3)
```

fricas [B] time = 0.63, size = 106, normalized size = 1.66

$$\frac{2\left(\left(4b^2x^2 - a^2 + 5ac + (3ab + 5bc)x\right)\sqrt{bx + a} - \left(4b^2x^2 + 5ac - c^2 + (5ab + 3bc)x\right)\sqrt{bx + c}\right)}{5\left(a^3b - 3a^2bc + 3abc^2 - bc^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 2/5*((4*b^2*x^2 - a^2 + 5*a*c + (3*a*b + 5*b*c)*x)*sqrt(b*x + a) - (4*b^2*x^2 + 5*a*c - c^2 + (5*a*b + 3*b*c)*x)*sqrt(b*x + c))/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

giac [B] time = 0.30, size = 427, normalized size = 6.67

$$\frac{2}{5} \left(\frac{4(a^2b^2 - 3a^2b^2c + 3ab^2c^2 - b^2c^3)(bx+a)}{a^2b^3 - 6a^2b^2c + 15a^2b^2c^2 - 20a^2b^2c^3 + 15a^2b^2c^4 - 6ab^3c^3 + b^3c^4} - \frac{3(a^2b^2 - 4a^2b^2c + 6a^2b^2c^2 - 4a^2b^2c^3 + b^2c^4)}{a^2b^3 - 6a^2b^2c + 15a^2b^2c^2 - 20a^2b^2c^3 + 15a^2b^2c^4 - 6ab^3c^3 + b^3c^4} - \frac{a^2b^2 - 5a^2b^2c + 10a^2b^2c^2 - 10a^2b^2c^3 + 5ab^3c^4 - b^2c^5}{a^2b^3 - 6a^2b^2c + 15a^2b^2c^2 - 20a^2b^2c^3 + 15a^2b^2c^4 - 6ab^3c^3 + b^3c^4} \right) \sqrt{bx+c} + \frac{2(4(bx+a)^2 - 5(bx+a)^2a + 5(bx+a)^2c)}{5(a^2b - 3a^2bc + 3abc^2 - bc^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] -2/5*((b*x + a)*(4*(a^3*b^2 - 3*a^2*b^2*c + 3*a*b^2*c^2 - b^2*c^3)*(b*x + a))/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6) - 3*(a^4*b^2 - 4*a^3*b^2*c + 6*a^2*b^2*c^2 - 4*a*b^2*c^3 + b^2*c^4)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6)) - (a^5*b^2 - 5*a^4*b^2*c + 10*a^3*b^2*c^2 - 10*a^2*b^2*c^3 + 5*a*b^2*c^4 - b^2*c^5)/(a^6*b^3 - 6*a^5*b^3*c + 15*a^4*b^3*c^2 - 20*a^3*b^3*c^3 + 15*a^2*b^3*c^4 - 6*a*b^3*c^5 + b^3*c^6))*sqrt(b*x + c) + 2/5*(4*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a + 5*(b*x + a)^(3/2)*c)/(a^3*b - 3*a^2*b*c + 3*a*b*c^2 - b*c^3)
```

maple [B] time = 0.01, size = 146, normalized size = 2.28

$$\frac{2(bx+a)^{\frac{3}{2}}a}{3(a-c)^3b} - \frac{2(bx+c)^{\frac{3}{2}}a}{(a-c)^3b} + \frac{2(bx+a)^{\frac{3}{2}}c}{(a-c)^3b} - \frac{2(bx+c)^{\frac{3}{2}}c}{3(a-c)^3b} + \frac{-\frac{8(bx+a)^{\frac{3}{2}}a}{3} + \frac{8(bx+a)^{\frac{5}{2}}}{5}}{(a-c)^3b} - \frac{8\left(-\frac{(bx+c)^{\frac{3}{2}}c}{3} + \frac{(bx+c)^{\frac{5}{2}}}{5}\right)}{(a-c)^3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)
```

[Out] $2/3*a*(b*x+a)^{(3/2)}/b/(a-c)^3+2/(a-c)^3*c*(b*x+a)^{(3/2)}/b-2/(a-c)^3*a*(b*x+c)^{(3/2)}/b-2/3*c*(b*x+c)^{(3/2)}/b/(a-c)^3+8/(a-c)^3/b*(-1/3*(b*x+a)^{(3/2)}*a+1/5*(b*x+a)^{(5/2)})-8/(a-c)^3/b*(-1/3*(b*x+c)^{(3/2)}*c+1/5*(b*x+c)^{(5/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sqrt{bx+a} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate((sqrt(b*x + a) + sqrt(b*x + c))^(-3), x)

mupad [B] time = 2.99, size = 252, normalized size = 3.94

$$\frac{\left(\frac{2(a^2+3ca)}{(a-c)^3} + \frac{2a\left(\frac{32ab}{5(a-c)^3} - \frac{2b(5a+3c)}{(a-c)^3}\right)}{3b}\right)\sqrt{a+bx}}{b} - \frac{\left(\frac{2c(3a+c)}{(a-c)^3} + \frac{2c\left(\frac{32bc}{5(a-c)^3} - \frac{2b(5a+5c)}{(a-c)^3}\right)}{3b}\right)\sqrt{c+bx}}{b} + \frac{8bx^2\sqrt{a+bx}}{5(a-c)^3} - \frac{8bx^2\sqrt{c+bx}}{5(a-c)^3} - \frac{x\left(\frac{32ab}{5(a-c)^3} - \frac{2b(5a+3c)}{(a-c)^3}\right)\sqrt{a+bx}}{3b} + \frac{x\left(\frac{32bc}{5(a-c)^3} - \frac{2b(5a+5c)}{(a-c)^3}\right)\sqrt{c+bx}}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(1/2) + (c + b*x)^(1/2))^3,x)

[Out] $((2*(3*a*c + a^2))/(a - c)^3 + (2*a*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3))/(3*b))*(a + b*x)^{(1/2)}/b - ((2*c*(3*a + c))/(a - c)^3 + (2*c*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3))/(3*b))*(c + b*x)^{(1/2)}/b + (8*b*x^2*(a + b*x)^{(1/2)})/(5*(a - c)^3) - (8*b*x^2*(c + b*x)^{(1/2)})/(5*(a - c)^3) - (x*((32*a*b)/(5*(a - c)^3) - (2*b*(5*a + 3*c))/(a - c)^3)*(a + b*x)^{(1/2)})/(3*b) + (x*((32*b*c)/(5*(a - c)^3) - (2*b*(3*a + 5*c))/(a - c)^3)*(c + b*x)^{(1/2)})/(3*b)$

sympy [A] time = 1.82, size = 384, normalized size = 6.00

$$\frac{\frac{2a}{\sqrt{a+bx}} + \frac{2c}{\sqrt{c+bx}}}{(\sqrt{a+bx})^3} \text{ for } b \neq 0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)

[Out] Piecewise((-2*a/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)) - 4*b*x/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)) - 2*c/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)) - 6*sqrt(a + b*x)*sqrt(b*x + c)/(5*a*b*sqrt(a + b*x) + 15*a*b*sqrt(b*x + c) + 20*b**2*x*sqrt(a + b*x) + 20*b**2*x*sqrt(b*x + c) + 15*b*c*sqrt(a + b*x) + 5*b*c*sqrt(b*x + c)), Ne(b, 0)), (x/(sqrt(a) + sqrt(c))**3, True))

$$3.191 \quad \int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$$

Optimal. Leaf size=157

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3} + \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} - \frac{8(bx+c)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3} - \frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3} + \frac{2\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Rubi [A] time = 0.23, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6689, 50, 63, 208}

$$\frac{8(a+bx)^{3/2}}{3(a-c)^3} + \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} - \frac{8(bx+c)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{bx+c}}{(a-c)^3} - \frac{2\sqrt{a}(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3} + \frac{2\sqrt{c}(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3),x]

[Out] (2*(a + 3*c)*Sqrt[a + b*x])/(a - c)^3 + (8*(a + b*x)^(3/2))/(3*(a - c)^3) - (2*(3*a + c)*Sqrt[c + b*x])/(a - c)^3 - (8*(c + b*x)^(3/2))/(3*(a - c)^3) - (2*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 + (2*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(\sqrt{a+bx} + \sqrt{c+bx})^3} dx &= \frac{\int \left(4b\sqrt{a+bx} + \frac{a\left(1+\frac{3c}{a}\right)\sqrt{a+bx}}{x} - 4b\sqrt{c+bx} - \frac{3a\left(1+\frac{c}{3a}\right)\sqrt{c+bx}}{x} \right) dx}{(a-c)^3} \\
&= \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{(3a+c) \int \frac{\sqrt{c+bx}}{x} dx}{(a-c)^3} + \frac{(a+3c) \int \frac{\sqrt{a+bx}}{x} dx}{(a-c)^3} \\
&= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} + \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{(c(3a+c) \operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{a}}\right) + (a+3c) \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c}}\right))}{(a-c)^3} \\
&= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} + \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{(2c(3a+c) \operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{a}}\right) + (a+3c) \operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c}}\right))}{(a-c)^3} \\
&= \frac{2(a+3c)\sqrt{a+bx}}{(a-c)^3} + \frac{8(a+bx)^{3/2}}{3(a-c)^3} - \frac{2(3a+c)\sqrt{c+bx}}{(a-c)^3} - \frac{8(c+bx)^{3/2}}{3(a-c)^3} - \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c+bx}}{\sqrt{a}}\right) + \sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{c}}\right)}{(a-c)^3}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 142, normalized size = 0.90

$$\frac{2(-9a\sqrt{bx+c} + 9c\sqrt{a+bx} - 3\sqrt{a}(a+3c)\operatorname{tanh}^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) + 3\sqrt{c}(3a+c)\operatorname{tanh}^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) + 7a\sqrt{a+bx} + 4bx\sqrt{a+bx} - 7c\sqrt{bx+c} - 4bx\sqrt{bx+c})}{3(a-c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3), x]

[Out] (2*(7*a*Sqrt[a + b*x] + 9*c*Sqrt[a + b*x] + 4*b*x*Sqrt[a + b*x] - 9*a*Sqrt[c + b*x] - 7*c*Sqrt[c + b*x] - 4*b*x*Sqrt[c + b*x] - 3*Sqrt[a]*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 3*Sqrt[c]*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(3*(a - c)^3)

IntegrateAlgebraic [B] time = 1.06, size = 320, normalized size = 2.04

$$\frac{2\sqrt{a+bx}(7a+4(bx+c)+5c)}{3(a-c)^3} - \frac{2(9a\sqrt{bx+c}+4(bx+c)^{3/2}+3c\sqrt{bx+c})}{3(a-c)^3} - \frac{2\sqrt{-(\sqrt{a}+\sqrt{c})^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a}-\sqrt{c})^3(\sqrt{a}+\sqrt{c})} - \frac{2\sqrt{-(\sqrt{a}-\sqrt{c})^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a}-\sqrt{c})(\sqrt{a}+\sqrt{c})^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[c + b*x])^3), x]

[Out] (2*Sqrt[a + b*x]*(7*a + 5*c + 4*(c + b*x)))/(3*(a - c)^3) - (2*(9*a*Sqrt[c + b*x] + 3*c*Sqrt[c + b*x] + 4*(c + b*x)^(3/2)))/(3*(a - c)^3) - (2*Sqrt[-(Sqrt[a] + Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a - 2*Sqrt[a]*Sqrt[c] - c]])/((Sqrt[a] - Sqrt[c])^3*(Sqrt[a] + Sqrt[c])) - (2*Sqrt[-(Sqrt[a] - Sqrt[c])^2]*ArcTan[Sqrt[a + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c] - Sqrt[c + b*x]/Sqrt[-a + 2*Sqrt[a]*Sqrt[c] - c]])/((Sqrt[a] - Sqrt[c])*(Sqrt[a] + Sqrt[c])^3)

fricas [A] time = 0.75, size = 516, normalized size = 3.29

$$\frac{2\sqrt{a+bx}(7a+4(bx+c)+5c)}{3(a-c)^3} - \frac{2(9a\sqrt{bx+c}+4(bx+c)^{3/2}+3c\sqrt{bx+c})}{3(a-c)^3} - \frac{2\sqrt{-(\sqrt{a}+\sqrt{c})^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{-2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a}-\sqrt{c})^3(\sqrt{a}+\sqrt{c})} - \frac{2\sqrt{-(\sqrt{a}-\sqrt{c})^2}\tan^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}-\frac{\sqrt{bx+c}}{\sqrt{2\sqrt{a}\sqrt{c}-a-c}}\right)}{(\sqrt{a}-\sqrt{c})(\sqrt{a}+\sqrt{c})^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")

[Out] [-1/3*(3*(a + 3*c)*sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*(3*a + c)*sqrt(c)*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(4*b*x

$$\begin{aligned}
& + 7*a + 9*c)*\text{sqrt}(b*x + a) + 2*(4*b*x + 9*a + 7*c)*\text{sqrt}(b*x + c))/(a^3 - 3* \\
& a^2*c + 3*a*c^2 - c^3), -1/3*(6*(3*a + c)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(b*x + c)*\text{sqrt} \\
& (-c)/c) + 3*(a + 3*c)*\text{sqrt}(a)*\log((b*x + 2*\text{sqrt}(b*x + a)*\text{sqrt}(a) + 2*a)/x) \\
& - 2*(4*b*x + 7*a + 9*c)*\text{sqrt}(b*x + a) + 2*(4*b*x + 9*a + 7*c)*\text{sqrt}(b*x + c \\
&))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 1/3*(6*\text{sqrt}(-a)*(a + 3*c)*\text{arctan}(\text{sqrt}(b \\
& *x + a)*\text{sqrt}(-a)/a) - 3*(3*a + c)*\text{sqrt}(c)*\log((b*x - 2*\text{sqrt}(b*x + c)*\text{sqrt}(c \\
&) + 2*c)/x) + 2*(4*b*x + 7*a + 9*c)*\text{sqrt}(b*x + a) - 2*(4*b*x + 9*a + 7*c)*\text{s} \\
& \text{qrt}(b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3), 2/3*(3*\text{sqrt}(-a)*(a + 3*c)*\text{ar} \\
& \text{ctan}(\text{sqrt}(b*x + a)*\text{sqrt}(-a)/a) - 3*(3*a + c)*\text{sqrt}(-c)*\text{arctan}(\text{sqrt}(b*x + c)* \\
& \text{sqrt}(-c)/c) + (4*b*x + 7*a + 9*c)*\text{sqrt}(b*x + a) - (4*b*x + 9*a + 7*c)*\text{sqrt}(\\
& b*x + c))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)]
\end{aligned}$$

giac [B] time = 2.91, size = 2652, normalized size = 16.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")

[Out]
$$\begin{aligned}
& -2/3*\text{sqrt}(b*x + c)*(4*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)*(b*x + a)/(a^6 - 6*a^ \\
& 5*c + 15*a^4*c^2 - 20*a^3*c^3 + 15*a^2*c^4 - 6*a*c^5 + c^6) + (5*a^4 - 8*a^ \\
& 3*c - 6*a^2*c^2 + 16*a*c^3 - 7*c^4)/(a^6 - 6*a^5*c + 15*a^4*c^2 - 20*a^3*c^ \\
& 3 + 15*a^2*c^4 - 6*a*c^5 + c^6)) + 2*(a^2 + 3*a*c)*\text{arctan}(\text{sqrt}(b*x + a)/\text{sqrt} \\
& (-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*\text{sqrt}(-a)) + 2/3*(4*(b*x + a)^(3/2)* \\
& a^6 + 3*\text{sqrt}(b*x + a)*a^7 - 24*(b*x + a)^(3/2)*a^5*c - 9*\text{sqrt}(b*x + a)*a^6* \\
& c + 60*(b*x + a)^(3/2)*a^4*c^2 - 9*\text{sqrt}(b*x + a)*a^5*c^2 - 80*(b*x + a)^(3/ \\
& 2)*a^3*c^3 + 75*\text{sqrt}(b*x + a)*a^4*c^3 + 60*(b*x + a)^(3/2)*a^2*c^4 - 135*\text{sq} \\
& \text{rt}(b*x + a)*a^3*c^4 - 24*(b*x + a)^(3/2)*a*c^5 + 117*\text{sqrt}(b*x + a)*a^2*c^5 \\
& + 4*(b*x + a)^(3/2)*c^6 - 51*\text{sqrt}(b*x + a)*a*c^6 + 9*\text{sqrt}(b*x + a)*c^7)/(a^ \\
& 9 - 9*a^8*c + 36*a^7*c^2 - 84*a^6*c^3 + 126*a^5*c^4 - 126*a^4*c^5 + 84*a^3* \\
& c^6 - 36*a^2*c^7 + 9*a*c^8 - c^9) - 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - \\
& 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 - 2*(3* \\
& a^2*c^2 + a*c^3 + (3*a*c^2 + c^3)*\text{sqrt}(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3) \\
&)^2*\text{sgn}(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 - (3*a^2*c + \\
& a*c^2)*\text{sqrt}(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c \\
& ^3 + 14*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3* \\
& c^3 - 6*a^2*c^4 - a*c^5 + c^6)*\text{sqrt}(a*c))*\text{abs}(-a^3 + 3*a^2*c - 3*a*c^2 + c^ \\
& 3)*\text{sgn}(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - (3*a^6*c - 11*a^5*c^2 + 14*a^4*c^3 \\
& - 6*a^3*c^4 - a^2*c^5 + a*c^6 + (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - 6*a^2* \\
& c^4 - a*c^5 + c^6)*\text{sqrt}(a*c))*\text{abs}(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) + (3*a^9* \\
& c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6*c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c \\
& ^7 - 2*a^2*c^8 + a*c^9 - (3*a^8*c - 14*a^7*c^2 + 22*a^6*c^3 - 6*a^5*c^4 - 2 \\
& 0*a^4*c^5 + 22*a^3*c^6 - 6*a^2*c^7 - 2*a*c^8 + c^9)*\text{sqrt}(a*c))*\text{sgn}(a^3 - 3* \\
& a^2*c + 3*a*c^2 - c^3) + (3*a^9 - 14*a^8*c + 22*a^7*c^2 - 6*a^6*c^3 - 20*a^ \\
& 5*c^4 + 22*a^4*c^5 - 6*a^3*c^6 - 2*a^2*c^7 + a*c^8)*\text{sqrt}(a*c))*\text{arctan}(-(\text{sqrt} \\
& (b*x + a) - \text{sqrt}(b*x + c))/\text{sqrt}(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + \text{sqrt}((a^ \\
& 4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + \\
& 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3))))/(a^3 - 3*a^2*c + 3*a*c^2 \\
& - c^3)))/((\text{sqrt}(-a)*a^8 - a^8*\text{sqrt}(-c) - 8*\text{sqrt}(-a)*a^7*c + 8*a^7*\text{sqrt}(-c)* \\
& c + 28*\text{sqrt}(-a)*a^6*c^2 - 28*a^6*\text{sqrt}(-c)*c^2 - 56*\text{sqrt}(-a)*a^5*c^3 + 56*a^ \\
& 5*\text{sqrt}(-c)*c^3 + 70*\text{sqrt}(-a)*a^4*c^4 - 70*a^4*\text{sqrt}(-c)*c^4 - 56*\text{sqrt}(-a)* \\
& 3*c^5 + 56*a^3*\text{sqrt}(-c)*c^5 + 28*\text{sqrt}(-a)*a^2*c^6 - 28*a^2*\text{sqrt}(-c)*c^6 - 8 \\
& *\text{sqrt}(-a)*a*c^7 + 8*a*\text{sqrt}(-c)*c^7 + \text{sqrt}(-a)*c^8 - \text{sqrt}(-c)*c^8)*\text{abs}(-a^3 \\
& + 3*a^2*c - 3*a*c^2 + c^3)) + 2*(3*a^9*c - 14*a^8*c^2 + 22*a^7*c^3 - 6*a^6* \\
& c^4 - 20*a^5*c^5 + 22*a^4*c^6 - 6*a^3*c^7 - 2*a^2*c^8 + a*c^9 + 2*(3*a^2*c^ \\
& 2 + a*c^3 + (3*a*c^2 + c^3)*\text{sqrt}(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*\text{sg} \\
& \text{n}(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 2*(3*a^2*c^2 + a*c^3 + (3*a^2*c + a*c^2) \\
& *\text{sqrt}(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2 - (3*a^5*c^2 - 11*a^4*c^3 + 1 \\
& 4*a^3*c^4 - 6*a^2*c^5 - a*c^6 + c^7 - (3*a^5*c - 11*a^4*c^2 + 14*a^3*c^3 - \\
& 6*a^2*c^4 - a*c^5 + c^6)*\text{sqrt}(a*c))*\text{abs}(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*\text{sgn}
\end{aligned}$$

$$(a^3 - 3a^2c + 3ac^2 - c^3) + (3a^6c - 11a^5c^2 + 14a^4c^3 - 6a^3c^4 - 3c^5 + a^2c^6 + ac^8 + (3a^5c - 11a^4c^2 + 14a^3c^3 - 6a^2c^4 - ac^5 + c^6) \sqrt{ac}) \cdot \text{abs}(-a^3 + 3a^2c - 3ac^2 + c^3) - (3a^9c - 14a^8c^2 + 22a^7c^3 - 6a^6c^4 - 20a^5c^5 + 22a^4c^6 - 6a^3c^7 - 2a^2c^8 + ac^9 + (3a^8c - 14a^7c^2 + 22a^6c^3 - 6a^5c^4 - 20a^4c^5 + 22a^3c^6 - 6a^2c^7 - 2ac^8 + c^9) \sqrt{ac}) \cdot \text{sgn}(a^3 - 3a^2c + 3ac^2 - c^3) + (3a^9 - 14a^8c + 22a^7c^2 - 6a^6c^3 - 20a^5c^4 + 22a^4c^5 - 6a^3c^6 - 2a^2c^7 + ac^8) \sqrt{ac} \cdot \arctan\left(\frac{-\sqrt{bx+a} - \sqrt{bx+c}}{\sqrt{-(a^4 - 2a^3c + 2ac^3 - c^4 - \sqrt{(a^4 - 2a^3c + 2ac^3 - c^4)^2 - (a^5 - 5a^4c + 10a^3c^2 - 10a^2c^3 + 5ac^4 - c^5)(a^3 - 3a^2c + 3ac^2 - c^3)}}}\right) \cdot \frac{1}{\sqrt{-(a^4 - 2a^3c + 2ac^3 - c^4 - \sqrt{(a^4 - 2a^3c + 2ac^3 - c^4)^2 - (a^5 - 5a^4c + 10a^3c^2 - 10a^2c^3 + 5ac^4 - c^5)(a^3 - 3a^2c + 3ac^2 - c^3)}}}}$$

maple [A] time = 0.01, size = 181, normalized size = 1.15

$$\frac{(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a})a}{(a-c)^3} - \frac{3(-2\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) + 2\sqrt{bx+c})a}{(a-c)^3} + \frac{3(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a})c}{(a-c)^3} - \frac{(-2\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) + 2\sqrt{bx+c})c}{(a-c)^3} + \frac{8(bx+a)^{\frac{3}{2}}}{3(a-c)^3} - \frac{8(bx+c)^{\frac{3}{2}}}{3(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)

[Out] $\frac{1}{(a-c)^3} \left(\frac{2(bx+a)^{3/2}}{(a-c)^3} - \frac{2a^{3/2} \operatorname{arctanh}\left(\frac{(bx+a)^{1/2}}{a^{1/2}}\right)}{(a-c)^3} + \frac{8}{3} \frac{(bx+a)^{3/2}}{(a-c)^3} - \frac{8}{3} \frac{(bx+c)^{3/2}}{(a-c)^3} + \frac{3}{(a-c)^3} \left(\frac{2(bx+a)^{3/2}}{(a-c)^3} - \frac{2a^{3/2} \operatorname{arctanh}\left(\frac{(bx+a)^{1/2}}{a^{1/2}}\right)}{(a-c)^3} \right) - \frac{3}{(a-c)^3} \left(\frac{2(bx+c)^{3/2}}{(a-c)^3} - \frac{2c^{3/2} \operatorname{arctanh}\left(\frac{(bx+c)^{1/2}}{c^{1/2}}\right)}{(a-c)^3} \right) - \frac{1}{(a-c)^3} \left(\frac{2(bx+c)^{3/2}}{(a-c)^3} - \frac{2c^{3/2} \operatorname{arctanh}\left(\frac{(bx+c)^{1/2}}{c^{1/2}}\right)}{(a-c)^3} \right) \right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{bx+a} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)

mupad [B] time = 27.72, size = 4060, normalized size = 25.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)

[Out] $\frac{((a^{1/2}(16a + 16c))/(3ac^2 - 3a^2c + a^3 - c^3) + (c^{1/2}(16a + 16c))/(3ac^2 - 3a^2c + a^3 - c^3)) \cdot ((a + b*x)^{1/2} - a^{1/2})}{((c + b*x)^{1/2} - c^{1/2}) + ((a^{1/2}(12a + 20c))/(3ac^2 - 3a^2c + a^3 - c^3) + (c^{1/2}(20a + 12c))/(3ac^2 - 3a^2c + a^3 - c^3)) \cdot ((a + b*x)^{1/2} - a^{1/2})^2} + \frac{((a^{1/2}(12a + 20c))/(3ac^2 - 3a^2c + a^3 - c^3) + (c^{1/2}(12a + 20c))/(3ac^2 - 3a^2c + a^3 - c^3)) \cdot ((a + b*x)^{1/2} - a^{1/2})^2}{((c + b*x)^{1/2} - c^{1/2})^2 + (a^{1/2}((28a)/3 + 12c))/(3ac^2 - 3a^2c + a^3 - c^3) + (c^{1/2}(12a + 20c)/3)/(3ac^2 - 3a^2c + a^3 - c^3)} + \frac{((a + b*x)^{1/2} - a^{1/2})^3}{((c + b*x)^{1/2} - c^{1/2})^3 + 1} + \frac{(\log(((a + b*x)^{1/2} - a^{1/2})/((c + b*x)^{1/2} - c^{1/2}))) \cdot (a \cdot (a^{1/2} + 3c^{1/2}) + c \cdot (3a^{1/2} + c^{1/2}))}{(3ac^2 - 3a^2c + a^3 - c^3) + 1}$

$$\begin{aligned}
 & \left(\operatorname{atan}\left(\left(\left(\left(a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}\right)\left(2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}\right)\right)^{1/2}\right)\right)\right)^{1/2} \cdot \left((6a^5c^{11/2} - 6a^{11/2}c + 2a^{3/2}c^5 - 2a^5c^{3/2} + 12a^3c^{7/2} - 12a^{7/2}c^3 - 16a^2c^{9/2} + 16a^{9/2}c^2) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) \right) \\
 & + \left(\left((a^{1/2}c^{15/2} - 5a^{3/2}c^{13/2} + 9a^{5/2}c^{11/2} - 5a^{7/2}c^{9/2} - 5a^{9/2}c^{7/2} + 9a^{11/2}c^{5/2} - 5a^{13/2}c^{3/2} + a^{15/2}c^{1/2}) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) \right) - (2((a + b^*x)^{1/2} - a^{1/2})) \cdot (a^9c + a^9c - 7a^2c^8 + 22a^3c^7 - 41a^4c^6 + 50a^5c^5 - 41a^6c^4 + 22a^7c^3 - 7a^8c^2) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \cdot \left((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}) \right)^{1/2} \\
 & \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right) - \left(2((a + b^*x)^{1/2} - a^{1/2}) \cdot (3a^{3/2}c^7 - 3a^7c^{3/2} + 8a^6c^{5/2} - 8a^{5/2}c^6 - 6a^5c^{7/2} + 6a^{7/2}c^5 + a^3c^{11/2} - a^{11/2}c^3) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) \cdot 1i \\
 & / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right) - \left(\left((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}) \right)^{1/2} \cdot \left(\left((a^{1/2}c^{15/2} - 5a^{3/2}c^{13/2} + 9a^{5/2}c^{11/2} - 5a^{7/2}c^{9/2} - 5a^{9/2}c^{7/2} + 9a^{11/2}c^{5/2} - 5a^{13/2}c^{3/2} + a^{15/2}c^{1/2}) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) \right) - (2((a + b^*x)^{1/2} - a^{1/2})) \cdot (a^9c + a^9c - 7a^2c^8 + 22a^3c^7 - 41a^4c^6 + 50a^5c^5 - 41a^6c^4 + 22a^7c^3 - 7a^8c^2) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \right) \cdot \left((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}) \right)^{1/2} \\
 & \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right) - \left(6a^5c^{11/2} - 6a^{11/2}c + 2a^{3/2}c^5 - 2a^5c^{3/2} + 12a^3c^{7/2} - 12a^{7/2}c^3 - 16a^2c^{9/2} + 16a^{9/2}c^2 \right) / \left(a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2 \right) + \left(2((a + b^*x)^{1/2} - a^{1/2}) \cdot (3a^{3/2}c^7 - 3a^7c^{3/2} + 8a^6c^{5/2} - 8a^{5/2}c^6 - 6a^5c^{7/2} + 6a^{7/2}c^5 + a^3c^{11/2} - a^{11/2}c^3) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) \cdot 1i \\
 & / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right) / \left((2(a^{1/2}c^{9/2} - 4a^{3/2}c^{7/2} + 6a^{5/2}c^{5/2} - 4a^{7/2}c^{3/2} + a^{9/2}c^{1/2})) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) - \left((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}) \right)^{1/2} \cdot \left((6a^5c^{11/2} - 6a^{11/2}c + 2a^{3/2}c^5 - 2a^5c^{3/2} + 12a^3c^{7/2} - 12a^{7/2}c^3 - 16a^2c^{9/2} + 16a^{9/2}c^2) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) \right) + \left((a^{1/2}c^{15/2} - 5a^{3/2}c^{13/2} + 9a^{5/2}c^{11/2} - 5a^{7/2}c^{9/2} - 5a^{9/2}c^{7/2} + 9a^{11/2}c^{5/2} - 5a^{13/2}c^{3/2} + a^{15/2}c^{1/2}) / (a^7c + a^7c - 6a^2c^6 + 15a^3c^5 - 20a^4c^4 + 15a^5c^3 - 6a^6c^2) - (2((a + b^*x)^{1/2} - a^{1/2})) \cdot (a^9c + a^9c - 7a^2c^8 + 22a^3c^7 - 41a^4c^6 + 50a^5c^5 - 41a^6c^4 + 22a^7c^3 - 7a^8c^2) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \right) \cdot \left((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}) \right)^{1/2} \\
 & \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right) - \left(2((a + b^*x)^{1/2} - a^{1/2}) \cdot (3a^{3/2}c^7 - 3a^7c^{3/2} + 8a^6c^{5/2} - 8a^{5/2}c^6 - 6a^5c^{7/2} + 6a^{7/2}c^5 + a^3c^{11/2} - a^{11/2}c^3) \right) / \left(((c + b^*x)^{1/2} - c^{1/2}) \cdot (a^2c^8 - 6a^3c^7 + 15a^4c^6 - 20a^5c^5 + 15a^6c^4 - 6a^7c^3 + a^8c^2) \right) \cdot \left(a^{1/2}c^3 - 3a^{5/2}c - 3ac^{5/2} + a^3c^{1/2} + 2a^2c^{3/2} + 2a^{3/2}c^2 \right) \cdot 1i \\
 & / \left(a^6c - a^6c - 5a^2c^5 + 10a^3c^4 - 10a^4c^3 + 5a^5c^2 \right)
 \end{aligned}$$

```

*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a
^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^
(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^
2) - (((a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
+ a^(3/2)*c^(1/2)))^(1/2)*(((a^(1/2)*c^(15/2) - 5*a^(3/2)*c^(13/2) + 9*a^
(5/2)*c^(11/2) - 5*a^(7/2)*c^(9/2) - 5*a^(9/2)*c^(7/2) + 9*a^(11/2)*c^(5/2)
- 5*a^(13/2)*c^(3/2) + a^(15/2)*c^(1/2))/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a
^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 - 6*a^6*c^2) - (2*((a + b*x)^(1/2) - a^(1/
2)))*(a*c^9 + a^9*c - 7*a^2*c^8 + 22*a^3*c^7 - 41*a^4*c^6 + 50*a^5*c^5 - 41*
a^6*c^4 + 22*a^7*c^3 - 7*a^8*c^2))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 -
6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(
(a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2) + a^(3
/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2)
+ 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 -
10*a^4*c^3 + 5*a^5*c^2) - (6*a*c^(11/2) - 6*a^(11/2)*c + 2*a^(3/2)*c^5 - 2
*a^5*c^(3/2) + 12*a^3*c^(7/2) - 12*a^(7/2)*c^3 - 16*a^2*c^(9/2) + 16*a^(9/2
)*c^2)/(a*c^7 + a^7*c - 6*a^2*c^6 + 15*a^3*c^5 - 20*a^4*c^4 + 15*a^5*c^3 -
6*a^6*c^2) + (2*((a + b*x)^(1/2) - a^(1/2))*(3*a^(3/2)*c^7 - 3*a^7*c^(3/2)
+ 8*a^6*c^(5/2) - 8*a^(5/2)*c^6 - 6*a^5*c^(7/2) + 6*a^(7/2)*c^5 + a^3*c^(11
/2) - a^(11/2)*c^3))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*c^8 - 6*a^3*c^7 + 15
*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^2)))*(a^(1/2)*c^3 -
3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2))/(
a*c^6 - a^6*c - 5*a^2*c^5 + 10*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2) + (4*((a +
b*x)^(1/2) - a^(1/2))*(6*a^3*c^4 - a^6*c - 5*a^2*c^5 - a*c^6 + 6*a^4*c^3 -
5*a^5*c^2 + 3*a^(3/2)*c^(11/2) + 4*a^(5/2)*c^(9/2) - 14*a^(7/2)*c^(7/2) +
4*a^(9/2)*c^(5/2) + 3*a^(11/2)*c^(3/2)))/(((c + b*x)^(1/2) - c^(1/2))*(a^2*
c^8 - 6*a^3*c^7 + 15*a^4*c^6 - 20*a^5*c^5 + 15*a^6*c^4 - 6*a^7*c^3 + a^8*c^
2))))*(a^(1/2)*c^(3/2) - 2*a*c + a^(3/2)*c^(1/2))*(2*a*c + a^(1/2)*c^(3/2)
+ a^(3/2)*c^(1/2)))^(1/2)*(a^(1/2)*c^3 - 3*a^(5/2)*c - 3*a*c^(5/2) + a^3*c
^(1/2) + 2*a^2*c^(3/2) + 2*a^(3/2)*c^2)*2i)/(a*c^6 - a^6*c - 5*a^2*c^5 + 10
*a^3*c^4 - 10*a^4*c^3 + 5*a^5*c^2)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)

[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)

$$3.192 \quad \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{c+bx})^3} dx$$

Optimal. Leaf size=162

$$\frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{8b\sqrt{bx+c}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3} + \frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3} - \frac{3b(3a+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3} - \frac{3b(a+3c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(c-a)^3}$$

Rubi [A] time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.38, number of steps used = 14, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6689, 47, 63, 208, 50}

$$\frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{8b\sqrt{bx+c}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{x(a-c)^3} + \frac{(3a+c)\sqrt{bx+c}}{x(a-c)^3} - \frac{b(a+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(a-c)^3} - \frac{8\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(a-c)^3} + \frac{b(3a+c)\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{\sqrt{c}(a-c)^3} + \frac{8b\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right)}{(a-c)^3}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3), x]

[Out] (8*b*Sqrt[a + b*x])/(a - c)^3 - ((a + 3*c)*Sqrt[a + b*x])/((a - c)^3*x) - (8*b*Sqrt[c + b*x])/(a - c)^3 + ((3*a + c)*Sqrt[c + b*x])/((a - c)^3*x) - (8*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(a - c)^3 - (b*(a + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(a - c)^3) + (8*b*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/(a - c)^3 + (b*(3*a + c)*ArcTanh[Sqrt[c + b*x]/Sqrt[c]])/((a - c)^3*Sqrt[c])

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6689

```
Int[(u_)*((e_)*Sqrt[(a_) + (b_)*(x_)^(n_)] + (f_)*Sqrt[(c_) + (d_)*
(x_)^(n_)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand
[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c,
d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{c+bx})^3} dx = \frac{\int \left(\frac{a(1+\frac{3c}{a})\sqrt{a+bx}}{x^2} + \frac{4b\sqrt{a+bx}}{x} - \frac{3a(1+\frac{c}{3a})\sqrt{c+bx}}{x^2} - \frac{4b\sqrt{c+bx}}{x} \right) dx}{(a-c)^3}$$

$$= \frac{(4b) \int \frac{\sqrt{a+bx}}{x} dx}{(a-c)^3} - \frac{(4b) \int \frac{\sqrt{c+bx}}{x} dx}{(a-c)^3} - \frac{(3a+c) \int \frac{\sqrt{c+bx}}{x^2} dx}{(a-c)^3} + \frac{(a+3c) \int \frac{\sqrt{a+bx}}{x^2} dx}{(a-c)^3}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x} - \frac{8b\sqrt{c+bx}}{(a-c)^3} + \frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x} + \frac{(4ab) \int \frac{1}{x^2} dx}{(a-c)^3}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x} - \frac{8b\sqrt{c+bx}}{(a-c)^3} + \frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x} + \frac{(8ab)}{(a-c)^3x} \quad (8a) \text{ Su}$$

$$= \frac{8b\sqrt{a+bx}}{(a-c)^3} - \frac{(a+3c)\sqrt{a+bx}}{(a-c)^3x} - \frac{8b\sqrt{c+bx}}{(a-c)^3} + \frac{(3a+c)\sqrt{c+bx}}{(a-c)^3x} + \frac{8\sqrt{a}bt}{(a-c)^3x}$$

Mathematica [A] time = 0.59, size = 187, normalized size = 1.15

$$b \left(\frac{(a+3c) \left(bx \sqrt{\frac{bx}{a}+1} \tanh^{-1} \left(\sqrt{\frac{bx}{a}+1} \right) + a+bx \right)}{bx \sqrt{a+bx}} + \frac{(3a+c) \left(bx \sqrt{\frac{bx}{c}+1} \tanh^{-1} \left(\sqrt{\frac{bx}{c}+1} \right) + bx+c \right)}{bx \sqrt{bx+c}} + 8\sqrt{a+bx} - 8\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right) - 8\sqrt{bx+c} + 8\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{bx+c}}{\sqrt{c}} \right) \right) / (a-c)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3), x]
```

```
[Out] (b*(8*Sqrt[a + b*x] - 8*Sqrt[c + b*x] - 8*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 8*Sqrt[c]*ArcTanh[Sqrt[c + b*x]/Sqrt[c]] - ((a + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])))/(b*x*Sqrt[a + b*x]) + ((3*a + c)*(c + b*x + b*x*Sqrt[1 + (b*x)/c]*ArcTanh[Sqrt[1 + (b*x)/c]]))/(b*x*Sqrt[c + b*x]))/(a - c)^3
```

IntegrateAlgebraic [B] time = 105.52, size = 8385, normalized size = 51.76

Result too large to show

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[c + b*x])^3), x]
```

```
[Out] Result too large to show
```

fricas [A] time = 0.91, size = 675, normalized size = 4.17

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] [-1/2*(3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), -1/2*(6*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + 3*(3*a*b*c + b*c^2)*sqrt(a)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) - 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) + 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), 1/2*(6*(3*a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(c)*x*log((b*x - 2*sqrt(b*x + c)*sqrt(c) + 2*c)/x) + 2*(8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) - 2*(8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x), (3*(3*a*b*c + b*c^2)*sqrt(-a)*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*(a^2*b + 3*a*b*c)*sqrt(-c)*x*arctan(sqrt(b*x + c)*sqrt(-c)/c) + (8*a*b*c*x - a^2*c - 3*a*c^2)*sqrt(b*x + a) - (8*a*b*c*x - 3*a^2*c - a*c^2)*sqrt(b*x + c))/((a^4*c - 3*a^3*c^2 + 3*a^2*c^3 - a*c^4)*x)]
```

giac [B] time = 39.98, size = 2594, normalized size = 16.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="giac")
```

```
[Out] 8*sqrt(b*x + a)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) - 8*sqrt(b*x + c)*b/(a^3 - 3*a^2*c + 3*a*c^2 - c^3) + 3*(3*a*b + b*c)*arctan(sqrt(b*x + a)/sqrt(-a))/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*sqrt(-a)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - 2*(a^2*c^2 + 3*a*c^3 - (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b + (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6*a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*sqrt(a*c))*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*sqrt(a*c))*b)*arctan(-(sqrt(b*x + a) - sqrt(b*x + c))/sqrt(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 + sqrt((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3))))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((sqrt(-a)*a^8*c - a^8*sqrt(-c)*c - 8*sqrt(-a)*a^7*c^2 + 8*a^7*sqrt(-c)*c^2 + 28*sqrt(-a)*a^6*c^3 - 28*a^6*sqrt(-c)*c^3 - 56*sqrt(-a)*a^5*c^4 + 56*a^5*sqrt(-c)*c^4 + 70*sqrt(-a)*a^4*c^5 - 70*a^4*sqrt(-c)*c^5 - 56*sqrt(-a)*a^3*c^6 + 56*a^3*sqrt(-c)*c^6 + 28*sqrt(-a)*a^2*c^7 - 28*a^2*sqrt(-c)*c^7 - 8*sqrt(-a)*a*c^8 + 8*a*sqrt(-c)*c^8 + sqrt(-a)*c^9 - sqrt(-c)*c^9)*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 3*(2*(a^2*c^2 + 3*a*c^3 + (a*c^2 + 3*c^3)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) + 2*(a^2*c^2 + 3*a*c^3 + (a^2*c + 3*a*c^2)*sqrt(a*c))*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)^2*b - (a^5*c^2 - a^4*c^3 - 6*a^3*c^4 + 14*a^2*c^5 - 11*a*c^6 + 3*c^7 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)*sgn(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^6*c - a^5*c^2 - 6*a^4*c^3 + 14*a^3*c^4 - 11*a^2*c^5 + 3*a*c^6 + (a^5*c - a^4*c^2 - 6*a^3*c^3 + 14*a^2*c^4 - 11*a*c^5 + 3*c^6)*sqrt(a*c))*b*abs(-a^3 + 3*a^2*c - 3*a*c^2 + c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 + (a^8*c - 2*a^7*c^2 - 6
```


$$\begin{aligned} & *a^6*c^3 + 22*a^5*c^4 - 20*a^4*c^5 - 6*a^3*c^6 + 22*a^2*c^7 - 14*a*c^8 + 3*c^9)*\text{sqrt}(a*c))*b*\text{sgn}(-2*a^3 + 6*a^2*c - 6*a*c^2 + 2*c^3) - (a^9*c - 2*a^8*c^2 - 6*a^7*c^3 + 22*a^6*c^4 - 20*a^5*c^5 - 6*a^4*c^6 + 22*a^3*c^7 - 14*a^2*c^8 + 3*a*c^9 - (a^9 - 2*a^8*c - 6*a^7*c^2 + 22*a^6*c^3 - 20*a^5*c^4 - 6*a^4*c^5 + 22*a^3*c^6 - 14*a^2*c^7 + 3*a*c^8)*\text{sqrt}(a*c))*b)*\text{arctan}(-(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))/\text{sqrt}(-(a^4 - 2*a^3*c + 2*a*c^3 - c^4 - \text{sqrt}((a^4 - 2*a^3*c + 2*a*c^3 - c^4)^2 - (a^5 - 5*a^4*c + 10*a^3*c^2 - 10*a^2*c^3 + 5*a*c^4 - c^5)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3))))/(a^3 - 3*a^2*c + 3*a*c^2 - c^3)))/((\text{sqrt}(-a)*a^8*c - a^8*\text{sqrt}(-c)*c - 8*\text{sqrt}(-a)*a^7*c^2 + 8*a^7*\text{sqrt}(-c)*c^2 + 28*\text{sqrt}(-a)*a^6*c^3 - 28*a^6*\text{sqrt}(-c)*c^3 - 56*\text{sqrt}(-a)*a^5*c^4 + 56*a^5*\text{sqrt}(-c)*c^4 + 70*\text{sqrt}(-a)*a^4*c^5 - 70*a^4*\text{sqrt}(-c)*c^5 - 56*\text{sqrt}(-a)*a^3*c^6 + 56*a^3*\text{sqrt}(-c)*c^6 + 28*\text{sqrt}(-a)*a^2*c^7 - 28*a^2*\text{sqrt}(-c)*c^7 - 8*\text{sqrt}(-a)*a*c^8 + 8*a*\text{sqrt}(-c)*c^8 + \text{sqrt}(-a)*c^9 - \text{sqrt}(-c)*c^9)*\text{abs}(-a^3 + 3*a^2*c - 3*a*c^2 + c^3)) - 2*(3*a*b*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))^3 + b*c*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))^3 - 3*a^2*b*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c)) + 2*a*b*c*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c)) + b*c^2*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c)))/(((\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))^4 - 2*a*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))^2 - 2*c*(\text{sqrt}(b*x + a) - \text{sqrt}(b*x + c))^2 + a^2 - 2*a*c + c^2)*(a^3 - 3*a^2*c + 3*a*c^2 - c^3)) - (\text{sqrt}(b*x + a)*a*b + 3*\text{sqrt}(b*x + a)*b*c)/((a^3 - 3*a^2*c + 3*a*c^2 - c^3)*b*x) \end{aligned}$$

maple [A] time = 0.02, size = 252, normalized size = 1.56

$$\frac{2\left(\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{c}}\right) - \sqrt{bx+a}}{2\sqrt{c}}\right)ab}{(a-c)^3} - \frac{6\left(\frac{\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) - \sqrt{bx+c}}{2\sqrt{c}}\right)ab}{(a-c)^3} + \frac{6\left(\frac{\text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{c}}\right) - \sqrt{bx+a}}{2\sqrt{c}}\right)bc}{(a-c)^3} - \frac{2\left(\frac{\text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) - \sqrt{bx+c}}{2\sqrt{c}}\right)bc}{(a-c)^3} + \frac{4(-2\sqrt{a} \text{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{c}}\right) + 2\sqrt{bx+a})b}{(a-c)^3} - \frac{4(-2\sqrt{c} \text{arctanh}\left(\frac{\sqrt{bx+c}}{\sqrt{c}}\right) + 2\sqrt{bx+c})b}{(a-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x)

[Out] $2/(a-c)^3*a*b*(-1/2*(b*x+a)^{(1/2)}/b/x-1/2*\text{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)})+6/(a-c)^3*c*b*(-1/2*(b*x+a)^{(1/2)}/b/x-1/2*\text{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)})/a^{(1/2)})-6/(a-c)^3*a*b*(-1/2*(b*x+c)^{(1/2)}/b/x-1/2/c^{(1/2)}*\text{arctanh}((b*x+c)^{(1/2)}/c^{(1/2)}))-2/(a-c)^3*c*b*(-1/2*(b*x+c)^{(1/2)}/b/x-1/2/c^{(1/2)}*\text{arctanh}((b*x+c)^{(1/2)}/c^{(1/2)}))+4/(a-c)^3*b*(2*(b*x+a)^{(1/2)}-2*a^{(1/2)}*\text{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)}))-4/(a-c)^3*b*(2*(b*x+c)^{(1/2)}-2*c^{(1/2)}*\text{arctanh}((b*x+c)^{(1/2)}/c^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(\sqrt{bx+a} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(b*x+c)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(b*x + c))^3), x)

mupad [B] time = 33.22, size = 4681, normalized size = 28.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (c + b*x)^(1/2))^3),x)

[Out] $(b*\text{atan}(((b*((a^{(1/2)}*c^{(3/2)} - 2*a*c + a^{(3/2)}*c^{(1/2)}))*(2*a*c + a^{(1/2)}*c^{(3/2)} + a^{(3/2)}*c^{(1/2)}))^{(1/2)}*((9*a^6*b*c^{(7/2)} - 9*a^{(7/2)}*b*c^6 - 24*a^5*b*c^{(9/2)} + 24*a^{(9/2)}*b*c^5 + 18*a^4*b*c^{(11/2)} - 18*a^{(11/2)}*b*c^4 - 3*a^2*b*c^{(15/2)} + 3*a^{(15/2)}*b*c^2)/(a^3*c^9 - 6*a^4*c^8 + 15*a^5*c^7 - 20*a^6*c^6 + 15*a^7*c^5 - 6*a^8*c^4 + a^9*c^3) + (((a + b*x)^{(1/2)} - a^{(1/2)}))* (6*a^{(3/2)}*b*c^8 - 6*a^8*b*c^{(3/2)} + 36*a^6*b*c^{(7/2)} - 36*a^{(7/2)}*b*c^6 -$

$$\begin{aligned}
& 48a^5b^2c^{9/2} + 48a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2}b^2c^4 \\
&) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6 \\
& 6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) - (3b((a^{5/2})c^{19/2} - 5a^{7/2} \\
& c^{17/2} + 9a^{9/2}c^{15/2} - 5a^{11/2}c^{13/2} - 5a^{13/2}c^{11/2} + 9a^{15/2} \\
& c^9 - 5a^{17/2}c^7 + a^{19/2}c^5)) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6 \\
& c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) \\
& - (((a + bx)^{1/2} - a^{1/2})(4a^2c^{10} - 28a^3c^9 + 88a^4c^8 - 164a^5 \\
& c^7 + 200a^6c^6 - 164a^7c^5 + 88a^8c^4 - 28a^9c^3 + 4a^{10}c^2)) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6 \\
& c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) * ((a^{1/2})c^{3/2} - 2ac + a^{3/2} \\
& c^{1/2}) * (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2})^{1/2} * (ac^{7/2} \\
& + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) \\
&) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \\
&) * (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + \\
& 2a^{5/2}c^2) * 3i) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6 \\
& c^3 - a^7c^2)) + (b((a^{1/2})c^{3/2} - 2ac + a^{3/2}c^{1/2}) * (2ac \\
& + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} * ((9a^6b^2c^{7/2} - 9a^{7/2}b^2 \\
& c^6 - 24a^5b^2c^{9/2} + 24a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2} \\
& b^2c^4 - 3a^2b^2c^{15/2} + 3a^{15/2}b^2c^2) / (a^3c^9 - 6a^4c^8 + 15a^5 \\
& c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) + (((a + bx)^{1/2} \\
& - a^{1/2}) * (6a^{3/2}b^2c^8 - 6a^8b^2c^{3/2} + 36a^6b^2c^{7/2} - 36a^{7/2} \\
& b^2c^6 - 48a^5b^2c^{9/2} + 48a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2} \\
& b^2c^4)) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5 \\
& c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) + (3b((a^{5/2})c^{19/2} - 5a^{7/2} \\
& c^{17/2} + 9a^{9/2}c^{15/2} - 5a^{11/2}c^{13/2} - 5a^{13/2}c^{11/2} + 9a^{15/2} \\
& c^9 - 5a^{17/2}c^7 + a^{19/2}c^5)) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6 \\
& c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) - (((a + bx)^{1/2} - a^{1/2})(4a^2c^{10} - 28a^3c^9 + 88a^4 \\
& c^8 - 164a^5c^7 + 200a^6c^6 - 164a^7c^5 + 88a^8c^4 - 28a^9c^3 + \\
& 4a^{10}c^2)) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6 \\
& c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) * ((a^{1/2})c^{3/2} - 2ac + a^{3/2} \\
& c^{1/2}) * (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2})^{1/2} * (ac^{7/2} + a^{7/2}c \\
& - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2 \\
& a^{5/2}c^2) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \\
&) * (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a \\
& ^{5/2}c^2) * 3i) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \\
&) / (((9a^{3/2}b^2c^{11/2}) / 2 - 18a^{5/2}b^2c^9 + 27a^{7/2}b^2c^7 - 18a^{9/2}b^2c^5 + (9a^{11/2}b^2 \\
& c^3) / 2) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) - (((a + bx)^{1/2} - a^{1/2}) * (72a^3b^2c^4 - 72 \\
& a^2b^2c^5 + 72a^4b^2c^3 - 72a^5b^2c^2 + 27a^{3/2}b^2c^{11/2} + \\
& 36a^{5/2}b^2c^9 - 126a^{7/2}b^2c^7 + 36a^{9/2}b^2c^5 + 27a^{11/2}b^2c^3)) / (((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) - (3b((a^{1/2})c^{3/2} - 2ac + a^{3/2}c^{1/2}) * (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} * ((9a^6b^2c^{7/2} - 9a^{7/2}b^2c^6 - 24a^5b^2c^{9/2} + 24a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2}b^2c^4 - 3a^2b^2c^{15/2} + 3a^{15/2}b^2c^2) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) + (((a + bx)^{1/2} - a^{1/2}) * (6a^{3/2}b^2c^8 - 6a^8b^2c^{3/2} + 36a^6b^2c^{7/2} - 36a^{7/2}b^2c^6 - 48a^5b^2c^{9/2} + 48a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2}b^2c^4)) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) - (3b((a^{5/2})c^{19/2} - 5a^{7/2}c^{17/2} + 9a^{9/2}c^{15/2} - 5a^{11/2}c^{13/2} - 5a^{13/2}c^{11/2} + 9a^{15/2}c^9 - 5a^{17/2}c^7 + a^{19/2}c^5)) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) - (((a + bx)^{1/2} - a^{1/2}) * (4a^2c^{10} - 28a^3c^9 + 88a^4c^8 - 164a^5c^7 + 200a^6c^6 - 164a^7c^5 + 88a^8c^4 - 28a^9c^3 + 4a^{10}c^2)) / (2((c + bx)^{1/2} - c^{1/2})(a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3))
\end{aligned}$$

$$\begin{aligned} & \sqrt[5]{-6a^8c^4 + a^9c^3}) \cdot ((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} \cdot (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \cdot (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \\ & + (3b \cdot ((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} \cdot ((9a^6b^2c^{7/2} - 9a^{7/2}b^2c^6 - 24a^5b^2c^{9/2} + 24a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2}b^2c^4 - 3a^2b^2c^{15/2} + 3a^{15/2}b^2c^2) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) + (((a + bx)^{1/2} - a^{1/2}) \cdot (6a^{3/2}b^2c^8 - 6a^8b^2c^{3/2} + 36a^6b^2c^{7/2} - 36a^{7/2}b^2c^6 - 48a^5b^2c^{9/2} + 48a^{9/2}b^2c^5 + 18a^4b^2c^{11/2} - 18a^{11/2}b^2c^4)) / (2((c + bx)^{1/2} - c^{1/2}) \cdot (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) + (3b \cdot ((a^{5/2}c^{19/2} - 5a^{7/2}c^{17/2} + 9a^{9/2}c^{15/2} - 5a^{11/2}c^{13/2} - 5a^{13/2}c^{11/2} + 9a^{15/2}c^9 - 5a^{17/2}c^7 + a^{19/2}c^5) / (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3) - ((a + bx)^{1/2} - a^{1/2}) \cdot (4a^2c^{10} - 28a^3c^9 + 88a^4c^8 - 164a^5c^7 + 200a^6c^6 - 164a^7c^5 + 88a^8c^4 - 28a^9c^3 + 4a^{10}c^2)) / (2((c + bx)^{1/2} - c^{1/2}) \cdot (a^3c^9 - 6a^4c^8 + 15a^5c^7 - 20a^6c^6 + 15a^7c^5 - 6a^8c^4 + a^9c^3)) \cdot ((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} \cdot (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \cdot (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) / (2(a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2)) \cdot ((a^{1/2}c^{3/2} - 2ac + a^{3/2}c^{1/2}) \cdot (2ac + a^{1/2}c^{3/2} + a^{3/2}c^{1/2}))^{1/2} \cdot (ac^{7/2} + a^{7/2}c - 3a^3c^{3/2} - 3a^{3/2}c^3 + 2a^2c^{5/2} + 2a^{5/2}c^2) \cdot 3i / (a^2c^7 - 5a^3c^6 + 10a^4c^5 - 10a^5c^4 + 5a^6c^3 - a^7c^2) - (\log(((a + bx)^{1/2} - a^{1/2}) / ((c + bx)^{1/2} - c^{1/2}))) \cdot (3a^2b^2c^{1/2} + 3a^{1/2}b^2c^2 + ac \cdot (9a^{1/2}b + 9b^2c^{1/2})) / (2a^2c^4 - 2a^4c - 6a^2c^3 + 6a^3c^2) - ((a^{1/2} \cdot ((3ab)/4 + (bc)/4)) / (ac^3 + 3a^3c - a^4 - 3a^2c^2) - (c^{1/2} \cdot ((ab)/4 + (3bc)/4)) / (3a^2c^3 + a^3c - c^4 - 3a^2c^2) - (((a^{1/2} \cdot ((3a^3b)/4 - (bc^3)/4 - (abc^2)/2 + 17a^2b^2c)) / (a^5c - a^2c^4 + 3a^3c^3 - 3a^4c^2) + (c^{1/2} \cdot ((a^3b)/4 - (3bc^3)/4 - 17abc^2 + (a^2b^2c)/2)) / (ac^5 - 3a^2c^4 + 3a^3c^3 - a^4c^2)) \cdot ((a + bx)^{1/2} - a^{1/2})^3 / ((c + bx)^{1/2} - c^{1/2})^3 + (((a^{1/2} \cdot ((bc^3)/4 - a^3b + (75ab^2c^2)/4 + 15a^2b^2c)) / (a^5c - a^2c^4 + 3a^3c^3 - 3a^4c^2) - (c^{1/2} \cdot ((a^3b)/4 - bc^3 + 15abc^2 + (75a^2b^2c)/4)) / (ac^5 - 3a^2c^4 + 3a^3c^3 - a^4c^2)) \cdot ((a + bx)^{1/2} - a^{1/2})^2 / ((c + bx)^{1/2} - c^{1/2})^2 + (((a^{1/2} \cdot ((a^2b)/4 - 2b^2c^2 + (67ab^2c)/4)) / (ac^4 - a^4c - 3a^2c^3 + 3a^3c^2) + (c^{1/2} \cdot ((bc^2)/4 - 2a^2b + (67ab^2c)/4)) / (ac^4 - a^4c - 3a^2c^3 + 3a^3c^2)) \cdot ((a + bx)^{1/2} - a^{1/2}) / ((c + bx)^{1/2} - c^{1/2}) / (((a + bx)^{1/2} - a^{1/2}) / ((c + bx)^{1/2} - c^{1/2})) - c^{1/2} + ((a + bx)^{1/2} - a^{1/2})^4 / ((c + bx)^{1/2} - c^{1/2})^4 - (((a + c) / (a^{1/2}c^{1/2}) - 1) \cdot ((a + bx)^{1/2} - a^{1/2})^2) / ((c + bx)^{1/2} - c^{1/2})^2 - (((a + c) / (a^{1/2}c^{1/2}) - 1) \cdot ((a + bx)^{1/2} - a^{1/2})^3) / ((c + bx)^{1/2} - c^{1/2})^3 - (b \cdot ((a + bx)^{1/2} - a^{1/2})) / (4a^{1/2}c^{1/2} \cdot (a^{1/2} - c^{1/2})^3 \cdot ((c + bx)^{1/2} - c^{1/2})) \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{bx+c})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/((b*x+a)**(1/2)+(b*x+c)**(1/2))**3,x)

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(b*x + c))**3), x)
```

$$3.193 \quad \int \frac{1}{\sqrt{x} + \sqrt{1+x}} dx$$

Optimal. Leaf size=21

$$\frac{2}{3}(x+1)^{3/2} - \frac{2x^{3/2}}{3}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2106, 30, 32}

$$\frac{2}{3}(x+1)^{3/2} - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] + Sqrt[1 + x])^(-1), x]

[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 32

Int[(a_) + (b_)*(x_)^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2106

Int[(u_)/((d_)*(x_)^(n_) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x_Symbol] := -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} + \sqrt{1+x}} dx &= - \int \sqrt{x} dx + \int \sqrt{1+x} dx \\ &= -\frac{2x^{3/2}}{3} + \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2} - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] + Sqrt[1 + x])^(-1), x]

[Out] (-2*x^(3/2))/3 + (2*(1 + x)^(3/2))/3

IntegrateAlgebraic [A] time = 0.10, size = 21, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2} - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x] + Sqrt[1 + x])^(-1), x]

[Out] $(-2x^{3/2})/3 + (2(1 + x)^{3/2})/3$

fricas [A] time = 0.64, size = 13, normalized size = 0.62

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] $2/3*(x + 1)^{3/2} - 2/3*x^{3/2}$

giac [A] time = 0.23, size = 13, normalized size = 0.62

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] $2/3*(x + 1)^{3/2} - 2/3*x^{3/2}$

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$-\frac{2x^{\frac{3}{2}}}{3} + \frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)+(x+1)^(1/2)),x)

[Out] $-2/3*x^{3/2}+2/3*(x+1)^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x + 1) + sqrt(x)), x)

mupad [B] time = 2.97, size = 21, normalized size = 1.00

$$\frac{2x\sqrt{x+1}}{3} + \frac{2\sqrt{x+1}}{3} - \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)^(1/2) + x^(1/2)),x)

[Out] $(2*x*(x + 1)^{1/2})/3 + (2*(x + 1)^{1/2})/3 - (2*x^{3/2})/3$

sympy [B] time = 0.94, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x+1}}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{4x}{3\sqrt{x} + 3\sqrt{x+1}} + \frac{2}{3\sqrt{x} + 3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x + 1)/(3*sqrt(x) + 3*sqrt(x + 1)) + 4*x/(3*sqrt(x) + 3*sqrt(x + 1)) + 2/(3*sqrt(x) + 3*sqrt(x + 1))
```

$$3.194 \quad \int \frac{1}{\sqrt{-1+x} + \sqrt{x}} dx$$

Optimal. Leaf size=21

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2106, 30, 32}

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x] + Sqrt[x])^(-1), x]

[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2106

Int[(u_)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x_Symbol] := -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x} + \sqrt{x}} dx &= - \int \sqrt{-1+x} dx + \int \sqrt{x} dx \\ &= -\frac{2}{3}(-1+x)^{3/2} + \frac{2x^{3/2}}{3} \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x] + Sqrt[x])^(-1), x]

[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3

IntegrateAlgebraic [A] time = 0.11, size = 21, normalized size = 1.00

$$\frac{2x^{3/2}}{3} - \frac{2}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[x])^(-1), x]

[Out] (-2*(-1 + x)^(3/2))/3 + (2*x^(3/2))/3

fricas [A] time = 0.67, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+x^(1/2)), x, algorithm="fricas")

[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)

giac [A] time = 0.19, size = 13, normalized size = 0.62

$$-\frac{2}{3}(x-1)^{\frac{3}{2}} + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+x^(1/2)), x, algorithm="giac")

[Out] -2/3*(x - 1)^(3/2) + 2/3*x^(3/2)

maple [A] time = 0.00, size = 14, normalized size = 0.67

$$\frac{2x^{\frac{3}{2}}}{3} - \frac{2(x-1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x-1)^(1/2)+x^(1/2)), x)

[Out] -2/3*(x-1)^(3/2)+2/3*x^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+x^(1/2)), x, algorithm="maxima")

[Out] integrate(1/(sqrt(x - 1) + sqrt(x)), x)

mupad [B] time = 2.94, size = 21, normalized size = 1.00

$$\frac{2\sqrt{x-1}}{3} - \frac{2x\sqrt{x-1}}{3} + \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^(1/2) + x^(1/2)), x)

[Out] (2*(x - 1)^(1/2))/3 - (2*x*(x - 1)^(1/2))/3 + (2*x^(3/2))/3

sympy [B] time = 0.39, size = 63, normalized size = 3.00

$$\frac{2\sqrt{x}\sqrt{x-1}}{3\sqrt{x} + 3\sqrt{x-1}} + \frac{4x}{3\sqrt{x} + 3\sqrt{x-1}} - \frac{2}{3\sqrt{x} + 3\sqrt{x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-1+x)**(1/2)+x**(1/2)),x)
```

```
[Out] 2*sqrt(x)*sqrt(x - 1)/(3*sqrt(x) + 3*sqrt(x - 1)) + 4*x/(3*sqrt(x) + 3*sqrt(x - 1)) - 2/(3*sqrt(x) + 3*sqrt(x - 1))
```

$$3.195 \quad \int \frac{1}{\sqrt{-1+x} + \sqrt{1+x}} dx$$

Optimal. Leaf size=23

$$\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {6689}

$$\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1), x]

[Out] -(-1 + x)^(3/2)/3 + (1 + x)^(3/2)/3

Rule 6689

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(a*e^2 - c*f^2)^m, Int[ExpandIntegrand[u/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x} + \sqrt{1+x}} dx &= -\left(\frac{1}{2} \int \left(\sqrt{-1+x} - \sqrt{1+x}\right) dx\right) \\ &= -\frac{1}{3}(-1+x)^{3/2} + \frac{1}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1), x]

[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3

IntegrateAlgebraic [A] time = 0.18, size = 23, normalized size = 1.00

$$\frac{1}{3}(x+1)^{3/2} - \frac{1}{3}(x-1)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[1 + x])^(-1), x]

[Out] -1/3*(-1 + x)^(3/2) + (1 + x)^(3/2)/3

fricas [A] time = 0.73, size = 15, normalized size = 0.65

$$\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)

giac [A] time = 0.19, size = 15, normalized size = 0.65

$$\frac{1}{3}(x+1)^{\frac{3}{2}} - \frac{1}{3}(x-1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] 1/3*(x + 1)^(3/2) - 1/3*(x - 1)^(3/2)

maple [A] time = 0.00, size = 16, normalized size = 0.70

$$-\frac{(x-1)^{\frac{3}{2}}}{3} + \frac{(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x-1)^(1/2)+(x+1)^(1/2)),x)

[Out] -1/3*(x-1)^(3/2)+1/3*(x+1)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x+1} + \sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x + 1) + sqrt(x - 1)), x)

mupad [B] time = 2.84, size = 15, normalized size = 0.65

$$\frac{(x+1)^{3/2}}{3} - \frac{(x-1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^(1/2) + (x + 1)^(1/2)),x)

[Out] (x + 1)^(3/2)/3 - (x - 1)^(3/2)/3

sympy [B] time = 0.41, size = 51, normalized size = 2.22

$$\frac{4x}{3\sqrt{x-1} + 3\sqrt{x+1}} + \frac{2\sqrt{x-1}\sqrt{x+1}}{3\sqrt{x-1} + 3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**(1/2)+(1+x)**(1/2)),x)

[Out] 4*x/(3*sqrt(x - 1) + 3*sqrt(x + 1)) + 2*sqrt(x - 1)*sqrt(x + 1)/(3*sqrt(x - 1) + 3*sqrt(x + 1))

$$3.196 \quad \int x^3 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx$$

Optimal. Leaf size=38

$$\frac{x^4}{2} + \frac{2}{5}(1-x^2)^{5/2} - \frac{2}{3}(1-x^2)^{3/2}$$

Rubi [A] time = 0.11, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6742, 266, 43}

$$\frac{x^4}{2} + \frac{2}{5}(1-x^2)^{5/2} - \frac{2}{3}(1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^3 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx &= \int \left(2x^3 + 2x^3\sqrt{1-x^2} \right) dx \\ &= \frac{x^4}{2} + 2 \int x^3\sqrt{1-x^2} dx \\ &= \frac{x^4}{2} + \text{Subst} \left(\int \sqrt{1-x} x dx, x, x^2 \right) \\ &= \frac{x^4}{2} + \text{Subst} \left(\int \left(\sqrt{1-x} - (1-x)^{3/2} \right) dx, x, x^2 \right) \\ &= \frac{x^4}{2} - \frac{2}{3}(1-x^2)^{3/2} + \frac{2}{5}(1-x^2)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 38, normalized size = 1.00

$$\frac{x^4}{2} + \frac{2}{5}(1-x^2)^{5/2} - \frac{2}{3}(1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^4/2 - (2*(1 - x^2)^(3/2))/3 + (2*(1 - x^2)^(5/2))/5

IntegrateAlgebraic [B] time = 0.20, size = 80, normalized size = 2.11

$$\frac{1}{2}((x+1)^4 - 4(x+1)^3 + 6(x+1)^2 - 4(x+1)) + \frac{2}{15}\sqrt{1-x}(3(x+1)^{9/2} - 12(x+1)^{7/2} + 17(x+1)^{5/2} - 10(x+1)^{3/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] (-4*(1 + x) + 6*(1 + x)^2 - 4*(1 + x)^3 + (1 + x)^4)/2 + (2*Sqrt[1 - x]*(-10*(1 + x)^(3/2) + 17*(1 + x)^(5/2) - 12*(1 + x)^(7/2) + 3*(1 + x)^(9/2)))/15

fricas [A] time = 0.81, size = 32, normalized size = 0.84

$$\frac{1}{2}x^4 + \frac{2}{15}(3x^4 - x^2 - 2)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")

[Out] 1/2*x^4 + 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)

giac [B] time = 0.22, size = 77, normalized size = 2.03

$$\frac{1}{2}x^4 + \frac{1}{60}((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{12}((2(3x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")

[Out] 1/2*x^4 + 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1)

maple [A] time = 0.01, size = 33, normalized size = 0.87

$$\frac{x^4}{2} + \frac{2\sqrt{x+1}\sqrt{-x+1}(x^2-1)(3x^2+2)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((1-x)^(1/2)+(x+1)^(1/2))^2,x)

[Out] 1/2*x^4+2/15*(x+1)^(1/2)*(1-x)^(1/2)*(x^2-1)*(3*x^2+2)

maxima [A] time = 1.31, size = 31, normalized size = 0.82

$$\frac{1}{2}x^4 - \frac{2}{5}(-x^2 + 1)^{\frac{3}{2}}x^2 - \frac{4}{15}(-x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")

[Out] 1/2*x^4 - 2/5*(-x^2 + 1)^(3/2)*x^2 - 4/15*(-x^2 + 1)^(3/2)

mupad [B] time = 3.02, size = 45, normalized size = 1.18

$$\frac{x^4}{2} - \frac{\sqrt{1-x}\left(-\frac{2x^5}{5} - \frac{2x^4}{5} + \frac{2x^3}{15} + \frac{2x^2}{15} + \frac{4x}{15} + \frac{4}{15}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)
```

```
[Out] x^4/2 - ((1 - x)^(1/2)*((4*x)/15 + (2*x^2)/15 + (2*x^3)/15 - (2*x^4)/5 - (2*x^5)/5 + 4/15))/(x + 1)^(1/2)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] Timed out
```

$$3.197 \quad \int x^2 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx$$

Optimal. Leaf size=48

$$\frac{2x^3}{3} - \frac{1}{4}\sqrt{1-x^2}x + \frac{1}{2}\sqrt{1-x^2}x^3 + \frac{1}{4}\sin^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {6742, 279, 321, 216}

$$\frac{1}{2}\sqrt{1-x^2}x^3 + \frac{2x^3}{3} - \frac{1}{4}\sqrt{1-x^2}x + \frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] (2*x^3)/3 - (x*Sqrt[1 - x^2])/4 + (x^3*Sqrt[1 - x^2])/2 + ArcSin[x]/4

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 279

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+n*p+1)), x] + Dist[(a*n*p)/(m+n*p+1), Int[(c*x)^m*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^2 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx &= \int \left(2x^2 + 2x^2\sqrt{1-x^2} \right) dx \\ &= \frac{2x^3}{3} + 2 \int x^2\sqrt{1-x^2} dx \\ &= \frac{2x^3}{3} + \frac{1}{2}x^3\sqrt{1-x^2} + \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx \\ &= \frac{2x^3}{3} - \frac{1}{4}x\sqrt{1-x^2} + \frac{1}{2}x^3\sqrt{1-x^2} + \frac{1}{4} \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{2x^3}{3} - \frac{1}{4}x\sqrt{1-x^2} + \frac{1}{2}x^3\sqrt{1-x^2} + \frac{1}{4}\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 0.88

$$\frac{1}{12} \left(-3\sqrt{1-x^2}x + \left(6\sqrt{1-x^2} + 8 \right) x^3 + 3\sin^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] (-3*x*Sqrt[1 - x^2] + x^3*(8 + 6*Sqrt[1 - x^2]) + 3*ArcSin[x])/12

IntegrateAlgebraic [C] time = 0.18, size = 101, normalized size = 2.10

$$\frac{2}{3} \left((x+1)^3 - 3(x+1)^2 + 3(x+1) \right) + \frac{1}{4} \sqrt{1-x} \left(2(x+1)^{7/2} - 6(x+1)^{5/2} + 5(x+1)^{3/2} - \sqrt{x+1} \right) + \frac{1}{2} i \log \left(\sqrt{1-x} - i\sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] (2*(3*(1 + x) - 3*(1 + x)^2 + (1 + x)^3))/3 + (Sqrt[1 - x]*(-Sqrt[1 + x] + 5*(1 + x)^(3/2) - 6*(1 + x)^(5/2) + 2*(1 + x)^(7/2)))/4 + (I/2)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.66, size = 51, normalized size = 1.06

$$\frac{2}{3} x^3 + \frac{1}{4} (2x^3 - x) \sqrt{x+1} \sqrt{-x+1} - \frac{1}{2} \arctan \left(\frac{\sqrt{x+1} \sqrt{-x+1} - 1}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")

[Out] 2/3*x^3 + 1/4*(2*x^3 - x)*sqrt(x + 1)*sqrt(-x + 1) - 1/2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [B] time = 0.23, size = 76, normalized size = 1.58

$$\frac{2}{3} x^3 + \frac{1}{12} \left((2(3x-10)(x+1)+43)(x+1) - 39 \right) \sqrt{x+1} \sqrt{-x+1} + \frac{1}{3} \left((2x-5)(x+1)+9 \right) \sqrt{x+1} \sqrt{-x+1} + \frac{1}{2} \arcsin \left(\frac{1}{2} \sqrt{2} \sqrt{x+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")

[Out] 2/3*x^3 + 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1) + 1/3*((2*x - 5)*(x + 1) + 9)*sqrt(x + 1)*sqrt(-x + 1) + 1/2*arcsin(1/2*sqrt(2)*sqrt(x + 1))

maple [A] time = 0.01, size = 59, normalized size = 1.23

$$\frac{2x^3}{3} + \frac{\sqrt{x+1} \sqrt{-x+1} \left(2\sqrt{-x^2+1} x^3 - \sqrt{-x^2+1} x + \arcsin(x) \right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)

[Out] 2/3*x^3+1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*x^3*(-x^2+1)^(1/2)-x*(-x^2+1)^(1/2)+arcsin(x))/(-x^2+1)^(1/2)

maxima [A] time = 1.40, size = 34, normalized size = 0.71

$$\frac{2}{3} x^3 - \frac{1}{2} (-x^2 + 1)^{\frac{3}{2}} x + \frac{1}{4} \sqrt{-x^2 + 1} x + \frac{1}{4} \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")

[Out] $2/3*x^3 - 1/2*(-x^2 + 1)^{(3/2)}*x + 1/4*\text{sqrt}(-x^2 + 1)*x + 1/4*\text{arcsin}(x)$

mupad [B] time = 14.09, size = 563, normalized size = 11.73

$$\frac{\frac{4(\sqrt{1-x}-1)}{\sqrt{x+1}} - \frac{28(\sqrt{1-x})^3}{(\sqrt{x+1})^3} + \frac{28(\sqrt{1-x})^5}{(\sqrt{x+1})^5} - \frac{4(\sqrt{1-x})^7}{(\sqrt{x+1})^7}}{\frac{4(\sqrt{1-x})^2}{(\sqrt{x+1})^2} + \frac{6(\sqrt{1-x})^4}{(\sqrt{x+1})^4} + \frac{4(\sqrt{1-x})^6}{(\sqrt{x+1})^6} + \frac{(\sqrt{1-x})^8}{(\sqrt{x+1})^8} + 1} - \text{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{3(\sqrt{1-x}-1)}{\sqrt{x+1}} + \frac{23(\sqrt{1-x})^3}{(\sqrt{x+1})^3} - \frac{333(\sqrt{1-x})^5}{(\sqrt{x+1})^5} + \frac{671(\sqrt{1-x})^7}{(\sqrt{x+1})^7} - \frac{671(\sqrt{1-x})^9}{(\sqrt{x+1})^9} + \frac{333(\sqrt{1-x})^{11}}{(\sqrt{x+1})^{11}} - \frac{23(\sqrt{1-x})^{13}}{(\sqrt{x+1})^{13}} - \frac{3(\sqrt{1-x})^{15}}{(\sqrt{x+1})^{15}} + \frac{2x^3}{3} - \frac{8(\sqrt{1-x})^2}{(\sqrt{x+1})^2} + \frac{28(\sqrt{1-x})^4}{(\sqrt{x+1})^4} + \frac{56(\sqrt{1-x})^6}{(\sqrt{x+1})^6} + \frac{70(\sqrt{1-x})^8}{(\sqrt{x+1})^8} + \frac{56(\sqrt{1-x})^{10}}{(\sqrt{x+1})^{10}} + \frac{28(\sqrt{1-x})^{12}}{(\sqrt{x+1})^{12}} + \frac{8(\sqrt{1-x})^{14}}{(\sqrt{x+1})^{14}} + \frac{(\sqrt{1-x})^{16}}{(\sqrt{x+1})^{16}} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)

[Out] $((4*((1-x)^{(1/2)} - 1))/((x+1)^{(1/2)} - 1) - (28*((1-x)^{(1/2)} - 1)^3)/((x+1)^{(1/2)} - 1)^3 + (28*((1-x)^{(1/2)} - 1)^5)/((x+1)^{(1/2)} - 1)^5 - (4*((1-x)^{(1/2)} - 1)^7)/((x+1)^{(1/2)} - 1)^7)/((4*((1-x)^{(1/2)} - 1)^2)/((x+1)^{(1/2)} - 1)^2 + (6*((1-x)^{(1/2)} - 1)^4)/((x+1)^{(1/2)} - 1)^4 + (4*((1-x)^{(1/2)} - 1)^6)/((x+1)^{(1/2)} - 1)^6 + ((1-x)^{(1/2)} - 1)^8/((x+1)^{(1/2)} - 1)^8 + 1) - \text{atan}(((1-x)^{(1/2)} - 1)/((x+1)^{(1/2)} - 1)) - ((3*((1-x)^{(1/2)} - 1))/((x+1)^{(1/2)} - 1) + (23*((1-x)^{(1/2)} - 1)^3)/((x+1)^{(1/2)} - 1)^3 - (333*((1-x)^{(1/2)} - 1)^5)/((x+1)^{(1/2)} - 1)^5 + (671*((1-x)^{(1/2)} - 1)^7)/((x+1)^{(1/2)} - 1)^7 - (671*((1-x)^{(1/2)} - 1)^9)/((x+1)^{(1/2)} - 1)^9 + (333*((1-x)^{(1/2)} - 1)^{11})/((x+1)^{(1/2)} - 1)^{11} - (23*((1-x)^{(1/2)} - 1)^{13})/((x+1)^{(1/2)} - 1)^{13} - (3*((1-x)^{(1/2)} - 1)^{15})/((x+1)^{(1/2)} - 1)^{15})/((8*((1-x)^{(1/2)} - 1)^2)/((x+1)^{(1/2)} - 1)^2 + (28*((1-x)^{(1/2)} - 1)^4)/((x+1)^{(1/2)} - 1)^4 + (56*((1-x)^{(1/2)} - 1)^6)/((x+1)^{(1/2)} - 1)^6 + (70*((1-x)^{(1/2)} - 1)^8)/((x+1)^{(1/2)} - 1)^8 + (56*((1-x)^{(1/2)} - 1)^{10})/((x+1)^{(1/2)} - 1)^{10} + (28*((1-x)^{(1/2)} - 1)^{12})/((x+1)^{(1/2)} - 1)^{12} + (8*((1-x)^{(1/2)} - 1)^{14})/((x+1)^{(1/2)} - 1)^{14} + ((1-x)^{(1/2)} - 1)^{16}/((x+1)^{(1/2)} - 1)^{16} + 1) + (2*x^3)/3$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*((1-x)**(1/2)+(1+x)**(1/2))**2,x)

[Out] Timed out

$$3.198 \quad \int x \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx$$

Optimal. Leaf size=19

$$x^2 - \frac{2}{3} (1-x^2)^{3/2}$$

Rubi [A] time = 0.05, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6742, 261}

$$x^2 - \frac{2}{3} (1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^2 - (2*(1 - x^2)^(3/2))/3

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx &= \int \left(2x + 2x\sqrt{1-x^2} \right) dx \\ &= x^2 + 2 \int x\sqrt{1-x^2} dx \\ &= x^2 - \frac{2}{3} (1-x^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 19, normalized size = 1.00

$$x^2 - \frac{2}{3} (1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x^2 - (2*(1 - x^2)^(3/2))/3

IntegrateAlgebraic [B] time = 0.14, size = 41, normalized size = 2.16

$$(x+1)^2 - 2(x+1) + \frac{2}{3}\sqrt{1-x} \left((x+1)^{5/2} - 2(x+1)^{3/2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] $-2*(1 + x) + (1 + x)^2 + (2*\text{Sqrt}[1 - x]*(-2*(1 + x)^{(3/2)} + (1 + x)^{(5/2)})) / 3$

fricas [A] time = 0.61, size = 23, normalized size = 1.21

$$x^2 + \frac{2}{3}(x^2 - 1)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")`

[Out] $x^2 + 2/3*(x^2 - 1)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1)$

giac [B] time = 0.25, size = 51, normalized size = 2.68

$$(x + 1)^2 + \frac{1}{3}((2x - 5)(x + 1) + 9)\sqrt{x+1}\sqrt{-x+1} + \sqrt{x+1}(x - 2)\sqrt{-x+1} - 2x - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")`

[Out] $(x + 1)^2 + 1/3*((2*x - 5)*(x + 1) + 9)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) + \text{sqrt}(x + 1)*(x - 2)*\text{sqrt}(-x + 1) - 2*x - 2$

maple [A] time = 0.00, size = 24, normalized size = 1.26

$$x^2 + \frac{2\sqrt{x+1}\sqrt{-x+1}(x^2-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((-x+1)^(1/2)+(x+1)^(1/2))^2,x)`

[Out] $x^2+2/3*(x+1)^{(1/2)*(-x+1)^{(1/2)}*(x^2-1)$

maxima [A] time = 1.34, size = 15, normalized size = 0.79

$$x^2 - \frac{2}{3}(-x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")`

[Out] $x^2 - 2/3*(-x^2 + 1)^{(3/2)}$

mupad [B] time = 2.98, size = 33, normalized size = 1.74

$$x^2 - \frac{\sqrt{1-x} \left(-\frac{2x^3}{3} - \frac{2x^2}{3} + \frac{2x}{3} + \frac{2}{3} \right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((x+1)^(1/2)+(1-x)^(1/2))^2,x)`

[Out] $x^2 - (((1-x)^{(1/2)}*((2*x)/3 - (2*x^2)/3 - (2*x^3)/3 + 2/3))/(x+1)^{(1/2)}$

sympy [A] time = 106.39, size = 110, normalized size = 5.79

$$\frac{x^3}{3} - x + \frac{(x+1)^3}{3} - 4 \left(\left(\frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\text{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \right) \text{ for } x \geq -1 \wedge x < 1 \right) + 4 \left(\left(\frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\text{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \right) \text{ for } x \geq -1 \wedge x < 1 \right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*((1-x)**(1/2)+(1+x)**(1/2))**2,x)
```

```
[Out] -x**3/3 - x + (x + 1)**3/3 - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 1
```

$$3.199 \quad \int \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx$$

Optimal. Leaf size=19

$$\sqrt{1-x^2}x + 2x + \sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6742, 195, 216}

$$\sqrt{1-x^2}x + 2x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] 2*x + x*Sqrt[1 - x^2] + ArcSin[x]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx &= \int \left(2 + 2\sqrt{1-x^2} \right) dx \\ &= 2x + 2 \int \sqrt{1-x^2} dx \\ &= 2x + x\sqrt{1-x^2} + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= 2x + x\sqrt{1-x^2} + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.95

$$x \left(\sqrt{1-x^2} + 2 \right) + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2,x]

[Out] x*(2 + Sqrt[1 - x^2]) + ArcSin[x]

IntegrateAlgebraic [C] time = 0.14, size = 59, normalized size = 3.11

$$2(x+1) + \sqrt{1-x} \left((x+1)^{3/2} - \sqrt{x+1} \right) + 2i \log \left(\sqrt{1-x} - i\sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2, x]

[Out] 2*(1 + x) + Sqrt[1 - x]*(-Sqrt[1 + x] + (1 + x)^(3/2)) + (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [B] time = 0.59, size = 40, normalized size = 2.11

$$\sqrt{x+1}x\sqrt{-x+1} + 2x - 2 \arctan \left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="fricas")

[Out] sqrt(x + 1)*x*sqrt(-x + 1) + 2*x - 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [B] time = 0.20, size = 48, normalized size = 2.53

$$\sqrt{x+1}(x-2)\sqrt{-x+1} + 2x + 2\sqrt{x+1}\sqrt{-x+1} + 2 \arcsin \left(\frac{1}{2} \sqrt{2} \sqrt{x+1} \right) + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="giac")

[Out] sqrt(x + 1)*(x - 2)*sqrt(-x + 1) + 2*x + 2*sqrt(x + 1)*sqrt(-x + 1) + 2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) + 2

maple [B] time = 0.01, size = 58, normalized size = 3.05

$$2x + \frac{\sqrt{(x+1)(-x+1)} \arcsin(x)}{\sqrt{-x+1} \sqrt{x+1}} - \sqrt{x+1} (-x+1)^{3/2} + \sqrt{-x+1} \sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^(1/2)+(-x+1)^(1/2))^2,x)

[Out] 2*x-(x+1)^(1/2)*(-x+1)^(3/2)+(-x+1)^(1/2)*(x+1)^(1/2)+((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)

maxima [A] time = 1.42, size = 17, normalized size = 0.89

$$\sqrt{-x^2+1}x + 2x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2,x, algorithm="maxima")

[Out] sqrt(-x^2 + 1)*x + 2*x + arcsin(x)

mupad [B] time = 7.72, size = 206, normalized size = 10.84

$$2x - 4 \operatorname{atan} \left(\frac{\sqrt{1-x} - 1}{\sqrt{x+1} - 1} \right) - \frac{\frac{4(\sqrt{1-x}-1)}{\sqrt{x+1}-1} - \frac{28(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3} + \frac{28(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5} - \frac{4(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7}}{\frac{4(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{6(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + \frac{4(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6} + \frac{(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)`

[Out] $2*x - 4*\operatorname{atan}\left(\frac{(1-x)^{1/2}-1}{(x+1)^{1/2}-1}\right) - \frac{(4*(1-x)^{1/2}-1)}{(x+1)^{1/2}-1} - \frac{(28*(1-x)^{1/2}-1)^3}{(x+1)^{1/2}-1} - \frac{3+(28*(1-x)^{1/2}-1)^5}{(x+1)^{1/2}-1} - \frac{(4*(1-x)^{1/2}-1)^7}{(x+1)^{1/2}-1} - \frac{(4*(1-x)^{1/2}-1)^2}{(x+1)^{1/2}-1} - \frac{(6*(1-x)^{1/2}-1)^4}{(x+1)^{1/2}-1} + \frac{(4*(1-x)^{1/2}-1)^4+(4*(1-x)^{1/2}-1)^6}{(x+1)^{1/2}-1} + \frac{(1-x)^{1/2}-1}{(x+1)^{1/2}-1} + 1$

sympy [A] time = 31.43, size = 44, normalized size = 2.32

$$2x + 4 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \quad \text{for } x \geq -1 \wedge x < 1 \right\} + 2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1-x)**(1/2)+(1+x)**(1/2))**2,x)`

[Out] $2*x + 4*\operatorname{Piecewise}\left(\frac{x*\sqrt{1-x}*\sqrt{x+1}}{4} + \operatorname{asin}\left(\frac{\sqrt{2}*\sqrt{x+1}}{2}\right)/2, (x \geq -1) \& (x < 1)\right) + 2$

$$3.200 \quad \int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx$$

Optimal. Leaf size=32

$$2\sqrt{1-x^2} - 2 \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2 \log(x)$$

Rubi [A] time = 0.09, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6742, 266, 50, 63, 206}

$$2\sqrt{1-x^2} - 2 \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x,x]

[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx &= \int \left(\frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} \right) dx \\
&= 2 \log(x) + 2 \int \frac{\sqrt{1-x^2}}{x} dx \\
&= 2 \log(x) + \text{Subst} \left(\int \frac{\sqrt{1-x}}{x} dx, x, x^2 \right) \\
&= 2\sqrt{1-x^2} + 2 \log(x) + \text{Subst} \left(\int \frac{1}{\sqrt{1-x}x} dx, x, x^2 \right) \\
&= 2\sqrt{1-x^2} + 2 \log(x) - 2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \\
&= 2\sqrt{1-x^2} - 2 \tanh^{-1}(\sqrt{1-x^2}) + 2 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 32, normalized size = 1.00

$$2\sqrt{1-x^2} - 2 \tanh^{-1}(\sqrt{1-x^2}) + 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x, x]

[Out] 2*Sqrt[1 - x^2] - 2*ArcTanh[Sqrt[1 - x^2]] + 2*Log[x]

IntegrateAlgebraic [C] time = 0.16, size = 75, normalized size = 2.34

$$2\sqrt{1-x}\sqrt{x+1} + 4 \log(\sqrt{1-x} - i\sqrt{x+1} + (1-i)) + 8 \tanh^{-1}((-1-i)\sqrt{1-x} - (1-i)\sqrt{x+1} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x, x]

[Out] 2*Sqrt[1 - x]*Sqrt[1 + x] + 8*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sqrt[1 + x]] + 4*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.47, size = 41, normalized size = 1.28

$$2\sqrt{x+1}\sqrt{-x+1} + 2 \log(x) + 2 \log\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="fricas")

[Out] 2*sqrt(x + 1)*sqrt(-x + 1) + 2*log(x) + 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616

423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086]2*ln(abs(sqrt(x+1)-1))+2*ln(sqrt(x+1)+1)+2*sqrt(x+1)*sqrt(-x+1)-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))

maple [A] time = 0.01, size = 51, normalized size = 1.59

$$2 \ln(x) + \frac{2\sqrt{x+1} \sqrt{-x+1} \left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \sqrt{-x^2+1} \right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^(1/2)+(x+1)^(1/2))^2/x,x)

[Out] 2*ln(x)+2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2)))

maxima [A] time = 1.53, size = 41, normalized size = 1.28

$$2\sqrt{-x^2+1} + 2\log(x) - 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x,x, algorithm="maxima")

[Out] 2*sqrt(-x^2 + 1) + 2*log(x) - 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

mupad [B] time = 4.10, size = 122, normalized size = 3.81

$$2 \ln\left(\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}-1\right) - 2 \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + 2 \ln(x) + \frac{16(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2 \left(\frac{2(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^(1/2)+(1-x)^(1/2))^2/x,x)

[Out] 2*log(((1-x)^(1/2)-1)^2/((x+1)^(1/2)-1)^2-1)-2*log(((1-x)^(1/2)-1)/((x+1)^(1/2)-1))+2*log(x)+(16*((1-x)^(1/2)-1)^2)/(((x+1)^(1/2)-1)^2*((2*((1-x)^(1/2)-1)^2)/((x+1)^(1/2)-1)^2+((1-x)^(1/2)-1)^4/((x+1)^(1/2)-1)^4+1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sqrt{1-x} + \sqrt{x+1})^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x,x)

[Out] Integral((sqrt(1-x)+sqrt(x+1))**2/x,x)

$$3.201 \quad \int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^2} dx$$

Optimal. Leaf size=26

$$-\frac{2\sqrt{1-x^2}}{x} - \frac{2}{x} - 2\sin^{-1}(x)$$

Rubi [A] time = 0.08, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {6742, 277, 216}

$$-\frac{2\sqrt{1-x^2}}{x} - \frac{2}{x} - 2\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]

[Out] -2/x - (2*Sqrt[1 - x^2])/x - 2*ArcSin[x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^2} dx &= \int \left(\frac{2}{x^2} + \frac{2\sqrt{1-x^2}}{x^2} \right) dx \\ &= -\frac{2}{x} + 2 \int \frac{\sqrt{1-x^2}}{x^2} dx \\ &= -\frac{2}{x} - \frac{2\sqrt{1-x^2}}{x} - 2 \int \frac{1}{\sqrt{1-x^2}} dx \\ &= -\frac{2}{x} - \frac{2\sqrt{1-x^2}}{x} - 2\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.85

$$-\frac{2\left(\sqrt{1-x^2} + x\sin^{-1}(x) + 1\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]

[Out] (-2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x

IntegrateAlgebraic [C] time = 0.16, size = 53, normalized size = 2.04

$$-\frac{2\sqrt{1-x}\sqrt{x+1}}{x} - \frac{2}{x} - 4i \log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^2,x]

[Out] -2/x - (2*Sqrt[1 - x]*Sqrt[1 + x])/x - (4*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.54, size = 44, normalized size = 1.69

$$\frac{2\left(2x \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) - \sqrt{x+1}\sqrt{-x+1} - 1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="fricas")

[Out] 2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1) - 1)/x

giac [B] time = 0.26, size = 149, normalized size = 5.73

$$-2\pi - \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} - \frac{2}{x} - 4 \arctan\left(\frac{\sqrt{x+1}\left(\frac{(\sqrt{2}-\sqrt{-x+1})^2}{x+1} - 1\right)}{2(\sqrt{2}-\sqrt{-x+1})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="giac")

[Out] -2*pi - 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))^2 - 4) - 2/x - 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))

maple [B] time = 0.02, size = 50, normalized size = 1.92

$$-\frac{2}{x} + \frac{2\left(-x \arcsin(x) - \sqrt{-x^2+1}\right)\sqrt{x+1}\sqrt{-x+1}}{\sqrt{-x^2+1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x)

[Out] -2/x+2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1/2)

maxima [A] time = 1.26, size = 24, normalized size = 0.92

$$-\frac{2\sqrt{-x^2+1}}{x} - \frac{2}{x} - 2 \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^2,x, algorithm="maxima")

[Out] -2*sqrt(-x^2 + 1)/x - 2/x - 2*arcsin(x)

mupad [B] time = 3.79, size = 120, normalized size = 4.62

$$8 \operatorname{atan} \left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} \right) - \frac{\frac{5(\sqrt{1-x}-1)^2}{2(\sqrt{x+1}-1)^2} - \frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3}} - \frac{\sqrt{1-x}-1}{2(\sqrt{x+1}-1)} - \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))^2/x^2,x)

[Out] 8*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((5*((1 - x)^(1/2) - 1)^2)/(2*((x + 1)^(1/2) - 1)^2) - 1/2)/(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1) - ((1 - x)^(1/2) - 1)^3/((x + 1)^(1/2) - 1)^3) - ((1 - x)^(1/2) - 1)/(2*((x + 1)^(1/2) - 1)) - 2/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sqrt{1-x} + \sqrt{x+1})^2}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**2,x)

[Out] Integral((sqrt(1 - x) + sqrt(x + 1))**2/x**2, x)

$$3.202 \quad \int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx$$

Optimal. Leaf size=34

$$-\frac{\sqrt{1-x^2}}{x^2} - \frac{1}{x^2} + \tanh^{-1}(\sqrt{1-x^2})$$

Rubi [A] time = 0.09, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6742, 266, 47, 63, 206}

$$-\frac{\sqrt{1-x^2}}{x^2} - \frac{1}{x^2} + \tanh^{-1}(\sqrt{1-x^2})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3,x]

[Out] -x^(-2) - Sqrt[1 - x^2]/x^2 + ArcTanh[Sqrt[1 - x^2]]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx &= \int \left(\frac{2}{x^3} + \frac{2\sqrt{1-x^2}}{x^3} \right) dx \\
&= -\frac{1}{x^2} + 2 \int \frac{\sqrt{1-x^2}}{x^3} dx \\
&= -\frac{1}{x^2} + \text{Subst} \left(\int \frac{\sqrt{1-x}}{x^2} dx, x, x^2 \right) \\
&= -\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-xx}} dx, x, x^2 \right) \\
&= -\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} + \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \\
&= -\frac{1}{x^2} - \frac{\sqrt{1-x^2}}{x^2} + \tanh^{-1}(\sqrt{1-x^2})
\end{aligned}$$

Mathematica [A] time = 0.04, size = 45, normalized size = 1.32

$$-\frac{1}{x^2\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}} - \frac{1}{x^2} + \tanh^{-1}(\sqrt{1-x^2})$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3, x]

[Out] -x^(-2) + 1/Sqrt[1 - x^2] - 1/(x^2*Sqrt[1 - x^2]) + ArcTanh[Sqrt[1 - x^2]]

IntegrateAlgebraic [C] time = 0.24, size = 54, normalized size = 1.59

$$-\frac{\sqrt{1-x}\sqrt{x+1}}{x^2} - \frac{1}{x^2} - 2i \tan^{-1}(x + i\sqrt{1-x}\sqrt{x+1})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])^2/x^3, x]

[Out] -x^(-2) - (Sqrt[1 - x]*Sqrt[1 + x])/x^2 - (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sqrt[1 + x]]

fricas [A] time = 0.40, size = 44, normalized size = 1.29

$$\frac{x^2 \log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + \sqrt{x+1}\sqrt{-x+1} + 1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3, x, algorithm="fricas")

[Out] -(x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/x^2

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3, x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086]
$$-(4*(2*\sqrt{x+1}/(-2*\sqrt{-x+1}+2*\sqrt{2}))-1/2*(-2*\sqrt{-x+1}+2*\sqrt{2}))/\sqrt{x+1})^3+16*(2*\sqrt{x+1}/(-2*\sqrt{-x+1}+2*\sqrt{2}))-1/2*(-2*\sqrt{-x+1}+2*\sqrt{2}))/\sqrt{x+1}))/((2*\sqrt{x+1}/(-2*\sqrt{-x+1}+2*\sqrt{2}))-1/2*(-2*\sqrt{-x+1}+2*\sqrt{2}))/\sqrt{x+1})^2-4)^2+\ln(\text{abs}(2*\sqrt{x+1}/(-2*\sqrt{-x+1}+2*\sqrt{2}))+2-1/2*(-2*\sqrt{-x+1}+2*\sqrt{2}))/\sqrt{x+1}))- \ln(\text{abs}(2*\sqrt{x+1}/(-2*\sqrt{-x+1}+2*\sqrt{2}))-2-1/2*(-2*\sqrt{-x+1}+2*\sqrt{2}))/\sqrt{x+1}))-1/x^2$$

maple [A] time = 0.02, size = 58, normalized size = 1.71

$$-\frac{1}{x^2} + \frac{\sqrt{x+1} \sqrt{-x+1} \left(x^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \sqrt{-x^2+1} \right)}{\sqrt{-x^2+1} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^(1/2)+(x+1)^(1/2))^2/x^3,x)

[Out] $-1/x^2+(x+1)^{(1/2)}*(-x+1)^{(1/2)}*(\operatorname{arctanh}(1/(-x^2+1)^{(1/2)}))*x^2-(-x^2+1)^{(1/2)}/x^2/(-x^2+1)^{(1/2)}$

maxima [A] time = 1.52, size = 54, normalized size = 1.59

$$-\sqrt{-x^2+1} - \frac{(-x^2+1)^{\frac{3}{2}}}{x^2} - \frac{1}{x^2} + \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))^2/x^3,x, algorithm="maxima")

[Out] $-\sqrt{-x^2+1} - (-x^2+1)^{(3/2)}/x^2 - 1/x^2 + \log(2*\sqrt{-x^2+1}/\text{abs}(x) + 2/\text{abs}(x))$

mupad [B] time = 4.88, size = 189, normalized size = 5.56

$$\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \ln\left(\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} - 1\right) + \frac{(\sqrt{1-x}-1)^2}{16(\sqrt{x+1}-1)^2} - \frac{\frac{(\sqrt{1-x}-1)^2}{8(\sqrt{x+1}-1)^2} + \frac{15(\sqrt{1-x}-1)^4}{16(\sqrt{x+1}-1)^4} - \frac{1}{16}}{\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} - \frac{2(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + \frac{(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6}} - \frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)^(1/2)+(1-x)^(1/2))^2/x^3,x)

[Out] $\log(((1-x)^{(1/2)}-1)/((x+1)^{(1/2)}-1)) - \log(((1-x)^{(1/2)}-1)^2/((x+1)^{(1/2)}-1)^2-1) + ((1-x)^{(1/2)}-1)^2/(16*((x+1)^{(1/2)}-1)^2) - (((1-x)^{(1/2)}-1)^2/(8*((x+1)^{(1/2)}-1)^2) + (15*((1-x)^{(1/2)}-1)^4)/(16*((x+1)^{(1/2)}-1)^4) - 1/16)/(((1-x)^{(1/2)}-1)^2/((x+1)^{(1/2)}-1)^2 - (2*((1-x)^{(1/2)}-1)^4)/((x+1)^{(1/2)}-1)^4 + ((1-x)^{(1/2)}-1)^6/((x+1)^{(1/2)}-1)^6) - 1/x^2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(\sqrt{1-x} + \sqrt{x+1})^2}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))**2/x**3,x)

[Out] Integral((sqrt(1-x) + sqrt(x+1))**2/x**3, x)

$$3.203 \quad \int \frac{x^3}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Optimal. Leaf size=147

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} - \frac{2(a+cx)^{7/2}}{7c^3(b-c)} + \frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2103, 43}

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} - \frac{2(a+cx)^{7/2}}{7c^3(b-c)} + \frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] (2*a^2*(a + b*x)^(3/2))/(3*b^3*(b - c)) - (4*a*(a + b*x)^(5/2))/(5*b^3*(b - c)) + (2*(a + b*x)^(7/2))/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^(3/2))/(3*(b - c)*c^3) + (4*a*(a + c*x)^(5/2))/(5*(b - c)*c^3) - (2*(a + c*x)^(7/2))/(7*(b - c)*c^3)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2103

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dist[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{\sqrt{a+bx} + \sqrt{a+cx}} dx &= \frac{\int x^2 \sqrt{a+bx} dx}{b-c} - \frac{\int x^2 \sqrt{a+cx} dx}{b-c} \\ &= \frac{\int \left(\frac{a^2 \sqrt{a+bx}}{b^2} - \frac{2a(a+bx)^{3/2}}{b^2} + \frac{(a+bx)^{5/2}}{b^2} \right) dx}{b-c} - \frac{\int \left(\frac{a^2 \sqrt{a+cx}}{c^2} - \frac{2a(a+cx)^{3/2}}{c^2} + \frac{(a+cx)^{5/2}}{c^2} \right) dx}{b-c} \\ &= \frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3(b-c)c^3} + \frac{4a(a+cx)^{5/2}}{5(b-c)c^3} - \frac{2(a+cx)^{7/2}}{7c^3(b-c)} \end{aligned}$$

Mathematica [A] time = 0.22, size = 147, normalized size = 1.00

$$\frac{2a^2(a+bx)^{3/2}}{3b^3(b-c)} - \frac{2a^2(a+cx)^{3/2}}{3c^3(b-c)} + \frac{2(a+bx)^{7/2}}{7b^3(b-c)} - \frac{4a(a+bx)^{5/2}}{5b^3(b-c)} - \frac{2(a+cx)^{7/2}}{7c^3(b-c)} + \frac{4a(a+cx)^{5/2}}{5c^3(b-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] $(2*a^2*(a + b*x)^{(3/2)})/(3*b^3*(b - c)) - (4*a*(a + b*x)^{(5/2)})/(5*b^3*(b - c)) + (2*(a + b*x)^{(7/2)})/(7*b^3*(b - c)) - (2*a^2*(a + c*x)^{(3/2)})/(3*(b - c)*c^3) + (4*a*(a + c*x)^{(5/2)})/(5*(b - c)*c^3) - (2*(a + c*x)^{(7/2)})/(7*(b - c)*c^3)$

IntegrateAlgebraic [A] time = 1.57, size = 208, normalized size = 1.41

$$\frac{2(35a^2(a+cx)^{3/2} + 15(a+cx)^{7/2} - 42a(a+cx)^{5/2})}{105c^3(b-c)} - \frac{2\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a(15a^2b^3 - 3a^2b^2c - 4a^2bc^2 - 8a^2c^3 - 45a^2b^3(a+cx) + 6a^2b^2c(a+cx) + 4a^2bc^2(a+cx) + 45ab^3(a+cx)^2 - 15b^3(a+cx)^3 - 3ab^2c(a+cx)^2)}}{105b^3c^3(b-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] $(-2*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*(15*a^3*b^3 - 3*a^3*b^2*c - 4*a^3*b*c^2 - 8*a^3*c^3 - 45*a^2*b^3*(a + c*x) + 6*a^2*b^2*c*(a + c*x) + 4*a^2*b*c^2*(a + c*x) + 45*a*b^3*(a + c*x)^2 - 3*a*b^2*c*(a + c*x)^2 - 15*b^3*(a + c*x)^3))/(105*b^3*(b - c)*c^3) - (2*(35*a^2*(a + c*x)^{(3/2)} - 42*a*(a + c*x)^{(5/2)} + 15*(a + c*x)^{(7/2)}))/(105*(b - c)*c^3)$

fricas [A] time = 0.41, size = 122, normalized size = 0.83

$$\frac{2\left(\left(15b^3c^3x^3 + 3ab^2c^3x^2 - 4a^2bc^3x + 8a^3c^3\right)\sqrt{bx+a} - \left(15b^3c^3x^3 + 3ab^3c^2x^2 - 4a^2b^3cx + 8a^3b^3\right)\sqrt{cx+a}\right)}{105\left(b^4c^3 - b^3c^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")

[Out] $2/105*((15*b^3*c^3*x^3 + 3*a*b^2*c^3*x^2 - 4*a^2*b*c^3*x + 8*a^3*c^3)*\text{sqrt}(b*x + a) - (15*b^3*c^3*x^3 + 3*a*b^3*c^2*x^2 - 4*a^2*b^3*c*x + 8*a^3*b^3)*\text{sqrt}(c*x + a))/(b^4*c^3 - b^3*c^4)$

giac [B] time = 0.43, size = 451, normalized size = 3.07

$$\frac{-2}{105} \frac{\sqrt{a^2 + (b+c)x + ac} \left(\frac{5(b^2c^2|b| - 2a^2c^2|b| + 3a^2c^2|b|)(b+c)}{2b^2c^2 - 3a^2c^2 + 3a^2c^2 - 3a^2c^2} + \frac{a^2c^2|b| - 17a^2c^2|b| + 31a^2c^2|b| - 15a^2c^2|b|}{2b^2c^2 - 3a^2c^2 + 3a^2c^2 - 3a^2c^2} \right) + \frac{4a^2b^3c^2|b| - 2a^2b^3c^2|b| - 53a^2b^3c^2|b| + 96a^2b^3c^2|b| - 45a^2b^3c^2|b|}{2b^2c^2 - 3a^2c^2 + 3a^2c^2 - 3a^2c^2} (b+c) + \frac{8a^2b^3c^2|b| - 12a^2b^3c^2|b| + 3a^2b^3c^2|b| - 17a^2b^3c^2|b| + 33a^2b^3c^2|b| - 15a^2b^3c^2|b|}{2b^2c^2 - 3a^2c^2 + 3a^2c^2 - 3a^2c^2} (b+c) \right)}{105(a^2 - b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out] $-2/105*\text{sqrt}(a*b^2 + (b*x + a)*b*c - a*b*c)*((3*(b*x + a)*(5*(b^17*c^5*\text{abs}(b) - 2*b^16*c^6*\text{abs}(b) + b^15*c^7*\text{abs}(b)))*(b*x + a)/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8) + (a*b^18*c^4*\text{abs}(b) - 17*a*b^17*c^5*\text{abs}(b) + 31*a*b^16*c^6*\text{abs}(b) - 15*a*b^15*c^7*\text{abs}(b)))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8)) - (4*a^2*b^19*c^3*\text{abs}(b) - 2*a^2*b^18*c^4*\text{abs}(b) - 53*a^2*b^17*c^5*\text{abs}(b) + 96*a^2*b^16*c^6*\text{abs}(b) - 45*a^2*b^15*c^7*\text{abs}(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8))*(b*x + a) + (8*a^3*b^20*c^2*\text{abs}(b) - 12*a^3*b^19*c^3*\text{abs}(b) + 3*a^3*b^18*c^4*\text{abs}(b) - 17*a^3*b^17*c^5*\text{abs}(b) + 33*a^3*b^16*c^6*\text{abs}(b) - 15*a^3*b^15*c^7*\text{abs}(b))/(b^23*c^5 - 3*b^22*c^6 + 3*b^21*c^7 - b^20*c^8)) + 2/105*(15*(b*x + a)^{(7/2)} - 42*(b*x + a)^{(5/2)}*a + 35*(b*x + a)^{(3/2)}*a^2)/(b^4 - b^3*c)$

maple [A] time = 0.00, size = 90, normalized size = 0.61

$$\frac{\frac{2(bx+a)^3 a^2}{3} - \frac{4(bx+a)^5 a}{5} + \frac{2(bx+a)^7}{7}}{(b-c)b^3} - \frac{2\left(\frac{(cx+a)^3 a^2}{3} - \frac{2(cx+a)^5 a}{5} + \frac{(cx+a)^7}{7}\right)}{(b-c)c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] $2/(b-c)/b^3*(1/3*(b*x+a)^{(3/2)}*a^2-2/5*(b*x+a)^{(5/2)}*a+1/7*(b*x+a)^{(7/2)})-2/(b-c)/c^3*(1/7*(c*x+a)^{(7/2)}-2/5*(c*x+a)^{(5/2)}*a+1/3*a^2*(c*x+a)^{(3/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{bx+a} + \sqrt{cx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a)), x)`

mupad [B] time = 2.92, size = 179, normalized size = 1.22

$$\frac{2x^3\sqrt{a+bx}}{7(b-c)} - \frac{2x^3\sqrt{a+cx}}{7(b-c)} + \frac{16a^3\sqrt{a+bx}}{105b^3(b-c)} - \frac{16a^3\sqrt{a+cx}}{105c^3(b-c)} + \frac{2ax^2\sqrt{a+bx}}{35b(b-c)} - \frac{8a^2x\sqrt{a+bx}}{105b^2(b-c)} - \frac{2ax^2\sqrt{a+cx}}{35c(b-c)} + \frac{8a^2x\sqrt{a+cx}}{105c^2(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)`

[Out] $(2*x^3*(a + b*x)^{(1/2)})/(7*(b - c)) - (2*x^3*(a + c*x)^{(1/2)})/(7*(b - c)) + (16*a^3*(a + b*x)^{(1/2)})/(105*b^3*(b - c)) - (16*a^3*(a + c*x)^{(1/2)})/(105*c^3*(b - c)) + (2*a*x^2*(a + b*x)^{(1/2)})/(35*b*(b - c)) - (8*a^2*x*(a + b*x)^{(1/2)})/(105*b^2*(b - c)) - (2*a*x^2*(a + c*x)^{(1/2)})/(35*c*(b - c)) + (8*a^2*x*(a + c*x)^{(1/2)})/(105*c^2*(b - c))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)`

[Out] `Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x)), x)`

$$3.204 \quad \int \frac{x^2}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Optimal. Leaf size=95

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(b-c)} - \frac{2(a+cx)^{5/2}}{5c^2(b-c)} + \frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2103, 43}

$$\frac{2(a+bx)^{5/2}}{5b^2(b-c)} - \frac{2a(a+bx)^{3/2}}{3b^2(b-c)} - \frac{2(a+cx)^{5/2}}{5c^2(b-c)} + \frac{2a(a+cx)^{3/2}}{3c^2(b-c)}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] (-2*a*(a + b*x)^(3/2))/(3*b^2*(b - c)) + (2*(a + b*x)^(5/2))/(5*b^2*(b - c)) + (2*a*(a + c*x)^(3/2))/(3*(b - c)*c^2) - (2*(a + c*x)^(5/2))/(5*(b - c)*c^2)

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 2103

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_.)]), x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dist[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{a+bx} + \sqrt{a+cx}} dx &= \frac{\int x\sqrt{a+bx} dx}{b-c} - \frac{\int x\sqrt{a+cx} dx}{b-c} \\ &= \frac{\int \left(-\frac{a\sqrt{a+bx}}{b} + \frac{(a+bx)^{3/2}}{b}\right) dx}{b-c} - \frac{\int \left(-\frac{a\sqrt{a+cx}}{c} + \frac{(a+cx)^{3/2}}{c}\right) dx}{b-c} \\ &= -\frac{2a(a+bx)^{3/2}}{3b^2(b-c)} + \frac{2(a+bx)^{5/2}}{5b^2(b-c)} + \frac{2a(a+cx)^{3/2}}{3(b-c)c^2} - \frac{2(a+cx)^{5/2}}{5(b-c)c^2} \end{aligned}$$

Mathematica [A] time = 0.19, size = 70, normalized size = 0.74

$$\frac{2 \left(\frac{3(a+bx)^{5/2}}{b^2} - \frac{5a(a+bx)^{3/2}}{b^2} - \frac{3(a+cx)^{5/2}}{c^2} + \frac{5a(a+cx)^{3/2}}{c^2} \right)}{15(b-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] $(2*((-5*a*(a + b*x)^(3/2))/b^2 + (3*(a + b*x)^(5/2))/b^2 + (5*a*(a + c*x)^(3/2))/c^2 - (3*(a + c*x)^(5/2))/c^2))/(15*(b - c))$

IntegrateAlgebraic [A] time = 1.04, size = 135, normalized size = 1.42

$$2\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a} \frac{(3a^2b^2 - a^2bc - 2a^2c^2 - 6ab^2(a+cx) + 3b^2(a+cx)^2 + abc(a+cx))}{15b^2c^2(b-c)} + \frac{2(5a(a+cx)^{3/2} - 3(a+cx)^{5/2})}{15c^2(b-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] $(2*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*(3*a^2*b^2 - a^2*b*c - 2*a^2*c^2 - 6*a*b^2*(a + c*x) + a*b*c*(a + c*x) + 3*b^2*(a + c*x)^2))/(15*b^2*(b - c)*c^2) + (2*(5*a*(a + c*x)^(3/2) - 3*(a + c*x)^(5/2)))/(15*(b - c)*c^2)$

fricas [A] time = 0.40, size = 92, normalized size = 0.97

$$\frac{2\left(\left(3b^2c^2x^2 + abc^2x - 2a^2c^2\right)\sqrt{bx+a} - \left(3b^2c^2x^2 + ab^2cx - 2a^2b^2\right)\sqrt{cx+a}\right)}{15\left(b^3c^2 - b^2c^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")

[Out] $2/15*((3*b^2*c^2*x^2 + a*b*c^2*x - 2*a^2*c^2)*\text{sqrt}(b*x + a) - (3*b^2*c^2*x^2 + a*b^2*c*x - 2*a^2*b^2)*\text{sqrt}(c*x + a))/(b^3*c^2 - b^2*c^3)$

giac [B] time = 0.39, size = 255, normalized size = 2.68

$$-\frac{2}{15}\sqrt{ab^2 + (bx+a)bc - abc}\left((bx+a)\left(\frac{3(b^9c^3|b| - b^8c^4|b|)(bx+a)}{b^{14}c^3 - 2b^{13}c^4 + b^{12}c^5} + \frac{ab^{10}c^2|b| - 7ab^9c^3|b| + 6ab^8c^4|b|}{b^{14}c^3 - 2b^{13}c^4 + b^{12}c^5}\right) - \frac{2a^2b^{11}c|b| - a^2b^{10}c^2|b| - 4a^2b^9c^3|b| + 3a^2b^8c^4|b|}{b^{14}c^3 - 2b^{13}c^4 + b^{12}c^5}\right) + \frac{2(3(bx+a)^{\frac{5}{2}} - 5(bx+a)^{\frac{3}{2}}a)}{15(b^3 - b^2c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out] $-2/15*\text{sqrt}(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*(3*(b^9*c^3*abs(b) - b^8*c^4*abs(b))*(b*x + a)/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + (a*b^10*c^2*a*bs(b) - 7*a*b^9*c^3*abs(b) + 6*a*b^8*c^4*abs(b)))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5)) - (2*a^2*b^11*c*abs(b) - a^2*b^10*c^2*abs(b) - 4*a^2*b^9*c^3*abs(b) + 3*a^2*b^8*c^4*abs(b))/(b^14*c^3 - 2*b^13*c^4 + b^12*c^5) + 2/15*(3*(b*x + a)^(5/2) - 5*(b*x + a)^(3/2)*a)/(b^3 - b^2*c)$

maple [A] time = 0.00, size = 66, normalized size = 0.69

$$\frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3} + \frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)b^2} - \frac{2\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3} + \frac{(cx+a)^{\frac{5}{2}}}{5}\right)}{(b-c)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] $2/(b-c)/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-2/(b-c)/c^2*(1/5*(c*x+a)^(5/2)-1/3*a*(c*x+a)^(3/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{bx+a} + \sqrt{cx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a)), x)

mupad [B] time = 2.86, size = 129, normalized size = 1.36

$$\frac{2x^2\sqrt{a+bx}}{5(b-c)} - \frac{2x^2\sqrt{a+cx}}{5(b-c)} - \frac{4a^2\sqrt{a+bx}}{15b^2(b-c)} + \frac{4a^2\sqrt{a+cx}}{15c^2(b-c)} + \frac{2ax\sqrt{a+bx}}{15b(b-c)} - \frac{2ax\sqrt{a+cx}}{15c(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)

[Out] (2*x^2*(a + b*x)^(1/2))/(5*(b - c)) - (2*x^2*(a + c*x)^(1/2))/(5*(b - c)) - (4*a^2*(a + b*x)^(1/2))/(15*b^2*(b - c)) + (4*a^2*(a + c*x)^(1/2))/(15*c^2*(b - c)) + (2*a*x*(a + b*x)^(1/2))/(15*b*(b - c)) - (2*a*x*(a + c*x)^(1/2))/(15*c*(b - c))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)

[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x)), x)

$$3.205 \quad \int \frac{x}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Optimal. Leaf size=47

$$\frac{2(a+bx)^{3/2}}{3b(b-c)} - \frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2103, 32}

$$\frac{2(a+bx)^{3/2}}{3b(b-c)} - \frac{2(a+cx)^{3/2}}{3c(b-c)}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] (2*(a + b*x)^(3/2))/(3*b*(b - c)) - (2*(a + c*x)^(3/2))/(3*(b - c)*c)

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] := Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rule 2103

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dist[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a+bx} + \sqrt{a+cx}} dx &= \frac{\int \sqrt{a+bx} dx}{b-c} - \frac{\int \sqrt{a+cx} dx}{b-c} \\ &= \frac{2(a+bx)^{3/2}}{3b(b-c)} - \frac{2(a+cx)^{3/2}}{3(b-c)c} \end{aligned}$$

Mathematica [A] time = 0.09, size = 39, normalized size = 0.83

$$\frac{2 \left(\frac{(a+bx)^{3/2}}{b} - \frac{(a+cx)^{3/2}}{c} \right)}{3(b-c)}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] (2*((a + b*x)^(3/2)/b - (a + c*x)^(3/2)/c))/(3*(b - c))

IntegrateAlgebraic [A] time = 0.71, size = 80, normalized size = 1.70

$$-\frac{2(a+cx)^{3/2}}{3c(b-c)} - \frac{2(-b(a+cx) + ab - ac)\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a}}{3bc(b-c)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x]),x]

[Out] $(-2*(a + c*x)^{(3/2)})/(3*(b - c)*c) - (2*(a*b - a*c - b*(a + c*x))*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])/(3*b*(b - c)*c)$

fricas [A] time = 0.41, size = 50, normalized size = 1.06

$$\frac{2 \left((bcx + ac)\sqrt{bx + a} - (bcx + ab)\sqrt{cx + a} \right)}{3(b^2c - bc^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")

[Out] $2/3*((b*c*x + a*c)*\text{sqrt}(b*x + a) - (b*c*x + a*b)*\text{sqrt}(c*x + a))/(b^2*c - b*c^2)$

giac [B] time = 0.32, size = 107, normalized size = 2.28

$$\frac{2 \left(\left(\frac{(bx+a)b^2c|b|}{b^5c-b^4c^2} + \frac{ab^3|b|-ab^2c|b|}{b^5c-b^4c^2} \right) \sqrt{ab^2 + (bx+a)bc - abc} - \frac{(bx+a)^{\frac{3}{2}}}{b-c} \right)}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out] $-2/3*(((b*x + a)*b^2*c*abs(b)/(b^5*c - b^4*c^2) + (a*b^3*abs(b) - a*b^2*c*a bs(b))/(b^5*c - b^4*c^2))*\text{sqrt}(a*b^2 + (b*x + a)*b*c - a*b*c) - (b*x + a)^{(3/2)/(b - c)})/b$

maple [A] time = 0.00, size = 40, normalized size = 0.85

$$\frac{2(bx + a)^{\frac{3}{2}}}{3(b - c)b} - \frac{2(cx + a)^{\frac{3}{2}}}{3(b - c)c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] $2/3*(b*x+a)^{(3/2)}/b/(b-c)-2/3*(c*x+a)^{(3/2)}/(b-c)/c$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{bx + a} + \sqrt{cx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a)), x)

mupad [B] time = 2.91, size = 79, normalized size = 1.68

$$\frac{2x\sqrt{a+bx}}{3(b-c)} - \frac{2x\sqrt{a+cx}}{3(b-c)} + \frac{2a\sqrt{a+bx}}{3b(b-c)} - \frac{2a\sqrt{a+cx}}{3c(b-c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2)),x)

[Out] $(2*x*(a + b*x)^{(1/2)})/(3*(b - c)) - (2*x*(a + c*x)^{(1/2)})/(3*(b - c)) + (2*a*(a + b*x)^{(1/2)})/(3*b*(b - c)) - (2*a*(a + c*x)^{(1/2)})/(3*c*(b - c))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + bx} + \sqrt{a + cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2)), x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x)), x)

$$3.206 \quad \int \frac{1}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Optimal. Leaf size=97

$$\frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Rubi [A] time = 0.07, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6690, 50, 63, 208}

$$\frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{b-c} + \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{b-c}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1), x]

[Out] (2*Sqrt[a + b*x])/(b - c) - (2*Sqrt[a + c*x])/(b - c) - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c) + (2*Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+bx} + \sqrt{a+cx}} dx &= \frac{\int \left(\frac{\sqrt{a+bx}}{x} - \frac{\sqrt{a+cx}}{x} \right) dx}{b-c} \\
&= \frac{\int \frac{\sqrt{a+bx}}{x} dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x} dx}{b-c} \\
&= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} + \frac{a \int \frac{1}{x\sqrt{a+bx}} dx}{b-c} - \frac{a \int \frac{1}{x\sqrt{a+cx}} dx}{b-c} \\
&= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} + \frac{(2a) \text{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx} \right)}{b(b-c)} - \frac{(2a) \text{Subst} \left(\int \frac{1}{\frac{-a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx} \right)}{b(b-c)} \\
&= \frac{2\sqrt{a+bx}}{b-c} - \frac{2\sqrt{a+cx}}{b-c} - \frac{2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right)}{b-c} + \frac{2\sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+cx}}{\sqrt{a}} \right)}{b-c}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 75, normalized size = 0.77

$$\frac{2 \left(\sqrt{a+bx} - \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right) - \sqrt{a+cx} + \sqrt{a} \tanh^{-1} \left(\frac{\sqrt{a+cx}}{\sqrt{a}} \right) \right)}{b-c}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1), x]

[Out] (2*(Sqrt[a + b*x] - Sqrt[a + c*x] - Sqrt[a]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + Sqrt[a]*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]))/(b - c)

IntegrateAlgebraic [B] time = 1.92, size = 372, normalized size = 3.84

$$\frac{\frac{2\sqrt{a+cx}}{b-c} + \frac{2\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a}}{b-c} - \frac{2\sqrt{a} \sqrt{-(\sqrt{b}-\sqrt{c})^2} (\sqrt{b}-\sqrt{c} \sqrt{\frac{b}{c}}) \tan^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{b}{c}} \sqrt{a+cx} - \sqrt{c} \sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a}}{\sqrt{a} \sqrt{2\sqrt{b}\sqrt{c}-b-c}} \right)}{\sqrt{b}(\sqrt{b}-\sqrt{c})^2(\sqrt{b}+\sqrt{c})} + \frac{2\sqrt{a} (\sqrt{c} \sqrt{\frac{b}{c}} + \sqrt{b}) \tanh^{-1} \left(\frac{\sqrt{c} \sqrt{\frac{b}{c}} \sqrt{a+cx} - \sqrt{c} \sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a}}{\sqrt{a}(\sqrt{b}+\sqrt{c})} \right)}{\sqrt{b}(\sqrt{b}-\sqrt{c})(\sqrt{b}+\sqrt{c})}}{b-c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-1), x]

[Out] (-2*Sqrt[a + c*x])/(b - c) + (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(b - c) - (2*Sqrt[a]*Sqrt[-(Sqrt[b] - Sqrt[c])^2]*(Sqrt[b] - Sqrt[b/c]*Sqrt[c])*ArcTan[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x] - Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*Sqrt[-b + 2*Sqrt[b]*Sqrt[c] - c])])/(Sqrt[b]*(Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])) + (2*Sqrt[a]*(Sqrt[b] + Sqrt[b/c]*Sqrt[c])*ArcTanh[(Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x])/(Sqrt[a]*(Sqrt[b] + Sqrt[c])) - (Sqrt[c]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(Sqrt[a]*(Sqrt[b] + Sqrt[c]))])/(Sqrt[b]*(Sqrt[b] - Sqrt[c])*(Sqrt[b] + Sqrt[c]))

fricas [A] time = 0.42, size = 158, normalized size = 1.63

$$\left[\frac{\sqrt{a} \log \left(\frac{bx+2\sqrt{bx+a}\sqrt{a+2a}}{x} \right) + \sqrt{a} \log \left(\frac{cx-2\sqrt{cx+a}\sqrt{a+2a}}{x} \right) - 2\sqrt{bx+a} + 2\sqrt{cx+a}}{b-c}, \frac{2 \left(\sqrt{-a} \arctan \left(\frac{\sqrt{bx+a}\sqrt{-a}}{a} \right) - \sqrt{-a} \arctan \left(\frac{\sqrt{cx+a}\sqrt{-a}}{a} \right) + \sqrt{bx+a} - \sqrt{cx+a} \right)}{b-c} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")

[Out] [-(sqrt(a)*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*sqrt(b*x + a) + 2*sqrt(c*x + a))/(b

- c), 2*(sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*arctan(sqrt(c*x + a)*sqrt(-a)/a) + sqrt(b*x + a) - sqrt(c*x + a))/(b - c)]

giac [B] time = 1.00, size = 1093, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out]
$$-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a} - \frac{-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right) + 2\sqrt{cx+a}}{b-c}$$

maple [A] time = 0.01, size = 73, normalized size = 0.75

$$\frac{-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a}}{b-c} - \frac{-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right) + 2\sqrt{cx+a}}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] $1/(b-c) * (2*(b*x+a)^(1/2) - 2*a^(1/2) * \operatorname{arctanh}((b*x+a)^(1/2)/a^(1/2))) - 1/(b-c) * (2*(c*x+a)^(1/2) - 2*a^(1/2) * \operatorname{arctanh}((c*x+a)^(1/2)/a^(1/2)))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{bx+a} + \sqrt{cx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(b*x + a) + sqrt(c*x + a)), x)

mupad [B] time = 4.33, size = 213, normalized size = 2.20

$$\frac{2\sqrt{a}c\left(\frac{2(\sqrt{a+bx}-\sqrt{a})}{\sqrt{a+cx}-\sqrt{a}} + \frac{\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)(\sqrt{a+bx}-\sqrt{a})^2}{(\sqrt{a+cx}-\sqrt{a})^2}\right) - 2\sqrt{a}b\left(\ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right) - \frac{2(\sqrt{a+bx}-\sqrt{a})}{\sqrt{a+cx}-\sqrt{a}} + 4\right)}{(b-c)\left(b - \frac{c(\sqrt{a+bx}-\sqrt{a})^2}{(\sqrt{a+cx}-\sqrt{a})^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2)), x)

[Out] $-(2*a^{(1/2)}*c*((2*((a + b*x)^{(1/2)} - a^{(1/2)}))/((a + c*x)^{(1/2)} - a^{(1/2)})))/((a + c*x)^{(1/2)} - a^{(1/2)}) + (\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)})))*((a + b*x)^{(1/2)} - a^{(1/2)})^2)/((a + c*x)^{(1/2)} - a^{(1/2)})^2 - 2*a^{(1/2)}*b*(\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)})) - (2*((a + b*x)^{(1/2)} - a^{(1/2)}))/((a + c*x)^{(1/2)} - a^{(1/2)} + 4)))/((b - c)*(b - (c*((a + b*x)^{(1/2)} - a^{(1/2)})^2)/((a + c*x)^{(1/2)} - a^{(1/2)})^2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+bx} + \sqrt{a+cx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2)), x)

[Out] Integral(1/(sqrt(a + b*x) + sqrt(a + c*x)), x)

$$3.207 \quad \int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})} dx$$

Optimal. Leaf size=103

$$-\frac{\sqrt{a+bx}}{x(b-c)} + \frac{\sqrt{a+cx}}{x(b-c)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)} + \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Rubi [A] time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {2103, 47, 63, 208}

$$-\frac{\sqrt{a+bx}}{x(b-c)} + \frac{\sqrt{a+cx}}{x(b-c)} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)} + \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]
```

```
[Out] -(Sqrt[a + b*x]/((b - c)*x)) + Sqrt[a + c*x]/((b - c)*x) - (b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]/(Sqrt[a]*(b - c))) + (c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]]/(Sqrt[a]*(b - c)))
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2103

```
Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]),
x_Symbol] := Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis
t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})} dx = \frac{\int \frac{\sqrt{a+bx}}{x^2} dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^2} dx}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{b \int \frac{1}{x\sqrt{a+bx}} dx}{2(b-c)} - \frac{c \int \frac{1}{x\sqrt{a+cx}} dx}{2(b-c)}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b-c} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a+cx}\right)}{b-c}$$

$$= -\frac{\sqrt{a+bx}}{(b-c)x} + \frac{\sqrt{a+cx}}{(b-c)x} - \frac{b \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)} + \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)}$$

Mathematica [A] time = 0.21, size = 135, normalized size = 1.31

$$\frac{-\frac{a}{\sqrt{a+bx}} - \frac{bx}{\sqrt{a+bx}} - \frac{bx\sqrt{\frac{bx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right)}{\sqrt{a+bx}} + \frac{a}{\sqrt{a+cx}} + \frac{cx}{\sqrt{a+cx}} + \frac{cx\sqrt{\frac{cx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right)}{\sqrt{a+cx}}}{bx - cx}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])), x]

[Out] $(-(a/\text{Sqrt}[a + b*x]) - (b*x)/\text{Sqrt}[a + b*x] + a/\text{Sqrt}[a + c*x] + (c*x)/\text{Sqrt}[a + c*x] - (b*x*\text{Sqrt}[1 + (b*x)/a]*\text{ArcTanH}[\text{Sqrt}[1 + (b*x)/a]])/\text{Sqrt}[a + b*x] + (c*x*\text{Sqrt}[1 + (c*x)/a]*\text{ArcTanH}[\text{Sqrt}[1 + (c*x)/a]])/\text{Sqrt}[a + c*x])/(b*x - c*x)$

IntegrateAlgebraic [B] time = 81.01, size = 4583, normalized size = 44.50

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])), x]

[Out] $(-(c*(5*a^2*b*\text{Sqrt}[a + c*x] - 3*a^2*c*\text{Sqrt}[a + c*x] - 6*a*b*(a + c*x)^{(3/2)}) - c*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b + a^2*c - 8*a*b*(a + c*x) + 2*a*c*(a + c*x) + 8*b*(a + c*x)^2) + \text{Sqrt}[b/c]*(-(c*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*(-2*a^2*c + 6*a*c*(a + c*x))) - c*(-4*a^2*b*\text{Sqrt}[a + c*x] + 2*a^2*c*\text{Sqrt}[a + c*x] + 12*a*b*(a + c*x)^{(3/2)} - 6*a*c*(a + c*x)^{(3/2)} - 8*b*(a + c*x)^{(5/2)})))/(-(\text{Sqrt}[b/c]*c*x*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*(-4*a*b*c*\text{Sqrt}[a + c*x] - 4*a*c^2*\text{Sqrt}[a + c*x] + 8*b*c*(a + c*x)^{(3/2)})) - c*x*(-(a^2*b^2) - 2*a^2*b*c + 3*a^2*c^2 + 8*a*b^2*(a + c*x) - 8*b^2*(a + c*x)^2) + (\text{Sqrt}[b]*\text{ArcTan}[(\text{Sqrt}[b/c]*\text{Sqrt}[c]*\text{Sqrt}[a + c*x])/(\text{Sqrt}[a]*\text{Sqrt}[-b + 2*\text{Sqrt}[b]*\text{Sqrt}[c] - c]) - (\text{Sqrt}[c]*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{Sqrt}[a]*\text{Sqrt}[-b + 2*\text{Sqrt}[b]*\text{Sqrt}[c] - c]) - (\text{Sqrt}[c]*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{Sqrt}[a]*\text{Sqrt}[-b + 2*\text{Sqrt}[b]*\text{Sqrt}[c] - c]) - (\text{Sqrt}[c]*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{Sqrt}[a]*\text{Sqrt}[-b + 2*\text{Sqrt}[b]*\text{Sqrt}[c] - c]) - (\text{Sqrt}[c]*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{Sqrt}[a]*\text{Sqrt}[-b + 2*\text{Sqrt}[b]*\text{Sqrt}[c] - c]) + ((b*\text{Sqrt}[a + c*x])/((\text{Sqrt}[b] - \text{Sqrt}[c])*\text{Sqrt}[c]) + (\text{Sqrt}[b/c]*\text{Sqrt}[c]*\text{Sqrt}[a + c*x])/(\text{Sqrt}[b] - \text{Sqrt}[c]) - (\text{Sqrt}[c]*\text{Sqrt}[a - (a*b)/c + (b*(a + c*x))/c])/(\text{Sqrt}[b] - \text{Sqrt}[c]) - (\text{Sqrt}[b/c]$

$$\frac{\text{sqrt}[c])]}{(2*(\text{sqrt}[b] + \text{sqrt}[c])) + (b*(a + c*x)*\text{ArcTanh}[(-(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x]) + \text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c]))*\text{sqrt}[c]) - (b*\text{sqrt}[b/c]*(a + c*x)*\text{ArcTanh}[(-(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x]) + \text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c]))*\text{sqrt}[c]) + (b*\text{sqrt}[a + c*x]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])* \text{ArcTanh}[(-(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x]) + \text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c]))*\text{sqrt}[c]) - (\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c]*\text{ArcTanh}[(-(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x]) + \text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/((-2*\text{sqrt}[a] - \text{sqrt}[b/c]*\text{sqrt}[a + c*x] + \text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])*(2*\text{sqrt}[a] - \text{sqrt}[b/c]*\text{sqrt}[a + c*x] + \text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])) + \text{ArcTanh}[(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c]))] - (\text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/\text{sqrt}[a] - (\text{sqrt}[b/c]*\text{sqrt}[c]*\text{ArcTanh}[(\text{sqrt}[b/c]*\text{sqrt}[c]*\text{sqrt}[a + c*x])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/\text{sqrt}[a] - (\text{sqrt}[c]*\text{sqrt}[a - (a*b)/c + (b*(a + c*x))/c])]/(\text{sqrt}[a]*(\text{sqrt}[b] + \text{sqrt}[c])))]/(\text{sqrt}[a]*\text{sqrt}[b])$$

fricas [A] time = 0.42, size = 182, normalized size = 1.77

$$\left[\frac{\sqrt{a} b x \log\left(\frac{b x + 2 \sqrt{b x + a} \sqrt{a + 2 a}}{x}\right) + \sqrt{a} c x \log\left(\frac{c x - 2 \sqrt{c x + a} \sqrt{a + 2 a}}{x}\right) + 2 \sqrt{b x + a} a - 2 \sqrt{c x + a} a - \sqrt{-a} b x \arctan\left(\frac{\sqrt{b x + a} \sqrt{-a}}{a}\right) - \sqrt{-a} c x \arctan\left(\frac{\sqrt{c x + a} \sqrt{-a}}{a}\right) - \sqrt{b x + a} a + \sqrt{c x + a} a}{2(a b - a c) x}, \frac{\sqrt{-a} b x \arctan\left(\frac{\sqrt{b x + a} \sqrt{-a}}{a}\right) - \sqrt{-a} c x \arctan\left(\frac{\sqrt{c x + a} \sqrt{-a}}{a}\right) - \sqrt{b x + a} a + \sqrt{c x + a} a}{(a b - a c) x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="fricas")

[Out] [-1/2*(sqrt(a)*b*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*sqrt(b*x + a)*a - 2*sqrt(c*x + a)*a)/((a*b - a*c)*x), (sqrt(-a)*b*x*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c*x*arctan(sqrt(c*x + a)*sqrt(-a)/a) - sqrt(b*x + a)*a + sqrt(c*x + a)*a)/((a*b - a*c)*x)]

giac [B] time = 12.67, size = 1402, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out] b*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*(b - c)) - 2*((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b^2*c*abs(b) - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))*a*b*c^2*abs(b) + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^3*c*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b^2 - 2*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*b*c + (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4)*(b - c)) - sqrt(b*x + a)/((b - c)*x) + (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*sqrt(-a*b*c)*abs(-a*b^2 + a*b*c)*abs(b)*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^4)*sqrt(-a)*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*sqrt(-a*b*c)*abs(b))*arctan(-(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/sqrt(-(a*b^3 - a*b*c^2 + sqrt((a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c))))/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c)) - (2*(a*b^3*c^2 - a*b^2*c^3)*(a*b^2 - a*b*c)^2*sqrt(-a)*abs(b)*sgn(-2*b + 2*c) + 2*(a*b^3*c - a*b^2*c^2)*(a*b^2 - a*b*c)^2*sqrt(-a*b*c)*abs(b) + (a^2*b^5*c - 3*a^2*b^

$4*c^2 + 3*a^2*b^3*c^3 - a^2*b^2*c^4)*\sqrt{-a*b*c}*abs(-a*b^2 + a*b*c)*abs(b)$
 $*sgn(-2*b + 2*c) + (a^2*b^6*c - 3*a^2*b^5*c^2 + 3*a^2*b^4*c^3 - a^2*b^3*c^4)*\sqrt{-a}*abs(-a*b^2 + a*b*c)*abs(b) + (a^3*b^7*c^2 - 2*a^3*b^6*c^3 + 2*a^3*b^4*c^5 - a^3*b^3*c^6)*\sqrt{-a}*abs(b)*sgn(-2*b + 2*c) + (a^3*b^7*c - 2*a^3*b^6*c^2 + 2*a^3*b^4*c^4 - a^3*b^3*c^5)*\sqrt{-a*b*c}*abs(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a*b^3 - a*b*c^2 - \sqrt{(a*b^3 - a*b*c^2)^2 - (a^2*b^5 - 3*a^2*b^4*c + 3*a^2*b^3*c^2 - a^2*b^2*c^3)*(b - c)}}/(b - c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^3*abs(-a*b^2 + a*b*c))$

maple [A] time = 0.01, size = 88, normalized size = 0.85

$$\frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx} \right) b}{b-c} - \frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{cx+a}}{2cx} \right) c}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] $2/(b-c)*b*(-1/2*(b*x+a)^{(1/2)}/b/x-1/2*\operatorname{arctanh}((b*x+a)^{(1/2)}/a^{(1/2)}))/a^{(1/2)}$
 $-2/(b-c)*c*(-1/2*(c*x+a)^{(1/2)}/x/c-1/2/a^{(1/2)}*\operatorname{arctanh}((c*x+a)^{(1/2)}/a^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{bx+a} + \sqrt{cx+a})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))), x)

mupad [B] time = 10.93, size = 1637, normalized size = 15.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)

[Out] $(2*a*b - 2*a*c + a*c*\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)})) - 2*a^{(1/2)}*b*(a + c*x)^{(1/2)} + 2*a^{(1/2)}*c*(a + b*x)^{(1/2)} + a*b*\operatorname{atan}((b^3*(a + b*x)^{(1/2)}*1i - b^3*(a + c*x)^{(1/2)}*1i + c^3*(a + b*x)^{(1/2)}*1i - a^{(1/2)}*c^3*1i + a^{(1/2)}*b*c^2*1i - b*c^2*(a + c*x)^{(1/2)}*1i)/(b^3*(a + b*x)^{(1/2)} - b^3*(a + c*x)^{(1/2)} - c^3*(a + b*x)^{(1/2)} + a^{(1/2)}*c^3 - a^{(1/2)}*b*c^2 + b*c^2*(a + c*x)^{(1/2)}))*2i - a*c*\operatorname{atan}((b^3*(a + b*x)^{(1/2)}*1i - b^3*(a + c*x)^{(1/2)}*1i + c^3*(a + b*x)^{(1/2)}*1i - a^{(1/2)}*c^3*1i + a^{(1/2)}*b*c^2*1i - b*c^2*(a + c*x)^{(1/2)}*1i)/(b^3*(a + b*x)^{(1/2)} - b^3*(a + c*x)^{(1/2)} - c^3*(a + b*x)^{(1/2)} + a^{(1/2)}*c^3 - a^{(1/2)}*b*c^2 + b*c^2*(a + c*x)^{(1/2)}))*2i + a*b*\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)})) + b*\operatorname{atan}((b^3*(a + b*x)^{(1/2)}*1i - b^3*(a + c*x)^{(1/2)}*1i + c^3*(a + b*x)^{(1/2)}*1i - a^{(1/2)}*c^3*1i + a^{(1/2)}*b*c^2*1i - b*c^2*(a + c*x)^{(1/2)}*1i)/(b^3*(a + b*x)^{(1/2)} - b^3*(a + c*x)^{(1/2)} - c^3*(a + b*x)^{(1/2)} + a^{(1/2)}*c^3 - a^{(1/2)}*b*c^2 + b*c^2*(a + c*x)^{(1/2)}))*2i + a*b*\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)})) + b*\operatorname{atan}((b^3*(a + b*x)^{(1/2)}*1i - b^3*(a + c*x)^{(1/2)}*1i + c^3*(a + b*x)^{(1/2)}*1i - a^{(1/2)}*c^3*1i + a^{(1/2)}*b*c^2*1i - b*c^2*(a + c*x)^{(1/2)}*1i)/(b^3*(a + b*x)^{(1/2)} - b^3*(a + c*x)^{(1/2)} - c^3*(a + b*x)^{(1/2)} + a^{(1/2)}*c^3 - a^{(1/2)}*b*c^2 + b*c^2*(a + c*x)^{(1/2)}))*2i + b*\log(((a + b*x)^{(1/2)} - a^{(1/2)})/((a + c*x)^{(1/2)} - a^{(1/2)}))$

```

*(a + b*x)^(1/2)*(a + c*x)^(1/2) + c*log(((a + b*x)^(1/2) - a^(1/2))/((a +
c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)*(a + c*x)^(1/2) - a^(1/2)*b*atan((b^
3*(a + b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^
(1/2)*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^
(1/2) - b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b
*c^2 + b*c^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i - a^(1/2)*b*atan((b^3*(a
+ b*x)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)
*c^3*1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2)
- b^3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2
+ b*c^2*(a + c*x)^(1/2)))*(a + c*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)
)^(1/2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*
1i + a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^
3*(a + c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c
^2*(a + c*x)^(1/2)))*(a + b*x)^(1/2)*2i + a^(1/2)*c*atan((b^3*(a + b*x)^(1/
2)*1i - b^3*(a + c*x)^(1/2)*1i + c^3*(a + b*x)^(1/2)*1i - a^(1/2)*c^3*1i +
a^(1/2)*b*c^2*1i - b*c^2*(a + c*x)^(1/2)*1i)/(b^3*(a + b*x)^(1/2) - b^3*(a
+ c*x)^(1/2) - c^3*(a + b*x)^(1/2) + a^(1/2)*c^3 - a^(1/2)*b*c^2 + b*c^2*(a
+ c*x)^(1/2)))*(a + c*x)^(1/2)*2i - a^(1/2)*b*log(((a + b*x)^(1/2) - a^(1/
2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2) - a^(1/2)*b*log(((a + b*x)
^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + c*x)^(1/2) - a^(1/2)*c*
log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*(a + b*x)^(1/2)
) - a^(1/2)*c*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*
(a + c*x)^(1/2))/(2*a^(1/2)*(b - c)*((a + b*x)^(1/2) - a^(1/2))*((a + c*x)^
(1/2) - a^(1/2)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)

[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))), x)

$$3.208 \quad \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{a+cx})} dx$$

Optimal. Leaf size=171

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{\sqrt{a+bx}}{2x^2(b-c)} + \frac{\sqrt{a+cx}}{2x^2(b-c)} - \frac{b\sqrt{a+bx}}{4ax(b-c)} + \frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Rubi [A] time = 0.11, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2103, 47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{\sqrt{a+bx}}{2x^2(b-c)} + \frac{\sqrt{a+cx}}{2x^2(b-c)} - \frac{b\sqrt{a+bx}}{4ax(b-c)} + \frac{c\sqrt{a+cx}}{4ax(b-c)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])),x]

[Out] -Sqrt[a + b*x]/(2*(b - c)*x^2) - (b*Sqrt[a + b*x])/(4*a*(b - c)*x) + Sqrt[a + c*x]/(2*(b - c)*x^2) + (c*Sqrt[a + c*x])/(4*a*(b - c)*x) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(3/2)*(b - c)) - (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(4*a^(3/2)*(b - c))

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2103

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dis

`t[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]`

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{a+cx})} dx &= \frac{\int \frac{\sqrt{a+bx}}{x^3} dx}{b-c} - \frac{\int \frac{\sqrt{a+cx}}{x^3} dx}{b-c} \\ &= -\frac{\sqrt{a+bx}}{2(b-c)x^2} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{b \int \frac{1}{x^2\sqrt{a+bx}} dx}{4(b-c)} - \frac{c \int \frac{1}{x^2\sqrt{a+cx}} dx}{4(b-c)} \\ &= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b^2 \int \frac{1}{x\sqrt{a+bx}} dx}{8a(b-c)} + \frac{c^2 \int \frac{1}{x\sqrt{a+cx}} dx}{8a(b-c)} \\ &= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} - \frac{b \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a(b-c)} \\ &= -\frac{\sqrt{a+bx}}{2(b-c)x^2} - \frac{b\sqrt{a+bx}}{4a(b-c)x} + \frac{\sqrt{a+cx}}{2(b-c)x^2} + \frac{c\sqrt{a+cx}}{4a(b-c)x} + \frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} - \frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{4a^{3/2}(b-c)} \end{aligned}$$

Mathematica [C] time = 0.09, size = 75, normalized size = 0.44

$$\frac{2c^2(a+cx)^{3/2} {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{cx}{a} + 1\right) - 2b^2(a+bx)^{3/2} {}_2F_1\left(\frac{3}{2}, 3; \frac{5}{2}; \frac{bx}{a} + 1\right)}{3a^3(b-c)}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])), x]`

[Out] `(-2*b^2*(a + b*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (b*x)/a] + 2*c^2*(a + c*x)^(3/2)*Hypergeometric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/(3*a^3*(b - c))`

IntegrateAlgebraic [F] time = 180.00, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] `IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])), x]`

[Out] \$Aborted

fricas [A] time = 0.45, size = 243, normalized size = 1.42

$$\left[\frac{\sqrt{a} b^2 x^2 \log\left(\frac{bx - 2\sqrt{bx+a}\sqrt{a+2a}}{x}\right) + \sqrt{a} c^2 x^2 \log\left(\frac{cx + 2\sqrt{cx+a}\sqrt{a+2a}}{x}\right) + 2(abx + 2a^2)\sqrt{bx+a} - 2(acx + 2a^2)\sqrt{cx+a}}{8(a^2b - a^2c)x^2}, \frac{\sqrt{-a} b^2 x^2 \arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right) - \sqrt{-a} c^2 x^2 \arctan\left(\frac{\sqrt{cx+a}\sqrt{-a}}{a}\right) + (abx + 2a^2)\sqrt{bx+a} - (acx + 2a^2)\sqrt{cx+a}}{4(a^2b - a^2c)x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)), x, algorithm="fricas")`

[Out] `[-1/8*(sqrt(a)*b^2*x^2*log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + sqrt(a)*c^2*x^2*log((c*x + 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(a*b*x + 2*a^2)*sqrt(b*x + a) - 2*(a*c*x + 2*a^2)*sqrt(c*x + a)]/((a^2*b - a^2*c)*x^2), -1/4*(sqrt(-a)*b^2*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - sqrt(-a)*c^2*x^2*arctan(sqrt(c*x + a)*sqrt(-a)/a)]`

$\text{ctan}(\sqrt{c*x + a}*\sqrt{-a}/a) + (a*b*x + 2*a^2)*\sqrt{b*x + a} - (a*c*x + 2*a^2)*\sqrt{c*x + a})/((a^2*b - a^2*c)*x^2]$

giac [B] time = 37.26, size = 1895, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="giac")

[Out]
$$-1/4*b^2*\arctan(\sqrt{b*x + a}/\sqrt{-a})/((a*b - a*c)*\sqrt{-a}) - 1/2*((\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^3*b^6*c^2*\text{abs}(b) - 3*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^3*b^5*c^3*\text{abs}(b) + 3*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^3*b^4*c^4*\text{abs}(b) - (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^3*b^3*c^5*\text{abs}(b) + 7*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^3*a^2*b^4*c^2*\text{abs}(b) - 10*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^3*a^2*b^3*c^3*\text{abs}(b) + 3*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^3*a^2*b^2*c^4*\text{abs}(b) + 7*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^5*a*b^2*c^2*\text{abs}(b) - 3*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^5*a*b*c^3*\text{abs}(b) + (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^7*c^2*\text{abs}(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a*b^2 - 2*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a*b*c + (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^4)^2*(a*b - a*c) - 1/4*((b*x + a)^(3/2)*b^2 + \sqrt{b*x + a}*a*b^2)/((a*b - a*c)*b^2*x^2) - 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*\sqrt{-a}*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*\sqrt{-a}*b*c*\text{abs}(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*\sqrt{-a}*b*c*\text{abs}(a^2*b^2 - a^2*b*c)*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*\sqrt{-a}*\text{abs}(a^2*b^2 - a^2*b*c)*\text{abs}(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*\sqrt{-a}*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*\sqrt{-a}*b*c*\text{abs}(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a^2*b^3 - a^2*b*c^2 + \sqrt{((a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c))})/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*\text{abs}(a^2*b^2 - a^2*b*c)) + 1/4*(2*(a*b^3*c^3 - a*b^2*c^4)*(a^2*b^2 - a^2*b*c)^2*\sqrt{-a}*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + 2*(a*b^3*c^2 - a*b^2*c^3)*(a^2*b^2 - a^2*b*c)^2*\sqrt{-a}*b*c*\text{abs}(b) + (a^3*b^5*c^2 - 3*a^3*b^4*c^3 + 3*a^3*b^3*c^4 - a^3*b^2*c^5)*\sqrt{-a}*b*c*\text{abs}(a^2*b^2 - a^2*b*c)*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + (a^3*b^6*c^2 - 3*a^3*b^5*c^3 + 3*a^3*b^4*c^4 - a^3*b^3*c^5)*\sqrt{-a}*\text{abs}(a^2*b^2 - a^2*b*c)*\text{abs}(b) + (a^5*b^7*c^3 - 2*a^5*b^6*c^4 + 2*a^5*b^4*c^6 - a^5*b^3*c^7)*\sqrt{-a}*\text{abs}(b)*\text{sgn}(8*a*b - 8*a*c) + (a^5*b^7*c^2 - 2*a^5*b^6*c^3 + 2*a^5*b^4*c^5 - a^5*b^3*c^6)*\sqrt{-a}*b*c*\text{abs}(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a^2*b^3 - a^2*b*c^2 - \sqrt{((a^2*b^3 - a^2*b*c^2)^2 - (a^3*b^5 - 3*a^3*b^4*c + 3*a^3*b^3*c^2 - a^3*b^2*c^3)*(a*b - a*c))})/(a*b - a*c)))/((b^8 - 5*b^7*c + 10*b^6*c^2 - 10*b^5*c^3 + 5*b^4*c^4 - b^3*c^5)*a^5*\text{abs}(a^2*b^2 - a^2*b*c))$$

maple [A] time = 0.01, size = 120, normalized size = 0.70

$$\frac{2 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^2} + \frac{\frac{(bx+a)^2}{8a} - \frac{\sqrt{bx+a}}{8}}{b^2x^2} \right) b^2}{b-c} - \frac{2 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^2} + \frac{\frac{(cx+a)^2}{8a} - \frac{\sqrt{cx+a}}{8}}{c^2x^2} \right) c^2}{b-c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x)

[Out] 2/(b-c)*b^2*((-1/8/a*(b*x+a)^(3/2)-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-2/(b-c)*c^2*((-1/8/a*(c*x+a)^(3/2)-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(\sqrt{bx+a} + \sqrt{cx+a})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))), x)

mupad [B] time = 11.85, size = 1610, normalized size = 9.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))),x)

[Out] ((a^(3/2)*b^3)/(16*(a^3*c^2 - a^3*b*c)) + (a^(3/2)*((a + b*x)^(1/2) - a^(1/2))^2*((b*c^2)/4 - (7*b^2*c)/16 + b^3/4))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))^2) - (a^(3/2)*((b^2*c)/16 + b^3/16)*((a + b*x)^(1/2) - a^(1/2)))/((a^3*c^2 - a^3*b*c)*((a + c*x)^(1/2) - a^(1/2))) + ((b^2/8 - c^2/8)*((a + b*x)^(1/2) - a^(1/2))^3)/(a^(3/2)*c*((a + c*x)^(1/2) - a^(1/2))^3)/((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 - ((b + c)*((a + b*x)^(1/2) - a^(1/2))^3)/(c*((a + c*x)^(1/2) - a^(1/2))^3) + (b*((a + b*x)^(1/2) - a^(1/2))^2)/(c*((a + c*x)^(1/2) - a^(1/2))^2) - (((c*(b + c))/(4*a^(3/2)*(b - c)) - (c*(b^2 - c^2))/(4*a^(3/2)*(b - c)^2))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2))))*(a^(3/2)*b^2 + a^(3/2)*c^2)/(8*a^3*b - 8*a^3*c) + (atan((((b + c)*((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - ((a + b*x)^(1/2) - a^(1/2))*((64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3) - ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - ((a + b*x)^(1/2) - a^(1/2))*((64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2))))*1i)/(8*a^3)))/(((b + c)*((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - ((a + b*x)^(1/2) - a^(1/2))*((64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - (b*c^4 - b^5)/(32*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((16*a^3*b^4 + 16*a^3*b*c^3)/(64*(a^6*c^3 - a^6*b*c^2)) + ((b + c)*((64*a^6*b^3 - 64*a^6*b*c^2)/(64*(a^6*c^3 - a^6*b*c^2)) - ((a + b*x)^(1/2) - a^(1/2))*((64*a^6*b^3 - 64*a^6*c^3 + 128*a^6*b*c^2 - 128*a^6*b^2*c))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) - ((8*a^3*b^4 + 8*a^3*c^4)*((a + b*x)^(1/2) - a^(1/2)))/(32*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2)))))/(8*a^3) + (((a + b*x)^(1/2) - a^(1/2))*((b*c^4 - b^4*c + b^2*c^3 - b^3*c^2))/(16*(a^6*c^3 - a^6*b*c^2))*((a + c*x)^(1/2) - a^(1/2))))*(a^(3/2)*b + a^(3/2)*c)*1i)/(4*a^3) + (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a^(3/2)*(b - c)*((a + c*x)^(1/2) - a^(1/2))^2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{a+cx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2)),x)
```

```
[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))), x)
```

$$3.209 \quad \int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=195

$$-\frac{a^3(b+c) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}} + \frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2} + \frac{ax^2}{(b-c)^2} - \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2}$$

Rubi [A] time = 0.35, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, number of rules / integrand size = 0.240, Rules used = {6690, 80, 50, 63, 217, 206}

$$\frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2c^2(b-c)} - \frac{a^3(b+c) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{4b^{5/2}c^{5/2}} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2c(b-c)^2} + \frac{ax^2}{(b-c)^2} - \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3bc(b-c)^2} + \frac{x^3(b+c)}{3(b-c)^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]

[Out] (a*x^2)/(b - c)^2 + ((b + c)*x^3)/(3*(b - c)^2) + (a^2*(b + c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(4*b^2*(b - c)*c^2) + (a*(b + c)*(a + b*x)^(3/2)*Sqrt[a + c*x])/(2*b^2*(b - c)^2*c) - (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*b*(b - c)^2*c) - (a^3*(b + c)*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(4*b^(5/2)*c^(5/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6690

Int[(u_)*((e_)*Sqrt[(a_) + (b_)*(x_)^(n_)]) + (f_)*Sqrt[(c_) + (d_)*(x_)^(n_)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx &= \frac{\int (2ax + b(1 + \frac{c}{b})x^2 - 2x\sqrt{a+bx}\sqrt{a+cx}) dx}{(b-c)^2} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} - \frac{2 \int x\sqrt{a+bx}\sqrt{a+cx} dx}{(b-c)^2} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} - \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c} + \frac{(a(b+c)) \int \sqrt{a+bx}\sqrt{a+cx}}{b(b-c)^2c} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c} - \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3b(b-c)^2c} + \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} + \frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)^2c} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} + \frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)^2c} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} + \frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)^2c} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c} \\ &= \frac{ax^2}{(b-c)^2} + \frac{(b+c)x^3}{3(b-c)^2} + \frac{a^2(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4b^2(b-c)^2c} + \frac{a(b+c)(a+bx)^{3/2}\sqrt{a+cx}}{2b^2(b-c)^2c} \end{aligned}$$

Mathematica [A] time = 0.67, size = 238, normalized size = 1.22

$$\frac{3a^4(c-b)^3(b+c)\sqrt{\frac{b(a+cx)}{a(b-c)}} \sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right) + b\sqrt{c}\left(a^2(3b^2-2bc+3c^2)\sqrt{a+bx}\sqrt{a+cx} + 4b^2c^2x^2(-2\sqrt{a+bx}\sqrt{a+cx} + bx+cx) - 2abcx(b\sqrt{a+bx}\sqrt{a+cx} + c\sqrt{a+bx}\sqrt{a+cx} - 6bcx)\right)}{12b^3c^{5/2}(b-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2, x]

[Out] (b*Sqrt[c]*(a^2*(3*b^2 - 2*b*c + 3*c^2)*Sqrt[a + b*x]*Sqrt[a + c*x] + 4*b^2*c^2*x^2*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - 2*a*b*c*x*(-6*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + (3*a^4*(-b + c)^3*(b + c)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c))]/(Sqrt[a*(b - c)]*Sqrt[a + c*x]))/(12*b^3*(b - c)^2*c^(5/2))

IntegrateAlgebraic [A] time = 1.65, size = 255, normalized size = 1.31

$$\frac{a^3\sqrt{b}(b+c)\log\left(\sqrt{\frac{b(a+cx)}{c}} - \frac{ab}{c} + a - \sqrt{\frac{b}{c}}\sqrt{a+cx}\right) + \sqrt{\frac{b(a+cx)}{c}} - \frac{ab}{c} + a\left(-3a^2b^2\sqrt{a+cx} + 3a^2c^2\sqrt{a+cx} - 8b^2(a+cx)^{3/2} + 14ab^2(a+cx)^{3/2} - 2abc(a+cx)^{3/2}\right) + \frac{3a^2b(a+cx) - 3a^2c(a+cx) + b(a+cx)^3 - 3ab(a+cx)^2 + c(a+cx)^3}{3c^3(b-c)^2}}{4b^3c^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]
```

```
[Out] (Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(-3*a^2*b^2*Sqrt[a + c*x] + 3*a^2*c^2*Sqrt[a + c*x] + 14*a*b^2*(a + c*x)^(3/2) - 2*a*b*c*(a + c*x)^(3/2) - 8*b^2*(a + c*x)^(5/2)))/(12*b^2*(b - c)^2*c^2) + (3*a^2*b*(a + c*x) - 3*a^2*c*(a + c*x) - 3*a*b*(a + c*x)^2 + b*(a + c*x)^3 + c*(a + c*x)^3)/(3*(b - c)^2*c^3) + (a^3*Sqrt[b/c]*(b + c)*Log[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/(4*b^3*c^2)
```

fricas [A] time = 0.43, size = 479, normalized size = 2.46

$$\frac{3a^2b^2c^2 + 8(b^2c^2 + 3(b^2c^2 - a^2b^2 + a^2c^2)\sqrt{bc})\log(ad^2 + 2abc + 2(2b - \sqrt{b})\sqrt{c}\sqrt{a+b}) - 2(8b^2c^2 - 3a^2b^2 - 3a^2c^2 + 2(ab^2 + a^2b^2))\sqrt{bc} + 12a^2b^2c^2 + 4(b^2c^2 + 3(b^2c^2 - a^2b^2 + a^2c^2)\sqrt{bc})\operatorname{arctan}\left(\frac{\sqrt{a+b}\sqrt{c}}{\sqrt{a+b} + \sqrt{c}}\right) - (8b^2c^2 - 3a^2b^2 - 3a^2c^2 + 2(ab^2 + a^2b^2))\sqrt{bc} + \sqrt{c}\sqrt{a+b}}{24(b^2c^2 - 3a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(24*a*b^3*c^3*x^2 + 8*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5), 1/12*(12*a*b^3*c^3*x^2 + 4*(b^4*c^3 + b^3*c^4)*x^3 + 3*(a^3*b^3 - a^3*b^2*c - a^3*b*c^2 + a^3*c^3)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (8*b^3*c^3*x^2 - 3*a^2*b^3*c + 2*a^2*b^2*c^2 - 3*a^2*b*c^3 + 2*(a*b^3*c^2 + a*b^2*c^3)*x)*sqrt(b*x + a)*sqrt(c*x + a))/(b^5*c^3 - 2*b^4*c^4 + b^3*c^5)]
```

giac [B] time = 3.47, size = 511, normalized size = 2.62

$$\frac{1}{12} \sqrt{ab^2 + bc} + \frac{1}{12} \sqrt{ab^2 + bc} \left(2 \log\left(\frac{4(b^2c^2 - 3a^2b^2) + 3a^2c^2}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + a \right) + \frac{ab^2c^2 - 10ab^2c^2 + 24a^2b^2c^2 - 22a^2c^3}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + 7a^2c^3 \right) - \frac{3(a^2b^2c^2 - 3a^2b^2c^2 + 2a^2b^2c^2 - 3a^2b^2c^2 + a^2b^2c^2)}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + \frac{(bc + a^2 - 3)(bc + a^2) + (bc + a^2 - 3)(bc + a^2) + 3(bc + a^2) + (a^2b^2 + a^2c^2)\log\left(\frac{\sqrt{bc}\sqrt{a+b} + \sqrt{ab^2 + bc}}{\sqrt{bc}\sqrt{a+b}}\right)}{3(b^2 - 2ab + a^2)} + \frac{(a^2b^2 + a^2c^2)\log\left(\frac{\sqrt{bc}\sqrt{a+b} + \sqrt{ab^2 + bc}}{\sqrt{bc}\sqrt{a+b}}\right)}{4\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
```

```
[Out] -1/12*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(2*(b*x + a)*(4*(b^11*c^4*abs(b) - 3*b^10*c^5*abs(b) + 3*b^9*c^6*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9) + (a*b^12*c^3*abs(b) - 10*a*b^11*c^4*abs(b) + 24*a*b^10*c^5*abs(b) - 22*a*b^9*c^6*abs(b) + 7*a*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9)) - 3*(a^2*b^13*c^2*abs(b) - 3*a^2*b^12*c^3*abs(b) + 2*a^2*b^11*c^4*abs(b) + 2*a^2*b^10*c^5*abs(b) - 3*a^2*b^9*c^6*abs(b) + a^2*b^8*c^7*abs(b))/(b^17*c^4 - 5*b^16*c^5 + 10*b^15*c^6 - 10*b^14*c^7 + 5*b^13*c^8 - b^12*c^9)*sqrt(b*x + a) + 1/3*((b*x + a)^3*b - 3*(b*x + a)*a^2*b + (b*x + a)^3*c - 3*(b*x + a)^2*a*c + 3*(b*x + a)*a^2*c)/(b^5 - 2*b^4*c + b^3*c^2) + 1/4*(a^3*b*abs(b) + a^3*c*abs(b))*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^3*c^2)
```

maple [B] time = 0.02, size = 517, normalized size = 2.65

$$\frac{b^2}{3(b^2 - a^2)} + \frac{c^2}{3(b^2 - a^2)} + \frac{a^2}{3(b^2 - a^2)} + \frac{\sqrt{a+b}\sqrt{c}}{12} \left(\frac{1}{2} \log\left(\frac{4(b^2c^2 - 3a^2b^2) + 3a^2c^2}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + a\right) + \frac{ab^2c^2 - 10ab^2c^2 + 24a^2b^2c^2 - 22a^2c^3}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + 7a^2c^3 \right) - \frac{3(a^2b^2c^2 - 3a^2b^2c^2 + 2a^2b^2c^2 - 3a^2b^2c^2 + a^2b^2c^2)}{b^2c^2 - 3a^2b^2 + 10a^2c^2} \sqrt{bc} + \frac{(bc + a^2 - 3)(bc + a^2) + (bc + a^2 - 3)(bc + a^2) + 3(bc + a^2) + (a^2b^2 + a^2c^2)\log\left(\frac{\sqrt{bc}\sqrt{a+b} + \sqrt{ab^2 + bc}}{\sqrt{bc}\sqrt{a+b}}\right)}{3(b^2 - 2ab + a^2)} + \frac{(a^2b^2 + a^2c^2)\log\left(\frac{\sqrt{bc}\sqrt{a+b} + \sqrt{ab^2 + bc}}{\sqrt{bc}\sqrt{a+b}}\right)}{4\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
```

```
[Out] 1/3*x^3/(b-c)^2*b+1/3*x^3/(b-c)^2*c+a*x^2/(b-c)^2-1/24/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(1/2)*(16*x^2*b^2*c^2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)+a*b+a*c)/(b*c)^(1/2))*a^3*b^3-3*ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b
```

$$c)^{(1/2)}+a*b+a*c)/(b*c)^{(1/2))*a^3*b^2*c-3*\ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}+a*b+a*c)/(b*c)^{(1/2))*a^3*b*c^2+3*\ln(1/2*(2*b*c*x+2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}+a*b+a*c)/(b*c)^{(1/2))*a^3*c^3+4*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}*x*a*b^2*c+4*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}*x*a*b*c^2-6*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}*a^2*b^2+4*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}*a^2*b*c-6*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}*(b*c)^{(1/2)}*a^2*c^2)/(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)}/b^2/c^2/(b*c)^{(1/2)}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)

mupad [B] time = 18.15, size = 1107, normalized size = 5.68

$$\frac{\int \frac{x^3}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx}{\int \frac{x^3}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)

[Out] (((a + b*x)^(1/2) - a^(1/2))^6*(128*a^3*b*c^3 + 128*a^3*b^3*c + (1312*a^3*b^2*c^2)/3)/((a + c*x)^(1/2) - a^(1/2))^6 - ((a + b*x)^(1/2) - a^(1/2))^7*(19*a^3*c^4 + 269*a^3*b*c^3 + 19*a^3*b^3*c + 269*a^3*b^2*c^2)/((a + c*x)^(1/2) - a^(1/2))^7 - ((a + b*x)^(1/2) - a^(1/2))^5*(19*a^3*b^4 + 19*a^3*b*c^3 + 269*a^3*b^3*c + 269*a^3*b^2*c^2)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^4*(64*a^3*b^4 + 192*a^3*b^3*c + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(64*a^3*c^4 + 192*a^3*b*c^3 + 64*a^3*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^8 + (16*a^3*b^4*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (16*a^3*c^4*((a + b*x)^(1/2) - a^(1/2))^10)/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)^(1/2) - a^(1/2))^11*(a^3*c^6 - a^3*b*c^5 - a^3*b^2*c^4 + a^3*b^3*c^3))/(2*b^2*((a + c*x)^(1/2) - a^(1/2))^11) - (((a + b*x)^(1/2) - a^(1/2))^3*(17*a^3*b^5 + 303*a^3*b^4*c + 17*a^3*b^2*c^3 + 303*a^3*b^3*c^2))/(6*c*((a + c*x)^(1/2) - a^(1/2))^3) - (((a + b*x)^(1/2) - a^(1/2))^9*(17*a^3*c^5 + 303*a^3*b*c^4 + 303*a^3*b^2*c^3 + 17*a^3*b^3*c^2))/(6*b*((a + c*x)^(1/2) - a^(1/2))^9) + ((a^3*b + a^3*c)*((a + b*x)^(1/2) - a^(1/2))*(b^5 - 2*b^4*c + b^3*c^2))/(2*c^2*((a + c*x)^(1/2) - a^(1/2)))/((b^8 - 2*b^7*c + b^6*c^2 + ((a + b*x)^(1/2) - a^(1/2))^12*(c^8 - 2*b*c^7 + b^2*c^6))/((a + c*x)^(1/2) - a^(1/2))^12 - (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^7*c + 6*b^5*c^3 - 12*b^6*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^10*(6*b*c^7 - 12*b^2*c^6 + 6*b^3*c^5))/((a + c*x)^(1/2) - a^(1/2))^10 + (((a + b*x)^(1/2) - a^(1/2))^4*(15*b^4*c^4 - 30*b^5*c^3 + 15*b^6*c^2))/((a + c*x)^(1/2) - a^(1/2))^4 + (((a + b*x)^(1/2) - a^(1/2))^8*(15*b^2*c^6 - 30*b^3*c^5 + 15*b^4*c^4))/((a + c*x)^(1/2) - a^(1/2))^8 - (((a + b*x)^(1/2) - a^(1/2))^6*(20*b^3*c^5 - 40*b^4*c^4 + 20*b^5*c^3))/((a + c*x)^(1/2) - a^(1/2))^6 + (x^3*(b + c))/(3*(b - c)^2) + (a*x^2)/(b - c)^2 - (a^3*atanh((c^(1/2))*((a + b*x)^(1/2) - a^(1/2))))/(b^(1/2)*((a + c*x)^(1/2) - a^(1/2)))*(b + c)/(2*b^(5/2)*c^(5/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)
```

$$3.210 \quad \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=142

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}} + \frac{2ax}{(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{x^2(b+c)}{2(b-c)^2}$$

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6690, 50, 63, 217, 206}

$$\frac{a^2 \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}} + \frac{2ax}{(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2bc(b-c)} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{x^2(b+c)}{2(b-c)^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]

[Out] (2*a*x)/(b - c)^2 + ((b + c)*x^2)/(2*(b - c)^2) - (a*Sqrt[a + b*x]*Sqrt[a + c*x])/(2*b*(b - c)*c) - ((a + b*x)^(3/2)*Sqrt[a + c*x])/(b*(b - c)^2) + (a^2*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(2*b^(3/2)*c^(3/2))

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_.) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx = \frac{\int (2a + b(1 + \frac{c}{b})x - 2\sqrt{a+bx}\sqrt{a+cx}) dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{2 \int \sqrt{a+bx}\sqrt{a+cx} dx}{(b-c)^2}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} - \frac{a \int \frac{\sqrt{a+bx}}{\sqrt{a+cx}} dx}{2b(b-c)}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2 \int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}} dx}{4bc}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}} dx, \sqrt{a+bx}\sqrt{a+cx}\right)}{4bc}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2 \operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}} dx, \sqrt{a+bx}\sqrt{a+cx}\right)}{4bc}$$

$$= \frac{2ax}{(b-c)^2} + \frac{(b+c)x^2}{2(b-c)^2} - \frac{a\sqrt{a+bx}\sqrt{a+cx}}{2b(b-c)c} - \frac{(a+bx)^{3/2}\sqrt{a+cx}}{b(b-c)^2} + \frac{a^2 \operatorname{tanh}^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{2b^{3/2}c^{3/2}}$$

Mathematica [A] time = 0.56, size = 177, normalized size = 1.25

$$\frac{b\sqrt{c} (bcx(-2\sqrt{a+bx}\sqrt{a+cx} + bx + cx) - a(b\sqrt{a+bx}\sqrt{a+cx} + c\sqrt{a+bx}\sqrt{a+cx} - 4bcx)) + \frac{(a(b-c))^{5/2} \sqrt{\frac{b(a+cx)}{a(b-c)}} \operatorname{sinh}^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a+cx}}}{2b^2c^{3/2}(b-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]

[Out] (b*Sqrt[c]*(b*c*x*(b*x + c*x - 2*Sqrt[a + b*x]*Sqrt[a + c*x]) - a*(-4*b*c*x + b*Sqrt[a + b*x]*Sqrt[a + c*x] + c*Sqrt[a + b*x]*Sqrt[a + c*x])) + ((a*(b - c))^(5/2)*Sqrt[(b*(a + c*x))/(a*(b - c))]*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]])/Sqrt[a + c*x])/(2*b^2*(b - c)^2*c^(3/2))

IntegrateAlgebraic [A] time = 0.94, size = 197, normalized size = 1.39

$$-\frac{a^2 \sqrt{\frac{b}{c}} \log\left(\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c}} + a - \sqrt{\frac{b}{c}} \sqrt{a+cx}\right)}{2b^2c} + \frac{b(a+cx)^2 - 2ab(a+cx) + c(a+cx)^2 + 2ac(a+cx)}{2c^2(b-c)^2} + \frac{\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c}} + a (-2b(a+cx)^{3/2} + ab\sqrt{a+cx} - ac\sqrt{a+cx})}{2bc(b-c)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]

[Out] (Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a*b*Sqrt[a + c*x] - a*c*Sqrt[a + c*x] - 2*b*(a + c*x)^(3/2)))/(2*b*(b - c)^2*c) + (-2*a*b*(a + c*x) + 2*a*c*(a + c*x) + b*(a + c*x)^2 + c*(a + c*x)^2)/(2*(b - c)^2*c^2) - (a^2*Sqrt[b/c]*Log[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/(2*b^2*c)

fricas [A] time = 0.44, size = 372, normalized size = 2.62

$$\frac{8a^2c^2x + 2(b^2c^2 + b^2c^2)x^2 + (b^2c^2 - 2a^2bc + a^2c^2)\sqrt{c} \log(ab^2 + 2abc + a^2 + 2(2bc + \sqrt{c}(b+c))\sqrt{bx+a}\sqrt{cx+a} + 2((2c + bc^2)x + 2(2bcx + ab + ac)\sqrt{c})) - 2(2b^2cx + ab^2 + abc^2)\sqrt{bx+a}\sqrt{cx+a} + 4a^2c^2x + (b^2c^2 + b^2c^2)x^2 - (b^2c^2 - 2a^2bc + a^2c^2)\sqrt{c} \operatorname{arctan}\left(\frac{\sqrt{c}\sqrt{bx+a}\sqrt{cx+a}}{bx+a}\right) - (2b^2c^2 + ab^2c + abc^2)\sqrt{bx+a}\sqrt{cx+a}}{4(b^2c^2 - 2b^2c^2 + b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")

[Out] [1/4*(8*a*b^2*c^2*x + 2*(b^3*c^2 + b^2*c^3)*x^2 + (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c + sqrt(b*c))*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x + 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) - 2*(2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4), 1/2*(4*a*b^2*c^2*x + (b^3*c^2 + b^2*c^3)*x^2 - (a^2*b^2 - 2*a^2*b*c + a^2*c^2)*sqrt(-b*c)*arctan((sqrt(-b*c))*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) - (2*b^2*c^2*x + a*b^2*c + a*b*c^2)*sqrt(b*x + a)*sqrt(c*x + a))/(b^4*c^2 - 2*b^3*c^3 + b^2*c^4)]

giac [B] time = 3.28, size = 272, normalized size = 1.92

$$-\frac{1}{2}\sqrt{ab^2+(bx+a)bc-abc}\sqrt{bx+a}\left(\frac{2(b^4c^2|b|-b^3c^3|b|)(bx+a)}{b^9c^2-3b^8c^3+3b^7c^4-b^6c^5}+\frac{ab^5c|b|-2ab^4c^2|b|+ab^3c^3|b|}{b^9c^2-3b^8c^3+3b^7c^4-b^6c^5}\right)-\frac{a^2|b|\log\left(\left|-\sqrt{bc}\sqrt{bx+a}+\sqrt{ab^2+(bx+a)bc-abc}\right|\right)}{2\sqrt{bc}b^2c}+\frac{(bx+a)^2b+2(bx+a)ab+(bx+a)^2c-2(bx+a)ac}{2(b^4-2b^3c+b^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")

[Out] -1/2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*sqrt(b*x + a)*(2*(b^4*c^2*abs(b) - b^3*c^3*abs(b))*(b*x + a)/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5) + (a*b^5*c*abs(b) - 2*a*b^4*c^2*abs(b) + a*b^3*c^3*abs(b))/(b^9*c^2 - 3*b^8*c^3 + 3*b^7*c^4 - b^6*c^5)) - 1/2*a^2*abs(b)*log(abs(-sqrt(b*c)*sqrt(b*x + a) + sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)))/(sqrt(b*c)*b^2*c) + 1/2*((b*x + a)^2*b + 2*(b*x + a)*a*b + (b*x + a)^2*c - 2*(b*x + a)*a*c)/(b^4 - 2*b^3*c + b^2*c^2)

maple [B] time = 0.01, size = 385, normalized size = 2.71

$$\frac{\sqrt{(bx+a)(cx+a)}a^2b\ln\left(\frac{bx+\frac{1}{2}a+\frac{1}{2}c}{\sqrt{bc}}+\sqrt{bcx^2+a^2+(ab+ac)x}\right)}{4(b-c)^2\sqrt{cx+a}\sqrt{bx+a}\sqrt{bc}}+\frac{\sqrt{(bx+a)(cx+a)}a^2c\ln\left(\frac{bx+\frac{1}{2}a+\frac{1}{2}c}{\sqrt{bc}}+\sqrt{bcx^2+a^2+(ab+ac)x}\right)}{4(b-c)^2\sqrt{cx+a}\sqrt{bx+a}\sqrt{bc}}-\frac{\sqrt{(bx+a)(cx+a)}a^2\ln\left(\frac{bx+\frac{1}{2}a+\frac{1}{2}c}{\sqrt{bc}}+\sqrt{bcx^2+a^2+(ab+ac)x}\right)}{2(b-c)^2\sqrt{cx+a}\sqrt{bx+a}\sqrt{bc}}+\frac{bx^2}{2(b-c)^2}+\frac{cx^2}{2(b-c)^2}+\frac{2ax}{(b-c)^2}+\frac{\sqrt{cx+a}\sqrt{bx+a}}{2(b-c)^2b}+\frac{\sqrt{cx+a}\sqrt{bx+a}}{2(b-c)^2c}+\frac{\sqrt{bx+a}(cx+a)^{\frac{3}{2}}}{(b-c)^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)

[Out] 1/2*x^2/(b-c)^2*b+1/2*x^2/(b-c)^2*c+2*a*x/(b-c)^2-1/(b-c)^2/c*(b*x+a)^(1/2)*(c*x+a)^(3/2)+1/2/(b-c)^2/c*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a-1/2/(b-c)^2/b*(c*x+a)^(1/2)*(b*x+a)^(1/2)*a+1/4/(b-c)^2/c*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2*b-1/2/(b-c)^2*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2+1/4/(b-c)^2*c/b*((b*x+a)*(c*x+a))^(1/2)/(c*x+a)^(1/2)/(b*x+a)^(1/2)*ln((1/2*a*b+1/2*a*c+b*c*x)/(b*c)^(1/2)+(b*c*x^2+(a*b+a*c)*x+a^2)^(1/2))/(b*c)^(1/2)*a^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)

mupad [B] time = 0.25, size = 129, normalized size = 0.91

$$\frac{2ax}{(b-c)^2} + \frac{x^2(b+c)}{2(b-c)^2} - \frac{2\left(\frac{x}{2} + \frac{ab+ac}{4bc}\right)\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{\ln(ab+ac+2bcx+2\sqrt{b}\sqrt{c}\sqrt{a+bx}\sqrt{a+cx})(ab-ac)^2}{4b^{3/2}c^{3/2}(b-c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)`

[Out] $(2ax)/(b-c)^2 + (x^2(b+c))/(2(b-c)^2) - (2(x/2 + (ab+ac)/(4bc)))(a+bx)^{1/2}(a+cx)^{1/2}/(b-c)^2 + (\log(ab+ac+2b^2cx + 2b^{1/2}c^{1/2}(a+bx)^{1/2}(a+cx)^{1/2}))(ab-ac)^2/(4b^{3/2}c^{3/2}(b-c)^2)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)`

[Out] `Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)`

$$3.211 \quad \int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=135

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} + \frac{4a \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a(b+c) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2} + \frac{x(b+c)}{(b-c)^2}$$

Rubi [A] time = 0.18, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {6690, 101, 157, 63, 217, 206, 93, 208}

$$-\frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} + \frac{4a \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{2a(b+c) \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{\sqrt{b}\sqrt{c}(b-c)^2} + \frac{x(b+c)}{(b-c)^2}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2,x]

[Out] ((b + c)*x)/(b - c)^2 - (2*Sqrt[a + b*x]*Sqrt[a + c*x])/(b - c)^2 + (4*a*ArcTanH[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (2*a*(b + c)*ArcTanH[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(Sqrt[b]*(b - c)^2*Sqrt[c]) + (2*a*Log[x])/(b - c)^2

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 93

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 101

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a*f) + b*n*(d*e - c*f))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))

Rule 157

Int((((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx = \frac{\int \left(b \left(1 + \frac{c}{b} \right) + \frac{2a}{x} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x} \right) dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} - \frac{2 \int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x} dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} + \frac{2 \int \frac{-a^2 - \frac{1}{2}a(b+c)x}{x\sqrt{a+bx}\sqrt{a+cx}} dx}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} - \frac{(2a^2) \int \frac{1}{x\sqrt{a+bx}\sqrt{a+cx}} dx}{(b-c)^2} - \frac{(a(b+c))}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} - \frac{(4a^2) \text{Subst} \left(\int \frac{1}{-a+ax^2} dx, x, \frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right)}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{4a \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right)}{(b-c)^2} + \frac{2a \log(x)}{(b-c)^2} - \frac{(2a(b+c)) \text{Su}}{(b-c)^2}$$

$$= \frac{(b+c)x}{(b-c)^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2} + \frac{4a \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right)}{(b-c)^2} - \frac{2a(b+c) \tanh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}} \right)}{\sqrt{b}(b-c)^2\sqrt{c}}$$

Mathematica [A] time = 0.86, size = 195, normalized size = 1.44

$$\frac{2(b+c)\sqrt{a(b-c)}(a+cx) \sinh^{-1} \left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}} \right)}{\sqrt{c}\sqrt{\frac{b(a+cx)}{a(b-c)}}} - (b-c) \left(-2cx\sqrt{a+bx} + bx\sqrt{a+cx} + 4a\sqrt{a+cx} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right) - 2a\sqrt{a+bx} + cx\sqrt{a+cx} + 2a \log(x)\sqrt{a+cx} \right)$$

$$(c-b)^3\sqrt{a+cx}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2, x]

[Out] ((2*Sqrt[a*(b - c)]*(b + c)*(a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]]/(Sqrt[c]*Sqrt[(b*(a + c*x))/(a*(b - c))]) - (b - c)*(-2*a*Sqrt[a + b*x] - 2*c*x*Sqrt[a + b*x] + b*x*Sqrt[a + c*x] + c*x*Sqrt[a + c*x] + 4*a*Sqrt[a + c*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]] + 2*a*Sqrt[a + c*x]*Log[x]))/((-b + c)^3*Sqrt[a + c*x])

IntegrateAlgebraic [B] time = 1.37, size = 434, normalized size = 3.21

$$\frac{(b+c)(a+cx) - 2\sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}} - \frac{ab}{b-c} + 2(ab\sqrt{\frac{a}{c}} + ac\sqrt{\frac{c}{a}} - 2ab)\log\left(\frac{\sqrt{b(a+cx)} - \frac{ab}{c} + a - \sqrt{\frac{a}{c}}\sqrt{a+cx}}{b(\sqrt{b-c}\sqrt{c})}\right) + 2a\log\left(\frac{-2b(a+cx) + ab - ac(b(a+cx) + \sqrt{a+cx}\sqrt{\frac{b(a+cx)}{c}} - \frac{ab}{c} + a)(2ab\sqrt{\frac{a}{c}} - 2bc\sqrt{\frac{c}{a}}(a+cx))}{(\sqrt{b-c}\sqrt{c})^2}\right) + \frac{4a\sqrt{c}\sqrt{\frac{a}{c}}\tanh^{-1}\left(\frac{\sqrt{b(a+cx)} + \sqrt{\frac{a}{c}}\sqrt{\frac{b(a+cx)}{c}}}{a\sqrt{b-c}}\right) + \frac{\sqrt{c}\sqrt{\frac{b(a+cx)}{c}}}{a\sqrt{b-c}}}{\sqrt{b-c}\sqrt{c}(\sqrt{b-c}\sqrt{c})^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^2, x]

[Out] ((b + c)*(a + c*x))/((b - c)^2*c) - (2*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(b - c)^2 + (4*a*Sqrt[b/c]*Sqrt[c]*ArcTanh[Sqrt[b]/Sqrt[c] - (Sqrt[b]*(a + c*x))/(a*Sqrt[c]) + (Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(a*Sqrt[b]))/(Sqrt[b]*(Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) + (2*(-2*a*b + a*b*Sqrt[b/c] + a*Sqrt[b/c]*c)*Log[-(Sqrt[b/c]*Sqrt[a + c*x]) + Sqrt[a - (a*b)/c + (b*(a + c*x))/c]])/(b*(Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) + (2*a*Log[(a*b - a*c - 2*b*(a + c*x))*(a*b - b*(a + c*x)) + Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(2*a*b*Sqrt[b/c]*c - 2*b*Sqrt[b/c]*c*(a + c*x)))/((Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2)

fricas [A] time = 0.45, size = 346, normalized size = 2.56

$$\frac{2abc\log(x) - 2abc\log\left(\frac{b(a+cx) + \sqrt{a+cx}\sqrt{b(a+cx)}}{b^2 - 2b^2c + bc^2}\right) - 2\sqrt{a+cx}\sqrt{b(a+cx)} + (ab+ac)\sqrt{bc}\log\left(\frac{ab^2 + 2abc + ac^2 + 2(2bc - \sqrt{bc}(b+c))\sqrt{a+cx} + 2(\sqrt{c} + bc^2) - 2(2bc + ab + ac)\sqrt{bc}}{b^2 - 2b^2c + bc^2}\right) + (\sqrt{c} + bc^2) + 2abc\log(x) - 2abc\log\left(\frac{b(a+cx) + \sqrt{a+cx}\sqrt{b(a+cx)}}{b^2 - 2b^2c + bc^2}\right) - 2\sqrt{a+cx}\sqrt{b(a+cx)} + 2(ab+ac)\sqrt{bc}\arctan\left(\frac{\sqrt{a+cx}\sqrt{b(a+cx)}}{a\sqrt{b-c}}\right) + (\sqrt{c} + bc^2)c}{b^2 - 2b^2c + bc^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2, x, algorithm="fricas")

[Out] [(2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + (a*b + a*c)*sqrt(b*c)*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c))*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b^2*c + b*c^2)*x]/(b^3*c - 2*b^2*c^2 + b*c^3), (2*a*b*c*log(x) - 2*a*b*c*log(-((b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) - 2*sqrt(b*x + a)*sqrt(c*x + a)*b*c + 2*(a*b + a*c)*sqrt(-b*c)*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b^2*c + b*c^2)*x]/(b^3*c - 2*b^2*c^2 + b*c^3)]

giac [B] time = 3.53, size = 306, normalized size = 2.27

$$\frac{\sqrt{bc}a(b+c)|b|\log\left(\frac{\sqrt{bc}\sqrt{bx+a} - \sqrt{ab^2+(bx+a)bc-abc}}{b^3c-2b^2c^2+bc^3}\right) + \frac{4\sqrt{bc}a|b|\arctan\left(\frac{ab^2+abc-\left(\sqrt{bc}\sqrt{bx+a} - \sqrt{ab^2+(bx+a)bc-abc}\right)^2}{2\sqrt{bc}ab}\right)}{(b^2-2bc+c^2)\sqrt{-bc}} + \frac{2ab\log(|bx|)}{b^2-2bc+c^2} - \frac{2\sqrt{ab^2+(bx+a)bc-abc}(b^2|b|-2bc|b|+c^2|b|)\sqrt{bx+a}}{b^5-4b^4c+6b^3c^2-4b^2c^3+bc^4} + \frac{(bx+a)b+(bx+a)c}{b^2-2bc+c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2, x, algorithm="giac")

[Out] (sqrt(b*c)*a*(b + c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))/((b^3*c - 2*b^2*c^2 + b*c^3) - 4*sqrt(b*c)*a*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b)))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)) + 2*a*b*log(abs(b*x))/((b^2 - 2*b*c + c^2) - 2*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(b^2*abs(b) - 2*b*c*abs(b) + c^2*abs(b))*sqrt(b*x + a)/(b^5 - 4*b^4*c + 6*b^3*c^2 - 4*b^2*c^3 + b*c^4) + ((b*x + a)*b + (b*x + a)*c)/(b^2 - 2*b*c + c^2))/b

maple [C] time = 0.02, size = 266, normalized size = 1.97

$$\frac{2a \ln(x)}{(b-c)^2} + \frac{bx}{(b-c)^2} + \frac{cx}{(b-c)^2} - \frac{\sqrt{bx+a} \sqrt{cx+a} \left(ab \operatorname{csgn}(a) \ln \left(\frac{2bx+ab+ac+2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}}{2\sqrt{bc}} \right) + ac \operatorname{csgn}(a) \ln \left(\frac{2cx+ab+ac+2\sqrt{bcx^2+abx+acx+a^2}\sqrt{bc}}{2\sqrt{bc}} \right) - 2\sqrt{bc} a \ln \left(\frac{(bx+cx+2a+2\sqrt{bcx^2+abx+acx+a^2}) \operatorname{csgn}(a)}{x} \right) + 2\sqrt{bcx^2+abx+acx+a^2} \sqrt{bc} \operatorname{csgn}(a) \right) \operatorname{csgn}(a)}{(b-c)^2 \sqrt{bcx^2+abx+acx+a^2} \sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)

[Out] x/(b-c)^2*b+x/(b-c)^2*c+2*a*ln(x)/(b-c)^2-1/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(1/2)*(csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)))/(b*c)^(1/2))*a*b+csgn(a)*ln(1/2*(2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*(b*c)^(1/2)))/(b*c)^(1/2))*a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*(b*c)^(1/2)-2*ln(a*(2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)+b*x+c*x+2*a)/x)*(b*c)^(1/2)*a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/(b*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^2, x)

mupad [B] time = 19.76, size = 5098, normalized size = 37.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)

[Out] (2*a*log(x))/(b^2 - 2*b*c + c^2) - (((4*a*c^2 + 4*a*b*c)*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + ((4*a*b^2 + 4*a*b*c)*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (16*a*b*c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)/(b^4 - 2*b^3*c + b^2*c^2 - ((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^3 + 2*b^3*c - 4*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^4*(c^4 - 2*b*c^3 + b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^4 - (2*a*log(((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(b^2 - 2*b*c + c^2) + (2*a*log(((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)))/(b - c)^2 + (x*(b + c))/(b - c)^2 + (a*atan(((a*(b*c)^(1/2)*(b + c))*((2*((a + b*x)^(1/2) - a^(1/2))*(32*a^3*b^2*c^10 - 64*a^3*b^3*c^9 + 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4))/((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7*c^5)))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^(1/2)*(b + c))*((4*(4*a^2*b^3*c^11 + 2*a^2*b^4*c^10 - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^10*c^4))/((b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(16*a^2*b^2*c^12 - 32*a^2*b^3*c^11 + 36*a^2*b^4*c^10 - 64*a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^10*c^4)))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^(1/2)*(b + c))*((4*(a*b^4*c^12 + 7*a*b^5*c^11 - 27*a*b^6*c^10 + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^10*c^6 + a*b^11*c^5)))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^(1/2) - a^(1/2))*(8*a*b^3*c^13 - 54*a*b^4*c^12 + 212*a*b^5*c^11 - 490*a*b^6*c^10 + 648*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^10*c^6 + 8*a*b^11*c^5))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c - 4*b

$$\begin{aligned}
& c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^{(1/2)}*(b + c)*((4*(4*b^5*c^{13} - b^4*c^{14} - \\
& 5*b^6*c^{12} + b^7*c^{11} + b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4 \\
& *b^{12}*c^6 - b^{13}*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a \\
& + b*x)^{(1/2)} - a^{(1/2)})*(4*b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - 300*b^6 \\
& *c^{12} + 516*b^7*c^{11} - 618*b^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11} \\
& *c^7 - 31*b^{12}*c^6 + 4*b^{13}*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3 \\
& *c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))/(b*c^3 + b \\
& ^3*c - 2*b^2*c^2))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2 \\
& *c^2) - (a*(b*c)^{(1/2)}*(b + c)*((4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3 \\
& *b^6*c^6 + 4*a^3*b^7*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2 \\
& *((a + b*x)^{(1/2)} - a^{(1/2)})*(32*a^3*b^2*c^{10} - 64*a^3*b^3*c^9 + 8*a^3*b^4* \\
& c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3*b^8*c^4)))/ \\
& ((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + \\
& (2*a*(b*c)^{(1/2)}*(b + c)*((4*(4*a^2*b^3*c^{11} + 2*a^2*b^4*c^{10} - 18*a^2*b^5*c^9 + \\
& 12*a^2*b^6*c^8 + 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4* \\
& a^2*b^{10}*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^ \\
& (1/2) - a^{(1/2)})*(16*a^2*b^2*c^{12} - 32*a^2*b^3*c^{11} + 36*a^2*b^4*c^{10} - 64* \\
& a^2*b^5*c^9 + 88*a^2*b^6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9 \\
& *c^5 + 16*a^2*b^{10}*c^4))/((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b* \\
& c^3 + c^4 + 6*b^2*c^2)) + (2*a*(b*c)^{(1/2)}*(b + c)*((2*((a + b*x)^{(1/2)} - a \\
& ^{(1/2)})*(8*a*b^3*c^{13} - 54*a*b^4*c^{12} + 212*a*b^5*c^{11} - 490*a*b^6*c^{10} + 6 \\
& 48*a*b^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^{10}*c^6 + 8*a*b^{11}*c^5 \\
&)))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) \\
&) - (4*(a*b^4*c^{12} + 7*a*b^5*c^{11} - 27*a*b^6*c^{10} + 19*a*b^7*c^9 + 19*a*b^8 \\
& *c^8 - 27*a*b^9*c^7 + 7*a*b^{10}*c^6 + a*b^{11}*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 \\
& + c^4 + 6*b^2*c^2) + (2*a*(b*c)^{(1/2)}*(b + c)*((4*(4*b^5*c^{13} - b^4*c^{14} - \\
& 5*b^6*c^{12} + b^7*c^{11} + b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4*b^{12} \\
& *c^6 - b^{13}*c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x) \\
& ^{(1/2)} - a^{(1/2)})*(4*b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - 300*b^6*c^{12} \\
& + 516*b^7*c^{11} - 618*b^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11}*c^7 \\
& - 31*b^{12}*c^6 + 4*b^{13}*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - \\
& 4*b*c^3 + c^4 + 6*b^2*c^2)))/(b*c^3 + b^3*c - 2*b^2*c^2))/(b*c^3 + b^3*c \\
& - 2*b^2*c^2))/(b*c^3 + b^3*c - 2*b^2*c^2))*2i)/(b*c^3 + b^3*c - 2*b^2*c^2) \\
&)/((4*((a + b*x)^{(1/2)} - a^{(1/2)})*(128*a^4*b^3*c^7 + 256*a^4*b^4*c^6 + 128* \\
& a^4*b^5*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + \\
& 6*b^2*c^2)) - (8*(16*a^4*b^3*c^7 + 56*a^4*b^4*c^6 + 56*a^4*b^5*c^5 + 16*a^ \\
& 4*b^6*c^4))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^{(1/2)}* \\
& (b + c)*((2*((a + b*x)^{(1/2)} - a^{(1/2)})*(32*a^3*b^2*c^{10} - 64*a^3*b^3*c^9 + \\
& 8*a^3*b^4*c^8 + 240*a^3*b^5*c^7 + 8*a^3*b^6*c^6 - 64*a^3*b^7*c^5 + 32*a^3* \\
& b^8*c^4))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b \\
& ^2*c^2)) - (4*(4*a^3*b^4*c^8 + 44*a^3*b^5*c^7 + 44*a^3*b^6*c^6 + 4*a^3*b^7* \\
& c^5))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*a*(b*c)^{(1/2)}*(b + c \\
&)*((4*(4*a^2*b^3*c^{11} + 2*a^2*b^4*c^{10} - 18*a^2*b^5*c^9 + 12*a^2*b^6*c^8 + \\
& 12*a^2*b^7*c^7 - 18*a^2*b^8*c^6 + 2*a^2*b^9*c^5 + 4*a^2*b^{10}*c^4))/(b^4 - 4 \\
& *b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(16*a^ \\
& 2*b^2*c^{12} - 32*a^2*b^3*c^{11} + 36*a^2*b^4*c^{10} - 64*a^2*b^5*c^9 + 88*a^2*b^ \\
& 6*c^8 - 64*a^2*b^7*c^7 + 36*a^2*b^8*c^6 - 32*a^2*b^9*c^5 + 16*a^2*b^{10}*c^4) \\
&)))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) \\
& + (2*a*(b*c)^{(1/2)}*(b + c)*((4*(a*b^4*c^{12} + 7*a*b^5*c^{11} - 27*a*b^6*c^{10} \\
& + 19*a*b^7*c^9 + 19*a*b^8*c^8 - 27*a*b^9*c^7 + 7*a*b^{10}*c^6 + a*b^{11}*c^5))/ \\
& (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) - (2*((a + b*x)^{(1/2)} - a^{(1/2)} \\
&)*(8*a*b^3*c^{13} - 54*a*b^4*c^{12} + 212*a*b^5*c^{11} - 490*a*b^6*c^{10} + 648*a*b \\
& ^7*c^9 - 490*a*b^8*c^8 + 212*a*b^9*c^7 - 54*a*b^{10}*c^6 + 8*a*b^{11}*c^5))/(((\\
& a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) + (2 \\
& *a*(b*c)^{(1/2)}*(b + c)*((4*(4*b^5*c^{13} - b^4*c^{14} - 5*b^6*c^{12} + b^7*c^{11} + \\
& b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4*b^{12}*c^6 - b^{13}*c^5))/(b^4 \\
& - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(4* \\
& b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - 300*b^6*c^{12} + 516*b^7*c^{11} - 618*b \\
& ^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11}*c^7 - 31*b^{12}*c^6 + 4*b^{13}
\end{aligned}$$

$$\frac{c^5)}{(((a + cx)^{(1/2)} - a^{(1/2)}) \cdot (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2))} / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) + (2a \cdot (bc)^{(1/2)} \cdot (b + c) \cdot ((4 \cdot (4a^3b^4c^8 + 44a^3b^5c^7 + 44a^3b^6c^6 + 4a^3b^7c^5)) / (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2) - (2 \cdot ((a + bx)^{(1/2)} - a^{(1/2)}) \cdot (32a^3b^2c^{10} - 64a^3b^3c^9 + 8a^3b^4c^8 + 240a^3b^5c^7 + 8a^3b^6c^6 - 64a^3b^7c^5 + 32a^3b^8c^4)) / (((a + cx)^{(1/2)} - a^{(1/2)}) \cdot (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2)) + (2a \cdot (bc)^{(1/2)} \cdot (b + c) \cdot ((4 \cdot (4a^2b^3c^{11} + 2a^2b^4c^{10} - 18a^2b^5c^9 + 12a^2b^6c^8 + 12a^2b^7c^7 - 18a^2b^8c^6 + 2a^2b^9c^5 + 4a^2b^{10}c^4)) / (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2) - (2 \cdot ((a + bx)^{(1/2)} - a^{(1/2)}) \cdot (16a^2b^2c^{12} - 32a^2b^3c^{11} + 36a^2b^4c^{10} - 64a^2b^5c^9 + 88a^2b^6c^8 - 64a^2b^7c^7 + 36a^2b^8c^6 - 32a^2b^9c^5 + 16a^2b^{10}c^4)) / (((a + cx)^{(1/2)} - a^{(1/2)}) \cdot (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2)) + (2a \cdot (bc)^{(1/2)} \cdot (b + c) \cdot ((2 \cdot ((a + bx)^{(1/2)} - a^{(1/2)}) \cdot (8a^3b^3c^{13} - 54a^3b^4c^{12} + 212a^3b^5c^{11} - 490a^3b^6c^{10} + 648a^3b^7c^9 - 490a^3b^8c^8 + 212a^3b^9c^7 - 54a^3b^{10}c^6 + 8a^3b^{11}c^5)) / (((a + cx)^{(1/2)} - a^{(1/2)}) \cdot (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2)) - (4 \cdot (a^3b^4c^{12} + 7a^3b^5c^{11} - 27a^3b^6c^{10} + 19a^3b^7c^9 + 19a^3b^8c^8 - 27a^3b^9c^7 + 7a^3b^{10}c^6 + a^3b^{11}c^5)) / (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2) + (2a \cdot (bc)^{(1/2)} \cdot (b + c) \cdot ((4 \cdot (4b^5c^{13} - b^4c^{14} - 5b^6c^{12} + b^7c^{11} + b^8c^{10} + b^9c^9 + b^{10}c^8 - 5b^{11}c^7 + 4b^{12}c^6 - b^{13}c^5)) / (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2) + (2 \cdot ((a + bx)^{(1/2)} - a^{(1/2)}) \cdot (4b^3c^{15} - 31b^4c^{14} + 120b^5c^{13} - 300b^6c^{12} + 516b^7c^{11} - 618b^8c^{10} + 516b^9c^9 - 300b^{10}c^8 + 120b^{11}c^7 - 31b^{12}c^6 + 4b^{13}c^5)) / (((a + cx)^{(1/2)} - a^{(1/2)}) \cdot (b^4 - 4b^3c - 4b^2c^2 + c^4 + 6b^2c^2))) / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) / (b^3c^3 + b^3c - 2b^2c^2)) \cdot (bc)^{(1/2)} \cdot (b + c) \cdot 4i) / (b^3c^3 + b^3c - 2b^2c^2)$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**2, x)

$$3.212 \quad \int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=138

$$-\frac{2a}{x(b-c)^2} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2} + \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2} + \frac{(b+c)\log(x)}{(b-c)^2}$$

Rubi [A] time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {6690, 97, 157, 63, 217, 206, 93, 208}

$$-\frac{2a}{x(b-c)^2} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x(b-c)^2} + \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{b}\sqrt{a+cx}}\right)}{(b-c)^2} + \frac{(b+c)\log(x)}{(b-c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2), x]
```

```
[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + b*x]*Sqrt[a + c*x])/((b - c)^2*x) + (2*(b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(b - c)^2 - (4*Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x])/(Sqrt[b]*Sqrt[a + c*x])])/(b - c)^2 + ((b + c)*Log[x])/(b - c)^2
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 93

```
Int((((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 97

```
Int(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p)/(b*(m + 1)), x] - Dist[1/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p - 1)*Simp[d*e*n + c*f*p + d*f*(n + p)*x, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[m, -1] && GtQ[n, 0] && GtQ[p, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 157

```
Int((((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_))) / ((a_.) + (b_.)*(x_)), x_Symbol] := Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx = \frac{\int \left(\frac{2a}{x^2} + \frac{b(1+\frac{c}{b})}{x} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^2} \right) dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{(b+c)\log(x)}{(b-c)^2} - \frac{2\int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^2} dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{(b+c)\log(x)}{(b-c)^2} - \frac{2\int \frac{\frac{1}{2}a(b+c)+bcx}{x\sqrt{a+bx}\sqrt{a+cx}} dx}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{(b+c)\log(x)}{(b-c)^2} - \frac{(2bc)\int \frac{1}{\sqrt{a+bx}\sqrt{a+cx}} dx}{(b-c)^2} - \frac{a(b+c)}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{(b+c)\log(x)}{(b-c)^2} - \frac{(4c)\text{Subst}\left[\int \frac{1}{\sqrt{a-\frac{ac}{b}+\frac{cx^2}{b}}} dx, \frac{a+bx}{b}\right]}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} + \frac{(b+c)\log(x)}{(b-c)^2} - \frac{a(b+c)}{(b-c)^2}$$

$$= -\frac{2a}{(b-c)^2x} + \frac{2\sqrt{a+bx}\sqrt{a+cx}}{(b-c)^2x} + \frac{2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2} - \frac{4\sqrt{b}\sqrt{c}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2}$$

Mathematica [A] time = 0.71, size = 178, normalized size = 1.29

$$\frac{2c\sqrt{a+bx} + \frac{2a(\sqrt{a+bx}-\sqrt{a+cx})}{x} + (b+c)\log(x)\sqrt{a+cx} - \frac{4b\sqrt{c}(a+cx)\sinh^{-1}\left(\frac{\sqrt{c}\sqrt{a+bx}}{\sqrt{a(b-c)}}\right)}{\sqrt{a(b-c)}\sqrt{\frac{b(a+cx)}{a(b-c)}}} + 2(b+c)\sqrt{a+cx}\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{(b-c)^2\sqrt{a+cx}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2), x]

[Out] (2*c*Sqrt[a + b*x] + (2*a*(Sqrt[a + b*x] - Sqrt[a + c*x]))/x - (4*b*Sqrt[c] * (a + c*x)*ArcSinh[(Sqrt[c]*Sqrt[a + b*x])/Sqrt[a*(b - c)]])/(Sqrt[a*(b - c)]) * Sqrt[(b*(a + c*x))/(a*(b - c))] + 2*(b + c)*Sqrt[a + c*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]] + (b + c)*Sqrt[a + c*x]*Log[x])/((b - c)^2*Sqrt[a + c*x])

IntegrateAlgebraic [B] time = 1.89, size = 378, normalized size = 2.74

$$\frac{2\sqrt{\frac{b}{c}}(b\sqrt{c} + c^2)\tanh^{-1}\left(\frac{-b(a+c)+\sqrt{\frac{b}{c}}\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}+\frac{ab}{c}+ab}}{a\sqrt{b}\sqrt{c}}\right) + \frac{(b+c)\log\left(-2bc^2x\sqrt{\frac{b}{c}}\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}-\frac{ab}{c}+a}-bcx(-2b(a+c)+ab-ac)\right)}{(\sqrt{b}-\sqrt{c})^2(\sqrt{b}+\sqrt{c})^2} - \frac{2a}{x(b-c)^2} + \frac{2\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}-\frac{ab}{c}+a}}{x(b-c)^2} - \frac{2(-2c\sqrt{\frac{b}{c}}+b+c)\log\left(\sqrt{\frac{b(a+c)}{c}-\frac{ab}{c}+a}-\sqrt{\frac{b}{c}}\sqrt{a+c}\right)}{(\sqrt{b}-\sqrt{c})^2(\sqrt{b}+\sqrt{c})^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-2), x]

[Out] (-2*a)/((b - c)^2*x) + (2*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/((b - c)^2*x) + (2*Sqrt[b/c]*(b*Sqrt[c] + c^(3/2))*ArcTanh[(a*b - b*(a + c*x) + Sqrt[b/c]*c*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(a*Sqrt[b]*Sqrt[c]))/(Sqrt[b]*(Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) - (2*(b + c - 2*Sqrt[b/c]*c)*Log[-(Sqrt[b/c]*Sqrt[a + c*x] + Sqrt[a - (a*b)/c + (b*(a + c*x))/c])]/((Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2) + ((b + c)*Log[-(b*c*x*(a*b - a*c - 2*b*(a + c*x)) - 2*b*Sqrt[b/c]*c^2*x*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])]/((Sqrt[b] - Sqrt[c])^2*(Sqrt[b] + Sqrt[c])^2)

fricas [A] time = 0.44, size = 317, normalized size = 2.30

$$\frac{2(b+c)x\log(-2(b+c)x\log\left(\frac{(b+c)\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}+\frac{ab}{c}+ab}}{a\sqrt{b}\sqrt{c}}\right) + 4\sqrt{bc}x\log(a^2+2abc+a^2+2(2bc-\sqrt{bc}(b+c))\sqrt{bx+a}\sqrt{cx+a}+2(b^2+bc^2)-2(2bca+ab+ac)\sqrt{bc}) + (b+c)x + 4\sqrt{bx+a}\sqrt{cx+a} - 4a - 2(b+c)\log(-2(b+c)x\log\left(\frac{(b+c)\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}+\frac{ab}{c}+ab}}{a\sqrt{b}\sqrt{c}}\right) + b\sqrt{bc}x\arctan\left(\frac{\sqrt{bc}\sqrt{a+c}\sqrt{\frac{b(a+c)}{c}-\frac{ab}{c}+a}}{bx}\right) + (b+c)x + 4\sqrt{bx+a}\sqrt{cx+a} - 4a)}{2(b^2-2bc+c^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")

[Out] [1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-(b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 4*sqrt(b*c)*x*log(a*b^2 + 2*a*b*c + a*c^2 + 2*(2*b*c - sqrt(b*c)*(b + c))*sqrt(b*x + a)*sqrt(c*x + a) + 2*(b^2*c + b*c^2)*x - 2*(2*b*c*x + a*b + a*c)*sqrt(b*c)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x), 1/2*(2*(b + c)*x*log(x) - 2*(b + c)*x*log(-(b + c)*x - 2*sqrt(b*x + a)*sqrt(c*x + a) + 2*a)/x) + 8*sqrt(-b*c)*x*arctan((sqrt(-b*c)*sqrt(b*x + a)*sqrt(c*x + a) - sqrt(-b*c)*a)/(b*c*x)) + (b + c)*x + 4*sqrt(b*x + a)*sqrt(c*x + a) - 4*a)/((b^2 - 2*b*c + c^2)*x)]

giac [B] time = 4.50, size = 438, normalized size = 3.17

$$\frac{2\sqrt{bc}b\log\left(\frac{\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}}{b^2-2b^2+c^2}\right) - \frac{2\sqrt{bc}(b+c)\operatorname{arctan}\left(\frac{a^2+abc-\sqrt{bc}\sqrt{bx+a}\sqrt{ab^2+(bx+a)bc-abc}}{2\sqrt{bc}ab}\right)}{(b^2-2bc+c^2)\sqrt{bc}b} + \frac{(b+c)\log(bx)}{b^2-2bc+c^2} + \frac{4\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2ab+c\|b-(b^2-2b^2+c^2)\sqrt{bc}a^2\|}{\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2-2(b^2+bc)\left(\sqrt{bc}\sqrt{bx+a}-\sqrt{ab^2+(bx+a)bc-abc}\right)^2a+(b^2-2b^2+c^2)ab^2} + \frac{(bx+a)b+ab+(bx+a)c-ac}{(b^2-2bc+c^2)bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")

[Out] 2*sqrt(b*c)*abs(b)*log((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(b^3 - 2*b^2*c + b*c^2) - 2*sqrt(b*c)*(b + c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((b^2 - 2*b*c + c^2)*sqrt(-b*c)*b) + (b + c)*log(abs(b*x))/(b^2 - 2*b*c + c^2) - 4*(sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a*(b + c)*abs(b) - (b^3 - 2*b^2*c + b*c^2)*sqrt(b*c)*a^2*abs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4 - 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)*(b^2 - 2*b*c + c^2)) - ((b*x + a)*b + a*b + (b*x + a)*c - a*c)/((b^2 - 2*b*c + c^2)*b*x)

maple [C] time = 0.02, size = 272, normalized size = 1.97

$$\frac{\frac{b \ln(x)}{(b-c)^2} + \frac{c \ln(x)}{(b-c)^2} - \frac{2a}{(b-c)^2 x} - \frac{\sqrt{bx+a} \sqrt{cx+a} \left(2bcx \operatorname{csign}(a) \ln \left(\frac{2bx+ab+ac+2\sqrt{bcx^2+abx+acx+a^2} \sqrt{bc}}{2\sqrt{bc}} \right) - \sqrt{bc} bx \ln \left(\frac{(bx+cx+2a+2\sqrt{bcx^2+abx+acx+a^2} \operatorname{csign}(a))}{x} \right) - \sqrt{bc} cx \ln \left(\frac{(bx+cx+2a+2\sqrt{bcx^2+abx+acx+a^2} \operatorname{csign}(a))}{x} \right) - 2\sqrt{bcx^2+abx+acx+a^2} \sqrt{bc} \operatorname{csign}(a) \right)}{(b-c)^2 \sqrt{bcx^2+abx+acx+a^2} \sqrt{bc} x}}{\operatorname{csign}(a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)`

[Out] $\frac{1}{(b-c)^2 b \ln(x) + 1/(b-c)^2 c \ln(x) - 2a/(b-c)^2/x - 1/(b-c)^2 (b*x+a)^{(1/2)} * (c*x+a)^{(1/2)} * (2*\operatorname{csign}(a) * \ln(1/2 * (2*b*c*x+a*b+a*c+2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)} * (b*c)^{(1/2)}) / (b*c)^{(1/2)}) * x*b*c - \ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)} * \operatorname{csign}(a)) * a/x) * x*b*(b*c)^{(1/2)} - \ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)} * \operatorname{csign}(a)) * a/x) * x*c*(b*c)^{(1/2)} - 2*(b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)} * (b*c)^{(1/2)} * \operatorname{csign}(a) * \operatorname{csign}(a) / (b*c*x^2+a*b*x+a*c*x+a^2)^{(1/2)} / x / (b*c)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")`

[Out] `integrate((sqrt(b*x + a) + sqrt(c*x + a))^(−2), x)`

mupad [B] time = 17.44, size = 4285, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^2,x)`

[Out]
$$\operatorname{atan}\left(\frac{((b*c)^{(1/2)} * ((4*(b*c)^{(1/2)} * ((4*(b^4*c^{12} + 16*b^5*c^{11} - 42*b^6*c^{10} + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^{10}*c^6 + b^{11}*c^5))) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)} * ((4*(4*b^5*c^{12} - 36*b^7*c^{10} + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^{11}*c^6))) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)} * ((4*(4*b^5*c^{13} - b^4*c^{14} - 5*b^6*c^{12} + b^7*c^{11} + b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4*b^{12}*c^6 - b^{13}*c^5))) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - a^{(1/2)}) * (4*b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - 300*b^6*c^{12} + 516*b^7*c^{11} - 618*b^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11}*c^7 - 31*b^{12}*c^6 + 4*b^{13}*c^5)) / (((a + c*x)^{(1/2)} - a^{(1/2)}) * (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))} / (b - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)}) * (4*b^3*c^{14} - 27*b^4*c^{13} + 99*b^5*c^{12} - 175*b^6*c^{11} + 99*b^7*c^{10} + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^{10}*c^7 - 27*b^{11}*c^6 + 4*b^{12}*c^5)) / (((a + c*x)^{(1/2)} - a^{(1/2)}) * (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))} / (b - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)}) * (73*b^4*c^{12} - 278*b^5*c^{11} + 503*b^6*c^{10} - 596*b^7*c^9 + 503*b^8*c^8 - 278*b^9*c^7 + 73*b^{10}*c^6)) / (((a + c*x)^{(1/2)} - a^{(1/2)}) * (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2))} / (b - c)^2 - (4*(4*b^5*c^{10} + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6)) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - a^{(1/2)}) * (65*b^4*c^{11} - 167*b^5*c^{10} + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6)) / (((a + c*x)^{(1/2)} - a^{(1/2)}) * (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)) * 4i) / (b - c)^2 - ((b*c)^{(1/2)} * ((4*(4*b^5*c^{10} + 24*b^6*c^9 + 40*b^7*c^8 + 24*b^8*c^7 + 4*b^9*c^6)) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)} * ((4*(b^4*c^{12} + 16*b^5*c^{11} - 42*b^6*c^{10} + 25*b^7*c^9 + 25*b^8*c^8 - 42*b^9*c^7 + 16*b^{10}*c^6 + b^{11}*c^5)) / (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)} * ((4*(b*c)^{(1/2)} * ((4*(4*b^5*c^{13} - b^4*c^{14} - 5*b^6*c^{12} + b^7*c^{11} + b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4*b^{12}*c^6 - b^{13}...$$

$$\begin{aligned}
& *c^5)/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - \\
& a^{(1/2)})*(4*b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - 300*b^6*c^{12} + 516*b^7* \\
& c^{11} - 618*b^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11}*c^7 - 31*b^{12}*c \\
& ^6 + 4*b^{13}*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c \\
& ^4 + 6*b^2*c^2)))/((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/ \\
& (b - c)^2 - (4*(4*b^5*c^{12} - 36*b^7*c^{10} + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^{11}*c^6))/ \\
& (b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(4*b^3*c^{14} - \\
& 27*b^4*c^{13} + 99*b^5*c^{12} - 175*b^6*c^{11} + 99*b^7*c^{10} + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^{10}*c^7 - \\
& 27*b^{11}*c^6 + 4*b^{12}*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/ \\
& (b - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(73*b^4*c^{12} - 278*b^5*c^{11} + 503*b^6*c^{10} - 596*b^7*c^9 + \\
& 503*b^8*c^8 - 278*b^9*c^7 + 73*b^{10}*c^6))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6* \\
& *b^2*c^2)))/((a + b*x)^{(1/2)} - a^{(1/2)})*(65*b^4*c^{11} - 167*b^5*c^{10} + 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + \\
& 65*b^9*c^6))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))*4i)/(b - \\
& c)^2)/((4*(b*c)^{(1/2)}*((4*(b*c)^{(1/2)}*((4*(b^4*c^{12} + 16*b^5*c^{11} - 42*b^6*c^{10} + 25*b^7*c^9 + 25*b^8*c^8 - \\
& 42*b^9*c^7 + 16*b^{10}*c^6 + b^{11}*c^5)))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)}*((4*(4*b^5*c^{12} - \\
& 36*b^7*c^{10} + 64*b^8*c^9 - 36*b^9*c^8 + 4*b^{11}*c^6)))/(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (4*(b*c)^{(1/2)}*((4*(4*b^5*c^{13} - \\
& b^4*c^{14} - 5*b^6*c^{12} + b^7*c^{11} + b^8*c^{10} + b^9*c^9 + b^{10}*c^8 - 5*b^{11}*c^7 + 4*b^{12}*c^6 - b^{13}*c^5)))/(b^4 - \\
& 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2) + (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(4*b^3*c^{15} - 31*b^4*c^{14} + 120*b^5*c^{13} - \\
& 300*b^6*c^{12} + 516*b^7*c^{11} - 618*b^8*c^{10} + 516*b^9*c^9 - 300*b^{10}*c^8 + 120*b^{11}*c^7 - 31*b^{12}*c^6 + 4*b^{13}*c^5))/ \\
& (((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - \\
& 4*b*c^3 + c^4 + 6*b^2*c^2)))/((a + b*x)^{(1/2)} - a^{(1/2)})*(4*b^3*c^{14} - 27*b^4*c^{13} + 99*b^5*c^{12} - 175*b^6*c^{11} + \\
& 99*b^7*c^{10} + 99*b^8*c^9 - 175*b^9*c^8 + 99*b^{10}*c^7 - 27*b^{11}*c^6 + 4*b^{12}*c^5))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - \\
& 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/ \\
& (b - c)^2 - (2*((a + b*x)^{(1/2)} - a^{(1/2)})*(73*b^4*c^{12} - 278*b^5*c^{11} + 503*b^6*c^{10} - 596*b^7*c^9 + 503*b^8*c^8 - \\
& 278*b^9*c^7 + 73*b^{10}*c^6))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/((a + \\
& c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + 6*b^2*c^2)))/((a + b*x)^{(1/2)} - a^{(1/2)})*(65*b^4*c^{11} - 167*b^5*c^{10} + \\
& 198*b^6*c^9 + 198*b^7*c^8 - 167*b^8*c^7 + 65*b^9*c^6))/(((a + c*x)^{(1/2)} - a^{(1/2)})*(b^4 - 4*b^3*c - 4*b*c^3 + c^4 + \\
& 6*b^2*c^2)))/((a + b*x)^{(1/2)} - a^{(1/2)})*(112*b^5*c^9
\end{aligned}$$

```

+ 224*b^6*c^8 + 112*b^7*c^7))/(((a + c*x)^(1/2) - a^(1/2))*(b^4 - 4*b^3*c
- 4*b*c^3 + c^4 + 6*b^2*c^2))))*(b*c)^(1/2)*8i)/(b - c)^2 - (((b*c + b^2)*(
(a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - b^2 + (((a + b*x)
^(1/2) - a^(1/2))^2*(b^2 - 3*b*c + c^2))/((a + c*x)^(1/2) - a^(1/2))^2)/(((
(a + b*x)^(1/2) - a^(1/2))^3*(2*b^2*c - 4*b*c^2 + 2*c^3))/((a + c*x)^(1/2)
- a^(1/2))^3 + (((a + b*x)^(1/2) - a^(1/2))^2*(2*b*c^2 + 2*b^2*c - 2*b^3 -
2*c^3))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))*(2*b*c
^2 - 4*b^2*c + 2*b^3))/((a + c*x)^(1/2) - a^(1/2))) + (log(((a + b*x)^(1/2)
- a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)))*(b + c))/(b - c)^2 + (log(x)*(b +
c))/(b^2 - 2*b*c + c^2) - (log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c
*((a + b*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))))/((a + c*x)^(1/2
) - a^(1/2))*(b + c))/(b^2 - 2*b*c + c^2) - (2*a)/(x*(b^2 - 2*b*c + c^2))
+ (c*((a + b*x)^(1/2) - a^(1/2)))/(2*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)

[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-2), x)

$$3.213 \quad \int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=123

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2} - \frac{a}{x^2(b-c)^2} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a} - \frac{b+c}{x(b-c)^2}$$

Rubi [A] time = 0.20, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6690, 94, 93, 208}

$$\frac{\sqrt{a+bx}(a+cx)^{3/2}}{ax^2(b-c)^2} - \frac{a}{x^2(b-c)^2} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2ax(b-c)} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{2a} - \frac{b+c}{x(b-c)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2), x]

[Out] -(a/((b - c)^2*x^2)) - (b + c)/((b - c)^2*x) + (Sqrt[a + b*x]*Sqrt[a + c*x])/(2*a*(b - c)*x) + (Sqrt[a + b*x]*(a + c*x)^(3/2))/(a*(b - c)^2*x^2) - ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]]/(2*a)

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})^2} dx &= \frac{\int \left(\frac{2a}{x^3} + \frac{b(1+\frac{c}{b})}{x^2} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^3} \right) dx}{(b-c)^2} \\
&= -\frac{a}{(b-c)^2 x^2} - \frac{b+c}{(b-c)^2 x} - \frac{2 \int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3} dx}{(b-c)^2} \\
&= -\frac{a}{(b-c)^2 x^2} - \frac{b+c}{(b-c)^2 x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2 x^2} - \frac{\int \frac{\sqrt{a+cx}}{x^2 \sqrt{a+bx}} dx}{2(b-c)} \\
&= -\frac{a}{(b-c)^2 x^2} - \frac{b+c}{(b-c)^2 x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2 x^2} + \frac{1}{4} \int \frac{1}{x\sqrt{a+bx}} dx \\
&= -\frac{a}{(b-c)^2 x^2} - \frac{b+c}{(b-c)^2 x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2 x^2} + \frac{1}{2} \operatorname{Subst} \left(\frac{1}{\sqrt{a+bx}}, \sqrt{a+bx} \right) \\
&= -\frac{a}{(b-c)^2 x^2} - \frac{b+c}{(b-c)^2 x} + \frac{\sqrt{a+bx}\sqrt{a+cx}}{2a(b-c)x} + \frac{\sqrt{a+bx}(a+cx)^{3/2}}{a(b-c)^2 x^2} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right)}{2a}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 109, normalized size = 0.89

$$\frac{-2a^2 - x^2(b-c)^2 \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}} \right) + 2a \left(\sqrt{a+bx}\sqrt{a+cx} - bx - cx \right) + x(b+c)\sqrt{a+bx}\sqrt{a+cx}}{2ax^2(b-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2), x]

[Out] (-2*a^2 + (b + c)*x*Sqrt[a + b*x]*Sqrt[a + c*x] + 2*a*(-(b*x) - c*x + Sqrt[a + b*x]*Sqrt[a + c*x]) - (b - c)^2*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(2*a*(b - c)^2*x^2)

IntegrateAlgebraic [B] time = 2.96, size = 250, normalized size = 2.03

$$\frac{-bc(a+cx) + abc - (c^2(a+cx))}{c^2 x^2 (b-c)^2} + \frac{\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a} (bc(a+cx)^{3/2} - abc\sqrt{a+cx} + c^2(a+cx)^{3/2} + ac^2\sqrt{a+cx})}{2ac^2 x^2 (b-c)^2} - \frac{\sqrt{c} \sqrt{\frac{b}{c}} \tanh^{-1} \left(\frac{\sqrt{b(a+cx)}}{a\sqrt{c}} + \frac{\sqrt{c} \sqrt{\frac{b}{c}} \sqrt{a+cx} \sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a}}{a\sqrt{b}} + \frac{\sqrt{b}}{\sqrt{c}} \right)}{2a\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(Sqrt[a + b*x] + Sqrt[a + c*x])^2), x]

[Out] (a*b*c - b*c*(a + c*x) - c^2*(a + c*x))/((b - c)^2*c^2*x^2) + (Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(-(a*b*c*Sqrt[a + c*x]) + a*c^2*Sqrt[a + c*x] + b*c*(a + c*x)^(3/2) + c^2*(a + c*x)^(3/2)))/(2*a*(b - c)^2*c^2*x^2) - (Sqrt[b/c]*Sqrt[c]*ArcTanh[Sqrt[b]/Sqrt[c] - (Sqrt[b]*(a + c*x))/(a*Sqrt[c]) + (Sqrt[b/c]*Sqrt[c]*Sqrt[a + c*x]*Sqrt[a - (a*b)/c + (b*(a + c*x))/c])/(a*Sqrt[b])])/(2*a*Sqrt[b])

fricas [A] time = 0.41, size = 126, normalized size = 1.02

$$\frac{4(b^2 - 2bc + c^2)x^2 \log \left(-\frac{(b+c)x - 2\sqrt{bx+a}\sqrt{cx+a} + 2a}{x} \right) + (b^2 + 2bc + c^2)x^2 + 8((b+c)x + 2a)\sqrt{bx+a}\sqrt{cx+a} - 16a^2 - 16(ab+ac)x}{16(ab^2 - 2abc + ac^2)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")

[Out] $1/16*(4*(b^2 - 2*b*c + c^2)*x^2*\log(-((b + c)*x - 2*\sqrt{b*x + a})*\sqrt{c*x + a} + 2*a)/x) + (b^2 + 2*b*c + c^2)*x^2 + 8*((b + c)*x + 2*a)*\sqrt{b*x + a})*\sqrt{c*x + a} - 16*a^2 - 16*(a*b + a*c)*x)/((a*b^2 - 2*a*b*c + a*c^2)*x^2)$

giac [B] time = 12.25, size = 532, normalized size = 4.33

$$\frac{\sqrt{b} \arctan\left(\frac{a^2 - ab - (\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})}{2\sqrt{bc}ab}\right)}{2\sqrt{bc}ab} + \frac{(b^2 + 6bc + c^2)\sqrt{b}(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})}{(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})^2} - \frac{(3b^4 + 5b^3c + 5b^2c^2 + 3bc^3)\sqrt{b}(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})}{(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})^2} + \frac{(3b^4 - 4b^3c - 4b^2c^2 + 3b^2c^3)\sqrt{b}(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})}{(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})^2} - \frac{(b^2 - 3b^2c + 2b^2c^2 - 3b^2c^3 + b^2c^4)\sqrt{b}(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})}{(\sqrt{b} \sqrt{bx+a} - \sqrt{c} \sqrt{cx+a})^2} + \frac{(bx + a)^2 + (bx + a)bc - abc}{(b^2 - 2bc + c^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")

[Out] $1/2*\sqrt{b*c}*abs(b)*\arctan(1/2*(a*b^2 + a*b*c - (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c}))^2)/(\sqrt{-b*c}*a*b)/(\sqrt{-b*c}*a*b) - ((b^2 + 6*b*c + c^2)*\sqrt{b*c}*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^6*abs(b) - (3*b^4 + 5*b^3*c + 5*b^2*c^2 + 3*b*c^3)*\sqrt{b*c}*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^4*a*abs(b) + (3*b^6 - 4*b^5*c + 2*b^4*c^2 - 4*b^3*c^3 + 3*b^2*c^4)*\sqrt{b*c}*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a^2*abs(b) - (b^8 - 3*b^7*c + 2*b^6*c^2 + 2*b^5*c^3 - 3*b^4*c^4 + b^3*c^5)*\sqrt{b*c})*a^3*abs(b))/(((\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^4 - 2*(b^2 + b*c)*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^2*(b^2 - 2*b*c + c^2)) - ((b*x + a)*b^2 + (b*x + a)*b*c - a*b*c)/((b^2 - 2*b*c + c^2)*b^2*x^2)$

maple [C] time = 0.01, size = 313, normalized size = 2.54

$$\frac{b}{(b-c)^2 x} - \frac{c}{(b-c)^2 x} - \frac{a}{(b-c)^2 x^2} + \frac{\sqrt{bx+a} \sqrt{cx+a} \left(-b^2 \ln\left(\frac{\ln(\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a)}{a}\right) + 2bc \ln\left(\frac{\ln(\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a)}{a}\right) - c^2 \ln\left(\frac{\ln(\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a)}{a}\right) + 2\sqrt{bx+a} \sqrt{cx+a} + acx + a^2 \operatorname{arcsinh}\left(\frac{a}{\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a}\right) + 2\sqrt{bx+a} \sqrt{cx+a} + abx + acx + a^2 \operatorname{arcsinh}\left(\frac{a}{\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a}\right) + 4\sqrt{bx+a} \sqrt{cx+a} + abx + a^2 \operatorname{arcsinh}\left(\frac{a}{\sqrt{bx+a} \sqrt{cx+a} + \sqrt{bx+a} \sqrt{cx+a} + a}\right) \right) \operatorname{csign}(a)}{4(b-c)^2 \sqrt{bx+a} \sqrt{cx+a} + abx + acx + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)

[Out] $-1/x/(b-c)^2*b-1/x/(b-c)^2*c-a/(b-c)^2/x^2+1/4/(b-c)^2*(b*x+a)^(1/2)*(c*x+a)^(1/2)/a*(-\ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*\operatorname{csign}(a))*a/x)*x^2*b^2+2*\ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*\operatorname{csign}(a))*a/x)*x^2*b*c-\ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*\operatorname{csign}(a))*a/x)*x^2*c^2+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*\operatorname{csign}(a)*x*b+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*\operatorname{csign}(a)*x*c+4*\operatorname{csign}(a)*a*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2))*\operatorname{csign}(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^2$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x*((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")

[Out] integrate(1/(x*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)

mupad [B] time = 12.32, size = 787, normalized size = 6.40

$$\frac{\ln\left(\frac{\sqrt{bx+a} - \sqrt{cx+a}}{\sqrt{bx+a} + \sqrt{cx+a}}\right)}{4a} + \frac{a^2}{2} + \frac{(\sqrt{bx+a})^2 \left(\frac{a^2}{2} + \frac{b^2}{2} - \frac{b^2}{2} \frac{(\sqrt{bx+a})^2}{(\sqrt{bx+a})^2}\right)}{(\sqrt{bx+a})^2} - \frac{(2b^2+2c^2)(\sqrt{bx+a} - \sqrt{cx+a})}{(\sqrt{bx+a})^2} + \frac{(\sqrt{bx+a})^2 \left(\frac{a^2}{2} + \frac{b^2}{2} - \frac{b^2}{2} \frac{(\sqrt{bx+a})^2}{(\sqrt{bx+a})^2}\right)}{(\sqrt{bx+a})^2} - \frac{(\sqrt{bx+a})^2 \left(\frac{a^2}{2} + \frac{b^2}{2} - \frac{b^2}{2} \frac{(\sqrt{bx+a})^2}{(\sqrt{bx+a})^2}\right)}{(\sqrt{bx+a})^2} - \frac{\ln\left(\frac{\sqrt{bx+a} - \sqrt{cx+a}}{\sqrt{bx+a} + \sqrt{cx+a}}\right)}{4a} + \frac{a^2 + (b+c)}{2} + \frac{c^2(\sqrt{bx+a} - \sqrt{cx+a})}{16a(b-c)^2(\sqrt{bx+a} - \sqrt{cx+a})} + \frac{c(b+c)(\sqrt{bx+a} - \sqrt{cx+a})}{8a(b-c)^2(\sqrt{bx+a} - \sqrt{cx+a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2),x)

```
[Out] log((((a + b*x)^(1/2) - (a + c*x)^(1/2))*(b - (c*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (b^4/2 + (((a + b*x)^(1/2) - a^(1/2))^4*(4*b*c^3 + 4*b^3*c - b^4/2 - c^4/2 + (3*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^4 - ((2*b^3*c + 2*b^4)*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - ((b*c^3 + b^2*c^2)*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(6*b^3*c + (5*b^4)/2 + (5*b^2*c^2)/2))/((a + c*x)^(1/2) - a^(1/2))^2 - (((a + b*x)^(1/2) - a^(1/2))^3*(b*c^3 + 6*b^3*c + b^4 + 6*b^2*c^2))/((a + c*x)^(1/2) - a^(1/2))^3)/((((a + b*x)^(1/2) - a^(1/2))^4*(8*a*b^4 + 8*a*c^4 - 48*a*b^2*c^2 + 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^4 - (((a + b*x)^(1/2) - a^(1/2))^3*(16*a*b^4 - 16*a*b^2*c^2 + 16*a*b*c^3 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^3 - (((a + b*x)^(1/2) - a^(1/2))^5*(16*a*c^4 - 16*a*b^2*c^2 - 16*a*b*c^3 + 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^5 + (((a + b*x)^(1/2) - a^(1/2))^2*(8*a*b^4 + 8*a*b^2*c^2 - 16*a*b^3*c))/((a + c*x)^(1/2) - a^(1/2))^2 + (((a + b*x)^(1/2) - a^(1/2))^6*(8*a*c^4 + 8*a*b^2*c^2 - 16*a*b*c^3))/((a + c*x)^(1/2) - a^(1/2))^6) - log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))/(4*a) - (a + x*(b + c))/(x^2*(b^2 - 2*b*c + c^2)) - (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(16*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2))^2) + (c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(8*a*(b - c)^2*((a + c*x)^(1/2) - a^(1/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)
```

```
[Out] Integral(1/(x*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)
```

$$3.214 \quad \int \frac{1}{x^2(\sqrt{a+bx} + \sqrt{a+cx})^2} dx$$

Optimal. Leaf size=174

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)} + \frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2} - \frac{2a}{3x^3(b-c)} - \frac{b+c}{2x^2(b-c)}$$

Rubi [A] time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6690, 96, 94, 93, 208}

$$\frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2x^3(b-c)^2} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2x^2(b-c)^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2x(b-c)} + \frac{(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right)}{4a^2} - \frac{2a}{3x^3(b-c)^2} - \frac{b+c}{2x^2(b-c)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2),x]

[Out] (-2*a)/(3*(b - c)^2*x^3) - (b + c)/(2*(b - c)^2*x^2) - ((b + c)*Sqrt[a + b*x]*Sqrt[a + c*x])/(4*a^2*(b - c)*x) - ((b + c)*Sqrt[a + b*x]*(a + c*x)^(3/2))/(2*a^2*(b - c)^2*x^2) + (2*(a + b*x)^(3/2)*(a + c*x)^(3/2))/(3*a^2*(b - c)^2*x^3) + ((b + c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a + c*x]])/(4*a^2)

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 94

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 96

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[(a*d*f*(m + 1) + b*c*f*(n + 1) + b*d*e*(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[Simplify[m + n + p + 3], 0] && (LtQ[m, -1] || SumSimplerQ[m, 1])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand

$[(u*x^{(m*n)})/(e*\text{Sqrt}[a + b*x^n] - f*\text{Sqrt}[c + d*x^n])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{ILtQ}[m, 0] \&\& \text{EqQ}[a*e^2 - c*f^2, 0]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (\sqrt{a+bx} + \sqrt{a+cx})^2} dx &= \frac{\int \left(\frac{2a}{x^4} + \frac{b(1+\frac{c}{b})}{x^3} - \frac{2\sqrt{a+bx}\sqrt{a+cx}}{x^4} \right) dx}{(b-c)^2} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} - \frac{2 \int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^4} dx}{(b-c)^2} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2 x^3} + \frac{(b+c) \int \frac{\sqrt{a+bx}\sqrt{a+cx}}{x^3} dx}{a(b-c)^2} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2 x^2} + \frac{2(a+bx)^{3/2}(a+cx)^{3/2}}{3a^2(b-c)^2 x^3} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2 x^2} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2 x^2} \\ &= -\frac{2a}{3(b-c)^2 x^3} - \frac{b+c}{2(b-c)^2 x^2} - \frac{(b+c)\sqrt{a+bx}\sqrt{a+cx}}{4a^2(b-c)x} - \frac{(b+c)\sqrt{a+bx}(a+cx)^{3/2}}{2a^2(b-c)^2 x^2} \end{aligned}$$

Mathematica [A] time = 0.18, size = 153, normalized size = 0.88

$$\frac{-8a^3 + a^2(8\sqrt{a+bx}\sqrt{a+cx} - 6bx - 6cx) + x^2(-3b^2 + 2bc - 3c^2)\sqrt{a+bx}\sqrt{a+cx} + 3x^3(b-c)^2(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a+cx}}\right) + 2ax(b+c)\sqrt{a+bx}\sqrt{a+cx}}{12a^2x^3(b-c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2), x]

[Out] $(-8a^3 + 2a(b+c)x\sqrt{a+bx}\sqrt{a+cx} + (-3b^2 + 2bc - 3c^2)x^2\sqrt{a+bx}\sqrt{a+cx} + a^2(-6b^2x - 6c^2x + 8\sqrt{a+bx}\sqrt{a+cx}) + 3(b-c)^2(b+c)x^3\text{ArcTanh}[\text{Sqrt}[a+bx]/\text{Sqrt}[a+cx]])/(12a^2(b-c)^2x^3)$

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[1/(x^2*(Sqrt[a + b*x] + Sqrt[a + c*x])^2), x]

[Out] \$Aborted

fricas [A] time = 0.41, size = 182, normalized size = 1.05

$$\frac{12(b^3 - b^2c - bc^2 + c^3)x^3 \log\left(-\frac{(b+c)x - 2\sqrt{bx+a}\sqrt{cx+a} + 2a}{x}\right) + (5b^3 + 3b^2c + 3bc^2 + 5c^3)x^3 + 64a^3 + 8((3b^2 - 2bc + 3c^2)x^2 - 8a^2 - 2(ab+ac)x)\sqrt{bx+a}\sqrt{cx+a} + 48(a^2b + a^2c)x}{96(a^2b^2 - 2a^2bc + a^2c^2)x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="fricas")
[Out] -1/96*(12*(b^3 - b^2*c - b*c^2 + c^3)*x^3*log(-(b + c)*x - 2*sqrt(b*x + a)
*sqrt(c*x + a) + 2*a)/x) + (5*b^3 + 3*b^2*c + 3*b*c^2 + 5*c^3)*x^3 + 64*a^3
+ 8*((3*b^2 - 2*b*c + 3*c^2)*x^2 - 8*a^2 - 2*(a*b + a*c)*x)*sqrt(b*x + a)*
sqrt(c*x + a) + 48*(a^2*b + a^2*c)*x)/((a^2*b^2 - 2*a^2*b*c + a^2*c^2)*x^3)
giac [B] time = 16.94, size = 802, normalized size = 4.61
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="giac")
[Out] -1/4*sqrt(b*c)*(b + c)*abs(b)*arctan(1/2*(a*b^2 + a*b*c - (sqrt(b*c)*sqrt(b
*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^2)/(sqrt(-b*c)*a*b))/((sqrt(-
b*c)*a^2*b) + 1/6*(3*(b^3 - b^2*c - b*c^2 + c^3)*sqrt(b*c)*(sqrt(b*c)*sqrt(
b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^10*abs(b) - 3*(5*b^5 + 22*b
^3*c^2 + 5*b*c^4)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x +
a)*b*c - a*b*c))^8*a*abs(b) + 2*(15*b^7 - b^6*c + 18*b^5*c^2 + 18*b^4*c^3 -
b^3*c^4 + 15*b^2*c^5)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b
*x + a)*b*c - a*b*c))^6*a^2*abs(b) - 6*(5*b^9 - 6*b^8*c - 5*b^7*c^2 + 12*b^
6*c^3 - 5*b^5*c^4 - 6*b^4*c^5 + 5*b^3*c^6)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x +
a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4*a^3*abs(b) + 3*(5*b^11 - 17*b^
10*c + 21*b^9*c^2 - 9*b^8*c^3 - 9*b^7*c^4 + 21*b^6*c^5 - 17*b^5*c^6 + 5*b^4*
c^7)*sqrt(b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*
c))^2*a^4*abs(b) - (3*b^13 - 20*b^12*c + 60*b^11*c^2 - 108*b^10*c^3 + 130*b
^9*c^4 - 108*b^8*c^5 + 60*b^7*c^6 - 20*b^6*c^7 + 3*b^5*c^8)*sqrt(b*c)*a^5*a
bs(b))/(((sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b*c))^4
- 2*(b^2 + b*c)*(sqrt(b*c)*sqrt(b*x + a) - sqrt(a*b^2 + (b*x + a)*b*c - a*b
*c))^2*a + (b^4 - 2*b^3*c + b^2*c^2)*a^2)^3*(b^2 - 2*b*c + c^2)*a) - 1/6*(3
*(b*x + a)*b^3 + a*b^3 + 3*(b*x + a)*b^2*c - 3*a*b^2*c)/((b^2 - 2*b*c + c^2
)*b^3*x^3)
maple [C] time = 0.02, size = 457, normalized size = 2.63
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x)
[Out] -1/2/x^2/(b-c)^2*b-1/2/x^2/(b-c)^2*c-2/3*a/(b-c)^2/x^3-1/24/(b-c)^2*(b*x+a)
^(1/2)*(c*x+a)^(1/2)/a^2*(-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b^3+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b^2*c+3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(1
/2)*csgn(a))*a/x)*x^3*b*c^2-3*ln((b*x+c*x+2*a+2*(b*c*x^2+a*b*x+a*c*x+a^2)^(
1/2)*csgn(a))*a/x)*x^3*c^3+6*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b^
2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*x^2*b*c+6*(b*c*x^2+a*b*x+a*c*x+
a^2)^(1/2)*csgn(a)*x^2*c^2-4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*b-
4*(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)*csgn(a)*a*x*c-16*(b*c*x^2+a*b*x+a*c*x+a^2
)^(1/2)*a^2*csgn(a)*csgn(a)/(b*c*x^2+a*b*x+a*c*x+a^2)^(1/2)/x^3
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x^2(\sqrt{bx+a} + \sqrt{cx+a})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^2,x, algorithm="maxima")
```

[Out] integrate(1/(x^2*(sqrt(b*x + a) + sqrt(c*x + a))^2), x)

mupad [B] time = 18.74, size = 1290, normalized size = 7.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*((a + b*x)^(1/2) + (a + c*x)^(1/2))^2), x)

[Out]
$$\frac{\log\left(\frac{(a + b*x)^{1/2} - a^{1/2}}{(a + c*x)^{1/2} - a^{1/2}}\right) * (b + c)}{(8*a^2) - \left(\frac{((a + b*x)^{1/2} - a^{1/2})^7 * (3*b^5*c - 15*b*c^5 + 3*c^6 + 3*b^2*c^4 + 3*b^3*c^3 - 15*b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^7} - \frac{((a + b*x)^{1/2} - a^{1/2})^5 * (26*b^5*c - b*c^5 - b^6 + 26*b^2*c^4 + 4*b^3*c^3 + 4*b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^5} - \frac{b^6}{3} + \frac{(b^5*c + b^6) * ((a + b*x)^{1/2} - a^{1/2})}{((a + c*x)^{1/2} - a^{1/2})} - \frac{((a + b*x)^{1/2} - a^{1/2})^8 * (c^6 - 6*b*c^5 + 7*b^2*c^4 - 6*b^3*c^3 + b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^8} + \frac{((a + b*x)^{1/2} - a^{1/2})^6 * (6*b*c^5 + 6*b^5*c - (5*b^6)/3 - (5*c^6)/3 + 30*b^2*c^4 - 24*b^3*c^3 + 30*b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^6} - \frac{((17*b^6)/3 + (17*b^3*c^3)/3) * ((a + b*x)^{1/2} - a^{1/2})^3}{((a + c*x)^{1/2} - a^{1/2})^3} + \frac{((a + b*x)^{1/2} - a^{1/2})^2 * (b^6 - 4*b^5*c + b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^2} + \frac{((a + b*x)^{1/2} - a^{1/2})^4 * (18*b^5*c + 5*b^6 + 5*b^2*c^4 + 18*b^3*c^3 - 6*b^4*c^2)}{((a + c*x)^{1/2} - a^{1/2})^4} - \frac{((a + b*x)^{1/2} - a^{1/2})^5 * (96*a^2*b^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 384*a^2*b^3*c^2)}{((a + c*x)^{1/2} - a^{1/2})^5} - \frac{((a + b*x)^{1/2} - a^{1/2})^8 * (96*a^2*c^5 - 96*a^2*b*c^4 - 96*a^2*b^2*c^3 + 96*a^2*b^3*c^2)}{((a + c*x)^{1/2} - a^{1/2})^8} - \frac{((a + b*x)^{1/2} - a^{1/2})^6 * (32*a^2*b^5 + 32*a^2*c^5 + 224*a^2*b*c^4 + 224*a^2*b^4*c - 256*a^2*b^2*c^3 - 256*a^2*b^3*c^2)}{((a + c*x)^{1/2} - a^{1/2})^6} - \frac{((a + b*x)^{1/2} - a^{1/2})^4 * (96*a^2*b^5 - 96*a^2*b^4*c + 96*a^2*b^2*c^3 - 96*a^2*b^3*c^2)}{((a + c*x)^{1/2} - a^{1/2})^4} + \frac{((a + b*x)^{1/2} - a^{1/2})^7 * (96*a^2*c^5 + 96*a^2*b*c^4 + 96*a^2*b^4*c - 384*a^2*b^2*c^3 + 96*a^2*b^3*c^2)}{((a + c*x)^{1/2} - a^{1/2})^7} + \frac{((a + b*x)^{1/2} - a^{1/2})^3 * (32*a^2*b^5 - 64*a^2*b^4*c + 32*a^2*b^2*c^3)}{((a + c*x)^{1/2} - a^{1/2})^3} + \frac{((a + b*x)^{1/2} - a^{1/2})^9 * (32*a^2*c^5 - 64*a^2*b*c^4 + 32*a^2*b^2*c^3)}{((a + c*x)^{1/2} - a^{1/2})^9} - \frac{((c*(8*b*c + 3*b^2 + 3*c^2))/(16*a^2*(b - c)^2) - (c*(17*b*c + 4*b^2 + 4*c^2))/(32*a^2*(b - c)^2)) * ((a + b*x)^{1/2} - a^{1/2})}{((a + c*x)^{1/2} - a^{1/2})} - \frac{\log\left(\frac{(a + b*x)^{1/2} - (a + c*x)^{1/2}}{(a + c*x)^{1/2} - a^{1/2}}\right) * (b - (c*((a + b*x)^{1/2} - a^{1/2})))}{((a + c*x)^{1/2} - a^{1/2})} * (b + c)}{(8*a^2) - \frac{(2*a)}{3} + x*(b/2 + c/2)} / (x^3*(b^2 - 2*b*c + c^2)) + \frac{c^3*((a + b*x)^{1/2} - a^{1/2})^3}{(96*a^2*(b - c)^2*((a + c*x)^{1/2} - a^{1/2})^3)}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (\sqrt{a + bx} + \sqrt{a + cx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**2,x)

[Out] Integral(1/(x**2*(sqrt(a + b*x) + sqrt(a + c*x))**2), x)

3.215 $\int \frac{x^4}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$

Optimal. Leaf size=277

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} - \frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3} + \frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} - \frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3} + \frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3} - \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3}$$

Rubi [A] time = 0.32, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6690, 43}

$$\frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} - \frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2a^2(3b+c)(a+cx)^{3/2}}{3c^3(b-c)^3} + \frac{8a^2(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{7/2}}{7b^3(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3} + \frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3} - \frac{8a(a+bx)^{5/2}}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]
```

```
[Out] (-8*a^2*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*a^2*(b + 3*c)*(a + b*x)^(3/2))/(3*b^3*(b - c)^3) + (8*a*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (4*a*(b + 3*c)*(a + b*x)^(5/2))/(5*b^3*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(7/2))/(7*b^3*(b - c)^3) + (8*a^2*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*a^2*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^3) - (8*a*(a + c*x)^(5/2))/(5*(b - c)^3*c^2) + (4*a*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^3) - (2*(3*b + c)*(a + c*x)^(7/2))/(7*(b - c)^3*c^3)
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]
```

Rule 6690

```
Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_.), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x^4}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx &= \frac{\int (4ax\sqrt{a+bx} + b(1 + \frac{3c}{b})x^2\sqrt{a+bx} - 4ax\sqrt{a+cx} - 3b(1 + \frac{c}{3b})x^2\sqrt{a+cx}) dx}{(b-c)^3} \\ &= \frac{(4a) \int x\sqrt{a+bx} dx}{(b-c)^3} - \frac{(4a) \int x\sqrt{a+cx} dx}{(b-c)^3} - \frac{(3b+c) \int x^2\sqrt{a+cx} dx}{(b-c)^3} + \frac{(b+3c) \int x^2\sqrt{a+bx} dx}{(b-c)^3} \\ &= \frac{(4a) \int (-\frac{a\sqrt{a+bx}}{b} + \frac{(a+bx)^{3/2}}{b}) dx}{(b-c)^3} - \frac{(4a) \int (-\frac{a\sqrt{a+cx}}{c} + \frac{(a+cx)^{3/2}}{c}) dx}{(b-c)^3} - \frac{(3b+c) \int x^2\sqrt{a+cx} dx}{(b-c)^3} + \frac{(b+3c) \int x^2\sqrt{a+bx} dx}{(b-c)^3} \\ &= -\frac{8a^2(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2a^2(b+3c)(a+bx)^{3/2}}{3b^3(b-c)^3} + \frac{8a(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{4a(b+3c)(a+bx)^{5/2}}{5b^3(b-c)^3} - \frac{2(3b+c)(a+cx)^{7/2}}{7c^3(b-c)^3} + \frac{4a(3b+c)(a+cx)^{5/2}}{5c^3(b-c)^3} - \frac{8a(a+cx)^{5/2}}{5c^2(b-c)^3} \end{aligned}$$

Mathematica [A] time = 0.39, size = 271, normalized size = 0.98

$$\frac{2(8a^2(b^4(-\sqrt{a+cx}) + 2b^3c\sqrt{a+cx} + c^4\sqrt{a+bx} - 2b^3c\sqrt{a+bx}) + 4a^2b^2cx(b^2\sqrt{a+cx} - 2b^2c\sqrt{a+cx} - c^2\sqrt{a+bx} + 2b^2c\sqrt{a+bx}) + 5b^4c^2x^3(-3b\sqrt{a+cx} + 3c\sqrt{a+bx} + b\sqrt{a+bx} - c\sqrt{a+cx}) + ab^2c^2x^2(-3b^2\sqrt{a+cx} + 3c^2\sqrt{a+bx} + 29bc(\sqrt{a+bx} - \sqrt{a+cx})))}{35b^3c^3(b-c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (2*(5*b^3*c^3*x^3*(b*Sqrt[a + b*x] + 3*c*Sqrt[a + b*x] - 3*b*Sqrt[a + c*x] - c*Sqrt[a + c*x]) + 4*a^2*b*c*x*(2*b*c^2*Sqrt[a + b*x] - c^3*Sqrt[a + b*x] + b^3*Sqrt[a + c*x] - 2*b^2*c*Sqrt[a + c*x]) + 8*a^3*(-2*b*c^3*Sqrt[a + b*x] + c^4*Sqrt[a + b*x] - b^4*Sqrt[a + c*x] + 2*b^3*c*Sqrt[a + c*x]) + a*b^2*c^2*x^2*(3*c^2*Sqrt[a + b*x] - 3*b^2*Sqrt[a + c*x] + 29*b*c*(Sqrt[a + b*x] - Sqrt[a + c*x])))/(35*b^3*(b - c)^3*c^3)

IntegrateAlgebraic [B] time = 61.52, size = 7868, normalized size = 28.40

Result too large to show

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^4/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] Result too large to show

fricas [A] time = 0.42, size = 225, normalized size = 0.81

$$\frac{2((16a^3bc^3 - 8a^3c^4 - 5(b^4c^3 + 3b^3c^4)x^3 - (29ab^2c^3 + 3ab^2c^4)x^2 - 4(2a^2b^2c^3 - a^2bc^4)x)\sqrt{bx+a} + (8a^3b^4 - 16a^3b^3c + 5(3b^4c^3 + b^3c^4)x^3 + (3ab^4c^2 + 29ab^2c^3)x^2 - 4(a^2b^4c - 2a^2b^3c^2)x)\sqrt{cx+a})}{35(b^6c^3 - 3b^5c^4 + 3b^4c^5 - b^3c^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")

[Out] -2/35*((16*a^3*b*c^3 - 8*a^3*c^4 - 5*(b^4*c^3 + 3*b^3*c^4)*x^3 - (29*a*b^3*c^3 + 3*a*b^2*c^4)*x^2 - 4*(2*a^2*b^2*c^3 - a^2*b*c^4)*x)*sqrt(b*x + a) + (8*a^3*b^4 - 16*a^3*b^3*c + 5*(3*b^4*c^3 + b^3*c^4)*x^3 + (3*a*b^4*c^2 + 29*a*b^3*c^3)*x^2 - 4*(a^2*b^4*c - 2*a^2*b^3*c^2)*x)*sqrt(c*x + a))/(b^6*c^3 - 3*b^5*c^4 + 3*b^4*c^5 - b^3*c^6)

giac [B] time = 5.58, size = 932, normalized size = 3.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out] -2/35*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*(((b*x + a)*(5*(3*b^22*c^5*abs(b) - 17*b^21*c^6*abs(b) + 39*b^20*c^7*abs(b) - 45*b^19*c^8*abs(b) + 25*b^18*c^9*abs(b) - 3*b^17*c^10*abs(b) - 3*b^16*c^11*abs(b) + b^15*c^12*abs(b))*(b*x + a)/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14) + (3*a*b^23*c^4*abs(b) - 34*a*b^22*c^5*abs(b) + 126*a*b^21*c^6*abs(b) - 210*a*b^20*c^7*abs(b) + 140*a*b^19*c^8*abs(b) + 42*a*b^18*c^9*abs(b) - 126*a*b^17*c^10*abs(b) + 74*a*b^16*c^11*abs(b) - 15*a*b^15*c^12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) - (4*a^2*b^24*c^3*abs(b) - 26*a^2*b^23*c^4*abs(b) + 85*a^2*b^22*c^5*abs(b) - 203*a^2*b^21*c^6*abs(b) + 385*a^2*b^20*c^7*abs(b) - 539*a^2*b^19*c^8*abs(b) + 511*a^2*b^18*c^9*abs(b) - 305*a^2*b^17*c^10*abs(b) + 103*a^2*b^16*c^11*abs(b) - 15*a^2*b^15*c^12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + (8*a^3*b^25*c^2*abs(b) - 60*a^3*b^24*c^3*abs(b) + 187*a^3*b^23*c^4*abs(b) - 296*a^3*b^22*c^5*abs(b) + 196*a^3*b^21*c^6*abs(b) + 112*a^3*b^20*c^7*abs(b) - 350*a^3*b^19*c^8*abs(b) + 328*a^3*b^18*c^9*abs(b) - 164*a^3*b^17*c^10*abs(b) + 44*a^3*b^16*c^11*abs(b) - 5*a^3*b^15*c^12*abs(b))/(b^29*c^5 - 9*b^28*c^6 + 36*b^27*c^7 - 84*b^26*c^8 + 126*b^25*c^9 - 126*b^24*c^10 + 84*b^23*c^11 - 36*b^22*c^12 + 9*b^21*c^13 - b^20*c^14)) + 2/35*(5*(b*x

$$+ a^{(7/2)} * b + 14 * (b * x + a)^{(5/2)} * a * b - 35 * (b * x + a)^{(3/2)} * a^2 * b + 15 * (b * x + a)^{(7/2)} * c - 42 * (b * x + a)^{(5/2)} * a * c + 35 * (b * x + a)^{(3/2)} * a^2 * c / (b^6 - 3 * b^5 * c + 3 * b^4 * c^2 - b^3 * c^3)$$

maple [A] time = 0.01, size = 246, normalized size = 0.89

$$\frac{8 \left(\frac{(bx+a)^{\frac{3}{2}} a}{3} + \frac{(bx+a)^{\frac{5}{2}}}{5} \right) a}{(b-c)^3 b^2} - \frac{8 \left(\frac{(cx+a)^{\frac{3}{2}} a}{3} + \frac{(cx+a)^{\frac{5}{2}}}{5} \right) a}{(b-c)^3 c^2} - \frac{6 \left(\frac{(cx+a)^{\frac{3}{2}} a^2}{3} - \frac{2(cx+a)^{\frac{5}{2}} a}{5} + \frac{(cx+a)^{\frac{7}{2}}}{7} \right) b}{(b-c)^3 c^3} + \frac{2(bx+a)^{\frac{3}{2}} a^2}{3} - \frac{4(bx+a)^{\frac{5}{2}} a}{5} + \frac{2(bx+a)^{\frac{7}{2}}}{7} + \frac{6 \left(\frac{(bx+a)^{\frac{3}{2}} a^2}{3} - \frac{2(bx+a)^{\frac{5}{2}} a}{5} + \frac{(bx+a)^{\frac{7}{2}}}{7} \right) c}{(b-c)^3 b^3} - \frac{2 \left(\frac{(cx+a)^{\frac{3}{2}} a^2}{3} - \frac{2(cx+a)^{\frac{5}{2}} a}{5} + \frac{(cx+a)^{\frac{7}{2}}}{7} \right)}{(b-c)^3 c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)

[Out] 2/(b-c)^3/b^2*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))+8/(b-c)^3*a/b^2*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))-8/(b-c)^3*a/c^2*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))+6/(b-c)^3*c/b^3*(1/3*(b*x+a)^(3/2)*a^2-2/5*(b*x+a)^(5/2)*a+1/7*(b*x+a)^(7/2))-6/(b-c)^3*b/c^3*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)-2/(b-c)^3/c^2*(1/7*(c*x+a)^(7/2)-2/5*(c*x+a)^(5/2)*a+1/3*(c*x+a)^(3/2)*a^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(\sqrt{bx+a} + \sqrt{cx+a})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x^4/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)

mupad [B] time = 3.34, size = 429, normalized size = 1.55

$$\frac{x^2 \left(\frac{12a(3bc)}{7(b-c)^2} - \frac{2a(3b+5c)}{(b-c)^2} \right) \sqrt{a+bx}}{5c} - \frac{2a \left(\frac{8a^2}{(b-c)^2} - \frac{4 \left(\frac{2a(3bc)}{(b-c)^2} - \frac{12a^2(3b+c)}{7(b-c)^2} \right) \sqrt{a+bx}}{3b} \right) \sqrt{a+bx}}{3b^2} + \frac{8a^2 \left(\frac{12a(3bc)}{(b-c)^2} - \frac{12a^2(3b+c)}{7(b-c)^2} \right) \sqrt{a+bx}}{5b} - \frac{2a \left(\frac{8a^2}{(b-c)^2} - \frac{4 \left(\frac{2a(3bc)}{(b-c)^2} - \frac{12a^2(3b+c)}{7(b-c)^2} \right) \sqrt{a+bx}}{3c} \right) \sqrt{a+bx}}{3c^2} - \frac{2a^2 \left(\frac{2a(3bc)}{(b-c)^2} - \frac{12a^2(3b+c)}{7(b-c)^2} \right) \sqrt{a+bx}}{5b} - \frac{8a^2 \left(\frac{12a(3bc)}{(b-c)^2} - \frac{12a^2(3b+c)}{7(b-c)^2} \right) \sqrt{a+bx}}{3c} - \frac{2a^3(3b+c) \sqrt{a+bx}}{7(b-c)^3} + \frac{2a^3(b^2+3cb) \sqrt{a+bx}}{7b(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)

[Out] (x^2*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3)*(a + c*x)^(1/2))/(5*c) - (2*a*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b^2) + (x*((8*a^2)/(b - c)^3 - (4*a*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3)))/(5*b))*(a + b*x)^(1/2))/(3*b) + (2*a*((8*a^2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c^2) + (x^2*((2*a*(5*b + 3*c))/(b - c)^3 - (12*a*(3*b*c + b^2))/(7*b*(b - c)^3))*(a + b*x)^(1/2))/(5*b) - (x*((8*a^2)/(b - c)^3 + (4*a*((12*a*(3*b + c))/(7*(b - c)^3) - (2*a*(3*b + 5*c))/(b - c)^3))/(5*c))*(a + c*x)^(1/2))/(3*c) - (2*x^3*(3*b + c)*(a + c*x)^(1/2))/(7*(b - c)^3) + (2*x^3*(3*b*c + b^2)*(a + b*x)^(1/2))/(7*b*(b - c)^3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**4/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)

[Out] Timed out

$$3.216 \quad \int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Optimal. Leaf size=163

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3} + \frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Rubi [A] time = 0.22, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6690, 43}

$$\frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} - \frac{2(3b+c)(a+cx)^{5/2}}{5c^2(b-c)^3} + \frac{2a(3b+c)(a+cx)^{3/2}}{3c^2(b-c)^3} + \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (8*a*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (2*a*(b + 3*c)*(a + b*x)^(3/2))/(3*b^2*(b - c)^3) + (2*(b + 3*c)*(a + b*x)^(5/2))/(5*b^2*(b - c)^3) - (8*a*(a + c*x)^(3/2))/(3*(b - c)^3*c) + (2*a*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^3*c^2) - (2*(3*b + c)*(a + c*x)^(5/2))/(5*(b - c)^3*c^2)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx &= \frac{\int \left(4a\sqrt{a+bx} + b\left(1 + \frac{3c}{b}\right)x\sqrt{a+bx} - 4a\sqrt{a+cx} - 3b\left(1 + \frac{c}{3b}\right)x\sqrt{a+cx} \right) dx}{(b-c)^3} \\ &= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} - \frac{(3b+c) \int x\sqrt{a+cx} dx}{(b-c)^3} + \frac{(b+3c) \int x\sqrt{a+bx} dx}{(b-c)^3} \\ &= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} - \frac{(3b+c) \int \left(-\frac{a\sqrt{a+cx}}{c} + \frac{(a+cx)^{3/2}}{c} \right) dx}{(b-c)^3} + \frac{(b+3c) \int x\sqrt{a+bx} dx}{(b-c)^3} \\ &= \frac{8a(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2a(b+3c)(a+bx)^{3/2}}{3b^2(b-c)^3} + \frac{2(b+3c)(a+bx)^{5/2}}{5b^2(b-c)^3} - \frac{8a(a+cx)^{3/2}}{3(b-c)^3c} + \frac{2(b+3c)(a+cx)^{5/2}}{5c^2(b-c)^3} \end{aligned}$$

Mathematica [A] time = 0.44, size = 120, normalized size = 0.74

$$\frac{2 \left(\frac{3(b+3c)(a+bx)^{5/2}}{b^2} - \frac{5a(b+3c)(a+bx)^{3/2}}{b^2} - \frac{3(3b+c)(a+cx)^{5/2}}{c^2} + \frac{5a(3b+c)(a+cx)^{3/2}}{c^2} + \frac{20a(a+bx)^{3/2}}{b} - \frac{20a(a+cx)^{3/2}}{c} \right)}{15(b-c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (2*((20*a*(a + b*x)^(3/2))/b - (5*a*(b + 3*c)*(a + b*x)^(3/2))/b^2 + (3*(b + 3*c)*(a + b*x)^(5/2))/b^2 - (20*a*(a + c*x)^(3/2))/c + (5*a*(3*b + c)*(a + c*x)^(3/2))/c^2 - (3*(3*b + c)*(a + c*x)^(5/2))/c^2))/(15*(b - c)^3)

IntegrateAlgebraic [A] time = 1.36, size = 197, normalized size = 1.21

$$\frac{2\sqrt{\frac{b(a+cx)}{c} - \frac{ab}{c} + a} (a^2b^3 - 4a^2b^2c + 5a^2bc^2 - 2a^2c^3 + b^3(a+cx)^2 - 2ab^3(a+cx) + 3b^2c(a+cx)^2 + ab^2c(a+cx) + abc^2(a+cx))}{5b^2c^2(b-c)^3} + \frac{2(-3b(a+cx)^{5/2} + 5ab(a+cx)^{3/2} - c(a+cx)^{5/2} - 5ac(a+cx)^{3/2})}{5c^2(b-c)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (2*Sqrt[a - (a*b)/c + (b*(a + c*x))/c]*(a^2*b^3 - 4*a^2*b^2*c + 5*a^2*b*c^2 - 2*a^2*c^3 - 2*a*b^3*(a + c*x) + a*b^2*c*(a + c*x) + a*b*c^2*(a + c*x) + b^3*(a + c*x)^2 + 3*b^2*c*(a + c*x)^2))/(5*b^2*(b - c)^3*c^2) + (2*(5*a*b*(a + c*x)^(3/2) - 5*a*c*(a + c*x)^(3/2) - 3*b*(a + c*x)^(5/2) - c*(a + c*x)^(5/2)))/(5*(b - c)^3*c^2)

fricas [A] time = 0.39, size = 167, normalized size = 1.02

$$\frac{2((6a^2bc^2 - 2a^2c^3 + (b^3c^2 + 3b^2c^3)x^2 + (7ab^2c^2 + abc^3)x)\sqrt{bx+a} + (2a^2b^3 - 6a^2b^2c - (3b^3c^2 + b^2c^3)x^2 - (ab^3c + 7ab^2c^2)x)\sqrt{cx+a})}{5(b^5c^2 - 3b^4c^3 + 3b^3c^4 - b^2c^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")

[Out] 2/5*((6*a^2*b*c^2 - 2*a^2*c^3 + (b^3*c^2 + 3*b^2*c^3)*x^2 + (7*a*b^2*c^2 + a*b*c^3)*x)*sqrt(b*x + a) + (2*a^2*b^3 - 6*a^2*b^2*c - (3*b^3*c^2 + b^2*c^3)*x^2 - (a*b^3*c + 7*a*b^2*c^2)*x)*sqrt(c*x + a))/(b^5*c^2 - 3*b^4*c^3 + 3*b^3*c^4 - b^2*c^5)

giac [B] time = 5.23, size = 480, normalized size = 2.94

$$\frac{2}{5} \sqrt{a^2 + (b+c)x} \left((bx+a) \left(\frac{(3b^3c^2b^3 - 8b^3c^2b^2c + 6b^3c^2b^2c^2 - b^3c^2b^2c^3)(bx+a)}{b^3c^2 - 6b^2c^3 + 15b^2c^4 - 20b^2c^5 + 15b^2c^6 - 6b^2c^7 + b^2c^8} + \frac{ab^3c^2b^3 - 2ab^3c^2b^2c + 8ab^3c^2b^2c^2 - 7ab^3c^2b^2c^3 + 2ab^3c^2b^2c^4}{b^3c^2 - 6b^2c^3 + 15b^2c^4 - 20b^2c^5 + 15b^2c^6 - 6b^2c^7 + b^2c^8} \right) + \frac{2a^2b^3c^2b^3 - 11a^2b^3c^2b^2c + 25a^2b^3c^2b^2c^2 - 30a^2b^3c^2b^2c^3 + 20a^2b^3c^2b^2c^4 - 7a^2b^3c^2b^2c^5 + a^2b^3c^2b^2c^6}{b^3c^2 - 6b^2c^3 + 15b^2c^4 - 20b^2c^5 + 15b^2c^6 - 6b^2c^7 + b^2c^8} \right) + \frac{2((bx+a)^{5/2}b + 5(bx+a)^{3/2}c - 5(bx+a)^{5/2}c)}{5(b^5 - 3b^4c + 3b^3c^2 - b^2c^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out] -2/5*sqrt(a*b^2 + (b*x + a)*b*c - a*b*c)*((b*x + a)*((3*b^12*c^3*abs(b) - 8*b^11*c^4*abs(b) + 6*b^10*c^5*abs(b) - b^8*c^7*abs(b))*(b*x + a)/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9) + (a*b^13*c^2*abs(b) - 2*a*b^12*c^3*abs(b) - 2*a*b^11*c^4*abs(b) + 8*a*b^10*c^5*abs(b) - 7*a*b^9*c^6*abs(b) + 2*a*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) - (2*a^2*b^14*c*abs(b) - 11*a^2*b^13*c^2*abs(b) + 25*a^2*b^12*c^3*abs(b) - 30*a^2*b^11*c^4*abs(b) + 20*a^2*b^10*c^5*abs(b) - 7*a^2*b^9*c^6*abs(b) + a^2*b^8*c^7*abs(b))/(b^18*c^3 - 6*b^17*c^4 + 15*b^16*c^5 - 20*b^15*c^6 + 15*b^14*c^7 - 6*b^13*c^8 + b^12*c^9)) + 2/5*((b*x + a)^(5/2)*b + 5*(b*x + a)^(3/2)*a*b + 3*(b*x + a)^(5/2)*c - 5*(b*x + a)^(3/2)*a*c)/(b^5 - 3*b^4*c + 3*b^3*c^2 - b^2*c^3)

maple [A] time = 0.00, size = 172, normalized size = 1.06

$$\frac{8(bx+a)^{\frac{3}{2}}a}{3(b-c)^3b} - \frac{8(cx+a)^{\frac{3}{2}}a}{3(b-c)^3c} - \frac{6\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3} + \frac{(cx+a)^{\frac{5}{2}}}{5}\right)b}{(b-c)^3c^2} + \frac{-\frac{2(bx+a)^{\frac{3}{2}}a}{3} + \frac{2(bx+a)^{\frac{5}{2}}}{5}}{(b-c)^3b} + \frac{6\left(-\frac{(bx+a)^{\frac{3}{2}}a}{3} + \frac{(bx+a)^{\frac{5}{2}}}{5}\right)c}{(b-c)^3b^2} - \frac{2\left(-\frac{(cx+a)^{\frac{3}{2}}a}{3} + \frac{(cx+a)^{\frac{5}{2}}}{5}\right)}{(b-c)^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
```

```
[Out] 2/(b-c)^3/b*(-1/3*(b*x+a)^(3/2)*a+1/5*(b*x+a)^(5/2))+8/3*a*(b*x+a)^(3/2)/b/
(b-c)^3-8/3*a*(c*x+a)^(3/2)/(b-c)^3/c+6/(b-c)^3*c/b^2*(-1/3*(b*x+a)^(3/2)*a
+1/5*(b*x+a)^(5/2))-6/(b-c)^3*b/c^2*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))
)-2/(b-c)^3/c*(-1/3*(c*x+a)^(3/2)*a+1/5*(c*x+a)^(5/2))
```

```
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{(\sqrt{bx+a} + \sqrt{cx+a})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)
```

```
mupad [B] time = 3.36, size = 268, normalized size = 1.64
```

$$\left(\frac{8a^2}{(b-c)^3} + \frac{2a\left(\frac{8a(b+3c)}{5(b-c)^2} - \frac{2a(5b+3c)}{(b-c)^2}\right)}{3b}\right)\sqrt{a+bx} - \left(\frac{8a^2}{(b-c)^3} + \frac{2a\left(\frac{8a(b+3c)}{5(b-c)^2} - \frac{2a(5b+3c)}{(b-c)^2}\right)}{3c}\right)\sqrt{a+cx} - x\left(\frac{8a(b+3c)}{5(b-c)^3} - \frac{2a(5b+3c)}{(b-c)^3}\right)\sqrt{a+bx} + x\left(\frac{8a(b+3c)}{5(b-c)^3} - \frac{2a(5b+3c)}{(b-c)^3}\right)\sqrt{a+cx} + \frac{2x^2(b+3c)\sqrt{a+bx}}{5(b-c)^3} - \frac{2x^2(3b+c)\sqrt{a+cx}}{5(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
```

```
[Out] (((8*a^2)/(b - c)^3 + (2*a*((8*a*(b + 3*c)))/(5*(b - c)^3) - (2*a*(5*b + 3*c)
))/((b - c)^3))/(3*b))*(a + b*x)^(1/2))/b - (((8*a^2)/(b - c)^3 + (2*a*((8*a
*(3*b + c)))/(5*(b - c)^3) - (2*a*(3*b + 5*c)))/(b - c)^3))/(3*c))*(a + c*x)^(
1/2))/c - (x*((8*a*(b + 3*c)))/(5*(b - c)^3) - (2*a*(5*b + 3*c))/(b - c)^3)
*(a + b*x)^(1/2))/(3*b) + (x*((8*a*(3*b + c)))/(5*(b - c)^3) - (2*a*(3*b + 5
*c))/(b - c)^3)*(a + c*x)^(1/2))/(3*c) + (2*x^2*(b + 3*c)*(a + b*x)^(1/2))/
(5*(b - c)^3) - (2*x^2*(3*b + c)*(a + c*x)^(1/2))/(5*(b - c)^3)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^3}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**3/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

$$3.217 \quad \int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Optimal. Leaf size=155

$$\frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{8a\sqrt{a+bx}}{(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {6690, 50, 63, 208}

$$\frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{8a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{8a\sqrt{a+bx}}{(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3c(b-c)^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (8*a*Sqrt[a + b*x])/(b - c)^3 + (2*(b + 3*c)*(a + b*x)^(3/2))/(3*b*(b - c)^3) - (8*a*Sqrt[a + c*x])/(b - c)^3 - (2*(3*b + c)*(a + c*x)^(3/2))/(3*(b - c)^3*c) - (8*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (8*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx &= \frac{\int \left(b \left(1 + \frac{3c}{b} \right) \sqrt{a+bx} + \frac{4a\sqrt{a+bx}}{x} - 3b \left(1 + \frac{c}{3b} \right) \sqrt{a+cx} - \frac{4a\sqrt{a+cx}}{x} \right) dx}{(b-c)^3} \\
&= \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(4a) \int \frac{\sqrt{a+bx}}{x} dx}{(b-c)^3} - \frac{(4a) \int \frac{\sqrt{a+cx}}{x} dx}{(b-c)^3} \\
&= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(4a^2)}{(b-c)^3} \\
&= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} + \frac{(8a^2)}{(b-c)^3} \\
&= \frac{8a\sqrt{a+bx}}{(b-c)^3} + \frac{2(b+3c)(a+bx)^{3/2}}{3b(b-c)^3} - \frac{8a\sqrt{a+cx}}{(b-c)^3} - \frac{2(3b+c)(a+cx)^{3/2}}{3(b-c)^3c} - \frac{8a^{3/2}}{(b-c)^3}
\end{aligned}$$

Mathematica [A] time = 0.27, size = 119, normalized size = 0.77

$$\frac{2 \left(-12a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+bx}}{\sqrt{a}} \right) + 12a^{3/2} \tanh^{-1} \left(\frac{\sqrt{a+cx}}{\sqrt{a}} \right) + \frac{(b+3c)(a+bx)^{3/2}}{b} - \frac{(3b+c)(a+cx)^{3/2}}{c} + 12a\sqrt{a+bx} - 12a\sqrt{a+cx} \right)}{3(b-c)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (2*(12*a*Sqrt[a + b*x] + ((b + 3*c)*(a + b*x)^(3/2))/b - 12*a*Sqrt[a + c*x] - ((3*b + c)*(a + c*x)^(3/2))/c - 12*a^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] + 12*a^(3/2)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(3*(b - c)^3)

IntegrateAlgebraic [F] time = 180.40, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] \$Aborted

fricas [A] time = 0.42, size = 321, normalized size = 2.07

$$\frac{2 \left(6a^2bc \log \left(\frac{\sqrt{a+bx}\sqrt{a+cx}}{a} \right) + 6a^2bc \log \left(\frac{\sqrt{a+bx}\sqrt{a+cx}}{a} \right) - (13abc + 3ac^2 + (b^2c + 3bc^2)x)\sqrt{bx+a} + (3ab^2 + 13abc + (3b^2c + bc^2)x)\sqrt{cx+a} \right) + 2 \left(12\sqrt{-a}abc \arctan \left(\frac{\sqrt{a+bx}}{a} \right) - 12\sqrt{-a}abc \arctan \left(\frac{\sqrt{a+cx}}{a} \right) + (13abc + 3ac^2 + (b^2c + 3bc^2)x)\sqrt{bx+a} - (3ab^2 + 13abc + (3b^2c + bc^2)x)\sqrt{cx+a} \right)}{3(b^2c - 3b^2c^2 + 3b^2c^3 - bc^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")

[Out] [-2/3*(6*a^(3/2)*b*c*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 6*a^(3/2)*b*c*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) + (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4), 2/3*(12*sqrt(-a)*a*b*c*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 12*sqrt(-a)*a*b*c*arctan(sqrt(c*x + a)*sqrt(-a)/a) + (13*a*b*c + 3*a*c^2 + (b^2*c + 3*b*c^2)*x)*sqrt(b*x + a) - (3*a*b^2 + 13*a*b*c + (3*b^2*c + b*c^2)*x)*sqrt(c*x + a))/(b^4*c - 3*b^3*c^2 + 3*b^2*c^3 - b*c^4)]

giac [B] time = 10.12, size = 2374, normalized size = 15.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -\frac{2}{3}\sqrt{a*b^2 + (b*x + a)*b*c - a*b*c}*((3*b^7*c*abs(b) - 8*b^6*c^2*abs(b) + 6*b^5*c^3*abs(b) - b^3*c^5*abs(b))*(b*x + a)/(b^{12}*c - 6*b^{11}*c^2 + 15*b^{10}*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7) + (3*a*b^8*abs(b) + a*b^7*c*abs(b) - 22*a*b^6*c^2*abs(b) + 30*a*b^5*c^3*abs(b) - 13*a*b^4*c^4*abs(b) + a*b^3*c^5*abs(b)))/(b^{12}*c - 6*b^{11}*c^2 + 15*b^{10}*c^3 - 20*b^9*c^4 + 15*b^8*c^5 - 6*b^7*c^6 + b^6*c^7)) + 8*a^2*\arctan(\sqrt{b*x + a})/\sqrt{-a} \\ & + \frac{2}{3}*((b*x + a)^{3/2}*b^9 + 12*\sqrt{b*x + a}*a*b^9 - 3*(b*x + a)^{3/2}*b^8*c - 72*\sqrt{b*x + a}*a*b^8*c - 3*(b*x + a)^{3/2}*b^7*c^2 + 180*\sqrt{b*x + a}*a*b^7*c^2 + 25*(b*x + a)^{3/2}*b^6*c^3 - 240*\sqrt{b*x + a}*a*b^6*c^3 - 45*(b*x + a)^{3/2}*b^5*c^4 + 180*\sqrt{b*x + a}*a*b^5*c^4 + 39*(b*x + a)^{3/2}*b^4*c^5 - 72*\sqrt{b*x + a}*a*b^4*c^5 - 17*(b*x + a)^{3/2}*b^3*c^6 + 12*\sqrt{b*x + a}*a*b^3*c^6 + 3*(b*x + a)^{3/2}*b^2*c^7)/(b^{12} - 9*b^{11}*c + 36*b^{10}*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9) - 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*\sqrt{-a*b*c}*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*\sqrt{-a*b*c}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*\sqrt{-a}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^{11}*c - 6*a^3*b^{10}*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^{11} - 6*a^3*b^{10}*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*\sqrt{-a*b*c}*abs(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4) + \sqrt{((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))})/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^{12} - 9*b^{11}*c + 36*b^{10}*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) + 8*(2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3*c - a*b^2*c^2)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^3 - a*b^2*c)*\sqrt{-a*b*c}*abs(b) + (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*\sqrt{-a*b*c}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^8 - 5*a^2*b^7*c + 10*a^2*b^6*c^2 - 10*a^2*b^5*c^3 + 5*a^2*b^4*c^4 - a^2*b^3*c^5)*\sqrt{-a}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^{11}*c - 6*a^3*b^{10}*c^2 + 14*a^3*b^9*c^3 - 14*a^3*b^8*c^4 + 14*a^3*b^6*c^6 - 14*a^3*b^5*c^7 + 6*a^3*b^4*c^8 - a^3*b^3*c^9)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^{11} - 6*a^3*b^{10}*c + 14*a^3*b^9*c^2 - 14*a^3*b^8*c^3 + 14*a^3*b^6*c^5 - 14*a^3*b^5*c^6 + 6*a^3*b^4*c^7 - a^3*b^3*c^8)*\sqrt{-a*b*c}*abs(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4) - \sqrt{((a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))})/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/(b^{12} - 9*b^{11}*c + 36*b^{10}*c^2 - 84*b^9*c^3 + 126*b^8*c^4 - 126*b^7*c^5 + 84*b^6*c^6 - 36*b^5*c^7 + 9*b^4*c^8 - b^3*c^9)*a*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) \end{aligned}$$

maple [A] time = 0.00, size = 148, normalized size = 0.95

$$\frac{4\left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a}\right)a}{(b-c)^3} - \frac{4\left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right) + 2\sqrt{cx+a}\right)a}{(b-c)^3} - \frac{2(cx+a)^{\frac{3}{2}}b}{(b-c)^3c} + \frac{2(bx+a)^{\frac{3}{2}}c}{(b-c)^3b} + \frac{2(bx+a)^{\frac{3}{2}}}{3(b-c)^3} - \frac{2(cx+a)^{\frac{3}{2}}}{3(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)

[Out] 2/3/(b-c)^3*(b*x+a)^(3/2)+2/(b-c)^3*c*(b*x+a)^(3/2)/b-2/(b-c)^3*b*(c*x+a)^(3/2)/c-2/3/(b-c)^3*(c*x+a)^(3/2)+4/(b-c)^3*a*(2*(b*x+a)^(1/2)-2*a^(1/2)*arc tanh((b*x+a)^(1/2)/a^(1/2)))-4/(b-c)^3*a*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{bx+a} + \sqrt{cx+a}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x^2/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)

mupad [B] time = 7.02, size = 762, normalized size = 4.92

$$\frac{4a^{3/2}b^4 - (4a^{3/2}c^4((4((a+b*x)^{1/2} - a^{1/2})^3)/((a+c*x)^{1/2} - a^{1/2})^3 - (15((a+b*x)^{1/2} - a^{1/2})^4)/((a+c*x)^{1/2} - a^{1/2})^4 + (24((a+b*x)^{1/2} - a^{1/2})^5)/((a+c*x)^{1/2} - a^{1/2})^5 + (6*\log(((a+b*x)^{1/2} - a^{1/2})/((a+c*x)^{1/2} - a^{1/2}))*((a+b*x)^{1/2} - a^{1/2})^6)/((a+c*x)^{1/2} - a^{1/2})^6))/3 - (4a^{3/2}b^2c^2((24((a+b*x)^{1/2} - a^{1/2}))/((a+c*x)^{1/2} - a^{1/2}) + (12*((a+b*x)^{1/2} - a^{1/2})^2)/((a+c*x)^{1/2} - a^{1/2})^2 + (12*((a+b*x)^{1/2} - a^{1/2})^3)/((a+c*x)^{1/2} - a^{1/2})^3 - (15((a+b*x)^{1/2} - a^{1/2})^4)/((a+c*x)^{1/2} - a^{1/2})^4 + (18*\log(((a+b*x)^{1/2} - a^{1/2})/((a+c*x)^{1/2} - a^{1/2}))*((a+b*x)^{1/2} - a^{1/2})^2)/((a+c*x)^{1/2} - a^{1/2})^2 - 3))/3 + (4a^{3/2}b^3c^3((6((a+b*x)^{1/2} - a^{1/2})^2)/((a+c*x)^{1/2} - a^{1/2})^2 - (12*((a+b*x)^{1/2} - a^{1/2})^3)/((a+c*x)^{1/2} - a^{1/2})^3 + (66*((a+b*x)^{1/2} - a^{1/2})^4)/((a+c*x)^{1/2} - a^{1/2})^4 - (24*((a+b*x)^{1/2} - a^{1/2})^5)/((a+c*x)^{1/2} - a^{1/2})^5 + (18*\log(((a+b*x)^{1/2} - a^{1/2})/((a+c*x)^{1/2} - a^{1/2}))*((a+b*x)^{1/2} - a^{1/2})^4)/((a+c*x)^{1/2} - a^{1/2})^4))/3 + (4a^{3/2}b^3c^3(6*\log(((a+b*x)^{1/2} - a^{1/2})/((a+c*x)^{1/2} - a^{1/2}))) - (24*((a+b*x)^{1/2} - a^{1/2}))/((a+c*x)^{1/2} - a^{1/2}) + (6*((a+b*x)^{1/2} - a^{1/2})^2)/((a+c*x)^{1/2} - a^{1/2})^2 - (4*((a+b*x)^{1/2} - a^{1/2})^3)/((a+c*x)^{1/2} - a^{1/2})^3 + 26))/3)/(c*(b-c)^3*(b - (c*((a+b*x)^{1/2} - a^{1/2})^2)/((a+c*x)^{1/2} - a^{1/2})^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)

[Out] (4*a^(3/2)*b^4 - (4*a^(3/2)*c^4*((4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^6)/((a + c*x)^(1/2) - a^(1/2))^6))/3 - (4*a^(3/2)*b^2*c^2*((24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (12*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (12*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 - (15*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - 3))/3 + (4*a^(3/2)*b^3*c^3*((6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (12*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + (66*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4 - (24*((a + b*x)^(1/2) - a^(1/2))^5)/((a + c*x)^(1/2) - a^(1/2))^5 + (18*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))*((a + b*x)^(1/2) - a^(1/2))^4)/((a + c*x)^(1/2) - a^(1/2))^4))/3 + (4*a^(3/2)*b^3*c^3*(6*log(((a + b*x)^(1/2) - a^(1/2))/((a + c*x)^(1/2) - a^(1/2)))) - (24*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + (6*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - (4*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3 + 26))/3)/(c*(b - c)^3*(b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2)^3)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(\sqrt{a+bx} + \sqrt{a+cx}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral(x**2/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)
```

$$3.218 \quad \int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Optimal. Leaf size=157

$$-\frac{4a\sqrt{a+bx}}{x(b-c)^3} + \frac{4a\sqrt{a+cx}}{x(b-c)^3} + \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} - \frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{6\sqrt{a}(b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Rubi [A] time = 0.22, antiderivative size = 223, normalized size of antiderivative = 1.42, number of steps used = 14, number of rules used = 5, integrand size = 23, number of rules / integrand size = 0.217, Rules used = {6690, 47, 63, 208, 50}

$$\frac{4a\sqrt{a+bx}}{x(b-c)^3} + \frac{4a\sqrt{a+cx}}{x(b-c)^3} + \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} - \frac{2\sqrt{a}(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} - \frac{4\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{4\sqrt{a}c\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3} + \frac{2\sqrt{a}(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{(b-c)^3}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3,x]

[Out] (2*(b + 3*c)*Sqrt[a + b*x])/(b - c)^3 - (4*a*Sqrt[a + b*x])/((b - c)^3*x) - (2*(3*b + c)*Sqrt[a + c*x])/(b - c)^3 + (4*a*Sqrt[a + c*x])/((b - c)^3*x) - (4*Sqrt[a]*b*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 - (2*Sqrt[a]*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(b - c)^3 + (4*Sqrt[a]*c*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3 + (2*Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(b - c)^3

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

Int[(u_.)*((e_.)*Sqrt[(a_.) + (b_.)*(x_)^(n_.)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)^(n_.)])^(m_), x_Symbol] :> Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand

$[(u*x^{(m*n)})/(e*\text{Sqrt}[a + b*x^n] - f*\text{Sqrt}[c + d*x^n])^m, x], x] /; \text{FreeQ}[\{a, b, c, d, e, f, n\}, x] \&\& \text{ILtQ}[m, 0] \&\& \text{EqQ}[a*e^2 - c*f^2, 0]$

Rubi steps

$$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^2} + \frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x} - \frac{4a\sqrt{a+cx}}{x^2} - \frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x} \right) dx}{(b-c)^3}$$

$$= \frac{(4a) \int \frac{\sqrt{a+bx}}{x^2} dx}{(b-c)^3} - \frac{(4a) \int \frac{\sqrt{a+cx}}{x^2} dx}{(b-c)^3} - \frac{(3b+c) \int \frac{\sqrt{a+cx}}{x} dx}{(b-c)^3} + \frac{(b+3c) \int \frac{\sqrt{a+bx}}{x} dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{(b-c)^3x} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} + \frac{4a\sqrt{a+cx}}{(b-c)^3x} + \frac{(2ab) \int \frac{1}{x} dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{(b-c)^3x} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} + \frac{4a\sqrt{a+cx}}{(b-c)^3x} + \frac{(4a) \int \frac{1}{x} dx}{(b-c)^3}$$

$$= \frac{2(b+3c)\sqrt{a+bx}}{(b-c)^3} - \frac{4a\sqrt{a+bx}}{(b-c)^3x} - \frac{2(3b+c)\sqrt{a+cx}}{(b-c)^3} + \frac{4a\sqrt{a+cx}}{(b-c)^3x} - \frac{4\sqrt{a} \ln|x|}{(b-c)^3}$$

Mathematica [A] time = 0.88, size = 192, normalized size = 1.22

$$\frac{2 \left(-(3b+c)\sqrt{a+cx} + (b+3c)\sqrt{a+bx} + \sqrt{a}(3b+c) \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right) - \sqrt{a}(b+3c) \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right) - \frac{2a\left(bx\sqrt{\frac{bx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right) + a+bx\right)}{x\sqrt{a+bx}} + \frac{2a\left(cx\sqrt{\frac{cx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right) + a+cx\right)}{x\sqrt{a+cx}} \right)}{(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3, x]
[Out] (2*((b + 3*c)*Sqrt[a + b*x] - (3*b + c)*Sqrt[a + c*x] - Sqrt[a]*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]] - (2*a*(a + b*x + b*x*Sqrt[1 + (b*x)/a])*ArcTanh[Sqrt[1 + (b*x)/a]]))/(x*Sqrt[a + b*x]) + Sqrt[a]*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]] + (2*a*(a + c*x + c*x*Sqrt[1 + (c*x)/a])*ArcTanh[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]))/(b - c)^3
```

IntegrateAlgebraic [F] time = 180.01, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x/(Sqrt[a + b*x] + Sqrt[a + c*x])^3, x]
[Out] $Aborted
```

fricas [A] time = 0.42, size = 260, normalized size = 1.66

$$\frac{\left(3\sqrt{a}(b+c)x \log\left(\frac{bx+2\sqrt{bx+a}\sqrt{a+2c}}{x}\right) + 3\sqrt{a}(b+c)x \log\left(\frac{cx-2\sqrt{cx+a}\sqrt{a+2c}}{x}\right) - 2((b+3c)x-2a)\sqrt{bx+a} + 2((3b+c)x-2a)\sqrt{cx+a} + 2\left(3\sqrt{-a}(b+c)x \arctan\left(\frac{\sqrt{bx+a}\sqrt{c}}{a}\right) - 3\sqrt{-a}(b+c)x \arctan\left(\frac{\sqrt{cx+a}\sqrt{c}}{a}\right) + ((b+3c)x-2a)\sqrt{bx+a} - ((3b+c)x-2a)\sqrt{cx+a}\right) \right)}{(b^3-3b^2c+3bc^2-c^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
[Out] [-(3*sqrt(a)*(b + c)*x*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqrt(a)*(b + c)*x*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) - 2*((b + 3*c)*
```

$$x - 2*a)*\sqrt{b*x + a} + 2*((3*b + c)*x - 2*a)*\sqrt{c*x + a})/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x), 2*(3*\sqrt{-a}*(b + c)*x*\arctan(\sqrt{b*x + a}*\sqrt{-a}/a) - 3*\sqrt{-a}*(b + c)*x*\arctan(\sqrt{c*x + a}*\sqrt{-a}/a) + ((b + 3*c)*x - 2*a)*\sqrt{b*x + a} - ((3*b + c)*x - 2*a)*\sqrt{c*x + a})/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x)]$$

giac [B] time = 78.70, size = 2318, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")

[Out]
$$\begin{aligned} & -2*(\sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*(3*b*abs(b) + c*abs(b))/(b^4 - 3*b^3*c + 3*b^2*c^2 - b*c^3) + 2*\sqrt{b*x + a}*a*b/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*x) - 3*(a*b^2 + a*b*c)*\arctan(\sqrt{b*x + a}/\sqrt{-a})/((b^3 - 3*b^2*c + 3*b*c^2 - c^3)*\sqrt{-a}) - (\sqrt{b*x + a}*b^2 + 3*\sqrt{b*x + a}*b*c)/(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 4*((\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^2*b^3*c*abs(b) - (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})*a^2*b^2*c^2*abs(b) + (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^3*a*b*c*abs(b))/((a^2*b^4 - 2*a^2*b^3*c + a^2*b^2*c^2 - 2*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a*b^2 - 2*(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^2*a*b*c + (\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})^4)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3)) + 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*\sqrt{-a*b*c}*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*\sqrt{-a*b*c}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*\sqrt{-a}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*\sqrt{-a*b*c}*abs(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 + \sqrt{(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))})/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9)*a^2*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) - 3*(2*(a*b^4*c - a*b^2*c^3)*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + 2*(a*b^4 - 3*a*b^3*c + 3*a*b^2*c^2 - a*b*c^3)^2*(a*b^4 - a*b^2*c^2)*\sqrt{-a*b*c}*abs(b) + (a^2*b^8 - 4*a^2*b^7*c + 5*a^2*b^6*c^2 - 5*a^2*b^4*c^4 + 4*a^2*b^3*c^5 - a^2*b^2*c^6)*\sqrt{-a*b*c}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^2*b^9 - 4*a^2*b^8*c + 5*a^2*b^7*c^2 - 5*a^2*b^5*c^4 + 4*a^2*b^4*c^5 - a^2*b^3*c^6)*\sqrt{-a}*abs(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)*abs(b) + (a^3*b^12*c - 5*a^3*b^11*c^2 + 8*a^3*b^10*c^3 - 14*a^3*b^8*c^5 + 14*a^3*b^7*c^6 - 8*a^3*b^5*c^8 + 5*a^3*b^4*c^9 - a^3*b^3*c^10)*\sqrt{-a}*abs(b)*sgn(b^3 - 3*b^2*c + 3*b*c^2 - c^3) + (a^3*b^12 - 5*a^3*b^11*c + 8*a^3*b^10*c^2 - 14*a^3*b^8*c^4 + 14*a^3*b^7*c^5 - 8*a^3*b^5*c^7 + 5*a^3*b^4*c^8 - a^3*b^3*c^9)*\sqrt{-a*b*c}*abs(b))*\arctan(-(\sqrt{b*c})*\sqrt{b*x + a} - \sqrt{a*b^2 + (b*x + a)*b*c - a*b*c})/\sqrt{-(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4 - \sqrt{(a*b^5 - 2*a*b^4*c + 2*a*b^2*c^3 - a*b*c^4)^2 - (a^2*b^7 - 5*a^2*b^6*c + 10*a^2*b^5*c^2 - 10*a^2*b^4*c^3 + 5*a^2*b^3*c^4 - a^2*b^2*c^5)*(b^3 - 3*b^2*c + 3*b*c^2 - c^3))})/(b^3 - 3*b^2*c + 3*b*c^2 - c^3)))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9))/((b^11 - 9*b^10*c + 36*b^9*c^2 - 84*b^8*c^3$$

$$3 + 126*b^7*c^4 - 126*b^6*c^5 + 84*b^5*c^6 - 36*b^4*c^7 + 9*b^3*c^8 - b^2*c^9) * a^2 * \text{abs}(-a*b^4 + 3*a*b^3*c - 3*a*b^2*c^2 + a*b*c^3)) / b$$

maple [A] time = 0.01, size = 237, normalized size = 1.51

$$\frac{8 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx} \right) ab - 8 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{cx+a}}{2cx} \right) ac + \left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a} \right) b - 3 \left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right) + 2\sqrt{cx+a} \right) b + 3 \left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) + 2\sqrt{bx+a} \right) c - \left(-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right) + 2\sqrt{cx+a} \right) c}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)

[Out] 1/(b-c)^3*b*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))+8/(b-c)^3*a*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2)))/a^(1/2))-8/(b-c)^3*a*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))+3/(b-c)^3*c*(2*(b*x+a)^(1/2)-2*a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-3/(b-c)^3*b*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))-1/(b-c)^3*c*(2*(c*x+a)^(1/2)-2*a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{bx+a} + \sqrt{cx+a})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")

[Out] integrate(x/(sqrt(b*x + a) + sqrt(c*x + a))^3, x)

mupad [B] time = 7.49, size = 559, normalized size = 3.56

$$\frac{2\sqrt{a} b^2 (\sqrt{a+cx} - \sqrt{a}) \left(\frac{3(\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} - \frac{2(\sqrt{bx+a} - \sqrt{a})^2}{(\sqrt{bx+a} - \sqrt{a})^2} + \frac{3 \ln\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) (\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} + 1 \right) - 2\sqrt{a} c^2 (\sqrt{a+cx} - \sqrt{a}) \left(\frac{2(\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} - \frac{(\sqrt{bx+a} - \sqrt{a})^2}{(\sqrt{bx+a} - \sqrt{a})^2} + \frac{3 \ln\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) (\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} \right) + 2\sqrt{a} bc (\sqrt{a+cx} - \sqrt{a}) \left(\frac{3(\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} - \frac{3 \ln\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) (\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}} \right) - \frac{3 \ln\left(\frac{\sqrt{bx+a} - \sqrt{a}}{\sqrt{bx+a} + \sqrt{a}}\right) (\sqrt{bx+a} - \sqrt{a})}{\sqrt{bx+a} - \sqrt{a}}}{(b-c)^3 (\sqrt{a+bx} - \sqrt{a}) \left(b - \frac{c(\sqrt{bx+a} - \sqrt{a})^2}{(\sqrt{bx+a} - \sqrt{a})^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)

[Out] (2*a^(1/2)*b^2*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) + 1) - 2*a^(1/2)*c^2*((a + c*x)^(1/2) - a^(1/2))*((2*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 - ((a + b*x)^(1/2) - a^(1/2))^4/((a + c*x)^(1/2) - a^(1/2))^4 + (3*log(((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3) + 2*a^(1/2)*b*c*((a + c*x)^(1/2) - a^(1/2))*((8*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (14*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2))^2 + (3*log(((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))*((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2)) - (3*log(((a + b*x)^(1/2) - a^(1/2)))/((a + c*x)^(1/2) - a^(1/2))))*((a + b*x)^(1/2) - a^(1/2))^3)/((a + c*x)^(1/2) - a^(1/2))^3)/((b - c)^3*((a + b*x)^(1/2) - a^(1/2))*((b - (c*((a + b*x)^(1/2) - a^(1/2))^2)/((a + c*x)^(1/2) - a^(1/2)))^2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)

[Out] Integral(x/(sqrt(a + b*x) + sqrt(a + c*x))**3, x)

$$3.219 \quad \int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Optimal. Leaf size=164

$$-\frac{2a\sqrt{a+bx}}{x^2(b-c)^3} + \frac{2a\sqrt{a+cx}}{x^2(b-c)^3} - \frac{(2b+3c)\sqrt{a+bx}}{x(b-c)^3} + \frac{(3b+2c)\sqrt{a+cx}}{x(b-c)^3} - \frac{3bc \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{3bc \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

Rubi [A] time = 0.18, antiderivative size = 275, normalized size of antiderivative = 1.68, number of steps used = 16, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {6690, 47, 51, 63, 208}

$$\frac{b^2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} - \frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} - \frac{2a\sqrt{a+bx}}{x^2(b-c)^3} + \frac{2a\sqrt{a+cx}}{x^2(b-c)^3} - \frac{b\sqrt{a+bx}}{x(b-c)^3} - \frac{(b+3c)\sqrt{a+bx}}{x(b-c)^3} + \frac{c\sqrt{a+cx}}{x(b-c)^3} + \frac{(3b+c)\sqrt{a+cx}}{x(b-c)^3} - \frac{b(b+3c)\tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3} + \frac{c(3b+c)\tanh^{-1}\left(\frac{\sqrt{a+cx}}{\sqrt{a}}\right)}{\sqrt{a}(b-c)^3}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3), x]

[Out] (-2*a*Sqrt[a + b*x])/((b - c)^3*x^2) - (b*Sqrt[a + b*x])/((b - c)^3*x) - ((b + 3*c)*Sqrt[a + b*x])/((b - c)^3*x) + (2*a*Sqrt[a + c*x])/((b - c)^3*x^2) + (c*Sqrt[a + c*x])/((b - c)^3*x) + ((3*b + c)*Sqrt[a + c*x])/((b - c)^3*x) + (b^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (b*(b + 3*c)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) - (c^2*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3) + (c*(3*b + c)*ArcTanh[Sqrt[a + c*x]/Sqrt[a]])/(Sqrt[a]*(b - c)^3)

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 6690

```
Int[(u_)*((e_)*Sqrt[(a_) + (b_)*(x_)^(n_)] + (f_)*Sqrt[(c_) + (d_)*
(x_)^(n_)])^(m_), x_Symbol] := Dist[(b*e^2 - d*f^2)^m, Int[ExpandIntegrand
[(u*x^(m*n))/(e*Sqrt[a + b*x^n] - f*Sqrt[c + d*x^n])^m, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n}, x] && ILtQ[m, 0] && EqQ[a*e^2 - c*f^2, 0]
```

Rubi steps

$$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx = \frac{\int \left(\frac{4a\sqrt{a+bx}}{x^3} + \frac{b\left(1+\frac{3c}{b}\right)\sqrt{a+bx}}{x^2} - \frac{4a\sqrt{a+cx}}{x^3} - \frac{3b\left(1+\frac{c}{3b}\right)\sqrt{a+cx}}{x^2} \right) dx}{(b-c)^3}$$

$$= \frac{(4a) \int \frac{\sqrt{a+bx}}{x^3} dx}{(b-c)^3} - \frac{(4a) \int \frac{\sqrt{a+cx}}{x^3} dx}{(b-c)^3} - \frac{(3b+c) \int \frac{\sqrt{a+cx}}{x^2} dx}{(b-c)^3} + \frac{(b+3c) \int \frac{\sqrt{a+bx}}{x^2} dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{(3b+c)\sqrt{a+cx}}{(b-c)^3x} + \frac{(ab) \int \frac{1}{x} dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(ab) \int \frac{1}{x} dx}{(b-c)^3}$$

$$= -\frac{2a\sqrt{a+bx}}{(b-c)^3x^2} - \frac{b\sqrt{a+bx}}{(b-c)^3x} - \frac{(b+3c)\sqrt{a+bx}}{(b-c)^3x} + \frac{2a\sqrt{a+cx}}{(b-c)^3x^2} + \frac{c\sqrt{a+cx}}{(b-c)^3x} + \frac{(ab) \int \frac{1}{x} dx}{(b-c)^3}$$

Mathematica [C] time = 0.27, size = 182, normalized size = 1.11

$$\frac{-\frac{8b^2(a+bx)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{bx}{a} + 1\right)}{a^2} + \frac{8c^2(a+cx)^{3/2} {}_2F_1\left(\frac{3}{2}, \frac{5}{2}; \frac{cx}{a} + 1\right)}{a^2} - \frac{3(b+3c)\left(bx\sqrt{\frac{bx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{bx}{a}+1}\right) + a+bx\right)}{x\sqrt{a+bx}} + \frac{3(3b+c)\left(cx\sqrt{\frac{cx}{a}+1} \tanh^{-1}\left(\sqrt{\frac{cx}{a}+1}\right) + a+cx\right)}{x\sqrt{a+cx}}}{3(b-c)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3), x]
[Out] ((-3*(b + 3*c)*(a + b*x + b*x*Sqrt[1 + (b*x)/a]*ArcTanh[Sqrt[1 + (b*x)/a]])
)/(x*Sqrt[a + b*x]) + (3*(3*b + c)*(a + c*x + c*x*Sqrt[1 + (c*x)/a]*ArcTanh
[Sqrt[1 + (c*x)/a]]))/(x*Sqrt[a + c*x]) - (8*b^2*(a + b*x)^(3/2)*Hypergeome
tric2F1[3/2, 3, 5/2, 1 + (b*x)/a])/a^2 + (8*c^2*(a + c*x)^(3/2)*Hypergeomet
ric2F1[3/2, 3, 5/2, 1 + (c*x)/a])/a^2)/(3*(b - c)^3)
```

IntegrateAlgebraic [F] time = 180.03, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(Sqrt[a + b*x] + Sqrt[a + c*x])^(-3), x]
[Out] $Aborted
```

fricas [A] time = 0.45, size = 297, normalized size = 1.81

$$\frac{3\sqrt{a}bc^2 \log\left(\frac{bx+\sqrt{bx+a}}{x}\right) + 3\sqrt{a}bc^2 \log\left(\frac{cx+\sqrt{cx+a}}{x}\right) + 2(2a^2 + (2ab+3ac)\sqrt{bx+a} - 2(2a^2 + (3ab+2ac)\sqrt{cx+a} - 3\sqrt{a}bcx^2 \arctan\left(\frac{\sqrt{bx+a}}{a}\right) - 3\sqrt{a}bcx^2 \arctan\left(\frac{\sqrt{cx+a}}{a}\right) - (2a^2 + (2ab+3ac)\sqrt{bx+a} + (2a^2 + (3ab+2ac)\sqrt{cx+a} + (ab^3 - 3ab^2c + 3abc^2 - ac^3)x^2)}{(ab^3 - 3ab^2c + 3abc^2 - ac^3)x^2}\right)}{2(ab^3 - 3ab^2c + 3abc^2 - ac^3)x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="fricas")
[Out] [-1/2*(3*sqrt(a)*b*c*x^2*log((b*x + 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x) + 3*sqrt(a)*b*c*x^2*log((c*x - 2*sqrt(c*x + a)*sqrt(a) + 2*a)/x) + 2*(2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) - 2*(2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2), (3*sqrt(-a)*b*c*x^2*arctan(sqrt(b*x + a)*sqrt(-a)/a) - 3*sqrt(-a)*b*c*x^2*arctan(sqrt(c*x + a)*sqrt(-a)/a) - (2*a^2 + (2*a*b + 3*a*c)*x)*sqrt(b*x + a) + (2*a^2 + (3*a*b + 2*a*c)*x)*sqrt(c*x + a))/((a*b^3 - 3*a*b^2*c + 3*a*b*c^2 - a*c^3)*x^2)]
giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="giac")
[Out] Timed out
maple [B] time = 0.02, size = 300, normalized size = 1.83
```

$$\frac{8 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}} + \frac{-(bx+a)^{\frac{3}{2}} - \sqrt{bx+a}}{b^2 x^2} \right) a b^2}{(b-c)^3} - \frac{8 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}} + \frac{-(cx+a)^{\frac{3}{2}} - \sqrt{cx+a}}{c^2 x^2} \right) a c^2}{(b-c)^3} + \frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx} \right) b^2}{(b-c)^3} + \frac{6 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{bx+a}}{2bx} \right) bc}{(b-c)^3} - \frac{6 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{cx+a}}{2cx} \right) bc}{(b-c)^3} - \frac{2 \left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{cx+a}}{\sqrt{a}}\right)}{2\sqrt{a}} - \frac{\sqrt{cx+a}}{2cx} \right) c^2}{(b-c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x)
[Out] 2/(b-c)^3*b^2*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))+8/(b-c)^3*a*b^2*((-1/8*(b*x+a)^(3/2)/a-1/8*(b*x+a)^(1/2))/b^2/x^2+1/8/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))-8/(b-c)^3*a*c^2*((-1/8*(c*x+a)^(3/2)/a-1/8*(c*x+a)^(1/2))/x^2/c^2+1/8/a^(3/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))+6/(b-c)^3*c*b*(-1/2*(b*x+a)^(1/2)/b/x-1/2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2))-6/(b-c)^3*b*c*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))-2/(b-c)^3*c^2*(-1/2*(c*x+a)^(1/2)/c/x-1/2/a^(1/2)*arctanh((c*x+a)^(1/2)/a^(1/2)))
maxima [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{(\sqrt{bx+a} + \sqrt{cx+a})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)^(1/2)+(c*x+a)^(1/2))^3,x, algorithm="maxima")
[Out] integrate((sqrt(b*x + a) + sqrt(c*x + a))^(-3), x)
mupad [B] time = 5.74, size = 287, normalized size = 1.75
```

$$\frac{c^2(\sqrt{a+bx}-\sqrt{a})^2}{4\sqrt{a}(b-c)^3(\sqrt{a+cx}-\sqrt{a})^2} - \frac{\left(\frac{\sqrt{a}b^2}{4(ab^3-3ab^2c+3abc^2-ac^3)} - \frac{\sqrt{a}(b^2+cb)(\sqrt{a+bx}-\sqrt{a})}{(\sqrt{a+cx}-\sqrt{a})(ab^3-3ab^2c+3abc^2-ac^3)}\right)(\sqrt{a+cx}-\sqrt{a})^2}{(\sqrt{a+bx}-\sqrt{a})^2} + \frac{3bc \ln\left(\frac{\sqrt{a+bx}-\sqrt{a}}{\sqrt{a+cx}-\sqrt{a}}\right)}{\sqrt{a}(b^3-3b^2c+3bc^2-c^3)} - \frac{c(b+c)(\sqrt{a+bx}-\sqrt{a})}{\sqrt{a}(b-c)^3(\sqrt{a+cx}-\sqrt{a})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*x)^(1/2) + (a + c*x)^(1/2))^3,x)
[Out] (c^2*((a + b*x)^(1/2) - a^(1/2))^2)/(4*a^(1/2)*(b - c)^3*((a + c*x)^(1/2) - a^(1/2))^2) - (((a^(1/2)*b^2)/(4*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)) - (a^(1/2)*(b*c + b^2)*((a + b*x)^(1/2) - a^(1/2))))/(((a + c*x)^(1/2) - a^(1/2))
```



```

1/2))*(a*b^3 - a*c^3 + 3*a*b*c^2 - 3*a*b^2*c)))*((a + c*x)^(1/2) - a^(1/2))
^2)/((a + b*x)^(1/2) - a^(1/2))^2 + (3*b*c*log(((a + b*x)^(1/2) - a^(1/2))/
((a + c*x)^(1/2) - a^(1/2))))/(a^(1/2)*(3*b*c^2 - 3*b^2*c + b^3 - c^3)) - (
c*(b + c)*((a + b*x)^(1/2) - a^(1/2)))/(a^(1/2)*(b - c)^3*((a + c*x)^(1/2)
- a^(1/2)))

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sqrt{a+bx} + \sqrt{a+cx})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((b*x+a)**(1/2)+(c*x+a)**(1/2))**3,x)
```

```
[Out] Integral((sqrt(a + b*x) + sqrt(a + c*x))**(-3), x)
```

$$3.220 \quad \int \sqrt{1-x} \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$$

Optimal. Leaf size=31

$$-\frac{x^2}{2} + \frac{1}{2}\sqrt{1-x^2}x + x + \frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.05, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6688, 195, 216}

$$-\frac{x^2}{2} + \frac{1}{2}\sqrt{1-x^2}x + x + \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] x - x^2/2 + (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned} \int \sqrt{1-x} \left(\sqrt{1-x} + \sqrt{1+x} \right) dx &= \int \left(1-x + \sqrt{1-x^2} \right) dx \\ &= x - \frac{x^2}{2} + \int \sqrt{1-x^2} dx \\ &= x - \frac{x^2}{2} + \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx \\ &= x - \frac{x^2}{2} + \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{x^2}{2} + \frac{1}{2}\sqrt{1-x^2}x + x + \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] $x - x^2/2 + (x\sqrt{1 - x^2})/2 + \text{ArcSin}[x]/2$

IntegrateAlgebraic [C] time = 0.14, size = 74, normalized size = 2.39

$$\frac{1}{2}\sqrt{1-x}\left((x+1)^{3/2} - \sqrt{x+1}\right) + \frac{1}{2}\left(4(x+1) - (x+1)^2\right) + i\log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x]*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] $(\text{Sqrt}[1 - x]*(-\text{Sqrt}[1 + x] + (1 + x)^{(3/2)}))/2 + (4*(1 + x) - (1 + x)^2)/2 + I*\text{Log}[\text{Sqrt}[1 - x] - I*\text{Sqrt}[1 + x]]$

fricas [A] time = 0.40, size = 44, normalized size = 1.42

$$-\frac{1}{2}x^2 + \frac{1}{2}\sqrt{x+1}x\sqrt{-x+1} + x - \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] $-1/2*x^2 + 1/2*\text{sqrt}(x + 1)*x*\text{sqrt}(-x + 1) + x - \arctan((\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) - 1)/x)$

giac [B] time = 0.61, size = 54, normalized size = 1.74

$$-\frac{1}{2}(x-1)^2 + \frac{1}{2}(x+2)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}\sqrt{-x+1} - \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{-x+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] $-1/2*(x - 1)^2 + 1/2*(x + 2)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) - \text{sqrt}(x + 1)*\text{sqrt}(-x + 1) - \arcsin(1/2*\text{sqrt}(2)*\text{sqrt}(-x + 1))$

maple [B] time = 0.00, size = 63, normalized size = 2.03

$$-\frac{x^2}{2} + x + \frac{\sqrt{(x+1)(-x+1)} \arcsin(x)}{2\sqrt{-x+1}\sqrt{x+1}} - \frac{\sqrt{x+1}(-x+1)^{3/2}}{2} + \frac{\sqrt{-x+1}\sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)*((-x+1)^(1/2)+(x+1)^(1/2)),x)

[Out] $x - 1/2*x^2 - 1/2*(x+1)^{(1/2)}*(-x+1)^{(3/2)} + 1/2*(-x+1)^{(1/2)}*(x+1)^{(1/2)} + 1/2*((x+1)*(-x+1))^{(1/2)}/(-x+1)^{(1/2)}/(x+1)^{(1/2)}*\arcsin(x)$

maxima [A] time = 1.49, size = 23, normalized size = 0.74

$$-\frac{1}{2}x^2 + \frac{1}{2}\sqrt{-x^2+1}x + x + \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] $-1/2*x^2 + 1/2*\text{sqrt}(-x^2 + 1)*x + x + 1/2*\arcsin(x)$

mupad [B] time = 8.12, size = 209, normalized size = 6.74

$$x - 2 \operatorname{atan} \left(\frac{\sqrt{1-x} - 1}{\sqrt{x+1} - 1} \right) - \frac{\frac{2(\sqrt{1-x}-1)}{\sqrt{x+1}-1} - \frac{14(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3} + \frac{14(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5} - \frac{2(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7}}{\frac{4(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{6(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + \frac{4(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6} + \frac{(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8} + 1} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x + 1)^(1/2) + (1 - x)^(1/2))*(1 - x)^(1/2), x)`

[Out] `x - 2*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - ((2*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (14*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (14*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (2*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1) - x^2/2`

sympy [A] time = 3.07, size = 48, normalized size = 1.55

$$-\frac{(1-x)^2}{2} - 2 \left(\left(-\frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{1-x}}{2}\right)}{2} \right) \text{ for } x \leq 1 \wedge x > -1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-x)**(1/2)*((1-x)**(1/2)+(1+x)**(1/2)), x)`

[Out] `-(1 - x)**2/2 - 2*Piecewise((-x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(1 - x)/2)/2, (x <= 1) & (x > -1))`

$$3.221 \quad \int x^3 \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$$

Optimal. Leaf size=38

$$-\frac{x^4}{2} - \frac{2}{5}(1-x^2)^{5/2} + \frac{2}{3}(1-x^2)^{3/2}$$

Rubi [A] time = 0.32, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6688, 6742, 266, 43}

$$-\frac{x^4}{2} - \frac{2}{5}(1-x^2)^{5/2} + \frac{2}{3}(1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -x^4/2 + (2*(1 - x^2)^(3/2))/3 - (2*(1 - x^2)^(5/2))/5

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x^3 \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx &= - \int x^3 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx \\ &= - \int \left(2x^3 + 2x^3\sqrt{1-x^2} \right) dx \\ &= -\frac{x^4}{2} - 2 \int x^3\sqrt{1-x^2} dx \\ &= -\frac{x^4}{2} - \text{Subst} \left(\int \sqrt{1-x} x dx, x, x^2 \right) \\ &= -\frac{x^4}{2} - \text{Subst} \left(\int \left(\sqrt{1-x} - (1-x)^{3/2} \right) dx, x, x^2 \right) \\ &= -\frac{x^4}{2} + \frac{2}{3}(1-x^2)^{3/2} - \frac{2}{5}(1-x^2)^{5/2} \end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.00

$$-\frac{x^4}{2} - \frac{2}{5}(1-x^2)^{5/2} + \frac{2}{3}(1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(-Sqrt[1-x]-Sqrt[1+x])*(Sqrt[1-x]+Sqrt[1+x]),x]

[Out] -1/2*x^4 + (2*(1-x^2)^(3/2))/3 - (2*(1-x^2)^(5/2))/5

IntegrateAlgebraic [B] time = 0.20, size = 82, normalized size = 2.16

$$\frac{1}{2}(-(x+1)^4 + 4(x+1)^3 - 6(x+1)^2 + 4(x+1)) - \frac{2}{15}\sqrt{1-x}(3(x+1)^{9/2} - 12(x+1)^{7/2} + 17(x+1)^{5/2} - 10(x+1)^{3/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(-Sqrt[1-x]-Sqrt[1+x])*(Sqrt[1-x]+Sqrt[1+x]),x]

[Out] (4*(1+x) - 6*(1+x)^2 + 4*(1+x)^3 - (1+x)^4)/2 - (2*Sqrt[1-x]*(-10*(1+x)^(3/2) + 17*(1+x)^(5/2) - 12*(1+x)^(7/2) + 3*(1+x)^(9/2)))/15

fricas [A] time = 0.39, size = 32, normalized size = 0.84

$$-\frac{1}{2}x^4 - \frac{2}{15}(3x^4 - x^2 - 2)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -1/2*x^4 - 2/15*(3*x^4 - x^2 - 2)*sqrt(x + 1)*sqrt(-x + 1)

giac [B] time = 0.54, size = 77, normalized size = 2.03

$$-\frac{1}{2}x^4 - \frac{1}{60}((2(3(4x-17)(x+1)+133)(x+1)-295)(x+1)+195)\sqrt{x+1}\sqrt{-x+1} - \frac{1}{12}((2(3x-10)(x+1)+43)(x+1)-39)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] -1/2*x^4 - 1/60*((2*(3*(4*x - 17)*(x + 1) + 133)*(x + 1) - 295)*(x + 1) + 195)*sqrt(x + 1)*sqrt(-x + 1) - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(-x + 1)

maple [A] time = 0.01, size = 33, normalized size = 0.87

$$-\frac{x^4}{2} - \frac{2\sqrt{x+1}\sqrt{-x+1}(x^2-1)(3x^2+2)}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)

[Out] -1/2*x^4-2/15*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)*(3*x^2+2)

maxima [A] time = 1.38, size = 31, normalized size = 0.82

$$-\frac{1}{2}x^4 + \frac{2}{5}(-x^2+1)^{\frac{3}{2}}x^2 + \frac{4}{15}(-x^2+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] -1/2*x^4 + 2/5*(-x^2 + 1)^(3/2)*x^2 + 4/15*(-x^2 + 1)^(3/2)

mupad [B] time = 3.06, size = 42, normalized size = 1.11

$$\sqrt{1-x} \left(\frac{4\sqrt{x+1}}{15} + \frac{2x^2\sqrt{x+1}}{15} - \frac{2x^4\sqrt{x+1}}{5} \right) - \frac{x^4}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x^3*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)

[Out] (1 - x)^(1/2)*((4*(x + 1)^(1/2))/15 + (2*x^2*(x + 1)^(1/2))/15 - (2*x^4*(x + 1)^(1/2))/5) - x^4/2

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)

[Out] Timed out

$$3.222 \quad \int x^2 \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$$

Optimal. Leaf size=48

$$-\frac{2x^3}{3} + \frac{1}{4}\sqrt{1-x^2}x - \frac{1}{2}\sqrt{1-x^2}x^3 - \frac{1}{4}\sin^{-1}(x)$$

Rubi [A] time = 0.24, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.119$, Rules used = {6688, 6742, 279, 321, 216}

$$-\frac{1}{2}\sqrt{1-x^2}x^3 - \frac{2x^3}{3} + \frac{1}{4}\sqrt{1-x^2}x - \frac{1}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2*(-Sqrt[1-x]-Sqrt[1+x])*(Sqrt[1-x]+Sqrt[1+x]),x]

[Out] (-2*x^3)/3 + (x*Sqrt[1-x^2])/4 - (x^3*Sqrt[1-x^2])/2 - ArcSin[x]/4

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 279

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+n*p+1)), x] + Dist[(a*n*p)/(m+n*p+1), Int[(c*x)^m*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[n, 0] && GtQ[p, 0] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 321

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^(n*(m-n+1)))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int x^2 \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx &= - \int x^2 \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx \\
&= - \int \left(2x^2 + 2x^2 \sqrt{1-x^2} \right) dx \\
&= -\frac{2x^3}{3} - 2 \int x^2 \sqrt{1-x^2} dx \\
&= -\frac{2x^3}{3} - \frac{1}{2} x^3 \sqrt{1-x^2} - \frac{1}{2} \int \frac{x^2}{\sqrt{1-x^2}} dx \\
&= -\frac{2x^3}{3} + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{2} x^3 \sqrt{1-x^2} - \frac{1}{4} \int \frac{1}{\sqrt{1-x^2}} dx \\
&= -\frac{2x^3}{3} + \frac{1}{4} x \sqrt{1-x^2} - \frac{1}{2} x^3 \sqrt{1-x^2} - \frac{1}{4} \sin^{-1}(x)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.90

$$\frac{1}{12} \left(3\sqrt{1-x^2} x - \left((6\sqrt{1-x^2} + 8) x^3 \right) - 3 \sin^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(-Sqrt[1-x]-Sqrt[1+x])*(Sqrt[1-x]+Sqrt[1+x]),x]

[Out] (3*x*Sqrt[1-x^2]-x^3*(8+6*Sqrt[1-x^2])-3*ArcSin[x])/12

IntegrateAlgebraic [C] time = 0.19, size = 99, normalized size = 2.06

$$-\frac{2}{3} \left((x+1)^3 - 3(x+1)^2 + 3(x+1) \right) + \frac{1}{4} \sqrt{1-x} \left(-2(x+1)^{7/2} + 6(x+1)^{5/2} - 5(x+1)^{3/2} + \sqrt{x+1} \right) - \frac{1}{2} i \log \left(\sqrt{1-x} - i\sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(-Sqrt[1-x]-Sqrt[1+x])*(Sqrt[1-x]+Sqrt[1+x]),x]

[Out] (-2*(3*(1+x)-3*(1+x)^2+(1+x)^3))/3+(Sqrt[1-x]*(Sqrt[1+x]-5*(1+x)^(3/2)+6*(1+x)^(5/2)-2*(1+x)^(7/2)))/4-(I/2)*Log[Sqrt[1-x]-I*Sqrt[1+x]]

fricas [A] time = 0.39, size = 51, normalized size = 1.06

$$-\frac{2}{3} x^3 - \frac{1}{4} (2x^3 - x) \sqrt{x+1} \sqrt{-x+1} + \frac{1}{2} \arctan \left(\frac{\sqrt{x+1} \sqrt{-x+1} - 1}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -2/3*x^3-1/4*(2*x^3-x)*sqrt(x+1)*sqrt(-x+1)+1/2*arctan((sqrt(x+1)*sqrt(-x+1)-1)/x)

giac [B] time = 0.47, size = 76, normalized size = 1.58

$$-\frac{2}{3} x^3 - \frac{1}{12} ((2(3x-10)(x+1)+43)(x+1)-39) \sqrt{x+1} \sqrt{-x+1} - \frac{1}{3} ((2x-5)(x+1)+9) \sqrt{x+1} \sqrt{-x+1} - \frac{1}{2} \arcsin \left(\frac{1}{2} \sqrt{2} \sqrt{x+1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] $-2/3*x^3 - 1/12*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) - 1/3*((2*x - 5)*(x + 1) + 9)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) - 1/2*\text{arcsin}(1/2*\text{sqrt}(2)*\text{sqrt}(x + 1))$

maple [A] time = 0.00, size = 59, normalized size = 1.23

$$\frac{2x^3}{3} - \frac{\sqrt{x+1} \sqrt{-x+1} \left(2\sqrt{-x^2+1} x^3 - \sqrt{-x^2+1} x + \arcsin(x) \right)}{4\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)`

[Out] $-2/3*x^3-1/4*(x+1)^(1/2)*(-x+1)^(1/2)*(2*(-x^2+1)^(1/2)*x^3-(-x^2+1)^(1/2)*x+\arcsin(x))/(-x^2+1)^(1/2)$

maxima [A] time = 1.50, size = 34, normalized size = 0.71

$$-\frac{2}{3}x^3 + \frac{1}{2}(-x^2 + 1)^{\frac{3}{2}}x - \frac{1}{4}\sqrt{-x^2 + 1}x - \frac{1}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $-2/3*x^3 + 1/2*(-x^2 + 1)^(3/2)*x - 1/4*\text{sqrt}(-x^2 + 1)*x - 1/4*\text{arcsin}(x)$

mupad [B] time = 10.39, size = 381, normalized size = 7.94

$$\text{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - \frac{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{35(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3} + \frac{273(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5} - \frac{715(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7} + \frac{715(\sqrt{1-x}-1)^9}{(\sqrt{x+1}-1)^9} - \frac{273(\sqrt{1-x}-1)^{11}}{(\sqrt{x+1}-1)^{11}} + \frac{35(\sqrt{1-x}-1)^{13}}{(\sqrt{x+1}-1)^{13}} - \frac{(\sqrt{1-x}-1)^{15}}{(\sqrt{x+1}-1)^{15}}}{\frac{8(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{28(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + \frac{56(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6} + \frac{70(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8} + \frac{56(\sqrt{1-x}-1)^{10}}{(\sqrt{x+1}-1)^{10}} + \frac{28(\sqrt{1-x}-1)^{12}}{(\sqrt{x+1}-1)^{12}} + \frac{8(\sqrt{1-x}-1)^{14}}{(\sqrt{x+1}-1)^{14}} + \frac{(\sqrt{1-x}-1)^{16}}{(\sqrt{x+1}-1)^{16}} + 1} - \frac{2x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x^2*((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)`

[Out] $\text{atan}\left(\frac{(1-x)^{1/2}-1}{(x+1)^{1/2}-1}\right) - \frac{((1-x)^{1/2}-1)/((x+1)^{1/2}-1) - (35*((1-x)^{1/2}-1)^3)/((x+1)^{1/2}-1)^3 + (273*((1-x)^{1/2}-1)^5)/((x+1)^{1/2}-1)^5 - (715*((1-x)^{1/2}-1)^7)/((x+1)^{1/2}-1)^7 + (715*((1-x)^{1/2}-1)^9)/((x+1)^{1/2}-1)^9 - (273*((1-x)^{1/2}-1)^{11})/((x+1)^{1/2}-1)^{11} + (35*((1-x)^{1/2}-1)^{13})/((x+1)^{1/2}-1)^{13} - ((1-x)^{1/2}-1)^{15}/((x+1)^{1/2}-1)^{15}}{(8*((1-x)^{1/2}-1)^2)/((x+1)^{1/2}-1)^2 + (28*((1-x)^{1/2}-1)^4)/((x+1)^{1/2}-1)^4 + (56*((1-x)^{1/2}-1)^6)/((x+1)^{1/2}-1)^6 + (70*((1-x)^{1/2}-1)^8)/((x+1)^{1/2}-1)^8 + (56*((1-x)^{1/2}-1)^{10})/((x+1)^{1/2}-1)^{10} + (28*((1-x)^{1/2}-1)^{12})/((x+1)^{1/2}-1)^{12} + (8*((1-x)^{1/2}-1)^{14})/((x+1)^{1/2}-1)^{14} + ((1-x)^{1/2}-1)^{16}/((x+1)^{1/2}-1)^{16} + 1} - (2*x^3)/3$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)`

[Out] Timed out

$$3.223 \quad \int x \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$$

Optimal. Leaf size=21

$$\frac{2}{3} (1-x^2)^{3/2} - x^2$$

Rubi [A] time = 0.11, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {6688, 6742, 261}

$$\frac{2}{3} (1-x^2)^{3/2} - x^2$$

Antiderivative was successfully verified.

[In] Int[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -x^2 + (2*(1 - x^2)^(3/2))/3

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int x \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx &= - \int x \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx \\ &= - \int \left(2x + 2x\sqrt{1-x^2} \right) dx \\ &= -x^2 - 2 \int x\sqrt{1-x^2} dx \\ &= -x^2 + \frac{2}{3} (1-x^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.00

$$\frac{2}{3} (1-x^2)^{3/2} - x^2$$

Antiderivative was successfully verified.

[In] Integrate[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -x^2 + (2*(1 - x^2)^(3/2))/3

IntegrateAlgebraic [B] time = 0.15, size = 43, normalized size = 2.05

$$-(x+1)^2 + 2(x+1) - \frac{2}{3}\sqrt{1-x} \left((x+1)^{5/2} - 2(x+1)^{3/2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] $2*(1 + x) - (1 + x)^2 - (2*\text{Sqrt}[1 - x]*(-2*(1 + x)^{(3/2)} + (1 + x)^{(5/2)}))/3$

fricas [A] time = 0.42, size = 25, normalized size = 1.19

$$-x^2 - \frac{2}{3}(x^2 - 1)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] $-x^2 - 2/3*(x^2 - 1)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1)$

giac [B] time = 0.39, size = 54, normalized size = 2.57

$$-(x+1)^2 - \frac{1}{3}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1} - \sqrt{x+1}(x-2)\sqrt{-x+1} + 2x+2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] $-(x+1)^2 - 1/3*((2*x-5)*(x+1)+9)*\text{sqrt}(x+1)*\text{sqrt}(-x+1) - \text{sqrt}(x+1)*(x-2)*\text{sqrt}(-x+1) + 2*x+2$

maple [A] time = 0.00, size = 26, normalized size = 1.24

$$-x^2 - \frac{2\sqrt{x+1}\sqrt{-x+1}(x^2-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)

[Out] $-x^2 - 2/3*(x+1)^{(1/2)}*(-x+1)^{(1/2)}*(x^2-1)$

maxima [A] time = 1.47, size = 17, normalized size = 0.81

$$-x^2 + \frac{2}{3}(-x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-(1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] $-x^2 + 2/3*(-x^2 + 1)^{(3/2)}$

mupad [B] time = 3.06, size = 25, normalized size = 1.19

$$-x^2 - \frac{2(x^2 - 1)\sqrt{1-x}\sqrt{x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x*((x+1)^(1/2)+(1-x)^(1/2))^2,x)

[Out] $-x^2 - (2*(x^2 - 1)*(1 - x)^{(1/2)}*(x + 1)^{(1/2)})/3$

sympy [A] time = 101.26, size = 110, normalized size = 5.24

$$\frac{x^3}{3} + x - \frac{(x+1)^3}{3} + 4 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \text{ for } x \geq -1 \wedge x < 1 \right\} - 4 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \text{ for } x \geq -1 \wedge x < 1 \right\} + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-(1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)`

[Out] `x**3/3 + x - (x + 1)**3/3 + 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 1`

$$3.224 \quad \int \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx$$

Optimal. Leaf size=22

$$-\sqrt{1-x^2}x - 2x - \sin^{-1}(x)$$

Rubi [A] time = 0.05, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {6688, 6742, 195, 216}

$$-\sqrt{1-x^2}x - 2x - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -2*x - x*Sqrt[1 - x^2] - ArcSin[x]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \left(-\sqrt{1-x} - \sqrt{1+x} \right) \left(\sqrt{1-x} + \sqrt{1+x} \right) dx &= - \int \left(\sqrt{1-x} + \sqrt{1+x} \right)^2 dx \\ &= - \int \left(2 + 2\sqrt{1-x^2} \right) dx \\ &= -2x - 2 \int \sqrt{1-x^2} dx \\ &= -2x - x\sqrt{1-x^2} - \int \frac{1}{\sqrt{1-x^2}} dx \\ &= -2x - x\sqrt{1-x^2} - \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 21, normalized size = 0.95

$$-x \left(\sqrt{1-x^2} + 2 \right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -(x*(2 + Sqrt[1 - x^2])) - ArcSin[x]

IntegrateAlgebraic [C] time = 0.14, size = 59, normalized size = 2.68

$$-2(x+1) + \sqrt{1-x} \left(\sqrt{x+1} - (x+1)^{3/2} \right) - 2i \log \left(\sqrt{1-x} - i\sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]),x]

[Out] -2*(1 + x) + Sqrt[1 - x]*(Sqrt[1 + x] - (1 + x)^(3/2)) - (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [B] time = 0.39, size = 41, normalized size = 1.86

$$-\sqrt{x+1}x\sqrt{-x+1} - 2x + 2 \arctan \left(\frac{\sqrt{x+1}\sqrt{-x+1} - 1}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -sqrt(x + 1)*x*sqrt(-x + 1) - 2*x + 2*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [B] time = 0.37, size = 49, normalized size = 2.23

$$-\sqrt{x+1}(x-2)\sqrt{-x+1} - 2x - 2\sqrt{x+1}\sqrt{-x+1} - 2 \arcsin \left(\frac{1}{2} \sqrt{2}\sqrt{x+1} \right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] -sqrt(x + 1)*(x - 2)*sqrt(-x + 1) - 2*x - 2*sqrt(x + 1)*sqrt(-x + 1) - 2*arcsin(1/2*sqrt(2)*sqrt(x + 1)) - 2

maple [B] time = 0.00, size = 59, normalized size = 2.68

$$-2x - \frac{\sqrt{(x+1)(-x+1)} \arcsin(x)}{\sqrt{-x+1}\sqrt{x+1}} + \sqrt{x+1}(-x+1)^{3/2} - \sqrt{-x+1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2)),x)

[Out] -2*x+(x+1)^(1/2)*(-x+1)^(3/2)-(-x+1)^(1/2)*(x+1)^(1/2)-((x+1)*(-x+1))^(1/2)/(-x+1)^(1/2)/(x+1)^(1/2)*arcsin(x)

maxima [A] time = 1.39, size = 20, normalized size = 0.91

$$-\sqrt{-x^2+1}x - 2x - \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] -sqrt(-x^2 + 1)*x - 2*x - arcsin(x)

mupad [B] time = 3.71, size = 205, normalized size = 9.32

$$4 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - 2x + \frac{\frac{4(\sqrt{1-x}-1)}{\sqrt{x+1}-1} - \frac{28(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3} + \frac{28(\sqrt{1-x}-1)^5}{(\sqrt{x+1}-1)^5} - \frac{4(\sqrt{1-x}-1)^7}{(\sqrt{x+1}-1)^7}}{\frac{4(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{6(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + \frac{4(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6} + \frac{(\sqrt{1-x}-1)^8}{(\sqrt{x+1}-1)^8} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x + 1)^(1/2) + (1 - x)^(1/2))^2,x)

[Out] 4*atan(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) - 2*x + ((4*((1 - x)^(1/2) - 1))/((x + 1)^(1/2) - 1) - (28*((1 - x)^(1/2) - 1)^3)/((x + 1)^(1/2) - 1)^3 + (28*((1 - x)^(1/2) - 1)^5)/((x + 1)^(1/2) - 1)^5 - (4*((1 - x)^(1/2) - 1)^7)/((x + 1)^(1/2) - 1)^7)/((4*((1 - x)^(1/2) - 1)^2)/((x + 1)^(1/2) - 1)^2 + (6*((1 - x)^(1/2) - 1)^4)/((x + 1)^(1/2) - 1)^4 + (4*((1 - x)^(1/2) - 1)^6)/((x + 1)^(1/2) - 1)^6 + ((1 - x)^(1/2) - 1)^8/((x + 1)^(1/2) - 1)^8 + 1)

sympy [A] time = 37.87, size = 46, normalized size = 2.09

$$-2x - 4 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \quad \text{for } x \geq -1 \wedge x < 1 \right\} \right) - 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2)),x)

[Out] -2*x - 4*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) - 2

$$3.225 \quad \int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x} dx$$

Optimal. Leaf size=32

$$-2\sqrt{1-x^2} + 2 \tanh^{-1}(\sqrt{1-x^2}) - 2 \log(x)$$

Rubi [A] time = 0.20, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6688, 6742, 266, 50, 63, 206}

$$-2\sqrt{1-x^2} + 2 \tanh^{-1}(\sqrt{1-x^2}) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]

[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x} dx &= - \int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x} dx \\
&= - \int \left(\frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} \right) dx \\
&= -2 \log(x) - 2 \int \frac{\sqrt{1-x^2}}{x} dx \\
&= -2 \log(x) - \text{Subst} \left(\int \frac{\sqrt{1-x}}{x} dx, x, x^2 \right) \\
&= -2\sqrt{1-x^2} - 2 \log(x) - \text{Subst} \left(\int \frac{1}{\sqrt{1-x}x} dx, x, x^2 \right) \\
&= -2\sqrt{1-x^2} - 2 \log(x) + 2 \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \\
&= -2\sqrt{1-x^2} + 2 \tanh^{-1}(\sqrt{1-x^2}) - 2 \log(x)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 32, normalized size = 1.00

$$-2\sqrt{1-x^2} + 2 \tanh^{-1}(\sqrt{1-x^2}) - 2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]

[Out] -2*Sqrt[1 - x^2] + 2*ArcTanh[Sqrt[1 - x^2]] - 2*Log[x]

IntegrateAlgebraic [C] time = 0.17, size = 75, normalized size = 2.34

$$-2\sqrt{1-x}\sqrt{x+1} - 4 \log(\sqrt{1-x} - i\sqrt{x+1} + (1-i)) - 8 \tanh^{-1}((-1-i)\sqrt{1-x} - (1-i)\sqrt{x+1} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x,x]

[Out] -2*Sqrt[1 - x]*Sqrt[1 + x] - 8*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sqrt[1 + x]] - 4*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.40, size = 41, normalized size = 1.28

$$-2 \sqrt{x+1} \sqrt{-x+1} - 2 \log(x) - 2 \log\left(\frac{\sqrt{x+1} \sqrt{-x+1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorithm="fricas")

[Out] -2*sqrt(x + 1)*sqrt(-x + 1) - 2*log(x) - 2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086]-2*ln(abs(sqrt(x+1)-1))-2*ln(sqrt(x+1)+1)-2*sqrt(x+1)*sqrt(-x+1)+2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))-2*ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))

maple [A] time = 0.01, size = 51, normalized size = 1.59

$$-2\ln(x) - \frac{2\sqrt{x+1}\sqrt{-x+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x,x)

[Out] -2*ln(x)-2*(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2)))

maxima [A] time = 1.95, size = 41, normalized size = 1.28

$$-2\sqrt{-x^2+1} - 2\log(x) + 2\log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x,x, algorithm="maxima")

[Out] -2*sqrt(-x^2 + 1) - 2*log(x) + 2*log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

mupad [B] time = 4.13, size = 122, normalized size = 3.81

$$2\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) - 2\ln\left(\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} - 1\right) - 2\ln(x) - \frac{16(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2\left(\frac{2(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2} + \frac{(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x+1)^(1/2)+(1-x)^(1/2))^2/x,x)

[Out] 2*log(((1-x)^(1/2)-1)/((x+1)^(1/2)-1))-2*log(((1-x)^(1/2)-1)^2/((x+1)^(1/2)-1)^2-1)-2*log(x)-(16*((1-x)^(1/2)-1)^2)/(((x+1)^(1/2)-1)^2*((2*((1-x)^(1/2)-1)^2)/((x+1)^(1/2)-1)^2+((1-x)^(1/2)-1)^4/((x+1)^(1/2)-1)^4+1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x} dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x,x)

[Out] -Integral(2/x, x) - Integral(2*sqrt(1-x)*sqrt(x+1)/x, x)

$$3.226 \quad \int \frac{(-\sqrt{1-x}-\sqrt{1+x})(\sqrt{1-x}+\sqrt{1+x})}{x^2} dx$$

Optimal. Leaf size=26

$$\frac{2\sqrt{1-x^2}}{x} + \frac{2}{x} + 2\sin^{-1}(x)$$

Rubi [A] time = 0.21, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6688, 6742, 277, 216}

$$\frac{2\sqrt{1-x^2}}{x} + \frac{2}{x} + 2\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]

[Out] 2/x + (2*Sqrt[1 - x^2])/x + 2*ArcSin[x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{(-\sqrt{1-x}-\sqrt{1+x})(\sqrt{1-x}+\sqrt{1+x})}{x^2} dx &= - \int \frac{(\sqrt{1-x}+\sqrt{1+x})^2}{x^2} dx \\ &= - \int \left(\frac{2}{x^2} + \frac{2\sqrt{1-x^2}}{x^2} \right) dx \\ &= \frac{2}{x} - 2 \int \frac{\sqrt{1-x^2}}{x^2} dx \\ &= \frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} + 2 \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{2}{x} + \frac{2\sqrt{1-x^2}}{x} + 2\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 0.85

$$\frac{2\left(\sqrt{1-x^2} + x \sin^{-1}(x) + 1\right)}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]

[Out] (2*(1 + Sqrt[1 - x^2] + x*ArcSin[x]))/x

IntegrateAlgebraic [C] time = 0.17, size = 53, normalized size = 2.04

$$\frac{2\sqrt{1-x}\sqrt{x+1}}{x} + \frac{2}{x} + 4i \log\left(\sqrt{1-x} - i\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^2,x]

[Out] 2/x + (2*Sqrt[1 - x]*Sqrt[1 + x])/x + (4*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.41, size = 44, normalized size = 1.69

$$\frac{2\left(2x \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) - \sqrt{x+1}\sqrt{-x+1} - 1\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algorithm="fricas")

[Out] -2*(2*x*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) - sqrt(x + 1)*sqrt(-x + 1) - 1)/x

giac [B] time = 0.46, size = 149, normalized size = 5.73

$$2\pi + \frac{8\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)}{\left(\frac{\sqrt{2}-\sqrt{-x+1}}{\sqrt{x+1}} - \frac{\sqrt{x+1}}{\sqrt{2}-\sqrt{-x+1}}\right)^2 - 4} + \frac{2}{x} + 4 \arctan\left(\frac{\sqrt{x+1}\left(\frac{(\sqrt{2}-\sqrt{-x+1})^2}{x+1} - 1\right)}{2(\sqrt{2}-\sqrt{-x+1})}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algorithm="giac")

[Out] 2*pi + 8*((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))/(((sqrt(2) - sqrt(-x + 1))/sqrt(x + 1) - sqrt(x + 1)/(sqrt(2) - sqrt(-x + 1)))^2 - 4) + 2/x + 4*arctan(1/2*sqrt(x + 1)*((sqrt(2) - sqrt(-x + 1))^2/(x + 1) - 1)/(sqrt(2) - sqrt(-x + 1)))

maple [B] time = 0.01, size = 50, normalized size = 1.92

$$\frac{2}{x} - \frac{2\left(-x \arcsin(x) - \sqrt{-x^2 + 1}\right)\sqrt{x+1}\sqrt{-x+1}}{\sqrt{-x^2 + 1}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^2,x)`

[Out] `2/x-2*(-arcsin(x)*x-(-x^2+1)^(1/2))*(x+1)^(1/2)*(-x+1)^(1/2)/x/(-x^2+1)^(1/2)`

maxima [A] time = 1.90, size = 24, normalized size = 0.92

$$\frac{2\sqrt{-x^2+1}}{x} + \frac{2}{x} + 2\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-(-1-x)^(1/2)-(1+x)^(1/2))*((-1-x)^(1/2)+(1+x)^(1/2))/x^2,x, algorithm="maxima")`

[Out] `2*sqrt(-x^2 + 1)/x + 2/x + 2*arcsin(x)`

mupad [B] time = 3.79, size = 118, normalized size = 4.54

$$\frac{\frac{5(\sqrt{1-x}-1)^2}{2(\sqrt{x+1}-1)^2} - \frac{1}{2}}{\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1} - \frac{(\sqrt{1-x}-1)^3}{(\sqrt{x+1}-1)^3}} - 8 \operatorname{atan}\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + \frac{\sqrt{1-x}-1}{2(\sqrt{x+1}-1)} + \frac{2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((x+1)^(1/2)+(1-x)^(1/2))^2/x^2,x)`

[Out] `((5*((1-x)^(1/2)-1)^2)/(2*((x+1)^(1/2)-1)^2)-1/2)/(((1-x)^(1/2)-1)/((x+1)^(1/2)-1)-((1-x)^(1/2)-1)^3/((x+1)^(1/2)-1)^3)-8*atan(((1-x)^(1/2)-1)/((x+1)^(1/2)-1))+((1-x)^(1/2)-1)/(2*((x+1)^(1/2)-1))+2/x`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x^2} dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-(-1-x)**(1/2)-(1+x)**(1/2))*((-1-x)**(1/2)+(1+x)**(1/2))/x**2,x)`

[Out] `-Integral(2/x**2, x) - Integral(2*sqrt(1-x)*sqrt(x+1)/x**2, x)`

$$3.227 \quad \int \frac{(-\sqrt{1-x}-\sqrt{1+x})(\sqrt{1-x}+\sqrt{1+x})}{x^3} dx$$

Optimal. Leaf size=33

$$\frac{\sqrt{1-x^2}}{x^2} + \frac{1}{x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Rubi [A] time = 0.22, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6688, 6742, 266, 47, 63, 206}

$$\frac{\sqrt{1-x^2}}{x^2} + \frac{1}{x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]

[Out] x^(-2) + Sqrt[1 - x^2]/x^2 - ArcTanh[Sqrt[1 - x^2]]

Rule 47

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{(-\sqrt{1-x} - \sqrt{1+x})(\sqrt{1-x} + \sqrt{1+x})}{x^3} dx &= -\int \frac{(\sqrt{1-x} + \sqrt{1+x})^2}{x^3} dx \\
&= -\int \left(\frac{2}{x^3} + \frac{2\sqrt{1-x^2}}{x^3} \right) dx \\
&= \frac{1}{x^2} - 2 \int \frac{\sqrt{1-x^2}}{x^3} dx \\
&= \frac{1}{x^2} - \text{Subst} \left(\int \frac{\sqrt{1-x}}{x^2} dx, x, x^2 \right) \\
&= \frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-xx}} dx, x, x^2 \right) \\
&= \frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} - \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \\
&= \frac{1}{x^2} + \frac{\sqrt{1-x^2}}{x^2} - \tanh^{-1}(\sqrt{1-x^2})
\end{aligned}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.39

$$\frac{1}{x^2\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} + \frac{1}{x^2} - \tanh^{-1}(\sqrt{1-x^2})$$

Antiderivative was successfully verified.

[In] Integrate[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]

[Out] x^(-2) - 1/Sqrt[1 - x^2] + 1/(x^2*Sqrt[1 - x^2]) - ArcTanh[Sqrt[1 - x^2]]

IntegrateAlgebraic [C] time = 0.25, size = 51, normalized size = 1.55

$$\frac{\sqrt{1-x}\sqrt{x+1}}{x^2} + \frac{1}{x^2} + 2i \tan^{-1}(x + i\sqrt{1-x}\sqrt{x+1})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-Sqrt[1 - x] - Sqrt[1 + x])*(Sqrt[1 - x] + Sqrt[1 + x]))/x^3,x]

[Out] x^(-2) + (Sqrt[1 - x]*Sqrt[1 + x])/x^2 + (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sqrt[1 + x]]

fricas [A] time = 0.40, size = 43, normalized size = 1.30

$$\frac{x^2 \log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + \sqrt{x+1}\sqrt{-x+1} + 1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algorithm="fricas")

[Out] (x^2*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + sqrt(x + 1)*sqrt(-x + 1) + 1)/x^2

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086] (4*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^3+16*(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))/((2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-4)^2-ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+1/x^2

maple [A] time = 0.02, size = 57, normalized size = 1.73

$$\frac{1}{x^2} - \frac{\sqrt{x+1} \sqrt{-x+1} \left(x^2 \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \sqrt{-x^2+1} \right)}{\sqrt{-x^2+1} x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-(-x+1)^(1/2)-(x+1)^(1/2))*((-x+1)^(1/2)+(x+1)^(1/2))/x^3,x)

[Out] 1/x^2-(x+1)^(1/2)*(-x+1)^(1/2)*(arctanh(1/(-x^2+1)^(1/2)))*x^2-(-x^2+1)^(1/2)/x^2/(-x^2+1)^(1/2)

maxima [A] time = 2.02, size = 51, normalized size = 1.55

$$\sqrt{-x^2+1} + \frac{(-x^2+1)^{\frac{3}{2}}}{x^2} + \frac{1}{x^2} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)^(1/2)-(1+x)^(1/2))*((1-x)^(1/2)+(1+x)^(1/2))/x^3,x, algorithm="maxima")

[Out] sqrt(-x^2 + 1) + (-x^2 + 1)^(3/2)/x^2 + 1/x^2 - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

mupad [B] time = 4.78, size = 186, normalized size = 5.64

$$\ln\left(\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}-1\right)-\ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right)-\frac{(\sqrt{1-x}-1)^2}{16(\sqrt{x+1}-1)^2}+\frac{\frac{(\sqrt{1-x}-1)^2}{8(\sqrt{x+1}-1)^2}+\frac{15(\sqrt{1-x}-1)^4}{16(\sqrt{x+1}-1)^4}-\frac{1}{16}}{\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}-\frac{2(\sqrt{1-x}-1)^4}{(\sqrt{x+1}-1)^4}+\frac{(\sqrt{1-x}-1)^6}{(\sqrt{x+1}-1)^6}}+\frac{1}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x+1)^(1/2)+(1-x)^(1/2))^2/x^3,x)

[Out] log(((1-x)^(1/2)-1)^2/((x+1)^(1/2)-1)^2-1)-log(((1-x)^(1/2)-1)/((x+1)^(1/2)-1))-((1-x)^(1/2)-1)^2/(16*((x+1)^(1/2)-1)^2)+(((1-x)^(1/2)-1)^2/(8*((x+1)^(1/2)-1)^2)+(15*((1-x)^(1/2)-1)^4)/(16*((x+1)^(1/2)-1)^4)-1/16)/(((1-x)^(1/2)-1)^2/((x+1)^(1/2)-1)^2-(2*((1-x)^(1/2)-1)^4)/((x+1)^(1/2)-1)^4+((1-x)^(1/2)-1)^6/((x+1)^(1/2)-1)^6)+1/x^2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2}{x^3} dx - \int \frac{2\sqrt{1-x}\sqrt{x+1}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1-x)**(1/2)-(1+x)**(1/2))*((1-x)**(1/2)+(1+x)**(1/2))/x**3,x)

[Out] -Integral(2/x**3, x) - Integral(2*sqrt(1 - x)*sqrt(x + 1)/x**3, x)

$$3.228 \quad \int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx$$

Optimal. Leaf size=28

$$\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + \log(x)$$

Rubi [A] time = 0.32, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.180$, Rules used = {2103, 6688, 14, 266, 50, 63, 206}

$$\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + \log(x)$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]),x]
```

```
[Out] Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]] + Log[x]
```

Rule 14

```
Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 50

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[ ((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2103

```
Int[(u_)/((e_)*Sqrt[(a_) + (b_)*(x_)] + (f_)*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Dist[c/(e*(b*c - a*d)), Int[(u*Sqrt[a + b*x])/x, x], x] - Dist[a/(f*(b*c - a*d)), Int[(u*Sqrt[c + d*x])/x, x], x] /; FreeQ[{a, b, c, d,
```

e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a*e^2 - c*f^2, 0]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{1-x} + \sqrt{1+x}}{-\sqrt{1-x} + \sqrt{1+x}} dx &= \frac{1}{2} \int \frac{\sqrt{1-x} (\sqrt{1-x} + \sqrt{1+x})}{x} dx + \frac{1}{2} \int \frac{\sqrt{1+x} (\sqrt{1-x} + \sqrt{1+x})}{x} dx \\
 &= \frac{1}{2} \int \frac{1-x + \sqrt{1-x^2}}{x} dx + \frac{1}{2} \int \frac{1+x + \sqrt{1-x^2}}{x} dx \\
 &= \frac{1}{2} \int \left(-1 + \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right) dx + \frac{1}{2} \int \left(1 + \frac{1}{x} + \frac{\sqrt{1-x^2}}{x} \right) dx \\
 &= \log(x) + 2 \left(\frac{1}{2} \int \frac{\sqrt{1-x^2}}{x} dx \right) \\
 &= \log(x) + 2 \left(\frac{1}{4} \text{Subst} \left(\int \frac{\sqrt{1-x}}{x} dx, x, x^2 \right) \right) \\
 &= \log(x) + 2 \left(\frac{\sqrt{1-x^2}}{2} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{\sqrt{1-x}x} dx, x, x^2 \right) \right) \\
 &= \log(x) + 2 \left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2} \right) \right) \\
 &= 2 \left(\frac{\sqrt{1-x^2}}{2} - \frac{1}{2} \tanh^{-1}(\sqrt{1-x^2}) \right) + \log(x)
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 48, normalized size = 1.71

$$\sqrt{1-x^2} - \tanh^{-1}(\sqrt{1-x^2}) + \log(x) + 2 \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \sin^{-1}(x)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]), x]

[Out] Sqrt[1 - x^2] + 2*ArcSin[Sqrt[1 - x]/Sqrt[2]] + ArcSin[x] - ArcTanh[Sqrt[1 - x^2]] + Log[x]

IntegrateAlgebraic [C] time = 0.18, size = 74, normalized size = 2.64

$$\sqrt{1-x}\sqrt{x+1} + 2 \log(\sqrt{1-x} - i\sqrt{x+1} + (1-i)) + 4 \tanh^{-1}((-1-i)\sqrt{1-x} - (1-i)\sqrt{x+1} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 - x] + Sqrt[1 + x])/(-Sqrt[1 - x] + Sqrt[1 + x]), x]

[Out] Sqrt[1 - x]*Sqrt[1 + x] + 4*ArcTanh[1 - (1 + I)*Sqrt[1 - x] - (1 - I)*Sqrt[1 + x]] + 2*Log[(1 - I) + Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [A] time = 0.38, size = 36, normalized size = 1.29

$$\sqrt{x+1}\sqrt{-x+1} + \log(x) + \log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] sqrt(x + 1)*sqrt(-x + 1) + log(x) + log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086]ln(abs(sqrt(x+1)-1))+ln(sqrt(x+1)+1)+sqrt(x+1)*sqrt(-x+1)-ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))

maple [A] time = 0.01, size = 48, normalized size = 1.71

$$\ln(x) + \frac{\sqrt{x+1}\sqrt{-x+1}\left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \sqrt{-x^2+1}\right)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)+(x+1)^(1/2))/(-(-x+1)^(1/2)+(x+1)^(1/2)),x)

[Out] ln(x)+(x+1)^(1/2)*(-x+1)^(1/2)/(-x^2+1)^(1/2)*((-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1} + \sqrt{-x+1}}{\sqrt{x+1} - \sqrt{-x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)^(1/2)+(1+x)^(1/2))/(-(1-x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] integrate((sqrt(x + 1) + sqrt(-x + 1))/(sqrt(x + 1) - sqrt(-x + 1)), x)

mupad [B] time = 4.49, size = 93, normalized size = 3.32

$$\ln\left(\frac{(\sqrt{1-x}-1)^2}{(\sqrt{x+1}-1)^2}-1\right) - \ln\left(\frac{\sqrt{1-x}-1}{\sqrt{x+1}-1}\right) + \ln(x) - \frac{8(x-2\sqrt{x+1}+2)(x+2\sqrt{1-x}-2)}{(2\sqrt{x+1}+2\sqrt{1-x}-4)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x + 1)^(1/2) + (1 - x)^(1/2))/((x + 1)^(1/2) - (1 - x)^(1/2)),x)
```

```
[Out] log(((1 - x)^(1/2) - 1)^2/((x + 1)^(1/2) - 1)^2 - 1) - log(((1 - x)^(1/2) - 1)/((x + 1)^(1/2) - 1)) + log(x) - (8*(x - 2*(x + 1)^(1/2) + 2)*(x + 2*(1 - x)^(1/2) - 2))/(2*(x + 1)^(1/2) + 2*(1 - x)^(1/2) - 4)^2
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$-\int \frac{\sqrt{1-x}}{\sqrt{1-x}-\sqrt{x+1}} dx - \int \frac{\sqrt{x+1}}{\sqrt{1-x}-\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)**(1/2)+(1+x)**(1/2))/(-(1-x)**(1/2)+(1+x)**(1/2)),x)
```

```
[Out] -Integral(sqrt(1 - x)/(sqrt(1 - x) - sqrt(x + 1)), x) - Integral(sqrt(x + 1)/(sqrt(1 - x) - sqrt(x + 1)), x)
```

$$3.229 \quad \int \frac{-\sqrt{-1+x} + \sqrt{1+x}}{\sqrt{-1+x} + \sqrt{1+x}} dx$$

Optimal. Leaf size=33

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{x-1}\sqrt{x+1}x + \frac{1}{2}\cosh^{-1}(x)$$

Rubi [A] time = 0.14, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2104, 6742, 38, 52}

$$\frac{x^2}{2} - \frac{1}{2}\sqrt{x-1}\sqrt{x+1}x + \frac{1}{2}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]

[Out] x^2/2 - (Sqrt[-1 + x]*x*Sqrt[1 + x])/2 + ArcCosh[x]/2

Rule 38

Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(m_), x_Symbol] := Simp[(x*(a + b*x)^m*(c + d*x)^m)/(2*m + 1), x] + Dist[(2*a*c*m)/(2*m + 1), Int[(a + b*x)^(m - 1)*(c + d*x)^(m - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a*d, 0] && IGtQ[m + 1/2, 0]

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] := Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 2104

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := -Dist[d/(e*(b*c - a*d)), Int[u*Sqrt[a + b*x], x], x] + Dist[b/(f*(b*c - a*d)), Int[u*Sqrt[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[b*e^2 - d*f^2, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-\sqrt{-1+x} + \sqrt{1+x}}{\sqrt{-1+x} + \sqrt{1+x}} dx &= -\left(\frac{1}{2} \int \sqrt{-1+x} (-\sqrt{-1+x} + \sqrt{1+x}) dx\right) + \frac{1}{2} \int \sqrt{1+x} (-\sqrt{-1+x} + \sqrt{1+x}) dx \\ &= \frac{1}{2} \int (1+x - \sqrt{-1+x}\sqrt{1+x}) dx - \frac{1}{2} \int (1-x + \sqrt{-1+x}\sqrt{1+x}) dx \\ &= \frac{x^2}{2} - 2 \left(\frac{1}{2} \int \sqrt{-1+x}\sqrt{1+x} dx\right) \\ &= \frac{x^2}{2} - 2 \left(\frac{1}{4} \sqrt{-1+x}x\sqrt{1+x} - \frac{1}{4} \int \frac{1}{\sqrt{-1+x}\sqrt{1+x}} dx\right) \\ &= \frac{x^2}{2} - 2 \left(\frac{1}{4} \sqrt{-1+x}x\sqrt{1+x} - \frac{1}{4} \cosh^{-1}(x)\right) \end{aligned}$$

Mathematica [A] time = 0.16, size = 58, normalized size = 1.76

$$\frac{1}{2} \left(x^2 - \sqrt{x-1} \sqrt{x+1} x + \frac{2(x-1) \sin^{-1} \left(\frac{\sqrt{1-x}}{\sqrt{2}} \right)}{\sqrt{-(x-1)^2}} + 1 \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]

[Out] (1 + x^2 - Sqrt[-1 + x]*x*Sqrt[1 + x] + (2*(-1 + x)*ArcSin[Sqrt[1 - x]/Sqrt[2]])/Sqrt[-(-1 + x)^2])/2

IntegrateAlgebraic [A] time = 0.14, size = 64, normalized size = 1.94

$$\frac{1}{2} \sqrt{x-1} \left(\sqrt{x+1} - (x+1)^{3/2} \right) + \frac{1}{2} \left((x+1)^2 - 2(x+1) \right) - \log \left(\sqrt{x-1} - \sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-Sqrt[-1 + x] + Sqrt[1 + x])/(Sqrt[-1 + x] + Sqrt[1 + x]),x]

[Out] (Sqrt[-1 + x]*(Sqrt[1 + x] - (1 + x)^(3/2)))/2 + (-2*(1 + x) + (1 + x)^2)/2 - Log[Sqrt[-1 + x] - Sqrt[1 + x]]

fricas [A] time = 0.41, size = 37, normalized size = 1.12

$$-\frac{1}{2} \sqrt{x+1} \sqrt{x-1} x + \frac{1}{2} x^2 - \frac{1}{2} \log \left(\sqrt{x+1} \sqrt{x-1} - x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -1/2*sqrt(x + 1)*sqrt(x - 1)*x + 1/2*x^2 - 1/2*log(sqrt(x + 1)*sqrt(x - 1) - x)

giac [A] time = 0.38, size = 41, normalized size = 1.24

$$\frac{1}{2} (x+1)^2 - \frac{1}{2} \sqrt{x+1} \sqrt{x-1} x - x - \log \left(\sqrt{x+1} - \sqrt{x-1} \right) - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="giac")

[Out] 1/2*(x + 1)^2 - 1/2*sqrt(x + 1)*sqrt(x - 1)*x - x - log(sqrt(x + 1) - sqrt(x - 1)) - 1

maple [B] time = 0.01, size = 62, normalized size = 1.88

$$\frac{x^2}{2} + \frac{\sqrt{(x-1)(x+1)} \ln \left(x + \sqrt{x^2-1} \right)}{2\sqrt{x+1} \sqrt{x-1}} - \frac{\sqrt{x-1} (x+1)^{3/2}}{2} + \frac{\sqrt{x-1} \sqrt{x+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((- (x-1)^(1/2) + (x+1)^(1/2)) / ((x-1)^(1/2) + (x+1)^(1/2)), x)

[Out] -1/2*(x-1)^(1/2)*(x+1)^(3/2)+1/2*(x-1)^(1/2)*(x+1)^(1/2)+1/2*((x-1)*(x+1))^(1/2)/(x+1)^(1/2)/(x-1)^(1/2)*ln(x+(x^2-1)^(1/2))+1/2*x^2

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1} - \sqrt{x-1}}{\sqrt{x+1} + \sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-1+x)^(1/2)+(1+x)^(1/2))/((-1+x)^(1/2)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] integrate((sqrt(x + 1) - sqrt(x - 1))/(sqrt(x + 1) + sqrt(x - 1)), x)

mupad [B] time = 10.85, size = 200, normalized size = 6.06

$$\operatorname{acosh}(x) - 2 \operatorname{atanh}\left(\frac{\sqrt{x-1} - i}{\sqrt{x+1} - 1}\right) + \frac{\frac{14(\sqrt{x-1}-i)^3}{(\sqrt{x+1}-1)^3} + \frac{14(\sqrt{x-1}-i)^5}{(\sqrt{x+1}-1)^5} + \frac{2(\sqrt{x-1}-i)^7}{(\sqrt{x+1}-1)^7} + \frac{2(\sqrt{x-1}-i)}{\sqrt{x+1}-1}}{1 + \frac{6(\sqrt{x-1}-i)^4}{(\sqrt{x+1}-1)^4} - \frac{4(\sqrt{x-1}-i)^6}{(\sqrt{x+1}-1)^6} + \frac{(\sqrt{x-1}-i)^8}{(\sqrt{x+1}-1)^8} - \frac{4(\sqrt{x-1}-i)^2}{(\sqrt{x+1}-1)^2}} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x - 1)^(1/2) - (x + 1)^(1/2))/((x - 1)^(1/2) + (x + 1)^(1/2)),x)

[Out] $\operatorname{acosh}(x) - 2 \operatorname{atanh}\left(\frac{(x-1)^{1/2} - 1i}{(x+1)^{1/2} - 1}\right) + \left(\frac{14((x-1)^{1/2} - 1i)^3}{((x+1)^{1/2} - 1)^3} + \frac{14((x-1)^{1/2} - 1i)^5}{((x+1)^{1/2} - 1)^5} + \frac{2((x-1)^{1/2} - 1i)^7}{((x+1)^{1/2} - 1)^7} + \frac{2((x-1)^{1/2} - 1i)}{(x+1)^{1/2} - 1}\right) / \left(1 + \frac{6((x-1)^{1/2} - 1i)^4}{((x+1)^{1/2} - 1)^4} - \frac{4((x-1)^{1/2} - 1i)^6}{((x+1)^{1/2} - 1)^6} + \frac{((x-1)^{1/2} - 1i)^8}{((x+1)^{1/2} - 1)^8} - \frac{4((x-1)^{1/2} - 1i)^2}{((x+1)^{1/2} - 1)^2}\right) + x^2/2$

sympy [A] time = 31.39, size = 226, normalized size = 6.85

$$-\frac{(x-1)^{5/2}}{4\sqrt{x+1}} - \frac{3(x-1)^{3/2}}{4\sqrt{x+1}} - \frac{\sqrt{x-1}}{2\sqrt{x+1}} + \frac{(x-1)^2}{4} + 2 \left(\left(\frac{(x+1)^2}{8} + \frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{5/2}}{8\sqrt{x-1}} + \frac{3(x+1)^{3/2}}{8\sqrt{x-1}} - \frac{\sqrt{x+1}}{4\sqrt{x-1}} \right) \text{ for } \frac{|x+1|}{2} > 1 \right) + \frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)}{2} \left(\frac{(x+1)^2}{8} - \frac{i \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{5/2}}{8\sqrt{1-x}} - \frac{3i(x+1)^{3/2}}{8\sqrt{1-x}} + \frac{i\sqrt{x+1}}{4\sqrt{1-x}} \right) \text{ otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-1+x)**(1/2)+(1+x)**(1/2))/((-1+x)**(1/2)+(1+x)**(1/2)),x)

[Out] $-(x-1)^{5/2}/(4\sqrt{x+1}) - 3(x-1)^{3/2}/(4\sqrt{x+1}) - \sqrt{x-1}/(2\sqrt{x+1}) + (x-1)^2/4 + 2 \operatorname{Piecewise}\left(\left(\frac{(x+1)^2}{8} + \frac{\operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} - \frac{(x+1)^{5/2}}{8\sqrt{x-1}} + \frac{3(x+1)^{3/2}}{8\sqrt{x-1}} - \frac{\sqrt{x+1}}{4\sqrt{x-1}}\right), \operatorname{Abs}(x+1)/2 > 1\right), \left(\frac{(x+1)^2}{8} - \frac{i \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{4} + \frac{i(x+1)^{5/2}}{8\sqrt{1-x}} - \frac{3i(x+1)^{3/2}}{8\sqrt{1-x}} + \frac{i\sqrt{x+1}}{4\sqrt{1-x}}\right), \operatorname{True}\right) + \operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{x-1}}{2}\right)/2$

$$3.230 \quad \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3 dx$$

Optimal. Leaf size=175

$$\frac{ad^3 f^2}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} + \frac{3ad^2 f^2 \log \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{2e} + \frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^4}{8e} + \frac{af^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^2}{4e}$$

Rubi [A] time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, number of rules / integrand size = 0.080, Rules used = {2117, 893}

$$\frac{ad^3 f^2}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} + \frac{3ad^2 f^2 \log \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{2e} + \frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^4}{8e} + \frac{af^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^2}{4e} + \frac{adf^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3,x]

[Out] -(a*d^3*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*d*f^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]))/e + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2)/(4*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4/(8*e) + (3*a*d^2*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3 dx &= \frac{\text{Subst} \left(\int \frac{x^3 (d^2 + af^2 - 2dx + x^2)}{(d-x)^2} dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{\text{Subst} \left(\int \left(2adf^2 + \frac{ad^3 f^2}{(d-x)^2} - \frac{3ad^2 f^2}{d-x} + af^2 x + x^3 \right) dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{ad^3 f^2}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{adf^2 \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{e} + \frac{af^2 \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2}{4e} \end{aligned}$$

Mathematica [A] time = 0.32, size = 158, normalized size = 0.90

$$\frac{-\frac{4ad^3f^2}{f\sqrt{a+\frac{e^2x^2}{f^2}+ex}} + 12ad^2f^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}+ex}\right) + \left(f\sqrt{a+\frac{e^2x^2}{f^2}+d+ex}\right)^4 + 2af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}+d+ex}\right)^2 + 8adf^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}+ex}\right)}{8e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3, x]

[Out] ((-4*a*d^3*f^2)/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 8*a*d*f^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + 2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2 + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^4 + 12*a*d^2*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(8*e)

IntegrateAlgebraic [A] time = 0.84, size = 179, normalized size = 1.02

$$\frac{3ad^2f^3\sqrt{\frac{e^2}{f^2}}\log\left(\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e^2}{f^2}}\right) + \sqrt{a+\frac{e^2x^2}{f^2}}(4adf^3+2aef^3x+3d^2efx+4de^2fx^2+2e^3fx^3)}{2e^2} + \frac{1}{2}(6adf^2x+3aef^2x^2+2d^3x+3d^2ex^2+4de^2x^3+2e^3x^4)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3, x]

[Out] (Sqrt[a + (e^2*x^2)/f^2]*(4*a*d*f^3 + 3*d^2*e*f*x + 2*a*e*f^3*x + 4*d*e^2*f*x^2 + 2*e^3*f*x^3))/(2*e) + (2*d^3*x + 6*a*d*f^2*x + 3*d^2*e*x^2 + 3*a*e*f^2*x^2 + 4*d*e^2*x^3 + 2*e^3*x^4)/2 - (3*a*d^2*Sqrt[e^2/f^2]*f^3*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*e^2)

fricas [A] time = 0.41, size = 161, normalized size = 0.92

$$\frac{2e^4x^4 + 4de^3x^3 - 3ad^2f^2\log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + 3(ae^2f^2 + d^2e^2)x^2 + 2(3ade^2f^2 + d^3e)x + (2e^3fx^3 + 4de^2fx^2 + 4adf^3 + (2aef^3 + 3d^2ef)x)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")

[Out] 1/2*(2*e^4*x^4 + 4*d*e^3*x^3 - 3*a*d^2*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a*e^2*f^2 + d^2*e^2)*x^2 + 2*(3*a*d*e*f^2 + d^3*e)*x + (2*e^3*f*x^3 + 4*d*e^2*f*x^2 + 4*a*d*f^3 + (2*a*e*f^3 + 3*d^2*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/e

giac [A] time = 0.45, size = 163, normalized size = 0.93

$$-\frac{3}{2}ad^2f|f|e^{(-1)}\log\left(-xe + \sqrt{af^2+x^2e^2}\right) + \frac{3}{2}af^2x^2e + 3adf^2x + x^4e^3 + 2dx^3e^2 + \frac{3}{2}d^2x^2e + d^3x + \frac{1}{2}\left(4adf|f|e^{(-1)} + \left(2\left(\frac{x|f|e^2}{f} + \frac{2d|f|e}{f}\right)x + \frac{(2af^4|f|e^4 + 3d^2f^2|f|e^4)e^{(-4)}}{f^3}\right)\right)\sqrt{af^2+x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")

[Out] -3/2*a*d^2*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 1/2*(4*a*d*f*abs(f)*e^(-1) + (2*(x*abs(f)*e^2/f + 2*d*abs(f)*e/f)*x + (2*a*f^4*abs(f)*e^4 + 3*d^2*f^2*abs(f)*e^4)*e^(-4)/f^3)*x)*sqrt(a*f^2 + x^2*e^2)

maple [A] time = 0.01, size = 175, normalized size = 1.00

$$e^3x^4 + \frac{3ae^2x^3}{2} + 2de^2x^3 + \frac{3ad^2f\ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}f^2} + \sqrt{\frac{e^2x^2}{f^2} + a}\right)}{2\sqrt{\frac{e^2}{f^2}}} + 3adf^2x + \frac{3d^2ex^2}{2} + d^3x + \frac{3\sqrt{\frac{e^2x^2}{f^2} + a}d^2fx}{2} + \left(\frac{e^2x^2}{f^2} + a\right)^{\frac{3}{2}}f^3x + \frac{d^4}{4e} + \frac{2\left(\frac{e^2x^2+af^2}{f^2}\right)^{\frac{3}{2}}df^3}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2))*f)^3,x

[Out] f^3*x*(e^2/f^2*x^2+a)^(3/2)+e^3*x^4+2*d*e^2*x^3+3/2*f^2*a*e*x^2+3*f^2*a*d*x+2*d/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+3/2*f*d^2*x*(e^2/f^2*x^2+a)^(1/2)+3/2*f*d^2*a*ln(e^2*x/f^2/(e^2/f^2)^(1/2)+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)+3/2*e*x^2*d^2+d^3*x+1/4/e*d^4

maxima [A] time = 0.94, size = 266, normalized size = 1.52

$$\frac{1}{4}e^3x^4 + \frac{3}{4e}\left(\frac{e^2}{f^2} + a\right)^{\frac{3}{2}}f^4 - \frac{3}{8}\left(\frac{e^2f^3 \operatorname{arsinh}\left(\frac{ex}{\sqrt{af}}\right) - 2\left(\frac{e^2}{f^2} + a\right)^{\frac{3}{2}}f^2x}{e^2} + \sqrt{\frac{e^2}{f^2} + a}dfx\right) + \frac{1}{8}\left(\frac{3e^2f \operatorname{arsinh}\left(\frac{ex}{\sqrt{af}}\right) + 2\left(\frac{e^2}{f^2} + a\right)^{\frac{3}{2}}x + 3\sqrt{\frac{e^2}{f^2} + a}ax\right)f^3 + d^3x + \frac{3}{2}\left(ef \operatorname{arsinh}\left(\frac{ex}{\sqrt{af}}\right) + \sqrt{\frac{e^2}{f^2} + a}x\right)f^2 + \left(e^2x^3 + \frac{2\left(\frac{e^2}{f^2} + a\right)^{\frac{3}{2}}f^3}{e} + \left(\frac{e^2}{f^2} + 3ax\right)f^2\right)d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")

[Out] 1/4*e^3*x^4 + 3/4*(e^2*x^2/f^2 + a)^2*f^4/e - 3/8*(a^2*f^3*arcsinh(e*x/(sqrt(a)*f)))/e^3 - 2*(e^2*x^2/f^2 + a)^(3/2)*f^2*x/e^2 + sqrt(e^2*x^2/f^2 + a)*a*f^2*x/e^2)*e^2*f + 1/8*(3*a^2*f*arcsinh(e*x/(sqrt(a)*f)))/e + 2*(e^2*x^2/f^2 + a)^(3/2)*x + 3*sqrt(e^2*x^2/f^2 + a)*a*x)*f^3 + d^3*x + 3/2*(e*x^2 + (a*f*arcsinh(e*x/(sqrt(a)*f)))/e + sqrt(e^2*x^2/f^2 + a)*x)*f*d^2 + (e^2*x^3 + 2*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + (e^2*x^3/f^2 + 3*a*x)*f^2)*d

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)

[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)

sympy [A] time = 10.46, size = 279, normalized size = 1.59

$$\frac{a^{\frac{3}{2}}f^3x\sqrt{1+\frac{e^2x^2}{af^2}}}{2} + \frac{a^{\frac{3}{2}}f^3x}{2\sqrt{1+\frac{e^2x^2}{af^2}}} + \frac{3\sqrt{a}d^2fx\sqrt{1+\frac{e^2x^2}{af^2}}}{2} + \frac{3\sqrt{a}e^2fx^3}{2\sqrt{1+\frac{e^2x^2}{af^2}}} + \frac{3ad^2f^2 \operatorname{asinh}\left(\frac{ex}{\sqrt{af}}\right)}{2e} + 3adf^2x + \frac{3ae^2x^2}{2} + d^3x + \frac{3d^2ex^2}{2} + 2de^2x^3 + 6def \begin{cases} \frac{\sqrt{ax^2}}{2} & \text{for } e^2 = 0 \\ \frac{f^2\left(a + \frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} + e^3x^4 + \frac{e^4x^5}{\sqrt{a}f\sqrt{1+\frac{e^2x^2}{af^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)

[Out] a**(3/2)*f**3*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + a**(3/2)*f**3*x/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*sqrt(a)*d**2*f*x*sqrt(1 + e**2*x**2/(a*f**2))/2 + 3*sqrt(a)*e**2*f*x**3/(2*sqrt(1 + e**2*x**2/(a*f**2))) + 3*a*d**2*f**2*asinh(e*x/(sqrt(a)*f))/(2*e) + 3*a*d*f**2*x + 3*a*e*f**2*x**2/2 + d**3*x + 3*d**2*e*x**2/2 + 2*d*e**2*x**3 + 6*d*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True)) + e**3*x**4 + e**4*x**5/(sqrt(a)*f*sqrt(1 + e**2*x**2/(a*f**2)))

$$3.231 \quad \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2 dx$$

Optimal. Leaf size=136

$$\frac{ad^2 f^2}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} + \frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^3}{6e} + \frac{ad f^2 \log \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{e} + \frac{af^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{2e}$$

Rubi [A] time = 0.10, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2117, 893}

$$\frac{ad^2 f^2}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} + \frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^3}{6e} + \frac{ad f^2 \log \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{e} + \frac{af^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2,x]

[Out] -(a*d^2*f^2)/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]))/(2*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/(6*e) + (a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2]))^(n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2 dx &= \frac{\text{Subst} \left(\int \frac{x^2(d^2 + af^2 - 2dx + x^2)}{(d-x)^2} dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{\text{Subst} \left(\int \left(af^2 + \frac{ad^2 f^2}{(d-x)^2} - \frac{2adf^2}{d-x} + x^2 \right) dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{ad^2 f^2}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{af^2 \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} + \frac{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3}{6e} \end{aligned}$$

Mathematica [A] time = 0.20, size = 128, normalized size = 0.94

$$\frac{\frac{ad^2f^2}{f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex} + \frac{1}{3}\left(f\sqrt{a+\frac{e^2x^2}{f^2}} + d + ex\right)^3 + 2adf^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}} + ex\right) + af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}} + ex\right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2, x]

[Out] ((a*d^2*f^2)/(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2]) + a*f^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^3/3 + 2*a*d*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)

IntegrateAlgebraic [A] time = 0.60, size = 132, normalized size = 0.97

$$\frac{1}{3}(3af^2x + 3d^2x + 3dex^2 + 2e^2x^3) + \frac{\sqrt{a + \frac{e^2x^2}{f^2}}(2af^3 + 3defx + 2e^2fx^2)}{3e} - \frac{adf^3\sqrt{\frac{e^2}{f^2}} \log\left(\sqrt{a + \frac{e^2x^2}{f^2}} - x\sqrt{\frac{e^2}{f^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2, x]

[Out] (Sqrt[a + (e^2*x^2)/f^2]*(2*a*f^3 + 3*d*e*f*x + 2*e^2*f*x^2))/(3*e) + (3*d^2*x + 3*a*f^2*x + 3*d*e*x^2 + 2*e^2*x^3)/3 - (a*d*Sqrt[e^2/f^2]*f^3*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/e^2

fricas [A] time = 0.40, size = 114, normalized size = 0.84

$$\frac{2e^3x^3 + 3de^2x^2 - 3adf^2 \log\left(-ex + f\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + 3(aef^2 + d^2e)x + (2e^2fx^2 + 2af^3 + 3defx)\sqrt{\frac{e^2x^2+af^2}{f^2}}}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")

[Out] 1/3*(2*e^3*x^3 + 3*d*e^2*x^2 - 3*a*d*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2))) + 3*(a*e*f^2 + d^2*e)*x + (2*e^2*f*x^2 + 2*a*f^3 + 3*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)/e

giac [A] time = 0.52, size = 103, normalized size = 0.76

$$-adf|f|e^{(-1)} \log\left(-xe + \sqrt{af^2 + x^2e^2}\right) + af^2x + \frac{2}{3}x^3e^2 + dx^2e + d^2x + \frac{1}{3}\left(2af|f|e^{(-1)} + \left(\frac{2x|f|e}{f} + \frac{3d|f|}{f}\right)x\right)\sqrt{af^2 + x^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")

[Out] -a*d*f*abs(f)*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x + 1/3*(2*a*f*abs(f)*e^(-1) + (2*x*abs(f)*e/f + 3*d*abs(f)/f)*x)*sqrt(a*f^2 + x^2*e^2)

maple [A] time = 0.01, size = 126, normalized size = 0.93

$$\frac{2e^2x^3}{3} + \frac{adf \ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}f^2} + \sqrt{\frac{e^2x^2}{f^2} + a}\right)}{\sqrt{\frac{e^2}{f^2}}} + af^2x + dex^2 + d^2x + \sqrt{\frac{e^2x^2}{f^2} + a}dfx + \frac{d^3}{3e} + \frac{2\left(\frac{e^2x^2+af^2}{f^2}\right)^{\frac{3}{2}}f^3}{3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)`

[Out] $f^2*a*x+2/3*e^2*x^3+2/3/e*f^3*((e^2*x^2+a*f^2)/f^2)^(3/2)+f*d*x*(e^2/f^2*x^2+a)^(1/2)+f*d*a*\ln(1/(e^2/f^2)^(1/2)*e^2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)+e*x^2*d+x*d^2+1/3/e*d^3$

maxima [A] time = 0.89, size = 99, normalized size = 0.73

$$\frac{1}{3}e^2x^3 + \frac{2\left(\frac{e^2x^2}{f^2} + a\right)^{\frac{3}{2}}f^3}{3e} + \frac{1}{3}\left(\frac{e^2x^3}{f^2} + 3ax\right)f^2 + d^2x + \left(ex^2 + \left(\frac{af \operatorname{arsinh}\left(\frac{ex}{\sqrt{af}}\right)}{e} + \sqrt{\frac{e^2x^2}{f^2} + ax} \right) f \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")`

[Out] $1/3*e^2*x^3 + 2/3*(e^2*x^2/f^2 + a)^(3/2)*f^3/e + 1/3*(e^2*x^3/f^2 + 3*a*x)*f^2 + d^2*x + (e*x^2 + (a*f*\operatorname{arcsinh}(e*x/(\operatorname{sqrt}(a)*f)))/e + \operatorname{sqrt}(e^2*x^2/f^2 + a)*x)*f*d$

mupad [B] time = 4.66, size = 210, normalized size = 1.54

$$\begin{cases} x(d + \sqrt{a}f)^2 & \text{if } e = 0 \\ x(d^2 + af^2) + \frac{2e^2x^3}{3} + dex^2 + \frac{2af^3\sqrt{a+\frac{e^2x^2}{f^2}}}{e} - \frac{2f\sqrt{a+\frac{e^2x^2}{f^2}}(2af^2-e^2x^2)}{3e} + dfx\sqrt{a+\frac{e^2x^2}{f^2}} + \frac{2adf\ln\left(x\sqrt{\frac{e^2}{f^2}} + \sqrt{a+\frac{e^2x^2}{f^2}}\right)}{\sqrt{\frac{e^2}{f^2}}} - \frac{ad^2\ln\left(2x\sqrt{\frac{e^2}{f^2}} + 2\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{f\left(\frac{e^2}{f^2}\right)^{3/2}} & \text{if } e \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)`

[Out] `piecewise(e == 0, x*(d + a^(1/2)*f)^2, e ~= 0, x*(a*f^2 + d^2) + (2*e^2*x^3)/3 + d*e*x^2 + (2*a*f^3*(a + (e^2*x^2)/f^2)^(1/2))/e - (2*f*(a + (e^2*x^2)/f^2)^(1/2)*(2*a*f^2 - e^2*x^2))/(3*e) + d*f*x*(a + (e^2*x^2)/f^2)^(1/2) + (2*a*d*f*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(e^2/f^2)^(1/2) - (a*d*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^2/f^2)^(3/2)))`

sympy [A] time = 4.48, size = 116, normalized size = 0.85

$$\sqrt{a}dfx\sqrt{1+\frac{e^2x^2}{af^2}} + \frac{adf^2\operatorname{asinh}\left(\frac{ex}{\sqrt{af}}\right)}{e} + af^2x + d^2x + dex^2 + \frac{2e^2x^3}{3} + 2ef\begin{cases} \frac{\sqrt{a}x^2}{2} & \text{for } e^2 = 0 \\ \frac{f^2\left(a+\frac{e^2x^2}{f^2}\right)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)`

[Out] `sqrt(a)*d*f*x*sqrt(1 + e**2*x**2/(a*f**2)) + a*d*f**2*asinh(e*x/(sqrt(a)*f))/e + a*f**2*x + d**2*x + d*e*x**2 + 2*e**2*x**3/3 + 2*e*f*Piecewise((sqrt(a)*x**2/2, Eq(e**2, 0)), (f**2*(a + e**2*x**2/f**2)**(3/2)/(3*e**2), True))`

$$3.232 \quad \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right) dx$$

Optimal. Leaf size=68

$$\frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} + \frac{a f^2 \tanh^{-1} \left(\frac{e x}{f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} + dx + \frac{e x^2}{2}$$

Rubi [A] time = 0.03, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {195, 217, 206}

$$\frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} + \frac{a f^2 \tanh^{-1} \left(\frac{e x}{f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} + dx + \frac{e x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2],x]

[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 + (a*f^2*ArcTanh[(e*x)/(f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right) dx &= dx + \frac{ex^2}{2} + f \int \sqrt{a + \frac{e^2 x^2}{f^2}} dx \\
&= dx + \frac{ex^2}{2} + \frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} + \frac{1}{2} (af) \int \frac{1}{\sqrt{a + \frac{e^2 x^2}{f^2}}} dx \\
&= dx + \frac{ex^2}{2} + \frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} + \frac{1}{2} (af) \operatorname{Subst} \left(\int \frac{1}{1 - \frac{e^2 x^2}{f^2}} dx, x, \frac{x}{\sqrt{a + \frac{e^2 x^2}{f^2}}} \right) \\
&= dx + \frac{ex^2}{2} + \frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} + \frac{af^2 \tanh^{-1} \left(\frac{ex}{f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 81, normalized size = 1.19

$$\frac{1}{2} f x \sqrt{\frac{af^2 + e^2 x^2}{f^2}} + \frac{af^2 \log \left(ef \sqrt{\frac{af^2 + e^2 x^2}{f^2}} + e^2 x \right)}{2e} + dx + \frac{ex^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2], x]

[Out] d*x + (e*x^2)/2 + (f*x*Sqrt[(a*f^2 + e^2*x^2)/f^2])/2 + (a*f^2*Log[e^2*x + e*f*Sqrt[(a*f^2 + e^2*x^2)/f^2]])/(2*e)

IntegrateAlgebraic [A] time = 0.28, size = 91, normalized size = 1.34

$$\frac{1}{2} f x \sqrt{a + \frac{e^2 x^2}{f^2}} - \frac{af^3 \sqrt{\frac{e^2}{f^2}} \log \left(\sqrt{a + \frac{e^2 x^2}{f^2}} - x \sqrt{\frac{e^2}{f^2}} \right)}{2e^2} + \frac{1}{2} (2dx + ex^2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2], x]

[Out] (2*d*x + e*x^2)/2 + (f*x*Sqrt[a + (e^2*x^2)/f^2])/2 - (a*Sqrt[e^2/f^2]*f^3*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*e^2)

fricas [A] time = 0.40, size = 74, normalized size = 1.09

$$\frac{e^2 x^2 - af^2 \log \left(-ex + f \sqrt{\frac{e^2 x^2 + af^2}{f^2}} \right) + efx \sqrt{\frac{e^2 x^2 + af^2}{f^2}} + 2dex}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2), x, algorithm="fricas")

[Out] 1/2*(e^2*x^2 - a*f^2*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)) + e*f*x*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d*e*x)/e

giac [A] time = 0.35, size = 65, normalized size = 0.96

$$\frac{1}{2}x^2e + dx - \frac{(af^2e^{(-1)} \log(|-xe + \sqrt{af^2 + x^2e^2}|) - \sqrt{af^2 + x^2e^2}x)|f|}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="giac")

[Out] 1/2*x^2*e + d*x - 1/2*(a*f^2*e^(-1)*log(abs(-x*e + sqrt(a*f^2 + x^2*e^2))) - sqrt(a*f^2 + x^2*e^2)*x)*abs(f)/f

maple [A] time = 0.00, size = 75, normalized size = 1.10

$$\frac{af \ln\left(\frac{e^2x}{\sqrt{\frac{e^2}{f^2}}f^2} + \sqrt{\frac{e^2x^2}{f^2} + a}\right)}{2\sqrt{\frac{e^2}{f^2}}} + \frac{ex^2}{2} + dx + \frac{\sqrt{\frac{e^2x^2}{f^2} + a}fx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f,x)

[Out] d*x+1/2*e*x^2+1/2*f*x*(e^2/f^2*x^2+a)^(1/2)+1/2*f*a*ln(1/(e^2/f^2)^(1/2)*e^2/f^2*x+(e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)

maxima [A] time = 0.87, size = 46, normalized size = 0.68

$$\frac{1}{2}ex^2 + \frac{1}{2}\left(\frac{af \operatorname{arsinh}\left(\frac{ex}{\sqrt{a}f}\right)}{e} + \sqrt{\frac{e^2x^2}{f^2} + a}x\right)f + dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")

[Out] 1/2*e*x^2 + 1/2*(a*f*arcsinh(e*x/(sqrt(a)*f)))/e + sqrt(e^2*x^2/f^2 + a)*x)*f + d*x

mupad [B] time = 3.89, size = 136, normalized size = 2.00

$$\begin{cases} x(d + \sqrt{a}f) & \text{if } e = 0 \\ dx + \frac{ex^2}{2} + \frac{fx\sqrt{a + \frac{e^2x^2}{f^2}}}{2} + \frac{ae^2 \ln\left(x\sqrt{\frac{e^2}{f^2} + \sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{f\left(\frac{e^2}{f^2}\right)^{3/2}} - \frac{ae^2 \ln\left(2x\sqrt{\frac{e^2}{f^2} + 2\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2f\left(\frac{e^2}{f^2}\right)^{3/2}} & \text{if } e \neq 0 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2),x)

[Out] piecewise(e == 0, x*(d + a^(1/2)*f), e != 0, d*x + (e*x^2)/2 + (f*x*(a + (e^2*x^2)/f^2)^(1/2))/2 + (a*e^2*log(x*(e^2/f^2)^(1/2) + (a + (e^2*x^2)/f^2)^(1/2)))/(f*(e^2/f^2)^(3/2)) - (a*e^2*log(2*x*(e^2/f^2)^(1/2) + 2*(a + (e^2*x^2)/f^2)^(1/2)))/(2*f*(e^2/f^2)^(3/2)))

sympy [A] time = 2.21, size = 54, normalized size = 0.79

$$dx + \frac{ex^2}{2} + f\left(\frac{\sqrt{a}x\sqrt{1 + \frac{e^2x^2}{af^2}}}{2} + \frac{af \operatorname{asinh}\left(\frac{ex}{\sqrt{a}f}\right)}{2e}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] d*x + e*x**2/2 + f*(sqrt(a)*x*sqrt(1 + e**2*x**2/(a*f**2)))/2 + a*f*asinh(e*  
x/(sqrt(a)*f))/(2*e)
```

$$3.233 \quad \int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=117

$$\frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e} + \frac{\left(\frac{af^2}{d^2}+1\right) \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e} - \frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Rubi [A] time = 0.09, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2117, 893}

$$\frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^2e} + \frac{\left(\frac{af^2}{d^2}+1\right) \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2e} - \frac{af^2}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1), x]

[Out] -(a*f^2)/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e) + ((1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}} dx &= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x} dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\ &= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d(d-x)^2} + \frac{af^2}{d^2(d-x)} + \frac{d^2+af^2}{d^2x}\right) dx, x, d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \\ &= -\frac{af^2}{2de\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)} - \frac{af^2 \log\left(ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2d^2e} + \frac{\left(1+\frac{af^2}{d^2}\right) \log\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)}{2e} \end{aligned}$$

Mathematica [A] time = 0.14, size = 109, normalized size = 0.93

$$\frac{-\frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^2} + \left(\frac{af^2}{d^2} + 1\right) \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}} + d + ex\right) + \frac{af^2}{d\left(f\left(-\sqrt{a+\frac{e^2x^2}{f^2}}\right)-ex\right)}}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1), x]

[Out] ((a*f^2)/(d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^2 + (1 + (a*f^2)/d^2)*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e)

IntegrateAlgebraic [B] time = 1.02, size = 395, normalized size = 3.38

$$\frac{(af^2+d)\log\left(\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}+dex}{4d^2e}\right) + (af^2+d)\log\left(\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}-dex}{4d^2e}\right) + (af^2+d)\log\left(\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}-x\sqrt{\frac{e}{f}}}{4d^2e}\right) + \sqrt{\frac{e}{f}}(af^2+d)\log\left(\frac{d\sqrt{a+\frac{e^2x^2}{f^2}}+af+dx\left(\sqrt{\frac{e}{f}}\right)}{4d^2e}\right) - \sqrt{\frac{e}{f}}(af^2+d)\log\left(\frac{f\sqrt{a+\frac{e^2x^2}{f^2}}+d-fx\sqrt{\frac{e}{f}}}{4d^2e}\right) + (af^2+d)\log\left(\frac{d\sqrt{a+\frac{e^2x^2}{f^2}}+d^2e-d^2efx\sqrt{\frac{e}{f}}}{4d^2e}\right) - \frac{f\sqrt{a+\frac{e^2x^2}{f^2}}}{2de} + \frac{x}{2d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-1), x]

[Out] x/(2*d) - (f*Sqrt[a + (e^2*x^2)/f^2])/(2*d*e) + ((-d^2*e) - d^2*Sqrt[e^2/f^2]*f - a*e*f^2 + a*Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]]/(4*d^2*e^2) + (Sqrt[e^2/f^2]*(d^2*f + a*f^3)*Log[a*f - d*Sqrt[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e^2) + ((d^2 + a*f^2)*Log[a*d*e*f - d^2*e*Sqrt[e^2/f^2]*x + d^2*e*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e) - (Sqrt[e^2/f^2]*(d^2*f + a*f^3)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e^2) + ((d^2 + a*f^2)*Log[d^3*e - d^2*e*Sqrt[e^2/f^2]*f*x + d^2*e*f*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^2*e)

fricas [A] time = 0.46, size = 187, normalized size = 1.60

$$\frac{2dex - 2df\sqrt{\frac{e^2x^2+af^2}{f^2}} + (af^2+d^2)\log\left(af^2-dex+df\sqrt{\frac{e^2x^2+af^2}{f^2}}\right) + (af^2+d^2)\log(-af^2+2dex+d^2) - (af^2+d^2)\log(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}}-d) + (af^2-d^2)\log(-ex+f\sqrt{\frac{e^2x^2+af^2}{f^2}})}{4d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)), x, algorithm="fricas")

[Out] 1/4*(2*d*e*x - 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + (a*f^2 + d^2)*log(a*f^2 - d*e*x + d*f*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a*f^2 + d^2)*log(-a*f^2 + 2*d*e*x + d^2) - (a*f^2 + d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) + (a*f^2 - d^2)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2)))/(d^2*e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

[undef, +∞, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)), x, algorithm="giac")

[Out] [undef, +Infinity, 1]

maple [B] time = 0.04, size = 1325, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f), x)

```
[Out] -1/4*f/d/e*(4*e^2*(x+1/2*(-a*f^2+d^2))/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2)-1/4*f/d^2*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2))/(e^2/f^2)^(1/2)*a+1/4*f*ln((1/2*e*(a*f^2-d^2)/f^2/d+e^2/f^2*(x+1/2*(-a*f^2+d^2)/d/e))/(e^2/f^2)^(1/2)+(e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/4*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2))/(e^2/f^2)^(1/2)+1/4*f^3/d^3/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2))*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2)))/(x+1/2*(-a*f^2+d^2)/d/e))*a^2+1/2*f/d/e/((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2))*ln((1/2*(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2+e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+1/2*((a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2)^(1/2)*(4*e^2*(x+1/2*(-a*f^2+d^2)/d/e)^2/f^2+4*e*(a*f^2-d^2)/f^2/d*(x+1/2*(-a*f^2+d^2)/d/e)+(a^2*f^4+2*a*d^2*f^2+d^4)/f^2/d^2^(1/2)))/(x+1/2*(-a*f^2+d^2)/d/e))+1/2*ln(a*f^2-2*d*e*x-d^2)/e+1/2/d*x+1/4/d^2/e*ln(-a*f^2+2*d*e*x+d^2)*a*f^2-1/4/e*ln(-a*f^2+2*d*e*x+d^2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ex + \sqrt{\frac{e^2x^2}{f^2} + af + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{d + ex + f \sqrt{a + \frac{e^2x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)),x)
```

```
[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f \sqrt{a + \frac{e^2x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2)),x)
```

```
[Out] Integral(1/(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)
```

$$3.234 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^2} dx$$

Optimal. Leaf size=151

$$\frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e} + \frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e} - \frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Rubi [A] time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2117, 893}

$$\frac{af^2}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)} - \frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{d^3e} + \frac{af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{d^3e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2), x]

[Out] -(a*f^2)/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e) + (a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(d^3*e)

Rule 893

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (c_.)*(x_.)^2]))^(n_.)^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2} dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^2} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^2(d-x)^2} + \frac{2af^2}{d^3(d-x)} + \frac{d^2+af^2}{d^2x^2} + \frac{2af^2}{d^3x}\right) dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} - \frac{1 + \frac{af^2}{d^2}}{2e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} - \frac{af^2 \log\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{d^3e}$$

Mathematica [A] time = 0.29, size = 141, normalized size = 0.93

$$\frac{2af^2 \log\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)}{d^3} - \frac{2af^2 \log\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)}{d^3} + \frac{af^2}{d^2\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)} + \frac{\frac{af^2}{d^2} + 1}{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}$$

2e

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2), x]

[Out] -1/2*((a*f^2)/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (1 + (a*f^2)/d^2)/(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (2*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3 - (2*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^3)/e

IntegrateAlgebraic [B] time = 1.70, size = 433, normalized size = 2.87

$$\frac{a^2 f^2 x - a d e f^2 x^2 + a^2 x + a^2 x^2}{d^2 (d^2 - a f^2) (-a f^2 + d^2 + 2 d e x)} + \frac{a f^2 \sqrt{\frac{a}{f^2}} \log\left(d \sqrt{a + \frac{e^2 x^2}{f^2}} + a f + d x \left(-\sqrt{\frac{a}{f^2}}\right)\right)}{2 d^3 e} - \frac{a f^2 \sqrt{\frac{a}{f^2}} \log\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d - f x \sqrt{\frac{a}{f^2}}\right)}{2 d^3 e} - \frac{a \left(e f^2 - f^3 \sqrt{\frac{a}{f^2}}\right) \log\left(\sqrt{a + \frac{e^2 x^2}{f^2}} - x \sqrt{\frac{a}{f^2}}\right)}{2 d^3 e} + \frac{\sqrt{a + \frac{e^2 x^2}{f^2}} (d e f x - a f^2)}{d^2 e (-a f^2 + d^2 + 2 d e x)} + \frac{a f^2 \log\left(d^2 e f \sqrt{a + \frac{e^2 x^2}{f^2}} + d^2 e - d^2 e f x \sqrt{\frac{a}{f^2}}\right)}{2 d^3 e} + \frac{a f^2 \log\left(d^2 e \sqrt{a + \frac{e^2 x^2}{f^2}} + a d^2 e f + d^2 (-d) x \sqrt{\frac{a}{f^2}}\right)}{2 d^3 e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-2), x]

[Out] -(((-(a*f^3) + d*e*f*x)*Sqrt[a + (e^2*x^2)/f^2])/(d^2*e*(d^2 - a*f^2 + 2*d*e*x))) + (d^4*x + a^2*f^4*x + d^3*e*x^2 - a*d*e*f^2*x^2)/(d^2*(d^2 - a*f^2)*(d^2 - a*f^2 + 2*d*e*x)) - (a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e^2) + (a*Sqrt[e^2/f^2]*f^3*Log[a*f - d*Sqrt[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e^2) + (a*f^2*Log[a*d^2*e*f - d^3*e*Sqrt[e^2/f^2]*x + d^3*e*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e) - (a*Sqrt[e^2/f^2]*f^3*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e^2) + (a*f^2*Log[d^4*e - d^3*e*Sqrt[e^2/f^2]*f*x + d^3*e*f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e)

fricas [B] time = 0.47, size = 284, normalized size = 1.88

$$\frac{a^2 f^4 - 2 d^2 e^2 x^2 + a d^2 f^2 - 2 d^2 e x + (a^2 f^4 - 2 a d e f^2 x - a d^2 f^2) \log\left(-a e f^2 x + 2 d e^2 x^2 + a d f^2 + (a f^3 - 2 d e f x) \sqrt{\frac{e^2 x^2 + a f^2}{f^2}}\right) + (a^2 f^4 - 2 a d e f^2 x - a d^2 f^2) \log\left(-a f^2 + 2 d e x + d^2\right) - (a^2 f^4 - 2 a d e f^2 x - a d^2 f^2) \log\left(-e x + f \sqrt{\frac{e^2 x^2 + a f^2}{f^2}} - d\right) - 2 (a d f^3 - d^2 e f x) \sqrt{\frac{e^2 x^2 + a f^2}{f^2}}}{2 (a d^2 e f^2 - 2 d^4 e^2 x - d^2 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")

[Out] 1/2*(a^2*f^4 - 2*d^2*e^2*x^2 + a*d^2*f^2 - 2*d^3*e*x + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*lo

$$g(-af^2 + 2d*ex + d^2) - (a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*\log(-ex + f*\sqrt{(e^2*x^2 + af^2)/f^2} - d) - 2*(a*d*f^3 - d^2*e*f*x)*\sqrt{(e^2*x^2 + af^2)/f^2})/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.04, size = 4136, normalized size = 27.39

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^2,x)

[Out]
$$\frac{1}{2} \frac{1}{d^2} x - \frac{1}{4} \frac{f}{e} \frac{1}{d^2} (4 * (x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} + \frac{1}{4} \frac{1}{f} \frac{1}{d} \ln \left(\frac{(x + \frac{1}{2} * (-af^2 + d^2) / d / e) * e^2 / f^2 + \frac{1}{2} * (af^2 - d^2) / d * e / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) + \frac{1}{4} \frac{1}{f} \frac{1}{e} \left(\frac{(a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2}{(e^2 / f^2)^{(1/2)}} * \ln \left(\frac{(af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{2} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2 + \frac{1}{2} * ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * (4 * (x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}}{(x + \frac{1}{2} * (-af^2 + d^2) / d / e)} + \frac{1}{2} \frac{1}{e} \frac{1}{d^3} \ln(-af^2 + 2 * d * e * x + d^2) * af^2 - \frac{1}{2} * af^2 / (-af^2 + 2 * d * e * x + d^2) / d / e + \frac{1}{4} * d^2 * f / e / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * (4 * (x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} - \frac{1}{4} * d^3 / f / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * \ln \left(\frac{(x + \frac{1}{2} * (-af^2 + d^2) / d / e) * e^2 / f^2 + \frac{1}{2} * (af^2 - d^2) / d * e / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) - d * f / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * x - \frac{1}{4} \frac{1}{e} \frac{1}{d^3} / (-af^2 + 2 * d * e * x + d^2) * a^2 * f^4 - \frac{1}{4} * f / d^3 * \ln \left(\frac{(x + \frac{1}{2} * (-af^2 + d^2) / d / e) * e^2 / f^2 + \frac{1}{2} * (af^2 - d^2) / d * e / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) * a^{-1/4} * d / (-af^2 + 2 * d * e * x + d^2) / e + d * f^3 / e^2 / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / (x - \frac{1}{2} / e / d * af^2 + \frac{1}{2} * d / e) * ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(3/2)} - \frac{3}{4} \frac{1}{d} * f^3 / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * \ln \left(\frac{(x + \frac{1}{2} * (-af^2 + d^2) / d / e) * e^2 / f^2 + \frac{1}{2} * (af^2 - d^2) / d * e / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) / (e^2 / f^2)^{(1/2)} * a^{-2} - \frac{3}{4} * d * f / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * \ln \left(\frac{(x + \frac{1}{2} * (-af^2 + d^2) / d / e) * e^2 / f^2 + \frac{1}{2} * (af^2 - d^2) / d * e / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * a^{-1/4} * d^4 / f / e / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln \left(\frac{(af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{2} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right) + \frac{1}{4} * f^3 / e / d^4 / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln \left(\frac{(af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{2} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2}{(e^2 / f^2)^{(1/2)} + ((x + \frac{1}{2} * (-af^2 + d^2) / d / e)^2 * e^2 / f^2 + (af^2 - d^2) * (x + \frac{1}{2} * (-af^2 + d^2) / d / e) / d * e / f^2 + \frac{1}{4} * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}} \right)$$

$$2) * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} / (x + 1/2 * (-a * f^2 + d^2) / d / e) * a^{2+1/2} * f / e / d^2 / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln(((a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/2 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}) / (x + 1/2 * (-a * f^2 + d^2) / d / e) * a^{-1/4} * f^5 / e / d^2 / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * a^{-2-1/4} * f^5 / d^3 / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * \ln(((x + 1/2 * (-a * f^2 + d^2) / d / e) * e^2 / f^2 + 1/2 * (a * f^2 - d^2) / d * e / f^2) / (e^2 / f^2)^{(1/2)} + ((x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/4 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}) / (e^2 / f^2)^{(1/2)} * a^{-3-f^3/d} / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) * ((x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/4 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * x * a + f^5 / e^2 / d / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / (x - 1/2 * e / d * a * f^2 + 1/2 * d / e) * ((x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/4 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(3/2)} * a + 1/4 * f^7 / e / d^4 / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln(((a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/2 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2 + 1/2 * ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}) / (x + 1/2 * (-a * f^2 + d^2) / d / e) * a^4 + 1/2 * d^2 * f^5 / e / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln(((a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/2 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2 + 1/2 * ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}) / (x + 1/2 * (-a * f^2 + d^2) / d / e) * a^{-3-1/2} * d^2 * f / e / (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * \ln(((a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + 1/2 * (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2 + 1/2 * ((a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)} * (4 * (x + 1/2 * (-a * f^2 + d^2) / d / e)^2 * e^2 / f^2 + 4 * (a * f^2 - d^2) * (x + 1/2 * (-a * f^2 + d^2) / d / e) / d * e / f^2 + (a^2 * f^4 + 2 * a * d^2 * f^2 + d^4) / d^2 / f^2)^{(1/2)}) / (x + 1/2 * (-a * f^2 + d^2) / d / e) * a$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a} f + d \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2,x)

[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**2,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-2), x)
```

$$3.235 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^3} dx$$

Optimal. Leaf size=193

$$\frac{3af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e} + \frac{3af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e} - \frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Rubi [A] time = 0.13, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2117, 893}

$$\frac{af^2}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{af^2}{d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)} - \frac{\frac{af^2}{d^2}+1}{4e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^2} - \frac{3af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)}{2d^4e} + \frac{3af^2 \log\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}{2d^4e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3), x]

[Out] -(a*f^2)/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(4*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (a*f^2)/(d^3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e) + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^4*e)

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^3} dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^3} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{af^2}{d^3(d-x)^2} + \frac{3af^2}{d^4(d-x)} + \frac{d^2+af^2}{d^2x^3} + \frac{2af^2}{d^3x^2} + \frac{3af^2}{d^4x}\right) dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= -\frac{af^2}{2d^3e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} - \frac{1 + \frac{af^2}{d^2}}{4e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^2} - \frac{af^2}{d^3e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}$$

Mathematica [A] time = 0.57, size = 180, normalized size = 0.93

$$\frac{\frac{3af^2 \log\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)}{d^4} + \frac{3af^2 \log\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)}{d^4} + \frac{af^2}{d^3\left(f\left(-\sqrt{a + \frac{e^2x^2}{f^2}}\right) - ex\right)} - \frac{2af^2}{d^3\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)} - \frac{\frac{af^2}{d^2} + 1}{2\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^2}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3), x]
```

```
[Out] ((a*f^2)/(d^3*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) - (1 + (a*f^2)/d^2)/(2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^2) - (2*a*f^2)/(d^3*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])) - (3*a*f^2*Log[e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4 + (3*a*f^2*Log[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/d^4)/(2*e)
```

IntegrateAlgebraic [B] time = 2.54, size = 528, normalized size = 2.74

$$\frac{\sqrt{a + \frac{e^2x^2}{f^2}} \left(-5af^3 + 5af^2e + 9af^2ex - 3af^2x^2 - 4af^2f^2 \right)}{2af^2 \left(-af^2 + af^2 + 2af^2 \right)} - \frac{3af^2e^2 - 9af^2ef^2 - 9af^2d^2f^2 + 3af^2f^2 + 3af^2f^2e + 3af^2f^2ex + 3af^2f^2x^2 + 4af^2f^2f^2 + af^2f^2 - 3af^2f^2f^2 - 9af^2f^2f^2 + 2af^2e + 5af^2e^2 + 4af^2e^2f^2}{2af^2 \left(-af^2 + af^2 + 2af^2 \right)} - \frac{3af^2\sqrt{a + \frac{e^2x^2}{f^2}} \log\left(\sqrt{a + \frac{e^2x^2}{f^2}} + d + \sqrt{a + \frac{e^2x^2}{f^2}}\right)}{4af^2} - \frac{3af^2 \left(f^2 - f^2 \right) \log\left(\sqrt{a + \frac{e^2x^2}{f^2}} - \sqrt{a + \frac{e^2x^2}{f^2}}\right)}{4af^2} - \frac{3af^2 \left(f^2 - f^2 \right) \log\left(\sqrt{a + \frac{e^2x^2}{f^2}} + d - f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{4af^2} - \frac{3af^2 \log\left(\sqrt{a + \frac{e^2x^2}{f^2}} + d + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{4af^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3), x]
```

```
[Out] (Sqrt[a + (e^2*x^2)/f^2]*(5*a*d^2*f^3 - 3*a^2*f^5 - 3*d^3*e*f*x + 9*a*d*e*f^3*x - 4*d^2*e^2*f*x^2))/(2*d^3*e*(d^2 - a*f^2 + 2*d*e*x)^2) + (2*d^8*x + a*d^6*f^2*x + 3*a^2*d^4*f^4*x - 9*a^3*d^2*f^6*x + 3*a^4*f^8*x + 5*d^7*e*x^2 - 3*a*d^5*e*f^2*x^2 + 15*a^2*d^3*e*f^4*x^2 - 9*a^3*d*e*f^6*x^2 + 4*d^6*e^2*x^3 - 8*a*d^4*e^2*f^2*x^3 + 4*a^2*d^2*e^2*f^4*x^3)/(2*d^3*(d^2 - a*f^2)^2*(d^2 - a*f^2 + 2*d*e*x)^2) - (3*a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[-(Sqrt[e^2/f^2]*x) + Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2) + (3*a*Sqrt[e^2/f^2]*f^3*Log[a*f - d*Sqrt[e^2/f^2]*x + d*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2) + (3*a*f^2*Log[a*d^3*e*f - d^4*e*Sqrt[e^2/f^2]*x + d^4*e*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e) + (3*a*(e*f^2 - Sqrt[e^2/f^2]*f^3)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(4*d^4*e^2)
```

fricas [B] time = 0.71, size = 536, normalized size = 2.78

$$\frac{5af^3 + 5af^2e - 6af^2ex - 3af^2x^2 + 2(af^2e^2 + 9af^2ef^2 - 2(af^2d^2 + af^2d^2e + af^2d^2ex + af^2d^2x^2) + 3(af^2e + 4af^2ef^2 - 2af^2d^2 + af^2d^2e - 4(af^2d^2 - af^2d^2f^2))\log\left(-af^2 + 2af^2 + af^2 + (af^2 - 2af^2)\sqrt{\frac{af^2}{d^2}}\right) + 3(af^2e + 4af^2ef^2 - 2af^2d^2 + af^2d^2e - 4(af^2d^2 - af^2d^2f^2))\log\left(-af^2 + 2af^2 + af^2 + 2af^2 + af^2\right) - 3(af^2e + 4af^2ef^2 - 2af^2d^2 + af^2d^2e - 4(af^2d^2 - af^2d^2f^2))\log\left(-af^2 + 2af^2 + af^2 + 2af^2 + af^2\right) - 3(af^2e + 4af^2ef^2 - 2af^2d^2 + af^2d^2e - 4(af^2d^2 - af^2d^2f^2))\log\left(-af^2 + 2af^2 + af^2 + 2af^2 + af^2\right) - 2(af^2d^2 + 4af^2d^2e - 5af^2d^2 - 3(af^2d^2 - af^2d^2f^2))\sqrt{\frac{af^2}{d^2}}}{4(af^2d^3 + 4af^2d^2e - 3af^2d^2f^2 - 4(af^2d^2 - af^2d^2f^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(5*a^3*f^6 + 8*d^3*e^3*x^3 - 6*a^2*d^2*f^4 - 3*a*d^4*f^2 + 2*(a*d^2*e^2*f^2 + 5*d^4*e^2)*x^2 - 2*(7*a^2*d*e*f^4 + a*d^3*e*f^2 - 2*d^5*e)*x + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*e*f^2*x + 2*d*e^2*x^2 + a*d*f^2 + (a*f^3 - 2*d*e*f*x)*sqrt((e^2*x^2 + a*f^2)/f^2)) + 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-a*f^2 + 2*d*e*x + d^2) - 3*(a^3*f^6 + 4*a*d^2*e^2*f^2*x^2 - 2*a^2*d^2*f^4 + a*d^4*f^2 - 4*(a^2*d*e*f^4 - a*d^3*e*f^2)*x)*log(-e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) - d) - 2*(3*a^2*d*f^5 + 4*d^3*e^2*f*x^2 - 5*a*d^3*f^3 - 3*(3*a*d^2*e*f^3 - d^4*e*f)*x)*sqrt((e^2*x^2 + a*f^2)/f^2))/(a^2*d^4*e*f^4 + 4*d^6*e^3*x^2 - 2*a*d^6*e*f^2 + d^8*e - 4*(a*d^5*e^2*f^2 - d^7*e^2)*x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

[Out] Timed out

maple [B] time = 0.05, size = 9721, normalized size = 50.37

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a} f + d \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3,x)
```

[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**3,x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3), x)
```

$$3.236 \quad \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal. Leaf size=225

$$\frac{5ad^{3/2}f^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e} - \frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e} + \frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{5/2}}{3e}$$

Rubi [A] time = 0.19, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, number of rules / integrand size = 0.185, Rules used = {2117, 897, 1257, 1810, 206}

$$\frac{ad^2f^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{5ad^{3/2}f^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2e} + \frac{\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e} + \frac{af^2\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{3e} + \frac{2adf^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2), x]

[Out] (2*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d^2*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/(3*e) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(7/2)/(7*e) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4))^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

$\text{Int}[(Pq_*)((a_) + (b_)*(x_)^2)^{(p_)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq_*(a + b*x^2)^p, x], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 2117

$\text{Int}[(g_ + (h_)*((d_) + (e_)*(x_) + (f_)*\text{Sqrt}[a_ + (c_)*(x_)^2])^{(n_)})^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*e), \text{Subst}[\text{Int}[(g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2)/(d - x)^2, x], x, d + e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, n\}, x] \ \&\& \ \text{EqQ}[e^2 - c*f^2, 0] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx &= \frac{\text{Subst} \left(\int \frac{x^{5/2}(d^2 + af^2 - 2dx + x^2)}{(d-x)^2} dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{\text{Subst} \left(\int \frac{x^6(d^2 + af^2 - 2dx^2 + x^4)}{(d-x)^2} dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{e} \\ &= -\frac{ad^2 f^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{\text{Subst} \left(\int \frac{-ad^2 f^2 - 2adf^2 x^2 - 2af^2 x^4 + 2dx^6 - 2x^8}{d-x^2} dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} \\ &= -\frac{ad^2 f^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{\text{Subst} \left(\int \left(4adf^2 + 2af^2 x^2 + 2x^6 - \frac{5ad^2}{d-x} \right) dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} \\ &= \frac{2adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{ad^2 f^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{af^2 \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}}{3e} \\ &= \frac{2adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{ad^2 f^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{af^2 \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}}{3e} \end{aligned}$$

Mathematica [A] time = 0.34, size = 213, normalized size = 0.95

$$\frac{-5ad^{3/2} f^2 \tanh^{-1} \left(\frac{\sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{a}} \right) - \frac{ad^2 f^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex} + \frac{2}{7} \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^{7/2} + \frac{2}{3} af^2 \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^{3/2} + 4adf^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2), x]

[Out] (4*a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d^2*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*a*f^2*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2))/3 + (2*(d + e*x + f*Sqrt[a

$(e^{2x^2}/f^2)^{(7/2)}/7 - 5ad^{(3/2)}f^2 \operatorname{ArcTanh}[\operatorname{Sqrt}[d + ex + f\operatorname{Sqrt}[a + (e^{2x^2}/f^2)]]/\operatorname{Sqrt}[d]]/(2e)$

IntegrateAlgebraic [A] time = 1.52, size = 277, normalized size = 1.23

$$\frac{(20a^2f^4 - 3ad^2f^2 + 152ade^2x + 76a^2f^2x^2 + 6d^3ex + 36d^2e^2x^2 + 72de^3x^3 + 48e^4x^4) \sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex} + \sqrt{a + \frac{e^2x^2}{f^2}} \sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex} (116adf^3 + 52ae^2f^2x + 6d^3f + 36d^2efx + 72d^2f^2x^2 + 48e^3fx^3)}{42ef\sqrt{a + \frac{e^2x^2}{f^2}} + 42e^2x} - \frac{5ad^{3/2}f^2 \operatorname{tanh}^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + ex + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2), x]

[Out] (Sqrt[a + (e^2*x^2)/f^2]*(6*d^3*f + 116*a*d*f^3 + 36*d^2*e*f*x + 52*a*e*f^3*x + 72*d*e^2*f*x^2 + 48*e^3*f*x^3)*Sqrt[d + ex + f*Sqrt[a + (e^2*x^2)/f^2]] + (-3*a*d^2*f^2 + 20*a^2*f^4 + 6*d^3*e*x + 152*a*d*e*f^2*x + 36*d^2*e^2*x^2 + 76*a*e^2*f^2*x^2 + 72*d*e^3*x^3 + 48*e^4*x^4)*Sqrt[d + ex + f*Sqrt[a + (e^2*x^2)/f^2]]/(42*e^2*x + 42*e*f*Sqrt[a + (e^2*x^2)/f^2]) - (5*a*d^(3/2)*f^2*ArcTanh[Sqrt[d + ex + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]]/(2*e)

fricas [A] time = 0.50, size = 416, normalized size = 1.85

$$\frac{105ad^3f^4 \log\left(\sqrt{d^2 - 2dex + 2d\sqrt{\frac{22ae^2x^2}{f^2}} + 2\sqrt{d}\sqrt{\frac{22ae^2x^2}{f^2}}}\sqrt{a + \sqrt{\frac{22ae^2x^2}{f^2}} + d}\right) + 2\left(24e^3x^3 + 36d^2e^2x^2 + 116ad^2e + 6d^3\right) + (32ae^2f^2 + 39d^2e)x + (24e^2f^2 + 20af^3 + 36d^2e - 3d^2f)\sqrt{\frac{22ae^2x^2}{f^2}}\sqrt{a + \sqrt{\frac{22ae^2x^2}{f^2}} + d}}{84e} - \frac{105e\sqrt{d}\operatorname{arctan}\left(\frac{\sqrt{d^2 - 2dex + 2d\sqrt{\frac{22ae^2x^2}{f^2}} + 2\sqrt{d}\sqrt{\frac{22ae^2x^2}{f^2}}}}{\sqrt{d}}\right)}{42e} + \frac{(24e^3x^3 + 36d^2e^2x^2 + 116ad^2e + 6d^3) + (32ae^2f^2 + 39d^2e)x + (24e^2f^2 + 20af^3 + 36d^2e - 3d^2f)\sqrt{\frac{22ae^2x^2}{f^2}}\sqrt{a + \sqrt{\frac{22ae^2x^2}{f^2}} + d}}{42e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+ex+f*(a+e^2*x^2/f^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] [1/84*(105*a*d^(3/2)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, 1/42*(105*a*sqrt(-d)*d*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (24*e^3*x^3 + 36*d*e^2*x^2 + 116*a*d*f^2 + 6*d^3 + (32*a*e*f^2 + 39*d^2*e)*x + (24*e^2*f*x^2 + 20*a*f^3 + 36*d*e*f*x - 3*d^2*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2x^2}{f^2} + af + d} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+ex+f*(a+e^2*x^2/f^2)^(1/2))^(5/2), x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + af} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2), x)

[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a} f + d \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)

[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(5/2), x)

$$3.237 \quad \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal. Leaf size=183

$$\frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^{5/2}}{5e} + \frac{af^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{e} - \frac{adf^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} - \frac{3a\sqrt{d} f^2 \tanh^{-1} \left(\frac{\sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{d}} \right)}{2e}$$

Rubi [A] time = 0.15, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1257, 1810, 206}

$$\frac{\left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^{5/2}}{5e} + \frac{af^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{e} - \frac{adf^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{2e \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex \right)} - \frac{3a\sqrt{d} f^2 \tanh^{-1} \left(\frac{\sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{d}} \right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2), x]

[Out] (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/e - (a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2)/(5*e) - (3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4))^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

$\text{Int}[(Pq_*)*((a_) + (b_)*(x_)^2)^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandIntegrand}[Pq*(a + b*x^2)^p, x], x] /; \text{FreeQ}\{a, b\}, x] \ \&\& \ \text{PolyQ}[Pq, x] \ \&\& \ \text{IGtQ}[p, -2]$

Rule 2117

$\text{Int}[(g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*\text{Sqrt}[a_ + (c_.)*(x_)^2])^{(n_.)}], x_Symbol] \rightarrow \text{Dist}[1/(2*e), \text{Subst}[\text{Int}[(g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2)/(d - x)^2, x], x, d + e*x + f*\text{Sqrt}[a + c*x^2]], x] /; \text{FreeQ}\{a, c, d, e, f, g, h, n\}, x] \ \&\& \ \text{EqQ}[e^2 - c*f^2, 0] \ \&\& \ \text{IntegerQ}[p]$

Rubi steps

$$\begin{aligned} \int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx &= \frac{\text{Subst} \left(\int \frac{x^{3/2}(d^2 + af^2 - 2dx + x^2)}{(d-x)^2} dx, x, d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)}{2e} \\ &= \frac{\text{Subst} \left(\int \frac{x^4(d^2 + af^2 - 2dx^2 + x^4)}{(d-x)^2} dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{e} \\ &= -\frac{adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} - \frac{\text{Subst} \left(\int \frac{adf^2 + 2af^2 x^2 - 2dx^4 + 2x^6}{d-x^2} dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} \\ &= -\frac{adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} - \frac{\text{Subst} \left(\int \left(-2af^2 - 2x^4 + \frac{3adf^2}{d-x^2} \right) dx, x, \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}} \right)}{2e} \\ &= \frac{af^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2}}{5e} \\ &= \frac{af^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{e} - \frac{adf^2 \sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}}{2e \left(ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)} + \frac{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{5/2}}{5e} \end{aligned}$$

Mathematica [A] time = 0.24, size = 175, normalized size = 0.96

$$\frac{\frac{2}{5} \left(f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex \right)^{5/2} + 2af^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex} - \frac{adf^2 \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{f \sqrt{a + \frac{e^2 x^2}{f^2}} + ex} - 3a\sqrt{d} f^2 \tanh^{-1} \left(\frac{\sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{d}} \right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]

[Out] (2*a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] - (a*d*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]))/(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) + (2*(d + e

*x + f*Sqrt[a + (e^2*x^2)/f^2])^(5/2))/5 - 3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]]/(2*e)

IntegrateAlgebraic [A] time = 1.37, size = 222, normalized size = 1.21

$$\frac{\sqrt{a + \frac{e^2 x^2}{f^2}} \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex} (12af^3 + 2d^2 f + 8defx + 8e^2 fx^2) + (-adf^2 + 16ae f^2 x + 2d^2 ex + 8de^2 x^2 + 8e^3 x^3) \sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{10ef \sqrt{a + \frac{e^2 x^2}{f^2}} + 10e^2 x} - \frac{3a\sqrt{d} f^2 \tanh^{-1}\left(\frac{\sqrt{f \sqrt{a + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2),x]

[Out] (Sqrt[a + (e^2*x^2)/f^2]*(2*d^2*f + 12*a*f^3 + 8*d*e*f*x + 8*e^2*f*x^2)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (-a*d*f^2) + 2*d^2*e*x + 16*a*e*f^2*x + 8*d*e^2*x^2 + 8*e^3*x^3)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/(10*e^2*x + 10*e*f*Sqrt[a + (e^2*x^2)/f^2]) - (3*a*Sqrt[d]*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*e)

fricas [A] time = 0.49, size = 337, normalized size = 1.84

$$\frac{15a\sqrt{d}f^2 \log\left(\frac{ef^2 - 2dex + 2d\sqrt{\frac{e^2x^2}{f^2} + d}}{2e}\right) + 2\left(\sqrt{d}ex - \sqrt{d}\sqrt{\frac{e^2x^2}{f^2} + d}\right)\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2} + d}} + 2\left(4e^2x^2 + 12af^2 + 9dex + 2d^2 + (4efx - df)\sqrt{\frac{e^2x^2}{f^2} + d}\right)\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2} + d}}}{20e} - \frac{15a\sqrt{-d}f^2 \arctan\left(\frac{\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2} + d}}}{d}\right) + (4e^2x^2 + 12af^2 + 9dex + 2d^2 + (4efx - df)\sqrt{\frac{e^2x^2}{f^2} + d})\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2} + d}}}{10e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out] [1/20*(15*a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d) + 2*(4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e, 1/10*(15*a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (4*e^2*x^2 + 12*a*f^2 + 9*d*e*x + 2*d^2 + (4*e*f*x - d*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/e]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a f + d} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{e^2 x^2}{f^2} + a f} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + af + d} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2),x)

[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(3/2), x)

$$3.238 \quad \int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=147

$$\frac{af^2\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{2e\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)} + \frac{\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^{3/2}}{3e} - \frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

Rubi [A] time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1257, 1153, 206}

$$\frac{af^2\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{2e\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)} + \frac{\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^{3/2}}{3e} - \frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2\sqrt{d}e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] -(a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)/(3*e) - (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*Sqrt[d]*e)

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 897

Int[((d_) + (e_)*(x_)^2)^(m_)*((f_) + (g_)*(x_)^2)^(n_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1153

Int[((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1257

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d

+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (c_.)*(x_.)^2])^(n_.))^p_.], x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \int \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}} dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{x}(d^2 + af^2 - 2dx + x^2)}{(d-x)^2} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e} \\
 &= \frac{\text{Subst}\left(\int \frac{x^2(d^2 + af^2 - 2dx + x^4)}{(d-x^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{e} \\
 &= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{-af^2 + 2dx^2 - 2x^4}{d-x^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2e} \\
 &= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(2x^2 - \frac{af^2}{d-x^2}\right) dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2e} \\
 &= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{(af^2)\text{Subst}\left(\int \frac{1}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2e} \\
 &= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{af^2 \tanh^{-1}\left(\frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2e}
 \end{aligned}$$

Mathematica [A] time = 0.33, size = 139, normalized size = 0.95

$$\frac{af^2\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{f\sqrt{a + \frac{e^2x^2}{f^2}} + ex} - \frac{2}{3}\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^{3/2} + \frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] $-1/2*((a*f^2*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]])/(e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]) - (2*(d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2])^(3/2))/3 + (a*f^2*\text{ArcTanh}[\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]]/\text{Sqrt}[d]])/\text{Sqrt}[d])/e$

IntegrateAlgebraic [A] time = 0.89, size = 184, normalized size = 1.25

$$\frac{\sqrt{a + \frac{e^2x^2}{f^2}} (2df + 4efx) \sqrt{f \sqrt{a + \frac{e^2x^2}{f^2}} + d + ex} + (-af^2 + 2dex + 4e^2x^2) \sqrt{f \sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{6ef \sqrt{a + \frac{e^2x^2}{f^2}} + 6e^2x} - \frac{af^2 \tanh^{-1} \left(\frac{\sqrt{f \sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}} \right)}{2\sqrt{d}e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] $((-(a*f^2) + 2*d*e*x + 4*e^2*x^2)*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]] + (2*d*f + 4*e*f*x)*\text{Sqrt}[a + (e^2*x^2)/f^2]*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]])/(6*e^2*x + 6*e*f*\text{Sqrt}[a + (e^2*x^2)/f^2]) - (a*f^2*\text{ArcTanh}[\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + (e^2*x^2)/f^2]]/\text{Sqrt}[d]])/(2*\text{Sqrt}[d]*e)$

fricas [A] time = 0.50, size = 301, normalized size = 2.05

$$\frac{3a\sqrt{d}f^2 \log \left(af^2 - 2dex + 2df \sqrt{\frac{e^2x^2}{f^2}} + 2 \left(\sqrt{d}ex - \sqrt{d}f \sqrt{\frac{e^2x^2}{f^2}} \right) \sqrt{ex + f \sqrt{\frac{e^2x^2}{f^2}} + d} \right) + 2 \left(5dex - df \sqrt{\frac{e^2x^2}{f^2}} + 2d^2 \right) \sqrt{ex + f \sqrt{\frac{e^2x^2}{f^2}} + d}}{12de} + \frac{3a\sqrt{-d}f^2 \arctan \left(\frac{\sqrt{ex + f \sqrt{\frac{e^2x^2}{f^2}} + d}}{d} \right) + (5dex - df \sqrt{\frac{e^2x^2}{f^2}} + 2d^2) \sqrt{ex + f \sqrt{\frac{e^2x^2}{f^2}} + d}}{6de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2), x, algorithm="fricas")

[Out] $[1/12*(3*a*\text{sqrt}(d)*f^2*\log(a*f^2 - 2*d*e*x + 2*d*f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + 2*(\text{sqrt}(d)*e*x - \text{sqrt}(d)*f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2))*\text{sqrt}(e*x + f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(5*d*e*x - d*f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*\text{sqrt}(e*x + f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + d))/(d*e), 1/6*(3*a*\text{sqrt}(-d)*f^2*\arctan(\text{sqrt}(e*x + f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + d))*\text{sqrt}(-d)/d) + (5*d*e*x - d*f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*\text{sqrt}(e*x + f*\text{sqrt}((e^2*x^2 + a*f^2)/f^2) + d))/(d*e)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2}} + af + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d + \sqrt{\frac{e^2x^2}{f^2}} + af} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2), x)

[Out] int((e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{\frac{e^2x^2}{f^2} + a}} f + d \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(e*x + sqrt(e^2*x^2/f^2 + a))*f + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d + ex + f \sqrt{a + \frac{e^2x^2}{f^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

[Out] int((d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f \sqrt{a + \frac{e^2x^2}{f^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)

[Out] Integral(sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)

$$3.239 \quad \int \frac{1}{\sqrt{d+ex+f}\sqrt{a+\frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=147

$$\frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Rubi [A] time = 0.11, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1157, 388, 206}

$$\frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2de\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} + \frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/e - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 897

Int[((d_) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m+1)-1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2

, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (c_.)*(x_.)^2])^(n_.))^p_.], x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} dx &= \frac{\text{Subst}\left(\int \frac{d^2 + af^2 - 2dx + x^2}{(d-x)^2\sqrt{x}} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{d^2 + af^2 - 2dx^2 + x^4}{(d-x^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{e} \\ &= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2de\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d^2 - af^2 + 2dx^2}{d-x^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2de} \\ &= \frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2de\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{(af^2)\text{Subst}\left(\int \frac{1}{d-x} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2d^{3/2}e} \\ &= \frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{e} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2de\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{af^2 \tanh^{-1}\left(\frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{\sqrt{d}}\right)}{2d^{3/2}e} \end{aligned}$$

Mathematica [A] time = 0.25, size = 143, normalized size = 0.97

$$\frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2d^{3/2}} + \frac{af^2\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{2d\left(f\left(-\sqrt{a + \frac{e^2x^2}{f^2}}\right) - ex\right)} + \sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] (Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*(-(e*x) - f*Sqrt[a + (e^2*x^2)/f^2])) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)))/e

IntegrateAlgebraic [A] time = 0.87, size = 171, normalized size = 1.16

$$\frac{af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{3/2}e} + \frac{2df\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex} + (2dex - af^2)\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2def\sqrt{a+\frac{e^2x^2}{f^2}} + 2de^2x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]], x]

[Out] ((-(a*f^2) + 2*d*e*x)*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + 2*d*f*Sqrt[a + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d*e^2*x + 2*d*e*f*Sqrt[a + (e^2*x^2)/f^2]) + (a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(3/2)*e)

fricas [A] time = 0.51, size = 298, normalized size = 2.03

$$\frac{a\sqrt{d}f^2\log\left(af^2 - 2dex + 2df\sqrt{\frac{e^2x^2}{f^2}} - 2\left(\sqrt{d}ex - \sqrt{d}f\sqrt{\frac{e^2x^2}{f^2}}\right)\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2}} + d}\right) + 2\left(dex - df\sqrt{\frac{e^2x^2}{f^2}} + 2d^2\right)\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2}} + d}}{4d^2e} - \frac{a\sqrt{-d}f^2\arctan\left(\frac{\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2}} + d}\sqrt{-d}}{d}\right) - \left(dex - df\sqrt{\frac{e^2x^2}{f^2}} + 2d^2\right)\sqrt{ex + f\sqrt{\frac{e^2x^2}{f^2}} + d}}{2d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2), x, algorithm="fricas")

[Out] [1/4*(a*sqrt(d)*f^2*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) + 2*(d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e), -1/2*(a*sqrt(-d)*f^2*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) - (d*e*x - d*f*sqrt((e^2*x^2 + a*f^2)/f^2) + 2*d^2)*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(d^2*e)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{\frac{e^2x^2}{f^2}} + af + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2), x, algorithm="giac")

[Out] integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + d + \sqrt{\frac{e^2x^2}{f^2}} + af}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(1/2), x)

[Out] $\int \frac{1}{\sqrt{e^2 x^2 + d + (e^2 x^2 + a)^{1/2}} \sqrt{e^2 x^2 + a}} dx$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{\frac{e^2 x^2}{f^2} + a}} \sqrt{f + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(1/sqrt(e*x + sqrt(e^2*x^2/f^2 + a)*f + d), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2),x)`

[Out] `int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(1/2),x)`

[Out] `Integral(1/sqrt(d + e*x + f*sqrt(a + e**2*x**2/f**2)), x)`

$$3.240 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{3/2}} dx$$

Optimal. Leaf size=158

$$\frac{3af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}$$

Rubi [A] time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1259, 453, 206}

$$-\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^2e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{\frac{af^2}{d^2}+1}{e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} + \frac{3af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{5/2}e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2), x]

[Out] -((1 + (a*f^2)/d^2)/(e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])) - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^2*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a+b*x^n)^(p+1)/(a*e^(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1)+1)/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m+1)-1)*((e*f-d*g)/e + (g*x^q)/e)^n*((c*d^2-b*d*e+a*e^2)/e^2 - ((2*c*d-b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d+e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f-d*g, 0] && NeQ[b^2-4*a*c, 0] && NeQ[c*d^2-b*d*e+a*e^2, 0] && IntegerQ[n, p] && FractionQ[m]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 2117

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{d^2 + af^2 - 2dx + x^2}{(d-x)^2x^{3/2}} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e} \\ &= \frac{\text{Subst}\left(\int \frac{d^2 + af^2 - 2dx^2 + x^4}{x^2(d-x^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{e} \\ &= \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} - \frac{\text{Subst}\left(\int \frac{-2d(d^2 + af^2) + (2d^2 - af^2)x^2}{x^2(d-x^2)} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2d^2e} \\ &= -\frac{d^2 + af^2}{d^2e\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{(3af^2)\text{Subst}\left(\int \frac{1}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2d^2e} \\ &= -\frac{d^2 + af^2}{d^2e\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^2e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{3af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{2d^2e} \end{aligned}$$

Mathematica [A] time = 0.43, size = 167, normalized size = 1.06

$$\frac{3af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}{\sqrt{d}}\right)}{d^{5/2}} + \frac{-2d^2\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right) - af^2\left(3f\sqrt{a + \frac{e^2x^2}{f^2}} + d + 3ex\right)}{2e\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + ex\right)\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2), x]
```

```
[Out] ((-2*d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2]) - a*f^2*(d + 3*e*x + 3*f*Sqrt[a + (e^2*x^2)/f^2]))/(d^2*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/d^(5/2))/(2*e)
```

IntegrateAlgebraic [A] time = 1.31, size = 194, normalized size = 1.23

$$\frac{3af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+dex}}{\sqrt{d}}\right)}{2d^{5/2}e} + \frac{(-3af^3 - 2d^2f)\sqrt{a+\frac{e^2x^2}{f^2}} - adf^2 - 3aef^2x - 2d^2ex}{2d^2e^2x\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+dex} + 2d^2ef\sqrt{a+\frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+dex}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-3/2), x]
```

```
[Out] (-a*d*f^2) - 2*d^2*e*x - 3*a*e*f^2*x + (-2*d^2*f - 3*a*f^3)*Sqrt[a + (e^2*x^2)/f^2])/(2*d^2*e^2*x*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]] + 2*d^2*e*f*Sqrt[a + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) + (3*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(5/2)*e)
```

fricas [A] time = 0.52, size = 487, normalized size = 3.08

$$\frac{3(d^2f^3 - 2ad^2f^2 - ad^2f^2)\sqrt{d}\log\left(\sqrt{d^2 + 2df\sqrt{\frac{2d^2e^2x^2}{f^2} + d}} - 2\left(\sqrt{d}e - \sqrt{f}\sqrt{\frac{2d^2e^2x^2}{f^2} + d}\right)\sqrt{d + f\sqrt{\frac{2d^2e^2x^2}{f^2} + d}}\right) - 2\left(2d^2e^2x^2 - 2ad^2f^2 - 2d^2e^2x - (3ad^2f^2 + d^2e^2) + (3ad^2f^2 - 2d^2e^2x + d^2e^2)\sqrt{\frac{2d^2e^2x^2}{f^2} + d}\right)}{4(ad^2f^2 - 2d^2e^2x - d^2e^2)} + \frac{3(d^2f^3 - 2ad^2f^2 - ad^2f^2)\sqrt{d}\operatorname{arctan}\left(\frac{\sqrt{d}\sqrt{\frac{2d^2e^2x^2}{f^2} + d}}{f}\right) + (2d^2e^2x^2 - 2ad^2f^2 - 2d^2e^2x - (3ad^2f^2 + d^2e^2) + (3ad^2f^2 - 2d^2e^2x + d^2e^2)\sqrt{\frac{2d^2e^2x^2}{f^2} + d})\sqrt{\frac{2d^2e^2x^2}{f^2} + d}}{2(ad^2f^2 - 2d^2e^2x - d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2), x, algorithm="fricas")
```

```
[Out] [1/4*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(d)*log(a*f^2 - 2*d*e*x + 2*d*f*sqrt((e^2*x^2 + a*f^2)/f^2) - 2*(sqrt(d)*e*x - sqrt(d)*f*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)) - 2*(2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e), -1/2*(3*(a^2*f^4 - 2*a*d*e*f^2*x - a*d^2*f^2)*sqrt(-d)*arctan(sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d)*sqrt(-d)/d) + (2*d^2*e^2*x^2 - 2*a*d^2*f^2 - 2*d^4 - (3*a*d*e*f^2 + d^3*e)*x + (3*a*d*f^3 - 2*d^2*e*f*x + d^3*f)*sqrt((e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((e^2*x^2 + a*f^2)/f^2) + d))/(a*d^3*e*f^2 - 2*d^4*e^2*x - d^5*e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2), x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e ,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1
```

, [0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [-49,85.3561567818,-64,-30]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [0,61.7937478349,0,0]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [56,62.4600259969,-13,46]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [-6,25.8388736797,81,18]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [63,31.8503101398,2,62]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [0,10.4309062702,0,0]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [65,39.1803401988,28,-44]Warning, choosing root of [1,0,%%{-2,[1,2,0,0]%%}+%%{-2,[0,1,0,0]%%}+%%{-2,[0,0,2,1]%%},0,%%{1,[2,4,0,0]%%}+%%{-2,[1,3,0,0]%%}+%%{2,[1,2,2,1]%%}+%%{1,[0,2,0,0]%%}+%%{-2,[0,1,2,1]%%}+%%{1,[0,0,4,2]%%}] at parameters values [91,88.2886286299,-21,88]Evaluation time: 0.64Done

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{\frac{e^2 x^2}{f^2} + a f} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a f} + d \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)`

[Out] `int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(3/2), x)`

[Out] `Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-3/2), x)`

$$3.241 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+\frac{e^2x^2}{f^2}}\right)^{5/2}} dx$$

Optimal. Leaf size=199

$$\frac{5af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e} - \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)}$$

Rubi [A] time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2117, 897, 1259, 1261, 206}

$$\frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{2d^3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+ex\right)} - \frac{2af^2}{d^3e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{\frac{af^2}{d^2}+1}{3e\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}} + \frac{5af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2), x]

[Out] -(1 + (a*f^2)/d^2)/(3*e*(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(3/2)) - (2*a*f^2)/(d^3*e*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]) - (a*f^2*Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]])/(2*d^3*e*(e*x + f*Sqrt[a + (e^2*x^2)/f^2])) + (5*a*f^2*ArcTanh[Sqrt[d + e*x + f*Sqrt[a + (e^2*x^2)/f^2]]/Sqrt[d]])/(2*d^(7/2)*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2117

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_.))^p, x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{5/2}} dx = \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx+x^2}{(d-x)^2x^{5/2}} dx, x, d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}{2e}$$

$$= \frac{\text{Subst}\left(\int \frac{d^2+af^2-2dx^2+x^4}{x^4(d-x^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{e}$$

$$= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^3e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \frac{2d^2(d^2+af^2)-2d(d^2-af^2)x^2+af^2x^4}{x^4(d-x^2)} dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2d^3e}$$

$$= -\frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^3e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)} + \frac{\text{Subst}\left(\int \left(\frac{2(d^3+adf^2)}{x^4} + \frac{4af^2}{x^2} + \frac{5af^2}{d-x^2}\right) dx, x, \sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}\right)}{2d^3e}$$

$$= -\frac{d^2 + af^2}{3d^2e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^3e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}$$

$$= -\frac{d^2 + af^2}{3d^2e\left(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)^{3/2}} - \frac{2af^2}{d^3e\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} - \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{2d^3e\left(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}\right)}$$

Mathematica [A] time = 0.60, size = 186, normalized size = 0.93

$$\frac{\frac{2d(af^2+d^2)}{3\left(f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}} + \frac{af^2\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex} + \frac{4af^2}{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}} - \frac{5af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{2d^3e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2), x]

[Out] $-\frac{1}{2} \cdot \frac{(2d(d^2 + af^2))}{(3(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}))^{\frac{3}{2}}} + \frac{4af^2}{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}} + \frac{af^2\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{(ex + f\sqrt{a + \frac{e^2x^2}{f^2}}) - (5af^2 \operatorname{ArcTanh}[\frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{\sqrt{d}}])/\sqrt{d}}}{d^3e}$

IntegrateAlgebraic [A] time = 1.41, size = 226, normalized size = 1.14

$$\frac{-15a^2f^4 + \sqrt{a + \frac{e^2x^2}{f^2}}(-20adf^3 - 30aef^3x - 2d^3f) - 3ad^2f^2 - 20ade^2f^2x - 30ae^2f^2x^2 - 2d^3ex}{6d^3e^2x\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^{3/2} + 6d^3ef\sqrt{a + \frac{e^2x^2}{f^2}}\left(f\sqrt{a + \frac{e^2x^2}{f^2}} + d + ex\right)^{3/2}} + \frac{5af^2 \tanh^{-1}\left(\frac{\sqrt{f\sqrt{a + \frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{d}}\right)}{2d^{7/2}e}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (e^2*x^2)/f^2])^(-5/2), x]

[Out] $(-3ad^2f^2 - 15a^2f^4 - 2d^3ex - 20a^2d^2ef^2x - 30a^2e^2f^2x^2 + (-2d^3f - 20a^2d^2f^3 - 30a^2e^2f^3x)\sqrt{a + \frac{e^2x^2}{f^2}})/(6d^3e^2x(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}})^{3/2} + 6d^3e^2f\sqrt{a + \frac{e^2x^2}{f^2}}(d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}})^{3/2}) + (5af^2 \operatorname{ArcTanh}[\frac{\sqrt{d + ex + f\sqrt{a + \frac{e^2x^2}{f^2}}}}{\sqrt{d}}])/(2d^{7/2}e)$

fricas [B] time = 0.54, size = 812, normalized size = 4.08



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2), x, algorithm="fricas")

[Out] $\frac{1}{12} \cdot (15(a^3f^6 + 4a^2d^2e^2f^2x^2 - 2a^2d^2f^4 + ad^4f^2 - 4(a^2d^2ef^4 - ad^3ef^2)x)\sqrt{d}) \log(af^2 - 2d^2ex + 2d^2f\sqrt{(e^2x^2 + af^2)/f^2}) - 2(\sqrt{d}ex - \sqrt{d}f\sqrt{(e^2x^2 + af^2)/f^2}) \sqrt{ex + f\sqrt{(e^2x^2 + af^2)/f^2} + d} + 2(12d^3e^3x^3 + 10a^2d^2f^4 - 16a^2d^4f^2 - 2d^6 - 8(5a^2d^2e^2f^2 - d^4e^2)x^2 + (15a^2d^2ef^4 - 46a^2d^3ef^2 - d^5e)x - (15a^2d^2f^5 + 12d^3e^2f^2x^2 - 22a^2d^3f^3 - d^5f - 8(5a^2d^2ef^3 - d^4ef)x)\sqrt{(e^2x^2 + af^2)/f^2}) \sqrt{ex + f\sqrt{(e^2x^2 + af^2)/f^2} + d})/(a^2d^4ef^4 + 4d^6e^3x^2 - 2a^2d^6ef^2 + d^8e - 4(a^2d^5e^2f^2 - d^7e^2)x), -\frac{1}{6} \cdot (15(a^3f^6 + 4a^2d^2e^2f^2x^2 - 2a^2d^2f^4 + ad^4f^2 - 4(a^2d^2ef^4 - ad^3ef^2)x)\sqrt{-d}) \arctan(\sqrt{ex + f\sqrt{(e^2x^2 + af^2)/f^2} + d})/\sqrt{-d} - (12d^3e^3x^3 + 10a^2d^2f^4 - 16a^2d^4f^2 - 2d^6 - 8(5a^2d^2e^2f^2 - d^4e^2)x^2 + (15a^2d^2ef^4 - 46a^2d^3ef^2 - d^5e)x - (15a^2d^2f^5 + 12d^3e^2f^2x^2 - 22a^2d^3f^3 - d^5f - 8(5a^2d^2ef^3 - d^4ef)x)\sqrt{(e^2x^2 + af^2)/f^2}) \sqrt{ex + f\sqrt{(e^2x^2 + af^2)/f^2} + d})/(a^2d^4ef^4 + 4d^6e^3x^2 - 2a^2d^6ef^2 + d^8e - 4(a^2d^5e^2f^2 - d^7e^2)x)$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{\frac{e^2 x^2}{f^2} + a} f \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

[Out] int(1/(e*x+d+(e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a} f + d \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a)*f + d)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2),x)

[Out] int(1/(d + e*x + f*(a + (e^2*x^2)/f^2)^(1/2))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + \frac{e^2 x^2}{f^2}} \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + e**2*x**2/f**2))**(-5/2), x)

$$3.242 \quad \int \sqrt{x - \sqrt{-4 + x^2}} \, dx$$

Optimal. Leaf size=41

$$\frac{1}{3} \left(x - \sqrt{x^2 - 4} \right)^{3/2} + \frac{4}{\sqrt{x - \sqrt{x^2 - 4}}}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2117, 14}

$$\frac{1}{3} \left(x - \sqrt{x^2 - 4} \right)^{3/2} + \frac{4}{\sqrt{x - \sqrt{x^2 - 4}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x - Sqrt[-4 + x^2]],x]

[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2117

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] :> Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \sqrt{x - \sqrt{-4 + x^2}} \, dx &= \frac{1}{2} \text{Subst} \left(\int \frac{-4 + x^2}{x^{3/2}} \, dx, x, x - \sqrt{-4 + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(-\frac{4}{x^{3/2}} + \sqrt{x} \right) \, dx, x, x - \sqrt{-4 + x^2} \right) \\ &= \frac{4}{\sqrt{x - \sqrt{-4 + x^2}}} + \frac{1}{3} \left(x - \sqrt{-4 + x^2} \right)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 40, normalized size = 0.98

$$\frac{2x^2 - 2\sqrt{x^2 - 4}x + 8}{3\sqrt{x - \sqrt{x^2 - 4}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x - Sqrt[-4 + x^2]],x]

[Out] (8 + 2*x^2 - 2*x*Sqrt[-4 + x^2])/(3*Sqrt[x - Sqrt[-4 + x^2]])

IntegrateAlgebraic [A] time = 0.05, size = 41, normalized size = 1.00

$$\frac{1}{3} \left(x - \sqrt{x^2 - 4} \right)^{3/2} + \frac{4}{\sqrt{x - \sqrt{x^2 - 4}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x - Sqrt[-4 + x^2]], x]

[Out] 4/Sqrt[x - Sqrt[-4 + x^2]] + (x - Sqrt[-4 + x^2])^(3/2)/3

fricas [A] time = 0.40, size = 26, normalized size = 0.63

$$\frac{2}{3} \left(2x + \sqrt{x^2 - 4} \right) \sqrt{x - \sqrt{x^2 - 4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2-4)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 2/3*(2*x + sqrt(x^2 - 4))*sqrt(x - sqrt(x^2 - 4))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x - \sqrt{x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2-4)^(1/2))^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \sqrt{x - \sqrt{x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2-4)^(1/2))^(1/2), x)

[Out] int((x-(x^2-4)^(1/2))^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x - \sqrt{x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2-4)^(1/2))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(x - sqrt(x^2 - 4)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x - \sqrt{x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - (x^2 - 4)^(1/2))^(1/2), x)

```
[Out] int((x - (x^2 - 4)^(1/2))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{x - \sqrt{x^2 - 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2-4)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(x - sqrt(x**2 - 4)), x)
```

$$3.243 \quad \int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}} dx$$

Optimal. Leaf size=69

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2} + c + ax}\right)^{3/2}}{3a} - \frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2} + c + ax}}}$$

Rubi [A] time = 0.06, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {2117, 14}

$$\frac{\left(b\sqrt{\frac{a^2x^2}{b^2} + c + ax}\right)^{3/2}}{3a} - \frac{b^2c}{a\sqrt{b\sqrt{\frac{a^2x^2}{b^2} + c + ax}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]

[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a^2*x^2)/b^2])^(3/2)/(3*a)

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2117

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}} dx &= \frac{\text{Subst}\left(\int \frac{b^2c+x^2}{x^{3/2}} dx, x, ax + b\sqrt{c + \frac{a^2x^2}{b^2}}\right)}{2a} \\ &= \frac{\text{Subst}\left(\int \left(\frac{b^2c}{x^{3/2}} + \sqrt{x}\right) dx, x, ax + b\sqrt{c + \frac{a^2x^2}{b^2}}\right)}{2a} \\ &= -\frac{b^2c}{a\sqrt{ax + b\sqrt{c + \frac{a^2x^2}{b^2}}}} + \frac{\left(ax + b\sqrt{c + \frac{a^2x^2}{b^2}}\right)^{3/2}}{3a} \end{aligned}$$

Mathematica [A] time = 0.06, size = 67, normalized size = 0.97

$$\frac{2 \left(abx \sqrt{\frac{a^2 x^2}{b^2} + c} + a^2 x^2 + b^2(-c) \right)}{3a \sqrt{b \sqrt{\frac{a^2 x^2}{b^2} + c} + ax}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]

[Out] (2*(-(b^2*c) + a^2*x^2 + a*b*x*Sqrt[c + (a^2*x^2)/b^2]))/(3*a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])

IntegrateAlgebraic [A] time = 0.66, size = 69, normalized size = 1.00

$$\frac{\left(b \sqrt{\frac{a^2 x^2}{b^2} + c} + ax \right)^{3/2}}{3a} - \frac{b^2 c}{a \sqrt{b \sqrt{\frac{a^2 x^2}{b^2} + c} + ax}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]],x]

[Out] -((b^2*c)/(a*Sqrt[a*x + b*Sqrt[c + (a^2*x^2)/b^2]])) + (a*x + b*Sqrt[c + (a^2*x^2)/b^2])^(3/2)/(3*a)

fricas [A] time = 0.41, size = 59, normalized size = 0.86

$$\frac{2 \left(2ax - b \sqrt{\frac{a^2 x^2 + b^2 c}{b^2}} \right) \sqrt{ax + b \sqrt{\frac{a^2 x^2 + b^2 c}{b^2}}}}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(2*a*x - b*sqrt((a^2*x^2 + b^2*c)/b^2))*sqrt(a*x + b*sqrt((a^2*x^2 + b^2*c)/b^2))/a

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2 x^2}{b^2} + c}} b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c))*b, x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2 x^2}{b^2} + c}} b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)

[Out] `int((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + \sqrt{\frac{a^2x^2}{b^2} + c}} b \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*(c+a^2*x^2/b^2)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(a*x + sqrt(a^2*x^2/b^2 + c))*b, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{ax + b \sqrt{c + \frac{a^2x^2}{b^2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2),x)`

[Out] `int((a*x + b*(c + (a^2*x^2)/b^2)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ax + b \sqrt{\frac{a^2x^2}{b^2} + c}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+b*(c+a**2*x**2/b**2)**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(a*x + b*sqrt(a**2*x**2/b**2 + c)), x)`

$$3.244 \quad \int \sqrt{1 + \sqrt{1 - x^2}} \, dx$$

Optimal. Leaf size=45

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}} - \frac{2x^3}{3(\sqrt{1-x^2}+1)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2129}

$$\frac{2x}{\sqrt{\sqrt{1-x^2}+1}} - \frac{2x^3}{3(\sqrt{1-x^2}+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[1 - x^2]], x]

[Out] (-2*x^3)/(3*(1 + Sqrt[1 - x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 - x^2]]

Rule 2129

Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] :> Simp[(2*b^2*d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]

Rubi steps

$$\int \sqrt{1 + \sqrt{1 - x^2}} \, dx = -\frac{2x^3}{3(1 + \sqrt{1 - x^2})^{3/2}} + \frac{2x}{\sqrt{1 + \sqrt{1 - x^2}}}$$

Mathematica [A] time = 0.10, size = 35, normalized size = 0.78

$$\frac{2x(\sqrt{1-x^2}+2)}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sqrt[1 - x^2]], x]

[Out] (2*x*(2 + Sqrt[1 - x^2]))/(3*Sqrt[1 + Sqrt[1 - x^2]])

IntegrateAlgebraic [A] time = 0.07, size = 56, normalized size = 1.24

$$\frac{2\sqrt{1-x^2}x}{3\sqrt{\sqrt{1-x^2}+1}} + \frac{4x}{3\sqrt{\sqrt{1-x^2}+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 - x^2]], x]

[Out] $(4*x)/(3*\text{Sqrt}[1 + \text{Sqrt}[1 - x^2]]) + (2*x*\text{Sqrt}[1 - x^2])/(3*\text{Sqrt}[1 + \text{Sqrt}[1 - x^2]])$

fricas [A] time = 0.42, size = 34, normalized size = 0.76

$$\frac{2 \left(x^2 - \sqrt{-x^2 + 1} + 1 \right) \sqrt{\sqrt{-x^2 + 1} + 1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="fricas")`

[Out] $2/3*(x^2 - \text{sqrt}(-x^2 + 1) + 1)*\text{sqrt}(\text{sqrt}(-x^2 + 1) + 1)/x$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{-x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="giac")`

[Out] `integrate(sqrt(sqrt(-x^2 + 1) + 1), x)`

maple [C] time = 0.04, size = 60, normalized size = 1.33

$$\frac{i \left(\frac{32i\sqrt{\pi} \sqrt{2} x^3 \cos\left(\frac{3\arcsin(x)}{2}\right)}{3} - \frac{8i\sqrt{\pi} \sqrt{2} \left(-\frac{4}{3}x^4 + \frac{2}{3}x^2 + \frac{2}{3}\right) \sin\left(\frac{3\arcsin(x)}{2}\right)}{\sqrt{-x^2+1}} \right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+(-x^2+1)^(1/2))^(1/2),x)`

[Out] $1/8*I/\text{Pi}^{(1/2)}*(32/3*I*\text{Pi}^{(1/2)}*2^{(1/2)}*x^3*\cos(3/2*\arcsin(x))-8*I*\text{Pi}^{(1/2)}*2^{(1/2)}*(-4/3*x^4+2/3*x^2+2/3)*\sin(3/2*\arcsin(x))/(-x^2+1)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{-x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-x^2+1)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(sqrt(-x^2 + 1) + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{1 - x^2} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((1 - x^2)^(1/2) + 1)^(1/2),x)`

[Out] `int(((1 - x^2)^(1/2) + 1)^(1/2), x)`

sympy [B] time = 1.28, size = 418, normalized size = 9.29

$$\begin{cases} \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}} + \frac{3\sqrt{2}ix\sqrt{x^2-1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}} + \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{-12i\pi\sqrt{x^2-1}\sqrt{i\sqrt{x^2-1}+1}-12\pi\sqrt{i\sqrt{x^2-1}+1}} & \text{for } |x^2| > 1 \\ \frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\sqrt{1-x^2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{1-x^2}\sqrt{\sqrt{1-x^2}+1}+12\pi\sqrt{\sqrt{1-x^2}+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(-x**2+1)**(1/2))**(1/2), x)

[Out] Piecewise((-sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*I*x*sqrt(x**2 - 1)*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)) + 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(-12*I*pi*sqrt(x**2 - 1)*sqrt(I*sqrt(x**2 - 1) + 1) - 12*pi*sqrt(I*sqrt(x**2 - 1) + 1)), Abs(x**2) > 1), (sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*sqrt(2)*x*sqrt(1 - x**2)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(1 - x**2)*sqrt(sqrt(1 - x**2) + 1) + 12*pi*sqrt(sqrt(1 - x**2) + 1)), True))

$$3.245 \quad \int \sqrt{1 + \sqrt{1 + x^2}} \, dx$$

Optimal. Leaf size=41

$$\frac{2x}{\sqrt{\sqrt{x^2+1}+1}} + \frac{2x^3}{3(\sqrt{x^2+1}+1)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2129}

$$\frac{2x^3}{3(\sqrt{x^2+1}+1)^{3/2}} + \frac{2x}{\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[1 + x^2]], x]

[Out] (2*x^3)/(3*(1 + Sqrt[1 + x^2])^(3/2)) + (2*x)/Sqrt[1 + Sqrt[1 + x^2]]

Rule 2129

Int[Sqrt[(a_) + (b_)*Sqrt[(c_) + (d_)*(x_)^2]], x_Symbol] := Simp[(2*b^2*d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]

Rubi steps

$$\int \sqrt{1 + \sqrt{1 + x^2}} \, dx = \frac{2x^3}{3(1 + \sqrt{1 + x^2})^{3/2}} + \frac{2x}{\sqrt{1 + \sqrt{1 + x^2}}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 1.07

$$\frac{2(\sqrt{x^2+1}-1)\sqrt{\sqrt{x^2+1}+1}(\sqrt{x^2+1}+2)}{3x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sqrt[1 + x^2]], x]

[Out] (2*(-1 + Sqrt[1 + x^2])*Sqrt[1 + Sqrt[1 + x^2]]*(2 + Sqrt[1 + x^2]))/(3*x)

IntegrateAlgebraic [A] time = 0.08, size = 50, normalized size = 1.22

$$\frac{2\sqrt{x^2+1}x}{3\sqrt{\sqrt{x^2+1}+1}} + \frac{4x}{3\sqrt{\sqrt{x^2+1}+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + x^2]], x]

[Out] (4*x)/(3*Sqrt[1 + Sqrt[1 + x^2]]) + (2*x*Sqrt[1 + x^2])/(3*Sqrt[1 + Sqrt[1 + x^2]])

fricas [A] time = 0.45, size = 28, normalized size = 0.68

$$\frac{2\left(x^2 + \sqrt{x^2 + 1} - 1\right)\sqrt{\sqrt{x^2 + 1} + 1}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="fricas")

[Out] 2/3*(x^2 + sqrt(x^2 + 1) - 1)*sqrt(sqrt(x^2 + 1) + 1)/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)

maple [C] time = 0.03, size = 55, normalized size = 1.34

$$\frac{\frac{32\sqrt{\pi}\sqrt{2}x^3\cosh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{3} - \frac{8\sqrt{\pi}\sqrt{2}\left(-\frac{4}{3}x^4 - \frac{2}{3}x^2 + \frac{2}{3}\right)\sinh\left(\frac{3\operatorname{arcsinh}(x)}{2}\right)}{\sqrt{x^2+1}}}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+(x^2+1)^(1/2))^(1/2),x)

[Out] -1/8/Pi^(1/2)*(-32/3*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(x))-8*Pi^(1/2)*2^(1/2)*(-4/3*x^4-2/3*x^2+2/3)*sinh(3/2*arcsinh(x)))/(x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)^(1/2)+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(x^2 + 1) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2 + 1)^(1/2) + 1)^(1/2),x)

[Out] int(((x^2 + 1)^(1/2) + 1)^(1/2), x)

sympy [B] time = 1.18, size = 197, normalized size = 4.80

$$\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\sqrt{x^2+1}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}} - \frac{3\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2+1}\sqrt{\sqrt{x^2+1}+1}+12\pi\sqrt{\sqrt{x^2+1}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)**(1/2)+1)**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*sqrt(x**2 + 1)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1)) - 3*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 1)*sqrt(sqrt(x**2 + 1) + 1) + 12*pi*sqrt(sqrt(x**2 + 1) + 1))
```

$$3.246 \quad \int \sqrt{5 + \sqrt{25 + x^2}} \, dx$$

Optimal. Leaf size=41

$$\frac{10x}{\sqrt{\sqrt{x^2 + 25} + 5}} + \frac{2x^3}{3(\sqrt{x^2 + 25} + 5)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2129}

$$\frac{2x^3}{3(\sqrt{x^2 + 25} + 5)^{3/2}} + \frac{10x}{\sqrt{\sqrt{x^2 + 25} + 5}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[5 + Sqrt[25 + x^2]], x]

[Out] (2*x^3)/(3*(5 + Sqrt[25 + x^2])^(3/2)) + (10*x)/Sqrt[5 + Sqrt[25 + x^2]]

Rule 2129

Int[Sqrt[(a_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2]], x_Symbol] := Simp[(2*b^2*d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]

Rubi steps

$$\int \sqrt{5 + \sqrt{25 + x^2}} \, dx = \frac{2x^3}{3(5 + \sqrt{25 + x^2})^{3/2}} + \frac{10x}{\sqrt{5 + \sqrt{25 + x^2}}}$$

Mathematica [A] time = 0.06, size = 44, normalized size = 1.07

$$\frac{2(\sqrt{x^2 + 25} - 5)\sqrt{\sqrt{x^2 + 25} + 5}(\sqrt{x^2 + 25} + 10)}{3x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[5 + Sqrt[25 + x^2]], x]

[Out] (2*(-5 + Sqrt[25 + x^2])*Sqrt[5 + Sqrt[25 + x^2]]*(10 + Sqrt[25 + x^2]))/(3*x)

IntegrateAlgebraic [A] time = 0.06, size = 50, normalized size = 1.22

$$\frac{2\sqrt{x^2 + 25}x}{3\sqrt{\sqrt{x^2 + 25} + 5}} + \frac{20x}{3\sqrt{\sqrt{x^2 + 25} + 5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[5 + Sqrt[25 + x^2]], x]

[Out] (20*x)/(3*Sqrt[5 + Sqrt[25 + x^2]]) + (2*x*Sqrt[25 + x^2])/(3*Sqrt[5 + Sqrt[25 + x^2]])

fricas [A] time = 0.43, size = 30, normalized size = 0.73

$$\frac{2(x^2 + 5\sqrt{x^2 + 25} - 25)\sqrt{\sqrt{x^2 + 25} + 5}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(x^2 + 5*sqrt(x^2 + 25) - 25)*sqrt(sqrt(x^2 + 25) + 5)/x

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2 + 25} + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)

maple [C] time = 0.02, size = 64, normalized size = 1.56

$$\frac{5\sqrt{5} \left(\frac{32\sqrt{\pi} \sqrt{2} x^3 \cosh\left(\frac{3 \operatorname{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{375} - \frac{8\sqrt{\pi} \sqrt{2} \left(-\frac{4}{1875}x^4 - \frac{2}{75}x^2 + \frac{2}{3}\right) \sinh\left(\frac{3 \operatorname{arcsinh}\left(\frac{x}{5}\right)}{2}\right)}{\sqrt{\frac{x^2}{25} + 1}} \right)}{8\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+(x^2+25)^(1/2))^(1/2),x)

[Out] -5/8*5^(1/2)/Pi^(1/2)*(-32/375*Pi^(1/2)*2^(1/2)*x^3*cosh(3/2*arcsinh(1/5*x)) - 8*Pi^(1/2)*2^(1/2)*(-4/1875*x^4 - 2/75*x^2 + 2/3)*sinh(3/2*arcsinh(1/5*x)))/(1/25*x^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x^2 + 25} + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+(x^2+25)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(x^2 + 25) + 5), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{x^2 + 25} + 5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2 + 25)^(1/2) + 5)^(1/2),x)

[Out] int(((x^2 + 25)^(1/2) + 5)^(1/2), x)

sympy [B] time = 1.22, size = 197, normalized size = 4.80

$$\frac{\sqrt{2}x^3\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2 + 25}\sqrt{\sqrt{x^2 + 25} + 5} + 60\pi\sqrt{\sqrt{x^2 + 25} + 5}} - \frac{15\sqrt{2}x\sqrt{x^2 + 25}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2 + 25}\sqrt{\sqrt{x^2 + 25} + 5} + 60\pi\sqrt{\sqrt{x^2 + 25} + 5}} - \frac{75\sqrt{2}x\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{12\pi\sqrt{x^2 + 25}\sqrt{\sqrt{x^2 + 25} + 5} + 60\pi\sqrt{\sqrt{x^2 + 25} + 5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((5+(x**2+25)**(1/2))**(1/2),x)
```

```
[Out] -sqrt(2)*x**3*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 15*sqrt(2)*x*sqrt(x**2 + 25)*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5)) - 75*sqrt(2)*x*gamma(-1/4)*gamma(1/4)/(12*pi*sqrt(x**2 + 25)*sqrt(sqrt(x**2 + 25) + 5) + 60*pi*sqrt(sqrt(x**2 + 25) + 5))
```

$$3.247 \quad \int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}} dx$$

Optimal. Leaf size=66

$$\frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2} + cx^2} + a}} + \frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2} + cx^2} + a\right)^{3/2}}$$

Rubi [A] time = 0.03, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {2129}

$$\frac{2b^2cx^3}{3\left(b\sqrt{\frac{a^2}{b^2} + cx^2} + a\right)^{3/2}} + \frac{2ax}{\sqrt{b\sqrt{\frac{a^2}{b^2} + cx^2} + a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]

[Out] (2*b^2*c*x^3)/(3*(a + b*Sqrt[a^2/b^2 + c*x^2])^(3/2)) + (2*a*x)/Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]]

Rule 2129

Int[Sqrt[(a_) + (b_)*Sqrt[(c_) + (d_)*(x_)^2]], x_Symbol] := Simp[(2*b^2*d*x^3)/(3*(a + b*Sqrt[c + d*x^2])^(3/2)), x] + Simp[(2*a*x)/Sqrt[a + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*c, 0]

Rubi steps

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}} dx = \frac{2b^2cx^3}{3\left(a + b\sqrt{\frac{a^2}{b^2} + cx^2}\right)^{3/2}} + \frac{2ax}{\sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}}}$$

Mathematica [A] time = 0.23, size = 55, normalized size = 0.83

$$\frac{2bx\sqrt{\frac{a^2}{b^2} + cx^2} + 4ax}{3\sqrt{b\sqrt{\frac{a^2}{b^2} + cx^2} + a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]

[Out] (4*a*x + 2*b*x*Sqrt[a^2/b^2 + c*x^2])/(3*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])

IntegrateAlgebraic [A] time = 6.79, size = 109, normalized size = 1.65

$$\frac{2a\sqrt{\frac{a^2}{b^2} + cx^2} \sqrt{b\sqrt{\frac{a^2}{b^2} + cx^2} + a}}{3bcx} + \frac{2(b^2cx^2 - a^2) \sqrt{b\sqrt{\frac{a^2}{b^2} + cx^2} + a}}{3b^2cx}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]],x]

[Out] (2*a*Sqrt[a^2/b^2 + c*x^2]*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])/(3*b*c*x) + (2*(-a^2 + b^2*c*x^2)*Sqrt[a + b*Sqrt[a^2/b^2 + c*x^2]])/(3*b^2*c*x)

fricas [A] time = 0.50, size = 70, normalized size = 1.06

$$\frac{2 \left(b^2 c x^2 + a b \sqrt{\frac{b^2 c x^2 + a^2}{b^2}} - a^2 \right) \sqrt{b \sqrt{\frac{b^2 c x^2 + a^2}{b^2}} + a}{3 b^2 c x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 2/3*(b^2*c*x^2 + a*b*sqrt((b^2*c*x^2 + a^2)/b^2) - a^2)*sqrt(b*sqrt((b^2*c*x^2 + a^2)/b^2) + a)/(b^2*c*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{c x^2 + \frac{a^2}{b^2}} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)

maple [F] time = 0.04, size = 0, normalized size = 0.00

$$\int \sqrt{a + \sqrt{c x^2 + \frac{a^2}{b^2}} b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)

[Out] int((a+b*(a^2/b^2+c*x^2)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{c x^2 + \frac{a^2}{b^2}} b + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(1/b^2*a^2+c*x^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(c*x^2 + a^2/b^2)*b + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{a + b \sqrt{c x^2 + \frac{a^2}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2),x)

[Out] int((a + b*(c*x^2 + a^2/b^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sqrt{\frac{a^2}{b^2} + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(a**2/b**2+c*x**2)**(1/2))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sqrt(a**2/b**2 + c*x**2)), x)
```

3.248 $\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3 dx$

Optimal. Leaf size=303

$$\frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^3}{32e^5 \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)} + \frac{3f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^2 \log \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)}{32e^5}$$

Rubi [A] time = 0.38, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, number of rules / integrand size = 0.071, Rules used = {2116, 893}

$$\frac{f^2 (4ae^2 - b^2 f^2) \left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^2}{16e^3} + \frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2) \left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + ex \right)}{8e^4} - \frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^3}{32e^5 \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)} + \frac{3f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^2 \log \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)}{32e^5} + \frac{\left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^4}{8e}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]
```

```
[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2)/(16*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^4/(8*e) - (f^2*(2*d*e - b*f^2)^3*(4*a*e^2 - b^2*f^2))/(32*e^5*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(32*e^5)
```

Rule 893

```
Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.^2))^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.^2)])^(n_.))^(p_.), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3 dx = 2 \operatorname{Subst} \left(\int \frac{x^3 (d^2 e - (bd - ae)f^2 - (2de - bf^2)x + ex^2)}{(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)$$

$$= 2 \operatorname{Subst} \left(\int \left(\frac{f^2 (2de - bf^2) (4ae^2 - b^2 f^2)}{16e^4} + \frac{f^2 (4ae^2 - b^2 f^2) x}{16e^3} + \frac{x^3}{4e} + \frac{f^2 (2de - bf^2) (4ae^2 - b^2 f^2) \left(ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)}{8e^4} + \frac{f^2 (4ae^2 - b^2 f^2)}{8e^4} \right) dx, x, d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)$$

Mathematica [A] time = 0.55, size = 276, normalized size = 0.91

$$\frac{2e^2 f^2 (4ae^2 - b^2 f^2) \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex \right)^2 + 4ef^2 (4ae^2 - b^2 f^2) (2de - bf^2) \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + ex \right) - \frac{f^2 (b^2 f^2 - 4ae^2) (bf^2 - 2de)^3}{2 \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + bf^2} - 3(b^2 f^2 - 4ae^2) (bf^3 - 2def) \log \left(-2e \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + ex \right) - bf^2 \right) + 4e^4 \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex \right)^4}{32e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]

[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 2*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^2 + 4*e^4*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^4 - (f^2*(-2*d*e + b*f^2)^3*(-4*a*e^2 + b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 3*(-4*a*e^2 + b^2*f^2)*(-2*d*e*f + b*f^3)^2*Log[-(b*f^2 - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]))]/(32*e^5)

IntegrateAlgebraic [B] time = 10.57, size = 782, normalized size = 2.58

$$\frac{\sqrt{a + bx + \frac{e^2 x^2}{f^2}} \left(12 b^2 d^2 e^2 f^3 + 32 a d^2 e^3 f^3 - 12 b^2 d^2 e^3 f^5 - 8 a^2 b e^2 f^5 + 3 b^3 f^7 + 24 d^2 e^4 f x + 8 b d e^3 f^3 x + 16 a e^4 f^3 x - 2 b^2 e^2 f^5 x + 32 d e^5 f x^2 + 8 b e^4 f^3 x^2 + 16 e^6 f x^3 \right) / (16 e^4) + (2 d^3 x + 6 a d f^2 x + 3 d^2 e x^2 + 3 b d f^2 x^2 + 3 a e f^2 x^2 + 4 d e^2 x^3 + 2 b e f^2 x^3 + 2 e^3 x^4) / 2 - (3 (16 a d^2 e^5 f^2 - 4 b^2 d^2 e^3 f^4 - 16 a b d e^4 f^4 - 4 b^2 d^2 e^2 \sqrt{e^2/f^2} f^5 - 16 a b d e^3 \sqrt{e^2/f^2} f^5 + 4 b^3 d e^2 f^6 + 4 a b^2 e^3 f^6 + 4 b^3 d e \sqrt{e^2/f^2} f^7 + 4 a b^2 e^2 \sqrt{e^2/f^2} f^7 - b^4 e f^8 - b^4 \sqrt{e^2/f^2} f^9) \operatorname{Log}[b f + 2 e \sqrt{e^2/f^2} x - 2 e \sqrt{a + b x + (e^2 x^2)/f^2}]) / (64 e^6) + (3 (16 a d^2 e^5 f^2 - 16 a d^2 e^4 \sqrt{e^2/f^2} f^3 - 4 b^2 d^2 e^3 f^4 - 16 a b d e^4 f^4 + 4 b^2 d^2 e^2 \sqrt{e^2/f^2} f^5 + 16 a b d e^3 \sqrt{e^2/f^2} f^5 + 4 b^3 d e^2 f^6 + 4 a b^2 e^3 f^6 - 4 b^3 d e \sqrt{e^2/f^2} f^7 - 4 a b^2 e^2 \sqrt{e^2/f^2} f^7 - b^4 e f^8 + b^4 \sqrt{e^2/f^2} f^9) \operatorname{Log}[b f - 2 e \sqrt{e^2/f^2} x + 2 e \sqrt{a + b x + (e^2 x^2)/f^2}]) / (64 e^6) - (3 a d^2 \sqrt{e^2/f^2} f^3 \operatorname{Log}[-(b e f) - 2 e^2 \sqrt{e^2/f^2} x + 2 e^2 \sqrt{a + b x + (e^2 x^2)/f^2}]) / (4 e^2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3,x]

[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(12*b^2*d^2*e^2*f^3 + 32*a*d^2*e^3*f^3 - 12*b^2*d^2*e^3*f^5 - 8*a^2*b*e^2*f^5 + 3*b^3*f^7 + 24*d^2*e^4*f*x + 8*b*d*e^3*f^3*x + 16*a*e^4*f^3*x - 2*b^2*e^2*f^5*x + 32*d*e^5*f*x^2 + 8*b*e^4*f^3*x^2 + 16*e^6*f*x^3))/(16*e^4) + (2*d^3*x + 6*a*d*f^2*x + 3*d^2*e*x^2 + 3*b*d*f^2*x^2 + 3*a*e*f^2*x^2 + 4*d*e^2*x^3 + 2*b*e*f^2*x^3 + 2*e^3*x^4)/2 - (3*(16*a*d^2*e^5*f^2 - 4*b^2*d^2*e^3*f^4 - 16*a*b*d*e^4*f^4 - 4*b^2*d^2*e^2*Sqrt[e^2/f^2]*f^5 - 16*a*b*d*e^3*Sqrt[e^2/f^2]*f^5 + 4*b^3*d*e^2*f^6 + 4*a*b^2*e^3*f^6 + 4*b^3*d*e*Sqrt[e^2/f^2]*f^7 + 4*a*b^2*e^2*Sqrt[e^2/f^2]*f^7 - b^4*e*f^8 - b^4*Sqrt[e^2/f^2]*f^9)*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2])/(64*e^6) + (3*(16*a*d^2*e^5*f^2 - 16*a*d^2*e^4*Sqrt[e^2/f^2]*f^3 - 4*b^2*d^2*e^3*f^4 - 16*a*b*d*e^4*f^4 + 4*b^2*d^2*e^2*Sqrt[e^2/f^2]*f^5 + 16*a*b*d*e^3*Sqrt[e^2/f^2]*f^5 + 4*b^3*d*e^2*f^6 + 4*a*b^2*e^3*f^6 - 4*b^3*d*e*Sqrt[e^2/f^2]*f^7 - 4*a*b^2*e^2*Sqrt[e^2/f^2]*f^7 - b^4*e*f^8 + b^4*Sqrt[e^2/f^2]*f^9)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2])/(64*e^6) - (3*a*d^2*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^2)

fricas [A] time = 0.81, size = 345, normalized size = 1.14

$$\frac{32e^4 + 32(b^2 f^2 + 2de)^2 + 48(e^2 + (bd^2 + ae^2)/f^2)^2 + 32(3abd^2 f^2 + d^3 e^2) + 3(b^4 f^4 - 16abd^2 f^2 - 4(b^2 de + ad^2 e)^2) + 4(b^2 d^2 + 4abd^2) f^4 \log \left(-bf^2 - 2e^2 + 2e \sqrt{\frac{2de + 2af^2}{f^2}} \right) + 2(3b^2 f^4 + 16d^2 f^2 - 4(3b^2 de + 2abd^2) f^2) + 4(3bd^2 + 8abd^2) f^2 + 8(b^2 f^2 + 4de f)^2 - 2(b^2 f^2 - 12d^2 f - 4(bd^2 + 2ae) f) \sqrt{\frac{2de + 2af^2}{f^2}}}{32e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
[Out] 1/32*(32*e^8*x^4 + 32*(b*e^6*f^2 + 2*d*e^7)*x^3 + 48*(d^2*e^6 + (b*d*e^5 + a*e^6)*f^2)*x^2 + 32*(3*a*d*e^5*f^2 + d^3*e^5)*x + 3*(b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*log(-b*f^2 - 2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 2*(3*b^3*e*f^7 + 16*e^7*f*x^3 - 4*(3*b^2*d*e^2 + 2*a*b*e^3)*f^5 + 4*(3*b*d^2*e^3 + 8*a*d*e^4)*f^3 + 8*(b*e^5*f^3 + 4*d*e^6*f)*x^2 - 2*(b^2*e^3*f^5 - 12*d^2*e^5*f - 4*(b*d*e^4 + 2*a*e^5)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)/e^5
```

giac [A] time = 0.37, size = 373, normalized size = 1.23

$\frac{b^4 f^8 - 16 a d^2 e^4 f^2 - 4 (b^3 d e + a b^2 e^2) f^6 + 4 (b^2 d^2 e^2 + 4 a b d e^3) f^4}{32} \log(-b f^2 - 2 e^2 x + 2 e f \sqrt{\frac{b f^2 x + e^2 x^2 + a f^2}{f^2}}) + \frac{2 (3 b^3 e f^7 + 16 e^7 f x^3 - 4 (3 b^2 d e^2 + 2 a b e^3) f^5 + 4 (3 b d^2 e^3 + 8 a d e^4) f^3 + 8 (b e^5 f^3 + 4 d e^6 f) x^2 - 2 (b^2 e^3 f^5 - 12 d^2 e^5 f - 4 (b d e^4 + 2 a e^5) f^3) x) \sqrt{\frac{b f^2 x + e^2 x^2 + a f^2}{f^2}}}{e^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
[Out] b*f^2*x^3*e + 3/2*b*d*f^2*x^2 + 3/2*a*f^2*x^2*e + 3*a*d*f^2*x + x^4*e^3 + 2*d*x^3*e^2 + 3/2*d^2*x^2*e + d^3*x + 3/32*(b^4*f^7*abs(f) - 4*b^3*d*f^5*abs(f)*e - 4*a*b^2*f^5*abs(f)*e^2 + 4*b^2*d^2*f^3*abs(f)*e^2 + 16*a*b*d*f^3*abs(f)*e^3 - 16*a*d^2*f*abs(f)*e^4)*e^(-5)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/16*sqrt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*(2*x*abs(f)*e^2/f + (b*f^4*abs(f)*e^6 + 4*d*f^2*abs(f)*e^7)*e^(-6)/f^3)*x - (b^2*f^6*abs(f)*e^4 - 4*b*d*f^4*abs(f)*e^5 - 8*a*f^4*abs(f)*e^6 - 12*d^2*f^2*abs(f)*e^6)*e^(-6)/f^3)*x + (3*b^3*f^8*abs(f)*e^2 - 12*b^2*d*f^6*abs(f)*e^3 - 8*a*b*f^6*abs(f)*e^4 + 12*b*d^2*f^4*abs(f)*e^4 + 32*a*d*f^4*abs(f)*e^5)*e^(-6)/f^3)
```

maple [B] time = 0.02, size = 685, normalized size = 2.26

$\frac{b^4 f^8 - 16 a d^2 e^4 f^2 - 4 (b^3 d e + a b^2 e^2) f^6 + 4 (b^2 d^2 e^2 + 4 a b d e^3) f^4}{32} \log(-b f^2 - 2 (x e - \sqrt{b f^2 x + a f^2 + x^2 e^2}) e) + \frac{1}{16} \sqrt{b f^2 x + a f^2 + x^2 e^2} (2 (4 (2 x \operatorname{abs}(f) e^2 / f + (b f^4 \operatorname{abs}(f) e^6 + 4 d f^2 \operatorname{abs}(f) e^7) e^{-6} / f^3) x - (b^2 f^6 \operatorname{abs}(f) e^4 - 4 b d f^4 \operatorname{abs}(f) e^5 - 8 a f^4 \operatorname{abs}(f) e^6 - 12 d^2 f^2 \operatorname{abs}(f) e^6) e^{-6} / f^3) x + (3 b^3 f^8 \operatorname{abs}(f) e^2 - 12 b^2 d f^6 \operatorname{abs}(f) e^3 - 8 a b f^6 \operatorname{abs}(f) e^4 + 12 b d^2 f^4 \operatorname{abs}(f) e^4 + 32 a d f^4 \operatorname{abs}(f) e^5) e^{-6} / f^3)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
[Out] 3/2*d^2*e*x^2+d^3*x+e^3*x^4-3/2*d/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a+1/4*d^4/e+2*d/e*f^3*(b*x+e^2/f^2*x^2+a)^(3/2)+3/2*f*d^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*f^5/e^2*a*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2-1/2*f^5/e^2*(b*x+e^2/f^2*x^2+a)^(3/2)*b+3/16*f^7/e^4*b^3*(b*x+e^2/f^2*x^2+a)^(1/2)+f^2*x^3*b*e+3/2*f^2*x^2*b*d+2*d*e^2*x^3+3/2*a*e*f^2*x^2+3*a*d*f^2*x+f^3*(b*x+e^2/f^2*x^2+a)^(3/2)*x-3/4*d/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a)^(1/2)+3/4*d^2/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+3/2*f*d^2*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-3/2*d/e*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x+3/8*d/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)-3/8*d^2/e^2*f^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2+3/8*f^5/e^2*b^2*(b*x+e^2/f^2*x^2+a)^(1/2)*x-3/32*f^7/e^4*b^4*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?` for more details) Is $b^2*f^2-4*a*e^2$ positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)

[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**3, x)

$$3.249 \quad \int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2 dx$$

Optimal. Leaf size=237

$$\frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^2}{16e^4 \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)} + \frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2) \log \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)}{8e^4}$$

Rubi [A] time = 0.24, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, number of rules / integrand size = 0.071, Rules used = {2116, 893}

$$\frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^2}{16e^4 \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)} + \frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2) \log \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)}{8e^4} + \frac{f^2 (4ae^2 - b^2 f^2) \left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + ex \right) \left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^3}{8e^3 + 6e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2,x]

[Out] (f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]))/(8*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^3/(6*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2))/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2])])/(8*e^4)

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2 dx &= 2 \operatorname{Subst} \left(\int \frac{x^2 (d^2 e - (bd - ae) f^2 - (2de - bf^2) x + ex^2)}{(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right) \\
&= 2 \operatorname{Subst} \left(\int \left(\frac{4ae^2 f^2 - b^2 f^4}{16e^3} + \frac{x^2}{4e} + \frac{(4ae^2 - b^2 f^2) (2def - bf^3)^2}{16e^3 (2de - bf^2 - 2ex)^2} - \frac{f^2 (2de - bf^2)}{8e} \right) dx, x, d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right) \\
&= \frac{f^2 (4ae^2 - b^2 f^2) \left(ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)}{8e^3} + \frac{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2}{6e}
\end{aligned}$$

Mathematica [A] time = 0.32, size = 213, normalized size = 0.90

$$\frac{6f^2(b^2f^2 - 4ae^2)(bf^2 - 2de) \log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)} + ex\right) - bf^2\right) + \frac{3(b^2f^2 - 4ae^2)(bf^2 - 2de)^2}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)} + ex\right) + bf^2} + 6ef^2(4ae^2 - b^2f^2)\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)} + ex\right) + 8e^3\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)} + d + ex\right)^3}{48e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2, x]

[Out] (6*e*f^2*(4*a*e^2 - b^2*f^2)*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 8*e^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^3 + (3*(-4*a*e^2 + b^2*f^2)*(-2*d*e*f + b*f^3)^2)/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 6*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])]/(48*e^4)

IntegrateAlgebraic [B] time = 24.46, size = 624, normalized size = 2.63

$$\frac{e^2 f^2 \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \left(6 b^2 d e f^3 + 8 a e^2 f^3 - 3 b^2 f^5 + 12 d e^3 f x + 2 b e^2 f^3 x + 8 e^4 f x^2 \right) + (6 d^2 x + 6 a f^2 x + 6 d e e x^2 + 3 b f^2 x^2 + 4 e^2 x^3) / 6 + \left((-8 a d e^4 f^2 + 2 b^2 d e^2 f^4 + 4 a b e^3 f^4 + 4 a b e^2 \operatorname{Sqrt}[e^2 / f^2] f^5 - b^3 e f^6 - b^3 \operatorname{Sqrt}[e^2 / f^2] f^7) \operatorname{Log}[b f + 2 e \operatorname{Sqrt}[e^2 / f^2] x - 2 e \operatorname{Sqrt}[a + b x + (e^2 x^2) / f^2] \right) / (16 e^5) + \left((8 a d e^4 f^2 - 8 a d e^3 \operatorname{Sqrt}[e^2 / f^2] f^3 - 2 b^2 d e^2 f^4 - 4 a b e^3 f^4 + 4 a b e^2 \operatorname{Sqrt}[e^2 / f^2] f^5 + b^3 e f^6 - b^3 \operatorname{Sqrt}[e^2 / f^2] f^7) \operatorname{Log}[b f - 2 e \operatorname{Sqrt}[e^2 / f^2] x + 2 e \operatorname{Sqrt}[a + b x + (e^2 x^2) / f^2] \right) / (16 e^5) - (a d \operatorname{Sqrt}[e^2 / f^2] f^3 \operatorname{Log}[-(b e f) - 2 e^2 \operatorname{Sqrt}[e^2 / f^2] x + 2 e^2 \operatorname{Sqrt}[a + b x + (e^2 x^2) / f^2]] / (2 e^2) + (b^2 d \operatorname{Sqrt}[e^2 / f^2] f^5 \operatorname{Log}[-(b e^3 f) - 2 e^4 \operatorname{Sqrt}[e^2 / f^2] x + 2 e^4 \operatorname{Sqrt}[a + b x + (e^2 x^2) / f^2]] / (8 e^4) + (b^2 d \operatorname{Sqrt}[e^2 / f^2] f^5 \operatorname{Log}[b e^3 f - 2 e^4 \operatorname{Sqrt}[e^2 / f^2] x + 2 e^4 \operatorname{Sqrt}[a + b x + (e^2 x^2) / f^2]] / (8 e^4)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2, x]

[Out] (Sqrt[a + b*x + (e^2*x^2)/f^2]*(6*b*d*e*f^3 + 8*a*e^2*f^3 - 3*b^2*f^5 + 12*d*e^3*f*x + 2*b*e^2*f^3*x + 8*e^4*f*x^2))/(12*e^3) + (6*d^2*x + 6*a*f^2*x + 6*d*e*e*x^2 + 3*b*f^2*x^2 + 4*e^2*x^3)/6 + ((-8*a*d*e^4*f^2 + 2*b^2*d*e^2*f^4 + 4*a*b*e^3*f^4 + 4*a*b*e^2*Sqrt[e^2/f^2]*f^5 - b^3*e*f^6 - b^3*Sqrt[e^2/f^2]*f^7)*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(16*e^5) + ((8*a*d*e^4*f^2 - 8*a*d*e^3*Sqrt[e^2/f^2]*f^3 - 2*b^2*d*e^2*f^4 - 4*a*b*e^3*f^4 + 4*a*b*e^2*Sqrt[e^2/f^2]*f^5 + b^3*e*f^6 - b^3*Sqrt[e^2/f^2]*f^7)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(16*e^5) - (a*d*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(2*e^2) + (b^2*d*Sqrt[e^2/f^2]*f^5*Log[-(b*e^3*f) - 2*e^4*Sqrt[e^2/f^2]*x + 2*e^4*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(8*e^4) + (b^2*d*Sqrt[e^2/f^2]*f^5*Log[b*e^3*f - 2*e^4*Sqrt[e^2/f^2]*x + 2*e^4*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(8*e^4)

fricas [A] time = 0.64, size = 219, normalized size = 0.92

$$\frac{16e^6x^3 + 12(b^4f^2 + 2de^5)x^2 + 24(ae^4f^2 + d^2e^4)x - 3(b^3f^6 + 8ade^3f^2 - 2(b^2de + 2abce^2)f^4) \log\left(-bf^2 - 2e^2x + 2ef\sqrt{\frac{bf^2x + e^2x^2 + af^2}{f^2}}\right) - 2(3b^2ef^5 - 8e^5fx^2 - 2(3bde^2 + 4ae^3)f^3 - 2(b^3f^3 + 6de^4f)x)\sqrt{\frac{bf^2x + e^2x^2 + af^2}{f^2}}}{24e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")


```
[Out] 1/24*(16*e^6*x^3 + 12*(b*e^4*f^2 + 2*d*e^5)*x^2 + 24*(a*e^4*f^2 + d^2*e^4)*
x - 3*(b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*log(-b*f^2 -
2*e^2*x + 2*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 2*(3*b^2*e*f^5 - 8
*e^5*f*x^2 - 2*(3*b*d*e^2 + 4*a*e^3)*f^3 - 2*(b*e^3*f^3 + 6*d*e^4*f)*x)*sq
rt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/e^4
```

giac [A] time = 0.30, size = 224, normalized size = 0.95

$$\frac{1}{2}bf^2x^2 + af^2x + \frac{2}{3}x^3e^2 + dx^2e + d^2x - \frac{1}{8}(b^3f^6 - 2b^2df^4 - 4abf^2|f|^2 + 8ad|f|^2)e^{-4} \log\left(\left| -bf^2 - 2\left(xe - \sqrt{bf^2x + af^2 + x^2e^2}\right) \right|\right) + \frac{1}{12}\sqrt{bf^2x + af^2 + x^2e^2}\left(2\left(\frac{4x|f|e}{f} + \frac{(bf^2|f|^2 + 6df|f|e^2)e^{-4}}{f^2}\right)x - \frac{(3b^2f^2|f|e - 6bdf^2|f|^2 - 8ad^2|f|e^2)e^{-4}}{f^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")
```

```
[Out] 1/2*b*f^2*x^2 + a*f^2*x + 2/3*x^3*e^2 + d*x^2*e + d^2*x - 1/8*(b^3*f^5*abs(f)
- 2*b^2*d*f^3*abs(f)*e - 4*a*b*f^3*abs(f)*e^2 + 8*a*d*f*abs(f)*e^3)*e^(-
4)*log(abs(-b*f^2 - 2*(x*e - sqrt(b*f^2*x + a*f^2 + x^2*e^2))*e)) + 1/12*sq
rt(b*f^2*x + a*f^2 + x^2*e^2)*(2*(4*x*abs(f)*e/f + (b*f^3*abs(f)*e^3 + 6*d*
f*abs(f)*e^4)*e^(-4)/f^2)*x - (3*b^2*f^5*abs(f)*e - 6*b*d*f^3*abs(f)*e^2 -
8*a*f^3*abs(f)*e^3)*e^(-4)/f^2)
```

maple [A] time = 0.01, size = 409, normalized size = 1.73

$$\frac{b^2f^2 \ln\left(\frac{\frac{1}{2}\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}{\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}\right) + abf^2 \ln\left(\frac{\frac{1}{2}\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}{\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}\right) + b^2d^2 \ln\left(\frac{\frac{1}{2}\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}{\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}\right) + \frac{bf^2x^2 + 2d^2x^2}{3} + \frac{adf \ln\left(\frac{\frac{1}{2}\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}{\sqrt{\frac{2x}{f^2} + \sqrt{bx + \frac{2x^2}{f^2} + a}}}\right) + af^2x - \sqrt{bx + \frac{2x^2}{f^2} + a} b^2f^2}{4e^3} + \frac{\sqrt{bx + \frac{2x^2}{f^2} + a} b^2f^2}{2e} + dx^2 + \frac{\sqrt{bx + \frac{2x^2}{f^2} + a} bdf^2}{2e^2} + d^2x + \sqrt{bx + \frac{2x^2}{f^2} + a} d^2x + \frac{d^2}{3e} + \frac{2\left(bx + \frac{2x^2}{f^2} + a\right)^{3/2} f^2}{3e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)
```

```
[Out] a*f^2*x+1/2*f^2*b*x^2+2/3*e^2*x^3+2/3*(b*x+e^2/f^2*x^2+a)^(3/2)/e*f^3-1/2/e
*f^3*b*(b*x+e^2/f^2*x^2+a)^(1/2)*x-1/4/e^3*f^5*b^2*(b*x+e^2/f^2*x^2+a)^(1/2
)-1/2/e*f^3*b*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2
))/(e^2/f^2)^(1/2)*a+1/8/e^3*f^5*b^3*ln((1/2*b+e^2/f^2*x)/(e^2/f^2)^(1/2)+(
b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)+f*d*(b*x+e^2/f^2*x^2+a)^(1/2)*x+
1/2*d/e^2*f^3*(b*x+e^2/f^2*x^2+a)^(1/2)*b+f*d*ln((1/2*b+e^2/f^2*x)/(e^2/f^2
)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*a-1/4*d/e^2*f^3*ln((1/2*b
+e^2/f^2*x)/(e^2/f^2)^(1/2)+(b*x+e^2/f^2*x^2+a)^(1/2))/(e^2/f^2)^(1/2)*b^2+
d*e*x^2+d^2*x+1/3*d^3/e
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see `assume?`
for more details)Is b^2*f^2-4*a*e^2 positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)
```

[Out] `int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)`

[Out] `Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**2, x)`

$$3.250 \quad \int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right) dx$$

Optimal. Leaf size=118

$$\frac{f^2 (4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{bf^2 + 2e^2 x}{2ef \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} \right)}{8e^3} + \frac{f (bf^2 + 2e^2 x) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} + dx + \frac{ex^2}{2}$$

Rubi [A] time = 0.06, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {612, 621, 206}

$$\frac{f^2 (4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{bf^2 + 2e^2 x}{2ef \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} \right)}{8e^3} + \frac{f (bf^2 + 2e^2 x) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} + dx + \frac{ex^2}{2}$$

Antiderivative was successfully verified.

[In] Int[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2], x]

[Out] d*x + (e*x^2)/2 + (f*(b*f^2 + 2*e^2*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^2) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(b*f^2 + 2*e^2*x)/(2*e*f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(8*e^3)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right) dx = dx + \frac{ex^2}{2} + f \int \sqrt{a + bx + \frac{e^2 x^2}{f^2}} dx$$

$$= dx + \frac{ex^2}{2} + \frac{f(bf^2 + 2e^2x) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} + \frac{1}{8} \left(f \left(4a - \frac{b^2 f^2}{e^2} \right) \right) \int \frac{1}{\sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx$$

$$= dx + \frac{ex^2}{2} + \frac{f(bf^2 + 2e^2x) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} + \frac{1}{4} \left(f \left(4a - \frac{b^2 f^2}{e^2} \right) \right) \text{Subst} \left(\int \frac{1}{\sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx, \frac{2ex + b}{2e}, \frac{4ae^2 - b^2 f^2}{4e^2} \right)$$

$$= dx + \frac{ex^2}{2} + \frac{f(bf^2 + 2e^2x) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} + \frac{f^2(4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{2ex + b}{2e} \sqrt{\frac{4ae^2 - b^2 f^2}{4e^2}} \right)}{8e^3}$$

Mathematica [A] time = 0.21, size = 120, normalized size = 1.02

$$\frac{1}{8} \left(\frac{(4ae^2 f^2 - b^2 f^4) \log \left(2e \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + ex \right) + bf^2 \right)}{e^3} + 4fx \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + \frac{2bf^3 \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)}}{e^2} + 8dx + 4ex^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2], x]

[Out] (8*d*x + 4*e*x^2 + (2*b*f^3*Sqrt[a + x*(b + (e^2*x)/f^2)]))/e^2 + 4*f*x*Sqrt[a + x*(b + (e^2*x)/f^2)] + ((4*a*e^2*f^2 - b^2*f^4)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/e^3)/8

IntegrateAlgebraic [B] time = 4.35, size = 339, normalized size = 2.87

$$\frac{b^2 f^5 \sqrt{\frac{e^2}{f^2}} \log \left(-2x \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + bf + 2ex \sqrt{\frac{e^2}{f^2}} \right)}{16e^4} - \frac{\sqrt{\frac{e^2}{f^2}} (4ae^2 f^2 - b^2 f^4) \log \left(2x \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + bf - 2ex \sqrt{\frac{e^2}{f^2}} \right)}{16e^4} + \frac{(4ae^2 f^2 - b^2 f^4) \tanh^{-1} \left(\frac{2x \sqrt{a + bx + \frac{e^2 x^2}{f^2}} - 2ex \sqrt{\frac{e^2}{f^2}}}{bf} \right)}{8e^3} + \frac{(bf^2 + 2e^2 fx) \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^2} - \frac{af^3 \sqrt{\frac{e^2}{f^2}} \log \left(2e^2 \sqrt{a + bx + \frac{e^2 x^2}{f^2}} - bef - 2e^2 x \sqrt{\frac{e^2}{f^2}} \right)}{4e^2} + \frac{1}{2} (2dx + ex^2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2], x]

[Out] (2*d*x + e*x^2)/2 + ((b*f^3 + 2*e^2*f*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/(4*e^2) + ((4*a*e^2*f^2 - b^2*f^4)*ArcTanh[(-2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2])/(b*f)])/(8*e^3) + (b^2*Sqrt[e^2/f^2]*f^5*Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4) - (Sqrt[e^2/f^2]*(4*a*e^2*f^3 - b^2*f^5)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(16*e^4) - (a*Sqrt[e^2/f^2]*f^3*Log[-(b*e*f) - 2*e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^2)

fricas [A] time = 0.61, size = 123, normalized size = 1.04

$$\frac{4e^4 x^2 + 8de^3 x + (b^2 f^4 - 4ae^2 f^2) \log \left(-bf^2 - 2e^2 x + 2ef \sqrt{\frac{bf^2 x + e^2 x^2 + af^2}{f^2}} \right) + 2(bef^3 + 2e^3 fx) \sqrt{\frac{bf^2 x + e^2 x^2 + af^2}{f^2}}}{8e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2), x, algorithm="fricas")

[Out] $\frac{1}{8}(4e^{4x^2} + 8de^{3x} + (b^2f^4 - 4ae^{2f^2})\log(-bf^2 - 2e^{2x} + 2e^f\sqrt{(bf^2x + e^{2x^2} + af^2)/f^2})) + 2*(b^2e^{3x} + 2e^{3f})\sqrt{(bf^2x + e^{2x^2} + af^2)/f^2})/e^3$

giac [A] time = 0.22, size = 111, normalized size = 0.94

$$\frac{1}{2}x^2e + dx + \frac{((b^2f^4 - 4af^2e^2)e^{(-3)}\log(|-bf^2 - 2(xe - \sqrt{bf^2x + af^2 + x^2e^2})e|) + 2\sqrt{bf^2x + af^2 + x^2e^2}(bf^2e^{(-2)} + 2x))|f|}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{2}x^2e + dx + \frac{1}{8}((b^2f^4 - 4a^2f^2e^2)e^{(-3)}\log(\text{abs}(-bf^2 - 2*(x*e - \sqrt{bf^2x + af^2 + x^2e^2}))*e)) + 2*\sqrt{bf^2x + af^2 + x^2e^2}*(b^2f^2e^{(-2)} + 2*x))*\text{abs}(f)/f$

maple [A] time = 0.01, size = 173, normalized size = 1.47

$$\frac{b^2f^3 \ln\left(\frac{\frac{b}{2} + \frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}} + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\right)}{8\sqrt{\frac{e^2}{f^2}} e^2} + \frac{af \ln\left(\frac{\frac{b}{2} + \frac{e^2x}{f^2}}{\sqrt{\frac{e^2}{f^2}}} + \sqrt{bx + \frac{e^2x^2}{f^2} + a}\right)}{2\sqrt{\frac{e^2}{f^2}}} + \frac{ex^2}{2} + \frac{\sqrt{bx + \frac{e^2x^2}{f^2} + a} bf^3}{4e^2} + dx + \frac{\sqrt{bx + \frac{e^2x^2}{f^2} + a} fx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f,x)`

[Out] $dx + \frac{1}{2}e^2x^2 + \frac{1}{2}f*(b*x + e^2/f^2*x^2 + a)^{(1/2)}*x + \frac{1}{4}e^2*f^3*(b*x + e^2/f^2*x^2 + a)^{(1/2)}*b + \frac{1}{2}f*\ln((1/2*b + e^2/f^2*x)/(e^2/f^2)^{(1/2)} + (b*x + e^2/f^2*x^2 + a)^{(1/2)})/(e^2/f^2)^{(1/2)}*a - \frac{1}{8}e^2*f^3*\ln((1/2*b + e^2/f^2*x)/(e^2/f^2)^{(1/2)} + (b*x + e^2/f^2*x^2 + a)^{(1/2)})/(e^2/f^2)^{(1/2)}*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")`

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(b^2*f^2-4*a*e^2>0)', see 'assume?' for more details) Is b^2*f^2-4*a*e^2 positive, negative or zero?

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2),x)`

[Out] `int(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2),x)
```

```
[Out] Integral(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2), x)
```

$$3.251 \quad \int \frac{1}{d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=215

$$\frac{f^2(4ae^2 - b^2f^2)}{2e(2de - bf^2)\left(2e\left(f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} + ex\right) + bf^2\right)} - \frac{f^2(4ae^2 - b^2f^2)\log\left(2e\left(f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} + ex\right) + bf^2\right)}{2e(2de - bf^2)^2}$$

Rubi [A] time = 0.19, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2116, 893}

$$\frac{f^2(4ae^2 - b^2f^2)}{2e(2de - bf^2)\left(2e\left(f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} + ex\right) + bf^2\right)} - \frac{f^2(4ae^2 - b^2f^2)\log\left(2e\left(f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} + ex\right) + bf^2\right)}{2e(2de - bf^2)^2} + \frac{2(aef^2 - bdf^2 + d^2e)\log\left(f\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d + ex\right)}{(2de - bf^2)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1), x]

[Out] -(f^2*(4*a*e^2 - b^2*f^2))/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2 - (f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]])/(2*e*(2*d*e - b*f^2)^2)

Rule 893

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.^2))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.^2)])^(n_.))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{x(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 2 \operatorname{Subst} \left(\int \left(\frac{d^2e - bdf^2 + aef^2}{(2de - bf^2)^2 x} + \frac{4ae^2f^2 - b^2f^4}{2(2de - bf^2)(2de - bf^2 - 2ex)^2} + \frac{2(d^2e - bdf^2 + aef^2)}{2(2de - bf^2 - 2ex)} \right) dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= -\frac{f^2(4ae^2 - b^2f^2)}{2e(2de - bf^2) \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{e^2x}{f^2}} \right) \right)} + \frac{2(d^2e - bdf^2 + aef^2)}{2(2de - bf^2 - 2ex)}$$

Mathematica [A] time = 0.20, size = 187, normalized size = 0.87

$$\frac{\frac{f^2(b^2f^2 - 4ae^2)(bf^2 - 2de)}{e \left(2e \left(f\sqrt{a + x \left(b + \frac{e^2x}{f^2} \right) + ex} \right) + bf^2 \right)} + \frac{f^2(b^2f^2 - 4ae^2) \log \left(-2e \left(f\sqrt{a + x \left(b + \frac{e^2x}{f^2} \right) + ex} \right) - bf^2 \right)}{e} + 4(aef^2 - bdf^2 + d^2e) \log \left(f\sqrt{a + x \left(b + \frac{e^2x}{f^2} \right) + d + ex} \right)}{2(bf^2 - 2de)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1), x]

[Out] (-((f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(e*(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2]))))) + 4*(d^2*e - b*d*f^2 + a*e*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] + (f^2*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2 - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2]))])/e)/(2*(-2*d*e + b*f^2)^2)

IntegrateAlgebraic [B] time = 2.44, size = 658, normalized size = 3.06

$$\frac{(-4a^2f^2 + 4ae^2f^2\sqrt{e^2x^2 + f^2a + bx}) \log\left(\frac{2e\sqrt{a + bx + \frac{e^2x^2}{f^2}} + bf^2 - 2de}{e}\right) + (b^2f^2 - 4ae^2) \log\left(\frac{2e\sqrt{a + bx + \frac{e^2x^2}{f^2}} + bf^2 - 2de}{e}\right) + 4(aef^2 - bdf^2 + d^2e) \log\left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right) + d + ex}\right)}{2(bf^2 - 2de)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-1), x]

[Out] (e*x)/(2*d*e - b*f^2) + (f*Sqrt[a + b*x + (e^2*x^2)/f^2])/(-2*d*e + b*f^2) - Log[b*f + 2*e*Sqrt[e^2/f^2]*x - 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(4*e) + ((-4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3 + b^2*e*f^4 - b^2*Sqrt[e^2/f^2]*f^5)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^2*(2*d*e - b*f^2)^2) - (Sqrt[e^2/f^2]*f*Log[-(b*e*f - 2*e^2*Sqrt[e^2/f^2]*x + 2*e^2*Sqrt[a + b*x + (e^2*x^2)/f^2])]/(4*e^2) + ((d^2*e^2 - d^2*e*Sqrt[e^2/f^2]*f - b*d*e*f^2 + a*e^2*f^2 + b*d*Sqrt[e^2/f^2]*f^3 - a*e*Sqrt[e^2/f^2]*f^3)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^2) - (b*d*Sqrt[e^2/f^2]*f^3*Log[b*d*f - 2*a*e*f + 2*d*e*Sqrt[e^2/f^2]*x - b*Sqrt[e^2/f^2]*f^2*x + (-2*d*e + b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^2) + ((d^2*e + d^2*Sqrt[e^2/f^2]*f - b*d*f^2 + a*e*f^2 + a*Sqrt[e^2/f^2]*f^3)*Log[b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*(2*d*e*x - b*f^2*x) + (-2*d*e + b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^2

fricas [A] time = 8.12, size = 371, normalized size = 1.73

$$\frac{2(b^2f^2 - 2de^2)x - 2(d^2e - (bde - ae^2)f^2) \log\left(\frac{(bd - 2ae)f^2 - (bf^2 - 2de)x + (bf^2 - 2de)f\sqrt{\frac{22ae^2d + a^2}{f^2}}}{2(d^2e - bdf^2 + aef^2)}\right) - 2(d^2e - (bde - ae^2)f^2) \log\left(\frac{2e\sqrt{a + bx + \frac{e^2x^2}{f^2}} + bf^2 - 2de}{e}\right) + (b^2f^2 - 4ae^2) \log\left(\frac{2e\sqrt{a + bx + \frac{e^2x^2}{f^2}} + bf^2 - 2de}{e}\right) + 4(aef^2 - bdf^2 + d^2e) \log\left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right) + d + ex}\right)}{2(bf^2 - 4bd^2f^2 + 4d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="fricas")

[Out]
$$-1/2*(2*(b*e^2*f^2 - 2*d*e^3)*x - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*\log((b*d - 2*a*e)*f^2 - (b*e*f^2 - 2*d*e^2)*x + (b*f^3 - 2*d*e*f)*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2}) - 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*\log(a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + (b^2*f^4 + 2*d^2*e^2 - 2*(b*d*e + a*e^2)*f^2)*\log(-b*f^2 - 2*e^2*x + 2*e*f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2}) + 2*(d^2*e^2 - (b*d*e - a*e^2)*f^2)*\log(-e*x + f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2}) - d) - 2*(b*e*f^3 - 2*d*e^2*f)*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2})/(b^2*e*f^4 - 4*b*d*e^2*f^2 + 4*d^2*e^3)$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$[undef, +\infty, 1]$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="giac")

[Out] [undef, +Infinity, 1]

maple [B] time = 0.06, size = 4918, normalized size = 22.87

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f),x)

[Out]
$$f/(b*f^2-2*d*e)*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}+1/2*f^3/(b*f^2-2*d*e)^2*\ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))))/(e^2/f^2)^{(1/2)}+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}/(e^2/f^2)^{(1/2)}*b^2-f/(b*f^2-2*d*e)^2*\ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))))/(e^2/f^2)^{(1/2)}+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}/(e^2/f^2)^{(1/2)}*b*d*e+1/f/(b*f^2-2*d*e)^2*\ln((-1/2*(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)+e^2/f^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))))/(e^2/f^2)^{(1/2)}+(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))+(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}/(e^2/f^2)^{(1/2)}*d^2*e^2-f^3/(b*f^2-2*d*e)^3/((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}*\ln((2*(a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))+2*((a^2*e^2*f^4-2*a*b*d*e*f^4+b^2*d^2*f^4+2*a*d^2*e^2*f^2-2*b*d^3*e*f^2+d^4*e^2)/f^2/(b*f^2-2*d*e)^2)^{(1/2)}*(e^2*(x+(a*f^2-d^2)/(b*f^2-2*d*e))^2/f^2-(-b^2*f^4+2*a*e^2*f^2+2*b*d*e*f^2-2*d^2*e^2)/f^2/(b*f^2-2*d*e)*(x+(a*f^2-d^2)/(b*f^2-2*d*e)))+(a^2*e^2*f^4-2*a*b$$

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)),x)

[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2)),x)

[Out] Integral(1/(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)

3.252
$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^2} dx$$

Optimal. Leaf size=266

$$\frac{2f^2(4ae^2 - b^2f^2) \log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex\right)}{(2de - bf^2)^3} - \frac{f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^2 \left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} - \frac{2f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3}$$

Rubi [A] time = 0.23, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2116, 893}

$$\frac{2f^2(4ae^2 - b^2f^2) \log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex\right)}{(2de - bf^2)^3} - \frac{f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^2 \left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} - \frac{2f^2(4ae^2 - b^2f^2) \log\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)}{(2de - bf^2)^3} - \frac{2(acf^2 - bdf^2 + d^2e)}{(2de - bf^2)^2 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2), x]
```

```
[Out] (-2*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (2*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^3 - (2*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]]))/(2*d*e - b*f^2)^3
```

Rule 893

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]))^(n_))^(p_), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{x^2(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 2 \operatorname{Subst} \left(\int \left(\frac{d^2e - bdf^2 + aef^2}{(2de - bf^2)^2 x^2} + \frac{4ae^2f^2 - b^2f^4}{(2de - bf^2)^3 x} + \frac{4ae^3f^2 - b^2e^2f^2}{(2de - bf^2)^2 (2de - bf^2 + 2ex)} \right) dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= -\frac{2(d^2e - bdf^2 + aef^2)}{(2de - bf^2)^2 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)} - \frac{f^2(4ae^3 - b^2e^2)}{(2de - bf^2)^2 \left(bf^2 + 2e\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)\right)}$$

Mathematica [A] time = 0.32, size = 237, normalized size = 0.89

$$\frac{\frac{f^2(b^2f^2 - 4ae^2)(bf^2 - 2de)}{2e\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)+ex}+bf^2} + 2f^2(b^2f^2 - 4ae^2)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)+d+ex}\right) - 2f^2(b^2f^2 - 4ae^2)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)+ex}\right) - bf^2\right) + \frac{2(2de-bf^2)(aef^2-bdf^2+d^2e)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)+d+ex}}}{(2de - bf^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2), x]

[Out] -(((2*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + 2*f^2*(-4*a*e^2 + b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - 2*f^2*(-4*a*e^2 + b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])))/(2*d*e - b*f^2)^3)

IntegrateAlgebraic [B] time = 5.25, size = 614, normalized size = 2.31

$$\frac{2(b^2d^2f^2 - 2abdf^2 + a^2e^2f^2 - 2abdf^2 + b^2d^2f^2 - 2abdf^2 + 2a^2e^2f^2)}{(d^2 - af^2)(2d - bf^2)(-d^2 - af^2 + 2d*ex - bf^2*ex)} \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} + f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} + f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-2), x]

[Out] (2*(b*d*f^3 - 2*a*e*f^3 + 2*d*e^2*f*x - b*e*f^3*x)*Sqrt[a + b*x + (e^2*x^2)/f^2])/((-2*d*e + b*f^2)^2*(-d^2 + a*f^2 - 2*d*e*x + b*f^2*x)) + (2*(2*d^4*e^2*x - 2*b*d^3*e*f^2*x + b^2*d^2*f^4*x - 2*a*b*d*e*f^4*x + 2*a^2*e^2*f^4*x + 2*d^3*e^3*x^2 - b*d^2*e^2*f^2*x^2 - 2*a*d*e^3*f^2*x^2 + a*b*e^2*f^4*x^2))/((d^2 - a*f^2)*(2*d*e - b*f^2)^2*(d^2 - a*f^2 + 2*d*e*x - b*f^2*x)) + ((-4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3 + b^2*e*f^4 - b^2*Sqrt[e^2/f^2]*f^5)*Log[b*f - 2*e*Sqrt[e^2/f^2]*x + 2*e*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^3) + ((4*a*e^3*f^2 - 4*a*e^2*Sqrt[e^2/f^2]*f^3 - b^2*e*f^4 + b^2*Sqrt[e^2/f^2]*f^5)*Log[d - Sqrt[e^2/f^2]*f*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^3) + ((4*a*e^3*f^2 + 4*a*e^2*Sqrt[e^2/f^2]*f^3 - b^2*e*f^4 - b^2*Sqrt[e^2/f^2]*f^5)*Log[b*d*f - 2*a*e*f + Sqrt[e^2/f^2]*(2*d*e*x - b*f^2*x) + (-2*d*e + b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(e*(2*d*e - b*f^2)^3)

fricas [B] time = 3.69, size = 826, normalized size = 3.11

$$\frac{2(b^2d^2f^2 - 2abdf^2 + a^2e^2f^2 - 2abdf^2 + b^2d^2f^2 - 2abdf^2 + 2a^2e^2f^2)}{(d^2 - af^2)(2d - bf^2)(-d^2 - af^2 + 2d*ex - bf^2*ex)} \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} + f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} - f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right) \left(\frac{4ae^2f^2\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)} \right) \log\left(\frac{b\sqrt{a + bx + \frac{e^2x^2}{f^2}} + f\sqrt{d - f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2d - bf^2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="fricas")

[Out]
$$-1/2*(a*b^2*f^6 + (3*b^2*d^2 - 14*a*b*d*e + 8*a^2*e^2)*f^4 - 2*(b*d^3*e - 4*a*d^2*e^2)*f^2 - 4*(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)*x^2 + (b^3*f^6 - 8*b^2*d*e*f^4 + 20*b*d^2*e^2*f^2 - 16*d^3*e^3)*x - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*\log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e - a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*\sqrt{((b*f^2*x + e^2*x^2 + a*f^2)/f^2)} - 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*\log(a*f^2 - d^2 + (b*f^2 - 2*d*e)*x) + 2*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*\log(-e*x + f*\sqrt{((b*f^2*x + e^2*x^2 + a*f^2)/f^2)} - d) - 4*((b^2*d - 2*a*b*e)*f^5 - 2*(b*d^2*e - 2*a*d*e^2)*f^3 - (b^2*e*f^5 - 4*b*d*e^2*f^3 + 4*d^2*e^3*f)*x)*\sqrt{((b*f^2*x + e^2*x^2 + a*f^2)/f^2)}/(a*b^3*f^8 + 8*d^5*e^3 - (b^3*d^2 + 6*a*b^2*d*e)*f^6 + 6*(b^2*d^3*e + 2*a*b*d^2*e^2)*f^4 - 4*(3*b*d^4*e^2 + 2*a*d^3*e^3)*f^2 + (b^4*f^8 - 8*b^3*d*e*f^6 + 24*b^2*d^2*e^2*f^4 - 32*b*d^3*e^3*f^2 + 16*d^4*e^4)*x)$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="giac")

[Out] Timed out

maple [B] time = 0.05, size = 58067, normalized size = 218.30

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^2,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2 x^2}{f^2} + af + d} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2,x)

[Out] `int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**2,x)`

[Out] `Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-2), x)`

$$3.253 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^3} dx$$

Optimal. Leaf size=330

$$\frac{2f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d + ex\right)} + \frac{6ef^2(4ae^2 - b^2f^2) \log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d + ex\right)}{(2de - bf^2)^4} - \frac{1}{(2de - bf^2)^3}$$

Rubi [A] time = 0.29, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2116, 893}

$$\frac{2f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d + ex\right)} + \frac{6ef^2(4ae^2 - b^2f^2) \log\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d + ex\right)}{(2de - bf^2)^4} - \frac{2ef^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \left(2e\left(f\sqrt{a+\frac{4(bf^2+e^2)}{f^2}} + ex\right) + bf^2\right)} - \frac{6ef^2(4ae^2 - b^2f^2) \log\left(2e\left(f\sqrt{a+\frac{4(bf^2+e^2)}{f^2}} + ex\right) + bf^2\right)}{(2de - bf^2)^4} - \frac{ae^2 - bf^2 + d^2e}{(2de - bf^2)^2 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d + ex\right)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3), x]

[Out] -((d^2*e - b*d*f^2 + a*e*f^2)/((2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^2) - (2*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])) - (2*e*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)^4 - (6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]])/(2*d*e - b*f^2)^4

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^3} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{x^3(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 2 \operatorname{Subst} \left(\int \left(\frac{d^2e - bdf^2 + aef^2}{(2de - bf^2)^2 x^3} + \frac{4ae^2f^2 - b^2f^4}{(2de - bf^2)^3 x^2} + \frac{3(4ae^3f^2 - b^2ef^4)}{(2de - bf^2)^4 x} \right) dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= -\frac{d^2e - bdf^2 + aef^2}{(2de - bf^2)^2 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^2} - \frac{2f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)}$$

Mathematica [A] time = 0.69, size = 300, normalized size = 0.91

$$\frac{\frac{2f^2(b^2f^2 - 4ae^2)(bf^2 - 2de)}{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex} + \frac{2ef^2(4ae^2 - b^2f^2)(2de - bf^2)}{2d\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex+bf^2} - 6ef^2(4ae^2 - b^2f^2)\log\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right) + 6ef^2(4ae^2 - b^2f^2)\log\left(-2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right) - bf^2\right) + \frac{(bf^2 - 2de)^2(af^2 - bf^2 + d^2e)}{\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex\right)^2}}{(bf^2 - 2de)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3), x]
[Out] -((((-2*d*e + b*f^2)^2*(d^2*e - b*d*f^2 + a*e*f^2))/(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^2 + (2*f^2*(-2*d*e + b*f^2)*(-4*a*e^2 + b^2*f^2))/(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (2*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2))/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) - 6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + 6*e*f^2*(4*a*e^2 - b^2*f^2)*Log[-(b*f^2) - 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])])/(-2*d*e + b*f^2)^4
```

IntegrateAlgebraic [F] time = 180.26, size = 0, normalized size = 0.00

\$Aborted

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3), x]
[Out] $Aborted
```

fricas [B] time = 32.34, size = 1954, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="fricas")
[Out] ((3*a*b^3*d - 4*a^2*b^2*e)*f^8 - (b^3*d^3 + 4*a*b^2*d^2*e + 10*a^2*b*d*e^2 - 20*a^3*e^3)*f^6 - 4*(b^2*d^4*e - 8*a*b*d^3*e^2 + 6*a^2*d^2*e^3)*f^4 - 4*(b^3*e^3*f^6 - 6*b^2*d*e^4*f^4 + 12*b*d^2*e^5*f^2 - 8*d^3*e^6)*x^3 + 2*(b*d^5*e^2 - 6*a*d^4*e^3)*f^2 - (b^4*e*f^8 - 2*a*b^2*e^3*f^6 - 40*d^4*e^5 - 2*(11*b^2*d^2*e^3 - 4*a*b*d*e^4)*f^4 + 8*(7*b*d^3*e^4 - a*d^2*e^5)*f^2)*x^2 + (16*d^5*e^4 + (3*b^4*d - 5*a*b^3*e)*f^8 - (7*b^3*d^2*e + 10*a*b^2*d*e^2 - 28*a^2*b*e^3)*f^6 + 2*(5*b^2*d^3*e^2 + 22*a*b*d^2*e^3 - 28*a^2*d*e^4)*f^4 - 8*(3*b*d^4*e^3 + a*d^3*e^4)*f^2)*x - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a
```

```

*b*d*e^4)*f^4)*x^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^
2*d*e^2 + 4*a^2*b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*
f^4)*x)*log(-4*a*d*e^2*f^2 - (b^2*d - 4*a*b*e)*f^4 + 4*(b*e^3*f^2 - 2*d*e^4
)*x^2 + (3*b^2*e*f^4 - 4*(2*b*d*e^2 - a*e^3)*f^2)*x - (b^2*f^5 - 4*(b*d*e -
a*e^2)*f^3 + 4*(b*e^2*f^3 - 2*d*e^3*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)
/f^2)) - 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2*a^3*e^3)*f
^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5*f^2 - 4*(b
^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x^2 + 2*(a*b
^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*b*e^3)*f^6
+ 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(a*f^2 - d^2 + (
b*f^2 - 2*d*e)*x) + 3*(a^2*b^2*e*f^8 - 4*a*d^4*e^3*f^2 - 2*(a*b^2*d^2*e + 2
*a^3*e^3)*f^6 + (b^2*d^4*e + 8*a^2*d^2*e^3)*f^4 + (b^4*e*f^8 - 16*a*d^2*e^5
*f^2 - 4*(b^3*d*e^2 + a*b^2*e^3)*f^6 + 4*(b^2*d^2*e^3 + 4*a*b*d*e^4)*f^4)*x
^2 + 2*(a*b^3*e*f^8 - 8*a*d^3*e^4*f^2 - (b^3*d^2*e + 2*a*b^2*d*e^2 + 4*a^2*
b*e^3)*f^6 + 2*(b^2*d^3*e^2 + 2*a*b*d^2*e^3 + 4*a^2*d*e^4)*f^4)*x)*log(-e*x
+ f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - d) - 2*(a*b^3*f^9 - 3*(a*b^2*d
*e + 2*a^2*b*e^2)*f^7 - 3*(b^2*d^3*e - 4*a*b*d^2*e^2 - 4*a^2*d*e^3)*f^5 + 2
*(3*b*d^4*e^2 - 10*a*d^3*e^3)*f^3 - 2*(b^3*e^2*f^7 - 6*b^2*d*e^3*f^5 + 12*b
*d^2*e^4*f^3 - 8*d^3*e^5*f)*x^2 + (b^4*f^9 + 12*d^4*e^4*f - 3*(b^3*d*e + 3*
a*b^2*e^2)*f^7 + 3*(b^2*d^2*e^2 + 12*a*b*d*e^3)*f^5 - 4*(2*b*d^3*e^3 + 9*a*
d^2*e^4)*f^3)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))/(a^2*b^4*f^12 + 16*
d^8*e^4 - 2*(a*b^4*d^2 + 4*a^2*b^3*d*e)*f^10 + (b^4*d^4 + 16*a*b^3*d^3*e +
24*a^2*b^2*d^2*e^2)*f^8 - 8*(b^3*d^5*e + 6*a*b^2*d^4*e^2 + 4*a^2*b*d^3*e^3)
*f^6 + 8*(3*b^2*d^6*e^2 + 8*a*b*d^5*e^3 + 2*a^2*d^4*e^4)*f^4 - 32*(b*d^7*e^
3 + a*d^6*e^4)*f^2 + (b^6*f^12 - 12*b^5*d*e*f^10 + 60*b^4*d^2*e^2*f^8 - 160
*b^3*d^3*e^3*f^6 + 240*b^2*d^4*e^4*f^4 - 192*b*d^5*e^5*f^2 + 64*d^6*e^6)*x^
2 + 2*(a*b^5*f^12 + 32*d^7*e^5 - (b^5*d^2 + 10*a*b^4*d*e)*f^10 + 10*(b^4*d^
3*e + 4*a*b^3*d^2*e^2)*f^8 - 40*(b^3*d^4*e^2 + 2*a*b^2*d^3*e^3)*f^6 + 80*(b
^2*d^5*e^3 + a*b*d^4*e^4)*f^4 - 16*(5*b*d^6*e^4 + 2*a*d^5*e^5)*f^2)*x)

```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 0.16, size = 295147, normalized size = 894.38

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^3,x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af + d} \right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3,x)

[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}\right)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**3,x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3), x)

$$3.254 \quad \int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx$$

Optimal. Leaf size=370

$$\frac{5f^2(4ae^2 - b^2f^2)(2de - bf^2)^{3/2} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}} \right)}{16\sqrt{2}e^{9/2}} + \frac{f^2(4ae^2 - b^2f^2)(2de - bf^2)\sqrt{f\sqrt{a+bx}}}{4e^4}$$

Rubi [A] time = 0.60, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2(4ae^2 - b^2f^2)\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{3/2}}{12e^3} + \frac{f^2(4ae^2 - b^2f^2)(2de - bf^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{4e^4} - \frac{f^2(4ae^2 - b^2f^2)(2de - bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{16e^4\left(2e\sqrt{f\sqrt{a+\frac{e^2x^2}{f^2}}+ex}+bf^2\right)} - \frac{5f^2(4ae^2 - b^2f^2)(2de - bf^2)^{3/2} \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex}}{\sqrt{2de-bf^2}} \right)}{16\sqrt{2}e^{9/2}} + \frac{\left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}}+d+ex\right)^{7/2}}{7e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2), x]

[Out] (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(4*e^4) + (f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2))/(12*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(7/2)/(7*e) - (f^2*(2*d*e - b*f^2)^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(16*e^4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (5*f^2*(2*d*e - b*f^2)^(3/2)*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]]/(16*Sqrt[2]*e^(9/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4))^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810

```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[Pq_
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c
_)*(x_)^2])^(n_))^(p_), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{5/2} dx = 2 \operatorname{Subst} \left(\int \frac{x^{5/2} (d^2 e - (bd - ae)f^2 - (2de - bf^2)x + ex^2)}{(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{x^6 (d^2 e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4)}{(-2de + bf^2 + 2ex^2)^2} dx, x, \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}} \right)$$

$$= - \frac{f^2 (2de - bf^2)^2 (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{16e^4 \left(bf^2 + 2e \left(ex + f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} \right) \right)}$$

$$= - \frac{f^2 (2de - bf^2)^2 (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{16e^4 \left(bf^2 + 2e \left(ex + f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} \right) \right)}$$

$$= \frac{f^2 (2de - bf^2) (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^4} + \frac{f^2 (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^4}$$

$$= \frac{f^2 (2de - bf^2) (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^4} + \frac{f^2 (4ae^2 - b^2 f^2) \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}}{4e^4}$$

Mathematica [A] time = 1.06, size = 357, normalized size = 0.96

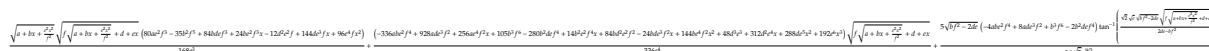
$$\frac{\frac{4}{3} e^2 f^2 (4ae^2 - b^2 f^2) \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex \right)^{3/2} + 4e f^2 (4ae^2 - b^2 f^2) (2de - bf^2) \sqrt{f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex} - \frac{5 \sqrt{c} f^2 (4ae^2 - b^2 f^2) (2de - bf^2)^{3/2} \operatorname{tanh}^{-1} \left(\frac{\sqrt{c} \sqrt{f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex}}{\sqrt{2de - bf^2}} \right)}{\sqrt{c}} - \frac{(4ae^2 f^2 - b^2 f^4) (bf^2 - 2de)^2 \sqrt{f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex}}{2 \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex \right)^2} + \frac{16}{7} e^4 \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2} \right)} + d + ex \right)^{7/2}}{16e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]
[Out] (4*e*f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b
+ (e^2*x)/f^2]]) + (4*e^2*f^2*(4*a*e^2 - b^2*f^2)*(d + e*x + f*Sqrt[a + x*
```

$(b + (e^{2x}/f^2))^{(3/2)}/3 + (16e^4(d + ex + f\sqrt{a + x(b + (e^{2x}/f^2))}^{(7/2)})/7 - ((-2d^2e + b^2f^2)^2(4ae^3f^2 - b^2ef^4)\sqrt{d + ex + f\sqrt{a + x(b + (e^{2x}/f^2))}})/(b^2f^2 + 2e(ex + f\sqrt{a + x(b + (e^{2x}/f^2))})) - (5\sqrt{e}f^2(2d^2e - b^2f^2)^{(3/2)}(4ae^2 - b^2f^2))\text{ArcTanh}[(\sqrt{2}\sqrt{e}\sqrt{d + ex + f\sqrt{a + x(b + (e^{2x}/f^2))}})]/\sqrt{2d^2e - b^2f^2})/\sqrt{2})/(16e^5)$

IntegrateAlgebraic [A] time = 5.05, size = 421, normalized size = 1.14



Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + ex + f*sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2),x]

[Out] $(\sqrt{a + b*x + (e^{2x^2})/f^2} * (-12d^2e^2f + 84b*d*e*f^3 + 80a*e^2*f^3 - 35b^2*f^5 + 144d*e^3*f*x + 24b*e^2*f^3*x + 96e^4*f*x^2) * \sqrt{d + ex + f\sqrt{a + b*x + (e^{2x^2})/f^2}}) / (168e^3) + ((48d^3e^3 + 84b*d^2e^2*f^2 + 928a*d*e^3*f^2 - 280b^2*d*e*f^4 - 336a*b*e^2*f^4 + 105b^3*f^6 + 312d^2e^4*x - 24b*d*e^3*f^2*x + 256a*e^4*f^2*x + 14b^2e^2*f^4*x + 288d*e^5*x^2 + 144b*e^4*f^2*x^2 + 192e^6*x^3) * \sqrt{d + ex + f\sqrt{a + b*x + (e^{2x^2})/f^2}}) / (336e^4) + (5\sqrt{2} * \sqrt{e} * \sqrt{d + ex + f\sqrt{a + b*x + (e^{2x^2})/f^2}}) * (8a*d*e^3*f^2 - 2b^2*d*e*f^4 - 4a*b*e^2*f^4 + b^3*f^6) * \text{ArcTan}[(\sqrt{2} * \sqrt{e} * \sqrt{d + ex + f\sqrt{a + b*x + (e^{2x^2})/f^2}})] / (2d^2e - b^2f^2) / (16\sqrt{2} * e^{(9/2)})$

fricas [A] time = 1.05, size = 923, normalized size = 2.49



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+ex+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")

[Out] $[1/672 * (105 * \sqrt{1/2} * (b^3 * f^6 + 8 * a * d * e^3 * f^2 - 2 * (b^2 * d * e + 2 * a * b * e^2) * f^4) * \sqrt{-(b * f^2 - 2 * d * e) / e} * \log(-b^2 * f^4 + 4 * (b * d * e - a * e^2) * f^2 - 4 * (b * e^2 * f^2 - 2 * d * e^3) * x + 4 * (2 * \sqrt{1/2} * e^2 * f * \sqrt{-(b * f^2 - 2 * d * e) / e} * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2} - \sqrt{1/2} * (b * e * f^2 + 2 * e^3 * x) * \sqrt{-(b * f^2 - 2 * d * e) / e}) * \sqrt{e * x + f * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2} + d} + 4 * (b * e * f^3 - 2 * d * e^2 * f) * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2}) + 2 * (105 * b^3 * f^6 + 192 * e^6 * x^3 + 48 * d^3 * e^3 - 56 * (5 * b^2 * d * e + 6 * a * b * e^2) * f^4 + 4 * (21 * b * d^2 * e^2 + 232 * a * d * e^3) * f^2 + 144 * (b * e^4 * f^2 + 2 * d * e^5) * x^2 + 2 * (7 * b^2 * e^2 * f^4 + 156 * d^2 * e^4 - 4 * (3 * b * d * e^3 - 32 * a * e^4) * f^2) * x - 2 * (35 * b^2 * e * f^5 - 96 * e^5 * f * x^2 + 12 * d^2 * e^3 * f - 4 * (21 * b * d * e^2 + 20 * a * e^3) * f^3 - 24 * (b * e^3 * f^3 + 6 * d * e^4 * f) * x) * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2}) * \sqrt{e * x + f * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2} + d}) / e^4, -1/336 * (105 * \sqrt{1/2} * (b^3 * f^6 + 8 * a * d * e^3 * f^2 - 2 * (b^2 * d * e + 2 * a * b * e^2) * f^4) * \sqrt{(b * f^2 - 2 * d * e) / e} * \arctan(2 * \sqrt{1/2} * \sqrt{e * x + f * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2} + d}) * e * \sqrt{(b * f^2 - 2 * d * e) / e} / (b * f^2 - 2 * d * e)) - (105 * b^3 * f^6 + 192 * e^6 * x^3 + 48 * d^3 * e^3 - 56 * (5 * b^2 * d * e + 6 * a * b * e^2) * f^4 + 4 * (21 * b * d^2 * e^2 + 232 * a * d * e^3) * f^2 + 144 * (b * e^4 * f^2 + 2 * d * e^5) * x^2 + 2 * (7 * b^2 * e^2 * f^4 + 156 * d^2 * e^4 - 4 * (3 * b * d * e^3 - 32 * a * e^4) * f^2) * x - 2 * (35 * b^2 * e * f^5 - 96 * e^5 * f * x^2 + 12 * d^2 * e^3 * f - 4 * (21 * b * d * e^2 + 20 * a * e^3) * f^3 - 24 * (b * e^3 * f^3 + 6 * d * e^4 * f) * x) * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2}) * \sqrt{e * x + f * \sqrt{(b * f^2 * x + e^2 * x^2 + a * f^2) / f^2} + d}) / e^4]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af + d} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{bx + \frac{e^2 x^2}{f^2} + a} f \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2 x^2}{f^2} + a} f + d \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)

[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(5/2), x)

$$3.255 \quad \int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{3/2} dx$$

Optimal. Leaf size=302

$$\frac{3f^2 (4ae^2 - b^2 f^2) \sqrt{2de - bf^2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{e} \sqrt{f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{2de - bf^2}} \right)}{8\sqrt{2} e^{7/2}} + \frac{f^2 (4ae^2 - b^2 f^2) \sqrt{f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{4e^3}$$

Rubi [A] time = 0.41, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1257, 1810, 208}

$$\frac{f^2 (4ae^2 - b^2 f^2) \sqrt{f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{4e^3} - \frac{f^2 (4ae^2 - b^2 f^2) (2de - bf^2) \sqrt{f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{8e^3 \left(2e \left(f \sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right)} - \frac{3f^2 (4ae^2 - b^2 f^2) \sqrt{2de - bf^2} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{e} \sqrt{f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{2de - bf^2}} \right)}{8\sqrt{2} e^{7/2}} + \frac{\left(f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^{5/2}}{5e}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2), x]

[Out] (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*e^3) + (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(5/2)/(5*e) - (f^2*(2*d*e - b*f^2)*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(8*e^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (3*f^2*Sqrt[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(8*Sqrt[2]*e^(7/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1257

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4))^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]

Rule 1810


```
Int[(Pq_)*((a_) + (b_)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2116

```
Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)^{3/2} dx = 2 \operatorname{Subst} \left(\int \frac{x^{3/2} (d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2)}{(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{x^4 (d^2e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4)}{(-2de + bf^2 + 2ex^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= \frac{f^2 (2de - bf^2) (4ae^2 - b^2f^2) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{8e^3 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)} - \frac{f^2 (2de - bf^2) (4ae^2 - b^2f^2) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{8e^3 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)}$$

$$= \frac{f^2 (4ae^2 - b^2f^2) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{4e^3} + \frac{(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}})}{5e}$$

$$= \frac{f^2 (4ae^2 - b^2f^2) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{4e^3} + \frac{(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}})}{5e}$$

Mathematica [A] time = 0.57, size = 291, normalized size = 0.96

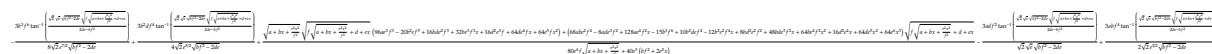
$$\frac{2ef^2(4ae^2 - b^2f^2) \sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex} - \frac{3\sqrt{e}f^2(4ae^2 - b^2f^2)\sqrt{2de - bf^2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{\sqrt{2de - bf^2}}\right)}{\sqrt{2}} - \frac{(4ae^3f^2 - b^2ef^4)(2de - bf^2) \sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{2\left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + ex\right) + bf^2} + \frac{8}{5}e^3 \left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex\right)^{5/2}}{8e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2), x]
[Out] (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) + (8*e^3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(5/2))/5 - ((2*d*e -
```

$$\frac{b*f^2*(4*a*e^3*f^2 - b^2*e*f^4)*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + x*(b + (e^2*x)/f^2)]]}{b*f^2 + 2*e*(e*x + f*\text{Sqrt}[a + x*(b + (e^2*x)/f^2)])} - (3*\text{Sqrt}[e]*f^2*\text{Sqrt}[2*d*e - b*f^2]*(4*a*e^2 - b^2*f^2)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + x*(b + (e^2*x)/f^2)])]/\text{Sqrt}[2*d*e - b*f^2])]/\text{Sqrt}[2])/ (8*e^4)$$

IntegrateAlgebraic [B] time = 3.68, size = 725, normalized size = 2.40

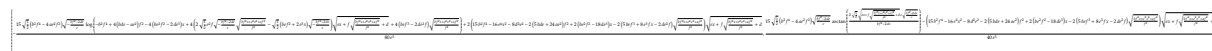


Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2),x]

[Out]
$$\begin{aligned} & (\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]*(16*d^2*e^3*f + 16*b*d*e^2*f^3 + 96*a*e^3*f^3 - 20*b^2*e*f^5 + 64*d*e^4*f*x + 32*b*e^3*f^3*x + 64*e^5*f*x^2)*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]] \\ & + (8*b*d^2*e^2*f^2 - 8*a*d*e^3*f^2 + 10*b^2*d*e*f^4 + 68*a*b*e^2*f^4 - 15*b^3*f^6 + 16*d^2*e^4*x + 48*b*d*e^3*f^2*x + 128*a*e^4*f^2*x - 12*b^2*e^2*f^4*x + 64*d*e^5*x^2 + 64*b*e^4*f^2*x^2 + 64*e^6*x^3)*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]] \\ &)/(40*e^3*(b*f^2 + 2*e^2*x) + 80*e^4*f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]) - (3*a*d*f^2*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[-2*d*e + b*f^2])*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[-2*d*e + b*f^2]) + (3*b^2*d*f^4*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[-2*d*e + b*f^2])*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(4*\text{Sqrt}[2]*e^(5/2)*\text{Sqrt}[-2*d*e + b*f^2]) + (3*a*b*f^4*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[-2*d*e + b*f^2])*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(2*\text{Sqrt}[2]*e^(3/2)*\text{Sqrt}[-2*d*e + b*f^2]) - (3*b^3*f^6*\text{ArcTan}[(\text{Sqrt}[2]*\text{Sqrt}[e]*\text{Sqrt}[-2*d*e + b*f^2])*\text{Sqrt}[d + e*x + f*\text{Sqrt}[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(8*\text{Sqrt}[2]*e^(7/2)*\text{Sqrt}[-2*d*e + b*f^2]) \end{aligned}$$

fricas [A] time = 0.83, size = 657, normalized size = 2.18



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [-1/80*(15*\text{sqrt}(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*\text{sqrt}(-(b*f^2 - 2*d*e)/e)*\log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 4*(2*\text{sqrt}(1/2)*e^2*f*\text{sqrt}(-(b*f^2 - 2*d*e)/e)*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - \text{sqrt}(1/2)*(b*e*f^2 + 2*e^3*x)*\text{sqrt}(-(b*f^2 - 2*d*e)/e))*\text{sqrt}(e*x + f*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + 2*(15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*\text{sqrt}(e*x + f*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3, 1/40*(15*\text{sqrt}(1/2)*(b^2*f^4 - 4*a*e^2*f^2)*\text{sqrt}((b*f^2 - 2*d*e)/e)*\text{arctan}(2*\text{sqrt}(1/2)*\text{sqrt}(e*x + f*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))*\text{sqrt}((b*f^2 - 2*d*e)/e)/(b*f^2 - 2*d*e) - (15*b^2*f^4 - 16*e^4*x^2 - 8*d^2*e^2 - 2*(5*b*d*e + 24*a*e^2)*f^2 + 2*(b*e^2*f^2 - 18*d*e^3)*x - 2*(5*b*e*f^3 + 8*e^3*f*x - 2*d*e^2*f)*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*\text{sqrt}(e*x + f*\text{sqrt}((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/e^3] \end{aligned}$$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af + d} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{bx + \frac{e^2 x^2}{f^2} + a} f \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{bx + \frac{e^2 x^2}{f^2} + a} f + d \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2),x)

[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}} \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2),x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(3/2), x)

$$3.256 \quad \int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=233

$$\frac{f^2 \left(4a - \frac{b^2 f^2}{e^2}\right) \sqrt{f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex} + f^2 (4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{e} \sqrt{f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{2de - bf^2}} \right) + \left(f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^{3/2}}{4 \left(2e \left(f\sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right) 4\sqrt{2} e^{5/2} \sqrt{2de - bf^2}}$$

Rubi [A] time = 0.30, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1257, 1153, 208}

$$\frac{f^2 \left(4a - \frac{b^2 f^2}{e^2}\right) \sqrt{f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex} + f^2 (4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{e} \sqrt{f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex}}{\sqrt{2de - bf^2}} \right) + \left(f\sqrt{a + bx + \frac{e^2 x^2}{f^2}} + d + ex \right)^{3/2}}{4 \left(2e \left(f\sqrt{a + \frac{x(bf^2 + e^2 x)}{f^2}} + ex \right) + bf^2 \right) 4\sqrt{2} e^{5/2} \sqrt{2de - bf^2} + 3e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]], x]

[Out] (d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)/(3*e) - (f^2*(4*a - (b^2*f^2)/e^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(4*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) - (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(4*Sqrt[2]*e^(5/2)*Sqrt[2*d*e - b*f^2])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1153

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 1257

Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*

```
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c
_.)*(x_)^2])^(n_.))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^
2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x
)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e,
f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} dx = 2 \operatorname{Subst} \left(\int \frac{\sqrt{x} (d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2)}{(-2de + bf^2 + 2ex)^2} dx, x, d + ex \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{x^2 (d^2e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4)}{(-2de + bf^2 + 2ex)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= -\frac{f^2 \left(4a - \frac{b^2f^2}{e^2}\right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}{4 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)} - \operatorname{Subst} \left(\int \frac{-ef^2(4ae^2 - b^2f^2)}{-2d} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= -\frac{f^2 \left(4a - \frac{b^2f^2}{e^2}\right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}{4 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)} - \operatorname{Subst} \left(\int \left(-4e^2x^2 - \frac{ef^2}{2}\right) dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= \frac{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2 \left(4a - \frac{b^2f^2}{e^2}\right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}{4 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)}$$

$$= \frac{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{3/2}}{3e} - \frac{f^2 \left(4a - \frac{b^2f^2}{e^2}\right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}{4 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}}\right)\right)}$$

Mathematica [A] time = 0.39, size = 223, normalized size = 0.96

$$\frac{\sqrt{e} f^2 (4ae^2 - b^2 f^2) \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{e} \sqrt{f \sqrt{a + x \left(b + \frac{e^2 x}{f^2}\right)} + d + ex}}{\sqrt{2de - bf^2}} \right)}{\sqrt{4de - 2bf^2}} + \frac{(b^2 e f^4 - 4ae^3 f^2) \sqrt{f \sqrt{a + x \left(b + \frac{e^2 x}{f^2}\right)} + d + ex}}{2e \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2}\right)} + ex\right) + bf^2} + \frac{4}{3} e^2 \left(f \sqrt{a + x \left(b + \frac{e^2 x}{f^2}\right)} + d + ex\right)^{3/2}}{4e^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]

[Out] $\frac{((4*e^2*(d + e*x + f*\sqrt{a + x*(b + (e^2*x)/f^2)}))^{3/2})/3 + ((-4*a*e^3*f^2 + b^2*e*f^4)*\sqrt{d + e*x + f*\sqrt{a + x*(b + (e^2*x)/f^2)}})/(b*f^2 + 2*e*(e*x + f*\sqrt{a + x*(b + (e^2*x)/f^2)})) - (\sqrt{e}*f^2*(4*a*e^2 - b^2*f^2)*\text{ArcTanh}[\frac{\sqrt{2}*\sqrt{e}*\sqrt{d + e*x + f*\sqrt{a + x*(b + (e^2*x)/f^2)}}}{\sqrt{2*d*e - b*f^2}}])/\sqrt{4*d*e - 2*b*f^2}}{(4*e^3)}$

IntegrateAlgebraic [A] time = 1.83, size = 404, normalized size = 1.73

$$\frac{b^2 f^4 \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{d+ex} \sqrt{\sqrt{a+bx+\frac{e^2 x^2}{f^2}}+d+ex}}{2de-3f^2}\right)}{4\sqrt{2}e^{3/2}\sqrt{bf^2-2de}} + \frac{(-4ae^2 f^2 + 3b^2 f^4 + 4bde f^2 + 12b^2 f^2 x + 8de^3 x + 16e^4 x^2) \sqrt{f\sqrt{a+bx+\frac{e^2 x^2}{f^2}}+d+ex} + \sqrt{a+bx+\frac{e^2 x^2}{f^2}} (4bde f^3 + 8de^2 f + 16e^3 f x) \sqrt{f\sqrt{a+bx+\frac{e^2 x^2}{f^2}}+d+ex}}{24e^3 f \sqrt{a+bx+\frac{e^2 x^2}{f^2}} + 12e^2 (bf^2 + 2e^2 x)}} - \frac{af^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{d+ex} \sqrt{\sqrt{a+bx+\frac{e^2 x^2}{f^2}}+d+ex}}{2de-3f^2}\right)}{\sqrt{2} \sqrt{e} \sqrt{bf^2-2de}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]

[Out] $\frac{((4*b*d*e*f^2 - 4*a*e^2*f^2 + 3*b^2*f^4 + 8*d*e^3*x + 12*b*e^2*f^2*x + 16*e^4*x^2)*\sqrt{d + e*x + f*\sqrt{a + b*x + (e^2*x^2)/f^2}} + (8*d*e^2*f + 4*b*e*f^3 + 16*e^3*f*x)*\sqrt{a + b*x + (e^2*x^2)/f^2}*\sqrt{d + e*x + f*\sqrt{a + b*x + (e^2*x^2)/f^2}})/(12*e^2*(b*f^2 + 2*e^2*x) + 24*e^3*f*\sqrt{a + b*x + (e^2*x^2)/f^2}) - (a*f^2*\text{ArcTan}[\frac{\sqrt{2}*\sqrt{e}*\sqrt{-2*d*e + b*f^2}*\sqrt{d + e*x + f*\sqrt{a + b*x + (e^2*x^2)/f^2}}}{(2*d*e - b*f^2)}])/\sqrt{2}*\sqrt{e}*\sqrt{-2*d*e + b*f^2}) + (b^2*f^4*\text{ArcTan}[\frac{\sqrt{2}*\sqrt{e}*\sqrt{-2*d*e + b*f^2}*\sqrt{d + e*x + f*\sqrt{a + b*x + (e^2*x^2)/f^2}}}{(2*d*e - b*f^2)}])/(4*\sqrt{2}*\sqrt{e}^{5/2}*\sqrt{-2*d*e + b*f^2})}$

fricas [A] time = 0.98, size = 692, normalized size = 2.97

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] $[-1/48*(3*(b^2*f^4 - 4*a*e^2*f^2)*\sqrt{-2*b*e*f^2 + 4*d*e^2})*\log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*\sqrt{-2*b*e*f^2 + 4*d*e^2})*e*f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} - \sqrt{-2*b*e*f^2 + 4*d*e^2}*(b*f^2 + 2*e^2*x))*\sqrt{e*x + f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} + d} + 4*(b*e*f^3 - 2*d*e^2*f)*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2}) - 4*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2})*\sqrt{e*x + f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} + d}]/(b*e^3*f^2 - 2*d*e^4), 1/24*(3*(b^2*f^4 - 4*a*e^2*f^2)*\sqrt{2*b*e*f^2 - 4*d*e^2})*\arctan(1/2*\sqrt{e*x + f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} + d}*(\sqrt{2*b*e*f^2 - 4*d*e^2})*f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} - \sqrt{2*b*e*f^2 - 4*d*e^2}*(e*x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x) + 2*(3*b^2*e*f^4 - 2*b*d*e^2*f^2 - 8*d^2*e^3 + 10*(b*e^3*f^2 - 2*d*e^4)*x - 2*(b*e^2*f^3 - 2*d*e^3*f)*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2})*\sqrt{e*x + f*\sqrt{(b*f^2*x + e^2*x^2 + a*f^2)/f^2} + d}]/(b*e^3*f^2 - 2*d*e^4)]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2 x^2}{f^2} + af + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a))*f + d), x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + a}} f dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2), x)

[Out] int((e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + a}} f + d dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

[Out] int((d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2), x)

[Out] Integral(sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)

$$3.257 \quad \int \frac{1}{\sqrt{d+ex+f}\sqrt{a+bx+\frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=244

$$\frac{f^2 \left(4ae - \frac{b^2f^2}{e}\right) \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{2(2de-bf^2) \left(2e \left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{f^2(4ae^2 - b^2f^2) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}(2de-bf^2)^{3/2}} + \frac{\sqrt{f}\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{e}$$

Rubi [A] time = 0.29, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1157, 388, 208}

$$\frac{f^2 \left(4ae - \frac{b^2f^2}{e}\right) \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{2(2de-bf^2) \left(2e \left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{f^2(4ae^2 - b^2f^2) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{2\sqrt{2}e^{3/2}(2de-bf^2)^{3/2}} + \frac{\sqrt{f}\sqrt{a+bx+\frac{e^2x^2}{f^2}}}{e}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]

[Out] Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/e - (f^2*(4*a*e - (b^2*f^2)/e)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]/(2*(2*d*e - b*f^2)*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1))/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 897

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m+1)-1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1157

Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x

, 0]], -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2])^(n_.))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\int \frac{1}{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{\sqrt{x} (-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4}{(-2de + bf^2 + 2ex^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= \frac{f^2 \left(4ae - \frac{b^2f^2}{e} \right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{2(2de - bf^2) \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)} + 2 \operatorname{Subst} \left(\int \frac{\frac{1}{4}(-8d^2)}{\dots} \right)$$

$$= \frac{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{e} - \frac{f^2 \left(4ae - \frac{b^2f^2}{e} \right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{2(2de - bf^2) \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)}$$

$$= \frac{\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{e} - \frac{f^2 \left(4ae - \frac{b^2f^2}{e} \right) \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{2(2de - bf^2) \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)}$$

Mathematica [A] time = 0.57, size = 238, normalized size = 0.98

$$\frac{f^2(b^2f^2 - 4ae^2) \sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{2e(2de - bf^2) \left(2e \left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + ex \right) + bf^2 \right)} + \frac{f^2(4ae^2 - b^2f^2) \tanh^{-1} \left(\frac{\sqrt{2}\sqrt{e} \sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{\sqrt{2de - bf^2}} \right)}{2\sqrt{2}e^{3/2}(2de - bf^2)^{3/2}} + \frac{\sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{e}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]], x]

[Out] Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]]/e + (f^2*(-4*a*e^2 + b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(2*e*(2*d*e - b*f^2)*(b

*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)^(3/2))

IntegrateAlgebraic [A] time = 1.71, size = 434, normalized size = 1.78

$$\frac{b^2 f^4 \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{b f^2 - 2 d e} \sqrt{f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} + d + e x}}{2 d e - b f^2}\right)}{2 \sqrt{2} e^{3/2} (2 d e - b f^2) \sqrt{b f^2 - 2 d e}} + \frac{(-4 a e^2 f^2 - b^2 f^4 + 4 b d e f^2 - 4 b e^2 f^2 x + 8 d e^3 x) \sqrt{f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} + d + e x} + (8 d e^2 f - 4 b e f^3) \sqrt{a + b x + \frac{e^2 x^2}{f^2}} \sqrt{f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} + d + e x}}{4 e^2 f (2 d e - b f^2) \sqrt{a + b x + \frac{e^2 x^2}{f^2}} + 2 e (2 d e - b f^2) (b f^2 + 2 e^2 x)}} + \frac{\sqrt{2} a \sqrt{e} f^2 \tan^{-1}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{b f^2 - 2 d e} \sqrt{f \sqrt{a + b x + \frac{e^2 x^2}{f^2}} + d + e x}}{2 d e - b f^2}\right)}{(2 d e - b f^2) \sqrt{b f^2 - 2 d e}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]],x]

[Out] ((4*b*d*e*f^2 - 4*a*e^2*f^2 - b^2*f^4 + 8*d*e^3*x - 4*b*e^2*f^2*x)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + (8*d*e^2*f - 4*b*e*f^3)*Sqrt[a + b*x + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*e*(2*d*e - b*f^2)*(b*f^2 + 2*e^2*x) + 4*e^2*f*(2*d*e - b*f^2)*Sqrt[a + b*x + (e^2*x^2)/f^2]) + (Sqrt[2]*a*Sqrt[e]*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2])*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/((2*d*e - b*f^2)*Sqrt[-2*d*e + b*f^2]) - (b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2])*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(2*Sqrt[2]*e^(3/2)*(2*d*e - b*f^2)*Sqrt[-2*d*e + b*f^2])

fricas [A] time = 1.14, size = 716, normalized size = 2.93

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="fricas")

[Out] [1/8*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x + 2*(2*sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(b^2*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4), 1/4*((b^2*f^4 - 4*a*e^2*f^2)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) + 2*(b^2*e*f^4 - 6*b*d*e^2*f^2 + 8*d^2*e^3 - 2*(b*e^3*f^2 - 2*d*e^4)*x + 2*(b*e^2*f^3 - 2*d*e^3*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(b^2*e^2*f^4 - 4*b*d*e^3*f^2 + 4*d^2*e^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e x + \sqrt{b x + \frac{e^2 x^2}{f^2} + a}} f + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + af}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)

[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af} + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2),x)

[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2)), x)

$$3.258 \quad \int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{3/2}} dx$$

Optimal. Leaf size=269

$$\frac{f^2(4ae^2 - b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{(2de - bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{3f^2(4ae^2 - b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}(2de - bf^2)^{5/2}} \quad (2de - bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}$$

Rubi [A] time = 0.37, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2116, 897, 1259, 453, 208}

$$\frac{f^2(4ae^2 - b^2f^2)\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{(2de - bf^2)^2\left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{3f^2(4ae^2 - b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{2}\sqrt{e}(2de - bf^2)^{5/2}} - \frac{4(aef^2 - bdf^2 + d^2e)}{(2de - bf^2)^2\sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2), x]

[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/((2*d*e - b*f^2)^2*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (3*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(Sqrt[2]*Sqrt[e]*(2*d*e - b*f^2)^(5/2))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 453

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m+1)*(a + b*x^n)^(p+1))/(a*e^(m+1)), x] + Dist[(a*d*(m+1) - b*c*(m+n*(p+1) + 1))/(a*e^n*(m+1)), Int[(e*x)^(m+n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (LtQ[n, 0] && GtQ[m+n, -1])) && !ILtQ[p, -1]

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m+1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]
```

Rule 2116

```
Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{3/2}} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{x^{3/2}(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4}{x^2(-2de + bf^2 + 2ex^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= -\frac{f^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^2 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)} - \operatorname{Subst} \left(\int \frac{8e^2(2de - bf^2)x}{(-2de + bf^2 + 2ex^2)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{(2de - bf^2)^2 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^2 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)}$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{(2de - bf^2)^2 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}} - \frac{f^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^2 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)}$$

Mathematica [A] time = 0.49, size = 257, normalized size = 0.96

$$\frac{2e^2(4ae^2f^2 - b^2f^4)\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{2e\left(f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+ex\right)+bf^2} + \frac{3e^{3/2}f^2(4ae^2 - b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}{\sqrt{2de-bf^2}}\right)}{\sqrt{de-\frac{bf^2}{2}}} - \frac{8e^2(aef^2 - bdf^2 + d^2e)}{\sqrt{f\sqrt{a+x\left(b+\frac{e^2x}{f^2}\right)}+d+ex}}$$

$$\frac{\hspace{10em}}{2e^2(bf^2 - 2de)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2), x]

[Out] ((-8*e^2*(d^2*e - b*d*f^2 + a*e*f^2))/Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]) - (2*e^2*(4*a*e^2*f^2 - b^2*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (3*e^(3/2)*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]]/Sqrt[d*e - (b*f^2)/2])/((2*e^2*(-2*d*e + b*f^2)^2)

IntegrateAlgebraic [A] time = 3.05, size = 458, normalized size = 1.70

$$\frac{(-12ae^2f^3 + b^2f^5 + 8bdef^3 - 8d^2e^2f)\sqrt{a + bx + \frac{e^2x^2}{f^2}} - 4abef^4 - 4ad^2f^2 - 12ae^3f^2x + 5b^2d^4f^4 + b^2ef^4x - 4bd^2ef^2 + 8bdc^2f^2x - 8d^2e^2x}{(2de - bf^2)^2(bf^2 + 2e^2x)\sqrt{f\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d + ex} + 2ef(2de - bf^2)^2\sqrt{a + bx + \frac{e^2x^2}{f^2}}\sqrt{f\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d + ex}} - \frac{3b^2f^4 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2 - 2de}\sqrt{f\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d + ex}}{2de - bf^2}\right)}{\sqrt{2}\sqrt{e}(bf^2 - 2de)^{5/2}} + \frac{6\sqrt{2}ae^{3/2}f^2 \tan^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{bf^2 - 2de}\sqrt{f\sqrt{a + bx + \frac{e^2x^2}{f^2}} + d + ex}}{2de - bf^2}\right)}{(bf^2 - 2de)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-3/2), x]

[Out] (-4*b*d^2*e*f^2 - 4*a*d*e^2*f^2 + 5*b^2*d*f^4 - 4*a*b*e*f^4 - 8*d^2*e^3*x + 8*b*d*e^2*f^2*x - 12*a*e^3*f^2*x + b^2*e*f^4*x + (-8*d^2*e^2*f + 8*b*d*e*f^3 - 12*a*e^2*f^3 + b^2*f^5)*Sqrt[a + b*x + (e^2*x^2)/f^2])/((2*d*e - b*f^2)^2*(b*f^2 + 2*e^2*x)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]] + 2*e*f*(2*d*e - b*f^2)^2*Sqrt[a + b*x + (e^2*x^2)/f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) + (6*Sqrt[2]*a*e^(3/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(-2*d*e + b*f^2)^(5/2) - (3*b^2*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2)]/(Sqrt[2]*Sqrt[e]*(-2*d*e + b*f^2)^(5/2))

fricas [B] time = 1.42, size = 1456, normalized size = 5.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2), x, algorithm="fricas")

[Out] [1/4*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(-2*b*e*f^2 + 4*d*e^2)*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(-2*b*e*f^2 + 4*d*e^2)*e*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(-2*b*e*f^2 + 4*d*e^2)*(b*f^2 + 2*e^2*x))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) + 4*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x), -1/2*(3*(a*b^2*f^6 + 4*a*d^2*e^2*f^2 - (b^2*d^2 + 4*a^2*e^2)*f^4 + (b^3*f^6 + 8*a*d*e^3*f^2 - 2*(b^2*d*e + 2*a*b*e^2)*f^4)*x)*sqrt(2*b*e*f^2 - 4*d*e^2)*arctan(1/2*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d)*(sqrt(2*b*e*f^2 - 4*d*e^2)*f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2*b*e*f^2 - 4*d*e^2)*(e*x + d))/(a*e*f^2 - d^2*e + (b*e*f^2 - 2*d*e^2)*x)) - 2*(a*b^2*e*f^6 - 8*d^4*e^3 - (5*b^2*d^2*e - 2*a*b*d*e^2)*f^4 + 2*(7*b*d^3*e^2 - 4*a*d^2*e^3)*f^2 + 2*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^2 + (b^3*e*f^6 - 4*d^3*e^4 - 2*(4*b^2*d*e^2 - 3*a*b*e^3)*f^4 + 2*(7*b*d^2*e^3 - 6*a*d*e^4)*f^2)*x + 2*(2*d^3*e^3*f + (2*b^2*d*e - 3

```
*a*b*e^2)*f^5 - (5*b*d^2*e^2 - 6*a*d*e^3)*f^3 - (b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2*e^4*f)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d))/(a*b^3*e*f^8 + 8*d^5*e^4 - (b^3*d^2*e + 6*a*b^2*d*e^2)*f^6 + 6*(b^2*d^3*e^2 + 2*a*b*d^2*e^3)*f^4 - 4*(3*b*d^4*e^3 + 2*a*d^3*e^4)*f^2 + (b^4*e*f^8 - 8*b^3*d*e^2*f^6 + 24*b^2*d^2*e^3*f^4 - 32*b*d^3*e^4*f^2 + 16*d^4*e^5)*x)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e
,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, cho
osing root of [1,0,%%{-2,[1,0,2,0,0]%%}+%%{-2,[0,1,2,1,0]%%}+%%{-2,[0,
0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%}],0,%%{1,[2,0,4,0,0]%%}+%%{2,[1,1,4,1,
0]%%}+%%{-2,[1,0,3,0,0]%%}+%%{2,[1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%}+%
%%{-2,[0,1,3,1,0]%%}+%%{2,[0,1,2,3,1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,[0
,0,1,2,1]%%}+%%{1,[0,0,0,4,2]%%}] at parameters values [-49,-86,61.79374
78349,-30,70]Warning, choosing root of [1,0,%%{-2,[1,0,2,0,0]%%}+%%{-2,[
0,1,2,1,0]%%}+%%{-2,[0,0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%}],0,%%{1,[2,0,4
,0,0]%%}+%%{2,[1,1,4,1,0]%%}+%%{-2,[1,0,3,0,0]%%}+%%{2,[1,0,2,2,1]%%
}+%%{1,[0,2,4,2,0]%%}+%%{-2,[0,1,3,1,0]%%}+%%{2,[0,1,2,3,1]%%}+%%{1,
[0,0,2,0,0]%%}+%%{-2,[0,0,1,2,1]%%}+%%{1,[0,0,0,4,2]%%}] at parameters
values [0,0,71.707969239,0,0]Warning, choosing root of [1,0,%%{-2,[1,0,2,
0,0]%%}+%%{-2,[0,1,2,1,0]%%}+%%{-2,[0,0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%
}],0,%%{1,[2,0,4,0,0]%%}+%%{2,[1,1,4,1,0]%%}+%%{-2,[1,0,3,0,0]%%}+%%
{2,[1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%}+%%{-2,[0,1,3,1,0]%%}+%%{2,[0,1,
2,3,1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,[0,0,1,2,1]%%}+%%{1,[0,0,0,4,2]%%
}] at parameters values [-9,-13,47.3295757947,24,49]Warning, choosing root
of [1,0,%%{-2,[1,0,2,0,0]%%}+%%{-2,[0,1,2,1,0]%%}+%%{-2,[0,0,1,0,0]%%
}+%%{-2,[0,0,0,2,1]%%}],0,%%{1,[2,0,4,0,0]%%}+%%{2,[1,1,4,1,0]%%}+%%
{-2,[1,0,3,0,0]%%}+%%{2,[1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%}+%%{-2,[0,1
,3,1,0]%%}+%%{2,[0,1,2,3,1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,[0,0,1,2,1]%%
}+%%{1,[0,0,0,4,2]%%}] at parameters values [81,18,54.7579903365,-33,-7
0]Warning, choosing root of [1,0,%%{-2,[1,0,2,0,0]%%}+%%{2,[0,1,2,1,0]%%
}+%%{-2,[0,0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%}],0,%%{1,[2,0,4,0,0]%%}+%%
{-2,[1,1,4,1,0]%%}+%%{-2,[1,0,3,0,0]%%}+%%{2,[1,0,2,2,1]%%}+%%{1,[0,
2,4,2,0]%%}+%%{2,[0,1,3,1,0]%%}+%%{-2,[0,1,2,3,1]%%}+%%{1,[0,0,2,0,0]
%%}+%%{-2,[0,0,1,2,1]%%}+%%{1,[0,0,0,4,2]%%}] at parameters values [62
,-37,8.05231268331,-23,65]Warning, choosing root of [1,0,%%{-2,[1,0,2,0,0]
%%}+%%{2,[0,1,2,1,0]%%}+%%{-2,[0,0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%}],0,
%%{1,[2,0,4,0,0]%%}+%%{-2,[1,1,4,1,0]%%}+%%{-2,[1,0,3,0,0]%%}+%%{2,[
1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%}+%%{2,[0,1,3,1,0]%%}+%%{-2,[0,1,2,3,
1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,[0,0,1,2,1]%%}+%%{1,[0,0,0,4,2]%%}]
at parameters values [0,0,64.3995612673,0,0]Warning, choosing root of [1,0,
%%{-2,[1,0,2,0,0]%%}+%%{2,[0,1,2,1,0]%%}+%%{-2,[0,0,1,0,0]%%}+%%{-2,
[0,0,0,2,1]%%}],0,%%{1,[2,0,4,0,0]%%}+%%{-2,[1,1,4,1,0]%%}+%%{-2,[1,0,
3,0,0]%%}+%%{2,[1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%}+%%{2,[0,1,3,1,0]%%
}+%%{-2,[0,1,2,3,1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,[0,0,1,2,1]%%}+%%{1
,[0,0,0,4,2]%%}] at parameters values [-22,93,94.1262030317,31,-21]Warning
, choosing root of [1,0,%%{-2,[1,0,2,0,0]%%}+%%{2,[0,1,2,1,0]%%}+%%{-2,
[0,0,1,0,0]%%}+%%{-2,[0,0,0,2,1]%%}],0,%%{1,[2,0,4,0,0]%%}+%%{-2,[1,1
,4,1,0]%%}+%%{-2,[1,0,3,0,0]%%}+%%{2,[1,0,2,2,1]%%}+%%{1,[0,2,4,2,0]%%
}+%%{2,[0,1,3,1,0]%%}+%%{-2,[0,1,2,3,1]%%}+%%{1,[0,0,2,0,0]%%}+%%{-2,
```

`-2, [0,0,1,2,1]%%}+%%{1, [0,0,0,4,2]%%}]` at parameters values `[-66,66,6.82 230772497,-23,79]` Evaluation time: 1.2Done

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + af} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2), x)`

[Out] `int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(3/2), x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af + d} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(3/2), x, algorithm="maxima")`

[Out] `integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)`

[Out] `int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(3/2), x)`

[Out] `Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-3/2), x)`

3.259
$$\int \frac{1}{\left(d+ex+f\sqrt{a+bx+\frac{e^2x^2}{f^2}}\right)^{5/2}} dx$$

Optimal. Leaf size=335

$$\frac{4f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}} - \frac{2ef^2(4ae^2 - b^2f^2) \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{(2de - bf^2)^3 \left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{5\sqrt{2} \sqrt{e} f^2 (4ae^2 - b^2f^2) \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{(2de - bf^2)^{7/2}} - \frac{4(aef^2 - bdf^2 + d^2e)}{3(2de - bf^2)^2 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex\right)^{3/2}}$$

Rubi [A] time = 0.50, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 30, number of rules / integrand size = 0.167, Rules used = {2116, 897, 1259, 1261, 208}

$$\frac{4f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}} - \frac{2ef^2(4ae^2 - b^2f^2) \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{(2de - bf^2)^3 \left(2e\left(f\sqrt{a+\frac{x(bf^2+e^2x)}{f^2}} + ex\right) + bf^2\right)} + \frac{5\sqrt{2} \sqrt{e} f^2 (4ae^2 - b^2f^2) \operatorname{arctanh}\left(\frac{\sqrt{2} \sqrt{e} \sqrt{f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex}}{\sqrt{2de-bf^2}}\right)}{(2de - bf^2)^{7/2}} - \frac{4(aef^2 - bdf^2 + d^2e)}{3(2de - bf^2)^2 \left(f\sqrt{a+bx+\frac{e^2x^2}{f^2}} + d+ex\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2), x]

[Out] (-4*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(2*d*e - b*f^2)^2*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (4*f^2*(4*a*e^2 - b^2*f^2))/((2*d*e - b*f^2)^3*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]]) - (2*e*f^2*(4*a*e^2 - b^2*f^2)*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/((2*d*e - b*f^2)^3*(b*f^2 + 2*e*(e*x + f*Sqrt[a + (x*(b*f^2 + e^2*x))/f^2]))) + (5*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/Sqrt[2*d*e - b*f^2]])/(2*d*e - b*f^2)^(7/2)

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 897

Int[((d_) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_)^(n_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1259

Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4))^p - ((c*d^2 - b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))]/(d + e*x^2)], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && ILtQ[m/2, 0]

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 2116

```
Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.))^p, x_Symbol] := Dist[2, Subst[Int[(g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}\right)^{5/2}} dx = 2 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 - (2de - bf^2)x + ex^2}{x^{5/2}(-2de + bf^2 + 2ex)^2} dx, x, d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{d^2e - (bd - ae)f^2 + (-2de + bf^2)x^2 + ex^4}{x^4(-2de + bf^2 + 2ex)^2} dx, x, \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}} \right)$$

$$= -\frac{2ef^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^3 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)} - \operatorname{Subst} \left(\int \frac{8e^2(2de - bf^2)}{\dots} \right)$$

$$= -\frac{2ef^2(4ae^2 - b^2f^2)\sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}{(2de - bf^2)^3 \left(bf^2 + 2e \left(ex + f\sqrt{a + \frac{x(bf^2 + e^2x)}{f^2}} \right) \right)} - \operatorname{Subst} \left(\int \left(-\frac{8e^2(2de - bf^2)}{\dots} \right) \right)$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{3(2de - bf^2)^2 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)^{3/2}} - \frac{4f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}$$

$$= -\frac{4(d^2e - bdf^2 + aef^2)}{3(2de - bf^2)^2 \left(d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}} \right)^{3/2}} - \frac{4f^2(4ae^2 - b^2f^2)}{(2de - bf^2)^3 \sqrt{d + ex + f\sqrt{a + bx + \frac{e^2x^2}{f^2}}}}$$

Mathematica [A] time = 0.86, size = 315, normalized size = 0.94

$$\frac{8f^2(b^2f^2 - 4ae^2)}{\sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}} + \frac{10\sqrt{2}\sqrt{e}f^2(4ae^2 - b^2f^2)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{\sqrt{2de - bf^2}}\right)}{\sqrt{2de - bf^2}} - \frac{4(4ae^3f^2 - b^2e^2f^4)\sqrt{f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex}}{2e\left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + ex\right) + bf^2} - \frac{8(2de - bf^2)(ae^2f^2 - bdf^2 + d^2e)}{3\left(f\sqrt{a + x\left(b + \frac{e^2x}{f^2}\right)} + d + ex\right)^{3/2}}$$

$$2(2de - bf^2)^3$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
[Out] ((-8*(2*d*e - b*f^2)*(d^2*e - b*d*f^2 + a*e*f^2))/(3*(d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])^(3/2)) + (8*f^2*(-4*a*e^2 + b^2*f^2))/Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]] - (4*(4*a*e^3*f^2 - b^2*e*f^4)*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/(b*f^2 + 2*e*(e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)])) + (10*Sqrt[2]*Sqrt[e]*f^2*(4*a*e^2 - b^2*f^2)*ArcTanh[(Sqrt[2]*Sqrt[e]*Sqrt[d + e*x + f*Sqrt[a + x*(b + (e^2*x)/f^2)]])/Sqrt[2*d*e - b*f^2]])/Sqrt[2*d*e - b*f^2])/(2*(2*d*e - b*f^2)^3)
```

IntegrateAlgebraic [A] time = 4.04, size = 605, normalized size = 1.81

$$\frac{-2(60d^2f^4 - 17d^2f^3 + 26abd^2f^4 + 80abd^2f^3 + 12ad^2f^2 + 80ad^2f^2 + 120ad^2f^2 - 4d^2f^2 - 21d^2f^2 - 9d^2f^2 - 14d^2f^2 - 30d^2f^2 + 80d^2f^2 - 120d^2f^2 + 8d^2f^2) - 2\sqrt{a + \frac{bx + d}{f}}(21abd^2f^4 + 80abd^2f^3 + 120ad^2f^2 - 4d^2f^2 - 14d^2f^2 - 30d^2f^2 - 120d^2f^2 + 8d^2f^2)}{3(2d - b)f^2(f^2 + 2e)\sqrt{\frac{bx + d}{f}} + e f(2d - b)f^2\sqrt{\frac{bx + d}{f}} + e f(2d - b)f^2\sqrt{\frac{bx + d}{f}}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(-5/2),x]
[Out] (-2*(8*d^3*e^3*f - 12*b*d^2*e^2*f^3 + 80*a*d*e^3*f^3 - 14*b^2*d*e*f^5 + 20*a*b*e^2*f^5 - 6*b^3*f^7 + 120*a*e^4*f^3*x - 30*b^2*e^2*f^5*x)*Sqrt[a + b*x + (e^2*x^2)/f^2] - 2*(4*b*d^3*e^2*f^2 + 12*a*d^2*e^3*f^2 - 9*b^2*d^2*e*f^4 + 28*a*b*d*e^2*f^4 + 60*a^2*e^3*f^4 - 4*b^3*d*f^6 - 17*a*b^2*e*f^6 + 8*d^3*e^4*x - 12*b*d^2*e^3*f^2*x + 80*a*d*e^4*f^2*x - 14*b^2*d*e^2*f^4*x + 80*a*b*e^3*f^4*x - 21*b^3*e*f^6*x + 120*a*e^5*f^2*x^2 - 30*b^2*e^3*f^4*x^2))/(3*(2*d*e - b*f^2)^3*(b*f^2 + 2*e^2*x)*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2) + 6*e*f*(2*d*e - b*f^2)^3*Sqrt[a + b*x + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2])^(3/2)) - (20*Sqrt[2]*a*e^(5/2)*f^2*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2])*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2))/(-2*d*e + b*f^2)^(7/2) + (5*Sqrt[2]*b^2*Sqrt[e]*f^4*ArcTan[(Sqrt[2]*Sqrt[e]*Sqrt[-2*d*e + b*f^2])*Sqrt[d + e*x + f*Sqrt[a + b*x + (e^2*x^2)/f^2]])/(2*d*e - b*f^2))/(-2*d*e + b*f^2)^(7/2)
```

fricas [B] time = 2.09, size = 2514, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="fricas")
[Out] [-1/6*(15*sqrt(2)*(a^2*b^2*f^8 - 4*a*d^4*e^2*f^2 - 2*(a*b^2*d^2 + 2*a^3*e^2)*f^6 + (b^2*d^4 + 8*a^2*d^2*e^2)*f^4 + (b^4*f^8 - 16*a*d^2*e^4*f^2 - 4*(b^3*d*e + a*b^2*e^2)*f^6 + 4*(b^2*d^2*e^2 + 4*a*b*d*e^3)*f^4)*x^2 + 2*(a*b^3*f^8 - 8*a*d^3*e^3*f^2 - (b^3*d^2 + 2*a*b^2*d*e + 4*a^2*b*e^2)*f^6 + 2*(b^2*d^3*e + 2*a*b*d^2*e^2 + 4*a^2*d*e^3)*f^4)*x)*sqrt(-e/(b*f^2 - 2*d*e))*log(-b^2*f^4 + 4*(b*d*e - a*e^2)*f^2 - 4*(b*e^2*f^2 - 2*d*e^3)*x - 2*(2*sqrt(2)*(b*e*f^3 - 2*d*e^2*f)*sqrt(-e/(b*f^2 - 2*d*e))*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) - sqrt(2)*(b^2*f^4 - 2*b*d*e*f^2 + 2*(b*e^2*f^2 - 2*d*e^3)*x)*sqrt(-e/(b*f^2 - 2*d*e)))*sqrt(e*x + f*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2) + d) + 4*(b*e*f^3 - 2*d*e^2*f)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2)) - 4*(4*d^5*e^2 + (8*a*b^2*d - 5*a^2*b*e)*f^6 - 2*(2*b^2*d^3 + a*b*d^2*e + 10*a^2*d*e^2)*f^4 - 6*(b^2*e^3*f^4 - 4*b*d*e^4*f^2 + 4*d^2*e^5)*x^3 - (9*b*d^4*e - 32*a*d^3*e^2)*f^2 + (3*b^3*e*f^6 - 16*d^3*e^4 + 4*(b^2*d*e^2 - 10*a*b*e^3)*f^4 - 4*(3*b*d^2*e^3 - 20*a*d*e^4)*f^2)*x^2 + 2*(d^4*e^3 + (4*b^3*d - a*b^2*e)*f^6 - (7*b^2*d^2*e + 6*a*b*d*e^2 + 15*a^2*e^3)*f^4 - 2*(5*b*d^3*e^2 - 23*a*d^2*e^3)*f^2)*x - 2*(3*a*b^2*f^7 + d^4*e^2*f - (b^2*d^2 + 2*a*b*d*e + 15*a^2*e^2)*f^5 - 2*(3*b*d^3*e - 11*a*d^2*e^2)*f^3 - 3*(b^2*e^2*f^5 - 4*b*d*e^3*f^3 + 4*d^2*e^4*f)*x^2 + (3*b^3*f^7 + 40*a*d*e^3*f^3 - 8*d^3*e^3*f - 4*(b^2*d*e + 5*a*b*e^2)*f^5)*x)*sqrt((b*f^2*x + e^2*x^2 + a*f^2)/f^2))*sqrt(
```

$$e^x + f\sqrt{\frac{b^2x^2 + e^2x^2 + af^2}{f^2}} + d) / (a^2b^3f^{10} - 8d^7e^3 - 2(a^2b^3d^2 + 3a^2b^2d^3e) f^8 + (b^3d^4 + 12a^2b^2d^3e + 12a^2b^2d^2e^2) f^6 - 2(3b^2d^5e + 12a^2b^2d^4e^2 + 4a^2d^3e^3) f^4 + 4(3b^2d^6e^2 + 4a^2d^5e^3) f^2 + (b^5f^{10} - 10b^4d^3e^2f^8 + 40b^3d^2e^2f^6 - 80b^2d^3e^3f^4 + 80b^2d^4e^4f^2 - 32d^5e^5) x^2 + 2(a^2b^4f^{10} - 16d^6e^4 - (b^4d^2 + 8a^2b^3d^3e) f^8 + 8(b^3d^3e + 3a^2b^2d^2e^2) f^6 - 8(3b^2d^4e^2 + 4a^2b^2d^3e^3) f^4 + 16(2b^2d^5e^3 + a^2d^4e^4) f^2) x), 1/3(15\sqrt{2})(a^2b^2f^8 - 4a^2d^4e^2f^2 - 2(a^2b^2d^2 + 2a^3e^2) f^6 + (b^2d^4 + 8a^2d^2e^2) f^4 + (b^4f^8 - 16a^2d^2e^4f^2 - 4(b^3d^2e + a^2b^2e^2) f^6 + 4(b^2d^2e^2 + 4a^2b^2d^2e^3) f^4) x^2 + 2(a^2b^3f^8 - 8a^2d^3e^3f^2 - (b^3d^2 + 2a^2b^2d^2e + 4a^2b^2e^2) f^6 + 2(b^2d^3e + 2a^2b^2d^2e^2 + 4a^2d^2e^3) f^4) x) \sqrt{e/(b^2f^2 - 2d^2e)}} \arctan(1/2(\sqrt{2})(b^2f^2 - 2d^2e) \sqrt{e/(b^2f^2 - 2d^2e)}) \sqrt{\frac{b^2x^2 + e^2x^2 + af^2}{f^2}} - \sqrt{2}(b^2d^2e - 2d^2e^2 + (b^2e^2f^2 - 2d^2e^2) x) \sqrt{e/(b^2f^2 - 2d^2e)}}) \sqrt{e^x + f\sqrt{\frac{b^2x^2 + e^2x^2 + af^2}{f^2}} + d) / (a^2b^3f^{10} - 8d^7e^3 - 2(a^2b^3d^2 + 3a^2b^2d^3e) f^8 + (b^3d^4 + 12a^2b^2d^3e + 12a^2b^2d^2e^2) f^6 - 2(3b^2d^5e + 12a^2b^2d^4e^2 + 4a^2d^3e^3) f^4 + 4(3b^2d^6e^2 + 4a^2d^5e^3) f^2 + (b^5f^{10} - 10b^4d^3e^2f^8 + 40b^3d^2e^2f^6 - 80b^2d^3e^3f^4 + 80b^2d^4e^4f^2 - 32d^5e^5) x^2 + 2(a^2b^4f^{10} - 16d^6e^4 - (b^4d^2 + 8a^2b^3d^3e) f^8 + 8(b^3d^3e + 3a^2b^2d^2e^2) f^6 - 8(3b^2d^4e^2 + 4a^2b^2d^3e^3) f^4 + 16(2b^2d^5e^3 + a^2d^4e^4) f^2) x)]]$$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="giac")

[Out] Timed out

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + d + \sqrt{bx + \frac{e^2x^2}{f^2} + af} \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

[Out] int(1/(e*x+d+(b*x+e^2/f^2*x^2+a)^(1/2)*f)^(5/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(ex + \sqrt{bx + \frac{e^2x^2}{f^2} + af} + d \right)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e^2*x^2/f^2)^(1/2))^(5/2),x, algorithm="maxima")

[Out] integrate((e*x + sqrt(b*x + e^2*x^2/f^2 + a)*f + d)^(-5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2),x)

[Out] int(1/(d + e*x + f*(a + b*x + (e^2*x^2)/f^2)^(1/2))^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(d + ex + f \sqrt{a + bx + \frac{e^2 x^2}{f^2}}\right)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(d+e*x+f*(a+b*x+e**2*x**2/f**2)**(1/2))**(5/2),x)

[Out] Integral((d + e*x + f*sqrt(a + b*x + e**2*x**2/f**2))**(-5/2), x)

$$3.260 \quad \int (a + x^2)^2 (x + \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=164

$$\frac{a^5 (\sqrt{a+x^2} + x)^{n-5}}{32(5-n)} - \frac{5a^4 (\sqrt{a+x^2} + x)^{n-3}}{32(3-n)} - \frac{5a^3 (\sqrt{a+x^2} + x)^{n-1}}{16(1-n)} + \frac{5a^2 (\sqrt{a+x^2} + x)^{n+1}}{16(n+1)} + \frac{5a (\sqrt{a+x^2} + x)^{n+3}}{32(n+3)}$$

Rubi [A] time = 0.11, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2122, 270}

$$\frac{a^5 (\sqrt{a+x^2} + x)^{n-5}}{32(5-n)} - \frac{5a^4 (\sqrt{a+x^2} + x)^{n-3}}{32(3-n)} - \frac{5a^3 (\sqrt{a+x^2} + x)^{n-1}}{16(1-n)} + \frac{5a^2 (\sqrt{a+x^2} + x)^{n+1}}{16(n+1)} + \frac{5a (\sqrt{a+x^2} + x)^{n+3}}{32(n+3)} + \frac{(\sqrt{a+x^2} + x)^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]

[Out] -(a^5*(x + Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x + Sqrt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x + Sqrt[a + x^2])^(-1 + n))/(16*(1 - n)) + (5*a^2*(x + Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x + Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x + Sqrt[a + x^2])^(5 + n)/(32*(5 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^2 (x + \sqrt{a + x^2})^n dx &= \frac{1}{32} \text{Subst} \left(\int x^{-6+n} (a + x^2)^5 dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{32} \text{Subst} \left(\int (a^5 x^{-6+n} + 5a^4 x^{-4+n} + 10a^3 x^{-2+n} + 10a^2 x^n + 5ax^{2+n} + x^{4+n}) dx, x, x + \sqrt{a + x^2} \right) \\ &= -\frac{a^5 (x + \sqrt{a + x^2})^{-5+n}}{32(5-n)} - \frac{5a^4 (x + \sqrt{a + x^2})^{-3+n}}{32(3-n)} - \frac{5a^3 (x + \sqrt{a + x^2})^{-1+n}}{16(1-n)} + \dots \end{aligned}$$

Mathematica [A] time = 0.38, size = 138, normalized size = 0.84

$$\frac{1}{32} (\sqrt{a+x^2} + x)^{n-5} \left(\frac{a^5}{n-5} + \frac{5a^4 (\sqrt{a+x^2} + x)^2}{n-3} + \frac{10a^3 (\sqrt{a+x^2} + x)^4}{n-1} + \frac{10a^2 (\sqrt{a+x^2} + x)^6}{n+1} + \frac{(\sqrt{a+x^2} + x)^{10}}{n+5} + \frac{5a (\sqrt{a+x^2} + x)^8}{n+3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]

```
[Out] ((x + Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x + Sqrt[a + x^2])^2)/(-3 + n) + (10*a^3*(x + Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x + Sqrt[a + x^2])^6)/(1 + n) + (5*a*(x + Sqrt[a + x^2])^8)/(3 + n) + (x + Sqrt[a + x^2])^10/(5 + n))/32
```

IntegrateAlgebraic [F] time = 0.28, size = 0, normalized size = 0.00

$$\int (a + x^2)^2 \left(x + \sqrt{a + x^2}\right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^2*(x + Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + x^2)^2*(x + Sqrt[a + x^2])^n, x]
```

fricas [A] time = 0.74, size = 158, normalized size = 0.96

$$\frac{(5(n^4 - 10n^2 + 9)x^5 + 10(an^4 - 16an^2 + 15a)x^3 + 5(a^2n^4 - 22a^2n^2 + 45a^2)x - (a^2n^5 - 30a^2n^3 + (n^5 - 10n^3 + 9n)x^4 + 149a^2n + 2(an^5 - 20an^3 + 19an)x^2)\sqrt{x^2 + a})(x + \sqrt{x^2 + a})^n}{n^6 - 35n^4 + 259n^2 - 225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4 - 22*a^2*n^2 + 45*a^2)*x - (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^2 \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)
```

maple [C] time = 0.09, size = 216, normalized size = 1.32

$$\frac{a^{2n+1}x^{n+3} \operatorname{hypergeom}\left(\left[\frac{n}{2}, -\frac{n}{2} + \frac{1}{2}, -\frac{n}{2} - \frac{3}{2}\right], \left[-n+1, -\frac{n}{2} - \frac{1}{2}\right], -\frac{a}{x^2}\right) + 2^na^{n+5} \operatorname{hypergeom}\left(\left[\frac{n}{2}, -\frac{n}{2} - \frac{5}{2}, \frac{n}{2} + \frac{1}{2}\right], \left[-n+1, -\frac{n}{2} - \frac{3}{2}\right], -\frac{a}{x^2}\right) + \frac{8\sqrt{\pi}\left(\frac{n+\frac{n}{2}+1}{2}\right)^{\frac{n}{2}}x^{n+1}\left(\sqrt{\frac{x^2+1}{2}}\right)^{n-1} + 4\sqrt{\pi}\sqrt{\frac{x^2+1}{2}}a^{\frac{n}{2}}x^{n+1}\left(\sqrt{\frac{x^2+1}{2}}\right)^{n-1}}{(n+1)(2n-2)!} + \frac{4\sqrt{\pi}\sqrt{\frac{x^2+1}{2}}a^{\frac{n}{2}}x^{n+1}\left(\sqrt{\frac{x^2+1}{2}}\right)^{n-1}}{(n+1)!}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x)
```

```
[Out] 2^n/(5+n)*x^(5+n)*hypergeom([-1/2*n, -1/2*n-5/2, 1/2-1/2*n], [1-n, -3/2-1/2*n], -a/x^2)+2^(n+1)*a/(3+n)*x^(3+n)*hypergeom([-1/2*n, 1/2-1/2*n, -3/2-1/2*n], [1-n, -1/2-1/2*n], -a/x^2)+1/4*a^(5/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)/n*x^(n+1)*a^(-1/2-1/2*n)*(a/x^2*n+n-1)/(2*n-2)*((1+a/x^2)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)/n*x^(n+1)*a^(-1/2-1/2*n)*(1+a/x^2)^(1/2)*((1+a/x^2)^(1/2)+1)^(n-1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^2 \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

[Out] integrate((x^2 + a)^2*(x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + a)^2 (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)

[Out] int((a + x^2)^2*(x + (a + x^2)^(1/2))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+a)**2*(x+(x**2+a)**(1/2))**n, x)

[Out] Timed out

$$3.261 \quad \int (a + x^2) (x + \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=108

$$-\frac{a^3 (\sqrt{a + x^2} + x)^{n-3}}{8(3-n)} - \frac{3a^2 (\sqrt{a + x^2} + x)^{n-1}}{8(1-n)} + \frac{3a (\sqrt{a + x^2} + x)^{n+1}}{8(n+1)} + \frac{(\sqrt{a + x^2} + x)^{n+3}}{8(n+3)}$$

Rubi [A] time = 0.06, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2122, 270}

$$-\frac{a^3 (\sqrt{a + x^2} + x)^{n-3}}{8(3-n)} - \frac{3a^2 (\sqrt{a + x^2} + x)^{n-1}}{8(1-n)} + \frac{3a (\sqrt{a + x^2} + x)^{n+1}}{8(n+1)} + \frac{(\sqrt{a + x^2} + x)^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)*(x + Sqrt[a + x^2])^n,x]

[Out] -(a^3*(x + Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x + Sqrt[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x + Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (x + Sqrt[a + x^2])^(3 + n)/(8*(3 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2) (x + \sqrt{a + x^2})^n dx &= \frac{1}{8} \text{Subst} \left(\int x^{-4+n} (a + x^2)^3 dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{8} \text{Subst} \left(\int (a^3 x^{-4+n} + 3a^2 x^{-2+n} + 3ax^n + x^{2+n}) dx, x, x + \sqrt{a + x^2} \right) \\ &= -\frac{a^3 (x + \sqrt{a + x^2})^{-3+n}}{8(3-n)} - \frac{3a^2 (x + \sqrt{a + x^2})^{-1+n}}{8(1-n)} + \frac{3a (x + \sqrt{a + x^2})^{1+n}}{8(1+n)} + \end{aligned}$$

Mathematica [A] time = 0.14, size = 92, normalized size = 0.85

$$\frac{1}{8} (\sqrt{a + x^2} + x)^{n-3} \left(\frac{a^3}{n-3} + \frac{3a^2 (\sqrt{a + x^2} + x)^2}{n-1} + \frac{(\sqrt{a + x^2} + x)^6}{n+3} + \frac{3a (\sqrt{a + x^2} + x)^4}{n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)*(x + Sqrt[a + x^2])^n,x]

[Out] ((x + Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x + Sqrt[a + x^2]))^2)/(-1 + n) + (3*a*(x + Sqrt[a + x^2])^4)/(1 + n) + (x + Sqrt[a + x^2])^6/(3 + n))/8

IntegrateAlgebraic [F] time = 0.24, size = 0, normalized size = 0.00

$$\int (a + x^2) \left(x + \sqrt{a + x^2} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)*(x + Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)*(x + Sqrt[a + x^2])^n, x]

fricas [A] time = 0.68, size = 78, normalized size = 0.72

$$\frac{\left(3(n^2 - 1)x^3 + 3(an^2 - 3a)x - (an^3 + (n^3 - n)x^2 - 7an)\sqrt{x^2 + a} \right) \left(x + \sqrt{x^2 + a} \right)^n}{n^4 - 10n^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x - (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a) \left(x + \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)

maple [C] time = 0.02, size = 167, normalized size = 1.55

$$\frac{2^n x^{n+3} \operatorname{hypergeom}\left(\left[\begin{matrix} -\frac{n}{2}, -\frac{n}{2} + \frac{1}{2}, -\frac{n}{2} - \frac{3}{2} \end{matrix}\right], \left[\begin{matrix} -n + 1, -\frac{n}{2} - \frac{1}{2} \end{matrix}\right], -\frac{a}{x^2}\right) + \left(\frac{8\sqrt{\pi} \left(\frac{n+\frac{a}{x^2}-1}{x^2}\right)^{\frac{n}{2}-\frac{1}{2}} x^{n+1} \left(\sqrt{\frac{a}{x^2}+1}\right)^{n-1}}{(n+1)(2n-2)n} + \frac{4\sqrt{\pi} \sqrt{\frac{a}{x^2}+1} a^{-\frac{n}{2}-\frac{1}{2}} x^{n+1} \left(\sqrt{\frac{a}{x^2}+1}\right)^{n-1}}{(n+1)n}\right) n a^{\frac{n}{2}+\frac{3}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)*(x+(x^2+a)^(1/2))^n,x)

[Out] 2^n/(n+3)*x^(n+3)*hypergeom([-1/2*n, -1/2*n+1/2, -1/2*n-3/2], [-n+1, -1/2*n-1/2], -a/x^2)+1/4*a^(3/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n-2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/x^2+1)^(1/2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a) \left(x + \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)*(x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + a) \left(x + \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)

[Out] int((a + x^2)*(x + (a + x^2)^(1/2))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+a)*(x+(x**2+a)**(1/2))**n, x)

[Out] Timed out

$$3.262 \quad \int \left(x + \sqrt{a + x^2}\right)^n dx$$

Optimal. Leaf size=52

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)} - \frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Rubi [A] time = 0.02, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2117, 14}

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n+1}}{2(n+1)} - \frac{a\left(\sqrt{a+x^2}+x\right)^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^n, x]

[Out] -(a*(x + Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x + Sqrt[a + x^2])^(1 + n)/(2*(1 + n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2117

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \left(x + \sqrt{a + x^2}\right)^n dx &= \frac{1}{2} \text{Subst} \left(\int x^{-2+n} (a + x^2) dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (ax^{-2+n} + x^n) dx, x, x + \sqrt{a + x^2} \right) \\ &= -\frac{a\left(x + \sqrt{a + x^2}\right)^{-1+n}}{2(1-n)} + \frac{\left(x + \sqrt{a + x^2}\right)^{1+n}}{2(1+n)} \end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.83

$$\frac{\left(\sqrt{a+x^2}+x\right)^{n-1} \left((n-1)x\left(\sqrt{a+x^2}+x\right)+an\right)}{n^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^n, x]

[Out] ((x + Sqrt[a + x^2])^(-1 + n)*(a*n + (-1 + n)*x*(x + Sqrt[a + x^2])))/(-1 + n^2)

IntegrateAlgebraic [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \left(x + \sqrt{a + x^2}\right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x + Sqrt[a + x^2])^n, x]

[Out] Defer[IntegrateAlgebraic][(x + Sqrt[a + x^2])^n, x]

fricas [A] time = 0.53, size = 32, normalized size = 0.62

$$\frac{\left(\sqrt{x^2 + a}n - x\right)\left(x + \sqrt{x^2 + a}\right)^n}{n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] (sqrt(x^2 + a)*n - x)*(x + sqrt(x^2 + a))^n/(n^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n, x)

maple [B] time = 0.02, size = 120, normalized size = 2.31

$$\frac{\left(\frac{8\sqrt{\pi}\left(n+\frac{an}{x^2}-1\right)a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}\right)^{n-1}}{(n+1)(2n-2)n} + \frac{4\sqrt{\pi}\sqrt{\frac{a}{x^2}+1}a^{-\frac{n}{2}-\frac{1}{2}}x^{n+1}\left(\sqrt{\frac{a}{x^2}+1}\right)^{n-1}}{(n+1)n}\right)na^{\frac{n}{2}+\frac{1}{2}}}{4\sqrt{\pi}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^n,x)

[Out] 1/4*a^(1/2+1/2*n)/Pi^(1/2)*n*(8*Pi^(1/2)/(n+1)*(n+a*n/x^2-1)/(2*n-2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1)+4*Pi^(1/2)/(n+1)*(a/x^2+1)^(1/2)/n*a^(-1/2*n-1/2)*x^(n+1)*((a/x^2+1)^(1/2)+1)^(n-1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(x + \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x + (a + x^2)^(1/2))^n, x)
```

```
[Out] int((x + (a + x^2)^(1/2))^n, x)
```

sympy [B] time = 2.75, size = 2147, normalized size = 41.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(x**2+a)**(1/2))**n,x)
```

```
[Out] Piecewise((-a**(9/2)*a**(n/2)*n**2*x*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))
)*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2)
+ 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) +
a**(9/2)*a**(n/2)*n*x*cosh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2
*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1
- n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - a**(7/2)*a**(n/2)*n**2*x**3*sqrt
(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + a**(7/2)*a**(n/2)*n*x**3*cosh(n*asinh(x/s
qrt(a)))*gamma(-n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 -
n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2
)) + 2*a**5*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1
- n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(
7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n/2)) - 2*a**5*a*
*(n/2)*n*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(
1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 -
n/2)) - 2*a**4*a**(n/2)*n*x**2*sqrt(a/x**2 + 1)*sinh(n*asinh(x/sqrt(a)) + a
sinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2
)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*ga
mma(1 - n/2)) + 4*a**4*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sq
rt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9/2)*gamma(1
- n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*gamma(1 - n
/2)) - 2*a**4*a**(n/2)*n*x**2*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) - 2*a**4*a**(n/2)*x**2*sqrt(a/x**2 + 1)*sinh(n*a
sinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1
- n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) -
2*a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**4*a**(n/2)*x**2*cosh(n*asinh(x/sqrt
(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2
*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2
)*x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*n*x**4*sqrt(a/x**2 + 1)*sinh(n*asin
h(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 -
n/2) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*
a**(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*x**4*cosh(n*asinh(x/sqrt(
a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*
a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*
x**2*gamma(1 - n/2)) - 2*a**3*a**(n/2)*x**4*sqrt(a/x**2 + 1)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2
) - 2*a**(9/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**
(7/2)*x**2*gamma(1 - n/2)) + 2*a**3*a**(n/2)*x**4*cosh(n*asinh(x/sqrt(a)) +
asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(9/2)*n**2*gamma(1 - n/2) - 2*a**(9
/2)*gamma(1 - n/2) + 2*a**(7/2)*n**2*x**2*gamma(1 - n/2) - 2*a**(7/2)*x**2*
gamma(1 - n/2)), Abs(x**2/a) > 1), (-2*a**(5/2)*a**(n/2)*n*x*sqrt(1 + x**2/a)
*sinh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n
**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)) + a**(5/2)*a**(n/2)*n*x*cos
h(n*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5
/2)*gamma(1 - n/2)) - 2*a**(5/2)*a**(n/2)*x*sqrt(1 + x**2/a)*sinh(n*asinh(x
/sqrt(a)) + asinh(x/sqrt(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2)
```

```

) - 2*a**(5/2)*gamma(1 - n/2)) - a**3*a**(n/2)*n**2*sqrt(1 + x**2/a)*sinh(n
*asinh(x/sqrt(a)))*gamma(-n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)
*gamma(1 - n/2)) + 2*a**3*a**(n/2)*n*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt
(a)))*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 -
n/2)) + 2*a**2*a**(n/2)*n*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))
*gamma(1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)
) + 2*a**2*a**(n/2)*x**2*cosh(n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))*gamma(
1 - n/2)/(2*a**(5/2)*n**2*gamma(1 - n/2) - 2*a**(5/2)*gamma(1 - n/2)), True
))

```

$$3.263 \quad \int (a + x^2)^2 (x - \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=176

$$\frac{a^5 (x - \sqrt{a + x^2})^{n-5}}{32(5-n)} - \frac{5a^4 (x - \sqrt{a + x^2})^{n-3}}{32(3-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^{n-1}}{16(1-n)} + \frac{5a^2 (x - \sqrt{a + x^2})^{n+1}}{16(n+1)} + \frac{5a (x - \sqrt{a + x^2})^{n+3}}{32(n+3)}$$

Rubi [A] time = 0.11, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 270}

$$\frac{a^5 (x - \sqrt{a + x^2})^{n-5}}{32(5-n)} - \frac{5a^4 (x - \sqrt{a + x^2})^{n-3}}{32(3-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^{n-1}}{16(1-n)} + \frac{5a^2 (x - \sqrt{a + x^2})^{n+1}}{16(n+1)} + \frac{5a (x - \sqrt{a + x^2})^{n+3}}{32(n+3)} + \frac{(x - \sqrt{a + x^2})^{n+5}}{32(n+5)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]

[Out] -(a^5*(x - Sqrt[a + x^2])^(-5 + n))/(32*(5 - n)) - (5*a^4*(x - Sqrt[a + x^2])^(-3 + n))/(32*(3 - n)) - (5*a^3*(x - Sqrt[a + x^2])^(-1 + n))/(16*(1 - n)) + (5*a^2*(x - Sqrt[a + x^2])^(1 + n))/(16*(1 + n)) + (5*a*(x - Sqrt[a + x^2])^(3 + n))/(32*(3 + n)) + (x - Sqrt[a + x^2])^(5 + n)/(32*(5 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^2 (x - \sqrt{a + x^2})^n dx &= \frac{1}{32} \text{Subst} \left(\int x^{-6+n} (a + x^2)^5 dx, x, x - \sqrt{a + x^2} \right) \\ &= \frac{1}{32} \text{Subst} \left(\int (a^5 x^{-6+n} + 5a^4 x^{-4+n} + 10a^3 x^{-2+n} + 10a^2 x^n + 5ax^{2+n} + x^{4+n}) dx, x, x - \sqrt{a + x^2} \right) \\ &= -\frac{a^5 (x - \sqrt{a + x^2})^{-5+n}}{32(5-n)} - \frac{5a^4 (x - \sqrt{a + x^2})^{-3+n}}{32(3-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^{-1+n}}{16(1-n)} + \end{aligned}$$

Mathematica [A] time = 0.39, size = 150, normalized size = 0.85

$$\frac{1}{32} (x - \sqrt{a + x^2})^{n-5} \left(\frac{a^5}{n-5} + \frac{5a^4 (x - \sqrt{a + x^2})^2}{n-3} + \frac{10a^3 (x - \sqrt{a + x^2})^4}{n-1} + \frac{10a^2 (x - \sqrt{a + x^2})^6}{n+1} + \frac{(x - \sqrt{a + x^2})^{10}}{n+5} + \frac{5a (x - \sqrt{a + x^2})^8}{n+3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]


```
[Out] ((x - Sqrt[a + x^2])^(-5 + n)*(a^5/(-5 + n) + (5*a^4*(x - Sqrt[a + x^2])^2)/(-3 + n) + (10*a^3*(x - Sqrt[a + x^2])^4)/(-1 + n) + (10*a^2*(x - Sqrt[a + x^2])^6)/(1 + n) + (5*a*(x - Sqrt[a + x^2])^8)/(3 + n) + (x - Sqrt[a + x^2])^10/(5 + n))/32
```

IntegrateAlgebraic [F] time = 0.36, size = 0, normalized size = 0.00

$$\int (a + x^2)^2 (x - \sqrt{a + x^2})^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + x^2)^2*(x - Sqrt[a + x^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + x^2)^2*(x - Sqrt[a + x^2])^n, x]
```

fricas [A] time = 0.53, size = 159, normalized size = 0.90

$$\frac{(5(n^4 - 10n^2 + 9)x^5 + 10(an^4 - 16an^2 + 15a)x^3 + 5(a^2n^4 - 22a^2n^2 + 45a^2)x + (a^2n^5 - 30a^2n^3 + (n^5 - 10n^3 + 9n)x^4 + 149a^2n + 2(an^5 - 20an^3 + 19an)x^2)\sqrt{x^2 + a})(x - \sqrt{x^2 + a})^n}{n^6 - 35n^4 + 259n^2 - 225}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")
```

```
[Out] -(5*(n^4 - 10*n^2 + 9)*x^5 + 10*(a*n^4 - 16*a*n^2 + 15*a)*x^3 + 5*(a^2*n^4 - 22*a^2*n^2 + 45*a^2)*x + (a^2*n^5 - 30*a^2*n^3 + (n^5 - 10*n^3 + 9*n)*x^4 + 149*a^2*n + 2*(a*n^5 - 20*a*n^3 + 19*a*n)*x^2)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^6 - 35*n^4 + 259*n^2 - 225)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^2 (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (x^2 + a)^2 (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

```
[Out] int((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^2 (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+a)^2*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((x^2 + a)^2*(x - sqrt(x^2 + a))^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x - \sqrt{x^2 + a})^n (x^2 + a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)`

[Out] `int((x - (a + x^2)^(1/2))^n*(a + x^2)^2, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2)^2 (x - \sqrt{a + x^2})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+a)**2*(x-(x**2+a)**(1/2))**n, x)`

[Out] `Integral((a + x**2)**2*(x - sqrt(a + x**2))**n, x)`

$$3.264 \quad \int (a + x^2) (x - \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=116

$$-\frac{a^3 (x - \sqrt{a + x^2})^{n-3}}{8(3-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^{n-1}}{8(1-n)} + \frac{3a (x - \sqrt{a + x^2})^{n+1}}{8(n+1)} + \frac{(x - \sqrt{a + x^2})^{n+3}}{8(n+3)}$$

Rubi [A] time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2122, 270}

$$-\frac{a^3 (x - \sqrt{a + x^2})^{n-3}}{8(3-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^{n-1}}{8(1-n)} + \frac{3a (x - \sqrt{a + x^2})^{n+1}}{8(n+1)} + \frac{(x - \sqrt{a + x^2})^{n+3}}{8(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)*(x - Sqrt[a + x^2])^n,x]

[Out] -(a^3*(x - Sqrt[a + x^2])^(-3 + n))/(8*(3 - n)) - (3*a^2*(x - Sqrt[a + x^2])^(-1 + n))/(8*(1 - n)) + (3*a*(x - Sqrt[a + x^2])^(1 + n))/(8*(1 + n)) + (x - Sqrt[a + x^2])^(3 + n)/(8*(3 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2) (x - \sqrt{a + x^2})^n dx &= \frac{1}{8} \text{Subst} \left(\int x^{-4+n} (a + x^2)^3 dx, x, x - \sqrt{a + x^2} \right) \\ &= \frac{1}{8} \text{Subst} \left(\int (a^3 x^{-4+n} + 3a^2 x^{-2+n} + 3ax^n + x^{2+n}) dx, x, x - \sqrt{a + x^2} \right) \\ &= -\frac{a^3 (x - \sqrt{a + x^2})^{-3+n}}{8(3-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^{-1+n}}{8(1-n)} + \frac{3a (x - \sqrt{a + x^2})^{1+n}}{8(1+n)} + \end{aligned}$$

Mathematica [A] time = 0.13, size = 100, normalized size = 0.86

$$\frac{1}{8} (x - \sqrt{a + x^2})^{n-3} \left(\frac{a^3}{n-3} + \frac{3a^2 (x - \sqrt{a + x^2})^2}{n-1} + \frac{(x - \sqrt{a + x^2})^6}{n+3} + \frac{3a (x - \sqrt{a + x^2})^4}{n+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)*(x - Sqrt[a + x^2])^n, x]

[Out] ((x - Sqrt[a + x^2])^(-3 + n)*(a^3/(-3 + n) + (3*a^2*(x - Sqrt[a + x^2])^2)/(-1 + n) + (3*a*(x - Sqrt[a + x^2])^4)/(1 + n) + (x - Sqrt[a + x^2])^6/(3 + n)))/8

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (a + x^2) \left(x - \sqrt{a + x^2} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)*(x - Sqrt[a + x^2])^n, x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)*(x - Sqrt[a + x^2])^n, x]

fricas [A] time = 0.68, size = 79, normalized size = 0.68

$$\frac{\left(3(n^2 - 1)x^3 + 3(an^2 - 3a)x + (an^3 + (n^3 - n)x^2 - 7an)\sqrt{x^2 + a} \right) \left(x - \sqrt{x^2 + a} \right)^n}{n^4 - 10n^2 + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(3*(n^2 - 1)*x^3 + 3*(a*n^2 - 3*a)*x + (a*n^3 + (n^3 - n)*x^2 - 7*a*n)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^4 - 10*n^2 + 9)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a) \left(x - \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (x^2 + a) \left(x - \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)*(x-(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a) \left(x - \sqrt{x^2 + a} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)*(x - sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x - \sqrt{x^2 + a} \right)^n (x^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2) \left(x - \sqrt{a + x^2} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)*(x-(x**2+a)**(1/2))**n, x)
```

```
[Out] Integral((a + x**2)*(x - sqrt(a + x**2))**n, x)
```

$$3.265 \quad \int (x - \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=56

$$\frac{(x - \sqrt{a + x^2})^{n+1}}{2(n+1)} - \frac{a(x - \sqrt{a + x^2})^{n-1}}{2(1-n)}$$

Rubi [A] time = 0.02, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2117, 14}

$$\frac{(x - \sqrt{a + x^2})^{n+1}}{2(n+1)} - \frac{a(x - \sqrt{a + x^2})^{n-1}}{2(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^n, x]

[Out] -(a*(x - Sqrt[a + x^2])^(-1 + n))/(2*(1 - n)) + (x - Sqrt[a + x^2])^(1 + n)/(2*(1 + n))

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int (x - \sqrt{a + x^2})^n dx &= \frac{1}{2} \text{Subst} \left(\int x^{-2+n} (a + x^2) dx, x, x - \sqrt{a + x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left(\int (ax^{-2+n} + x^n) dx, x, x - \sqrt{a + x^2} \right) \\ &= -\frac{a(x - \sqrt{a + x^2})^{-1+n}}{2(1-n)} + \frac{(x - \sqrt{a + x^2})^{1+n}}{2(1+n)} \end{aligned}$$

Mathematica [A] time = 0.06, size = 50, normalized size = 0.89

$$\frac{1}{2} (x - \sqrt{a + x^2})^{n-1} \left(\frac{(x - \sqrt{a + x^2})^2}{n+1} + \frac{a}{n-1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^n, x]

[Out] $((x - \sqrt{a + x^2})^{-1 + n} * (a / (-1 + n) + (x - \sqrt{a + x^2})^2 / (1 + n))) / 2$

IntegrateAlgebraic [F] time = 0.20, size = 0, normalized size = 0.00

$$\int (x - \sqrt{a + x^2})^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x - Sqrt[a + x^2])^n, x]

[Out] Defer[IntegrateAlgebraic][(x - Sqrt[a + x^2])^n, x]

fricas [A] time = 0.58, size = 33, normalized size = 0.59

$$-\frac{(\sqrt{x^2 + a}n + x)(x - \sqrt{x^2 + a})^n}{n^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(sqrt(x^2 + a)*n + x)*(x - sqrt(x^2 + a))^n/(n^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^n, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^n,x)

[Out] int((x-(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n, x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(x - \sqrt{a + x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((x - sqrt(a + x**2))**n, x)
```


$$3.266 \quad \int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=187

$$\frac{a^6 (\sqrt{a+x^2} + x)^{n-6}}{64(6-n)} - \frac{3a^5 (\sqrt{a+x^2} + x)^{n-4}}{32(4-n)} - \frac{15a^4 (\sqrt{a+x^2} + x)^{n-2}}{64(2-n)} + \frac{5a^3 (\sqrt{a+x^2} + x)^n}{16n} + \frac{15a^2 (\sqrt{a+x^2} + x)^{n+2}}{64(n+2)}$$

Rubi [A] time = 0.12, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, number of rules / integrand size = 0.087, Rules used = {2122, 270}

$$\frac{a^6 (\sqrt{a+x^2} + x)^{n-6}}{64(6-n)} - \frac{3a^5 (\sqrt{a+x^2} + x)^{n-4}}{32(4-n)} - \frac{15a^4 (\sqrt{a+x^2} + x)^{n-2}}{64(2-n)} + \frac{5a^3 (\sqrt{a+x^2} + x)^n}{16n} + \frac{15a^2 (\sqrt{a+x^2} + x)^{n+2}}{64(n+2)} + \frac{3a (\sqrt{a+x^2} + x)^{n+4}}{32(n+4)} + \frac{(\sqrt{a+x^2} + x)^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]

[Out] -(a^6*(x + Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) - (3*a^5*(x + Sqrt[a + x^2])^(-4 + n))/(32*(4 - n)) - (15*a^4*(x + Sqrt[a + x^2])^(-2 + n))/(64*(2 - n)) + (5*a^3*(x + Sqrt[a + x^2])^n)/(16*n) + (15*a^2*(x + Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) + (3*a*(x + Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) + (x + Sqrt[a + x^2])^(6 + n)/(64*(6 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx &= \frac{1}{64} \text{Subst} \left(\int x^{-7+n} (a + x^2)^6 dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{64} \text{Subst} \left(\int (a^6 x^{-7+n} + 6a^5 x^{-5+n} + 15a^4 x^{-3+n} + 20a^3 x^{-1+n} + 15a^2 x^{1+n} + 6a x^{3+n} + x^{5+n}) dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{a^6 (x + \sqrt{a + x^2})^{-6+n}}{64(6-n)} - \frac{3a^5 (x + \sqrt{a + x^2})^{-4+n}}{32(4-n)} - \frac{15a^4 (x + \sqrt{a + x^2})^{-2+n}}{64(2-n)} + \frac{5a^3 (x + \sqrt{a + x^2})^n}{16n} + \frac{15a^2 (x + \sqrt{a + x^2})^{n+2}}{64(n+2)} + \frac{3a (x + \sqrt{a + x^2})^{n+4}}{32(n+4)} + \frac{(x + \sqrt{a + x^2})^{n+6}}{64(n+6)} \end{aligned}$$

Mathematica [A] time = 0.38, size = 157, normalized size = 0.84

$$\frac{1}{64} (\sqrt{a+x^2} + x)^n \left(\frac{a^6}{(n-6)(\sqrt{a+x^2} + x)^6} + \frac{6a^5}{(n-4)(\sqrt{a+x^2} + x)^4} + \frac{15a^4}{(n-2)(\sqrt{a+x^2} + x)^2} + \frac{20a^3}{n} + \frac{15a^2(\sqrt{a+x^2} + x)^2}{n+2} + \frac{6a(\sqrt{a+x^2} + x)^4}{n+4} + \frac{(\sqrt{a+x^2} + x)^6}{n+6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]

[Out] ((x + Sqrt[a + x^2])^n*((20*a^3)/n + a^6/((-6 + n)*(x + Sqrt[a + x^2])^6) + (6*a^5)/((-4 + n)*(x + Sqrt[a + x^2])^4) + (15*a^4)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (15*a^2*(x + Sqrt[a + x^2])^2)/(2 + n) + (6*a*(x + Sqrt[a + x^2])^4)/(4 + n) + (x + Sqrt[a + x^2])^6/(6 + n))/64

IntegrateAlgebraic [F] time = 0.26, size = 0, normalized size = 0.00

$$\int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)^(5/2)*(x + Sqrt[a + x^2])^n, x]

fricas [A] time = 0.75, size = 201, normalized size = 1.07

$$\frac{(a^3 n^6 - 50 a^3 n^4 + (n^6 - 20 n^4 + 64 n^2) x^6 + 544 a^3 n^2 + 3(a n^6 - 30 a n^4 + 104 a n^2) x^4 - 720 a^3 + 3(a^2 n^6 - 40 a^2 n^4 + 264 a^2 n^2) x^2 - 6((n^5 - 20 n^3 + 64 n) x^5 + 2(a n^5 - 30 a n^3 + 104 a n) x^3 + (a^2 n^5 - 40 a^2 n^3 + 264 a^2 n) x) \sqrt{x^2 + a})(x + \sqrt{x^2 + a})^n}{n^7 - 56 n^5 + 784 n^3 - 2304 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] (a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2*n^2)*x^2 - 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)^(5/2)*(x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + a)^{5/2} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n,x)

[Out] int((a + x^2)^(5/2)*(x + (a + x^2)^(1/2))^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2)^{5/2} (x + \sqrt{a + x^2})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+a)**(5/2)*(x+(x**2+a)**(1/2))**n,x)

[Out] Integral((a + x**2)**(5/2)*(x + sqrt(a + x**2))**n, x)

$$3.267 \quad \int (a + x^2)^{3/2} (x + \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=131

$$\frac{a^4 (\sqrt{a+x^2} + x)^{n-4}}{16(4-n)} - \frac{a^3 (\sqrt{a+x^2} + x)^{n-2}}{4(2-n)} + \frac{3a^2 (\sqrt{a+x^2} + x)^n}{8n} + \frac{a (\sqrt{a+x^2} + x)^{n+2}}{4(n+2)} + \frac{(\sqrt{a+x^2} + x)^{n+4}}{16(n+4)}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 270}

$$\frac{a^4 (\sqrt{a+x^2} + x)^{n-4}}{16(4-n)} - \frac{a^3 (\sqrt{a+x^2} + x)^{n-2}}{4(2-n)} + \frac{3a^2 (\sqrt{a+x^2} + x)^n}{8n} + \frac{a (\sqrt{a+x^2} + x)^{n+2}}{4(n+2)} + \frac{(\sqrt{a+x^2} + x)^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]

[Out] -(a^4*(x + Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) - (a^3*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (3*a^2*(x + Sqrt[a + x^2])^n)/(8*n) + (a*(x + Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) + (x + Sqrt[a + x^2])^(4 + n)/(16*(4 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^{3/2} (x + \sqrt{a + x^2})^n dx &= \frac{1}{16} \text{Subst} \left(\int x^{-5+n} (a + x^2)^4 dx, x, x + \sqrt{a + x^2} \right) \\ &= \frac{1}{16} \text{Subst} \left(\int (a^4 x^{-5+n} + 4a^3 x^{-3+n} + 6a^2 x^{-1+n} + 4ax^{1+n} + x^{3+n}) dx, x, x + \sqrt{a + x^2} \right) \\ &= -\frac{a^4 (x + \sqrt{a + x^2})^{-4+n}}{16(4-n)} - \frac{a^3 (x + \sqrt{a + x^2})^{-2+n}}{4(2-n)} + \frac{3a^2 (x + \sqrt{a + x^2})^n}{8n} + \frac{a (x + \sqrt{a + x^2})^{n+2}}{4(n+2)} + \frac{(x + \sqrt{a + x^2})^{n+4}}{16(n+4)} \end{aligned}$$

Mathematica [A] time = 0.23, size = 111, normalized size = 0.85

$$\frac{1}{16} (\sqrt{a+x^2} + x)^n \left(\frac{a^4}{(n-4)(\sqrt{a+x^2} + x)^4} + \frac{4a^3}{(n-2)(\sqrt{a+x^2} + x)^2} + \frac{6a^2}{n} + \frac{4a(\sqrt{a+x^2} + x)^2}{n+2} + \frac{(\sqrt{a+x^2} + x)^4}{n+4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]

[Out] ((x + Sqrt[a + x^2])^n*((6*a^2)/n + a^4/((-4 + n)*(x + Sqrt[a + x^2])^4) + (4*a^3)/((-2 + n)*(x + Sqrt[a + x^2])^2) + (4*a*(x + Sqrt[a + x^2])^2)/(2 + n) + (x + Sqrt[a + x^2])^4/(4 + n))/16

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int (a + x^2)^{3/2} (x + \sqrt{a + x^2})^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)^(3/2)*(x + Sqrt[a + x^2])^n, x]

fricas [A] time = 0.54, size = 110, normalized size = 0.84

$$\frac{(a^2 n^4 + (n^4 - 4n^2)x^4 - 16a^2 n^2 + 2(an^4 - 10an^2)x^2 + 24a^2 - 4((n^3 - 4n)x^3 + (an^3 - 10an)x)\sqrt{x^2 + a})(x + \sqrt{x^2 + a})^n}{n^5 - 20n^3 + 64n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] (a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a^2 - 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x + sqrt(x^2 + a))^n/(n^5 - 20*n^3 + 64*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)^(3/2)*(x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x^2 + a)^{3/2} (x + \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)`

[Out] `int((a + x^2)^(3/2)*(x + (a + x^2)^(1/2))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2)^{\frac{3}{2}} \left(x + \sqrt{a + x^2} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+a)**(3/2)*(x+(x**2+a)**(1/2))**n, x)`

[Out] `Integral((a + x**2)**(3/2)*(x + sqrt(a + x**2))**n, x)`

$$3.268 \quad \int \sqrt{a+x^2} (x + \sqrt{a+x^2})^n dx$$

Optimal. Leaf size=75

$$-\frac{a^2 (\sqrt{a+x^2} + x)^{n-2}}{4(2-n)} + \frac{a (\sqrt{a+x^2} + x)^n}{2n} + \frac{(\sqrt{a+x^2} + x)^{n+2}}{4(n+2)}$$

Rubi [A] time = 0.08, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 270}

$$-\frac{a^2 (\sqrt{a+x^2} + x)^{n-2}}{4(2-n)} + \frac{a (\sqrt{a+x^2} + x)^n}{2n} + \frac{(\sqrt{a+x^2} + x)^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]

[Out] -(a^2*(x + Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) + (a*(x + Sqrt[a + x^2])^n)/(2*n) + (x + Sqrt[a + x^2])^(2 + n)/(4*(2 + n))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_.), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{a+x^2} (x + \sqrt{a+x^2})^n dx &= \frac{1}{4} \text{Subst} \left(\int x^{-3+n} (a+x^2)^2 dx, x, x + \sqrt{a+x^2} \right) \\ &= \frac{1}{4} \text{Subst} \left(\int (a^2 x^{-3+n} + 2ax^{-1+n} + x^{1+n}) dx, x, x + \sqrt{a+x^2} \right) \\ &= -\frac{a^2 (x + \sqrt{a+x^2})^{-2+n}}{4(2-n)} + \frac{a (x + \sqrt{a+x^2})^n}{2n} + \frac{(x + \sqrt{a+x^2})^{2+n}}{4(2+n)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 65, normalized size = 0.87

$$\frac{1}{4} (\sqrt{a+x^2} + x)^n \left(\frac{a^2}{(n-2)(\sqrt{a+x^2} + x)^2} + \frac{(\sqrt{a+x^2} + x)^2}{n+2} + \frac{2a}{n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]

[Out] ((x + Sqrt[a + x^2])^n*((2*a)/n + a^2/((-2 + n)*(x + Sqrt[a + x^2])^2) + (x + Sqrt[a + x^2])^2/(2 + n)))/4

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2} \left(x + \sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a + x^2]*(x + Sqrt[a + x^2])^n, x]

fricas [A] time = 0.86, size = 48, normalized size = 0.64

$$\frac{\left(n^2x^2 + an^2 - 2\sqrt{x^2 + a}nx - 2a\right)\left(x + \sqrt{x^2 + a}\right)^n}{n^3 - 4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] (n^2*x^2 + a*n^2 - 2*sqrt(x^2 + a)*n*x - 2*a)*(x + sqrt(x^2 + a))^n/(n^3 - 4*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x + \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x + \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x + \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x+(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + a)*(x + sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^2+a} \left(x + \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)`

[Out] `int((a + x^2)^(1/2)*(x + (a + x^2)^(1/2))^n, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2} \left(x + \sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+a)**(1/2)*(x+(x**2+a)**(1/2))**n, x)`

[Out] `Integral(sqrt(a + x**2)*(x + sqrt(a + x**2))**n, x)`

$$3.269 \quad \int \frac{(x + \sqrt{a+x^2})^n}{\sqrt{a+x^2}} dx$$

Optimal. Leaf size=17

$$\frac{(\sqrt{a+x^2} + x)^n}{n}$$

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {2122, 30}

$$\frac{(\sqrt{a+x^2} + x)^n}{n}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2], x]

[Out] (x + Sqrt[a + x^2])^n/n

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(x + \sqrt{a+x^2})^n}{\sqrt{a+x^2}} dx &= \text{Subst} \left(\int x^{-1+n} dx, x, x + \sqrt{a+x^2} \right) \\ &= \frac{(x + \sqrt{a+x^2})^n}{n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 1.00

$$\frac{(\sqrt{a+x^2} + x)^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2], x]

[Out] (x + Sqrt[a + x^2])^n/n

IntegrateAlgebraic [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{\left(\sqrt{a+x^2}+x\right)^n}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] (x + Sqrt[a + x^2])^n/n

fricas [A] time = 0.66, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + a))^n/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)

maple [F] time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)

[Out] int((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x+\sqrt{x^2+a}\right)^n}{\sqrt{x^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 + a))^n/sqrt(x^2 + a), x)

mupad [B] time = 3.00, size = 15, normalized size = 0.88

$$\frac{\left(x+\sqrt{x^2+a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + (a + x^2)^(1/2))^n/(a + x^2)^(1/2), x)`

[Out] `(x + (a + x^2)^(1/2))^n/n`

sympy [B] time = 2.64, size = 311, normalized size = 18.29

$$\left\{ \begin{array}{l} \frac{\sqrt{a} a^{\frac{n}{2}} \sinh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n x \sqrt{\frac{a}{x^2} + 1}} - \frac{2 a^{\frac{n}{2}} \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \Gamma\left(1 - \frac{n}{2}\right)}{n^2 \Gamma\left(-\frac{n}{2}\right)} + \frac{a^{\frac{n}{2}} x \cosh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} n} - \frac{a^{\frac{n}{2}} x \sinh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} n \sqrt{\frac{a}{x^2} + 1}} \quad \text{for } \left|\frac{x^2}{a}\right| > 1 \\ \frac{a^{\frac{n}{2}} \sinh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{n \sqrt{1 + \frac{x^2}{a}}} - \frac{2 a^{\frac{n}{2}} \cosh\left(n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right) \Gamma\left(1 - \frac{n}{2}\right)}{n^2 \Gamma\left(-\frac{n}{2}\right)} - \frac{a^{\frac{n}{2}} x^2 \sinh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{a n \sqrt{1 + \frac{x^2}{a}}} + \frac{a^{\frac{n}{2}} x \cosh\left(-n \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right) + \operatorname{asinh}\left(\frac{x}{\sqrt{a}}\right)\right)}{\sqrt{a} n} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(x**2+a)**(1/2))**n/(x**2+a)**(1/2), x)`

[Out] `Piecewise((-sqrt(a)*a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(n*x*sqrt(a/x**2 + 1)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) + a**(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n) - a**(n/2)*x*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n*sqrt(a/x**2 + 1)), Abs(x**2/a) > 1), (-a**(n/2)*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(n*sqrt(1 + x**2/a)) - 2*a**(n/2)*cosh(n*asinh(x/sqrt(a)))*gamma(1 - n/2)/(n**2*gamma(-n/2)) - a**(n/2)*x**2*sinh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(a*n*sqrt(1 + x**2/a)) + a**(n/2)*x*cosh(-n*asinh(x/sqrt(a)) + asinh(x/sqrt(a)))/(sqrt(a)*n), True)`

$$3.270 \quad \int (a + x^2)^{5/2} (x - \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=201

$$\frac{a^6 (x - \sqrt{a + x^2})^{n-6}}{64(6-n)} + \frac{3a^5 (x - \sqrt{a + x^2})^{n-4}}{32(4-n)} + \frac{15a^4 (x - \sqrt{a + x^2})^{n-2}}{64(2-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^n}{16n} - \frac{15a^2 (x - \sqrt{a + x^2})^{n+2}}{64(n+2)}$$

Rubi [A] time = 0.11, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 270}

$$\frac{a^6 (x - \sqrt{a + x^2})^{n-6}}{64(6-n)} + \frac{3a^5 (x - \sqrt{a + x^2})^{n-4}}{32(4-n)} + \frac{15a^4 (x - \sqrt{a + x^2})^{n-2}}{64(2-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^n}{16n} - \frac{15a^2 (x - \sqrt{a + x^2})^{n+2}}{64(n+2)} - \frac{3a (x - \sqrt{a + x^2})^{n+4}}{32(n+4)} - \frac{(x - \sqrt{a + x^2})^{n+6}}{64(n+6)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]

[Out] (a^6*(x - Sqrt[a + x^2])^(-6 + n))/(64*(6 - n)) + (3*a^5*(x - Sqrt[a + x^2])^(-4 + n))/(32*(4 - n)) + (15*a^4*(x - Sqrt[a + x^2])^(-2 + n))/(64*(2 - n)) - (5*a^3*(x - Sqrt[a + x^2])^n)/(16*n) - (15*a^2*(x - Sqrt[a + x^2])^(2 + n))/(64*(2 + n)) - (3*a*(x - Sqrt[a + x^2])^(4 + n))/(32*(4 + n)) - (x - Sqrt[a + x^2])^(6 + n)/(64*(6 + n))

Rule 270

Int[((c_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2])^(n_.), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^{5/2} (x - \sqrt{a + x^2})^n dx &= -\left(\frac{1}{64} \text{Subst}\left(\int x^{-7+n} (a + x^2)^6 dx, x, x - \sqrt{a + x^2}\right)\right) \\ &= -\left(\frac{1}{64} \text{Subst}\left(\int (a^6 x^{-7+n} + 6a^5 x^{-5+n} + 15a^4 x^{-3+n} + 20a^3 x^{-1+n} + 15a^2 x^{1+n} + 6a x^{3+n} + x^{5+n}) dx, x, x - \sqrt{a + x^2}\right)\right) \\ &= \frac{a^6 (x - \sqrt{a + x^2})^{-6+n}}{64(6-n)} + \frac{3a^5 (x - \sqrt{a + x^2})^{-4+n}}{32(4-n)} + \frac{15a^4 (x - \sqrt{a + x^2})^{-2+n}}{64(2-n)} - \frac{5a^3 (x - \sqrt{a + x^2})^n}{16n} - \frac{15a^2 (x - \sqrt{a + x^2})^{n+2}}{64(n+2)} - \frac{3a (x - \sqrt{a + x^2})^{n+4}}{32(n+4)} - \frac{(x - \sqrt{a + x^2})^{n+6}}{64(n+6)} \end{aligned}$$

Mathematica [A] time = 0.49, size = 173, normalized size = 0.86

$$\frac{1}{64} (x - \sqrt{a + x^2})^n \left(-\frac{a^6}{(n-6)(x - \sqrt{a + x^2})^6} - \frac{6a^5}{(n-4)(x - \sqrt{a + x^2})^4} - \frac{15a^4}{(n-2)(x - \sqrt{a + x^2})^2} - \frac{20a^3}{n} - \frac{15a^2 (x - \sqrt{a + x^2})^2}{n+2} - \frac{6a (x - \sqrt{a + x^2})^4}{n+4} - \frac{(x - \sqrt{a + x^2})^6}{n+6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]

[Out] ((x - Sqrt[a + x^2])^n*((-20*a^3)/n - a^6/((-6 + n)*(x - Sqrt[a + x^2]))^6) - (6*a^5)/((-4 + n)*(x - Sqrt[a + x^2])^4) - (15*a^4)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (15*a^2*(x - Sqrt[a + x^2])^2)/(2 + n) - (6*a*(x - Sqrt[a + x^2])^4)/(4 + n) - (x - Sqrt[a + x^2])^6/(6 + n))/64

IntegrateAlgebraic [F] time = 0.29, size = 0, normalized size = 0.00

$$\int (a + x^2)^{5/2} \left(x - \sqrt{a + x^2}\right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)^(5/2)*(x - Sqrt[a + x^2])^n, x]

fricas [A] time = 0.82, size = 204, normalized size = 1.01

$$\frac{(a^3n^6 - 50a^3n^4 + (n^6 - 20n^4 + 64n^2)x^6 + 544a^3n^2 + 3(a^2n^6 - 30a^2n^4 + 104an^2)x^4 - 720a^3 + 3(a^2n^6 - 40a^2n^4 + 264a^2n^2)x^2 + 6((n^5 - 20n^3 + 64n)x^5 + 2(an^5 - 30an^3 + 104an)x^3 + (a^2n^5 - 40a^2n^3 + 264a^2n)x)\sqrt{x^2 + a})(x - \sqrt{x^2 + a})^n}{n^7 - 56n^5 + 784n^3 - 2304n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(a^3*n^6 - 50*a^3*n^4 + (n^6 - 20*n^4 + 64*n^2)*x^6 + 544*a^3*n^2 + 3*(a*n^6 - 30*a*n^4 + 104*a*n^2)*x^4 - 720*a^3 + 3*(a^2*n^6 - 40*a^2*n^4 + 264*a^2*n^2)*x^2 + 6*((n^5 - 20*n^3 + 64*n)*x^5 + 2*(a*n^5 - 30*a*n^3 + 104*a*n)*x^3 + (a^2*n^5 - 40*a^2*n^3 + 264*a^2*n)*x)*sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n/(n^7 - 56*n^5 + 784*n^3 - 2304*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{5}{2}} \left(x - \sqrt{x^2 + a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(5/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)^(5/2)*(x - sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(x - \sqrt{x^2 + a}\right)^n (x^2 + a)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)

[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(5/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2)^{\frac{5}{2}} \left(x - \sqrt{a + x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+a)**(5/2)*(x-(x**2+a)**(1/2))**n, x)

[Out] Integral((a + x**2)**(5/2)*(x - sqrt(a + x**2))**n, x)

$$3.271 \quad \int (a + x^2)^{3/2} (x - \sqrt{a + x^2})^n dx$$

Optimal. Leaf size=141

$$\frac{a^4 (x - \sqrt{a + x^2})^{n-4}}{16(4-n)} + \frac{a^3 (x - \sqrt{a + x^2})^{n-2}}{4(2-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^n}{8n} - \frac{a (x - \sqrt{a + x^2})^{n+2}}{4(n+2)} - \frac{(x - \sqrt{a + x^2})^{n+4}}{16(n+4)}$$

Rubi [A] time = 0.09, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 270}

$$\frac{a^4 (x - \sqrt{a + x^2})^{n-4}}{16(4-n)} + \frac{a^3 (x - \sqrt{a + x^2})^{n-2}}{4(2-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^n}{8n} - \frac{a (x - \sqrt{a + x^2})^{n+2}}{4(n+2)} - \frac{(x - \sqrt{a + x^2})^{n+4}}{16(n+4)}$$

Antiderivative was successfully verified.

[In] Int[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]

[Out] (a^4*(x - Sqrt[a + x^2])^(-4 + n))/(16*(4 - n)) + (a^3*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (3*a^2*(x - Sqrt[a + x^2])^n)/(8*n) - (a*(x - Sqrt[a + x^2])^(2 + n))/(4*(2 + n)) - (x - Sqrt[a + x^2])^(4 + n)/(16*(4 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int (a + x^2)^{3/2} (x - \sqrt{a + x^2})^n dx &= -\left(\frac{1}{16} \text{Subst}\left(\int x^{-5+n} (a + x^2)^4 dx, x, x - \sqrt{a + x^2}\right)\right) \\ &= -\left(\frac{1}{16} \text{Subst}\left(\int (a^4 x^{-5+n} + 4a^3 x^{-3+n} + 6a^2 x^{-1+n} + 4ax^{1+n} + x^{3+n}) dx, x, x - \sqrt{a + x^2}\right)\right) \\ &= \frac{a^4 (x - \sqrt{a + x^2})^{-4+n}}{16(4-n)} + \frac{a^3 (x - \sqrt{a + x^2})^{-2+n}}{4(2-n)} - \frac{3a^2 (x - \sqrt{a + x^2})^n}{8n} - \frac{a (x - \sqrt{a + x^2})^{n+2}}{4(n+2)} - \frac{(x - \sqrt{a + x^2})^{n+4}}{16(n+4)} \end{aligned}$$

Mathematica [A] time = 0.25, size = 123, normalized size = 0.87

$$\frac{1}{16} (x - \sqrt{a + x^2})^n \left(-\frac{a^4}{(n-4)(x - \sqrt{a + x^2})^4} - \frac{4a^3}{(n-2)(x - \sqrt{a + x^2})^2} - \frac{6a^2}{n} - \frac{4a(x - \sqrt{a + x^2})^2}{n+2} - \frac{(x - \sqrt{a + x^2})^4}{n+4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]

[Out] ((x - Sqrt[a + x^2])^n*((-6*a^2)/n - a^4/((-4 + n)*(x - Sqrt[a + x^2])^4) - (4*a^3)/((-2 + n)*(x - Sqrt[a + x^2])^2) - (4*a*(x - Sqrt[a + x^2])^2)/(2 + n) - (x - Sqrt[a + x^2])^4/(4 + n))/16

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int (a + x^2)^{3/2} (x - \sqrt{a + x^2})^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + x^2)^(3/2)*(x - Sqrt[a + x^2])^n, x]

fricas [A] time = 0.57, size = 113, normalized size = 0.80

$$\frac{(a^2n^4 + (n^4 - 4n^2)x^4 - 16a^2n^2 + 2(an^4 - 10an^2)x^2 + 24a^2 + 4((n^3 - 4n)x^3 + (an^3 - 10an)x)\sqrt{x^2 + a})(x - \sqrt{x^2 + a})^n}{n^5 - 20n^3 + 64n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(a^2*n^4 + (n^4 - 4*n^2)*x^4 - 16*a^2*n^2 + 2*(a*n^4 - 10*a*n^2)*x^2 + 24*a^2 + 4*((n^3 - 4*n)*x^3 + (a*n^3 - 10*a*n)*x)*sqrt(x^2 + a))*(x - sqrt(x^2 + a))^n/(n^5 - 20*n^3 + 64*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (x^2 + a)^{\frac{3}{2}} (x - \sqrt{x^2 + a})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(3/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((x^2 + a)^(3/2)*(x - sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (x - \sqrt{x^2 + a})^n (x^2 + a)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + x^2)^{\frac{3}{2}} (x - \sqrt{a + x^2})^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(3/2)*(x-(x**2+a)**(1/2))**n,x)
```

```
[Out] Integral((a + x**2)**(3/2)*(x - sqrt(a + x**2))**n, x)
```

$$3.272 \quad \int \sqrt{a+x^2} (x - \sqrt{a+x^2})^n dx$$

Optimal. Leaf size=81

$$\frac{a^2 (x - \sqrt{a+x^2})^{n-2}}{4(2-n)} - \frac{a (x - \sqrt{a+x^2})^n}{2n} - \frac{(x - \sqrt{a+x^2})^{n+2}}{4(n+2)}$$

Rubi [A] time = 0.08, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 270}

$$\frac{a^2 (x - \sqrt{a+x^2})^{n-2}}{4(2-n)} - \frac{a (x - \sqrt{a+x^2})^n}{2n} - \frac{(x - \sqrt{a+x^2})^{n+2}}{4(n+2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]

[Out] (a^2*(x - Sqrt[a + x^2])^(-2 + n))/(4*(2 - n)) - (a*(x - Sqrt[a + x^2])^n)/(2*n) - (x - Sqrt[a + x^2])^(2 + n)/(4*(2 + n))

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{a+x^2} (x - \sqrt{a+x^2})^n dx &= -\left(\frac{1}{4} \text{Subst}\left(\int x^{-3+n} (a+x^2)^2 dx, x, x - \sqrt{a+x^2}\right)\right) \\ &= -\left(\frac{1}{4} \text{Subst}\left(\int (a^2x^{-3+n} + 2ax^{-1+n} + x^{1+n}) dx, x, x - \sqrt{a+x^2}\right)\right) \\ &= \frac{a^2 (x - \sqrt{a+x^2})^{-2+n}}{4(2-n)} - \frac{a (x - \sqrt{a+x^2})^n}{2n} - \frac{(x - \sqrt{a+x^2})^{2+n}}{4(2+n)} \end{aligned}$$

Mathematica [A] time = 0.07, size = 73, normalized size = 0.90

$$\frac{1}{4} (x - \sqrt{a+x^2})^n \left(-\frac{a^2}{(n-2)(x - \sqrt{a+x^2})^2} - \frac{(x - \sqrt{a+x^2})^2}{n+2} - \frac{2a}{n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]

[Out] ((x - Sqrt[a + x^2])^n*((-2*a)/n - a^2/((-2 + n)*(x - Sqrt[a + x^2])^2) - (x - Sqrt[a + x^2])^2/(2 + n)))/4

IntegrateAlgebraic [F] time = 0.25, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2} \left(x - \sqrt{a+x^2}\right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a + x^2]*(x - Sqrt[a + x^2])^n, x]

fricas [A] time = 0.83, size = 51, normalized size = 0.63

$$\frac{\left(n^2x^2 + an^2 + 2\sqrt{x^2+a}nx - 2a\right)\left(x - \sqrt{x^2+a}\right)^n}{n^3 - 4n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="fricas")

[Out] -(n^2*x^2 + a*n^2 + 2*sqrt(x^2 + a)*n*x - 2*a)*(x - sqrt(x^2 + a))^n/(n^3 - 4*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x - \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="giac")

[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x - \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)

[Out] int((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^2+a} \left(x - \sqrt{x^2+a}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+a)^(1/2)*(x-(x^2+a)^(1/2))^n,x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + a)*(x - sqrt(x^2 + a))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x - \sqrt{x^2+a}\right)^n \sqrt{x^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

```
[Out] int((x - (a + x^2)^(1/2))^n*(a + x^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+x^2} \left(x - \sqrt{a+x^2}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+a)**(1/2)*(x-(x**2+a)**(1/2))**n, x)
```

```
[Out] Integral(sqrt(a + x**2)*(x - sqrt(a + x**2))**n, x)
```

$$3.273 \quad \int \frac{(x - \sqrt{a+x^2})^n}{\sqrt{a+x^2}} dx$$

Optimal. Leaf size=20

$$-\frac{(x - \sqrt{a+x^2})^n}{n}$$

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {2122, 30}

$$-\frac{(x - \sqrt{a+x^2})^n}{n}$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2], x]

[Out] -((x - Sqrt[a + x^2])^n/n)

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2122

Int[((g_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (c_)*(x_)^2])^(n_), x_Symbol] :> Dist[(1*(i/c)^m)/(2^(2*m + 1)*e*f^(2*m)), Subst[Int[(x^n*(d^2 + a*f^2 - 2*d*x + x^2)^(2*m + 1))/(-d + x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int \frac{(x - \sqrt{a+x^2})^n}{\sqrt{a+x^2}} dx &= -\text{Subst}\left(\int x^{-1+n} dx, x, x - \sqrt{a+x^2}\right) \\ &= -\frac{(x - \sqrt{a+x^2})^n}{n} \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 1.00

$$-\frac{(x - \sqrt{a+x^2})^n}{n}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2], x]

[Out] -((x - Sqrt[a + x^2])^n/n)

IntegrateAlgebraic [A] time = 0.04, size = 20, normalized size = 1.00

$$\frac{\left(x - \sqrt{a + x^2}\right)^n}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[a + x^2])^n/Sqrt[a + x^2],x]

[Out] -((x - Sqrt[a + x^2])^n/n)

fricas [A] time = 0.60, size = 18, normalized size = 0.90

$$\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="fricas")

[Out] -(x - sqrt(x^2 + a))^n/n

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)

maple [F] time = 0.06, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)

[Out] int((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(x - \sqrt{x^2 + a}\right)^n}{\sqrt{x^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^2+a)^(1/2))^n/(x^2+a)^(1/2),x, algorithm="maxima")

[Out] integrate((x - sqrt(x^2 + a))^n/sqrt(x^2 + a), x)

mupad [B] time = 3.19, size = 18, normalized size = 0.90

$$\frac{\left(x - \sqrt{x^2 + a}\right)^n}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - (a + x^2)^(1/2))^n/(a + x^2)^(1/2), x)`

[Out] `-(x - (a + x^2)^(1/2))^n/n`

sympy [A] time = 1.57, size = 36, normalized size = 1.80

$$\begin{cases} \frac{(x - \sqrt{a+x^2})^n}{n} & \text{for } n \neq 0 \\ \operatorname{asinh}\left(x\sqrt{\frac{1}{a}}\right) & \text{for } a > 0 \\ \operatorname{acosh}\left(x\sqrt{-\frac{1}{a}}\right) & \text{for } a < 0 \end{cases} \quad \text{otherwise}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x-(x**2+a)**(1/2))**n/(x**2+a)**(1/2), x)`

[Out] `Piecewise((- (x - sqrt(a + x**2))**n/n, Ne(n, 0)), (Piecewise((asinh(x*sqrt(1/a)), a > 0), (acosh(x*sqrt(-1/a)), a < 0)), True))`

$$3.274 \quad \int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=365

$$\frac{(d^2 - af^2)^5 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-5}}{32ef^4(5-n)} - \frac{5(d^2 - af^2)^4 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-3}}{32ef^4(3-n)} + \frac{5(d^2 - af^2)^3}{32ef^4(n+5)}$$

Rubi [A] time = 0.47, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2121, 12, 270}

$$\frac{(d^2 - af^2)^5 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-5}}{32ef^4(5-n)} - \frac{5(d^2 - af^2)^4 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-3}}{32ef^4(3-n)} + \frac{5(d^2 - af^2)^3 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{16ef^4(1-n)} + \frac{5(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{16ef^4(n+1)} - \frac{5(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+3}}{32ef^4(n+3)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+5}}{32ef^4(n+5)}$$

Antiderivative was successfully verified.

[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d^2 - a*f^2)^5*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-5 + n))/(32*e*f^4*(5 - n)) - (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(32*e*f^4*(3 - n)) + (5*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(16*e*f^4*(1 - n)) + (5*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(16*e*f^4*(1 + n)) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n))/(32*e*f^4*(3 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(5 + n)/(32*e*f^4*(5 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2121

Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned}
\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx &= \frac{2 \operatorname{Subst} \left(\int \frac{x^{-6+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^5}{64e^6} dx, x, d + ex \right)}{f^4} \\
&= \frac{\operatorname{Subst} \left(\int x^{-6+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^5 dx, x, d + ex \right)}{32e^6 f^4} \\
&= \frac{\operatorname{Subst} \left(\int \left(-e^5 (d^2 - af^2)^5 x^{-6+n} + 5e^5 (d^2 - af^2)^4 \left(-ae + \frac{2d^2e}{f^2} \right) f^2 x^{-5+n} \right. \right. \\
&\quad \left. \left. + 10e^5 (d^2 - af^2)^3 \left(-ae + \frac{2d^2e}{f^2} \right)^2 f^2 x^{-4+n} + 10e^5 (d^2 - af^2)^2 \left(-ae + \frac{2d^2e}{f^2} \right) f^2 x^{-3+n} \right. \right. \\
&\quad \left. \left. + 5e^5 (d^2 - af^2) \left(-ae + \frac{2d^2e}{f^2} \right)^2 f^2 x^{-2+n} + e^5 \left(-ae + \frac{2d^2e}{f^2} \right)^2 f^2 x^{-1+n} \right) dx, x, d + ex \right)}{32e^6 f^4} \\
&= \frac{(d^2 - af^2)^5 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-5+n}}{32ef^4(5-n)}
\end{aligned}$$

Mathematica [A] time = 2.91, size = 280, normalized size = 0.77

$$\frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^{n-5} \left(-\frac{5(d^2-af^2) \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^8}{n+3} + \frac{10(d^2-af^2)^2 \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^6}{n+1} - \frac{10(d^2-af^2)^3 \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^4}{n-1} + \frac{5(d^2-af^2)^4 \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n-3} - \frac{(d^2-af^2)^5}{n-5} + \frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^{10}}{n+5} \right)}{32ef^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-5 + n))*(-(d^2 - a*f^2)^5/(-5 + n)) + (5*(d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(-3 + n) - (10*(d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4)/(-1 + n) + (10*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6)/(1 + n) - (5*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^8)/(3 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^10/(5 + n))/(32*e*f^4)

IntegrateAlgebraic [F] time = 4.00, size = 0, normalized size = 0.00

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]

fricas [A] time = 0.66, size = 654, normalized size = 1.79

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

```
[Out] -(5*a^2*d*f^4*n^4 + 225*a^2*d*f^4 - 300*a*d^3*f^2 + 5*(e^5*n^4 - 10*e^5*n^2
+ 9*e^5)*x^5 + 120*d^5 + 25*(d*e^4*n^4 - 10*d*e^4*n^2 + 9*d*e^4)*x^4 + 10*
(15*a*e^3*f^2 + 30*d^2*e^3 + (a*e^3*f^2 + 4*d^2*e^3)*n^4 - 2*(8*a*e^3*f^2 +
17*d^2*e^3)*n^2)*x^3 - 10*(11*a^2*d*f^4 - 6*a*d^3*f^2)*n^2 + 10*(45*a*d*e^
2*f^2 + (3*a*d*e^2*f^2 + 2*d^3*e^2)*n^4 - 2*(24*a*d*e^2*f^2 + d^3*e^2)*n^2)
*x^2 + 5*(45*a^2*e*f^4 + (a^2*e*f^4 + 4*a*d^2*e*f^2)*n^4 - 2*(11*a^2*e*f^4
+ 26*a*d^2*e*f^2 - 12*d^4*e)*n^2)*x - (a^2*f^5*n^5 + (e^4*f*n^5 - 10*e^4*f*
n^3 + 9*e^4*f*n)*x^4 - 10*(3*a^2*f^5 - 2*a*d^2*f^3)*n^3 + 4*(d*e^3*f*n^5 -
10*d*e^3*f*n^3 + 9*d*e^3*f*n)*x^3 + 2*((a*e^2*f^3 + 2*d^2*e^2*f)*n^5 - 10*(
2*a*e^2*f^3 + d^2*e^2*f)*n^3 + (19*a*e^2*f^3 + 8*d^2*e^2*f)*n)*x^2 + (149*a
^2*f^5 - 260*a*d^2*f^3 + 120*d^4*f)*n + 4*(a*d*e*f^3*n^5 - 10*(2*a*d*e*f^3
- d^3*e*f)*n^3 + (19*a*d*e*f^3 - 10*d^3*e*f)*n)*x)*sqrt((e^2*x^2 + a*f^2 +
2*d*e*x)/f^2))*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*f^4
*n^6 - 35*e*f^4*n^4 + 259*e*f^4*n^2 - 225*e*f^4)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2} \right)^2 \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="giac")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2} \right)^2 \left(ex + d + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

```
[Out] int((a+2*d*e*x/f^2+e^2/f^2*x^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2/f^2*x^2)^(1/2
))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2} \right)^2 \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^2*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2
)^(1/2))^n,x, algorithm="maxima")
```

```
[Out] integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^2*(e*x + sqrt(e^2*x^2/f^2 + a + 2
*d*e*x/f^2)*f + d)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f \sqrt{a + \frac{e^2 x^2}{f^2} + \frac{2 dex}{f^2}} + ex \right)^n \left(a + \frac{e^2 x^2}{f^2} + \frac{2 dex}{f^2} \right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^2,x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^2, x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**2*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```

$$3.275 \quad \int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=239

$$\frac{(d^2 - af^2)^3 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-3}}{8ef^2(3-n)} - \frac{3(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{8ef^2(1-n)} - \frac{3(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{8ef^2(n+1)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+3}}{8ef^2(n+3)}$$

Rubi [A] time = 0.25, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 54, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {2121, 12, 270}

$$\frac{(d^2 - af^2)^3 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-3}}{8ef^2(3-n)} - \frac{3(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{8ef^2(1-n)} - \frac{3(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{8ef^2(n+1)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+3}}{8ef^2(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-3 + n))/(8*e*f^2*(3 - n)) - (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(8*e*f^2*(1 - n)) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n))/(8*e*f^2*(1 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(3 + n)/(8*e*f^2*(3 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2121

Int[((g_.) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_.) + (e_)*(x_) + (f_)*Sqrt[(a_.) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx = \frac{2 \operatorname{Subst} \left(\int \frac{x^{-4+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^3}{16e^4} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{f^2}$$

$$= \frac{\operatorname{Subst} \left(\int x^{-4+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^3 dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{8e^4 f^2}$$

$$= \frac{\operatorname{Subst} \left(\int \left(-e^3 (d^2 - af^2)^3 x^{-4+n} + 3e^3 (d^2 - af^2)^2 \left(-ae + \frac{2d^2e}{f^2} \right) f^2 x^{-4+n} + 3e^3 (d^2 - af^2) \left(-ae + \frac{2d^2e}{f^2} \right) f^2 ex x^{-4+n} + 3e^3 (d^2 - af^2) ex^2 x^{-4+n} \right) dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{8e^4 f^2}$$

$$= \frac{(d^2 - af^2)^3 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-3+n}}{8ef^2(3 - n)}$$

Mathematica [A] time = 0.61, size = 186, normalized size = 0.78

$$\frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^{n-3} \left(-\frac{3(d^2-af^2) \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^4}{n+1} + \frac{3(d^2-af^2)^2 \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n-1} - \frac{(d^2-af^2)^3}{n-3} + \frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^6}{n+3} \right)}{8ef^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-3 + n)*(-(d^2 - a*f^2)^3/(-3 + n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(-1 + n) - (3*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4)/(1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^6/(3 + n))/ (8*e*f^2)

IntegrateAlgebraic [F] time = 2.80, size = 0, normalized size = 0.00

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]

fricas [A] time = 0.71, size = 239, normalized size = 1.00

$$\frac{(3ad^2n^2 - 9adf^2 + 3(e^3n^2 - e^3)x^3 + 6d^3 + 9(d^2n^2 - d^2e)x^2 - 3(3aef^2 - (aef^2 + 2d^2e)n^2)x - (af^3n^3 + (e^2fn^3 - e^2fn)x^2 - (7af^3 - 6d^2f)n + 2(defn^3 - defn)x) \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}}) \left(ex + f \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} + d \right)^n}{ef^2n^4 - 10ef^2n^2 + 9ef^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] $-(3*a*d*f^2*n^2 - 9*a*d*f^2 + 3*(e^3*n^2 - e^3)*x^3 + 6*d^3 + 9*(d*e^2*n^2 - d*e^2)*x^2 - 3*(3*a*e*f^2 - (a*e*f^2 + 2*d^2*e)*n^2)*x - (a*f^3*n^3 + (e^2*f*n^3 - e^2*f*n)*x^2 - (7*a*f^3 - 6*d^2*f)*n + 2*(d*e*f*n^3 - d*e*f*n)*x)*\sqrt{(e^2*x^2 + a*f^2 + 2*d*e*x)/f^2}*(e*x + f*\sqrt{(e^2*x^2 + a*f^2 + 2*d*e*x)/f^2} + d)^n/(e*f^2*n^4 - 10*e*f^2*n^2 + 9*e*f^2)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right) \left(e x + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")`

[Out] `integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right) \left(e x + d + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

[Out] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right) \left(e x + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")`

[Out] `integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f \sqrt{a + \frac{e^2 x^2}{f^2} + \frac{2 d e x}{f^2}} + e x \right) \left(a + \frac{e^2 x^2}{f^2} + \frac{2 d e x}{f^2} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2), x)`

[Out] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Timed out
```


$$3.276 \quad \int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=107

$$\frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{2e(1-n)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{2e(n+1)}$$

Rubi [A] time = 0.09, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2116, 12, 14}

$$\frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{2e(1-n)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]]^n,x]

[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2116

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx &= 2 \operatorname{Subst} \left(\int \frac{x^{-2+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)}{4e^2} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right) \\
&= \frac{\operatorname{Subst} \left(\int x^{-2+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right) dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{2e^2} \\
&= \frac{\operatorname{Subst} \left(\int \left(-e \left(d^2 - af^2 \right) x^{-2+n} + ex^n \right) dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{2e^2} \\
&= \frac{\left(d^2 - af^2 \right) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-1+n}}{2e(1-n)} + \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{2e(1+n)}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 89, normalized size = 0.83

$$\frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^{n-1} \left(\frac{af^2-d^2}{n-1} + \frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n+1} \right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]]^n, x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n)))/(2*e)

IntegrateAlgebraic [F] time = 1.39, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]]^n, x]

[Out] Defer[IntegrateAlgebraic] [(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]]^n, x]

fricas [A] time = 0.79, size = 80, normalized size = 0.75

$$\frac{\left(fn \sqrt{\frac{e^2x^2+af^2+2dex}{f^2}} - ex - d \right) \left(ex + f \sqrt{\frac{e^2x^2+af^2+2dex}{f^2}} + d \right)^n}{en^2 - e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2dex}{f^2} f + d} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2dex}{f^2} f} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)

[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2dex}{f^2} f + d} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + f \sqrt{a + \frac{e^2 x^2}{f^2} + \frac{2dex}{f^2}} + ex \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n,x)

[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)

[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)

$$3.277 \quad \int \left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}} \right)^n dx$$

Optimal. Leaf size=107

$$\frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{2e(1-n)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{2e(n+1)}$$

Rubi [A] time = 0.13, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2118, 2116, 12, 14}

$$\frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-1}}{2e(1-n)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+1}}{2e(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]

[Out] ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-1 + n))/(2*e*(1 - n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(1 + n)/(2*e*(1 + n))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)] /; FreeQ[{a, b}, x] && InverseFunctionQ[v]

Rule 2116

Int[((g_) + (h_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_))^(p_), x_Symbol] := Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rule 2118

Int[((g_) + (h_)*((u_) + (f_)*Sqrt[v_])^(n_))^(p_), x_Symbol] := Int[(g + h*(ExpandToSum[u, x] + f*Sqrt[ExpandToSum[v, x]])^n)^p, x] /; FreeQ[{f, g, h, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x]) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
\int \left(d + ex + f \sqrt{\frac{af^2 + ex(2d + ex)}{f^2}} \right)^n dx &= \int \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx \\
&= 2 \operatorname{Subst} \left(\int \frac{x^{-2+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)}{4e^2} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right) \\
&= \frac{\operatorname{Subst} \left(\int x^{-2+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right) dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{2e^2} \\
&= \frac{\operatorname{Subst} \left(\int \left(-e(d^2 - af^2)x^{-2+n} + ex^n \right) dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{2e^2} \\
&= \frac{(d^2 - af^2) \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-1+n}}{2e(1-n)} + \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{2e(1-n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 89, normalized size = 0.83

$$\frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^{n-1} \left(\frac{af^2 - d^2}{n-1} + \frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n+1} \right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^(-1 + n)*((-d^2 + a*f^2)/(-1 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(1 + n))/(2*e)

IntegrateAlgebraic [F] time = 0.89, size = 0, normalized size = 0.00

$$\int \left(d + ex + f \sqrt{\frac{af^2 + ex(2d + ex)}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n,x]

[Out] Defer[IntegrateAlgebraic] [(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n, x]

fricas [A] time = 0.70, size = 80, normalized size = 0.75

$$\frac{\left(fn \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} - ex - d \right) \left(ex + f \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} + d \right)^n}{en^2 - e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] (f*n*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) - e*x - d)*(e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n/(e*n^2 - e)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + f \sqrt{\frac{af^2 + (ex + 2d)ex}{f^2}} + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="giac")

[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n, x)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{\frac{af^2 + (ex + 2d)ex}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)

[Out] int((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(ex + d + \sqrt{af^2 + (ex + 2d)ex} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n,x, algorithm="maxima")

[Out] integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + ex + f \sqrt{\frac{af^2 + ex(2d + ex)}{f^2}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n,x)

[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n,x)

[Out] Timed out

$$3.278 \quad \int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^{3/2} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=297

$$\frac{(d^2 - af^2)^4 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-4}}{16ef^3(4-n)} + \frac{(d^2 - af^2)^3 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2}}{4ef^3(2-n)} + \frac{3(d^2 - af^2)^2}{16ef^3(n+4)}$$

Rubi [A] time = 0.42, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2121, 12, 270}

$$\frac{(d^2 - af^2)^4 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-4}}{16ef^3(4-n)} + \frac{(d^2 - af^2)^3 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2}}{4ef^3(2-n)} + \frac{3(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^n}{8ef^3n} - \frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+2}}{4ef^3(n+2)} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+4}}{16ef^3(n+4)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^4*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-4 + n))/(16*e*f^3*(4 - n)) + ((d^2 - a*f^2)^3*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f^3*(2 - n)) + (3*(d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(8*e*f^3*n) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*e*f^3*(2 + n)) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(4 + n)/(16*e*f^3*(4 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 2121

```
Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^{3/2} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx = \frac{2 \operatorname{Subst} \left(\int \frac{x^{-5+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^4}{32e^5} dx, x, d + ex \right)}{f^3}$$

$$= \frac{\operatorname{Subst} \left(\int x^{-5+n} \left(d^2e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^4 dx \right)}{16e^5 f^3}$$

$$= \frac{\operatorname{Subst} \left(\int \left(e^4 (d^2 - af^2)^4 x^{-5+n} - 4e^4 (d^2 - af^2)^3 \right) dx \right)}{16e^5 f^3}$$

$$= - \frac{(d^2 - af^2)^4 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-4+n}}{16ef^3(4 - n)}$$

Mathematica [A] time = 1.20, size = 228, normalized size = 0.77

$$\frac{\left(f \sqrt{a + \frac{cx(2d+ex)}{f^2}} + d + ex \right)^n \left(\frac{(d^2-af^2)^4}{(n-4) \left(f \sqrt{a + \frac{cx(2d+ex)}{f^2}} + d + ex \right)^4} - \frac{4(d^2-af^2)^3}{(n-2) \left(f \sqrt{a + \frac{cx(2d+ex)}{f^2}} + d + ex \right)^2} - \frac{4(d^2-af^2) \left(f \sqrt{a + \frac{cx(2d+ex)}{f^2}} + d + ex \right)^2}{n+2} + \frac{6(d^2-af^2)^2}{n} + \frac{\left(f \sqrt{a + \frac{cx(2d+ex)}{f^2}} + d + ex \right)^4}{n+4} \right)}{16ef^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((6*(d^2 - a*f^2)^2)/n + (d^2 - a*f^2)^4/((-4 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4) - (4*(d^2 - a*f^2)^3)/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) - (4*(d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2)/(2 + n) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^4/(4 + n))/(16*e*f^3)
```

IntegrateAlgebraic [F] time = 7.97, size = 0, normalized size = 0.00

$$\int \left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2} \right)^{3/2} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2)^(3/2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]
```

fricas [A] time = 0.70, size = 377, normalized size = 1.27

$$\frac{\left(\frac{e^2 f^4 n^4 + 24 e^2 f^3 - 48 a d f^2 + (e^4 n^4 - 4 e^4 n^2) x^2 + 24 d^4 + 4 (d e^4 n^4 - 4 d e^4 n^2) x^2 - 4 (4 e^2 f^4 - 3 a d f^2) n^2 + 2 (5 a e^2 f^2 + 2 d^2 e^2) n^4 - 2 (5 a e^2 f^2 + d^2 e^2) x^2 + 4 (a d e^2 f^4 n^2 - 2 (5 a d e^2 f^2 - 3 d^2 e^2) n^2) - 4 (a d f^4 n^2 + (e^2 f^4 n^2 - 4 e^2 f^4 n) x^2 + 3 (d e^2 f^4 n^2 - 4 d e^2 f^4 n) x^2 - 2 (5 a d f^2 - 3 d^2 f^2) n + ((e^2 f^2 + 2 d e^2 f^2) n^2 - 2 (5 a e^2 f^2 + d e^2 f^2) n) \sqrt{\frac{2 d^2 + 2 d e x + e^2 x^2}{f^2}} \right) \left(d + e x + f \sqrt{a + \frac{2 d e x}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n}{e^2 f^3 n^2 - 20 e^2 f^3 n + 64 e^2 f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")
```


[Out] $(a^2 f^4 n^4 + 24 a^2 f^4 - 48 a d^2 f^2 + (e^4 n^4 - 4 e^4 n^2) x^4 + 24 d^4 + 4 (d e^3 n^4 - 4 d e^3 n^2) x^3 - 4 (4 a^2 f^4 - 3 a d^2 f^2) n^2 + 2 ((a e^2 f^2 + 2 d^2 e^2) n^4 - 2 (5 a e^2 f^2 + d^2 e^2) n^2) x^2 + 4 (a d e f^2 n^4 - 2 (5 a d e f^2 - 3 d^3 e) n^2) x - 4 (a d f^3 n^3 + (e^3 f n^3 - 4 e^3 f n) x^3 + 3 (d e^2 f n^3 - 4 d e^2 f n) x^2 - 2 (5 a d f^3 - 3 d^3 f) n + ((a e f^3 + 2 d^2 e f) n^3 - 2 (5 a e f^3 + d^2 e f) n) x) \sqrt{(e^2 x^2 + a f^2 + 2 d e x) / f^2} (e x + f \sqrt{(e^2 x^2 + a f^2 + 2 d e x) / f^2} + d)^n / (e f^3 n^5 - 20 e f^3 n^3 + 64 e f^3 n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right)^{\frac{3}{2}} \left(e x + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")`

[Out] `integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right)^{\frac{3}{2}} \left(e x + d + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

[Out] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(3/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2} \right)^{\frac{3}{2}} \left(e x + \sqrt{\frac{e^2 x^2}{f^2} + a + \frac{2 d e x}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(3/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")`

[Out] `integrate((e^2*x^2/f^2 + a + 2*d*e*x/f^2)^(3/2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f \sqrt{a + \frac{e^2 x^2}{f^2} + \frac{2 d e x}{f^2}} + e x \right)^n \left(a + \frac{e^2 x^2}{f^2} + \frac{2 d e x}{f^2} \right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(3/2),x)`

[Out] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(3/2), x)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(3/2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)

[Out] Timed out

$$3.279 \quad \int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=171

$$\frac{(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2}}{4ef(2-n)} - \frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^n}{2efn} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+2}}{4ef(n+2)}$$

Rubi [A] time = 0.32, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2121, 12, 270}

$$\frac{(d^2 - af^2)^2 \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2}}{4ef(2-n)} - \frac{(d^2 - af^2) \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^n}{2efn} + \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+2}}{4ef(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]
```

```
[Out] -((d^2 - a*f^2)^2*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)) - ((d^2 - a*f^2)*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n) + (d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n)/(4*e*f*(2 + n))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 270

```
Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]
```

Rule 2121

```
Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx &= \frac{2 \operatorname{Subst} \left(\int \frac{x^{-3+n} \left(d^2 e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^2}{8e^3} dx, x, d + ex \right)}{f} \\
&= \frac{\operatorname{Subst} \left(\int x^{-3+n} \left(d^2 e - \left(-ae + \frac{2d^2e}{f^2} \right) f^2 + ex^2 \right)^2 dx \right)}{4e^3 f} \\
&= \frac{\operatorname{Subst} \left(\int \left(e^2 (d^2 - af^2)^2 x^{-3+n} - 2e^2 (d^2 - af^2) x^{-2+n} \right) dx \right)}{4e^3} \\
&= -\frac{(d^2 - af^2)^2 \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^{-2+n}}{4ef(2-n)}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 135, normalized size = 0.79

$$\frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^n \left(\frac{(d^2 - af^2)^2}{(n-2) \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2} + \frac{2(af^2 - d^2)}{n} + \frac{\left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n+2} \right)}{4ef}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] ((d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n)))/(4*e*f)

IntegrateAlgebraic [F] time = 3.83, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]

fricas [A] time = 0.78, size = 122, normalized size = 0.71

$$\frac{\left(e^2n^2x^2 + af^2n^2 + 2den^2x - 2af^2 + 2d^2 - 2(efnx + dfn) \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} \right) \left(ex + f \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} + d \right)^n}{efn^3 - 4efn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] $(e^{2n^2x^2} + af^{2n^2} + 2d*en^2*x - 2*af^2 + 2*d^2 - 2*(ef*n*x + d*f*n)*\sqrt{(e^{2x^2} + af^2 + 2d*ex)/f^2})*(ex + f*\sqrt{(e^{2x^2} + af^2 + 2d*ex)/f^2} + d)^n/(ef*n^3 - 4*ef*n)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")`

[Out] `integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

[Out] `int((e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n,x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")`

[Out] `integrate(sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d + f \sqrt{a + \frac{e^2x^2}{f^2} + \frac{2dex}{f^2}} + ex \right)^n \sqrt{a + \frac{e^2x^2}{f^2} + \frac{2dex}{f^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2), x)`

[Out] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n,x)
```

```
[Out] Integral(sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2)*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)
```

$$3.280 \quad \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}} dx$$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Rubi [A] time = 0.25, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2], x]

[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2121

Int[((g_.) + (h_.)*(x_) + (i_.)*(x_)^2)^(m_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_.), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rubi steps

$$\begin{aligned} \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}} dx &= (2f) \text{Subst} \left(\int \frac{x^{-1+n}}{2e} dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}} \right) \\ &= \frac{f \text{Subst} \left(\int x^{-1+n} dx, x, d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}} \right)}{e} \\ &= \frac{f\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{en} \end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 0.88

$$\frac{f \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^n}{en}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2], x]

[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)

IntegrateAlgebraic [A] time = 0.70, size = 43, normalized size = 1.05

$$\frac{f \left(f \sqrt{\frac{af^2+2dex+e^2x^2}{f^2}} + d + ex \right)^n}{en}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2], x]

[Out] (f*(d + e*x + f*Sqrt[(a*f^2 + 2*d*e*x + e^2*x^2)/f^2])^n)/(e*n)

fricas [A] time = 0.72, size = 41, normalized size = 1.00

$$\frac{\left(ex + f \sqrt{\frac{e^2x^2+af^2+2dex}{f^2}} + d \right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2), x, algorithm="fricas")

[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2), x, algorithm="giac")

[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2), x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f \right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2),x)`

[Out] `int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2),x)`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n}{\sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2),x, algorithm="maxima")`

[Out] `integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2), x)`

mupad [B] time = 3.08, size = 41, normalized size = 1.00

$$\frac{f \left(d + ex + f \sqrt{\frac{e^2x^2 + 2dex + af^2}{f^2}} \right)^n}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2),x)`

[Out] `(f*(d + e*x + f*((a*f^2 + e^2*x^2 + 2*d*e*x)/f^2)^(1/2))^n)/(e*n)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2),x)`

[Out] `Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2), x)`

$$3.281 \quad \int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}} dx$$

Optimal. Leaf size=41

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Rubi [A] time = 0.44, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2127, 2121, 12, 30}

$$\frac{f\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2], x]
```

```
[Out] (f*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(e*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2121

```
Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1)/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2127

```
Int[((u_) + (f_)*((j_) + (k_)*Sqrt[v_]))^(n_)*(w_)^(m_), x_Symbol] := Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v, x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v, w}, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0] || EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}} dx &= \int \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}} dx \\
&= (2f) \text{ Subst} \left(\int \frac{x^{-1+n}}{2e} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right) \\
&= \frac{f \text{ Subst} \left(\int x^{-1+n} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)}{e} \\
&= \frac{f \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{en}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.88

$$\frac{f \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex\right)^n}{en}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2], x]

[Out] (f*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n)

IntegrateAlgebraic [A] time = 0.57, size = 43, normalized size = 1.05

$$\frac{f \left(f \sqrt{\frac{af^2 + 2dex + e^2x^2}{f^2}} + d + ex\right)^n}{en}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2], x]

[Out] (f*(d + e*x + f*Sqrt[(a*f^2 + 2*d*e*x + e^2*x^2)/f^2])^n)/(e*n)

fricas [A] time = 0.57, size = 41, normalized size = 1.00

$$\frac{\left(ex + f \sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} + d\right)^n f}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2), x, algorithm="fricas")

[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f/(e*n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + f \sqrt{\frac{af^2 + (ex+2d)ex}{f^2}} + d\right)^n}{\sqrt{\frac{af^2 + (ex+2d)ex}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2),x, algorithm="giac")

[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2), x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{af^2 + (ex + 2d)ex}{f^2}} f \right)^n}{\sqrt{\frac{af^2 + (ex + 2d)ex}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2),x)

[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$f \int \frac{\left(ex + d + \sqrt{af^2 + (ex + 2d)ex} \right)^n}{\sqrt{af^2 + (ex + 2d)ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2),x, algorithm="maxima")

[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2 + (e*x + 2*d)*e*x), x)

mupad [B] time = 3.08, size = 39, normalized size = 0.95

$$\frac{f \left(d + ex + f \sqrt{\frac{af^2 + ex(2d + ex)}{f^2}} \right)^n}{en}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2),x)

[Out] (f*(d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n)/(e*n)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))^n/((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2),x)

[Out] Timed out

3.282

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Optimal. Leaf size=327

$$\frac{(d^2 - af^2)^2 \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2} (d^2 - af^2) \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^n}{4ef(2-n) \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + 2efn \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

Rubi [A] time = 0.62, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2123, 2121, 12, 270}

$$\frac{(d^2 - af^2)^2 \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n-2} (d^2 - af^2) \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^n + \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + d + ex \right)^{n+2}}{4ef(2-n) \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + 2efn \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} + 4ef(n+2) \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] -((d^2 - a*f^2)^2*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(-2 + n))/(4*e*f*(2 - n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) - ((d^2 - a*f^2)*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n)/(2*e*f*n*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]) + (Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^(2 + n))/(4*e*f*(2 + n)*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 270

Int[((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] && IGtQ[p, 0]

Rule 2121

Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rule 2123

Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[((i/c)^(m - 1/2)*Sqrt[g + h*x + i*x^2])/Sqrt[a + b*x + c*x^2], Int[(a + b*x + c*x^2)^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f,

$g, h, i, n\}, x]$ && EqQ[$e^2 - c*f^2, 0]$ && EqQ[$c*g - a*i, 0]$ && EqQ[$c*h - b$
 $*i, 0]$ && IGtQ[$m + 1/2, 0]$ && !GtQ[$i/c, 0]$

Rubi steps

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx = \frac{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \int \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

$$= \frac{\left(2\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \right) \text{Subst} \left(\int \frac{x^{-3+n} \left(d^2 e - \left(-ae^2 + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2} \right) \right)}{8} dx \right)}{f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

$$= \frac{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \text{Subst} \left(\int x^{-3+n} \left(d^2 e - \left(-ae^2 + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2} \right) \right) dx \right)}{4e^3 f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

$$= \frac{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \text{Subst} \left(\int \left(e^2 (d^2 - af^2) \right)^2 dx \right)}{4ef(2-n)\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

$$= \frac{(d^2 - af^2)^2 \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n}{4ef(2-n)\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}}$$

Mathematica [A] time = 0.23, size = 175, normalized size = 0.54

$$\frac{\sqrt{g \left(a + \frac{ex(2d+ex)}{f^2} \right)} \left(f\sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^n \left(\frac{(d^2 - af^2)^2}{(n-2) \left(f\sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2} + \frac{2(af^2 - d^2)}{n} + \frac{\left(f\sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex \right)^2}{n+2} \right)}{4ef\sqrt{a + \frac{ex(2d+ex)}{f^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] (Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n*((2*(-d^2 + a*f^2))/n + (d^2 - a*f^2)^2/((-2 + n)*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2) + (d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^2/(2 + n))/(4*e*f*Sqrt[a + (e*x*(2*d + e*x))/f^2])

IntegrateAlgebraic [F] time = 0.90, size = 0, normalized size = 0.00

$$\int \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \right)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n,x]

[Out] Defer[IntegrateAlgebraic][Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n, x]

fricas [A] time = 0.75, size = 231, normalized size = 0.71

$$\frac{\left(2e^3nx^3 + 6de^2nx^2 + 2adf^2n + 2(aef^2 + 2d^2e)nx - (e^2fn^2x^2 + af^3n^2 + 2defn^2x - 2af^3 + 2d^2f)\sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}}\right)\left(ex + f\sqrt{\frac{e^2x^2 + af^2 + 2dex}{f^2}} + d\right)^n \sqrt{\frac{e^2gx^2 + af^2g + 2degx}{f^2}}}{ae^2n^3 - 4ae^2n + (e^3n^3 - 4e^3n)x^2 + 2(de^2n^3 - 4de^2n)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="fricas")

[Out] $-(2e^3n^3x^3 + 6d^2e^2n^2x^2 + 2a^2d^2f^2n + 2(a^2ef^2 + 2d^2e^2)n^2x - (e^2fn^2x^2 + af^3n^2 + 2d^2efn^2x - 2a^2f^3 + 2d^2e^2f))\sqrt{(e^2x^2 + af^2 + 2d^2e^2x)/f^2} * (ex + f\sqrt{(e^2x^2 + af^2 + 2d^2e^2x)/f^2} + d)^n \sqrt{(e^2gx^2 + af^2g + 2degx)/f^2} / (a^2ef^2n^3 - 4a^2e^2f^2n + (e^3n^3 - 4e^3n)x^2 + 2(d^2e^2n^3 - 4d^2e^2n)x)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="giac")

[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2))*f + d)^n, x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2g x^2}{f^2} + ag + \frac{2degx}{f^2}} \left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2))*f)^n,x)

[Out] int((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2))*f)^n,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2degx}{f^2}} \left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2)*(d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n,x, algorithm="maxima")

[Out] integrate(sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2)*(e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2))*f + d)^n, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(d + f \sqrt{a + \frac{e^2 x^2}{f^2} + \frac{2dex}{f^2}} + ex \right)^n \sqrt{ag + \frac{e^2 g x^2}{f^2} + \frac{2degx}{f^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)`

[Out] `int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n*(a*g + (e^2*g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{g \left(a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2} \right)} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2 x^2}{f^2}} \right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2)*(d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n, x)`

[Out] `Integral(sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2))*(d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n, x)`

$$3.283 \quad \int \frac{\left(d+ex+f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}} dx$$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Rubi [A] time = 0.53, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 62, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]

[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 2121

Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])

Rule 2125

Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[((i/c)^(m + 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && ILtQ[m - 1/2, 0] && !GtQ[i/c, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} dx &= \frac{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}} dx}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(2f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right) \text{Subst}\left(\int \frac{x^{-1+n}}{2e} dx, x, d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right) \text{Subst}\left(\int x^{-1+n} dx, x, d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)}{e\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{en\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 76, normalized size = 0.82

$$\frac{f\sqrt{a + \frac{ex(2d+ex)}{f^2}} \left(f\sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex\right)^n}{en\sqrt{g\left(a + \frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]

[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])

IntegrateAlgebraic [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]

[Out] Defer[IntegrateAlgebraic] [(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2], x]

fricas [A] time = 0.67, size = 117, normalized size = 1.26

$$\frac{\left(ex + f\sqrt{\frac{e^2x^2+af^2+2dex}{f^2}} + d\right)^n f^3 \sqrt{\frac{e^2gx^2+af^2g+2degx}{f^2}} \sqrt{\frac{e^2x^2+af^2+2dex}{f^2}}}{e^3gnx^2 + aef^2gn + 2de^2gnx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2),x, algorithm="fricas")
```

```
[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2degx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f \right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2degx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/f^2*g*x)^(1/2),x)
```

```
[Out] int((e*x+d+(e^2/f^2*x^2+a+2*d*e/f^2*x)^(1/2)*f)^n/(e^2/f^2*g*x^2+a*g+2*d*e/f^2*g*x)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + \sqrt{\frac{e^2x^2}{f^2} + a + \frac{2dex}{f^2}} f + d \right)^n}{\sqrt{\frac{e^2gx^2}{f^2} + ag + \frac{2degx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f^2+e^2*x^2/f^2)^(1/2))^n/(a*g+2*d*e*g*x/f^2+e^2*g*x^2/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*x + sqrt(e^2*x^2/f^2 + a + 2*d*e*x/f^2)*f + d)^n/sqrt(e^2*g*x^2/f^2 + a*g + 2*d*e*g*x/f^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + f \sqrt{a + \frac{e^2x^2}{f^2} + \frac{2dex}{f^2}} + ex \right)^n}{\sqrt{ag + \frac{e^2gx^2}{f^2} + \frac{2degx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

```
[Out] int((d + f*(a + (e^2*x^2)/f^2 + (2*d*e*x)/f^2)^(1/2) + e*x)^n/(a*g + (e^2*g*x^2)/f^2 + (2*d*e*g*x)/f^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{g\left(a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*(a+2*d*e*x/f**2+e**2*x**2/f**2)**(1/2))**n/(a*g+2*d*e*g*x/f**2+e**2*g*x**2/f**2)**(1/2), x)
```

```
[Out] Integral((d + e*x + f*sqrt(a + 2*d*e*x/f**2 + e**2*x**2/f**2))**n/sqrt(g*(a + 2*d*e*x/f**2 + e**2*x**2/f**2)), x)
```

$$3.284 \quad \int \frac{\left(d+ex+f\sqrt{\frac{af^2+ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g+egx(2d+ex)}{f^2}}} dx$$

Optimal. Leaf size=93

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Rubi [A] time = 0.74, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 60, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2127, 2125, 2121, 12, 30}

$$\frac{f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}\left(f\sqrt{a+\frac{2dex}{f^2}+\frac{e^2x^2}{f^2}}+d+ex\right)^n}{en\sqrt{ag+\frac{2degx}{f^2}+\frac{e^2gx^2}{f^2}}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g*x*(2*d + e*x))/f^2], x]
```

```
[Out] (f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2]*(d + e*x + f*Sqrt[a + (2*d*e*x)/f^2 + (e^2*x^2)/f^2])^n/(e*n*Sqrt[a*g + (2*d*e*g*x)/f^2 + (e^2*g*x^2)/f^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]
```

Rule 2121

```
Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[(2*(i/c)^m)/f^(2*m), Subst[Int[(x^n*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)^(2*m + 1))/(-2*d*e + b*f^2 + 2*e*x)^(2*(m + 1)), x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && IntegerQ[2*m] && (IntegerQ[m] || GtQ[i/c, 0])
```

Rule 2125

```
Int[((g_) + (h_)*(x_) + (i_)*(x_)^2)^(m_)*((d_) + (e_)*(x_) + (f_) * Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2])^(n_), x_Symbol] := Dist[((i/c)^(m + 1/2)*Sqrt[a + b*x + c*x^2])/Sqrt[g + h*x + i*x^2], Int[(a + b*x + c*x^2)^m*(d + e*x + f*Sqrt[a + b*x + c*x^2])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n}, x] && EqQ[e^2 - c*f^2, 0] && EqQ[c*g - a*i, 0] && EqQ[c*h - b*i, 0] && ILtQ[m - 1/2, 0] && !GtQ[i/c, 0]
```

Rule 2127

```
Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*(w_)^(m_.), x_Symbol] :>
Int[ExpandToSum[w, x]^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v,
x]])^n, x] /; FreeQ[{f, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[{v,
w}, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[{v, w}, x] && (EqQ[j, 0]
|| EqQ[f, 1])) && EqQ[Coefficient[u, x, 1]^2 - Coefficient[v, x, 2]*f^2*k^
2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g + egx(2d+ex)}{f^2}}} dx &= \int \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} dx \\
&= \frac{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \int \frac{\left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{\sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}} dx}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(2f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right) \text{Subst}\left(\int \frac{x^{-1+n}}{2e} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)}{\sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{\left(f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right) \text{Subst}\left(\int x^{-1+n} dx, x, d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)}{e \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}} \\
&= \frac{f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}} \left(d + ex + f \sqrt{a + \frac{2dex}{f^2} + \frac{e^2x^2}{f^2}}\right)^n}{en \sqrt{ag + \frac{2degx}{f^2} + \frac{e^2gx^2}{f^2}}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 0.82

$$\frac{f \sqrt{a + \frac{ex(2d+ex)}{f^2}} \left(f \sqrt{a + \frac{ex(2d+ex)}{f^2}} + d + ex\right)^n}{en \sqrt{g \left(a + \frac{ex(2d+ex)}{f^2}\right)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g
+ e*g*x*(2*d + e*x))/f^2], x]
```

```
[Out] (f*Sqrt[a + (e*x*(2*d + e*x))/f^2]*(d + e*x + f*Sqrt[a + (e*x*(2*d + e*x))/
f^2])^n)/(e*n*Sqrt[g*(a + (e*x*(2*d + e*x))/f^2)])
```

IntegrateAlgebraic [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{af^2g + egx(2d+ex)}{f^2}}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g*x*(2*d + e*x))/f^2], x]

[Out] Defer[IntegrateAlgebraic] [(d + e*x + f*Sqrt[(a*f^2 + e*x*(2*d + e*x))/f^2])^n/Sqrt[(a*f^2*g + e*g*x*(2*d + e*x))/f^2], x]

fricas [A] time = 0.66, size = 117, normalized size = 1.26

$$\frac{\left(ex + f \sqrt{\frac{e^2 x^2 + a f^2 + 2 d e x}{f^2}} + d \right)^n f^3 \sqrt{\frac{e^2 g x^2 + a f^2 g + 2 d e g x}{f^2}} \sqrt{\frac{e^2 x^2 + a f^2 + 2 d e x}{f^2}}}{e^3 g n x^2 + a e f^2 g n + 2 d e^2 g n x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2), x, algorithm="fricas")

[Out] (e*x + f*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2) + d)^n*f^3*sqrt((e^2*g*x^2 + a*f^2*g + 2*d*e*g*x)/f^2)*sqrt((e^2*x^2 + a*f^2 + 2*d*e*x)/f^2)/(e^3*g*n*x^2 + a*e*f^2*g*n + 2*d*e^2*g*n*x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + f \sqrt{\frac{a f^2 + (ex + 2d)ex}{f^2}} + d \right)^n}{\sqrt{\frac{a f^2 g + (ex + 2d)egx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2), x, algorithm="giac")

[Out] integrate((e*x + f*sqrt((a*f^2 + (e*x + 2*d)*e*x)/f^2) + d)^n/sqrt((a*f^2*g + (e*x + 2*d)*e*g*x)/f^2), x)

maple [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\left(ex + d + \sqrt{\frac{a f^2 + (ex + 2d)ex}{f^2}} f \right)^n}{\sqrt{\frac{a f^2 g + (ex + 2d)egx}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2), x)

[Out] int((e*x+d+((a*f^2+(e*x+2*d)*e*x)/f^2)^(1/2)*f)^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$f \int \frac{\left(ex + d + \sqrt{a f^2 + (ex + 2d)ex} \right)^n}{\sqrt{a f^2 g + (ex + 2d)egx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f^2+e*x*(e*x+2*d))/f^2)^(1/2))^n/((a*f^2*g+e*g*x*(e*x+2*d))/f^2)^(1/2),x, algorithm="maxima")
```

```
[Out] f*integrate((e*x + d + sqrt(a*f^2 + (e*x + 2*d)*e*x))^n/sqrt(a*f^2*g + (e*x + 2*d)*e*g*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(d + ex + f \sqrt{\frac{af^2 + ex(2d+ex)}{f^2}}\right)^n}{\sqrt{\frac{agf^2 + egx(2d+ex)}{f^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x*(2*d + e*x))/f^2)^(1/2),x)
```

```
[Out] int((d + e*x + f*((a*f^2 + e*x*(2*d + e*x))/f^2)^(1/2))^n/((a*f^2*g + e*g*x*(2*d + e*x))/f^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d+e*x+f*((a*f**2+e*x*(e*x+2*d))/f**2)**(1/2))**n/((a*f**2*g+e*g*x*(e*x+2*d))/f**2)**(1/2),x)
```

```
[Out] Timed out
```


$$3.285 \quad \int \frac{e^{-2fx^2}}{e^2 + 4dfx^2 + 4efx^2 + 4f^2x^4} dx$$

Optimal. Leaf size=81

$$\frac{\log(2\sqrt{-d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{-d}\sqrt{f}} - \frac{\log(-2\sqrt{-d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{-d}\sqrt{f}}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {6, 1164, 628}

$$\frac{\log(2\sqrt{-d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{-d}\sqrt{f}} - \frac{\log(-2\sqrt{-d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{-d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] -Log[e - 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f]) + Log[e + 2*Sqrt[-d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[-d]*Sqrt[f])

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1164

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{e - 2fx^2}{e^2 + 4dfx^2 + 4efx^2 + 4f^2x^4} dx &= \int \frac{e - 2fx^2}{e^2 + 4(d + e)fx^2 + 4f^2x^4} dx \\ &= -\frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}} + 2x}{-\frac{e}{2f} - \frac{\sqrt{-d}x}{\sqrt{f}} - x^2} dx}{4\sqrt{-d}\sqrt{f}} - \frac{\int \frac{\frac{\sqrt{-d}}{\sqrt{f}} - 2x}{-\frac{e}{2f} + \frac{\sqrt{-d}x}{\sqrt{f}} - x^2} dx}{4\sqrt{-d}\sqrt{f}} \\ &= -\frac{\log(e - 2\sqrt{-d}\sqrt{f}x + 2fx^2)}{4\sqrt{-d}\sqrt{f}} + \frac{\log(e + 2\sqrt{-d}\sqrt{f}x + 2fx^2)}{4\sqrt{-d}\sqrt{f}} \end{aligned}$$

Mathematica [B] time = 0.12, size = 191, normalized size = 2.36

$$\frac{(\sqrt{d}\sqrt{d+2e}-d-2e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-\sqrt{d}\sqrt{d+2e+d+e}}}\right) - (\sqrt{d}\sqrt{d+2e+d+2e})\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{\sqrt{d}\sqrt{d+2e+d+e}}}\right)}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{d+2e}}$$

Antiderivative was successfully verified.

[In] Integrate[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] (-(((d - 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]])/Sqrt[d + e - Sqrt[d]*Sqrt[d + 2*e]]) - ((d + 2*e + Sqrt[d]*Sqrt[d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/Sqrt[d + e + Sqrt[d]*Sqrt[d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[f]*Sqrt[d + 2*e])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fx^2}{e^2 + 4dfx^2 + 4efx^2 + 4f^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 + 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

fricas [A] time = 0.48, size = 141, normalized size = 1.74

$$\left[\frac{\sqrt{-df} \log\left(\frac{4f^2x^4 - 4(d-e)fx^2 + e^2 + 4(2fx^3 + ex)\sqrt{-df}}{4f^2x^4 + 4(d+e)fx^2 + e^2}\right)}{4df}, \frac{\sqrt{df} \arctan\left(\frac{\sqrt{df}x}{d}\right) - \sqrt{df} \arctan\left(\frac{(2fx^3 + (2d+e)x)\sqrt{df}}{de}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*sqrt(-d*f))/(4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^3 + (2*d + e)*x)*sqrt(d*f)/(d*e)))/(d*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command: INPUT:sage2:=int(sage0,x);OUTPUT:index.cc index_m operator + Error: Bad Argument Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc index_m operator + Error: Bad Argument ValueDone

maple [B] time = 0.05, size = 394, normalized size = 4.86

$$\frac{\sqrt{2} df \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df - ef + \sqrt{(d+2e)d f^2}}}\right)}{4\sqrt{(d+2e)d f^2} \sqrt{-df - ef + \sqrt{(d+2e)d f^2}}} - \frac{\sqrt{2} df \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{df + ef + \sqrt{(d+2e)d f^2}}}\right)}{4\sqrt{(d+2e)d f^2} \sqrt{df + ef + \sqrt{(d+2e)d f^2}}} - \frac{\sqrt{2} ef \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df - ef + \sqrt{(d+2e)d f^2}}}\right)}{2\sqrt{(d+2e)d f^2} \sqrt{-df - ef + \sqrt{(d+2e)d f^2}}} - \frac{\sqrt{2} ef \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{df + ef + \sqrt{(d+2e)d f^2}}}\right)}{2\sqrt{(d+2e)d f^2} \sqrt{df + ef + \sqrt{(d+2e)d f^2}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df - ef + \sqrt{(d+2e)d f^2}}}\right)}{4\sqrt{-df - ef + \sqrt{(d+2e)d f^2}}} - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{df + ef + \sqrt{(d+2e)d f^2}}}\right)}{4\sqrt{df + ef + \sqrt{(d+2e)d f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2), x)

[Out] $-1/4*f/(d*f^2*(d+2*e))^{(1/2)}*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})*d-1/2*f/(d*f^2*(d+2*e))^{(1/2)}*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})*e-1/4*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctan}(f*x*2^{(1/2)}/(d*f+f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})-1/4*f/(d*f^2*(d+2*e))^{(1/2)}*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})*d-1/2*f/(d*f^2*(d+2*e))^{(1/2)}*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})*e+1/4*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)}*\operatorname{arctanh}(f*x*2^{(1/2)}/(-d*f-f*e+(d*f^2*(d+2*e))^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2fx^2 - e}{4f^2x^4 + 4dfx^2 + 4efx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x^2+e)/(4*f^2*x^4+4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="maxima")

[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)

mupad [B] time = 3.11, size = 50, normalized size = 0.62

$$\frac{\operatorname{atan}\left(\frac{2f^{3/2}x^3+2d\sqrt{f}x+e\sqrt{f}x}{\sqrt{d}e}\right) - \operatorname{atan}\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 + 4*d*f*x^2 + 4*e*f*x^2), x)

[Out] $(\operatorname{atan}((2*f^{(3/2)}*x^3 + 2*d*f^{(1/2)}*x + e*f^{(1/2)}*x)/(d^{(1/2)}*e)) - \operatorname{atan}((f^{(1/2)}*x)/d^{(1/2)}))/(2*d^{(1/2)}*f^{(1/2)})$

sympy [A] time = 0.56, size = 70, normalized size = 0.86

$$\frac{\sqrt{-\frac{1}{df}} \log\left(-dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4} - \frac{\sqrt{-\frac{1}{df}} \log\left(dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x**2+e)/(4*f**2*x**4+4*d*f*x**2+4*e*f*x**2+e**2), x)

[Out] $\operatorname{sqrt}(-1/(d*f))*\log(-d*x*\operatorname{sqrt}(-1/(d*f)) + e/(2*f) + x**2)/4 - \operatorname{sqrt}(-1/(d*f))*\log(d*x*\operatorname{sqrt}(-1/(d*f)) + e/(2*f) + x**2)/4$

$$3.286 \quad \int \frac{e^{-2fx^2}}{e^2 - 4dfx^2 + 4efx^2 + 4f^2x^4} dx$$

Optimal. Leaf size=73

$$\frac{\log(2\sqrt{d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{d}\sqrt{f}} - \frac{\log(-2\sqrt{d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.04, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$, Rules used = {6, 1164, 628}

$$\frac{\log(2\sqrt{d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{d}\sqrt{f}} - \frac{\log(-2\sqrt{d}\sqrt{f}x + e + 2fx^2)}{4\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] -Log[e - 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f]) + Log[e + 2*Sqrt[d]*Sqrt[f]*x + 2*f*x^2]/(4*Sqrt[d]*Sqrt[f])

Rule 6

Int[(u_)*((w_) + (a_)*(v_) + (b_)*(v_))^(p_), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 1164

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :> With[{q = Rt[(-2*d)/e - b/c, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x], x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] && !GtQ[b^2 - 4*a*c, 0]

Rubi steps

$$\begin{aligned} \int \frac{e - 2fx^2}{e^2 - 4dfx^2 + 4efx^2 + 4f^2x^4} dx &= \int \frac{e - 2fx^2}{e^2 + (-4d + 4e)fx^2 + 4f^2x^4} dx \\ &= \int \frac{\frac{\sqrt{d}}{\sqrt{f}} + 2x}{-\frac{e}{2f} - \frac{\sqrt{d}x}{\sqrt{f}} - x^2} dx - \int \frac{\frac{\sqrt{d}}{\sqrt{f}} - 2x}{-\frac{e}{2f} + \frac{\sqrt{d}x}{\sqrt{f}} - x^2} dx \\ &= -\frac{\log(e - 2\sqrt{d}\sqrt{f}x + 2fx^2)}{4\sqrt{d}\sqrt{f}} + \frac{\log(e + 2\sqrt{d}\sqrt{f}x + 2fx^2)}{4\sqrt{d}\sqrt{f}} \end{aligned}$$

Mathematica [C] time = 0.14, size = 233, normalized size = 3.19

$$\frac{(\sqrt{d}\sqrt{2e-d}-id+2ie)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{-i\sqrt{d}\sqrt{2e-d}-d+e}}\right) - (\sqrt{d}\sqrt{2e-d}+id-2ie)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{f}x}{\sqrt{i\sqrt{d}\sqrt{2e-d}-d+e}}\right)}{2\sqrt{2}\sqrt{d}\sqrt{f}\sqrt{2e-d}}$$

Antiderivative was successfully verified.

[In] Integrate[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] (-((((-I)*d + (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e - I*Sqrt[d]*Sqrt[-d + 2*e]]) - ((I*d - (2*I)*e + Sqrt[d]*Sqrt[-d + 2*e])*ArcTan[(Sqrt[2]*Sqrt[f]*x)/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]]])/Sqrt[-d + e + I*Sqrt[d]*Sqrt[-d + 2*e]])/(2*Sqrt[2]*Sqrt[d]*Sqrt[-d + 2*e]*Sqrt[f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fx^2}{e^2 - 4dfx^2 + 4efx^2 + 4f^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

[Out] IntegrateAlgebraic[(e - 2*f*x^2)/(e^2 - 4*d*f*x^2 + 4*e*f*x^2 + 4*f^2*x^4), x]

fricas [A] time = 0.65, size = 146, normalized size = 2.00

$$\left[\frac{\sqrt{df} \log\left(\frac{4f^2x^4 + 4(d+e)fx^2 + e^2 + 4(2fx^3 + ex)\sqrt{df}}{4f^2x^4 - 4(d-e)fx^2 + e^2}\right)}{4df}, \frac{\sqrt{-df} \arctan\left(\frac{\sqrt{-df}x}{d}\right) - \sqrt{-df} \arctan\left(\frac{(2fx^3 - (2d-e)x)\sqrt{-df}}{de}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log((4*f^2*x^4 + 4*(d + e)*f*x^2 + e^2 + 4*(2*f*x^3 + e*x)*sqrt(d*f))/(4*f^2*x^4 - 4*(d - e)*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^3 - (2*d - e)*x)*sqrt(-d*f)/(d*e)))/(d*f)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:index.cc index_m operator + Error: Bad Argument Valueindex.cc index_m operator + Error: Bad Argument Valueindex.cc index_m operator + Error: Bad Argument ValueDone

maple [B] time = 0.05, size = 394, normalized size = 5.40

$$\frac{\sqrt{2} df \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{4\sqrt{(d-2e)d f^2} \sqrt{-df+ef+\sqrt{(d-2e)d f^2}}} + \frac{\sqrt{2} df \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{4\sqrt{(d-2e)d f^2} \sqrt{-df+ef+\sqrt{(d-2e)d f^2}}} - \frac{\sqrt{2} ef \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{2\sqrt{(d-2e)d f^2} \sqrt{-df+ef+\sqrt{(d-2e)d f^2}}} - \frac{\sqrt{2} ef \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{2\sqrt{(d-2e)d f^2} \sqrt{-df+ef+\sqrt{(d-2e)d f^2}}} + \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{4\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}} - \frac{\sqrt{2} \operatorname{arctan}\left(\frac{\sqrt{2} fx}{\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}\right)}{4\sqrt{-df+ef+\sqrt{(d-2e)d f^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2), x)
[Out] 1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*a
rctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f^2*(d
-2*e))^(1/2)*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1
/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e+1/4*2^(1/2)/(d*f-e*f+(d*f^2*(d
-2*e))^(1/2))^(1/2)*arctanh(f*x*2^(1/2)/(d*f-e*f+(d*f^2*(d-2*e))^(1/2))^(1/
2))+1/4*f/(d*f^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1
/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*d-1/2*f/(d*f
^2*(d-2*e))^(1/2)*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x
*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/2))^(1/2))*e-1/4*2^(1/2)/(-d*f+e*f+(d
*f^2*(d-2*e))^(1/2))^(1/2)*arctan(f*x*2^(1/2)/(-d*f+e*f+(d*f^2*(d-2*e))^(1/
2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2fx^2 - e}{4f^2x^4 - 4dfx^2 + 4efx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x^2+e)/(4*f^2*x^4-4*d*f*x^2+4*e*f*x^2+e^2), x, algorithm="ma
xima")
[Out] -integrate((2*f*x^2 - e)/(4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2 + e^2), x)
```

mupad [B] time = 3.14, size = 28, normalized size = 0.38

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x}{2fx^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e - 2*f*x^2)/(e^2 + 4*f^2*x^4 - 4*d*f*x^2 + 4*e*f*x^2), x)
[Out] atanh((2*d^(1/2)*f^(1/2)*x)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))
```

sympy [A] time = 0.58, size = 63, normalized size = 0.86

$$-\frac{\sqrt{\frac{1}{df}} \log\left(-dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4} + \frac{\sqrt{\frac{1}{df}} \log\left(dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^2\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-2*f*x**2+e)/(4*f**2*x**4-4*d*f*x**2+4*e*f*x**2+e**2), x)
[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4 + sqrt(1/(d*f))*l
og(d*x*sqrt(1/(d*f)) + e/(2*f) + x**2)/4
```

$$3.287 \quad \int \frac{e^{-4fx^3}}{e^2 + 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2093

Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n}, x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B*c + 2*A*d*(n - 1), 0]

Rubi steps

$$\int \frac{e - 4fx^3}{e^2 + 4dfx^2 + 4efx^3 + 4f^2x^6} dx = (2e^2) \text{Subst}\left(\int \frac{1}{e^2 + 16de^2fx^2} dx, x, \frac{x}{2e + 4fx^3}\right) = \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.06, size = 87, normalized size = 2.29

$$\frac{\text{RootSum}\left[4\#1^6f^2 + 4\#1^3ef + 4\#1^2df + e^2\&, \frac{4\#1^3f\log(x-\#1)-e\log(x-\#1)}{6\#1^5f+3\#1^2e+2\#1d}\&\right]}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] -1/4*RootSum[e^2 + 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 &, (-e*Log[x - #1]) + 4*f*Log[x - #1]*#1^3)/(2*d*#1 + 3*e*#1^2 + 6*f*#1^5) &]/f

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 4fx^3}{e^2 + 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 + 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

fricas [B] time = 0.66, size = 153, normalized size = 4.03

$$\left[-\frac{\sqrt{-df} \log\left(\frac{4f^2x^6 + 4efx^3 - 4dfx^2 + e^2 + 4(2fx^4 + ex)\sqrt{-df}}{4f^2x^6 + 4efx^3 + 4dfx^2 + e^2}\right)}{4df}, -\frac{\sqrt{df} \arctan\left(\frac{\sqrt{df}x^2}{d}\right) - \sqrt{df} \arctan\left(\frac{(2fx^5 + ex^2 + 2dx)\sqrt{df}}{de}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2 + 4*(2*f*x^4 + e*x)*sqrt(-d*f))/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqrt(d*f)*arctan(sqrt(d*f)*x^2/d) - sqrt(d*f)*arctan((2*f*x^5 + e*x^2 + 2*d*x)*sqrt(d*f)/(d*e)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4fx^3 - e}{4f^2x^6 + 4efx^3 + 4dfx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="giac")

[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)

maple [C] time = 0.01, size = 70, normalized size = 1.84

$$\frac{(-4\text{RootOf}(4f^2_Z^6 + 4ef_Z^3 + 4df_Z^2 + e^2)^3 f + e) \ln(-\text{RootOf}(4f^2_Z^6 + 4ef_Z^3 + 4df_Z^2 + e^2) + x)}{4f(6f\text{RootOf}(4f^2_Z^6 + 4ef_Z^3 + 4df_Z^2 + e^2)^5 + 3e\text{RootOf}(4f^2_Z^6 + 4ef_Z^3 + 4df_Z^2 + e^2)^2 + 2d\text{RootOf}(4f^2_Z^6 + 4ef_Z^3 + 4df_Z^2 + e^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x)

[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e+2*_R*d)*ln(-_R+x), _R=RootOf(4*_Z^6*f^2+4*_Z^3*e*f+4*_Z^2*d*f+e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4fx^3 - e}{4f^2x^6 + 4efx^3 + 4dfx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3+4*d*f*x^2+e^2),x, algorithm="maxima")

[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2), x)

mupad [B] time = 3.32, size = 54, normalized size = 1.42

$$\frac{\operatorname{atan}\left(\frac{2f^{3/2}x^5 + 2d\sqrt{f}xe + \sqrt{f}x^2}{\sqrt{d}e}\right) - \operatorname{atan}\left(\frac{\sqrt{f}x^2}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 + 4*d*f*x^2 + 4*e*f*x^3), x)

[Out] (atan((2*f^(3/2)*x^5 + 2*d*f^(1/2)*x + e*f^(1/2)*x^2)/(d^(1/2)*e)) - atan((f^(1/2)*x^2)/d^(1/2)))/(2*d^(1/2)*f^(1/2))

sympy [B] time = 0.75, size = 70, normalized size = 1.84

$$\frac{\sqrt{-\frac{1}{df}} \log\left(-dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}} \log\left(dx\sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3+4*d*f*x**2+e**2), x)

[Out] sqrt(-1/(d*f))*log(-d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4 - sqrt(-1/(d*f))*log(d*x*sqrt(-1/(d*f)) + e/(2*f) + x**3)/4

$$3.288 \quad \int \frac{e^{-4fx^3}}{e^2 - 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2093

Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)*(x_)^(n2_)), x_Symbol] := Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n}, x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B*c + 2*A*d*(n - 1), 0]

Rubi steps

$$\int \frac{e - 4fx^3}{e^2 - 4dfx^2 + 4efx^3 + 4f^2x^6} dx = (2e^2) \text{Subst}\left(\int \frac{1}{e^2 - 16de^2fx^2} dx, x, \frac{x}{2e + 4fx^3}\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.06, size = 87, normalized size = 2.29

$$\frac{\text{RootSum}\left[4\#1^6 f^2 + 4\#1^3 ef - 4\#1^2 df + e^2 \&, \frac{4\#1^3 f \log(x-\#1) - e \log(x-\#1)}{6\#1^5 f + 3\#1^2 e - 2\#1 d} \&\right]}{4f}$$

Antiderivative was successfully verified.

[In] Integrate[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] -1/4*RootSum[e^2 - 4*d*f*#1^2 + 4*e*f*#1^3 + 4*f^2*#1^6 &, (-e*Log[x - #1]) + 4*f*Log[x - #1]*#1^3)/(-2*d*#1 + 3*e*#1^2 + 6*f*#1^5) &]/f

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 4fx^3}{e^2 - 4dfx^2 + 4efx^3 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[(e - 4*f*x^3)/(e^2 - 4*d*f*x^2 + 4*e*f*x^3 + 4*f^2*x^6), x]

fricas [B] time = 0.61, size = 155, normalized size = 4.08

$$\left[\frac{\sqrt{df} \log\left(\frac{4f^2x^6 + 4efx^3 + 4dfx^2 + e^2 + 4(2fx^4 + ex)\sqrt{df}}{4f^2x^6 + 4efx^3 - 4dfx^2 + e^2}\right)}{4df}, \sqrt{-df} \arctan\left(\frac{\sqrt{-df}x^2}{d}\right) - \sqrt{-df} \arctan\left(\frac{(2fx^5 + ex^2 - 2dx)\sqrt{-df}}{de}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^2 + e^2 + 4*(2*f*x^4 + e*x)*sqrt(d*f))/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arctan(sqrt(-d*f)*x^2/d) - sqrt(-d*f)*arctan((2*f*x^5 + e*x^2 - 2*d*x)*sqrt(-d*f)/(d*e)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{4fx^3 - e}{4f^2x^6 + 4efx^3 - 4dfx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="giac")

[Out] integrate(-(4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)

maple [C] time = 0.01, size = 70, normalized size = 1.84

$$\frac{(-4\text{RootOf}(4f^2Z^6 + 4efZ^3 - 4dfZ^2 + e^2)^3 f + e) \ln(-\text{RootOf}(4f^2Z^6 + 4efZ^3 - 4dfZ^2 + e^2) + x)}{4f(6f\text{RootOf}(4f^2Z^6 + 4efZ^3 - 4dfZ^2 + e^2)^5 + 3e\text{RootOf}(4f^2Z^6 + 4efZ^3 - 4dfZ^2 + e^2)^2 - 2d\text{RootOf}(4f^2Z^6 + 4efZ^3 - 4dfZ^2 + e^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x)

[Out] 1/4/f*sum((-4*_R^3*f+e)/(6*_R^5*f+3*_R^2*e-2*_R*d)*ln(-_R+x),_R=RootOf(4*_Z^6*f^2+4*_Z^3*e*f-4*_Z^2*d*f+e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4fx^3 - e}{4f^2x^6 + 4efx^3 - 4dfx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x^3+e)/(4*f^2*x^6+4*e*f*x^3-4*d*f*x^2+e^2),x, algorithm="maxima")

[Out] -integrate((4*f*x^3 - e)/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^2 + e^2), x)

mupad [B] time = 3.12, size = 67, normalized size = 1.76

$$\frac{\operatorname{atanh}\left(\frac{-32fd^2x+27e^3+54fe^2x^3}{16d^{3/2}e\sqrt{f}+32d^{3/2}f^{3/2}x^3-54\sqrt{d}e^2\sqrt{f}x}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 4*f*x^3)/(e^2 + 4*f^2*x^6 - 4*d*f*x^2 + 4*e*f*x^3), x)

[Out] -atanh((27*e^3 - 32*d^2*f*x + 54*e^2*f*x^3)/(16*d^(3/2)*e*f^(1/2) + 32*d^(3/2)*f^(3/2)*x^3 - 54*d^(1/2)*e^2*f^(1/2)*x))/(2*d^(1/2)*f^(1/2))

sympy [A] time = 0.74, size = 63, normalized size = 1.66

$$-\frac{\sqrt{\frac{1}{df}} \log\left(-dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}} \log\left(dx\sqrt{\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4*f*x**3+e)/(4*f**2*x**6+4*e*f*x**3-4*d*f*x**2+e**2), x)

[Out] -sqrt(1/(d*f))*log(-d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4 + sqrt(1/(d*f))*log(d*x*sqrt(1/(d*f)) + e/(2*f) + x**3)/4

$$3.289 \quad \int \frac{e^{-2f(-1+n)x^n}}{e^2 + 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Optimal. Leaf size=38

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2093, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x)/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2093

Int(((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n}, x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B*c + 2*A*d*(n - 1), 0]

Rubi steps

$$\int \frac{e^{-2f(-1+n)x^n}}{e^2 + 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx = -\left((e^2(1-n)) \text{Subst}\left(\int \frac{1}{e^2 + 4de^2f(-1+n)^2x^2} dx, x, \frac{x}{e(-1+n) + 2f(-1+n)x^n}\right)\right)$$

$$= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{-2f(-1+n)x^n}}{e^2 + 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

IntegrateAlgebraic [F] time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{e - 2f(-1 + n)x^n}{e^2 + 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(e - 2*f*(-1 + n)*x^n)/(e^2 + 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

fricas [A] time = 0.75, size = 144, normalized size = 3.79

$$\left[-\frac{\sqrt{-df} \log\left(-\frac{4dfx^2 - 4f^2x^{2n} - 4\sqrt{-df}ex - e^2 - 4(2\sqrt{-df}fx + ef)x^n}{4dfx^2 + 4f^2x^{2n} + 4efx^n + e^2}\right)}{4df}, -\frac{\sqrt{df} \arctan\left(\frac{2\sqrt{df}fx^n + \sqrt{df}e}{2dfx}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*sqrt(-d*f)*e*x - e^2 - 4*(2*sqrt(-d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*f*x))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2f(n-1)x^n - e}{4dfx^2 + 4f^2x^{2n} + 4efx^n + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(n-1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x, algorithm="giac")

[Out] integrate(-(2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)

maple [B] time = 0.07, size = 78, normalized size = 2.05

$$\frac{\ln\left(x^n + \frac{-2dfx + \sqrt{-df}e}{2\sqrt{-df}f}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^n + \frac{2dfx + \sqrt{-df}e}{2\sqrt{-df}f}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e-2*f*(n-1)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)), x)

[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)+1/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(-d*f)^(1/2))/(-d*f)^(1/2)/f)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{2f(n-1)x^n - e}{4dfx^2 + 4f^2x^{2n} + 4efx^n + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2+4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="maxima")

[Out] -integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)

mupad [B] time = 3.42, size = 196, normalized size = 5.16

$$\frac{\ln\left(-\frac{e+2fx^n-2fnx^n}{4f^2} - \frac{e^2n-4dfx^2+4dfnx^2+2efnx^n}{8\sqrt{-d}f^{5/2}x}\right)}{4\sqrt{-d}\sqrt{f}} - \frac{\operatorname{atan}\left(\frac{x(8dfn^2-16dfn+8df)}{4\sqrt{d}\sqrt{f}(e^n-e^{n^2})}\right)}{2\sqrt{d}\sqrt{f}} - \frac{\ln\left(\frac{e^{2n}-4dfx^2+4dfnx^2+2efnx^n}{8\sqrt{-d}f^{5/2}x} - \frac{e+2fx^n-2fnx^n}{4f^2}\right)}{4\sqrt{-d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^2 + 4*e*f*x^n),x)

[Out] log(- (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2) - (e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x))/(4*(-d)^(1/2)*f^(1/2)) - atan((x*(8*d*f - 16*d*f*n + 8*d*f*n^2))/(4*d^(1/2)*f^(1/2)*(e*n - e*n^2)))/(2*d^(1/2)*f^(1/2)) - log((e^2*n - 4*d*f*x^2 + 4*d*f*n*x^2 + 2*e*f*n*x^n)/(8*(-d)^(1/2)*f^(5/2)*x) - (e + 2*f*x^n - 2*f*n*x^n)/(4*f^2))/(4*(-d)^(1/2)*f^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{e - 2fnx^n + 2fx^n}{4dfx^2 + e^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x**n)/(e**2+4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)), x)

[Out] Integral((e - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2 + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)

$$3.290 \quad \int \frac{e^{-2f(-1+n)x^n}}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2093, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] ArcTanh[(2*sqrt[d]*sqrt[f]*x)/(e + 2*f*x^n)]/(2*sqrt[d]*sqrt[f])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2093

Int[((A_) + (B_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^(n_) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[A^2*(n - 1), Subst[Int[1/(a + A^2*b*(n - 1)^2*x^2), x], x, x/(A*(n - 1) - B*x^n)], x] /; FreeQ[{a, b, c, d, A, B, n}, x] && EqQ[n2, 2*n] && NeQ[n, 2] && EqQ[a*B^2 - A^2*d*(n - 1)^2, 0] && EqQ[B*c + 2*A*d*(n - 1), 0]

Rubi steps

$$\int \frac{e^{-2f(-1+n)x^n}}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx = -\left((e^2(1-n)) \text{Subst}\left(\int \frac{1}{e^2 - 4de^2f(-1+n)^2x^2} dx, x, \frac{x}{e(-1+n) + 2f(-1+n)x^n}\right)\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{e^{-2f(-1+n)x^n}}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] Integrate[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

IntegrateAlgebraic [F] time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{e - 2f(-1 + n)x^n}{e^2 - 4dfx^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(e - 2*f*(-1 + n)*x^n)/(e^2 - 4*d*f*x^2 + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

fricas [A] time = 0.42, size = 144, normalized size = 3.79

$$\left[\frac{\sqrt{df} \log\left(-\frac{4dfx^2 + 4f^2x^{2n} + 4\sqrt{df}ex + e^2 + 4(2\sqrt{df}fx + ef)x^n}{4dfx^2 - 4f^2x^{2n} - 4efx^n - e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{2\sqrt{-df}fx^n + \sqrt{-df}e}{2dfx}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2 + 4*f^2*x^(2*n) + 4*sqrt(d*f)*e*x + e^2 + 4*(2*sqrt(d*f)*f*x + e*f)*x^n)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n + sqrt(-d*f)*e)/(d*f*x))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2f(n-1)x^n - e}{4dfx^2 - 4f^2x^{2n} - 4efx^n - e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="giac")

[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)

maple [B] time = 0.07, size = 72, normalized size = 1.89

$$-\frac{\ln\left(x^n + \frac{-2dfx + \sqrt{df}e}{2\sqrt{df}f}\right)}{4\sqrt{df}} + \frac{\ln\left(x^n + \frac{2dfx + \sqrt{df}e}{2\sqrt{df}f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e-2*f*(n-1)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)), x)

[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)-1/4/(d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x+e*(d*f)^(1/2))/(d*f)^(1/2)/f)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2f(n-1)x^n - e}{4dfx^2 - 4f^2x^{2n} - 4efx^n - e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x^n)/(e^2-4*d*f*x^2+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="maxima")

[Out] integrate((2*f*(n - 1)*x^n - e)/(4*d*f*x^2 - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)

mupad [B] time = 5.30, size = 139, normalized size = 3.66

$$\frac{\ln\left(\frac{(e+2fx^n+2\sqrt{d}\sqrt{f}x)(en+2\sqrt{d}\sqrt{f}x-2\sqrt{d}\sqrt{f}nx)}{x}\right)}{4\sqrt{d}\sqrt{f}} - \frac{\ln\left(\frac{(e+2fx^n-2\sqrt{d}\sqrt{f}x)(en-2\sqrt{d}\sqrt{f}x+2\sqrt{d}\sqrt{f}nx)}{x}\right)}{4\sqrt{d}\sqrt{f}} - \frac{\operatorname{atan}\left(\frac{2\sqrt{d}\sqrt{f}x(n1i-i)}{en}\right)1i}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e - 2*f*x^n*(n - 1))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^2 + 4*e*f*x^n),x)

[Out] log(((e + 2*f*x^n + 2*d^(1/2)*f^(1/2)*x)*(e*n + 2*d^(1/2)*f^(1/2)*x - 2*d^(1/2)*f^(1/2)*n*x)/x)/(4*d^(1/2)*f^(1/2)) - log(((e + 2*f*x^n - 2*d^(1/2)*f^(1/2)*x)*(e*n - 2*d^(1/2)*f^(1/2)*x + 2*d^(1/2)*f^(1/2)*n*x)/x)/(4*d^(1/2)*f^(1/2)) - (atan((2*d^(1/2)*f^(1/2)*x*(n*1i - 1i))/(e*n))*1i)/(2*d^(1/2)*f^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}} dx - \int \frac{2fx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}} dx - \int \left(-\frac{2fnx^n}{4dfx^2 - e^2 - 4efx^n - 4f^2x^{2n}}\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e-2*f*(-1+n)*x**n)/(e**2-4*d*f*x**2+4*e*f*x**n+4*f**2*x**(2*n)), x)

[Out] -Integral(e/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**n/(4*d*f*x**2 - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)

$$3.291 \quad \int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4} dx$$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{de}}\right)}{4\sqrt{de}\sqrt{f}}$$

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6, 1107, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{de}}\right)}{4\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] := Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4} dx &= \int \frac{x}{e^2 + 4efx^2 + 4(df + f^2)x^4} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{e^2 + 4efx + 4(df + f^2)x^2} dx, x, x^2\right) \\ &= -\text{Subst}\left(\int \frac{1}{-16de^2f - x^2} dx, x, 4f(e + 2(d + f)x^2)\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{f}(e + 2(d + f)x^2)}{\sqrt{de}}\right)}{4\sqrt{de}\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^2(d+f)+e)}{\sqrt{de}}\right)}{4\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{e^2 + 4efx^2 + 4dfx^4 + 4f^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 + 4*d*f*x^4 + 4*f^2*x^4), x]

fricas [A] time = 0.39, size = 155, normalized size = 3.69

$$\left[\frac{\sqrt{-df} \log\left(\frac{4(d^2f+2df^2+f^3)x^4 - de^2 + e^2f + 4(df+ef^2)x^2 - 2(de+ef)x^2 + e^2}{4(df+f^2)x^4 + 4efx^2 + e^2}\right) \sqrt{df} \arctan\left(\frac{(2(d+f)x^2+e)\sqrt{df}}{de}\right)}{8def}, \frac{\sqrt{df} \arctan\left(\frac{(2(d+f)x^2+e)\sqrt{df}}{de}\right)}{4def} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [-1/8*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^4 - d*e^2 + e^2*f + 4*(d*e*f + e*f^2)*x^2 - 2*(2*(d*e + e*f)*x^2 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x^4 + 4*e*f*x^2 + e^2))/(d*e*f), 1/4*sqrt(d*f)*arctan((2*(d + f)*x^2 + e)*sqrt(d*f)/(d*e))/(d*e*f)]

giac [A] time = 10.58, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{(2dfx^2+2f^2x^2+fe)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] 1/4*arctan((2*d*f*x^2 + 2*f^2*x^2 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)

maple [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{4ef+2(4df+4f^2)x^2}{4\sqrt{df}e}\right)}{4\sqrt{df}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x)

[Out] 1/4/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^2+4*e*f)/(d*f)^(1/2)/e)

maxima [A] time = 2.17, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2(df+f^2)x^2+ef}{\sqrt{dfe}}\right)}{4\sqrt{dfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")

[Out] 1/4*arctan((2*(d*f + f^2)*x^2 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)

mupad [B] time = 3.10, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\sqrt{f}+2f^{3/2}x^2+2d\sqrt{f}x^2}{\sqrt{d}e}\right)}{4\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e^2 + 4*f^2*x^4 + 4*d*f*x^4 + 4*e*f*x^2),x)

[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^2 + 2*d*f^(1/2)*x^2)/(d^(1/2)*e))/(4*d^(1/2)*e*f^(1/2))

sympy [B] time = 0.62, size = 78, normalized size = 1.86

$$\frac{\sqrt{-\frac{1}{df}} \log\left(x^2 + \frac{-de\sqrt{-\frac{1}{df}} + e}{2d+2f}\right)}{8} + \frac{\sqrt{-\frac{1}{df}} \log\left(x^2 + \frac{de\sqrt{-\frac{1}{df}} + e}{2d+2f}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)

[Out] (-sqrt(-1/(d*f))*log(x**2 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f)))/8 + sqrt(-1/(d*f))*log(x**2 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/8)/e

$$3.292 \quad \int \frac{x}{e^2 + 4efx^2 - 4dfx^4 + 4f^2x^4} dx$$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^2(d-f))}{\sqrt{de}}\right)}{4\sqrt{d}e\sqrt{f}}$$

Rubi [A] time = 0.07, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6, 1107, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^2(d-f))}{\sqrt{de}}\right)}{4\sqrt{d}e\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^2))/(Sqrt[d]*e)]/(4*Sqrt[d]*e*Sqrt[f])

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{e^2 + 4efx^2 - 4dfx^4 + 4f^2x^4} dx &= \int \frac{x}{e^2 + 4efx^2 + (-4df + 4f^2)x^4} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{e^2 + 4efx + (-4df + 4f^2)x^2} dx, x, x^2\right) \\ &= -\text{Subst}\left(\int \frac{1}{16de^2f - x^2} dx, x, 4f(e - 2(d - f)x^2)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e+2(-d+f)x^2)}{\sqrt{de}}\right)}{4\sqrt{d}e\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.05

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}(-2dx^2+e+2fx^2)}{\sqrt{de}}\right)}{4\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] -1/4*ArcTanh[(Sqrt[f]*(e - 2*d*x^2 + 2*f*x^2))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{e^2 + 4efx^2 - 4dfx^4 + 4f^2x^4} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4), x]

[Out] IntegrateAlgebraic[x/(e^2 + 4*e*f*x^2 - 4*d*f*x^4 + 4*f^2*x^4), x]

fricas [A] time = 0.41, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df} \log\left(-\frac{4(d^2f-2df^2+f^3)x^4+de^2+e^2f-4(def-ef^2)x^2+2(de-ef)x^2-e^2)\sqrt{df}}{4(df-f^2)x^4-4efx^2-e^2}\right)}{8def}, \frac{\sqrt{-df} \arctan\left(-\frac{(2(d-f)x^2-e)\sqrt{-df}}{de}\right)}{4def} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [1/8*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^4 + d*e^2 + e^2*f - 4*(d*e*f - e*f^2)*x^2 + 2*(2*(d*e - e*f)*x^2 - e^2)*sqrt(d*f)))/(4*(d*f - f^2)*x^4 - 4*e*f*x^2 - e^2)/(d*e*f), 1/4*sqrt(-d*f)*arctan(-(2*(d - f)*x^2 - e)*sqrt(-d*f)/(d*e))/(d*e*f)]

giac [A] time = 10.05, size = 41, normalized size = 0.93

$$\frac{\arctan\left(\frac{2dfx^2-2f^2x^2-fe}{\sqrt{-dfe^2}}\right)}{4\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] -1/4*arctan((2*d*f*x^2 - 2*f^2*x^2 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)

maple [A] time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{-4ef+2(4df-4f^2)x^2}{4\sqrt{df}e}\right)}{4\sqrt{df}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2), x)

[Out] $1/4/(d*f)^{(1/2)}/e*\operatorname{arctanh}(1/4*(2*(4*d*f-4*f^2)*x^2-4*e*f)/(d*f)^{(1/2)}/e)$

maxima [A] time = 1.87, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2(df-f^2)x^2-ef+\sqrt{df}e}{2(df-f^2)x^2-ef-\sqrt{df}e}\right)}{8\sqrt{df}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-4*d*f*x^4+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")`

[Out] $1/8*\log((2*(d*f - f^2)*x^2 - e*f + \operatorname{sqrt}(d*f)*e)/(2*(d*f - f^2)*x^2 - e*f - \operatorname{sqrt}(d*f)*e))/(\operatorname{sqrt}(d*f)*e)$

mupad [B] time = 3.11, size = 199, normalized size = 4.52

$$\operatorname{atanh}\left(\frac{\frac{16d^{3/2}f^{3/2}x^2}{\frac{8ef^3}{d}-32f^3x^2-16ef^2+16df^2x^2+8def+\frac{16f^4x^2}{d}} - \frac{32\sqrt{d}f^{5/2}x^2}{\frac{8ef^3}{d}-32f^3x^2-16ef^2+16df^2x^2+8def+\frac{16f^4x^2}{d}} + \frac{16f^{7/2}x^2}{\sqrt{d}\left(\frac{8ef^3}{d}-32f^3x^2-16ef^2+16df^2x^2+8def+\frac{16f^4x^2}{d}\right)}}{4\sqrt{d}e\sqrt{f}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e^2 + 4*f^2*x^4 - 4*d*f*x^4 + 4*e*f*x^2),x)`

[Out] $\operatorname{atanh}\left(\frac{(16*d^{(3/2)}*f^{(3/2)}*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) - (32*d^{(1/2)}*f^{(5/2)}*x^2)/((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d) + (16*f^{(7/2)}*x^2)/(d^{(1/2)}*((8*e*f^3)/d - 32*f^3*x^2 - 16*e*f^2 + 16*d*f^2*x^2 + 8*d*e*f + (16*f^4*x^2)/d))}{(4*d^{(1/2)}*e*f^{(1/2)})}\right)$

sympy [A] time = 0.66, size = 75, normalized size = 1.70

$$\frac{\frac{\sqrt{\frac{1}{df}} \log\left(x^2 + \frac{-de\sqrt{\frac{1}{df}} - e}{2d-2f}\right)}{8} - \frac{\sqrt{\frac{1}{df}} \log\left(x^2 + \frac{de\sqrt{\frac{1}{df}} - e}{2d-2f}\right)}{8}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(-4*d*f*x**4+4*f**2*x**4+4*e*f*x**2+e**2),x)`

[Out] $-(\operatorname{sqrt}(1/(d*f))*\log(x**2 + (-d*e*\operatorname{sqrt}(1/(d*f)) - e)/(2*d - 2*f)))/8 - \operatorname{sqrt}(1/(d*f))*\log(x**2 + (d*e*\operatorname{sqrt}(1/(d*f)) - e)/(2*d - 2*f))/8)/e$

$$3.293 \quad \int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6} dx$$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4+4dfx^6} dx = (3e^2) \text{Subst}\left(\int \frac{1}{e^2+36de^2fx^2} dx, x, \frac{x^3}{3e+6fx^2}\right) = \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[4\#1^6df + 4\#1^4f^2 + 4\#1^2ef + e^2\&, \frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6),x]

[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 + 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 + 3*d*#1^4) &]/(8*f)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6), x]

[Out] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^6), x]

fricas [B] time = 0.41, size = 208, normalized size = 5.20

$$\left[\frac{\sqrt{-df} \log\left(\frac{4dfx^6 - 4f^2x^4 - 4efx^2 - e^2 - 4(2fx^5 + ex^3)\sqrt{-df}}{4dfx^6 + 4f^2x^4 + 4efx^2 + e^2}\right)}{4df}, \frac{\sqrt{df} \arctan\left(\frac{\sqrt{df}x}{f}\right) - \sqrt{df} \arctan\left(\frac{2(2dfx^5 - (de - 2f^2)x^3 + efx)\sqrt{df}}{de^2}\right) + \sqrt{df} \arctan\left(\frac{(2dfx^3 - (de - 2f^2)x)\sqrt{df}}{def}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log((4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2 - 4*(2*f*x^5 + e*x^3)*sqrt(-d*f))/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2))/(d*f), 1/2*(sqrt(d*f)*arctan(sqrt(d*f)*x/f) - sqrt(d*f)*arctan(2*(2*d*f*x^5 - (d*e - 2*f^2)*x^3 + e*f*x)*sqrt(d*f)/(d*e^2)) + sqrt(d*f)*arctan((2*d*f*x^3 - (d*e - 2*f^2)*x)*sqrt(d*f)/(d*e*f)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2fx^2 + 3e)x^2}{4dfx^6 + 4f^2x^4 + 4efx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)

maple [C] time = 0.15, size = 74, normalized size = 1.85

$$\frac{(2\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2)^4 f + 3\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2)^2 e) \ln(-\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2) + x)}{8f(3d\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2)^5 + 2f\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2)^3 + e\text{RootOf}(4df_Z^6 + 4f^2_Z^4 + 4ef_Z^2 + e^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x)

[Out] 1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d+2*_R^3*f+_R*e)*ln(-_R+x), _R=RootOf(4*_Z^6*d*f+4*_Z^4*f^2+4*_Z^2*e*f+e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2fx^2 + 3e)x^2}{4dfx^6 + 4f^2x^4 + 4efx^2 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")

[Out] integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2), x)

mupad [B] time = 3.21, size = 278, normalized size = 6.95

$$\frac{\operatorname{atan}\left(\frac{2f^2x+2dfx^3-dex}{\sqrt{d}e\sqrt{f}}\right) - \operatorname{atan}\left(\frac{1984d^{3/2}f^{9/2}x^3}{432d^2e^2f^2-128de f^4} + \frac{1728d^{5/2}f^{7/2}x^5}{432d^2e^2f^2-128de f^4} + \frac{512\sqrt{d}f^{13/2}x^3}{128d^2f^4-432d^2e^3f^2} + \frac{512d^{3/2}f^{11/2}x^5}{128d^2f^4-432d^2e^3f^2} - \frac{256\sqrt{d}f^{11/2}x}{432d^2e^2f^2-128de f^4} + \frac{864d^{3/2}ef^{7/2}x}{432d^2e^2f^2-128de f^4} - \frac{864d^{5/2}ef^{5/2}x^3}{432d^2e^2f^2-128de f^4}\right)}{2\sqrt{d}\sqrt{f}} + \operatorname{atan}\left(\frac{\sqrt{d}x}{\sqrt{f}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 + 4*d*f*x^6 + 4*e*f*x^2),x)

[Out] (atan((2*f^2*x + 2*d*f*x^3 - d*e*x)/(d^(1/2)*e*f^(1/2))) - atan((1984*d^(3/2)*f^(9/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (1728*d^(5/2)*f^(7/2)*x^5)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (512*d^(1/2)*f^(13/2)*x^3)/(128*d*e^2*f^4 - 432*d^2*e^3*f^2) + (512*d^(3/2)*f^(11/2)*x^5)/(128*d*e^2*f^4 - 432*d^2*e^3*f^2) - (256*d^(1/2)*f^(11/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) + (864*d^(3/2)*e*f^(7/2)*x)/(432*d^2*e^2*f^2 - 128*d*e*f^4) - (864*d^(5/2)*e*f^(5/2)*x^3)/(432*d^2*e^2*f^2 - 128*d*e*f^4)) + atan((d^(1/2)*x)/f^(1/2)))/(2*d^(1/2)*f^(1/2))

sympy [B] time = 1.13, size = 90, normalized size = 2.25

$$-\frac{\sqrt{-\frac{1}{df}} \log\left(-\frac{e\sqrt{-\frac{1}{df}}}{2} - fx^2\sqrt{-\frac{1}{df}} + x^3\right)}{4} + \frac{\sqrt{-\frac{1}{df}} \log\left(\frac{e\sqrt{-\frac{1}{df}}}{2} + fx^2\sqrt{-\frac{1}{df}} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*f*x**2+3*e)/(4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)

[Out] -sqrt(-1/(d*f))*log(-e*sqrt(-1/(d*f))/2 - f*x**2*sqrt(-1/(d*f)) + x**3)/4 + sqrt(-1/(d*f))*log(e*sqrt(-1/(d*f))/2 + f*x**2*sqrt(-1/(d*f)) + x**3)/4

$$3.294 \quad \int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.13, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 42, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^3)/(e + 2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] & EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^2(3e+2fx^2)}{e^2+4efx^2+4f^2x^4-4dfx^6} dx = (3e^2) \text{Subst}\left(\int \frac{1}{e^2-36de^2fx^2} dx, x, \frac{x^3}{3e+6fx^2}\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.05, size = 85, normalized size = 2.12

$$\frac{\text{RootSum}\left[-4\#1^6df + 4\#1^4f^2 + 4\#1^2ef + e^2\&, \frac{2\#1^3f\log(x-\#1)+3\#1e\log(x-\#1)}{-3\#1^4d+2\#1^2f+e}\&\right]}{8f}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6),x]

[Out] RootSum[e^2 + 4*e*f*#1^2 + 4*f^2*#1^4 - 4*d*f*#1^6 & , (3*e*Log[x - #1]*#1 + 2*f*Log[x - #1]*#1^3)/(e + 2*f*#1^2 - 3*d*#1^4) &]/(8*f)

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (3e + 2fx^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6), x]

[Out] IntegrateAlgebraic[(x^2*(3*e + 2*f*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^6), x]

fricas [B] time = 0.41, size = 213, normalized size = 5.32

$$\left[\frac{\sqrt{df} \log\left(\frac{4dfx^6 + 4f^2x^4 + 4efx^2 + e^2 + 4(2fx^5 + ex^3)\sqrt{df}}{4dfx^6 - 4f^2x^4 - 4efx^2 - e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{\sqrt{-df}x}{f}\right) - \sqrt{-df} \arctan\left(\frac{2(2dfx^5 - (de+2f^2)x^3 - efx)\sqrt{-df}}{d^2}\right) + \sqrt{-df} \arctan\left(\frac{(2dfx^3 - (de+2f^2)x)\sqrt{-df}}{def}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log((4*d*f*x^6 + 4*f^2*x^4 + 4*e*f*x^2 + e^2 + 4*(2*f*x^5 + e*x^3)*sqrt(d*f)))/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2))/(d*f), -1/2*(sqrt(-d*f)*arctan(sqrt(-d*f)*x/f) - sqrt(-d*f)*arctan(2*(2*d*f*x^5 - (d*e + 2*f^2)*x^3 - e*f*x)*sqrt(-d*f)/(d*e^2)) + sqrt(-d*f)*arctan((2*d*f*x^3 - (d*e + 2*f^2)*x)*sqrt(-d*f)/(d*e*f)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2fx^2 + 3e)x^2}{4dfx^6 - 4f^2x^4 - 4efx^2 - e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x, algorithm="giac")

[Out] integrate(-(2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x)

maple [C] time = 0.14, size = 77, normalized size = 1.92

$$\frac{(2\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2)^4 f + 3\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2)^2 e) \ln(-\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2) + x)}{8f(3d\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2)^5 - 2f\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2)^3 - e\text{RootOf}(4df_Z^6 - 4f^2_Z^4 - 4ef_Z^2 - e^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2), x)

[Out] -1/8/f*sum((2*_R^4*f+3*_R^2*e)/(3*_R^5*d-2*_R^3*f-_R*e)*ln(-_R+x), _R=RootOf(4*_Z^6*d*f-4*_Z^4*f^2-4*_Z^2*e*f-e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2fx^2 + 3e)x^2}{4dfx^6 - 4f^2x^4 - 4efx^2 - e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(2*f*x^2+3*e)/(-4*d*f*x^6+4*f^2*x^4+4*e*f*x^2+e^2),x, algorithm="maxima")

[Out] -integrate((2*f*x^2 + 3*e)*x^2/(4*d*f*x^6 - 4*f^2*x^4 - 4*e*f*x^2 - e^2), x)

mupad [B] time = 3.14, size = 30, normalized size = 0.75

$$\frac{\operatorname{atanh}\left(\frac{2\sqrt{d}\sqrt{f}x^3}{2fx^2+e}\right)}{2\sqrt{d}\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(3*e + 2*f*x^2))/(e^2 + 4*f^2*x^4 - 4*d*f*x^6 + 4*e*f*x^2),x)

[Out] atanh((2*d^(1/2)*f^(1/2)*x^3)/(e + 2*f*x^2))/(2*d^(1/2)*f^(1/2))

sympy [B] time = 1.12, size = 80, normalized size = 2.00

$$-\frac{\sqrt{\frac{1}{df}} \log\left(-\frac{e\sqrt{\frac{1}{df}}}{2} - fx^2\sqrt{\frac{1}{df}} + x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}} \log\left(\frac{e\sqrt{\frac{1}{df}}}{2} + fx^2\sqrt{\frac{1}{df}} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(2*f*x**2+3*e)/(-4*d*f*x**6+4*f**2*x**4+4*e*f*x**2+e**2),x)

[Out] -sqrt(1/(d*f))*log(-e*sqrt(1/(d*f))/2 - f*x**2*sqrt(1/(d*f)) + x**3)/4 + sqrt(1/(d*f))*log(e*sqrt(1/(d*f))/2 + f*x**2*sqrt(1/(d*f)) + x**3)/4

$$3.295 \quad \int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^2 + 2m} dx$$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^(2 + 2*m)), x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*(1 - m^2)*x^(1 + m))/((1 - m)*(1 + m)*(e + 2*f*x^2))]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\begin{aligned} \int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^2 + 2m} dx &= -\left((e^2(1-m)(1+m)) \text{Subst}\left(\int \frac{1}{e^2 + 4de^2f(-1+m)^2(1+m)^2x^2} dx, \right.\right. \\ &= \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-1 + m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 + 4*d*f*x^(2 + 2*m)), x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1+m))/(e+2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])

IntegrateAlgebraic [F] time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 + 4dfx^{2+2m}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1+m)+2*f*(-1+m)*x^2))/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1+m)+2*f*(-1+m)*x^2))/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)), x]

fricas [A] time = 0.44, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{-df} \log\left(-\frac{4f^2x^4 - 4dfx^2x^{2m} + 4efx^2 + 4(2fx^3 + ex)\sqrt{-df}x^m + e^2}{4f^2x^4 + 4dfx^2x^{2m} + 4efx^2 + e^2}\right)}{4df}, \frac{\sqrt{df} \arctan\left(\frac{(2fx^2 + e)\sqrt{df}}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^4 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + 4*(2*f*x^3 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^4 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^2 + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^2 + e)*sqrt(d*f)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-1)x^2 + e(m+1))x^m}{4f^2x^4 + 4efx^2 + 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),x, algorithm="giac")

[Out] integrate((2*f*(m-1)*x^2 + e*(m+1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m+2) + e^2), x)

maple [B] time = 0.09, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m - \frac{(2fx^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^m + \frac{(2fx^2+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)), x)

[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/2)*ln(x^m-1/2*(2*f*x^2+e)*(-d*f)^(1/2)/d/f/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-1)x^2 + e(m+1))x^m}{4f^2x^4 + 4efx^2 + 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4+4*d*f*x^(2+2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (2 f (m - 1) x^2 + e (m + 1))}{e^2 + 4 f^2 x^4 + 4 e f x^2 + 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 + 4*d*f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4+4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

$$3.296 \quad \int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^{2+2m}} dx$$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.22, antiderivative size = 61, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{m+1}}{(1-m)(m+1)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1+m) + 2*f*(-1+m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^(2+2*m)), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*(1-m^2)*x^(1+m))/((1-m)*(1+m)*(e+2*f*x^2))]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2094

Int[((x_)^(m_)*((A_) + (B_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(k_) + (c_)*(x_)^(n_) + (d_)*(x_)^(n2_)), x_Symbol] := Dist[(A^2*(m-n+1))/(m+1), Subst[Int[1/(a+A^2*b*(m-n+1)^2*x^2), x], x, x^(m+1)/(A*(m-n+1)+B*(m+1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m+1)] && EqQ[a*B^2*(m+1)^2 - A^2*d*(m-n+1)^2, 0] & EqQ[B*c*(m+1) - 2*A*d*(m-n+1), 0]

Rubi steps

$$\begin{aligned} \int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^{2+2m}} dx &= -\left((e^2(1-m)(1+m)) \text{Subst}\left(\int \frac{1}{e^2 - 4de^2f(-1+m)^2(1+m)^2x^2} dx, x, \frac{x^{m+1}}{e+2fx^2}\right)\right) \\ &= \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}(1-m^2)x^{1+m}}{(1-m)(1+m)(e+2fx^2)}\right)}{2\sqrt{d}\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^2}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^m*(e*(1+m) + 2*f*(-1+m)*x^2))/(e^2 + 4*e*f*x^2 + 4*f^2*x^4 - 4*d*f*x^(2+2*m)), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1+m))/(e+2*f*x^2)]/(2*Sqrt[d]*Sqrt[f])

IntegrateAlgebraic [F] time = 0.39, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(-1+m)x^2)}{e^2 + 4efx^2 + 4f^2x^4 - 4dfx^{2+2m}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1+m)+2*f*(-1+m)*x^2))/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1+m)+2*f*(-1+m)*x^2))/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)), x]

fricas [A] time = 0.43, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df} \log\left(-\frac{4f^2x^4+4dfx^2x^{2m}+4efx^2+4(2fx^3+ex)\sqrt{df}x^m+e^2}{4f^2x^4-4dfx^2x^{2m}+4efx^2+e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{(2fx^2+e)\sqrt{-df}}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^4+4*d*f*x^2*x^(2*m)+4*e*f*x^2+4*(2*f*x^3+e*x)*sqrt(d*f)*x^m+e^2)/(4*f^2*x^4-4*d*f*x^2*x^(2*m)+4*e*f*x^2+e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^2+e)*sqrt(-d*f)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-1)x^2 + e(m+1))x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),x, algorithm="giac")

[Out] integrate((2*f*(m-1)*x^2+e*(m+1))*x^m/(4*f^2*x^4+4*e*f*x^2-4*d*f*x^(2*m+2)+e^2), x)

maple [B] time = 0.09, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m - \frac{(2fx^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}} + \frac{\ln\left(x^m + \frac{(2fx^2+e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)), x)

[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*ln(x^m-1/2*(2*f*x^2+e)*(d*f)^(1/2)/d/f/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-1)x^2 + e(m+1))x^m}{4f^2x^4 + 4efx^2 - 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-1+m)*x^2)/(e^2+4*e*f*x^2+4*f^2*x^4-4*d*f*x^(2+2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 1)*x^2 + e*(m + 1))*x^m/(4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (2f(m-1)x^2 + e(m+1))}{e^2 + 4f^2x^4 + 4efx^2 - 4dfx^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^2*(m - 1)))/(e^2 + 4*f^2*x^4 + 4*e*f*x^2 - 4*d*f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-1+m)*x**2)/(e**2+4*e*f*x**2+4*f**2*x**4-4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

$$3.297 \quad \int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx$$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_.)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3+4dfx^4+4f^2x^6} dx = -\left((2e^2) \text{Subst}\left(\int \frac{1}{e^2+16de^2fx^2} dx, x, \frac{x^2}{-2e-4fx^3}\right)\right) = \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 2.15

$$\frac{\text{RootSum}\left[4\#1^6f^2 + 4\#1^4df + 4\#1^3ef + e^2\&, \frac{\#1^3f \log(x-\#1)-e \log(x-\#1)}{6\#1^4f+4\#1^2d+3\#1e}\&\right]}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6),x]

[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 + 4*d*f*#1^4 + 4*f^2*#1^6 &, (-e*Log[x - #1] + f*Log[x - #1]*#1^3)/(3*e*#1 + 4*d*#1^2 + 6*f*#1^4) &]/f

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 + 4dfx^4 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 + 4*d*f*x^4 + 4*f^2*x^6), x]

fricas [B] time = 0.44, size = 153, normalized size = 3.82

$$\left[\frac{\sqrt{-df} \log\left(\frac{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2 + 4(2fx^5 + ex^2)\sqrt{-df}}{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2}\right)}{4df}, \frac{\sqrt{df} \arctan\left(\frac{\sqrt{df}x}{d}\right) - \sqrt{df} \arctan\left(\frac{(2fx^4 + 2dx^2 + ex)\sqrt{df}}{de}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log((4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 + e*x^2)*sqrt(-d*f))/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(sqrt(d*f)*arctan(sqrt(d*f)*x/d) - sqrt(d*f)*arctan((2*f*x^4 + 2*d*x^2 + e*x)*sqrt(d*f)/(d*e)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2(fx^3 - e)x}{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="giac")

[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)

maple [C] time = 0.01, size = 74, normalized size = 1.85

$$\frac{(\text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2)^4 f - \text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2)e) \ln(-\text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2) + x)}{2f(6f\text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2)^5 + 4d\text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2)^3 + 3e\text{RootOf}(4f^2Z^6 + 4dfZ^4 + 4efZ^3 + e^2)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2), x)

[Out] -1/2/f*sum((R^4*f - R*e)/(6*R^5*f + 4*R^3*d + 3*R^2*e)*ln(-R+x), R=RootOf(4*_Z^6*f^2+4*_Z^4*d*f+4*_Z^3*e*f+e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{(fx^3 - e)x}{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6+4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="maxima")

[Out] $-2 \int (f x^3 - e) x / (4 f^2 x^6 + 4 d f x^4 + 4 e f x^3 + e^2), x$

mupad [B] time = 3.49, size = 233, normalized size = 5.82

$$\frac{\operatorname{atan}\left(\frac{128 d^{7/2} \sqrt{f} x^2}{64 d^3 e + 729 f e^3} - \frac{216 d^{3/2} e^2 \sqrt{f}}{64 d^3 e + 729 f e^3} + \frac{128 d^{5/2} f^{3/2} x^4}{64 d^3 e + 729 f e^3} + \frac{216 d^{3/2} e \sqrt{f}}{64 d^3 + 729 f e^2} + \frac{729 d^{3/2} e^2 f^{3/2} x}{64 d^5 + 729 f d^2 e^2} + \frac{1458 d^{3/2} e f^{5/2} x^4}{64 d^5 + 729 f d^2 e^2} + \frac{64 d^{5/2} e \sqrt{f} x}{64 d^3 e + 729 f e^3} + \frac{1458 d^{3/2} e f^{3/2} x^2}{64 d^4 + 729 f d e^2}\right) - \operatorname{atan}\left(\frac{\sqrt{f} x}{\sqrt{d}}\right)}{2 \sqrt{d} \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x(2e - 2fx^3))/(e^2 + 4f^2x^6 + 4dfox^4 + 4eefx^3), x$

[Out] $(\operatorname{atan}((128 d^{7/2} f^{1/2} x^2)/(64 d^3 e + 729 e^3 f) - (216 d^{3/2} e^2 f^{1/2})/(64 d^3 e + 729 e^3 f) + (128 d^{5/2} f^{3/2} x^4)/(64 d^3 e + 729 e^3 f) + (216 d^{3/2} e f^{1/2})/(729 e^2 f + 64 d^3) + (729 d^{3/2} e^2 f^{3/2} x)/(64 d^5 + 729 d^2 e^2 f) + (1458 d^{3/2} e f^{5/2} x^4)/(64 d^5 + 729 d^2 e^2 f) + (64 d^{5/2} e f^{1/2} x)/(64 d^3 e + 729 e^3 f) + (1458 d^{3/2} e f^{3/2} x^2)/(64 d^4 + 729 d e^2 f)) - \operatorname{atan}((f^{1/2} x)/d^{1/2}))/ (2 d^{1/2} f^{1/2})$

sympy [B] time = 1.12, size = 73, normalized size = 1.82

$$\frac{\sqrt{-\frac{1}{df}} \log\left(-dx^2 \sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4} - \frac{\sqrt{-\frac{1}{df}} \log\left(dx^2 \sqrt{-\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\int (x(-2fx^3 + 2e))/(4f^2x^6 + 4dfox^4 + 4eefx^3 + e^2), x$

[Out] $\sqrt{-1/(d*f)} * \log(-d*x**2 * \sqrt{-1/(d*f)} + e/(2*f) + x**3)/4 - \sqrt{-1/(d*f)} * \log(d*x**2 * \sqrt{-1/(d*f)} + e/(2*f) + x**3)/4$

$$3.298 \quad \int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.09, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^2)/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2094

Int[((x_)^(m_)*((A_) + (B_)*(x_)^(n_)))/((a_) + (b_)*(x_)^(k_) + (c_)*(x_)^(n_) + (d_)*(x_)^(n2_)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] & EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x(2e-2fx^3)}{e^2+4efx^3-4dfx^4+4f^2x^6} dx = -\left((2e^2) \text{Subst}\left(\int \frac{1}{e^2-16de^2fx^2} dx, x, \frac{x^2}{-2e-4fx^3}\right)\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^2}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [C] time = 0.05, size = 86, normalized size = 2.15

$$\frac{\text{RootSum}\left[4\#1^6f^2 - 4\#1^4df + 4\#1^3ef + e^2 \&, \frac{\#1^3f \log(x-\#1) - e \log(x-\#1)}{6\#1^4f - 4\#1^2d + 3\#1e} \&\right]}{2f}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6),x]

[Out] -1/2*RootSum[e^2 + 4*e*f*#1^3 - 4*d*f*#1^4 + 4*f^2*#1^6 &, (-e*Log[x - #1]) + f*Log[x - #1]*#1^3)/(3*e*#1 - 4*d*#1^2 + 6*f*#1^4) &]/f

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(2e - 2fx^3)}{e^2 + 4efx^3 - 4dfx^4 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[(x*(2*e - 2*f*x^3))/(e^2 + 4*e*f*x^3 - 4*d*f*x^4 + 4*f^2*x^6), x]

fricas [B] time = 0.42, size = 155, normalized size = 3.88

$$\left[\frac{\sqrt{df} \log\left(\frac{4f^2x^6 + 4dfx^4 + 4efx^3 + e^2 + 4(2fx^5 + ex^2)\sqrt{df}}{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{\sqrt{-df}x}{d}\right) - \sqrt{-df} \arctan\left(\frac{(2fx^4 - 2dx^2 + ex)\sqrt{-df}}{de}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log((4*f^2*x^6 + 4*d*f*x^4 + 4*e*f*x^3 + e^2 + 4*(2*f*x^5 + e*x^2)*sqrt(d*f))/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2))/(d*f), -1/2*(sqrt(-d*f)*arctan(sqrt(-d*f)*x/d) - sqrt(-d*f)*arctan((2*f*x^4 - 2*d*x^2 + e*x)*sqrt(-d*f)/(d*e)))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{2(fx^3 - e)x}{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="giac")

[Out] integrate(-2*(f*x^3 - e)*x/(4*f^2*x^6 - 4*d*f*x^4 + 4*e*f*x^3 + e^2), x)

maple [C] time = 0.01, size = 74, normalized size = 1.85

$$\frac{\left(\text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2)\right)^4 f - \text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2) e \ln\left(-\text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2) + x\right)}{2f\left(6f\text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2)^5 - 4d\text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2)^3 + 3e\text{RootOf}(4f^2_Z^6 - 4df_Z^4 + 4ef_Z^3 + e^2)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2), x)

[Out] -1/2/f*sum((R^4*f - R*e)/(6*R^5*f - 4*R^3*d + 3*R^2*e)*ln(-R+x), R=RootOf(4*_Z^6*f^2 - 4*_Z^4*d*f + 4*_Z^3*e*f + e^2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-2 \int \frac{(fx^3 - e)x}{4f^2x^6 - 4dfx^4 + 4efx^3 + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-2*f*x^3+2*e)/(4*f^2*x^6-4*d*f*x^4+4*e*f*x^3+e^2), x, algorithm="maxima")

[Out] $-2 \int (f x^3 - e) x / (4 f^2 x^6 - 4 d f x^4 + 4 e f x^3 + e^2), x$

mupad [B] time = 3.38, size = 67, normalized size = 1.68

$$-\frac{\operatorname{atanh}\left(\frac{27 e^2 \sqrt{f} + 54 e f^{3/2} x^3 - 16 d^2 \sqrt{f} x^2}{8 d^{3/2} e + 16 d^{3/2} f x^3 - 54 \sqrt{d} e f x^2}\right)}{2 \sqrt{d} \sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{int}((x(2e - 2fx^3))/(e^2 + 4f^2x^6 - 4dfx^4 + 4efx^3), x)$

[Out] $-\operatorname{atanh}\left(\frac{27e^2f^{1/2} + 54ef^{3/2}x^3 - 16d^2f^{1/2}x^2}{(8d^{3/2})e + 16d^{3/2}fx^3 - 54d^{1/2}efx^2}\right)/(2d^{1/2}f^{1/2})$

sympy [A] time = 1.12, size = 66, normalized size = 1.65

$$-\frac{\sqrt{\frac{1}{df}} \log\left(-dx^2 \sqrt{\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4} + \frac{\sqrt{\frac{1}{df}} \log\left(dx^2 \sqrt{\frac{1}{df}} + \frac{e}{2f} + x^3\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\operatorname{integrate}(x(-2fx^3+2e)/(4f^2x^6-4dfx^4+4efx^3+e^2), x)$

[Out] $-\sqrt{1/(d*f)}*\log(-d*x**2*\sqrt{1/(d*f)} + e/(2*f) + x**3)/4 + \sqrt{1/(d*f)}*\log(d*x**2*\sqrt{1/(d*f)} + e/(2*f) + x**3)/4$

$$3.299 \quad \int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6} dx$$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^3(d+f)+e)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Rubi [A] time = 0.06, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6, 1352, 618, 204}

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^3(d+f)+e)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6),x]

[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])

Rule 6

Int[(u_)*((w_) + (a_)*(v_) + (b_)*(v_))^(p_), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_)) + (b_)*(x_)^(n_)^(p_), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6} dx &= \int \frac{x^2}{e^2 + 4efx^3 + 4(df + f^2)x^6} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{e^2 + 4efx + 4(df + f^2)x^2} dx, x, x^3\right) \\ &= -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{-16de^2f - x^2} dx, x, 4f(e + 2(d + f)x^3)\right)\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{f}(e+2(d+f)x^3)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{f}(2x^3(d+f)+e)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6), x]

[Out] ArcTan[(Sqrt[f]*(e + 2*(d + f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{e^2 + 4efx^3 + 4dfx^6 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 + 4*d*f*x^6 + 4*f^2*x^6), x]

fricas [A] time = 0.41, size = 155, normalized size = 3.69

$$\left[\frac{\sqrt{-df} \log\left(\frac{4(d^2f+2df^2+f^3)x^6+4(def+ef^2)x^3-de^2+e^2f-2(2(de+ef)x^3+e^2)\sqrt{-df}}{4(df+f^2)x^6+4efx^3+e^2}\right)}{12def}, \frac{\sqrt{df} \arctan\left(\frac{(2(d+f)x^3+e)\sqrt{df}}{de}\right)}{6def} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x, algorithm="fricas")

[Out] [-1/12*sqrt(-d*f)*log((4*(d^2*f + 2*d*f^2 + f^3)*x^6 + 4*(d*e*f + e*f^2)*x^3 - d*e^2 + e^2*f - 2*(2*(d*e + e*f)*x^3 + e^2)*sqrt(-d*f))/(4*(d*f + f^2)*x^6 + 4*e*f*x^3 + e^2))/(d*e*f), 1/6*sqrt(d*f)*arctan((2*(d + f)*x^3 + e)*sqrt(d*f)/(d*e))/(d*e*f)]

giac [A] time = 10.13, size = 38, normalized size = 0.90

$$\frac{\arctan\left(\frac{(2dfx^3+2f^2x^3+fe)e^{(-1)}}{\sqrt{df}}\right)e^{(-1)}}{6\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x, algorithm="giac")

[Out] 1/6*arctan((2*d*f*x^3 + 2*f^2*x^3 + f*e)*e^(-1)/sqrt(d*f))*e^(-1)/sqrt(d*f)

maple [A] time = 0.00, size = 42, normalized size = 1.00

$$\frac{\arctan\left(\frac{2(4df+4f^2)x^3+4ef}{4\sqrt{df}e}\right)}{6\sqrt{df}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x)

[Out] 1/6/(d*f)^(1/2)/e*arctan(1/4*(2*(4*d*f+4*f^2)*x^3+4*e*f)/(d*f)^(1/2)/e)

maxima [A] time = 1.42, size = 36, normalized size = 0.86

$$\frac{\arctan\left(\frac{2(df+f^2)x^3+ef}{\sqrt{dfe}}\right)}{6\sqrt{dfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")

[Out] 1/6*arctan((2*(d*f + f^2)*x^3 + e*f)/(sqrt(d*f)*e))/(sqrt(d*f)*e)

mupad [B] time = 3.43, size = 42, normalized size = 1.00

$$\frac{\operatorname{atan}\left(\frac{e\sqrt{f}+2f^{3/2}x^3+2d\sqrt{f}x^3}{\sqrt{d}e}\right)}{6\sqrt{d}e\sqrt{f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e^2 + 4*f^2*x^6 + 4*d*f*x^6 + 4*e*f*x^3),x)

[Out] atan((e*f^(1/2) + 2*f^(3/2)*x^3 + 2*d*f^(1/2)*x^3)/(d^(1/2)*e))/(6*d^(1/2)*e*f^(1/2))

sympy [B] time = 0.79, size = 78, normalized size = 1.86

$$\frac{\sqrt{-\frac{1}{df}} \log\left(x^3 + \frac{-de\sqrt{-\frac{1}{df}} + e}{2d+2f}\right)}{12} + \frac{\sqrt{-\frac{1}{df}} \log\left(x^3 + \frac{de\sqrt{-\frac{1}{df}} + e}{2d+2f}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)

[Out] (-sqrt(-1/(d*f))*log(x**3 + (-d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12 + sqrt(-1/(d*f))*log(x**3 + (d*e*sqrt(-1/(d*f)) + e)/(2*d + 2*f))/12)/e

$$3.300 \quad \int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6} dx$$

Optimal. Leaf size=44

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^3(d-f))}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Rubi [A] time = 0.06, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6, 1352, 618, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2x^3(d-f))}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6),x]

[Out] -ArcTanh[(Sqrt[f]*(e - 2*(d - f)*x^3))/(Sqrt[d]*e)]/(6*Sqrt[d]*e*Sqrt[f])

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6} dx &= \int \frac{x^2}{e^2 + 4efx^3 + (-4df + 4f^2)x^6} dx \\ &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{e^2 + 4efx + (-4df + 4f^2)x^2} dx, x, x^3\right) \\ &= -\left(\frac{2}{3} \text{Subst}\left(\int \frac{1}{16de^2f - x^2} dx, x, 4f(e - 2(d - f)x^3)\right)\right) \\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{f}(e-2(d-f)x^3)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 46, normalized size = 1.05

$$\frac{\tanh^{-1}\left(\frac{\sqrt{f}(-2dx^3+e+2fx^3)}{\sqrt{de}}\right)}{6\sqrt{de}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6), x]

[Out] -1/6*ArcTanh[(Sqrt[f]*(e - 2*d*x^3 + 2*f*x^3))/(Sqrt[d]*e)]/(Sqrt[d]*e*Sqrt[f])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{e^2 + 4efx^3 - 4dfx^6 + 4f^2x^6} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6), x]

[Out] IntegrateAlgebraic[x^2/(e^2 + 4*e*f*x^3 - 4*d*f*x^6 + 4*f^2*x^6), x]

fricas [A] time = 0.41, size = 168, normalized size = 3.82

$$\left[\frac{\sqrt{df} \log\left(-\frac{4(d^2f-2df^2+f^3)x^6-4(def-ef^2)x^3+de^2+e^2f+2(de-ef)x^3-e^2)\sqrt{df}}{4(df-f^2)x^6-4efx^3-e^2}\right)}{12def}, \frac{\sqrt{-df} \arctan\left(-\frac{(2(d-f)x^3-e)\sqrt{-df}}{de}\right)}{6def} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x, algorithm="fricas")

[Out] [1/12*sqrt(d*f)*log(-(4*(d^2*f - 2*d*f^2 + f^3)*x^6 - 4*(d*e*f - e*f^2)*x^3 + d*e^2 + e^2*f + 2*(2*(d*e - e*f)*x^3 - e^2)*sqrt(d*f))/(4*(d*f - f^2)*x^6 - 4*e*f*x^3 - e^2))/(d*e*f), 1/6*sqrt(-d*f)*arctan(-(2*(d - f)*x^3 - e)*sqrt(-d*f)/(d*e))/(d*e*f)]

giac [A] time = 9.22, size = 41, normalized size = 0.93

$$\frac{\arctan\left(\frac{2dfx^3-2f^2x^3-fe}{\sqrt{-dfe^2}}\right)}{6\sqrt{-dfe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x, algorithm="giac")

[Out] -1/6*arctan((2*d*f*x^3 - 2*f^2*x^3 - f*e)/sqrt(-d*f*e^2))/sqrt(-d*f*e^2)

maple [A] time = 0.00, size = 42, normalized size = 0.95

$$\frac{\operatorname{arctanh}\left(\frac{2(4df-4f^2)x^3-4ef}{4\sqrt{df}e}\right)}{6\sqrt{df}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2), x)

[Out] $1/6/(d*f)^{(1/2)}/e*\operatorname{arctanh}(1/4*(2*(4*d*f-4*f^2)*x^3-4*e*f)/(d*f)^{(1/2)}/e)$

maxima [A] time = 1.54, size = 67, normalized size = 1.52

$$\frac{\log\left(\frac{2(df-f^2)x^3-ef+\sqrt{dfe}}{2(df-f^2)x^3-ef-\sqrt{dfe}}\right)}{12\sqrt{dfe}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-4*d*f*x^6+4*f^2*x^6+4*e*f*x^3+e^2),x, algorithm="maxima")`

[Out] $1/12*\log((2*(d*f - f^2)*x^3 - e*f + \operatorname{sqrt}(d*f)*e)/(2*(d*f - f^2)*x^3 - e*f - \operatorname{sqrt}(d*f)*e))/(\operatorname{sqrt}(d*f)*e)$

mupad [B] time = 3.53, size = 923, normalized size = 20.98

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e^2 + 4*f^2*x^6 - 4*d*f*x^6 + 4*e*f*x^3),x)`

[Out] $(\operatorname{atan}(\frac{(x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^{(1/2)}*e*f^{(1/2))})/(d^{(1/2)}*e*f^{(1/2))}) * i)}{d^{(1/2)}*e*f^{(1/2)}} + ((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^{(1/2)}*e*f^{(1/2))})/(d^{(1/2)}*e*f^{(1/2))}) * i)}{d^{(1/2)}*e*f^{(1/2)}})/((x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) + (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 - ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^{(1/2)}*e*f^{(1/2))})/(d^{(1/2)}*e*f^{(1/2))}) - (x^3*(96*d*f^5 - 32*f^6 - 96*d^2*f^4 + 32*d^3*f^3) - (x^3*(64*e*f^7 + 192*d^2*e*f^5 - 64*d^3*e*f^4 - 192*d*e*f^6) + 16*e^2*f^6 - 48*d*e^2*f^5 + 48*d^2*e^2*f^4 - 16*d^3*e^2*f^3 + ((x^3*(384*e^2*f^8 - 1152*d*e^2*f^7 + 1152*d^2*e^2*f^6 - 384*d^3*e^2*f^5))/12 + 16*e^3*f^7 - 48*d*e^3*f^6 + 48*d^2*e^3*f^5 - 16*d^3*e^3*f^4)/(d^{(1/2)}*e*f^{(1/2))})/(d^{(1/2)}*e*f^{(1/2))}) * i)}{6*d^{(1/2)}*e*f^{(1/2)}})$

sympy [A] time = 0.85, size = 75, normalized size = 1.70

$$\frac{\frac{\sqrt{\frac{1}{df}} \log\left(x^3 + \frac{-de\sqrt{\frac{1}{df}} - e}{2d-2f}\right)}{12} - \frac{\sqrt{\frac{1}{df}} \log\left(x^3 + \frac{de\sqrt{\frac{1}{df}} - e}{2d-2f}\right)}{12}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-4*d*f*x**6+4*f**2*x**6+4*e*f*x**3+e**2),x)`

[Out] $-(\operatorname{sqrt}(1/(d*f))*\log(x**3 + (-d*e*\operatorname{sqrt}(1/(d*f)) - e)/(2*d - 2*f)))/12 - \operatorname{sqrt}(1/(d*f))*\log(x**3 + (d*e*\operatorname{sqrt}(1/(d*f)) - e)/(2*d - 2*f))/12)/e$

$$3.301 \quad \int \frac{x^m (e(1+m) + 2f(-2+m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 + 4dfx^{2+2m}} dx$$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d*f*x^(2 + 2*m)),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^m (e(1+m) + 2f(-2+m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 + 4dfx^{2+2m}} dx = -\left((e^2(2-m)(1+m)) \text{Subst}\left(\int \frac{1}{e^2 + 4de^2f(-2+m)^2(1+m)^2x^2} dx, \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}\right)\right)$$

Mathematica [A] time = 0.32, size = 42, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 + 4*d*f*x^(2 + 2*m)),x]

[Out] ArcTan[(2*sqrt[d]*sqrt[f]*x^(1+m))/(e+2*f*x^3)]/(2*sqrt[d]*sqrt[f])

IntegrateAlgebraic [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(-2+m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 + 4dfx^{2+2m}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1+m)+2*f*(-2+m)*x^3))/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1+m)+2*f*(-2+m)*x^3))/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)), x]

fricas [A] time = 0.45, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{-df} \log\left(-\frac{4f^2x^6 - 4dfx^2x^{2m} + 4efx^3 + 4(2fx^4 + ex)\sqrt{-df}x^m + e^2}{4f^2x^6 + 4dfx^2x^{2m} + 4efx^3 + e^2}\right)}{4df}, \frac{\sqrt{df} \arctan\left(\frac{(2fx^3 + e)\sqrt{df}}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log(-(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4 + e*x)*sqrt(-d*f)*x^m + e^2)/(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(d*f)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-2)x^3 + e(m+1))x^m}{4f^2x^6 + 4efx^3 + 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x, algorithm="giac")

[Out] integrate((2*f*(m-2)*x^3 + e*(m+1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m+2) + e^2), x)

maple [B] time = 0.05, size = 78, normalized size = 1.86

$$\frac{\ln\left(x^m - \frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^m + \frac{(2fx^3+e)\sqrt{-df}}{2dfx}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)), x)

[Out] -1/4/(-d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)+1/4/(-d*f)^(1/2)*ln(x^m-1/2*(2*f*x^3+e)*(-d*f)^(1/2)/d/f/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-2)x^3 + e(m+1))x^m}{4f^2x^6 + 4efx^3 + 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6+4*d*f*x^(2+2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (2 f (m - 2) x^3 + e (m + 1))}{e^2 + 4 f^2 x^6 + 4 e f x^3 + 4 d f x^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 + 4*d*f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6+4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

$$3.302 \quad \int \frac{x^m (e(1+m) + 2f(-2+m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 - 4dfx^{2+2m}} dx$$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.22, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 51, $\frac{\text{number of rules}}{\text{integrand size}} = 0.039$, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*f*x^(2 + 2*m)), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^m (e(1+m) + 2f(-2+m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 - 4dfx^{2+2m}} dx = -\left((e^2(2-m)(1+m)) \text{Subst}\left(\int \frac{1}{e^2 - 4de^2f(-2+m)^2(1+m)^2x^2} dx, x, \frac{x^{m+1}}{e+2fx^3}\right)\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^3}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*f*x^(2 + 2*m)), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^3)]/(2*Sqrt[d]*Sqrt[f])

IntegrateAlgebraic [F] time = 0.57, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1 + m) + 2f(-2 + m)x^3)}{e^2 + 4efx^3 + 4f^2x^6 - 4dfx^{2+2m}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*f*x^(2 + 2*m)), x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1 + m) + 2*f*(-2 + m)*x^3))/(e^2 + 4*e*f*x^3 + 4*f^2*x^6 - 4*d*f*x^(2 + 2*m)), x]

fricas [A] time = 0.52, size = 146, normalized size = 3.48

$$\left[\frac{\sqrt{df} \log\left(-\frac{4f^2x^6 + 4dfx^2x^{2m} + 4efx^3 + 4(2fx^4 + ex)\sqrt{df}x^m + e^2}{4f^2x^6 - 4dfx^2x^{2m} + 4efx^3 + e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{(2fx^3 + e)\sqrt{-df}}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log(-(4*f^2*x^6 + 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + 4*(2*f*x^4 + e*x)*sqrt(d*f)*x^m + e^2)/(4*f^2*x^6 - 4*d*f*x^2*x^(2*m) + 4*e*f*x^3 + e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*f*x^3 + e)*sqrt(-d*f)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-2)x^3 + e(m+1))x^m}{4f^2x^6 + 4efx^3 - 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)), x, algorithm="giac")

[Out] integrate((2*f*(m-2)*x^3 + e*(m+1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m+2) + e^2), x)

maple [B] time = 0.06, size = 74, normalized size = 1.76

$$-\frac{\ln\left(x^m - \frac{(2fx^3 + e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}} + \frac{\ln\left(x^m + \frac{(2fx^3 + e)\sqrt{df}}{2dfx}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)), x)

[Out] 1/4/(d*f)^(1/2)*ln(x^m+1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)-1/4/(d*f)^(1/2)*ln(x^m-1/2*(2*f*x^3+e)*(d*f)^(1/2)/d/f/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-2)x^3 + e(m+1))x^m}{4f^2x^6 + 4efx^3 - 4dfx^{2m+2} + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(-2+m)*x^3)/(e^2+4*e*f*x^3+4*f^2*x^6-4*d*f*x^(2+2*m)),x, algorithm="maxima")
```

```
[Out] integrate((2*f*(m - 2)*x^3 + e*(m + 1))*x^m/(4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m + 2) + e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (2f(m-2)x^3 + e(m+1))}{e^2 + 4f^2x^6 + 4efx^3 - 4dfx^{2m+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m + 2)),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^3*(m - 2)))/(e^2 + 4*f^2*x^6 + 4*e*f*x^3 - 4*d*f*x^(2*m + 2)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(-2+m)*x**3)/(e**2+4*e*f*x**3+4*f**2*x**6-4*d*f*x**(2+2*m)),x)
```

```
[Out] Timed out
```

$$3.303 \quad \int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Optimal. Leaf size=42

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.25, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2094, 205}

$$\frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1+m) + 2*f*(1+m-n)*x^n))/(e^2 + 4*d*f*x^(2+2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] ArcTan[(2*Sqrt[d]*Sqrt[f]*x^(1+m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] :> Dist[(A^2*(m-n+1))/(m+1), Subst[Int[1/(a + A^2*b*(m-n+1)^2*x^2), x], x, x^(m+1)/(A*(m-n+1) + B*(m+1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m+1)] && EqQ[a*B^2*(m+1)^2 - A^2*d*(m-n+1)^2, 0] && EqQ[B*c*(m+1) - 2*A*d*(m-n+1), 0]

Rubi steps

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx = (e^2(1+m)(1+m-n)) \text{Subst}\left(\int \frac{1}{e^2 + 4de^2f(1+m)^2(1+m-n)^2x^2}\right) \\ = \frac{\tan^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.47, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] Integrate[(x^m*(e*(1+m) + 2*f*(1+m-n)*x^n))/(e^2 + 4*d*f*x^(2+2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)),x]

[Out] Integrate[(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

IntegrateAlgebraic [F] time = 0.23, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 + 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

fricas [A] time = 1.11, size = 165, normalized size = 3.93

$$\left[-\frac{\sqrt{-df} \log\left(-\frac{4dfx^2x^{2m}-4\sqrt{-df}exx^m-4f^2x^{2n}-e^2-4(2\sqrt{-df}fxx^m+ef)x^n}{4dfx^2x^{2m}+4f^2x^{2n}+4efx^n+e^2}\right)}{4df}, -\frac{\sqrt{df} \arctan\left(\frac{2\sqrt{df}fx^n+\sqrt{df}e}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="fricas")

[Out] [-1/4*sqrt(-d*f)*log(-(4*d*f*x^2*x^(2*m) - 4*sqrt(-d*f)*e*x*x^m - 4*f^2*x^(2*n) - e^2 - 4*(2*sqrt(-d*f)*f*x*x^m + e*f)*x^n)/(4*d*f*x^2*x^(2*m) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2))/(d*f), -1/2*sqrt(d*f)*arctan(1/2*(2*sqrt(d*f)*f*x^n + sqrt(d*f)*e)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-n+1)x^n + e(m+1))x^m}{4dfx^{2m+2} + 4f^2x^{2n} + 4efx^n + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="giac")

[Out] integrate((2*f*(m-n+1)*x^n + e*(m+1))*x^m/(4*d*f*x^(2*m+2) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)

maple [B] time = 0.09, size = 84, normalized size = 2.00

$$\frac{\ln\left(x^n + \frac{-2dfxx^m + \sqrt{-df}e}{2\sqrt{-df}f}\right)}{4\sqrt{-df}} - \frac{\ln\left(x^n + \frac{2dfxx^m + \sqrt{-df}e}{2\sqrt{-df}f}\right)}{4\sqrt{-df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x)

[Out] -1/4/(-d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)+1/4/(-d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(-d*f)^(1/2)*e)/(-d*f)^(1/2)/f)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2f(m-n+1)x^n + e(m+1))x^m}{4dfx^{2m+2} + 4f^2x^{2n} + 4efx^n + e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2+4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="maxima")

[Out] integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) + 4*f^2*x^(2*n) + 4*e*f*x^n + e^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (e(m+1) + 2fx^n(m-n+1))}{e^2 + 4f^2x^{2n} + 4dfx^{2m+2} + 4efx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^(2*m + 2) + 4*e*f*x^n),x)

[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) + 4*d*f*x^(2*m + 2) + 4*e*f*x^n), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m (em + e + 2fmx^n - 2fnx^n + 2fx^n)}{4dfx^2x^{2m} + e^2 + 4efx^n + 4f^2x^{2n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2+4*d*f*x**(2+2*m)+4*e*f*x**n+4*f**2*x**(2*n)),x)

[Out] Integral(x**m*(e*m + e + 2*f*m*x**n - 2*f*n*x**n + 2*f*x**n)/(4*d*f*x**2*x**(2*m) + e**2 + 4*e*f*x**n + 4*f**2*x**(2*n)), x)

$$3.304 \quad \int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Rubi [A] time = 0.24, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 56, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {2094, 208}

$$\frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{m+1}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

[In] Int[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] ArcTanh[(2*Sqrt[d]*Sqrt[f]*x^(1 + m))/(e + 2*f*x^n)]/(2*Sqrt[d]*Sqrt[f])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2094

Int[((x_)^(m_.)*((A_) + (B_.)*(x_)^(n_.)))/((a_) + (b_.)*(x_)^(k_.) + (c_.)*(x_)^(n_.) + (d_.)*(x_)^(n2_)), x_Symbol] := Dist[(A^2*(m - n + 1))/(m + 1), Subst[Int[1/(a + A^2*b*(m - n + 1)^2*x^2), x], x, x^(m + 1)/(A*(m - n + 1) + B*(m + 1)*x^n)], x] /; FreeQ[{a, b, c, d, A, B, m, n}, x] && EqQ[n2, 2*n] && EqQ[k, 2*(m + 1)] && EqQ[a*B^2*(m + 1)^2 - A^2*d*(m - n + 1)^2, 0] && EqQ[B*c*(m + 1) - 2*A*d*(m - n + 1), 0]

Rubi steps

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx = (e^2(1+m)(1+m-n)) \text{Subst}\left(\int \frac{1}{e^2 - 4de^2f(1+m)^2(1+m-n)^2x^2} dx\right) = \frac{\tanh^{-1}\left(\frac{2\sqrt{d}\sqrt{f}x^{1+m}}{e+2fx^n}\right)}{2\sqrt{d}\sqrt{f}}$$

Mathematica [F] time = 0.50, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] Integrate[(x^m*(e*(1 + m) + 2*f*(1 + m - n)*x^n))/(e^2 - 4*d*f*x^(2 + 2*m) + 4*e*f*x^n + 4*f^2*x^(2*n)), x]

[Out] Integrate[(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

IntegrateAlgebraic [F] time = 0.22, size = 0, normalized size = 0.00

$$\int \frac{x^m (e(1+m) + 2f(1+m-n)x^n)}{e^2 - 4dfx^{2+2m} + 4efx^n + 4f^2x^{2n}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

[Out] Defer[IntegrateAlgebraic] [(x^m*(e*(1+m)+2*f*(1+m-n)*x^n))/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x]

fricas [A] time = 0.43, size = 165, normalized size = 3.93

$$\left[\frac{\sqrt{df} \log\left(-\frac{4dfx^{2m+4}\sqrt{df}exx^m+4f^2x^{2n}+e^2+4(2\sqrt{df}fxx^m+ef)x^n}{4dfx^{2m}-4f^2x^{2n}-4efx^n-e^2}\right)}{4df}, -\frac{\sqrt{-df} \arctan\left(\frac{2\sqrt{-df}fx^n+\sqrt{-df}e}{2dfxx^m}\right)}{2df} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="fricas")

[Out] [1/4*sqrt(d*f)*log(-(4*d*f*x^2*x^(2*m)+4*sqrt(d*f)*e*x*x^m+4*f^2*x^(2*n)+e^2+4*(2*sqrt(d*f)*f*x*x^m+e*f)*x^n)/(4*d*f*x^2*x^(2*m)-4*f^2*x^(2*n)-4*e*f*x^n-e^2))/(d*f), -1/2*sqrt(-d*f)*arctan(1/2*(2*sqrt(-d*f)*f*x^n+sqrt(-d*f)*e)/(d*f*x*x^m))/(d*f)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{(2f(m-n+1)x^n+e(m+1))x^m}{4dfx^{2m+2}-4f^2x^{2n}-4efx^n-e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x, algorithm="giac")

[Out] integrate(-(2*f*(m-n+1)*x^n+e*(m+1))*x^m/(4*d*f*x^(2*m+2)-4*f^2*x^(2*n)-4*e*f*x^n-e^2), x)

maple [B] time = 0.08, size = 78, normalized size = 1.86

$$-\frac{\ln\left(x^n+\frac{-2dfxx^m+\sqrt{df}e}{2\sqrt{df}f}\right)}{4\sqrt{df}}+\frac{\ln\left(x^n+\frac{2dfxx^m+\sqrt{df}e}{2\sqrt{df}f}\right)}{4\sqrt{df}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(e*(m+1)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)), x)

[Out] 1/4/(d*f)^(1/2)*ln(x^n+1/2*(2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)-1/4/(d*f)^(1/2)*ln(x^n+1/2*(-2*d*f*x*x^m+(d*f)^(1/2)*e)/(d*f)^(1/2)/f)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{(2f(m-n+1)x^n+e(m+1))x^m}{4dfx^{2m+2}-4f^2x^{2n}-4efx^n-e^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*(e*(1+m)+2*f*(1+m-n)*x^n)/(e^2-4*d*f*x^(2+2*m)+4*e*f*x^n+4*f^2*x^(2*n)),x, algorithm="maxima")
```

```
[Out] -integrate((2*f*(m - n + 1)*x^n + e*(m + 1))*x^m/(4*d*f*x^(2*m + 2) - 4*f^2*x^(2*n) - 4*e*f*x^n - e^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^m (e (m + 1) + 2 f x^n (m - n + 1))}{e^2 + 4 f^2 x^{2n} - 4 d f x^{2m+2} + 4 e f x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^(2*m + 2) + 4*e*f*x^n),x)
```

```
[Out] int((x^m*(e*(m + 1) + 2*f*x^n*(m - n + 1)))/(e^2 + 4*f^2*x^(2*n) - 4*d*f*x^(2*m + 2) + 4*e*f*x^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{e x^m}{4 d f x^{2+2 m}-e^2-4 e f x^n-4 f^2 x^{2 n}} dx - \int \frac{e m x^m}{4 d f x^{2+2 m}-e^2-4 e f x^n-4 f^2 x^{2 n}} dx - \int \frac{2 f x^m x^n}{4 d f x^{2+2 m}-e^2-4 e f x^n-4 f^2 x^{2 n}} dx - \int \frac{2 f m x^m x^n}{4 d f x^{2+2 m}-e^2-4 e f x^n-4 f^2 x^{2 n}} dx - \int \left(-\frac{2 f n x^m x^n}{4 d f x^{2+2 m}-e^2-4 e f x^n-4 f^2 x^{2 n}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*(e*(1+m)+2*f*(1+m-n)*x**n)/(e**2-4*d*f*x**(2+2*m)+4*e*f*x**n+4*f**2*x**(2*n)),x)
```

```
[Out] -Integral(e*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(e*m*x**m/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(2*f*m*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x) - Integral(-2*f*n*x**m*x**n/(4*d*f*x**2*x**(2*m) - e**2 - 4*e*f*x**n - 4*f**2*x**(2*n)), x)
```

$$3.305 \quad \int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=134

$$-\frac{d(a+bx^2)^{3/2}}{3b^3c^2} + \frac{(ac^2-d^2)^2 \log(c\sqrt{a+bx^2}+d)}{b^3c^5} + \frac{d\sqrt{a+bx^2}(2ac^2-d^2)}{b^3c^4} + \frac{(a+bx^2)^2}{4b^3c} - \frac{x^2(2ac^2-d^2)}{2b^2c^3}$$

Rubi [A] time = 0.37, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 697}

$$-\frac{x^2(2ac^2-d^2)}{2b^2c^3} + \frac{d\sqrt{a+bx^2}(2ac^2-d^2)}{b^3c^4} + \frac{(ac^2-d^2)^2 \log(c\sqrt{a+bx^2}+d)}{b^3c^5} - \frac{d(a+bx^2)^{3/2}}{3b^3c^2} + \frac{(a+bx^2)^2}{4b^3c}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]

[Out] -((2*a*c^2 - d^2)*x^2)/(2*b^2*c^3) + (d*(2*a*c^2 - d^2)*Sqrt[a + b*x^2])/(b^3*c^4) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) + (a + b*x^2)^2/(4*b^3*c) + ((a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[1/n, Subst[Int[x^((m+1)/n-1)/(c+d*x+e*Sqrt[a+b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m+1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{ac+bcx^2+d\sqrt{a+bx^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}} dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{(a-x^2)^2}{d+cx} dx, x, \sqrt{a+bx^2} \right)}{b^3} \\ &= \frac{\text{Subst} \left(\int \left(\frac{2ac^2d-d^3}{c^4} - \frac{(2ac^2-d^2)x}{c^3} - \frac{dx^2}{c^2} + \frac{x^3}{c} + \frac{(ac^2-d^2)^2}{c^4(d+cx)} \right) dx, x, \sqrt{a+bx^2} \right)}{b^3} \\ &= -\frac{(2ac^2-d^2)x^2}{2b^2c^3} + \frac{d(2ac^2-d^2)\sqrt{a+bx^2}}{b^3c^4} - \frac{d(a+bx^2)^{3/2}}{3b^3c^2} + \frac{(a+bx^2)^2}{4b^3c} + \frac{(a+bx^2)^2}{4b^3c} \end{aligned}$$

Mathematica [A] time = 0.22, size = 126, normalized size = 0.94

$$\frac{c \left(a \left(20c^2d\sqrt{a+bx^2} - 6bc^3x^2 \right) + 2bcdx^2 \left(3d - 2c\sqrt{a+bx^2} \right) - 12d^3\sqrt{a+bx^2} + 3b^2c^3x^4 \right) + 12(d^2-ac^2)^2 \log(c\sqrt{a+bx^2}+d)}{12b^3c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]

[Out] (c*(3*b^2*c^3*x^4 - 12*d^3*Sqrt[a + b*x^2] + 2*b*c*d*x^2*(3*d - 2*c*Sqrt[a + b*x^2])) + a*(-6*b*c^3*x^2 + 20*c^2*d*Sqrt[a + b*x^2])) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a + b*x^2]]/(12*b^3*c^5)

IntegrateAlgebraic [A] time = 0.17, size = 142, normalized size = 1.06

$$\frac{(a^2c^4 - 2ac^2d^2 + d^4)\log\left(c\sqrt{a + bx^2} + d\right)}{b^3c^5} + \frac{-3a^2c^2 - 2abc^2x^2 + 2ad^2 + b^2c^2x^4 + 2bd^2x^2}{4b^3c^3} + \frac{\sqrt{a + bx^2}(5ac^2d - bc^2dx^2 - 3d^3)}{3b^3c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]

[Out] (Sqrt[a + b*x^2]*(5*a*c^2*d - 3*d^3 - b*c^2*d*x^2))/(3*b^3*c^4) + (-3*a^2*c^2 + 2*a*d^2 - 2*a*b*c^2*x^2 + 2*b*d^2*x^2 + b^2*c^2*x^4)/(4*b^3*c^3) + ((a^2*c^4 - 2*a*c^2*d^2 + d^4)*Log[d + c*Sqrt[a + b*x^2]])/(b^3*c^5)

fricas [A] time = 0.61, size = 233, normalized size = 1.74

$$\frac{3b^2c^4x^4 - 6(ab^2c^4 - bc^2d^2)x^2 + 6(a^2c^4 - 2ac^2d^2 + d^4)\log(bc^2x^2 + ac^2 - d^2) + 3(a^2c^4 - 2ac^2d^2 + d^4)\log\left(\frac{bx^2 + a + \sqrt{bx^2 + a}}{x}\right) - 3(a^2c^4 - 2ac^2d^2 + d^4)\log\left(\frac{-bx^2 + a + \sqrt{bx^2 + a}}{x}\right) - 4(bc^3dx^2 - 5ac^3d + 3cd^3)\sqrt{bx^2 + a}}{12b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x, algorithm="fricas")

[Out] 1/12*(3*b^2*c^4*x^4 - 6*(a*b*c^4 - b*c^2*d^2)*x^2 + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(b*c^2*x^2 + a*c^2 - d^2) + 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a))*c*d + d^2)/x^2) - 3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a))*c*d + d^2)/x^2) - 4*(b*c^3*d*x^2 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^2 + a))/(b^3*c^5)

giac [A] time = 0.35, size = 155, normalized size = 1.16

$$\frac{(a^2c^4 - 2ac^2d^2 + d^4)\log\left(\sqrt{bx^2 + a} + d\right)}{b^3c^5} + \frac{3(bx^2 + a)^2b^9c^3 - 12(bx^2 + a)ab^9c^3 - 4(bx^2 + a)^{\frac{3}{2}}b^9c^2d + 24\sqrt{bx^2 + a}ab^9c^2d + 6(bx^2 + a)b^9cd^2 - 12\sqrt{bx^2 + a}b^9d^3}{12b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x, algorithm="giac")

[Out] (a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^2 + a)*c + d))/(b^3*c^5) + 1/12*(3*(b*x^2 + a)^2*b^9*c^3 - 12*(b*x^2 + a)*a*b^9*c^3 - 4*(b*x^2 + a)^(3/2)*b^9*c^2*d + 24*sqrt(b*x^2 + a)*a*b^9*c^2*d + 6*(b*x^2 + a)*b^9*c*d^2 - 12*sqrt(b*x^2 + a)*b^9*d^3)/(b^12*c^4)

maple [B] time = 0.09, size = 4947, normalized size = 36.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x)

[Out] -a/c^3/b^3*d^2*ln(b*c^2*x^2+a*c^2-d^2)-1/3*d*(b*x^2+a)^(3/2)/b^3/c^2-1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2)*a*d^3+1/b^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))/c^2*d^5/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)

$$\begin{aligned}
& /c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)})/(x-(-(a*c^2-d^2) \\
&)*b*c^2)^{(1/2)}/b/c^2)*a+1/2/b^3/c^5*d^4*\ln(b*c^2*x^2+a*c^2-d^2)-1/2*a/c/b^ \\
& 2*x^2+1/2*a^2/c/b^3*\ln(b*c^2*x^2+a*c^2-d^2)+1/2/b^2/c^3*d^2*x^2-1/2/b^2/((- \\
& a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)* \\
& b*c^2)^{(1/2)})*d^3/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/ \\
& c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+2*(1/c^2*d^2)^{(1/2)}*(b*(x-(-(a \\
& *c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x-(-(a*c^ \\
& 2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)})/(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/ \\
& b/c^2)*a^2-1/2/b^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(\\
& 1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})/c^4*d^7/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2*d^ \\
& 2+2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+2*(\\
& 1/c^2*d^2)^{(1/2)}*(b*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2+2*(-(a*c^2-d^2)* \\
& b*c^2)^{(1/2)}/c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)})/(x- \\
& (-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)-1/2*d/b^2*c^2*a^2/((-a*b)^{(1/2)}*c^2+(-(a \\
& *c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*(b*(x \\
& +(-a*b)^{(1/2)}/b)^2-2*(-a*b)^{(1/2)}*(x+(-a*b)^{(1/2)}/b))^(1/2)-1/2*d/b^2*c^2*a \\
& ^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2 \\
& -d^2)*b*c^2)^{(1/2)})*(b*(x+(-a*b)^{(1/2)}/b)^2+2*(-a*b)^{(1/2)}*(x+(-a*b)^{(1/2)}/ \\
& b))^(1/2)+1/2*d/b^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(\\
& 1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*c^2*(b*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b \\
& /c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^ \\
& 2)+1/c^2*d^2)^{(1/2)}*a^2-1/2*d/b^(5/2)/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2 \\
&)^(1/2))/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*(-(a*c^2-d^2)*b*c^2) \\
& ^{(1/2)}*\ln((-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/ \\
& b/c^2))/b^(1/2)+b*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2-2*(-(a*c^2-d^2)*b \\
& *c^2)^{(1/2)}/c^2*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)}*a^2- \\
& 1/2/b^(5/2)/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2 \\
& -(-(a*c^2-d^2)*b*c^2)^{(1/2)})/c^4*(-(a*c^2-d^2)*b*c^2)^{(1/2)}*\ln((-(-(a*c^2-d \\
& ^2)*b*c^2)^{(1/2)}/c^2+b*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2))/b^(1/2)+b*(x+ \\
& (-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x+(- \\
& a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)}*d^5-1/2/b^2/((-a*b)^{(1/2)}* \\
& c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2) \\
&))*d^3/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x+ \\
& (-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+2*(1/c^2*d^2)^{(1/2)}*(b*(x+(-(a*c^2-d^2)*b \\
& *c^2)^{(1/2)}/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x+(-(a*c^2-d^2)*b*c^ \\
& 2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)})/(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)*a^2 \\
& -1/2/b^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(- \\
& (a*c^2-d^2)*b*c^2)^{(1/2)})/c^4*d^7/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2*d^2-2*(-(a*c^ \\
& 2-d^2)*b*c^2)^{(1/2)}/c^2*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+2*(1/c^2*d^2)^ \\
& (1/2)*b*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2) \\
&)/c^2*(x+(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)})/(x+(-(a*c^2-d^ \\
& 2)*b*c^2)^{(1/2)}/b/c^2)+1/2*d/b^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1 \\
& /2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*c^2*(b*(x-(-(a*c^2-d^2)* \\
& b*c^2)^{(1/2)}/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x-(-(a*c^2-d^2)*b*c \\
& ^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)}*a^2+1/2*d/b^(5/2)/((-a*b)^{(1/2)}*c^2+(-(a* \\
& c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*(-(a*c \\
& ^2-d^2)*b*c^2)^{(1/2)}*\ln((-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2+b*(x-(-(a*c^2-d^2)* \\
& b*c^2)^{(1/2)}/b/c^2))/b^(1/2)+b*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2+2*(- \\
& (a*c^2-d^2)*b*c^2)^{(1/2)}/c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2 \\
&)^(1/2)*a^2+1/2/b^(5/2)/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a \\
& *b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})/c^4*(-(a*c^2-d^2)*b*c^2)^{(1/2)}*\ln \\
& (((-(a*c^2-d^2)*b*c^2)^{(1/2)}/c^2+b*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2))/b^ \\
& (1/2)+b*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2+2*(-(a*c^2-d^2)*b*c^2)^{(1/2) \\
&)/c^2*(x-(-(a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)+1/c^2*d^2)^{(1/2)}*d^5+1/4/b/c*x^ \\
& 4+1/2*d/b^(5/2)*c^2*a^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a* \\
& b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*(-a*b)^{(1/2)}*\ln((b*(x+(-a*b)^{(1/2) \\
& /b)-(-a*b)^{(1/2)})/b^(1/2)+b*(x+(-a*b)^{(1/2)}/b)^2-2*(-a*b)^{(1/2)}*(x+(-a*b) \\
& ^{(1/2)}/b))^(1/2)-1/2*d/b^(5/2)*c^2*a^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^ \\
& 2)^{(1/2)})/((-a*b)^{(1/2)}*c^2-(-(a*c^2-d^2)*b*c^2)^{(1/2)})*(-a*b)^{(1/2)}*\ln((b*
\end{aligned}$$

$(x - (-a*b)^{(1/2)/b}) + (-a*b)^{(1/2)/b} + (b*(x - (-a*b)^{(1/2)/b})^2 + 2*(-a*b)^{(1/2)/b} * (x - (-a*b)^{(1/2)/b}))^{(1/2)} + 1/b^{(5/2)} / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / c^2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} * \ln(((-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 + b * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)) / b^{(1/2)} + (b * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 - 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} * a * d^3 + 1 / b^2 / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / c^2 * d^5 / (1 / c^2 * d^2)^{(1/2)} * \ln((2 / c^2 * d^2 - 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2) + 2 * (1 / c^2 * d^2)^{(1/2)} * (b * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 - 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} / (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2) * a - 1 / b^{(5/2)} / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / c^2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} * \ln(((a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 + b * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)) / b^{(1/2)} + (b * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 + 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} * a * d^3 + 1 / 2 / b^2 / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / c^2 * (b * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 - 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x + (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} * d^5 - 1 / b^2 / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) * (b * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 + 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} * a * d^3 + 1 / 2 / b^2 / (((-a*b)^{(1/2)/b} * c^2 + (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / (((-a*b)^{(1/2)/b} * c^2 - (-a*c^2 - d^2) * b * c^2)^{(1/2)}) / c^2 * (b * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 + 2 * (-a*c^2 - d^2) * b * c^2)^{(1/2)} / c^2 * (x - (-a*c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2 + 1 / c^2 * d^2)^{(1/2)} * d^5$

maxima [A] time = 0.65, size = 125, normalized size = 0.93

$$\frac{3(bx^2+a)^2c^3 - 4(bx^2+a)^{\frac{3}{2}}c^2d - 6(2ac^3 - cd^2)(bx^2+a) + 12(2ac^2d - d^3)\sqrt{bx^2+a}}{c^4} + \frac{12(a^2c^4 - 2ac^2d^2 + d^4)\log(\sqrt{bx^2+a} + d)}{c^5}$$

12b³

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] 1/12*((3*(b*x^2 + a)^2*c^3 - 4*(b*x^2 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2)*(b*x^2 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^2 + a))/c^4 + 12*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^2 + a)*c + d)/c^5)/b^3

mupad [B] time = 3.64, size = 167, normalized size = 1.25

$$\frac{x^4}{4bc} - \sqrt{bx^2+a} \left(\frac{d^3}{b^3c^4} - \frac{2ad}{b^3c^2} \right) - \frac{d(bx^2+a)^{3/2}}{3b^3c^2} - \frac{x^2(a^2-d^2)}{2b^2c^3} + \frac{\operatorname{atanh}\left(\frac{c\sqrt{bx^2+a}}{d}\right)(a^2-d^2)^2}{b^3c^5} + \frac{\ln(bc^2x^2+ac^2-d^2)(a^2c^4-2ac^2d^2+d^4)}{2b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)

[Out] x^4/(4*b*c) - (a + b*x^2)^(1/2)*(d^3/(b^3*c^4) - (2*a*d)/(b^3*c^2)) - (d*(a + b*x^2)^(3/2))/(3*b^3*c^2) - (x^2*(a*c^2 - d^2))/(2*b^2*c^3) + (atanh((c*(a + b*x^2)^(1/2))/d)*(a*c^2 - d^2)^2)/(b^3*c^5) + (log(a*c^2 - d^2 + b*c^2*x^2)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(2*b^3*c^5)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{ac + bcx^2 + d\sqrt{a + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(x**5/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)
```

```
[Out] Integral(x**5/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

$$3.306 \quad \int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=69

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2} - \frac{(ac^2-d^2)\log(c\sqrt{a+bx^2}+d)}{b^2c^3} + \frac{x^2}{2bc}$$

Rubi [A] time = 0.21, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 697}

$$-\frac{(ac^2-d^2)\log(c\sqrt{a+bx^2}+d)}{b^2c^3} - \frac{d\sqrt{a+bx^2}}{b^2c^2} + \frac{x^2}{2bc}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] x^2/(2*b*c) - (d*Sqrt[a + b*x^2])/(b^2*c^2) - ((a*c^2 - d^2)*Log[d + c*Sqrt[a + b*x^2]])/(b^2*c^3)

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^((m+1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m+1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{ac+bcx^2+d\sqrt{a+bx^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}} dx, x, x^2 \right) \\ &= \frac{\text{Subst} \left(\int \frac{-a+x^2}{d+cx} dx, x, \sqrt{a+bx^2} \right)}{b^2} \\ &= \frac{\text{Subst} \left(\int \left(-\frac{d}{c^2} + \frac{x}{c} + \frac{-ac^2+d^2}{c^2(d+cx)} \right) dx, x, \sqrt{a+bx^2} \right)}{b^2} \\ &= \frac{x^2}{2bc} - \frac{d\sqrt{a+bx^2}}{b^2c^2} - \frac{(ac^2-d^2)\log(d+c\sqrt{a+bx^2})}{b^2c^3} \end{aligned}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 0.94

$$-\frac{d\sqrt{a+bx^2}}{c^2} - \frac{(ac^2-d^2)\log(c\sqrt{a+bx^2}+d)}{c^3} + \frac{bx^2}{2c}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] ((b*x^2)/(2*c) - (d*Sqrt[a + b*x^2])/c^2 - ((a*c^2 - d^2)*Log[d + c*Sqrt[a + b*x^2]])/c^3)/b^2

IntegrateAlgebraic [A] time = 0.08, size = 71, normalized size = 1.03

$$-\frac{d\sqrt{a+bx^2}}{b^2c^2} + \frac{(d^2-ac^2)\log\left(c\sqrt{a+bx^2}+d\right)}{b^2c^3} + \frac{a+bx^2}{2b^2c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] -((d*Sqrt[a + b*x^2])/(b^2*c^2)) + (a + b*x^2)/(2*b^2*c) + ((-(a*c^2) + d^2)*Log[d + c*Sqrt[a + b*x^2]])/(b^2*c^3)

fricas [B] time = 0.67, size = 161, normalized size = 2.33

$$\frac{2bc^2x^2 - 4\sqrt{bx^2+acd} - 2(ac^2-d^2)\log(bc^2x^2+ac^2-d^2) - (ac^2-d^2)\log\left(-\frac{bc^2x^2+ac^2+2\sqrt{bx^2+acd}+d^2}{x^2}\right) + (ac^2-d^2)\log\left(-\frac{bc^2x^2+ac^2-2\sqrt{bx^2+acd}+d^2}{x^2}\right)}{4b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] 1/4*(2*b*c^2*x^2 - 4*sqrt(b*x^2 + a)*c*d - 2*(a*c^2 - d^2)*log(b*c^2*x^2 + a*c^2 - d^2) - (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) + (a*c^2 - d^2)*log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2))/(b^2*c^3)

giac [A] time = 0.40, size = 72, normalized size = 1.04

$$-\frac{\frac{2(ac^2-d^2)\log\left(\left|\sqrt{bx^2+ac}+d\right|\right)}{bc^3} - \frac{(bx^2+a)bc-2\sqrt{bx^2+ac}bd}{b^2c^2}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out] -1/2*(2*(a*c^2 - d^2)*log(abs(sqrt(b*x^2 + a)*c + d))/(b*c^3) - ((b*x^2 + a)*b*c - 2*sqrt(b*x^2 + a)*b*d)/(b^2*c^2))/b

maple [B] time = 0.04, size = 3410, normalized size = 49.42

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out] -1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b*(b*(x+(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2*a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/b^(3/2)*(-a*b)^(1/2)*ln((b*(x+(-a*b)^(1/2)/b)+(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] 1/2*((b*x^2 + a)*c - 2*sqrt(b*x^2 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b*x^2 + a)*c + d)/c^3)/b^2

mupad [B] time = 3.50, size = 123, normalized size = 1.78

$$\frac{x^2}{2bc} - \frac{d\sqrt{bx^2+a}}{b^2c^2} + \frac{\operatorname{atanh}\left(\frac{c(ac^2-d^2)\sqrt{bx^2+a}}{d^3-ac^2d}\right)(ac^2-d^2)}{b^2c^3} - \frac{\ln(bc^2x^2+ac^2-d^2)(ac^2-d^2)}{2b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)

[Out] x^2/(2*b*c) - (d*(a + b*x^2)^(1/2))/(b^2*c^2) + (atanh((c*(a*c^2 - d^2)*(a + b*x^2)^(1/2))/(d^3 - a*c^2*d))*(a*c^2 - d^2))/(b^2*c^3) - (log(a*c^2 - d^2 + b*c^2*x^2)*(a*c^2 - d^2))/(2*b^2*c^3)

sympy [A] time = 6.51, size = 88, normalized size = 1.28

$$\left\{ \begin{array}{ll} \frac{(ac^2-d^2) \left(\begin{array}{ll} \frac{\sqrt{a+bx^2}}{d} & \text{for } c = 0 \\ \frac{\log(c\sqrt{a+bx^2}+d)}{c} & \text{otherwise} \end{array} \right)}{\frac{a+bx^2}{2bc} - \frac{d\sqrt{a+bx^2}}{bc^2}} \frac{1}{bc^2} & \text{for } b \neq 0 \\ \frac{x^4}{2(2\sqrt{a}d+2ac)} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)

[Out] Piecewise((((a + b*x**2)/(2*b*c) - d*sqrt(a + b*x**2)/(b*c**2) - (a*c**2 - d**2)*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, True))/(b*c**2))/b, Ne(b, 0)), (x**4/(2*(2*sqrt(a)*d + 2*a*c)), True))

$$3.307 \quad \int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=23

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Rubi [A] time = 0.09, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2155, 31}

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

[In] Int[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2155

Int[(x_)^{(m_)/((c_) + (d_.)*(x_)^{(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^{((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x])}], x], x, xⁿ], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]}}

Rubi steps

$$\begin{aligned} \int \frac{x}{ac+bcx^2+d\sqrt{a+bx^2}} dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}} dx, x, x^2\right) \\ &= \frac{\text{Subst}\left(\int \frac{1}{d+cx} dx, x, \sqrt{a+bx^2}\right)}{b} \\ &= \frac{\log\left(d+c\sqrt{a+bx^2}\right)}{bc} \end{aligned}$$

Mathematica [A] time = 0.02, size = 23, normalized size = 1.00

$$\frac{\log\left(c\sqrt{a+bx^2}+d\right)}{bc}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] Log[d + c*Sqrt[a + b*x^2]]/(b*c)

IntegrateAlgebraic [A] time = 0.04, size = 26, normalized size = 1.13

$$\frac{\log\left(bc\sqrt{a+bx^2}+bd\right)}{bc}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]),x]

[Out] Log[b*d + b*c*Sqrt[a + b*x^2]]/(b*c)

fricas [B] time = 0.66, size = 105, normalized size = 4.57

$$\frac{2 \log\left(\frac{bc^2x^2 + ac^2 - d^2}{x^2}\right) + \log\left(\frac{-bc^2x^2 + ac^2 + 2\sqrt{bx^2 + a}cd + d^2}{x^2}\right) - \log\left(\frac{-bc^2x^2 + ac^2 - 2\sqrt{bx^2 + a}cd + d^2}{x^2}\right)}{4bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] 1/4*(2*log(b*c^2*x^2 + a*c^2 - d^2) + log(-(b*c^2*x^2 + a*c^2 + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2) - log(-(b*c^2*x^2 + a*c^2 - 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2))/(b*c)

giac [A] time = 0.32, size = 22, normalized size = 0.96

$$\frac{\log\left(\left|\sqrt{bx^2 + a}c + d\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out] log(abs(sqrt(b*x^2 + a)*c + d))/(b*c)

maple [B] time = 0.04, size = 1931, normalized size = 83.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out] 1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-a*b)^(1/2)/b)-(-a*b)^(1/2))/b^(1/2)+(b*(x+(-a*b)^(1/2)/b)^2-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))/b^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x-(-a*b)^(1/2)/b)^2+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))/b^(1/2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(b*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2+1/c^2*d^2)^(1/2)+1/2*d/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*c^2-d^2)*b*c^2)^(1/2)*ln((-(-a*c^2-d^2)*b*c^2)^(1/2)/c^2+b*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))/b^(1/2)+(b*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2+1/c^2*d^2)^(1/2))/b^(1/2)+1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+2*(1/c^2*d^2)^(1/2)*(b*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2*(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/c^2*d^2)^(1/2))/(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)-1/2*d*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))

$$\frac{1}{2}c^2 + (-ac^2 - d^2)bc^2)^{1/2} / (-(-ab)^{1/2}c^2 + (-ac^2 - d^2)bc^2)^{1/2} * (b(x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2)^2 + 2 * (-ac^2 - d^2)bc^2)^{1/2} / c^2 * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2 + 1/c^2 * d^2)^{1/2} - 1/2 * d / ((-ab)^{1/2}c^2 + (-ac^2 - d^2)bc^2)^{1/2} * (x - (-ac^2 - d^2)bc^2)^{1/2} / (-(-ab)^{1/2}c^2 + (-ac^2 - d^2)bc^2)^{1/2} * (-ac^2 - d^2)bc^2)^{1/2} * \ln(((-ac^2 - d^2)bc^2)^{1/2} / c^2 + b * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2) / b^{1/2} + (b * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2)^2 + 2 * (-ac^2 - d^2)bc^2)^{1/2} / c^2 * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2 + 1/c^2 * d^2)^{1/2} / b^{1/2} + 1/2 / ((-ab)^{1/2}c^2 + (-ac^2 - d^2)bc^2)^{1/2} * d^3 / (1/c^2 * d^2)^{1/2} * \ln((2/c^2 * d^2 + 2 * (-ac^2 - d^2)bc^2)^{1/2} / c^2 * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2) + 2 * (1/c^2 * d^2)^{1/2} * (b * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2)^2 + 2 * (-ac^2 - d^2)bc^2)^{1/2} / c^2 * (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2 + 1/c^2 * d^2)^{1/2} / (x - (-ac^2 - d^2)bc^2)^{1/2} / b/c^2) + 1/2 / b/c * \ln(b * c^2 * x^2 + a * c^2 - d^2)$$

maxima [A] time = 0.71, size = 21, normalized size = 0.91

$$\frac{\log(\sqrt{bx^2 + ac + d})}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] log(sqrt(b*x^2 + a)*c + d)/(b*c)

mupad [B] time = 3.47, size = 45, normalized size = 1.96

$$\frac{\operatorname{atanh}\left(\frac{c\sqrt{bx^2+a}}{d}\right) + \frac{\ln(bc^2x^2+ac^2-d^2)}{2}}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2),x)

[Out] (atanh((c*(a + b*x^2)^(1/2))/d) + log(a*c^2 - d^2 + b*c^2*x^2)/2)/(b*c)

sympy [A] time = 4.48, size = 29, normalized size = 1.26

$$\frac{\begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c = 0 \\ \frac{\log(c\sqrt{a+bx^2}+d)}{c} & \text{otherwise} \end{cases}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)

[Out] Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, True))/b

$$3.308 \quad \int \frac{1}{x(ac+bcx^2+d\sqrt{a+bx^2})} dx$$

Optimal. Leaf size=88

$$-\frac{c \log(c\sqrt{a+bx^2}+d)}{ac^2-d^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}(ac^2-d^2)} + \frac{c \log(x)}{ac^2-d^2}$$

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{c \log(c\sqrt{a+bx^2}+d)}{ac^2-d^2} + \frac{d \tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{\sqrt{a}(ac^2-d^2)} + \frac{c \log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x]))/(a*c^2 - d^2) - (c*Log[d + c*Sqrt[a + b*x^2]]/(a*c^2 - d^2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d

, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(ac + bcx^2 + d\sqrt{a + bx^2})} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x(ac + bcx + d\sqrt{a + bx})} dx, x, x^2 \right) \\
 &= \text{Subst} \left(\int \frac{1}{(d + cx)(-a + x^2)} dx, x, \sqrt{a + bx^2} \right) \\
 &= -\frac{c^2 \text{Subst} \left(\int \frac{1}{d+cx} dx, x, \sqrt{a + bx^2} \right) + \text{Subst} \left(\int \frac{d-cx}{-a+x^2} dx, x, \sqrt{a + bx^2} \right)}{ac^2 - d^2} \\
 &= -\frac{c \log(d + c\sqrt{a + bx^2})}{ac^2 - d^2} + \frac{c \text{Subst} \left(\int \frac{x}{-a+x^2} dx, x, \sqrt{a + bx^2} \right) - d \text{Subst} \left(\int \frac{1}{-a+x^2} dx, x, \sqrt{a + bx^2} \right)}{ac^2 - d^2} \\
 &= \frac{d \tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a}} \right) + \frac{c \log(x)}{ac^2 - d^2} - \frac{c \log(d + c\sqrt{a + bx^2})}{ac^2 - d^2}}{\sqrt{a} (ac^2 - d^2)}
 \end{aligned}$$

Mathematica [A] time = 0.13, size = 107, normalized size = 1.22

$$\frac{(\sqrt{a}c - d) \log(\sqrt{a} - \sqrt{a + bx^2}) + (\sqrt{a}c + d) \log(\sqrt{a + bx^2} + \sqrt{a}) - 2\sqrt{a}c \log(c\sqrt{a + bx^2} + d)}{2\sqrt{a} (ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^2]] + (Sqrt[a]*c + d)*Log[Sqrt[a] + Sqrt[a + b*x^2]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^2]])/(2*Sqrt[a]*(a*c^2 - d^2))

IntegrateAlgebraic [A] time = 0.10, size = 95, normalized size = 1.08

$$\frac{c \log(bx^2)}{2(ac^2 - d^2)} - \frac{c \log(c\sqrt{a + bx^2} + d)}{ac^2 - d^2} + \frac{d \tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a}} \right)}{\sqrt{a} (ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] (d*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]]/(Sqrt[a]*(a*c^2 - d^2)) + (c*Log[b*x^2])/(2*(a*c^2 - d^2)) - (c*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2))

fricas [A] time = 0.88, size = 316, normalized size = 3.59

$$\left[\frac{2ac \log(bx^2 + a) - 4ac \log(x) + ac \log\left(\frac{bx^2 + a + \sqrt{bx^2 + a}}{2}\right) - ac \log\left(\frac{bx^2 + a - \sqrt{bx^2 + a}}{2}\right) + 2\sqrt{a}d \log\left(\frac{bx^2 + a + \sqrt{bx^2 + a}}{2}\right)}{4(a^2c^2 - a^2d^2)}, \frac{2ac \log(bx^2 + a) - 4ac \log(x) + ac \log\left(\frac{bx^2 + a + \sqrt{bx^2 + a}}{2}\right) - ac \log\left(\frac{bx^2 + a - \sqrt{bx^2 + a}}{2}\right) + 4\sqrt{a}d \arctan\left(\frac{\sqrt{a}}{\sqrt{bx^2 + a}}\right)}{4(a^2c^2 - a^2d^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] [-1/4*(2*a*c*log(b*c^2*x^2 + a*c^2 - d^2) - 4*a*c*log(x) + a*c*log(-(b*c^2*x^2 + a*c^2 - d^2)/x^2) - a*c*log(-(b*c^2*x^2 + a*c^2 - d^2)/x^2) + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2 - a*c*log(-(b*c^2*x^2 + a*c^2 - d^2)/x^2) + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2 - a*c*log(-(b*c^2*x^2 + a*c^2 - d^2)/x^2) + 2*sqrt(b*x^2 + a)*c*d + d^2)/x^2]

$$\begin{aligned} &^2 - 2*\sqrt{b*x^2 + a}*c*d + d^2)/x^2) + 2*\sqrt{a}*d*\log(-(b*x^2 - 2*\sqrt{b} \\ &*x^2 + a)*\sqrt{a} + 2*a)/x^2))/(a^2*c^2 - a*d^2), -1/4*(2*a*c*\log(b*c^2*x^2 \\ &+ a*c^2 - d^2) - 4*a*c*\log(x) + a*c*\log(-(b*c^2*x^2 + a*c^2 + 2*\sqrt{b*x^2} \\ &+ a)*c*d + d^2)/x^2) - a*c*\log(-(b*c^2*x^2 + a*c^2 - 2*\sqrt{b*x^2 + a}*c*d \\ &+ d^2)/x^2) + 4*\sqrt{-a}*d*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}))/ (a^2*c^2 - a* \\ &d^2)] \end{aligned}$$

giac [A] time = 0.34, size = 94, normalized size = 1.07

$$-\frac{c^2 \log\left(\left|\sqrt{bx^2 + a}c + d\right|\right)}{ac^3 - cd^2} + \frac{c \log(bx^2)}{2(ac^2 - d^2)} - \frac{d \arctan\left(\frac{\sqrt{bx^2 + a}}{\sqrt{-a}}\right)}{(ac^2 - d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out] -c^2*log(abs(sqrt(b*x^2 + a)*c + d))/(a*c^3 - c*d^2) + 1/2*c*log(b*x^2)/(a*c^2 - d^2) - d*arctan(sqrt(b*x^2 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt(-a))

maple [B] time = 0.04, size = 2175, normalized size = 24.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out]
$$\begin{aligned} &-1/2*a*c^3/(a*c^2-d^2)/d^2*\ln(b*c^2*x^2+a*c^2-d^2)+c*\ln(x)/(a*c^2-d^2)+1/2* \\ &c/d^2*\ln(b*c^2*x^2+a*c^2-d^2)+1/2*d*b*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2) \\ &*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/ \\ &2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(1/2)*c^2/a/((-a \\ &*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b \\ &*c^2)^(1/2))*(-a*b)^(1/2)*\ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((\\ &x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))+1/2*d*b*c^2 \\ &/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2 \\ &-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/ \\ &b))^(1/2)+1/2*d*b^(1/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)) \\ &/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*\ln(((x-(-a*b)^(1 \\ &2)/b)*b+(-a*b)^(1/2))/b^(1/2)+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x- \\ &-a*b)^(1/2)/b))^(1/2)-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d \\ &^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*c^ \\ &2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(- \\ &a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)+1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((- \\ &-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2) \\ &*b*c^2)^(1/2))*(-(a*c^2-d^2)*b*c^2)^(1/2)*\ln(((x+(-a*c^2-d^2)*b*c^2)^(1/2) \\ &/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-a*c^2-d^2)*b*c^2)^(\\ &1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-a*c^2-d^2)*b* \\ &c^2)^(1/2)/b/c^2)/c^2)^(1/2))+1/2*b*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a* \\ &c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1 \\ &/c^2*d^2)^(1/2)*\ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-a*c^2-d^2) \\ &*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2))*((x+(-a*c^2-d^2)*b*c^2)^(1/2) \\ &/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-a*c^2-d^2)*b*c^2)^(\\ &1/2)/b/c^2)/c^2)^(1/2))/(x+(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))+d/a^(1/2)/(a \\ &*c^2-d^2)*\ln((2*a+2*(b*x^2+a)^(1/2)*a^(1/2))/x)-d/a/(a*c^2-d^2)*(b*x^2+a)^(\\ &1/2)-1/2*d*b*c^4/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/ \\ &((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-a*c^2-d^2)*b*c^2)^(1/2) \\ &/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-a*c^2-d^2)*b*c^2) \\ &^(1/2)/b/c^2)/c^2)^(1/2)-1/2*d*b^(1/2)*c^2/(a*c^2-d^2)/((-a*b)^(1/2)*c^2+(- \\ &(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(- \\ &a*c^2-d^2)*b*c^2)^(1/2)*\ln(((x-(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-a*c^2 \\ &-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1 \end{aligned}$$

$$\frac{1}{c^2 d^2 + 2 * (- (a * c^2 - d^2) * b * c^2)^{(1/2)} * (x - (- (a * c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2) / c^2)^{(1/2)} + 1/2 * b * c^2 / (a * c^2 - d^2) / ((- a * b)^{(1/2)} * c^2 + (- (a * c^2 - d^2) * b * c^2)^{(1/2)}) / ((- a * b)^{(1/2)} * c^2 - (- (a * c^2 - d^2) * b * c^2)^{(1/2)}) * d^3 / ((1 / c^2 * d^2)^{(1/2)} * \ln((2 / c^2 * d^2 + 2 * (- (a * c^2 - d^2) * b * c^2)^{(1/2)} * (x - (- (a * c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2) / c^2 + 2 * (1 / c^2 * d^2)^{(1/2)} * ((x - (- (a * c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2)^2 * b + 1 / c^2 * d^2 + 2 * (- (a * c^2 - d^2) * b * c^2)^{(1/2)} * (x - (- (a * c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2) / c^2)^{(1/2)}) / (x - (- (a * c^2 - d^2) * b * c^2)^{(1/2)} / b / c^2))}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

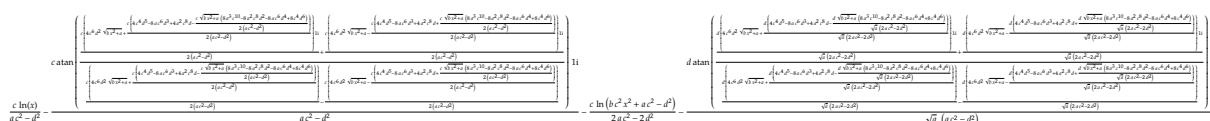
$$\int \frac{1}{(bcx^2 + ac + \sqrt{bx^2 + a}d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x), x)

mupad [B] time = 3.95, size = 1270, normalized size = 14.43



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)

[Out]
$$\frac{(c * \log(x)) / (a * c^2 - d^2) - (c * \operatorname{atan}(((c * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} + (c * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d - (c * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (2 * (a * c^2 - d^2)))) / (2 * (a * c^2 - d^2))) * i) / (2 * (a * c^2 - d^2)) + (c * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} - (c * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d + (c * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (2 * (a * c^2 - d^2)))) / (2 * (a * c^2 - d^2))) * i) / (2 * (a * c^2 - d^2)) / ((c * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} + (c * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d - (c * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (2 * (a * c^2 - d^2)))) / (2 * (a * c^2 - d^2))) / (2 * (a * c^2 - d^2)) - (c * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} - (c * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d + (c * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (2 * (a * c^2 - d^2)))) / (2 * (a * c^2 - d^2))) * i) / (a * c^2 - d^2) - (c * \log(a * c^2 - d^2 + b * c^2 * x^2)) / (2 * a * c^2 - 2 * d^2) - (d * \operatorname{atan}(((d * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} + (d * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d - (d * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)))) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2))) * i) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)) + (d * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} - (d * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d + (d * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)))) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2))) * i) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)) / ((d * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} + (d * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d - (d * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)))) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2))) - (d * (4 * c^6 * d^2 * (a + b * x^2)^{(1/2)} - (d * (4 * c^4 * d^5 - 8 * a * c^6 * d^3 + 4 * a^2 * c^8 * d + (d * (a + b * x^2)^{(1/2)} * (8 * a^3 * c^{10} + 8 * c^4 * d^6 - 8 * a * c^6 * d^4 - 8 * a^2 * c^8 * d^2)) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2)))) / (a^{(1/2)} * (2 * a * c^2 - 2 * d^2))) * i) / (a^{(1/2)} * (a * c^2 - d^2))$$

sympy [A] time = 10.50, size = 88, normalized size = 1.00

$$\frac{c^2 \begin{cases} \frac{\sqrt{a+bx^2}}{d} & \text{for } c = 0 \\ \frac{\log(c\sqrt{a+bx^2}+d)}{c} & \text{otherwise} \end{cases}}{ac^2 - d^2} - \frac{-\frac{c \log(-bx^2)}{2} + \frac{d \operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a}}\right)}{\sqrt{-a}}}{ac^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)), x)

[Out] -c**2*Piecewise((sqrt(a + b*x**2)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**2) + d)/c, True))/(a*c**2 - d**2) - (-c*log(-b*x**2)/2 + d*atan(sqrt(a + b*x**2)/sqrt(-a))/sqrt(-a))/(a*c**2 - d**2)

$$3.309 \quad \int \frac{1}{x^3(ac+bcx^2+d\sqrt{a+bx^2})} dx$$

Optimal. Leaf size=151

$$\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}(ac^2-d^2)^2} - \frac{ac-d\sqrt{a+bx^2}}{2ax^2(ac^2-d^2)} + \frac{bc^3\log(c\sqrt{a+bx^2}+d)}{(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2}$$

Rubi [A] time = 0.35, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2155, 741, 801, 635, 206, 260}

$$\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a}}\right)}{2a^{3/2}(ac^2-d^2)^2} - \frac{ac-d\sqrt{a+bx^2}}{2ax^2(ac^2-d^2)} + \frac{bc^3\log(c\sqrt{a+bx^2}+d)}{(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a*c + b*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] -(a*c - d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*d*(3*a*c^2 - d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x])/((a*c^2 - d^2)^2 + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 801

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 2155

```
Int[(x_)^(m_)/((c_) + (d_)*(x_)^(n_) + (e_)*Sqrt[(a_) + (b_)*(x_)^(n_)
), x_Symbol] := Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a +
b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d
, 0] && IntegerQ[(m + 1)/n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (ac + bcx^2 + d\sqrt{a + bx^2})} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{x^2 (ac + bcx + d\sqrt{a + bx})} dx, x, x^2 \right) \\ &= b \text{Subst} \left(\int \frac{1}{(d + cx)(a - x^2)^2} dx, x, \sqrt{a + bx^2} \right) \\ &= \frac{ac - d\sqrt{a + bx^2}}{2a(ac^2 - d^2)x^2} - \frac{b \text{Subst} \left(\int \frac{-2ac^2 + d^2 + cdx}{(d + cx)(a - x^2)} dx, x, \sqrt{a + bx^2} \right)}{2a(ac^2 - d^2)} \\ &= \frac{ac - d\sqrt{a + bx^2}}{2a(ac^2 - d^2)x^2} - \frac{b \text{Subst} \left(\int \left(-\frac{2ac^4}{(ac^2 - d^2)(d + cx)} + \frac{3ac^2d - d^3 - 2ac^3x}{(ac^2 - d^2)(a - x^2)} \right) dx, x, \sqrt{a + bx^2} \right)}{2a(ac^2 - d^2)} \\ &= \frac{ac - d\sqrt{a + bx^2}}{2a(ac^2 - d^2)x^2} + \frac{bc^3 \log(d + c\sqrt{a + bx^2})}{(ac^2 - d^2)^2} - \frac{b \text{Subst} \left(\int \frac{3ac^2d - d^3 - 2ac^3x}{a - x^2} dx, x, \sqrt{a + bx^2} \right)}{2a(ac^2 - d^2)} \\ &= \frac{ac - d\sqrt{a + bx^2}}{2a(ac^2 - d^2)x^2} + \frac{bc^3 \log(d + c\sqrt{a + bx^2})}{(ac^2 - d^2)^2} + \frac{(bc^3) \text{Subst} \left(\int \frac{x}{a - x^2} dx, x, \sqrt{a + bx^2} \right)}{(ac^2 - d^2)^2} \\ &= \frac{ac - d\sqrt{a + bx^2}}{2a(ac^2 - d^2)x^2} - \frac{bd(3ac^2 - d^2) \tanh^{-1} \left(\frac{\sqrt{a + bx^2}}{\sqrt{a}} \right)}{2a^{3/2}(ac^2 - d^2)^2} - \frac{bc^3 \log(x)}{(ac^2 - d^2)^2} + \frac{bc^3}{(ac^2 - d^2)^2} \end{aligned}$$

Mathematica [A] time = 1.08, size = 291, normalized size = 1.93

$$\frac{\sqrt{a} \left(-d^2 c^3 \sqrt{a + bx^2} + d^2 c^2 d + 2abc^3 x^2 \sqrt{a + bx^2} \tanh^{-1} \left(\frac{c\sqrt{a + bx^2}}{d} \right) - 2abc^3 x^2 \log(x) \sqrt{a + bx^2} + bdx^2 \sqrt{\frac{bx^2}{a} + 1} (ac^2 - d^2) \tanh^{-1} \left(\sqrt{\frac{bx^2}{a} + 1} \right) + abc^2 dx^2 + abc^3 x^2 \sqrt{a + bx^2} \log(ac^2 + bc^2 x^2 - d^2) + acd^2 \sqrt{a + bx^2} - ad^3 - bc^3 x^2 \right)}{x^2 \sqrt{a + bx^2}} + \frac{2bd(d^2 - 2ac^2) \tanh^{-1} \left(\frac{\sqrt{a + bx^2}}{\sqrt{a}} \right)}{2a^{3/2}(d^2 - ac^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] (2*b*d*(-2*a*c^2 + d^2)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]] + (Sqrt[a]*(a^2*c^2*d - a*d^3 + a*b*c^2*d*x^2 - b*d^3*x^2 - a^2*c^3*Sqrt[a + b*x^2] + a*c*d^2*Sqrt[a + b*x^2] + 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*ArcTanh[(c*Sqrt[a + b*x^2])/d] + b*d*(a*c^2 - d^2)*x^2*Sqrt[1 + (b*x^2)/a]*ArcTanh[Sqrt[1 + (b*x^2)/a]]) - 2*a*b*c^3*x^2*Sqrt[a + b*x^2]*Log[x] + a*b*c^3*x^2*Sqrt[a + b*x^2]*Log[a*c^2 - d^2 + b*c^2*x^2]))/(x^2*Sqrt[a + b*x^2]))/(2*a^(3/2)*(-(a*c^2) + d^2)^2)

IntegrateAlgebraic [A] time = 0.26, size = 172, normalized size = 1.14

$$\frac{b(3ac^2d - d^3) \tanh^{-1} \left(\frac{\sqrt{a + bx^2}}{\sqrt{a}} \right)}{2a^{3/2}(ac^2 - d^2)^2} + \frac{d\sqrt{a + bx^2}}{2ax^2(ac^2 - d^2)} - \frac{bc^3 \log(bx^2)}{2(ac^2 - d^2)^2} + \frac{bc^3 \log(c\sqrt{a + bx^2} + d)}{(ac^2 - d^2)^2} - \frac{c}{2x^2(ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2))),x]

[Out]
$$-1/2*c/((a*c^2 - d^2)*x^2) + (d*Sqrt[a + b*x^2])/(2*a*(a*c^2 - d^2)*x^2) - (b*(3*a*c^2*d - d^3)*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a]])/(2*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[b*x^2])/(2*(a*c^2 - d^2)^2) + (b*c^3*Log[d + c*Sqrt[a + b*x^2]])/(a*c^2 - d^2)^2$$

fricas [A] time = 1.80, size = 530, normalized size = 3.51

$$\frac{2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a}) + 2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a})}{4(a^2c^2-d^2)^2} - \frac{2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a}) + 2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a})}{4(a^2c^2-d^2)^2} - \frac{2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a}) + 2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a})}{4(a^2c^2-d^2)^2} - \frac{2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a}) + 2\sqrt{a}\log(\sqrt{bx^2+a}) - 2\sqrt{a}\log(\sqrt{bx^2+a})}{4(a^2c^2-d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out]
$$\begin{aligned} & [1/4*(2*a^2*b*c^3*x^2*\log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*\log(x) \\ & + a^2*b*c^3*x^2*\log(-(b*c^2*x^2 + a*c^2 + 2*\sqrt{b*x^2 + a})*c*d + d^2)/x^2 \\ &) - a^2*b*c^3*x^2*\log(-(b*c^2*x^2 + a*c^2 - 2*\sqrt{b*x^2 + a})*c*d + d^2)/x^2 \\ & - 2*a^3*c^3 + 2*a^2*c*d^2 - (3*a*b*c^2*d - b*d^3)*\sqrt{a}*x^2*\log(-(b*x^2 + 2*\sqrt{b*x^2 + a})*\sqrt{a} + 2*a)/x^2) + 2*(a^2*c^2*d - a*d^3)*\sqrt{b*x^2 + a} \\ &)]/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2), 1/4*(2*a^2*b*c^3*x^2*\log(b*c^2*x^2 + a*c^2 - d^2) - 4*a^2*b*c^3*x^2*\log(x) + a^2*b*c^3*x^2*\log(-(b*c^2*x^2 + a*c^2 + 2*\sqrt{b*x^2 + a})*c*d + d^2)/x^2) - a^2*b*c^3*x^2*\log(-(b*c^2*x^2 + a*c^2 - 2*\sqrt{b*x^2 + a})*c*d + d^2)/x^2) - 2*a^3*c^3 + 2*a^2*c*d^2 + 2*(3*a*b*c^2*d - b*d^3)*\sqrt{-a}*x^2*\arctan(\sqrt{-a}/\sqrt{b*x^2 + a}) + 2*(a^2*c^2*d - a*d^3)*\sqrt{b*x^2 + a}]/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^2)] \end{aligned}$$

giac [A] time = 0.37, size = 210, normalized size = 1.39

$$\frac{bc^4 \log\left(\left|\sqrt{bx^2 + a} + c + d\right|\right)}{a^2c^5 - 2ac^3d^2 + cd^4} - \frac{bc^3 \log(-bx^2)}{2(a^2c^4 - 2ac^2d^2 + d^4)} + \frac{(3abc^2d - bd^3) \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a}}\right)}{2(a^3c^4 - 2a^2c^2d^2 + ad^4)\sqrt{-a}} - \frac{a^2bc^3 - abcd^2 - (abc^2d - bd^3)\sqrt{bx^2 + a}}{2(ac^2 - d^2)^2 abx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out]
$$b*c^4*\log(\text{abs}(\sqrt{b*x^2 + a}*c + d))/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/2*b*c^3*\log(-b*x^2)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/2*(3*a*b*c^2*d - b*d^3)*\arctan(\sqrt{b*x^2 + a}/\sqrt{-a})/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*\sqrt{-a}) - 1/2*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*\sqrt{b*x^2 + a})/((a*c^2 - d^2)^2*a*b*x^2)$$

maple [B] time = 0.05, size = 2459, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out]
$$\begin{aligned} & 1/2*a*c^5*b/(a*c^2-d^2)^2/d^2*\ln(b*c^2*x^2+a*c^2-d^2)-1/2*c/(a*c^2-d^2)/x^2 \\ & -2*b*c^3*\ln(x)/(a*c^2-d^2)^2+1/a*c*b/(a*c^2-d^2)^2*\ln(x)*d^2-1/2*b*c^3/(a*c^2-d^2)/d^2*\ln(b*c^2*x^2+a*c^2-d^2)+b*c/a/(a*c^2-d^2)*\ln(x)-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*(-a*b)^(1/2)*\ln(((x+(-a*b)^(1/2)/b)*b-(-a*b)^(1/2))/b^(1/2)+((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2))-1/2*d*b^2*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)-1/2*d*b^(3/2)*c^2/a^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2)) \end{aligned}$$

$$\begin{aligned}
& b*c^2)^{(1/2))*(-a*b)^{(1/2)}*\ln(((x-(-a*b)^{(1/2)}/b)*b+(-a*b)^{(1/2)})/b^{(1/2)}+(\\
& (x-(-a*b)^{(1/2)}/b)^2*b+2*(-a*b)^{(1/2)}*(x-(-a*b)^{(1/2)}/b))^{(1/2)}+1/2*d/a^2/ \\
& (a*c^2-d^2)/x^2*(b*x^2+a)^{(3/2)}+1/2*d/a^{(3/2)}/(a*c^2-d^2)*b*\ln((2*a+2*(b*x^ \\
& 2+a)^{(1/2)}*a^{(1/2)})/x)-1/2*d/a^2/(a*c^2-d^2)*b*(b*x^2+a)^{(1/2)}+1/2*d*b^2*c^ \\
& 6/(a*c^2-d^2)^2/((-a*b)^{(1/2)}*c^2+(-a*c^2-d^2)*b*c^2)^{(1/2)}/((-a*b)^{(1/2)} \\
& *c^2-(-a*c^2-d^2)*b*c^2)^{(1/2)}*((x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+ \\
& 1/c^2*d^2-2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2 \\
& /c^2)^{(1/2)}-1/2*d*b^{(3/2)}*c^4/(a*c^2-d^2)^2/((-a*b)^{(1/2)}*c^2+(-a*c^2-d^2) \\
& *b*c^2)^{(1/2)}/((-a*b)^{(1/2)}*c^2-(-a*c^2-d^2)*b*c^2)^{(1/2)}*(-a*c^2-d^2)* \\
& b*c^2)^{(1/2)}*\ln(((x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)*b-(-a*c^2-d^2)*b*c^2 \\
&)^{(1/2)}/c^2)/b^{(1/2)}+(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2-2* \\
& (-a*c^2-d^2)*b*c^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)} \\
& -1/2*b^2*c^4/(a*c^2-d^2)^2/((-a*b)^{(1/2)}*c^2+(-a*c^2-d^2)*b*c^2)^{(1/2)}/((\\
& -a*b)^{(1/2)}*c^2-(-a*c^2-d^2)*b*c^2)^{(1/2)})*d^3/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2 \\
& *d^2-2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2+ \\
& 2*(1/c^2*d^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2-2*(\\
& -a*c^2-d^2)*b*c^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)}/ \\
& (x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2))-2*d*b/a^{(1/2)}/(a*c^2-d^2)^2*\ln((2*a+2 \\
& *(b*x^2+a)^{(1/2)}*a^{(1/2)})/x)*c^2+2*d*b/a/(a*c^2-d^2)^2*(b*x^2+a)^{(1/2)}*c^2+ \\
& b/a^{(3/2)}/(a*c^2-d^2)^2*\ln((2*a+2*(b*x^2+a)^{(1/2)}*a^{(1/2)})/x)*d^3-b/a^2/(a* \\
& c^2-d^2)^2*(b*x^2+a)^{(1/2)}*d^3+1/2*d*b^2*c^6/(a*c^2-d^2)^2/((-a*b)^{(1/2)}*c^ \\
& 2+(-a*c^2-d^2)*b*c^2)^{(1/2)}/((-a*b)^{(1/2)}*c^2-(-a*c^2-d^2)*b*c^2)^{(1/2)} \\
& *((x-(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2+2*(-a*c^2-d^2)*b*c^2) \\
& ^{(1/2)}*(x-(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)}+1/2*d*b^{(3/2)}*c^4/(a \\
& *c^2-d^2)^2/((-a*b)^{(1/2)}*c^2+(-a*c^2-d^2)*b*c^2)^{(1/2)}/((-a*b)^{(1/2)}*c^2 \\
& -(-a*c^2-d^2)*b*c^2)^{(1/2)}*(-a*c^2-d^2)*b*c^2)^{(1/2)}*\ln(((x-(-a*c^2-d^2) \\
&)*b*c^2)^{(1/2)}/b/c^2)*b+(-a*c^2-d^2)*b*c^2)^{(1/2)}/c^2)/b^{(1/2)}+(x-(-a*c^ \\
& 2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2+2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x-(- \\
& (a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)}-1/2*b^2*c^4/(a*c^2-d^2)^2/((-a* \\
& b)^{(1/2)}*c^2+(-a*c^2-d^2)*b*c^2)^{(1/2)}/((-a*b)^{(1/2)}*c^2-(-a*c^2-d^2)*b* \\
& c^2)^{(1/2)})*d^3/(1/c^2*d^2)^{(1/2)}*\ln((2/c^2*d^2+2*(-a*c^2-d^2)*b*c^2)^{(1/2) \\
&)*(x-(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2+2*(1/c^2*d^2)^{(1/2)}*((x-(-a*c^2 \\
& -d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2+2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x-(- \\
& a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)}/(x-(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/ \\
& c^2))
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bcx^2 + ac + \sqrt{bx^2 + ad})x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^3), x)

mupad [B] time = 5.56, size = 4602, normalized size = 30.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)

[Out] (atan(((((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/(16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^(1/2)*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)))^(1/2)*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 -

$$\begin{aligned}
& 4*a^6*c^6*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)})/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^{(1/2)}*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) - (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/((16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^{(1/2)}*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2)))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)})/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^{(1/2)}*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*1i)/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))/(((b^3*c^8*d^5)/2 - (3*a*b^3*c^10*d^3)/2)/(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/((16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - ((a + b*x^2)^{(1/2)}*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2)))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)})/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) + ((a + b*x^2)^{(1/2)}*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)})/(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2) + (((((12*a^2*b*c^6*d^9 - 28*a^3*b*c^8*d^7 + 32*a^4*b*c^10*d^5 - 18*a^5*b*c^12*d^3 - 2*a*b*c^4*d^11 + 4*a^6*b*c^14*d)/((16*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + ((a + b*x^2)^{(1/2)}*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2)))/(512*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)})/(16*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - ((a + b*x^2)^{(1/2)}*(b^2*c^6*d^6 - 6*a*b^2*c^8*d^4 + 13*a^2*b^2*c^10*d^2))/(32*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))*(16*(b^2*d^6 - 6*a*b^2*c^2*d^4 + 9*a^2*b^2*c^4*d^2)*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2))^{(1/2)}*1i)/(8*(a^7*c^8 + a^3*d^8 - 4*a^4*c^2*d^6 + 6*a^5*c^4*d^4 - 4*a^6*c^6*d^2)) - c/(2*x^2*(a*c^2 - d^2)) - (b*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) + (b*c^3*log(a*c^2 - d^2 + b*c^2*x^2))/(2*d^4 + 2*a^2*c^4 - 4*a*c^2*d^2) - (d*(a + b*x^2)^{(1/2)})/(2*x^2*(a*d^2 - a^2*c^2)) + (b*c^3*a*tan(((c^3*((a + b*x^2)^{(1/2)}*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/
\end{aligned}$$

$$4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2) - (c^3*(a + b*x^2)^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))/(2*(a*c^2 - d^2)^2)*1i)/(2*(a*c^2 - d^2)^2) + (c^3*((a + b*x^2)^{(1/2)}*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))/(2*(a*c^2 - d^2)^2)*1i)/(2*(a*c^2 - d^2)^2))/((c^8*d^5 - 3*a*c^10*d^3)/(2*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*((a + b*x^2)^{(1/2)}*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) + (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) - (c^3*(a + b*x^2)^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^2 - d^2)^2) + (c^3*((a + b*x^2)^{(1/2)}*(c^6*d^6 - 6*a*c^8*d^4 + 13*a^2*c^10*d^2))/(2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)) - (c^3*((8*a*c^4*d^11 - 16*a^6*c^14*d - 48*a^2*c^6*d^9 + 112*a^3*c^8*d^7 - 128*a^4*c^10*d^5 + 72*a^5*c^12*d^3)/(4*(a^5*c^6 - a^2*d^6 + 3*a^3*c^2*d^4 - 3*a^4*c^4*d^2)) + (c^3*(a + b*x^2)^{(1/2)}*(16*a^7*c^14 + 16*a^2*c^4*d^10 - 48*a^3*c^6*d^8 + 32*a^4*c^8*d^6 + 32*a^5*c^10*d^4 - 48*a^6*c^12*d^2))/(4*(a*c^2 - d^2)^2*(a^4*c^4 + a^2*d^4 - 2*a^3*c^2*d^2)))/(2*(a*c^2 - d^2)^2)))/(2*(a*c^2 - d^2)^2))*1i)/(a*c^2 - d^2)^2$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (ac + bcx^2 + d\sqrt{a + bx^2})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)

[Out] Integral(1/(x**3*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)

$$3.310 \quad \int \frac{x^2}{ac+bcx^2+d\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=147

$$\frac{\sqrt{ac^2-d^2} \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2} - \frac{\sqrt{ac^2-d^2} \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2} - \frac{d \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2} + \frac{x}{bc}$$

Rubi [A] time = 0.24, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {2156, 321, 205, 483, 217, 206, 377}

$$\frac{\sqrt{ac^2-d^2} \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2} - \frac{\sqrt{ac^2-d^2} \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{b^{3/2}c^2} - \frac{d \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{a+bx^2}}\right)}{b^{3/2}c^2} + \frac{x}{bc}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]

[Out] x/(b*c) - (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(b^(3/2)*c^2) + (Sqrt[a*c^2 - d^2]*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(b^(3/2)*c^2) - (d*ArcTanh[(Sqrt[b]*x)/Sqrt[a + b*x^2]])/(b^(3/2)*c^2)

Rule 205

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 321

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 377

Int[((a_) + (b_)*(x_)^(n_))^(p_)/((c_) + (d_)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 483

Int[(((e_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))^(q_))/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Dist[e^n/b, Int[(e*x)^(m-n)*(c+d*x^n)^q, x], x] - Dist[(a*e^n)/b, Int[((e*x)^(m-n)*(c+d*x^n)^q)/(a+b*x^n), x], x] /; Free

$Q[\{a, b, c, d, e, m, q\}, x] \ \&\& \ \text{NeQ}[b*c - a*d, 0] \ \&\& \ \text{IGtQ}[n, 0] \ \&\& \ \text{LeQ}[n, m, 2*n - 1] \ \&\& \ \text{IntBinomialQ}[a, b, c, d, e, m, n, -1, q, x]$

Rule 2156

$\text{Int}[(u_)/((c_)+(d_)*(x_)^{(n_)}+(e_)*\text{Sqrt}[(a_)+(b_)*(x_)^{(n_)}]), x_]$
 Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[b*c - a*d, 0]

Rubi steps

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}} dx = (ac) \int \frac{x^2}{a^2c^2 - ad^2 + abc^2x^2} dx - (ad) \int \frac{x^2}{\sqrt{a + bx^2} (a^2c^2 - ad^2 + abc^2x^2)} dx$$

$$= \frac{x}{bc} - \frac{d \int \frac{1}{\sqrt{a+bx^2}} dx}{bc^2} - \frac{(a(ac^2 - d^2)) \int \frac{1}{a^2c^2 - ad^2 + abc^2x^2} dx}{bc} + \frac{(ad(ac^2 - d^2)) \int \frac{1}{\sqrt{a+bx^2}} dx}{bc^2}$$

$$= \frac{x}{bc} - \frac{\sqrt{ac^2 - d^2} \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{b^{3/2}c^2} - \frac{d \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{a+bx^2}}\right)}{bc^2} + \frac{(ad(ac^2 - d^2)) \int \frac{1}{\sqrt{a+bx^2}} dx}{bc^2}$$

$$= \frac{x}{bc} - \frac{\sqrt{ac^2 - d^2} \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{b^{3/2}c^2} + \frac{\sqrt{ac^2 - d^2} \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2 - d^2} \sqrt{a+bx^2}}\right)}{b^{3/2}c^2} - \frac{d \tan^{-1}\left(\frac{x}{\sqrt{a+bx^2}}\right)}{bc^2}$$

Mathematica [A] time = 0.27, size = 157, normalized size = 1.07

$$\frac{\sqrt{ac^2 - d^2} \left(\sqrt{b}cx - d \log\left(\sqrt{b} \sqrt{a + bx^2} + bx\right) \right) + (ac^2 - d^2) \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2} \sqrt{ac^2 - d^2}}\right) + (d^2 - ac^2) \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{b^{3/2}c^2 \sqrt{ac^2 - d^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]
 [Out] ((-(a*c^2) + d^2)*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + (a*c^2 - d^2)*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])] + Sqrt[a*c^2 - d^2]*(Sqrt[b]*c*x - d*Log[b*x + Sqrt[b]*Sqrt[a + b*x^2]]))/(b^(3/2)*c^2*Sqrt[a*c^2 - d^2])

IntegrateAlgebraic [A] time = 0.39, size = 136, normalized size = 0.93

$$\frac{2\sqrt{ac^2 - d^2} \tan^{-1}\left(\frac{c\sqrt{a+bx^2}}{\sqrt{ac^2 - d^2}} - \frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}} + \frac{d}{\sqrt{ac^2 - d^2}}\right)}{b^{3/2}c^2} + \frac{d \log\left(\sqrt{a + bx^2} - \sqrt{b}x\right)}{b^{3/2}c^2} + \frac{x}{bc}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a*c + b*c*x^2 + d*Sqrt[a + b*x^2]), x]
 [Out] x/(b*c) + (2*Sqrt[a*c^2 - d^2]*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2] + (c*Sqrt[a + b*x^2])/Sqrt[a*c^2 - d^2]]/(b^(3/2)*c^2) + (d*Log[-(Sqrt[b]*x) + Sqrt[a + b*x^2]])/(b^(3/2)*c^2)

fricas [A] time = 0.99, size = 1168, normalized size = 7.95



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] [1/4*(4*b*c*x + 2*sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/4*(4*b*c*x + 4*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) + b*sqrt(-(a*c^2 - d^2)/b)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*((a*b^2*c^2*d - 2*b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)*sqrt(b*x^2 + a)*sqrt(-(a*c^2 - d^2)/b))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*b*sqrt(-(a*c^2 - d^2)/b)*log((b*c^2*x^2 - 2*b*c*x*sqrt(-(a*c^2 - d^2)/b) - a*c^2 + d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) - b*sqrt((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x)) + sqrt(b)*d*log(-2*b*x^2 + 2*sqrt(b*x^2 + a)*sqrt(b)*x - a))/(b^2*c^2), 1/2*(2*b*c*x + 2*b*sqrt((a*c^2 - d^2)/b)*arctan(-b*c*x*sqrt((a*c^2 - d^2)/b)/(a*c^2 - d^2)) + 2*sqrt(-b)*d*arctan(sqrt(-b)*x/sqrt(b*x^2 + a)) - b*sqrt((a*c^2 - d^2)/b)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt((a*c^2 - d^2)/b)/((a*b*c^2*d - b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x)))/(b^2*c^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument Value

maple [B] time = 0.04, size = 3485, normalized size = 23.71

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out]
$$\frac{1}{2}d*c^2*a/(-a*b)^{(1/2)}/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/(-(-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})*((x+(-a*b)^{(1/2)}/b)^2*b-2*(-a*b)^{(1/2)}*(x+(-a*b)^{(1/2)}/b))^{(1/2)}-1/2*d*c^2*a/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/(-(-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})*ln(((x+(-a*b)^{(1/2)}/b)*b-(-a*b)^{(1/2)}/b^{(1/2)}+(x+(-a*b)^{(1/2)}/b)^2*b-2*(-a*b)^{(1/2)}*(x+(-a*b)^{(1/2)}/b))^{(1/2)})/b^{(1/2)}-1/2*d*c^2*a/(-a*b)^{(1/2)}/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/(-(-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})*((x-(-a*b)^{(1/2)}/b)^2*b+2*(-a*b)^{(1/2)}*(x-(-a*b)^{(1/2)}/b))^{(1/2)}-1/2*d*c^2*a/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/(-(-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})*ln(((x-(-a*b)^{(1/2)}/b)*b+(-a*b)^{(1/2)}/b^{(1/2)}+(x-(-a*b)^{(1/2)}/b)^2*b+2*(-a*b)^{(1/2)}*(x-(-a*b)^{(1/2)}/b))^{(1/2)})/b^{(1/2)}-1/2*d*c^4/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/(-(-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)^{(1/2)})*((x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^{(1/2)}*(x+(-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2)^{(1/2)}*a+1/2*c^2/((-a*b)^{(1/2)}*c^2+(-(a*c^2-d^2)*b*c^2)$$

```
^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-((a*c^2-d^2)*b*c^2)^(1/2)*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*d^3+1/2*d*c^2/(((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*a-1/2/(((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b-(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*d^3+1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2))*((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2-2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)))/((x+(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)+1/2*d*c^4/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-((a*c^2-d^2)*b*c^2)^(1/2))*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*a-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-((a*c^2-d^2)*b*c^2)^(1/2))*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*a-1/2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)*b+(-(a*c^2-d^2)*b*c^2)^(1/2)/c^2)/b^(1/2)+((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/b^(1/2)*d^3-1/2*c^2/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2)))/(-((a*c^2-d^2)*b*c^2)^(1/2))*d^3/(1/c^2*d^2)^(1/2)*ln((2/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2+2*(1/c^2*d^2)^(1/2))*((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2))/((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)^2*b+1/c^2*d^2+2*(-(a*c^2-d^2)*b*c^2)^(1/2)*(x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2)/c^2)^(1/2)))/((x-(-(a*c^2-d^2)*b*c^2)^(1/2)/b/c^2))-a/b/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))+x/b/c+1/b/c^2*d^2/((a*c^2-d^2)*b)^(1/2)*arctan(x*b*c/((a*c^2-d^2)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{bcx^2 + ac + \sqrt{bx^2 + ad}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")
```

[Out] integrate(x^2/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{ac + d\sqrt{bx^2 + a} + bcx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)

[Out] int(x^2/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{ac + bcx^2 + d\sqrt{a + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)), x)

[Out] Integral(x**2/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)

$$3.311 \quad \int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=103

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Rubi [A] time = 0.07, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2156, 205, 377}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

[In] Int[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1), x]

[Out] ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]]/(Sqrt[b]*Sqrt[a*c^2 - d^2]) - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])]/(Sqrt[b]*Sqrt[a*c^2 - d^2])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 2156

Int[(u_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{ac+bcx^2+d\sqrt{a+bx^2}} dx &= (ac) \int \frac{1}{a^2c^2-ad^2+abc^2x^2} dx - (ad) \int \frac{1}{\sqrt{a+bx^2}(a^2c^2-ad^2+abc^2x^2)} dx \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - (ad) \text{Subst}\left(\int \frac{1}{a^2c^2-ad^2-(-a^2bc^2+b(a^2c^2-ad^2))x^2} dx\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{ac^2-d^2}\sqrt{a+bx^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}} \end{aligned}$$

Mathematica [A] time = 0.13, size = 83, normalized size = 0.81

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right) - \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1), x]
```

```
[Out] (ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] - ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

IntegrateAlgebraic [A] time = 0.33, size = 93, normalized size = 0.90

$$-\frac{2 \tan^{-1}\left(\frac{c\sqrt{a+bx^2}}{\sqrt{ac^2-d^2}} - \frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}} + \frac{d}{\sqrt{ac^2-d^2}}\right)}{\sqrt{b}\sqrt{ac^2-d^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])^(-1), x]
```

```
[Out] (-2*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2] + (c*Sqrt[a + b*x^2])/Sqrt[a*c^2 - d^2])/(Sqrt[b]*Sqrt[a*c^2 - d^2])
```

fricas [B] time = 0.65, size = 510, normalized size = 4.95

$$\left[\frac{\sqrt{-abc^2 + bd^2} \log\left(\frac{c^4 x^4 - 2c^3 d x^3 + (c^2 d^2 + 8bd^2)x^2 + 2(c^2 d^2 - 5c^2 d^2 + 4abd^2)x + d^4}{b^2 x^4 + 2c^2 d^2 + 4bd^2}\right) + 2\sqrt{-abc^2 + bd^2} \log\left(\frac{bc^2 x^2 - c^2 d^2 - \sqrt{-abc^2 + bd^2} cx + d^2}{b^2 x^2 + ac^2 - d^2}\right)}{4(abc^2 - bd^2)} - \sqrt{abc^2 - bd^2} \arctan\left(\frac{\sqrt{abc^2 - bd^2} cx}{ac^2 - d^2}\right) - \sqrt{abc^2 - bd^2} \arctan\left(\frac{(c^2 x^2 - d^2 + (abc^2 - 2bd^2)x) \sqrt{abc^2 - bd^2}}{2((abc^2 - bd^2)x^2 + (c^2 d^2 - abd^2))}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a*b*c^2 + b*d^2)*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 - 4*sqrt(-a*b*c^2 + b*d^2)*((a*b*c^2*d - 2*b*d^3)*x^3 + (a^2*c^2*d - a*d^3)*x)*sqrt(b*x^2 + a))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2)) + 2*sqrt(-a*b*c^2 + b*d^2)*log((b*c^2*x^2 - a*c^2 - 2*sqrt(-a*b*c^2 + b*d^2)*c*x + d^2)/(b*c^2*x^2 + a*c^2 - d^2)))/(a*b*c^2 - b*d^2), -1/2*(2*sqrt(a*b*c^2 - b*d^2)*arctan(-sqrt(a*b*c^2 - b*d^2)*c*x/(a*c^2 - d^2)) - sqrt(a*b*c^2 - b*d^2)*arctan(1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(a*b*c^2 - b*d^2)*sqrt(b*x^2 + a)/((a*b^2*c^2*d - b^2*d^3)*x^3 + (a^2*b*c^2*d - a*b*d^3)*x)))/(a*b*c^2 - b*d^2)]
```

giac [A] time = 0.36, size = 107, normalized size = 1.04

$$\frac{\arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}} + \frac{\arctan\left(\frac{(\sqrt{b}x-\sqrt{bx^2+a})^2 c^2+ac^2-2d^2}{2\sqrt{ac^2-d^2}d}\right)}{\sqrt{ac^2-d^2}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x, algorithm="giac")
```

```
[Out] arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/sqrt(a*b*c^2 - b*d^2) + arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/(sqrt(a*c^2 - d^2)*sqrt(b))
```

maple [B] time = 0.04, size = 1995, normalized size = 19.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)), x)
```

```
[Out] -1/2*d*b*c^2/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/(-(-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b
```

$$\begin{aligned} &)^{(1/2)} * (x + (-a*b)^{(1/2)}/b)^{(1/2)} + 1/2*d*b^{(1/2)}*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * \ln(((x + (-a*b)^{(1/2)}/b)*b - (-a*b)^{(1/2)})/b^{(1/2)} + ((x + (-a*b)^{(1/2)}/b)^2*b - 2*(-a*b)^{(1/2)}*(x + (-a*b)^{(1/2)}/b))^{(1/2)} + 1/2*d*b*c^2/(-a*b)^{(1/2)} / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)}) \\ &* ((x - (-a*b)^{(1/2)}/b)^2*b + 2*(-a*b)^{(1/2)}*(x - (-a*b)^{(1/2)}/b))^{(1/2)} + 1/2*d*b^{(1/2)}*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * \ln(((x - (-a*b)^{(1/2)}/b)*b + (-a*b)^{(1/2)})/b^{(1/2)} + (x - (-a*b)^{(1/2)}/b)^2*b + 2*(-a*b)^{(1/2)}*(x - (-a*b)^{(1/2)}/b))^{(1/2)} + 1/2*d*b*c^4 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * ((x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 - 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} - 1/2*d*b^{(1/2)}*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * \ln(((x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)*b - (-a*c^2-d^2)*b*c^2)^{(1/2)}/c^2/b^{(1/2)} + ((x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 - 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} - 1/2*b*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-a*c^2-d^2)*b*c^2)^{(1/2)} * d^3 / (1/c^2*d^2)^{(1/2)} * \ln((2/c^2*d^2 - 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2 + 2*(1/c^2*d^2)^{(1/2)}*((x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 - 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} / (x + (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2) - 1/2*d*b*c^4 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-a*c^2-d^2)*b*c^2)^{(1/2)} * ((x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 + 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} - 1/2*d*b^{(1/2)}*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * \ln(((x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)*b + (-a*c^2-d^2)*b*c^2)^{(1/2)}/c^2/b^{(1/2)} + ((x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 + 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} + 1/2*b*c^2 / ((-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} / (-(-a*b)^{(1/2)}*c^2 + (-a*c^2-d^2)*b*c^2)^{(1/2)} * d^3 / (1/c^2*d^2)^{(1/2)} * \ln((2/c^2*d^2 + 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)/c^2 + 2*(1/c^2*d^2)^{(1/2)}*((x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2)^2*b + 1/c^2*d^2 + 2*(-a*c^2-d^2)*b*c^2)^{(1/2)}*(x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2/c^2)^{(1/2)} / (x - (-a*c^2-d^2)*b*c^2)^{(1/2)}/b/c^2) + 1 / ((a*c^2-d^2)*b)^{(1/2)} * \arctan(x*b*c / ((a*c^2-d^2)*b)^{(1/2)}) \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{bcx^2 + ac + \sqrt{bx^2 + a}d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(b*c*x^2 + a*c + sqrt(b*x^2 + a)*d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\left\{ \begin{array}{ll} \frac{\operatorname{atan}\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}} - \frac{dx}{\sqrt{a}(ac^2-d^2)} & \text{if } b = 0 \vee d = 0 \\ \frac{\operatorname{atan}\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}} - \frac{d \operatorname{atan}\left(\frac{x\sqrt{abc^2-b(ac^2-d^2)}}{\sqrt{ac^2-d^2}\sqrt{bx^2+a}}\right)}{\sqrt{-(ac^2-d^2)}(b(ac^2-d^2)-abc^2)} & \text{if } 0 < bd^2 \\ \frac{\operatorname{atan}\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2-bd^2}} - \frac{d \ln\left(\frac{\sqrt{(ac^2-d^2)(bx^2+a)}+x\sqrt{b(ac^2-d^2)-abc^2}}{\sqrt{(ac^2-d^2)(bx^2+a)}-x\sqrt{b(ac^2-d^2)-abc^2}}\right)}{2\sqrt{(ac^2-d^2)}(b(ac^2-d^2)-abc^2)} & \text{if } bd^2 < 0 \\ \int \frac{1}{ac+d\sqrt{bx^2+a}+bcx^2} dx & \text{if } bd^2 \notin \mathbb{R} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x)
```

```
[Out] piecewise(b == 0 | d == 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*x)/(a^(1/2)*(a*c^2 - d^2)), 0 < b*d^2, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*atan((x*(- b*(a*c^2 - d^2) + a*b*c^2)^(1/2))/((a*c^2 - d^2)^(1/2)*(a + b*x^2)^(1/2))))/(-(a*c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2), b*d^2 < 0, atan((b*c*x)/(- b*d^2 + a*b*c^2)^(1/2))/(- b*d^2 + a*b*c^2)^(1/2) - (d*log((((a*c^2 - d^2)*(a + b*x^2))^(1/2) + x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))/(((a*c^2 - d^2)*(a + b*x^2))^(1/2) - x*(b*(a*c^2 - d^2) - a*b*c^2)^(1/2))))/(2*((a*c^2 - d^2)*(b*(a*c^2 - d^2) - a*b*c^2))^(1/2)), ~in(b*d^2, 'real'), int(1/(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{ac + bcx^2 + d\sqrt{a + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)), x)
```

```
[Out] Integral(1/(a*c + b*c*x**2 + d*sqrt(a + b*x**2)), x)
```

$$3.312 \quad \int \frac{1}{x^2(ac+bcx^2+d\sqrt{a+bx^2})} dx$$

Optimal. Leaf size=160

$$\frac{d\sqrt{a+bx^2}}{ax(ac^2-d^2)} + \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}} - \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}} - \frac{c}{x(ac^2-d^2)}$$

Rubi [A] time = 0.24, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2156, 325, 205, 480, 12, 377}

$$\frac{d\sqrt{a+bx^2}}{ax(ac^2-d^2)} + \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a+bx^2}\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}} - \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2-d^2}}\right)}{(ac^2-d^2)^{3/2}} - \frac{c}{x(ac^2-d^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])),x]

[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) - (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2) + (Sqrt[b]*c^2*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*c^2 - d^2)^(3/2)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 325

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^(p+1))/(a*c*(m+1)), x] - Dist[(b*(m+n*(p+1)+1))/(a*c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 480

Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Simp[((e*x)^(m+1)*(a+b*x^n)^(p+1)*(c+d*x^n)^(q+1))/(a*c*e*(m+1)), x] - Dist[1/(a*c*e^n*(m+1)), Int[(e*x)^(m+n)*(a+b*x^n)^p*(c+d*x^n)^q*Simp[(b*c+a*d)*(m+n+1)+n*(b*c*p+a*d*q)+b*d*(m+n*(p+q+2)+1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]

Rule 2156

Int[(u_)/((c_) + (d_)*(x_)^(n_) + (e_)*Sqrt[(a_) + (b_)*(x_)^(n_)]), x_ Symbol] := Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 (ac + bcx^2 + d\sqrt{a + bx^2})} dx &= (ac) \int \frac{1}{x^2 (a^2c^2 - ad^2 + abc^2x^2)} dx - (ad) \int \frac{1}{x^2 \sqrt{a + bx^2} (a^2c^2 - ad^2 + abc^2x^2)} dx \\ &= -\frac{c}{(ac^2 - d^2)x} + \frac{d\sqrt{a + bx^2}}{a(ac^2 - d^2)x} - \frac{(abc^3) \int \frac{1}{a^2c^2 - ad^2 + abc^2x^2} dx}{ac^2 - d^2} + \frac{d \int \frac{1}{\sqrt{a + bx^2} (a^2c^2 - ad^2 + abc^2x^2)} dx}{a(ac^2 - d^2)} \\ &= -\frac{c}{(ac^2 - d^2)x} + \frac{d\sqrt{a + bx^2}}{a(ac^2 - d^2)x} - \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{(ac^2 - d^2)^{3/2}} + \frac{(abc^2d) \int \frac{1}{\sqrt{a + bx^2} (a^2c^2 - ad^2 + abc^2x^2)} dx}{ac^2 - d^2} \\ &= -\frac{c}{(ac^2 - d^2)x} + \frac{d\sqrt{a + bx^2}}{a(ac^2 - d^2)x} - \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{(ac^2 - d^2)^{3/2}} + \frac{(abc^2d) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx^2} (a^2c^2 - ad^2 + abc^2x^2)} dx\right)}{ac^2 - d^2} \\ &= -\frac{c}{(ac^2 - d^2)x} + \frac{d\sqrt{a + bx^2}}{a(ac^2 - d^2)x} - \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{(ac^2 - d^2)^{3/2}} + \frac{\sqrt{b}c^2 \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{(ac^2 - d^2)^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.34, size = 139, normalized size = 0.87

$$\frac{\sqrt{ac^2 - d^2} (d\sqrt{a + bx^2} - ac) + a\sqrt{b}c^2x \tan^{-1}\left(\frac{\sqrt{b}dx}{\sqrt{a + bx^2}\sqrt{ac^2 - d^2}}\right) - a\sqrt{b}c^2x \tan^{-1}\left(\frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}}\right)}{ax(ac^2 - d^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])), x]

[Out] (Sqrt[a*c^2 - d^2]*(-(a*c) + d*Sqrt[a + b*x^2]) - a*Sqrt[b]*c^2*x*ArcTan[(Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2]] + a*Sqrt[b]*c^2*x*ArcTan[(Sqrt[b]*d*x)/(Sqrt[a*c^2 - d^2]*Sqrt[a + b*x^2])])/(a*(a*c^2 - d^2)^(3/2)*x)

IntegrateAlgebraic [A] time = 0.52, size = 148, normalized size = 0.92

$$\frac{d\sqrt{a + bx^2}}{ax(ac^2 - d^2)} + \frac{2\sqrt{b}c^2 \tan^{-1}\left(\frac{c\sqrt{a + bx^2}}{\sqrt{ac^2 - d^2}} - \frac{\sqrt{b}cx}{\sqrt{ac^2 - d^2}} + \frac{d}{\sqrt{ac^2 - d^2}}\right)}{(ac^2 - d^2)^{3/2}} - \frac{c}{x(ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a*c + b*c*x^2 + d*Sqrt[a + b*x^2])), x]

[Out] -(c/((a*c^2 - d^2)*x)) + (d*Sqrt[a + b*x^2])/(a*(a*c^2 - d^2)*x) + (2*Sqrt[b]*c^2*x*ArcTan[d/Sqrt[a*c^2 - d^2] - (Sqrt[b]*c*x)/Sqrt[a*c^2 - d^2] + (c*Sqrt[a + b*x^2])/Sqrt[a*c^2 - d^2]])/(a*c^2 - d^2)^(3/2)

fricas [A] time = 0.75, size = 581, normalized size = 3.63

$$\frac{\frac{ac^2\sqrt{\frac{b}{ac^2-d^2}} \log\left(\frac{(a^2-2bx+bx^2+ad)(a^2-2bx+bx^2+ad)^2+4((a^2-2bx+bx^2+ad)^2+4ad^2)\sqrt{\frac{b}{ac^2-d^2}}}{2(a^2-2bx+bx^2+ad)^2}\right) + 2ac^2\sqrt{\frac{b}{ac^2-d^2}} \log\left(\frac{(a^2-2bx+bx^2+ad)\sqrt{\frac{b}{ac^2-d^2}}}{2(a^2-2bx+bx^2+ad)}\right) + 4ac - 4\sqrt{bd^2+ad} - 2ac^2\sqrt{\frac{b}{ac^2-d^2}} \arctan\left(\frac{a}{\sqrt{2d^2}}\right) - ac^2\sqrt{\frac{b}{ac^2-d^2}} \arctan\left(\frac{(a^2-2bx+bx^2+ad)\sqrt{\frac{b}{ac^2-d^2}}}{2(a^2-2bx+bx^2+ad)}\right) + 2ac - 2\sqrt{bd^2+ad}}{4(a^2-2bx+bx^2+ad)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="fricas")

[Out] [-1/4*(a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4 + (a^2*b^2*c^4 - 8*a*b^2*c^2*d^2 + 8*b^2*d^4)*x^4 + 2*(a^3*b*c^4 - 5*a^2*b*c^2*d^2 + 4*a*b*d^4)*x^2 + 4*((a^2*b*c^4*d - 3*a*b*c^2*d^3 + 2*b*d^5)*x^3 + (a^3*c^4*d - 2*a^2*c^2*d^3 + a*d^5)*x)*sqrt(b*x^2 + a)*sqrt(-b/(a*c^2 - d^2)))/(b^2*c^4*x^4 + a^2*c^4 - 2*a*c^2*d^2 + d^4 + 2*(a*b*c^4 - b*c^2*d^2)*x^2) + 2*a*c^2*x*sqrt(-b/(a*c^2 - d^2))*log((b*c^2*x^2 - a*c^2 + 2*(a*c^3 - c*d^2)*x*sqrt(-b/(a*c^2 - d^2)) + d^2)/(b*c^2*x^2 + a*c^2 - d^2)) + 4*a*c - 4*sqrt(b*x^2 + a)*d/((a^2*c^2 - a*d^2)*x), -1/2*(2*a*c^2*x*sqrt(b/(a*c^2 - d^2))*arctan(c*x*sqrt(b/(a*c^2 - d^2))) - a*c^2*x*sqrt(b/(a*c^2 - d^2))*arctan(-1/2*(a^2*c^2 - a*d^2 + (a*b*c^2 - 2*b*d^2)*x^2)*sqrt(b*x^2 + a)*sqrt(b/(a*c^2 - d^2))/(b^2*d*x^3 + a*b*d*x)) + 2*a*c - 2*sqrt(b*x^2 + a)*d/((a^2*c^2 - a*d^2)*x)]

giac [A] time = 0.46, size = 211, normalized size = 1.32

$$-b^{\frac{3}{2}}d \left(\frac{c^2 \arctan\left(\frac{(\sqrt{bx-\sqrt{bx^2+a}})^2 c^2 + ac^2 - 2d^2}{2\sqrt{ac^2-d^2}d}\right)}{(abc^2 - bd^2)\sqrt{ac^2 - d^2}d} + \frac{2}{(abc^2 - bd^2)\left(\left(\sqrt{bx-\sqrt{bx^2+a}}\right)^2 - a\right)} \right) - \frac{bc^2 \arctan\left(\frac{bcx}{\sqrt{abc^2-bd^2}}\right)}{\sqrt{abc^2 - bd^2}(ac^2 - d^2)} - \frac{c}{(ac^2 - d^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="giac")

[Out] -b^(3/2)*d*(c^2*arctan(1/2*((sqrt(b)*x - sqrt(b*x^2 + a))^2*c^2 + a*c^2 - 2*d^2)/(sqrt(a*c^2 - d^2)*d))/((a*b*c^2 - b*d^2)*sqrt(a*c^2 - d^2)*d) + 2/(((a*b*c^2 - b*d^2)*((sqrt(b)*x - sqrt(b*x^2 + a))^2 - a))) - b*c^2*arctan(b*c*x/sqrt(a*b*c^2 - b*d^2))/(sqrt(a*b*c^2 - b*d^2)*(a*c^2 - d^2)) - c/((a*c^2 - d^2)*x)

maple [B] time = 0.05, size = 2289, normalized size = 14.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x)

[Out] b*c^2/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d^2)*b)^(1/2)*b*c*x)-c/(a*c^2-d^2)/x-a*c^4/(a*c^2-d^2)*b/d^2/((a*c^2-d^2)*b)^(1/2)*arctan(1/((a*c^2-d^2)*b)^(1/2)*b*c*x)-1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x+(-a*b)^(1/2)/b)^2*b-2*(-a*b)^(1/2)*(x+(-a*b)^(1/2)/b))^(1/2)+d/a^2/(a*c^2-d^2)/x*(b*x^2+a)^(3/2)-d/a^2/(a*c^2-d^2)*b*x*(b*x^2+a)^(1/2)-d/a/(a*c^2-d^2)*b^(1/2)*ln(b^(1/2)*x+(b*x^2+a)^(1/2))+1/2*d*b^2*c^2/a/(-a*b)^(1/2)/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2)+1/2*d*b^(3/2)*c^2/a/((-a*b)^(1/2)*c^2+(-(a*c^2-d^2)*b*c^2)^(1/2))/((-a*b)^(1/2)*c^2-(-(a*c^2-d^2)*b*c^2)^(1/2))*ln(((x-(-a*b)^(1/2)/b)*b+(-a*b)^(1/2))/b^(1/2))+((x-(-a*b)^(1/2)/b)^2*b+2*(-a*b)^(1/2)*(x-(-a*b)^(1/2)/b))^(1/2))+1/2*d

$$\frac{b^2 c^6}{(a c^2 - d^2)} \frac{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} \frac{((x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 - 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2} - 1/2 d b^{3/2} c^4 / (a c^2 - d^2)}{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} * \ln \left(\frac{(x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2 * b - (-a c^2 - d^2) b c^2)^{1/2} / c^2}{b^{1/2}} + \frac{(x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 - 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2}}{(x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2) - 1/2 d b^2 c^6 / (a c^2 - d^2)} \right) \frac{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} * d^3 / (1/c^2 d^2)^{1/2} * \ln \left(\frac{(2/c^2 d^2 - 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2 + 2 * (1/c^2 d^2)^{1/2} * ((x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 - 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2}}{(x + (-a c^2 - d^2) b c^2)^{1/2} / b c^2) - 1/2 d b^2 c^6 / (a c^2 - d^2)} \right) \frac{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} * \ln \left(\frac{(x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 + 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2} - 1/2 d b^{3/2} c^4 / (a c^2 - d^2)}{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} * \ln \left(\frac{(x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2 * b + (-a c^2 - d^2) b c^2)^{1/2} / c^2}{b^{1/2}} + \frac{(x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 + 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2}}{(x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2) + 1/2 d b^2 c^4 / (a c^2 - d^2)} \right) \frac{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} * d^3 / (1/c^2 d^2)^{1/2} * \ln \left(\frac{(2/c^2 d^2 + 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2 + 2 * (1/c^2 d^2)^{1/2} * ((x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2)^2 b + 1/c^2 d^2 + 2 * (-a c^2 - d^2) b c^2)^{1/2} * (x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2 / c^2)^{1/2}}{(x - (-a c^2 - d^2) b c^2)^{1/2} / b c^2) \right) \frac{((-a b)^{1/2} c^2 + (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}} \frac{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}{((-a b)^{1/2} c^2 - (-a c^2 - d^2) b c^2)^{1/2}}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bcx^2 + ac + \sqrt{bx^2 + a}d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a*c+b*c*x^2+d*(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/((b*c*x^2 + a*c + sqrt(b*x^2 + a)*d)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 (ac + d\sqrt{bx^2 + a} + bcx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)),x)

[Out] int(1/(x^2*(a*c + d*(a + b*x^2)^(1/2) + b*c*x^2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (ac + bcx^2 + d\sqrt{a + bx^2})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a*c+b*c*x**2+d*(b*x**2+a)**(1/2)),x)

[Out] Integral(1/(x**2*(a*c + b*c*x**2 + d*sqrt(a + b*x**2))), x)

$$3.313 \quad \int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=140

$$-\frac{2d(a+bx^3)^{3/2}}{9b^3c^2} + \frac{2(ac^2-d^2)^2 \log(c\sqrt{a+bx^3}+d)}{3b^3c^5} + \frac{2d\sqrt{a+bx^3}(2ac^2-d^2)}{3b^3c^4} + \frac{(a+bx^3)^2}{6b^3c} - \frac{x^3(2ac^2-d^2)}{3b^2c^3}$$

Rubi [A] time = 0.30, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 697}

$$-\frac{x^3(2ac^2-d^2)}{3b^2c^3} + \frac{2d\sqrt{a+bx^3}(2ac^2-d^2)}{3b^3c^4} + \frac{2(ac^2-d^2)^2 \log(c\sqrt{a+bx^3}+d)}{3b^3c^5} - \frac{2d(a+bx^3)^{3/2}}{9b^3c^2} + \frac{(a+bx^3)^2}{6b^3c}$$

Antiderivative was successfully verified.

[In] Int[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]

[Out] -((2*a*c^2 - d^2)*x^3)/(3*b^2*c^3) + (2*d*(2*a*c^2 - d^2)*Sqrt[a + b*x^3])/(3*b^3*c^4) - (2*d*(a + b*x^3)^(3/2))/(9*b^3*c^2) + (a + b*x^3)^2/(6*b^3*c) + (2*(a*c^2 - d^2)^2*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[1/n, Subst[Int[x^((m+1)/n-1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m+1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^8}{ac+bcx^3+d\sqrt{a+bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{ac+bcx+d\sqrt{a+bx}} dx, x, x^3 \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{(a-x^2)^2}{d+cx} dx, x, \sqrt{a+bx^3} \right)}{3b^3} \\ &= \frac{2 \text{Subst} \left(\int \left(\frac{2ac^2d-d^3}{c^4} - \frac{(2ac^2-d^2)x}{c^3} - \frac{dx^2}{c^2} + \frac{x^3}{c} + \frac{(ac^2-d^2)^2}{c^4(d+cx)} \right) dx, x, \sqrt{a+bx^3} \right)}{3b^3} \\ &= -\frac{(2ac^2-d^2)x^3}{3b^2c^3} + \frac{2d(2ac^2-d^2)\sqrt{a+bx^3}}{3b^3c^4} - \frac{2d(a+bx^3)^{3/2}}{9b^3c^2} + \frac{(a+bx^3)^2}{6b^3c} + \end{aligned}$$

Mathematica [A] time = 0.20, size = 126, normalized size = 0.90

$$\frac{c \left(a \left(20c^2d\sqrt{a+bx^3} - 6bc^3x^3 \right) + 2bcdx^3 \left(3d - 2c\sqrt{a+bx^3} \right) - 12d^3\sqrt{a+bx^3} + 3b^2c^3x^6 \right) + 12(d^2-ac^2)^2 \log(c\sqrt{a+bx^3}+d)}{18b^3c^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] (c*(3*b^2*c^3*x^6 - 12*d^3*Sqrt[a + b*x^3] + 2*b*c*d*x^3*(3*d - 2*c*Sqrt[a + b*x^3])) + a*(-6*b*c^3*x^3 + 20*c^2*d*Sqrt[a + b*x^3])) + 12*(-(a*c^2) + d^2)^2*Log[d + c*Sqrt[a + b*x^3]]/(18*b^3*c^5)

IntegrateAlgebraic [A] time = 0.15, size = 145, normalized size = 1.04

$$\frac{2(a^2c^4 - 2ac^2d^2 + d^4) \log\left(c\sqrt{a + bx^3} + d\right)}{3b^3c^5} + \frac{-3a^2c^2 - 2abc^2x^3 + 2ad^2 + b^2c^2x^6 + 2bd^2x^3}{6b^3c^3} + \frac{2\sqrt{a + bx^3} (5ac^2d - bc^2dx^3 - 3d^3)}{9b^3c^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^8/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] (2*Sqrt[a + b*x^3]*(5*a*c^2*d - 3*d^3 - b*c^2*d*x^3))/(9*b^3*c^4) + (-3*a^2*c^2 + 2*a*d^2 - 2*a*b*c^2*x^3 + 2*b*d^2*x^3 + b^2*c^2*x^6)/(6*b^3*c^3) + (2*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^3*c^5)

fricas [A] time = 0.54, size = 191, normalized size = 1.36

$$\frac{3b^2c^4x^6 - 6(abc^4 - bc^2d^2)x^3 + 6(a^2c^4 - 2ac^2d^2 + d^4) \log(bc^2x^3 + ac^2 - d^2) + 6(a^2c^4 - 2ac^2d^2 + d^4) \log(\sqrt{bx^3 + ac} + d) - 6(a^2c^4 - 2ac^2d^2 + d^4) \log(\sqrt{bx^3 + ac} - d) - 4(bc^2dx^3 - 5ac^2d + 3cd^3)\sqrt{bx^3 + a}}{18b^3c^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")

[Out] 1/18*(3*b^2*c^4*x^6 - 6*(a*b*c^4 - b*c^2*d^2)*x^3 + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(b*c^2*x^3 + a*c^2 - d^2) + 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d) - 6*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c - d) - 4*(b*c^3*d*x^3 - 5*a*c^3*d + 3*c*d^3)*sqrt(b*x^3 + a))/(b^3*c^5)

giac [A] time = 0.39, size = 156, normalized size = 1.11

$$\frac{2(a^2c^4 - 2ac^2d^2 + d^4) \log\left(\sqrt{bx^3 + a} + d\right)}{3b^3c^5} + \frac{3(bx^3 + a)^2b^9c^3 - 12(bx^3 + a)ab^9c^3 - 4(bx^3 + a)^{\frac{3}{2}}b^9c^2d + 24\sqrt{bx^3 + a}ab^9c^2d + 6(bx^3 + a)b^9cd^2 - 12\sqrt{bx^3 + a}b^9d^3}{18b^{12}c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")

[Out] 2/3*(a^2*c^4 - 2*a*c^2*d^2 + d^4)*log(abs(sqrt(b*x^3 + a)*c + d))/(b^3*c^5) + 1/18*(3*(b*x^3 + a)^2*b^9*c^3 - 12*(b*x^3 + a)*a*b^9*c^3 - 4*(b*x^3 + a)^(3/2)*b^9*c^2*d + 24*sqrt(b*x^3 + a)*a*b^9*c^2*d + 6*(b*x^3 + a)*b^9*c*d^2 - 12*sqrt(b*x^3 + a)*b^9*d^3)/(b^12*c^4)

maple [C] time = 0.18, size = 1473, normalized size = 10.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)

[Out] -2/9*d*(b*x^3+a)^(3/2)/b^3/c^2+4/3*d/b^3/c^2*(b*x^3+a)^(1/2)*a-2/3/b^3/c^4*d^3*(b*x^3+a)^(1/2)-1/3*I/b^5/d*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3)))/((-a*b^2)^(1/3))^(1/2)*(b*(x-1/b*((-a*b^2)^(1/3)))/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))^(1/2)*(-1/2*I*b*(2*x+1/b*((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3)))/((-a*b^2)^(1/3))^(1/2)/(b*x^3+a)^(1/2)*(I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)+2*_alpha^2*b^2-(-a*b^2)^(1/3)*_alpha*b-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2/b*((-a*b^2)^(1/3))-1/2*I*3^(1/2)/b*((-a*b^2)^(1/3)))*3^(1/2)*b/((-a*b^2)^(1/3))^(1/2),-1/2/b*c^2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha+I*3^(1/2)*a*b-3*(-a*b^2)^(2/3)*_alpha-3*a*b)

$/d^2, (I^3^{(1/2)}/b*(-a*b^2)^{(1/3)} / (-3/2/b*(-a*b^2)^{(1/3)} + 1/2*I^3^{(1/2)}/b*(-a*b^2)^{(1/3)}))^{(1/2)}, _alpha = \text{RootOf}(_Z^3*b*c^2+a*c^2-d^2)*a^2+2/3*I*d/b^5/c^2*2^{(1/2)}*\text{sum}((-a*b^2)^{(1/3)}*(1/2*I*b*(2*x+1/b*((-a*b^2)^{(1/3)}-I^3^{(1/2)}*(-a*b^2)^{(1/3)})))/(-a*b^2)^{(1/3)})^{(1/2)}*(b*(x-1/b*(-a*b^2)^{(1/3)})/(-3*(-a*b^2)^{(1/3)}+I^3^{(1/2)}*(-a*b^2)^{(1/3)}))^{(1/2)}*(-1/2*I*b*(2*x+1/b*((-a*b^2)^{(1/3)}+I^3^{(1/2)}*(-a*b^2)^{(1/3)})))/(-a*b^2)^{(1/3)}^{(1/2)}/(b*x^3+a)^{(1/2)}*(I*(-a*b^2)^{(1/3)}*3^{(1/2)}*_alpha*b-I*(-a*b^2)^{(2/3)}*3^{(1/2)}+2*_alpha^2*b^2-(-a*b^2)^{(1/3)}*_alpha*b-(-a*b^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-a*b^2)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-a*b^2)^{(1/3)})*3^{(1/2)}*b/(-a*b^2)^{(1/3)})^{(1/2)}, -1/2/b*c^2*(2*I*(-a*b^2)^{(1/3)}*3^{(1/2)}*_alpha^2*b-I*(-a*b^2)^{(2/3)}*3^{(1/2)}*_alpha+I^3^{(1/2)}*a*b-3*(-a*b^2)^{(2/3)}*_alpha-3*a*b)/d^2, (I^3^{(1/2)}/b*(-a*b^2)^{(1/3)} / (-3/2/b*(-a*b^2)^{(1/3)} + 1/2*I^3^{(1/2)}/b*(-a*b^2)^{(1/3)}))^{(1/2)}, _alpha = \text{RootOf}(_Z^3*b*c^2+a*c^2-d^2)*a-1/3*I/b^5/c^4*d^3*2^{(1/2)}*\text{sum}((-a*b^2)^{(1/3)}*(1/2*I*b*(2*x+1/b*((-a*b^2)^{(1/3)}-I^3^{(1/2)}*(-a*b^2)^{(1/3)})))/(-a*b^2)^{(1/3)})^{(1/2)}*(b*(x-1/b*(-a*b^2)^{(1/3)})/(-3*(-a*b^2)^{(1/3)}+I^3^{(1/2)}*(-a*b^2)^{(1/3)}))^{(1/2)}*(-1/2*I*b*(2*x+1/b*((-a*b^2)^{(1/3)}+I^3^{(1/2)}*(-a*b^2)^{(1/3)})))/(-a*b^2)^{(1/3)}^{(1/2)}/(b*x^3+a)^{(1/2)}*(I*(-a*b^2)^{(1/3)}*3^{(1/2)}*_alpha*b-I*(-a*b^2)^{(2/3)}*3^{(1/2)}+2*_alpha^2*b^2-(-a*b^2)^{(1/3)}*_alpha*b-(-a*b^2)^{(2/3)})*\text{EllipticPi}(1/3*3^{(1/2)}*(I*(x+1/2/b*(-a*b^2)^{(1/3)}-1/2*I^3^{(1/2)}/b*(-a*b^2)^{(1/3)})*3^{(1/2)}*b/(-a*b^2)^{(1/3)})^{(1/2)}, -1/2/b*c^2*(2*I*(-a*b^2)^{(1/3)}*3^{(1/2)}*_alpha^2*b-I*(-a*b^2)^{(2/3)}*3^{(1/2)}*_alpha+I^3^{(1/2)}*a*b-3*(-a*b^2)^{(2/3)}*_alpha-3*a*b)/d^2, (I^3^{(1/2)}/b*(-a*b^2)^{(1/3)} / (-3/2/b*(-a*b^2)^{(1/3)} + 1/2*I^3^{(1/2)}/b*(-a*b^2)^{(1/3)}))^{(1/2)}, _alpha = \text{RootOf}(_Z^3*b*c^2+a*c^2-d^2))-1/3*a/c/b^2*x^3+1/3*a^2/c/b^3*\ln(b*c^2*x^3+a*c^2-d^2)-2/3*a/c^3/b^3*d^2*\ln(b*c^2*x^3+a*c^2-d^2)+1/6/b/c*x^6+1/3/b^2/c^3*x^3*d^2+1/3/b^3/c^5*d^4*\ln(b*c^2*x^3+a*c^2-d^2)$

maxima [A] time = 0.78, size = 125, normalized size = 0.89

$$\frac{3(bx^3+a)^2c^3-4(bx^3+a)^2c^2d-6(2ac^3-cd^2)(bx^3+a)+12(2ac^2d-d^3)\sqrt{bx^3+a}}{c^4} + \frac{12(a^2c^4-2ac^2d^2+d^4)\log(\sqrt{bx^3+a}+d)}{c^5}$$

18b³

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
[Out] 1/18*((3*(b*x^3 + a)^2*c^3 - 4*(b*x^3 + a)^(3/2)*c^2*d - 6*(2*a*c^3 - c*d^2)
)*(b*x^3 + a) + 12*(2*a*c^2*d - d^3)*sqrt(b*x^3 + a))/c^4 + 12*(a^2*c^4 - 2
*a*c^2*d^2 + d^4)*log(sqrt(b*x^3 + a)*c + d)/c^5/b^3
```

mupad [B] time = 3.74, size = 200, normalized size = 1.43

$$\frac{\left(\frac{2d(a^2-d^2)}{b^2c^4} + \frac{4ad}{3b^2c^2}\right)\sqrt{bx^3+a}}{3b} + \frac{x^6}{6bc} - \frac{x^3(a^2-d^2)}{3b^2c^3} + \frac{\ln\left(\frac{d+c\sqrt{bx^3+a}}{d-c\sqrt{bx^3+a}}\right)(a^2-d^2)^2}{3b^3c^5} + \frac{\ln(b^2c^2x^3+a^2-d^2)(a^2c^4-2ac^2d^2+d^4)}{3b^3c^5} - \frac{2dx^3\sqrt{bx^3+a}}{9b^2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)
[Out] (((2*d*(a*c^2 - d^2))/(b^2*c^4) + (4*a*d)/(3*b^2*c^2))*(a + b*x^3)^(1/2))/(
3*b) + x^6/(6*b*c) - (x^3*(a*c^2 - d^2))/(3*b^2*c^3) + (log((d + c*(a + b*x
^3)^(1/2))/(d - c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2)^2)/(3*b^3*c^5) + (log(a
*c^2 - d^2 + b*c^2*x^3)*(d^4 + a^2*c^4 - 2*a*c^2*d^2))/(3*b^3*c^5) - (2*d*x
^3*(a + b*x^3)^(1/2))/(9*b^2*c^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

$$3.314 \quad \int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=73

$$-\frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2(ac^2-d^2)\log(c\sqrt{a+bx^3}+d)}{3b^2c^3} + \frac{x^3}{3bc}$$

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 697}

$$-\frac{2(ac^2-d^2)\log(c\sqrt{a+bx^3}+d)}{3b^2c^3} - \frac{2d\sqrt{a+bx^3}}{3b^2c^2} + \frac{x^3}{3bc}$$

Antiderivative was successfully verified.

[In] Int[x^5/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]), x]

[Out] x^3/(3*b*c) - (2*d*Sqrt[a + b*x^3])/(3*b^2*c^2) - (2*(a*c^2 - d^2)*Log[d + c*Sqrt[a + b*x^3]])/(3*b^2*c^3)

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]) , x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^5}{ac+bcx^3+d\sqrt{a+bx^3}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x}{ac+bcx+d\sqrt{a+bx}} dx, x, x^3 \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{-a+x^2}{d+cx} dx, x, \sqrt{a+bx^3} \right)}{3b^2} \\ &= \frac{2 \text{Subst} \left(\int \left(-\frac{d}{c^2} + \frac{x}{c} + \frac{-ac^2+d^2}{c^2(d+cx)} \right) dx, x, \sqrt{a+bx^3} \right)}{3b^2} \\ &= \frac{x^3}{3bc} - \frac{2d\sqrt{a+bx^3}}{3b^2c^2} - \frac{2(ac^2-d^2)\log(d+c\sqrt{a+bx^3})}{3b^2c^3} \end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.86

$$\frac{(2d^2 - 2ac^2)\log(c\sqrt{a+bx^3}+d) + c(bc^3 - 2d\sqrt{a+bx^3})}{3b^2c^3}$$

Antiderivative was successfully verified.

$$\frac{1}{3} + I \cdot 3^{1/2} \cdot (-a \cdot b^2)^{1/3} \cdot b^{1/2} \cdot (-1/2 \cdot I \cdot (2 \cdot x + ((-a \cdot b^2)^{1/3} + I \cdot 3^{1/2}) \cdot (-a \cdot b^2)^{1/3}) / b) / (-a \cdot b^2)^{1/3} \cdot b^{1/2} / (b \cdot x^3 + a)^{1/2} \cdot (2 \cdot \alpha^2 \cdot b^2 + I \cdot (-a \cdot b^2)^{1/3} \cdot 3^{1/2} \cdot \alpha \cdot b - (-a \cdot b^2)^{1/3} \cdot \alpha \cdot b - I \cdot (-a \cdot b^2)^{2/3} \cdot 3^{1/2} - (-a \cdot b^2)^{2/3}) \cdot \text{EllipticPi}(1/3 \cdot 3^{1/2} \cdot (I \cdot (x + 1/2 \cdot (-a \cdot b^2)^{1/3}) / b - 1/2 \cdot I \cdot 3^{1/2} \cdot (-a \cdot b^2)^{1/3} / b) \cdot 3^{1/2} / (-a \cdot b^2)^{1/3} \cdot b^{1/2}), -1/2 \cdot (2 \cdot I \cdot (-a \cdot b^2)^{1/3} \cdot 3^{1/2} \cdot \alpha^2 \cdot b + I \cdot 3^{1/2} \cdot a \cdot b - 3 \cdot a \cdot b - I \cdot (-a \cdot b^2)^{2/3} \cdot 3^{1/2} \cdot \alpha - 3 \cdot (-a \cdot b^2)^{2/3} \cdot \alpha) / b \cdot c^2 / d^2, (I \cdot 3^{1/2} \cdot (-a \cdot b^2)^{1/3} / (-3/2 \cdot (-a \cdot b^2)^{1/3} / b + 1/2 \cdot I \cdot 3^{1/2} \cdot (-a \cdot b^2)^{1/3} / b) / b)^{1/2}), \alpha = \text{RootOf}(_Z^3 \cdot b \cdot c^2 + a \cdot c^2 - d^2) - 1/3 \cdot a \cdot c / b^2 \cdot \ln(b \cdot c^2 \cdot x^3 + a \cdot c^2 - d^2) + 1/3 \cdot x^3 / b \cdot c + 1/3 \cdot b^2 / c^3 \cdot d^2 \cdot \ln(b \cdot c^2 \cdot x^3 + a \cdot c^2 - d^2)$$

maxima [A] time = 0.58, size = 62, normalized size = 0.85

$$\frac{\frac{(bx^3+a)c-2\sqrt{bx^3+ad}}{c^2} - \frac{2(ac^2-d^2)\log(\sqrt{bx^3+ad})}{c^3}}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")

[Out] 1/3*((b*x^3 + a)*c - 2*sqrt(b*x^3 + a)*d)/c^2 - 2*(a*c^2 - d^2)*log(sqrt(b*x^3 + a)*c + d)/c^3)/b^2

mupad [B] time = 3.56, size = 119, normalized size = 1.63

$$\frac{x^3}{3bc} - \frac{2d\sqrt{bx^3+a}}{3b^2c^2} + \frac{\ln\left(\frac{d-c\sqrt{bx^3+a}}{d+c\sqrt{bx^3+a}}\right)(ac^2-d^2)}{3b^2c^3} - \frac{\ln(bc^2x^3+ac^2-d^2)(ac^2-d^2)}{3b^2c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3),x)

[Out] x^3/(3*b*c) - (2*d*(a + b*x^3)^(1/2))/(3*b^2*c^2) + (log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x^3)^(1/2)))*(a*c^2 - d^2))/(3*b^2*c^3) - (log(a*c^2 - d^2 + b*c^2*x^3)*(a*c^2 - d^2))/(3*b^2*c^3)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**5/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)

[Out] Timed out

$$3.315 \quad \int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx$$

Optimal. Leaf size=26

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Rubi [A] time = 0.11, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {2155, 31}

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{ac+bcx^3+d\sqrt{a+bx^3}} dx &= \frac{1}{3} \text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}} dx, x, x^3\right) \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{d+cx} dx, x, \sqrt{a+bx^3}\right)}{3b} \\ &= \frac{2\log\left(d+c\sqrt{a+bx^3}\right)}{3bc} \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.00

$$\frac{2\log\left(c\sqrt{a+bx^3}+d\right)}{3bc}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]

[Out] (2*Log[d + c*Sqrt[a + b*x^3]])/(3*b*c)

IntegrateAlgebraic [A] time = 0.05, size = 29, normalized size = 1.12

$$\frac{2\log\left(bc\sqrt{a+bx^3}+bd\right)}{3bc}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^2/(a*c + b*c*x^3 + d*Sqrt[a + b*x^3]),x]
```

```
[Out] (2*Log[b*d + b*c*Sqrt[a + b*x^3]])/(3*b*c)
```

fricas [B] time = 0.72, size = 61, normalized size = 2.35

$$\frac{\log(bc^2x^3 + ac^2 - d^2) + \log(\sqrt{bx^3 + ac} + d) - \log(\sqrt{bx^3 + ac} - d)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*(log(b*c^2*x^3 + a*c^2 - d^2) + log(sqrt(b*x^3 + a)*c + d) - log(sqrt(b*x^3 + a)*c - d))/(b*c)
```

giac [A] time = 0.32, size = 23, normalized size = 0.88

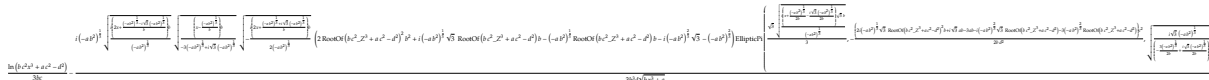
$$\frac{2 \log(|\sqrt{bx^3 + ac} + d|)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] 2/3*log(abs(sqrt(b*x^3 + a)*c + d))/(b*c)
```

maple [C] time = 0.03, size = 455, normalized size = 17.50



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] -1/3*I/d/b^3*2^(1/2)*sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2))*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-((-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2))*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2))*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2))*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2))*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2))*(-a*b^2)^(1/3)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))+1/3/b/c*ln(b*c^2*x^3+a*c^2-d^2)
```

maxima [A] time = 0.56, size = 22, normalized size = 0.85

$$\frac{2 \log(\sqrt{bx^3 + ac} + d)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] 2/3*log(sqrt(b*x^3 + a)*c + d)/(b*c)
```

mupad [B] time = 3.51, size = 60, normalized size = 2.31

$$\frac{\ln\left(\frac{d+c\sqrt{bx^3+a}}{d-c\sqrt{bx^3+a}}\right) + \ln(bc^2x^3 + ac^2 - d^2)}{3bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3), x)`

[Out] `(log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))) + log(a*c^2 - d^2 + b*c^2*x^3))/(3*b*c)`

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)), x)`

[Out] Timed out

$$3.316 \quad \int \frac{1}{x(ac+bcx^3+d\sqrt{a+bx^3})} dx$$

Optimal. Leaf size=93

$$-\frac{2c \log\left(c\sqrt{a+bx^3}+d\right)}{3(ac^2-d^2)} + \frac{2d \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}(ac^2-d^2)} + \frac{c \log(x)}{ac^2-d^2}$$

Rubi [A] time = 0.22, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2155, 706, 31, 635, 207, 260}

$$-\frac{2c \log\left(c\sqrt{a+bx^3}+d\right)}{3(ac^2-d^2)} + \frac{2d \tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3\sqrt{a}(ac^2-d^2)} + \frac{c \log(x)}{ac^2-d^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]]/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[x])/ (a*c^2 - d^2) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 706

Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d - c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d

, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(ac + bcx^3 + d\sqrt{a + bx^3})} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x(ac + bcx + d\sqrt{a + bx})} dx, x, x^3 \right) \\
 &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{(d + cx)(-a + x^2)} dx, x, \sqrt{a + bx^3} \right) \\
 &= -\frac{2 \text{Subst} \left(\int \frac{d-cx}{-a+x^2} dx, x, \sqrt{a + bx^3} \right)}{3(ac^2 - d^2)} - \frac{(2c^2) \text{Subst} \left(\int \frac{1}{d+cx} dx, x, \sqrt{a + bx^3} \right)}{3(ac^2 - d^2)} \\
 &= -\frac{2c \log(d + c\sqrt{a + bx^3})}{3(ac^2 - d^2)} + \frac{(2c) \text{Subst} \left(\int \frac{x}{-a+x^2} dx, x, \sqrt{a + bx^3} \right)}{3(ac^2 - d^2)} - \frac{(2d) \text{Subst} \left(\int \frac{1}{d+cx} dx, x, \sqrt{a + bx^3} \right)}{3(ac^2 - d^2)} \\
 &= \frac{2d \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}(ac^2 - d^2)} + \frac{c \log(x)}{ac^2 - d^2} - \frac{2c \log(d + c\sqrt{a + bx^3})}{3(ac^2 - d^2)}
 \end{aligned}$$

Mathematica [A] time = 0.11, size = 107, normalized size = 1.15

$$\frac{(\sqrt{a}c - d) \log(\sqrt{a} - \sqrt{a + bx^3}) + (\sqrt{a}c + d) \log(\sqrt{a + bx^3} + \sqrt{a}) - 2\sqrt{a}c \log(c\sqrt{a + bx^3} + d)}{3\sqrt{a}(ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out] ((Sqrt[a]*c - d)*Log[Sqrt[a] - Sqrt[a + b*x^3]] + (Sqrt[a]*c + d)*Log[Sqrt[a] + Sqrt[a + b*x^3]] - 2*Sqrt[a]*c*Log[d + c*Sqrt[a + b*x^3]])/(3*Sqrt[a]*(a*c^2 - d^2))

IntegrateAlgebraic [A] time = 0.10, size = 100, normalized size = 1.08

$$\frac{c \log(bx^3)}{3(ac^2 - d^2)} - \frac{2c \log(c\sqrt{a + bx^3} + d)}{3(ac^2 - d^2)} + \frac{2d \tanh^{-1} \left(\frac{\sqrt{a+bx^3}}{\sqrt{a}} \right)}{3\sqrt{a}(ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out] (2*d*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*Sqrt[a]*(a*c^2 - d^2)) + (c*Log[b*x^3])/(3*(a*c^2 - d^2)) - (2*c*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2))

fricas [A] time = 0.93, size = 232, normalized size = 2.49

$$\left[\frac{ac \log(bc^2x^3 + ac^2 - d^2) + ac \log(\sqrt{bx^3 + ac + d}) - ac \log(\sqrt{bx^3 + ac - d}) - 3ac \log(x) - \sqrt{a}d \log\left(\frac{bx^3 + 2\sqrt{bx^3 + ac + d}}{3}\right)}{3(a^2c^2 - ad^2)}, \frac{ac \log(bc^2x^3 + ac^2 - d^2) + ac \log(\sqrt{bx^3 + ac + d}) - ac \log(\sqrt{bx^3 + ac - d}) - 3ac \log(x) + 2\sqrt{-a}d \arctan\left(\frac{\sqrt{bx^3 + ac}}{a}\right)}{3(a^2c^2 - ad^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")

```
[Out] [-1/3*(a*c*log(b*c^2*x^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) -
a*c*log(sqrt(b*x^3 + a)*c - d) - 3*a*c*log(x) - sqrt(a)*d*log((b*x^3 + 2*sq
rt(b*x^3 + a)*sqrt(a) + 2*a)/x^3))/(a^2*c^2 - a*d^2), -1/3*(a*c*log(b*c^2*x
^3 + a*c^2 - d^2) + a*c*log(sqrt(b*x^3 + a)*c + d) - a*c*log(sqrt(b*x^3 + a
)*c - d) - 3*a*c*log(x) + 2*sqrt(-a)*d*arctan(sqrt(b*x^3 + a)*sqrt(-a)/a))/
(a^2*c^2 - a*d^2)]
```

giac [A] time = 0.35, size = 94, normalized size = 1.01

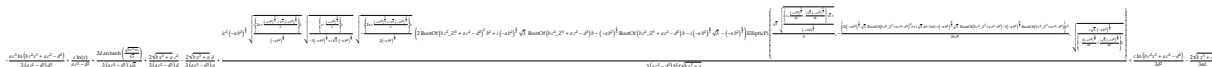
$$-\frac{2c^2 \log\left(\left|\sqrt{bx^3 + a}c + d\right|\right)}{3(ac^3 - cd^2)} + \frac{c \log(bx^3)}{3(ac^2 - d^2)} - \frac{2d \arctan\left(\frac{\sqrt{bx^3 + a}}{\sqrt{-a}}\right)}{3(ac^2 - d^2)\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")
```

```
[Out] -2/3*c^2*log(abs(sqrt(b*x^3 + a)*c + d))/(a*c^3 - c*d^2) + 1/3*c*log(b*x^3)
/(a*c^2 - d^2) - 2/3*d*arctan(sqrt(b*x^3 + a)/sqrt(-a))/((a*c^2 - d^2)*sqrt
(-a))
```

maple [C] time = 0.06, size = 636, normalized size = 6.84



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)
```

```
[Out] 1/(a*c^2-d^2)*c*ln(x)-1/3*a*c^3/(a*c^2-d^2)/d^2*ln(b*c^2*x^3+a*c^2-d^2)+1/3
*c/d^2*ln(b*c^2*x^3+a*c^2-d^2)-2/3/a/d*(b*x^3+a)^(1/2)-2/3*d/a/(a*c^2-d^2)*
(b*x^3+a)^(1/2)+2/3*d*arctanh((b*x^3+a)^(1/2)/a^(1/2))/(a*c^2-d^2)/a^(1/2)+
2/3*c^2/(a*c^2-d^2)/d*(b*x^3+a)^(1/2)+1/3*I/b^2*c^2/(a*c^2-d^2)/d*2^(1/2)*s
um((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2))*(-a*b^2)^(1/3))/b)/
(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*
(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2))*(-a*b^2)^(1
/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(
1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a
*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2
))*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2), -1/2*(2*I*(-a*b^2)^(1/3
))*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*
(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2, (I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(
1/3)/b+1/2*I*3^(1/2))*(-a*b^2)^(1/3)/b)/b)^(1/2)), _alpha=RootOf(_Z^3*b*c^2+a
*c^2-d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bcx^3 + ac + \sqrt{bx^3 + a}d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x), x)
```

mupad [B] time = 4.31, size = 156, normalized size = 1.68

$$\frac{c \ln(x)}{a c^2 - d^2} + \frac{c \ln\left(\frac{d-c \sqrt{b x^3+a}}{d+c \sqrt{b x^3+a}}\right)}{3\left(a c^2 - d^2\right)} - \frac{c \ln\left(b c^2 x^3 + a c^2 - d^2\right)}{3 a c^2 - 3 d^2} + \frac{d \ln\left(\frac{\left(\sqrt{b x^3+a}-\sqrt{a}\right)\left(\sqrt{b x^3+a}+\sqrt{a}\right)^3}{x^6}\right)}{3 \sqrt{a}\left(a c^2 - d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)
```

```
[Out] (c*log(x))/(a*c^2 - d^2) + (c*log((d - c*(a + b*x^3)^(1/2))/(d + c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^2)) - (c*log(a*c^2 - d^2 + b*c^2*x^3))/(3*a*c^2 - 3*d^2) + (d*log((((a + b*x^3)^(1/2) - a^(1/2))*((a + b*x^3)^(1/2) + a^(1/2))^3)/x^6))/(3*a^(1/2)*(a*c^2 - d^2))
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)
```

```
[Out] Timed out
```

$$3.317 \quad \int \frac{1}{x^4(ac+bcx^3+d\sqrt{a+bx^3})} dx$$

Optimal. Leaf size=154

$$\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}(ac^2-d^2)^2} - \frac{ac-d\sqrt{a+bx^3}}{3ax^3(ac^2-d^2)} + \frac{2bc^3\log(c\sqrt{a+bx^3}+d)}{3(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2}$$

Rubi [A] time = 0.30, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {2155, 741, 801, 635, 206, 260}

$$\frac{bd(3ac^2-d^2)\tanh^{-1}\left(\frac{\sqrt{a+bx^3}}{\sqrt{a}}\right)}{3a^{3/2}(ac^2-d^2)^2} - \frac{ac-d\sqrt{a+bx^3}}{3ax^3(ac^2-d^2)} + \frac{2bc^3\log(c\sqrt{a+bx^3}+d)}{3(ac^2-d^2)^2} - \frac{bc^3\log(x)}{(ac^2-d^2)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out] -(a*c - d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*d*(3*a*c^2 - d^2)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[x])/(a*c^2 - d^2)^2 + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^2)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 741

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 801

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_)*(x_)^(n_) + (e_)*Sqrt[(a_) + (b_)*(x_)^(n_)], x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^4 (ac + bcx^3 + d\sqrt{a + bx^3})} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1}{x^2 (ac + bcx + d\sqrt{a + bx})} dx, x, x^3 \right) \\
 &= \frac{1}{3} (2b) \text{Subst} \left(\int \frac{1}{(d + cx) (a - x^2)^2} dx, x, \sqrt{a + bx^3} \right) \\
 &= -\frac{ac - d\sqrt{a + bx^3}}{3a (ac^2 - d^2) x^3} - \frac{b \text{Subst} \left(\int \frac{-2ac^2 + d^2 + cdx}{(d + cx)(a - x^2)} dx, x, \sqrt{a + bx^3} \right)}{3a (ac^2 - d^2)} \\
 &= -\frac{ac - d\sqrt{a + bx^3}}{3a (ac^2 - d^2) x^3} - \frac{b \text{Subst} \left(\int \left(-\frac{2ac^4}{(ac^2 - d^2)(d + cx)} + \frac{3ac^2d - d^3 - 2ac^3x}{(ac^2 - d^2)(a - x^2)} \right) dx, x, \sqrt{a + bx^3} \right)}{3a (ac^2 - d^2)} \\
 &= -\frac{ac - d\sqrt{a + bx^3}}{3a (ac^2 - d^2) x^3} + \frac{2bc^3 \log(d + c\sqrt{a + bx^3})}{3 (ac^2 - d^2)^2} - \frac{b \text{Subst} \left(\int \frac{3ac^2d - d^3 - 2ac^3x}{a - x^2} dx, x, \sqrt{a + bx^3} \right)}{3a (ac^2 - d^2)} \\
 &= -\frac{ac - d\sqrt{a + bx^3}}{3a (ac^2 - d^2) x^3} + \frac{2bc^3 \log(d + c\sqrt{a + bx^3})}{3 (ac^2 - d^2)^2} + \frac{(2bc^3) \text{Subst} \left(\int \frac{x}{a - x^2} dx, x, \sqrt{a + bx^3} \right)}{3 (ac^2 - d^2)^2} \\
 &= -\frac{ac - d\sqrt{a + bx^3}}{3a (ac^2 - d^2) x^3} - \frac{bd (3ac^2 - d^2) \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3a^{3/2} (ac^2 - d^2)^2} - \frac{bc^3 \log(x)}{(ac^2 - d^2)^2} + \frac{2bc^3 \log(d + c\sqrt{a + bx^3})}{3 (ac^2 - d^2)^2}
 \end{aligned}$$

Mathematica [A] time = 0.67, size = 307, normalized size = 1.99

$$\frac{\sqrt{a} (-a^2c^3\sqrt{a + bx^3} + a^2c^2d + 2abc^3x\sqrt{a + bx^3} \tanh^{-1}(\frac{\sqrt{a+bx^3}}{d}) - 3abc^3x^3 \log(x)\sqrt{a + bx^3} + bdx^3 \sqrt{\frac{bx^3}{a} + 1} (ac^2 - d^2) \tanh^{-1}(\sqrt{\frac{bx^3}{a} + 1}) + abc^2dx^3 + abc^3x^3\sqrt{a + bx^3} \log(ac^2 + bc^2x^3 - d^2) + acd^2\sqrt{a + bx^3} - ad^3 - bd^2x^3) - 2bdx^3\sqrt{a + bx^3} (2ac^2 - d^2) \tanh^{-1}(\frac{\sqrt{a+bx^3}}{d})}{3a^{3/2}x^3\sqrt{a + bx^3} (d^2 - ac^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out] (-2*b*d*(2*a*c^2 - d^2)*x^3*Sqrt[a + b*x^3]*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a] + Sqrt[a]*(a^2*c^2*d - a*d^3 + a*b*c^2*d*x^3 - b*d^3*x^3 - a^2*c^3*Sqrt[a + b*x^3] + a*c*d^2*Sqrt[a + b*x^3] + 2*a*b*c^3*x^3*Sqrt[a + b*x^3]*ArcTanh[(c*Sqrt[a + b*x^3])/d] + b*d*(a*c^2 - d^2)*x^3*Sqrt[1 + (b*x^3)/a]*ArcTanh[Sqrt[1 + (b*x^3)/a]] - 3*a*b*c^3*x^3*Sqrt[a + b*x^3]*Log[x] + a*b*c^3*x^3*Sqrt[a + b*x^3]*Log[a*c^2 - d^2 + b*c^2*x^3])/(3*a^(3/2)*(-(a*c^2) + d^2)^2*x^3*Sqrt[a + b*x^3])

IntegrateAlgebraic [A] time = 0.22, size = 175, normalized size = 1.14

$$-\frac{b(3ac^2d - d^3) \tanh^{-1} \left(\frac{\sqrt{a + bx^3}}{\sqrt{a}} \right)}{3a^{3/2} (ac^2 - d^2)^2} + \frac{d\sqrt{a + bx^3}}{3ax^3 (ac^2 - d^2)} - \frac{bc^3 \log(bx^3)}{3(ac^2 - d^2)^2} + \frac{2bc^3 \log(c\sqrt{a + bx^3} + d)}{3(ac^2 - d^2)^2} - \frac{c}{3x^3 (ac^2 - d^2)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^4*(a*c + b*c*x^3 + d*Sqrt[a + b*x^3])),x]

[Out]
$$-1/3*c/((a*c^2 - d^2)*x^3) + (d*Sqrt[a + b*x^3])/(3*a*(a*c^2 - d^2)*x^3) - (b*(3*a*c^2*d - d^3)*ArcTanh[Sqrt[a + b*x^3]/Sqrt[a]])/(3*a^(3/2)*(a*c^2 - d^2)^2) - (b*c^3*Log[b*x^3])/(3*(a*c^2 - d^2)^2) + (2*b*c^3*Log[d + c*Sqrt[a + b*x^3]])/(3*(a*c^2 - d^2)^2)$$

fricas [A] time = 0.80, size = 445, normalized size = 2.89

$$\frac{2abc^2 \log(\sqrt{bx^3+a}) + 2abc^2 \log(\sqrt{bx^3+a}) - 2abc^2 \log(\sqrt{bx^3+a}) - 6abc^2 \log(\sqrt{bx^3+a}) - 2abc^2 \log(\sqrt{bx^3+a}) - 2abc^2 \log(\sqrt{bx^3+a}) + 2abc^2 \log(\sqrt{bx^3+a}) + 2abc^2 \log(\sqrt{bx^3+a}) + 2abc^2 \log(\sqrt{bx^3+a}) + 2abc^2 \log(\sqrt{bx^3+a})}{3(a^4 - 2a^3c^2 + ad^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="fricas")

[Out]
$$[1/6*(2*a^2*b*c^3*x^3*\log(b*c^2*x^3 + a*c^2 - d^2) + 2*a^2*b*c^3*x^3*\log(\sqrt{bx^3 + a})*c + d) - 2*a^2*b*c^3*x^3*\log(\sqrt{bx^3 + a})*c - d) - 6*a^2*b*c^3*x^3*\log(x) - 2*a^3*c^3 - (3*a*b*c^2*d - b*d^3)*\sqrt{a}*x^3*\log((b*x^3 + 2*\sqrt{bx^3 + a})*\sqrt{a} + 2*a)/x^3) + 2*a^2*c*d^2 + 2*(a^2*c^2*d - a*d^3)*\sqrt{bx^3 + a})/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3), 1/3*(a^2*b*c^3*x^3*\log(b*c^2*x^3 + a*c^2 - d^2) + a^2*b*c^3*x^3*\log(\sqrt{bx^3 + a})*c + d) - a^2*b*c^3*x^3*\log(\sqrt{bx^3 + a})*c - d) - 3*a^2*b*c^3*x^3*\log(x) - a^3*c^3 + (3*a*b*c^2*d - b*d^3)*\sqrt{-a}*x^3*\arctan(\sqrt{bx^3 + a}*\sqrt{-a})/a) + a^2*c*d^2 + (a^2*c^2*d - a*d^3)*\sqrt{bx^3 + a})/((a^4*c^4 - 2*a^3*c^2*d^2 + a^2*d^4)*x^3)]$$

giac [A] time = 0.34, size = 211, normalized size = 1.37

$$\frac{2bc^4 \log\left(\sqrt{bx^3+ac+d}\right)}{3(a^2c^5-2ac^3d^2+cd^4)} - \frac{bc^3 \log(-bx^3)}{3(a^2c^4-2ac^2d^2+d^4)} + \frac{(3abc^2d-bd^3) \arctan\left(\frac{\sqrt{bx^3+a}}{\sqrt{-a}}\right)}{3(a^3c^4-2a^2c^2d^2+ad^4)\sqrt{-a}} - \frac{a^2bc^3-abcd^2-(abc^2d-bd^3)\sqrt{bx^3+a}}{3(ac^2-d^2)^2abx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="giac")

[Out]
$$2/3*b*c^4*\log(\text{abs}(\sqrt{bx^3 + a})*c + d)/(a^2*c^5 - 2*a*c^3*d^2 + c*d^4) - 1/3*b*c^3*\log(-b*x^3)/(a^2*c^4 - 2*a*c^2*d^2 + d^4) + 1/3*(3*a*b*c^2*d - b*d^3)*\arctan(\sqrt{bx^3 + a}/\sqrt{-a})/((a^3*c^4 - 2*a^2*c^2*d^2 + a*d^4)*\sqrt{-a}) - 1/3*(a^2*b*c^3 - a*b*c*d^2 - (a*b*c^2*d - b*d^3)*\sqrt{bx^3 + a})/((a*c^2 - d^2)^2*a*b*x^3)$$

maple [C] time = 0.07, size = 863, normalized size = 5.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x)

[Out]
$$-1/3*c/(a*c^2-d^2)/x^3-2/(a*c^2-d^2)^2*b*c^3*\ln(x)+1/(a*c^2-d^2)^2/a*b*c*d^2*\ln(x)+1/3*a*c^5*b/(a*c^2-d^2)^2/d^2*\ln(b*c^2*x^3+a*c^2-d^2)+1/(a*c^2-d^2)/a*b*c*\ln(x)-1/3*b*c^3/(a*c^2-d^2)/d^2*\ln(b*c^2*x^3+a*c^2-d^2)+2/3*b/d/a^2*(b*x^3+a)^(1/2)+1/3*d/a/(a*c^2-d^2)*(b*x^3+a)^(1/2)/x^3+1/3*d/a^(3/2)/(a*c^2-d^2)*b*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))+4/3*d*b/a/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*c^2-2/3*b/a^2/(a*c^2-d^2)^2*(b*x^3+a)^(1/2)*d^3-4/3*d*b/a^(1/2)/(a*c^2-d^2)^2*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))*c^2+2/3*b/a^(3/2)/(a*c^2-d^2)^2*\operatorname{arctanh}((b*x^3+a)^(1/2)/a^(1/2))*d^3-2/3*b*c^4/(a*c^2-d^2)^2/d*(b*x^3+a)^(1/2)-1/3*I/b*c^4/(a*c^2-d^2)^2/d^2*(1/2)*\sum((-a*b^2)^(1/3)*(1/2*I*(2*x+((-a*b^2)^(1/3)-I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)*((x-(-a*b^2)^(1/3)/b)/(-3*(-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))*b)^(1/2)*(-1/2*I*(2*x+((-a*b^2)^(1/3)+I*3^(1/2)*(-a*b^2)^(1/3))/b)/(-a*b^2)^(1/3)*b)^(1/2)/(b$$

```
*x^3+a)^(1/2)*(2*_alpha^2*b^2+I*(-a*b^2)^(1/3)*3^(1/2)*_alpha*b-(-a*b^2)^(1/3)*_alpha*b-I*(-a*b^2)^(2/3)*3^(1/2)-(-a*b^2)^(2/3))*EllipticPi(1/3*3^(1/2)*(I*(x+1/2*(-a*b^2)^(1/3)/b-1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)*3^(1/2)/(-a*b^2)^(1/3)*b)^(1/2),-1/2*(2*I*(-a*b^2)^(1/3)*3^(1/2)*_alpha^2*b+I*3^(1/2)*a*b-3*a*b-I*(-a*b^2)^(2/3)*3^(1/2)*_alpha-3*(-a*b^2)^(2/3)*_alpha)/b*c^2/d^2,(I*3^(1/2)*(-a*b^2)^(1/3)/(-3/2*(-a*b^2)^(1/3)/b+1/2*I*3^(1/2)*(-a*b^2)^(1/3)/b)/b)^(1/2)),_alpha=RootOf(_Z^3*b*c^2+a*c^2-d^2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bcx^3 + ac + \sqrt{bx^3 + a}d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^4/(a*c+b*c*x^3+d*(b*x^3+a)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/((b*c*x^3 + a*c + sqrt(b*x^3 + a)*d)*x^4), x)

mupad [B] time = 5.46, size = 248, normalized size = 1.61

$$\frac{bc^3 \ln(bc^2x^3 + ac^2 - d^2)}{3a^2c^4 - 6ac^2d^2 + 3d^4} - \frac{bc^3 \ln(x)}{a^2c^4 - 2ac^2d^2 + d^4} - \frac{c}{3x^3(a^2 - d^2)} + \frac{bc^3 \ln\left(\frac{d+c\sqrt{bx^3+a}}{d-c\sqrt{bx^3+a}}\right)}{3(a^2 - d^2)^2} + \frac{d\sqrt{bx^3+a}}{3ax^3(a^2 - d^2)} + \frac{bd \ln\left(\frac{(\sqrt{bx^3+a}-\sqrt{a})^3(\sqrt{bx^3+a}+\sqrt{a})}{x^6}\right)}{6a^{3/2}(a^2 - d^2)^2} (3a^2 - d^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4*(a*c + d*(a + b*x^3)^(1/2) + b*c*x^3)),x)

[Out] (b*c^3*log(a*c^2 - d^2 + b*c^2*x^3))/(3*d^4 + 3*a^2*c^4 - 6*a*c^2*d^2) - (b*c^3*log(x))/(d^4 + a^2*c^4 - 2*a*c^2*d^2) - c/(3*x^3*(a*c^2 - d^2)) + (b*c^3*log((d + c*(a + b*x^3)^(1/2))/(d - c*(a + b*x^3)^(1/2))))/(3*(a*c^2 - d^2)^2) + (d*(a + b*x^3)^(1/2))/(3*a*x^3*(a*c^2 - d^2)) + (b*d*log((((a + b*x^3)^(1/2) - a^(1/2))^3*((a + b*x^3)^(1/2) + a^(1/2))))/x^6*(3*a*c^2 - d^2))/(6*a^(3/2)*(a*c^2 - d^2)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**4/(a*c+b*c*x**3+d*(b*x**3+a)**(1/2)),x)

[Out] Timed out

$$3.318 \quad \int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}} dx$$

Optimal. Leaf size=27

$$\frac{2 \log(c\sqrt{a+bx^n} + d)}{bcn}$$

Rubi [A] time = 0.11, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {2155, 31}

$$\frac{2 \log(c\sqrt{a+bx^n} + d)}{bcn}$$

Antiderivative was successfully verified.

[In] Int[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]), x]

[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2155

Int[(x_)^(m_)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x]), x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+n}}{ac+bcx^n+d\sqrt{a+bx^n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{ac+bcx+d\sqrt{a+bx}} dx, x, x^n\right)}{n} \\ &= \frac{2 \text{Subst}\left(\int \frac{1}{d+cx} dx, x, \sqrt{a+bx^n}\right)}{bn} \\ &= \frac{2 \log(d + c\sqrt{a+bx^n})}{bcn} \end{aligned}$$

Mathematica [A] time = 0.06, size = 27, normalized size = 1.00

$$\frac{2 \log(c\sqrt{a+bx^n} + d)}{bcn}$$

Antiderivative was successfully verified.

[In] Integrate[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]), x]

[Out] (2*Log[d + c*Sqrt[a + b*x^n]])/(b*c*n)

IntegrateAlgebraic [A] time = 0.08, size = 32, normalized size = 1.19

$$\frac{2 \log(bcn\sqrt{a+bx^n} + bdn)}{bcn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(-1 + n)/(a*c + b*c*x^n + d*Sqrt[a + b*x^n]),x]

[Out] (2*Log[b*d*n + b*c*n*Sqrt[a + b*x^n]])/(b*c*n)

fricas [A] time = 0.70, size = 25, normalized size = 0.93

$$\frac{2 \log(\sqrt{bx^n + ac + d})}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="fricas")

[Out] 2*log(sqrt(b*x^n + a)*c + d)/(b*c*n)

giac [A] time = 0.37, size = 26, normalized size = 0.96

$$\frac{2 \log(|\sqrt{bx^n + ac + d}|)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="giac")

[Out] 2*log(abs(sqrt(b*x^n + a)*c + d))/(b*c*n)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{x^{n-1}}{bcx^n + ac + \sqrt{bx^n + ad}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)

[Out] int(x^(n-1)/(b*c*x^n+a*c+(b*x^n+a)^(1/2)*d),x)

maxima [B] time = 0.65, size = 61, normalized size = 2.26

$$-\frac{\log\left(\frac{bx^n+a}{b}\right)}{bcn} + \frac{2 \log\left(\frac{bcx^n+ac+\sqrt{bx^n+ad}}{d}\right)}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(-1+n)/(a*c+b*c*x^n+d*(a+b*x^n)^(1/2)),x, algorithm="maxima")

[Out] -log((b*x^n + a)/b)/(b*c*n) + 2*log((b*c*x^n + a*c + sqrt(b*x^n + a)*d)/d)/(b*c*n)

mupad [B] time = 3.35, size = 25, normalized size = 0.93

$$\frac{2 \ln(d + c \sqrt{a + bx^n})}{bcn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(n - 1)/(a*c + d*(a + b*x^n)^(1/2) + b*c*x^n),x)

[Out] (2*log(d + c*(a + b*x^n)^(1/2)))/(b*c*n)

sympy [A] time = 33.63, size = 32, normalized size = 1.19

$$\frac{2 \left(\begin{array}{ll} \frac{\sqrt{a+bx^n}}{d} & \text{for } c = 0 \\ \frac{\log(c\sqrt{a+bx^n}+d)}{c} & \text{otherwise} \end{array} \right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(-1+n)/(a*c+b*c*x**n+d*(a+b*x**n)**(1/2)),x)

[Out] 2*Piecewise((sqrt(a + b*x**n)/d, Eq(c, 0)), (log(c*sqrt(a + b*x**n) + d)/c, True))/(b*n)

$$3.319 \quad \int \frac{1}{\sqrt{x} + 4x^{3/2}} dx$$

Optimal. Leaf size=8

$$\tan^{-1}(2\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 63, 203}

$$\tan^{-1}(2\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] + 4*x^(3/2))^(-1), x]

[Out] ArcTan[2*Sqrt[x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} + 4x^{3/2}} dx &= \int \frac{1}{\sqrt{x}(1 + 4x)} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1 + 4x^2} dx, x, \sqrt{x}\right) \\ &= \tan^{-1}(2\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$\tan^{-1}(2\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] + 4*x^(3/2))^(-1), x]

[Out] ArcTan[2*Sqrt[x]]

IntegrateAlgebraic [A] time = 0.01, size = 8, normalized size = 1.00

$$\tan^{-1}(2\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x] + 4*x^(3/2))^(-1), x]

[Out] ArcTan[2*Sqrt[x]]

fricas [A] time = 0.55, size = 6, normalized size = 0.75

$$\arctan(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^(3/2)+x^(1/2)), x, algorithm="fricas")

[Out] arctan(2*sqrt(x))

giac [A] time = 0.32, size = 6, normalized size = 0.75

$$\arctan(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^(3/2)+x^(1/2)), x, algorithm="giac")

[Out] arctan(2*sqrt(x))

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$\arctan(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4*x^(3/2)+x^(1/2)), x)

[Out] arctan(2*x^(1/2))

maxima [A] time = 1.28, size = 6, normalized size = 0.75

$$\arctan(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x^(3/2)+x^(1/2)), x, algorithm="maxima")

[Out] arctan(2*sqrt(x))

mupad [B] time = 0.05, size = 6, normalized size = 0.75

$$\operatorname{atan}(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) + 4*x^(3/2)), x)

[Out] atan(2*x^(1/2))

sympy [A] time = 0.22, size = 7, normalized size = 0.88

$$\operatorname{atan}(2\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4*x**(3/2)+x**(1/2)), x)

[Out] atan(2*sqrt(x))

$$3.320 \quad \int \frac{1}{\sqrt{x-x^{5/2}}} dx$$

Optimal. Leaf size=13

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1593, 329, 212, 206, 203}

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] - x^(5/2))^(-1), x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m+1)-1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q-p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q-p]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} - x^{5/2}} dx &= \int \frac{1}{\sqrt{x} (1 - x^2)} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{1}{1 - x^4} dx, x, \sqrt{x} \right) \\
&= \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{x} \right) \\
&= \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] - x^(5/2))^(-1), x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x] - x^(5/2))^(-1), x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

fricas [B] time = 0.68, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(5/2)+x^(1/2)), x, algorithm="fricas")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

giac [B] time = 0.35, size = 22, normalized size = 1.69

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(|\sqrt{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(5/2)+x^(1/2)), x, algorithm="giac")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}(\sqrt{x}) + \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(5/2)+x^(1/2)), x)

[Out] arctan(x^(1/2))+arctanh(x^(1/2))

maxima [B] time = 1.41, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(5/2)+x^(1/2)),x, algorithm="maxima")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

mupad [B] time = 3.08, size = 9, normalized size = 0.69

$$\operatorname{atan}(\sqrt{x}) + \operatorname{atanh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) - x^(5/2)),x)

[Out] atan(x^(1/2)) + atanh(x^(1/2))

sympy [B] time = 0.40, size = 26, normalized size = 2.00

$$-\frac{\log(\sqrt{x} - 1)}{2} + \frac{\log(\sqrt{x} + 1)}{2} + \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(5/2)+x**(1/2)),x)

[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))

$$3.321 \quad \int \frac{1}{-\sqrt[4]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=27

$$2\sqrt{x} + 4\sqrt[4]{x} + 4\log(1 - \sqrt[4]{x})$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 266, 43}

$$2\sqrt{x} + 4\sqrt[4]{x} + 4\log(1 - \sqrt[4]{x})$$

Antiderivative was successfully verified.

[In] Int[(-x^(1/4) + Sqrt[x])^(-1), x]

[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt[4]{x} + \sqrt{x}} dx &= \int \frac{1}{(-1 + \sqrt[4]{x}) \sqrt[4]{x}} dx \\ &= 4 \text{Subst} \left(\int \frac{x^2}{-1 + x} dx, x, \sqrt[4]{x} \right) \\ &= 4 \text{Subst} \left(\int \left(1 + \frac{1}{-1 + x} + x \right) dx, x, \sqrt[4]{x} \right) \\ &= 4\sqrt[4]{x} + 2\sqrt{x} + 4\log(1 - \sqrt[4]{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$2\sqrt{x} + 4\sqrt[4]{x} + 4\log(1 - \sqrt[4]{x})$$

Antiderivative was successfully verified.

[In] Integrate[(-x^(1/4) + Sqrt[x])^(-1), x]

[Out] 4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 - x^(1/4)]

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 0.93

$$2\sqrt[4]{x}(\sqrt[4]{x} + 2) + 4\log(\sqrt[4]{x} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-x^(1/4) + Sqrt[x])^(-1), x]

[Out] 2*(2 + x^(1/4))*x^(1/4) + 4*Log[-1 + x^(1/4)]

fricas [A] time = 0.48, size = 19, normalized size = 0.70

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)), x, algorithm="fricas")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)

giac [A] time = 0.34, size = 20, normalized size = 0.74

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(\left|x^{\frac{1}{4}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)), x, algorithm="giac")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(abs(x^(1/4) - 1))

maple [A] time = 0.01, size = 20, normalized size = 0.74

$$4\ln\left(x^{\frac{1}{4}} - 1\right) + 2\sqrt{x} + 4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(1/4)+x^(1/2)), x)

[Out] 2*x^(1/2)+4*x^(1/4)+4*ln(x^(1/4)-1)

maxima [A] time = 0.65, size = 19, normalized size = 0.70

$$2\sqrt{x} + 4x^{\frac{1}{4}} + 4\log\left(x^{\frac{1}{4}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/4)+x^(1/2)), x, algorithm="maxima")

[Out] 2*sqrt(x) + 4*x^(1/4) + 4*log(x^(1/4) - 1)

mupad [B] time = 3.10, size = 19, normalized size = 0.70

$$4\ln\left(x^{1/4} - 1\right) + 2\sqrt{x} + 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) - x^(1/4)), x)

[Out] 4*log(x^(1/4) - 1) + 2*x^(1/2) + 4*x^(1/4)

sympy [A] time = 0.24, size = 22, normalized size = 0.81

$$4\sqrt[4]{x} + 2\sqrt{x} + 4\log(\sqrt[4]{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(1/4)+x**(1/2)),x)

[Out] 4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) - 1)

$$3.322 \quad \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=32

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log(\sqrt[6]{x} + 1)$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x^(1/3) + Sqrt[x])^(-1), x]

[Out] 6*x^(1/6) - 3*x^(1/3) + 2*Sqrt[x] - 6*Log[1 + x^(1/6)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx &= \int \frac{1}{(1 + \sqrt[6]{x}) \sqrt[3]{x}} dx \\ &= 6 \operatorname{Subst} \left(\int \frac{x^3}{1+x} dx, x, \sqrt[6]{x} \right) \\ &= 6 \operatorname{Subst} \left(\int \left(1 + \frac{1}{-1-x} - x + x^2 \right) dx, x, \sqrt[6]{x} \right) \\ &= 6\sqrt[6]{x} - 3\sqrt[3]{x} + 2\sqrt{x} - 6\log(1 + \sqrt[6]{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6\log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(1/3) + Sqrt[x])^(-1), x]

[Out] $6x^{1/6} - 3x^{1/3} + 2\sqrt{x} - 6\log[1 + x^{1/6}]$

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.03

$$(2\sqrt[3]{x} - 3\sqrt[6]{x} + 6)\sqrt{x} - 6\log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(1/3) + Sqrt[x])^(-1), x]

[Out] $(6 - 3x^{1/6} + 2x^{1/3})x^{1/6} - 6\log[1 + x^{1/6}]$

fricas [A] time = 0.71, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log\left(x^{1/6} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)), x, algorithm="fricas")

[Out] $2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(x^{1/6} + 1)$

giac [A] time = 0.33, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log\left(x^{1/6} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)), x, algorithm="giac")

[Out] $2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(x^{1/6} + 1)$

maple [B] time = 0.03, size = 92, normalized size = 2.88

$-\ln(x-1) + \ln(\sqrt{x}-1) - \ln(\sqrt{x}+1) - 2\ln\left(x^{1/6}+1\right) - 2\ln\left(x^{1/3}-1\right) + 2\ln\left(x^{1/6}-1\right) + \ln\left(x^{1/3}-x^{1/6}+1\right) - \ln\left(x^{1/3}+x^{1/6}+1\right) + \ln\left(x^{2/3}+x^{1/3}+1\right) + 2\sqrt{x} - 3x^{1/3} + 6x^{1/6}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/3)+x^(1/2)), x)

[Out] $2\ln(x^{1/6}-1) - \ln(x^{1/3}+x^{1/6}+1) + \ln(1-x^{1/6}+x^{1/3}) - 2\ln(1+x^{1/6}) + 2x^{1/2} + \ln(-1+x^{1/2}) - \ln(1+x^{1/2}) + 6x^{1/6} - \ln(x-1) - 2\ln(x^{1/3}-1) + \ln(x^{2/3}+x^{1/3}+1) - 3x^{1/3}$

maxima [A] time = 0.70, size = 24, normalized size = 0.75

$$2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log\left(x^{1/6} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/3)+x^(1/2)), x, algorithm="maxima")

[Out] $2\sqrt{x} - 3x^{1/3} + 6x^{1/6} - 6\log(x^{1/6} + 1)$

mupad [B] time = 0.03, size = 24, normalized size = 0.75

$$2\sqrt{x} - 6\ln(x^{1/6} + 1) - 3x^{1/3} + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) + x^(1/3)), x)

[Out] $2x^{1/2} - 6\log(x^{1/6} + 1) - 3x^{1/3} + 6x^{1/6}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[3]{x} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**(1/3)+x**(1/2)),x)

[Out] Integral(1/(x**(1/3) + sqrt(x)), x)

$$3.323 \quad \int \frac{1}{\sqrt[4]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=25

$$2\sqrt{x} - 4\sqrt[4]{x} + 4 \log(\sqrt[4]{x} + 1)$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$2\sqrt{x} - 4\sqrt[4]{x} + 4 \log(\sqrt[4]{x} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x^(1/4) + Sqrt[x])^(-1), x]

[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{x} + \sqrt{x}} dx &= \int \frac{1}{(1 + \sqrt[4]{x}) \sqrt[4]{x}} dx \\ &= 4 \text{Subst} \left(\int \frac{x^2}{1 + x} dx, x, \sqrt[4]{x} \right) \\ &= 4 \text{Subst} \left(\int \left(-1 + x + \frac{1}{1 + x} \right) dx, x, \sqrt[4]{x} \right) \\ &= -4\sqrt[4]{x} + 2\sqrt{x} + 4 \log(1 + \sqrt[4]{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$2\sqrt{x} - 4\sqrt[4]{x} + 4 \log(\sqrt[4]{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(1/4) + Sqrt[x])^(-1), x]

[Out] -4*x^(1/4) + 2*Sqrt[x] + 4*Log[1 + x^(1/4)]

IntegrateAlgebraic [A] time = 0.02, size = 25, normalized size = 1.00

$$2\sqrt[4]{x}(\sqrt[4]{x}-2)+4\log(\sqrt[4]{x}+1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(1/4) + Sqrt[x])^(-1), x]

[Out] 2*(-2 + x^(1/4))*x^(1/4) + 4*Log[1 + x^(1/4)]

fricas [A] time = 0.58, size = 19, normalized size = 0.76

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/2)), x, algorithm="fricas")

[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)

giac [A] time = 0.34, size = 19, normalized size = 0.76

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/2)), x, algorithm="giac")

[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)

maple [A] time = 0.01, size = 20, normalized size = 0.80

$$4\ln\left(x^{\frac{1}{4}}+1\right)+2\sqrt{x}-4x^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/4)+x^(1/2)), x)

[Out] -4*x^(1/4)+4*ln(1+x^(1/4))+2*x^(1/2)

maxima [A] time = 0.69, size = 19, normalized size = 0.76

$$2\sqrt{x}-4x^{\frac{1}{4}}+4\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/2)), x, algorithm="maxima")

[Out] 2*sqrt(x) - 4*x^(1/4) + 4*log(x^(1/4) + 1)

mupad [B] time = 0.03, size = 19, normalized size = 0.76

$$4\ln\left(x^{1/4}+1\right)+2\sqrt{x}-4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) + x^(1/4)), x)

[Out] 4*log(x^(1/4) + 1) + 2*x^(1/2) - 4*x^(1/4)

sympy [A] time = 0.24, size = 22, normalized size = 0.88

$$-4\sqrt[4]{x} + 2\sqrt{x} + 4\log(\sqrt[4]{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**(1/4)+x**(1/2)),x)

[Out] -4*x**(1/4) + 2*sqrt(x) + 4*log(x**(1/4) + 1)

$$3.324 \quad \int \frac{1}{-\sqrt[3]{x} + x^{2/3}} dx$$

Optimal. Leaf size=20

$$3\sqrt[3]{x} + 3\log(1 - \sqrt[3]{x})$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1593, 266, 43}

$$3\sqrt[3]{x} + 3\log(1 - \sqrt[3]{x})$$

Antiderivative was successfully verified.

[In] Int[(-x^(1/3) + x^(2/3))^(-1), x]

[Out] 3*x^(1/3) + 3*Log[1 - x^(1/3)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt[3]{x} + x^{2/3}} dx &= \int \frac{1}{(-1 + \sqrt[3]{x}) \sqrt[3]{x}} dx \\ &= 3 \text{Subst} \left(\int \frac{x}{-1 + x} dx, x, \sqrt[3]{x} \right) \\ &= 3 \text{Subst} \left(\int \left(1 + \frac{1}{-1 + x} \right) dx, x, \sqrt[3]{x} \right) \\ &= 3\sqrt[3]{x} + 3\log(1 - \sqrt[3]{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.90

$$3 \left(\sqrt[3]{x} + \log(1 - \sqrt[3]{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-x^(1/3) + x^(2/3))^(-1), x]

[Out] 3*(x^(1/3) + Log[1 - x^(1/3)])

IntegrateAlgebraic [A] time = 0.02, size = 18, normalized size = 0.90

$$3\sqrt[3]{x} + 3 \log(\sqrt[3]{x} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-x^(1/3) + x^(2/3))^(−1), x]

[Out] 3*x^(1/3) + 3*Log[-1 + x^(1/3)]

fricas [A] time = 0.69, size = 14, normalized size = 0.70

$$3x^{\frac{1}{3}} + 3 \log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)), x, algorithm="fricas")

[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)

giac [A] time = 0.43, size = 15, normalized size = 0.75

$$3x^{\frac{1}{3}} + 3 \log\left(\left|x^{\frac{1}{3}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)), x, algorithm="giac")

[Out] 3*x^(1/3) + 3*log(abs(x^(1/3) - 1))

maple [A] time = 0.00, size = 15, normalized size = 0.75

$$3 \ln\left(x^{\frac{1}{3}} - 1\right) + 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^(1/3)+x^(2/3)), x)

[Out] 3*x^(1/3)+3*ln(x^(1/3)-1)

maxima [A] time = 0.70, size = 14, normalized size = 0.70

$$3x^{\frac{1}{3}} + 3 \log\left(x^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^(1/3)+x^(2/3)), x, algorithm="maxima")

[Out] 3*x^(1/3) + 3*log(x^(1/3) - 1)

mupad [B] time = 0.08, size = 14, normalized size = 0.70

$$3 \ln\left(x^{1/3} - 1\right) + 3x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(x^(1/3) - x^(2/3)), x)

[Out] 3*log(x^(1/3) - 1) + 3*x^(1/3)

sympy [A] time = 0.16, size = 15, normalized size = 0.75

$$3\sqrt[3]{x} + 3\log(\sqrt[3]{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**(1/3)+x**(2/3)),x)

[Out] 3*x**(1/3) + 3*log(x**(1/3) - 1)

$$3.325 \quad \int \frac{1}{\frac{1}{\sqrt[4]{x}} + \sqrt{x}} dx$$

Optimal. Leaf size=62

$$2\sqrt{x} + \frac{4}{3} \log(\sqrt[4]{x} + 1) - \frac{2}{3} \log(\sqrt{x} - \sqrt[4]{x} + 1) + \frac{4 \tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {1593, 341, 321, 292, 31, 634, 618, 204, 628}

$$2\sqrt{x} + \frac{4}{3} \log(\sqrt[4]{x} + 1) - \frac{2}{3} \log(\sqrt{x} - \sqrt[4]{x} + 1) + \frac{4 \tan^{-1}\left(\frac{1-2\sqrt[4]{x}}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1/4) + Sqrt[x])^(-1), x]

[Out] 2*Sqrt[x] + (4*ArcTan[(1 - 2*x^(1/4))/Sqrt[3]])/Sqrt[3] + (4*Log[1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 292

Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] := -Dist[(3*Rt[a, 3]*Rt[b, 3])^(-1), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]*Rt[b, 3]), Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 341

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m+1)-1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 628

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Simp}[(d*\text{Log}[\text{RemoveContent}[a + b*x + c*x^2, x]])/b, x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{EqQ}[2*c*d - b*e, 0]$

Rule 634

$\text{Int}[(d_.) + (e_.)*(x_.)]/((a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2), x_Symbol] \rightarrow \text{Dist}[(2*c*d - b*e)/(2*c), \text{Int}[1/(a + b*x + c*x^2), x], x] + \text{Dist}[e/(2*c), \text{Int}[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; \text{FreeQ}[\{a, b, c, d, e\}, x] \&\& \text{NeQ}[2*c*d - b*e, 0] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{!NiceSqrtQ}[b^2 - 4*a*c]$

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}], x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q - p)})^n, x] /; \text{FreeQ}[\{a, b, p, q\}, x] \&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q - p]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{1}{\sqrt[4]{x}} + \sqrt{x}} dx &= \int \frac{\sqrt[4]{x}}{1 + x^{3/4}} dx \\ &= 4 \text{Subst} \left(\int \frac{x^4}{1 + x^3} dx, x, \sqrt[4]{x} \right) \\ &= 2\sqrt{x} - 4 \text{Subst} \left(\int \frac{x}{1 + x^3} dx, x, \sqrt[4]{x} \right) \\ &= 2\sqrt{x} + \frac{4}{3} \text{Subst} \left(\int \frac{1}{1 + x} dx, x, \sqrt[4]{x} \right) - \frac{4}{3} \text{Subst} \left(\int \frac{1 + x}{1 - x + x^2} dx, x, \sqrt[4]{x} \right) \\ &= 2\sqrt{x} + \frac{4}{3} \log(1 + \sqrt[4]{x}) - \frac{2}{3} \text{Subst} \left(\int \frac{-1 + 2x}{1 - x + x^2} dx, x, \sqrt[4]{x} \right) - 2 \text{Subst} \left(\int \frac{1}{1 - x + x^2} dx, x, \sqrt[4]{x} \right) \\ &= 2\sqrt{x} + \frac{4}{3} \log(1 + \sqrt[4]{x}) - \frac{2}{3} \log(1 - \sqrt[4]{x} + \sqrt{x}) + 4 \text{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, -1 + 2\sqrt[4]{x} \right) \\ &= 2\sqrt{x} + \frac{4 \tan^{-1} \left(\frac{1 - 2\sqrt[4]{x}}{\sqrt{3}} \right)}{\sqrt{3}} + \frac{4}{3} \log(1 + \sqrt[4]{x}) - \frac{2}{3} \log(1 - \sqrt[4]{x} + \sqrt{x}) \end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.39

$$-2\sqrt{x} \left({}_2F_1 \left(\frac{2}{3}, 1; \frac{5}{3}; -x^{3/4} \right) - 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1/4) + Sqrt[x])^(-1), x]

[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[2/3, 1, 5/3, -x^(3/4)])

IntegrateAlgebraic [A] time = 0.05, size = 65, normalized size = 1.05

$$2\sqrt{x} + \frac{4}{3} \log(\sqrt[4]{x} + 1) - \frac{2}{3} \log(\sqrt{x} - \sqrt[4]{x} + 1) + \frac{4 \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[4]{x}}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1/4) + Sqrt[x])^(-1), x]

[Out] 2*Sqrt[x] + (4*ArcTan[1/Sqrt[3] - (2*x^(1/4))/Sqrt[3]]/Sqrt[3] + (4*Log[1 + x^(1/4)])/3 - (2*Log[1 - x^(1/4) + Sqrt[x]])/3

fricas [A] time = 0.64, size = 47, normalized size = 0.76

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{4}}-\frac{1}{3}\sqrt{3}\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^(1/4)+x^(1/2)), x, algorithm="fricas")

[Out] -4/3*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/4) - 1/3*sqrt(3)) + 2*sqrt(x) - 2/3*log(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)

giac [A] time = 0.33, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^(1/4)+x^(1/2)), x, algorithm="giac")

[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)

maple [A] time = 0.01, size = 46, normalized size = 0.74

$$-\frac{4\sqrt{3}\arctan\left(\frac{\left(2x^{\frac{1}{4}}-1\right)\sqrt{3}}{3}\right)}{3}+\frac{4\ln\left(x^{\frac{1}{4}}+1\right)}{3}-\frac{2\ln\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)}{3}+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^(1/4)+x^(1/2)), x)

[Out] 2*x^(1/2)-2/3*ln(1-x^(1/4)+x^(1/2))-4/3*3^(1/2)*arctan(1/3*(2*x^(1/4)-1)*3^(1/2))+4/3*ln(x^(1/4)+1)

maxima [A] time = 1.53, size = 45, normalized size = 0.73

$$-\frac{4}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{4}}-1\right)\right)+2\sqrt{x}-\frac{2}{3}\log\left(\sqrt{x}-x^{\frac{1}{4}}+1\right)+\frac{4}{3}\log\left(x^{\frac{1}{4}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^(1/4)+x^(1/2)), x, algorithm="maxima")

[Out] -4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/4) - 1)) + 2*sqrt(x) - 2/3*log(sqrt(x) - x^(1/4) + 1) + 4/3*log(x^(1/4) + 1)

mupad [B] time = 3.08, size = 73, normalized size = 1.18

$$\frac{4\ln(16x^{1/4}+16)}{3}+\ln\left(9\left(-\frac{2}{3}+\frac{\sqrt{3}2i}{3}\right)^2+16x^{1/4}\right)\left(-\frac{2}{3}+\frac{\sqrt{3}2i}{3}\right)-\ln\left(9\left(\frac{2}{3}+\frac{\sqrt{3}2i}{3}\right)^2+16x^{1/4}\right)\left(\frac{2}{3}+\frac{\sqrt{3}2i}{3}\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/2) + 1/x^(1/4)),x)`

[Out] $(4 \cdot \log(16x^{1/4} + 16))/3 + \log(9((3^{1/2} \cdot 2i)/3 - 2/3)^2 + 16x^{1/4}) \cdot ((3^{1/2} \cdot 2i)/3 - 2/3) - \log(9((3^{1/2} \cdot 2i)/3 + 2/3)^2 + 16x^{1/4}) \cdot ((3^{1/2} \cdot 2i)/3 + 2/3) + 2x^{1/2}$

sympy [A] time = 0.64, size = 68, normalized size = 1.10

$$2\sqrt{x} + \frac{4 \log(\sqrt[4]{x} + 1)}{3} - \frac{2 \log(-4\sqrt[4]{x} + 4\sqrt{x} + 4)}{3} - \frac{4\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt[4]{x}}{3} - \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1/x**(1/4)+x**(1/2)),x)`

[Out] $2\sqrt{x} + 4 \cdot \log(x^{1/4} + 1)/3 - 2 \cdot \log(-4x^{1/4} + 4\sqrt{x} + 4)/3 - 4\sqrt{3} \cdot \operatorname{atan}(2\sqrt{3}x^{1/4}/3 - \sqrt{3}/3)/3$

$$3.326 \quad \int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Optimal. Leaf size=73

$$\frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log\left(\sqrt[12]{x} + 1\right)$$

Rubi [A] time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$\frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log\left(\sqrt[12]{x} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(x^(1/4) + x^(1/3))^(-1), x]

[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 + 12*Log[1 + x^(1/12)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}} dx &= \int \frac{1}{(1 + \sqrt[12]{x}) \sqrt[4]{x}} dx \\ &= 12 \operatorname{Subst} \left(\int \frac{x^8}{1+x} dx, x, \sqrt[12]{x} \right) \\ &= 12 \operatorname{Subst} \left(\int \left(-1 + x - x^2 + x^3 - x^4 + x^5 - x^6 + x^7 + \frac{1}{1+x} \right) dx, x, \sqrt[12]{x} \right) \\ &= -12 \sqrt[12]{x} + 6\sqrt[6]{x} - 4\sqrt[4]{x} + 3\sqrt[3]{x} - \frac{12x^{5/12}}{5} + 2\sqrt{x} - \frac{12x^{7/12}}{7} + \frac{3x^{2/3}}{2} + 12 \log\left(1 + \sqrt[12]{x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 73, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log\left(\sqrt[12]{x} + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^(1/4) + x^(1/3))^(−1), x]

[Out] $-12x^{1/12} + 6x^{1/6} - 4x^{1/4} + 3x^{1/3} - (12x^{5/12})/5 + 2\sqrt{x}$
 $[x] - (12x^{7/12})/7 + (3x^{2/3})/2 + 12\text{Log}[1 + x^{1/12}]$

IntegrateAlgebraic [A] time = 0.04, size = 72, normalized size = 0.99

$$\frac{1}{70} (105x^{2/3} - 120x^{7/12} - 168x^{5/12} + 140\sqrt{x} + 210\sqrt[3]{x} - 280\sqrt[4]{x} + 420\sqrt[6]{x} - 840\sqrt[12]{x}) + 12 \log(\sqrt[12]{x} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(1/4) + x^(1/3))^(−1), x]

[Out] $(-840x^{1/12} + 420x^{1/6} - 280x^{1/4} + 210x^{1/3} - 168x^{5/12} + 140\sqrt{x} - 120x^{7/12} + 105x^{2/3})/70 + 12\text{Log}[1 + x^{1/12}]$

fricas [A] time = 0.61, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12 \log(x^{1/12} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="fricas")

[Out] $3/2*x^{2/3} - 12/7*x^{7/12} + 2*\text{sqrt}(x) - 12/5*x^{5/12} + 3*x^{1/3} - 4*x^{1/4} + 6*x^{1/6} - 12*x^{1/12} + 12*\text{log}(x^{1/12} + 1)$

giac [A] time = 0.32, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12 \log(x^{1/12} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="giac")

[Out] $3/2*x^{2/3} - 12/7*x^{7/12} + 2*\text{sqrt}(x) - 12/5*x^{5/12} + 3*x^{1/3} - 4*x^{1/4} + 6*x^{1/6} - 12*x^{1/12} + 12*\text{log}(x^{1/12} + 1)$

maple [B] time = 0.12, size = 173, normalized size = 2.37

$\ln(x-1) + \ln(\sqrt{x}-1) - \ln(\sqrt{x}+1) + 2\ln(x^{1/3}+1) - 2\ln(x^{1/3}-1) + 4\ln(x^{1/6}+1) + 2\ln(x^{1/6}-1) - 2\ln(x^{1/4}-1) + 2\ln(x^{1/4}+1) - 4\ln(x^{1/12}-1) + \ln(x^{1/12}+1) - 2\ln(x^{1/12}-x^{1/6}+1) - \ln(x^{1/12}+x^{1/6}+1) + 2\ln(x^{1/12}+x^{1/6}+1) - \ln(x^{1/12}+x^{1/6}+1) + \frac{3x^{1/12}}{2} - \frac{12x^{1/12}}{7} + 2\sqrt{x} - \frac{12x^{5/12}}{5} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/4)+x^(1/3)), x)

[Out] $\ln(x-1) - 2*\ln(x^{1/6}+1) + \ln(x^{1/3}-x^{1/6}+1) + 2*\ln(x^{1/6}-1) - \ln(x^{1/3}+x^{1/6}+1) + \ln(x^{1/2}-1) - \ln(x^{1/2}+1) - 12/7*x^{7/12} - 12/5*x^{5/12} - 12*x^{1/12} - 2*\ln(x^{1/4}-1) + 2*\ln(x^{1/4}+1) + 6*x^{1/6} + 3/2*x^{2/3} + 3*x^{1/3} + 2*x^{1/2} - 4*x^{1/4} - 2*\ln(1-x^{1/12}+x^{1/6}) - \ln(x^{2/3}+x^{1/3}+1) + 4*\ln(1+x^{1/12}) + 2*\ln(x^{1/3}-1) - 4*\ln(x^{1/12}-1) + 2*\ln(x^{1/6}+x^{1/12}+1)$

maxima [A] time = 0.70, size = 49, normalized size = 0.67

$$\frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12 \log(x^{1/12} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/4)+x^(1/3)),x, algorithm="maxima")

[Out] $3/2*x^{(2/3)} - 12/7*x^{(7/12)} + 2*\text{sqrt}(x) - 12/5*x^{(5/12)} + 3*x^{(1/3)} - 4*x^{(1/4)} + 6*x^{(1/6)} - 12*x^{(1/12)} + 12*\log(x^{(1/12)} + 1)$

mupad [B] time = 0.04, size = 49, normalized size = 0.67

$$12 \ln(x^{1/12} + 1) + 2\sqrt{x} + 3x^{1/3} - 4x^{1/4} + \frac{3x^{2/3}}{2} + 6x^{1/6} - 12x^{1/12} - \frac{12x^{5/12}}{5} - \frac{12x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x^(1/3) + x^(1/4)), x)`

[Out] $12*\log(x^{(1/12)} + 1) + 2*x^{(1/2)} + 3*x^{(1/3)} - 4*x^{(1/4)} + (3*x^{(2/3)})/2 + 6*x^{(1/6)} - 12*x^{(1/12)} - (12*x^{(5/12)})/5 - (12*x^{(7/12)})/7$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**(1/4)+x**(1/3)), x)`

[Out] `Integral(1/(x**(1/4) + x**(1/3)), x)`

$$3.327 \quad \int \frac{1}{\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[4]{x}}} dx$$

Optimal. Leaf size=130

$$\frac{4x^{5/4}}{5} - \frac{6x^{7/6}}{7} + \frac{12x^{13/12}}{13} + \frac{12x^{11/12}}{11} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} - \frac{3x^{2/3}}{2} + \frac{12x^{7/12}}{7} + \frac{12x^{5/12}}{5} - x - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12$$

Rubi [A] time = 0.05, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1593, 266, 43}

$$\frac{4x^{5/4}}{5} - \frac{6x^{7/6}}{7} + \frac{12x^{13/12}}{13} + \frac{12x^{11/12}}{11} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} - \frac{3x^{2/3}}{2} + \frac{12x^{7/12}}{7} + \frac{12x^{5/12}}{5} - x - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12 \log\left(\frac{\sqrt[12]{x}}{x} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(x^(-1/3) + x^(-1/4))^(-1), x]

[Out] 12*x^(1/12) - 6*x^(1/6) + 4*x^(1/4) - 3*x^(1/3) + (12*x^(5/12))/5 - 2*sqrt[x] + (12*x^(7/12))/7 - (3*x^(2/3))/2 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(11/12))/11 - x + (12*x^(13/12))/13 - (6*x^(7/6))/7 + (4*x^(5/4))/5 - 12*Log[1 + x^(1/12)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{1}{\sqrt[3]{x}} + \frac{1}{\sqrt[4]{x}}} dx &= \int \frac{\sqrt[3]{x}}{1 + \sqrt[12]{x}} dx \\ &= 12 \text{Subst}\left(\int \frac{x^{15}}{1 + x} dx, x, \sqrt[12]{x}\right) \\ &= 12 \text{Subst}\left(\int \left(1 + \frac{1}{-1 - x} - x + x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - x^9 + x^{10} - x^{11} + x^{12} - x^{13} + \dots\right) dx, x, \sqrt[12]{x}\right) \\ &= 12 \sqrt[12]{x} - 6\sqrt[6]{x} + 4\sqrt[4]{x} - 3\sqrt[3]{x} + \frac{12x^{5/12}}{5} - 2\sqrt{x} + \frac{12x^{7/12}}{7} - \frac{3x^{2/3}}{2} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{11/12}}{11} - \dots \end{aligned}$$

Mathematica [A] time = 0.04, size = 130, normalized size = 1.00

$$\frac{4x^{5/4}}{5} - \frac{6x^{7/6}}{7} + \frac{12x^{13/12}}{13} + \frac{12x^{11/12}}{11} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} - \frac{3x^{2/3}}{2} + \frac{12x^{7/12}}{7} + \frac{12x^{5/12}}{5} - x - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} - 6\sqrt[6]{x} + 12\sqrt[12]{x} - 12 \log\left(\frac{\sqrt[12]{x}}{x} + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^{-1/3} + x^{-1/4})⁻¹, x]

[Out] $12x^{1/12} - 6x^{1/6} + 4x^{1/4} - 3x^{1/3} + (12x^{5/12})/5 - 2\sqrt{x} + (12x^{7/12})/7 - (3x^{2/3})/2 + (4x^{3/4})/3 - (6x^{5/6})/5 + (12x^{11/12})/11 - x + (12x^{13/12})/13 - (6x^{7/6})/7 + (4x^{5/4})/5 - 12\log[1 + x^{1/12}]$

IntegrateAlgebraic [A] time = 0.05, size = 117, normalized size = 0.90

$\frac{24024x^{5/4} - 25740x^{7/6} + 27720x^{13/12} + 32760x^{11/12} - 36036x^{5/6} + 40040x^{3/4} - 45045x^{2/3} + 51480x^{7/12} + 72072x^{5/12} - 30030x - 60060\sqrt{x} - 90090\sqrt[3]{x} + 120120\sqrt[4]{x} - 180180\sqrt[5]{x} + 360360\sqrt[6]{x} - 12\log(\sqrt[12]{x} + 1)}{30030}$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^{-1/3} + x^{-1/4})⁻¹, x]

[Out] $(360360x^{1/12} - 180180x^{1/6} + 120120x^{1/4} - 90090x^{1/3} + 72072x^{5/12} - 60060\sqrt{x} + 51480x^{7/12} - 45045x^{2/3} + 40040x^{3/4} - 36036x^{5/6} + 32760x^{11/12} - 30030x + 27720x^{13/12} - 25740x^{7/6} + 24024x^{5/4})/30030 - 12\log[1 + x^{1/12}]$

fricas [A] time = 0.79, size = 76, normalized size = 0.58

$\frac{4}{5}(x+5)x^{1/4} - \frac{6}{7}(x+7)x^{1/6} + \frac{12}{13}(x+13)x^{1/12} - x + \frac{12}{11}x^{11/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} - \frac{3}{2}x^{2/3} + \frac{12}{7}x^{7/12} - 2\sqrt{x} + \frac{12}{5}x^{5/12} - 3x^{1/3} - 12\log\left(x^{1/12} + 1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^{1/3}+1/x^{1/4}), x, algorithm="fricas")

[Out] $4/5*(x + 5)*x^{1/4} - 6/7*(x + 7)*x^{1/6} + 12/13*(x + 13)*x^{1/12} - x + 12/11*x^{11/12} - 6/5*x^{5/6} + 4/3*x^{3/4} - 3/2*x^{2/3} + 12/7*x^{7/12} - 2*\sqrt{x} + 12/5*x^{5/12} - 3*x^{1/3} - 12*\log(x^{1/12} + 1)$

giac [A] time = 0.36, size = 82, normalized size = 0.63

$\frac{4}{5}x^{5/4} - \frac{6}{7}x^{7/6} + \frac{12}{13}x^{13/12} - x + \frac{12}{11}x^{11/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} - \frac{3}{2}x^{2/3} + \frac{12}{7}x^{7/12} - 2\sqrt{x} + \frac{12}{5}x^{5/12} - 3x^{1/3} + 4x^{1/4} - 6x^{1/6} + 12x^{1/12} - 12\log\left(x^{1/12} + 1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^{1/3}+1/x^{1/4}), x, algorithm="giac")

[Out] $4/5*x^{5/4} - 6/7*x^{7/6} + 12/13*x^{13/12} - x + 12/11*x^{11/12} - 6/5*x^{5/6} + 4/3*x^{3/4} - 3/2*x^{2/3} + 12/7*x^{7/12} - 2*\sqrt{x} + 12/5*x^{5/12} - 3*x^{1/3} + 4*x^{1/4} - 6*x^{1/6} + 12*x^{1/12} - 12*\log(x^{1/12} + 1)$

maple [A] time = 0.00, size = 83, normalized size = 0.64

$\frac{4x^{5/4}}{5} - \frac{6x^{7/6}}{7} + \frac{12x^{13/12}}{13} - x - 12\ln\left(x^{1/12} + 1\right) + \frac{12x^{11/12}}{11} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} - \frac{3x^{2/3}}{2} + \frac{12x^{7/12}}{7} - 2\sqrt{x} + \frac{12x^{5/12}}{5} - 3x^{1/3} + 4x^{1/4} - 6x^{1/6} + 12x^{1/12}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^{1/3}+1/x^{1/4}), x)

[Out] $12x^{1/12} - 6x^{1/6} + 4x^{1/4} - 3x^{1/3} + 12/5x^{5/12} + 12/7x^{7/12} - 3/2x^{2/3} + 4/3x^{3/4} - 6/5x^{5/6} + 12/11x^{11/12} - x + 12/13x^{13/12} - 6/7x^{7/6} + 4/5x^{5/4} - 12*\ln(x^{1/12} + 1) - 2*x^{1/2}$

maxima [A] time = 0.73, size = 82, normalized size = 0.63

$\frac{4}{5}x^{5/4} - \frac{6}{7}x^{7/6} + \frac{12}{13}x^{13/12} - x + \frac{12}{11}x^{11/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} - \frac{3}{2}x^{2/3} + \frac{12}{7}x^{7/12} - 2\sqrt{x} + \frac{12}{5}x^{5/12} - 3x^{1/3} + 4x^{1/4} - 6x^{1/6} + 12x^{1/12} - 12\log\left(x^{1/12} + 1\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x^(1/3)+1/x^(1/4)),x, algorithm="maxima")

[Out] $4/5*x^{5/4} - 6/7*x^{7/6} + 12/13*x^{13/12} - x + 12/11*x^{11/12} - 6/5*x^{5/6} + 4/3*x^{3/4} - 3/2*x^{2/3} + 12/7*x^{7/12} - 2*\sqrt{x} + 12/5*x^{5/12} - 3*x^{1/3} + 4*x^{1/4} - 6*x^{1/6} + 12*x^{1/12} - 12*\log(x^{1/12} + 1)$

mupad [B] time = 0.15, size = 82, normalized size = 0.63

$$4x^{1/4} - 12 \ln(x^{1/12} + 1) - 2\sqrt{x} - 3x^{1/3} - x - \frac{3x^{2/3}}{2} - 6x^{1/6} + \frac{4x^{3/4}}{3} + \frac{4x^{5/4}}{5} - \frac{6x^{5/6}}{5} + 12x^{1/12} - \frac{6x^{7/6}}{7} + \frac{12x^{5/12}}{5} + \frac{12x^{7/12}}{7} + \frac{12x^{11/12}}{11} + \frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x^(1/3) + 1/x^(1/4)),x)

[Out] $4*x^{1/4} - 12*\log(x^{1/12} + 1) - 2*x^{1/2} - 3*x^{1/3} - x - (3*x^{2/3})/2 - 6*x^{1/6} + (4*x^{3/4})/3 + (4*x^{5/4})/5 - (6*x^{5/6})/5 + 12*x^{1/12} - (6*x^{7/6})/7 + (12*x^{5/12})/5 + (12*x^{7/12})/7 + (12*x^{11/12})/11 + (12*x^{13/12})/13$

sympy [A] time = 3.08, size = 121, normalized size = 0.93

$$\frac{12x^{13}}{13} + \frac{12x^{11}}{11} + \frac{12x^7}{7} + \frac{12x^5}{5} + 12\sqrt[12]{x} - \frac{6x^7}{7} - \frac{6x^5}{5} - 6\sqrt[6]{x} + \frac{4x^5}{5} + \frac{4x^3}{3} + 4\sqrt[4]{x} - \frac{3x^2}{2} - 3\sqrt[3]{x} - 2\sqrt{x} - x - 12\log(\sqrt[12]{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x**(1/3)+1/x**(1/4)),x)

[Out] $12*x^{13/12}/13 + 12*x^{11/12}/11 + 12*x^{7/12}/7 + 12*x^{5/12}/5 + 12*x^{1/12} - 6*x^{7/6}/7 - 6*x^{5/6}/5 - 6*x^{1/6} + 4*x^{5/4}/5 + 4*x^{3/4}/3 + 4*x^{1/4} - 3*x^{2/3}/2 - 3*x^{1/3} - 2*\sqrt{x} - x - 12*\log(x^{1/12} + 1)$

$$3.328 \quad \int \frac{1}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx$$

Optimal. Leaf size=200

$$2\sqrt{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 + \sqrt{5}) \log(2\sqrt[3]{x} - \sqrt{5} \sqrt[6]{x} + \sqrt[6]{x} + 2) - \frac{3}{10} (1 - \sqrt{5}) \log(2\sqrt[3]{x} + \sqrt{5} \sqrt[6]{x} + \sqrt[6]{x} + 1)$$

Rubi [A] time = 0.40, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1593, 341, 321, 294, 634, 618, 204, 628, 31}

$$2\sqrt{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 + \sqrt{5}) \log(2\sqrt[3]{x} - \sqrt{5} \sqrt[6]{x} + \sqrt[6]{x} + 2) - \frac{3}{10} (1 - \sqrt{5}) \log(2\sqrt[3]{x} + \sqrt{5} \sqrt[6]{x} + \sqrt[6]{x} + 1) + \frac{3}{5} \sqrt{2(5 - \sqrt{5})} \tan^{-1} \left(\frac{4\sqrt[6]{x} - \sqrt{5} + 1}{\sqrt{2(5 + \sqrt{5})}} \right) - \frac{3}{5} \sqrt{2(5 + \sqrt{5})} \tan^{-1} \left(\frac{1}{2} \sqrt{\frac{1}{10}(5 + \sqrt{5})} (4\sqrt[6]{x} + \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Int[(-x^(-1/3) + Sqrt[x])^(-1), x]

[Out] 2*Sqrt[x] + (3*Sqrt[2*(5 - Sqrt[5])]*ArcTan[(1 - Sqrt[5] + 4*x^(1/6))/Sqrt[2*(5 + Sqrt[5])]])/5 - (3*Sqrt[2*(5 + Sqrt[5])]*ArcTan[(Sqrt[(5 + Sqrt[5])/10]*(1 + Sqrt[5] + 4*x^(1/6)))/2])/5 + (6*Log[1 - x^(1/6)])/5 - (3*(1 + Sqrt[5])*Log[2 + x^(1/6) - Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10 - (3*(1 - Sqrt[5])*Log[2 + x^(1/6) + Sqrt[5]*x^(1/6) + 2*x^(1/3)])/10

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 294

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r*Cos[((2*k - 1)*m*Pi)/n] + s*Cos[((2*k - 1)*(m + 1)*Pi)/n]*x)/(r^2 + 2*r*s*Cos[((2*k - 1)*Pi)/n]*x + s^2*x^2), x]; (r^(m + 1)*Int[1/(r - s*x), x])/(a*n*s^m) - Dist[(2*(-r)^(m + 1))/(a*n*s^m), Sum[u, {k, 1, (n - 1)/2}], x, x] /; FreeQ[{a, b}, x] && IGtQ[(n - 1)/2, 0] && IGtQ[m, 0] && LtQ[m, n - 1] && NegQ[a/b]

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 341

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx &= \int \frac{\sqrt[3]{x}}{-1 + x^{5/6}} dx \\
&= 6 \operatorname{Subst} \left(\int \frac{x^7}{-1 + x^5} dx, x, \sqrt[6]{x} \right) \\
&= 2\sqrt{x} + 6 \operatorname{Subst} \left(\int \frac{x^2}{-1 + x^5} dx, x, \sqrt[6]{x} \right) \\
&= 2\sqrt{x} - \frac{6}{5} \operatorname{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[6]{x} \right) - \frac{12}{5} \operatorname{Subst} \left(\int \frac{\frac{1}{4}(-1 - \sqrt{5}) + \frac{1}{4}(1 + \sqrt{5})x}{1 + \frac{1}{2}(1 - \sqrt{5})x + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 2\sqrt{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) + \frac{3 \operatorname{Subst} \left(\int \frac{1}{1 + \frac{1}{2}(1 - \sqrt{5})x + x^2} dx, x, \sqrt[6]{x} \right)}{\sqrt{5}} - \frac{3 \operatorname{Subst} \left(\int \frac{1}{1 + \frac{1}{2}(1 + \sqrt{5})x + x^2} dx, x, \sqrt[6]{x} \right)}{\sqrt{5}} \\
&= 2\sqrt{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 + \sqrt{5}) \log(2 + \sqrt[6]{x} - \sqrt{5} \sqrt[6]{x} + 2\sqrt[3]{x}) - \frac{3}{10} (1 - \sqrt{5}) \log(2 - \sqrt[6]{x} + \sqrt{5} \sqrt[6]{x} + 2\sqrt[3]{x}) \\
&= 2\sqrt{x} + 6 \sqrt{\frac{2}{5(5 + \sqrt{5})}} \tan^{-1} \left(\frac{1 - \sqrt{5} + 4\sqrt[6]{x}}{\sqrt{2(5 + \sqrt{5})}} \right) - \frac{3}{5} \sqrt{2(5 + \sqrt{5})} \tan^{-1} \left(\frac{1}{2} \sqrt{\frac{1}{10}(5 + \sqrt{5})} (2 + \sqrt[6]{x} - \sqrt{5} \sqrt[6]{x} + 2\sqrt[3]{x}) \right)
\end{aligned}$$

Mathematica [C] time = 0.01, size = 22, normalized size = 0.11

$$-2\sqrt{x} \left({}_2F_1 \left(\frac{3}{5}, 1; \frac{8}{5}; x^{5/6} \right) - 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-x^{-1/3} + Sqrt[x])⁻¹, x]

[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/5, 1, 8/5, x^(5/6)]])

IntegrateAlgebraic [C] time = 0.07, size = 126, normalized size = 0.63

$$-\frac{6}{5}\text{RootSum}\left[\#1^4 + \#1^3 + \#1^2 + \#1 + 1 \&, \frac{\#1^3 \log(\sqrt[6]{x} - \#1) + 2\#1^2 \log(\sqrt[6]{x} - \#1) - 2\#1 \log(\sqrt[6]{x} - \#1) - \log(\sqrt[6]{x} - \#1)}{4\#1^3 + 3\#1^2 + 2\#1 + 1} \& \right] + 2\sqrt{x} + \frac{6}{5} \log(\sqrt[6]{x} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-x^{-1/3} + Sqrt[x])⁻¹, x]

[Out] 2*Sqrt[x] + (6*Log[-1 + x^{(1/6)]])/5 - (6*RootSum[1 + #1 + #1² + #1³ + #1⁴ &, (-Log[x^(1/6) - #1] - 2*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1² + Log[x^(1/6) - #1]*#1³)/(1 + 2*#1 + 3*#1² + 4*#1³) &])/5}

fricas [B] time = 2.68, size = 638, normalized size = 3.19

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1/x^(1/3)+x^(1/2))),x, algorithm="fricas")

[Out] 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 3*sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² - 3*sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90)*(sqrt(5) - 1) + 72*x^(1/6) + 36) - 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)² + 36*x^(1/6)) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-9/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)² + 36*x^(1/6)) + 2*sqrt(x) + 6/5*log(x^(1/6) - 1)

giac [A] time = 1.12, size = 139, normalized size = 0.70

$$\frac{3}{5}\sqrt{-2\sqrt{5}+10} \arctan\left(\frac{\sqrt{5}-4x^{\frac{1}{6}}-1}{\sqrt{2\sqrt{5}+10}}\right) - \frac{3}{5}\sqrt{2\sqrt{5}+10} \arctan\left(\frac{\sqrt{5}+4x^{\frac{1}{6}}+1}{\sqrt{-2\sqrt{5}+10}}\right) + \frac{3}{10}\sqrt{5} \log\left(\frac{1}{2}x^{\frac{1}{6}}(\sqrt{5}+1)+x^{\frac{1}{3}}+1\right) - \frac{3}{10}\sqrt{5} \log\left(-\frac{1}{2}x^{\frac{1}{6}}(\sqrt{5}-1)+x^{\frac{1}{3}}+1\right) + 2\sqrt{x} - \frac{3}{10} \log\left(x^{\frac{2}{3}} + \sqrt{x} + x^{\frac{1}{3}} + x^{\frac{1}{6}} + 1\right) + \frac{6}{5} \log\left(\left|x^{\frac{1}{6}} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1/x^(1/3)+x^(1/2))),x, algorithm="giac")

[Out] 3/5*sqrt(-2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5) + 10)) - 3/5*sqrt(2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) + 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1) - 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + 2*sqrt(x) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) - 1))

maple [A] time = 0.04, size = 175, normalized size = 0.88

$$\frac{12\sqrt{5} \arctan\left(\frac{4x^{\frac{1}{6}}+1-\sqrt{5}}{\sqrt{10+2\sqrt{5}}}\right) - 12\sqrt{5} \arctan\left(\frac{4x^{\frac{1}{6}}+1+\sqrt{5}}{\sqrt{10-2\sqrt{5}}}\right) + 6\ln(x^{\frac{1}{3}}-1) - 3\sqrt{5} \ln(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}x^{\frac{1}{6}}+2) - 3\ln(2x^{\frac{1}{3}}+x^{\frac{1}{6}}-\sqrt{5}x^{\frac{1}{6}}+2) + 3\sqrt{5} \ln(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}x^{\frac{1}{6}}+2) - 3\ln(2x^{\frac{1}{3}}+x^{\frac{1}{6}}+\sqrt{5}x^{\frac{1}{6}}+2)}{5\sqrt{10+2\sqrt{5}} - 5\sqrt{10-2\sqrt{5}} + 5} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-1/x^(1/3)+x^(1/2)),x)

[Out] $2*x^{(1/2)}+6/5*\ln(x^{(1/6)}-1)+3/10*\ln(2+x^{(1/6)}+2*x^{(1/3)}+x^{(1/6)}*5^{(1/2)})*5^{(1/2)}-3/10*\ln(2+x^{(1/6)}+2*x^{(1/3)}+x^{(1/6)}*5^{(1/2)})-12/5/(10-2*5^{(1/2)})^{(1/2)}*\arctan((1+4*x^{(1/6)}+5^{(1/2)})/(10-2*5^{(1/2)})^{(1/2)})*5^{(1/2)}-3/10*\ln(2+x^{(1/6)}+2*x^{(1/3)}-x^{(1/6)}*5^{(1/2)})*5^{(1/2)}-3/10*\ln(2+x^{(1/6)}+2*x^{(1/3)}-x^{(1/6)}*5^{(1/2)})+12/5/(10+2*5^{(1/2)})^{(1/2)}*\arctan((1+4*x^{(1/6)}-5^{(1/2)})/(10+2*5^{(1/2)})^{(1/2)})*5^{(1/2)}$

maxima [B] time = 1.39, size = 272, normalized size = 1.36

$$\frac{6}{5}(-1)^{\frac{3}{5}}\log\left((-1)^{\frac{1}{5}}+x^{\frac{1}{6}}\right) - \frac{6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}+(-1)^{\frac{1}{5}}-4x^{\frac{1}{6}}}\right) + 6\sqrt{5}(-1)^{\frac{3}{5}}\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}-(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{2\sqrt{5}-10}} + 2\sqrt{x} + \frac{6\log\left(-x^{\frac{1}{6}}\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}+2(-1)^{\frac{1}{5}}+2x^{\frac{1}{6}}\right) - 6\log\left(x^{\frac{1}{6}}\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}}+2(-1)^{\frac{1}{5}}+2x^{\frac{1}{6}}\right)}{5\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")

[Out] $-6/5*(-1)^{(3/5)}*\log((-1)^{(1/5)} + x^{(1/6)}) - 6/5*\sqrt{5}*(-1)^{(3/5)}*\log((\sqrt{5}*(-1)^{(1/5)} + (-1)^{(1/5)}*\sqrt{2*\sqrt{5}-10}) + (-1)^{(1/5)} - 4*x^{(1/6)})/(\sqrt{5}*(-1)^{(1/5)} - (-1)^{(1/5)}*\sqrt{2*\sqrt{5}-10}) + (-1)^{(1/5)} - 4*x^{(1/6)})/\sqrt{2*\sqrt{5}-10} + 6/5*\sqrt{5}*(-1)^{(3/5)}*\log((\sqrt{5}*(-1)^{(1/5)} - (-1)^{(1/5)}*\sqrt{2*\sqrt{5}-10}) - (-1)^{(1/5)} + 4*x^{(1/6)})/(\sqrt{5}*(-1)^{(1/5)} + (-1)^{(1/5)}*\sqrt{2*\sqrt{5}-10}) - (-1)^{(1/5)} + 4*x^{(1/6)})/\sqrt{2*\sqrt{5}-10} + 2*\sqrt{x} + 6/5*\log(-x^{(1/6)}*(\sqrt{5}*(-1)^{(1/5)} + (-1)^{(1/5)} + 2*(-1)^{(1/5)} + 2*x^{(1/6)})) + 2*(-1)^{(2/5)} + 2*x^{(1/3)})/(\sqrt{5}*(-1)^{(2/5)} + (-1)^{(2/5)}) - 6/5*\log(x^{(1/6)}*(\sqrt{5}*(-1)^{(1/5)} - (-1)^{(1/5)} + 2*(-1)^{(1/5)} + 2*x^{(1/6)})) + 2*(-1)^{(2/5)} + 2*x^{(1/3)})/(\sqrt{5}*(-1)^{(2/5)} - (-1)^{(2/5)})$

mupad [B] time = 0.12, size = 223, normalized size = 1.12

$$\frac{6\ln(1296x^{1/6}-1296)}{5} - \ln\left(-750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} - \frac{3\sqrt{5}}{10} + \frac{3}{10}\right) - \ln\left(750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} - \frac{3\sqrt{5}}{10} + \frac{3}{10}\right) + \ln\left(-750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} + \frac{3\sqrt{5}}{10} - \frac{3}{10}\right) - \ln\left(750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} + \frac{3\sqrt{5}}{10} - \frac{3}{10}\right) + \ln\left(-750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} + \frac{3\sqrt{5}}{10} + \frac{3}{10}\right) - \ln\left(750x^{1/6}\right) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} + \frac{3\sqrt{5}}{10} + \frac{3}{10}\right) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2) - 1/x^(1/3)),x)

[Out] $(6*\log(1296*x^{(1/6)} - 1296))/5 - \log(-750*x^{(1/6)}*((3*2^{(1/2)}*(-5^{(1/2)} - 5)^{(1/2)})/10 - (3*5^{(1/2)})/10 + 3/10)^3 - 1296)*((3*2^{(1/2)}*(-5^{(1/2)} - 5)^{(1/2)})/10 - (3*5^{(1/2)})/10 + 3/10) + \log(750*x^{(1/6)}*((3*2^{(1/2)}*(-5^{(1/2)} - 5)^{(1/2)})/10 + (3*5^{(1/2)})/10 - 3/10)^3 - 1296)*((3*2^{(1/2)}*(-5^{(1/2)} - 5)^{(1/2)})/10 + (3*5^{(1/2)})/10 - 3/10) - \log(-750*x^{(1/6)}*((3*5^{(1/2)})/10 - (3*2^{(1/2)}*(5^{(1/2)} - 5)^{(1/2)})/10 + 3/10)^3 - 1296)*((3*5^{(1/2)})/10 - (3*2^{(1/2)}*(5^{(1/2)} - 5)^{(1/2)})/10 + 3/10) - \log(-750*x^{(1/6)}*((3*5^{(1/2)})/10 + (3*2^{(1/2)}*(5^{(1/2)} - 5)^{(1/2)})/10 + 3/10)^3 - 1296)*((3*5^{(1/2)})/10 + (3*2^{(1/2)}*(5^{(1/2)} - 5)^{(1/2)})/10 + 3/10) + 2*x^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{(\sqrt[6]{x}-1)\left(\sqrt[6]{x}+x^{\frac{2}{3}}+\sqrt[3]{x}+\sqrt{x}+1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1/x**(1/3)+x**(1/2)),x)

[Out] Integral(x**(1/3)/((x**(1/6) - 1)*(x**(1/6) + x**(2/3) + x**(1/3) + sqrt(x) + 1)), x)

$$3.329 \quad \int \frac{\sqrt{x}}{x+x^2} dx$$

Optimal. Leaf size=8

$$2 \tan^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {647, 63, 203}

$$2 \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(x + x^2), x]

[Out] 2*ArcTan[Sqrt[x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 647

Int[((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[1/e^p, Int[(e*x)^(m + p)*(b + c*x)^p, x], x] /; FreeQ[{b, c, e, m}, x] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{x+x^2} dx &= \int \frac{1}{\sqrt{x}(1+x)} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right) \\ &= 2 \tan^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2 \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(x + x^2), x]

[Out] 2*ArcTan[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.02, size = 8, normalized size = 1.00

$$2 \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(x + x^2),x]

[Out] 2*ArcTan[Sqrt[x]]

fricas [A] time = 0.64, size = 6, normalized size = 0.75

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^2+x),x, algorithm="fricas")

[Out] 2*arctan(sqrt(x))

giac [A] time = 0.29, size = 6, normalized size = 0.75

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^2+x),x, algorithm="giac")

[Out] 2*arctan(sqrt(x))

maple [A] time = 0.00, size = 7, normalized size = 0.88

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^2+x),x)

[Out] 2*arctan(x^(1/2))

maxima [A] time = 2.00, size = 6, normalized size = 0.75

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^2+x),x, algorithm="maxima")

[Out] 2*arctan(sqrt(x))

mupad [B] time = 0.14, size = 6, normalized size = 0.75

$$2 \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x + x^2),x)

[Out] 2*atan(x^(1/2))

sympy [A] time = 0.31, size = 7, normalized size = 0.88

$$2 \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(x**2+x),x)

[Out] 2*atan(sqrt(x))

$$3.330 \quad \int \frac{x}{4\sqrt{x}+x} dx$$

Optimal. Leaf size=19

$$x - 8\sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1584, 266, 43}

$$x - 8\sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] Int[x/(4*Sqrt[x] + x),x]

[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x}{4\sqrt{x}+x} dx &= \int \frac{\sqrt{x}}{4+\sqrt{x}} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{x^2}{4+x} dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int \left(-4 + x + \frac{16}{4+x} \right) dx, x, \sqrt{x} \right) \\ &= -8\sqrt{x} + x + 32 \log(4 + \sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$x - 8\sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] Integrate[x/(4*Sqrt[x] + x),x]

[Out] -8*Sqrt[x] + x + 32*Log[4 + Sqrt[x]]

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 1.26

$$\sqrt{x} (\sqrt{x} - 8) + 32 \log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(4*Sqrt[x] + x),x]

[Out] (-8 + Sqrt[x])*Sqrt[x] + 32*Log[4 + Sqrt[x]]

fricas [A] time = 0.90, size = 15, normalized size = 0.79

$$x - 8 \sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+4*x^(1/2)),x, algorithm="fricas")

[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)

giac [A] time = 0.39, size = 15, normalized size = 0.79

$$x - 8 \sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+4*x^(1/2)),x, algorithm="giac")

[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$x + 32 \ln(\sqrt{x} + 4) - 8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+4*x^(1/2)),x)

[Out] x+32*ln(4+x^(1/2))-8*x^(1/2)

maxima [A] time = 0.87, size = 15, normalized size = 0.79

$$x - 8 \sqrt{x} + 32 \log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+4*x^(1/2)),x, algorithm="maxima")

[Out] x - 8*sqrt(x) + 32*log(sqrt(x) + 4)

mupad [B] time = 0.04, size = 15, normalized size = 0.79

$$x + 32 \ln(\sqrt{x} + 4) - 8\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + 4*x^(1/2)),x)

[Out] x + 32*log(x^(1/2) + 4) - 8*x^(1/2)

sympy [A] time = 0.17, size = 17, normalized size = 0.89

$$-8\sqrt{x} + x + 32 \log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+4*x**(1/2)),x)

[Out] -8*sqrt(x) + x + 32*log(sqrt(x) + 4)

$$3.331 \quad \int \frac{\sqrt{x}}{\sqrt[3]{x+x}} dx$$

Optimal. Leaf size=108

$$2\sqrt{x} - \frac{3 \log(\sqrt[3]{x} - \sqrt{2} \sqrt[6]{x} + 1)}{2\sqrt{2}} + \frac{3 \log(\sqrt[3]{x} + \sqrt{2} \sqrt[6]{x} + 1)}{2\sqrt{2}} + \frac{3 \tan^{-1}(1 - \sqrt{2} \sqrt[6]{x})}{\sqrt{2}} - \frac{3 \tan^{-1}(\sqrt{2} \sqrt[6]{x} + 1)}{\sqrt{2}}$$

Rubi [A] time = 0.08, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {1584, 341, 321, 329, 297, 1162, 617, 204, 1165, 628}

$$2\sqrt{x} - \frac{3 \log(\sqrt[3]{x} - \sqrt{2} \sqrt[6]{x} + 1)}{2\sqrt{2}} + \frac{3 \log(\sqrt[3]{x} + \sqrt{2} \sqrt[6]{x} + 1)}{2\sqrt{2}} + \frac{3 \tan^{-1}(1 - \sqrt{2} \sqrt[6]{x})}{\sqrt{2}} - \frac{3 \tan^{-1}(\sqrt{2} \sqrt[6]{x} + 1)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(x^(1/3) + x), x]

[Out] 2*Sqrt[x] + (3*ArcTan[1 - Sqrt[2]*x^(1/6)]/Sqrt[2] - (3*ArcTan[1 + Sqrt[2]*x^(1/6)]/Sqrt[2] - (3*Log[1 - Sqrt[2]*x^(1/6) + x^(1/3)]/(2*Sqrt[2]) + (3*Log[1 + Sqrt[2]*x^(1/6) + x^(1/3)]/(2*Sqrt[2]))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 297

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[a/b, 2]], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*s), Int[(r + s*x^2)/(a + b*x^4), x], x] - Dist[1/(2*s), Int[(r - s*x^2)/(a + b*x^4), x], x] /; FreeQ[{a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] & AtomQ[SplitProduct[SumBaseQ, b]]))

Rule 321

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a + b*x^n)^(p+1))/(b*(m+n*p+1)), x] - Dist[(a*c^n*(m-n+1))/(b*(m+n*p+1)), Int[(c*x)^(m-n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n-1] && NeQ[m+n*p+1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 329

Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m+1)-1)*(a + (b*x^(k*n)))^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 341

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m+1)-1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1584

```
Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol]
:= Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x]
&& IntegerQ[n] && PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{\sqrt[3]{x} + x} dx &= \int \frac{\sqrt[6]{x}}{1 + x^{2/3}} dx \\
&= 3 \operatorname{Subst} \left(\int \frac{x^{5/2}}{1 + x^2} dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{x} - 3 \operatorname{Subst} \left(\int \frac{\sqrt{x}}{1 + x^2} dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{x} - 6 \operatorname{Subst} \left(\int \frac{x^2}{1 + x^4} dx, x, \sqrt[6]{x} \right) \\
&= 2\sqrt{x} + 3 \operatorname{Subst} \left(\int \frac{1 - x^2}{1 + x^4} dx, x, \sqrt[6]{x} \right) - 3 \operatorname{Subst} \left(\int \frac{1 + x^2}{1 + x^4} dx, x, \sqrt[6]{x} \right) \\
&= 2\sqrt{x} - \frac{3}{2} \operatorname{Subst} \left(\int \frac{1}{1 - \sqrt{2}x + x^2} dx, x, \sqrt[6]{x} \right) - \frac{3}{2} \operatorname{Subst} \left(\int \frac{1}{1 + \sqrt{2}x + x^2} dx, x, \sqrt[6]{x} \right) - \frac{3 \operatorname{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, 1 - \sqrt{2} \sqrt[6]{x} \right)}{\sqrt{2}} \\
&= 2\sqrt{x} - \frac{3 \log(1 - \sqrt{2} \sqrt[6]{x} + \sqrt[3]{x})}{2\sqrt{2}} + \frac{3 \log(1 + \sqrt{2} \sqrt[6]{x} + \sqrt[3]{x})}{2\sqrt{2}} - \frac{3 \operatorname{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, 1 - \sqrt{2} \sqrt[6]{x} \right)}{\sqrt{2}} \\
&= 2\sqrt{x} + \frac{3 \tan^{-1}(1 - \sqrt{2} \sqrt[6]{x})}{\sqrt{2}} - \frac{3 \tan^{-1}(1 + \sqrt{2} \sqrt[6]{x})}{\sqrt{2}} - \frac{3 \log(1 - \sqrt{2} \sqrt[6]{x} + \sqrt[3]{x})}{2\sqrt{2}} + \frac{3 \log(1 + \sqrt{2} \sqrt[6]{x} + \sqrt[3]{x})}{2\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 0.01, size = 24, normalized size = 0.22

$$-2\sqrt{x} \left({}_2F_1 \left(\frac{3}{4}, 1; \frac{7}{4}; -x^{2/3} \right) - 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(x^(1/3) + x), x]

[Out] -2*Sqrt[x]*(-1 + Hypergeometric2F1[3/4, 1, 7/4, -x^(2/3)])

IntegrateAlgebraic [A] time = 0.10, size = 69, normalized size = 0.64

$$2\sqrt{x} - \frac{3 \tan^{-1} \left(\frac{\sqrt[3]{x} - 1}{\frac{\sqrt{2} - \sqrt{2}}{\sqrt{x}}} \right)}{\sqrt{2}} + \frac{3 \tanh^{-1} \left(\frac{\sqrt{2} \sqrt[6]{x}}{\sqrt[3]{x} + 1} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(x^(1/3) + x), x]

[Out] 2*Sqrt[x] - (3*ArcTan[(-1/Sqrt[2]) + x^(1/3)/Sqrt[2]]/x^(1/6)]/Sqrt[2] + (3*ArcTanh[(Sqrt[2]*x^(1/6))/(1 + x^(1/3))])/Sqrt[2]

fricas [A] time = 0.62, size = 120, normalized size = 1.11

$$3\sqrt{2} \arctan \left(\sqrt{2} \sqrt{\sqrt{2}x^{\frac{1}{6}} + x^{\frac{1}{3}} + 1} - \sqrt{2}x^{\frac{1}{6}} - 1 \right) + 3\sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \sqrt{-4\sqrt{2}x^{\frac{1}{6}} + 4x^{\frac{1}{3}} + 4} - \sqrt{2}x^{\frac{1}{6}} + 1 \right) + \frac{3}{4} \sqrt{2} \log(4\sqrt{2}x^{\frac{1}{6}} + 4x^{\frac{1}{3}} + 4) - \frac{3}{4} \sqrt{2} \log(-4\sqrt{2}x^{\frac{1}{6}} + 4x^{\frac{1}{3}} + 4) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/3)+x), x, algorithm="fricas")

[Out] 3*sqrt(2)*arctan(sqrt(2)*sqrt(sqrt(2)*x^(1/6) + x^(1/3) + 1) - sqrt(2)*x^(1/6) - 1) + 3*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) - sqrt(2)*x^(1/6) + 1) + 3/4*sqrt(2)*log(4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) - 3/4*sqrt(2)*log(-4*sqrt(2)*x^(1/6) + 4*x^(1/3) + 4) + 2*sqrt(x)

giac [A] time = 0.44, size = 83, normalized size = 0.77

$$-\frac{3}{2} \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} (\sqrt{2} + 2x^{\frac{1}{6}}) \right) - \frac{3}{2} \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} (\sqrt{2} - 2x^{\frac{1}{6}}) \right) + \frac{3}{4} \sqrt{2} \log(\sqrt{2}x^{\frac{1}{6}} + x^{\frac{1}{3}} + 1) - \frac{3}{4} \sqrt{2} \log(-\sqrt{2}x^{\frac{1}{6}} + x^{\frac{1}{3}} + 1) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/3)+x), x, algorithm="giac")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)

maple [A] time = 0.00, size = 71, normalized size = 0.66

$$-\frac{3\sqrt{2} \arctan \left(\sqrt{2} x^{\frac{1}{6}} - 1 \right)}{2} - \frac{3\sqrt{2} \arctan \left(\sqrt{2} x^{\frac{1}{6}} + 1 \right)}{2} - \frac{3\sqrt{2} \ln \left(\frac{x^{\frac{1}{3}} - \sqrt{2} x^{\frac{1}{6}} + 1}{x^{\frac{1}{3}} + \sqrt{2} x^{\frac{1}{6}} + 1} \right)}{4} + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/3)+x), x)

[Out] 2*x^(1/2)-3/2*arctan(1+x^(1/6)*2^(1/2))*2^(1/2)-3/2*arctan(-1+x^(1/6)*2^(1/2))*2^(1/2)-3/4*2^(1/2)*ln((1+x^(1/3)-x^(1/6)*2^(1/2))/(1+x^(1/3)+x^(1/6)*2^(1/2)))

maxima [A] time = 1.96, size = 83, normalized size = 0.77

$$-\frac{3}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2x^{\frac{1}{6}}\right)\right)-\frac{3}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2x^{\frac{1}{6}}\right)\right)+\frac{3}{4}\sqrt{2}\log\left(\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)-\frac{3}{4}\sqrt{2}\log\left(-\sqrt{2}x^{\frac{1}{6}}+x^{\frac{1}{3}}+1\right)+2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/3)+x),x, algorithm="maxima")

[Out] -3/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*x^(1/6))) - 3/2*sqrt(2)*arctan(-1/2*sqrt(2)*(sqrt(2) - 2*x^(1/6))) + 3/4*sqrt(2)*log(sqrt(2)*x^(1/6) + x^(1/3) + 1) - 3/4*sqrt(2)*log(-sqrt(2)*x^(1/6) + x^(1/3) + 1) + 2*sqrt(x)

mupad [B] time = 0.08, size = 42, normalized size = 0.39

$$2\sqrt{x} + \sqrt{2}\operatorname{atan}\left(\sqrt{2}x^{1/6}\left(\frac{1}{2} - \frac{1}{2}i\right)\right)\left(-\frac{3}{2} + \frac{3}{2}i\right) + \sqrt{2}\operatorname{atan}\left(\sqrt{2}x^{1/6}\left(\frac{1}{2} + \frac{1}{2}i\right)\right)\left(-\frac{3}{2} - \frac{3}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x + x^(1/3)),x)

[Out] 2*x^(1/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 + 1i/2))*(3/2 + 3i/2) - 2^(1/2)*atan(2^(1/2)*x^(1/6)*(1/2 - 1i/2))*(3/2 - 3i/2)

sympy [A] time = 2.18, size = 110, normalized size = 1.02

$$2\sqrt{x} - \frac{3\sqrt{2}\log(-4\sqrt{2}\sqrt[6]{x} + 4\sqrt[3]{x} + 4)}{4} + \frac{3\sqrt{2}\log(4\sqrt{2}\sqrt[6]{x} + 4\sqrt[3]{x} + 4)}{4} - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}\sqrt[6]{x} - 1)}{2} - \frac{3\sqrt{2}\operatorname{atan}(\sqrt{2}\sqrt[6]{x} + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(x**(1/3)+x),x)

[Out] 2*sqrt(x) - 3*sqrt(2)*log(-4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 + 3*sqrt(2)*log(4*sqrt(2)*x**(1/6) + 4*x**(1/3) + 4)/4 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) - 1)/2 - 3*sqrt(2)*atan(sqrt(2)*x**(1/6) + 1)/2

$$3.332 \quad \int \frac{\sqrt[3]{x}}{\sqrt[4]{x} + \sqrt{x}} dx$$

Optimal. Leaf size=76

$$\frac{6x^{5/6}}{5} - \frac{12x^{7/12}}{7} + 3\sqrt[3]{x} - 12\sqrt[12]{x} + 6 \log(\sqrt[12]{x} + 1) - 2 \log(\sqrt[4]{x} + 1) - 4\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {1584, 341, 50, 58, 618, 204, 31}

$$\frac{6x^{5/6}}{5} - \frac{12x^{7/12}}{7} + 3\sqrt[3]{x} - 12\sqrt[12]{x} + 6 \log(\sqrt[12]{x} + 1) - 2 \log(\sqrt[4]{x} + 1) - 4\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[x^(1/3)/(x^(1/4) + Sqrt[x]), x]

[Out] -12*x^(1/12) + 3*x^(1/3) - (12*x^(7/12))/7 + (6*x^(5/6))/5 - 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] + 6*Log[1 + x^(1/12)] - 2*Log[1 + x^(1/4)]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 58

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] := With[{q = Rt[-((b*c - a*d)/b), 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (Dist[3/(2*b*q), Subst[Int[1/(q^2 - q*x + x^2), x], x, (c + d*x)^(1/3)], x] + Dist[3/(2*b*q^2), Subst[Int[1/(q + x), x], x, (c + d*x)^(1/3)], x]) /; FreeQ[{a, b, c, d}, x] && NegQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 341

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x] \&\& \text{NeQ}[b^2 - 4*a*c, 0]$

Rule 1584

$\text{Int}[(u_.)*(x_.)^{(m_.)}*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol]$
 $\rightarrow \text{Int}[u*x^{(m+n*p)}*(a+b*x^{(q-p)})^n, x] /; \text{FreeQ}\{a, b, m, p, q\}, x]$
 $\&\& \text{IntegerQ}[n] \&\& \text{PosQ}[q-p]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{x}}{\sqrt[4]{x} + \sqrt{x}} dx &= \int \frac{\sqrt[12]{x}}{1 + \sqrt[4]{x}} dx \\ &= 4 \text{Subst} \left(\int \frac{x^{10/3}}{1+x} dx, x, \sqrt[4]{x} \right) \\ &= \frac{6x^{5/6}}{5} - 4 \text{Subst} \left(\int \frac{x^{7/3}}{1+x} dx, x, \sqrt[4]{x} \right) \\ &= -\frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4 \text{Subst} \left(\int \frac{x^{4/3}}{1+x} dx, x, \sqrt[4]{x} \right) \\ &= 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4 \text{Subst} \left(\int \frac{\sqrt[3]{x}}{1+x} dx, x, \sqrt[4]{x} \right) \\ &= -12 \sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 4 \text{Subst} \left(\int \frac{1}{x^{2/3}(1+x)} dx, x, \sqrt[4]{x} \right) \\ &= -12 \sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 2 \log(1 + \sqrt[4]{x}) + 6 \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[12]{x} \right) + 6 \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \sqrt[12]{x} \right) \\ &= -12 \sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} + 6 \log(1 + \sqrt[12]{x}) - 2 \log(1 + \sqrt[4]{x}) - 12 \text{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \sqrt[12]{x} \right) \\ &= -12 \sqrt[12]{x} + 3\sqrt[3]{x} - \frac{12x^{7/12}}{7} + \frac{6x^{5/6}}{5} - 4\sqrt{3} \tan^{-1} \left(\frac{1 - 2\sqrt[12]{x}}{\sqrt{3}} \right) + 6 \log(1 + \sqrt[12]{x}) - 2 \log(1 + \sqrt[4]{x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 83, normalized size = 1.09

$$\frac{6x^{5/6}}{5} - \frac{12x^{7/12}}{7} + 3\sqrt[3]{x} - 12 \sqrt[12]{x} + 4 \log(\sqrt[12]{x} + 1) - 2 \log(\sqrt[6]{x} - \sqrt[12]{x} + 1) + 4\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[12]{x} - 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^(1/3)/(x^(1/4) + Sqrt[x]),x]

[Out] $-12*x^{(1/12)} + 3*x^{(1/3)} - (12*x^{(7/12)})/7 + (6*x^{(5/6)})/5 + 4*\text{Sqrt}[3]*\text{ArcTan}[-1 + 2*x^{(1/12)})/\text{Sqrt}[3]] + 4*\text{Log}[1 + x^{(1/12)}] - 2*\text{Log}[1 - x^{(1/12)} + x^{(1/6)}]$

IntegrateAlgebraic [A] time = 0.08, size = 87, normalized size = 1.14

$$\frac{3}{35} (14x^{5/6} - 20x^{7/12} + 35\sqrt[3]{x} - 140\sqrt[12]{x}) + 4 \log(\sqrt[12]{x} + 1) - 2 \log(\sqrt[6]{x} - \sqrt[12]{x} + 1) - 4\sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[12]{x}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^(1/3)/(x^(1/4) + Sqrt[x]),x]

[Out] $(3*(-140*x^{(1/12)} + 35*x^{(1/3)} - 20*x^{(7/12)} + 14*x^{(5/6)}))/35 - 4*\text{Sqrt}[3]*\text{ArcTan}[1/\text{Sqrt}[3] - (2*x^{(1/12)})/\text{Sqrt}[3]] + 4*\text{Log}[1 + x^{(1/12)}] - 2*\text{Log}[1 - x^{(1/12)} + x^{(1/6)}]$

fricas [A] time = 0.72, size = 62, normalized size = 0.82

$$4\sqrt{3} \arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{12}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="fricas")

[Out] 4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 6/5*x^(5/6) - 12/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12) + 1)

giac [A] time = 0.34, size = 60, normalized size = 0.79

$$4\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="giac")

[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12) + 1)

maple [A] time = 0.01, size = 61, normalized size = 0.80

$$4\sqrt{3} \arctan\left(\frac{\left(2x^{\frac{1}{12}} - 1\right)\sqrt{3}}{3}\right) + 4\ln\left(x^{\frac{1}{12}} + 1\right) - 2\ln\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + \frac{6x^{\frac{5}{6}}}{5} - \frac{12x^{\frac{7}{12}}}{7} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/3)/(x^(1/4)+x^(1/2)),x)

[Out] 6/5*x^(5/6)-12/7*x^(7/12)+3*x^(1/3)-12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)+4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))+4*ln(x^(1/12)+1)

maxima [A] time = 1.36, size = 60, normalized size = 0.79

$$4\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{12}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} - \frac{12}{7}x^{\frac{7}{12}} + 3x^{\frac{1}{3}} - 12x^{\frac{1}{12}} - 2\log\left(x^{\frac{1}{6}} - x^{\frac{1}{12}} + 1\right) + 4\log\left(x^{\frac{1}{12}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/3)/(x^(1/4)+x^(1/2)),x, algorithm="maxima")

[Out] 4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 6/5*x^(5/6) - 12/7*x^(7/12) + 3*x^(1/3) - 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) + 4*log(x^(1/12) + 1)

mupad [B] time = 3.08, size = 78, normalized size = 1.03

$$4\ln(144x^{1/12} + 144) - \ln(18 - 36x^{1/12} + \sqrt{3}18i)(2 + \sqrt{3}2i) + \ln(36x^{1/12} - 18 + \sqrt{3}18i)(-2 + \sqrt{3}2i) + 3x^{1/3} + \frac{6x^{5/6}}{5} - 12x^{1/12} - \frac{12x^{7/12}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/3)/(x^(1/2) + x^(1/4)),x)

[Out] 4*log(144*x^(1/12) + 144) - log(3^(1/2)*18i - 36*x^(1/12) + 18)*(3^(1/2)*2i + 2) + log(3^(1/2)*18i + 36*x^(1/12) - 18)*(3^(1/2)*2i - 2) + 3*x^(1/3) + (6*x^(5/6))/5 - 12*x^(1/12) - (12*x^(7/12))/7

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{x}}{\sqrt[4]{x} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/3)/(x**(1/4)+x**(1/2)),x)

[Out] Integral(x**(1/3)/(x**(1/4) + sqrt(x)), x)

$$3.333 \quad \int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Optimal. Leaf size=119

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log$$

Rubi [A] time = 0.05, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1584, 266, 43}

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log(\sqrt[12]{x} + 1)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(x^(1/4) + x^(1/3)), x]

[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}} dx &= \int \frac{\sqrt[4]{x}}{1 + \sqrt[12]{x}} dx \\ &= 12 \text{Subst} \left(\int \frac{x^{14}}{1 + x} dx, x, \sqrt[12]{x} \right) \\ &= 12 \text{Subst} \left(\int \left(-1 + x - x^2 + x^3 - x^4 + x^5 - x^6 + x^7 - x^8 + x^9 - x^{10} + x^{11} - x^{12} + x^{13} + \frac{1}{1 + x} \right) dx, x, \sqrt[12]{x} \right) \\ &= -12 \sqrt[12]{x} + 6\sqrt[6]{x} - 4\sqrt[4]{x} + 3\sqrt[3]{x} - \frac{12x^{5/12}}{5} + 2\sqrt{x} - \frac{12x^{7/12}}{7} + \frac{3x^{2/3}}{2} - \frac{4x^{3/4}}{3} + \frac{6x^{5/6}}{5} - \frac{12x^{11/12}}{11} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log(\sqrt[12]{x} + 1) \end{aligned}$$

Mathematica [A] time = 0.03, size = 119, normalized size = 1.00

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} - \frac{12x^{5/12}}{5} + x + 2\sqrt{x} + 3\sqrt[3]{x} - 4\sqrt[4]{x} + 6\sqrt[6]{x} - 12\sqrt[12]{x} + 12 \log(\sqrt[12]{x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(x^(1/4) + x^(1/3)),x]

[Out] -12*x^(1/12) + 6*x^(1/6) - 4*x^(1/4) + 3*x^(1/3) - (12*x^(5/12))/5 + 2*Sqrt[x] - (12*x^(7/12))/7 + (3*x^(2/3))/2 - (4*x^(3/4))/3 + (6*x^(5/6))/5 - (12*x^(11/12))/11 + x - (12*x^(13/12))/13 + (6*x^(7/6))/7 + 12*Log[1 + x^(1/12)]

IntegrateAlgebraic [A] time = 0.07, size = 110, normalized size = 0.92

$$\frac{25740x^{7/6} - 27720x^{13/12} - 32760x^{11/12} + 36036x^{5/6} - 40040x^{3/4} + 45045x^{2/3} - 51480x^{7/12} - 72072x^{5/12} + 30030x + 60060\sqrt{x} + 90090\sqrt[3]{x} - 120120\sqrt[4]{x} + 180180\sqrt[5]{x} - 360360\sqrt[6]{x}}{30030} + 12\log(\sqrt[6]{x} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(x^(1/4) + x^(1/3)),x]

[Out] (-360360*x^(1/12) + 180180*x^(1/6) - 120120*x^(1/4) + 90090*x^(1/3) - 72072*x^(5/12) + 60060*Sqrt[x] - 51480*x^(7/12) + 45045*x^(2/3) - 40040*x^(3/4) + 36036*x^(5/6) - 32760*x^(11/12) + 30030*x - 27720*x^(13/12) + 25740*x^(7/6))/30030 + 12*Log[1 + x^(1/12)]

fricas [A] time = 0.57, size = 71, normalized size = 0.60

$$\frac{6}{7}(x+7)x^{1/6} - \frac{12}{13}(x+13)x^{1/12} + x - \frac{12}{11}x^{11/12} + \frac{6}{5}x^{5/6} - \frac{4}{3}x^{3/4} + \frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 12\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="fricas")

[Out] 6/7*(x + 7)*x^(1/6) - 12/13*(x + 13)*x^(1/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(1/4) + 12*log(x^(1/12) + 1)

giac [A] time = 0.46, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{7/6} - \frac{12}{13}x^{13/12} + x - \frac{12}{11}x^{11/12} + \frac{6}{5}x^{5/6} - \frac{4}{3}x^{3/4} + \frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="giac")

[Out] 6/7*x^(7/6) - 12/13*x^(13/12) + x - 12/11*x^(11/12) + 6/5*x^(5/6) - 4/3*x^(3/4) + 3/2*x^(2/3) - 12/7*x^(7/12) + 2*sqrt(x) - 12/5*x^(5/12) + 3*x^(1/3) - 4*x^(1/4) + 6*x^(1/6) - 12*x^(1/12) + 12*log(x^(1/12) + 1)

maple [A] time = 0.00, size = 76, normalized size = 0.64

$$\frac{6x^{7/6}}{7} - \frac{12x^{13/12}}{13} + x + 12\ln\left(x^{1/12} + 1\right) - \frac{12x^{11/12}}{11} + \frac{6x^{5/6}}{5} - \frac{4x^{3/4}}{3} + \frac{3x^{2/3}}{2} - \frac{12x^{7/12}}{7} + 2\sqrt{x} - \frac{12x^{5/12}}{5} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/4)+x^(1/3)),x)

[Out] -12*x^(1/12)+6*x^(1/6)-4*x^(1/4)+3*x^(1/3)-12/5*x^(5/12)-12/7*x^(7/12)+3/2*x^(2/3)-4/3*x^(3/4)+6/5*x^(5/6)-12/11*x^(11/12)+x-12/13*x^(13/12)+6/7*x^(7/6)+12*ln(x^(1/12)+1)+2*x^(1/2)

maxima [A] time = 0.53, size = 75, normalized size = 0.63

$$\frac{6}{7}x^{7/6} - \frac{12}{13}x^{13/12} + x - \frac{12}{11}x^{11/12} + \frac{6}{5}x^{5/6} - \frac{4}{3}x^{3/4} + \frac{3}{2}x^{2/3} - \frac{12}{7}x^{7/12} + 2\sqrt{x} - \frac{12}{5}x^{5/12} + 3x^{1/3} - 4x^{1/4} + 6x^{1/6} - 12x^{1/12} + 12\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(x^(1/4)+x^(1/3)),x, algorithm="maxima")

[Out] $6/7*x^{7/6} - 12/13*x^{13/12} + x - 12/11*x^{11/12} + 6/5*x^{5/6} - 4/3*x^{3/4} + 3/2*x^{2/3} - 12/7*x^{7/12} + 2*\sqrt{x} - 12/5*x^{5/12} + 3*x^{1/3} - 4*x^{1/4} + 6*x^{1/6} - 12*x^{1/12} + 12*\log(x^{1/12} + 1)$

mupad [B] time = 0.15, size = 75, normalized size = 0.63

$$x + 12 \ln(x^{1/12} + 1) + 2\sqrt{x} + 3x^{1/3} - 4x^{1/4} + \frac{3x^{2/3}}{2} + 6x^{1/6} - \frac{4x^{3/4}}{3} + \frac{6x^{5/6}}{5} - 12x^{1/12} + \frac{6x^{7/6}}{7} - \frac{12x^{5/12}}{5} - \frac{12x^{7/12}}{7} - \frac{12x^{11/12}}{11} - \frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/3) + x^(1/4)),x)

[Out] $x + 12*\log(x^{1/12} + 1) + 2*x^{1/2} + 3*x^{1/3} - 4*x^{1/4} + (3*x^{2/3})/2 + 6*x^{1/6} - (4*x^{3/4})/3 + (6*x^{5/6})/5 - 12*x^{1/12} + (6*x^{7/6})/7 - (12*x^{5/12})/5 - (12*x^{7/12})/7 - (12*x^{11/12})/11 - (12*x^{13/12})/13$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{\sqrt[4]{x} + \sqrt[3]{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(x**(1/4)+x**(1/3)),x)

[Out] Integral(sqrt(x)/(x**(1/4) + x**(1/3)), x)

$$3.334 \quad \int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx$$

Optimal. Leaf size=201

$$x + 6\sqrt[6]{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 - \sqrt{5}) \log(2\sqrt[3]{x} - \sqrt{5}\sqrt[6]{x} + \sqrt[6]{x} + 2) - \frac{3}{10} (1 + \sqrt{5}) \log(2\sqrt[3]{x} + \sqrt{5}\sqrt[6]{x} + \sqrt[6]{x} + 2)$$

Rubi [A] time = 0.22, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1584, 341, 302, 202, 634, 618, 204, 628, 31}

$$x + 6\sqrt[6]{x} + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 - \sqrt{5}) \log(2\sqrt[3]{x} - \sqrt{5}\sqrt[6]{x} + \sqrt[6]{x} + 2) - \frac{3}{10} (1 + \sqrt{5}) \log(2\sqrt[3]{x} + \sqrt{5}\sqrt[6]{x} + \sqrt[6]{x} + 2) - \frac{3}{5} \sqrt{2(5 + \sqrt{5})} \tan^{-1} \left(\frac{4\sqrt[6]{x} - \sqrt{5} + 1}{\sqrt{2(5 + \sqrt{5})}} \right) - \frac{3}{5} \sqrt{2(5 - \sqrt{5})} \tan^{-1} \left(\frac{1}{\sqrt{2(5 + \sqrt{5})}} (4\sqrt[6]{x} + \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(-x^(-1/3) + Sqrt[x]),x]

[Out] $6x^{1/6} + x - (3\sqrt{2(5 + \sqrt{5})}) \text{ArcTan}[(1 - \sqrt{5} + 4x^{1/6})/\sqrt{2(5 + \sqrt{5})}]/5 - (3\sqrt{2(5 - \sqrt{5})}) \text{ArcTan}[(\sqrt{5} + \sqrt{2(5 + \sqrt{5})})/10 * (1 + \sqrt{5} + 4x^{1/6})]/5 + (6 \log[1 - x^{1/6}])/5 - (3(1 - \sqrt{5}) \log[2 + x^{1/6} - \sqrt{5}x^{1/6} + 2x^{1/3}])/10 - (3(1 + \sqrt{5}) \log[2 + x^{1/6} + \sqrt{5}x^{1/6} + 2x^{1/3}])/10$

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 202

Int[((a_) + (b_.)*(x_)^(n_))^(m_), x_Symbol] := Module[{r = Numerator[Rt[-(a/b), n]], s = Denominator[Rt[-(a/b), n]], k, u}, Simp[u = Int[(r + s*Cos[(2*k - 1)*Pi/n]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*Pi/n]*x + s^2*x^2), x]; (r * Int[1/(r - s*x), x])/(a*n) + Dist[(2*r)/(a*n), Sum[u, {k, 1, (n - 1)/2}], x], x] /; FreeQ[{a, b}, x] && IGtQ[(n - 3)/2, 0] && NegQ[a/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(m_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 341

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(m_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},

$x]$ && NeQ[$b^2 - 4ac$, 0]

Rule 628

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]]/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1584

Int[(u_)*(x_)^(m_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x}}{-\frac{1}{\sqrt[3]{x}} + \sqrt{x}} dx &= \int \frac{x^{5/6}}{-1 + x^{5/6}} dx \\
 &= 6 \operatorname{Subst}\left(\int \frac{x^{10}}{-1 + x^5} dx, x, \sqrt[6]{x}\right) \\
 &= 6 \operatorname{Subst}\left(\int \left(1 + x^5 + \frac{1}{-1 + x^5}\right) dx, x, \sqrt[6]{x}\right) \\
 &= 6\sqrt[6]{x} + x + 6 \operatorname{Subst}\left(\int \frac{1}{-1 + x^5} dx, x, \sqrt[6]{x}\right) \\
 &= 6\sqrt[6]{x} + x - \frac{6}{5} \operatorname{Subst}\left(\int \frac{1}{1 - x} dx, x, \sqrt[6]{x}\right) - \frac{12}{5} \operatorname{Subst}\left(\int \frac{1 + \frac{1}{4}(1 - \sqrt{5})x}{1 + \frac{1}{2}(1 - \sqrt{5})x + x^2} dx, x, \sqrt[6]{x}\right) \\
 &= 6\sqrt[6]{x} + x + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{1}{10} (3(1 - \sqrt{5})) \operatorname{Subst}\left(\int \frac{\frac{1}{2}(1 - \sqrt{5}) + 2x}{1 + \frac{1}{2}(1 - \sqrt{5})x + x^2} dx, x, \sqrt[6]{x}\right) \\
 &= 6\sqrt[6]{x} + x + \frac{6}{5} \log(1 - \sqrt[6]{x}) - \frac{3}{10} (1 - \sqrt{5}) \log(2 + \sqrt[6]{x} - \sqrt{5} \sqrt[6]{x} + 2\sqrt[3]{x}) - \frac{3}{10} (1 + \sqrt{5}) \\
 &= 6\sqrt[6]{x} + x - \frac{3}{5} \sqrt{2(5 + \sqrt{5})} \tan^{-1}\left(\frac{1 - \sqrt{5} + 4\sqrt[6]{x}}{\sqrt{2(5 + \sqrt{5})}}\right) - \frac{3}{5} \sqrt{2(5 - \sqrt{5})} \tan^{-1}\left(\frac{1}{2} \sqrt{\frac{1}{10}} (5 + \sqrt{5})\right)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 29, normalized size = 0.14

$$-6\sqrt[6]{x} {}_2F_1\left(\frac{1}{5}, 1; \frac{6}{5}; x^{5/6}\right) + x + 6\sqrt[6]{x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(-x^(-1/3) + Sqrt[x]), x]

[Out] 6*x^(1/6) + x - 6*x^(1/6)*Hypergeometric2F1[1/5, 1, 6/5, x^(5/6)]

IntegrateAlgebraic [C] time = 0.08, size = 127, normalized size = 0.63

$$-\frac{6}{5}\text{RootSum}\left[\#1^4 + \#1^3 + \#1^2 + \#1 + 1 \&, \frac{\#1^3 \log(\sqrt[6]{x} - \#1) + 2\#1^2 \log(\sqrt[6]{x} - \#1) + 3\#1 \log(\sqrt[6]{x} - \#1) + 4 \log(\sqrt[6]{x} - \#1)}{4\#1^3 + 3\#1^2 + 2\#1 + 1} \&\right] + x + 6\sqrt[6]{x} + \frac{6}{5} \log(\sqrt[6]{x} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(-x^(-1/3) + Sqrt[x]), x]

[Out] 6*x^(1/6) + x + (6*Log[-1 + x^(1/6)])/5 - (6*RootSum[1 + #1 + #1^2 + #1^3 + #1^4 &, (4*Log[x^(1/6) - #1] + 3*Log[x^(1/6) - #1]*#1 + 2*Log[x^(1/6) - #1]*#1^2 + Log[x^(1/6) - #1]*#1^3)/(1 + 2*#1 + 3*#1^2 + 4*#1^3) &]/5

fricas [B] time = 2.83, size = 547, normalized size = 2.72

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)), x, algorithm="fricas")

[Out] -3/10*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)*log(3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 3/10*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)*log(-3/2*sqrt(2)*sqrt(sqrt(5) - 5) + 3/2*sqrt(5) + 6*x^(1/6) + 3/2) + 1/10*(3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) + sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + 1/10*(3*sqrt(5) + sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) - 3)*log(-3*sqrt(5) - sqrt(-27/4*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) + 1)^2 + 9/2*(sqrt(2)*sqrt(sqrt(5) - 5) + sqrt(5) - 3)*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1) - 27/4*(sqrt(2)*sqrt(sqrt(5) - 5) - sqrt(5) - 1)^2 + 18*sqrt(2)*sqrt(sqrt(5) - 5) + 18*sqrt(5) - 90) + 12*x^(1/6) + 3) + x + 6*x^(1/6) + 6/5*log(x^(1/6) - 1)

giac [A] time = 1.20, size = 140, normalized size = 0.70

$$\frac{3}{5}\sqrt{2\sqrt{5} + 10} \arctan\left(\frac{\sqrt{5} - 4x^{\frac{1}{6}} - 1}{\sqrt{2\sqrt{5} + 10}}\right) - \frac{3}{5}\sqrt{-2\sqrt{5} + 10} \arctan\left(\frac{\sqrt{5} + 4x^{\frac{1}{6}} + 1}{\sqrt{-2\sqrt{5} + 10}}\right) - \frac{3}{10}\sqrt{5} \log\left(\frac{1}{2}x^{\frac{1}{6}}(\sqrt{5} + 1) + x^{\frac{1}{3}} + 1\right) + \frac{3}{10}\sqrt{5} \log\left(-\frac{1}{2}x^{\frac{1}{6}}(\sqrt{5} - 1) + x^{\frac{1}{3}} + 1\right) + x + 6x^{\frac{1}{6}} - \frac{3}{10} \log(x^{\frac{1}{3}} + \sqrt{x} + x^{\frac{1}{6}} + 1) + \frac{6}{5} \log(|x^{\frac{1}{6}} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)), x, algorithm="giac")

[Out] -3/5*sqrt(2*sqrt(5) + 10)*arctan(-(sqrt(5) - 4*x^(1/6) - 1)/sqrt(2*sqrt(5) + 10)) - 3/5*sqrt(-2*sqrt(5) + 10)*arctan((sqrt(5) + 4*x^(1/6) + 1)/sqrt(-2*sqrt(5) + 10)) - 3/10*sqrt(5)*log(1/2*x^(1/6)*(sqrt(5) + 1) + x^(1/3) + 1) + 3/10*sqrt(5)*log(-1/2*x^(1/6)*(sqrt(5) - 1) + x^(1/3) + 1) + x + 6*x^(1/6) - 3/10*log(x^(2/3) + sqrt(x) + x^(1/3) + x^(1/6) + 1) + 6/5*log(abs(x^(1/6) - 1))

maple [A] time = 0.03, size = 242, normalized size = 1.20

$$x - \frac{6 \arctan\left(\frac{4x^{\frac{1}{6}} + 1 - \sqrt{5}}{\sqrt{10 + 2\sqrt{5}}}\right)}{\sqrt{10 + 2\sqrt{5}}} - \frac{6\sqrt{5} \arctan\left(\frac{4x^{\frac{1}{6}} + 1 - \sqrt{5}}{\sqrt{10 + 2\sqrt{5}}}\right)}{5\sqrt{10 + 2\sqrt{5}}} - \frac{6 \arctan\left(\frac{4x^{\frac{1}{6}} + 1 + \sqrt{5}}{\sqrt{10 - 2\sqrt{5}}}\right)}{\sqrt{10 - 2\sqrt{5}}} + \frac{6\sqrt{5} \arctan\left(\frac{4x^{\frac{1}{6}} + 1 + \sqrt{5}}{\sqrt{10 - 2\sqrt{5}}}\right)}{5\sqrt{10 - 2\sqrt{5}}} + \frac{6 \ln(x^{\frac{1}{3}} - 1)}{5} + \frac{3\sqrt{5} \ln(2x^{\frac{1}{6}} + x^{\frac{1}{3}} - \sqrt{5}x^{\frac{1}{6}} + 2)}{10} - \frac{3 \ln(2x^{\frac{1}{6}} + x^{\frac{1}{3}} - \sqrt{5}x^{\frac{1}{6}} + 2)}{10} - \frac{3\sqrt{5} \ln(2x^{\frac{1}{6}} + x^{\frac{1}{3}} + \sqrt{5}x^{\frac{1}{6}} + 2)}{10} - \frac{3 \ln(2x^{\frac{1}{6}} + x^{\frac{1}{3}} + \sqrt{5}x^{\frac{1}{6}} + 2)}{10} + 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x)

[Out] $x+6x^{1/6}+6/5\ln(x^{1/6}-1)-3/10\sqrt{5}^{1/2}\ln(2x^{1/3}+x^{1/6}+5^{1/2})x^{1/6}+2-3/10\ln(2x^{1/3}+x^{1/6}+5^{1/2})x^{1/6}+2-6/(10-2\sqrt{5}^{1/2})^{1/2}\arctan((4x^{1/6}+1+5^{1/2})/(10-2\sqrt{5}^{1/2})^{1/2})+6/5/(10-2\sqrt{5}^{1/2})^{1/2}5^{1/2}\arctan((4x^{1/6}+1+5^{1/2})/(10-2\sqrt{5}^{1/2})^{1/2})+3/10\sqrt{5}^{1/2}\ln(2x^{1/3}+x^{1/6}-5^{1/2})x^{1/6}+2-3/10\ln(2x^{1/3}+x^{1/6}-5^{1/2})x^{1/6}+2-6/(10+2\sqrt{5}^{1/2})^{1/2}\arctan((4x^{1/6}+1-5^{1/2})/(10+2\sqrt{5}^{1/2})^{1/2})-6/5/(10+2\sqrt{5}^{1/2})^{1/2}5^{1/2}\arctan((4x^{1/6}+1-5^{1/2})/(10+2\sqrt{5}^{1/2})^{1/2})$

maxima [B] time = 1.52, size = 293, normalized size = 1.46

$$\frac{3\sqrt{5}(-1)^{\frac{1}{5}}(\sqrt{5}-1)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{2}\sqrt{5-10}} - \frac{3\sqrt{5}(-1)^{\frac{1}{5}}(\sqrt{5}+1)\log\left(\frac{\sqrt{5}(-1)^{\frac{1}{5}}(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}{\sqrt{5}(-1)^{\frac{1}{5}}(-1)^{\frac{1}{5}}\sqrt{2\sqrt{5}-10}(-1)^{\frac{1}{5}}+4x^{\frac{1}{6}}}\right)}{5\sqrt{2}\sqrt{5-10}} - \frac{6}{5}(-1)^{\frac{1}{5}}\log((-1)^{\frac{1}{5}}+x) - \frac{3(\sqrt{5}+3)\log(-x^{\frac{1}{6}}(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}})+2(-1)^{\frac{1}{5}}+2x^{\frac{1}{3}})}{5(\sqrt{5}(-1)^{\frac{1}{5}}+(-1)^{\frac{1}{5}})} - \frac{3(\sqrt{5}-3)\log(x^{\frac{1}{6}}(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}})+2(-1)^{\frac{1}{5}}+2x^{\frac{1}{3}})}{5(\sqrt{5}(-1)^{\frac{1}{5}}-(-1)^{\frac{1}{5}})} + 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-1/x^(1/3)+x^(1/2)),x, algorithm="maxima")

[Out] $-3/5\sqrt{5}\sqrt{5}*(-1)^{1/5}*(\sqrt{5}-1)*\log((\sqrt{5}*(-1)^{1/5}+(-1)^{1/5})*\sqrt{2*\sqrt{5}-10}+(-1)^{1/5}-4*x^{1/6})/(\sqrt{5}*(-1)^{1/5}-(-1)^{1/5})*\sqrt{2*\sqrt{5}-10}+(-1)^{1/5}-4*x^{1/6})/(\sqrt{5}*(-1)^{1/5}-(-1)^{1/5})*\sqrt{2*\sqrt{5}-10}-3/5\sqrt{5}\sqrt{5}*(-1)^{1/5}*(\sqrt{5}+1)*\log((\sqrt{5}*(-1)^{1/5}-(-1)^{1/5})*\sqrt{-2*\sqrt{5}-10}-(-1)^{1/5}+4*x^{1/6})/(\sqrt{5}*(-1)^{1/5}+(-1)^{1/5})*\sqrt{-2*\sqrt{5}-10}-(-1)^{1/5}+4*x^{1/6})/(\sqrt{5}*(-1)^{1/5}+(-1)^{1/5})*\sqrt{-2*\sqrt{5}-10}-6/5*(-1)^{1/5}*\log((-1)^{1/5}+x^{1/6})+x-3/5*(\sqrt{5}+3)*\log(-x^{1/6}*(\sqrt{5}*(-1)^{1/5}+(-1)^{1/5}))+2*(-1)^{2/5}+2*x^{1/3})/(\sqrt{5}*(-1)^{4/5}+(-1)^{4/5})-3/5*(\sqrt{5}-3)*\log(x^{1/6}*(\sqrt{5}*(-1)^{1/5}-(-1)^{1/5}))+2*(-1)^{2/5}+2*x^{1/3})/(\sqrt{5}*(-1)^{4/5}-(-1)^{4/5})+6*x^{1/6}$

mupad [B] time = 0.06, size = 208, normalized size = 1.03

$$x + \frac{6 \ln(1296x^{1/6} - 1296)}{5} - \ln(270\sqrt{2}\sqrt{\sqrt{5}-5} - 270\sqrt{5} + 1080x^{1/6} + 270) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} - \frac{3\sqrt{5}}{10} \right) + \ln(270\sqrt{2}\sqrt{\sqrt{5}-5} + 270\sqrt{5} - 1080x^{1/6} - 270) \left(\frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} + \frac{3\sqrt{5}}{10} \right) - \ln(270\sqrt{5} + 1080x^{1/6} - 270\sqrt{2}\sqrt{\sqrt{5}-5} + 270) \left(\frac{3\sqrt{5}}{10} - \frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} \right) - \ln(270\sqrt{5} + 1080x^{1/6} + 270\sqrt{2}\sqrt{\sqrt{5}-5} + 270) \left(\frac{3\sqrt{5}}{10} + \frac{3\sqrt{2}\sqrt{\sqrt{5}-5}}{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^(1/2) - 1/x^(1/3)),x)

[Out] $x + (6*\log(1296*x^{1/6} - 1296))/5 - \log(270*2^{1/2}*(-5^{1/2} - 5)^{1/2} - 270*5^{1/2} + 1080*x^{1/6} + 270)*((3*2^{1/2}*(-5^{1/2} - 5)^{1/2})/10 - (3*5^{1/2})/10 + 3/10) + \log(270*2^{1/2}*(-5^{1/2} - 5)^{1/2} + 270*5^{1/2} - 1080*x^{1/6} - 270)*((3*2^{1/2}*(-5^{1/2} - 5)^{1/2})/10 + (3*5^{1/2})/10 - 3/10) + 6*x^{1/6} - \log(270*5^{1/2} + 1080*x^{1/6} - 270*2^{1/2}*(5^{1/2} - 5)^{1/2} + 270)*((3*5^{1/2})/10 - (3*2^{1/2}*(5^{1/2} - 5)^{1/2})/10 + 3/10) - \log(270*5^{1/2} + 1080*x^{1/6} + 270*2^{1/2}*(5^{1/2} - 5)^{1/2} + 270)*((3*5^{1/2})/10 + (3*2^{1/2}*(5^{1/2} - 5)^{1/2})/10 + 3/10)$

sympy [A] time = 24.15, size = 311, normalized size = 1.55

$$6\sqrt{x} + x + \frac{6 \log(\sqrt{x} - 1)}{5} - \frac{3\sqrt{5} \log(8\sqrt{x} + 8\sqrt{5}\sqrt{x} + 16\sqrt{x} + 16)}{10} - \frac{3 \log(8\sqrt{x} + 8\sqrt{5}\sqrt{x} + 16\sqrt{x} + 16)}{10} - \frac{3 \log(-8\sqrt{5}\sqrt{x} + 8\sqrt{x} + 16\sqrt{x} + 16)}{10} + \frac{3\sqrt{5} \log(-8\sqrt{5}\sqrt{x} + 8\sqrt{x} + 16\sqrt{x} + 16)}{10} - \frac{3\sqrt{2}\sqrt{5-\sqrt{5}} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt{x} + \sqrt{5}}{\sqrt{5-\sqrt{5}}}, \frac{\sqrt{5}}{2\sqrt{5-\sqrt{5}}}\right) + \frac{\sqrt{10}}{2\sqrt{5-\sqrt{5}}}}{5} - \frac{3\sqrt{2}\sqrt{5+5} \operatorname{atan}\left(\frac{2\sqrt{2}\sqrt{x} - \sqrt{5}}{\sqrt{5+\sqrt{5}}}, \frac{\sqrt{5}}{2\sqrt{5+\sqrt{5}}}\right) + \frac{\sqrt{10}}{2\sqrt{5+\sqrt{5}}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(-1/x**(1/3)+x**(1/2)),x)

[Out] $6*x^{1/6} + x + 6*\log(x^{1/6} - 1)/5 - 3*\sqrt{5}*\log(8*x^{1/6} + 8*\sqrt{5}*x^{1/6} + 16*x^{1/6} + 16)/10 - 3*\log(8*x^{1/6} + 8*\sqrt{5}*x^{1/6} + 16*x^{1/6} + 16)/10 - 3*\log(-8*\sqrt{5}*x^{1/6} + 8*x^{1/6} + 16*x^{1/6} + 16)/10 + 3*\sqrt{5}*\log(-8*\sqrt{5}*x^{1/6} + 8*x^{1/6} + 16*x^{1/6} + 16)/10 + 3*\sqrt{2}*\sqrt{5 - \sqrt{5}}*\operatorname{atan}(2*\sqrt{2}*x^{1/6}/\sqrt{5 - \sqrt{5}} + \sqrt{2}/(2*\sqrt{5 - \sqrt{5}}))/5 - 3*\sqrt{2}*\sqrt{5 + \sqrt{5}}*\operatorname{atan}(2*\sqrt{2}*x^{1/6}/\sqrt{5 + \sqrt{5}} + \sqrt{2}/(2*\sqrt{5 + \sqrt{5}}))/5$

$$3.335 \quad \int \frac{\sqrt{b - \frac{a}{x}} x^m}{\sqrt{a - bx}} dx$$

Optimal. Leaf size=36

$$\frac{2x^{m+1} \sqrt{b - \frac{a}{x}}}{(2m + 1)\sqrt{a - bx}}$$

Rubi [A] time = 0.04, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {515, 23, 30}

$$\frac{2x^{m+1} \sqrt{b - \frac{a}{x}}}{(2m + 1)\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x^(1 + m))/((1 + 2*m)*Sqrt[a - b*x])

Rule 23

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_.)*((c_.) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}} x^m}{\sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{x^{-\frac{1}{2}+m} \sqrt{-a+bx}}{\sqrt{a-bx}} dx}{\sqrt{-a+bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int x^{-\frac{1}{2}+m} dx}{\sqrt{a-bx}} \\ &= \frac{2\sqrt{b - \frac{a}{x}} x^{1+m}}{(1+2m)\sqrt{a-bx}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 0.97

$$\frac{x^{m+1} \sqrt{b - \frac{a}{x}}}{\left(m + \frac{1}{2}\right) \sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]

[Out] (Sqrt[b - a/x]*x^(1 + m))/((1/2 + m)*Sqrt[a - b*x])

IntegrateAlgebraic [F] time = 1.73, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x}} x^m}{\sqrt{a - bx}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x],x]

[Out] Defer[IntegrateAlgebraic] [(Sqrt[b - a/x]*x^m)/Sqrt[a - b*x], x]

fricas [A] time = 0.62, size = 44, normalized size = 1.22

$$\frac{2 \sqrt{-bx + a} x x^m \sqrt{\frac{bx-a}{x}}}{2 am - (2 bm + b)x + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(-b*x + a)*x*x^m*sqrt((b*x - a)/x)/(2*a*m - (2*b*m + b)*x + a)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(x)]
 Warning, integration of abs or sign assumes constant sign by intervals (cor
 rect if the argument is real):Check [abs(t_nostep)]Undef/Unsigned Inf encou
 ntered in limitLimit: Max order reached or unable to make series expansion
 Error: Bad Argument Value

maple [A] time = 0.01, size = 36, normalized size = 1.00

$$\frac{2 \sqrt{-\frac{bx+a}{x}} x^{m+1}}{(2m + 1) \sqrt{-bx + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)

[Out] 2*x^(m+1)/(1+2*m)*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)

maxima [C] time = 0.71, size = 15, normalized size = 0.42

$$\frac{2\sqrt{x}x^m}{2im+i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x)*x^m/(2*I*m + I)

mupad [B] time = 3.26, size = 32, normalized size = 0.89

$$\frac{2x^{m+1}\sqrt{b-\frac{a}{x}}}{(2m+1)\sqrt{a-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^m*(b - a/x)^(1/2))/(a - b*x)^(1/2),x)

[Out] (2*x^(m + 1)*(b - a/x)^(1/2))/((2*m + 1)*(a - b*x)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**m*(b-a/x)**(1/2)/(-b*x+a)**(1/2),x)

[Out] Integral(x**m*sqrt(-a/x + b)/sqrt(a - b*x), x)

$$3.336 \quad \int \frac{\sqrt{b - \frac{a}{x}} x^2}{\sqrt{a - bx}} dx$$

Optimal. Leaf size=29

$$\frac{2x^3 \sqrt{b - \frac{a}{x}}}{5\sqrt{a - bx}}$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {515, 23, 30}

$$\frac{2x^3 \sqrt{b - \frac{a}{x}}}{5\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x], x]

[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])

Rule 23

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_.)*((c_.) + (d_.)*(x_)^(mn_.))^(q_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}} x^2}{\sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{x^{3/2} \sqrt{-a+bx}}{\sqrt{a-bx}} dx}{\sqrt{-a+bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int x^{3/2} dx}{\sqrt{a-bx}} \\ &= \frac{2\sqrt{b - \frac{a}{x}} x^3}{5\sqrt{a-bx}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^3 \sqrt{b - \frac{a}{x}}}{5\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x^3)/(5*Sqrt[a - b*x])

IntegrateAlgebraic [A] time = 10.33, size = 32, normalized size = 1.10

$$\frac{2x^2\sqrt{a-bx}}{5\sqrt{-\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x^2)/Sqrt[a - b*x],x]

[Out] (-2*x^2*Sqrt[a - b*x])/(5*Sqrt[-((a - b*x)/x)])

fricas [A] time = 0.70, size = 35, normalized size = 1.21

$$-\frac{2\sqrt{-bx+a}x^3\sqrt{\frac{bx-a}{x}}}{5(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")

[Out] -2/5*sqrt(-b*x + a)*x^3*sqrt((b*x - a)/x)/(b*x - a)

giac [B] time = 0.38, size = 76, normalized size = 2.62

$$\frac{2\sqrt{-ab}a^2|b|\operatorname{sgn}(x)}{5b^4} - \frac{2\left(\sqrt{-ab}a^2 - \frac{((bx-a)b+ab)^2\sqrt{-(bx-a)b-ab}}{b^2}\right)|b|\operatorname{sgn}(x)}{5b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/5*sqrt(-a*b)*a^2*abs(b)*sgn(x)/b^4 - 2/5*(sqrt(-a*b)*a^2 - ((b*x - a)*b + a*b)^2*sqrt(-(b*x - a)*b - a*b)/b^2)*abs(b)*sgn(x)/b^4

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{-bx+a}{x}}x^3}{5\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x)

[Out] 2/5*x^3*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)

maxima [C] time = 0.81, size = 5, normalized size = 0.17

$$-\frac{2}{5}ix^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] -2/5*I*x^(5/2)

mupad [B] time = 3.10, size = 23, normalized size = 0.79

$$\frac{2x^3 \sqrt{b - \frac{a}{x}}}{5\sqrt{a - bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(b - a/x)^(1/2))/(a - b*x)^(1/2), x)

[Out] (2*x^3*(b - a/x)^(1/2))/(5*(a - b*x)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b-a/x)**(1/2)/(-b*x+a)**(1/2), x)

[Out] Integral(x**2*sqrt(-a/x + b)/sqrt(a - b*x), x)

$$3.337 \quad \int \frac{\sqrt{b - \frac{a}{x}} x}{\sqrt{a - bx}} dx$$

Optimal. Leaf size=29

$$\frac{2x^2 \sqrt{b - \frac{a}{x}}}{3\sqrt{a - bx}}$$

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {515, 23, 30}

$$\frac{2x^2 \sqrt{b - \frac{a}{x}}}{3\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x]*x)/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])

Rule 23

Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}} x}{\sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{\sqrt{x} \sqrt{-a+bx}}{\sqrt{a-bx}} dx}{\sqrt{-a+bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \sqrt{x} dx}{\sqrt{a-bx}} \\ &= \frac{2\sqrt{b - \frac{a}{x}} x^2}{3\sqrt{a-bx}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$\frac{2x^2 \sqrt{b - \frac{a}{x}}}{3\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x]*x)/Sqrt[a - b*x], x]

[Out] (2*Sqrt[b - a/x]*x^2)/(3*Sqrt[a - b*x])

IntegrateAlgebraic [A] time = 7.22, size = 30, normalized size = 1.03

$$\frac{2x\sqrt{a-bx}}{3\sqrt{-\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[b - a/x]*x)/Sqrt[a - b*x], x]

[Out] (-2*x*Sqrt[a - b*x])/(3*Sqrt[-((a - b*x)/x)])

fricas [A] time = 0.56, size = 35, normalized size = 1.21

$$\frac{2\sqrt{-bx+a}x^2\sqrt{\frac{bx-a}{x}}}{3(bx-a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2), x, algorithm="fricas")

[Out] -2/3*sqrt(-b*x + a)*x^2*sqrt((b*x - a)/x)/(b*x - a)

giac [B] time = 0.49, size = 56, normalized size = 1.93

$$\frac{2\sqrt{-ab}a|b|\operatorname{sgn}(x)}{3b^3} - \frac{2\left(\sqrt{-ab}a + \frac{(-bx-a)b-ab)^{\frac{3}{2}}}{b}\right)|b|\operatorname{sgn}(x)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2), x, algorithm="giac")

[Out] 2/3*sqrt(-a*b)*a*abs(b)*sgn(x)/b^3 - 2/3*(sqrt(-a*b)*a + (-b*x - a)*b - a*b)^(3/2)/b*abs(b)*sgn(x)/b^3

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{2\sqrt{-\frac{bx+a}{x}}x^2}{3\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2), x)

[Out] 2/3*x^2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)

maxima [C] time = 0.73, size = 5, normalized size = 0.17

$$-\frac{2}{3}ix^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x)^(1/2)/(-b*x+a)^(1/2), x, algorithm="maxima")

[Out] -2/3*I*x^(3/2)

mupad [B] time = 3.06, size = 23, normalized size = 0.79

$$\frac{2x^2 \sqrt{b - \frac{a}{x}}}{3\sqrt{a - bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b - a/x)^(1/2))/(a - b*x)^(1/2), x)

[Out] (2*x^2*(b - a/x)^(1/2))/(3*(a - b*x)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x)**(1/2)/(-b*x+a)**(1/2), x)

[Out] Integral(x*sqrt(-a/x + b)/sqrt(a - b*x), x)

$$3.338 \quad \int \frac{\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}} dx$$

Optimal. Leaf size=25

$$\frac{2x\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Rubi [A] time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {435, 23, 30}

$$\frac{2x\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x]/Sqrt[a - b*x], x]

[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 435

Int[((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[((a + b*x^n)^p*(d + c*x^n)^q/x^(n*q), x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{\sqrt{-a+bx}}{\sqrt{x} \sqrt{a-bx}} dx}{\sqrt{-a + bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{1}{\sqrt{x}} dx}{\sqrt{a - bx}} \\ &= \frac{2\sqrt{b - \frac{a}{x}} x}{\sqrt{a - bx}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{2x\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x]/Sqrt[a - b*x],x]

[Out] (2*Sqrt[b - a/x]*x)/Sqrt[a - b*x]

IntegrateAlgebraic [A] time = 5.48, size = 27, normalized size = 1.08

$$\frac{2\sqrt{a-bx}}{\sqrt{\frac{a-bx}{x}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[b - a/x]/Sqrt[a - b*x],x]

[Out] (-2*Sqrt[a - b*x])/Sqrt[-((a - b*x)/x)]

fricas [A] time = 0.77, size = 33, normalized size = 1.32

$$\frac{2\sqrt{-bx+a}x\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-b*x + a)*x*sqrt((b*x - a)/x)/(b*x - a)

giac [B] time = 0.40, size = 51, normalized size = 2.04

$$\frac{2\left(\sqrt{-(bx-a)b-ab}-\sqrt{-ab}\right)|b|\operatorname{sgn}(x)}{b^2} + \frac{2\sqrt{-ab}|b|\operatorname{sgn}(x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="giac")

[Out] 2*(sqrt(-(b*x - a)*b - a*b) - sqrt(-a*b))*abs(b)*sgn(x)/b^2 + 2*sqrt(-a*b)*abs(b)*sgn(x)/b^2

maple [A] time = 0.00, size = 25, normalized size = 1.00

$$\frac{2\sqrt{\frac{-bx+a}{x}}x}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x)^(1/2)/(-b*x+a)^(1/2),x)

[Out] 2*x*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)

maxima [C] time = 0.67, size = 5, normalized size = 0.20

$$-2i\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] -2*I*sqrt(x)

mupad [B] time = 3.04, size = 21, normalized size = 0.84

$$\frac{2x\sqrt{b-\frac{a}{x}}}{\sqrt{a-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b - a/x)^(1/2)/(a - b*x)^(1/2), x)

[Out] (2*x*(b - a/x)^(1/2))/(a - b*x)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x} + b}}{\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)**(1/2)/(-b*x+a)**(1/2), x)

[Out] Integral(sqrt(-a/x + b)/sqrt(a - b*x), x)

$$3.339 \quad \int \frac{\sqrt{b - \frac{a}{x}}}{x\sqrt{a - bx}} dx$$

Optimal. Leaf size=24

$$-\frac{2\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]

Rule 23

Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}}}{x\sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{\sqrt{-a+bx}}{x^{3/2}\sqrt{a-bx}} dx}{\sqrt{-a + bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{1}{x^{3/2}} dx}{\sqrt{a - bx}} \\ &= -\frac{2\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/Sqrt[a - b*x]

IntegrateAlgebraic [A] time = 2.62, size = 27, normalized size = 1.12

$$-\frac{2\sqrt{-\frac{a-bx}{x}}}{\sqrt{a-bx}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[b - a/x]/(x*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[-((a - b*x)/x)])/Sqrt[a - b*x]

fricas [A] time = 0.63, size = 32, normalized size = 1.33

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{bx-a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x - a)

giac [B] time = 0.43, size = 42, normalized size = 1.75

$$\frac{2\left(\frac{b^3}{\sqrt{-(bx-a)b-ab}} - \frac{b^3}{\sqrt{-ab}}\right)|b|\operatorname{sgn}(x)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="giac")

[Out] 2*(b^3/sqrt(-(b*x - a)*b - a*b) - b^3/sqrt(-a*b))*abs(b)*sgn(x)/b^3

maple [A] time = 0.00, size = 24, normalized size = 1.00

$$-\frac{2\sqrt{-\frac{-bx+a}{x}}}{\sqrt{-bx+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x)

[Out] -2*(-(-b*x+a)/x)^(1/2)/(-b*x+a)^(1/2)

maxima [C] time = 0.86, size = 5, normalized size = 0.21

$$\frac{2i}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] 2*I/sqrt(x)

mupad [B] time = 3.08, size = 20, normalized size = 0.83

$$-\frac{2\sqrt{b - \frac{a}{x}}}{\sqrt{a - bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b - a/x)^(1/2)/(x*(a - b*x)^(1/2)), x)`

[Out] `-(2*(b - a/x)^(1/2))/(a - b*x)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x} + b}}{x\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b-a/x)**(1/2)/x/(-b*x+a)**(1/2), x)`

[Out] `Integral(sqrt(-a/x + b)/(x*sqrt(a - b*x)), x)`

$$3.340 \quad \int \frac{\sqrt{b - \frac{a}{x}}}{x^2 \sqrt{a - bx}} dx$$

Optimal. Leaf size=29

$$-\frac{2\sqrt{b - \frac{a}{x}}}{3x\sqrt{a - bx}}$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {515, 23, 30}

$$-\frac{2\sqrt{b - \frac{a}{x}}}{3x\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_)*((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x}}}{x^2 \sqrt{a - bx}} dx &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{\sqrt{-a+bx}}{x^{5/2} \sqrt{a-bx}} dx}{\sqrt{-a+bx}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x}} \sqrt{x}\right) \int \frac{1}{x^{5/2}} dx}{\sqrt{a - bx}} \\ &= -\frac{2\sqrt{b - \frac{a}{x}}}{3x\sqrt{a - bx}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\sqrt{b - \frac{a}{x}}}{3x\sqrt{a - bx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]

[Out] (-2*Sqrt[b - a/x])/(3*x*Sqrt[a - b*x])

IntegrateAlgebraic [A] time = 4.63, size = 29, normalized size = 1.00

$$\frac{2\left(-\frac{a-bx}{x}\right)^{3/2}}{3(a-bx)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[b - a/x]/(x^2*Sqrt[a - b*x]),x]

[Out] (2*(-((a - b*x)/x))^(3/2))/(3*(a - b*x)^(3/2))

fricas [A] time = 0.72, size = 35, normalized size = 1.21

$$\frac{2\sqrt{-bx+a}\sqrt{\frac{bx-a}{x}}}{3(bx^2-ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="fricas")

[Out] 2/3*sqrt(-b*x + a)*sqrt((b*x - a)/x)/(b*x^2 - a*x)

giac [B] time = 0.51, size = 60, normalized size = 2.07

$$\frac{2\left(\frac{b^5}{((bx-a)b+ab)\sqrt{-(bx-a)b-ab}} - \frac{b^4}{\sqrt{-ab a}}\right)|b|\operatorname{sgn}(x)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="giac")

[Out] 2/3*(b^5/(((b*x - a)*b + a*b)*sqrt(-(b*x - a)*b - a*b)) - b^4/(sqrt(-a*b)*a)))*abs(b)*sgn(x)/b^3

maple [A] time = 0.00, size = 27, normalized size = 0.93

$$-\frac{2\sqrt{-\frac{bx+a}{x}}}{3\sqrt{-bx+a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x)

[Out] -2/3*(-(-b*x+a)/x)^(1/2)/x/(-b*x+a)^(1/2)

maxima [C] time = 0.84, size = 5, normalized size = 0.17

$$\frac{2i}{3x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x)^(1/2)/x^2/(-b*x+a)^(1/2),x, algorithm="maxima")

[Out] $2/3 \int 1/x^{3/2}$

mupad [B] time = 3.10, size = 23, normalized size = 0.79

$$-\frac{2\sqrt{b-\frac{a}{x}}}{3x\sqrt{a-bx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b - a/x)^(1/2)/(x^2*(a - b*x)^(1/2)), x)`

[Out] $-(2*(b - a/x)^{(1/2)})/(3*x*(a - b*x)^{(1/2)})$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x} + b}}{x^2\sqrt{a - bx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b-a/x)**(1/2)/x**2/(-b*x+a)**(1/2), x)`

[Out] `Integral(sqrt(-a/x + b)/(x**2*sqrt(a - b*x)), x)`

$$3.341 \quad \int \frac{\sqrt{b - \frac{a}{x^2}} x^m}{\sqrt{a - bx^2}} dx$$

Optimal. Leaf size=33

$$\frac{x^{m+1} \sqrt{b - \frac{a}{x^2}}}{m\sqrt{a - bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {515, 23, 30}

$$\frac{x^{m+1} \sqrt{b - \frac{a}{x^2}}}{m\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_)*((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}} x^m}{\sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{x^{-1+m} \sqrt{-a+bx^2}}{\sqrt{a-bx^2}} dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int x^{-1+m} dx}{\sqrt{a-bx^2}} \\ &= \frac{\sqrt{b - \frac{a}{x^2}} x^{1+m}}{m\sqrt{a-bx^2}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$\frac{x^{m+1} \sqrt{b - \frac{a}{x^2}}}{m\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x^(1 + m))/(m*Sqrt[a - b*x^2])

IntegrateAlgebraic [F] time = 0.49, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}} x^m}{\sqrt{a - bx^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2], x]

[Out] Defer[IntegrateAlgebraic] [(Sqrt[b - a/x^2]*x^m)/Sqrt[a - b*x^2], x]

fricas [A] time = 0.88, size = 44, normalized size = 1.33

$$\frac{\sqrt{-bx^2 + a} x x^m \sqrt{\frac{bx^2 - a}{x^2}}}{bm x^2 - am}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] -sqrt(-b*x^2 + a)*x*x^m*sqrt((b*x^2 - a)/x^2)/(b*m*x^2 - a*m)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}} x^m}{\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)*x^m/sqrt(-b*x^2 + a), x)

maple [A] time = 0.00, size = 35, normalized size = 1.06

$$\frac{\sqrt{-\frac{bx^2+a}{x^2}} x^{m+1}}{\sqrt{-bx^2 + a} m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x)

[Out] x^(m+1)/m*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)

maxima [C] time = 1.07, size = 8, normalized size = 0.24

$$-\frac{i x^m}{m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^m*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="maxima")

[Out] -I*x^m/m

mupad [B] time = 3.36, size = 29, normalized size = 0.88

$$\frac{x^{m+1} \sqrt{b - \frac{a}{x^2}}}{m \sqrt{a - b x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^m*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)`

[Out] `(x^(m + 1)*(b - a/x^2)^(1/2))/(m*(a - b*x^2)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^m \sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - b x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2), x)`

[Out] `Integral(x**m*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)`

$$3.342 \quad \int \frac{\sqrt{b - \frac{a}{x^2}} x^2}{\sqrt{a - bx^2}} dx$$

Optimal. Leaf size=31

$$\frac{x^3 \sqrt{b - \frac{a}{x^2}}}{2\sqrt{a - bx^2}}$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {515, 23, 30}

$$\frac{x^3 \sqrt{b - \frac{a}{x^2}}}{2\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_)*((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}} x^2}{\sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{x\sqrt{-a+bx^2}}{\sqrt{a-bx^2}} dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int x dx}{\sqrt{a - bx^2}} \\ &= \frac{\sqrt{b - \frac{a}{x^2}} x^3}{2\sqrt{a - bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{x^3 \sqrt{b - \frac{a}{x^2}}}{2\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x^3)/(2*Sqrt[a - b*x^2])

IntegrateAlgebraic [A] time = 0.12, size = 43, normalized size = 1.39

$$\frac{x\sqrt{b - \frac{a}{x^2}}(a - bx^2)^{3/2}}{2b(bx^2 - a)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x*(a - b*x^2)^(3/2))/(2*b*(-a + b*x^2))

fricas [A] time = 0.55, size = 41, normalized size = 1.32

$$-\frac{\sqrt{-bx^2 + a}x^3\sqrt{\frac{bx^2 - a}{x^2}}}{2(bx^2 - a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] -1/2*sqrt(-b*x^2 + a)*x^3*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)

giac [C] time = 0.33, size = 15, normalized size = 0.48

$$-\frac{ibx^2 - ia}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="giac")

[Out] -1/2*(I*b*x^2 - I*a)/b

maple [A] time = 0.00, size = 31, normalized size = 1.00

$$\frac{\sqrt{-\frac{bx^2 + a}{x^2}}x^3}{2\sqrt{-bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x)

[Out] 1/2*x^3*(-(-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)

maxima [C] time = 1.02, size = 5, normalized size = 0.16

$$-\frac{1}{2}ix^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="maxima")

[Out] -1/2*I*x^2

mupad [B] time = 3.14, size = 25, normalized size = 0.81

$$\frac{x^3 \sqrt{b - \frac{a}{x^2}}}{2 \sqrt{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)

[Out] (x^3*(b - a/x^2)^(1/2))/(2*(a - b*x^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2), x)

[Out] Integral(x**2*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)

$$3.343 \quad \int \frac{\sqrt{b - \frac{a}{x^2}} x}{\sqrt{a - bx^2}} dx$$

Optimal. Leaf size=28

$$\frac{x^2 \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {515, 23, 8}

$$\frac{x^2 \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 23

Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] := Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 515

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}} x}{\sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{\sqrt{-a+bx^2}}{\sqrt{a-bx^2}} dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int 1 dx}{\sqrt{a - bx^2}} \\ &= \frac{\sqrt{b - \frac{a}{x^2}} x^2}{\sqrt{a - bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x^2 \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]

[Out] (Sqrt[b - a/x^2]*x^2)/Sqrt[a - b*x^2]

IntegrateAlgebraic [A] time = 0.27, size = 26, normalized size = 0.93

$$-\frac{\sqrt{a - bx^2}}{\sqrt{b - \frac{a}{x^2}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[b - a/x^2]*x)/Sqrt[a - b*x^2],x]

[Out] -(Sqrt[a - b*x^2]/Sqrt[b - a/x^2])

fricas [A] time = 0.66, size = 41, normalized size = 1.46

$$-\frac{\sqrt{-bx^2 + a} x^2 \sqrt{\frac{bx^2 - a}{x^2}}}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-b*x^2 + a)*x^2*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}} x}{\sqrt{-bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)*x/sqrt(-b*x^2 + a), x)

maple [A] time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{bx^2 - a}{x^2}} \sqrt{-bx^2 + a} x^2}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x)

[Out] -((b*x^2-a)/x^2)^(1/2)*x^2/(b*x^2-a)*(-b*x^2+a)^(1/2)

maxima [C] time = 1.12, size = 7, normalized size = 0.25

$$-i \sqrt{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] -I*sqrt(x^2)

mupad [B] time = 3.10, size = 27, normalized size = 0.96

$$\frac{\sqrt{bx^2 - a} \sqrt{x^2}}{\sqrt{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(b - a/x^2)^(1/2))/(a - b*x^2)^(1/2), x)

[Out] ((b*x^2 - a)^(1/2)*(x^2)^(1/2))/(a - b*x^2)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x \sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2), x)

[Out] Integral(x*sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)

$$3.344 \quad \int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx$$

Optimal. Leaf size=28

$$\frac{x \log(x) \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Rubi [A] time = 0.02, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {435, 23, 29}

$$\frac{x \log(x) \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 29

Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]

Rule 435

Int[((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[((a + b*x^n)^p*(d + c*x^n)^q/x^(n*q), x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{\sqrt{-a+bx^2}}{x\sqrt{a-bx^2}} dx}{\sqrt{-a+bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{1}{x} dx}{\sqrt{a - bx^2}} \\ &= \frac{\sqrt{b - \frac{a}{x^2}} x \log(x)}{\sqrt{a - bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{x \log(x) \sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]

[Out] (Sqrt[b - a/x^2]*x*Log[x])/Sqrt[a - b*x^2]

IntegrateAlgebraic [F] time = 0.93, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[b - a/x^2]/Sqrt[a - b*x^2], x]

fricas [B] time = 0.67, size = 51, normalized size = 1.82

$$-\arctan\left(\frac{\sqrt{-bx^2 + a}(x^3 + x)\sqrt{\frac{bx^2 - a}{x^2}}}{bx^4 - (a + b)x^2 + a}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] -arctan(sqrt(-b*x^2 + a)*(x^3 + x)*sqrt((b*x^2 - a)/x^2)/(b*x^4 - (a + b)*x^2 + a))

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x)::OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(x)] Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_nostep)]Sign error %%%{b,2%%}Limit: Max order reached or unable to make series expansion Error: Bad Argument Value

maple [A] time = 0.01, size = 42, normalized size = 1.50

$$-\frac{\sqrt{\frac{bx^2 - a}{x^2}} \sqrt{-bx^2 + a} x \ln(x)}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2), x)

[Out] -((b*x^2-a)/x^2)^(1/2)*x/(b*x^2-a)*(-b*x^2+a)^(1/2)*ln(x)

maxima [C] time = 1.04, size = 4, normalized size = 0.14

$$-i \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/(-b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] -I*log(x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)

[Out] int((b - a/x^2)^(1/2)/(a - b*x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x**2)**(1/2)/(-b*x**2+a)**(1/2), x)

[Out] Integral(sqrt(-a/x**2 + b)/sqrt(a - b*x**2), x)

$$3.345 \quad \int \frac{\sqrt{b - \frac{a}{x^2}}}{x\sqrt{a - bx^2}} dx$$

Optimal. Leaf size=26

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]

[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])

Rule 23

Int[(u_.)*((a_) + (b_.)*(v_))^(m_)*((c_) + (d_.)*(v_))^(n_), x_Symbol] :> Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[(x^(n*FracPart[q])*(c + d*x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}}}{x\sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{\sqrt{-a + bx^2}}{x^2\sqrt{a - bx^2}} dx}{\sqrt{-a + bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{1}{x^2} dx}{\sqrt{a - bx^2}} \\ &= -\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]

[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])

IntegrateAlgebraic [A] time = 0.27, size = 26, normalized size = 1.00

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2]),x]

[Out] -(Sqrt[b - a/x^2]/Sqrt[a - b*x^2])

fricas [A] time = 0.56, size = 41, normalized size = 1.58

$$\frac{\sqrt{-bx^2 + a}(x - 1)\sqrt{\frac{bx^2 - a}{x^2}}}{bx^2 - a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="fricas")

[Out] -sqrt(-b*x^2 + a)*(x - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^2 - a)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{-bx^2 + a}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b - a/x^2)/(sqrt(-b*x^2 + a)*x), x)

maple [A] time = 0.01, size = 28, normalized size = 1.08

$$-\frac{\sqrt{\frac{-bx^2+a}{x^2}}}{\sqrt{-bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x)

[Out] -((-b*x^2+a)/x^2)^(1/2)/(-b*x^2+a)^(1/2)

maxima [C] time = 1.11, size = 7, normalized size = 0.27

$$\frac{i}{\sqrt{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x/(-b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] I/sqrt(x^2)

mupad [B] time = 3.34, size = 22, normalized size = 0.85

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{\sqrt{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b - a/x^2)^(1/2)/(x*(a - b*x^2)^(1/2)), x)`

[Out] `-(b - a/x^2)^(1/2)/(a - b*x^2)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{x\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b-a/x**2)**(1/2)/x/(-b*x**2+a)**(1/2), x)`

[Out] `Integral(sqrt(-a/x**2 + b)/(x*sqrt(a - b*x**2)), x)`

$$3.346 \quad \int \frac{\sqrt{b - \frac{a}{x^2}}}{x^2 \sqrt{a - bx^2}} dx$$

Optimal. Leaf size=31

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{2x\sqrt{a - bx^2}}$$

Rubi [A] time = 0.03, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {515, 23, 30}

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{2x\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]),x]

[Out] -Sqrt[b - a/x^2]/(2*x*Sqrt[a - b*x^2])

Rule 23

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(a + b*v)^m/(c + d*v)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[b*c - a*d, 0] && !(IntegerQ[m] || IntegerQ[n] || GtQ[b/d, 0])

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 515

Int[(x_)^(m_)*((c_) + (d_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b - \frac{a}{x^2}}}{x^2 \sqrt{a - bx^2}} dx &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{\sqrt{-a + bx^2}}{x^3 \sqrt{a - bx^2}} dx}{\sqrt{-a + bx^2}} \\ &= \frac{\left(\sqrt{b - \frac{a}{x^2}} x\right) \int \frac{1}{x^3} dx}{\sqrt{a - bx^2}} \\ &= -\frac{\sqrt{b - \frac{a}{x^2}}}{2x\sqrt{a - bx^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{2x\sqrt{a - bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]), x]

[Out] -1/2*Sqrt[b - a/x^2]/(x*Sqrt[a - b*x^2])

IntegrateAlgebraic [F] time = 0.94, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{b - \frac{a}{x^2}}}{x^2 \sqrt{a - bx^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]), x]

[Out] Defer[IntegrateAlgebraic][Sqrt[b - a/x^2]/(x^2*Sqrt[a - b*x^2]), x]

fricas [A] time = 0.85, size = 44, normalized size = 1.42

$$-\frac{\sqrt{-bx^2 + a}(x^2 - 1)\sqrt{\frac{bx^2 - a}{x^2}}}{2(bx^3 - ax)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] -1/2*sqrt(-b*x^2 + a)*(x^2 - 1)*sqrt((b*x^2 - a)/x^2)/(b*x^3 - a*x)

giac [C] time = 0.37, size = 5, normalized size = 0.16

$$\frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2), x, algorithm="giac")

[Out] 1/2*I/x^2

maple [A] time = 0.00, size = 31, normalized size = 1.00

$$-\frac{\sqrt{\frac{-bx^2+a}{x^2}}}{2\sqrt{-bx^2+a}x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2), x)

[Out] -1/2*(-(-b*x^2+a)/x^2)^(1/2)/x/(-b*x^2+a)^(1/2)

maxima [C] time = 1.06, size = 5, normalized size = 0.16

$$\frac{i}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b-a/x^2)^(1/2)/x^2/(-b*x^2+a)^(1/2), x, algorithm="maxima")

[Out] 1/2*I/x^2

mupad [B] time = 3.46, size = 25, normalized size = 0.81

$$-\frac{\sqrt{b - \frac{a}{x^2}}}{2x\sqrt{a - bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b - a/x^2)^(1/2)/(x^2*(a - b*x^2)^(1/2)), x)`

[Out] `-(b - a/x^2)^(1/2)/(2*x*(a - b*x^2)^(1/2))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{a}{x^2} + b}}{x^2\sqrt{a - bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b-a/x**2)**(1/2)/x**2/(-b*x**2+a)**(1/2), x)`

[Out] `Integral(sqrt(-a/x**2 + b)/(x**2*sqrt(a - b*x**2)), x)`

$$3.347 \quad \int \frac{-1+x^3}{(-4x+x^4)^{2/3}} dx$$

Optimal. Leaf size=15

$$\frac{3}{4} \sqrt[3]{x^4 - 4x}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$\frac{3}{4} \sqrt[3]{x^4 - 4x}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(-4*x + x^4)^(2/3), x]

[Out] (3*(-4*x + x^4)^(1/3))/4

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{-1+x^3}{(-4x+x^4)^{2/3}} dx = \frac{3}{4} \sqrt[3]{-4x+x^4}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4} \sqrt[3]{x(x^3 - 4)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(-4*x + x^4)^(2/3), x]

[Out] (3*(x*(-4 + x^3))^(1/3))/4

IntegrateAlgebraic [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{3}{4} \sqrt[3]{x^4 - 4x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^3)/(-4*x + x^4)^(2/3), x]

[Out] (3*(-4*x + x^4)^(1/3))/4

fricas [A] time = 0.65, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^4 - 4x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="fricas")

[Out] 3/4*(x^4 - 4*x)^(1/3)

giac [A] time = 0.32, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^4 - 4x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="giac")

[Out] 3/4*(x^4 - 4*x)^(1/3)

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{3(x^3 - 4)x}{4(x^4 - 4x)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3-1)/(x^4-4*x)^(2/3),x)

[Out] 3/4*x*(x^3-4)/(x^4-4*x)^(2/3)

maxima [A] time = 0.89, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^4 - 4x)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^4-4*x)^(2/3),x, algorithm="maxima")

[Out] 3/4*(x^4 - 4*x)^(1/3)

mupad [B] time = 3.52, size = 11, normalized size = 0.73

$$\frac{3(x^4 - 4x)^{1/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3 - 1)/(x^4 - 4*x)^(2/3),x)

[Out] (3*(x^4 - 4*x)^(1/3))/4

sympy [A] time = 0.20, size = 12, normalized size = 0.80

$$\frac{3\sqrt[3]{x^4 - 4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3-1)/(x**4-4*x)**(2/3),x)

[Out] 3*(x**4 - 4*x)**(1/3)/4

$$3.348 \quad \int (2 - x^2) \sqrt[4]{6x - x^3} dx$$

Optimal. Leaf size=17

$$\frac{4}{15} (6x - x^3)^{5/4}$$

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\frac{4}{15} (6x - x^3)^{5/4}$$

Antiderivative was successfully verified.

[In] Int[(2 - x^2)*(6*x - x^3)^(1/4), x]

[Out] (4*(6*x - x^3)^(5/4))/15

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2 - x^2) \sqrt[4]{6x - x^3} dx = \frac{4}{15} (6x - x^3)^{5/4}$$

Mathematica [C] time = 0.05, size = 72, normalized size = 4.24

$$\frac{4\sqrt[4]{-x(x^2 - 6)} \left(5x^3 {}_2F_1\left(-\frac{1}{4}, \frac{13}{8}; \frac{21}{8}; \frac{x^2}{6}\right) - 26x {}_2F_1\left(-\frac{1}{4}, \frac{5}{8}; \frac{13}{8}; \frac{x^2}{6}\right) \right)}{65\sqrt[4]{1 - \frac{x^2}{6}}}$$

Antiderivative was successfully verified.

[In] Integrate[(2 - x^2)*(6*x - x^3)^(1/4), x]

[Out] (-4*(-(x*(-6 + x^2)))^(1/4)*(-26*x*Hypergeometric2F1[-1/4, 5/8, 13/8, x^2/6] + 5*x^3*Hypergeometric2F1[-1/4, 13/8, 21/8, x^2/6]))/(65*(1 - x^2/6)^(1/4))

IntegrateAlgebraic [A] time = 0.03, size = 17, normalized size = 1.00

$$\frac{4}{15} (6x - x^3)^{5/4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 - x^2)*(6*x - x^3)^(1/4), x]

[Out] (4*(6*x - x^3)^(5/4))/15

fricas [A] time = 0.62, size = 20, normalized size = 1.18

$$-\frac{4}{15} (x^3 - 6x)(-x^3 + 6x)^{\frac{1}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="fricas")

[Out] -4/15*(x^3 - 6*x)*(-x^3 + 6*x)^(1/4)

giac [A] time = 0.33, size = 13, normalized size = 0.76

$$\frac{4}{15} (-x^3 + 6x)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="giac")

[Out] 4/15*(-x^3 + 6*x)^(5/4)

maple [A] time = 0.00, size = 20, normalized size = 1.18

$$-\frac{4(-x^3 + 6x)^{\frac{1}{4}}(x^2 - 6)x}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+2)*(-x^3+6*x)^(1/4),x)

[Out] -4/15*(-x^3+6*x)^(1/4)*x*(x^2-6)

maxima [A] time = 0.88, size = 13, normalized size = 0.76

$$\frac{4}{15} (-x^3 + 6x)^{\frac{5}{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+2)*(-x^3+6*x)^(1/4),x, algorithm="maxima")

[Out] 4/15*(-x^3 + 6*x)^(5/4)

mupad [B] time = 3.16, size = 19, normalized size = 1.12

$$-\frac{4x(x^2 - 6)(6x - x^3)^{1/4}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 2)*(6*x - x^3)^(1/4),x)

[Out] -(4*x*(x^2 - 6)*(6*x - x^3)^(1/4))/15

sympy [B] time = 0.27, size = 31, normalized size = 1.82

$$-\frac{4x^3\sqrt[4]{-x^3 + 6x}}{15} + \frac{8x\sqrt[4]{-x^3 + 6x}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+2)*(-x**3+6*x)**(1/4),x)

[Out] -4*x**3*(-x**3 + 6*x)**(1/4)/15 + 8*x*(-x**3 + 6*x)**(1/4)/5

$$3.349 \quad \int (1 + x^4) \sqrt{5x + x^5} dx$$

Optimal. Leaf size=15

$$\frac{2}{15} (x^5 + 5x)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1588}

$$\frac{2}{15} (x^5 + 5x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(5*x + x^5)^(3/2))/15

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (1 + x^4) \sqrt{5x + x^5} dx = \frac{2}{15} (5x + x^5)^{3/2}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{15} (x(x^4 + 5))^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(x*(5 + x^4))^(3/2))/15

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{2}{15} (x^5 + 5x)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x^4)*Sqrt[5*x + x^5],x]

[Out] (2*(5*x + x^5)^(3/2))/15

fricas [A] time = 0.46, size = 11, normalized size = 0.73

$$\frac{2}{15} (x^5 + 5x)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="fricas")

[Out] 2/15*(x^5 + 5*x)^(3/2)

giac [A] time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{15} (x^5 + 5x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="giac")

[Out] 2/15*(x^5 + 5*x)^(3/2)

maple [A] time = 0.01, size = 18, normalized size = 1.20

$$\frac{2(x^4 + 5)\sqrt{x^5 + 5x}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4+1)*(x^5+5*x)^(1/2),x)

[Out] 2/15*x*(x^4+5)*(x^5+5*x)^(1/2)

maxima [A] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{15} (x^5 + 5x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4+1)*(x^5+5*x)^(1/2),x, algorithm="maxima")

[Out] 2/15*(x^5 + 5*x)^(3/2)

mupad [B] time = 3.10, size = 11, normalized size = 0.73

$$\frac{2(x^5 + 5x)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x + x^5)^(1/2)*(x^4 + 1),x)

[Out] (2*(5*x + x^5)^(3/2))/15

sympy [B] time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5 + 5x}}{15} + \frac{2x\sqrt{x^5 + 5x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4+1)*(x**5+5*x)**(1/2),x)

[Out] 2*x**5*sqrt(x**5 + 5*x)/15 + 2*x*sqrt(x**5 + 5*x)/3

$$3.350 \quad \int (2 + 5x^4) \sqrt{2x + x^5} dx$$

Optimal. Leaf size=15

$$\frac{2}{3} (x^5 + 2x)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1588}

$$\frac{2}{3} (x^5 + 2x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(2 + 5*x^4)*Sqrt[2*x + x^5],x]

[Out] (2*(2*x + x^5)^(3/2))/3

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int (2 + 5x^4) \sqrt{2x + x^5} dx = \frac{2}{3} (2x + x^5)^{3/2}$$

Mathematica [A] time = 0.03, size = 15, normalized size = 1.00

$$\frac{2}{3} (x(x^4 + 2))^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[(2 + 5*x^4)*Sqrt[2*x + x^5],x]

[Out] (2*(x*(2 + x^4))^(3/2))/3

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$\frac{2}{3} (x^5 + 2x)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + 5*x^4)*Sqrt[2*x + x^5],x]

[Out] (2*(2*x + x^5)^(3/2))/3

fricas [A] time = 0.69, size = 11, normalized size = 0.73

$$\frac{2}{3} (x^5 + 2x)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="fricas")

[Out] 2/3*(x^5 + 2*x)^(3/2)

giac [A] time = 0.40, size = 11, normalized size = 0.73

$$\frac{2}{3} (x^5 + 2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="giac")

[Out] 2/3*(x^5 + 2*x)^(3/2)

maple [A] time = 0.00, size = 18, normalized size = 1.20

$$\frac{2(x^4 + 2)\sqrt{x^5 + 2x}x}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5*x^4+2)*(x^5+2*x)^(1/2),x)

[Out] 2/3*x*(x^4+2)*(x^5+2*x)^(1/2)

maxima [A] time = 0.87, size = 11, normalized size = 0.73

$$\frac{2}{3} (x^5 + 2x)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x^4+2)*(x^5+2*x)^(1/2),x, algorithm="maxima")

[Out] 2/3*(x^5 + 2*x)^(3/2)

mupad [B] time = 3.11, size = 11, normalized size = 0.73

$$\frac{2(x^5 + 2x)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^5)^(1/2)*(5*x^4 + 2),x)

[Out] (2*(2*x + x^5)^(3/2))/3

sympy [B] time = 0.26, size = 31, normalized size = 2.07

$$\frac{2x^5\sqrt{x^5 + 2x}}{3} + \frac{4x\sqrt{x^5 + 2x}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5*x**4+2)*(x**5+2*x)**(1/2),x)

[Out] 2*x**5*sqrt(x**5 + 2*x)/3 + 4*x*sqrt(x**5 + 2*x)/3

$$3.351 \quad \int \frac{x+3x^2}{\sqrt{x^2+2x^3}} dx$$

Optimal. Leaf size=13

$$\sqrt{2x^3 + x^2}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {1588}

$$\sqrt{2x^3 + x^2}$$

Antiderivative was successfully verified.

[In] Int[(x + 3*x^2)/Sqrt[x^2 + 2*x^3], x]

[Out] Sqrt[x^2 + 2*x^3]

Rule 1588

Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q + 1)*Qq^(m + 1))/((p + m*q + 1)*Coeff[Qq, x, q]), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])] /; FreeQ[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{x + 3x^2}{\sqrt{x^2 + 2x^3}} dx = \sqrt{x^2 + 2x^3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\sqrt{x^2(2x + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x + 3*x^2)/Sqrt[x^2 + 2*x^3], x]

[Out] Sqrt[x^2*(1 + 2*x)]

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 1.00

$$\sqrt{2x^3 + x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + 3*x^2)/Sqrt[x^2 + 2*x^3], x]

[Out] Sqrt[x^2 + 2*x^3]

fricas [A] time = 0.69, size = 11, normalized size = 0.85

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="fricas")

[Out] $\sqrt{2x^3 + x^2}$

giac [A] time = 0.36, size = 11, normalized size = 0.85

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="giac")`

[Out] $\sqrt{2x^3 + x^2}$

maple [A] time = 0.00, size = 21, normalized size = 1.62

$$\frac{(2x + 1)x^2}{\sqrt{2x^3 + x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x^2+x)/(2*x^3+x^2)^(1/2),x)`

[Out] $x^2(2x+1)/(2x^3+x^2)^{(1/2)}$

maxima [A] time = 0.88, size = 11, normalized size = 0.85

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x^2+x)/(2*x^3+x^2)^(1/2),x, algorithm="maxima")`

[Out] $\sqrt{2x^3 + x^2}$

mupad [B] time = 3.23, size = 10, normalized size = 0.77

$$|x|\sqrt{2x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 3*x^2)/(x^2 + 2*x^3)^(1/2),x)`

[Out] $\text{abs}(x)(2x + 1)^{(1/2)}$

sympy [A] time = 0.16, size = 10, normalized size = 0.77

$$\sqrt{2x^3 + x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3*x**2+x)/(2*x**3+x**2)**(1/2),x)`

[Out] $\sqrt{2x^3 + x^2}$

$$3.352 \quad \int \frac{2 + \sqrt[3]{1-5x}}{3 + \sqrt[3]{1-5x}} dx$$

Optimal. Leaf size=44

$$x + \frac{3}{10}(1-5x)^{2/3} - \frac{9}{5}\sqrt[3]{1-5x} + \frac{27}{5} \log(\sqrt[3]{1-5x} + 3)$$

Rubi [A] time = 0.02, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {431, 376, 77}

$$x + \frac{3}{10}(1-5x)^{2/3} - \frac{9}{5}\sqrt[3]{1-5x} + \frac{27}{5} \log(\sqrt[3]{1-5x} + 3)$$

Antiderivative was successfully verified.

[In] Int[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)), x]

[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)^(1/3)])/5

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 376

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]

Rule 431

Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{2 + \sqrt[3]{1-5x}}{3 + \sqrt[3]{1-5x}} dx &= -\left(\frac{1}{5} \text{Subst}\left(\int \frac{2 + \sqrt[3]{x}}{3 + \sqrt[3]{x}} dx, x, 1-5x\right)\right) \\ &= -\left(\frac{3}{5} \text{Subst}\left(\int \frac{x^2(2+x)}{3+x} dx, x, \sqrt[3]{1-5x}\right)\right) \\ &= -\left(\frac{3}{5} \text{Subst}\left(\int \left(3-x+x^2-\frac{9}{3+x}\right) dx, x, \sqrt[3]{1-5x}\right)\right) \\ &= -\frac{9}{5}\sqrt[3]{1-5x} + \frac{3}{10}(1-5x)^{2/3} + x + \frac{27}{5} \log(3 + \sqrt[3]{1-5x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$x + \frac{3}{10}(1-5x)^{2/3} - \frac{9}{5}\sqrt[3]{1-5x} + \frac{27}{5}\log\left(\sqrt[3]{1-5x} + 3\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]

[Out] (-9*(1 - 5*x)^(1/3))/5 + (3*(1 - 5*x)^(2/3))/10 + x + (27*Log[3 + (1 - 5*x)^(1/3)])/5

IntegrateAlgebraic [A] time = 0.03, size = 54, normalized size = 1.23

$$\frac{27}{5}\log\left(\sqrt[3]{1-5x} + 3\right) - \frac{1}{10}\left(2(1-5x)^{2/3} - 3\sqrt[3]{1-5x} + 18\right)\sqrt[3]{1-5x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + (1 - 5*x)^(1/3))/(3 + (1 - 5*x)^(1/3)),x]

[Out] -1/10*((18 - 3*(1 - 5*x)^(1/3) + 2*(1 - 5*x)^(2/3))*(1 - 5*x)^(1/3)) + (27*Log[3 + (1 - 5*x)^(1/3)])/5

fricas [A] time = 0.52, size = 32, normalized size = 0.73

$$x + \frac{3}{10}(-5x+1)^{2/3} - \frac{9}{5}(-5x+1)^{1/3} + \frac{27}{5}\log\left((-5x+1)^{1/3} + 3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="fricas")

[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3) + 3)

giac [A] time = 0.48, size = 33, normalized size = 0.75

$$x + \frac{3}{10}(-5x+1)^{2/3} - \frac{9}{5}(-5x+1)^{1/3} + \frac{27}{5}\log\left((-5x+1)^{1/3} + 3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="giac")

[Out] x + 3/10*(-5*x + 1)^(2/3) - 9/5*(-5*x + 1)^(1/3) + 27/5*log((-5*x + 1)^(1/3) + 3) - 1/5

maple [A] time = 0.00, size = 34, normalized size = 0.77

$$x + \frac{27\ln\left(3 + (-5x+1)^{1/3}\right)}{5} - \frac{1}{5} + \frac{3(-5x+1)^{2/3}}{10} - \frac{9(-5x+1)^{1/3}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x)

[Out] -1/5+x+3/10*(1-5*x)^(2/3)-9/5*(1-5*x)^(1/3)+27/5*ln(3+(1-5*x)^(1/3))

maxima [A] time = 0.84, size = 33, normalized size = 0.75

$$x + \frac{3}{10}(-5x+1)^{2/3} - \frac{9}{5}(-5x+1)^{1/3} + \frac{27}{5}\log\left((-5x+1)^{1/3} + 3\right) - \frac{1}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(1-5*x)^(1/3))/(3+(1-5*x)^(1/3)),x, algorithm="maxima")

[Out] $x + \frac{3}{10}(-5x + 1)^{2/3} - \frac{9}{5}(-5x + 1)^{1/3} + \frac{27}{5}\log((-5x + 1)^{1/3} + 3) - \frac{1}{5}$

mupad [B] time = 0.11, size = 32, normalized size = 0.73

$$x + \frac{27 \ln\left((1 - 5x)^{1/3} + 3\right)}{5} - \frac{9(1 - 5x)^{1/3}}{5} + \frac{3(1 - 5x)^{2/3}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1 - 5*x)^(1/3) + 2)/((1 - 5*x)^(1/3) + 3),x)

[Out] $x + \frac{27\log((1 - 5x)^{1/3} + 3)}{5} - \frac{9(1 - 5x)^{1/3}}{5} + \frac{3(1 - 5x)^{2/3}}{10}$

sympy [A] time = 0.20, size = 39, normalized size = 0.89

$$x + \frac{3(1 - 5x)^{2/3}}{10} - \frac{9\sqrt[3]{1 - 5x}}{5} + \frac{27\log\left(\sqrt[3]{1 - 5x} + 3\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(1-5*x)**(1/3))/(3+(1-5*x)**(1/3)),x)

[Out] $x + \frac{3(1 - 5x)^{2/3}}{10} - \frac{9(1 - 5x)^{1/3}}{5} + \frac{27\log((1 - 5x)^{1/3} + 3)}{5}$

$$3.353 \quad \int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx$$

Optimal. Leaf size=21

$$x + 4\sqrt{x} + 4 \log(1 - \sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {376, 77}

$$x + 4\sqrt{x} + 4 \log(1 - \sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x])/(-1 + Sqrt[x]), x]

[Out] 4*Sqrt[x] + x + 4*Log[1 - Sqrt[x]]

Rule 77

Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 376

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^(p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1+\sqrt{x}}{-1+\sqrt{x}} dx &= 2 \text{Subst} \left(\int \frac{x(1+x)}{-1+x} dx, x, \sqrt{x} \right) \\ &= 2 \text{Subst} \left(\int \left(2 + \frac{2}{-1+x} + x \right) dx, x, \sqrt{x} \right) \\ &= 4\sqrt{x} + x + 4 \log(1 - \sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.95

$$x + 4 \left(\sqrt{x} + \log(1 - \sqrt{x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[x])/(-1 + Sqrt[x]), x]

[Out] x + 4*(Sqrt[x] + Log[1 - Sqrt[x]])

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 1.14

$$\sqrt{x} (\sqrt{x} + 4) + 4 \log(\sqrt{x} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[x])/(-1 + Sqrt[x]),x]

[Out] (4 + Sqrt[x])*Sqrt[x] + 4*Log[-1 + Sqrt[x]]

fricas [A] time = 0.58, size = 15, normalized size = 0.71

$$x + 4\sqrt{x} + 4 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="fricas")

[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)

giac [A] time = 0.31, size = 16, normalized size = 0.76

$$x + 4\sqrt{x} + 4 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="giac")

[Out] x + 4*sqrt(x) + 4*log(abs(sqrt(x) - 1))

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$x + 4 \ln(\sqrt{x} - 1) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)+1)/(x^(1/2)-1),x)

[Out] x+4*x^(1/2)+4*ln(x^(1/2)-1)

maxima [A] time = 0.88, size = 15, normalized size = 0.71

$$x + 4\sqrt{x} + 4 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(-1+x^(1/2)),x, algorithm="maxima")

[Out] x + 4*sqrt(x) + 4*log(sqrt(x) - 1)

mupad [B] time = 3.01, size = 15, normalized size = 0.71

$$x + 4 \ln(\sqrt{x} - 1) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2) + 1)/(x^(1/2) - 1),x)

[Out] x + 4*log(x^(1/2) - 1) + 4*x^(1/2)

sympy [A] time = 0.15, size = 17, normalized size = 0.81

$$4\sqrt{x} + x + 4 \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x**(1/2))/(-1+x**(1/2)),x)

[Out] 4*sqrt(x) + x + 4*log(sqrt(x) - 1)

$$3.354 \quad \int \frac{1 - \sqrt{2+3x}}{1 + \sqrt{2+3x}} dx$$

Optimal. Leaf size=33

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2} + 1)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {431, 376, 77}

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2} + 1)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]), x]

[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3

Rule 77

Int[((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))

Rule 376

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]

Rule 431

Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1 - \sqrt{2+3x}}{1 + \sqrt{2+3x}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{1 - \sqrt{x}}{1 + \sqrt{x}} dx, x, 2 + 3x \right) \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{(1-x)x}{1+x} dx, x, \sqrt{2+3x} \right) \\ &= \frac{2}{3} \text{Subst} \left(\int \left(2 - x - \frac{2}{1+x} \right) dx, x, \sqrt{2+3x} \right) \\ &= -x + \frac{4}{3}\sqrt{2+3x} - \frac{4}{3}\log(1 + \sqrt{2+3x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]), x]

[Out] -x + (4*Sqrt[2 + 3*x])/3 - (4*Log[1 + Sqrt[2 + 3*x]])/3

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.24

$$-\frac{1}{3}\sqrt{3x+2}(\sqrt{3x+2}-4) - \frac{4}{3}\log(\sqrt{3x+2}+1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[2 + 3*x])/(1 + Sqrt[2 + 3*x]), x]

[Out] -1/3*(Sqrt[2 + 3*x]*(-4 + Sqrt[2 + 3*x])) - (4*Log[1 + Sqrt[2 + 3*x]])/3

fricas [A] time = 0.70, size = 25, normalized size = 0.76

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2}+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)), x, algorithm="fricas")

[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1)

giac [A] time = 0.33, size = 26, normalized size = 0.79

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2}+1) - \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)), x, algorithm="giac")

[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3

maple [A] time = 0.01, size = 27, normalized size = 0.82

$$-x - \frac{4\ln(1 + \sqrt{3x+2})}{3} + \frac{4\sqrt{3x+2}}{3} - \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-(3*x+2)^(1/2))/(1+(3*x+2)^(1/2)), x)

[Out] 4/3*(3*x+2)^(1/2)-x-2/3-4/3*ln(1+(3*x+2)^(1/2))

maxima [A] time = 0.88, size = 26, normalized size = 0.79

$$-x + \frac{4}{3}\sqrt{3x+2} - \frac{4}{3}\log(\sqrt{3x+2}+1) - \frac{2}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(2+3*x)^(1/2))/(1+(2+3*x)^(1/2)), x, algorithm="maxima")

[Out] -x + 4/3*sqrt(3*x + 2) - 4/3*log(sqrt(3*x + 2) + 1) - 2/3

mupad [B] time = 3.10, size = 25, normalized size = 0.76

$$\frac{4\sqrt{3x+2}}{3} - \frac{4\ln(\sqrt{3x+2}+1)}{3} - x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-((3*x + 2)^(1/2) - 1)/((3*x + 2)^(1/2) + 1),x)`

[Out] $(4*(3*x + 2)^{(1/2)})/3 - (4*\log((3*x + 2)^{(1/2)} + 1))/3 - x$

sympy [A] time = 0.17, size = 27, normalized size = 0.82

$$-x + \frac{4\sqrt{3x+2}}{3} - \frac{4\log(\sqrt{3x+2} + 1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(2+3*x)**(1/2))/(1+(2+3*x)**(1/2)),x)`

[Out] $-x + 4*\text{sqrt}(3*x + 2)/3 - 4*\log(\text{sqrt}(3*x + 2) + 1)/3$

$$3.355 \quad \int \frac{-1 + \sqrt{a+bx}}{1 + \sqrt{a+bx}} dx$$

Optimal. Leaf size=33

$$-\frac{4\sqrt{a+bx}}{b} + \frac{4\log(\sqrt{a+bx} + 1)}{b} + x$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {431, 376, 77}

$$-\frac{4\sqrt{a+bx}}{b} + \frac{4\log(\sqrt{a+bx} + 1)}{b} + x$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]),x]

[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_)^(n_.))*((e_.) + (f_.)*(x_)^(p_.)), x_Symbol]
:> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0] && ILtQ[p, 0])
|| EqQ[p, 1] || (IGtQ[p, 0] && (!IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

Rule 376

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
:> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(a + b*x^(g*n))^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]
/; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && FractionQ[n]
```

Rule 431

```
Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol]
:> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x]
/; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned} \int \frac{-1 + \sqrt{a+bx}}{1 + \sqrt{a+bx}} dx &= \frac{\text{Subst}\left(\int \frac{-1 + \sqrt{x}}{1 + \sqrt{x}} dx, x, a + bx\right)}{b} \\ &= \frac{2 \text{Subst}\left(\int \frac{(-1+x)x}{1+x} dx, x, \sqrt{a+bx}\right)}{b} \\ &= \frac{2 \text{Subst}\left(\int \left(-2 + x + \frac{2}{1+x}\right) dx, x, \sqrt{a+bx}\right)}{b} \\ &= x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log(1 + \sqrt{a+bx})}{b} \end{aligned}$$

Mathematica [A] time = 0.02, size = 33, normalized size = 1.00

$$-\frac{4\sqrt{a+bx}}{b} + \frac{4\log(\sqrt{a+bx}+1)}{b} + x$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]), x]

[Out] x - (4*Sqrt[a + b*x])/b + (4*Log[1 + Sqrt[a + b*x]])/b

IntegrateAlgebraic [A] time = 0.04, size = 44, normalized size = 1.33

$$\frac{\sqrt{a+bx}(\sqrt{a+bx}-4)}{b} + \frac{4\log(b\sqrt{a+bx}+b)}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + Sqrt[a + b*x])/(1 + Sqrt[a + b*x]), x]

[Out] (Sqrt[a + b*x]*(-4 + Sqrt[a + b*x]))/b + (4*Log[b + b*Sqrt[a + b*x]])/b

fricas [A] time = 0.60, size = 29, normalized size = 0.88

$$\frac{bx - 4\sqrt{bx+a} + 4\log(\sqrt{bx+a}+1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)), x, algorithm="fricas")

[Out] (b*x - 4*sqrt(b*x + a) + 4*log(sqrt(b*x + a) + 1))/b

giac [A] time = 0.36, size = 38, normalized size = 1.15

$$\frac{4\log(\sqrt{bx+a}+1)}{b} + \frac{(bx+a)b - 4\sqrt{bx+a}b}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)), x, algorithm="giac")

[Out] 4*log(sqrt(b*x + a) + 1)/b + ((b*x + a)*b - 4*sqrt(b*x + a)*b)/b^2

maple [A] time = 0.00, size = 35, normalized size = 1.06

$$x + \frac{a}{b} + \frac{4\ln(1 + \sqrt{bx+a})}{b} - \frac{4\sqrt{bx+a}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)), x)

[Out] -4*(b*x+a)^(1/2)/b+x+a/b+4*ln(1+(b*x+a)^(1/2))/b

maxima [A] time = 1.05, size = 30, normalized size = 0.91

$$\frac{bx+a-4\sqrt{bx+a}+4\log(\sqrt{bx+a}+1)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+(b*x+a)^(1/2))/(1+(b*x+a)^(1/2)), x, algorithm="maxima")

[Out] $(b*x + a - 4*\sqrt{b*x + a} + 4*\log(\sqrt{b*x + a} + 1))/b$

mupad [B] time = 3.02, size = 29, normalized size = 0.88

$$x + \frac{4 \ln(\sqrt{a+bx} + 1)}{b} - \frac{4\sqrt{a+bx}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a + b*x)^(1/2) - 1)/((a + b*x)^(1/2) + 1), x)`

[Out] $x + (4*\log((a + b*x)^{(1/2)} + 1))/b - (4*(a + b*x)^{(1/2)})/b$

sympy [A] time = 0.44, size = 42, normalized size = 1.27

$$\begin{cases} x - \frac{4\sqrt{a+bx}}{b} + \frac{4\log(\sqrt{a+bx}+1)}{b} & \text{for } b \neq 0 \\ \frac{x(\sqrt{a}-1)}{\sqrt{a}+1} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(b*x+a)**(1/2))/(1+(b*x+a)**(1/2)), x)`

[Out] `Piecewise((x - 4*sqrt(a + b*x)/b + 4*log(sqrt(a + b*x) + 1)/b, Ne(b, 0)), (x*(sqrt(a) - 1)/(sqrt(a) + 1), True))`

$$3.356 \quad \int \frac{a+bnx^{-1+n}}{ax+bx^n} dx$$

Optimal. Leaf size=10

$$\log(ax + bx^n)$$

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.70, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1593, 514, 446, 72}

$$\log(ax^{1-n} + b) + n \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*n*x^(-1 + n))/(a*x + b*x^n), x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 514

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{a + bnx^{-1+n}}{ax + bx^n} dx &= \int \frac{x^{-n}(a + bnx^{-1+n})}{b + ax^{1-n}} dx \\ &= \int \frac{bn + ax^{1-n}}{x(b + ax^{1-n})} dx \\ &= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)} dx, x, x^{1-n}\right)}{1-n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{n}{x} + \frac{a-an}{b+ax}\right) dx, x, x^{1-n}\right)}{1-n} \\ &= n \log(x) + \log(b + ax^{1-n}) \end{aligned}$$

Mathematica [A] time = 0.03, size = 17, normalized size = 1.70

$$\log(ax^{1-n} + b) + n \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*n*x^(-1 + n))/(a*x + b*x^n), x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]

IntegrateAlgebraic [A] time = 0.05, size = 10, normalized size = 1.00

$$\log(ax + bx^n)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*n*x^(-1 + n))/(a*x + b*x^n), x]

[Out] Log[a*x + b*x^n]

fricas [A] time = 0.69, size = 10, normalized size = 1.00

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n), x, algorithm="fricas")

[Out] log(a*x + b*x^n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnx^{n-1} + a}{ax + bx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n), x, algorithm="giac")

[Out] integrate((b*n*x^(n - 1) + a)/(a*x + b*x^n), x)

maple [A] time = 0.02, size = 13, normalized size = 1.30

$$\ln(ax + b e^{n \ln(x)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*n*x^(n-1))/(a*x+b*x^n), x)

[Out] ln(a*x+b*exp(n*ln(x)))

maxima [A] time = 0.87, size = 10, normalized size = 1.00

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(a*x+b*x^n), x, algorithm="maxima")

[Out] log(a*x + b*x^n)

mupad [B] time = 3.27, size = 10, normalized size = 1.00

$$\ln(bx^n + ax)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*n*x^(n - 1))/(b*x^n + a*x), x)`

[Out] `log(b*x^n + a*x)`

sympy [A] time = 6.96, size = 32, normalized size = 3.20

$$\begin{cases} \log\left(x + \frac{bx^n}{a}\right) & \text{for } a \neq 0 \\ n\left(\frac{n^2 \log(x)}{n^2-n} - \frac{n \log(x)}{n^2-n}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*n*x**(-1+n))/(a*x+b*x**n), x)`

[Out] `Piecewise((log(x + b*x**n/a), Ne(a, 0)), (n*(n**2*log(x)/(n**2 - n) - n*log(x)/(n**2 - n)), True))`

$$3.357 \quad \int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx$$

Optimal. Leaf size=17

$$\log(ax^{1-n} + b) + n \log(x)$$

Rubi [A] time = 0.04, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {514, 446, 72}

$$\log(ax^{1-n} + b) + n \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]

[Out] n*Log[x] + Log[b + a*x^(1 - n)]

Rule 72

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 514

Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])

Rubi steps

$$\begin{aligned} \int \frac{x^{-n}(a+bnx^{-1+n})}{b+ax^{1-n}} dx &= \int \frac{bn+ax^{1-n}}{x(b+ax^{1-n})} dx \\ &= \frac{\text{Subst}\left(\int \frac{bn+ax}{x(b+ax)} dx, x, x^{1-n}\right)}{1-n} \\ &= \frac{\text{Subst}\left(\int \left(\frac{n}{x} + \frac{a-an}{b+ax}\right) dx, x, x^{1-n}\right)}{1-n} \\ &= n \log(x) + \log(b+ax^{1-n}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\log(ax^{1-n} + b) + n \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]

[Out] $n \cdot \text{Log}[x] + \text{Log}[b + a \cdot x^{1-n}]$

IntegrateAlgebraic [A] time = 0.08, size = 33, normalized size = 1.94

$$\log(ax^{1-n} + b) - \frac{n \log((n-1)x^{1-n})}{n-1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*n*x^(-1 + n))/(x^n*(b + a*x^(1 - n))),x]

[Out] -((n*Log[(-1 + n)*x^(1 - n)])/(-1 + n)) + Log[b + a*x^(1 - n)]

fricas [A] time = 0.71, size = 10, normalized size = 0.59

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="fricas")

[Out] log(a*x + b*x^n)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bnx^{n-1} + a}{(ax^{-n+1} + b)x^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="giac")

[Out] integrate((b*n*x^(n - 1) + a)/((a*x^(-n + 1) + b)*x^n), x)

maple [A] time = 0.03, size = 13, normalized size = 0.76

$$\ln(ax + b e^{n \ln(x)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*n*x^(n-1))/(x^n)/(b+a*x^(-n+1)),x)

[Out] ln(a*x+b*exp(n*ln(x)))

maxima [B] time = 0.95, size = 86, normalized size = 5.06

$$bn \left(\frac{\log(x)}{b} - \frac{n \log(x)}{b(n-1)} + \frac{\log\left(\frac{ax+bx^n}{b}\right)}{b(n-1)} \right) + a \left(\frac{n \log(x)}{a(n-1)} - \frac{\log\left(\frac{ax+bx^n}{b}\right)}{a(n-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*n*x^(-1+n))/(x^n)/(b+a*x^(1-n)),x, algorithm="maxima")

[Out] b*n*(log(x)/b - n*log(x)/(b*(n - 1)) + log((a*x + b*x^n)/b)/(b*(n - 1))) + a*(n*log(x)/(a*(n - 1)) - log((a*x + b*x^n)/b)/(a*(n - 1)))

mupad [B] time = 3.37, size = 39, normalized size = 2.29

$$\frac{\ln(b + a x^{1-n}) - 2n \operatorname{atanh}\left(\frac{2a x^{1-n}}{b} + 1\right)}{n-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*n*x^(n - 1))/(x^n*(b + a*x^(1 - n))), x)
```

```
[Out] -(log(b + a*x^(1 - n)) - 2*n*atanh((2*a*x^(1 - n))/b + 1))/(n - 1)
```

sympy [A] time = 53.07, size = 8, normalized size = 0.47

$$\log(ax + bx^n)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*n*x**(-1+n))/(x**n)/(b+a*x**(1-n)), x)
```

```
[Out] log(a*x + b*x**n)
```

3.358

$$\int x (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (2ad + (3bd + 3ae + bdm$$

Optimal. Leaf size=37

$$x^2 (a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 176, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$x^2 (a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d + 3*a
*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q =
Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Q
q^(m + 1)*Rr^(n + 1))/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r])
, x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q
]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr
+ (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])]] /; FreeQ[{m, n}, x] && P
olyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int x (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (2ad + (3bd + 3ae + bdm + aen)x + (4cd + 4be + 4af + 2cdm + be$$

Mathematica [A] time = 0.35, size = 34, normalized size = 0.92

$$x^2(a + x(b + cx))^{m+1}(d + x(e + x(f + gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + (3*b*d
+ 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b*e*m + b*
e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b
*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)
*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] x^2*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

IntegrateAlgebraic [F] time = 1.57, size = 0, normalized size = 0.00

$$\int x (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (2ad + (3bd + 3ae + bdm + aen)x + (4cd + 4be + 4af + 2cdm + bdm + be$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d
+ (3*b*d + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*d*m + b
*e*m + b*e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f*m + c*
e*n + 2*b*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*c*f*n +
3*b*g*n)*x^4 + c*g*(7 + 2*m + 3*n)*x^5),x]
```

```
[Out] Defer[IntegrateAlgebraic][x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n
*(2*a*d + (3*b*d + 3*a*e + b*d*m + a*e*n)*x + (4*c*d + 4*b*e + 4*a*f + 2*c*
d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (5*c*e + 5*b*f + 5*a*g + 2*c*e*m + b*f
*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (6*c*f + 6*b*g + 2*c*f*m + b*g*m + 2*
c*f*n + 3*b*g*n)*x^4 + c*g*(7 + 2*m + 3*n)*x^5), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$x^2 (c x^2 + b x + a)^{m+1} (g x^3 + f x^2 + e x + d)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d
)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*m+2*b*
f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b
*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x)
```

[Out] $x^2(c x^2 + b x + a)^{m+1}(g x^3 + f x^2 + e x + d)^{n+1}$

maxima [B] time = 1.76, size = 97, normalized size = 2.62

$$(c g x^7 + (c f + b g) x^6 + (c e + b f + a g) x^5 + (c d + b e + a f) x^4 + a d x^2 + (b d + a e) x^3) e^{(n \log(g x^3 + f x^2 + e x + d) + m \log(c x^2 + b x + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(2*a*d+(a*e*n+b*d*m+3*a*e
+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x^2+(3*a*g*n+b*f*
m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f
*n+6*b*g+6*c*f)*x^4+c*g*(7+2*m+3*n)*x^5),x, algorithm="maxima")
```



```
[Out] (c*g*x^7 + (c*f + b*g)*x^6 + (c*e + b*f + a*g)*x^5 + (c*d + b*e + a*f)*x^4 + a*d*x^2 + (b*d + a*e)*x^3)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*x + a))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(2*a*d + x^4*(6*b*g + 6*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(4*a*f + 4*b*e + 4*c*d + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(3*a*e + 3*b*d + b*d*m + a*e*n) + x^3*(5*a*g + 5*b*f + 5*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 7)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(2*a*d+(a*e*n+b*d*m+3*a*e+3*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+4*a*f+4*b*e+4*c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+5*a*g+5*b*f+5*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+6*b*g+6*c*f)*x**4+c*g*(7+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

3.359

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (ad + (2bd + 2ae + bdm + aen)x + (3cd + 3be + 3af + 2cdm + bem + bcn + 2afn)x^2 + (4ce + 4bf + 4ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (5cf + 5bg + 2cfm + bgn + 2cfm + 3agn)x^4 + cg(6 + 2m + 3n)x^5) dx$$

Optimal. Leaf size=35

$$x(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Rubi [A] time = 0.10, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 174, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$x(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (ad + (2bd + 2ae + bdm + aen)x + (3cd + 3be + 3af + 2cdm + bem + bcn + 2afn)x^2 + (4ce + 4bf + 4ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (5cf + 5bg + 2cfm + bgn + 2cfm + 3agn)x^4 + cg(6 + 2m + 3n)x^5) dx$$

Mathematica [A] time = 0.33, size = 32, normalized size = 0.91

$$x(a + x(b + cx))^{m+1} (d + x(e + x(f + gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5),x]
```

```
[Out] x*(a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

IntegrateAlgebraic [F] time = 1.54, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (ad + (2bd + 2ae + bdm + aen)x + (3cd + 3be + 3af + 2cdm + bem + bcn + 2afn)x^2 + (4ce + 4bf + 4ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (5cf + 5bg + 2cfm + bgn + 2cfm + 3agn)x^4 + cg(6 + 2m + 3n)x^5) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + (
2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m + b*e*m
+ b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m + c*e*n
+ 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b
*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5), x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(
a*d + (2*b*d + 2*a*e + b*d*m + a*e*n)*x + (3*c*d + 3*b*e + 3*a*f + 2*c*d*m
+ b*e*m + b*e*n + 2*a*f*n)*x^2 + (4*c*e + 4*b*f + 4*a*g + 2*c*e*m + b*f*m +
c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (5*c*f + 5*b*g + 2*c*f*m + b*g*m + 2*c*f*
n + 3*b*g*n)*x^4 + c*g*(6 + 2*m + 3*n)*x^5), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5), x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5), x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.03, size = 36, normalized size = 1.03

$$x(c x^2 + b x + a)^{m+1} (g x^3 + f x^2 + e x + d)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b*d)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+
2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5*b*g+5
*c*f)*x^4+c*g*(6+2*m+3*n)*x^5), x)
```

[Out] $x(c x^2 + b x + a)^{m+1} (g x^3 + f x^2 + e x + d)^{n+1}$

maxima [B] time = 1.73, size = 95, normalized size = 2.71

$$(c g x^6 + (c f + b g) x^5 + (c e + b f + a g) x^4 + (c d + b e + a f) x^3 + a d x + (b d + a e) x^2) e^{(n \log(g x^3 + f x^2 + e x + d) + m \log(c x^2 + b x + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(a*d+(a*e*n+b*d*m+2*a*e+2*b
*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x^2+(3*a*g*n+b*f*m+2*
b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+5
*b*g+5*c*f)*x^4+c*g*(6+2*m+3*n)*x^5), x, algorithm="maxima")
```

```
[Out] (c*g*x^6 + (c*f + b*g)*x^5 + (c*e + b*f + a*g)*x^4 + (c*d + b*e + a*f)*x^3
+ a*d*x + (b*d + a*e)*x^2)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2
+ b*x + a))
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.03

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*d + x^4*(5*b*g + 5*c
*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*f + 3*b*e + 3*c*d + b*
e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x*(2*a*e + 2*b*d + b*d*m + a*e*n) + x^3*
(4*a*g + 4*b*f + 4*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g
*x^5*(2*m + 3*n + 6)),x)
```

```
[Out] \text{Hanged}
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(a*d+(a*e*n+b*d*m+2*a*
e+2*b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+3*a*f+3*b*e+3*c*d)*x**2+(3*a*g*n+b*
f*m+2*b*f*n+2*c*e*m+c*e*n+4*a*g+4*b*f+4*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*
c*f*n+5*b*g+5*c*f)*x**4+c*g*(6+2*m+3*n)*x**5),x)
```

```
[Out] Timed out
```

3.360

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (bd + ae + bdm + aen + (2cd + 2be + 2af + 2cdm + bem + ben + 2aef + 2bfg + 2cgm + 2cfn + 3bgn)x^2 + (4cf + 4bg + 2cfm + bgm + 2cfn + 3bgn)x^3 + cg(5 + 2m + 3n)x^4) dx$$

Optimal. Leaf size=34

$$(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Rubi [A] time = 0.12, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 164, $\frac{\text{number of rules}}{\text{integrand size}} = 0.006$, Rules used = {1590}

$$(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)*x^4), x]
```

```
[Out] (a + b*x + c*x^2)^(1 + m)*(d + e*x + f*x^2 + g*x^3)^(1 + n)
```

Rule 1590

```
Int[(Pp_)*(Qq_)^(m_.)*(Rr_)^(n_.), x_Symbol] := With[{p = Expon[Pp, x], q = Expon[Qq, x], r = Expon[Rr, x]}, Simp[(Coeff[Pp, x, p]*x^(p - q - r + 1)*Qq^(m + 1)*Rr^(n + 1)]/((p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]), x] /; NeQ[p + m*q + n*r + 1, 0] && EqQ[(p + m*q + n*r + 1)*Coeff[Qq, x, q]*Coeff[Rr, x, r]*Pp, Coeff[Pp, x, p]*x^(p - q - r)*((p - q - r + 1)*Qq*Rr + (m + 1)*x*Rr*D[Qq, x] + (n + 1)*x*Qq*D[Rr, x])] /; FreeQ[{m, n}, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && PolyQ[Rr, x] && NeQ[m, -1] && NeQ[n, -1]
```

Rubi steps

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (bd + ae + bdm + aen + (2cd + 2be + 2af + 2cdm + bem + ben + 2aef + 2bfg + 2cgm + 2cfn + 3bgn)x^2 + (4cf + 4bg + 2cfm + bgm + 2cfn + 3bgn)x^3 + cg(5 + 2m + 3n)x^4) dx$$

Mathematica [A] time = 0.31, size = 31, normalized size = 0.91

$$(a + x(b + cx))^{m+1} (d + x(e + x(f + gx)))^{n+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*(5 + 2*m + 3*n)*x^4), x]
```

```
[Out] (a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n)
```

IntegrateAlgebraic [F] time = 1.24, size = 0, normalized size = 0.00

$$\int (a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (bd + ae + bdm + aen + (2cd + 2be + 2af + 2cdm + bem + ben + 2aef + 2bfg + 2cgm + 2cfn + 3bgn)x^2 + (4cf + 4bg + 2cfm + bgm + 2cfn + 3bgn)x^3 + cg(5 + 2m + 3n)x^4) dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(b*d + a
*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*n + 2*a
*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*
g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3 + c*g*
(5 + 2*m + 3*n)*x^4),x]
```

```
[Out] Defer[IntegrateAlgebraic] [(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(
b*d + a*e + b*d*m + a*e*n + (2*c*d + 2*b*e + 2*a*f + 2*c*d*m + b*e*m + b*e*
n + 2*a*f*n)*x + (3*c*e + 3*b*f + 3*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n
+ 3*a*g*n)*x^2 + (4*c*f + 4*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^3
+ c*g*(5 + 2*m + 3*n)*x^4), x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.03, size = 35, normalized size = 1.03

$$(cx^2 + bx + a)^{m+1} (gx^3 + fx^2 + ex + d)^{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e
*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+
3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^3+c*g*
(5+2*m+3*n)*x^4),x)
```

[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)

maxima [B] time = 1.70, size = 92, normalized size = 2.71

$$(cgx^5 + (cf + bg)x^4 + (ce + bf + ag)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x)e^{(n \log(gx^3 + fx^2 + ex + d) + m \log(cx^2 + bx + a))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(b*d+a*e+b*d*m+a*e*n+(2*a*f
*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+
c*e*n+3*a*g+3*b*f+3*c*e)*x^2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x^
3+c*g*(5+2*m+3*n)*x^4),x, algorithm="maxima")
```

[Out] $(c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2 + a*d + (b*d + a*e)*x)*e^{(n*\log(g*x^3 + f*x^2 + e*x + d) + m*\log(c*x^2 + b*x + a))}$

mupad [B] time = 9.78, size = 148, normalized size = 4.35

$(g x^3 + f x^2 + e x + d)^n (x^4 (b g + c f) (c x^2 + b x + a)^m + x^2 (c x^2 + b x + a)^m (a f + b e + c d) + x^3 (c x^2 + b x + a)^m (a g + b f + c e) + a d (c x^2 + b x + a)^m + x (a e + b d) (c x^2 + b x + a)^m + c g x^5 (c x^2 + b x + a)^m)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*e + b*d + x^3*(4*b*g + 4*c*f + b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) + x^2*(3*a*g + 3*b*f + 3*c*e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + x*(2*a*f + 2*b*e + 2*c*d + b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + b*d*m + a*e*n + c*g*x^4*(2*m + 3*n + 5)), x)$

[Out] $(d + e*x + f*x^2 + g*x^3)^n*(x^4*(b*g + c*f)*(a + b*x + c*x^2)^m + x^2*(a + b*x + c*x^2)^m*(a*f + b*e + c*d) + x^3*(a + b*x + c*x^2)^m*(a*g + b*f + c*e) + a*d*(a + b*x + c*x^2)^m + x*(a*e + b*d)*(a + b*x + c*x^2)^m + c*g*x^5*(a + b*x + c*x^2)^m)$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(b*d+a*e+b*d*m+a*e*n+(2*a*f*n+b*e*m+b*e*n+2*c*d*m+2*a*f+2*b*e+2*c*d)*x+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+3*a*g+3*b*f+3*c*e)*x**2+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+4*b*g+4*c*f)*x**3+c*g*(5+2*m+3*n)*x**4), x)$

[Out] Timed out

3.361

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2c$$

Optimal. Leaf size=37

$$\frac{(a+bx+cx^2)^{m+1} (d+ex+fx^2+gx^3)^{n+1}}{x}$$

Rubi [F] time = 3.37, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfm+3agn)x^3+(3cf+3bg+2cfm+bgm+2cfm+3agn)x^4+cg(4+2m+3n)x^5)}{x^2} dx$$

Verification is not applicable to the result.

[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x^5))/x^2,x]

[Out] (c*(d + 2*d*m) + b*e*(1 + m + n) + a*f*(1 + 2*n))*Defer[Int] [(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - a*d*Defer[Int] [(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n/x^2, x] + (b*d*m + a*e*n)*Defer[Int] [(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n/x, x] + (c*e*(2 + 2*m + n) + b*f*(2 + m + 2*n) + a*g*(2 + 3*n))*Defer[Int] [x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + (c*f*(3 + 2*m + 2*n) + b*g*(3 + m + 3*n))*Defer[Int] [x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(4 + 2*m + 3*n)*Defer[Int] [x^3*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]

Rubi steps

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2c$$

Mathematica [A] time = 0.81, size = 34, normalized size = 0.92

$$\frac{(a+x(b+cx))^{m+1}(d+x(e+x(f+gx)))^{n+1}}{x}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4 + 2*m + 3*n)*x^5))/x^2,x]

[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x

IntegrateAlgebraic [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-ad+(bdm+aen)x+(cd+be+af+2cdm+bem+ben+2afn)x^2+(2ce+2bf+2ag+2cem+bfm+cen+2bfm+3agn)x^3+(3cf+3bg+2cfm+bgm+2cfm+3agn)x^4+cg(4+2m+3n)x^5)}{x^2} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-(a*d)
+ (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n + 2*a*f*n)
)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*
n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(4
+ 2*m + 3*n)*x^5)/x^2,x]
```

```
[Out] Defer[IntegrateAlgebraic][((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*
(-(a*d) + (b*d*m + a*e*n)*x + (c*d + b*e + a*f + 2*c*d*m + b*e*m + b*e*n +
2*a*f*n)*x^2 + (2*c*e + 2*b*f + 2*a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n +
3*a*g*n)*x^3 + (3*c*f + 3*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 +
c*g*(4 + 2*m + 3*n)*x^5))/x^2, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="fricas")
```

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

sage0x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="giac")
```

[Out] *sage0x*

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{(cx^2 + bx + a)^{m+1} (gx^3 + fx^2 + ex + d)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*f*n+b*
e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+2*a
*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c*g*(4+
2*m+3*n)*x^5)/x^2,x)
```

[Out] $(c*x^2+b*x+a)^{(m+1)}*(g*x^3+f*x^2+e*x+d)^{(n+1)}/x$

maxima [B] time = 1.76, size = 95, normalized size = 2.57

$$\frac{(cgx^5 + (cf + bg)x^4 + (ce + bf + ag)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x)e^{(n \log(gx^3 + fx^2 + ex + d) + m \log(cx^2 + bx + a))}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-a*d+(a*e*n+b*d*m)*x+(2*a*
f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e
```

```
*n+2*a*g+2*b*f+2*c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f)*x^4+c
*g*(4+2*m+3*n)*x^5)/x^2,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x
```

mupad [B] time = 9.68, size = 37, normalized size = 1.00

$$\frac{(cx^2 + bx + a)^{m+1} (gx^3 + fx^2 + ex + d)^{n+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(3*b*g + 3*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - a*d + x^2*(a*f + b*e + c*d + b*e*m +
2*c*d*m + 2*a*f*n + b*e*n) + x*(b*d*m + a*e*n) + x^3*(2*a*g + 2*b*f + 2*c*
e + b*f*m + 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 4))
)/x^2,x)
```

```
[Out] ((a + b*x + c*x^2)^(m + 1)*(d + e*x + f*x^2 + g*x^3)^(n + 1))/x
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-a*d+(a*e*n+b*d*m)*x+
(2*a*f*n+b*e*m+b*e*n+2*c*d*m+a*f+b*e+c*d)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e
*m+c*e*n+2*a*g+2*b*f+2*c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+3*b*g+3*c*f
)*x**4+c*g*(4+2*m+3*n)*x**5)/x**2,x)
```

```
[Out] Timed out
```

3.362

$$\int \frac{(a+bx+cx^2)^m (d+ex+fx^2+gx^3)^n (-2ad+(-bd-ae+bdm+aen)x+(2cdm+bem+ben+2afn)x^2+(ce+bf+ag+2cem+bfm+cen+2bfm+3agn)x^3+(2cf+2bg+2cfm+bgm+2cfm+3bgn)x^4+cg(3+2m+3n)x^5)}{x^3} dx$$

Optimal. Leaf size=37

$$\frac{(a + bx + cx^2)^{m+1} (d + ex + fx^2 + gx^3)^{n+1}}{x^2}$$

Rubi [F] time = 3.00, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (-2ad + (-bd - ae + bdm + aen)x + (2cdm + bem + ben + 2afn)x^2 + (ce + bf + ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (2cf + 2bg + 2cfm + bgm + 2cfm + 3bgn)x^4 + cg(3 + 2m + 3n)x^5)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] Int[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3, x]
```

```
[Out] (c*e*(1 + 2*m + n) + b*f*(1 + m + 2*n) + a*g*(1 + 3*n))*Defer[Int][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] - 2*a*d*Defer[Int][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n/x^3, x] - (b*d*(1 - m) + a*e*(1 - n))*Defer[Int][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n/x^2, x] + (2*c*d*m + 2*a*f*n + b*e*(m + n))*Defer[Int][(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n/x, x] + (2*c*f*(1 + m + n) + b*g*(2 + m + 3*n))*Defer[Int][x*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x] + c*g*(3 + 2*m + 3*n)*Defer[Int][x^2*(a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n, x]
```

Rubi steps

$$\int \frac{(a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (-2ad + (-bd - ae + bdm + aen)x + (2cdm + bem + ben + 2afn)x^2 + (ce + bf + ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (2cf + 2bg + 2cfm + bgm + 2cfm + 3bgn)x^4 + cg(3 + 2m + 3n)x^5)}{x^3} dx$$

Mathematica [A] time = 1.15, size = 34, normalized size = 0.92

$$\frac{(a + x(b + cx))^{m+1} (d + x(e + x(f + gx)))^{n+1}}{x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d + (-b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 + (2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m + 3*n)*x^5))/x^3, x]
```

```
[Out] ((a + x*(b + c*x))^(1 + m)*(d + x*(e + x*(f + g*x)))^(1 + n))/x^2
```

IntegrateAlgebraic [F] time = 2.37, size = 0, normalized size = 0.00

$$\int \frac{(a + bx + cx^2)^m (d + ex + fx^2 + gx^3)^n (-2ad + (-bd - ae + bdm + aen)x + (2cdm + bem + ben + 2afn)x^2 + (ce + bf + ag + 2cem + bfm + cen + 2bfm + 3agn)x^3 + (2cf + 2bg + 2cfm + bgm + 2cfm + 3bgn)x^4 + cg(3 + 2m + 3n)x^5)}{x^3} dx$$

Verification is not applicable to the result.

```
[In] IntegrateAlgebraic[((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(-2*a*d
+ (-(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a*f*n)*x
^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)*x^3 +
(2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 + 2*m +
3*n)*x^5))/x^3,x]
```

```
[Out] Defer[IntegrateAlgebraic](((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*
(-2*a*d + (-(b*d) - a*e + b*d*m + a*e*n)*x + (2*c*d*m + b*e*m + b*e*n + 2*a
*f*n)*x^2 + (c*e + b*f + a*g + 2*c*e*m + b*f*m + c*e*n + 2*b*f*n + 3*a*g*n)
*x^3 + (2*c*f + 2*b*g + 2*c*f*m + b*g*m + 2*c*f*n + 3*b*g*n)*x^4 + c*g*(3 +
2*m + 3*n)*x^5))/x^3, x]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="fricas")
```

[Out] Timed out

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="giac")
```

[Out] Timed out

maple [A] time = 0.03, size = 38, normalized size = 1.03

$$\frac{(cx^2 + bx + a)^{m+1} (gx^3 + fx^2 + ex + d)^{n+1}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*d)*x+(
2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n+a*g+b
*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*m+3*n)
*x^5)/x^3,x)
```

[Out] (c*x^2+b*x+a)^(m+1)*(g*x^3+f*x^2+e*x+d)^(n+1)/x^2

maxima [B] time = 1.81, size = 95, normalized size = 2.57

$$\frac{(cgx^5 + (cf + bg)x^4 + (ce + bf + ag)x^3 + (cd + be + af)x^2 + ad + (bd + ae)x)e^{(n \log(gx^3 + fx^2 + ex + d) + m \log(cx^2 + bx + a))}}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)^m*(g*x^3+f*x^2+e*x+d)^n*(-2*a*d+(a*e*n+b*d*m-a*e-b*
d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x^2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m+c*e*n
```

```
+a*g+b*f+c*e)*x^3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x^4+c*g*(3+2*
m+3*n)*x^5)/x^3,x, algorithm="maxima")
```

```
[Out] (c*g*x^5 + (c*f + b*g)*x^4 + (c*e + b*f + a*g)*x^3 + (c*d + b*e + a*f)*x^2
+ a*d + (b*d + a*e)*x)*e^(n*log(g*x^3 + f*x^2 + e*x + d) + m*log(c*x^2 + b*
x + a))/x^2
```

mupad [B] time = 9.22, size = 146, normalized size = 3.95

$$(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n (af+be+cd+cgx^3+agx+bfxc+ce+bgx^2+cfx^2) + \frac{(ae+bd)(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x} + \frac{ad(cx^2+bx+a)^m (gx^3+fx^2+ex+d)^n}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(x^4*(2*b*g + 2*c*f +
b*g*m + 2*c*f*m + 3*b*g*n + 2*c*f*n) - 2*a*d - x*(a*e + b*d - b*d*m - a*e*n
) + x^2*(b*e*m + 2*c*d*m + 2*a*f*n + b*e*n) + x^3*(a*g + b*f + c*e + b*f*m
+ 2*c*e*m + 3*a*g*n + 2*b*f*n + c*e*n) + c*g*x^5*(2*m + 3*n + 3)))/x^3,x)
```

```
[Out] (a + b*x + c*x^2)^m*(d + e*x + f*x^2 + g*x^3)^n*(a*f + b*e + c*d + c*g*x^3
+ a*g*x + b*f*x + c*e*x + b*g*x^2 + c*f*x^2) + ((a*e + b*d)*(a + b*x + c*x^
2)^m*(d + e*x + f*x^2 + g*x^3)^n)/x + (a*d*(a + b*x + c*x^2)^m*(d + e*x + f
*x^2 + g*x^3)^n)/x^2
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**m*(g*x**3+f*x**2+e*x+d)**n*(-2*a*d+(a*e*n+b*d*m-a
*e-b*d)*x+(2*a*f*n+b*e*m+b*e*n+2*c*d*m)*x**2+(3*a*g*n+b*f*m+2*b*f*n+2*c*e*m
+c*e*n+a*g+b*f+c*e)*x**3+(b*g*m+3*b*g*n+2*c*f*m+2*c*f*n+2*b*g+2*c*f)*x**4+c
*g*(3+2*m+3*n)*x**5)/x**3,x)
```

```
[Out] Timed out
```

$$3.363 \quad \int x^3 \left(a + b\sqrt{c + dx} \right)^2 dx$$

Optimal. Leaf size=185

$$\frac{c^2(3a^2 - b^2c)(c + dx)^2}{2d^4} + \frac{(a^2 - 3b^2c)(c + dx)^4}{4d^4} - \frac{c(a^2 - b^2c)(c + dx)^3}{d^4} - \frac{a^2c^3x}{d^3} - \frac{4abc^3(c + dx)^{3/2}}{3d^4} + \frac{12abc^2(c + dx)^{5/2}}{5d^4}$$

Rubi [A] time = 0.25, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{c^2(3a^2 - b^2c)(c + dx)^2}{2d^4} + \frac{(a^2 - 3b^2c)(c + dx)^4}{4d^4} - \frac{c(a^2 - b^2c)(c + dx)^3}{d^4} - \frac{a^2c^3x}{d^3} + \frac{12abc^2(c + dx)^{5/2}}{5d^4} - \frac{4abc^3(c + dx)^{3/2}}{3d^4} + \frac{4ab(c + dx)^{9/2}}{9d^4} - \frac{12abc(c + dx)^{7/2}}{7d^4} + \frac{b^2(c + dx)^5}{5d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*sqrt[c + d*x])^2,x]

[Out] -((a^2*c^3*x)/d^3) - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (c^2*(3*a^2 - b^2*c)*(c + d*x)^2)/(2*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (c*(a^2 - b^2*c)*(c + d*x)^3)/d^4 - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4) + ((a^2 - 3*b^2*c)*(c + d*x)^4)/(4*d^4) + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (b^2*(c + d*x)^5)/(5*d^4)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x^3 \left(a + b\sqrt{c + dx} \right)^2 dx &= \frac{\text{Subst} \left(\int \left(a + b\sqrt{x} \right)^2 (-c + x)^3 dx, x, c + dx \right)}{d^4} \\ &= \frac{2 \text{Subst} \left(\int x(a + bx)^2 (-c + x^2)^3 dx, x, \sqrt{c + dx} \right)}{d^4} \\ &= \frac{2 \text{Subst} \left(\int \left(-a^2c^3x - 2abc^3x^2 - c^2(-3a^2 + b^2c)x^3 + 6abc^2x^4 + 3c(-a^2 + b^2c)x^5 - \dots \right) dx, x, \sqrt{c + dx} \right)}{d^4} \\ &= -\frac{a^2c^3x}{d^3} - \frac{4abc^3(c + dx)^{3/2}}{3d^4} + \frac{c^2(3a^2 - b^2c)(c + dx)^2}{2d^4} + \frac{12abc^2(c + dx)^{5/2}}{5d^4} - \frac{c(a^2 - b^2c)(c + dx)^3}{d^4} \end{aligned}$$

Mathematica [A] time = 0.31, size = 88, normalized size = 0.48

$$\frac{a^2x^4}{4} + \frac{4ab\sqrt{c+dx}(-16c^4 + 8c^3dx - 6c^2d^2x^2 + 5cd^3x^3 + 35d^4x^4)}{315d^4} + \frac{1}{20}b^2x^4(5c + 4dx)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^2,x]

[Out] (a^2*x^4)/4 + (b^2*x^4*(5*c + 4*d*x))/20 + (4*a*b*Sqrt[c + d*x]*(-16*c^4 + 8*c^3*d*x - 6*c^2*d^2*x^2 + 5*c*d^3*x^3 + 35*d^4*x^4))/(315*d^4)

IntegrateAlgebraic [A] time = 0.07, size = 171, normalized size = 0.92

$$\frac{(c+dx)(-1260a^2c^3+1890a^2c^2(c+dx)+315a^2(c+dx)^3-1260a^2c(c+dx)^2-1680abc^3\sqrt{c+dx}+3024abc^2(c+dx)^{3/2}+560ab(c+dx)^{7/2}-2160abc(c+dx)^{5/2}-630b^2c^3(c+dx)+1260b^2c^2(c+dx)^2+252b^2(c+dx)^4-945b^2c(c+dx)^3)}{1260d^4}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(a + b*Sqrt[c + d*x])^2,x]

[Out] ((c + d*x)*(-1260*a^2*c^3 - 1680*a*b*c^3*Sqrt[c + d*x] + 1890*a^2*c^2*(c + d*x) - 630*b^2*c^3*(c + d*x) + 3024*a*b*c^2*(c + d*x)^(3/2) - 1260*a^2*c*(c + d*x)^2 + 1260*b^2*c^2*(c + d*x)^2 - 2160*a*b*c*(c + d*x)^(5/2) + 315*a^2*(c + d*x)^3 - 945*b^2*c*(c + d*x)^3 + 560*a*b*(c + d*x)^(7/2) + 252*b^2*(c + d*x)^4)/(1260*d^4)

fricas [A] time = 1.14, size = 94, normalized size = 0.51

$$\frac{252b^2d^5x^5 + 315(b^2c + a^2)d^4x^4 + 16(35abd^4x^4 + 5abcd^3x^3 - 6abc^2d^2x^2 + 8abc^3dx - 16abc^4)\sqrt{dx+c}}{1260d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/1260*(252*b^2*d^5*x^5 + 315*(b^2*c + a^2)*d^4*x^4 + 16*(35*a*b*d^4*x^4 + 5*a*b*c*d^3*x^3 - 6*a*b*c^2*d^2*x^2 + 8*a*b*c^3*d*x - 16*a*b*c^4)*sqrt(d*x + c))/d^4

giac [A] time = 0.41, size = 151, normalized size = 0.82

$$\frac{252b^2d^5x^5 + 315b^2cdx^4 + 315a^2dx^4 + \frac{144(5(dx+c)^7-21(dx+c)^5c+35(dx+c)^3c^2-35\sqrt{dx+c}c^3)abc}{d^3} + \frac{16(35(dx+c)^9-180(dx+c)^7c+378(dx+c)^5c^2-420(dx+c)^3c^3+315\sqrt{dx+c}c^4)ab}{d^3}}{1260d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 1/1260*(252*b^2*d^2*x^5 + 315*b^2*c*d*x^4 + 315*a^2*d*x^4 + 144*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b*c/d^3 + 16*(35*(d*x + c)^(9/2) - 180*(d*x + c)^(7/2)*c + 378*(d*x + c)^(5/2)*c^2 - 420*(d*x + c)^(3/2)*c^3 + 315*sqrt(d*x + c)*c^4)*a*b/d^3)/d

maple [A] time = 0.00, size = 78, normalized size = 0.42

$$\frac{a^2x^4}{4} + \left(\frac{1}{5}dx^5 + \frac{1}{4}cx^4\right)b^2 + \frac{4\left(-\frac{(dx+c)^3c^3}{3} + \frac{3(dx+c)^5c^2}{5} - \frac{3(dx+c)^7c}{7} + \frac{(dx+c)^9}{9}\right)ab}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*(d*x+c)^(1/2))^2,x)

[Out] b^2*(1/5*d*x^5+1/4*c*x^4)+4*a*b/d^4*(1/9*(d*x+c)^(9/2)-3/7*c*(d*x+c)^(7/2)+3/5*c^2*(d*x+c)^(5/2)-1/3*c^3*(d*x+c)^(3/2))+1/4*a^2*x^4

maxima [A] time = 0.88, size = 151, normalized size = 0.82

$$\frac{252(dx+c)^5b^2+560(dx+c)^4ab-2160(dx+c)^3abc+3024(dx+c)^2abc^2-1680(dx+c)abc^3-1260(dx+c)a^2c^3-315(3b^2c-a^2)(dx+c)^4+1260(b^2c^2-a^2c)(dx+c)^3-630(b^2c^3-3a^2c^2)(dx+c)^2}{1260d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/1260*(252*(d*x + c)^5*b^2 + 560*(d*x + c)^(9/2)*a*b - 2160*(d*x + c)^(7/2)*a*b*c + 3024*(d*x + c)^(5/2)*a*b*c^2 - 1680*(d*x + c)^(3/2)*a*b*c^3 - 1260*(d*x + c)*a^2*c^3 - 315*(3*b^2*c - a^2)*(d*x + c)^4 + 1260*(b^2*c^2 - a^2*c)*(d*x + c)^3 - 630*(b^2*c^3 - 3*a^2*c^2)*(d*x + c)^2)/d^4

mupad [B] time = 3.37, size = 167, normalized size = 0.90

$$\frac{b^2(c+dx)^5}{5d^4} - \frac{(6b^2c-2a^2)(c+dx)^4}{8d^4} + \frac{(6a^2c^2-2b^2c^3)(c+dx)^3}{4d^4} - \frac{a^2c^3x}{d^3} + \frac{4ab(c+dx)^{9/2}}{9d^4} + \frac{c(b^2c-a^2)(c+dx)^3}{d^4} - \frac{4abc^3(c+dx)^{3/2}}{3d^4} + \frac{12abc^2(c+dx)^{5/2}}{5d^4} - \frac{12abc(c+dx)^{7/2}}{7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b*(c + d*x)^(1/2))^2,x)

[Out] (b^2*(c + d*x)^5)/(5*d^4) - ((6*b^2*c - 2*a^2)*(c + d*x)^4)/(8*d^4) + ((6*a^2*c^2 - 2*b^2*c^3)*(c + d*x)^2)/(4*d^4) - (a^2*c^3*x)/d^3 + (4*a*b*(c + d*x)^(9/2))/(9*d^4) + (c*(b^2*c - a^2)*(c + d*x)^3)/d^4 - (4*a*b*c^3*(c + d*x)^(3/2))/(3*d^4) + (12*a*b*c^2*(c + d*x)^(5/2))/(5*d^4) - (12*a*b*c*(c + d*x)^(7/2))/(7*d^4)

sympy [A] time = 7.02, size = 139, normalized size = 0.75

$$\left\{ \begin{array}{l} \frac{\frac{a^2dx^4}{4} + \frac{4ab\left(-\frac{c^3(c+dx)^3}{3} + \frac{3c^2(c+dx)^5}{5} - \frac{3c(c+dx)^7}{7} + \frac{(c+dx)^9}{9}\right)}{d^3} + \frac{2b^2\left(-\frac{c^3(c+dx)^2}{4} + \frac{c^2(c+dx)^3}{2} - \frac{3c(c+dx)^4}{8} + \frac{(c+dx)^5}{10}\right)}{d^3}}{d} \quad \text{for } d \neq 0 \\ \frac{x^4(a+b\sqrt{c})^2}{4} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise(((a**2*d*x**4/4 + 4*a*b*(-c**3*(c + d*x)**(3/2)/3 + 3*c**2*(c + d*x)**(5/2)/5 - 3*c*(c + d*x)**(7/2)/7 + (c + d*x)**(9/2)/9)/d**3 + 2*b**2*(-c**3*(c + d*x)**2/4 + c**2*(c + d*x)**3/2 - 3*c*(c + d*x)**4/8 + (c + d*x)**5/10)/d**3)/d, Ne(d, 0)), (x**4*(a + b*sqrt(c))**2/4, True))

$$3.364 \quad \int x^2 \left(a + b\sqrt{c + dx} \right)^2 dx$$

Optimal. Leaf size=138

$$\frac{(a^2 - 2b^2c)(c + dx)^3}{3d^3} - \frac{c(2a^2 - b^2c)(c + dx)^2}{2d^3} + \frac{a^2c^2x}{d^2} + \frac{4abc^2(c + dx)^{3/2}}{3d^3} + \frac{4ab(c + dx)^{7/2}}{7d^3} - \frac{8abc(c + dx)^{5/2}}{5d^3} + \frac{b^2(c + dx)^4}{4d^3}$$

Rubi [A] time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{(a^2 - 2b^2c)(c + dx)^3}{3d^3} - \frac{c(2a^2 - b^2c)(c + dx)^2}{2d^3} + \frac{a^2c^2x}{d^2} + \frac{4abc^2(c + dx)^{3/2}}{3d^3} + \frac{4ab(c + dx)^{7/2}}{7d^3} - \frac{8abc(c + dx)^{5/2}}{5d^3} + \frac{b^2(c + dx)^4}{4d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*Sqrt[c + d*x])^2,x]

[Out] (a^2*c^2*x)/d^2 + (4*a*b*c^2*(c + d*x)^(3/2))/(3*d^3) - (c*(2*a^2 - b^2*c)*(c + d*x)^2)/(2*d^3) - (8*a*b*c*(c + d*x)^(5/2))/(5*d^3) + ((a^2 - 2*b^2*c)*(c + d*x)^3)/(3*d^3) + (4*a*b*(c + d*x)^(7/2))/(7*d^3) + (b^2*(c + d*x)^4)/(4*d^3)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x^2 \left(a + b\sqrt{c + dx} \right)^2 dx &= \frac{\text{Subst} \left(\int (a + b\sqrt{x})^2 (-c + x)^2 dx, x, c + dx \right)}{d^3} \\ &= \frac{2 \text{Subst} \left(\int x(a + bx)^2 (-c + x^2)^2 dx, x, \sqrt{c + dx} \right)}{d^3} \\ &= \frac{2 \text{Subst} \left(\int (a^2c^2x + 2abc^2x^2 + c(-2a^2 + b^2c)x^3 - 4abcx^4 + (a^2 - 2b^2c)x^5 + 2ab^2c^2x^6) dx, x, \sqrt{c + dx} \right)}{d^3} \\ &= \frac{a^2c^2x}{d^2} + \frac{4abc^2(c + dx)^{3/2}}{3d^3} - \frac{c(2a^2 - b^2c)(c + dx)^2}{2d^3} - \frac{8abc(c + dx)^{5/2}}{5d^3} + \frac{(a^2 - 2b^2c)(c + dx)^4}{4d^3} \end{aligned}$$

Mathematica [A] time = 0.19, size = 77, normalized size = 0.56

$$\frac{a^2x^3}{3} + \frac{4ab\sqrt{c+dx}(8c^3 - 4c^2dx + 3cd^2x^2 + 15d^3x^3)}{105d^3} + \frac{1}{12}b^2x^3(4c + 3dx)$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^2,x]

[Out] (a^2*x^3)/3 + (b^2*x^3*(4*c + 3*d*x))/12 + (4*a*b*Sqrt[c + d*x]*(8*c^3 - 4*c^2*d*x + 3*c*d^2*x^2 + 15*d^3*x^3))/(105*d^3)

IntegrateAlgebraic [A] time = 0.06, size = 125, normalized size = 0.91

$$\frac{(c+dx)(420a^2c^2+140a^2(c+dx)^2-420a^2c(c+dx)+560abc^2\sqrt{c+dx}+240ab(c+dx)^{5/2}-672abc(c+dx)^{3/2}+210b^2c^2(c+dx)+105b^2(c+dx)^3-280b^2c(c+dx)^2)}{420d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*(a + b*Sqrt[c + d*x])^2,x]

[Out] ((c + d*x)*(420*a^2*c^2 + 560*a*b*c^2*Sqrt[c + d*x] - 420*a^2*c*(c + d*x) + 210*b^2*c^2*(c + d*x) - 672*a*b*c*(c + d*x)^(3/2) + 140*a^2*(c + d*x)^2 - 280*b^2*c*(c + d*x)^2 + 240*a*b*(c + d*x)^(5/2) + 105*b^2*(c + d*x)^3)/(420*d^3)

fricas [A] time = 1.21, size = 81, normalized size = 0.59

$$\frac{105b^2d^4x^4 + 140(b^2c + a^2)d^3x^3 + 16(15abd^3x^3 + 3abcd^2x^2 - 4abc^2dx + 8abc^3)\sqrt{dx+c}}{420d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/420*(105*b^2*d^4*x^4 + 140*(b^2*c + a^2)*d^3*x^3 + 16*(15*a*b*d^3*x^3 + 3*a*b*c*d^2*x^2 - 4*a*b*c^2*d*x + 8*a*b*c^3)*sqrt(d*x + c))/d^3

giac [A] time = 0.43, size = 127, normalized size = 0.92

$$\frac{105b^2d^2x^4 + 140b^2cdx^3 + 140a^2dx^3 + \frac{112(3(dx+c)^{\frac{5}{2}}-10(dx+c)^{\frac{3}{2}}c+15\sqrt{dx+c}c^2)abc}{d^2} + \frac{48(5(dx+c)^{\frac{7}{2}}-21(dx+c)^{\frac{5}{2}}c+35(dx+c)^{\frac{3}{2}}c^2-35\sqrt{dx+c}c^3)ab}{d^2}}{420d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 1/420*(105*b^2*d^2*x^4 + 140*b^2*c*d*x^3 + 140*a^2*d*x^3 + 112*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b*c/d^2 + 48*(5*(d*x + c)^(7/2) - 21*(d*x + c)^(5/2)*c + 35*(d*x + c)^(3/2)*c^2 - 35*sqrt(d*x + c)*c^3)*a*b/d^2)/d

maple [A] time = 0.00, size = 66, normalized size = 0.48

$$\frac{a^2x^3}{3} + \left(\frac{1}{4}dx^4 + \frac{1}{3}cx^3\right)b^2 + \frac{4\left(\frac{(dx+c)^{\frac{3}{2}}c^2}{3} - \frac{2(dx+c)^{\frac{5}{2}}c}{5} + \frac{(dx+c)^{\frac{7}{2}}}{7}\right)ab}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+b*(d*x+c)^(1/2))^2,x)

[Out] $b^2*(1/4*x^4*d+1/3*c*x^3)+4*a*b/d^3*(1/7*(d*x+c)^{(7/2)}-2/5*(d*x+c)^{(5/2)}*c+1/3*c^2*(d*x+c)^{(3/2)})+1/3*a^2*x^3$

maxima [A] time = 0.88, size = 112, normalized size = 0.81

$$\frac{105(dx+c)^4b^2 + 240(dx+c)^{\frac{7}{2}}ab - 672(dx+c)^{\frac{5}{2}}abc + 560(dx+c)^{\frac{3}{2}}abc^2 + 420(dx+c)a^2c^2 - 140(2b^2c - a^2)(dx+c)^3 + 210(b^2c^2 - 2a^2c)(dx+c)^2}{420d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] $1/420*(105*(d*x + c)^4*b^2 + 240*(d*x + c)^{(7/2)}*a*b - 672*(d*x + c)^{(5/2)}*a*b*c + 560*(d*x + c)^{(3/2)}*a*b*c^2 + 420*(d*x + c)*a^2*c^2 - 140*(2*b^2*c - a^2)*(d*x + c)^3 + 210*(b^2*c^2 - 2*a^2*c)*(d*x + c)^2)/d^3$

mupad [B] time = 0.06, size = 124, normalized size = 0.90

$$\frac{b^2(c+dx)^4}{4d^3} - \frac{(4a^2c - 2b^2c^2)(c+dx)^2}{4d^3} - \frac{(4b^2c - 2a^2)(c+dx)^3}{6d^3} + \frac{a^2c^2x}{d^2} + \frac{4ab(c+dx)^{7/2}}{7d^3} + \frac{4abc^2(c+dx)^{3/2}}{3d^3} - \frac{8abc(c+dx)^{5/2}}{5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*(c + d*x)^(1/2))^2,x)

[Out] $(b^2*(c + d*x)^4)/(4*d^3) - ((4*a^2*c - 2*b^2*c^2)*(c + d*x)^2)/(4*d^3) - ((4*b^2*c - 2*a^2)*(c + d*x)^3)/(6*d^3) + (a^2*c^2*x)/d^2 + (4*a*b*(c + d*x)^{(7/2)})/(7*d^3) + (4*a*b*c^2*(c + d*x)^{(3/2)})/(3*d^3) - (8*a*b*c*(c + d*x)^{(5/2)})/(5*d^3)$

sympy [A] time = 5.97, size = 110, normalized size = 0.80

$$\left\{ \begin{array}{ll} \frac{\frac{a^2dx^3}{3} + \frac{4ab\left(\frac{c^2(c+dx)^{\frac{3}{2}}}{3} - \frac{2c(c+dx)^{\frac{5}{2}}}{5} + \frac{(c+dx)^{\frac{7}{2}}}{7}\right)}{d^2} + \frac{2b^2\left(\frac{c^2(c+dx)^2}{4} - \frac{c(c+dx)^3}{3} + \frac{(c+dx)^4}{8}\right)}{d^2}}{d} & \text{for } d \neq 0 \\ \frac{x^3(a+b\sqrt{c})^2}{3} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise(((a**2*d*x**3/3 + 4*a*b*(c**2*(c + d*x)**(3/2)/3 - 2*c*(c + d*x)**(5/2)/5 + (c + d*x)**(7/2)/7)/d**2 + 2*b**2*(c**2*(c + d*x)**2/4 - c*(c + d*x)**3/3 + (c + d*x)**4/8)/d**2)/d, Ne(d, 0)), (x**3*(a + b*sqrt(c))**2/3, True))

$$3.365 \quad \int x \left(a + b\sqrt{c + dx} \right)^2 dx$$

Optimal. Leaf size=89

$$\frac{(a^2 - b^2c)(c + dx)^2}{2d^2} - \frac{a^2cx}{d} + \frac{4ab(c + dx)^{5/2}}{5d^2} - \frac{4abc(c + dx)^{3/2}}{3d^2} + \frac{b^2(c + dx)^3}{3d^2}$$

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{(a^2 - b^2c)(c + dx)^2}{2d^2} - \frac{a^2cx}{d} + \frac{4ab(c + dx)^{5/2}}{5d^2} - \frac{4abc(c + dx)^{3/2}}{3d^2} + \frac{b^2(c + dx)^3}{3d^2}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*Sqrt[c + d*x])^2,x]

[Out] -((a^2*c*x)/d) - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2) + ((a^2 - b^2*c)*(c + d*x)^2)/(2*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) + (b^2*(c + d*x)^3)/(3*d^2)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x \left(a + b\sqrt{c + dx} \right)^2 dx &= \frac{\text{Subst} \left(\int \left(a + b\sqrt{x} \right)^2 (-c + x) dx, x, c + dx \right)}{d^2} \\ &= \frac{2 \text{Subst} \left(\int x(a + bx)^2 (-c + x^2) dx, x, \sqrt{c + dx} \right)}{d^2} \\ &= \frac{2 \text{Subst} \left(\int \left(-a^2cx - 2abcx^2 + (a^2 - b^2c)x^3 + 2abx^4 + b^2x^5 \right) dx, x, \sqrt{c + dx} \right)}{d^2} \\ &= -\frac{a^2cx}{d} - \frac{4abc(c + dx)^{3/2}}{3d^2} + \frac{(a^2 - b^2c)(c + dx)^2}{2d^2} + \frac{4ab(c + dx)^{5/2}}{5d^2} + \frac{b^2(c + dx)^3}{3d^2} \end{aligned}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.71

$$\frac{1}{30} \left(15a^2x^2 + \frac{8ab\sqrt{c + dx} (-2c^2 + cdx + 3d^2x^2)}{d^2} + 5b^2x^2(3c + 2dx) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*Sqrt[c + d*x])^2,x]

[Out] (15*a^2*x^2 + 5*b^2*x^2*(3*c + 2*d*x) + (8*a*b*Sqrt[c + d*x]*(-2*c^2 + c*d*x + 3*d^2*x^2)))/d^2)/30

IntegrateAlgebraic [A] time = 0.05, size = 79, normalized size = 0.89

$$\frac{(c + dx) \left(15a^2(c + dx) - 30a^2c + 24ab(c + dx)^{3/2} - 40abc\sqrt{c + dx} + 10b^2(c + dx)^2 - 15b^2c(c + dx) \right)}{30d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(a + b*Sqrt[c + d*x])^2,x]

[Out] ((c + d*x)*(-30*a^2*c - 40*a*b*c*Sqrt[c + d*x] + 15*a^2*(c + d*x) - 15*b^2*c*(c + d*x) + 24*a*b*(c + d*x)^(3/2) + 10*b^2*(c + d*x)^2))/(30*d^2)

fricas [A] time = 1.26, size = 67, normalized size = 0.75

$$\frac{10 b^2 d^3 x^3 + 15 (b^2 c + a^2) d^2 x^2 + 8 (3 a b d^2 x^2 + a b c d x - 2 a b c^2) \sqrt{d x + c}}{30 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/30*(10*b^2*d^3*x^3 + 15*(b^2*c + a^2)*d^2*x^2 + 8*(3*a*b*d^2*x^2 + a*b*c*d*x - 2*a*b*c^2)*sqrt(d*x + c))/d^2

giac [A] time = 0.51, size = 131, normalized size = 1.47

$$\frac{10 b^2 d^2 x^3 + \frac{40 \left((d x + c)^{\frac{3}{2}} - 3 \sqrt{d x + c} \right) a b c}{d} + \frac{15 \left((d x + c)^2 - 2 (d x + c) c \right) b^2 c}{d} + \frac{15 \left((d x + c)^2 - 2 (d x + c) c \right) a^2}{d} + \frac{8 \left(3 (d x + c)^{\frac{5}{2}} - 10 (d x + c)^{\frac{3}{2}} c + 15 \sqrt{d x + c} c^2 \right) a b}{d}}{30 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 1/30*(10*b^2*d^2*x^3 + 40*((d*x + c)^(3/2) - 3*sqrt(d*x + c))*a*b*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*b^2*c/d + 15*((d*x + c)^2 - 2*(d*x + c)*c)*a^2/d + 8*(3*(d*x + c)^(5/2) - 10*(d*x + c)^(3/2)*c + 15*sqrt(d*x + c)*c^2)*a*b/d)/d

maple [A] time = 0.00, size = 54, normalized size = 0.61

$$\frac{a^2 x^2}{2} + \left(\frac{1}{3} d x^3 + \frac{1}{2} c x^2 \right) b^2 + \frac{4 \left(-\frac{(d x + c)^{\frac{3}{2}} c}{3} + \frac{(d x + c)^{\frac{5}{2}}}{5} \right) a b}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a+b*(d*x+c)^(1/2))^2,x)

[Out] b^2*(1/3*d*x^3+1/2*c*x^2)+4*a*b/d^2*(1/5*(d*x+c)^(5/2)-1/3*(d*x+c)^(3/2)*c)+1/2*a^2*x^2

maxima [A] time = 0.88, size = 72, normalized size = 0.81

$$\frac{10 (d x + c)^3 b^2 + 24 (d x + c)^{\frac{5}{2}} a b - 40 (d x + c)^{\frac{3}{2}} a b c - 30 (d x + c) a^2 c - 15 (b^2 c - a^2) (d x + c)^2}{30 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/30*(10*(d*x + c)^3*b^2 + 24*(d*x + c)^(5/2)*a*b - 40*(d*x + c)^(3/2)*a*b*c - 30*(d*x + c)*a^2*c - 15*(b^2*c - a^2)*(d*x + c)^2)/d^2

mupad [B] time = 0.03, size = 79, normalized size = 0.89

$$\frac{b^2(c+dx)^3}{3d^2} - \frac{(2b^2c - 2a^2)(c+dx)^2}{4d^2} + \frac{4ab(c+dx)^{5/2}}{5d^2} - \frac{a^2cx}{d} - \frac{4abc(c+dx)^{3/2}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(a + b*(c + d*x)^(1/2))^2,x)

[Out] (b^2*(c + d*x)^3)/(3*d^2) - ((2*b^2*c - 2*a^2)*(c + d*x)^2)/(4*d^2) + (4*a*b*(c + d*x)^(5/2))/(5*d^2) - (a^2*c*x)/d - (4*a*b*c*(c + d*x)^(3/2))/(3*d^2)

sympy [A] time = 4.77, size = 94, normalized size = 1.06

$$\left\{ \begin{array}{l} \frac{2a^2\left(-\frac{c(dx)}{2} + \frac{(c+dx)^2}{4}\right) + 4ab\left(-\frac{c(dx)^{\frac{3}{2}}}{3} + \frac{(c+dx)^{\frac{5}{2}}}{5}\right) + 2b^2\left(-\frac{c(dx)^2}{4} + \frac{(c+dx)^3}{6}\right)}{d} \quad \text{for } d \neq 0 \\ \frac{x^2(a+b\sqrt{c})^2}{2} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise(((2*a**2*(-c*(c + d*x)/2 + (c + d*x)**2/4)/d + 4*a*b*(-c*(c + d*x)**(3/2)/3 + (c + d*x)**(5/2)/5)/d + 2*b**2*(-c*(c + d*x)**2/4 + (c + d*x)**3/6)/d)/d, Ne(d, 0)), (x**2*(a + b*sqrt(c))**2/2, True))

$$3.366 \quad \int (a + b\sqrt{c + dx})^2 dx$$

Optimal. Leaf size=41

$$a^2x + \frac{4ab(c + dx)^{3/2}}{3d} + \frac{b^2(c + dx)^2}{2d}$$

Rubi [A] time = 0.03, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 190, 43}

$$a^2x + \frac{4ab(c + dx)^{3/2}}{3d} + \frac{b^2(c + dx)^2}{2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^2,x]

[Out] a^2*x + (4*a*b*(c + d*x)^(3/2))/(3*d) + (b^2*(c + d*x)^2)/(2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int (a + b\sqrt{c + dx})^2 dx &= \frac{\text{Subst}\left(\int (a + b\sqrt{x})^2 dx, x, c + dx\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int x(a + bx)^2 dx, x, \sqrt{c + dx}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int (a^2x + 2abx^2 + b^2x^3) dx, x, \sqrt{c + dx}\right)}{d} \\ &= a^2x + \frac{4ab(c + dx)^{3/2}}{3d} + \frac{b^2(c + dx)^2}{2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.98

$$\frac{6a^2dx + 8ab(c + dx)^{3/2} + 3b^2(c + dx)^2}{6d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^2,x]

[Out] (6*a^2*d*x + 8*a*b*(c + d*x)^(3/2) + 3*b^2*(c + d*x)^2)/(6*d)

IntegrateAlgebraic [A] time = 0.02, size = 41, normalized size = 1.00

$$\frac{(c + dx)(6a^2 + 8ab\sqrt{c + dx} + 3b^2(c + dx))}{6d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2,x]

[Out] ((c + d*x)*(6*a^2 + 8*a*b*Sqrt[c + d*x] + 3*b^2*(c + d*x)))/(6*d)

fricas [A] time = 0.75, size = 49, normalized size = 1.20

$$\frac{3b^2d^2x^2 + 6(b^2c + a^2)dx + 8(abdx + abc)\sqrt{dx + c}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/6*(3*b^2*d^2*x^2 + 6*(b^2*c + a^2)*d*x + 8*(a*b*d*x + a*b*c)*sqrt(d*x + c))/d

giac [B] time = 0.36, size = 82, normalized size = 2.00

$$\frac{6(dx + c)b^2c + 24\sqrt{dx + c}abc + 6(dx + c)a^2 + 8\left((dx + c)^{\frac{3}{2}} - 3\sqrt{dx + c}c\right)ab + 3\left((dx + c)^2 - 2(dx + c)c\right)b^2}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 1/6*(6*(d*x + c)*b^2*c + 24*sqrt(d*x + c)*a*b*c + 6*(d*x + c)*a^2 + 8*((d*x + c)^(3/2) - 3*sqrt(d*x + c)*c)*a*b + 3*((d*x + c)^2 - 2*(d*x + c)*c)*b^2)/d

maple [A] time = 0.00, size = 35, normalized size = 0.85

$$a^2x + \left(\frac{1}{2}dx^2 + cx\right)b^2 + \frac{4(dx + c)^{\frac{3}{2}}ab}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(d*x+c)^(1/2))^2,x)

[Out] b^2*(1/2*d*x^2+c*x)+4/3*a*b*(d*x+c)^(3/2)/d+a^2*x

maxima [A] time = 0.83, size = 35, normalized size = 0.85

$$\frac{1}{2}(dx^2 + 2cx)b^2 + a^2x + \frac{4(dx + c)^{\frac{3}{2}}ab}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/2*(d*x^2 + 2*c*x)*b^2 + a^2*x + 4/3*(d*x + c)^(3/2)*a*b/d

mupad [B] time = 0.05, size = 36, normalized size = 0.88

$$\frac{3b^2(c+dx)^2 + 8ab(c+dx)^{3/2} + 6a^2dx}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^2, x)

[Out] (3*b^2*(c + d*x)^2 + 8*a*b*(c + d*x)^(3/2) + 6*a^2*d*x)/(6*d)

sympy [A] time = 0.21, size = 68, normalized size = 1.66

$$\begin{cases} a^2x + \frac{4abc\sqrt{c+dx}}{3d} + \frac{4abx\sqrt{c+dx}}{3} + b^2cx + \frac{b^2dx^2}{2} & \text{for } d \neq 0 \\ x(a + b\sqrt{c})^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**2, x)

[Out] Piecewise((a**2*x + 4*a*b*c*sqrt(c + d*x)/(3*d) + 4*a*b*x*sqrt(c + d*x)/3 + b**2*c*x + b**2*d*x**2/2, Ne(d, 0)), (x*(a + b*sqrt(c))**2, True))

$$3.367 \quad \int \frac{(a+b\sqrt{c+dx})^2}{x} dx$$

Optimal. Leaf size=57

$$\log(x)(a^2 + b^2c) + 4ab\sqrt{c+dx} - 4ab\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + b^2dx$$

Rubi [A] time = 0.07, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {371, 1398, 801, 635, 207, 260}

$$\log(x)(a^2 + b^2c) + 4ab\sqrt{c+dx} - 4ab\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + b^2dx$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^2/x,x]

[Out] b^2*d*x + 4*a*b*Sqrt[c + d*x] - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^p_.*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{(a + b\sqrt{c + dx})^2}{x} dx &= \text{Subst} \left(\int \frac{(a + b\sqrt{x})^2}{-c + x} dx, x, c + dx \right) \\
&= 2 \text{Subst} \left(\int \frac{x(a + bx)^2}{-c + x^2} dx, x, \sqrt{c + dx} \right) \\
&= 2 \text{Subst} \left(\int \left(2ab + b^2x + \frac{2abc + (a^2 + b^2c)x}{-c + x^2} \right) dx, x, \sqrt{c + dx} \right) \\
&= b^2dx + 4ab\sqrt{c + dx} + 2 \text{Subst} \left(\int \frac{2abc + (a^2 + b^2c)x}{-c + x^2} dx, x, \sqrt{c + dx} \right) \\
&= b^2dx + 4ab\sqrt{c + dx} + (4abc) \text{Subst} \left(\int \frac{1}{-c + x^2} dx, x, \sqrt{c + dx} \right) + (2(a^2 + b^2c)) \text{Subst} \left(\int \frac{x}{-c + x^2} dx, x, \sqrt{c + dx} \right) \\
&= b^2dx + 4ab\sqrt{c + dx} - 4ab\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right) + (a^2 + b^2c) \log(x)
\end{aligned}$$

Mathematica [A] time = 0.16, size = 79, normalized size = 1.39

$$b(4a\sqrt{c + dx} + bdx) + (a - b\sqrt{c})^2 \log(\sqrt{c + dx} + \sqrt{c}) + (a + b\sqrt{c})^2 \log(\sqrt{c} - \sqrt{c + dx})$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^2/x,x]

[Out] b*(b*d*x + 4*a*Sqrt[c + d*x]) + (a + b*Sqrt[c])^2*Log[Sqrt[c] - Sqrt[c + d*x]] + (a - b*Sqrt[c])^2*Log[Sqrt[c] + Sqrt[c + d*x]]

IntegrateAlgebraic [A] time = 0.06, size = 67, normalized size = 1.18

$$(a^2 + b^2c) \log(-dx) + b\sqrt{c + dx} (4a + b\sqrt{c + dx}) - 4ab\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x,x]

[Out] b*Sqrt[c + d*x]*(4*a + b*Sqrt[c + d*x]) - 4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + (a^2 + b^2*c)*Log[-(d*x)]

fricas [A] time = 1.00, size = 118, normalized size = 2.07

$$\left[b^2dx + 2ab\sqrt{c} \log\left(\frac{dx - 2\sqrt{dx+c}\sqrt{c} + 2c}{x}\right) + 4\sqrt{dx+c}ab + (b^2c + a^2) \log(x), b^2dx + 4ab\sqrt{-c} \arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + 4\sqrt{dx+c}ab + (b^2c + a^2) \log(x) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="fricas")

[Out] [b^2*d*x + 2*a*b*sqrt(c)*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x), b^2*d*x + 4*a*b*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 4*sqrt(d*x + c)*a*b + (b^2*c + a^2)*log(x)]

giac [A] time = 0.42, size = 59, normalized size = 1.04

$$\frac{4abc \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + (dx + c)b^2 + 4\sqrt{dx + c}ab + (b^2c + a^2) \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="giac")

[Out] $4*a*b*c*\arctan(\sqrt{d*x+c}/\sqrt{-c})/\sqrt{-c} + (d*x+c)*b^2 + 4*\sqrt{d*x+c}*a*b + (b^2*c+a^2)*\log(d*x)$

maple [A] time = 0.01, size = 51, normalized size = 0.89

$$b^2c \ln(x) + b^2dx - 4ab\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right) + a^2 \ln(x) + 4\sqrt{dx+c} ab$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(d*x+c)^(1/2))^2/x,x)

[Out] $\ln(x)*b^2*c+b^2*d*x-4*a*b*\operatorname{arctanh}((d*x+c)^(1/2)/c^(1/2))*c^(1/2)+4*a*b*(d*x+c)^(1/2)+a^2*\ln(x)$

maxima [A] time = 1.93, size = 70, normalized size = 1.23

$$2ab\sqrt{c} \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right) + (dx+c)b^2 + 4\sqrt{dx+c} ab + (b^2c+a^2) \log(dx)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x,x, algorithm="maxima")

[Out] $2*a*b*\sqrt{c}*\log((\sqrt{d*x+c}-\sqrt{c})/(\sqrt{d*x+c}+\sqrt{c})) + (d*x+c)*b^2 + 4*\sqrt{d*x+c}*a*b + (b^2*c+a^2)*\log(d*x)$

mupad [B] time = 0.09, size = 130, normalized size = 2.28

$\ln((2a^2+2cb^2)\sqrt{c+dx}-2(a+b\sqrt{c})^2\sqrt{c+dx+4abc})(a+b\sqrt{c})^2 + \ln((2a^2+2cb^2)\sqrt{c+dx}-2(a-b\sqrt{c})^2\sqrt{c+dx+4abc})(a-b\sqrt{c})^2 + 4ab\sqrt{c+dx} + b^2dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^2/x,x)

[Out] $\log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a + b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a + b*c^(1/2))^2 + \log((2*b^2*c + 2*a^2)*(c + d*x)^(1/2) - 2*(a - b*c^(1/2))^2*(c + d*x)^(1/2) + 4*a*b*c)*(a - b*c^(1/2))^2 + 4*a*b*(c + d*x)^(1/2) + b^2*d*x$

sympy [A] time = 27.29, size = 65, normalized size = 1.14

$$a^2 \log(x) - 2ab \left(-\frac{2c \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - 2\sqrt{c+dx} \right) + b^2c \log(x) + b^2dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**2/x,x)

[Out] $a**2*\log(x) - 2*a*b*(-2*c*\operatorname{atan}(\sqrt{c+d*x}/\sqrt{-c})/\sqrt{-c} - 2*\sqrt{c+d*x}) + b**2*c*\log(x) + b**2*d*x$

$$3.368 \quad \int \frac{(a+b\sqrt{c+dx})^2}{x^2} dx$$

Optimal. Leaf size=54

$$-\frac{(a+b\sqrt{c+dx})^2}{x} - \frac{2abd \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d \log(x)$$

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {371, 1398, 819, 635, 207, 260}

$$-\frac{(a+b\sqrt{c+dx})^2}{x} - \frac{2abd \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}} + b^2d \log(x)$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^2/x^2,x]

[Out] -((a + b*Sqrt[c + d*x])^2/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*d*Log[x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol]
:= With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))
]^(q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{(a + b\sqrt{c + dx})^2}{x^2} dx = d \operatorname{Subst} \left(\int \frac{(a + b\sqrt{x})^2}{(-c + x)^2} dx, x, c + dx \right)$$

$$= (2d) \operatorname{Subst} \left(\int \frac{x(a + bx)^2}{(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)$$

$$= -\frac{(a + b\sqrt{c + dx})^2}{x} - \frac{d \operatorname{Subst} \left(\int \frac{-2abc - 2b^2cx}{-c + x^2} dx, x, \sqrt{c + dx} \right)}{c}$$

$$= -\frac{(a + b\sqrt{c + dx})^2}{x} + (2abd) \operatorname{Subst} \left(\int \frac{1}{-c + x^2} dx, x, \sqrt{c + dx} \right) + (2b^2d) \operatorname{Subst} \left(\int \frac{1}{-c + x^2} dx, x, \sqrt{c + dx} \right)$$

$$= -\frac{(a + b\sqrt{c + dx})^2}{x} - \frac{2abd \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{\sqrt{c}} + b^2d \log(x)$$

Mathematica [B] time = 0.20, size = 161, normalized size = 2.98

$$\frac{\sqrt{c} (a^4 + 2a^3b\sqrt{c + dx} - 2ab^3c\sqrt{c + dx} - b^4c(c + 2dx)) + bdx(a + b\sqrt{c})(a - b\sqrt{c})^2 \log(\sqrt{c + dx} + \sqrt{c}) + bdx(b\sqrt{c} - a)(a + b\sqrt{c})^2 \log(\sqrt{c} - \sqrt{c + dx})}{\sqrt{c}x(b^2c - a^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^2, x]
[Out] (Sqrt[c]*(a^4 + 2*a^3*b*Sqrt[c + d*x] - 2*a*b^3*c*Sqrt[c + d*x] - b^4*c*(c + 2*d*x)) + b*(-a + b*Sqrt[c])*(a + b*Sqrt[c])^2*d*x*Log[Sqrt[c] - Sqrt[c + d*x]] + b*(a - b*Sqrt[c])^2*(a + b*Sqrt[c])*d*x*Log[Sqrt[c] + Sqrt[c + d*x]])/(Sqrt[c]*(-a^2 + b^2*c)*x)
```

IntegrateAlgebraic [A] time = 0.15, size = 64, normalized size = 1.19

$$-\frac{a^2 + 2ab\sqrt{c + dx} + b^2c}{x} - \frac{2abd \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{\sqrt{c}} + b^2d \log(-dx)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x^2, x]
[Out] -((a^2 + b^2*c + 2*a*b*Sqrt[c + d*x])/x) - (2*a*b*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/Sqrt[c] + b^2*d*Log[-(d*x)]
```

fricas [A] time = 1.00, size = 147, normalized size = 2.72

$$\left[\frac{b^2cdx \log(x) + ab\sqrt{c} dx \log\left(\frac{dx - 2\sqrt{dx+c}\sqrt{c} + 2c}{x}\right) - b^2c^2 - 2\sqrt{dx+c}abc - a^2c}{cx}, \frac{b^2cdx \log(x) + 2ab\sqrt{-c} dx \arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) - b^2c^2 - 2\sqrt{dx+c}abc - a^2c}{cx} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2, x, algorithm="fricas")
```

[Out] $[(b^2*c*d*x*\log(x) + a*b*\sqrt{c}*d*x*\log((d*x - 2*\sqrt{d*x + c})*\sqrt{c} + 2*c)/x) - b^2*c^2 - 2*\sqrt{d*x + c}*a*b*c - a^2*c)/(c*x), (b^2*c*d*x*\log(x) + 2*a*b*\sqrt{-c}*d*x*\arctan(\sqrt{d*x + c})*\sqrt{-c}/c) - b^2*c^2 - 2*\sqrt{d*x + c}*a*b*c - a^2*c)/(c*x)]$

giac [A] time = 0.44, size = 80, normalized size = 1.48

$$\frac{b^2 d^2 \log(dx) + \frac{2abd^2 \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{b^2 cd^2 + 2\sqrt{dx+c}abd^2 + a^2 d^2}{dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="giac")

[Out] $(b^2*d^2*\log(d*x) + 2*a*b*d^2*\arctan(\sqrt{d*x + c}/\sqrt{-c}))/\sqrt{-c} - (b^2*c*d^2 + 2*\sqrt{d*x + c}*a*b*d^2 + a^2*d^2)/(d*x))/d$

maple [A] time = 0.02, size = 60, normalized size = 1.11

$$b^2 d \ln(x) - \frac{2abd \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 c}{x} - \frac{a^2}{x} - \frac{2\sqrt{dx+c} ab}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(d*x+c)^(1/2))^2/x^2,x)

[Out] $-b^2*c/x + b^2*d*\ln(x) - 2*a*b/x*(d*x+c)^(1/2) - 2*a*b*d*\operatorname{arctanh}((d*x+c)^(1/2)/c^(1/2))/c^(1/2) - a^2/x$

maxima [A] time = 1.97, size = 73, normalized size = 1.35

$$\left(b^2 \log(dx) + \frac{ab \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{\sqrt{c}} - \frac{b^2 c + 2\sqrt{dx+c} ab + a^2}{dx} \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x^2,x, algorithm="maxima")

[Out] $(b^2*\log(d*x) + a*b*\log((\sqrt{d*x + c} - \sqrt{c})/(\sqrt{d*x + c} + \sqrt{c}))/\sqrt{c} - (b^2*c + 2*\sqrt{d*x + c}*a*b + a^2)/(d*x))*d$

mupad [B] time = 0.12, size = 131, normalized size = 2.43

$$bd \ln\left(2bd\left(b + \frac{a}{\sqrt{c}}\right)\sqrt{c+dx} - 2b^2d\sqrt{c+dx} - 2abd\right)\left(b + \frac{a}{\sqrt{c}}\right) - \frac{a^2d + b^2cd + 2abd\sqrt{c+dx}}{dx} + bd \ln\left(2bd\left(b - \frac{a}{\sqrt{c}}\right)\sqrt{c+dx} - 2b^2d\sqrt{c+dx} - 2abd\right)\left(b - \frac{a}{\sqrt{c}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^2/x^2,x)

[Out] $b*d*\log(2*b*d*(b + a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2*a*b*d*(b + a/c^(1/2))) - (a^2*d + b^2*c*d + 2*a*b*d*(c + d*x)^(1/2))/(d*x) + b*d*\log(2*b*d*(b - a/c^(1/2))*(c + d*x)^(1/2) - 2*b^2*d*(c + d*x)^(1/2) - 2*a*b*d*(b - a/c^(1/2)))$

sympy [B] time = 87.77, size = 139, normalized size = 2.57

$$-\frac{a^2}{x} - abcd\sqrt{\frac{1}{c^3}} \log\left(-c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + abcd\sqrt{\frac{1}{c^3}} \log\left(c^2\sqrt{\frac{1}{c^3}} + \sqrt{c+dx}\right) + \frac{4abd \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{2ab\sqrt{c+dx}}{x} - \frac{b^2c}{x} + b^2d \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(d*x+c)**(1/2))**2/x**2,x)
```

```
[Out] -a**2/x - a*b*c*d*sqrt(c**(-3))*log(-c**2*sqrt(c**(-3)) + sqrt(c + d*x)) +  
a*b*c*d*sqrt(c**(-3))*log(c**2*sqrt(c**(-3)) + sqrt(c + d*x)) + 4*a*b*d*ata  
n(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) - 2*a*b*sqrt(c + d*x)/x - b**2*c/x + b**  
2*d*log(x)
```


$$3.369 \quad \int \frac{(a+b\sqrt{c+dx})^2}{x^3} dx$$

Optimal. Leaf size=80

$$\frac{abd^2 \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}} - \frac{(a+b\sqrt{c+dx})^2}{2x^2} - \frac{bd(a\sqrt{c+dx}+bc)}{2cx}$$

Rubi [A] time = 0.07, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {371, 1398, 821, 12, 639, 207}

$$\frac{abd^2 \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}} - \frac{(a+b\sqrt{c+dx})^2}{2x^2} - \frac{bd(a\sqrt{c+dx}+bc)}{2cx}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^2/x^3,x]

[Out] -(b*d*(b*c + a*Sqrt[c + d*x]))/(2*c*x) - (a + b*Sqrt[c + d*x])^2/(2*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 639

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[(d*(2*p + 3))/(2*a*(p + 1)), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 821

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n

)^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b\sqrt{c + dx})^2}{x^3} dx &= d^2 \operatorname{Subst} \left(\int \frac{(a + b\sqrt{x})^2}{(-c + x)^3} dx, x, c + dx \right) \\
 &= (2d^2) \operatorname{Subst} \left(\int \frac{x(a + bx)^2}{(-c + x^2)^3} dx, x, \sqrt{c + dx} \right) \\
 &= -\frac{(a + b\sqrt{c + dx})^2}{2x^2} - \frac{d^2 \operatorname{Subst} \left(\int \frac{-2bc(a + bx)}{(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{2c} \\
 &= -\frac{(a + b\sqrt{c + dx})^2}{2x^2} + (bd^2) \operatorname{Subst} \left(\int \frac{a + bx}{(-c + x^2)^2} dx, x, \sqrt{c + dx} \right) \\
 &= -\frac{bd(bc + a\sqrt{c + dx})}{2cx} - \frac{(a + b\sqrt{c + dx})^2}{2x^2} - \frac{(abd^2) \operatorname{Subst} \left(\int \frac{1}{-c + x^2} dx, x, \sqrt{c + dx} \right)}{2c} \\
 &= -\frac{bd(bc + a\sqrt{c + dx})}{2cx} - \frac{(a + b\sqrt{c + dx})^2}{2x^2} + \frac{abd^2 \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{2c^{3/2}}
 \end{aligned}$$

Mathematica [B] time = 0.35, size = 221, normalized size = 2.76

$$\frac{-2\sqrt{c}(a^6c + a^5b\sqrt{c+dx}(2c+dx) + a^4b^2(-c^2+2cdx+3d^2x^2) - 2a^3b^3c\sqrt{c+dx}(2c+dx) - a^2b^4c(c^2+4cdx+2d^2x^2) + ab^5c^2\sqrt{c+dx}(2c+dx) + b^6c^2(c+dx)^2) - abd^2 \log(\sqrt{c} - \sqrt{c+dx}) + abd^2 \log(\sqrt{c+dx} + \sqrt{c})}{x^2(a^2 - b^2c)^2} \cdot \frac{1}{4c^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^2/x^3, x]

[Out] ((-2*Sqrt[c]*(a^6*c + b^6*c^2*(c + d*x)^2 + a^5*b*Sqrt[c + d*x]*(2*c + d*x) - 2*a^3*b^3*c*Sqrt[c + d*x]*(2*c + d*x) + a*b^5*c^2*Sqrt[c + d*x]*(2*c + d*x) - a^2*b^4*c*(c^2 + 4*c*d*x + 2*d^2*x^2) + a^4*b^2*(-c^2 + 2*c*d*x + 3*d^2*x^2)))/((a^2 - b^2*c)^2*x^2) - a*b*d^2*Log[Sqrt[c] - Sqrt[c + d*x]] + a*b*d^2*Log[Sqrt[c] + Sqrt[c + d*x]])/(4*c^(3/2))

IntegrateAlgebraic [A] time = 0.23, size = 93, normalized size = 1.16

$$\frac{a^2(-c) - ab(c + dx)^{3/2} - abc\sqrt{c + dx} + b^2c^2 - 2b^2c(c + dx)}{2cx^2} + \frac{abd^2 \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{2c^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^2/x^3, x]

[Out] (-a^2*c + b^2*c^2 - a*b*c*Sqrt[c + d*x] - 2*b^2*c*(c + d*x) - a*b*(c + d*x)^(3/2))/(2*c*x^2) + (a*b*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2))

fricas [A] time = 0.85, size = 181, normalized size = 2.26

$$\left[\frac{ab\sqrt{c}d^2x^2 \log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c+2c}}{x}\right) - 4b^2c^2dx - 2b^2c^3 - 2a^2c^2 - 2(abcdx + 2abc^2)\sqrt{dx+c}}{4c^2x^2}, \frac{ab\sqrt{-c}d^2x^2 \arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + 2b^2c^2dx + b^2c^3 + a^2c^2 + (abcdx + 2abc^2)\sqrt{dx+c}}{2c^2x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="fricas")

[Out] [1/4*(a*b*sqrt(c)*d^2*x^2*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 4*b^2*c^2*d*x - 2*b^2*c^3 - 2*a^2*c^2 - 2*(a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c))/(c^2*x^2), -1/2*(a*b*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c) + 2*b^2*c^2*d*x + b^2*c^3 + a^2*c^2 + (a*b*c*d*x + 2*a*b*c^2)*sqrt(d*x + c))/(c^2*x^2)]

giac [A] time = 0.36, size = 105, normalized size = 1.31

$$\frac{\frac{abd^3 \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \frac{2(dx+c)b^2cd^3 - b^2c^2d^3 + (dx+c)^2abd^3 + \sqrt{dx+c}abcd^3 + a^2cd^3}{cd^2x^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="giac")

[Out] -1/2*(a*b*d^3*arctan(sqrt(d*x + c)/sqrt(-c))/(sqrt(-c)*c) + (2*(d*x + c)*b^2*c*d^3 - b^2*c^2*d^3 + (d*x + c)^(3/2)*a*b*d^3 + sqrt(d*x + c)*a*b*c*d^3 + a^2*c*d^3)/(c*d^2*x^2))/d

maple [A] time = 0.02, size = 81, normalized size = 1.01

$$4 \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{8c^{\frac{3}{2}}} + \frac{\frac{(dx+c)^{\frac{3}{2}}}{8c} - \frac{\sqrt{dx+c}}{8}}{d^2x^2} \right) ab d^2 + \left(-\frac{d}{x} - \frac{c}{2x^2} \right) b^2 - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(d*x+c)^(1/2))^2/x^3,x)

[Out] b^2*(-d/x-1/2*c/x^2)+4*a*b*d^2*((-1/8/c*(d*x+c)^(3/2)-1/8*(d*x+c)^(1/2))/x^2/d^2+1/8/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2)))-1/2*a^2/x^2

maxima [A] time = 1.96, size = 113, normalized size = 1.41

$$-\frac{1}{4} \left(\frac{ab \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{c^{\frac{3}{2}}} + \frac{2 \left(2(dx+c)b^2c - b^2c^2 + (dx+c)^{\frac{3}{2}}ab + \sqrt{dx+c}abc + a^2c \right)}{(dx+c)^2c - 2(dx+c)c^2 + c^3} \right) d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^2/x^3,x, algorithm="maxima")

[Out] -1/4*(a*b*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/c^(3/2) + 2*(2*(d*x + c)*b^2*c - b^2*c^2 + (d*x + c)^(3/2)*a*b + sqrt(d*x + c)*a*b*c + a^2*c)/((d*x + c)^2*c - 2*(d*x + c)*c^2 + c^3))*d^2

mupad [B] time = 3.40, size = 80, normalized size = 1.00

$$\frac{ab d^2 \operatorname{atanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^{3/2}} - \frac{b^2 c}{2x^2} - \frac{b^2 d}{x} - \frac{ab \sqrt{c+dx}}{2x^2} - \frac{ab(c+dx)^{3/2}}{2cx^2} - \frac{a^2}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^2/x^3,x)

[Out] (a*b*d^2*atanh((c + d*x)^(1/2)/c^(1/2)))/(2*c^(3/2)) - (b^2*c)/(2*x^2) - (b^2*d)/x - (a*b*(c + d*x)^(1/2))/(2*x^2) - (a*b*(c + d*x)^(3/2))/(2*c*x^2) - a^2/(2*x^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**2/x**3,x)

[Out] Timed out

$$3.370 \quad \int x^3 \sqrt{a + b\sqrt{c + dx}} \, dx$$

Optimal. Leaf size=326

$$\frac{12(7a^2 - b^2c)(a + b\sqrt{c + dx})^{13/2}}{13b^8d^4} - \frac{20a(7a^2 - 3b^2c)(a + b\sqrt{c + dx})^{11/2}}{11b^8d^4} - \frac{12a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{9/2}}{7b^8d^4}$$

Rubi [A] time = 0.24, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {371, 1398, 772}

$$\frac{4(-30a^2b^2c + 35a^4 + 3b^4c^2)(a + b\sqrt{c + dx})^{9/2}}{9b^8d^4} + \frac{12(7a^2 - b^2c)(a + b\sqrt{c + dx})^{13/2}}{13b^8d^4} - \frac{20a(7a^2 - 3b^2c)(a + b\sqrt{c + dx})^{11/2}}{11b^8d^4} - \frac{12a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{9/2}}{7b^8d^4} + \frac{4(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^8d^4} - \frac{4a(a^2 - b^2c)^3(a + b\sqrt{c + dx})^{3/2}}{3b^8d^4} + \frac{4(a + b\sqrt{c + dx})^{17/2}}{17b^8d^4} - \frac{28a(a + b\sqrt{c + dx})^{15/2}}{15b^8d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] $(-4*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^{3/2})/(3*b^8*d^4) + (4*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^{5/2})/(5*b^8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^{7/2})/(7*b^8*d^4) + (4*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^{9/2})/(9*b^8*d^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^{11/2})/(11*b^8*d^4) + (12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^{13/2})/(13*b^8*d^4) - (28*a*(a + b*Sqrt[c + d*x])^{15/2})/(15*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^{17/2})/(17*b^8*d^4)$

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\int x^3 \sqrt{a + b\sqrt{c + dx}} dx = \frac{\text{Subst}\left(\int \sqrt{a + b\sqrt{x}} (-c + x)^3 dx, x, c + dx\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int x\sqrt{a + bx} (-c + x^2)^3 dx, x, \sqrt{c + dx}\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int \left(-\frac{a(a^2 - b^2c)^3 \sqrt{a+bx}}{b^7} - \frac{(-7a^2 + b^2c)(-a^2 + b^2c)^2 (a+bx)^{3/2}}{b^7} - \frac{3(7a^5 - 10a^3b^2c + 3ab^4c^2)(a+bx)^{5/2}}{b^7}\right) dx, x, \sqrt{c + dx}\right)}{d^4}$$

$$= -\frac{4a(a^2 - b^2c)^3 (a + b\sqrt{c + dx})^{3/2}}{3b^8d^4} + \frac{4(a^2 - b^2c)^2 (7a^2 - b^2c) (a + b\sqrt{c + dx})^{5/2}}{5b^8d^4} - \frac{12(7a^5 - 10a^3b^2c + 3ab^4c^2)(a + b\sqrt{c + dx})^{7/2}}{7b^8d^4}$$

Mathematica [A] time = 0.42, size = 232, normalized size = 0.71

$$\frac{4(a + b\sqrt{c + dx})^{3/2} (-14336a^7 + 21504a^6b\sqrt{c + dx} + 3840a^5b^2(10c - 7dx) - 640a^4b^3(104c - 49dx)\sqrt{c + dx} - 48a^3b^4(616c^2 - 1080c*d*x + 735d^2*x^2) + 24a^2b^5\sqrt{c + dx} (2960c^2 - 2716c*d*x + 1617d^2*x^2) + 6ab^6(320c^3 - 3936c^2*d*x + 5754cd^2*x^2 - 7007d^3*x^3) - 231b^7\sqrt{c + dx} (128c^3 - 160c^2*d*x + 180c*d^2*x^2 - 195d^3*x^3))}{765765b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + b*Sqrt[c + d*x]], x]
[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-14336*a^7 + 3840*a^5*b^2*(10*c - 7*d*x) + 21504*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(104*c - 49*d*x)*Sqrt[c + d*x] - 48*a^3*b^4*(616*c^2 - 1080*c*d*x + 735*d^2*x^2) + 24*a^2*b^5*Sqrt[c + d*x]*(2960*c^2 - 2716*c*d*x + 1617*d^2*x^2) + 6*a*b^6*(320*c^3 - 3936*c^2*d*x + 5754*c*d^2*x^2 - 7007*d^3*x^3) - 231*b^7*Sqrt[c + d*x]*(128*c^3 - 160*c^2*d*x + 180*c*d^2*x^2 - 195*d^3*x^3)))/(765765*b^8*d^4)
```

IntegrateAlgebraic [A] time = 0.21, size = 313, normalized size = 0.96

$$\frac{4(a + b\sqrt{c + dx})^{3/2} (14336a^7 - 65280a^5b^2c + 116688a^3b^4c^2 - 102102a^2b^6c^3 - 21504a^6b\sqrt{c + dx} + 97920a^4b^3c\sqrt{c + dx} - 175032a^2b^5c^2\sqrt{c + dx} + 153153b^7c^3\sqrt{c + dx} + 26880a^5b^2(c + d*x) - 122400a^3b^4c(c + d*x) + 218790a^2b^6c^2(c + d*x) - 31360a^4b^3(c + d*x)^{3/2} + 142800a^2b^5c(c + d*x)^{3/2} - 255255b^7c^2(c + d*x)^{3/2} + 35280a^3b^4(c + d*x)^2 - 160650a^2b^6c(c + d*x)^2 - 38808a^2b^5(c + d*x)^{5/2} + 176715b^7c(c + d*x)^{5/2} + 42042a^2b^6(c + d*x)^3 - 45045b^7(c + d*x)^{7/2})}{765765b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3*Sqrt[a + b*Sqrt[c + d*x]], x]
[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(14336*a^7 - 65280*a^5*b^2*c + 116688*a^3*b^4*c^2 - 102102*a^2*b^6*c^3 - 21504*a^6*b*Sqrt[c + d*x] + 97920*a^4*b^3*c*Sqrt[c + d*x] - 175032*a^2*b^5*c^2*Sqrt[c + d*x] + 153153*b^7*c^3*Sqrt[c + d*x] + 26880*a^5*b^2*(c + d*x) - 122400*a^3*b^4*c*(c + d*x) + 218790*a^2*b^6*c^2*(c + d*x) - 31360*a^4*b^3*(c + d*x)^(3/2) + 142800*a^2*b^5*c*(c + d*x)^(3/2) - 255255*b^7*c^2*(c + d*x)^(3/2) + 35280*a^3*b^4*(c + d*x)^2 - 160650*a^2*b^6*c*(c + d*x)^2 - 38808*a^2*b^5*(c + d*x)^(5/2) + 176715*b^7*c*(c + d*x)^(5/2) + 42042*a^2*b^6*(c + d*x)^3 - 45045*b^7*(c + d*x)^(7/2)))/(765765*b^8*d^4)
```

fricas [A] time = 1.16, size = 286, normalized size = 0.88

$$\frac{4(45045b^7d^4 - 29568b^8c^4 + 72960a^2b^6c^3 - 96128a^4b^4c^2 + 59904a^6b^2c - 14336a^8 + 231(15b^8c - 14a^2b^6)d^3x^3 - 28(165b^8c^2 - 291a^2b^6c + 140a^4b^4)d^2x^2 + 32(231b^8c^3 - 555a^2b^6c^2 + 520a^4b^4c - 168a^6b^2)d*x + (3003a^2b^7d^3x^3 - 27648a^3b^5c^2 - 28160a^5b^3c + 7168a^7b - 35(2a^2c - d^2b^2)x^2 + 32(47ab^2d - 417a^2c + 140d^2b^2)x) \sqrt{dx + c}}{765765b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2), x, algorithm="fricas")
[Out] 4/765765*(45045*b^8*d^4*x^4 - 29568*b^8*c^4 + 72960*a^2*b^6*c^3 - 96128*a^4*b^4*c^2 + 59904*a^6*b^2*c - 14336*a^8 + 231*(15*b^8*c - 14*a^2*b^6)*d^3*x^3 - 28*(165*b^8*c^2 - 291*a^2*b^6*c + 140*a^4*b^4)*d^2*x^2 + 32*(231*b^8*c^3 - 555*a^2*b^6*c^2 + 520*a^4*b^4*c - 168*a^6*b^2)*d*x + (3003*a*b^7*d^3*x^3 - 27648*a^3*b^5*c^2 + 41472*a^3*b^5*c^2 - 28160*a^5*b^3*c + 7168*a^7*b - 35
```

$28*(2*a*b^7*c - a^3*b^5)*d^2*x^2 + 32*(417*a*b^7*c^2 - 417*a^3*b^5*c + 140*a^5*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)$

giac [B] time = 0.63, size = 915, normalized size = 2.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out]
$$\begin{aligned} & -4/765765*(17*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 810 \\ & 81*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + \\ & 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24 \\ & 255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/b^7*d^3 + (153153 \\ & *(sqrt(d*x + c)*b + a)^(5/2)*b^6*c^3 - 510510*(sqrt(d*x + c)*b + a)^(3/2)*a*b^6*c^3 + 765765*sqrt(sqrt(d*x + c)*b + a)*a^2*b^6*c^3 - 255255*(sqrt(d*x + c)*b + a)^(9/2)*b^4*c^2 + 1312740*(sqrt(d*x + c)*b + a)^(7/2)*a*b^4*c^2 - 2756754*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^4*c^2 + 3063060*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^4*c^2 - 2297295*sqrt(sqrt(d*x + c)*b + a)*a^4*b^4*c^2 + 1 \\ & 76715*(sqrt(d*x + c)*b + a)^(13/2)*b^2*c - 1253070*(sqrt(d*x + c)*b + a)^(11/2)*a*b^2*c + 3828825*(sqrt(d*x + c)*b + a)^(9/2)*a^2*b^2*c - 6563700*(sqrt(d*x + c)*b + a)^(7/2)*a^3*b^2*c + 6891885*(sqrt(d*x + c)*b + a)^(5/2)*a^4*b^2*c - 4594590*(sqrt(d*x + c)*b + a)^(3/2)*a^5*b^2*c + 2297295*sqrt(sqrt(d*x + c)*b + a)*a^6*b^2*c - 45045*(sqrt(d*x + c)*b + a)^(17/2) + 408408*(sqrt(d*x + c)*b + a)^(15/2)*a - 1649340*(sqrt(d*x + c)*b + a)^(13/2)*a^2 + 38 \\ & 98440*(sqrt(d*x + c)*b + a)^(11/2)*a^3 - 5955950*(sqrt(d*x + c)*b + a)^(9/2)*a^4 + 6126120*(sqrt(d*x + c)*b + a)^(7/2)*a^5 - 4288284*(sqrt(d*x + c)*b + a)^(5/2)*a^6 + 2042040*(sqrt(d*x + c)*b + a)^(3/2)*a^7 - 765765*sqrt(sqrt(d*x + c)*b + a)*a^8)/(b^7*d^3)/(b*d) \end{aligned}$$

maple [A] time = 0.01, size = 383, normalized size = 1.17

$$\frac{-\frac{28(a^5 b^3 c^2 + 140 a^3 b^5 c^2 - 417 a b^7 c^2) d^2 x^2 + 32(417 a b^7 c^2 - 417 a^3 b^5 c + 140 a^5 b^3) d x + 28 a^5 b^3 c^2}{15} \sqrt{d x + c} \sqrt{\sqrt{d x + c} b + a}}{b^8 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x)

[Out]
$$\begin{aligned} & 4/d^4/b^8*(1/17*(a+b*(d*x+c)^(1/2))^(17/2)-7/15*a*(a+b*(d*x+c)^(1/2))^(15/2) \\ &)+1/13*(-3*b^2*c+21*a^2)*(a+b*(d*x+c)^(1/2))^(13/2)+1/11*(-8*(-b^2*c+a^2)*a \\ & -2*a*(-2*b^2*c+6*a^2)-(-3*b^2*c+15*a^2)*a)*(a+b*(d*x+c)^(1/2))^(11/2)+1/9*(\\ & (-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2-(-8*(-b^2*c \\ & +a^2)*a-2*a*(-2*b^2*c+6*a^2))*a)*(a+b*(d*x+c)^(1/2))^(9/2)+1/7*(-6*(-b^2*c+ \\ & a^2)^2*a-((-b^2*c+a^2)*(-2*b^2*c+6*a^2)+8*a^2*(-b^2*c+a^2)+(-b^2*c+a^2)^2)* \\ & a)*(a+b*(d*x+c)^(1/2))^(7/2)+1/5*((-b^2*c+a^2)^3+6*(-b^2*c+a^2)^2*a^2)*(a+b \\ & *(d*x+c)^(1/2))^(5/2)-1/3*(-b^2*c+a^2)^3*a*(a+b*(d*x+c)^(1/2))^(3/2) \end{aligned}$$

maxima [A] time = 0.96, size = 268, normalized size = 0.82

$$\frac{4(45045(\sqrt{d x + c})^2 - 357357(\sqrt{d x + c})^4 + 170715(d x - 7 d)(\sqrt{d x + c})^6 + 348075(3 a^2 c - 7 d)(\sqrt{d x + c})^8 + 85085(3 a^2 c^2 - 20 a^2 b^2 c + 35 a^2)(\sqrt{d x + c})^{10} - 328185(3 a^2 c^2 - 10 a^2 b^2 c + 7 d)(\sqrt{d x + c})^{12} - 153153(b^2 c^2 - 9 a^2 b^2 c + 15 a^2 d^2 - 7 d)(\sqrt{d x + c})^{14} + 255255(a^2 c^2 - 3 a^2 b^2 c - d)(\sqrt{d x + c})^{16})}{765765 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

```
[Out] 4/765765*(45045*(sqrt(d*x + c)*b + a)^(17/2) - 357357*(sqrt(d*x + c)*b + a)^(15/2)*a - 176715*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(13/2) + 348075*(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(11/2) + 85085*(3*b^4*c^2 - 30*a^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(9/2) - 328185*(3*a*b^4*c^2 - 10*a^3*b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(7/2) - 153153*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(5/2) + 255255*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*(sqrt(d*x + c)*b + a)^(3/2))/(b^8*d^4)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \sqrt{a + b\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

```
[Out] int(x^3*(a + b*(c + d*x)^(1/2))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{a + b\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**(1/2), x)
```

```
[Out] Integral(x**3*sqrt(a + b*sqrt(c + d*x)), x)
```


$$3.371 \quad \int x^2 \sqrt{a + b\sqrt{c + dx}} dx$$

Optimal. Leaf size=224

$$\frac{8(5a^2 - b^2c)(a + b\sqrt{c + dx})^{9/2}}{9b^6d^3} - \frac{8a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{7/2}}{7b^6d^3} - \frac{4a(a^2 - b^2c)^2(a + b\sqrt{c + dx})^{3/2}}{3b^6d^3} + \frac{4(5a^4 - 6a^2b^2c + b^4c^2)(a + b\sqrt{c + dx})^{11/2}}{11b^6d^3}$$

Rubi [A] time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {371, 1398, 772}

$$\frac{4(-6a^2b^2c + 5a^4 + b^4c^2)(a + b\sqrt{c + dx})^{5/2}}{5b^6d^3} + \frac{8(5a^2 - b^2c)(a + b\sqrt{c + dx})^{9/2}}{9b^6d^3} - \frac{8a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{7/2}}{7b^6d^3} - \frac{4a(a^2 - b^2c)^2(a + b\sqrt{c + dx})^{3/2}}{3b^6d^3} + \frac{4(a + b\sqrt{c + dx})^{13/2}}{13b^6d^3} - \frac{20a(a + b\sqrt{c + dx})^{11/2}}{11b^6d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2*sqrt[a + b*sqrt[c + d*x]],x]

[Out] (-4*a*(a^2 - b^2*c)^2*(a + b*sqrt[c + d*x])^(3/2))/(3*b^6*d^3) + (4*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*sqrt[c + d*x])^(5/2))/(5*b^6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*sqrt[c + d*x])^(7/2))/(7*b^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*sqrt[c + d*x])^(9/2))/(9*b^6*d^3) - (20*a*(a + b*sqrt[c + d*x])^(11/2))/(11*b^6*d^3) + (4*(a + b*sqrt[c + d*x])^(13/2))/(13*b^6*d^3)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x^2 \sqrt{a + b\sqrt{c + dx}} dx &= \frac{\text{Subst}\left(\int \sqrt{a + b\sqrt{x}} (-c + x)^2 dx, x, c + dx\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int x\sqrt{a + bx} (-c + x^2)^2 dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{a(a^2 - b^2c)^2 \sqrt{a + bx}}{b^5} + \frac{(5a^4 - 6a^2b^2c + b^4c^2)(a + bx)^{3/2}}{b^5} - \frac{2(5a^3 - 3ab^2c)(a + bx)^{5/2}}{b^5} - \frac{2(-5a^2 + 5a^2 - b^2c)(a + bx)^{7/2}}{b^5}\right) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= -\frac{4a(a^2 - b^2c)^2(a + b\sqrt{c + dx})^{3/2}}{3b^6d^3} + \frac{4(5a^4 - 6a^2b^2c + b^4c^2)(a + b\sqrt{c + dx})^{5/2}}{5b^6d^3} - \end{aligned}$$

Mathematica [A] time = 0.24, size = 147, normalized size = 0.66

$$\frac{4(a + b\sqrt{c + dx})^{3/2}(-1280a^5 + 1920a^4b\sqrt{c + dx} + 32a^3b^2(68c - 75dx) + 16a^2b^3\sqrt{c + dx}(175dx - 254c) - 6ab^4(96c^2 - 380cdx + 525d^2x^2) + 77b^5\sqrt{c + dx}(32c^2 - 40cdx + 45d^2x^2))}{45045b^6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-1280*a^5 + 32*a^3*b^2*(68*c - 75*d*x) + 1920*a^4*b*Sqrt[c + d*x] + 16*a^2*b^3*Sqrt[c + d*x]*(-254*c + 175*d*x) + 77*b^5*Sqrt[c + d*x]*(32*c^2 - 40*c*d*x + 45*d^2*x^2) - 6*a*b^4*(96*c^2 - 380*c*d*x + 525*d^2*x^2)))/(45045*b^6*d^3)

IntegrateAlgebraic [A] time = 0.13, size = 185, normalized size = 0.83

$$\frac{4(a + b\sqrt{c + dx})^{3/2}(1280a^5 - 1920a^4b\sqrt{c + dx} + 2400a^3b^2(c + dx) - 4576a^2b^3c - 2800a^2b^3(c + dx)^{3/2} + 6864a^2b^3c\sqrt{c + dx} + 6006ab^4c^2 + 3150ab^4(c + dx)^2 - 8580ab^4c(c + dx) - 9009b^5c^2\sqrt{c + dx} - 3465b^5(c + dx)^{5/2} + 10010b^5(c + dx)^{3/2})}{45045b^6d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(1280*a^5 - 4576*a^3*b^2*c + 6006*a*b^4*c^2 - 1920*a^4*b*Sqrt[c + d*x] + 6864*a^2*b^3*c*Sqrt[c + d*x] - 9009*b^5*c^2*Sqrt[c + d*x] + 2400*a^3*b^2*(c + d*x) - 8580*a*b^4*c*(c + d*x) - 2800*a^2*b^3*(c + d*x)^(3/2) + 10010*b^5*c*(c + d*x)^(3/2) + 3150*a*b^4*(c + d*x)^2 - 3465*b^5*(c + d*x)^(5/2)))/(45045*b^6*d^3)

fricas [A] time = 1.01, size = 184, normalized size = 0.82

$$\frac{4(3465b^6d^3x^3 + 2464b^6c^3 - 4640a^2b^4c^2 + 4096a^4b^2c - 1280a^6 + 35(11b^6c - 10a^2b^4)d^2x^2 - 8(77b^6c^2 - 127a^2b^4c + 60a^4b^2)dx + (315ab^5d^2x^2 + 1888ab^5c^2 - 1888a^3b^3c + 640a^5b - 400(2ab^5c - a^3b^3)dx)\sqrt{dx + c})\sqrt{\sqrt{dx + c} + a}}{45045b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/45045*(3465*b^6*d^3*x^3 + 2464*b^6*c^3 - 4640*a^2*b^4*c^2 + 4096*a^4*b^2*c - 1280*a^6 + 35*(11*b^6*c - 10*a^2*b^4)*d^2*x^2 - 8*(77*b^6*c^2 - 127*a^2*b^4*c + 60*a^4*b^2)*d*x + (315*a*b^5*d^2*x^2 + 1888*a*b^5*c^2 - 1888*a^3*b^3*c + 640*a^5*b - 400*(2*a*b^5*c - a^3*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^6*d^3)

giac [B] time = 0.46, size = 549, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 4/45045*(13*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x + c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 6930*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 - 6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)*a/(b^5*d^2) + (9009*(sqrt(d*x + c)*b + a)^(5/2)*b^4*c^2 - 30030*(sqrt(d*x + c)*b + a)^(3/2)*a*b^4*c^2 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^2*b^4*c^2 - 10010*(sqrt(d*x + c)*b + a)^(9/2)*b^2*c + 51480*(sqrt(d*x + c)*b + a)^(7/2)*a*b^2*c - 108108*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^2*c + 120120*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^2*c - 90090*sqrt(sqrt(d*x + c)*b + a)*a^4*b^2*c + 3465*(sqrt(d*x + c)*b + a)^(13/2) - 24570*(sqrt(d*x + c)*b + a)^(11/2)*a + 75075*(sqrt(d*x + c)*b + a)^(9/2))

$$2) * a^2 - 128700 * (\sqrt{d*x + c}) * b + a)^{(7/2)} * a^3 + 135135 * (\sqrt{d*x + c}) * b + a)^{(5/2)} * a^4 - 90090 * (\sqrt{d*x + c}) * b + a)^{(3/2)} * a^5 + 45045 * \sqrt{(\sqrt{d*x + c}) * b + a} * a^6) / (b^5 * d^2)) / (b * d)$$

maple [A] time = 0.00, size = 183, normalized size = 0.82

$$\frac{-\frac{20(a+\sqrt{dx+c}b)^{\frac{11}{2}}}{11} - \frac{4(-b^2c+a^2)^2(a+\sqrt{dx+c}b)^{\frac{3}{2}}}{3} + \frac{4(a+\sqrt{dx+c}b)^{\frac{13}{2}}}{13} + \frac{4(-2b^2c+10a^2)(a+\sqrt{dx+c}b)^{\frac{9}{2}}}{9} + \frac{4(-4(-b^2c+a^2)a-(-2b^2c+6a^2)a)(a+\sqrt{dx+c}b)^{\frac{7}{2}}}{7} + \frac{4(4(-b^2c+a^2)a^2+(-b^2c+a^2)^2)(a+\sqrt{dx+c}b)^{\frac{5}{2}}}{5}}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+(d*x+c)^(1/2)*b)^(1/2), x)

[Out] 4/d^3/b^6*(1/13*(a+(d*x+c)^(1/2)*b)^(13/2)-5/11*a*(a+(d*x+c)^(1/2)*b)^(11/2)+1/9*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(-4*(-b^2*c+a^2)*a-(-2*b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*((-b^2*c+a^2)^2+4*(-b^2*c+a^2)*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)^(3/2))

maxima [A] time = 0.94, size = 167, normalized size = 0.75

$$\frac{4\left(3465(\sqrt{dx+cb+a})^{\frac{13}{2}}-20475(\sqrt{dx+cb+a})^{\frac{11}{2}}a-10010(b^2c-5a^2)(\sqrt{dx+cb+a})^{\frac{9}{2}}+12870(3ab^2c-5a^3)(\sqrt{dx+cb+a})^{\frac{7}{2}}+9009(b^4c^2-6a^2b^2c+5a^4)(\sqrt{dx+cb+a})^{\frac{5}{2}}-15015(ab^4c^2-2a^3b^2c+a^5)(\sqrt{dx+cb+a})^{\frac{3}{2}}\right)}{45045b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^(1/2), x, algorithm="maxima")

[Out] 4/45045*(3465*(sqrt(d*x + c))*b + a)^(13/2) - 20475*(sqrt(d*x + c))*b + a)^(11/2)*a - 10010*(b^2*c - 5*a^2)*(sqrt(d*x + c))*b + a)^(9/2) + 12870*(3*a*b^2*c - 5*a^3)*(sqrt(d*x + c))*b + a)^(7/2) + 9009*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sqrt(d*x + c))*b + a)^(5/2) - 15015*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(sqrt(d*x + c))*b + a)^(3/2))/(b^6*d^3)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \sqrt{a + b \sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)

[Out] int(x^2*(a + b*(c + d*x)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a + b \sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**(1/2), x)

[Out] Integral(x**2*sqrt(a + b*sqrt(c + d*x)), x)

$$3.372 \quad \int x \sqrt{a + b\sqrt{c + dx}} \, dx$$

Optimal. Leaf size=133

$$\frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2} - \frac{4a(a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} + \frac{4(a + b\sqrt{c + dx})^{9/2}}{9b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{7/2}}{7b^4d^2}$$

Rubi [A] time = 0.10, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2} - \frac{4a(a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} + \frac{4(a + b\sqrt{c + dx})^{9/2}}{9b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{7/2}}{7b^4d^2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[a + b*Sqrt[c + d*x]], x]

[Out] (-4*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) + (4*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^4*d^2)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x \sqrt{a + b\sqrt{c + dx}} \, dx &= \frac{\text{Subst}\left(\int \sqrt{a + b\sqrt{x}} (-c + x) \, dx, x, c + dx\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int x \sqrt{a + bx} (-c + x^2) \, dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{(-a^3 + ab^2c)\sqrt{a+bx}}{b^3} + \frac{(3a^2 - b^2c)(a+bx)^{3/2}}{b^3} - \frac{3a(a+bx)^{5/2}}{b^3} + \frac{(a+bx)^{7/2}}{b^3}\right) dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= -\frac{4a(a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} + \frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{7/2}}{7b^4d^2} \end{aligned}$$

Mathematica [A] time = 0.14, size = 84, normalized size = 0.63

$$\frac{4(a + b\sqrt{c + dx})^{3/2}(-16a^3 + 24a^2b\sqrt{c + dx} + 6ab^2(2c - 5dx) + 7b^3\sqrt{c + dx}(5dx - 4c))}{315b^4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-16*a^3 + 6*a*b^2*(2*c - 5*d*x) + 24*a^2*b*Sqrt[c + d*x] + 7*b^3*Sqrt[c + d*x]*(-4*c + 5*d*x)))/(315*b^4*d^2)

IntegrateAlgebraic [A] time = 0.07, size = 95, normalized size = 0.71

$$\frac{4(a + b\sqrt{c + dx})^{3/2}(16a^3 - 24a^2b\sqrt{c + dx} + 30ab^2(c + dx) - 42ab^2c - 35b^3(c + dx)^{3/2} + 63b^3c\sqrt{c + dx})}{315b^4d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*(a + b*Sqrt[c + d*x])^(3/2)*(16*a^3 - 42*a*b^2*c - 24*a^2*b*Sqrt[c + d*x] + 63*b^3*c*Sqrt[c + d*x] + 30*a*b^2*(c + d*x) - 35*b^3*(c + d*x)^(3/2)))/(315*b^4*d^2)

fricas [A] time = 0.91, size = 103, normalized size = 0.77

$$\frac{4(35b^4d^2x^2 - 28b^4c^2 + 36a^2b^2c - 16a^4 + (7b^4c - 6a^2b^2)dx + (5ab^3dx - 16ab^3c + 8a^3b)\sqrt{dx + c})\sqrt{\sqrt{dx + c}b + a}}{315b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/315*(35*b^4*d^2*x^2 - 28*b^4*c^2 + 36*a^2*b^2*c - 16*a^4 + (7*b^4*c - 6*a^2*b^2)*d*x + (5*a*b^3*d*x - 16*a*b^3*c + 8*a^3*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)

giac [B] time = 0.37, size = 279, normalized size = 2.10

$$\frac{4\left(\frac{35(\sqrt{dx+c})^{\frac{5}{2}}d^2-105\sqrt{dx+c}b+d^2c-15(\sqrt{dx+c})^{\frac{7}{2}}+63(\sqrt{dx+c}b+a)^{\frac{5}{2}}d-105(\sqrt{dx+c}b+a)^{\frac{3}{2}}d^2+105\sqrt{dx+c}b+a^{\frac{5}{2}}}{b^4d} + \frac{63(\sqrt{dx+c}b+a)^{\frac{5}{2}}d^2-210(\sqrt{dx+c}b+a)^{\frac{3}{2}}d^2+315\sqrt{dx+c}b+a^{\frac{5}{2}}d^2-35(\sqrt{dx+c}b+a)^{\frac{9}{2}}+180(\sqrt{dx+c}b+a)^{\frac{7}{2}}d-378(\sqrt{dx+c}b+a)^{\frac{5}{2}}d^2+420(\sqrt{dx+c}b+a)^{\frac{3}{2}}d^2-315\sqrt{dx+c}b+a^{\frac{5}{2}}}{b^5d}\right)}{315bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] -4/315*(3*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^3)*a/(b^3*d) + (63*(sqrt(d*x + c)*b + a)^(5/2)*b^2*c - 210*(sqrt(d*x + c)*b + a)^(3/2)*a*b^2*c + 315*sqrt(sqrt(d*x + c)*b + a)*a^2*b^2*c - 35*(sqrt(d*x + c)*b + a)^(9/2) + 180*(sqrt(d*x + c)*b + a)^(7/2)*a - 378*(sqrt(d*x + c)*b + a)^(5/2)*a^2 + 420*(sqrt(d*x + c)*b + a)^(3/2)*a^3 - 315*sqrt(sqrt(d*x + c)*b + a)*a^4)/(b^3*d)/(b*d)

maple [A] time = 0.00, size = 94, normalized size = 0.71

$$\frac{-\frac{12(a + \sqrt{dx+c}b)^{\frac{7}{2}}a}{7} - \frac{4(-b^2c+a^2)(a + \sqrt{dx+c}b)^{\frac{3}{2}}a}{3} + \frac{4(a + \sqrt{dx+c}b)^{\frac{9}{2}}}{9} + \frac{4(-b^2c+3a^2)(a + \sqrt{dx+c}b)^{\frac{5}{2}}}{5}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+(d*x+c)^(1/2)*b)^(1/2),x)`

[Out] $4/d^2/b^4*(1/9*(a+(d*x+c)^(1/2)*b)^(9/2)-3/7*a*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-b^2*c+3*a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*(-b^2*c+a^2)*a*(a+(d*x+c)^(1/2)*b)^(3/2))$

maxima [A] time = 0.94, size = 93, normalized size = 0.70

$$\frac{4 \left(35 (\sqrt{dx+cb+a})^{\frac{9}{2}} - 135 (\sqrt{dx+cb+a})^{\frac{7}{2}} a - 63 (b^2c - 3a^2) (\sqrt{dx+cb+a})^{\frac{5}{2}} + 105 (ab^2c - a^3) (\sqrt{dx+cb+a})^{\frac{3}{2}} \right)}{315 b^4 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $4/315*(35*(\text{sqrt}(d*x + c)*b + a)^(9/2) - 135*(\text{sqrt}(d*x + c)*b + a)^(7/2)*a - 63*(b^2*c - 3*a^2)*(\text{sqrt}(d*x + c)*b + a)^(5/2) + 105*(a*b^2*c - a^3)*(\text{sqrt}(d*x + c)*b + a)^(3/2))/(b^4*d^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{a + b \sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*(c + d*x)^(1/2))^(1/2),x)`

[Out] `int(x*(a + b*(c + d*x)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{a + b \sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*(d*x+c)**(1/2))**(1/2),x)`

[Out] `Integral(x*sqrt(a + b*sqrt(c + d*x)), x)`

$$3.373 \quad \int \sqrt{a + b\sqrt{c + dx}} \, dx$$

Optimal. Leaf size=56

$$\frac{4(a + b\sqrt{c + dx})^{5/2}}{5b^2d} - \frac{4a(a + b\sqrt{c + dx})^{3/2}}{3b^2d}$$

Rubi [A] time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {247, 190, 43}

$$\frac{4(a + b\sqrt{c + dx})^{5/2}}{5b^2d} - \frac{4a(a + b\sqrt{c + dx})^{3/2}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sqrt[c + d*x]], x]

[Out] (-4*a*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d) + (4*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \sqrt{a + b\sqrt{c + dx}} \, dx &= \frac{\text{Subst}\left(\int \sqrt{a + b\sqrt{x}} \, dx, x, c + dx\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int x\sqrt{a + bx} \, dx, x, \sqrt{c + dx}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{a\sqrt{a+bx}}{b} + \frac{(a+bx)^{3/2}}{b}\right) dx, x, \sqrt{c + dx}\right)}{d} \\ &= -\frac{4a(a + b\sqrt{c + dx})^{3/2}}{3b^2d} + \frac{4(a + b\sqrt{c + dx})^{5/2}}{5b^2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.77

$$\frac{4(a + b\sqrt{c + dx})^{3/2} (3b\sqrt{c + dx} - 2a)}{15b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*(a + b*Sqrt[c + d*x])^(3/2)*(-2*a + 3*b*Sqrt[c + d*x]))/(15*b^2*d)

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 1.00

$$-\frac{4\sqrt{a + b\sqrt{c + dx}} (2a^2 - ab\sqrt{c + dx} - 3b^2(c + dx))}{15b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(2*a^2 - a*b*Sqrt[c + d*x] - 3*b^2*(c + d*x)))/(15*b^2*d)

fricas [A] time = 1.02, size = 50, normalized size = 0.89

$$\frac{4(3b^2dx + 3b^2c + \sqrt{dx + c}ab - 2a^2)\sqrt{\sqrt{dx + c}b + a}}{15b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/15*(3*b^2*d*x + 3*b^2*c + sqrt(d*x + c)*a*b - 2*a^2)*sqrt(sqrt(d*x + c)*b + a)/(b^2*d)

giac [B] time = 0.36, size = 99, normalized size = 1.77

$$\frac{4\left(\frac{5\left(\sqrt{dx+cb+a}\right)^3-3\sqrt{\sqrt{dx+cb+a}}a}{b} + \frac{3\left(\sqrt{dx+cb+a}\right)^5-10\left(\sqrt{dx+cb+a}\right)^3a+15\sqrt{\sqrt{dx+cb+a}}a^2}{b}\right)}{15bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 4/15*(5*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)*a/b + (3*(sqrt(d*x + c)*b + a)^(5/2) - 10*(sqrt(d*x + c)*b + a)^(3/2)*a + 15*sqrt(sqrt(d*x + c)*b + a)*a^2)/b)/(b*d)

maple [A] time = 0.00, size = 41, normalized size = 0.73

$$\frac{-\frac{4(a+\sqrt{dx+c}b)^2a}{3} + \frac{4(a+\sqrt{dx+c}b)^5}{5}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+(d*x+c)^(1/2)*b)^(1/2),x)

[Out] 4/d/b^2*(1/5*(a+(d*x+c)^(1/2)*b)^(5/2)-1/3*a*(a+(d*x+c)^(1/2)*b)^(3/2))

maxima [A] time = 0.91, size = 43, normalized size = 0.77

$$\frac{4 \left(\frac{3(\sqrt{dx+c}b+a)^5}{b^2} - \frac{5(\sqrt{dx+c}b+a)^3 a}{b^2} \right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] 4/15*(3*(sqrt(d*x + c)*b + a)^(5/2)/b^2 - 5*(sqrt(d*x + c)*b + a)^(3/2)*a/b^2)/d

mupad [B] time = 3.39, size = 44, normalized size = 0.79

$$\frac{4(a + b\sqrt{c + dx})^{5/2}}{5b^2d} - \frac{4a(a + b\sqrt{c + dx})^{3/2}}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^(1/2),x)

[Out] (4*(a + b*(c + d*x)^(1/2))^(5/2))/(5*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(sqrt(a + b*sqrt(c + d*x)), x)

$$3.374 \quad \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} dx$$

Optimal. Leaf size=116

$$4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}} \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}} \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Rubi [A] time = 0.16, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {371, 1398, 825, 827, 1166, 207}

$$4\sqrt{a+b\sqrt{c+dx}} - 2\sqrt{a-b\sqrt{c}} \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) - 2\sqrt{a+b\sqrt{c}} \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x,x]

[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 825

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x]]/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]

Rule 827

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne

$Q[c*d^2 - a*e^2, 0] \ \&\& \ \text{PosQ}[b^2 - 4*a*c]$

Rule 1398

$\text{Int}[(a_ + (c_)*(x_)^{(n2_)})^{(p_)}*((d_ + (e_)*(x_)^{(n_)})^{(q_)}), x_Symbol] :> \text{With}[\{g = \text{Denominator}[n]\}, \text{Dist}[g, \text{Subst}[\text{Int}[x^{(g-1)}*(d + e*x^{(g*n)})^{(q)}*(a + c*x^{(2*g*n)})^{(p)}, x], x, x^{(1/g)}], x]] /; \text{FreeQ}[\{a, c, d, e, p, q\}, x] \ \&\& \ \text{EqQ}[n2, 2*n] \ \&\& \ \text{FractionQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + b\sqrt{c + dx}}}{x} dx &= \text{Subst} \left(\int \frac{\sqrt{a + b\sqrt{x}}}{-c + x} dx, x, c + dx \right) \\ &= 2 \text{Subst} \left(\int \frac{x\sqrt{a + bx}}{-c + x^2} dx, x, \sqrt{c + dx} \right) \\ &= 4\sqrt{a + b\sqrt{c + dx}} + 2 \text{Subst} \left(\int \frac{bc + ax}{\sqrt{a + bx}(-c + x^2)} dx, x, \sqrt{c + dx} \right) \\ &= 4\sqrt{a + b\sqrt{c + dx}} + 4 \text{Subst} \left(\int \frac{-a^2 + b^2c + ax^2}{a^2 - b^2c - 2ax^2 + x^4} dx, x, \sqrt{a + b\sqrt{c + dx}} \right) \\ &= 4\sqrt{a + b\sqrt{c + dx}} + (2(a - b\sqrt{c})) \text{Subst} \left(\int \frac{1}{-a + b\sqrt{c} + x^2} dx, x, \sqrt{a + b\sqrt{c + dx}} \right) \\ &= 4\sqrt{a + b\sqrt{c + dx}} - 2\sqrt{a - b\sqrt{c}} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a - b\sqrt{c}}} \right) - 2\sqrt{a + b\sqrt{c}} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a + b\sqrt{c}}} \right) \end{aligned}$$

Mathematica [A] time = 0.13, size = 116, normalized size = 1.00

$$4\sqrt{a + b\sqrt{c + dx}} - 2\sqrt{a - b\sqrt{c}} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a - b\sqrt{c}}} \right) - 2\sqrt{a + b\sqrt{c}} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a + b\sqrt{c}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x,x]

[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] - 2*Sqrt[a - b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]] - 2*Sqrt[a + b*Sqrt[c]]*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]

IntegrateAlgebraic [A] time = 0.33, size = 147, normalized size = 1.27

$$4\sqrt{a + b\sqrt{c + dx}} + 2\sqrt{b\sqrt{c} - a} \tan^{-1} \left(\frac{\sqrt{b\sqrt{c} - a} \sqrt{a + b\sqrt{c + dx}}}{a - b\sqrt{c}} \right) + 2\sqrt{-a - b\sqrt{c}} \tan^{-1} \left(\frac{\sqrt{-a - b\sqrt{c}} \sqrt{a + b\sqrt{c + dx}}}{a + b\sqrt{c}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x,x]

[Out] 4*Sqrt[a + b*Sqrt[c + d*x]] + 2*Sqrt[-a + b*Sqrt[c]]*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])] + 2*Sqrt[-a - b*Sqrt[c]]*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])]

fricas [B] time = 0.96, size = 194, normalized size = 1.67

$$-\sqrt{a+\sqrt{b^2c}} \log\left(2\sqrt{\sqrt{dx+cb+a}+2\sqrt{a+\sqrt{b^2c}}}\right) + \sqrt{a+\sqrt{b^2c}} \log\left(2\sqrt{\sqrt{dx+cb+a}-2\sqrt{a+\sqrt{b^2c}}}\right) - \sqrt{a-\sqrt{b^2c}} \log\left(2\sqrt{\sqrt{dx+cb+a}+2\sqrt{a-\sqrt{b^2c}}}\right) + \sqrt{a-\sqrt{b^2c}} \log\left(2\sqrt{\sqrt{dx+cb+a}-2\sqrt{a-\sqrt{b^2c}}}\right) + 4\sqrt{\sqrt{dx+cb+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="fricas")

[Out] -sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a + sqrt(b^2*c))) + sqrt(a + sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a + sqrt(b^2*c))) - sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) + 2*sqrt(a - sqrt(b^2*c))) + sqrt(a - sqrt(b^2*c))*log(2*sqrt(sqrt(d*x + c)*b + a) - 2*sqrt(a - sqrt(b^2*c))) + 4*sqrt(sqrt(d*x + c)*b + a)

giac [A] time = 0.49, size = 150, normalized size = 1.29

$$\frac{2 \left(2 \sqrt{\sqrt{dx+cb+a}b} - \frac{(b^3c-a^2b) \arctan\left(\frac{\sqrt{\sqrt{dx+cb+a}}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{(b\sqrt{c+a})\sqrt{b\sqrt{c-a}}} + \frac{(b^3c-a^2b) \arctan\left(\frac{\sqrt{\sqrt{dx+cb+a}}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{(b\sqrt{c-a})\sqrt{-b\sqrt{c-a}}} \right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="giac")

[Out] 2*(2*sqrt(sqrt(d*x + c)*b + a)*b - (b^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c))))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^3*c - a^2*b)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c) - a)*sqrt(-b*sqrt(c) - a))/b

maple [B] time = 0.04, size = 221, normalized size = 1.91

$$\frac{2b^2c \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\sqrt{-a-\sqrt{b^2c}}} - \frac{2b^2c \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{b^2c}\sqrt{-a+\sqrt{b^2c}}} + \frac{2a \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}} + \frac{2a \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}} + 4\sqrt{a+\sqrt{dx+c}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x,x)

[Out] 4*(a+(d*x+c)^(1/2)*b)^(1/2)+2/(b^2*c)^(1/2)/((-b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((-b^2*c)^(1/2)-a)^(1/2))*b^2*c+2/((-b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((-b^2*c)^(1/2)-a)^(1/2))*a-2/((b^2*c)^(1/2)/((-b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*b^2*c+2/((b^2*c)^(1/2)-a)^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/((b^2*c)^(1/2)-a)^(1/2))*a

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+cb+a}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x,x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(c + d*x)^(1/2))^(1/2)/x,x)`

[Out] `int((a + b*(c + d*x)^(1/2))^(1/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(d*x+c)**(1/2))**(1/2)/x,x)`

[Out] `Integral(sqrt(a + b*sqrt(c + d*x))/x, x)`

$$3.375 \quad \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^2} dx$$

Optimal. Leaf size=137

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}} - \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Rubi [A] time = 0.17, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {371, 1398, 821, 12, 708, 1093, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}}{x} + \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a-b\sqrt{c}}} - \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]

[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*Sqrt[a + b*Sqrt[c]]*Sqrt[c])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 708

Int[1/(Sqrt[(d_) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2*e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 821

Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q_.], x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2} dx &= d \operatorname{Subst} \left(\int \frac{\sqrt{a + b\sqrt{x}}}{(-c + x)^2} dx, x, c + dx \right) \\
&= (2d) \operatorname{Subst} \left(\int \frac{x\sqrt{a + bx}}{(-c + x^2)^2} dx, x, \sqrt{c + dx} \right) \\
&= -\frac{\sqrt{a + b\sqrt{c + dx}}}{x} - \frac{d \operatorname{Subst} \left(\int -\frac{bc}{2\sqrt{a+bx}(-c+x^2)} dx, x, \sqrt{c + dx} \right)}{c} \\
&= -\frac{\sqrt{a + b\sqrt{c + dx}}}{x} + \frac{1}{2}(bd) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + bx}(-c + x^2)} dx, x, \sqrt{c + dx} \right) \\
&= -\frac{\sqrt{a + b\sqrt{c + dx}}}{x} + (b^2d) \operatorname{Subst} \left(\int \frac{1}{a^2 - b^2c - 2ax^2 + x^4} dx, x, \sqrt{a + b\sqrt{c + dx}} \right) \\
&= -\frac{\sqrt{a + b\sqrt{c + dx}}}{x} + \frac{(bd) \operatorname{Subst} \left(\int \frac{1}{-a-b\sqrt{c}+x^2} dx, x, \sqrt{a + b\sqrt{c + dx}} \right)}{2\sqrt{c}} - \frac{(bd) \operatorname{Subst} \left(\int \frac{1}{-a-b\sqrt{c}+x^2} dx, x, \sqrt{a + b\sqrt{c + dx}} \right)}{2\sqrt{c}} \\
&= -\frac{\sqrt{a + b\sqrt{c + dx}}}{x} + \frac{bd \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}} \right)}{2\sqrt{a-b\sqrt{c}} \sqrt{c}} - \frac{bd \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}} \right)}{2\sqrt{a+b\sqrt{c}} \sqrt{c}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 181, normalized size = 1.32

$$\frac{(a - b\sqrt{c}) \left(2\sqrt{c} (a + b\sqrt{c}) \sqrt{a + b\sqrt{c + dx}} + bdx\sqrt{a + b\sqrt{c}} \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}} \right) \right) - bdx\sqrt{a - b\sqrt{c}} (a + b\sqrt{c}) \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}} \right)}{2\sqrt{c}x(b^2c - a^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]
```

```
[Out] (- (b*Sqrt[a - b*Sqrt[c]]*(a + b*Sqrt[c])*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]) + (a - b*Sqrt[c])*(2*(a + b*Sqrt[c])*Sqrt[c]*Sqrt[a + b*Sqrt[c + d*x]] + b*Sqrt[a + b*Sqrt[c]]*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[c]*(-a^2 + b^2*c)*x)
```

IntegrateAlgebraic [A] time = 0.51, size = 168, normalized size = 1.23

$$-\frac{\sqrt{a + b\sqrt{c + dx}}}{x} + \frac{bd \tan^{-1}\left(\frac{\sqrt{b\sqrt{c} - a} \sqrt{a + b\sqrt{c + dx}}}{a - b\sqrt{c}}\right)}{2\sqrt{c} \sqrt{b\sqrt{c} - a}} - \frac{bd \tan^{-1}\left(\frac{\sqrt{-a - b\sqrt{c}} \sqrt{a + b\sqrt{c + dx}}}{a + b\sqrt{c}}\right)}{2\sqrt{c} \sqrt{-a - b\sqrt{c}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x^2,x]

[Out] -(Sqrt[a + b*Sqrt[c + d*x]]/x) + (b*d*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])])/(2*Sqrt[-a + b*Sqrt[c]]*Sqrt[c]) - (b*d*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])])/(2*Sqrt[-a - b*Sqrt[c]]*Sqrt[c])

fricas [B] time = 1.08, size = 1003, normalized size = 7.32

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="fricas")

[Out] -1/4*(x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 + (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)) - x*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)) + x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 + (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)) - x*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c))*log(sqrt(sqrt(d*x + c)*b + a)*b^4*d^3 - (b^4*c*d^2 + sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(a*b^2*c^2 - a^3*c))*sqrt(-(a*b^2*d^2 - sqrt(b^6*d^4/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))*(b^2*c^2 - a^2*c))/(b^2*c^2 - a^2*c)) + 4*sqrt(sqrt(d*x + c)*b + a)/x

giac [B] time = 0.66, size = 232, normalized size = 1.69

$$\frac{2\sqrt{\sqrt{dx+cb+a}b^3d^2}}{b^2c - (\sqrt{dx+cb+a})^2 + 2(\sqrt{dx+cb+a})a - a^2} - \frac{(b^3cd^2|b|+ab^3\sqrt{c}d^2) \arctan\left(\frac{\sqrt{\sqrt{dx+cb+a}}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\left(\frac{3}{bc^2+ac}\right)\sqrt{b\sqrt{c}-a}|b|} + \frac{(b^3cd^2|b|-ab^3\sqrt{c}d^2) \arctan\left(\frac{\sqrt{\sqrt{dx+cb+a}}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\left(\frac{3}{bc^2-ac}\right)\sqrt{-b\sqrt{c}-a}|b|}$$

2bd

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="giac")

[Out] 1/2*(2*sqrt(sqrt(d*x + c)*b + a)*b^3*d^2/(b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2) - (b^3*c*d^2*abs(b) + a*b^3*sqrt(c)*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*c^(3/2) + a*c)*sqrt(b*sqrt(c) - a)*abs(b)) + (b^3*c*d^2*abs(b) - a*b^3*sqrt(c)*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*c^(3/2) - a*c)*sqrt(-b*sqrt(c) - a)*abs(b)))/(b*d)

maple [A] time = 0.03, size = 151, normalized size = 1.10

$$\frac{b^2 d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}} b}{\sqrt{-a-\sqrt{b^2c}}}\right)}{2\sqrt{b^2c} \sqrt{-a-\sqrt{b^2c}}} - \frac{b^2 d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}} b}{\sqrt{-a+\sqrt{b^2c}}}\right)}{2\sqrt{b^2c} \sqrt{-a+\sqrt{b^2c}}} - \frac{\sqrt{a+\sqrt{dx+c}} b b^2 d}{-b^2c + (dx+c) b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^2,x)

[Out] $-d*b^2*(a+(d*x+c)^{(1/2)*b}^{(1/2)})/((d*x+c)*b^2-b^2*c)+1/2*d*b^2/(b^2*c)^{(1/2)}/(-a-(b^2*c)^{(1/2)})^{(1/2)}*\arctan((a+(d*x+c)^{(1/2)*b}^{(1/2)})/(-a-(b^2*c)^{(1/2)})^{(1/2)})-1/2*d*b^2/(b^2*c)^{(1/2)}/(-a+(b^2*c)^{(1/2)})^{(1/2)}*\arctan((a+(d*x+c)^{(1/2)*b}^{(1/2)})/(-a+(b^2*c)^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c} b + a}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^2,x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2,x)

[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b\sqrt{c + dx}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**2,x)

[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**2, x)

$$3.376 \quad \int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3} dx$$

Optimal. Leaf size=224

$$\frac{bd(bc - a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8cx(a^2 - b^2c)} - \frac{bd^2(2a - 3b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}(a - b\sqrt{c})^{3/2}} + \frac{bd^2(2a + 3b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}(a + b\sqrt{c})^{3/2}}$$

Rubi [A] time = 0.43, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {371, 1398, 821, 12, 741, 827, 1166, 207}

$$\frac{bd(bc - a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8cx(a^2 - b^2c)} - \frac{bd^2(2a - 3b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}(a - b\sqrt{c})^{3/2}} + \frac{bd^2(2a + 3b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}(a + b\sqrt{c})^{3/2}} - \frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]
```

```
[Out] -Sqrt[a + b*Sqrt[c + d*x]]/(2*x^2) + (b*d*(b*c - a*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]]/(8*c*(a^2 - b^2*c)*x) - (b*(2*a - 3*b*Sqrt[c])*d^2*ArcTan h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(16*(a - b*Sqrt[c])^(3/2) *c^(3/2)) + (b*(2*a + 3*b*Sqrt[c])*d^2*ArcTan h[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(16*(a + b*Sqrt[c])^(3/2)*c^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan h[Rt[b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 741

```
Int[((d_) + (e_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp [((d + e*x)^(m + 1)*(a*e + c*d*x)*(a + c*x^2)^(p + 1))/(2*a*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Simp [c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_)^(m_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^m*(a + c*x^2)^(p + 1)*(a*g - c*f*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1), x], x]
```

```
(p + 1)*Simp[a*e*g*m - c*d*f*(2*p + 3) - c*e*f*(m + 2*p + 3)*x, x], x], x]
/; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && G
tQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 827

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)),
x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x
^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && N
eQ[c*d^2 + a*e^2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symb
ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3} dx &= d^2 \operatorname{Subst} \left(\int \frac{\sqrt{a+b\sqrt{x}}}{(-c+x)^3} dx, x, c+dx \right) \\
&= (2d^2) \operatorname{Subst} \left(\int \frac{x\sqrt{a+bx}}{(-c+x^2)^3} dx, x, \sqrt{c+dx} \right) \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} - \frac{d^2 \operatorname{Subst} \left(\int -\frac{bc}{2\sqrt{a+bx}(-c+x^2)^2} dx, x, \sqrt{c+dx} \right)}{2c} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{1}{4} (bd^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+bx}(-c+x^2)^2} dx, x, \sqrt{c+dx} \right) \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd(bc-a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)x} + \frac{(bd^2) \operatorname{Subst} \left(\int \frac{\frac{1}{2}(-2a^2+3b^2c)}{\sqrt{a+bx}(-c+x^2)} dx, x, \sqrt{c+dx} \right)}{8c(a^2-b^2c)} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd(bc-a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)x} + \frac{(bd^2) \operatorname{Subst} \left(\int \frac{\frac{a^2b}{2} + \frac{1}{2}b(-2a^2)}{a^2-b^2c} dx, x, \sqrt{c+dx} \right)}{4c} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd(bc-a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)x} + \frac{(b(2a-3b\sqrt{c})d^2) \operatorname{Subst} \left(\int \frac{1}{\sqrt{a+bx}(-c+x^2)} dx, x, \sqrt{c+dx} \right)}{16c} \\
&= -\frac{\sqrt{a+b\sqrt{c+dx}}}{2x^2} + \frac{bd(bc-a\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{8c(a^2-b^2c)x} - \frac{b(2a-3b\sqrt{c})d^2 \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}} \right)}{16(a-b\sqrt{c})^{3/2} c}
\end{aligned}$$

Mathematica [A] time = 0.42, size = 258, normalized size = 1.15

$$\frac{-(a-b\sqrt{c}) \left(2\sqrt{c}(a+b\sqrt{c})\sqrt{a+b\sqrt{c+dx}}(4a^2c+abd^2\sqrt{c+dx}-b^2c(dx+dx))-bd^2x^2\sqrt{a+b\sqrt{c}}(2a^2+ab\sqrt{c}-3b^2c)\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)-bd^2x^2(2a-3b\sqrt{c})\sqrt{a-b\sqrt{c}}(a+b\sqrt{c})^2\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right) \right)}{16c^{3/2}x^2(a^2-b^2c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]

[Out] $(-(b*(2*a - 3*b*\sqrt{c}))*\sqrt{a - b*\sqrt{c}}*(a + b*\sqrt{c})^2*d^2*x^2*\operatorname{ArcTanh}[\sqrt{a + b*\sqrt{c + d*x}}/\sqrt{a - b*\sqrt{c}}]) - (a - b*\sqrt{c})*(2*(a + b*\sqrt{c}))*\sqrt{c}*\sqrt{a + b*\sqrt{c + d*x}}*(4*a^2*c + a*b*d*x*\sqrt{c + d*x} - b^2*c*(4*c + d*x)) - b*\sqrt{a + b*\sqrt{c}}*(2*a^2 + a*b*\sqrt{c} - 3*b^2*c)*d^2*x^2*\operatorname{ArcTanh}[\sqrt{a + b*\sqrt{c + d*x}}/\sqrt{a + b*\sqrt{c}}])/(16*c^{3/2}*(a^2 - b^2*c)^2*x^2)$

IntegrateAlgebraic [A] time = 3.08, size = 310, normalized size = 1.38

$$\frac{\sqrt{a+b\sqrt{c+dx}}(-4a^2cd^2-abd^2(c+dx)^{3/2}+abcd^2\sqrt{c+dx}+3b^2c^2d^2+b^2cd^2(c+dx))}{8cd^2x^2(b^2c-a^2)} - \frac{d^2(2ab-3b^2\sqrt{c})\tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{16c^{3/2}(a-b\sqrt{c})\sqrt{b\sqrt{c}-a}} + \frac{d^2(2ab+3b^2\sqrt{c})\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{16c^{3/2}\sqrt{-a-b\sqrt{c}}(a+b\sqrt{c})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*Sqrt[c + d*x]]/x^3,x]

[Out] $-1/8*(\sqrt{a + b*\sqrt{c + d*x}}*(-4*a^2*c*d^2 + 3*b^2*c^2*d^2 + a*b*c*d^2*\sqrt{c + d*x} + b^2*c*d^2*(c + d*x) - a*b*d^2*(c + d*x)^{3/2}))/c*(-a^2 + b^2*c)$

$$\begin{aligned} &^2*c)*d^2*x^2) - ((2*a*b - 3*b^2*\text{Sqrt}[c])*d^2*\text{ArcTan}[(\text{Sqrt}[-a + b*\text{Sqrt}[c]]* \\ &\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]])/(a - b*\text{Sqrt}[c]))/(16*(a - b*\text{Sqrt}[c])* \text{Sqrt}[-a + \\ &b*\text{Sqrt}[c]]*c^{(3/2)}) + ((2*a*b + 3*b^2*\text{Sqrt}[c])*d^2*\text{ArcTan}[(\text{Sqrt}[-a - b*\text{Sqrt}[\\ &c]]*\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]])/(a + b*\text{Sqrt}[c]))/(16*\text{Sqrt}[-a - b*\text{Sqrt}[c]]* \\ &(a + b*\text{Sqrt}[c])*c^{(3/2)}) \end{aligned}$$

fricas [B] time = 1.31, size = 2856, normalized size = 12.75

result too large to display

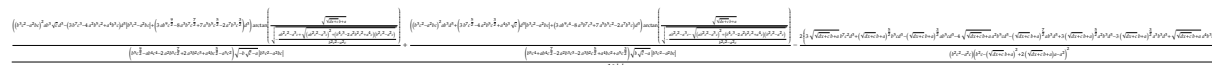
Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="fricas")

[Out]
$$\frac{1}{32} * ((b^2*c^2 - a^2*c)*x^2*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*\log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*\text{sqrt}(\text{sqrt}(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)) - (b^2*c^2 - a^2*c)*x^2*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*\log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*\text{sqrt}(\text{sqrt}(d*x + c)*b + a)*d^6 - ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 - 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 + (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)) + (b^2*c^2 - a^2*c)*x^2*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*\log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*\text{sqrt}(\text{sqrt}(d*x + c)*b + a)*d^6 + ((27*b^10*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)) - (b^2*c^2 - a^2*c)*x^2*\text{sqrt}(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*\text{sqrt}((81*b^14*c^2 - 90*a^2*b^12*c + 25*a^4*b^10)*d^8/(b^12*c^9 - 6*a^2*b^10*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^10*b^2*c^4 + a^12*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3))*\log((81*b^10*c^2 - 81*a^2*b^8*c + 20*a^4*b^6)*\text{sqrt}(\text{sqrt}(d*x + c)$$

$$*b + a)*d^6 - ((27*b^{10}*c^4 - 24*a^2*b^8*c^3 + 5*a^4*b^6*c^2)*d^4 + 2*(2*a*b^8*c^7 - 7*a^3*b^6*c^6 + 9*a^5*b^4*c^5 - 5*a^7*b^2*c^4 + a^9*c^3)*sqrt((81*b^{14}*c^2 - 90*a^2*b^{12}*c + 25*a^4*b^{10})*d^8/(b^{12}*c^9 - 6*a^2*b^{10}*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^{10}*b^2*c^4 + a^{12}*c^3))) * sqrt(-((15*a*b^6*c^2 - 15*a^3*b^4*c + 4*a^5*b^2)*d^4 - (b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)*sqrt((81*b^{14}*c^2 - 90*a^2*b^{12}*c + 25*a^4*b^{10})*d^8/(b^{12}*c^9 - 6*a^2*b^{10}*c^8 + 15*a^4*b^8*c^7 - 20*a^6*b^6*c^6 + 15*a^8*b^4*c^5 - 6*a^{10}*b^2*c^4 + a^{12}*c^3))))/(b^6*c^6 - 3*a^2*b^4*c^5 + 3*a^4*b^2*c^4 - a^6*c^3)) - 4*(b^2*c*d*x - sqrt(d*x + c)*a*b*d*x + 4*b^2*c^2 - 4*a^2*c)*sqrt(sqrt(d*x + c)*b + a)/((b^2*c^2 - a^2*c)*x^2)$$

giac [B] time = 1.10, size = 895, normalized size = 4.00

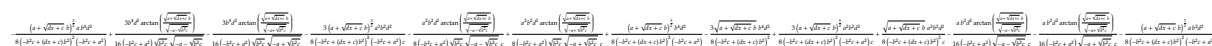


Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="giac")

$$[Out] 1/16*(((b^3*c^2 - a^2*b*c)^2*a*b^3*sqrt(c)*d^3 - (3*b^7*c^3 - 4*a^2*b^5*c^2 + a^4*b^3*c)*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^(9/2) - 8*a^3*b^7*c^(7/2) + 7*a^5*b^5*c^(5/2) - 2*a^7*b^3*c^(3/2))*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c + sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c))))/(b^2*c^2 - a^2*c)))/(b^5*c^(9/2) - a*b^4*c^4 - 2*a^2*b^3*c^(7/2) + 2*a^3*b^2*c^3 + a^4*b*c^(5/2) - a^5*c^2)*sqrt(-b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) + ((b^3*c^2 - a^2*b*c)^2*a*b^3*d^3 + (3*b^7*c^(5/2) - 4*a^2*b^5*c^(3/2) + a^4*b^3*sqrt(c))*d^3*abs(b^3*c^2 - a^2*b*c) + (3*a*b^9*c^4 - 8*a^3*b^7*c^3 + 7*a^5*b^5*c^2 - 2*a^7*b^3*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c^2 - a^3*c - sqrt((a*b^2*c^2 - a^3*c)^2 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c^2 - a^2*c))))/(b^2*c^2 - a^2*c)))/(b^5*c^4 + a*b^4*c^(7/2) - 2*a^2*b^3*c^3 - 2*a^3*b^2*c^(5/2) + a^4*b*c^2 + a^5*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^3*c^2 - a^2*b*c)) - 2*(3*sqrt(sqrt(d*x + c)*b + a)*b^7*c^2*d^3 + (sqrt(d*x + c)*b + a)^(5/2)*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(3/2)*a*b^5*c*d^3 - 4*sqrt(sqrt(d*x + c)*b + a)*a^2*b^5*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a*b^3*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/2)*a^2*b^3*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^3*b^3*d^3 + sqrt(sqrt(d*x + c)*b + a)*a^4*b^3*d^3)/(b^2*c^2 - a^2*c)*(b^2*c - sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2)/(b*d)$$

maple [B] time = 0.04, size = 784, normalized size = 3.50



Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+(d*x+c)^(1/2)*b)^(1/2)/x^3,x)

$$[Out] -1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)+3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(5/2)*a^2-1/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*a/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-3/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2*a^3/c/(-b^2*c+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-3/8*b^4*d^2/(-b^2*c+(d*x+c)*b^2)^2*(a+(d*x+c)^(1/2)*b)^(1/2)+1/8*b^2*d^2/(-b^2*c+(d*x+c)*b^2)^2/c*(a+(d*x+c)^(1/2)*b)^(1/2)*a^2+3/16*b^4*d^2/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))-1/16*b^2*d^2/c/(-b^2*c+a^2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))*a-1/8*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))*a^2-3/16*b^4*d^2/(-b^2*c+a^2)/(b^2*c)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))$$

$$\frac{\sqrt{\frac{1}{2}}}{(-a+(b^2*c)^{\frac{1}{2}})^{\frac{1}{2}}}-\frac{1}{16*b^2*d^2/c/(-b^2*c+a^2)/(-a+(b^2*c)^{\frac{1}{2}})^{\frac{1}{2}}*\arctan((a+(d*x+c)^{\frac{1}{2}}*b)^{\frac{1}{2}}/(-a+(b^2*c)^{\frac{1}{2}})^{\frac{1}{2}})*a+1/8*b^2*d^2/c/(-b^2*c+a^2)/(b^2*c)^{\frac{1}{2}}/(-a+(b^2*c)^{\frac{1}{2}})^{\frac{1}{2}}*\arctan((a+(d*x+c)^{\frac{1}{2}}*b)^{\frac{1}{2}}/(-a+(b^2*c)^{\frac{1}{2}})^{\frac{1}{2}})*a^2}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{dx+c}b+a}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^(1/2)/x^3,x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(d*x + c)*b + a)/x^3, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3,x)

[Out] int((a + b*(c + d*x)^(1/2))^(1/2)/x^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+b\sqrt{c+dx}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)**(1/2))**(1/2)/x**3,x)

[Out] Integral(sqrt(a + b*sqrt(c + d*x))/x**3, x)

$$3.377 \quad \int \frac{x^3}{a+b\sqrt{c+dx}} dx$$

Optimal. Leaf size=230

$$\frac{2a(a^2 - b^2c)^3 \log(a + b\sqrt{c + dx})}{b^8 d^4} + \frac{2(a^2 - b^2c)^3 \sqrt{c + dx}}{b^7 d^4} - \frac{a(a^2 - 3b^2c)(c + dx)^2}{2b^4 d^4} + \frac{2(a^2 - 3b^2c)(c + dx)^{5/2}}{5b^3 d^4} - \frac{a}{b^3 d^4}$$

Rubi [A] time = 0.26, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2(-3a^2b^2c + a^4 + 3b^4c^2)(c + dx)^{3/2}}{3b^5d^4} - \frac{ax(-3a^2b^2c + a^4 + 3b^4c^2)}{b^6d^3} + \frac{2(a^2 - 3b^2c)(c + dx)^{3/2}}{5b^3d^4} - \frac{a(a^2 - 3b^2c)(c + dx)^2}{2b^4d^4} + \frac{2(a^2 - b^2c)^3 \sqrt{c + dx}}{b^7d^4} - \frac{2a(a^2 - b^2c)^3 \log(a + b\sqrt{c + dx})}{b^8d^4} - \frac{a(c + dx)^3}{3b^2d^4} + \frac{2(c + dx)^{7/2}}{7bd^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*Sqrt[c + d*x]),x]

[Out] -((a*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3)) + (2*(a^2 - b^2*c)^3*Sqrt[c + d*x])/(b^7*d^4) + (2*(a^4 - 3*a^2*b^2*c + 3*b^4*c^2)*(c + d*x)^(3/2))/(3*b^5*d^4) - (a*(a^2 - 3*b^2*c)*(c + d*x)^2)/(2*b^4*d^4) + (2*(a^2 - 3*b^2*c)*(c + d*x)^(5/2))/(5*b^3*d^4) - (a*(c + d*x)^3)/(3*b^2*d^4) + (2*(c + d*x)^(7/2))/(7*b*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x^3}{a + b\sqrt{c + dx}} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{a+b\sqrt{x}} dx, x, c + dx\right)}{d^4} \\ &= \frac{2 \text{Subst}\left(\int \frac{x^{(-c+x^2)^3}}{a+bx} dx, x, \sqrt{c + dx}\right)}{d^4} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{(-a^2+b^2c)^3}{b^7} - \frac{a(a^4-3a^2b^2c+3b^4c^2)x}{b^6} + \frac{(a^4-3a^2b^2c+3b^4c^2)x^2}{b^5} - \frac{a(a^2-3b^2c)x^3}{b^4} - \frac{(-a^2+3b^2c)x^4}{b^3}\right) dx, x, \sqrt{c + dx}\right)}{d^4} \\ &= -\frac{a(a^4 - 3a^2b^2c + 3b^4c^2)x}{b^6d^3} + \frac{2(a^2 - b^2c)^3 \sqrt{c + dx}}{b^7d^4} + \frac{2(a^4 - 3a^2b^2c + 3b^4c^2)(c + dx)^{3/2}}{3b^5d^4} \end{aligned}$$

Mathematica [A] time = 0.20, size = 213, normalized size = 0.93

$$\frac{b(420a^6\sqrt{c+dx} - 210a^5b\sqrt{c+dx} - 140a^4b^2(8c-dx)\sqrt{c+dx} - 105a^3b^3dx(dx-4c) + 84a^2b^4\sqrt{c+dx}(11c^2-3cdx+d^2x^2) - 35ab^5dx(6c^2-3cdx+2d^2x^2) + 12b^6\sqrt{c+dx}(-16c^3+8c^2dx-6cd^2x^2+5d^3x^3)) - 420a(a^2-b^2c)^3\log(a+b\sqrt{c+dx})}{210b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (b*(-210*a^5*b*d*x - 105*a^3*b^3*d*x*(-4*c + d*x) + 420*a^6*Sqrt[c + d*x] - 140*a^4*b^2*(8*c - d*x)*Sqrt[c + d*x] + 84*a^2*b^4*Sqrt[c + d*x]*(11*c^2 - 3*c*d*x + d^2*x^2) - 35*a*b^5*d*x*(6*c^2 - 3*c*d*x + 2*d^2*x^2) + 12*b^6*Sqrt[c + d*x]*(-16*c^3 + 8*c^2*d*x - 6*c*d^2*x^2 + 5*d^3*x^3)) - 420*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(210*b^8*d^4)
```

IntegrateAlgebraic [A] time = 0.15, size = 268, normalized size = 1.17

$$\frac{\sqrt{c+dx}(420a^6 - 210a^5b\sqrt{c+dx} + 140a^4b^2(c+dx) - 1260a^3b^3c + 105a^2b^4(c+dx)^2 + 630ab^5c\sqrt{c+dx} + 1260a^2b^4(c+dx)^2 - 420a^3b^3c + 420a^2b^4(c+dx)^2 + 60b^6(c+dx)^3 - 252b^6c(c+dx)^2) - 2a(a^2-b^2c)^3\log(a+b\sqrt{c+dx})}{210b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/(a + b*Sqrt[c + d*x]),x]
```

```
[Out] (Sqrt[c + d*x]*(420*a^6 - 1260*a^4*b^2*c + 1260*a^2*b^4*c^2 - 420*b^6*c^3 - 210*a^5*b*Sqrt[c + d*x] + 630*a^3*b^3*c*Sqrt[c + d*x] - 630*a*b^5*c^2*Sqrt[c + d*x] + 140*a^4*b^2*(c + d*x) - 420*a^2*b^4*c*(c + d*x) + 420*b^6*c^2*(c + d*x) - 105*a^3*b^3*(c + d*x)^(3/2) + 315*a*b^5*c*(c + d*x)^(3/2) + 84*a^2*b^4*(c + d*x)^2 - 252*b^6*c*(c + d*x)^2 - 70*a*b^5*(c + d*x)^(5/2) + 60*b^6*(c + d*x)^3))/(210*b^7*d^4) - (2*a*(a^2 - b^2*c)^3*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)
```

fricas [A] time = 0.87, size = 228, normalized size = 0.99

$$\frac{70ab^6d^3x^3 - 105(ab^6c - a^2b^4)d^2x^2 + 210(ab^6c^2 - 2a^2b^4c + a^2b^2)d^2x - 420(ab^6c^3 - 3a^2b^4c^2 + 3a^2b^2c - a^2)\log(\sqrt{dx+c}b+a) - 4(15b^7d^3x^3 - 48b^7c^3 + 231a^2b^5c^2 - 280a^4b^3c + 105a^6b - 3(6b^7c - 7a^2b^5)d^2x^2 + (24b^7c^2 - 63a^2b^5c + 35a^4b^3)d^2x)\sqrt{dx+c}}{210b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/210*(70*a*b^6*d^3*x^3 - 105*(a*b^6*c - a^3*b^4)*d^2*x^2 + 210*(a*b^6*c^2 - 2*a^3*b^4*c + a^5*b^2)*d*x - 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(sqrt(d*x + c)*b + a) - 4*(15*b^7*d^3*x^3 - 48*b^7*c^3 + 231*a^2*b^5*c^2 - 280*a^4*b^3*c + 105*a^6*b - 3*(6*b^7*c - 7*a^2*b^5)*d^2*x^2 + (24*b^7*c^2 - 63*a^2*b^5*c + 35*a^4*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4)
```

giac [A] time = 0.37, size = 341, normalized size = 1.48

$$\frac{2(ab^6c^2 - 3a^3b^4c + 3a^5b^2)\log(\sqrt{dx+c}b+a) - 80ab^6 - 252ab^6c + 630ab^6c^2 + 420ab^6c^3 - 420\sqrt{dx+c}b^7c^3 - 70ab^6c^3 + 335ab^6c^2 + 630ab^6c - 630ab^6 + 60ab^6c^2 + 84ab^6 + 630ab^6c^2 - 420ab^6 + 630ab^6c^2 + 1260\sqrt{dx+c}b^7c^3 - 105ab^6c^3 + 630ab^6c^2 + 630ab^6c + 140ab^6 + 630ab^6c^2 - 1260\sqrt{dx+c}b^7c^3 - 210ab^6 + 420\sqrt{dx+c}b^7c^3}{210b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(abs(sqrt(d*x + c)*b + a))/(b^8*d^4) + 1/210*(60*(d*x + c)^(7/2)*b^6*d^24 - 252*(d*x + c)^(5/2)*b^6*c*d^24 + 420*(d*x + c)^(3/2)*b^6*c^2*d^24 - 420*sqrt(d*x + c)*b^6*c^3*d^24 - 70*(d*x + c)^3*a*b^5*d^24 + 315*(d*x + c)^2*a*b^5*c*d^24 - 630*(d*x + c)*a*b^5*c^2*d^24 + 84*(d*x + c)^(5/2)*a^2*b^4*d^24 - 420*(d*x + c)^(3/2)*a^2*b^4*c*d^24 + 1260*sqrt(d*x + c)*a^2*b^4*c^2*d^24 - 105*(d*x + c)^2*a^3*b^3*d^24 + 630*(d*x + c)*a^3*b^3*c*d^24 + 140*(d*x + c)^(3/2)*a^4*b^2*d^24 - 1260*sqrt(d*x + c)*a^4*b^2*c*d^24 - 210*(d*x + c)*a^5*b*d^24 + 420*sqrt(d*x + c)*a^6*d^24)/(b^7*d^28)
```

maple [A] time = 0.01, size = 394, normalized size = 1.71

$$\frac{ax^3}{3b^2d^4} - \frac{ax^2}{2b^2d^3} + \frac{a^2x}{2b^2d^2} - \frac{2ac^2 \ln(e + \sqrt{dx+c})}{b^2d^4} - \frac{a^2c^2x}{b^2d^4} - \frac{6a^2c^2 \ln(a + \sqrt{dx+c})}{b^2d^4} - \frac{2a^2cx}{b^2d^4} - \frac{11a^2c^2}{6b^2d^4} - \frac{2\sqrt{dx+c}c^3}{b^2d^4} + \frac{6a^2c \ln(e + \sqrt{dx+c})}{b^2d^4} - \frac{a^2x}{b^2d^4} - \frac{5a^2c^2}{2b^2d^4} + \frac{6\sqrt{dx+c}d^2c^2}{b^2d^4} + \frac{2(dx+c)^2c^2}{b^2d^4} - \frac{2a^2 \ln(a + \sqrt{dx+c})}{b^2d^4} - \frac{a^2c}{b^2d^4} - \frac{6\sqrt{dx+c}d^2c}{b^2d^4} - \frac{2(dx+c)^2c}{b^2d^4} - \frac{6(dx+c)^2c}{5b^2d^4} - \frac{2\sqrt{dx+c}d^2c}{b^2d^4} - \frac{2(dx+c)^2d^2}{3b^2d^4} - \frac{2(dx+c)^2d^2}{5b^2d^4} - \frac{2(dx+c)^2}{7b^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+(d*x+c)^(1/2)*b), x)
[Out] -11/6/d^4/b^2*a*c^3-6/5/d^4/b*(d*x+c)^(5/2)*c+2/5/d^4/b^3*(d*x+c)^(5/2)*a^2
+2/d^4/b*(d*x+c)^(3/2)*c^2-2/d^4/b*(d*x+c)^(1/2)*c^3+2/3/d^4/b^5*(d*x+c)^(3
/2)*a^4+2/7*(d*x+c)^(7/2)/b/d^4+2/d^4/b^7*(d*x+c)^(1/2)*a^6-1/d^3/b^6*x*a^5
-1/2/d^2/b^4*x^2*a^3+6/d^4/b^3*(d*x+c)^(1/2)*a^2*c^2-6/d^4/b^5*(d*x+c)^(1/2
)*a^4*c-1/3/d/b^2*x^3*a-2/d^4/b^3*(d*x+c)^(3/2)*a^2*c+1/2/d^2/b^2*x^2*a*c-1
/d^3/b^2*x*a*c^2+2/d^3/b^4*x*a^3*c-1/d^4/b^6*a^5*c+5/2/d^4/b^4*a^3*c^2+2/d^
4*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3-6/d^4*a^3/b^4*ln(a+(d*x+c)^(1/2)*b)*c^2+6
/d^4*a^5/b^6*ln(a+(d*x+c)^(1/2)*b)*c-2/d^4*a^7/b^8*ln(a+(d*x+c)^(1/2)*b)
```

maxima [A] time = 0.90, size = 243, normalized size = 1.06

$$\frac{60(dx+c)^7b^6-70(dx+c)^3ab^5-84(3b^6c-a^2b^4)(dx+c)^5+105(3ab^5c-a^3b^3)(dx+c)^2+140(3b^6c^2-3a^2b^4c+a^4b^2)(dx+c)^3-210(3ab^5c^2-3a^3b^3c+a^5b)(dx+c)-420(b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6)\sqrt{dx+c}+420(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7)\log(\sqrt{dx+c}b+a)}{210d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2)), x, algorithm="maxima")
[Out] 1/210*((60*(d*x + c)^(7/2)*b^6 - 70*(d*x + c)^3*a*b^5 - 84*(3*b^6*c - a^2*b
^4)*(d*x + c)^(5/2) + 105*(3*a*b^5*c - a^3*b^3)*(d*x + c)^2 + 140*(3*b^6*c^
2 - 3*a^2*b^4*c + a^4*b^2)*(d*x + c)^(3/2) - 210*(3*a*b^5*c^2 - 3*a^3*b^3*c
+ a^5*b)*(d*x + c) - 420*(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt
(d*x + c))/b^7 + 420*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*log(s
qrt(d*x + c)*b + a)/b^8)/d^4
```

mupad [B] time = 0.07, size = 317, normalized size = 1.38

$$\frac{2(c+d*x)^{7/2}}{7b^4d^4} - \left(\frac{a^2 \left(\frac{6c}{b^2d^4} - \frac{2a^2}{b^2d^4} \right) - \frac{6c^2}{b^2d^4}}{b^2} \right) \sqrt{c+d*x} - \left(\frac{6c}{3b^2d^4} - \frac{2a^2}{b^2d^4} \right) (c+d*x)^{3/2} - \left(\frac{6c}{5b^4d^4} - \frac{2a^2}{5b^4d^4} \right) (c+d*x)^{5/2} + \frac{a \left(\frac{6c}{b^2d^4} - \frac{2a^2}{b^2d^4} \right) (c+d*x)^2}{4b} - \frac{a(c+d*x)^3}{3b^2d^4} - \frac{\ln(a+b\sqrt{c+d*x}) (2a^7-6a^5b^2c+6a^3b^4c^2-2ab^6c^3)}{b^8d^4} + \frac{a dx \left(\frac{6c}{b^2d^4} - \frac{2a^2}{b^2d^4} \right) - \frac{6c^2}{b^2d^4}}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + b*(c + d*x)^(1/2)), x)
[Out] (2*(c + d*x)^(7/2))/(7*b*d^4) - ((a^2*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d
^4)))/b^2 - (6*c^2)/(b*d^4)))/b^2 + (2*c^3)/(b*d^4))*(c + d*x)^(1/2) - ((a^
2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/(3*b^2) - (2*c^2)/(b*d^4))*(c + d*x)
^(3/2) - ((6*c)/(5*b*d^4) - (2*a^2)/(5*b^3*d^4))*(c + d*x)^(5/2) + (a*((6*c
)/(b*d^4) - (2*a^2)/(b^3*d^4))*(c + d*x)^2)/(4*b) - (a*(c + d*x)^3)/(3*b^2*
d^4) - (log(a + b*(c + d*x)^(1/2))*(2*a^7 - 6*a^5*b^2*c - 2*a*b^6*c^3 + 6*a
^3*b^4*c^2))/(b^8*d^4) + (a*d*x*((a^2*((6*c)/(b*d^4) - (2*a^2)/(b^3*d^4)))/
b^2 - (6*c^2)/(b*d^4)))/(2*b)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{a + b\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+b*(d*x+c)**(1/2)), x)
[Out] Integral(x**3/(a + b*sqrt(c + d*x)), x)
```

$$3.378 \quad \int \frac{x^2}{a+b\sqrt{c+dx}} dx$$

Optimal. Leaf size=151

$$-\frac{2a(a^2 - b^2c)^2 \log(a + b\sqrt{c + dx})}{b^6d^3} + \frac{2(a^2 - b^2c)^2 \sqrt{c + dx}}{b^5d^3} - \frac{ax(a^2 - 2b^2c)}{b^4d^2} + \frac{2(a^2 - 2b^2c)(c + dx)^{3/2}}{3b^3d^3} - \frac{a(c + dx)^2}{2b^2d^3} + \frac{2(c + dx)^{5/2}}{5bd^3}$$

Rubi [A] time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2(a^2 - 2b^2c)(c + dx)^{3/2}}{3b^3d^3} + \frac{2(a^2 - b^2c)^2 \sqrt{c + dx}}{b^5d^3} - \frac{ax(a^2 - 2b^2c)}{b^4d^2} - \frac{2a(a^2 - b^2c)^2 \log(a + b\sqrt{c + dx})}{b^6d^3} - \frac{a(c + dx)^2}{2b^2d^3} + \frac{2(c + dx)^{5/2}}{5bd^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*Sqrt[c + d*x]),x]

[Out] -((a*(a^2 - 2*b^2*c)*x)/(b^4*d^2)) + (2*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^5*d^3) + (2*(a^2 - 2*b^2*c)*(c + d*x)^(3/2))/(3*b^3*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (2*(c + d*x)^(5/2))/(5*b*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{a+b\sqrt{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{a+b\sqrt{x}} dx, x, c+dx\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)^2}{a+bx} dx, x, \sqrt{c+dx}\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{(-a^2+b^2c)^2}{b^5} - \frac{a(a^2-2b^2c)x}{b^4} - \frac{(-a^2+2b^2c)x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^3} \\ &= -\frac{a(a^2 - 2b^2c)x}{b^4d^2} + \frac{2(a^2 - b^2c)^2 \sqrt{c + dx}}{b^5d^3} + \frac{2(a^2 - 2b^2c)(c + dx)^{3/2}}{3b^3d^3} - \frac{a(c + dx)^2}{2b^2d^3} + \frac{2(c + dx)^{5/2}}{5bd^3} \end{aligned}$$

Mathematica [A] time = 0.13, size = 138, normalized size = 0.91

$$\frac{b(60a^4\sqrt{c+dx} - 30a^3bdx - 20a^2b^2(5c-dx)\sqrt{c+dx} - 15ab^3dx(dx-2c) + 4b^4\sqrt{c+dx}(8c^2-4cdx+3d^2x^2)) - 60a(a^2-b^2c)^2 \log(a+b\sqrt{c+dx})}{30b^6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*sqrt[c + d*x]), x]

[Out] (b*(-30*a^3*b*d*x - 15*a*b^3*d*x*(-2*c + d*x) + 60*a^4*sqrt[c + d*x] - 20*a^2*b^2*(5*c - d*x)*sqrt[c + d*x] + 4*b^4*sqrt[c + d*x]*(8*c^2 - 4*c*d*x + 3*d^2*x^2)) - 60*a*(a^2 - b^2*c)^2*Log[a + b*sqrt[c + d*x]])/(30*b^6*d^3)

IntegrateAlgebraic [A] time = 0.13, size = 160, normalized size = 1.06

$$\frac{\sqrt{c+dx}(60a^4 - 30a^3b\sqrt{c+dx} + 20a^2b^2(c+dx) - 120a^2b^2c - 15ab^3(c+dx)^{3/2} + 60ab^3c\sqrt{c+dx} + 60b^4c^2 + 12b^4(c+dx)^2 - 40b^4c(c+dx)) - 2a(a^2-b^2c)^2 \log(a+b\sqrt{c+dx})}{30b^5d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a + b*sqrt[c + d*x]), x]

[Out] (sqrt[c + d*x]*(60*a^4 - 120*a^2*b^2*c + 60*b^4*c^2 - 30*a^3*b*sqrt[c + d*x] + 60*a*b^3*c*sqrt[c + d*x] + 20*a^2*b^2*(c + d*x) - 40*b^4*c*(c + d*x) - 15*a*b^3*(c + d*x)^(3/2) + 12*b^4*(c + d*x)^2))/(30*b^5*d^3) - (2*a*(a^2 - b^2*c)^2*Log[a + b*sqrt[c + d*x]])/(b^6*d^3)

fricas [A] time = 0.83, size = 138, normalized size = 0.91

$$\frac{15ab^4d^2x^2 - 30(ab^4c - a^3b^2)dx + 60(ab^4c^2 - 2a^3b^2c + a^5) \log(\sqrt{dx+c}b+a) - 4(3b^5d^2x^2 + 8b^5c^2 - 25a^2b^3c + 15a^4b - (4b^5c - 5a^2b^3)dx)\sqrt{dx+c}}{30b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2)), x, algorithm="fricas")

[Out] -1/30*(15*a*b^4*d^2*x^2 - 30*(a*b^4*c - a^3*b^2)*d*x + 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a) - 4*(3*b^5*d^2*x^2 + 8*b^5*c^2 - 25*a^2*b^3*c + 15*a^4*b - (4*b^5*c - 5*a^2*b^3)*d*x)*sqrt(d*x + c))/(b^6*d^3)

giac [A] time = 0.41, size = 198, normalized size = 1.31

$$\frac{2(ab^4c^2 - 2a^3b^2c + a^5) \log(\sqrt{dx+c}b+a)}{b^6d^3} + \frac{12(dx+c)^{5/2}b^4d^{12} - 40(dx+c)^{3/2}b^4cd^{12} + 60\sqrt{dx+c}b^4d^{12} - 15(dx+c)^2ab^3d^{12} + 60(dx+c)ab^3cd^{12} + 20(dx+c)^3a^2b^2d^{12} - 120\sqrt{dx+c}a^2b^2cd^{12} - 30(dx+c)a^3bd^{12} + 60\sqrt{dx+c}a^4d^{12}}{30b^5d^{15}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2)), x, algorithm="giac")

[Out] -2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(abs(sqrt(d*x + c)*b + a))/(b^6*d^3) + 1/30*(12*(d*x + c)^(5/2)*b^4*d^12 - 40*(d*x + c)^(3/2)*b^4*c*d^12 + 60*sqrt(d*x + c)*b^4*c^2*d^12 - 15*(d*x + c)^2*a*b^3*d^12 + 60*(d*x + c)*a*b^3*c*d^12 + 20*(d*x + c)^(3/2)*a^2*b^2*d^12 - 120*sqrt(d*x + c)*a^2*b^2*c*d^12 - 30*(d*x + c)*a^3*b*d^12 + 60*sqrt(d*x + c)*a^4*d^12)/(b^5*d^15)

maple [A] time = 0.01, size = 235, normalized size = 1.56

$$\frac{ax^2}{2b^2d} - \frac{2a^2c \ln(a + \sqrt{dx+c}b)}{b^2d^3} + \frac{acx}{b^2d^2} + \frac{4a^3c \ln(a + \sqrt{dx+c}b)}{b^4d^3} - \frac{a^3x}{b^4d^2} + \frac{3ac^2}{2b^2d^3} + \frac{2\sqrt{dx+c}c^2}{b^4d^3} - \frac{2a^5 \ln(a + \sqrt{dx+c}b)}{b^6d^3} - \frac{a^3c}{b^4d^3} - \frac{4\sqrt{dx+c}a^2c}{b^5d^3} - \frac{4(dx+c)^{3/2}c}{3bd^3} + \frac{2\sqrt{dx+c}a^4}{b^5d^3} + \frac{2(dx+c)^{5/2}a^2}{3b^3d^3} + \frac{2(dx+c)^{5/2}}{5bd^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+(d*x+c)^(1/2)*b), x)

[Out] 2/5*(d*x+c)^(5/2)/b/d^3-1/2/d/b^2*x^2*a+1/d^2/b^2*x*a*c+3/2/d^3/b^2*a*c^2-4/3/d^3/b*(d*x+c)^(3/2)*c+2/3/d^3/b^3*(d*x+c)^(3/2)*a^2+2/d^3/b*(d*x+c)^(1/2)*c^2-1/d^2/b^4*x*a^3-1/d^3/b^4*a^3*c-4/d^3/b^3*(d*x+c)^(1/2)*a^2*c+2/d^3/b

$\int \frac{5(d*x+c)^{1/2} * a^4 - 2/d^3 * a/b^2 * \ln(a+(d*x+c)^{1/2} * b) * c^2 + 4/d^3 * a^3/b^4 * \ln(a+(d*x+c)^{1/2} * b) * c - 2/d^3 * a^5/b^6 * \ln(a+(d*x+c)^{1/2} * b)}{30 d^3} dx$

maxima [A] time = 0.88, size = 148, normalized size = 0.98

$$\frac{12(dx+c)^{5/2}b^4 - 15(dx+c)^2ab^3 - 20(2b^4c - a^2b^2)(dx+c)^{3/2} + 30(2ab^3c - a^3b)(dx+c) + 60(b^4c^2 - 2a^2b^2c + a^4)\sqrt{dx+c}}{b^5} - \frac{60(ab^4c^2 - 2a^3b^2c + a^5)\log(\sqrt{dx+c}b+a)}{b^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] 1/30*((12*(d*x + c)^(5/2)*b^4 - 15*(d*x + c)^2*a*b^3 - 20*(2*b^4*c - a^2*b^2)*(d*x + c)^(3/2) + 30*(2*a*b^3*c - a^3*b)*(d*x + c) + 60*(b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(d*x + c))/b^5 - 60*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*log(sqrt(d*x + c)*b + a)/b^6)/d^3

mupad [B] time = 3.21, size = 184, normalized size = 1.22

$$\frac{2(c+dx)^{5/2}}{5bd^3} - \left(\frac{a^2 \left(\frac{4c}{bd^3} - \frac{2a^2}{b^3d^3} \right)}{b^2} - \frac{2c^2}{bd^3} \right) \sqrt{c+dx} - \left(\frac{4c}{3bd^3} - \frac{2a^2}{3b^3d^3} \right) (c+dx)^{3/2} - \frac{\ln(a+b\sqrt{c+dx}) (2a^5 - 4a^3b^2c + 2ab^4c^2)}{b^6d^3} - \frac{a(c+dx)^2}{2b^2d^3} + \frac{adx \left(\frac{4c}{bd^3} - \frac{2a^2}{b^3d^3} \right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*(c + d*x)^(1/2)),x)

[Out] (2*(c + d*x)^(5/2))/(5*b*d^3) - ((a^2*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/b^2 - (2*c^2)/(b*d^3))*(c + d*x)^(1/2) - ((4*c)/(3*b*d^3) - (2*a^2)/(3*b^3*d^3))*(c + d*x)^(3/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^5 - 4*a^3*b^2*c + 2*a*b^4*c^2))/(b^6*d^3) - (a*(c + d*x)^2)/(2*b^2*d^3) + (a*d*x*((4*c)/(b*d^3) - (2*a^2)/(b^3*d^3)))/(2*b)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{a + b\sqrt{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(a+b*(d*x+c)**(1/2)),x)

[Out] Integral(x**2/(a + b*sqrt(c + d*x)), x)

$$3.379 \quad \int \frac{x}{a+b\sqrt{c+dx}} dx$$

Optimal. Leaf size=90

$$-\frac{2a(a^2 - b^2c) \log(a + b\sqrt{c + dx})}{b^4d^2} + \frac{2(a^2 - b^2c) \sqrt{c + dx}}{b^3d^2} - \frac{ax}{b^2d} + \frac{2(c + dx)^{3/2}}{3bd^2}$$

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{2(a^2 - b^2c) \sqrt{c + dx}}{b^3d^2} - \frac{2a(a^2 - b^2c) \log(a + b\sqrt{c + dx})}{b^4d^2} - \frac{ax}{b^2d} + \frac{2(c + dx)^{3/2}}{3bd^2}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*Sqrt[c + d*x]),x]

[Out] -((a*x)/(b^2*d)) + (2*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^3*d^2) + (2*(c + d*x)^(3/2))/(3*b*d^2) - (2*a*(a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x}{a+b\sqrt{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{a+b\sqrt{x}} dx, x, c+dx\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)}{a+bx} dx, x, \sqrt{c+dx}\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{a^2-b^2c}{b^3} - \frac{ax}{b^2} + \frac{x^2}{b} + \frac{-a^3+ab^2c}{b^3(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^2} \\ &= -\frac{ax}{b^2d} + \frac{2(a^2 - b^2c) \sqrt{c + dx}}{b^3d^2} + \frac{2(c + dx)^{3/2}}{3bd^2} - \frac{2a(a^2 - b^2c) \log(a + b\sqrt{c + dx})}{b^4d^2} \end{aligned}$$

Mathematica [A] time = 0.07, size = 82, normalized size = 0.91

$$\frac{b(6a^2\sqrt{c+dx} - 3abdx + 2b^2(dx-2c)\sqrt{c+dx}) - 6(a^3 - ab^2c)\log(a + b\sqrt{c+dx})}{3b^4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*Sqrt[c + d*x]), x]

[Out] (b*(-3*a*b*d*x + 6*a^2*Sqrt[c + d*x] + 2*b^2*(-2*c + d*x)*Sqrt[c + d*x]) - 6*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(3*b^4*d^2)

IntegrateAlgebraic [A] time = 0.07, size = 88, normalized size = 0.98

$$\frac{\sqrt{c+dx}(6a^2 - 3ab\sqrt{c+dx} + 2b^2(c+dx) - 6b^2c)}{3b^3d^2} - \frac{2(a^3 - ab^2c)\log(a + b\sqrt{c+dx})}{b^4d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b*Sqrt[c + d*x]), x]

[Out] (Sqrt[c + d*x]*(6*a^2 - 6*b^2*c - 3*a*b*Sqrt[c + d*x] + 2*b^2*(c + d*x)))/(3*b^3*d^2) - (2*(a^3 - a*b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)

fricas [A] time = 0.92, size = 71, normalized size = 0.79

$$\frac{3ab^2dx - 6(ab^2c - a^3)\log(\sqrt{dx+c}b + a) - 2(b^3dx - 2b^3c + 3a^2b)\sqrt{dx+c}}{3b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2)), x, algorithm="fricas")

[Out] -1/3*(3*a*b^2*d*x - 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a) - 2*(b^3*d*x - 2*b^3*c + 3*a^2*b)*sqrt(d*x + c))/(b^4*d^2)

giac [A] time = 0.33, size = 105, normalized size = 1.17

$$\frac{6(ab^2c - a^3)\log(\sqrt{dx+c}b + a)}{b^4d} + \frac{2(dx+c)^{\frac{3}{2}}b^2d^2 - 6\sqrt{dx+c}b^2cd^2 - 3(dx+c)abd^2 + 6\sqrt{dx+c}a^2d^2}{b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2)), x, algorithm="giac")

[Out] 1/3*(6*(a*b^2*c - a^3)*log(abs(sqrt(d*x + c)*b + a))/(b^4*d) + (2*(d*x + c)^(3/2)*b^2*d^2 - 6*sqrt(d*x + c)*b^2*c*d^2 - 3*(d*x + c)*a*b*d^2 + 6*sqrt(d*x + c)*a^2*d^2)/(b^3*d^3))/d

maple [A] time = 0.00, size = 116, normalized size = 1.29

$$\frac{2ac\ln(a + \sqrt{dx+c}b)}{b^2d^2} - \frac{ax}{b^2d} - \frac{2a^3\ln(a + \sqrt{dx+c}b)}{b^4d^2} - \frac{ac}{b^2d^2} - \frac{2\sqrt{dx+c}c}{bd^2} + \frac{2\sqrt{dx+c}a^2}{b^3d^2} + \frac{2(dx+c)^{\frac{3}{2}}}{3bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+(d*x+c)^(1/2)*b), x)

[Out] 2/3*(d*x+c)^(3/2)/b/d^2 - a*x/b^2/d - 1/d^2/b^2*a*c - 2/d^2/b*(d*x+c)^(1/2)*c + 2/d^2/b^3*(d*x+c)^(1/2)*a^2 + 2/d^2*a/b^2*ln(a+(d*x+c)^(1/2)*b)*c - 2/d^2*a^3/b^4*ln(a+(d*x+c)^(1/2)*b)

maxima [A] time = 0.88, size = 81, normalized size = 0.90

$$\frac{\frac{2(dx+c)^{\frac{3}{2}}b^2 - 3(dx+c)ab - 6(b^2c - a^2)\sqrt{dx+c}}{b^3} + \frac{6(ab^2c - a^3)\log(\sqrt{dx+c}b+a)}{b^4}}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] 1/3*((2*(d*x + c)^(3/2)*b^2 - 3*(d*x + c)*a*b - 6*(b^2*c - a^2)*sqrt(d*x + c))/b^3 + 6*(a*b^2*c - a^3)*log(sqrt(d*x + c)*b + a)/b^4)/d^2

mupad [B] time = 0.05, size = 89, normalized size = 0.99

$$\frac{2(c+dx)^{3/2}}{3bd^2} - \left(\frac{2c}{bd^2} - \frac{2a^2}{b^3d^2} \right) \sqrt{c+dx} - \frac{\ln(a+b\sqrt{c+dx})(2a^3-2ab^2c)}{b^4d^2} - \frac{ax}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*(c + d*x)^(1/2)),x)

[Out] (2*(c + d*x)^(3/2))/(3*b*d^2) - ((2*c)/(b*d^2) - (2*a^2)/(b^3*d^2))*(c + d*x)^(1/2) - (log(a + b*(c + d*x)^(1/2))*(2*a^3 - 2*a*b^2*c))/(b^4*d^2) - (a*x)/(b^2*d)

sympy [A] time = 4.72, size = 109, normalized size = 1.21

$$\left\{ \begin{array}{l} \left(\frac{a(a^2-b^2c) \left(\begin{array}{l} \frac{\sqrt{c+dx}}{a} \quad \text{for } b=0 \\ \frac{\log(a+b\sqrt{c+dx})}{b} \quad \text{otherwise} \end{array} \right)}{b^3d} + \frac{(c+dx)^{\frac{3}{2}}}{3bd} + \frac{(a^2-b^2c)\sqrt{c+dx}}{b^3d} \right) \\ \frac{a(c+dx)}{2b^2d} \\ \hline d \quad \text{for } d \neq 0 \\ \frac{x^2}{2(a+b\sqrt{c})} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)**(1/2)),x)

[Out] Piecewise((2*(-a*(c + d*x)/(2*b**2*d) - a*(a**2 - b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True)))/(b**3*d) + (c + d*x)**(3/2)/(3*b*d) + (a**2 - b**2*c)*sqrt(c + d*x)/(b**3*d))/d, Ne(d, 0)), (x**2/(2*(a + b*sqrt(c))), True))

$$3.380 \quad \int \frac{1}{a+b\sqrt{c+dx}} dx$$

Optimal. Leaf size=41

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a \log(a+b\sqrt{c+dx})}{b^2d}$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 190, 43}

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a \log(a+b\sqrt{c+dx})}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^(-1), x]

[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a + b*Sqrt[c + d*x]])/(b^2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{a+b\sqrt{c+dx}} dx &= \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}} dx, x, c+dx\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \frac{x}{a+bx} dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \left(\frac{1}{b} - \frac{a}{b(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2\sqrt{c+dx}}{bd} - \frac{2a \log(a+b\sqrt{c+dx})}{b^2d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{c+dx}}{b} - \frac{a \log(a+b\sqrt{c+dx})}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^(-1),x]

[Out] (2*(Sqrt[c + d*x]/b - (a*Log[a + b*Sqrt[c + d*x]])/b^2))/d

IntegrateAlgebraic [A] time = 0.03, size = 47, normalized size = 1.15

$$\frac{2\sqrt{c+dx}}{bd} - \frac{2a \log(abd + b^2d\sqrt{c+dx})}{b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^(-1),x]

[Out] (2*Sqrt[c + d*x])/(b*d) - (2*a*Log[a*b*d + b^2*d*Sqrt[c + d*x]])/(b^2*d)

fricas [A] time = 0.81, size = 33, normalized size = 0.80

$$-\frac{2(a \log(\sqrt{dx+c}b+a) - \sqrt{dx+c}b)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] -2*(a*log(sqrt(d*x + c)*b + a) - sqrt(d*x + c)*b)/(b^2*d)

giac [A] time = 0.35, size = 38, normalized size = 0.93

$$-\frac{2a \log(|\sqrt{dx+c}b+a|)}{b^2d} + \frac{2\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] -2*a*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*sqrt(d*x + c)/(b*d)

maple [B] time = 0.01, size = 87, normalized size = 2.12

$$\frac{a \ln(-a + \sqrt{dx+c}b)}{b^2d} - \frac{a \ln(a + \sqrt{dx+c}b)}{b^2d} - \frac{a \ln(b^2dx + b^2c - a^2)}{b^2d} + \frac{2\sqrt{dx+c}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+(d*x+c)^(1/2)*b),x)

[Out] 2*(d*x+c)^(1/2)/b/d+1/b^2/d*a*ln(-a+(d*x+c)^(1/2)*b)-a*ln(a+(d*x+c)^(1/2)*b)/b^2/d-a*ln(b^2*d*x+b^2*c-a^2)/b^2/d

maxima [A] time = 0.89, size = 35, normalized size = 0.85

$$-\frac{2\left(\frac{a \log(\sqrt{dx+c}b+a)}{b^2} - \frac{\sqrt{dx+c}}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] -2*(a*log(sqrt(d*x + c)*b + a)/b^2 - sqrt(d*x + c)/b)/d

mupad [B] time = 0.05, size = 33, normalized size = 0.80

$$\frac{2 \left(a \ln \left(a + b \sqrt{c + dx} \right) - b \sqrt{c + dx} \right)}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^(1/2)), x)

[Out] -(2*(a*log(a + b*(c + d*x)^(1/2)) - b*(c + d*x)^(1/2)))/(b^2*d)

sympy [A] time = 0.55, size = 49, normalized size = 1.20

$$\begin{cases} \frac{x}{a} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{a+b\sqrt{c}} & \text{for } d = 0 \\ -\frac{2a \log\left(\frac{a}{b} + \sqrt{c+dx}\right)}{b^2 d} + \frac{2\sqrt{c+dx}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**(1/2)), x)

[Out] Piecewise((x/a, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c)), Eq(d, 0)), (-2*a*log(a/b + sqrt(c + d*x))/(b**2*d) + 2*sqrt(c + d*x)/(b*d), True))

$$3.381 \quad \int \frac{1}{x(a+b\sqrt{c+dx})} dx$$

Optimal. Leaf size=82

$$-\frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c} + \frac{2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c} + \frac{a \log(x)}{a^2-b^2c}$$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {371, 1398, 801, 635, 206, 260}

$$-\frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c} + \frac{2b\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{a^2-b^2c} + \frac{a \log(x)}{a^2-b^2c}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*Sqrt[c + d*x])),x]

[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]/(a^2 - b^2*c) + (a*Log[x])/(a^2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(a+b\sqrt{c+dx})} dx &= \text{Subst} \left(\int \frac{1}{(a+b\sqrt{x})(-c+x)} dx, x, c+dx \right) \\
&= 2 \text{Subst} \left(\int \frac{x}{(a+bx)(-c+x^2)} dx, x, \sqrt{c+dx} \right) \\
&= 2 \text{Subst} \left(\int \left(-\frac{ab}{(a^2-b^2c)(a+bx)} + \frac{bc-ax}{(a^2-b^2c)(c-x^2)} \right) dx, x, \sqrt{c+dx} \right) \\
&= -\frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c} + \frac{2 \text{Subst} \left(\int \frac{bc-ax}{c-x^2} dx, x, \sqrt{c+dx} \right)}{a^2-b^2c} \\
&= -\frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c} - \frac{(2a) \text{Subst} \left(\int \frac{x}{c-x^2} dx, x, \sqrt{c+dx} \right)}{a^2-b^2c} + \frac{(2bc) \text{Subst} \left(\int \frac{1}{c-x^2} dx, x, \sqrt{c+dx} \right)}{a^2-b^2c} \\
&= \frac{2b\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx}}{\sqrt{c}} \right)}{a^2-b^2c} + \frac{a \log(x)}{a^2-b^2c} - \frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c}
\end{aligned}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 0.74

$$\frac{-2a \log(a+b\sqrt{c+dx}) + a \log(dx) + 2b\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx}}{\sqrt{c}} \right)}{a^2-b^2c}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*Sqrt[c + d*x])),x]

[Out] (2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + a*Log[d*x] - 2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)

IntegrateAlgebraic [A] time = 0.08, size = 86, normalized size = 1.05

$$\frac{a \log(-dx)}{a^2-b^2c} - \frac{2a \log(a+b\sqrt{c+dx})}{a^2-b^2c} - \frac{2b\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c+dx}}{\sqrt{c}} \right)}{b^2c-a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*Sqrt[c + d*x])),x]

[Out] (-2*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(-a^2 + b^2*c) + (a*Log[-(d*x)])/(-a^2 - b^2*c) - (2*a*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)

fricas [A] time = 0.65, size = 125, normalized size = 1.52

$$\left[\frac{b\sqrt{c} \log\left(\frac{dx-2\sqrt{dx+c}\sqrt{c+2c}}{x}\right) + 2a \log(\sqrt{dx+c}b+a) - a \log(x)}{b^2c-a^2}, \frac{2b\sqrt{-c} \arctan\left(\frac{\sqrt{dx+c}\sqrt{-c}}{c}\right) + 2a \log(\sqrt{dx+c}b+a) - a \log(x)}{b^2c-a^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] [(b*sqr(c)*log((d*x - 2*sqr(d*x + c)*sqr(c) + 2*c)/x) + 2*a*log(sqr(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2), (2*b*sqr(-c)*arctan(sqr(d*x + c)*sqr(-c)/c) + 2*a*log(sqr(d*x + c)*b + a) - a*log(x))/(b^2*c - a^2)]

giac [A] time = 0.34, size = 88, normalized size = 1.07

$$\frac{2ab \log\left(\left|\sqrt{dx+c}b+a\right|\right)}{b^3c-a^2b} + \frac{2bc \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^2c-a^2)\sqrt{-c}} - \frac{a \log(dx)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] 2*a*b*log(abs(sqrt(d*x + c)*b + a))/(b^3*c - a^2*b) + 2*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^2*c - a^2)*sqrt(-c)) - a*log(d*x)/(b^2*c - a^2)

maple [A] time = 0.01, size = 77, normalized size = 0.94

$$\frac{2b\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{-b^2c+a^2} + \frac{a \ln(dx)}{-b^2c+a^2} - \frac{2a \ln(a + \sqrt{dx+c}b)}{-b^2c+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+(d*x+c)^(1/2)*b),x)

[Out] 1/(-b^2*c+a^2)*a*ln(d*x)+2*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)-2*a*ln(a+(d*x+c)^(1/2)*b)/(-b^2*c+a^2)

maxima [A] time = 2.03, size = 95, normalized size = 1.16

$$\frac{b\sqrt{c} \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^2c-a^2} - \frac{a \log(dx)}{b^2c-a^2} + \frac{2a \log(\sqrt{dx+c}b+a)}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^2*c - a^2) - a*log(d*x)/(b^2*c - a^2) + 2*a*log(sqrt(d*x + c)*b + a)/(b^2*c - a^2)

mupad [B] time = 3.28, size = 181, normalized size = 2.21

$$\frac{\ln(2b^3c^{3/2}-2b^3c\sqrt{c+dx}-6ab^2c+6ab^2\sqrt{c}\sqrt{c+dx})}{a+b\sqrt{c}} + \frac{\ln(-2b^3c^{3/2}-2b^3c\sqrt{c+dx}-6ab^2c-6ab^2\sqrt{c}\sqrt{c+dx})}{a-b\sqrt{c}} + \frac{2a \ln(4b^5c^2\sqrt{c+dx}-36a^3b^2c+4ab^4c^2-36a^2b^3c\sqrt{c+dx})}{b^2c-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^(1/2))),x)

[Out] log(2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c + 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a + b*c^(1/2)) + log(- 2*b^3*c^(3/2) - 2*b^3*c*(c + d*x)^(1/2) - 6*a*b^2*c - 6*a*b^2*c^(1/2)*(c + d*x)^(1/2))/(a - b*c^(1/2)) + (2*a*log(4*b^5*c^2*(c + d*x)^(1/2) - 36*a^3*b^2*c + 4*a*b^4*c^2 - 36*a^2*b^3*c*(c + d*x)^(1/2)))/(b^2*c - a^2)

sympy [A] time = 13.01, size = 85, normalized size = 1.04

$$\frac{2ab \left(\begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b = 0 \\ \frac{\log(a+b\sqrt{c+dx})}{b} & \text{otherwise} \end{cases} \right)}{a^2 - b^2c} - \frac{2 \left(-\frac{a \log(-dx)}{2} + \frac{bc \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} \right)}{a^2 - b^2c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(d*x+c)**(1/2)),x)
```

```
[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b,
True))/(a**2 - b**2*c) - 2*(-a*log(-d*x)/2 + b*c*atan(sqrt(c + d*x)/sqrt(-c
))/sqrt(-c))/(a**2 - b**2*c)
```

$$3.382 \quad \int \frac{1}{x^2(a+b\sqrt{c+dx})} dx$$

Optimal. Leaf size=130

$$-\frac{a-b\sqrt{c+dx}}{x(a^2-b^2c)} + \frac{ab^2d \log(x)}{(a^2-b^2c)^2} - \frac{2ab^2d \log(a+b\sqrt{c+dx})}{(a^2-b^2c)^2} + \frac{bd(a^2+b^2c) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)^2}$$

Rubi [A] time = 0.18, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{a-b\sqrt{c+dx}}{x(a^2-b^2c)} + \frac{ab^2d \log(x)}{(a^2-b^2c)^2} - \frac{2ab^2d \log(a+b\sqrt{c+dx})}{(a^2-b^2c)^2} + \frac{bd(a^2+b^2c) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])),x]
```

```
[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + (b*(a^2 + b^2*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^2) + (a*b^2*d*Log[x])/(a^2 - b^2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
```


$(2*a*c*(p + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{Simp}[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; \text{FreeQ}\{a, c, d, e, f, g\}, x \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p] \mid\mid \text{IntegersQ}[2*m, 2*p])$

Rule 1398

$\text{Int}[(a + (c_*)*(x_)^{(n2_*)})^{(p_*)}*((d_) + (e_*)*(x_)^{(n_*)})^{(q_*)}, x_Symbol] :> \text{With}\{g = \text{Denominator}[n]\}, \text{Dist}[g, \text{Subst}[\text{Int}[x^{(g - 1)}*(d + e*x^{(g*n)})^{(q)}*(a + c*x^{(2*g*n)})^{(p)}, x], x, x^{(1/g)}], x] /; \text{FreeQ}\{a, c, d, e, p, q\}, x \&\& \text{EqQ}[n2, 2*n] \&\& \text{FractionQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2(a + b\sqrt{c + dx})} dx &= d \text{Subst} \left(\int \frac{1}{(a + b\sqrt{x})(-c + x)^2} dx, x, c + dx \right) \\ &= (2d) \text{Subst} \left(\int \frac{x}{(a + bx)(-c + x^2)^2} dx, x, \sqrt{c + dx} \right) \\ &= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x} + \frac{d \text{Subst} \left(\int \frac{-abc + b^2cx}{(a + bx)(-c + x^2)} dx, x, \sqrt{c + dx} \right)}{c(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x} + \frac{d \text{Subst} \left(\int \left(-\frac{2ab^3c}{(a^2 - b^2c)(a + bx)} - \frac{bc(a^2 + b^2c - 2abx)}{(-a^2 + b^2c)(c - x^2)} \right) dx, x, \sqrt{c + dx} \right)}{c(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x} - \frac{2ab^2d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{(bd) \text{Subst} \left(\int \frac{a^2 + b^2c - 2abx}{c - x^2} dx, x, \sqrt{c + dx} \right)}{(a^2 - b^2c)^2} \\ &= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x} - \frac{2ab^2d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} - \frac{(2ab^2d) \text{Subst} \left(\int \frac{x}{c - x^2} dx, x, \sqrt{c + dx} \right)}{(a^2 - b^2c)^2} \\ &= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x} + \frac{b(a^2 + b^2c)d \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{\sqrt{c}(a^2 - b^2c)^2} + \frac{ab^2d \log(x)}{(a^2 - b^2c)^2} - \frac{2ab^2d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} \end{aligned}$$

Mathematica [A] time = 0.19, size = 144, normalized size = 1.11

$$\frac{\sqrt{c} \left(-(a^2 - b^2c)(a - b\sqrt{c + dx}) - ab^2dx \log(a^2 - b^2(c + dx)) + ab^2dx \log(x) + bdx(a^2 + b^2c) \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right) - 2ab^2\sqrt{c} dx \tanh^{-1} \left(\frac{b\sqrt{c + dx}}{a} \right) \right)}{\sqrt{c}x(a^2 - b^2c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])),x]

[Out] $(-2*a*b^2*\text{Sqrt}[c]*d*x*\text{ArcTanh}[(b*\text{Sqrt}[c + d*x])/a] + b*(a^2 + b^2*c)*d*x*\text{ArcTanh}[\text{Sqrt}[c + d*x]/\text{Sqrt}[c]] + \text{Sqrt}[c]*(-(a^2 - b^2*c)*(a - b*\text{Sqrt}[c + d*x])) + a*b^2*d*x*\text{Log}[x] - a*b^2*d*x*\text{Log}[a^2 - b^2*(c + d*x)])/(\text{Sqrt}[c]*(a^2 - b^2*c)^2*x)$

IntegrateAlgebraic [A] time = 0.27, size = 135, normalized size = 1.04

$$\frac{a - b\sqrt{c + dx}}{x(a^2 - b^2c)} + \frac{ab^2d \log(-dx)}{(a^2 - b^2c)^2} - \frac{2ab^2d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{d(a^2b + b^3c) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(b^2c - a^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a + b*Sqrt[c + d*x])),x]

[Out] -((a - b*Sqrt[c + d*x])/((a^2 - b^2*c)*x)) + ((a^2*b + b^3*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(-a^2 + b^2*c)^2) + (a*b^2*d*Log[-(d*x)])/(a^2 - b^2*c)^2 - (2*a*b^2*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2

fricas [A] time = 0.86, size = 283, normalized size = 2.18

$$\frac{4ab^2cdx \log(\sqrt{dx+c}b+a) - 2ab^2cdx \log(x) - 2ab^2c^2 - (b^3c+a^2b)\sqrt{c}dx \log\left(\frac{dx+2\sqrt{dx+c}\sqrt{c}+c}{x}\right) + 2a^3c + 2(b^3c-a^2b)\sqrt{dx+c} - 2ab^2cdx \log(\sqrt{dx+c}b+a) - ab^2cdx \log(x) - ab^2c^2 + (b^3c+a^2b)\sqrt{-c}dx \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right) + a^3c + (b^3c-a^2b)\sqrt{dx+c}}{2(b^4c^3-2a^2b^2c^2+a^4c)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] [-1/2*(4*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - 2*a*b^2*c*d*x*log(x) - 2*a*b^2*c^2 - (b^3*c + a^2*b)*sqrt(c)*d*x*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) + 2*a^3*c + 2*(b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x), -(2*a*b^2*c*d*x*log(sqrt(d*x + c)*b + a) - a*b^2*c*d*x*log(x) - a*b^2*c^2 + (b^3*c + a^2*b)*sqrt(-c)*d*x*arctan(sqrt(d*x + c)*sqrt(-c)/c) + a^3*c + (b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x)]

giac [A] time = 0.46, size = 191, normalized size = 1.47

$$\frac{2ab^3d \log(|\sqrt{dx+c}b+a|)}{b^5c^2 - 2a^2b^3c + a^4b} + \frac{ab^2d \log(-dx)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{(b^3cd + a^2bd) \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^4c^2 - 2a^2b^2c + a^4)\sqrt{-c}} + \frac{ab^2cd - a^3d - (b^3cd - a^2bd)\sqrt{dx+c}}{(b^2c - a^2)^2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] -2*a*b^3*d*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) + a*b^2*d*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - (b^3*c*d + a^2*b*d)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (a*b^2*c*d - a^3*d - (b^3*c*d - a^2*b*d)*sqrt(d*x + c))/((b^2*c - a^2)^2*d*x)

maple [A] time = 0.02, size = 216, normalized size = 1.66

$$\frac{b^3\sqrt{c}d \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(b^2c+a^2)^2} + \frac{ab^2d \ln(dx)}{(b^2c+a^2)^2} - \frac{2ab^2d \ln(a + \sqrt{dx+c}b)}{(b^2c+a^2)^2} + \frac{a^2bd \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(b^2c+a^2)^2\sqrt{c}} + \frac{ab^2c}{(b^2c+a^2)^2x} - \frac{\sqrt{dx+c}b^3c}{(b^2c+a^2)^2x} - \frac{a^3}{(b^2c+a^2)^2x} + \frac{\sqrt{dx+c}a^2b}{(b^2c+a^2)^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+(d*x+c)^(1/2)*b),x)

[Out] -1/((-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b^3*c+1/((-b^2*c+a^2)^2/x*(d*x+c)^(1/2)*b*a^2+1/((-b^2*c+a^2)^2/x*a*b^2*c-1/((-b^2*c+a^2)^2/x*a^3+d/((-b^2*c+a^2)^2*a*b^2*ln(d*x)+d/((-b^2*c+a^2)^2*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*b^3+d/((-b^2*c+a^2)^2*b/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^2-2*a*b^2*d*ln(a+(d*x+c)^(1/2)*b))/((-b^2*c+a^2)^2)

maxima [A] time = 2.04, size = 191, normalized size = 1.47

$$\frac{1}{2} \left(\frac{2ab^2 \log(dx)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{4ab^2 \log(\sqrt{dx+c}b+a)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{(b^3c+a^2b) \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{(b^4c^2 - 2a^2b^2c + a^4)\sqrt{c}} + \frac{2(\sqrt{dx+c}b-a)}{b^2c^2 - a^2c - (b^2c - a^2)(dx+c)} \right) d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] $\frac{1}{2} \frac{(2ab^2 \log(dx) / (b^4c^2 - 2a^2b^2c + a^4) - 4ab^2 \log(\sqrt{dx+c} * b + a) / (b^4c^2 - 2a^2b^2c + a^4) - (b^3c + a^2b) * \log((\sqrt{dx+c} - \sqrt{c}) / (\sqrt{dx+c} + \sqrt{c}))) / ((b^4c^2 - 2a^2b^2c + a^4) * \sqrt{c}) + 2 * (\sqrt{dx+c} * b - a) / (b^2c^2 - a^2c - (b^2c - a^2) * (dx+c))}{d}$

mupad [B] time = 3.59, size = 220, normalized size = 1.69

$$\frac{\ln(\sqrt{c+dx} - \sqrt{c}) (4ab^2cd - b\sqrt{c}d(2a^2 + 2cb^2))}{4a^4c - 8a^2b^2c^2 + 4b^4c^3} + \frac{\ln(\sqrt{c+dx} + \sqrt{c}) (4ab^2cd + b\sqrt{c}d(2a^2 + 2cb^2))}{4a^4c - 8a^2b^2c^2 + 4b^4c^3} + \frac{\frac{ad}{b^2c-a^2} - \frac{bd\sqrt{c+dx}}{b^2c-a^2}}{dx} - \frac{2ab^2d \ln(a + b\sqrt{c+dx})}{(b^2c - a^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))),x)

[Out] $(\log((c + dx)^{1/2} - c^{1/2}) * (4ab^2cd - b^2c^{1/2} * d * (2b^2c + 2a^2)) / (4a^4c + 4b^4c^3 - 8a^2b^2c^2) + (\log((c + dx)^{1/2} + c^{1/2}) * (4ab^2cd + b^2c^{1/2} * d * (2b^2c + 2a^2))) / (4a^4c + 4b^4c^3 - 8a^2b^2c^2) + ((a*d) / (b^2c - a^2) - (b*d * (c + dx)^{1/2}) / (b^2c - a^2)) / (d * x) - (2ab^2d * \log(a + b * (c + dx)^{1/2})) / (b^2c - a^2)^2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 (a + b\sqrt{c + dx})} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2)),x)

[Out] Integral(1/(x**2*(a + b*sqrt(c + d*x))), x)

$$3.383 \quad \int \frac{1}{x^3(a+b\sqrt{c+dx})} dx$$

Optimal. Leaf size=204

$$\frac{a - b\sqrt{c + dx}}{2x^2(a^2 - b^2c)} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4cx(a^2 - b^2c)^2} + \frac{ab^4d^2 \log(x)}{(a^2 - b^2c)^3} - \frac{2ab^4d^2 \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} - \frac{bd^2(a^4 - 6a^2b^2c)}{4c^{3/2}}$$

Rubi [A] time = 0.28, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$-\frac{bd^2(-6a^2b^2c + a^4 - 3b^4c^2) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}(a^2 - b^2c)^3} + \frac{ab^4d^2 \log(x)}{(a^2 - b^2c)^3} - \frac{2ab^4d^2 \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} - \frac{a - b\sqrt{c + dx}}{2x^2(a^2 - b^2c)} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4cx(a^2 - b^2c)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])),x]

[Out] -(a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2) - (b*d*(4*a*b*c - (a^2 + 3*b^2*c)*Sqrt[c + d*x]))/(4*c*(a^2 - b^2*c)^2*x) - (b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(a^2 - b^2*c)^3) + (a*b^4*d^2*Log[x])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 823

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a

$*e*g)*x*(a + c*x^2)^{(p + 1)}/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + \text{Dist}[1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), \text{Int}[(d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{Simp}[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x] /; \text{FreeQ}[\{a, c, d, e, f, g\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{LtQ}[p, -1] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p] \mid\mid \text{IntegersQ}[2*m, 2*p])$

Rule 1398

$\text{Int}[(a + c*x^2)^{(n2)}*(x)^{(n1)}*((d + e*x)^m*(a + c*x^2)^{(p + 1)}*\text{Simp}[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x], x] /; \text{FreeQ}[\{a, c, d, e, p, q\}, x] \&\& \text{EqQ}[n2, 2*n] \&\& \text{FractionQ}[n]$

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3(a + b\sqrt{c + dx})} dx &= d^2 \text{Subst} \left(\int \frac{1}{(a + b\sqrt{x})(-c + x)^3} dx, x, c + dx \right) \\ &= (2d^2) \text{Subst} \left(\int \frac{x}{(a + bx)(-c + x^2)^3} dx, x, \sqrt{c + dx} \right) \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} + \frac{d^2 \text{Subst} \left(\int \frac{-abc + 3b^2cx}{(a + bx)(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{2c(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4c(a^2 - b^2c)^2x} + \frac{d^2 \text{Subst} \left(\int \frac{abc(a^2 - 5b^2c) + b^2c(-c + x^2)}{(a + bx)(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{4c^2(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4c(a^2 - b^2c)^2x} + \frac{d^2 \text{Subst} \left(\int \left(-\frac{8ab^5c^2}{(a^2 - b^2c)(a + bx)} \right) dx, x, \sqrt{c + dx} \right)}{4c^2(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4c(a^2 - b^2c)^2x} - \frac{2ab^4d^2 \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4c(a^2 - b^2c)^2x} - \frac{2ab^4d^2 \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2} - \frac{bd(4abc - (a^2 + 3b^2c)\sqrt{c + dx})}{4c(a^2 - b^2c)^2x} - \frac{b(a^4 - 6a^2b^2c - 3b^4c^2)d^2 \log(a + b\sqrt{c + dx})}{4c^{3/2}(a^2 - b^2c)^3} \end{aligned}$$

Mathematica [A] time = 0.43, size = 228, normalized size = 1.12

$$\frac{bd^2x^2(a^4 - 6a^2b^2c - 3b^4c^2) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) + \sqrt{c}(4ab^4cd^2x^2 \log(a^2 - b^2(c + dx)) + (a^2 - b^2c)(2a^3c - a^2b\sqrt{c + dx}(2c + dx) - 2ab^2c(c - 2dx) + b^3c(2c - 3dx)\sqrt{c + dx}) - 4ab^4cd^2x^2 \log(x)) + 8ab^4c^3d^2x^2 \tanh^{-1}\left(\frac{b\sqrt{c+dx}}{a}\right)}{4c^{3/2}x^2(b^2c - a^2)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])),x]

[Out] (8*a*b^4*c^(3/2)*d^2*x^2*ArcTanh[(b*Sqrt[c + d*x])/a] + b*(a^4 - 6*a^2*b^2*c - 3*b^4*c^2)*d^2*x^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]] + Sqrt[c]*((a^2 - b^2

c)(2*a^3*c - 2*a*b^2*c*(c - 2*d*x) + b^3*c*(2*c - 3*d*x)*Sqrt[c + d*x] - a^2*b*Sqrt[c + d*x]*(2*c + d*x)) - 4*a*b^4*c*d^2*x^2*Log[x] + 4*a*b^4*c*d^2*x^2*Log[a^2 - b^2*(c + d*x)])))/(4*c^(3/2)*(-a^2 + b^2*c)^3*x^2)

IntegrateAlgebraic [A] time = 0.44, size = 237, normalized size = 1.16

$$\frac{ab^4d^2 \log(-dx)}{(a^2 - b^2c)^3} - \frac{2ab^4d^2 \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} + \frac{d^2(a^4b - 6a^2b^3c - 3b^5c^2) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4c^{3/2}(b^2c - a^2)^3} + \frac{-2a^3c + a^2b(c + dx)^{3/2} + a^2bc\sqrt{c + dx} + 6ab^2c^2 - 4ab^2c(c + dx) - 5b^3c^2\sqrt{c + dx} + 3b^3c(c + dx)^{3/2}}{4cx^2(b^2c - a^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*Sqrt[c + d*x])),x]

[Out] (-2*a^3*c + 6*a*b^2*c^2 + a^2*b*c*Sqrt[c + d*x] - 5*b^3*c^2*Sqrt[c + d*x] - 4*a*b^2*c*(c + d*x) + a^2*b*(c + d*x)^(3/2) + 3*b^3*c*(c + d*x)^(3/2))/(4*c*(-a^2 + b^2*c)^2*x^2) + ((a^4*b - 6*a^2*b^3*c - 3*b^5*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(4*c^(3/2)*(-a^2 + b^2*c)^3) + (a*b^4*d^2*Log[-(d*x)])/(a^2 - b^2*c)^3 - (2*a*b^4*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3

fricas [A] time = 2.15, size = 534, normalized size = 2.62

$$\frac{8ab^4d^2 \log(\sqrt{c+dx}) - 8ab^4d^2 \log(a + b\sqrt{c+dx}) + 4ab^4d^2 \log\left(\frac{a + b\sqrt{c+dx}}{a^2 - b^2c}\right) - 8ab^4d^2 \log\left(\frac{a + b\sqrt{c+dx}}{a^2 - b^2c}\right) \operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) - 8ab^4d^2 \log\left(\frac{a + b\sqrt{c+dx}}{a^2 - b^2c}\right) \operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) - 8ab^4d^2 \log\left(\frac{a + b\sqrt{c+dx}}{a^2 - b^2c}\right) \operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right) - 8ab^4d^2 \log\left(\frac{a + b\sqrt{c+dx}}{a^2 - b^2c}\right) \operatorname{arctanh}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{4(b^2c - a^2)^3 c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] [1/8*(16*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 8*a*b^4*c^2*d^2*x^2*log(x) + 4*a*b^4*c^4 - 8*a^3*b^2*c^3 + 4*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(c)*d^2*x^2*log((d*x - 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 8*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - 2*(2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2), 1/4*(8*a*b^4*c^2*d^2*x^2*log(sqrt(d*x + c)*b + a) - 4*a*b^4*c^2*d^2*x^2*log(x) + 2*a*b^4*c^4 - 4*a^3*b^2*c^3 + 2*a^5*c^2 + (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*sqrt(-c)*d^2*x^2*arctan(sqrt(d*x + c)*sqrt(-c)/c) - 4*(a*b^4*c^3 - a^3*b^2*c^2)*d*x - (2*b^5*c^4 - 4*a^2*b^3*c^3 + 2*a^4*b*c^2 - (3*b^5*c^3 - 2*a^2*b^3*c^2 - a^4*b*c)*d*x)*sqrt(d*x + c))/((b^6*c^5 - 3*a^2*b^4*c^4 + 3*a^4*b^2*c^3 - a^6*c^2)*x^2)]

giac [A] time = 0.46, size = 375, normalized size = 1.84

$$\frac{2ab^4d^2 \log(\sqrt{dx + c} + a)}{b^7c^5 - 3a^2b^5c^2 + 3a^4b^3c - a^6b} + \frac{ab^4d^2 \log(dx)}{b^7c^5 - 3a^2b^5c^2 + 3a^4b^3c - a^6b} + \frac{(3b^5c^2d^2 + 6a^2b^3cd^2 - a^4bd^2) \operatorname{arctan}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4(b^7c^4 - 3a^2b^4c^3 + 3a^4b^2c^2 - a^6c) \sqrt{c}} + \frac{6ab^4c^2d^2 - 8a^3b^2c^2d^2 + 2a^5cd^2 + (3b^5c^2d^2 - 2a^2b^3cd^2 - a^4bd^2)(dx + c)^3 - 4(ab^4c^2d^2 - a^2b^2cd^2)(dx + c) - (5b^5c^3d^2 - 6a^2b^3c^2d^2 + a^4bcd^2)\sqrt{dx + c}}{4(b^7c^5 - 3a^2b^5c^2 + 3a^4b^3c - a^6b)c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] 2*a*b^5*d^2*log(abs(sqrt(d*x + c)*b + a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) - a*b^4*d^2*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) + 1/4*(3*b^5*c^2*d^2 + 6*a^2*b^3*c*d^2 - a^4*b*d^2)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(-c)) + 1/4*(6*a*b^4*c^3*d^2 - 8*a^3*b^2*c^2*d^2 + 2*a^5*c*d^2 + (3*b^5*c^2*d^2 - 2*a^2*b^3*c*d^2 - a^4*b*d^2)*(d*x + c)^(3/2) - 4*(a*b^4*c^2*d^2 - a^3*b^2*c*d^2)*(d*x + c) - (5*b^5*c^3*d^2 - 6*a^2*b^3*c^2*d^2 + a^4*b*c*d^2)*sqrt(d*x + c))/((b^2*c - a^2)^3*c*d^2*x^2)

maple [B] time = 0.02, size = 459, normalized size = 2.25

$$\frac{3b^5 \sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4(-b^2c + a^2)^2} + \frac{ab^4d^2 \ln(dx)}{(-b^2c + a^2)} - \frac{2ab^4d^2 \ln(a + \sqrt{dx+c} + b)}{(-b^2c + a^2)} + \frac{3a^2b^3d^2 \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{2(-b^2c + a^2) \sqrt{c}} - \frac{a^4b^2d^2 \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{4(-b^2c + a^2)^2 c^2} + \frac{a^4b^4d^2}{(-b^2c + a^2)^2 x} - \frac{a^4b^4d^2}{(-b^2c + a^2)^2 x} + \frac{a^4b^4d^2}{(-b^2c + a^2)^2 x} + \frac{5\sqrt{dx+c} b^5 c^2}{4(-b^2c + a^2)^2 x^2} - \frac{a^4 b^4 c}{(-b^2c + a^2)^2 x^2} + \frac{3\sqrt{dx+c} a^2 b^4 c}{2(-b^2c + a^2)^2 x^2} - \frac{3(dx + c)^{3/2} b^5 c}{4(-b^2c + a^2)^2 x^2} - \frac{a^4}{2(-b^2c + a^2)^2 x^2} + \frac{\sqrt{dx+c} a^4 b}{4(-b^2c + a^2)^2 x^2} + \frac{(dx + c)^{3/2} a^4 b}{4(-b^2c + a^2)^2 c x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+(d*x+c)^(1/2)*b),x)

[Out]
$$-3/4/(-b^2*c+a^2)^3/x^2*b^5*c*(d*x+c)^{(3/2)}+1/2/(-b^2*c+a^2)^3/x^2*b^3*(d*x+c)^{(3/2)}*a^2+1/4/(-b^2*c+a^2)^3/x^2*b/c*(d*x+c)^{(3/2)}*a^4+d/(-b^2*c+a^2)^3/x*a*b^4*c-1/2/(-b^2*c+a^2)^3/x^2*b^4*a*c^2-d/(-b^2*c+a^2)^3/x*a^3*b^2+1/(-b^2*c+a^2)^3/x^2*b^2*a^3*c-3/2/(-b^2*c+a^2)^3/x^2*(d*x+c)^{(1/2)}*a^2*b^3*c+1/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^{(1/2)}*b*a^4+5/4/(-b^2*c+a^2)^3/x^2*(d*x+c)^{(1/2)}*c^2*b^5-1/2/(-b^2*c+a^2)^3/x^2*a^5+d^2/(-b^2*c+a^2)^3*b^4*a*ln(d*x)+3/4*d^2/(-b^2*c+a^2)^3*b^5*c^{(1/2)}*arctanh((d*x+c)^{(1/2)}/c^{(1/2)})+3/2*d^2/(-b^2*c+a^2)^3*b^3/c^{(1/2)}*arctanh((d*x+c)^{(1/2)}/c^{(1/2)})*a^2-1/4*d^2/(-b^2*c+a^2)^3*b/c^{(3/2)}*arctanh((d*x+c)^{(1/2)}/c^{(1/2)})*a^4-2*a*b^4*d^2*ln(a+(d*x+c)^{(1/2)}*b)/(-b^2*c+a^2)^3$$

maxima [A] time = 2.08, size = 367, normalized size = 1.80

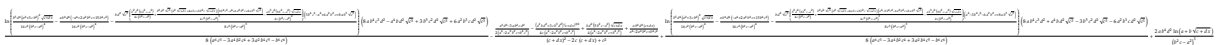
$$\frac{1}{8} \left(\frac{8ab^4 \log(dx)}{b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6} - \frac{16ab^4 \log(\sqrt{dx+cb+a})}{b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6} - \frac{(3b^5c^2 + 6a^2b^3c - a^4b) \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{(b^6c^4 - 3a^2b^4c^3 + 3a^4b^2c^2 - a^6c)\sqrt{c}} + \frac{2(4(dx+c)ab^2c - 6ab^2c^2 + 2a^3c - (3b^3c + a^2b)(dx+c)^2 + (5b^3c^2 - a^2bc)\sqrt{dx+c})}{b^4c^5 - 2a^2b^2c^4 + a^4c^3 + (b^4c^3 - 2a^2b^2c^2 + a^4c)(dx+c)^2 - 2(b^4c^4 - 2a^2b^2c^3 + a^4c^2)(dx+c)} \right) d^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out]
$$-1/8*(8*a*b^4*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - 16*a*b^4*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) - (3*b^5*c^2 + 6*a^2*b^3*c - a^4*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt(c)) + 2*(4*(d*x + c)*a*b^2*c - 6*a*b^2*c^2 + 2*a^3*c - (3*b^3*c + a^2*b)*(d*x + c)^(3/2) + (5*b^3*c^2 - a^2*b*c)*sqrt(d*x + c))/(b^4*c^5 - 2*a^2*b^2*c^4 + a^4*c^3 + (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(d*x + c)^2 - 2*(b^4*c^4 - 2*a^2*b^2*c^3 + a^4*c^2)*(d*x + c))*d^2$$

mupad [B] time = 5.01, size = 1094, normalized size = 5.36



Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))),x)

[Out]
$$(\log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^{(1/2)})/(16*c^2*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b*d^2*(c^3)^{(1/2)}*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*(c^3)^{(1/2)}*(a^2*(c + d*x)^{(1/2)} + 4*a*b*c + 3*b^2*c*(c + d*x)^{(1/2)})*(3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^{(1/2)}))/(4*c^3*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^{(1/2)})/(2*c*(b^2*c - a^2)^2))*(3*b^4*c^2 - a^4 + 6*a^2*b^2*c + 8*a*b^3*(c^3)^{(1/2)}))/(8*c^3*(b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 - a^4*b*d^2*(c^3)^{(1/2)} + 3*b^5*c^2*d^2*(c^3)^{(1/2)} + 6*a^2*b^3*c*d^2*(c^3)^{(1/2)}))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c^5)) - ((a^3*d^2 - 3*a*b^2*c*d^2)/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - ((a^2*b*d^2 + 3*b^3*c*d^2)*(c + d*x)^{(3/2)})/(4*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*d^2*(5*b^2*c - a^2)*(c + d*x)^{(1/2)})/(4*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (a*b^2*d^2*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/((c + d*x)^2 - 2*c*(c + d*x) + c^2) + (\log((b^5*d^4*(3*b^2*c + a^2)^2*(c + d*x)^{(1/2)})/(16*c^2*(b^2*c - a^2)^4) - (a*b^4*d^4*(15*b^4*c^2 - a^4 + 2*a^2*b^2*c))/(16*c^2*(b^2*c - a^2)^4) - (b*d^2*(c^3)^{(1/2)}*((b^2*d^2*(3*b^2*c - a^2))/(4*c*(b^2*c - a^2)) + (b^2*d^2*(c^3)^{(1/2)}*(a^2*(c + d*x)^{(1/2)} + 4*a*b*c + 3*b^2*c*(c + d*x)^{(1/2)})*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^{(1/2)}))/(4*c^3*(b^2*c - a^2)^3) - (a*b^3*d^2*(9*b^2*c - a^2)*(c + d*x)^{(1/2)})/(2*c*(b^2*c - a^2)^2))*(a^4 - 3*b^4*c^2 - 6*a^2*b^2*c + 8*a*b^3*(c^3)^{(1/2)}))/(8*c^3*(b^2*c - a^2)^3))*(8*a*b^4*c^3*d^2 + a^4*b*d^2*(c^3)^{(1/2)} - 3*b^5*c^2*d^2*(c^3)^{(1/2)} - 6*a^2*b^3*c*d^2*(c^3)^{(1/2)}))/(8*(a^6*c^3 - b^6*c^6 - 3*a^4*b^2*c^4 + 3*a^2*b^4*c^5)) + (2*a*b^4*d^2*log(a + b*(c + d*x)^{(1/2)}))/(b^2*c - a^2)^3$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2)),x)

[Out] Timed out

$$3.384 \quad \int \frac{x^3}{(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=240

$$\frac{2a(a^2 - b^2c)^3}{b^8d^4(a + b\sqrt{c + dx})} + \frac{2(7a^2 - b^2c)(a^2 - b^2c)^2 \log(a + b\sqrt{c + dx})}{b^8d^4} - \frac{12a(a^2 - b^2c)^2 \sqrt{c + dx}}{b^7d^4} - \frac{4a(2a^2 - 3b^2c)}{3b^5d^4}$$

Rubi [A] time = 0.28, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{x(-9a^2b^2c + 5a^4 + 3b^4c^2)}{b^6d^3} + \frac{2a(a^2 - b^2c)^3}{b^8d^4(a + b\sqrt{c + dx})} - \frac{12a(a^2 - b^2c)^2 \sqrt{c + dx}}{b^7d^4} + \frac{3(a^2 - b^2c)(c + dx)^2}{2b^4d^4} - \frac{4a(2a^2 - 3b^2c)(c + dx)^{3/2}}{3b^5d^4} + \frac{2(7a^2 - b^2c)(a^2 - b^2c)^2 \log(a + b\sqrt{c + dx})}{b^8d^4} - \frac{4a(c + dx)^{5/2}}{5b^3d^4} + \frac{(c + dx)^3}{3b^2d^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/(a + b*Sqrt[c + d*x])^2,x]

[Out] ((5*a^4 - 9*a^2*b^2*c + 3*b^4*c^2)*x)/(b^6*d^3) - (12*a*(a^2 - b^2*c)^2*Sqrt[c + d*x])/(b^7*d^4) - (4*a*(2*a^2 - 3*b^2*c)*(c + d*x)^(3/2))/(3*b^5*d^4) + (3*(a^2 - b^2*c)*(c + d*x)^2)/(2*b^4*d^4) - (4*a*(c + d*x)^(5/2))/(5*b^3*d^4) + (c + d*x)^3/(3*b^2*d^4) + (2*a*(a^2 - b^2*c)^3)/(b^8*d^4*(a + b*Sqrt[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\int \frac{x^3}{(a + b\sqrt{c + dx})^2} dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{(a+b\sqrt{x})^2} dx, x, c + dx\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)^3}{(a+bx)^2} dx, x, \sqrt{c + dx}\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int \left(-\frac{6a(a^2-b^2c)^2}{b^7} + \frac{(5a^4-9a^2b^2c+3b^4c^2)x}{b^6} - \frac{2a(2a^2-3b^2c)x^2}{b^5} - \frac{3(-a^2+b^2c)x^3}{b^4} - \frac{2ax^4}{b^3} + \frac{x^5}{b^2} - \right)}{d^4}$$

$$= \frac{(5a^4 - 9a^2b^2c + 3b^4c^2)x}{b^6d^3} - \frac{12a(a^2 - b^2c)^2 \sqrt{c + dx}}{b^7d^4} - \frac{4a(2a^2 - 3b^2c)(c + dx)^{3/2}}{3b^5d^4} + \frac{3}{b^2}$$

Mathematica [A] time = 0.27, size = 273, normalized size = 1.14

$$\frac{96a^7 - 324a^6b\sqrt{c+dx} - 6a^5b^2(102c + 35dx) + 2a^4b^3\sqrt{c+dx}(284c + 35dx) + a^3b^4(856c^2 + 380cdx - 35d^2x^2) - 3a^2b^5\sqrt{c+dx}(76c^2 + 36cdx - 7d^2x^2) + 60(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c+dx})\log(a + b\sqrt{c+dx}) - ab^6(324c^3 + 162c^2dx - 33cd^2x^2 + 14d^3x^3) + 5b^7dx\sqrt{c+dx}(6c^2 - 3cdx + 2d^2x^2)}{30b^8d^4(a + b\sqrt{c+dx})}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(a + b*Sqrt[c + d*x])^2,x]

[Out] (96*a^7 - 324*a^6*b*Sqrt[c + d*x] - 6*a^5*b^2*(102*c + 35*d*x) + 2*a^4*b^3*Sqrt[c + d*x]*(284*c + 35*d*x) + a^3*b^4*(856*c^2 + 380*c*d*x - 35*d^2*x^2) - 3*a^2*b^5*Sqrt[c + d*x]*(76*c^2 + 36*c*d*x - 7*d^2*x^2) + 5*b^7*d*x*Sqrt[c + d*x]*(6*c^2 - 3*c*d*x + 2*d^2*x^2) - a*b^6*(324*c^3 + 162*c^2*d*x - 33*c*d^2*x^2 + 14*d^3*x^3) + 60*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(30*b^8*d^4*(a + b*Sqrt[c + d*x]))

IntegrateAlgebraic [A] time = 0.17, size = 341, normalized size = 1.42

$$\frac{2(a^2 - b^2c)^2(a^2 - b^2c)\log(a + b\sqrt{c + dx})}{30b^8d^4} + \frac{60a^7 - 360a^6b\sqrt{c + dx} - 210a^5b^2(c + dx) - 180a^4b^3\sqrt{c + dx} + 720a^3b^4(c + dx)^{3/2} + 180a^2b^5\sqrt{c + dx} - 35d^2b^6(c + dx)^{3/2} + 450a^2b^6(c + dx)^{5/2} - 360a^2b^6c\sqrt{c + dx} + 21a^2b^6(c + dx)^{7/2} - 150a^2b^6(c + dx)^{9/2} - 60ab^7c^2 - 270ab^7c(c + dx) - 14ab^7(c + dx)^3 + 75ab^7(c + dx)^5 + 90b^8c^2(c + dx)^{3/2} + 10b^8c^2(c + dx)^{5/2} - 45b^8c^2(c + dx)^{7/2}}{30b^8d^4(a + b\sqrt{c + dx})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3/(a + b*Sqrt[c + d*x])^2,x]

[Out] (60*a^7 - 180*a^5*b^2*c + 180*a^3*b^4*c^2 - 60*a*b^6*c^3 - 360*a^6*b*Sqrt[c + d*x] + 720*a^4*b^3*c*Sqrt[c + d*x] - 360*a^2*b^5*c^2*Sqrt[c + d*x] - 210*a^5*b^2*(c + d*x) + 450*a^3*b^4*c*(c + d*x) - 270*a*b^6*c^2*(c + d*x) + 70*a^4*b^3*(c + d*x)^(3/2) - 150*a^2*b^5*c*(c + d*x)^(3/2) + 90*b^7*c^2*(c + d*x)^(3/2) - 35*a^3*b^4*(c + d*x)^2 + 75*a*b^6*c*(c + d*x)^2 + 21*a^2*b^5*(c + d*x)^(5/2) - 45*b^7*c*(c + d*x)^(5/2) - 14*a*b^6*(c + d*x)^3 + 10*b^7*(c + d*x)^(7/2))/(30*b^8*d^4*(a + b*Sqrt[c + d*x])) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^8*d^4)

fricas [A] time = 0.93, size = 392, normalized size = 1.63

$$\frac{10a^7d^4 + 55a^6b^8c^4 + 195a^4b^6c^3 + 30a^2b^8c^2 - 60a^8 - 5(b^8c - 7a^2b^6)d^3x^3 + 15(b^8c^2 - 8a^2b^6c + 7a^4b^4)d^2x^2 + 5(17b^8c^3 - 87a^2b^6c^2 + 96a^4b^4c - 60b^8c^2 + 10a^2b^6c^2 + 21a^4b^4c^2 - 22a^6b^2c^2 - 7a^8 - 9a^2b^6c + 15a^4b^4c - 7a^6b^2c) \log(\sqrt{a + b\sqrt{c + dx}}) - 4(6a^7d^2 + 81a^6b^8c^2 - 271a^5b^8c^2 + 295a^4b^8c^2 - 105a^3b^8c^2 - 2(6a^7c - 7a^5b^8c + 2(2a^6c^2 - 81a^4b^6c + 35a^2b^4c^2) \sqrt{a + b\sqrt{c + dx}})}{30b^8d^4(a + b\sqrt{c + dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/30*(10*b^8*d^4*x^4 + 55*b^8*c^4 - 220*a^2*b^6*c^3 + 195*a^4*b^4*c^2 + 30*a^6*b^2*c - 60*a^8 - 5*(b^8*c - 7*a^2*b^6)*d^3*x^3 + 15*(b^8*c^2 - 8*a^2*b^6*c + 7*a^4*b^4)*d^2*x^2 + 5*(17*b^8*c^3 - 87*a^2*b^6*c^2 + 96*a^4*b^4*c -

$$30*a^6*b^2)*d*x - 60*(b^8*c^4 - 10*a^2*b^6*c^3 + 24*a^4*b^4*c^2 - 22*a^6*b^2*c + 7*a^8 + (b^8*c^3 - 9*a^2*b^6*c^2 + 15*a^4*b^4*c - 7*a^6*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(6*a*b^7*d^3*x^3 + 81*a*b^7*c^3 - 271*a^3*b^5*c^2 + 295*a^5*b^3*c - 105*a^7*b - 2*(6*a*b^7*c - 7*a^3*b^5)*d^2*x^2 + 2*(24*a*b^7*c^2 - 61*a^3*b^5*c + 35*a^5*b^3)*d*x)*sqrt(d*x + c))/(b^10*d^5*x + (b^10*c - a^2*b^8)*d^4)$$

giac [A] time = 0.48, size = 324, normalized size = 1.35

$$\frac{2(b^8c^3 - 9a^2b^6c^2 + 15a^4b^4c - 7a^6b^2)*d*x + 7a^8 + (b^8c^3 - 9a^2b^6c^2 + 15a^4b^4c - 7a^6b^2)*d*x}{\sqrt{dx+c} \cdot b + a} - \frac{4(6ab^7d^3x^3 + 81ab^7c^3 - 271a^3b^5c^2 + 295a^5b^3c - 105a^7b - 2(6ab^7c - 7a^3b^5)d^2x^2 + 2(24ab^7c^2 - 61a^3b^5c + 35a^5b^3)d*x)}{\sqrt{dx+c}}}{b^{10}d^5x + (b^{10}c - a^2b^8)d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

$$[Out] -2*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(abs(sqrt(d*x + c)*b + a))/(b^8*d^4) - 2*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/((sqrt(d*x + c)*b + a)*b^8*d^4) + 1/30*(10*(d*x + c)^3*b^10*d^20 - 45*(d*x + c)^2*b^10*c*d^20 + 90*(d*x + c)*b^10*c^2*d^20 - 24*(d*x + c)^(5/2)*a*b^9*d^20 + 120*(d*x + c)^(3/2)*a*b^9*c*d^20 - 360*sqrt(d*x + c)*a*b^9*c^2*d^20 + 45*(d*x + c)^2*a^2*b^8*d^20 - 270*(d*x + c)*a^2*b^8*c*d^20 - 80*(d*x + c)^(3/2)*a^3*b^7*d^20 + 720*sqrt(d*x + c)*a^3*b^7*c*d^20 + 150*(d*x + c)*a^4*b^6*d^20 - 360*sqrt(d*x + c)*a^5*b^5*d^20)/(b^12*d^24)$$

maple [A] time = 0.01, size = 416, normalized size = 1.73

$$\frac{d^2}{30d} + \frac{2a^2c^2}{25d^2} + \frac{2a^2c}{(c + \sqrt{dx+c})^{3/2}} + \frac{2a^2\ln(a + \sqrt{dx+c})}{b^2d} + \frac{2c}{b^2d} + \frac{6a^2c}{(c + \sqrt{dx+c})^{3/2}} + \frac{18a^2\ln(a + \sqrt{dx+c})}{10d^2} + \frac{6a^2c}{b^2d} + \frac{11c^2}{40d^2} + \frac{6a^2c}{(c + \sqrt{dx+c})^{3/2}} + \frac{30a^2\ln(a + \sqrt{dx+c})}{10d^2} + \frac{6a^2c}{b^2d} + \frac{13c^2}{20d^2} + \frac{12\sqrt{dx+c}d^2}{10d^2} + \frac{2a^2}{(c + \sqrt{dx+c})^{3/2}} + \frac{14a^2\ln(a + \sqrt{dx+c})}{30d} + \frac{6a^2c}{b^2d} + \frac{24\sqrt{dx+c}d^2}{10d^2} + \frac{4a^2c^2}{30d} + \frac{12\sqrt{dx+c}d^2}{10d^2} + \frac{8a^2c^2}{30d^2} + \frac{4a^2c^2}{30d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+(d*x+c)^(1/2)*b)^2,x)

$$[Out] 1/3/d/b^2*x^3-1/2/d^2/b^2*x^2*c+1/d^3/b^2*x*c^2+11/6/d^4/b^2*c^3-4/5*a*(d*x+c)^(5/2)/b^3/d^4+3/2/d^2/b^4*x^2*a^2-6/d^3/b^4*x*a^2*c-15/2/d^4/b^4*a^2*c^2+4/d^4/b^3*(d*x+c)^(3/2)*a*c-8/3/d^4/b^5*(d*x+c)^(3/2)*a^3-12/d^4/b^3*a*c^2*(d*x+c)^(1/2)+5/d^3/b^6*x*a^4+5/d^4/b^6*a^4*c+24/d^4/b^5*a^3*c*(d*x+c)^(1/2)-12/d^4/b^7*a^5*(d*x+c)^(1/2)-2/d^4*a/b^2/(a+(d*x+c)^(1/2)*b)*c^3+6/d^4*a^3/b^4/(a+(d*x+c)^(1/2)*b)*c^2-6/d^4*a^5/b^6/(a+(d*x+c)^(1/2)*b)*c+2/d^4*a^7/b^8/(a+(d*x+c)^(1/2)*b)-2/d^4/b^2*ln(a+(d*x+c)^(1/2)*b)*c^3+18/d^4/b^4*ln(a+(d*x+c)^(1/2)*b)*a^2*c^2-30/d^4/b^6*ln(a+(d*x+c)^(1/2)*b)*a^4*c+14/d^4/b^8*ln(a+(d*x+c)^(1/2)*b)*a^6$$

maxima [A] time = 0.93, size = 251, normalized size = 1.05

$$\frac{60(ab^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^7) + 60(b^6c^3 - 9a^2b^4c^2 + 15a^4b^2c - 7a^6)}{\sqrt{dx+c} b^8} - \frac{10(dx+c)^3 b^5 - 24(dx+c)^2 ab^4 - 45(b^5c - a^2b^3)(dx+c)^2 + 40(3ab^4c - 2a^3b^2)(dx+c)^2 + 30(3b^5c^2 - 9a^2b^3c + 5a^4b)(dx+c) - 360(ab^4c^2 - 2a^3b^2c + a^5)\sqrt{dx+c}}{b^7} + \frac{60(b^6c^3 - 9a^2b^4c^2 + 15a^4b^2c - 7a^6)}{b^8} \log(\sqrt{dx+c} b + a)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

$$[Out] -1/30*(60*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)/(sqrt(d*x + c)*b^9 + a*b^8) - (10*(d*x + c)^3*b^5 - 24*(d*x + c)^(5/2)*a*b^4 - 45*(b^5*c - a^2*b^3)*(d*x + c)^2 + 40*(3*a*b^4*c - 2*a^3*b^2)*(d*x + c)^(3/2) + 30*(3*b^5*c^2 - 9*a^2*b^3*c + 5*a^4*b)*(d*x + c) - 360*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(d*x + c))/b^7 + 60*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*log(sqrt(d*x + c)*b + a)/b^8)/d^4$$

mupad [B] time = 0.09, size = 461, normalized size = 1.92

$$\left(\frac{4a^2}{33b^8d} + \frac{2a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{(c+dx)^{3/2}} - \left(\frac{3c}{33b^8d} - \frac{3a^2}{24b^8d}\right)(c+dx)^2 \left[\frac{2a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b} + \frac{2a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b} + \frac{2a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b} \right] + \frac{a^2\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b^2} \right) \sqrt{c+dx} + \frac{(c+dx)^2}{3b^2d^2} + \frac{2(a^2-3a^2b^2c+3a^2b^4c^2-a^4b^2c)}{b(b^2a^2\sqrt{c+dx}+a^2b^2d)} + dx \left[\frac{a^2\left(\frac{4a^2}{3b} - \frac{6a^2}{b^2d}\right)}{2b^2} + \frac{a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b} + \frac{2a\left(\frac{2a^2}{3b} - \frac{6a^2}{b^2d}\right)}{b} \right] + \frac{2a^2}{b^2d^2} \ln(a+b\sqrt{c+dx}) - \frac{14a^6-30a^4b^2c+18a^2b^4c^2-2b^6c^3}{b^8d^2} - \frac{4a(c+dx)^{3/2}}{3b^8d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b*(c + d*x)^(1/2))^2,x)`

[Out]
$$\begin{aligned} & \left(\frac{4a^3}{3b^5d^4} + \frac{2a((6c)/(b^2d^4) - (6a^2)/(b^4d^4))}{3b} \right) (c + dx)^{3/2} - \left(\frac{3c}{2b^2d^4} - \frac{3a^2}{2b^4d^4} \right) (c + dx)^2 - \left(\frac{2a((a^2((6c)/(b^2d^4) - (6a^2)/(b^4d^4)))/b^2 - (2a((4a^3)/(b^5d^4) + (2a((6c)/(b^2d^4) - (6a^2)/(b^4d^4)))/b))/b + (6c^2)/(b^2d^4))}{b} + \frac{a^2((4a^3)/(b^5d^4) + (2a((6c)/(b^2d^4) - (6a^2)/(b^4d^4)))/b))/b^2 \right) (c + dx)^{1/2} + \frac{(c + dx)^3}{3b^2d^4} + \frac{2(a^7 - 3a^5b^2c - ab^6c^3 + 3a^3b^4c^2)}{b(b^8d^4(c + dx)^{1/2} + ab^7d^4)} + \frac{dx((a^2((6c)/(b^2d^4) - (6a^2)/(b^4d^4)))/(2b^2) - (a((4a^3)/(b^5d^4) + (2a((6c)/(b^2d^4) - (6a^2)/(b^4d^4)))/b))/b + (3c^2)/(b^2d^4))}{b^8d^4} + \frac{\log(a + b(c + d*x)^{1/2})(14a^6 - 2b^6c^3 - 30a^4b^2c + 18a^2b^4c^2)}{b^8d^4} - \frac{4a(c + d*x)^{5/2}}{5b^3d^4} \end{aligned}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + b\sqrt{c + dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+b*(d*x+c)**(1/2))**2,x)`

[Out] `Integral(x**3/(a + b*sqrt(c + d*x))**2, x)`

$$3.385 \quad \int \frac{x^2}{(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=166

$$\frac{2a(a^2 - b^2c)^2}{b^6d^3(a + b\sqrt{c + dx})} - \frac{8a(a^2 - b^2c)\sqrt{c + dx}}{b^5d^3} + \frac{x(3a^2 - 2b^2c)}{b^4d^2} + \frac{2(5a^4 - 6a^2b^2c + b^4c^2)\log(a + b\sqrt{c + dx})}{b^6d^3} - \frac{4}{b^6d^3}$$

Rubi [A] time = 0.17, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2(-6a^2b^2c + 5a^4 + b^4c^2)\log(a + b\sqrt{c + dx})}{b^6d^3} + \frac{2a(a^2 - b^2c)^2}{b^6d^3(a + b\sqrt{c + dx})} - \frac{8a(a^2 - b^2c)\sqrt{c + dx}}{b^5d^3} + \frac{x(3a^2 - 2b^2c)}{b^4d^2} - \frac{4a(c + dx)^{3/2}}{3b^3d^3} + \frac{(c + dx)^2}{2b^2d^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/(a + b*Sqrt[c + d*x])^2,x]

[Out] ((3*a^2 - 2*b^2*c)*x)/(b^4*d^2) - (8*a*(a^2 - b^2*c)*Sqrt[c + d*x])/(b^5*d^3) - (4*a*(c + d*x)^(3/2))/(3*b^3*d^3) + (c + d*x)^2/(2*b^2*d^3) + (2*a*(a^2 - b^2*c)^2)/(b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(a+b\sqrt{c+dx})^2} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{(a+b\sqrt{x})^2} dx, x, c+dx\right)}{d^3} \\
&= \frac{2 \text{Subst}\left(\int \frac{x(-c+x)^2}{(a+bx)^2} dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= \frac{2 \text{Subst}\left(\int \left(-\frac{4a(a^2-b^2c)}{b^5} - \frac{(-3a^2+2b^2c)x}{b^4} - \frac{2ax^2}{b^3} + \frac{x^3}{b^2} - \frac{a(a^2-b^2c)^2}{b^5(a+bx)^2} + \frac{5a^4-6a^2b^2c+b^4c^2}{b^5(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= \frac{(3a^2-2b^2c)x}{b^4d^2} - \frac{8a(a^2-b^2c)\sqrt{c+dx}}{b^5d^3} - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{(c+dx)^2}{2b^2d^3} + \frac{2a(a^2-b^2c)}{b^6d^3(a+b\sqrt{c+dx})}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 185, normalized size = 1.11

$$\frac{16a^5 - 44a^4b\sqrt{c+dx} - 2a^3b^2(38c+15dx) + 2a^2b^3\sqrt{c+dx}(18c+5dx) + 12(5a^4 - 6a^2b^2c + b^4c^2)(a+b\sqrt{c+dx})\log(a+b\sqrt{c+dx}) + ab^4(52c^2 + 26cdx - 5d^2x^2) + 3b^5dx(dx-2c)\sqrt{c+dx}}{6b^6d^3(a+b\sqrt{c+dx})}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(a + b*Sqrt[c + d*x])^2,x]

[Out] (16*a^5 - 44*a^4*b*Sqrt[c + d*x] + 3*b^5*d*x*(-2*c + d*x)*Sqrt[c + d*x] + 2*a^2*b^3*Sqrt[c + d*x]*(18*c + 5*d*x) - 2*a^3*b^2*(38*c + 15*d*x) + a*b^4*(52*c^2 + 26*c*d*x - 5*d^2*x^2) + 12*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(6*b^6*d^3*(a + b*Sqrt[c + d*x]))

IntegrateAlgebraic [A] time = 0.14, size = 211, normalized size = 1.27

$$\frac{2(5a^4 - 6a^2b^2c + b^4c^2)\log(a+b\sqrt{c+dx})}{b^6d^3} + \frac{12a^5 - 48a^4b\sqrt{c+dx} - 30a^3b^2(c+dx) - 24a^2b^3c\sqrt{c+dx} + 12ab^4c^2 - 5ab^4(c+dx)^2 + 36ab^4c(c+dx) + 3b^5(c+dx)^{5/2} - 12b^5c(c+dx)^{3/2}}{6b^6d^3(a+b\sqrt{c+dx})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(a + b*Sqrt[c + d*x])^2,x]

[Out] (12*a^5 - 24*a^3*b^2*c + 12*a*b^4*c^2 - 48*a^4*b*Sqrt[c + d*x] + 48*a^2*b^3*c*Sqrt[c + d*x] - 30*a^3*b^2*(c + d*x) + 36*a*b^4*c*(c + d*x) + 10*a^2*b^3*(c + d*x)^(3/2) - 12*b^5*c*(c + d*x)^(3/2) - 5*a*b^4*(c + d*x)^2 + 3*b^5*(c + d*x)^(5/2))/(6*b^6*d^3*(a + b*Sqrt[c + d*x])) + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*Log[a + b*Sqrt[c + d*x]])/(b^6*d^3)

fricas [A] time = 0.84, size = 269, normalized size = 1.62

$$\frac{3b^6d^3x^3 - 9b^6c^3 + 15a^2b^4c^2 + 6a^4b^2c - 12a^6 - 3(b^6c - 5a^2b^4)d^2x^2 - 3(5b^6c^2 - 14a^2b^4c + 6a^4b^2)dx + 12(b^6c^3 - 7a^2b^4c^2 + 11a^4b^2c - 5a^6 + (b^6c - 6a^2b^4c + 5a^4b^2)dx)\log(\sqrt{dx+c}+a) - 4(2ab^5d^2x^2 - 13ab^5c + 28a^3b^3c - 15a^5b - 2(4ab^5c - 5a^3b^3)dx)\sqrt{dx+c}}{6(b^6d^3x + (b^6c - a^2b^4)d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 1/6*(3*b^6*d^3*x^3 - 9*b^6*c^3 + 15*a^2*b^4*c^2 + 6*a^4*b^2*c - 12*a^6 - 3*(b^6*c - 5*a^2*b^4)*d^2*x^2 - 3*(5*b^6*c^2 - 14*a^2*b^4*c + 6*a^4*b^2)*d*x + 12*(b^6*c^3 - 7*a^2*b^4*c^2 + 11*a^4*b^2*c - 5*a^6 + (b^6*c^2 - 6*a^2*b^4*c + 5*a^4*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 4*(2*a*b^5*d^2*x^2 - 13*a*b^5*c^2 + 28*a^3*b^3*c - 15*a^5*b - 2*(4*a*b^5*c - 5*a^3*b^3)*d*x)*sqrt(d*x + c))/(b^8*d^4*x + (b^8*c - a^2*b^6)*d^3)

giac [A] time = 0.37, size = 191, normalized size = 1.15

$$\frac{2(b^4c^2 - 6a^2b^2c + 5a^4)\log(\sqrt{dx+cb+a})}{b^6d^3} + \frac{2(ab^4c^2 - 2a^3b^2c + a^5)}{(\sqrt{dx+cb+a})b^6d^3} + \frac{3(dx+c)^2b^6d^9 - 12(dx+c)b^6cd^9 - 8(dx+c)^3ab^5d^9 + 48\sqrt{dx+c}ab^5cd^9 + 18(dx+c)a^2b^4d^9 - 48\sqrt{dx+c}a^3b^3d^9}{6b^5d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

$$[Out] \frac{2*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*\log(\text{abs}(\text{sqrt}(d*x + c)*b + a))/(b^6*d^3) + 2*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/((\text{sqrt}(d*x + c)*b + a)*b^6*d^3) + 1/6*(3*(d*x + c)^2*b^6*d^9 - 12*(d*x + c)*b^6*c*d^9 - 8*(d*x + c)^{(3/2)}*a*b^5*d^9 + 48*\text{sqrt}(d*x + c)*a*b^5*c*d^9 + 18*(d*x + c)*a^2*b^4*d^9 - 48*\text{sqrt}(d*x + c)*a^3*b^3*d^9)/(b^8*d^{12})$$

maple [A] time = 0.01, size = 253, normalized size = 1.52

$$\frac{x^2}{2b^2d} + \frac{2ac^2}{(a + \sqrt{dx+c})b^2d^3} + \frac{2c^2 \ln(a + \sqrt{dx+c})}{b^2d^3} - \frac{cx}{b^2d^2} - \frac{4a^3c}{(a + \sqrt{dx+c})b^4d^3} - \frac{12c^2 \ln(a + \sqrt{dx+c})}{b^4d^3} + \frac{3a^2x}{b^4d^2} - \frac{3c^2}{2b^2d^3} + \frac{2a^5}{(a + \sqrt{dx+c})b^6d^3} + \frac{10a^4 \ln(a + \sqrt{dx+c})}{b^6d^3} + \frac{3a^2c}{b^6d^3} + \frac{8\sqrt{dx+c}ac}{b^6d^3} - \frac{8\sqrt{dx+c}a^3}{b^6d^3} - \frac{4(dx+c)^{3/2}a}{3b^5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a+(d*x+c)^(1/2)*b)^2,x)

$$[Out] \frac{1/2/d/b^2*x^2 - 1/d^2/b^2*x*c - 3/2/d^3/b^2*c^2 - 4/3*a*(d*x+c)^{(3/2)}/b^3/d^3 + 3/d^2/b^4*x*a^2 + 3/d^3/b^4*a^2*c + 8/d^3/b^3*a*c*(d*x+c)^{(1/2)} - 8/d^3/b^5*a^3*(d*x+c)^{(1/2)} + 2/d^3*a/b^2/(a+(d*x+c)^{(1/2)*b})*c^2 - 4/d^3*a^3/b^4/(a+(d*x+c)^{(1/2)*b})*c + 2/d^3*a^5/b^6/(a+(d*x+c)^{(1/2)*b}) + 2/d^3/b^2*\ln(a+(d*x+c)^{(1/2)*b})*c^2 - 12/d^3/b^4*\ln(a+(d*x+c)^{(1/2)*b})*a^2*c + 10/d^3/b^6*\ln(a+(d*x+c)^{(1/2)*b})*a^4$$

maxima [A] time = 0.93, size = 158, normalized size = 0.95

$$\frac{12(ab^4c^2 - 2a^3b^2c + a^5)}{\sqrt{dx+c}b^7 + ab^6} + \frac{3(dx+c)^2b^3 - 8(dx+c)^2ab^2 - 6(2b^3c - 3a^2b)(dx+c) + 48(ab^2c - a^3)\sqrt{dx+c}}{b^5} + \frac{12(b^4c^2 - 6a^2b^2c + 5a^4)\log(\sqrt{dx+c}b+a)}{b^6}$$

$6d^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

$$[Out] \frac{1}{6}*(12*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)/(\text{sqrt}(d*x + c)*b^7 + a*b^6) + (3*(d*x + c)^2*b^3 - 8*(d*x + c)^{(3/2)}*a*b^2 - 6*(2*b^3*c - 3*a^2*b)*(d*x + c) + 48*(a*b^2*c - a^3)*\text{sqrt}(d*x + c))/b^5 + 12*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*\log(\text{sqrt}(d*x + c)*b + a)/b^6)/d^3$$

mapad [B] time = 3.20, size = 197, normalized size = 1.19

$$\left(\frac{4a^3}{b^5d^3} + \frac{2a\left(\frac{4c}{b^2d^3} - \frac{6a^2}{b^4d^3}\right)}{b}\right)\sqrt{c+dx} + \frac{2(a^5 - 2a^3b^2c + ab^4c^2)}{b(b^6d^3\sqrt{c+dx} + ab^5d^3)} + \frac{(c+dx)^2}{2b^2d^3} - dx\left(\frac{2c}{b^2d^3} - \frac{3a^2}{b^4d^3}\right) - \frac{4a(c+dx)^{3/2}}{3b^3d^3} + \frac{\ln(a+b\sqrt{c+dx})(10a^4 - 12a^2b^2c + 2b^4c^2)}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(a + b*(c + d*x)^(1/2))^2,x)

$$[Out] \left(\frac{4*a^3}{b^5*d^3} + \frac{2*a*((4*c)/(b^2*d^3) - (6*a^2)/(b^4*d^3))}{b}\right)*(c + d*x)^{(1/2)} + \frac{2*(a^5 - 2*a^3*b^2*c + a*b^4*c^2)}{b*(b^6*d^3*(c + d*x)^{(1/2)} + a*b^5*d^3)} + \frac{(c + d*x)^2}{2*b^2*d^3} - d*x*\left(\frac{2*c}{b^2*d^3} - \frac{3*a^2}{b^4*d^3}\right) - \frac{4*a*(c + d*x)^{(3/2)}}{3*b^3*d^3} + \frac{\log(a + b*(c + d*x)^{(1/2)})*(10*a^4 + 2*b^4*c^2 - 12*a^2*b^2*c)}{b^6*d^3}$$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(a + b\sqrt{c + dx})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(x**2/(a + b*sqrt(c + d*x))**2, x)
```


$$3.386 \quad \int \frac{x}{(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=95

$$\frac{2a(a^2 - b^2c)}{b^4d^2(a + b\sqrt{c + dx})} + \frac{2(3a^2 - b^2c)\log(a + b\sqrt{c + dx})}{b^4d^2} - \frac{4a\sqrt{c + dx}}{b^3d^2} + \frac{x}{b^2d}$$

Rubi [A] time = 0.09, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{2a(a^2 - b^2c)}{b^4d^2(a + b\sqrt{c + dx})} + \frac{2(3a^2 - b^2c)\log(a + b\sqrt{c + dx})}{b^4d^2} - \frac{4a\sqrt{c + dx}}{b^3d^2} + \frac{x}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*Sqrt[c + d*x])^2,x]

[Out] x/(b^2*d) - (4*a*Sqrt[c + d*x])/(b^3*d^2) + (2*a*(a^2 - b^2*c))/(b^4*d^2*(a + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x}{(a + b\sqrt{c + dx})^2} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{(a+b\sqrt{x})^2} dx, x, c + dx\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)}{(a+bx)^2} dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{2a}{b^3} + \frac{x}{b^2} + \frac{-a^3+ab^2c}{b^3(a+bx)^2} + \frac{3a^2-b^2c}{b^3(a+bx)}\right) dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= \frac{x}{b^2d} - \frac{4a\sqrt{c + dx}}{b^3d^2} + \frac{2a(a^2 - b^2c)}{b^4d^2(a + b\sqrt{c + dx})} + \frac{2(3a^2 - b^2c)\log(a + b\sqrt{c + dx})}{b^4d^2} \end{aligned}$$

Mathematica [A] time = 0.09, size = 112, normalized size = 1.18

$$\frac{2a^3 + 2(3a^2 - b^2c)(a + b\sqrt{c + dx}) \log(a + b\sqrt{c + dx}) - 4a^2b\sqrt{c + dx} - 3ab^2(2c + dx) + b^3dx\sqrt{c + dx}}{b^4d^2(a + b\sqrt{c + dx})}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*Sqrt[c + d*x])^2,x]

[Out] (2*a^3 - 4*a^2*b*Sqrt[c + d*x] + b^3*d*x*Sqrt[c + d*x] - 3*a*b^2*(2*c + d*x) + 2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2*(a + b*Sqrt[c + d*x]))

IntegrateAlgebraic [A] time = 0.08, size = 109, normalized size = 1.15

$$\frac{2(3a^2 - b^2c) \log(a + b\sqrt{c + dx})}{b^4d^2} + \frac{2a^3 - 4a^2b\sqrt{c + dx} - 3ab^2(c + dx) - 2ab^2c + b^3(c + dx)^{3/2}}{b^4d^2(a + b\sqrt{c + dx})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b*Sqrt[c + d*x])^2,x]

[Out] (2*a^3 - 2*a*b^2*c - 4*a^2*b*Sqrt[c + d*x] - 3*a*b^2*(c + d*x) + b^3*(c + d*x)^(3/2))/(b^4*d^2*(a + b*Sqrt[c + d*x])) + (2*(3*a^2 - b^2*c)*Log[a + b*Sqrt[c + d*x]])/(b^4*d^2)

fricas [A] time = 0.96, size = 163, normalized size = 1.72

$$\frac{b^4d^2x^2 + b^4c^2 + a^2b^2c - 2a^4 + (2b^4c - a^2b^2)dx - 2(b^4c^2 - 4a^2b^2c + 3a^4 + (b^4c - 3a^2b^2)dx) \log(\sqrt{dx + c}b + a) - 2(2ab^3dx + 3ab^3c - 3a^3b)\sqrt{dx + c}}{b^6d^3x + (b^6c - a^2b^4)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] (b^4*d^2*x^2 + b^4*c^2 + a^2*b^2*c - 2*a^4 + (2*b^4*c - a^2*b^2)*d*x - 2*(b^4*c^2 - 4*a^2*b^2*c + 3*a^4 + (b^4*c - 3*a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) - 2*(2*a*b^3*d*x + 3*a*b^3*c - 3*a^3*b)*sqrt(d*x + c))/(b^6*d^3*x + (b^6*c - a^2*b^4)*d^2)

giac [A] time = 0.41, size = 102, normalized size = 1.07

$$\frac{\frac{2(b^2c - 3a^2) \log(|\sqrt{dx + c}b + a|)}{b^4d} - \frac{(dx + c)b^2d - 4\sqrt{dx + c}abd}{b^4d^2} + \frac{2(ab^2c - a^3)}{(\sqrt{dx + c}b + a)b^4d}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] -(2*(b^2*c - 3*a^2)*log(abs(sqrt(d*x + c)*b + a)))/(b^4*d) - ((d*x + c)*b^2*d - 4*sqrt(d*x + c)*a*b*d)/(b^4*d^2) + 2*(a*b^2*c - a^3)/((sqrt(d*x + c)*b + a)*b^4*d)/d

maple [A] time = 0.01, size = 125, normalized size = 1.32

$$-\frac{2ac}{(a + \sqrt{dx + c}b)b^2d^2} - \frac{2c \ln(a + \sqrt{dx + c}b)}{b^2d^2} + \frac{x}{b^2d} + \frac{2a^3}{(a + \sqrt{dx + c}b)b^4d^2} + \frac{6a^2 \ln(a + \sqrt{dx + c}b)}{b^4d^2} + \frac{c}{b^2d^2} - \frac{4\sqrt{dx + c}a}{b^3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+(d*x+c)^(1/2)*b)^2,x)

[Out] $x/b^2/d+1/d^2/b^2*c-4*a*(d*x+c)^{(1/2)}/b^3/d^2-2/d^2*a/b^2/(a+(d*x+c)^{(1/2)*b)*c+2/d^2*a^3/b^4/(a+(d*x+c)^{(1/2)*b})-2/d^2/b^2*\ln(a+(d*x+c)^{(1/2)*b)*c+6/d^2/b^4*\ln(a+(d*x+c)^{(1/2)*b)*a^2$

maxima [A] time = 1.05, size = 90, normalized size = 0.95

$$\frac{\frac{2(ab^2c-a^3)}{\sqrt{dx+c}b^5+ab^4} - \frac{(dx+c)b-4\sqrt{dx+ca}}{b^3} + \frac{2(b^2c-3a^2)\log(\sqrt{dx+c}b+a)}{b^4}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] $-(2*(a*b^2*c - a^3)/(\sqrt{d*x + c}*b^5 + a*b^4) - ((d*x + c)*b - 4*\sqrt{d*x + c})*a)/b^3 + 2*(b^2*c - 3*a^2)*\log(\sqrt{d*x + c}*b + a)/b^4/d^2$

mupad [B] time = 0.06, size = 98, normalized size = 1.03

$$\frac{x}{b^2 d} + \frac{2(a^3 - a b^2 c)}{b(b^4 d^2 \sqrt{c + dx} + a b^3 d^2)} - \frac{\ln(a + b \sqrt{c + dx})(2 b^2 c - 6 a^2)}{b^4 d^2} - \frac{4 a \sqrt{c + dx}}{b^3 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*(c + d*x)^(1/2))^2,x)

[Out] $x/(b^2*d) + (2*(a^3 - a*b^2*c))/(b*(b^4*d^2*(c + d*x)^{(1/2)} + a*b^3*d^2)) - (\log(a + b*(c + d*x)^{(1/2)})*(2*b^2*c - 6*a^2))/(b^4*d^2) - (4*a*(c + d*x)^{(1/2))/(b^3*d^2)$

sympy [A] time = 42.38, size = 131, normalized size = 1.38

$$\left\{ \begin{array}{l} \left(\frac{a(a^2-b^2c) \begin{cases} \frac{\sqrt{c+dx}}{a^2} & \text{for } b=0 \\ -\frac{1}{b(a+b\sqrt{c+dx})} & \text{otherwise} \end{cases}}{b^3d} - \frac{2a\sqrt{c+dx}}{b^3d} + \frac{c+dx}{2b^2d} + \frac{(3a^2-b^2c) \begin{cases} \frac{\sqrt{c+dx}}{a} & \text{for } b=0 \\ \frac{\log(a+b\sqrt{c+dx})}{b} & \text{otherwise} \end{cases}}{b^3d} \right) \\ \hline d \end{array} \right. \begin{array}{l} \text{for } d \neq 0 \\ \\ \text{otherwise} \end{array}$$

$$\left\{ \begin{array}{l} \frac{x^2}{2(a+b\sqrt{c})^2} \\ \hline \end{array} \right. \begin{array}{l} \\ \\ \text{otherwise} \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)**(1/2))**2,x)

[Out] $\text{Piecewise}((2*(-a*(a**2 - b**2*c)*\text{Piecewise}((\sqrt{c + d*x})/a**2, \text{Eq}(b, 0)), (-1/(b*(a + b*\sqrt{c + d*x}))), \text{True}))/b**3*d - 2*a*\sqrt{c + d*x}/(b**3*d) + (c + d*x)/(2*b**2*d) + (3*a**2 - b**2*c)*\text{Piecewise}((\sqrt{c + d*x})/a, \text{Eq}(b, 0)), (\log(a + b*\sqrt{c + d*x}))/b, \text{True}))/b**3*d)/d, \text{Ne}(d, 0)), (x**2/(2*(a + b*\sqrt{c})**2), \text{True}))$

$$3.387 \quad \int \frac{1}{(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=47

$$\frac{2a}{b^2d(a+b\sqrt{c+dx})} + \frac{2\log(a+b\sqrt{c+dx})}{b^2d}$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 190, 43}

$$\frac{2a}{b^2d(a+b\sqrt{c+dx})} + \frac{2\log(a+b\sqrt{c+dx})}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^(-2), x]

[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a + b*Sqrt[c + d*x]])/(b^2*d)

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{(a+b\sqrt{c+dx})^2} dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+b\sqrt{x})^2} dx, x, c+dx\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \frac{x}{(a+bx)^2} dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2} + \frac{1}{b(a+bx)}\right) dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2a}{b^2d(a+b\sqrt{c+dx})} + \frac{2\log(a+b\sqrt{c+dx})}{b^2d} \end{aligned}$$

Mathematica [A] time = 0.03, size = 40, normalized size = 0.85

$$\frac{2 \left(\frac{a}{a+b\sqrt{c+dx}} + \log(a + b\sqrt{c+dx}) \right)}{b^2 d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^(-2), x]

[Out] (2*(a/(a + b*Sqrt[c + d*x]) + Log[a + b*Sqrt[c + d*x]]))/(b^2*d)

IntegrateAlgebraic [A] time = 0.04, size = 53, normalized size = 1.13

$$\frac{2a}{b^2 d (a + b\sqrt{c + dx})} + \frac{2 \log(abd + b^2 d\sqrt{c + dx})}{b^2 d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^(-2), x]

[Out] (2*a)/(b^2*d*(a + b*Sqrt[c + d*x])) + (2*Log[a*b*d + b^2*d*Sqrt[c + d*x]])/(b^2*d)

fricas [A] time = 0.82, size = 75, normalized size = 1.60

$$\frac{2(\sqrt{dx+c}ab - a^2 + (b^2dx + b^2c - a^2)\log(\sqrt{dx+c}b + a))}{b^4d^2x + (b^4c - a^2b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] 2*(sqrt(d*x + c)*a*b - a^2 + (b^2*d*x + b^2*c - a^2)*log(sqrt(d*x + c)*b + a))/(b^4*d^2*x + (b^4*c - a^2*b^2)*d)

giac [A] time = 0.34, size = 44, normalized size = 0.94

$$\frac{2 \log(|\sqrt{dx+c}b + a|)}{b^2 d} + \frac{2a}{(\sqrt{dx+c}b + a)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] 2*log(abs(sqrt(d*x + c)*b + a))/(b^2*d) + 2*a/((sqrt(d*x + c)*b + a)*b^2*d)

maple [B] time = 0.02, size = 142, normalized size = 3.02

$$-\frac{2a^2}{(b^2dx + b^2c - a^2)b^2d} + \frac{a}{(-a + \sqrt{dx+c})b^2d} + \frac{a}{(a + \sqrt{dx+c})b^2d} - \frac{\ln(-a + \sqrt{dx+c}b)}{b^2d} + \frac{\ln(a + \sqrt{dx+c}b)}{b^2d} + \frac{\ln(b^2dx + b^2c - a^2)}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+(d*x+c)^(1/2)*b)^2,x)

[Out] -2*a^2/(b^2*d*x+b^2*c-a^2)/b^2/d+ln(b^2*d*x+b^2*c-a^2)/b^2/d+a/b^2/d/(-a+(d*x+c)^(1/2)*b)-1/b^2/d*ln(-a+(d*x+c)^(1/2)*b)+a/b^2/d/(a+(d*x+c)^(1/2)*b)+ln(a+(d*x+c)^(1/2)*b)/b^2/d

maxima [A] time = 0.87, size = 43, normalized size = 0.91

$$\frac{2 \left(\frac{a}{\sqrt{dx+c} b^3 + ab^2} + \frac{\log(\sqrt{dx+c} b + a)}{b^2} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 2*(a/(sqrt(d*x + c)*b^3 + a*b^2) + log(sqrt(d*x + c)*b + a)/b^2)/d

mupad [B] time = 0.05, size = 43, normalized size = 0.91

$$\frac{2 \ln(a + b \sqrt{c + dx})}{b^2 d} + \frac{2 a}{b^2 (a d + b d \sqrt{c + dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^(1/2))^2,x)

[Out] (2*log(a + b*(c + d*x)^(1/2)))/(b^2*d) + (2*a)/(b^2*(a*d + b*d*(c + d*x)^(1/2)))

sympy [A] time = 1.09, size = 124, normalized size = 2.64

$$\begin{cases} \frac{x}{a^2} & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ \frac{x}{(a+b\sqrt{c})^2} & \text{for } d = 0 \\ \frac{2a \log\left(\frac{a}{b} + \sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} + \frac{2a}{ab^2d+b^3d\sqrt{c+dx}} + \frac{2b\sqrt{c+dx} \log\left(\frac{a}{b} + \sqrt{c+dx}\right)}{ab^2d+b^3d\sqrt{c+dx}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**(1/2))**2,x)

[Out] Piecewise((x/a**2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x/(a + b*sqrt(c))**2, Eq(d, 0)), (2*a*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*a/(a*b**2*d + b**3*d*sqrt(c + d*x)) + 2*b*sqrt(c + d*x)*log(a/b + sqrt(c + d*x))/(a*b**2*d + b**3*d*sqrt(c + d*x)), True))

$$3.388 \quad \int \frac{1}{x(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=129

$$\frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} - \frac{2(a^2 + b^2c) \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{4ab\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2 - b^2c)^2} + \frac{\log(x)(a^2 + b^2c)}{(a^2 - b^2c)^2}$$

Rubi [A] time = 0.12, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {371, 1398, 801, 635, 206, 260}

$$\frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} - \frac{2(a^2 + b^2c) \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{4ab\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{(a^2 - b^2c)^2} + \frac{\log(x)(a^2 + b^2c)}{(a^2 - b^2c)^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*Sqrt[c + d*x])^2), x]

[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[x])/(a^2 - b^2*c)^2 - (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^p_.*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q},

x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\int \frac{1}{x(a + b\sqrt{c + dx})^2} dx = \text{Subst} \left(\int \frac{1}{(a + b\sqrt{x})^2 (-c + x)} dx, x, c + dx \right)$$

$$= 2 \text{Subst} \left(\int \frac{x}{(a + bx)^2 (-c + x^2)} dx, x, \sqrt{c + dx} \right)$$

$$= 2 \text{Subst} \left(\int \left(-\frac{ab}{(a^2 - b^2c)(a + bx)^2} - \frac{b(a^2 + b^2c)}{(a^2 - b^2c)^2(a + bx)} + \frac{2abc - (a^2 + b^2c)x}{(a^2 - b^2c)^2(c - x^2)} \right) dx, \right.$$

$$= \frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} - \frac{2(a^2 + b^2c) \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{2 \text{Subst} \left(\int \frac{2abc - (a^2 + b^2c)x}{c - x^2} \right)}{(a^2 - b^2c)^2}$$

$$= \frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} - \frac{2(a^2 + b^2c) \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{(4abc) \text{Subst} \left(\int \frac{1}{c - x^2} \right)}{(a^2 - b^2c)^2}$$

$$= \frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} + \frac{4ab\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{(a^2 - b^2c)^2} + \frac{(a^2 + b^2c) \log(x)}{(a^2 - b^2c)^2} - \frac{2(a^2 + b^2c)}{(a^2 - b^2c)^2}$$

Mathematica [A] time = 0.28, size = 164, normalized size = 1.27

$$\frac{2a^3 - 2(a^2 + b^2c)(a + b\sqrt{c + dx}) \log(a + b\sqrt{c + dx}) - 2ab^2c + (a - b\sqrt{c})^2 \log(\sqrt{c} - \sqrt{c + dx})(a + b\sqrt{c + dx}) + (a + b\sqrt{c})^2 \log(\sqrt{c + dx} + \sqrt{c})(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2(a + b\sqrt{c + dx})}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*Sqrt[c + d*x]))^2), x]

[Out] (2*a^3 - 2*a*b^2*c + (a - b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] - Sqrt[c + d*x]] + (a + b*Sqrt[c])^2*(a + b*Sqrt[c + d*x])*Log[Sqrt[c] + Sqrt[c + d*x]] - 2*(a^2 + b^2*c)*(a + b*Sqrt[c + d*x])*Log[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x]))

IntegrateAlgebraic [A] time = 0.15, size = 132, normalized size = 1.02

$$\frac{2a}{(a^2 - b^2c)(a + b\sqrt{c + dx})} + \frac{(a^2 + b^2c) \log(-dx)}{(a^2 - b^2c)^2} - \frac{2(a^2 + b^2c) \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} + \frac{4ab\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{(a^2 - b^2c)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*Sqrt[c + d*x]))^2), x]

[Out] (2*a)/((a^2 - b^2*c)*(a + b*Sqrt[c + d*x])) + (4*a*b*Sqrt[c]*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(a^2 - b^2*c)^2 + ((a^2 + b^2*c)*Log[-(d*x)])/(a^2 - b^2*c)^2 - (2*(a^2 + b^2*c)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2

fricas [A] time = 1.04, size = 444, normalized size = 3.44

$$\frac{2a^2\sqrt{c} - 2a^2 + 2(ab^2dx + ab^2c - a^2b)\sqrt{c} \log\left(\frac{a + b\sqrt{c + dx}}{a + b\sqrt{c}}\right) - 2(b^2c - a^4 + (b^2c + a^2b^2)dx) \log(\sqrt{c + dx} + a) + (b^2c - a^4 + (b^2c + a^2b^2)dx) \log(x) - 2(ab^2c - a^2b)\sqrt{c} \arctan\left(\frac{\sqrt{c + dx}}{\sqrt{c}}\right) - 2(b^2c - a^4 + (b^2c + a^2b^2)dx) \log(\sqrt{c + dx} + a) + (b^2c - a^4 + (b^2c + a^2b^2)dx) \log(x) - 2(ab^2c - a^2b)\sqrt{c}}{b^2c^3 - 3a^2b^2c^2 + 3a^2b^2c - a^4 + (b^2c^2 - 2a^2b^2c + a^2b^2)dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out] [(2*a^2*b^2*c - 2*a^4 + 2*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(c)*log((d*x + 2*sqrt(d*x + c)*sqrt(c) + 2*c)/x) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x), (2*a^2*b^2*c - 2*a^4 - 4*(a*b^3*d*x + a*b^3*c - a^3*b)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) - 2*(b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(sqrt(d*x + c)*b + a) + (b^4*c^2 - a^4 + (b^4*c + a^2*b^2)*d*x)*log(x) - 2*(a*b^3*c - a^3*b)*sqrt(d*x + c))/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6 + (b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*x)]

giac [A] time = 0.41, size = 174, normalized size = 1.35

$$\frac{4abc \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right)}{(b^4c^2 - 2a^2b^2c + a^4)\sqrt{-c}} + \frac{(b^2c + a^2)\log(-dx)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{2(b^3c + a^2b)\log(|\sqrt{dx+c}b + a|)}{b^5c^2 - 2a^2b^3c + a^4b} - \frac{2(ab^2c - a^3)}{(b^2c - a^2)^2(\sqrt{dx+c}b + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] -4*a*b*c*arctan(sqrt(d*x + c)/sqrt(-c))/((b^4*c^2 - 2*a^2*b^2*c + a^4)*sqrt(-c)) + (b^2*c + a^2)*log(-d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^3*c + a^2*b)*log(abs(sqrt(d*x + c)*b + a))/(b^5*c^2 - 2*a^2*b^3*c + a^4*b) - 2*(a*b^2*c - a^3)/((b^2*c - a^2)^2*(sqrt(d*x + c)*b + a))

maple [A] time = 0.01, size = 161, normalized size = 1.25

$$\frac{b^2c \ln(dx)}{(-b^2c + a^2)^2} - \frac{2b^2c \ln(a + \sqrt{dx+c}b)}{(-b^2c + a^2)^2} + \frac{4ab\sqrt{c} \operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c + a^2)^2} + \frac{a^2 \ln(dx)}{(-b^2c + a^2)^2} - \frac{2a^2 \ln(a + \sqrt{dx+c}b)}{(-b^2c + a^2)^2} + \frac{2a}{(-b^2c + a^2)(a + \sqrt{dx+c}b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+(d*x+c)^(1/2)*b)^2,x)

[Out] 1/(-b^2*c+a^2)^2*ln(d*x)*b^2*c+1/(-b^2*c+a^2)^2*ln(d*x)*a^2+4*a*b*arctanh((d*x+c)^(1/2)/c^(1/2))*c^(1/2)/(-b^2*c+a^2)^2+2*a/(-b^2*c+a^2)/(a+(d*x+c)^(1/2)*b)-2/(-b^2*c+a^2)^2*ln(a+(d*x+c)^(1/2)*b)*b^2*c-2/(-b^2*c+a^2)^2*ln(a+(d*x+c)^(1/2)*b)*a^2

maxima [A] time = 1.99, size = 176, normalized size = 1.36

$$\frac{2ab\sqrt{c} \log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{b^4c^2 - 2a^2b^2c + a^4} + \frac{(b^2c + a^2)\log(dx)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{2(b^2c + a^2)\log(\sqrt{dx+c}b + a)}{b^4c^2 - 2a^2b^2c + a^4} - \frac{2a}{ab^2c - a^3 + (b^3c - a^2b)\sqrt{dx+c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] -2*a*b*sqrt(c)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/(b^4*c^2 - 2*a^2*b^2*c + a^4) + (b^2*c + a^2)*log(d*x)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*(b^2*c + a^2)*log(sqrt(d*x + c)*b + a)/(b^4*c^2 - 2*a^2*b^2*c + a^4) - 2*a/(a*b^2*c - a^3 + (b^3*c - a^2*b)*sqrt(d*x + c))

mupad [B] time = 3.51, size = 125, normalized size = 0.97

$$\frac{\ln(\sqrt{c+dx}-\sqrt{c})}{(a+b\sqrt{c})^2} + \ln(a+b\sqrt{c+dx}) \left(\frac{2}{b^2c-a^2} - \frac{4b^2c}{(b^2c-a^2)^2} \right) + \frac{\ln(\sqrt{c+dx}+\sqrt{c})}{(a-b\sqrt{c})^2} - \frac{2a}{(b^2c-a^2)(a+b\sqrt{c+dx})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^(1/2))^2),x)

[Out] log((c + d*x)^(1/2) - c^(1/2))/(a + b*c^(1/2))^2 + log(a + b*(c + d*x)^(1/2)) * (2/(b^2*c - a^2) - (4*b^2*c)/(b^2*c - a^2)^2) + log((c + d*x)^(1/2) + c^(1/2))/(a - b*c^(1/2))^2 - (2*a)/((b^2*c - a^2)*(a + b*(c + d*x)^(1/2)))

sympy [A] time = 50.51, size = 153, normalized size = 1.19

$$\frac{2ab \left(\begin{matrix} \frac{\sqrt{c+dx}}{a^2} & \text{for } b = 0 \\ -\frac{1}{b(a+b\sqrt{c+dx})} & \text{otherwise} \end{matrix} \right)}{a^2 - b^2c} - \frac{2b(a^2 + b^2c) \left(\begin{matrix} \frac{\sqrt{c+dx}}{a} & \text{for } b = 0 \\ \frac{\log(a+b\sqrt{c+dx})}{b} & \text{otherwise} \end{matrix} \right)}{(a^2 - b^2c)^2} - \frac{2 \left(\frac{2abc \operatorname{atan}\left(\frac{\sqrt{c+dx}}{\sqrt{-c}}\right)}{\sqrt{-c}} + \left(-\frac{a^2}{2} - \frac{b^2c}{2}\right) \log(-dx) \right)}{(a^2 - b^2c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**2,x)

[Out] -2*a*b*Piecewise((sqrt(c + d*x)/a**2, Eq(b, 0)), (-1/(b*(a + b*sqrt(c + d*x))), True))/(a**2 - b**2*c) - 2*b*(a**2 + b**2*c)*Piecewise((sqrt(c + d*x)/a, Eq(b, 0)), (log(a + b*sqrt(c + d*x))/b, True))/(a**2 - b**2*c)**2 - 2*(2*a*b*c*atan(sqrt(c + d*x)/sqrt(-c))/sqrt(-c) + (-a**2/2 - b**2*c/2)*log(-d*x))/(a**2 - b**2*c)**2

$$3.389 \quad \int \frac{1}{x^2(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=202

$$\frac{4ab^2d}{(a^2-b^2c)^2(a+b\sqrt{c+dx})} - \frac{a-b\sqrt{c+dx}}{x(a^2-b^2c)(a+b\sqrt{c+dx})} + \frac{b^2d \log(x)(3a^2+b^2c)}{(a^2-b^2c)^3} - \frac{2b^2d(3a^2+b^2c) \log(a+b\sqrt{c+dx})}{(a^2-b^2c)^3}$$

Rubi [A] time = 0.25, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$\frac{4ab^2d}{(a^2-b^2c)^2(a+b\sqrt{c+dx})} - \frac{a-b\sqrt{c+dx}}{x(a^2-b^2c)(a+b\sqrt{c+dx})} + \frac{b^2d \log(x)(3a^2+b^2c)}{(a^2-b^2c)^3} - \frac{2b^2d(3a^2+b^2c) \log(a+b\sqrt{c+dx})}{(a^2-b^2c)^3} + \frac{2abd(a^2+3b^2c) \tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(a^2-b^2c)^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(a + b*Sqrt[c + d*x])^2), x]
```

```
[Out] (4*a*b^2*d)/((a^2 - b^2*c)^2*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])
/((a^2 - b^2*c)*x*(a + b*Sqrt[c + d*x])) + (2*a*b*(a^2 + 3*b^2*c)*d*ArcTanh
[Sqrt[c + d*x]/Sqrt[c]])/(Sqrt[c]*(a^2 - b^2*c)^3) + (b^2*(3*a^2 + b^2*c)*d
*Log[x])/(a^2 - b^2*c)^3 - (2*b^2*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]
])/((a^2 - b^2*c)^3)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[Simp
lifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /;
FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 801

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))/((a_) + (c_.)*(x_)^2),
x_Symbol] := Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
```

```
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symb
ol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q},
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^2 (a + b\sqrt{c + dx})^2} dx &= d \operatorname{Subst} \left(\int \frac{1}{(a + b\sqrt{x})^2 (-c + x)^2} dx, x, c + dx \right) \\
&= (2d) \operatorname{Subst} \left(\int \frac{x}{(a + bx)^2 (-c + x^2)^2} dx, x, \sqrt{c + dx} \right) \\
&= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x(a + b\sqrt{c + dx})} + \frac{d \operatorname{Subst} \left(\int \frac{-2abc + 2b^2cx}{(a + bx)^2 (-c + x^2)} dx, x, \sqrt{c + dx} \right)}{c(a^2 - b^2c)} \\
&= -\frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x(a + b\sqrt{c + dx})} + \frac{d \operatorname{Subst} \left(\int \left(-\frac{4ab^3c}{(a^2 - b^2c)(a + bx)^2} - \frac{2b^3c(3a^2 + b^2c)}{(-a^2 + b^2c)^2(a + bx)} + \frac{2bc(a^2 - b^2c)}{(-a^2 + b^2c)^2(a + bx)} \right) dx, x, \sqrt{c + dx} \right)}{c(a^2 - b^2c)} \\
&= \frac{4ab^2d}{(a^2 - b^2c)^2(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x(a + b\sqrt{c + dx})} - \frac{2b^2(3a^2 + b^2c)d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} \\
&= \frac{4ab^2d}{(a^2 - b^2c)^2(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x(a + b\sqrt{c + dx})} - \frac{2b^2(3a^2 + b^2c)d \log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^2} \\
&= \frac{4ab^2d}{(a^2 - b^2c)^2(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{(a^2 - b^2c)x(a + b\sqrt{c + dx})} + \frac{2ab(a^2 + 3b^2c)d \tanh^{-1} \left(\frac{\sqrt{c + dx}}{\sqrt{c}} \right)}{\sqrt{c}(a^2 - b^2c)}
\end{aligned}$$

Mathematica [A] time = 0.76, size = 230, normalized size = 1.14

$$\frac{\sqrt{c} \left(2b^2 \sqrt{c} d (3a^2 + b^2c) \log(a + b\sqrt{c + dx}) + \frac{\sqrt{c} (a^2 - b^2c) (a^3 - a^2b\sqrt{c + dx} - ab^2(c + 4dx) + b^3c\sqrt{c + dx})}{x(a + b\sqrt{c + dx})} - bd(a + b\sqrt{c})^3 \log(\sqrt{c + dx} + \sqrt{c}) \right)}{(a^2 - b^2c)^2} + \frac{(ab\sqrt{c}d - b^2cd) \log(\sqrt{c} - \sqrt{c + dx})}{(a + b\sqrt{c})^2}$$

$$c(b^2c - a^2)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(a + b*Sqrt[c + d*x])^2), x]
```

```
[Out] (((a*b*Sqrt[c]*d - b^2*c*d)*Log[Sqrt[c] - Sqrt[c + d*x]])/(a + b*Sqrt[c])^2
+ (Sqrt[c]*((Sqrt[c]*(a^2 - b^2*c)*(a^3 - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt
[c + d*x] - a*b^2*(c + 4*d*x)))/(x*(a + b*Sqrt[c + d*x])) - b*(a + b*Sqrt[c
```

])^3*d*Log[Sqrt[c] + Sqrt[c + d*x]] + 2*b^2*Sqrt[c]*(3*a^2 + b^2*c)*d*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^2)/(c*(-a^2 + b^2*c))

IntegrateAlgebraic [A] time = 0.38, size = 212, normalized size = 1.05

$$\frac{\log(-dx)(3a^2b^2d + b^4cd)}{(a^2 - b^2c)^3} - \frac{2(3a^2b^2d + b^4cd)\log(a + b\sqrt{c + dx})}{(a^2 - b^2c)^3} - \frac{a^3 - a^2b\sqrt{c + dx} - 4ab^2(c + dx) + 3ab^2c + b^3c\sqrt{c + dx}}{x(a^2 - b^2c)^2(a + b\sqrt{c + dx})} - \frac{2d(a^3b + 3ab^3c)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{\sqrt{c}(b^2c - a^2)^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*(a + b*Sqrt[c + d*x])^2), x]
 [Out] -((a^3 + 3*a*b^2*c - a^2*b*Sqrt[c + d*x] + b^3*c*Sqrt[c + d*x] - 4*a*b^2*(c + d*x))/((a^2 - b^2*c)^2*x*(a + b*Sqrt[c + d*x]))) - (2*(a^3*b + 3*a*b^3*c)*d*ArcTanh[Sqrt[c + d*x]/Sqrt[c]]/(Sqrt[c]*(-a^2 + b^2*c)^3) + ((3*a^2*b^2*d + b^4*c*d)*Log[-(d*x)])/(a^2 - b^2*c)^3 - (2*(3*a^2*b^2*d + b^4*c*d)*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^3

fricas [B] time = 1.63, size = 854, normalized size = 4.23



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")
 [Out] [-(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - ((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(c)*log((d*x - 2*sqrt(d*x + c))*sqrt(c) + 2*c)/x) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x), -(b^6*c^4 - a^2*b^4*c^3 - a^4*b^2*c^2 + a^6*c + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x - 2*((3*a*b^5*c + a^3*b^3)*d^2*x^2 + (3*a*b^5*c^2 - 2*a^3*b^3*c - a^5*b)*d*x)*sqrt(-c)*arctan(sqrt(d*x + c)*sqrt(-c)/c) - 2*((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(sqrt(d*x + c)*b + a) + ((b^6*c^2 + 3*a^2*b^4*c)*d^2*x^2 + (b^6*c^3 + 2*a^2*b^4*c^2 - 3*a^4*b^2*c)*d*x)*log(x) - 2*(a*b^5*c^3 - 2*a^3*b^3*c^2 + a^5*b*c + 2*(a*b^5*c^2 - a^3*b^3*c)*d*x)*sqrt(d*x + c))/((b^8*c^4 - 3*a^2*b^6*c^3 + 3*a^4*b^4*c^2 - a^6*b^2*c)*d*x^2 + (b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*x)]

giac [A] time = 0.41, size = 311, normalized size = 1.54

$$\frac{(b^4cd + 3a^2b^2d)\log(-dx)}{b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6} + \frac{2(b^5cd + 3a^2b^3d)\log(|-\sqrt{dx + c}b - a|)}{b^7c^3 - 3a^2b^5c^2 + 3a^4b^3c - a^6b} + \frac{2(3ab^3cd + a^3bd)\arctan\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(b^6c^3 - 3a^2b^4c^2 + 3a^4b^2c - a^6)\sqrt{c}} - \frac{\sqrt{dx + c}b^3cd - 4(dx + c)ab^2d + 3ab^2cd - \sqrt{dx + c}a^2bd + a^3d}{(b^4c^2 - 2a^2b^2c + a^4)((dx + c)^2b - \sqrt{dx + c}bc + (dx + c)a - ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")
 [Out] -(b^4*c*d + 3*a^2*b^2*d)*log(-d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6) + 2*(b^5*c*d + 3*a^2*b^3*d)*log(abs(-sqrt(d*x + c)*b - a))/(b^7*c^3 - 3*a^2*b^5*c^2 + 3*a^4*b^3*c - a^6*b) + 2*(3*a*b^3*c*d + a^3*b*d)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*sqrt(-c)) - (sqrt(d*x + c)*b^3*c*d - 4*(d*x + c)*a*b^2*d + 3*a*b^2*c*d - sqrt(d*x + c)*a^2*b*d + a^3*d)/((b^4*c^2 - 2*a^2*b^2*c + a^4)*((d*x + c)^(3/2)*b - sqrt(d*x + c)*b*c + (d*x + c)*a - a*c))

maple [A] time = 0.02, size = 312, normalized size = 1.54

$$\frac{b^4cd\ln(dx)}{(-b^2c + a^2)^3} - \frac{2b^4cd\ln(a + \sqrt{dx + c}b)}{(-b^2c + a^2)^3} + \frac{6ab^3\sqrt{c}d\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c + a^2)^3} + \frac{3a^2b^3d\ln(dx)}{(-b^2c + a^2)^3} - \frac{6a^2b^2d\ln(a + \sqrt{dx + c}b)}{(-b^2c + a^2)^3} + \frac{2a^3bd\operatorname{arctanh}\left(\frac{\sqrt{dx+c}}{\sqrt{c}}\right)}{(-b^2c + a^2)^3\sqrt{c}} + \frac{b^4c^2}{(-b^2c + a^2)^3x} - \frac{2\sqrt{dx + c}ab^3c}{(-b^2c + a^2)^3x} + \frac{2ab^2d}{(-b^2c + a^2)^2(a + \sqrt{dx + c}b)} - \frac{a^4}{(-b^2c + a^2)^3x} + \frac{2\sqrt{dx + c}a^3b}{(-b^2c + a^2)^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^2,x)
```

```
[Out] -2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*a*b^3*c+2/(-b^2*c+a^2)^3/x*(d*x+c)^(1/2)*
a^3*b+1/(-b^2*c+a^2)^3/x*b^4*c^2-1/(-b^2*c+a^2)^3/x*a^4+d/(-b^2*c+a^2)^3*ln
(d*x)*b^4*c+3*d/(-b^2*c+a^2)^3*ln(d*x)*b^2*a^2+6*d/(-b^2*c+a^2)^3*c^(1/2)*a
rctanh((d*x+c)^(1/2)/c^(1/2))*a*b^3+2*d/(-b^2*c+a^2)^3*b/c^(1/2)*arctanh((d
*x+c)^(1/2)/c^(1/2))*a^3+2*a*b^2*d/(-b^2*c+a^2)^2/(a+(d*x+c)^(1/2)*b)-2*d*b
^4/(-b^2*c+a^2)^3*ln(a+(d*x+c)^(1/2)*b)*c-6*d*b^2/(-b^2*c+a^2)^3*ln(a+(d*x+
c)^(1/2)*b)*a^2
```

maxima [A] time = 2.01, size = 367, normalized size = 1.82

$$-\frac{\left(\frac{(b^4c+3a^2b^2)\log(dx)}{b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6}-\frac{2(b^4c+3a^2b^2)\log(\sqrt{dx+cb+a})}{b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6}-\frac{(3ab^3c+a^3b)\log\left(\frac{\sqrt{dx+c}-\sqrt{c}}{\sqrt{dx+c}+\sqrt{c}}\right)}{(b^6c^3-3a^2b^4c^2+3a^4b^2c-a^6)\sqrt{c}}+\frac{4(dx+c)ab^2-3ab^2c-a^3-(b^3c-a^2b)\sqrt{dx+c}}{ab^4c^3-2a^2b^2c^2+a^5c-(b^5c^2-2a^2b^3c+a^4b)(dx+c)^2-(ab^4c^2-2a^3b^2c+a^5)(dx+c)+(b^5c^3-2a^2b^3c^2+a^4bc)\sqrt{dx+c}}\right)}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] -d*((b^4*c + 3*a^2*b^2)*log(d*x)/(b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a
^6) - 2*(b^4*c + 3*a^2*b^2)*log(sqrt(d*x + c)*b + a)/(b^6*c^3 - 3*a^2*b^4*c
^2 + 3*a^4*b^2*c - a^6) - (3*a*b^3*c + a^3*b)*log((sqrt(d*x + c) - sqrt(c))
/(sqrt(d*x + c) + sqrt(c)))/((b^6*c^3 - 3*a^2*b^4*c^2 + 3*a^4*b^2*c - a^6)*
sqrt(c)) + (4*(d*x + c)*a*b^2 - 3*a*b^2*c - a^3 - (b^3*c - a^2*b)*sqrt(d*x
+ c))/(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - (b^5*c^2 - 2*a^2*b^3*c + a^4*b)*
(d*x + c)^(3/2) - (a*b^4*c^2 - 2*a^3*b^2*c + a^5)*(d*x + c) + (b^5*c^3 - 2*
a^2*b^3*c^2 + a^4*b*c)*sqrt(d*x + c))
```

mupad [B] time = 0.73, size = 275, normalized size = 1.36

$$\frac{bd \ln(\sqrt{c+dx} + \sqrt{c})}{a^3\sqrt{c} - b^3c^2 + 3ab^2c^{3/2} - 3a^2bc} - \frac{\frac{ad(a^2+3c^2)}{(b^2c-a^2)^2} + \frac{bd\sqrt{c+dx}}{b^2c-a^2} - \frac{4a^2d(c+dx)}{a^4-2a^2b^2c+b^4c^2}}{b(c+dx)^{3/2} - ac + a(c+dx) - bc\sqrt{c+dx}} - \frac{bd \ln(\sqrt{c+dx} - \sqrt{c})}{a^3\sqrt{c} + b^3c^2 + 3ab^2c^{3/2} + 3a^2bc} - \ln(a + b\sqrt{c+dx}) \left(\frac{6b^2d}{(b^2c-a^2)^2} - \frac{8b^4cd}{(b^2c-a^2)^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^2),x)
```

```
[Out] (b*d*log((c + d*x)^(1/2) + c^(1/2)))/(a^3*c^(1/2) - b^3*c^2 + 3*a*b^2*c^(3/2)
- 3*a^2*b*c) - ((a*d*(3*b^2*c + a^2))/(b^2*c - a^2)^2 + (b*d*(c + d*x)^(
1/2))/(b^2*c - a^2) - (4*a*b^2*d*(c + d*x))/(a^4 + b^4*c^2 - 2*a^2*b^2*c))/
(b*(c + d*x)^(3/2) - a*c + a*(c + d*x) - b*c*(c + d*x)^(1/2)) - (b*d*log((c
+ d*x)^(1/2) - c^(1/2)))/(a^3*c^(1/2) + b^3*c^2 + 3*a*b^2*c^(3/2) + 3*a^2*
b*c) - log(a + b*(c + d*x)^(1/2))*((6*b^2*d)/(b^2*c - a^2)^2 - (8*b^4*c*d)/
(b^2*c - a^2)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

$$3.390 \quad \int \frac{1}{x^3(a+b\sqrt{c+dx})^2} dx$$

Optimal. Leaf size=306

$$\frac{ab^2d^2(a^2+11b^2c)}{2c(a^2-b^2c)^3(a+b\sqrt{c+dx})} - \frac{a-b\sqrt{c+dx}}{2x^2(a^2-b^2c)(a+b\sqrt{c+dx})} - \frac{bd(3abc-(a^2+2b^2c)\sqrt{c+dx})}{2cx(a^2-b^2c)^2(a+b\sqrt{c+dx})} + \frac{b^4d^2\log(x)}{(a^2-b^2c)^4}$$

Rubi [A] time = 0.40, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {371, 1398, 823, 801, 635, 206, 260}

$$\frac{abd^2(-10a^2b^2c+a^4-15b^4c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2c^3(a^2-b^2c)^4} + \frac{ab^2d^2(a^2+11b^2c)}{2c(a^2-b^2c)^3(a+b\sqrt{c+dx})} + \frac{b^4d^2\log(x)(5a^2+b^2c)}{(a^2-b^2c)^4} - \frac{2b^4d^2(5a^2+b^2c)\log(a+b\sqrt{c+dx})}{(a^2-b^2c)^4} - \frac{a-b\sqrt{c+dx}}{2x^2(a^2-b^2c)(a+b\sqrt{c+dx})} - \frac{bd(3abc-(a^2+2b^2c)\sqrt{c+dx})}{2cx(a^2-b^2c)^2(a+b\sqrt{c+dx})}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]

[Out] (a*b^2*(a^2 + 11*b^2*c)*d^2)/(2*c*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])) - (a - b*Sqrt[c + d*x])/(2*(a^2 - b^2*c)*x^2*(a + b*Sqrt[c + d*x])) - (b*d*(3*a*b*c - (a^2 + 2*b^2*c)*Sqrt[c + d*x]))/(2*c*(a^2 - b^2*c)^2*x*(a + b*Sqrt[c + d*x])) - (a*b*(a^4 - 10*a^2*b^2*c - 15*b^4*c^2)*d^2*ArcTanh[Sqrt[c + d*x]/Sqrt[c]])/(2*c^(3/2)*(a^2 - b^2*c)^4) + (b^4*(5*a^2 + b^2*c)*d^2*Log[x])/(a^2 - b^2*c)^4 - (2*b^4*(5*a^2 + b^2*c)*d^2*Log[a + b*Sqrt[c + d*x]])/(a^2 - b^2*c)^4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 801

Int((((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^(q*(a + c*x^(2*g*n)))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^3 (a + b\sqrt{c + dx})^2} dx &= d^2 \text{Subst} \left(\int \frac{1}{(a + b\sqrt{x})^2 (-c + x)^3} dx, x, c + dx \right) \\ &= (2d^2) \text{Subst} \left(\int \frac{x}{(a + bx)^2 (-c + x^2)^3} dx, x, \sqrt{c + dx} \right) \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} + \frac{d^2 \text{Subst} \left(\int \frac{-2abc + 4b^2cx}{(a + bx)^2(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{2c(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} - \frac{bd(3abc - (a^2 + 2b^2c)\sqrt{c + dx})}{2c(a^2 - b^2c)^2 x(a + b\sqrt{c + dx})} + \frac{d^2 \text{Subst} \left(\int \frac{x}{(a + bx)^2(-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{2c(a^2 - b^2c)} \\ &= -\frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} - \frac{bd(3abc - (a^2 + 2b^2c)\sqrt{c + dx})}{2c(a^2 - b^2c)^2 x(a + b\sqrt{c + dx})} + \frac{d^2 \text{Subst} \left(\int \frac{x}{(a + bx)^2(-c + x^2)} dx, x, \sqrt{c + dx} \right)}{2c(a^2 - b^2c)} \\ &= \frac{ab^2(a^2 + 11b^2c)d^2}{2c(a^2 - b^2c)^3(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} - \frac{bd(3abc - (a^2 + 2b^2c)\sqrt{c + dx})}{2c(a^2 - b^2c)^2 x(a + b\sqrt{c + dx})} \\ &= \frac{ab^2(a^2 + 11b^2c)d^2}{2c(a^2 - b^2c)^3(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} - \frac{bd(3abc - (a^2 + 2b^2c)\sqrt{c + dx})}{2c(a^2 - b^2c)^2 x(a + b\sqrt{c + dx})} \\ &= \frac{ab^2(a^2 + 11b^2c)d^2}{2c(a^2 - b^2c)^3(a + b\sqrt{c + dx})} - \frac{a - b\sqrt{c + dx}}{2(a^2 - b^2c)x^2(a + b\sqrt{c + dx})} - \frac{bd(3abc - (a^2 + 2b^2c)\sqrt{c + dx})}{2c(a^2 - b^2c)^2 x(a + b\sqrt{c + dx})} \end{aligned}$$

Mathematica [A] time = 0.83, size = 401, normalized size = 1.31

$$\frac{d^2 \left(\frac{2b\sqrt{c}(a^2 + 2b^2c) \left((b\sqrt{c} - a) \log(\sqrt{c} - \sqrt{c + dx}) + (a + b\sqrt{c}) \log(\sqrt{c + dx} + \sqrt{c}) - 2b\sqrt{c} \log(a + b\sqrt{c + dx}) \right) - abc(a^2 + 11b^2c) \left(\frac{2b \left(\frac{b^2c - a^2}{a + b\sqrt{c + dx}} + 2a \log(a + b\sqrt{c + dx}) \right)}{(a^2 - b^2c)^2} + \frac{\log(\sqrt{c} - \sqrt{c + dx}) - \log(\sqrt{c + dx} + \sqrt{c})}{\sqrt{(a + b\sqrt{c})^2}} - \frac{\log(a - b\sqrt{c})}{\sqrt{(a - b\sqrt{c})^2}} \right) \right)}{2c(a^2 - b^2c)} + \frac{bd(a^2\sqrt{c + dx} - 3abc + 2b^2c\sqrt{c + dx})}{x(a^2 - b^2c)(a + b\sqrt{c + dx})} - \frac{c(a - b\sqrt{c + dx})}{x^2(a + b\sqrt{c + dx})} \right)}{2c(a^2 - b^2c)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]

[Out]
$$\frac{-((c*(a - b*\sqrt{c + d*x}))/x^2*(a + b*\sqrt{c + d*x}))) + (b*d*(-3*a*b*c + a^2*\sqrt{c + d*x} + 2*b^2*c*\sqrt{c + d*x}))/((a^2 - b^2*c)*x*(a + b*\sqrt{c + d*x})) + (d^2*((2*b*\sqrt{c}*(a^2 + 2*b^2*c)*((-a + b*\sqrt{c}))*\text{Log}[\sqrt{c} - \sqrt{c + d*x}] + (a + b*\sqrt{c}))*\text{Log}[\sqrt{c} + \sqrt{c + d*x}] - 2*b*\sqrt{c}*\text{Log}[a + b*\sqrt{c + d*x}]])/(-a^2 + b^2*c) - a*b*c*(a^2 + 11*b^2*c)*(\text{Log}[\sqrt{c} - \sqrt{c + d*x}]/((a + b*\sqrt{c})^2*\sqrt{c}) - \text{Log}[\sqrt{c} + \sqrt{c + d*x}]/((a - b*\sqrt{c})^2*\sqrt{c})) + (2*b*((-a^2 + b^2*c)/(a + b*\sqrt{c + d*x}) + 2*a*\text{Log}[a + b*\sqrt{c + d*x}]))/(a^2 - b^2*c)^2)}{(2*c*(a^2 - b^2*c))}$$

IntegrateAlgebraic [A] time = 0.82, size = 358, normalized size = 1.17

$$\frac{\log(-dx)(5a^2b^2d^2 + b^2cd^2) - 2(5a^2b^2d^2 + b^2cd^2)\log(a + b\sqrt{c+dx})}{(a^2 - b^2c)^4} - \frac{d^2(a^2b - 10a^2b^2c - 15ab^2c^2)\tanh^{-1}\left(\frac{\sqrt{c+dx}}{\sqrt{c}}\right)}{2a^2(b^2c - a^2)^4} - \frac{a^2(-c) + a^2b(c + dx)^{3/2} + 6a^2b^2c^2 + a^2b^2(c + dx)^2 - 5a^2b^2c(c + dx) - 3a^2b^2c^2\sqrt{c + dx} + a^2b^2c(c + dx)^{3/2} + 7ab^3c^2 - 19ab^3c^2(c + dx) + 11ab^3c^2(c + dx)^2 + 3b^3c^2\sqrt{c + dx} - 2b^3c^2(c + dx)^{3/2}}{2c^2(b^2c - a^2)^4(a + b\sqrt{c+dx})}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^3*(a + b*Sqrt[c + d*x])^2),x]

[Out]
$$\frac{-1/2*(-(a^5*c) + 6*a^3*b^2*c^2 + 7*a*b^4*c^3 - 3*a^2*b^3*c^2*\sqrt{c + d*x} + 3*b^5*c^3*\sqrt{c + d*x} - 5*a^3*b^2*c*(c + d*x) - 19*a*b^4*c^2*(c + d*x) + a^4*b*(c + d*x)^{3/2} + a^2*b^3*c*(c + d*x)^{3/2} - 2*b^5*c^2*(c + d*x)^{3/2} + a^3*b^2*c*(c + d*x)^2 + 11*a*b^4*c*(c + d*x)^2)/(c*(-a^2 + b^2*c)^3*x^2*(a + b*\sqrt{c + d*x})) - ((a^5*b - 10*a^3*b^3*c - 15*a*b^5*c^2)*d^2*\text{ArcTanh}[\sqrt{c + d*x}/\sqrt{c}])/(2*c^{3/2}*(-a^2 + b^2*c)^4) + ((5*a^2*b^4*d^2 + b^6*c*d^2)*\text{Log}[-(d*x)])/(a^2 - b^2*c)^4 - (2*(5*a^2*b^4*d^2 + b^6*c*d^2)*\text{Log}[a + b*\sqrt{c + d*x}])/(a^2 - b^2*c)^4}$$

fricas [B] time = 3.96, size = 1252, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} &[-1/4*(2*b^8*c^6 - 4*a^2*b^6*c^5 + 4*a^6*b^2*c^3 - 2*a^8*c^2 - 4*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - 2*(b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x - ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*\sqrt{c}*\log((d*x + 2*\sqrt{d*x + c})*\sqrt{c} + 2*c)/x + 8*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*\log(\sqrt{d*x + c}*b + a) - 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*\log(x) - 2*(2*a*b^7*c^5 - 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*\sqrt{d*x + c}]/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2), -1/2*(b^8*c^6 - 2*a^2*b^6*c^5 + 2*a^6*b^2*c^3 - a^8*c^2 - 2*(b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2 - (b^8*c^5 + 3*a^2*b^6*c^4 - 9*a^4*b^4*c^3 + 5*a^6*b^2*c^2)*d*x + ((15*a*b^7*c^2 + 10*a^3*b^5*c - a^5*b^3)*d^3*x^3 + (15*a*b^7*c^3 - 5*a^3*b^5*c^2 - 11*a^5*b^3*c + a^7*b)*d^2*x^2)*\sqrt{-c}*\arctan(\sqrt{d*x + c}*\sqrt{-c}/c) + 4*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*\log(\sqrt{d*x + c}*b + a) - 2*((b^8*c^3 + 5*a^2*b^6*c^2)*d^3*x^3 + (b^8*c^4 + 4*a^2*b^6*c^3 - 5*a^4*b^4*c^2)*d^2*x^2)*\log(x) - (2*a*b^7*c^5 - 6*a^3*b^5*c^4 + 6*a^5*b^3*c^3 - 2*a^7*b*c^2 - (11*a*b^7*c^3 - 10*a^3*b^5*c^2 - a^5*b^3*c)*d^2*x^2 - (5*a*b^7*c^4 - 9*a^3*b^5*c^3 + 3*a^5*b^3*c^2 + a^7*b*c)*d*x)*\sqrt{d*x + c}]/((b^10*c^6 - 4*a^2*b^8*c^5 + 6*a^4*b^6*c^4 - 4*a^6*b^4*c^3 + a^8*b^2*c^2)*d*x^3 + (b^10*c^7 - 5*a^2*b^8*c^6 + 10*a^4*b^6*c^5 - 10*a^6*b^4*c^4 + 5*a^8*b^2*c^3 - a^10*c^2)*x^2)] \end{aligned}$$

giac [A] time = 0.50, size = 521, normalized size = 1.70

$$\frac{(b^2d^2 + 5a^2b^2)\log(-dx)}{b^4c^4 - 4a^2b^2c^2 + 6a^4b^2c^2} - \frac{2(b^2d^2 + 5a^2b^2)\log(\sqrt{dx+c+a})}{b^4c^4 - 4a^2b^2c^2 + 6a^4b^2c^2} + \frac{(15ab^2c^2d^2 + 10a^3b^3c^2d^2 - a^5b^5d^2)\arctan(\frac{\sqrt{dx+c}}{\sqrt{-c}})}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} - \frac{7ab^4c^2d^2 - 7a^3b^4c^2d^2 + a^5c^2d^2 + (11ab^2c^2d^2 - 10a^3b^3c^2d^2 - a^5b^5c^2d^2)(dx+c)^2 - (2b^7c^3d^2 - 3a^2b^5c^2d^2 + a^6b^7d^2)(dx+c)^2 - (19ab^6c^2d^2 - 14a^3b^4c^2d^2 - 5a^5b^2c^2d^2)(dx+c) + 3(b^7c^4d^2 - 2a^2b^5c^4d^2 + a^4b^3c^4d^2)\sqrt{dx+c}}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="giac")

[Out] (b^6*c*d^2 + 5*a^2*b^4*d^2)*log(-d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - 2*(b^7*c*d^2 + 5*a^2*b^5*d^2)*log(abs(sqrt(d*x + c)*b + a))/(b^9*c^4 - 4*a^2*b^7*c^3 + 6*a^4*b^5*c^2 - 4*a^6*b^3*c + a^8*b) - 1/2*(15*a*b^5*c^2*d^2 + 10*a^3*b^3*c*d^2 - a^5*b*d^2)*arctan(sqrt(d*x + c)/sqrt(-c))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*sqrt(-c)) - 1/2*(7*a*b^6*c^4*d^2 - a^3*b^4*c^3*d^2 - 7*a^5*b^2*c^2*d^2 + a^7*c*d^2 + (11*a*b^6*c^2*d^2 - 10*a^3*b^4*c*d^2 - a^5*b^2*d^2)*(d*x + c)^2 - (2*b^7*c^3*d^2 - 3*a^2*b^5*c^2*d^2 + a^6*b*d^2)*(d*x + c)^(3/2) - (19*a*b^6*c^3*d^2 - 14*a^3*b^4*c^2*d^2 - 5*a^5*b^2*c*d^2)*(d*x + c) + 3*(b^7*c^4*d^2 - 2*a^2*b^5*c^3*d^2 + a^4*b^3*c^2*d^2)*sqrt(d*x + c))/(b^2*c - a^2)^4*(sqrt(d*x + c)*b + a)*c*d^2*x^2)

maple [B] time = 0.02, size = 610, normalized size = 1.99

$$\frac{b^2d^2 \ln(dx)}{(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} - \frac{2b^2d^2 \ln(a + \sqrt{dx+c})}{(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} + \frac{15ab^2c^2d^2 + 10a^3b^3c^2d^2 - a^5b^5d^2}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} \arctan\left(\frac{\sqrt{dx+c}}{\sqrt{-c}}\right) - \frac{7ab^4c^2d^2 - 7a^3b^4c^2d^2 + a^5c^2d^2 + (11ab^2c^2d^2 - 10a^3b^3c^2d^2 - a^5b^5c^2d^2)(dx+c)^2 - (2b^7c^3d^2 - 3a^2b^5c^2d^2 + a^6b^7d^2)(dx+c)^2 - (19ab^6c^2d^2 - 14a^3b^4c^2d^2 - 5a^5b^2c^2d^2)(dx+c) + 3(b^7c^4d^2 - 2a^2b^5c^4d^2 + a^4b^3c^4d^2)\sqrt{dx+c}}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^2,x)

[Out] -7/2/(-b^2*c+a^2)^4/x^2*a*b^5*c*(d*x+c)^(3/2)+3/(-b^2*c+a^2)^4/x^2*a^3*b^3*(d*x+c)^(3/2)+1/2/(-b^2*c+a^2)^4/x^2*a^5*b/c*(d*x+c)^(3/2)+d/(-b^2*c+a^2)^4/x*b^6*c^2-1/2/(-b^2*c+a^2)^4/x^2*c^3*b^6+2*d/(-b^2*c+a^2)^4/x*a^2*b^4*c+1/2/(-b^2*c+a^2)^4/x^2*b^4*a^2*c^2-3*d/(-b^2*c+a^2)^4/x*a^4*b^2+1/2/(-b^2*c+a^2)^4/x^2*b^2*a^4*c+9/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*a*b^5*c^2-5/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*b^3*a^3*c+1/2/(-b^2*c+a^2)^4/x^2*(d*x+c)^(1/2)*b*a^5-1/2/(-b^2*c+a^2)^4/x^2*a^6+d^2/(-b^2*c+a^2)^4*b^6*c*ln(d*x)+5*d^2/(-b^2*c+a^2)^4*b^4*ln(d*x)*a^2+15/2*d^2/(-b^2*c+a^2)^4*b^5*c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a+5*d^2/(-b^2*c+a^2)^4*b^3/c^(1/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^3-1/2*d^2/(-b^2*c+a^2)^4*b/c^(3/2)*arctanh((d*x+c)^(1/2)/c^(1/2))*a^5+2*d^2*b^4/(-b^2*c+a^2)^3*a/(a+(d*x+c)^(1/2)*b)-2*d^2*b^6/(-b^2*c+a^2)^4*ln(a+(d*x+c)^(1/2)*b)*c-10*d^2*b^4/(-b^2*c+a^2)^4*ln(a+(d*x+c)^(1/2)*b)*a^2

maxima [B] time = 2.09, size = 659, normalized size = 2.15

$$\frac{1}{2} \left(\frac{4(b^2c^2 + 5a^2b^2)\log(dx)}{b^4c^4 - 4a^2b^2c^2 + 6a^4b^2c^2} - \frac{8(b^2c^2 + 5a^2b^2)\log(\sqrt{dx+c+a})}{b^4c^4 - 4a^2b^2c^2 + 6a^4b^2c^2} + \frac{(15ab^2c^2d^2 + 10a^3b^3c^2d^2 - a^5b^5d^2)\log(\frac{\sqrt{dx+c}}{\sqrt{-c}})}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} - \frac{7ab^4c^2d^2 - 7a^3b^4c^2d^2 + a^5c^2d^2 + (11ab^2c^2d^2 - 10a^3b^3c^2d^2 - a^5b^5c^2d^2)(dx+c)^2 - (2b^7c^3d^2 - 3a^2b^5c^2d^2 + a^6b^7d^2)(dx+c)^2 - (19ab^6c^2d^2 - 14a^3b^4c^2d^2 - 5a^5b^2c^2d^2)(dx+c) + 3(b^7c^4d^2 - 2a^2b^5c^4d^2 + a^4b^3c^4d^2)\sqrt{dx+c}}{2(b^2c^2 - 4a^2b^2c^2 + 6a^4b^2c^2 + a^6c^2)\sqrt{-c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^2,x, algorithm="maxima")

[Out] 1/4*d^2*(4*(b^6*c + 5*a^2*b^4)*log(d*x)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - 8*(b^6*c + 5*a^2*b^4)*log(sqrt(d*x + c)*b + a)/(b^8*c^4 - 4*a^2*b^6*c^3 + 6*a^4*b^4*c^2 - 4*a^6*b^2*c + a^8) - (15*a*b^5*c^2 + 10*a^3*b^3*c - a^5*b)*log((sqrt(d*x + c) - sqrt(c))/(sqrt(d*x + c) + sqrt(c)))/((b^8*c^5 - 4*a^2*b^6*c^4 + 6*a^4*b^4*c^3 - 4*a^6*b^2*c^2 + a^8*c)*sqrt(c)) - 2*(7*a*b^4*c^3 + 6*a^3*b^2*c^2 - a^5*c + (11*a*b^4*c + a^3*b^2)*(d*x + c)^2 - (2*b^5*c^2 - a^2*b^3*c - a^4*b)*(d*x + c)^(3/2) - (19*a*b^4*c^2 + 5*a^3*b^2*c)*(d*x + c) + 3*(b^5*c^3 - a^2*b^3*c^2)*sqrt(d*x + c))/(a*b^6*c^6 - 3*a^3*b^4*c^5 + 3*a^5*b^2*c^4 - a^7*c^3 + (b^7*c^4 - 3*a^2*b^5*c^3 + 3*a^4*b^3*c^2 - a^6*b*c)*(d*x + c)^(5/2) + (a*b^6*c^4 - 3*a^3*b^4*c^3 + 3*a^5*b^2*c^2 - a^7*c)*(d*x + c)^2 - 2*(b^7*c^5 - 3*a^2*b^5*c^4 + 3*a^4*b^3*c^3 - a^6*b*c^2)*(d*x + c)^(3/2) - 2*(a*b^6*c^5 - 3*a^3*b^4*c^4 + 3*a^5*c

$b^2*c^3 - a^7*c^2)*(d*x + c) + (b^7*c^6 - 3*a^2*b^5*c^5 + 3*a^4*b^3*c^4 - a^6*b*c^3)*\text{sqrt}(d*x + c))$

mupad [B] time = 5.96, size = 1441, normalized size = 4.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(1/(x^3*(a + b*(c + d*x)^{(1/2)})^2), x)$

[Out]
$$\begin{aligned} &(((5*a^3*b^2*d^2 + 19*a*b^4*c*d^2)*(c + d*x))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + ((a^3*b^2*d^2 + 11*a*b^4*c*d^2)*(c + d*x)^2)/(2*c*(a^6 - b^6*c^3 - 3*a^4*b^2*c + 3*a^2*b^4*c^2)) - (a*(7*b^4*c^2*d^2 - a^4*d^2 + 6*a^2*b^2*c*d^2))/(2*(b^2*c - a^2)*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) + (b*(a^2*d^2 + 2*b^2*c*d^2)*(c + d*x)^{(3/2)})/(2*c*(a^4 + b^4*c^2 - 2*a^2*b^2*c)) - (3*b^3*c*d^2*(c + d*x)^{(1/2)})/(2*(a^4 + b^4*c^2 - 2*a^2*b^2*c)))/(a*(c + d*x)^2 + b*(c + d*x)^{(5/2)} + a*c^2 - 2*a*c*(c + d*x) - 2*b*c*(c + d*x)^{(3/2)} + b*c^2*(c + d*x)^{(1/2)}) + \log(a + b*(c + d*x)^{(1/2)})*((10*b^4*d^2)/(b^2*c - a^2)^3 - (12*b^6*c*d^2)/(b^2*c - a^2)^4) + (\log((a*b^4*d^4*(a^6 - 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^2*d^2*(a^2*(c + d*x)^{(1/2)} + 4*a*b*c + 3*b^2*c*(c + d*x)^{(1/2)})*(a^5*(c^3)^{(1/2)} + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^{(1/2)} - 15*a*b^4*c^2*(c^3)^{(1/2)})))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^{(1/2))*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(a^5*(c^3)^{(1/2)} + 4*b^5*c^4 + 20*a^2*b^3*c^3 - 10*a^3*b^2*c*(c^3)^{(1/2)} - 15*a*b^4*c^2*(c^3)^{(1/2)})))/(4*c^3*(b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^{(1/2)})/(4*c^2*(b^2*c - a^2)^6)) * (4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 + a^5*b*d^2*(c^3)^{(1/2)} - 10*a^3*b^3*c*d^2*(c^3)^{(1/2)} - 15*a*b^5*c^2*d^2*(c^3)^{(1/2)})))/(4*(a^8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) + (\log((a*b^4*d^4*(a^6 - 44*b^6*c^3 + 2*a^4*b^2*c - 103*a^2*b^4*c^2))/(4*c^2*(b^2*c - a^2)^6) - (b*d^2*((b^2*d^2*(a^2*(c + d*x)^{(1/2)} + 4*a*b*c + 3*b^2*c*(c + d*x)^{(1/2)})*(4*b^5*c^4 - a^5*(c^3)^{(1/2)} + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^{(1/2)} + 15*a*b^4*c^2*(c^3)^{(1/2)})))/(2*c^3*(b^2*c - a^2)^4) - (b^3*d^2*(c + d*x)^{(1/2))*(6*b^4*c^2 - a^4 + 19*a^2*b^2*c))/(c*(b^2*c - a^2)^3) + (a*b^2*d^2*(7*b^2*c - a^2))/(2*c*(b^2*c - a^2)^2))*(4*b^5*c^4 - a^5*(c^3)^{(1/2)} + 20*a^2*b^3*c^3 + 10*a^3*b^2*c*(c^3)^{(1/2)} + 15*a*b^4*c^2*(c^3)^{(1/2)})))/(4*c^3*(b^2*c - a^2)^4) + (a^2*b^5*d^4*(11*b^2*c + a^2)^2*(c + d*x)^{(1/2)})/(4*c^2*(b^2*c - a^2)^6)) * (4*b^6*c^4*d^2 + 20*a^2*b^4*c^3*d^2 - a^5*b*d^2*(c^3)^{(1/2)} + 10*a^3*b^3*c*d^2*(c^3)^{(1/2)} + 15*a*b^5*c^2*d^2*(c^3)^{(1/2)})))/(4*(a^8*c^3 + b^8*c^7 - 4*a^6*b^2*c^4 + 6*a^4*b^4*c^5 - 4*a^2*b^6*c^6)) \end{aligned}$$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}(1/x**3/(a+b*(d*x+c)**(1/2))**2, x)$

[Out] Timed out

3.391 $\int \frac{x^3}{\sqrt{a+b\sqrt{c+dx}}} dx$

Optimal. Leaf size=324

$$\frac{12(7a^2 - b^2c)(a + b\sqrt{c + dx})^{11/2}}{11b^8d^4} - \frac{20a(7a^2 - 3b^2c)(a + b\sqrt{c + dx})^{9/2}}{9b^8d^4} - \frac{12a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{7/2}}{5b^8d^4}$$

Rubi [A] time = 0.23, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {371, 1398, 772}

$$\frac{4(-30a^2b^2c + 35a^4 + 3b^4c^2)(a + b\sqrt{c + dx})^{7/2}}{7b^8d^4} + \frac{12(7a^2 - b^2c)(a + b\sqrt{c + dx})^{11/2}}{11b^8d^4} - \frac{20a(7a^2 - 3b^2c)(a + b\sqrt{c + dx})^{9/2}}{9b^8d^4} - \frac{12a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{7/2}}{5b^8d^4} + \frac{4(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^8d^4} - \frac{4a(a^2 - b^2c)^3\sqrt{a + b\sqrt{c + dx}}}{b^8d^4} + \frac{4(a + b\sqrt{c + dx})^{15/2}}{15b^8d^4} - \frac{28a(a + b\sqrt{c + dx})^{13/2}}{13b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^3*Sqrt[a + b*Sqrt[c + d*x]]/(b^8*d^4) + (4*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^8*d^4) - (12*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^8*d^4) + (4*(3*5*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^8*d^4) - (20*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^8*d^4) + (12*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^8*d^4) - (28*a*(a + b*Sqrt[c + d*x])^(13/2))/(13*b^8*d^4) + (4*(a + b*Sqrt[c + d*x])^(15/2))/(15*b^8*d^4)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a + b\sqrt{c + dx}}} dx = \frac{\text{Subst}\left(\int \frac{(-c+x)^3}{\sqrt{a+b\sqrt{x}}} dx, x, c + dx\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)^3}{\sqrt{a+bx}} dx, x, \sqrt{c + dx}\right)}{d^4}$$

$$= \frac{2 \text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^3}{b^7\sqrt{a+bx}} - \frac{(-7a^2+b^2c)(-a^2+b^2c)^2\sqrt{a+bx}}{b^7} - \frac{3(7a^5-10a^3b^2c+3ab^4c^2)(a+bx)^{3/2}}{b^7} + \frac{(35a^7-14336a^5b^2c+7168a^4b^3c^2+768a^3b^4c^3+61776a^2b^5c^4+24960a^2b^3c^2+30030ab^6c^3-7168a^6b^3c^2+61776a^3b^4c^2+24960a^4b^3c^2+30030a^5b^2c^2+14336a^7)}{45045b^8d^4}\right)}{d^4} + \frac{(35a^7-14336a^5b^2c+7168a^4b^3c^2+768a^3b^4c^3+61776a^2b^5c^4+24960a^2b^3c^2+30030ab^6c^3-7168a^6b^3c^2+61776a^3b^4c^2+24960a^4b^3c^2+30030a^5b^2c^2+14336a^7)}{45045b^8d^4}$$

$$= -\frac{4a(a^2 - b^2c)^3 \sqrt{a + b\sqrt{c + dx}}}{b^8d^4} + \frac{4(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^8d^4} - \frac{12(35a^7 - 14336a^5b^2c + 7168a^4b^3c^2 + 768a^3b^4c^3 + 61776a^2b^5c^4 + 24960a^2b^3c^2 + 30030ab^6c^3 - 7168a^6b^3c^2 + 61776a^3b^4c^2 + 24960a^4b^3c^2 + 30030a^5b^2c^2 + 14336a^7)}{45045b^8d^4}$$

Mathematica [A] time = 0.35, size = 232, normalized size = 0.72

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(-14336a^7+7168a^6b\sqrt{c+dx}+768a^5b^2(58c-7d)x-640a^4b^3(32c-7d)x\sqrt{c+dx}-16a^3b^4(2936c^2-680cdx+245d^2x^2)+24a^2b^5\sqrt{c+dx}(784c^2-356cdx+147d^2x^2)+6ab^6(2880c^3-928c^2dx+658cd^2x^2-539d^3x^3)-39b^7\sqrt{c+dx}(128c^3-96c^2dx+84cd^2x^2-77d^3x^3))}{45045b^8d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-14336*a^7 + 768*a^5*b^2*(58*c - 7*d*x) + 7168*a^6*b*Sqrt[c + d*x] - 640*a^4*b^3*(32*c - 7*d*x)*Sqrt[c + d*x] + 24*a^2*b^5*Sqrt[c + d*x]*(784*c^2 - 356*c*d*x + 147*d^2*x^2) - 16*a^3*b^4*(2936*c^2 - 680*c*d*x + 245*d^2*x^2) + 6*a*b^6*(2880*c^3 - 928*c^2*d*x + 658*c*d^2*x^2 - 539*d^3*x^3) - 39*b^7*Sqrt[c + d*x]*(128*c^3 - 96*c^2*d*x + 84*c*d^2*x^2 - 77*d^3*x^3)))/(45045*b^8*d^4)
```

IntegrateAlgebraic [A] time = 0.19, size = 313, normalized size = 0.97

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(-14336a^7-49920a^5b^2c+61776a^3b^4c^2-30030a^2b^6c^3-7168a^6b^3c^2+61776a^3b^4c^2+24960a^4b^3c^2+30030a^5b^2c^2+14336a^7)-4480a^4b^3(c+d*x)^{3/2}+15600a^2b^5c(c+d*x)^{3/2}-19305b^7c^2(c+d*x)^{3/2}+3920a^3b^4(c+d*x)^2-13650a^2b^6c(c+d*x)^2-3528a^2b^5(c+d*x)^{5/2}+12285b^7c(c+d*x)^{5/2}+3234a^2b^6(c+d*x)^3-3003b^7(c+d*x)^{7/2}}{45045b^8d^4}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x^3/Sqrt[a + b*Sqrt[c + d*x]],x]
[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(14336*a^7 - 49920*a^5*b^2*c + 61776*a^3*b^4*c^2 - 30030*a^2*b^6*c^3 - 7168*a^6*b^3*c^2 + 61776*a^3*b^4*c^2 + 24960*a^4*b^3*c^2 + 30030*a^5*b^2*c^2 + 14336*a^7) - 4480*a^4*b^3*(c + d*x)^(3/2) + 15600*a^2*b^5*c*(c + d*x)^(3/2) - 19305*b^7*c^2*(c + d*x)^(3/2) + 3920*a^3*b^4*(c + d*x)^2 - 13650*a^2*b^6*c*(c + d*x)^2 - 3528*a^2*b^5*(c + d*x)^(5/2) + 12285*b^7*c*(c + d*x)^(5/2) + 3234*a^2*b^6*(c + d*x)^3 - 3003*b^7*(c + d*x)^(7/2))/(45045*b^8*d^4)
```

fricas [A] time = 1.06, size = 231, normalized size = 0.71

$$\frac{4(3234ab^6d^3-17280ab^6c^3+46976a^3b^4c^2-44544a^5b^2c+14336a^7-28(141ab^6c-140a^3b^4)c^2+64(87ab^6c-170a^3b^4c+84a^5b^2)c^2+32(3003b^7d^3-4992b^7c^3+18816a^2b^5c^2-20480a^2b^3c+7168a^2b-252(13b^7c-14a^2b^5)d^2+32(117b^7c-267a^2b^5c+140a^2b^3)d)\sqrt{dx+c}\sqrt{dx+cb+a}}{45045b^8d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
[Out] -4/45045*(3234*a^2*b^6*d^3*x^3 - 17280*a^2*b^6*c^3 + 46976*a^3*b^4*c^2 - 44544*a^5*b^2*c + 14336*a^7 - 28*(141*a*b^6*c - 140*a^3*b^4)*d^2*x^2 + 64*(87*a*b^6*c^2 - 170*a^3*b^4*c + 84*a^5*b^2)*d*x - (3003*b^7*d^3*x^3 - 4992*b^7*c^3
```

$$+ 18816*a^2*b^5*c^2 - 20480*a^4*b^3*c + 7168*a^6*b - 252*(13*b^7*c - 14*a^2*b^5)*d^2*x^2 + 32*(117*b^7*c^2 - 267*a^2*b^5*c + 140*a^4*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^8*d^4)$$

giac [A] time = 0.42, size = 409, normalized size = 1.26

$\frac{4(3003(\sqrt{dx+cb+a})^{\frac{15}{2}} - 24255(\sqrt{dx+cb+a})^{\frac{13}{2}} + 12285(b^2c-7a^2)(\sqrt{dx+cb+a})^{\frac{11}{2}} + 25025(3ab^2c-7a^3)(\sqrt{dx+cb+a})^{\frac{9}{2}} + 6435(3b^4c^2-30a^2b^2c+35a^4)(\sqrt{dx+cb+a})^{\frac{7}{2}} - 27027(3ab^2c-10a^3b^2c+7a^5)(\sqrt{dx+cb+a})^{\frac{5}{2}} - 15015(b^6c^3-9a^2b^4c^2+15a^4b^2c^2-7a^6)(\sqrt{dx+cb+a})^{\frac{3}{2}} + 45045(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7)\sqrt{dx+cb+a})}{45045b^8d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] -4/45045*(15015*(sqrt(d*x + c)*b + a)^(3/2)*b^6*c^3 - 45045*sqrt(sqrt(d*x + c)*b + a)*a*b^6*c^3 - 19305*(sqrt(d*x + c)*b + a)^(7/2)*b^4*c^2 + 81081*(sqrt(d*x + c)*b + a)^(5/2)*a*b^4*c^2 - 135135*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^4*c^2 + 135135*sqrt(sqrt(d*x + c)*b + a)*a^3*b^4*c^2 + 12285*(sqrt(d*x + c)*b + a)^(11/2)*b^2*c - 75075*(sqrt(d*x + c)*b + a)^(9/2)*a*b^2*c + 193050*(sqrt(d*x + c)*b + a)^(7/2)*a^2*b^2*c - 270270*(sqrt(d*x + c)*b + a)^(5/2)*a^3*b^2*c + 225225*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^2*c - 135135*sqrt(sqrt(d*x + c)*b + a)*a^5*b^2*c - 3003*(sqrt(d*x + c)*b + a)^(15/2) + 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 85995*(sqrt(d*x + c)*b + a)^(11/2)*a^2 + 175175*(sqrt(d*x + c)*b + a)^(9/2)*a^3 - 225225*(sqrt(d*x + c)*b + a)^(7/2)*a^4 + 189189*(sqrt(d*x + c)*b + a)^(5/2)*a^5 - 105105*(sqrt(d*x + c)*b + a)^(3/2)*a^6 + 45045*sqrt(sqrt(d*x + c)*b + a)*a^7)/(b^8*d^4)

maple [A] time = 0.00, size = 383, normalized size = 1.18

$\frac{28(a\sqrt{dx+c})^{\frac{15}{2}} - 4(-b^2c+a^2)\sqrt{dx+c} + \frac{4(a\sqrt{dx+c})^{\frac{13}{2}}}{15} + \frac{4(-2b^2c+21a^2)(a\sqrt{dx+c})^{\frac{11}{2}}}{11} + \frac{4(-4b^2c+a^2)(-2b^2c+15a^2)(a\sqrt{dx+c})^{\frac{9}{2}}}{9} + \frac{4(4b^2c+a^2)(-4b^2c+15a^2)(-2b^2c+6a^2)(a\sqrt{dx+c})^{\frac{7}{2}}}{7} + \frac{4(-4b^2c+a^2)(-4b^2c+15a^2)(-2b^2c+6a^2)(a\sqrt{dx+c})^{\frac{5}{2}}}{5} + \frac{4(4b^2c+a^2)(-4b^2c+15a^2)(-2b^2c+6a^2)(a\sqrt{dx+c})^{\frac{3}{2}}}{3} + \frac{4(4b^2c+a^2)(-4b^2c+15a^2)(-2b^2c+6a^2)(a\sqrt{dx+c})^{\frac{1}{2}}}{3}}{45045b^8d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)

[Out] 4/d^4/b^8*(1/15*(a+(d*x+c)^(1/2)*b)^(15/2)-7/13*a*(a+(d*x+c)^(1/2)*b)^(13/2)+1/11*(-3*b^2*c+21*a^2)*(a+(d*x+c)^(1/2)*b)^(11/2)+1/9*(-8*(-b^2*c+a^2)*a-2*(-2*b^2*c+6*a^2)*a-(-3*b^2*c+15*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(9/2)+1/7*(8*(-b^2*c+a^2)*a^2-(-8*(-b^2*c+a^2)*a-2*(-2*b^2*c+6*a^2)*a)*a+(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-6*(-b^2*c+a^2)^2*a-(8*(-b^2*c+a^2)*a^2+(-b^2*c+a^2)*(-2*b^2*c+6*a^2)+(-b^2*c+a^2)^2)*a)*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(6*(-b^2*c+a^2)^2*a^2+(-b^2*c+a^2)^3)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^3*a*(a+(d*x+c)^(1/2)*b)^(1/2))

maxima [A] time = 0.94, size = 268, normalized size = 0.83

$\frac{4(3003(\sqrt{dx+cb+a})^{\frac{15}{2}} - 24255(\sqrt{dx+cb+a})^{\frac{13}{2}} + 12285(b^2c-7a^2)(\sqrt{dx+cb+a})^{\frac{11}{2}} + 25025(3ab^2c-7a^3)(\sqrt{dx+cb+a})^{\frac{9}{2}} + 6435(3b^4c^2-30a^2b^2c+35a^4)(\sqrt{dx+cb+a})^{\frac{7}{2}} - 27027(3ab^2c-10a^3b^2c+7a^5)(\sqrt{dx+cb+a})^{\frac{5}{2}} - 15015(b^6c^3-9a^2b^4c^2+15a^4b^2c-7a^6)(\sqrt{dx+cb+a})^{\frac{3}{2}} + 45045(ab^6c^3-3a^3b^4c^2+3a^5b^2c-a^7)\sqrt{dx+cb+a})}{45045b^8d^4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] 4/45045*(3003*(sqrt(d*x + c)*b + a)^(15/2) - 24255*(sqrt(d*x + c)*b + a)^(13/2)*a - 12285*(b^2*c - 7*a^2)*(sqrt(d*x + c)*b + a)^(11/2) + 25025*(3*a*b^2*c - 7*a^3)*(sqrt(d*x + c)*b + a)^(9/2) + 6435*(3*b^4*c^2 - 30*a^2*b^2*c + 35*a^4)*(sqrt(d*x + c)*b + a)^(7/2) - 27027*(3*a*b^4*c^2 - 10*a^3*b^2*c + 7*a^5)*(sqrt(d*x + c)*b + a)^(5/2) - 15015*(b^6*c^3 - 9*a^2*b^4*c^2 + 15*a^4*b^2*c - 7*a^6)*(sqrt(d*x + c)*b + a)^(3/2) + 45045*(a*b^6*c^3 - 3*a^3*b^4*c^2 + 3*a^5*b^2*c - a^7)*sqrt(sqrt(d*x + c)*b + a))/(b^8*d^4)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a + b\sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)`

[Out] `int(x^3/(a + b*(c + d*x)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a + b\sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a+b*(d*x+c)**(1/2))**(1/2), x)`

[Out] `Integral(x**3/sqrt(a + b*sqrt(c + d*x)), x)`

$$3.392 \quad \int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=222

$$\frac{8(5a^2 - b^2c)(a + b\sqrt{c + dx})^{7/2}}{7b^6d^3} - \frac{8a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^6d^3} - \frac{4a(a^2 - b^2c)^2\sqrt{a + b\sqrt{c + dx}}}{b^6d^3} + \frac{4(5a^4 - 6a^2b^2c + b^4c^2)(a + b\sqrt{c + dx})^{3/2}}{3b^6d^3} - \frac{8(5a^2 - b^2c)(a + b\sqrt{c + dx})^{7/2}}{7b^6d^3} - \frac{8a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^6d^3} - \frac{4a(a^2 - b^2c)^2\sqrt{a + b\sqrt{c + dx}}}{b^6d^3} + \frac{4(a + b\sqrt{c + dx})^{11/2}}{11b^6d^3} - \frac{20a(a + b\sqrt{c + dx})^{9/2}}{9b^6d^3}$$

Rubi [A] time = 0.16, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {371, 1398, 772}

$$\frac{4(-6a^2b^2c + 5a^4 + b^4c^2)(a + b\sqrt{c + dx})^{3/2}}{3b^6d^3} + \frac{8(5a^2 - b^2c)(a + b\sqrt{c + dx})^{7/2}}{7b^6d^3} - \frac{8a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{5/2}}{5b^6d^3} - \frac{4a(a^2 - b^2c)^2\sqrt{a + b\sqrt{c + dx}}}{b^6d^3} + \frac{4(a + b\sqrt{c + dx})^{11/2}}{11b^6d^3} - \frac{20a(a + b\sqrt{c + dx})^{9/2}}{9b^6d^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + b*Sqrt[c + d*x]],x]
```

```
[Out] (-4*a*(a^2 - b^2*c)^2*Sqrt[a + b*Sqrt[c + d*x]]/(b^6*d^3) + (4*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^6*d^3) - (8*a*(5*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^6*d^3) + (8*(5*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^6*d^3) - (20*a*(a + b*Sqrt[c + d*x])^(9/2))/(9*b^6*d^3) + (4*(a + b*Sqrt[c + d*x])^(11/2))/(11*b^6*d^3)
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}} dx &= \frac{\text{Subst}\left(\int \frac{(-c+x)^2}{\sqrt{a+b\sqrt{x}}} dx, x, c+dx\right)}{d^3} \\
&= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)^2}{\sqrt{a+bx}} dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= \frac{2 \text{Subst}\left(\int \left(-\frac{a(a^2-b^2c)^2}{b^5\sqrt{a+bx}} + \frac{(5a^4-6a^2b^2c+b^4c^2)\sqrt{a+bx}}{b^5} - \frac{2(5a^3-3ab^2c)(a+bx)^{3/2}}{b^5} - \frac{2(-5a^2+b^2c)(a+bx)^{5/2}}{b^5}\right) dx, x, \sqrt{c+dx}\right)}{d^3} \\
&= -\frac{4a(a^2-b^2c)^2\sqrt{a+b\sqrt{c+dx}}}{b^6d^3} + \frac{4(5a^4-6a^2b^2c+b^4c^2)(a+b\sqrt{c+dx})^{3/2}}{3b^6d^3} - \frac{8a(5a^3-3ab^2c)(a+b\sqrt{c+dx})^{5/2}}{3b^6d^3} + \frac{8a(-5a^2+b^2c)(a+b\sqrt{c+dx})^{7/2}}{3b^6d^3}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 147, normalized size = 0.66

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(-1280a^5+640a^4b\sqrt{c+dx}+96a^3b^2(28c-5dx)-16a^2b^3(74c-25dx)\sqrt{c+dx}-2ab^4(736c^2-244cdx+175d^2x^2)+15b^5\sqrt{c+dx}(32c^2-24cdx+21d^2x^2))}{3465b^6d^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/Sqrt[a + b*Sqrt[c + d*x]], x]

[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-1280*a^5 + 96*a^3*b^2*(28*c - 5*d*x) + 640*a^4*b*Sqrt[c + d*x] - 16*a^2*b^3*(74*c - 25*d*x)*Sqrt[c + d*x] + 15*b^5*Sqrt[c + d*x]*(32*c^2 - 24*c*d*x + 21*d^2*x^2) - 2*a*b^4*(736*c^2 - 244*c*d*x + 175*d^2*x^2)))/(3465*b^6*d^3)

IntegrateAlgebraic [A] time = 0.12, size = 185, normalized size = 0.83

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(1280a^5-640a^4b\sqrt{c+dx}+480a^3b^2(c+dx)-3168a^3b^2c-400a^2b^3(c+dx)^{3/2}+1584a^2b^3c\sqrt{c+dx}+2310ab^4c^2+350ab^4(c+dx)^2-1188ab^4c(c+dx)-1155b^5c^2\sqrt{c+dx}-315b^5(c+dx)^{3/2}+990b^5c(c+dx)^{3/2})}{3465b^6d^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/Sqrt[a + b*Sqrt[c + d*x]], x]

[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(1280*a^5 - 3168*a^3*b^2*c + 2310*a*b^4*c^2 - 640*a^4*b*Sqrt[c + d*x] + 1584*a^2*b^3*c*Sqrt[c + d*x] - 1155*b^5*c^2*Sqrt[c + d*x] + 480*a^3*b^2*(c + d*x) - 1188*a*b^4*c*(c + d*x) - 400*a^2*b^3*(c + d*x)^(3/2) + 990*b^5*c*(c + d*x)^(3/2) + 350*a*b^4*(c + d*x)^2 - 315*b^5*c*(c + d*x)^(5/2)))/(3465*b^6*d^3)

fricas [A] time = 1.01, size = 140, normalized size = 0.63

$$\frac{4(350ab^4d^2x^2+1472ab^4c^2-2688a^3b^2c+1280a^5-8(61ab^4c-60a^3b^2)dx-(315b^5d^2x^2+480b^5c^2-1184a^2b^3c+640a^4b-40(9b^5c-10a^2b^3)dx)\sqrt{dx+c})\sqrt{\sqrt{dx+c}b+aa}}{3465b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2), x, algorithm="fricas")

[Out] -4/3465*(350*a*b^4*d^2*x^2 + 1472*a*b^4*c^2 - 2688*a^3*b^2*c + 1280*a^5 - 8*(61*a*b^4*c - 60*a^3*b^2)*d*x - (315*b^5*d^2*x^2 + 480*b^5*c^2 - 1184*a^2*b^3*c + 640*a^4*b - 40*(9*b^5*c - 10*a^2*b^3)*d*x)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^6*d^3)

giac [A] time = 0.33, size = 238, normalized size = 1.07

$$\frac{4(1155(\sqrt{dx+c}b+a)^2b^4d^2-3465\sqrt{dx+c}b+aa^2b^4c-990(\sqrt{dx+c}b+a)^2b^3c+4158(\sqrt{dx+c}b+a)^2b^2c-6930(\sqrt{dx+c}b+a)^2b^2c-6930(\sqrt{dx+c}b+a)^2b^2c+315(\sqrt{dx+c}b+a)^2b^2c-1925(\sqrt{dx+c}b+a)^2b^2c+4950(\sqrt{dx+c}b+a)^2b^2c-6930(\sqrt{dx+c}b+a)^2b^2c+5775(\sqrt{dx+c}b+a)^2b^2c-3465\sqrt{dx+c}b+aa^2)}{3465b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 4/3465*(1155*(sqrt(d*x + c)*b + a)^(3/2)*b^4*c^2 - 3465*sqrt(sqrt(d*x + c)*
b + a)*a*b^4*c^2 - 990*(sqrt(d*x + c)*b + a)^(7/2)*b^2*c + 4158*(sqrt(d*x +
c)*b + a)^(5/2)*a*b^2*c - 6930*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^2*c + 693
0*sqrt(sqrt(d*x + c)*b + a)*a^3*b^2*c + 315*(sqrt(d*x + c)*b + a)^(11/2) -
1925*(sqrt(d*x + c)*b + a)^(9/2)*a + 4950*(sqrt(d*x + c)*b + a)^(7/2)*a^2 -
6930*(sqrt(d*x + c)*b + a)^(5/2)*a^3 + 5775*(sqrt(d*x + c)*b + a)^(3/2)*a^
4 - 3465*sqrt(sqrt(d*x + c)*b + a)*a^5)/(b^6*d^3)
```

```
maple [A] time = 0.00, size = 183, normalized size = 0.82
```

$$\frac{-\frac{20(a+\sqrt{dx+c})^{\frac{9}{2}}}{9} - 4(-b^2c+a^2)^2\sqrt{a+\sqrt{dx+c}} + \frac{4(a+\sqrt{dx+c})^{\frac{11}{2}}}{11} + \frac{4(-2b^2c+10a^2)(a+\sqrt{dx+c})^{\frac{7}{2}}}{7} + \frac{4(-4(-b^2c+a^2)a-(-2b^2c+6a^2)a)(a+\sqrt{dx+c})^{\frac{5}{2}}}{5} + \frac{4(4(-b^2c+a^2)a^2+(-b^2c+a^2)^2)(a+\sqrt{dx+c})^{\frac{3}{2}}}{3}}{b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 4/d^3/b^6*(1/11*(a+(d*x+c)^(1/2)*b)^(11/2)-5/9*a*(a+(d*x+c)^(1/2)*b)^(9/2)+
1/7*(-2*b^2*c+10*a^2)*(a+(d*x+c)^(1/2)*b)^(7/2)+1/5*(-4*(-b^2*c+a^2)*a-(-2*
b^2*c+6*a^2)*a)*(a+(d*x+c)^(1/2)*b)^(5/2)+1/3*(4*(-b^2*c+a^2)*a^2+(-b^2*c+a
^2)^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-(-b^2*c+a^2)^2*a*(a+(d*x+c)^(1/2)*b)^(1/2)
)
```

```
maxima [A] time = 0.92, size = 167, normalized size = 0.75
```

$$\frac{4\left(315(\sqrt{dx+cb+a})^{\frac{11}{2}}-1925(\sqrt{dx+cb+a})^{\frac{9}{2}}a-990(b^2c-5a^2)(\sqrt{dx+cb+a})^{\frac{7}{2}}+1386(3ab^2c-5a^3)(\sqrt{dx+cb+a})^{\frac{5}{2}}+1155(b^4c^2-6a^2b^2c+5a^4)(\sqrt{dx+cb+a})^{\frac{3}{2}}-3465(ab^4c^2-2a^3b^2c+a^5)\sqrt{\sqrt{dx+cb+a}}\right)}{3465b^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")
```

```
[Out] 4/3465*(315*(sqrt(d*x + c)*b + a)^(11/2) - 1925*(sqrt(d*x + c)*b + a)^(9/2)
*a - 990*(b^2*c - 5*a^2)*(sqrt(d*x + c)*b + a)^(7/2) + 1386*(3*a*b^2*c - 5*
a^3)*(sqrt(d*x + c)*b + a)^(5/2) + 1155*(b^4*c^2 - 6*a^2*b^2*c + 5*a^4)*(sq
rt(d*x + c)*b + a)^(3/2) - 3465*(a*b^4*c^2 - 2*a^3*b^2*c + a^5)*sqrt(sqrt(d
*x + c)*b + a))/(b^6*d^3)
```

```
mupad [F] time = 0.00, size = -1, normalized size = -0.00
```

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2),x)
```

```
[Out] int(x^2/(a + b*(c + d*x)^(1/2))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x^2}{\sqrt{a+b\sqrt{c+dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a + b*sqrt(c + d*x)), x)
```

$$3.393 \quad \int \frac{x}{\sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=131

$$\frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} - \frac{4a(a^2 - b^2c)\sqrt{a + b\sqrt{c + dx}}}{b^4d^2} + \frac{4(a + b\sqrt{c + dx})^{7/2}}{7b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2}$$

Rubi [A] time = 0.09, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} - \frac{4a(a^2 - b^2c)\sqrt{a + b\sqrt{c + dx}}}{b^4d^2} + \frac{4(a + b\sqrt{c + dx})^{7/2}}{7b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2}$$

Antiderivative was successfully verified.

[In] Int[x/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*a*(a^2 - b^2*c)*Sqrt[a + b*Sqrt[c + d*x]]/(b^4*d^2) + (4*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^4*d^2) - (12*a*(a + b*Sqrt[c + d*x])^(5/2))/(5*b^4*d^2) + (4*(a + b*Sqrt[c + d*x])^(7/2))/(7*b^4*d^2)

Rule 371

Int[((a_) + (b_.)*(v_)^(n_.))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{a + b\sqrt{c + dx}}} dx &= \frac{\text{Subst}\left(\int \frac{-c+x}{\sqrt{a+b\sqrt{x}}} dx, x, c + dx\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \frac{x(-c+x^2)}{\sqrt{a+bx}} dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= \frac{2 \text{Subst}\left(\int \left(\frac{-a^3+ab^2c}{b^3\sqrt{a+bx}} + \frac{(3a^2-b^2c)\sqrt{a+bx}}{b^3} - \frac{3a(a+bx)^{3/2}}{b^3} + \frac{(a+bx)^{5/2}}{b^3}\right) dx, x, \sqrt{c + dx}\right)}{d^2} \\ &= -\frac{4a(a^2 - b^2c)\sqrt{a + b\sqrt{c + dx}}}{b^4d^2} + \frac{4(3a^2 - b^2c)(a + b\sqrt{c + dx})^{3/2}}{3b^4d^2} - \frac{12a(a + b\sqrt{c + dx})^{5/2}}{5b^4d^2} \end{aligned}$$

Mathematica [A] time = 0.10, size = 84, normalized size = 0.64

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(-48a^3+24a^2b\sqrt{c+dx}+2ab^2(26c-9dx)+5b^3\sqrt{c+dx}(3dx-4c))}{105b^4d^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*Sqrt[a + b*Sqrt[c + d*x]]*(-48*a^3 + 2*a*b^2*(26*c - 9*d*x) + 24*a^2*b*Sqrt[c + d*x] + 5*b^3*Sqrt[c + d*x]*(-4*c + 3*d*x)))/(105*b^4*d^2)

IntegrateAlgebraic [A] time = 0.06, size = 95, normalized size = 0.73

$$\frac{4\sqrt{a+b\sqrt{c+dx}}(48a^3-24a^2b\sqrt{c+dx}+18ab^2(c+dx)-70ab^2c-15b^3(c+dx)^{3/2}+35b^3c\sqrt{c+dx})}{105b^4d^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*Sqrt[a + b*Sqrt[c + d*x]]*(48*a^3 - 70*a*b^2*c - 24*a^2*b*Sqrt[c + d*x] + 35*b^3*c*Sqrt[c + d*x] + 18*a*b^2*(c + d*x) - 15*b^3*(c + d*x)^(3/2)))/(105*b^4*d^2)

fricas [A] time = 1.02, size = 71, normalized size = 0.54

$$\frac{4(18ab^2dx-52ab^2c+48a^3-(15b^3dx-20b^3c+24a^2b)\sqrt{dx+c})\sqrt{\sqrt{dx+c}b+a}}{105b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -4/105*(18*a*b^2*d*x - 52*a*b^2*c + 48*a^3 - (15*b^3*d*x - 20*b^3*c + 24*a^2*b)*sqrt(d*x + c))*sqrt(sqrt(d*x + c)*b + a)/(b^4*d^2)

giac [A] time = 0.37, size = 115, normalized size = 0.88

$$\frac{4\left(35(\sqrt{dx+cb+a})^{\frac{3}{2}}b^2c-105\sqrt{\sqrt{dx+cb+a}ab^2c}-15(\sqrt{dx+cb+a})^{\frac{7}{2}}+63(\sqrt{dx+cb+a})^{\frac{5}{2}}a-105(\sqrt{dx+cb+a})^{\frac{3}{2}}a^2+105\sqrt{\sqrt{dx+cb+a}aa^3}\right)}{105b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] -4/105*(35*(sqrt(d*x + c)*b + a)^(3/2)*b^2*c - 105*sqrt(sqrt(d*x + c)*b + a)*a*b^2*c - 15*(sqrt(d*x + c)*b + a)^(7/2) + 63*(sqrt(d*x + c)*b + a)^(5/2)*a - 105*(sqrt(d*x + c)*b + a)^(3/2)*a^2 + 105*sqrt(sqrt(d*x + c)*b + a)*a^3)/(b^4*d^2)

maple [A] time = 0.00, size = 94, normalized size = 0.72

$$\frac{-\frac{12(a+\sqrt{dx+cb})^{\frac{5}{2}}a}{5}-4(-b^2c+a^2)\sqrt{a+\sqrt{dx+cb}}ba+\frac{4(a+\sqrt{dx+cb})^{\frac{7}{2}}}{7}+\frac{4(-b^2c+3a^2)(a+\sqrt{dx+cb})^{\frac{3}{2}}}{3}}{b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a+(d*x+c)^(1/2)*b)^(1/2),x)

[Out] $4/d^2/b^4*(1/7*(a+(d*x+c)^{(1/2)}*b)^{(7/2)}-3/5*(a+(d*x+c)^{(1/2)}*b)^{(5/2)}*a+1/3*(-b^2*c+3*a^2)*(a+(d*x+c)^{(1/2)}*b)^{(3/2)}-(-b^2*c+a^2)*a*(a+(d*x+c)^{(1/2)}*b)^{(1/2)})$

maxima [A] time = 0.93, size = 93, normalized size = 0.71

$$\frac{4 \left(15 (\sqrt{dx+cb+a})^{\frac{7}{2}} - 63 (\sqrt{dx+cb+a})^{\frac{5}{2}} a - 35 (b^2c - 3a^2) (\sqrt{dx+cb+a})^{\frac{3}{2}} + 105 (ab^2c - a^3) \sqrt{\sqrt{dx+cb+a}} \right)}{105 b^4 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $4/105*(15*(\text{sqrt}(d*x + c)*b + a)^{(7/2)} - 63*(\text{sqrt}(d*x + c)*b + a)^{(5/2)}*a - 35*(b^2*c - 3*a^2)*(\text{sqrt}(d*x + c)*b + a)^{(3/2)} + 105*(a*b^2*c - a^3)*\text{sqrt}(\text{sqrt}(d*x + c)*b + a))/(b^4*d^2)$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(a + b*(c + d*x)^(1/2))^(1/2),x)`

[Out] `int(x/(a + b*(c + d*x)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a+b*(d*x+c)**(1/2))**(1/2),x)`

[Out] `Integral(x/sqrt(a + b*sqrt(c + d*x)), x)`

$$3.394 \quad \int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=54

$$\frac{4(a+b\sqrt{c+dx})^{3/2}}{3b^2d} - \frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Rubi [A] time = 0.03, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {247, 190, 43}

$$\frac{4(a+b\sqrt{c+dx})^{3/2}}{3b^2d} - \frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*a*Sqrt[a + b*Sqrt[c + d*x]]/(b^2*d) + (4*(a + b*Sqrt[c + d*x])^(3/2))/(3*b^2*d))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{a+b\sqrt{c+dx}}} dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b\sqrt{x}}} dx, x, c+dx\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \frac{x}{\sqrt{a+bx}} dx, x, \sqrt{c+dx}\right)}{d} \\ &= \frac{2\text{Subst}\left(\int \left(-\frac{a}{b\sqrt{a+bx}} + \frac{\sqrt{a+bx}}{b}\right) dx, x, \sqrt{c+dx}\right)}{d} \\ &= -\frac{4a\sqrt{a+b\sqrt{c+dx}}}{b^2d} + \frac{4(a+b\sqrt{c+dx})^{3/2}}{3b^2d} \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 0.78

$$\frac{4(b\sqrt{c+dx} - 2a)\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (4*(-2*a + b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)

IntegrateAlgebraic [A] time = 0.04, size = 43, normalized size = 0.80

$$\frac{4(2a - b\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{3b^2d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a + b*Sqrt[c + d*x]],x]

[Out] (-4*(2*a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/(3*b^2*d)

fricas [A] time = 1.02, size = 34, normalized size = 0.63

$$\frac{4\sqrt{\sqrt{dx+c}b+a}(\sqrt{dx+c}b-2a)}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 4/3*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - 2*a)/(b^2*d)

giac [A] time = 0.44, size = 38, normalized size = 0.70

$$\frac{4\left(\left(\sqrt{dx+c}b+a\right)^{\frac{3}{2}}-3\sqrt{\sqrt{dx+c}b+a}a\right)}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2) - 3*sqrt(sqrt(d*x + c)*b + a)*a)/(b^2*d)

maple [A] time = 0.01, size = 41, normalized size = 0.76

$$\frac{-4\sqrt{a+\sqrt{dx+c}b}a+\frac{4(a+\sqrt{dx+c}b)^{\frac{3}{2}}}{3}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+(d*x+c)^(1/2)*b)^(1/2),x)

[Out] 4/d/b^2*(1/3*(a+(d*x+c)^(1/2)*b)^(3/2)-a*(a+(d*x+c)^(1/2)*b)^(1/2))

maxima [A] time = 0.86, size = 42, normalized size = 0.78

$$\frac{4\left(\frac{(\sqrt{dx+c}b+a)^{\frac{3}{2}}}{b^2}-\frac{3\sqrt{\sqrt{dx+c}b+a}a}{b^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] 4/3*((sqrt(d*x + c)*b + a)^(3/2)/b^2 - 3*sqrt(sqrt(d*x + c)*b + a)*a/b^2)/d

mupad [B] time = 3.26, size = 44, normalized size = 0.81

$$\frac{4(a + b\sqrt{c + dx})^{3/2}}{3b^2d} - \frac{4a\sqrt{a + b\sqrt{c + dx}}}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a + b*(c + d*x)^(1/2))^(1/2),x)

[Out] (4*(a + b*(c + d*x)^(1/2))^(3/2))/(3*b^2*d) - (4*a*(a + b*(c + d*x)^(1/2))^(1/2))/(b^2*d)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(a + b*sqrt(c + d*x)), x)

$$3.395 \quad \int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=97

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Rubi [A] time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {371, 1398, 827, 1166, 207}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{\sqrt{a-b\sqrt{c}}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/Sqrt[a - b*Sqrt[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a + b*Sqrt[c]]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 827

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1166

Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n)], x], x, x^g]]

))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x\sqrt{a+b\sqrt{c+dx}}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{a+b\sqrt{x}}(-c+x)} dx, x, c+dx \right) \\
 &= 2 \text{Subst} \left(\int \frac{x}{\sqrt{a+bx}(-c+x^2)} dx, x, \sqrt{c+dx} \right) \\
 &= 4 \text{Subst} \left(\int \frac{-a+x^2}{a^2-b^2c-2ax^2+x^4} dx, x, \sqrt{a+b\sqrt{c+dx}} \right) \\
 &= 2 \text{Subst} \left(\int \frac{1}{-a-b\sqrt{c}+x^2} dx, x, \sqrt{a+b\sqrt{c+dx}} \right) + 2 \text{Subst} \left(\int \frac{1}{-a+b\sqrt{c}+x^2} dx, x, \sqrt{a+b\sqrt{c+dx}} \right) \\
 &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}} \right)}{\sqrt{a-b\sqrt{c}}} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}} \right)}{\sqrt{a+b\sqrt{c}}}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 97, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}} \right)}{\sqrt{a-b\sqrt{c}}} - \frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}} \right)}{\sqrt{a+b\sqrt{c}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]]/Sqrt[a - b*Sqrt[c]] - (2*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/Sqrt[a + b*Sqrt[c]])

IntegrateAlgebraic [A] time = 0.22, size = 128, normalized size = 1.32

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}} \right)}{\sqrt{b\sqrt{c}-a}} - \frac{2 \tan^{-1} \left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}} \right)}{\sqrt{-a-b\sqrt{c}}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] (-2*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c])]/Sqrt[-a + b*Sqrt[c]] - (2*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c])])/Sqrt[-a - b*Sqrt[c]])

fricas [B] time = 1.11, size = 743, normalized size = 7.66



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)*sqrt(-((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) - sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a)) + sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2))*log(-4*((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) + a)*sqrt(((b^2*c - a^2)*sqrt(b^2*c/(b^4*c^2 - 2*a^2*b^2*c + a^4)) - a)/(b^2*c - a^2)) + 4*sqrt(sqrt(d*x + c)*b + a))

giac [A] time = 0.50, size = 140, normalized size = 1.44

$$\frac{2 \left(\frac{(b^2 \sqrt{c} |b| + ab^2) \arctan\left(\frac{\sqrt{dx+c}b+a}{\sqrt{-a+\sqrt{b^2c}}}\right)}{(b\sqrt{c}+a)\sqrt{b\sqrt{c}-a}} + \frac{(b^2 \sqrt{c} |b| - ab^2) \arctan\left(\frac{\sqrt{dx+c}b+a}{\sqrt{-a-\sqrt{b^2c}}}\right)}{(b\sqrt{c}-a)\sqrt{-b\sqrt{c}-a}} \right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

[Out] 2*((b^2*sqrt(c)*abs(b) + a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a + sqrt(b^2*c)))/((b*sqrt(c) + a)*sqrt(b*sqrt(c) - a)) + (b^2*sqrt(c)*abs(b) - a*b^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-a - sqrt(b^2*c)))/((b*sqrt(c) - a)*sqrt(-b*sqrt(c) - a)))/b^2

maple [A] time = 0.02, size = 92, normalized size = 0.95

$$\frac{2 \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{\sqrt{-a-\sqrt{b^2c}}} + \frac{2 \arctan\left(\frac{\sqrt{a+\sqrt{dx+c}b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{\sqrt{-a+\sqrt{b^2c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+(d*x+c)^(1/2)*b)^(1/2),x)

[Out] 2/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))+2/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c}b+ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)),x)

[Out] int(1/(x*(a + b*(c + d*x)^(1/2))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*sqrt(c + d*x))), x)

$$3.396 \quad \int \frac{1}{x^2 \sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=163

$$-\frac{\sqrt{a+b\sqrt{c+dx}}(a-b\sqrt{c+dx})}{x(a^2-b^2c)} - \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}(a-b\sqrt{c})^{3/2}} + \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}(a+b\sqrt{c})^{3/2}}$$

Rubi [A] time = 0.20, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {371, 1398, 823, 827, 1166, 207}

$$-\frac{\sqrt{a+b\sqrt{c+dx}}(a-b\sqrt{c+dx})}{x(a^2-b^2c)} - \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{2\sqrt{c}(a-b\sqrt{c})^{3/2}} + \frac{bd \tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{2\sqrt{c}(a+b\sqrt{c})^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] -(((a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/((a^2 - b^2*c)*x)) - (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b*Sqrt[c]]])/(2*(a - b*Sqrt[c])^(3/2)*Sqrt[c]) + (b*d*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]])/(2*(a + b*Sqrt[c])^(3/2)*Sqrt[c])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 823

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 827

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
- q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^p_.*((d_) + (e_.)*(x_)^(n_.))^q_.), x_Symb
ol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x^2 \sqrt{a + b\sqrt{c + dx}}} dx &= d \operatorname{Subst} \left(\int \frac{1}{\sqrt{a + b\sqrt{x}} (-c + x)^2} dx, x, c + dx \right) \\ &= (2d) \operatorname{Subst} \left(\int \frac{x}{\sqrt{a + bx} (-c + x^2)^2} dx, x, \sqrt{c + dx} \right) \\ &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{(a^2 - b^2c)x} + \frac{d \operatorname{Subst} \left(\int \frac{-\frac{1}{2}abc + \frac{1}{2}b^2cx}{\sqrt{a + bx} (-c + x^2)} dx, x, \sqrt{c + dx} \right)}{c(a^2 - b^2c)} \\ &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{(a^2 - b^2c)x} + \frac{(2d) \operatorname{Subst} \left(\int \frac{-ab^2c + \frac{1}{2}b^2cx^2}{a^2 - b^2c - 2ax^2 + x^4} dx, x, \sqrt{a + b\sqrt{c + dx}} \right)}{c(a^2 - b^2c)} \\ &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{(a^2 - b^2c)x} + \frac{(bd) \operatorname{Subst} \left(\int \frac{1}{-a + b\sqrt{c + x^2}} dx, x, \sqrt{a + b\sqrt{c + dx}} \right)}{2(a - b\sqrt{c}) \sqrt{c}} \\ &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{(a^2 - b^2c)x} - \frac{bd \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a - b\sqrt{c}}} \right)}{2(a - b\sqrt{c})^{3/2} \sqrt{c}} + \frac{bd \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a + b\sqrt{c}}} \right)}{2(a + b\sqrt{c})^{3/2} \sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.25, size = 216, normalized size = 1.33

$$\frac{\sqrt{a - b\sqrt{c}} \left(bdx(a - b\sqrt{c}) \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a + b\sqrt{c}}} \right) - 2\sqrt{c} \sqrt{a + b\sqrt{c}} (a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}} \right) - bdx(a + b\sqrt{c})^{3/2} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a - b\sqrt{c}}} \right)}{2\sqrt{c}x\sqrt{a - b\sqrt{c}} \sqrt{a + b\sqrt{c}} (a^2 - b^2c)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]), x]
```

```
[Out] (-(b*(a + b*Sqrt[c])^(3/2)*d*x*ArcTanh[Sqrt[a + b*Sqrt[c + d*x]]/Sqrt[a - b
*Sqrt[c]]]) + Sqrt[a - b*Sqrt[c]]*(-2*Sqrt[a + b*Sqrt[c]]*Sqrt[c]*(a - b*Sqr
t[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]] + b*(a - b*Sqrt[c])*d*x*ArcTanh[Sqrt
[a + b*Sqrt[c + d*x]]/Sqrt[a + b*Sqrt[c]]]))/(2*Sqrt[a - b*Sqrt[c]]*Sqrt[a
+ b*Sqrt[c]]*Sqrt[c]*(a^2 - b^2*c)*x)
```

IntegrateAlgebraic [A] time = 0.66, size = 200, normalized size = 1.23

$$\frac{\sqrt{a+b\sqrt{c+dx}}(ad-bd\sqrt{c+dx})}{dx(a^2-b^2c)} + \frac{bd \tan^{-1}\left(\frac{\sqrt{b\sqrt{c}-a}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{2\sqrt{c}(b\sqrt{c}-a)^{3/2}} - \frac{bd \tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{2\sqrt{c}(-a-b\sqrt{c})^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x^2*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] -((Sqrt[a + b*Sqrt[c + d*x]]*(a*d - b*d*Sqrt[c + d*x]))/((a^2 - b^2*c)*d*x) + (b*d*ArcTan[(Sqrt[-a + b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a - b*Sqrt[c]))/(2*(-a + b*Sqrt[c])^(3/2)*Sqrt[c]) - (b*d*ArcTan[(Sqrt[-a - b*Sqrt[c]]*Sqrt[a + b*Sqrt[c + d*x]])/(a + b*Sqrt[c]))/(2*(-a - b*Sqrt[c])^(3/2)*Sqrt[c])

fricas [B] time = 1.17, size = 2493, normalized size = 15.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/4*((b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 - (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) + (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) *log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 + (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - (b^2*c - a^2)*x*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/((b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)))

```
*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))*log((b^6*c + 3*a^2*b^4)*sqrt(sqrt(d*x + c)*b + a)*d^3 - (2*(a*b^6*c^2 + 3*a^3*b^4*c)*d^2 + (b^8*c^5 - 2*a^2*b^6*c^4 + 2*a^6*b^2*c^2 - a^8*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c)))*sqrt(-((3*a*b^4*c + a^3*b^2)*d^2 - (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*sqrt((b^10*c^2 + 6*a^2*b^8*c + 9*a^4*b^6)*d^4/(b^12*c^7 - 6*a^2*b^10*c^6 + 15*a^4*b^8*c^5 - 20*a^6*b^6*c^4 + 15*a^8*b^4*c^3 - 6*a^10*b^2*c^2 + a^12*c))))/(b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c))) - 4*sqrt(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b - a)/((b^2*c - a^2)*x)
```

giac [B] time = 0.76, size = 654, normalized size = 4.01

$$\frac{\left(\frac{\sqrt{a+bx}}{2bx}\right) \arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a-\sqrt{b^2c}}}\right) + \frac{\left(\frac{\sqrt{a+bx}}{2bx}\right) \arctan\left(\frac{\sqrt{a+bx}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{2bd} + \frac{2\left(\sqrt{a+bx}\right)^2 \sqrt{-a-\sqrt{b^2c}}}{\left(b^2c - \sqrt{a+bx}\right)^2 + 2\left(\sqrt{a+bx}\right)^2 \sqrt{-a-\sqrt{b^2c}}}}{\left(\frac{\sqrt{a+bx}}{2bx}\right) \sqrt{-a-\sqrt{b^2c}} + \frac{\left(\frac{\sqrt{a+bx}}{2bx}\right) \sqrt{-a+\sqrt{b^2c}}}{2bd} + \frac{2\left(\sqrt{a+bx}\right)^2 \sqrt{-a-\sqrt{b^2c}}}{\left(b^2c - \sqrt{a+bx}\right)^2 + 2\left(\sqrt{a+bx}\right)^2 \sqrt{-a-\sqrt{b^2c}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")

```
[Out] 1/2*(((b^3*c - a^2*b)^2*b^4*c^(3/2)*d^2 - 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 + sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2))))/(b^2*c - a^2)))/((b^5*c^(7/2) + a*b^4*c^3 - 2*a^2*b^3*c^(5/2) - 2*a^3*b^2*c^2 + a^4*b*c^(3/2) + a^5*c)*sqrt(b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + ((b^3*c - a^2*b)^2*b^4*c^(3/2)*d^2 + 2*(a*b^6*c^2 - a^3*b^4*c)*d^2*abs(-b^3*c + a^2*b) + (a^2*b^8*c^(5/2) - 2*a^4*b^6*c^(3/2) + a^6*b^4*sqrt(c))*d^2)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^2*c - a^3 - sqrt((a*b^2*c - a^3)^2 + (b^4*c^2 - 2*a^2*b^2*c + a^4)*(b^2*c - a^2))))/(b^2*c - a^2)))/((b^5*c^(7/2) - a*b^4*c^3 - 2*a^2*b^3*c^(5/2) + 2*a^3*b^2*c^2 + a^4*b*c^(3/2) - a^5*c)*sqrt(-b*sqrt(c) - a)*abs(-b^3*c + a^2*b)) + 2*((sqrt(d*x + c)*b + a)^(3/2)*b^4*d^2 - 2*sqrt(sqrt(d*x + c)*b + a)*a*b^4*d^2)/((b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)*(b^2*c - a^2))/((b^2*d)
```

maple [B] time = 0.03, size = 265, normalized size = 1.63

$$\frac{2\sqrt{b^2c} d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c} b}}{\sqrt{-a-\sqrt{b^2c}}}\right)}{(-4a - 4\sqrt{b^2c})\sqrt{-a-\sqrt{b^2c}} c} - \frac{2\sqrt{b^2c} d \arctan\left(\frac{\sqrt{a+\sqrt{dx+c} b}}{\sqrt{-a+\sqrt{b^2c}}}\right)}{(-4a + 4\sqrt{b^2c})\sqrt{-a+\sqrt{b^2c}} c} - \frac{2\sqrt{b^2c} \sqrt{a+\sqrt{dx+c} b} d}{(-4a - 4\sqrt{b^2c})\left(-\sqrt{dx+c} b + \sqrt{b^2c}\right) c} - \frac{2\sqrt{b^2c} \sqrt{a+\sqrt{dx+c} b} d}{(-4a + 4\sqrt{b^2c})\left(\sqrt{dx+c} b + \sqrt{b^2c}\right) c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(a+(d*x+c)^(1/2)*b)^(1/2),x)

```
[Out] -2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(-4*(b^2*c)^(1/2)-4*a)/(-(d*x+c)^(1/2)*b+(b^2*c)^(1/2))+2*d*(b^2*c)^(1/2)/c/(-4*(b^2*c)^(1/2)-4*a)/(-a-(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^(1/2))-2*d*(b^2*c)^(1/2)/c*(a+(d*x+c)^(1/2)*b)^(1/2)/(4*(b^2*c)^(1/2)-4*a)/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))-2*d*(b^2*c)^(1/2)/c/(4*(b^2*c)^(1/2)-4*a)/(-a+(b^2*c)^(1/2))^(1/2)*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a+(b^2*c)^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c} b + a x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^2/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(sqrt(d*x + c)*b + a)*x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x^2 \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)),x)

[Out] int(1/(x^2*(a + b*(c + d*x)^(1/2))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**2/(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(1/(x**2*sqrt(a + b*sqrt(c + d*x))), x)

$$3.397 \quad \int \frac{1}{x^3 \sqrt{a+b\sqrt{c+dx}}} dx$$

Optimal. Leaf size=261

$$\frac{(a-b\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{2x^2(a^2-b^2c)} - \frac{bd\sqrt{a+b\sqrt{c+dx}}(6abc-(a^2+5b^2c)\sqrt{c+dx})}{8cx(a^2-b^2c)^2} + \frac{bd^2(2a-5b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}(a-b\sqrt{c})}$$

Rubi [A] time = 0.48, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {371, 1398, 823, 827, 1166, 207}

$$\frac{(a-b\sqrt{c+dx})\sqrt{a+b\sqrt{c+dx}}}{2x^2(a^2-b^2c)} - \frac{bd\sqrt{a+b\sqrt{c+dx}}(6abc-(a^2+5b^2c)\sqrt{c+dx})}{8cx(a^2-b^2c)^2} + \frac{bd^2(2a-5b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a-b\sqrt{c}}}\right)}{16c^{3/2}(a-b\sqrt{c})^2} - \frac{bd^2(2a+5b\sqrt{c})\tanh^{-1}\left(\frac{\sqrt{a+b\sqrt{c+dx}}}{\sqrt{a+b\sqrt{c}}}\right)}{16c^{3/2}(a+b\sqrt{c})^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*sqrt[a + b*sqrt[c + d*x]]),x]

[Out] -((a - b*sqrt[c + d*x])*sqrt[a + b*sqrt[c + d*x]]/(2*(a^2 - b^2*c)*x^2) - (b*d*sqrt[a + b*sqrt[c + d*x]]*(6*a*b*c - (a^2 + 5*b^2*c)*sqrt[c + d*x]))/(8*c*(a^2 - b^2*c)^2*x) + (b*(2*a - 5*b*sqrt[c])*d^2*ArcTanh[sqrt[a + b*sqrt[c + d*x]]/sqrt[a - b*sqrt[c]]])/(16*(a - b*sqrt[c])^(5/2)*c^(3/2)) - (b*(2*a + 5*b*sqrt[c])*d^2*ArcTanh[sqrt[a + b*sqrt[c + d*x]]/sqrt[a + b*sqrt[c]]])/(16*(a + b*sqrt[c])^(5/2)*c^(3/2))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 823

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a*e*g)*(m + 2*p + 4)*x, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])

Rule 827

Int[((f_.) + (g_.)*(x_))/(sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1166

Int[((d_) + (e_)*(x_)^2)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] :
 > With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
 Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]

Rule 1398

Int[((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symb
 ol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n
))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q},
 x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x^3 \sqrt{a + b\sqrt{c + dx}}} dx &= d^2 \text{Subst} \left(\int \frac{1}{\sqrt{a + b\sqrt{x}} (-c + x)^3} dx, x, c + dx \right) \\
 &= (2d^2) \text{Subst} \left(\int \frac{x}{\sqrt{a + bx} (-c + x^2)^3} dx, x, \sqrt{c + dx} \right) \\
 &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{2(a^2 - b^2c)x^2} + \frac{d^2 \text{Subst} \left(\int \frac{-\frac{1}{2}abc + \frac{5}{2}b^2cx}{\sqrt{a + bx} (-c + x^2)^2} dx, x, \sqrt{c + dx} \right)}{2c(a^2 - b^2c)} \\
 &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{2(a^2 - b^2c)x^2} - \frac{bd\sqrt{a + b\sqrt{c + dx}} (6abc - (a^2 + 5b^2c)\sqrt{c + dx})}{8c(a^2 - b^2c)^2 x} \\
 &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{2(a^2 - b^2c)x^2} - \frac{bd\sqrt{a + b\sqrt{c + dx}} (6abc - (a^2 + 5b^2c)\sqrt{c + dx})}{8c(a^2 - b^2c)^2 x} \\
 &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{2(a^2 - b^2c)x^2} - \frac{bd\sqrt{a + b\sqrt{c + dx}} (6abc - (a^2 + 5b^2c)\sqrt{c + dx})}{8c(a^2 - b^2c)^2 x} \\
 &= -\frac{(a - b\sqrt{c + dx}) \sqrt{a + b\sqrt{c + dx}}}{2(a^2 - b^2c)x^2} - \frac{bd\sqrt{a + b\sqrt{c + dx}} (6abc - (a^2 + 5b^2c)\sqrt{c + dx})}{8c(a^2 - b^2c)^2 x}
 \end{aligned}$$

Mathematica [A] time = 0.75, size = 281, normalized size = 1.08

$$\frac{\frac{8(a^2 - b^2c)(a - b\sqrt{c + dx})\sqrt{a + b\sqrt{c + dx}}}{x^2} - \frac{2bd\sqrt{a + b\sqrt{c + dx}}(a^2\sqrt{c + dx} - 6abc + 5b^2c\sqrt{c + dx})}{cx} + \frac{bd^2 \left((a - b\sqrt{c})^{5/2} (2a + 5b\sqrt{c}) \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a + b\sqrt{c}}} \right) - (2a - 5b\sqrt{c})(a + b\sqrt{c})^{5/2} \tanh^{-1} \left(\frac{\sqrt{a + b\sqrt{c + dx}}}{\sqrt{a - b\sqrt{c}}} \right) \right)}{c^{3/2} \sqrt{a - b\sqrt{c}} \sqrt{a + b\sqrt{c}}}}{16(a^2 - b^2c)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]

[Out] -1/16*((8*(a^2 - b^2*c)*(a - b*Sqrt[c + d*x])*Sqrt[a + b*Sqrt[c + d*x]])/x^2 - (2*b*d*Sqrt[a + b*Sqrt[c + d*x]]*(-6*a*b*c + a^2*Sqrt[c + d*x] + 5*b^2*c

$$c*\text{Sqrt}[c + d*x]))/(c*x) + (b*d^2*(-((2*a - 5*b*\text{Sqrt}[c])*(a + b*\text{Sqrt}[c])^(5/2)*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]]/\text{Sqrt}[a - b*\text{Sqrt}[c]]]) + (a - b*\text{Sqrt}[c])^(5/2)*(2*a + 5*b*\text{Sqrt}[c])*\text{ArcTanh}[\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]]/\text{Sqrt}[a + b*\text{Sqrt}[c]]])))/(\text{Sqrt}[a - b*\text{Sqrt}[c]]*\text{Sqrt}[a + b*\text{Sqrt}[c]]*c^(3/2)))/(a^2 - b^2*c)^2$$

IntegrateAlgebraic [A] time = 2.01, size = 332, normalized size = 1.27

$$\frac{\sqrt{a + b\sqrt{c + dx}} (-4a^3cd^2 + a^2bd^2(c + dx)^{3/2} + 3a^2bd^2\sqrt{c + dx} + 10ab^2c^2d^2 - 6ab^2cd^2(c + dx) - 9b^3c^2d^2\sqrt{c + dx} + 5b^3cd^2(c + dx)^{3/2})}{8cd^2x^2(b^2c - a^2)^2} + \frac{d^2(2ab - 5b^2\sqrt{c})\tan^{-1}\left(\frac{\sqrt{b\sqrt{c-a}}\sqrt{a+b\sqrt{c+dx}}}{a-b\sqrt{c}}\right)}{16c^{3/2}(b\sqrt{c} - a)^{5/2}} - \frac{d^2(2ab + 5b^2\sqrt{c})\tan^{-1}\left(\frac{\sqrt{-a-b\sqrt{c}}\sqrt{a+b\sqrt{c+dx}}}{a+b\sqrt{c}}\right)}{16c^{3/2}(-a - b\sqrt{c})^{5/2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x^3*Sqrt[a + b*Sqrt[c + d*x]]),x]
```

$$[\text{Out}] (\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]]*(-4*a^3*c*d^2 + 10*a*b^2*c^2*d^2 + 3*a^2*b*c*d^2*\text{Sqrt}[c + d*x] - 9*b^3*c^2*d^2*\text{Sqrt}[c + d*x] - 6*a*b^2*c*d^2*(c + d*x) + a^2*b*d^2*(c + d*x)^(3/2) + 5*b^3*c*d^2*(c + d*x)^(3/2)))/(8*c*(-a^2 + b^2*c)^2*d^2*x^2) + ((2*a*b - 5*b^2*\text{Sqrt}[c])*d^2*\text{ArcTan}[(\text{Sqrt}[-a + b*\text{Sqrt}[c]]*\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]])/(a - b*\text{Sqrt}[c])])/(16*(-a + b*\text{Sqrt}[c])^(5/2)*c^(3/2)) - ((2*a*b + 5*b^2*\text{Sqrt}[c])*d^2*\text{ArcTan}[(\text{Sqrt}[-a - b*\text{Sqrt}[c]]*\text{Sqrt}[a + b*\text{Sqrt}[c + d*x]])/(a + b*\text{Sqrt}[c])])/(16*(-a - b*\text{Sqrt}[c])^(5/2)*c^(3/2))$$

fricas [B] time = 2.41, size = 4390, normalized size = 16.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="fricas")
```

$$[\text{Out}] -1/32*((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*\text{sqrt}(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*\text{sqrt}((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3))))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*\text{log}((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6)*\text{sqrt}(\text{sqrt}(d*x + c)*b + a)*d^6 + ((325*a*b^12*c^5 + 1977*a^3*b^10*c^4 - 609*a^5*b^8*c^3 + 35*a^7*b^6*c^2)*d^4 - (5*b^14*c^10 - 16*a^2*b^12*c^9 + 3*a^4*b^10*c^8 + 50*a^6*b^8*c^7 - 85*a^8*b^6*c^6 + 60*a^10*b^4*c^5 - 19*a^12*b^2*c^4 + 2*a^14*c^3)*\text{sqrt}((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3))))*\text{sqrt}(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*\text{sqrt}((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3))))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)) - (b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*x^2*\text{sqrt}(-((105*a*b^8*c^3 + 70*a^3*b^6*c^2 - 35*a^5*b^4*c + 4*a^7*b^2)*d^4 + (b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3)*\text{sqrt}((625*b^18*c^4 + 7700*a^2*b^16*c^3 + 21966*a^4*b^14*c^2 - 10780*a^6*b^12*c + 1225*a^8*b^10)*d^8/(b^20*c^13 - 10*a^2*b^18*c^12 + 45*a^4*b^16*c^11 - 120*a^6*b^14*c^10 + 210*a^8*b^12*c^9 - 252*a^10*b^10*c^8 + 210*a^12*b^8*c^7 - 120*a^14*b^6*c^6 + 45*a^16*b^4*c^5 - 10*a^18*b^2*c^4 + a^20*c^3))))/(b^10*c^8 - 5*a^2*b^8*c^7 + 10*a^4*b^6*c^6 - 10*a^6*b^4*c^5 + 5*a^8*b^2*c^4 - a^10*c^3))*\text{log}((625*b^12*c^3 + 3750*a^2*b^10*c^2 - 1491*a^4*b^8*c + 140*a^6*b^6$$

$$\frac{b^6c^6 + 45a^{16}b^4c^5 - 10a^{18}b^2c^4 + a^{20}c^3)}{(b^{10}c^8 - 5a^2b^8c^7 + 10a^4b^6c^6 - 10a^6b^4c^5 + 5a^8b^2c^4 - a^{10}c^3)} + \frac{4(6ab^2cdx - 4a^2b^2c^2 + 4a^3c + (4b^3c^2 - 4a^2bc - (5b^3c + a^2b)dx)\sqrt{dx+c})\sqrt{\sqrt{dx+c}b+a}}{(b^4c^3 - 2a^2b^2c^2 + a^4c)x^2}$$

giac [B] time = 1.27, size = 1303, normalized size = 4.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 - (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c + sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 - a*b^8*c^(11/2) - 4*a^2*b^7*c^5 + 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 - 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 + 4*a^7*b^2*c^(5/2) + a^8*b*c^2 - a^9*c^(3/2))*sqrt(-b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) + (((b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)^2*(5*b^6*c + a^2*b^4)*d^3 + (13*a*b^10*c^(7/2) - 27*a^3*b^8*c^(5/2) + 15*a^5*b^6*c^(3/2) - a^7*b^4*sqrt(c))*d^3*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c) + 2*(4*a^2*b^14*c^6 - 17*a^4*b^12*c^5 + 28*a^6*b^10*c^4 - 22*a^8*b^8*c^3 + 8*a^10*b^6*c^2 - a^12*b^4*c)*d^3)*arctan(sqrt(sqrt(d*x + c)*b + a)/sqrt(-(a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c - sqrt((a*b^4*c^3 - 2*a^3*b^2*c^2 + a^5*c)^2 + (b^6*c^4 - 3*a^2*b^4*c^3 + 3*a^4*b^2*c^2 - a^6*c)*(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/(b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)))/((b^9*c^6 + a*b^8*c^(11/2) - 4*a^2*b^7*c^5 - 4*a^3*b^6*c^(9/2) + 6*a^4*b^5*c^4 + 6*a^5*b^4*c^(7/2) - 4*a^6*b^3*c^3 - 4*a^7*b^2*c^(5/2) + a^8*b*c^2 + a^9*c^(3/2))*sqrt(b*sqrt(c) - a)*abs(b^5*c^3 - 2*a^2*b^3*c^2 + a^4*b*c)) - 2*(9*(sqrt(d*x + c)*b + a)^(3/2)*b^8*c^2*d^3 - 19*sqrt(sqrt(d*x + c)*b + a)*a*b^8*c^2*d^3 - 5*(sqrt(d*x + c)*b + a)^(7/2)*b^6*c*d^3 + 21*(sqrt(d*x + c)*b + a)^(5/2)*a*b^6*c*d^3 - 30*(sqrt(d*x + c)*b + a)^(3/2)*a^2*b^6*c*d^3 + 18*sqrt(sqrt(d*x + c)*b + a)*a^3*b^6*c*d^3 - (sqrt(d*x + c)*b + a)^(7/2)*a^2*b^4*d^3 + 3*(sqrt(d*x + c)*b + a)^(5/2)*a^3*b^4*d^3 - 3*(sqrt(d*x + c)*b + a)^(3/2)*a^4*b^4*d^3 + sqrt(sqrt(d*x + c)*b + a)*a^5*b^4*d^3)/((b^4*c^3 - 2*a^2*b^2*c^2 + a^4*c)*(b^2*c - (sqrt(d*x + c)*b + a)^2 + 2*(sqrt(d*x + c)*b + a)*a - a^2)^2)/(b^2*d)
```

maple [B] time = 0.10, size = 840, normalized size = 3.22



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(a+(d*x+c)^(1/2)*b)^(1/2),x)
```

```
[Out] 5/16*b^2*d^2/c/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)+1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)*a-7/16*b^2*d^2/c/(((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)-1/8*b^2*d^2/c/(b^2*c)^(1/2)/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))^2/((b^2*c)^(1/2)+a)*(a+(d*x+c)^(1/2)*b)^(1/2)*a+5/16*b^2*d^2/c/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^2/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^2/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))+1/8*b^2*d^2/c/(b^2*c)^(1/2)/(b^2*c+2*a*(b^2*c)^(1/2)+a^2)/(-a-(b^2*c)^(1/2))^2/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))*arctan((a+(d*x+c)^(1/2)*b)^(1/2)/(-a-(b^2*c)^(1/2))^2/((d*x+c)^(1/2)*b-(b^2*c)^(1/2))+1/8*b^2*d^2/c/((d*x+c)^(1/2)*b+(b^2*c)^(1/2))^2/(b^2*c-2*a*(b^2*c)^(1/2)+a^2)*(a+(d*x+c)^(1/2)*b)^(3/2)-1/8*b^2*d^2/c/(b^2*c)
```

$$\begin{aligned} & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \\ & \frac{1}{\sqrt{dx+c} \sqrt{b+ax^3}} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{dx+c} b + ax^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^3/(a+b*(d*x+c)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(sqrt(dx + c)*b + a)*x^3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3 \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)),x)

[Out] int(1/(x^3*(a + b*(c + d*x)^(1/2))^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 \sqrt{a + b \sqrt{c + dx}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**3/(a+b*(d*x+c)**(1/2))**(1/2),x)

[Out] Integral(1/(x**3*sqrt(a + b*sqrt(c + d*x))), x)

3.398 $\int x^3 (a + b\sqrt{c + dx})^p dx$

Optimal. Leaf size=350

$$-\frac{2a(a^2 - b^2c)^3(a + b\sqrt{c + dx})^{p+1}}{b^8d^4(p + 1)} + \frac{2(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c + dx})^{p+2}}{b^8d^4(p + 2)} - \frac{6a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{p+3}}{b^8d^4(p + 3)}$$

Rubi [A] time = 0.28, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2(-30a^2b^2c + 35a^4 + 3b^4c^2)(a + b\sqrt{c + dx})^{p+4}}{b^8d^4(p + 4)} - \frac{2a(a^2 - b^2c)^3(a + b\sqrt{c + dx})^{p+1}}{b^8d^4(p + 1)} + \frac{2(a^2 - b^2c)^2(7a^2 - b^2c)(a + b\sqrt{c + dx})^{p+2}}{b^8d^4(p + 2)} - \frac{6a(7a^2 - 3b^2c)(a^2 - b^2c)(a + b\sqrt{c + dx})^{p+3}}{b^8d^4(p + 3)} - \frac{10a(7a^2 - 3b^2c)(a + b\sqrt{c + dx})^{p+5}}{b^8d^4(p + 5)} + \frac{6(7a^2 - b^2c)(a + b\sqrt{c + dx})^{p+6}}{b^8d^4(p + 6)} - \frac{14a(a + b\sqrt{c + dx})^{p+7}}{b^8d^4(p + 7)} + \frac{2(a + b\sqrt{c + dx})^{p+8}}{b^8d^4(p + 8)}$$

Antiderivative was successfully verified.

[In] Int[x^3*(a + b*Sqrt[c + d*x])^p,x]

[Out] (-2*a*(a^2 - b^2*c)^3*(a + b*Sqrt[c + d*x])^(1 + p))/(b^8*d^4*(1 + p)) + (2*(a^2 - b^2*c)^2*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(2 + p))/(b^8*d^4*(2 + p)) - (6*a*(7*a^2 - 3*b^2*c)*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(3 + p))/(b^8*d^4*(3 + p)) + (2*(35*a^4 - 30*a^2*b^2*c + 3*b^4*c^2)*(a + b*Sqrt[c + d*x])^(4 + p))/(b^8*d^4*(4 + p)) - (10*a*(7*a^2 - 3*b^2*c)*(a + b*Sqrt[c + d*x])^(5 + p))/(b^8*d^4*(5 + p)) + (6*(7*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^(6 + p))/(b^8*d^4*(6 + p)) - (14*a*(a + b*Sqrt[c + d*x])^(7 + p))/(b^8*d^4*(7 + p)) + (2*(a + b*Sqrt[c + d*x])^(8 + p))/(b^8*d^4*(8 + p))

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int x^3 \left(a + b\sqrt{c + dx} \right)^p dx &= \frac{\text{Subst} \left(\int (a + b\sqrt{x})^p (-c + x)^3 dx, x, c + dx \right)}{d^4} \\
&= \frac{2 \text{Subst} \left(\int x(a + bx)^p (-c + x^2)^3 dx, x, \sqrt{c + dx} \right)}{d^4} \\
&= \frac{2 \text{Subst} \left(\int \left(-\frac{a(a^2 - b^2c)^3 (a + bx)^p}{b^7} - \frac{(-7a^2 + b^2c)(-a^2 + b^2c)^2 (a + bx)^{1+p}}{b^7} - \frac{3(7a^5 - 10a^3b^2c + 3ab^4c^2)(a + bx)^{2+p}}{b^7} \right) dx, x, \sqrt{c + dx} \right)}{d^4} \\
&= -\frac{2a(a^2 - b^2c)^3 (a + b\sqrt{c + dx})^{1+p}}{b^8 d^4 (1 + p)} + \frac{2(a^2 - b^2c)^2 (7a^2 - b^2c) (a + b\sqrt{c + dx})^{2+p}}{b^8 d^4 (2 + p)}
\end{aligned}$$

Mathematica [A] time = 0.85, size = 555, normalized size = 1.59

Antiderivative was successfully verified.

[In] Integrate[x^3*(a + b*Sqrt[c + d*x])^p,x]

[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(5040*a^7 - 5040*a^6*b*(1 + p)*Sqrt[c + d*x] + 360*a^5*b^2*(6*c*(-7 + p + p^2) + 7*d*(2 + 3*p + p^2)*x) - 120*a^4*b^3*(1 + p)*Sqrt[c + d*x]*(2*c*(-63 - 5*p + 2*p^2) + 7*d*(6 + 5*p + p^2)*x) + 6*a^3*b^4*(8*c^2*(315 - 124*p - 139*p^2 - 14*p^3 + p^4) + 40*c*d*(-42 - 61*p - 16*p^2 + 4*p^3 + p^4)*x + 35*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^2) - 6*a^2*b^5*(1 + p)*Sqrt[c + d*x]*(-24*c^2*(-105 - 24*p + 5*p^2 + p^3) + 4*c*d*(-420 - 386*p - 94*p^2 - p^3 + p^4)*x + 7*d^2*(120 + 154*p + 71*p^2 + 14*p^3 + p^4)*x^2) - b^7*(105 + 176*p + 86*p^2 + 16*p^3 + p^4)*Sqrt[c + d*x]*(-48*c^3 + 24*c^2*d*(2 + p)*x - 6*c*d^2*(8 + 6*p + p^2)*x^2 + d^3*(48 + 44*p + 12*p^2 + p^3)*x^3) + a*b^6*(48*c^3*(-105 + 103*p + 138*p^2 + 38*p^3 + 3*p^4) - 24*c^2*d*(-210 - 283*p - 21*p^2 + 74*p^3 + 24*p^4 + 2*p^5)*x + 6*c*d^2*(-840 - 1726*p - 1151*p^2 - 265*p^3 + 10*p^4 + 11*p^5 + p^6)*x^2 + 7*d^3*(720 + 1764*p + 1624*p^2 + 735*p^3 + 175*p^4 + 21*p^5 + p^6)*x^3)))/(b^8*d^4*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p)*(7 + p)*(8 + p))

IntegrateAlgebraic [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b\sqrt{c + dx} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^3*(a + b*Sqrt[c + d*x])^p,x]

[Out] Defer[IntegrateAlgebraic][x^3*(a + b*Sqrt[c + d*x])^p, x]

fricas [B] time = 1.33, size = 1416, normalized size = 4.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] -2*(5040*b^8*c^4 - 20160*a^2*b^6*c^3 + 30240*a^4*b^4*c^2 - 20160*a^6*b^2*c + 5040*a^8 + 48*(b^8*c^4 + 6*a^2*b^6*c^3 + a^4*b^4*c^2)*p^4 - (b^8*d^4*p^7 + 28*b^8*d^4*p^6 + 322*b^8*d^4*p^5 + 1960*b^8*d^4*p^4 + 6769*b^8*d^4*p^3 + 13132*b^8*d^4*p^2 + 13068*b^8*d^4*p + 5040*b^8*d^4)*x^4 + 384*(2*b^8*c^4 + 7*a^2*b^6*c^3 - 3*a^4*b^4*c^2)*p^3 - (b^8*c*d^3*p^7 + (22*b^8*c - 7*a^2*b^6

$$\begin{aligned} &)d^3p^6 + 5(38b^8c - 21a^2b^6)d^3p^5 + 5(164b^8c - 119a^2b^6) \\ & *d^3p^4 + (1849b^8c - 1575a^2b^6)d^3p^3 + 2(1019b^8c - 959a^2b^6) \\ & *d^3p^2 + 840(b^8c - a^2b^6)d^3p) *x^3 + 48(86b^8c^4 + 81a^2b^6 \\ & *c^3 - 124a^4b^4c^2 + 45a^6b^2c) *p^2 + 6(18b^8c^2d^2p^5 + (b^8c \\ & ^2 + a^2b^6c) *d^2p^6 + (118b^8c^2 - 95a^2b^6c + 35a^4b^4) *d^2p^4 \\ & + 6(58b^8c^2 - 80a^2b^6c + 35a^4b^4) *d^2p^3 + (457b^8c^2 - 806a \\ & ^2b^6c + 385a^4b^4) *d^2p^2 + 210(b^8c^2 - 2a^2b^6c + a^4b^4) *d^ \\ & 2p) *x^2 + 192(44b^8c^4 - 71a^2b^6c^3 + 54a^4b^4c^2 - 15a^6b^2c \\ &) *p - 24((b^8c^3 + 3a^2b^6c^2) *d *p^5 + 2(8b^8c^3 + 9a^2b^6c^2 - \\ & 5a^4b^4c) *d *p^4 + (86b^8c^3 - 57a^2b^6c^2 + 15a^4b^4c) *d *p^3 + (\\ & 176b^8c^3 - 387a^2b^6c^2 + 340a^4b^4c - 105a^6b^2) *d *p^2 + 105(b \\ & ^8c^3 - 3a^2b^6c^2 + 3a^4b^4c - a^6b^2) *d *p) *x + (192(a *b^7c^3 + \\ & a^3b^5c^2) *p^4 + 96(27a *b^7c^3 + 2a^3b^5c^2 - 5a^5b^3c) *p^3 - (a \\ & *b^7d^3p^7 + 21a *b^7d^3p^6 + 175a *b^7d^3p^5 + 735a *b^7d^3p^4 + 1 \\ & 624a *b^7d^3p^3 + 1764a *b^7d^3p^2 + 720a *b^7d^3p) *x^3 + 192(56a *b \\ & ^7c^3 - 49a^3b^5c^2 + 15a^5b^3c) *p^2 + 6(2a *b^7c *d^2p^6 + (33a * \\ & b^7c - 7a^3b^5) *d^2p^5 + 10(20a *b^7c - 7a^3b^5) *d^2p^4 + 5(111a \\ & *b^7c - 49a^3b^5) *d^2p^3 + 2(349a *b^7c - 175a^3b^5) *d^2p^2 + 24(\\ & 13a *b^7c - 7a^3b^5) *d^2p) *x^2 + 48(279a *b^7c^3 - 511a^3b^5c^2 + \\ & 385a^5b^3c - 105a^7b) *p - 24(((3a *b^7c^2 + a^3b^5c) *d *p^5 + 2(21a \\ & *b^7c^2 - 5a^3b^5c) *d *p^4 + (192a *b^7c^2 - 135a^3b^5c + 35a^5b^ \\ & 3) *d *p^3 + (327a *b^7c^2 - 320a^3b^5c + 105a^5b^3) *d *p^2 + 2(87a *b^ \\ & 7c^2 - 98a^3b^5c + 35a^5b^3) *d *p) *x) *sqrt(d *x + c) * (sqrt(d *x + c) *b \\ & + a) ^p / (b^8d^4p^8 + 36b^8d^4p^7 + 546b^8d^4p^6 + 4536b^8d^4p^5 + \\ & 22449b^8d^4p^4 + 67284b^8d^4p^3 + 118124b^8d^4p^2 + 109584b^8d^ \\ & 4p + 40320b^8d^4) \end{aligned}$$

giac [B] time = 0.97, size = 5699, normalized size = 16.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")

[Out]
$$\begin{aligned} & -2*((\sqrt{d*x + c}) *b + a)^2 *(\sqrt{d*x + c}) *b + a)^p *b^6 *c^3 *p^7 - (\sqrt{d*x} \\ & + c) *b + a) *(\sqrt{d*x + c}) *b + a)^p *a *b^6 *c^3 *p^7 + 34 *(\sqrt{d*x + c}) *b + \\ & a)^2 *(\sqrt{d*x + c}) *b + a)^p *b^6 *c^3 *p^6 - 35 *(\sqrt{d*x + c}) *b + a) *(\sqrt{d \\ & *x + c}) *b + a)^p *a *b^6 *c^3 *p^6 - 3 *(\sqrt{d*x + c}) *b + a)^4 *(\sqrt{d*x + c}) *b \\ & + a)^p *b^4 *c^2 *p^7 + 9 *(\sqrt{d*x + c}) *b + a)^3 *(\sqrt{d*x + c}) *b + a)^p *a *b \\ & ^4 *c^2 *p^7 - 9 *(\sqrt{d*x + c}) *b + a)^2 *(\sqrt{d*x + c}) *b + a)^p *a^2 *b^4 *c^2 * \\ & p^7 + 3 *(\sqrt{d*x + c}) *b + a) *(\sqrt{d*x + c}) *b + a)^p *a^3 *b^4 *c^2 *p^7 + 478 \\ & *(\sqrt{d*x + c}) *b + a)^2 *(\sqrt{d*x + c}) *b + a)^p *b^6 *c^3 *p^5 - 511 *(\sqrt{d*x \\ & + c}) *b + a) *(\sqrt{d*x + c}) *b + a)^p *a *b^6 *c^3 *p^5 - 96 *(\sqrt{d*x + c}) *b + \\ & a)^4 *(\sqrt{d*x + c}) *b + a)^p *b^4 *c^2 *p^6 + 297 *(\sqrt{d*x + c}) *b + a)^3 *(\sqrt{ \\ & d*x + c}) *b + a)^p *a *b^4 *c^2 *p^6 - 306 *(\sqrt{d*x + c}) *b + a)^2 *(\sqrt{d*x \\ & + c}) *b + a)^p *a^2 *b^4 *c^2 *p^6 + 105 *(\sqrt{d*x + c}) *b + a) *(\sqrt{d*x + c}) *b \\ & + a)^p *a^3 *b^4 *c^2 *p^6 + 3 *(\sqrt{d*x + c}) *b + a)^6 *(\sqrt{d*x + c}) *b + a)^p * \\ & b^2 *c *p^7 - 15 *(\sqrt{d*x + c}) *b + a)^5 *(\sqrt{d*x + c}) *b + a)^p *a *b^2 *c *p^7 \\ & + 30 *(\sqrt{d*x + c}) *b + a)^4 *(\sqrt{d*x + c}) *b + a)^p *a^2 *b^2 *c *p^7 - 30 *(\sqrt{ \\ & d*x + c}) *b + a)^3 *(\sqrt{d*x + c}) *b + a)^p *a^3 *b^2 *c *p^7 + 15 *(\sqrt{d*x + \\ & c}) *b + a)^2 *(\sqrt{d*x + c}) *b + a)^p *a^4 *b^2 *c *p^7 - 3 *(\sqrt{d*x + c}) *b + a \\ &) *(\sqrt{d*x + c}) *b + a)^p *a^5 *b^2 *c *p^7 + 3580 *(\sqrt{d*x + c}) *b + a)^2 *(\sqrt{ \\ & d*x + c}) *b + a)^p *b^6 *c^3 *p^4 - 4025 *(\sqrt{d*x + c}) *b + a) *(\sqrt{d*x + c}) \\ & *b + a)^p *a *b^6 *c^3 *p^4 - 1254 *(\sqrt{d*x + c}) *b + a)^4 *(\sqrt{d*x + c}) *b + a \\ &)^p *b^4 *c^2 *p^5 + 4023 *(\sqrt{d*x + c}) *b + a)^3 *(\sqrt{d*x + c}) *b + a)^p *a *b^ \\ & 4 *c^2 *p^5 - 4302 *(\sqrt{d*x + c}) *b + a)^2 *(\sqrt{d*x + c}) *b + a)^p *a^2 *b^4 *c^ \\ & 2 *p^5 + 1533 *(\sqrt{d*x + c}) *b + a) *(\sqrt{d*x + c}) *b + a)^p *a^3 *b^4 *c^2 *p^5 \\ & + 90 *(\sqrt{d*x + c}) *b + a)^6 *(\sqrt{d*x + c}) *b + a)^p *b^2 *c *p^6 - 465 *(\sqrt{ \\ & d*x + c}) *b + a)^5 *(\sqrt{d*x + c}) *b + a)^p *a *b^2 *c *p^6 + 960 *(\sqrt{d*x + c}) * \\ & b + a)^4 *(\sqrt{d*x + c}) *b + a)^p *a^2 *b^2 *c *p^6 - 990 *(\sqrt{d*x + c}) *b + a)^ \end{aligned}$$

$$\begin{aligned}
& 3*(\sqrt{d*x + c}*b + a)^p*a^3*b^2*c*p^6 + 510*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^4*b^2*c*p^6 - 105*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^5*b^2*c*p^6 - (\sqrt{d*x + c}*b + a)^8*(\sqrt{d*x + c}*b + a)^p*p^7 + 7*(\sqrt{d*x + c}*b + a)^7*(\sqrt{d*x + c}*b + a)^p*a*p^7 - 21*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*a^2*p^7 + 35*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a^3*p^7 - 35*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^4*p^7 + 21*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^5*p^7 - 7*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^6*p^7 + (\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^7*p^7 + 15289*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*b^6*c^3*p^3 - 18424*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a*b^6*c^3*p^3 - 8592*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*b^4*c^2*p^4 + 28755*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a*b^4*c^2*p^4 - 32220*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^2*b^4*c^2*p^4 + 12075*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^3*b^4*c^2*p^4 + 1098*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*b^2*c*p^5 - 5865*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a*b^2*c*p^5 + 12540*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^2*b^2*c*p^5 - 13410*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^3*b^2*c*p^5 + 7170*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^4*b^2*c*p^5 - 1533*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^5*b^2*c*p^5 - 28*(\sqrt{d*x + c}*b + a)^8*(\sqrt{d*x + c}*b + a)^p*p^6 + 203*(\sqrt{d*x + c}*b + a)^7*(\sqrt{d*x + c}*b + a)^p*a*p^6 - 630*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*a^2*p^6 + 1085*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a^3*p^6 - 1120*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^4*p^6 + 693*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^5*p^6 - 238*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^6*p^6 + 35*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^7*p^6 + 36706*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*b^6*c^3*p^2 - 48860*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a*b^6*c^3*p^2 - 32979*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*b^4*c^2*p^3 + 115776*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a*b^4*c^2*p^3 - 137601*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^2*b^4*c^2*p^3 + 55272*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^3*b^4*c^2*p^3 + 7020*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*b^2*c*p^4 - 38715*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a*b^2*c*p^4 + 85920*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^2*b^2*c*p^4 - 95850*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^3*b^2*c*p^4 + 53700*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^4*b^2*c*p^4 - 12075*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^5*b^2*c*p^4 - 322*(\sqrt{d*x + c}*b + a)^8*(\sqrt{d*x + c}*b + a)^p*p^5 + 2401*(\sqrt{d*x + c}*b + a)^7*(\sqrt{d*x + c}*b + a)^p*a*p^5 - 7686*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*a^2*p^5 + 13685*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a^3*p^5 - 14630*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^4*p^5 + 9387*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^5*p^5 - 3346*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^6*p^5 + 511*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^7*p^5 + 44712*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*b^6*c^3*p - 69264*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a*b^6*c^3*p - 69936*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*b^4*c^2*p^2 + 258228*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a*b^4*c^2*p^2 - 330354*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^2*b^4*c^2*p^2 + 146580*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^3*b^4*c^2*p^2 + 25227*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*b^2*c*p^3 - 143160*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a*b^2*c*p^3 + 329790*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^2*b^2*c*p^3 - 385920*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^3*b^2*c*p^3 + 229335*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^4*b^2*c*p^3 - 55272*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^5*b^2*c*p^3 - 1960*(\sqrt{d*x + c}*b + a)^8*(\sqrt{d*x + c}*b + a)^p*p^4 + 14945*(\sqrt{d*x + c}*b + a)^7*(\sqrt{d*x + c}*b + a)^p*a*p^4 - 49140*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*a^2*p^4 + 90335*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a^3*p^4 - 100240*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^4*p^4
\end{aligned}$$

$$\begin{aligned}
& 4 + 67095*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^5*p^4 - 25060*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^6*p^4 + 4025*(\sqrt{d*x + c}) * b + a*(\sqrt{d*x + c}*b + a)^p*a^7*p^4 + 20160*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*b^6*c^3 - 40320*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}) * b + a)^p*a*b^6*c^3 - 74628*(\sqrt{d*x + c}*b + a)^4*(\sqrt{d*x + c}*b + a)^p * b^4*c^2*p + 288432*(\sqrt{d*x + c}*b + a)^3*(\sqrt{d*x + c}*b + a)^p*a*b^4*c^2*p - 402408*(\sqrt{d*x + c}*b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^2*b^4*c^2*p + 207792*(\sqrt{d*x + c}*b + a)*(\sqrt{d*x + c}*b + a)^p*a^3*b^4*c^2*p + 50490*(\sqrt{d*x + c}*b + a)^6*(\sqrt{d*x + c}*b + a)^p*b^2*c*p^2 - 293460*(\sqrt{d*x + c}*b + a)^5*(\sqrt{d*x + c}*b + a)^p*a*b^2*c*p^2 + 699360*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}*b + a)^p*a^2*b^2*c*p^2 - 860760*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}*b + a)^p*a^3*b^2*c*p^2 + 550590*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}*b + a)^p*a^4*b^2*c*p^2 - 146580*(\sqrt{d*x + c}) * b + a*(\sqrt{d*x + c}*b + a)^p*a^5*b^2*c*p^2 - 6769*(\sqrt{d*x + c}) * b + a)^8*(\sqrt{d*x + c}) * b + a)^p*p^3 + 52528*(\sqrt{d*x + c}) * b + a)^7*(\sqrt{d*x + c}) * b + a)^p * a*p^3 - 176589*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*a^2*p^3 + 334040*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a^3*p^3 - 384755*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^4*p^3 + 270144*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^5*p^3 - 107023*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^6*p^3 + 18424*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^7*p^3 - 30240*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p * b^4*c^2 + 120960*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a*b^4*c^2 - 181440*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^2*b^4*c^2 + 120960*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^3*b^4*c^2 + 51432*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*b^2*c*p - 304560*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a*b^2*c*p + 746280*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^2*b^2*c*p - 961440*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^3*b^2*c*p + 670680*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^4*b^2*c*p - 207792*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^5*b^2*c*p - 13132*(\sqrt{d*x + c}) * b + a)^8*(\sqrt{d*x + c}) * b + a)^p*p^2 + 103292*(\sqrt{d*x + c}) * b + a)^7*(\sqrt{d*x + c}) * b + a)^p*a*p^2 - 353430*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*a^2*p^2 + 684740*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a^3*p^2 - 815920*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^4*p^2 + 602532*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^5*p^2 - 256942*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^6*p^2 + 48860*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^7*p^2 + 20160*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*b^2*c - 120960*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a*b^2*c + 302400*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^2*b^2*c - 403200*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^3*b^2*c + 302400*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^4*b^2*c - 120960*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^5*b^2*c - 13068*(\sqrt{d*x + c}) * b + a)^8*(\sqrt{d*x + c}) * b + a)^p * p + 103824*(\sqrt{d*x + c}) * b + a)^7*(\sqrt{d*x + c}) * b + a)^p*a*p - 360024*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*a^2*p + 710640*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a^3*p - 870660*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^4*p + 673008*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^5*p - 312984*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^6 * p + 69264*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^7*p - 5040*(\sqrt{d*x + c}) * b + a)^8*(\sqrt{d*x + c}) * b + a)^p + 40320*(\sqrt{d*x + c}) * b + a)^7*(\sqrt{d*x + c}) * b + a)^p*a - 141120*(\sqrt{d*x + c}) * b + a)^6*(\sqrt{d*x + c}) * b + a)^p*a^2 + 282240*(\sqrt{d*x + c}) * b + a)^5*(\sqrt{d*x + c}) * b + a)^p*a^3 - 352800*(\sqrt{d*x + c}) * b + a)^4*(\sqrt{d*x + c}) * b + a)^p*a^4 + 282240*(\sqrt{d*x + c}) * b + a)^3*(\sqrt{d*x + c}) * b + a)^p*a^5 - 141120*(\sqrt{d*x + c}) * b + a)^2*(\sqrt{d*x + c}) * b + a)^p*a^6 + 40320*(\sqrt{d*x + c}) * b + a)*(\sqrt{d*x + c}) * b + a)^p*a^7)/((b^6*d^3*p^8 + 36*b^6*d^3*p^7 + 546*b^6*d^3*p^6 + 4536*b^6*d^3*p^5 + 22449*b^6*d^3*p^4 + 67284*b^6*d^3*p^3 + 118124*b^6*d^3*p^2 + 109584*b^6*d^3*p + 40320*b^6*d^3)*b^2*d)
\end{aligned}$$

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int x^3 \left(a + \sqrt{dx + c} b \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)

[Out] int(x^3*(a+(d*x+c)^(1/2)*b)^p,x)

maxima [B] time = 1.13, size = 728, normalized size = 2.08

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")

[Out]
$$\frac{-2*((d*x + c)*b^2*(p + 1) + \sqrt{d*x + c}*a*b*p - a^2)*(sqrt{d*x + c}*b + a)^p*c^3/((p^2 + 3*p + 2)*b^2) - 3*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^{3/2}*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt{d*x + c}*a^3*b*p - 6*a^4)*(sqrt{d*x + c}*b + a)^p*c^2/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4) + 3*((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^{5/2}*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 + 3*p^2 + 2*p)*(d*x + c)^{3/2}*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 120*sqrt{d*x + c}*a^5*b*p - 120*a^6)*(sqrt{d*x + c}*b + a)^p*c/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6) - ((p^7 + 28*p^6 + 322*p^5 + 1960*p^4 + 6769*p^3 + 13132*p^2 + 13068*p + 5040)*(d*x + c)^4*b^8 + (p^7 + 21*p^6 + 175*p^5 + 735*p^4 + 1624*p^3 + 1764*p^2 + 720*p)*(d*x + c)^{7/2}*a*b^7 - 7*(p^6 + 15*p^5 + 85*p^4 + 225*p^3 + 274*p^2 + 120*p)*(d*x + c)^3*a^2*b^6 + 42*(p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^{5/2}*a^3*b^5 - 210*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^4*b^4 + 840*(p^3 + 3*p^2 + 2*p)*(d*x + c)^{3/2}*a^5*b^3 - 2520*(p^2 + p)*(d*x + c)*a^6*b^2 + 5040*sqrt{d*x + c}*a^7*b*p - 5040*a^8)*(sqrt{d*x + c}*b + a)^p/((p^8 + 36*p^7 + 546*p^6 + 4536*p^5 + 22449*p^4 + 67284*p^3 + 118124*p^2 + 109584*p + 40320)*b^8))/d^4$$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^3 \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(a + b*(c + d*x)^(1/2))^p,x)

[Out] int(x^3*(a + b*(c + d*x)^(1/2))^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(a+b*(d*x+c)**(1/2))**p,x)

[Out] Integral(x**3*(a + b*sqrt(c + d*x))**p, x)

3.399 $\int x^2 (a + b\sqrt{c + dx})^p dx$

Optimal. Leaf size=242

$$\frac{2a(a^2 - b^2c)^2 (a + b\sqrt{c + dx})^{p+1}}{b^6 d^3 (p+1)} - \frac{4a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{p+3}}{b^6 d^3 (p+3)} + \frac{4(5a^2 - b^2c)(a + b\sqrt{c + dx})^{p+4}}{b^6 d^3 (p+4)} + \frac{2(5a^2 - b^2c)^2 (a + b\sqrt{c + dx})^{p+6}}{b^6 d^3 (p+6)}$$

Rubi [A] time = 0.18, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {371, 1398, 772}

$$\frac{2(-6a^2b^2c + 5a^4 + b^4c^2)(a + b\sqrt{c + dx})^{p+2}}{b^6 d^3 (p+2)} - \frac{2a(a^2 - b^2c)^2 (a + b\sqrt{c + dx})^{p+1}}{b^6 d^3 (p+1)} - \frac{4a(5a^2 - 3b^2c)(a + b\sqrt{c + dx})^{p+3}}{b^6 d^3 (p+3)} + \frac{4(5a^2 - b^2c)(a + b\sqrt{c + dx})^{p+4}}{b^6 d^3 (p+4)} - \frac{10a(a + b\sqrt{c + dx})^{p+5}}{b^6 d^3 (p+5)} + \frac{2(a + b\sqrt{c + dx})^{p+6}}{b^6 d^3 (p+6)}$$

Antiderivative was successfully verified.

[In] Int[x^2*(a + b*sqrt[c + d*x])^p,x]

[Out] (-2*a*(a^2 - b^2*c)^2*(a + b*sqrt[c + d*x])^(1 + p))/(b^6*d^3*(1 + p)) + (2*(5*a^4 - 6*a^2*b^2*c + b^4*c^2)*(a + b*sqrt[c + d*x])^(2 + p))/(b^6*d^3*(2 + p)) - (4*a*(5*a^2 - 3*b^2*c)*(a + b*sqrt[c + d*x])^(3 + p))/(b^6*d^3*(3 + p)) + (4*(5*a^2 - b^2*c)*(a + b*sqrt[c + d*x])^(4 + p))/(b^6*d^3*(4 + p)) - (10*a*(a + b*sqrt[c + d*x])^(5 + p))/(b^6*d^3*(5 + p)) + (2*(a + b*sqrt[c + d*x])^(6 + p))/(b^6*d^3*(6 + p))

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x^2 (a + b\sqrt{c + dx})^p dx &= \frac{\text{Subst}\left(\int (a + b\sqrt{x})^p (-c + x)^2 dx, x, c + dx\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int x(a + bx)^p (-c + x^2)^2 dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{a(a^2 - b^2c)^2 (a + bx)^p}{b^5} + \frac{(5a^4 - 6a^2b^2c + b^4c^2)(a + bx)^{1+p}}{b^5} - \frac{2(5a^3 - 3ab^2c)(a + bx)^{2+p}}{b^5} - \frac{2(-5a^2 + 5a^2 - b^2c)^2 (a + bx)^{3+p}}{b^5}\right) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= -\frac{2a(a^2 - b^2c)^2 (a + b\sqrt{c + dx})^{1+p}}{b^6 d^3 (1 + p)} + \frac{2(5a^4 - 6a^2b^2c + b^4c^2)(a + b\sqrt{c + dx})^{2+p}}{b^6 d^3 (2 + p)} - \frac{2(5a^3 - 3ab^2c)(a + b\sqrt{c + dx})^{3+p}}{b^6 d^3 (3 + p)} - \frac{2(-5a^2 + 5a^2 - b^2c)^2 (a + b\sqrt{c + dx})^{4+p}}{b^6 d^3 (4 + p)} \end{aligned}$$

Mathematica [A] time = 0.37, size = 284, normalized size = 1.17

$$\frac{2(a + b\sqrt{c+dx})^{p+1} (120a^5 - 120a^4b(1+p)\sqrt{c+dx} + 12a^3b^2(4c(p^2 + p - 5) + 5d(p^2 + 3p + 2)) - 4a^2b^3(p+1)\sqrt{c+dx}(2(p^2 - 4p - 30) + 5d(p^2 + 5p + 6))x + ad^2(2p^2 + 12p - 15) + 4ad(p^4 + 4p^3 - 10p^2 - 43p - 30)x + 5d^2(p^4 + 10p^3 + 35p^2 + 50p + 24)x^2 - d^3(p^4 + 23p + 15)\sqrt{c+dx}(6c^2 - 4cdp + 21c + d^2(p^2 + 6p + 8))x^2)}{b^6d^3(p+1)(p+2)(p+3)(p+4)(p+5)(p+6)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2*(a + b*Sqrt[c + d*x])^p,x]

[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(120*a^5 - 120*a^4*b*(1 + p)*Sqrt[c + d*x] + 12*a^3*b^2*(4*c*(-5 + p + p^2) + 5*d*(2 + 3*p + p^2)*x) - 4*a^2*b^3*(1 + p)*Sqrt[c + d*x]*(2*c*(-30 - 4*p + p^2) + 5*d*(6 + 5*p + p^2)*x) - b^5*(15 + 23*p + 9*p^2 + p^3)*Sqrt[c + d*x]*(8*c^2 - 4*c*d*(2 + p)*x + d^2*(8 + 6*p + p^2)*x^2) + a*b^4*(-8*c^2*(-15 + 10*p + 12*p^2 + 2*p^3) + 4*c*d*(-30 - 43*p - 10*p^2 + 4*p^3 + p^4)*x + 5*d^2*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^2))/(b^6*d^3*(1 + p)*(2 + p)*(3 + p)*(4 + p)*(5 + p)*(6 + p))

IntegrateAlgebraic [F] time = 0.20, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b\sqrt{c + dx} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x^2*(a + b*Sqrt[c + d*x])^p,x]

[Out] Defer[IntegrateAlgebraic][x^2*(a + b*Sqrt[c + d*x])^p, x]

fricas [B] time = 1.17, size = 712, normalized size = 2.94

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] 2*(120*b^6*c^3 - 360*a^2*b^4*c^2 + 360*a^4*b^2*c - 120*a^6 + 8*(b^6*c^3 + 3*a^2*b^4*c^2)*p^3 + (b^6*d^3*p^5 + 15*b^6*d^3*p^4 + 85*b^6*d^3*p^3 + 225*b^6*d^3*p^2 + 274*b^6*d^3*p + 120*b^6*d^3)*x^3 + 24*(3*b^6*c^3 + 3*a^2*b^4*c^2 - 2*a^4*b^2*c)*p^2 + (b^6*c*d^2*p^5 + (11*b^6*c - 5*a^2*b^4)*d^2*p^4 + (41*b^6*c - 30*a^2*b^4)*d^2*p^3 + (61*b^6*c - 55*a^2*b^4)*d^2*p^2 + 30*(b^6*c - a^2*b^4)*d^2*p)*x^2 + 8*(23*b^6*c^3 - 24*a^2*b^4*c^2 + 9*a^4*b^2*c)*p - 4*((b^6*c^2 + a^2*b^4*c)*d*p^4 + 3*(3*b^6*c^2 - a^2*b^4*c)*d*p^3 + (23*b^6*c^2 - 34*a^2*b^4*c + 15*a^4*b^2)*d*p^2 + 15*(b^6*c^2 - 2*a^2*b^4*c + a^4*b^2)*d*p)*x + (8*(3*a*b^5*c^2 + a^3*b^3*c)*p^3 + 24*(7*a*b^5*c^2 - 3*a^3*b^3*c)*p^2 + (a*b^5*d^2*p^5 + 10*a*b^5*d^2*p^4 + 35*a*b^5*d^2*p^3 + 50*a*b^5*d^2*p^2 + 24*a*b^5*d^2*p)*x^2 + 8*(33*a*b^5*c^2 - 40*a^3*b^3*c + 15*a^5*b)*p - 4*(2*a*b^5*c*d*p^4 + 5*(3*a*b^5*c - a^3*b^3)*d*p^3 + (31*a*b^5*c - 15*a^3*b^3)*d*p^2 + 2*(9*a*b^5*c - 5*a^3*b^3)*d*p)*x)*sqrt(d*x + c)*(sqrt(d*x + c)*b + a)^p/(b^6*d^3*p^6 + 21*b^6*d^3*p^5 + 175*b^6*d^3*p^4 + 735*b^6*d^3*p^3 + 1624*b^6*d^3*p^2 + 1764*b^6*d^3*p + 720*b^6*d^3)

giac [B] time = 0.59, size = 2511, normalized size = 10.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")

[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^5 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^5 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^4*c^2*p^4 - 20*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^4*c^2*p^4 - 2*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*b^2*c*p^5 + 6*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*b^2*

$$\begin{aligned}
& c^p^5 - 6*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^2*b^2*c^p^5 + 2 \\
& *(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*b^2*c^p^5 + 137*(\text{sqrt}(d*x \\
& + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*b^4*c^2*p^3 - 155*(\text{sqrt}(d*x + c)*b \\
& + a)*(\text{sqrt}(d*x + c)*b + a)^p*a*b^4*c^2*p^3 - 34*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^p*b^2*c^p^4 + 108*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c) \\
& *b + a)^p*a*b^2*c^p^4 - 114*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p \\
& *a^2*b^2*c^p^4 + 40*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*b^2*c \\
& *p^4 + (\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a)^p*p^5 - 5*(\text{sqrt}(d*x + \\
& c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a*p^5 + 10*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^p*a^2*p^5 - 10*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)*b + \\
& a)^p*a^3*p^5 + 5*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^4*p^5 - \\
& (\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^5*p^5 + 461*(\text{sqrt}(d*x + c) \\
& *b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*b^4*c^2*p^2 - 580*(\text{sqrt}(d*x + c)*b + a)* \\
& (\text{sqrt}(d*x + c)*b + a)^p*a*b^4*c^2*p^2 - 214*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt}(d*x \\
& + c)*b + a)^p*b^2*c^p^3 + 726*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)*b + \\
& a)^p*a*b^2*c^p^3 - 822*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^2* \\
& b^2*c^p^3 + 310*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*b^2*c^p^3 \\
& + 15*(\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a)^p*p^4 - 80*(\text{sqrt}(d*x + \\
& c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a*p^4 + 170*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^p*a^2*p^4 - 180*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)*b \\
& + a)^p*a^3*p^4 + 95*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^4*p^4 \\
& - 20*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^5*p^4 + 702*(\text{sqrt}(d*x \\
& + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*b^4*c^2*p - 1044*(\text{sqrt}(d*x + c)*b + \\
& a)*(\text{sqrt}(d*x + c)*b + a)^p*a*b^4*c^2*p - 614*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^p*b^2*c^p^2 + 2232*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)* \\
& b + a)^p*a*b^2*c^p^2 - 2766*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p \\
& *a^2*b^2*c^p^2 + 1160*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*b^2 \\
& *c^p^2 + 85*(\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a)^p*p^3 - 475*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a*p^3 + 1070*(\text{sqrt}(d*x + c)*b + \\
& a)^4*(\text{sqrt}(d*x + c)*b + a)^p*a^2*p^3 - 1210*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d \\
& *x + c)*b + a)^p*a^3*p^3 + 685*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a) \\
&)^p*a^4*p^3 - 155*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^5*p^3 + 3 \\
& 60*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*b^4*c^2 - 720*(\text{sqrt}(d*x \\
& + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a*b^4*c^2 - 792*(\text{sqrt}(d*x + c)*b + a)^4 \\
& *(\text{sqrt}(d*x + c)*b + a)^p*b^2*c^p + 3048*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + \\
& c)*b + a)^p*a*b^2*c^p - 4212*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a) \\
& ^p*a^2*b^2*c^p + 2088*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*b^2 \\
& *c^p + 225*(\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a)^p*p^2 - 1300*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a*p^2 + 3070*(\text{sqrt}(d*x + c)*b + \\
& a)^4*(\text{sqrt}(d*x + c)*b + a)^p*a^2*p^2 - 3720*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d \\
& *x + c)*b + a)^p*a^3*p^2 + 2305*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + \\
& a)^p*a^4*p^2 - 580*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^5*p^2 - \\
& 360*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt}(d*x + c)*b + a)^p*b^2*c + 1440*(\text{sqrt}(d*x \\
& + c)*b + a)^3*(\text{sqrt}(d*x + c)*b + a)^p*a*b^2*c - 2160*(\text{sqrt}(d*x + c)*b + a)^ \\
& 2*(\text{sqrt}(d*x + c)*b + a)^p*a^2*b^2*c + 1440*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x \\
& + c)*b + a)^p*a^3*b^2*c + 274*(\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a) \\
& ^p*p - 1620*(\text{sqrt}(d*x + c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a*p + 3960*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt}(d*x + c)*b + a)^p*a^2*p - 5080*(\text{sqrt}(d*x + c)*b + \\
& a)^3*(\text{sqrt}(d*x + c)*b + a)^p*a^3*p + 3510*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d* \\
& x + c)*b + a)^p*a^4*p - 1044*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p* \\
& a^5*p + 120*(\text{sqrt}(d*x + c)*b + a)^6*(\text{sqrt}(d*x + c)*b + a)^p - 720*(\text{sqrt}(d*x \\
& + c)*b + a)^5*(\text{sqrt}(d*x + c)*b + a)^p*a + 1800*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt} \\
& (\text{sqrt}(d*x + c)*b + a)^p*a^2 - 2400*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)*b + \\
& a)^p*a^3 + 1800*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^4 - 720*(\\
& \text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^5)/((b^4*d^2*p^6 + 21*b^4*d^ \\
& 2*p^5 + 175*b^4*d^2*p^4 + 735*b^4*d^2*p^3 + 1624*b^4*d^2*p^2 + 1764*b^4*d^2 \\
& *p + 720*b^4*d^2)*b^2*d)
\end{aligned}$$

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + \sqrt{dx + c} b \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)

[Out] int(x^2*(a+(d*x+c)^(1/2)*b)^p,x)

maxima [A] time = 1.06, size = 402, normalized size = 1.66

$$\frac{2 \left(\frac{(dx+cp)^2 + \sqrt{dx+c} dp - 2 \sqrt{dx+c} a}{(p^2+2p+2)d^2} - \frac{2 \left((p^4+9p^2+11p+6)(dx+cp)^2 + (p^4+3p^2+2p)(dx+cp) + \frac{1}{2} p^2 \right)}{(p^4+10p^2+35p+24)d^2} \sqrt{dx+c} + \frac{\left((p^4+15p^4+85p^3+225p^2+274p+120)(dx+cp)^2 + (p^4+10p^4+35p^3+50p^2+24p)(dx+cp) + \frac{1}{2} p^2 \right)}{(p^4+21p^4+175p^3+1624p^2+1764p+720)d^2} \sqrt{dx+c} \right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")

[Out] 2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)^p*c^2/((p^2 + 3*p + 2)*b^2) - 2*((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p*c/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4) + ((p^5 + 15*p^4 + 85*p^3 + 225*p^2 + 274*p + 120)*(d*x + c)^3*b^6 + (p^5 + 10*p^4 + 35*p^3 + 50*p^2 + 24*p)*(d*x + c)^(5/2)*a*b^5 - 5*(p^4 + 6*p^3 + 11*p^2 + 6*p)*(d*x + c)^2*a^2*b^4 + 20*(p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a^3*b^3 - 60*(p^2 + p)*(d*x + c)*a^4*b^2 + 120*sqrt(d*x + c)*a^5*b*p - 120*a^6)*(sqrt(d*x + c)*b + a)^p/((p^6 + 21*p^5 + 175*p^4 + 735*p^3 + 1624*p^2 + 1764*p + 720)*b^6))/d^3

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int x^2 \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(a + b*(c + d*x)^(1/2))^p,x)

[Out] int(x^2*(a + b*(c + d*x)^(1/2))^p, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2*(a+b*(d*x+c)**(1/2))**p,x)

[Out] Integral(x**2*(a + b*sqrt(c + d*x))**p, x)

3.400 $\int x \left(a + b\sqrt{c + dx} \right)^p dx$

Optimal. Leaf size=145

$$-\frac{2a(a^2 - b^2c)(a + b\sqrt{c + dx})^{p+1}}{b^4d^2(p+1)} + \frac{2(3a^2 - b^2c)(a + b\sqrt{c + dx})^{p+2}}{b^4d^2(p+2)} - \frac{6a(a + b\sqrt{c + dx})^{p+3}}{b^4d^2(p+3)} + \frac{2(a + b\sqrt{c + dx})^{p+4}}{b^4d^2(p+4)}$$

Rubi [A] time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$-\frac{2a(a^2 - b^2c)(a + b\sqrt{c + dx})^{p+1}}{b^4d^2(p+1)} + \frac{2(3a^2 - b^2c)(a + b\sqrt{c + dx})^{p+2}}{b^4d^2(p+2)} - \frac{6a(a + b\sqrt{c + dx})^{p+3}}{b^4d^2(p+3)} + \frac{2(a + b\sqrt{c + dx})^{p+4}}{b^4d^2(p+4)}$$

Antiderivative was successfully verified.

[In] Int[x*(a + b*Sqrt[c + d*x])^p,x]

[Out] $(-2*a*(a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^{(1 + p)})/(b^4*d^2*(1 + p)) + (2*(3*a^2 - b^2*c)*(a + b*Sqrt[c + d*x])^{(2 + p)})/(b^4*d^2*(2 + p)) - (6*a*(a + b*Sqrt[c + d*x])^{(3 + p)})/(b^4*d^2*(3 + p)) + (2*(a + b*Sqrt[c + d*x])^{(4 + p)})/(b^4*d^2*(4 + p))$

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^(q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int x \left(a + b\sqrt{c + dx} \right)^p dx &= \frac{\text{Subst} \left(\int \left(a + b\sqrt{x} \right)^p (-c + x) dx, x, c + dx \right)}{d^2} \\ &= \frac{2 \text{Subst} \left(\int x(a + bx)^p (-c + x^2) dx, x, \sqrt{c + dx} \right)}{d^2} \\ &= \frac{2 \text{Subst} \left(\int \left(\frac{(-a^3 + ab^2c)(a+bx)^p}{b^3} + \frac{(3a^2 - b^2c)(a+bx)^{1+p}}{b^3} - \frac{3a(a+bx)^{2+p}}{b^3} + \frac{(a+bx)^{3+p}}{b^3} \right) dx, x, \sqrt{c + dx} \right)}{d^2} \\ &= -\frac{2a(a^2 - b^2c)(a + b\sqrt{c + dx})^{1+p}}{b^4d^2(1+p)} + \frac{2(3a^2 - b^2c)(a + b\sqrt{c + dx})^{2+p}}{b^4d^2(2+p)} - \frac{6a(a + b\sqrt{c + dx})^{3+p}}{b^4d^2(3+p)} + \frac{2(a + b\sqrt{c + dx})^{4+p}}{b^4d^2(4+p)} \end{aligned}$$

Mathematica [A] time = 0.16, size = 128, normalized size = 0.88

$$\frac{2(a + b\sqrt{c + dx})^{p+1} (6a^3 - 6a^2b(p+1)\sqrt{c + dx} + ab^2(2c(p^2 + p - 3) + 3d(p^2 + 3p + 2)x) - b^3(p^2 + 4p + 3)\sqrt{c + dx}(d(p+2)x - 2c))}{b^4d^2(p+1)(p+2)(p+3)(p+4)}$$

Antiderivative was successfully verified.

[In] Integrate[x*(a + b*Sqrt[c + d*x])^p,x]

[Out] (-2*(a + b*Sqrt[c + d*x])^(1 + p)*(6*a^3 - 6*a^2*b*(1 + p)*Sqrt[c + d*x] - b^3*(3 + 4*p + p^2)*Sqrt[c + d*x]*(-2*c + d*(2 + p)*x) + a*b^2*(2*c*(-3 + p + p^2) + 3*d*(2 + 3*p + p^2)*x)))/(b^4*d^2*(1 + p)*(2 + p)*(3 + p)*(4 + p))

IntegrateAlgebraic [F] time = 0.16, size = 0, normalized size = 0.00

$$\int x \left(a + b\sqrt{c + dx} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x*(a + b*Sqrt[c + d*x])^p,x]

[Out] Defer[IntegrateAlgebraic][x*(a + b*Sqrt[c + d*x])^p, x]

fricas [B] time = 1.04, size = 294, normalized size = 2.03

$$\frac{2(6b^4c^2 - 12a^2b^2c + 6a^4 + 2(b^4c^2 + a^2b^2c)p^2 - (b^4d^2p^3 + 6b^4d^2p^2 + 11b^4d^2p + 6b^4d^2)x^2 + 4(2b^4c^2 - a^2b^2c)p - (b^4cd^3 + (4b^4c - 3a^2b^2)d^2p + 3(b^4c - a^2b^2)d^2p)x + (4ab^3c^2 + 2(5ab^3c - 3a^3b)p - (ab^3d^3 + 3ab^3d^2 + 2ab^3d)p)x)\sqrt{dx + c}(\sqrt{dx + c} + a)^p}{b^4d^2p^4 + 10b^4d^2p^3 + 35b^4d^2p^2 + 50b^4d^2p + 24b^4d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] -2*(6*b^4*c^2 - 12*a^2*b^2*c + 6*a^4 + 2*(b^4*c^2 + a^2*b^2*c)*p^2 - (b^4*d^2*p^3 + 6*b^4*d^2*p^2 + 11*b^4*d^2*p + 6*b^4*d^2)*x^2 + 4*(2*b^4*c^2 - a^2*b^2*c)*p - (b^4*c*d*p^3 + (4*b^4*c - 3*a^2*b^2)*d*p^2 + 3*(b^4*c - a^2*b^2)*d*p)*x + (4*a*b^3*c*p^2 + 2*(5*a*b^3*c - 3*a^3*b)*p - (a*b^3*d*p^3 + 3*a*b^3*d*p^2 + 2*a*b^3*d*p)*x)*sqrt(d*x + c)*(sqrt(d*x + c)*b + a)^p/(b^4*d^2*p^4 + 10*b^4*d^2*p^3 + 35*b^4*d^2*p^2 + 50*b^4*d^2*p + 24*b^4*d^2)

giac [B] time = 0.44, size = 806, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")

[Out] -2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^3 - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^3 + 8*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p^2 - 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p^2 - (sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^3 + 3*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^3 - 3*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^3 + (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p^3 + 19*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c*p - 26*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c*p - 6*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p^2 + 21*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p^2 - 24*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p^2 + 9*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a^3*p^2 + 12*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*b^2*c - 24*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*b^2*c - 11*(sqrt(d*x + c)*b + a)^4*(sqrt(d*x + c)*b + a)^p*p + 42*(sqrt(d*x + c)*b + a)^3*(sqrt(d*x + c)*b + a)^p*a*p - 57*(sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*a^2*p + 26*(

$\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3*p - 6*(\text{sqrt}(d*x + c)*b + a)^4*(\text{sqrt}(d*x + c)*b + a)^p + 24*(\text{sqrt}(d*x + c)*b + a)^3*(\text{sqrt}(d*x + c)*b + a)^p*a - 36*(\text{sqrt}(d*x + c)*b + a)^2*(\text{sqrt}(d*x + c)*b + a)^p*a^2 + 24*(\text{sqrt}(d*x + c)*b + a)*(\text{sqrt}(d*x + c)*b + a)^p*a^3)/((b^2*p^4 + 10*b^2*p^3 + 35*b^2*p^2 + 50*b^2*p + 24*b^2)*b^2*d^2)$

maple [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + \sqrt{dx + c} b \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a+(d*x+c)^(1/2)*b)^p,x)`

[Out] `int(x*(a+(d*x+c)^(1/2)*b)^p,x)`

maxima [A] time = 0.99, size = 187, normalized size = 1.29

$$\frac{2 \left(\frac{((dx+c)b^2(p+1)+\sqrt{dx+c}abp-a^2)(\sqrt{dx+c}b+a)^p c}{(p^2+3p+2)b^2} - \frac{((p^3+6p^2+11p+6)(dx+c)^2b^4+(p^3+3p^2+2p)(dx+c)^{\frac{3}{2}}ab^3-3(p^2+p)(dx+c)a^2b^2+6\sqrt{dx+c}a^3bp-6a^4)(\sqrt{dx+c}b+a)^p}{(p^4+10p^3+35p^2+50p+24)b^4} \right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")`

[Out] `-2*(((d*x + c)*b^2*(p + 1) + sqrt(d*x + c)*a*b*p - a^2)*(sqrt(d*x + c)*b + a)^p*c/((p^2 + 3*p + 2)*b^2) - ((p^3 + 6*p^2 + 11*p + 6)*(d*x + c)^2*b^4 + (p^3 + 3*p^2 + 2*p)*(d*x + c)^(3/2)*a*b^3 - 3*(p^2 + p)*(d*x + c)*a^2*b^2 + 6*sqrt(d*x + c)*a^3*b*p - 6*a^4)*(sqrt(d*x + c)*b + a)^p/((p^4 + 10*p^3 + 35*p^2 + 50*p + 24)*b^4))/d^2`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a + b*(c + d*x)^(1/2))^p,x)`

[Out] `int(x*(a + b*(c + d*x)^(1/2))^p, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \left(a + b \sqrt{c + dx} \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a+b*(d*x+c)**(1/2))**p,x)`

[Out] `Integral(x*(a + b*sqrt(c + d*x))**p, x)`

$$3.401 \quad \int (a + b\sqrt{c + dx})^p dx$$

Optimal. Leaf size=62

$$\frac{2(a + b\sqrt{c + dx})^{p+2}}{b^2d(p + 2)} - \frac{2a(a + b\sqrt{c + dx})^{p+1}}{b^2d(p + 1)}$$

Rubi [A] time = 0.04, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 190, 43}

$$\frac{2(a + b\sqrt{c + dx})^{p+2}}{b^2d(p + 2)} - \frac{2a(a + b\sqrt{c + dx})^{p+1}}{b^2d(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sqrt[c + d*x])^p, x]

[Out] (-2*a*(a + b*Sqrt[c + d*x])^(1 + p))/(b^2*d*(1 + p)) + (2*(a + b*Sqrt[c + d*x])^(2 + p))/(b^2*d*(2 + p))

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int (a + b\sqrt{c + dx})^p dx &= \frac{\text{Subst}\left(\int (a + b\sqrt{x})^p dx, x, c + dx\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int x(a + bx)^p dx, x, \sqrt{c + dx}\right)}{d} \\ &= \frac{2 \text{Subst}\left(\int \left(-\frac{a+bx}{b} + \frac{(a+bx)^{1+p}}{b}\right) dx, x, \sqrt{c + dx}\right)}{d} \\ &= -\frac{2a(a + b\sqrt{c + dx})^{1+p}}{b^2d(1 + p)} + \frac{2(a + b\sqrt{c + dx})^{2+p}}{b^2d(2 + p)} \end{aligned}$$

Mathematica [A] time = 0.04, size = 53, normalized size = 0.85

$$\frac{2(a + b\sqrt{c + dx})^{p+1} (b(p + 1)\sqrt{c + dx} - a)}{b^2d(p + 1)(p + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*Sqrt[c + d*x])^p,x]

[Out] (2*(a + b*Sqrt[c + d*x])^(1 + p)*(-a + b*(1 + p)*Sqrt[c + d*x]))/(b^2*d*(1 + p)*(2 + p))

IntegrateAlgebraic [F] time = 0.12, size = 0, normalized size = 0.00

$$\int \left(a + b\sqrt{c + dx} \right)^p dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*Sqrt[c + d*x])^p,x]

[Out] Defer[IntegrateAlgebraic] [(a + b*Sqrt[c + d*x])^p, x]

fricas [A] time = 0.99, size = 81, normalized size = 1.31

$$\frac{2 \left(b^2 c p + \sqrt{dx + c} a b p + b^2 c - a^2 + (b^2 d p + b^2 d) x \right) (\sqrt{dx + c} b + a)^p}{b^2 d p^2 + 3 b^2 d p + 2 b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="fricas")

[Out] 2*(b^2*c*p + sqrt(d*x + c)*a*b*p + b^2*c - a^2 + (b^2*d*p + b^2*d)*x)*(sqrt(d*x + c)*b + a)^p/(b^2*d*p^2 + 3*b^2*d*p + 2*b^2*d)

giac [B] time = 0.44, size = 129, normalized size = 2.08

$$\frac{2 \left((\sqrt{dx + c} b + a)^2 (\sqrt{dx + c} b + a)^p p - (\sqrt{dx + c} b + a) (\sqrt{dx + c} b + a)^p a p + (\sqrt{dx + c} b + a)^2 (\sqrt{dx + c} b + a)^p - 2 (\sqrt{dx + c} b + a) (\sqrt{dx + c} b + a)^p a \right)}{(p^2 + 3p + 2)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="giac")

[Out] 2*((sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p*p - (sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a*p + (sqrt(d*x + c)*b + a)^2*(sqrt(d*x + c)*b + a)^p - 2*(sqrt(d*x + c)*b + a)*(sqrt(d*x + c)*b + a)^p*a)/((p^2 + 3*p + 2)*b^2*d)

maple [F] time = 0.01, size = 0, normalized size = 0.00

$$\int \left(a + \sqrt{dx + c} b \right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+(d*x+c)^(1/2)*b)^p,x)

[Out] int((a+(d*x+c)^(1/2)*b)^p,x)

maxima [A] time = 0.93, size = 60, normalized size = 0.97

$$\frac{2 \left((dx + c) b^2 (p + 1) + \sqrt{dx + c} a b p - a^2 \right) (\sqrt{dx + c} b + a)^p}{(p^2 + 3p + 2)b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(d*x+c)^(1/2))^p,x, algorithm="maxima")

[Out] $2*((d*x + c)*b^2*(p + 1) + \sqrt{d*x + c}*a*b*p - a^2)*(\sqrt{d*x + c}*b + a)^p / ((p^2 + 3*p + 2)*b^2*d)$

mupad [B] time = 3.58, size = 146, normalized size = 2.35

$$\left\{ \begin{array}{ll} -\frac{2a \ln(a+b\sqrt{c+dx}) - 2b\sqrt{c+dx}}{b^2 d} & \text{if } p = -1 \\ \frac{2\left(\ln(a+b\sqrt{c+dx}) + \frac{a}{a+b\sqrt{c+dx}}\right)}{b^2 d} & \text{if } p = -2 \\ \frac{4(a+b\sqrt{c+dx})^{p+2}}{b^2 d(2p+4)} - \frac{4a(a+b\sqrt{c+dx})^{p+1}}{b^2 d(2p+2)} & \text{if } p \neq -1 \wedge p \neq -2 \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(c + d*x)^(1/2))^p, x)`

[Out] `piecewise(p == -1, -(2*a*log(a + b*(c + d*x)^(1/2)) - 2*b*(c + d*x)^(1/2))/(b^2*d), p == -2, (2*(log(a + b*(c + d*x)^(1/2)) + a/(a + b*(c + d*x)^(1/2)))/(b^2*d), p ~= -1 & p ~= -2, (4*(a + b*(c + d*x)^(1/2))^(p + 2))/(b^2*d*(2*p + 4)) - (4*a*(a + b*(c + d*x)^(1/2))^(p + 1))/(b^2*d*(2*p + 2)))`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sqrt{c + dx})^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(d*x+c)**(1/2))**p, x)`

[Out] `Integral((a + b*sqrt(c + d*x))**p, x)`

$$3.402 \quad \int \frac{(a+b(cx)^n)^{5/2}}{x} dx$$

Optimal. Leaf size=93

$$-\frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a^2 \sqrt{a+b(cx)^n}}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n}$$

Rubi [A] time = 0.08, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2a^2 \sqrt{a+b(cx)^n}}{n} - \frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a(a+b(cx)^n)^{3/2}}{3n} + \frac{2(a+b(cx)^n)^{5/2}}{5n}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c*x)^n)^(5/2)/x, x]

[Out] (2*a^2*sqrt[a + b*(c*x)^n])/n + (2*a*(a + b*(c*x)^n)^(3/2))/(3*n) + (2*(a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b(cx)^n)^{5/2}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{5/2}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{(a + bx^n)^{5/2}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{5/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2(a + b(cx)^n)^{5/2}}{5n} + \frac{a \text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a(a + b(cx)^n)^{3/2}}{3n} + \frac{2(a + b(cx)^n)^{5/2}}{5n} + \frac{a^2 \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a^2\sqrt{a + b(cx)^n}}{n} + \frac{2a(a + b(cx)^n)^{3/2}}{3n} + \frac{2(a + b(cx)^n)^{5/2}}{5n} + \frac{a^3 \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a^2\sqrt{a + b(cx)^n}}{n} + \frac{2a(a + b(cx)^n)^{3/2}}{3n} + \frac{2(a + b(cx)^n)^{5/2}}{5n} + \frac{(2a^3) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, (cx)^n\right)}{bn} \\
 &= \frac{2a^2\sqrt{a + b(cx)^n}}{n} + \frac{2a(a + b(cx)^n)^{3/2}}{3n} + \frac{2(a + b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 77, normalized size = 0.83

$$\frac{2\sqrt{a + b(cx)^n} (23a^2 + 11ab(cx)^n + 3b^2(cx)^{2n}) - 30a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c*x)^n)^(5/2)/x, x]

[Out] (2*Sqrt[a + b*(c*x)^n]*(23*a^2 + 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(15*n)

IntegrateAlgebraic [A] time = 0.32, size = 88, normalized size = 0.95

$$\frac{2(15a^2\sqrt{a + b(cx)^n} + 3(a + b(cx)^n)^{5/2} + 5a(a + b(cx)^n)^{3/2})}{15n} - \frac{2a^{5/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*(c*x)^n)^(5/2)/x, x]

[Out] (2*(15*a^2*Sqrt[a + b*(c*x)^n] + 5*a*(a + b*(c*x)^n)^(3/2) + 3*(a + b*(c*x)^n)^(5/2)))/(15*n) - (2*a^(5/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 1.02, size = 164, normalized size = 1.76

$$\left[\frac{15a^2 \log\left(\frac{(cx)^{5n} b - 2\sqrt{(cx)^{2n} b + a} \sqrt{a+2a}}{(cx)^n}\right) + 2(11(cx)^n ab + 3(cx)^{2n} b^2 + 23a^2)\sqrt{(cx)^n b + a}}{15n}, \frac{2(15\sqrt{-a} a^2 \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + (11(cx)^n ab + 3(cx)^{2n} b^2 + 23a^2)\sqrt{(cx)^n b + a})}{15n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")

[Out] [1/15*(15*a^(5/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a)/n, 2/15*(15*sqrt(-a)*a^2*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (11*(c*x)^n*a*b + 3*(c*x)^(2*n)*b^2 + 23*a^2)*sqrt((c*x)^n*b + a))/n]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")

[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)

maple [A] time = 0.01, size = 70, normalized size = 0.75

$$\frac{-2a^{\frac{5}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right) + 2\sqrt{b(cx)^n+a} a^2 + \frac{2(b(cx)^n+a)^{\frac{3}{2}} a}{3} + \frac{2(b(cx)^n+a)^{\frac{5}{2}}}{5}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*(c*x)^n)^(5/2)/x,x)

[Out] 1/n*(2/5*(a+b*(c*x)^n)^(5/2)+2/3*a*(a+b*(c*x)^n)^(3/2)+2*(a+b*(c*x)^n)^(1/2))*a^2-2*a^(5/2)*arctanh((a+b*(c*x)^n)^(1/2)/a^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b + a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")

[Out] integrate(((c*x)^n*b + a)^(5/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b(cx)^n)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c*x)^n)^(5/2)/x,x)

[Out] int((a + b*(c*x)^n)^(5/2)/x, x)

sympy [A] time = 92.66, size = 122, normalized size = 1.31

$$\begin{cases} \frac{2a^3 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right) + 2a^2 \sqrt{a+b(cx)^n} + \frac{2a(a+b(cx)^n)^{\frac{3}{2}}}{3} + \frac{2(a+b(cx)^n)^{\frac{5}{2}}}{5}}{n} & \text{for } n \neq 0 \\ -\left(-a^2 \sqrt{a+b} - 2ab \sqrt{a+b} - b^2 \sqrt{a+b}\right) \log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise(((2*a**3*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) + 2*a**2*sqrt(a + b*(c*x)**n) + 2*a*(a + b*(c*x)**n)**(3/2)/3 + 2*(a + b*(c*x)**n)**(5/2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(a + b) - 2*a*b*sqrt(a + b) - b**2*sqrt(a + b))*log(c*x), True))
```

$$3.403 \quad \int \frac{(a+b(cx)^n)^{3/2}}{x} dx$$

Optimal. Leaf size=70

$$-\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n}$$

Rubi [A] time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 50, 63, 208}

$$-\frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n} + \frac{2a\sqrt{a+b(cx)^n}}{n} + \frac{2(a+b(cx)^n)^{3/2}}{3n}$$

Antiderivative was successfully verified.

[In] Int[(a + b*(c*x)^n)^(3/2)/x, x]

[Out] (2*a*Sqrt[a + b*(c*x)^n])/n + (2*(a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[(d*x)/c]^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(a + b(cx)^n)^{3/2}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c(a+bx^n)^{3/2}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{(a + bx^n)^{3/2}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2(a + b(cx)^n)^{3/2}}{3n} + \frac{a \text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a\sqrt{a + b(cx)^n}}{n} + \frac{2(a + b(cx)^n)^{3/2}}{3n} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a\sqrt{a + b(cx)^n}}{n} + \frac{2(a + b(cx)^n)^{3/2}}{3n} + \frac{(2a^2) \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a + b(cx)^n}\right)}{bn} \\
 &= \frac{2a\sqrt{a + b(cx)^n}}{n} + \frac{2(a + b(cx)^n)^{3/2}}{3n} - \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 0.87

$$\frac{2\sqrt{a + b(cx)^n} (4a + b(cx)^n) - 6a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{3n}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*(c*x)^n)^(3/2)/x,x]

[Out] (2*Sqrt[a + b*(c*x)^n]*(4*a + b*(c*x)^n) - 6*a^(3/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(3*n)

IntegrateAlgebraic [A] time = 0.30, size = 68, normalized size = 0.97

$$\frac{2\left((a + b(cx)^n)^{3/2} + 3a\sqrt{a + b(cx)^n}\right)}{3n} - \frac{2a^{3/2} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*(c*x)^n)^(3/2)/x,x]

[Out] (2*(3*a*Sqrt[a + b*(c*x)^n] + (a + b*(c*x)^n)^(3/2))/(3*n) - (2*a^(3/2)*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 0.97, size = 130, normalized size = 1.86

$$\left[\frac{3a^{\frac{3}{2}} \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a} \sqrt{a+2a}}{(cx)^n}\right) + 2((cx)^n b + 4a)\sqrt{(cx)^n b + a}}{3n}, \frac{2\left(3\sqrt{-a} a \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + ((cx)^n b + 4a)\sqrt{(cx)^n b + a}\right)}{3n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")

[Out] [1/3*(3*a^(3/2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n, 2/3*(3*sqrt(-a)*a*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + ((c*x)^n*b + 4*a)*sqrt((c*x)^n*b + a))/n]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")

[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)

maple [A] time = 0.00, size = 54, normalized size = 0.77

$$\frac{-2a^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right) + 2\sqrt{b(cx)^n+a} a + \frac{2(b(cx)^n+a)^{\frac{3}{2}}}{3}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*(c*x)^n+a)^(3/2)/x,x)

[Out] 1/n*(2/3*(b*(c*x)^n+a)^(3/2)+2*a*(b*(c*x)^n+a)^(1/2)-2*a^(3/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b + a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")

[Out] integrate(((c*x)^n*b + a)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + b(cx)^n)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*(c*x)^n)^(3/2)/x,x)

[Out] int((a + b*(c*x)^n)^(3/2)/x, x)

sympy [A] time = 66.15, size = 102, normalized size = 1.46

$$\left\{ \begin{array}{l} \frac{\left(-a \left(-\frac{2a \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{\sqrt{-a}} - 2\sqrt{a+b(cx)^n} \right) - b \left(\begin{array}{l} -\sqrt{a} (cx)^n \text{ for } b = 0 \\ -\frac{2(a+b(cx)^n)^{\frac{3}{2}}}{3b} \text{ otherwise} \end{array} \right) \right)}{n} \text{ for } n \neq 0 \\ (a\sqrt{a+b} + b\sqrt{a+b}) \log(x) \text{ otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((a*(-2*a*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/sqrt(-a) - 2*sqrt(a + b*(c*x)**n)) - b*Piecewise((-sqrt(a)*(c*x)**n, Eq(b, 0)), (-2*(a + b*(c*x)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((a*sqrt(a + b) + b*sqrt(a + b))*log(x), True))
```

$$3.404 \quad \int \frac{\sqrt{a+b(cx)^n}}{x} dx$$

Optimal. Leaf size=49

$$\frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Rubi [A] time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b*(c*x)^n]/x, x]

[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a+b(cx)^n}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{a+bx^n}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2\sqrt{a+b(cx)^n}}{n} + \frac{(2a) \text{Subst}\left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b(cx)^n}\right)}{bn} \\
 &= \frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 0.96

$$\frac{2\sqrt{a+b(cx)^n} - 2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b*(c*x)^n]/x, x]

[Out] (2*Sqrt[a + b*(c*x)^n] - 2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

IntegrateAlgebraic [A] time = 0.28, size = 49, normalized size = 1.00

$$\frac{2\sqrt{a+b(cx)^n}}{n} - \frac{2\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + b*(c*x)^n]/x, x]

[Out] (2*Sqrt[a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 0.94, size = 103, normalized size = 2.10

$$\left[\frac{\sqrt{a} \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a} \sqrt{a+2a}}{(cx)^n}\right) + 2\sqrt{(cx)^n b + a}}{n}, \frac{2\left(\sqrt{-a} \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + \sqrt{(cx)^n b + a}\right)}{n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*(c*x)^n)^(1/2)/x, x, algorithm="fricas")

[Out] $\left[\frac{\sqrt{a} \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a} \sqrt{a} + 2a}{(cx)^n} + \frac{2\sqrt{(cx)^n b + a}}{n}\right) + 2\sqrt{-a} \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + \sqrt{(cx)^n b + a}}{n}\right]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")`

[Out] `integrate(sqrt((c*x)^n*b + a)/x, x)`

maple [A] time = 0.00, size = 40, normalized size = 0.82

$$\frac{-2\sqrt{a} \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right) + 2\sqrt{b(cx)^n+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*(c*x)^n+a)^(1/2)/x,x)`

[Out] `1/n*(2*(b*(c*x)^n+a)^(1/2)-2*a^(1/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2)))`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b + a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt((c*x)^n*b + a)/x, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a + b (cx)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*(c*x)^n)^(1/2)/x,x)`

[Out] `int((a + b*(c*x)^n)^(1/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + b (cx)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*(c*x)**n)**(1/2)/x,x)`

[Out] `Integral(sqrt(a + b*(c*x)**n)/x, x)`

$$3.405 \quad \int \frac{1}{x\sqrt{a+b(cx)^n}} dx$$

Optimal. Leaf size=30

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {367, 12, 266, 63, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*x)^n]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]/(Sqrt[a]*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(cx)^n}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}} dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}} dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{n} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b(cx)^n}\right)}{bn} \\
&= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*(c*x)^n]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]/(Sqrt[a]*n)

IntegrateAlgebraic [A] time = 0.27, size = 30, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*x)^n]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]]/(Sqrt[a]*n)

fricas [A] time = 0.72, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^{n-2} \sqrt{(cx)^n b + a} \sqrt{a+2a}}{(cx)^n}\right)}{\sqrt{a} n}, \frac{2 \sqrt{-a} \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right)}{an} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(1/2), x, algorithm="fricas")

[Out] [log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n)/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a)/(a*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b + a} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n+a)^(1/2),x)

[Out] -2*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))/n/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b + a x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x)^n*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x \sqrt{a + b (cx)^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*x)^n)^(1/2)),x)

[Out] int(1/(x*(a + b*(c*x)^n)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b (cx)^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*x)**n)), x)

$$3.406 \quad \int \frac{1}{x(a+b(cx)^n)^{3/2}} dx$$

Optimal. Leaf size=52

$$\frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c*x)^n)^(3/2)),x]

[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(3/2)*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(a+b(cx)^n)^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{3/2}} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{1}{x(a+bx^n)^{3/2}} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{an} \\
 &= \frac{2}{an\sqrt{a+b(cx)^n}} + \frac{2 \text{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b(cx)^n}\right)}{abn} \\
 &= \frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 0.79

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; \frac{b(cx)^n}{a} + 1\right)}{an\sqrt{a+b(cx)^n}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c*x)^n)^(3/2)), x]

[Out] (2*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*(c*x)^n)/a])/(a*n*Sqrt[a + b*(c*x)^n])

IntegrateAlgebraic [A] time = 0.31, size = 52, normalized size = 1.00

$$\frac{2}{an\sqrt{a+b(cx)^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*(c*x)^n)^(3/2)), x]

[Out] 2/(a*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(3/2)*n)

fricas [A] time = 0.89, size = 164, normalized size = 3.15

$$\left[\frac{\left((cx)^n \sqrt{a} b + a^{\frac{3}{2}}\right) \log\left(\frac{(cx)^{n-2} \sqrt{(cx)^n b + a} \sqrt{a+2a}}{(cx)^n}\right) + 2 \sqrt{(cx)^n b + a} a}{(cx)^n a^2 b n + a^3 n}, \frac{2 \left(\left((cx)^n \sqrt{-a} b + \sqrt{-a} a\right) \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + \sqrt{(cx)^n b + a} a\right)}{(cx)^n a^2 b n + a^3 n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(3/2), x, algorithm="fricas")

[Out] [(((c*x)^n*sqrt(a)*b + a^(3/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n), 2*((c*x)^n*sqrt(-a)*b + sqrt(-a)*a)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a + sqrt((c*x)^n*b + a)*a)/((c*x)^n*a^2*b*n + a^3*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)

maple [A] time = 0.01, size = 43, normalized size = 0.83

$$\frac{-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}} + \frac{2}{\sqrt{b(cx)^n+a} a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n+a)^(3/2),x)

[Out] 1/n*(2/a/(b*(c*x)^n+a)^(1/2)-2/a^(3/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b + a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(((c*x)^n*b + a)^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x(a + b(cx)^n)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*x)^n)^(3/2)),x)

[Out] int(1/(x*(a + b*(c*x)^n)^(3/2)), x)

sympy [A] time = 11.43, size = 48, normalized size = 0.92

$$\frac{2}{an\sqrt{a + b(cx)^n}} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{-a}}\right)}{an\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)**n)**(3/2),x)

[Out] 2/(a*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a*n*sqrt(-a))

$$3.407 \quad \int \frac{1}{x(a+b(cx)^n)^{5/2}} dx$$

Optimal. Leaf size=75

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2}{3an(a+b(cx)^n)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {367, 12, 266, 51, 63, 208}

$$\frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n} + \frac{2}{3an(a+b(cx)^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(a + b*(c*x)^n)^(5/2)),x]

[Out] 2/(3*a*n*(a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[a + b*(c*x)^n]) - (2*ArcTan h[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match Q[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(a+b(cx)^n)^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x(a+bx^n)^{5/2}} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{1}{x(a+bx^n)^{5/2}} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2}{3an(a+b(cx)^n)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}} dx, x, (cx)^n\right)}{an} \\
 &= \frac{2}{3an(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^n\right)}{a^2n} \\
 &= \frac{2}{3an(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} + \frac{2\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b(cx)^n}\right)}{a^2bn} \\
 &= \frac{2}{3an(a+b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{a+b(cx)^n}} - \frac{2\text{tanh}^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 43, normalized size = 0.57

$$\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; \frac{b(cx)^n}{a} + 1\right)}{3an(a+b(cx)^n)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(a + b*(c*x)^n)^(5/2)),x]

[Out] (2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*(c*x)^n)/a])/(3*a*n*(a + b*(c*x)^n)^(3/2))

IntegrateAlgebraic [A] time = 0.35, size = 67, normalized size = 0.89

$$\frac{2(3(a+b(cx)^n)+a)}{3a^2n(a+b(cx)^n)^{3/2}} - \frac{2\text{tanh}^{-1}\left(\frac{\sqrt{a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(a + b*(c*x)^n)^(5/2)),x]

[Out] (2*(a + 3*(a + b*(c*x)^n)))/(3*a^2*n*(a + b*(c*x)^n)^(3/2)) - (2*ArcTanh[Sqrt[a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)

fricas [B] time = 0.93, size = 262, normalized size = 3.49

$$\left[\frac{3\left(2(cx)^n a^2 b + (cx)^{2n} \sqrt{a} b^2 + a^{\frac{5}{2}}\right) \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b + a} \sqrt{a}}{(cx)^n}\right) + 2\left(3(cx)^n ab + 4a^2\right) \sqrt{(cx)^n b + a}}{3\left(2(cx)^n a^4 bn + (cx)^{2n} a^3 b^2 n + a^5 n\right)}, \frac{2\left(3\left(2(cx)^n \sqrt{-a} ab + (cx)^{2n} \sqrt{-a} b^2 + \sqrt{-a} a^2\right) \arctan\left(\frac{\sqrt{(cx)^n b + a} \sqrt{-a}}{a}\right) + \left(3(cx)^n ab + 4a^2\right) \sqrt{(cx)^n b + a}\right)}{3\left(2(cx)^n a^4 bn + (cx)^{2n} a^3 b^2 n + a^5 n\right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="fricas")

[Out] [1/3*(3*(2*(c*x)^n*a^(3/2)*b + (c*x)^(2*n)*sqrt(a)*b^2 + a^(5/2))*log(((c*x)^n*b - 2*sqrt((c*x)^n*b + a)*sqrt(a) + 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b + 4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n), 2/3*(3*(2*(c*x)^n*sqrt(-a)*a*b + (c*x)^(2*n)*sqrt(-a)*b^2 + sqrt(-a)*a^2)*arctan(sqrt((c*x)^n*b + a)*sqrt(-a)/a) + (3*(c*x)^n*a*b + 4*a^2)*sqrt((c*x)^n*b + a))/(2*(c*x)^n*a^4*b*n + (c*x)^(2*n)*a^3*b^2*n + a^5*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b + a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)

maple [A] time = 0.01, size = 59, normalized size = 0.79

$$\frac{\frac{2}{3(b(cx)^n + a)^{\frac{3}{2}} a} - \frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^n + a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} + \frac{2}{\sqrt{b(cx)^n + a} a^2}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n+a)^(5/2),x)

[Out] 1/n*(-2/a^(5/2)*arctanh((b*(c*x)^n+a)^(1/2)/a^(1/2))+2/a^2/(b*(c*x)^n+a)^(1/2)+2/3/a/(b*(c*x)^n+a)^(3/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b + a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(((c*x)^n*b + a)^(5/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x (a + b (cx)^n)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*x)^n)^(5/2)),x)

[Out] int(1/(x*(a + b*(c*x)^n)^(5/2)), x)

sympy [A] time = 16.79, size = 70, normalized size = 0.93

$$\frac{2}{3an (a + b (cx)^n)^{\frac{3}{2}}} + \frac{2}{a^2 n \sqrt{a + b (cx)^n}} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{a + b (cx)^n}}{\sqrt{-a}}\right)}{a^2 n \sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*x)**n)**(5/2),x)
```

```
[Out] 2/(3*a*n*(a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(a + b*(c*x)**n)) + 2*atan(sqrt(a + b*(c*x)**n)/sqrt(-a))/(a**2*n*sqrt(-a))
```

$$3.408 \quad \int \frac{(-a+b(cx)^n)^{5/2}}{x} dx$$

Optimal. Leaf size=101

$$-\frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n} + \frac{2a^2\sqrt{b(cx)^n-a}}{n} - \frac{2a(b(cx)^n-a)^{3/2}}{3n} + \frac{2(b(cx)^n-a)^{5/2}}{5n}$$

Rubi [A] time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^2\sqrt{b(cx)^n-a}}{n} - \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n-a}}{\sqrt{a}}\right)}{n} - \frac{2a(b(cx)^n-a)^{3/2}}{3n} + \frac{2(b(cx)^n-a)^{5/2}}{5n}$$

Antiderivative was successfully verified.

[In] Int[(-a + b*(c*x)^n)^(5/2)/x,x]

[Out] (2*a^2*sqrt[-a + b*(c*x)^n])/n - (2*a*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*(-a + b*(c*x)^n)^(5/2))/(5*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(-a + b(cx)^n)^{5/2}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{5/2}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{(-a + bx^n)^{5/2}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{5/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2(-a + b(cx)^n)^{5/2}}{5n} - \frac{a \text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= -\frac{2a(-a + b(cx)^n)^{3/2}}{3n} + \frac{2(-a + b(cx)^n)^{5/2}}{5n} + \frac{a^2 \text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a^2\sqrt{-a + b(cx)^n}}{n} - \frac{2a(-a + b(cx)^n)^{3/2}}{3n} + \frac{2(-a + b(cx)^n)^{5/2}}{5n} - \frac{a^3 \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2a^2\sqrt{-a + b(cx)^n}}{n} - \frac{2a(-a + b(cx)^n)^{3/2}}{3n} + \frac{2(-a + b(cx)^n)^{5/2}}{5n} - \frac{(2a^3) \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, (cx)^n\right)}{bn} \\
 &= \frac{2a^2\sqrt{-a + b(cx)^n}}{n} - \frac{2a(-a + b(cx)^n)^{3/2}}{3n} + \frac{2(-a + b(cx)^n)^{5/2}}{5n} - \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.08, size = 81, normalized size = 0.80

$$\frac{2\sqrt{b(cx)^n - a} (23a^2 - 11ab(cx)^n + 3b^2(cx)^{2n}) - 30a^{5/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{15n}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b*(c*x)^n)^(5/2)/x, x]

[Out] (2*Sqrt[-a + b*(c*x)^n]*(23*a^2 - 11*a*b*(c*x)^n + 3*b^2*(c*x)^(2*n)) - 30*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(15*n)

IntegrateAlgebraic [A] time = 0.33, size = 96, normalized size = 0.95

$$\frac{2(15a^2\sqrt{b(cx)^n - a} + 3(b(cx)^n - a)^{5/2} - 5a(b(cx)^n - a)^{3/2})}{15n} - \frac{2a^{5/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-a + b*(c*x)^n)^(5/2)/x, x]

[Out] (2*(15*a^2*Sqrt[-a + b*(c*x)^n] - 5*a*(-a + b*(c*x)^n)^(3/2) + 3*(-a + b*(c*x)^n)^(5/2))/(15*n) - (2*a^(5/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 0.93, size = 169, normalized size = 1.67

$$\left[\frac{15\sqrt{-a} a^2 \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b - a} \sqrt{-a} - 2a}{(cx)^n}\right) - 2(11(cx)^n ab - 3(cx)^{2n} b^2 - 23a^2)\sqrt{(cx)^n b - a}}{15n}, \frac{2(15a^{5/2} \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) + (11(cx)^n ab - 3(cx)^{2n} b^2 - 23a^2)\sqrt{(cx)^n b - a})}{15n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="fricas")

[Out] [1/15*(15*sqrt(-a)*a^2*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) - 2*(11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n, -2/15*(15*a^(5/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + (11*(c*x)^n*a*b - 3*(c*x)^(2*n)*b^2 - 23*a^2)*sqrt((c*x)^n*b - a))/n]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b - a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="giac")

[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)

maple [A] time = 0.01, size = 86, normalized size = 0.85

$$-\frac{2a^{\frac{5}{2}} \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{b(cx)^n - a} a^2}{n} - \frac{2(b(cx)^n - a)^{\frac{3}{2}} a}{3n} + \frac{2(b(cx)^n - a)^{\frac{5}{2}}}{5n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a+b*(c*x)^n)^(5/2)/x,x)

[Out] -2/3*a*(-a+b*(c*x)^n)^(3/2)/n+2/5*(-a+b*(c*x)^n)^(5/2)/n-2*a^(5/2)*arctan((-a+b*(c*x)^n)^(1/2)/a^(1/2))/n+2*a^2*(-a+b*(c*x)^n)^(1/2)/n

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b - a)^{\frac{5}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(5/2)/x,x, algorithm="maxima")

[Out] integrate(((c*x)^n*b - a)^(5/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b(cx)^n - a)^{5/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*(c*x)^n - a)^(5/2)/x,x)

[Out] int((b*(c*x)^n - a)^(5/2)/x, x)

sympy [A] time = 85.59, size = 114, normalized size = 1.13

$$\begin{cases} \frac{-2a^{\frac{5}{2}} \operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) + 2a^2\sqrt{-a+b(cx)^n} - \frac{2a(-a+b(cx)^n)^{\frac{3}{2}}}{3} + \frac{2(-a+b(cx)^n)^{\frac{5}{2}}}{5}}{n} & \text{for } n \neq 0 \\ -\left(-a^2\sqrt{-a+b} + 2ab\sqrt{-a+b} - b^2\sqrt{-a+b}\right) \log(cx) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(5/2)/x,x)
```

```
[Out] Piecewise((( -2*a**(5/2)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) + 2*a**2*sqrt(-a + b*(c*x)**n) - 2*a*(-a + b*(c*x)**n)**(3/2)/3 + 2*(-a + b*(c*x)**n)**(5/2)/5)/n, Ne(n, 0)), (-(-a**2*sqrt(-a + b) + 2*a*b*sqrt(-a + b) - b**2*sqrt(-a + b))*log(c*x), True))
```


$$3.409 \quad \int \frac{(-a+b(cx)^n)^{3/2}}{x} dx$$

Optimal. Leaf size=76

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} - \frac{2a\sqrt{b(cx)^n - a}}{n} + \frac{2(b(cx)^n - a)^{3/2}}{3n}$$

Rubi [A] time = 0.06, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} - \frac{2a\sqrt{b(cx)^n - a}}{n} + \frac{2(b(cx)^n - a)^{3/2}}{3n}$$

Antiderivative was successfully verified.

[In] Int[(-a + b*(c*x)^n)^(3/2)/x,x]

[Out] (-2*a*Sqrt[-a + b*(c*x)^n])/n + (2*(-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(-a + b(cx)^n)^{3/2}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c(-a+bx^n)^{3/2}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{(-a + bx^n)^{3/2}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{(-a+bx)^{3/2}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2(-a + b(cx)^n)^{3/2}}{3n} - \frac{a \text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= -\frac{2a\sqrt{-a + b(cx)^n}}{n} + \frac{2(-a + b(cx)^n)^{3/2}}{3n} + \frac{a^2 \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= -\frac{2a\sqrt{-a + b(cx)^n}}{n} + \frac{2(-a + b(cx)^n)^{3/2}}{3n} + \frac{(2a^2) \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + b(cx)^n}\right)}{bn} \\
 &= -\frac{2a\sqrt{-a + b(cx)^n}}{n} + \frac{2(-a + b(cx)^n)^{3/2}}{3n} + \frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.87

$$\frac{6a^{3/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right) - 2(4a - b(cx)^n) \sqrt{b(cx)^n - a}}{3n}$$

Antiderivative was successfully verified.

[In] Integrate[(-a + b*(c*x)^n)^(3/2)/x, x]

[Out] (-2*(4*a - b*(c*x)^n)*Sqrt[-a + b*(c*x)^n] + 6*a^(3/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]/(3*n)

IntegrateAlgebraic [A] time = 0.29, size = 76, normalized size = 1.00

$$\frac{2a^{3/2} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} - \frac{2(3a\sqrt{b(cx)^n - a} - (b(cx)^n - a)^{3/2})}{3n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-a + b*(c*x)^n)^(3/2)/x, x]

[Out] (-2*(3*a*Sqrt[-a + b*(c*x)^n] - (-a + b*(c*x)^n)^(3/2))/(3*n) + (2*a^(3/2)*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 0.93, size = 135, normalized size = 1.78

$$\left[\frac{3\sqrt{-a} a \log\left(\frac{(cx)^n b + 2\sqrt{(cx)^n b - a} \sqrt{-a - 2a}}{(cx)^n}\right) + 2\sqrt{(cx)^n b - a} ((cx)^n b - 4a)}{3n}, \frac{2\left(3a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) + \sqrt{(cx)^n b - a} ((cx)^n b - 4a)\right)}{3n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="fricas")

[Out] [1/3*(3*sqrt(-a)*a*log(((c*x)^n*b + 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/((c*x)^n) + 2*sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n, 2/3*(3*a^(3/2)*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) + sqrt((c*x)^n*b - a)*((c*x)^n*b - 4*a))/n]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b - a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="giac")

[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)

maple [A] time = 0.01, size = 65, normalized size = 0.86

$$\frac{2a^{\frac{3}{2}} \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} - \frac{2\sqrt{b(cx)^n - a} a}{n} + \frac{2(b(cx)^n - a)^{\frac{3}{2}}}{3n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*(c*x)^n-a)^(3/2)/x,x)

[Out] 2/3*(b*(c*x)^n-a)^(3/2)/n+2*a^(3/2)*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n-2*a*(b*(c*x)^n-a)^(1/2)/n

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{((cx)^n b - a)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(3/2)/x,x, algorithm="maxima")

[Out] integrate(((c*x)^n*b - a)^(3/2)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(b(cx)^n - a)^{3/2}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*(c*x)^n - a)^(3/2)/x,x)

[Out] int((b*(c*x)^n - a)^(3/2)/x, x)

sympy [A] time = 70.06, size = 95, normalized size = 1.25

$$\left\{ \begin{array}{l} \frac{a \left(2\sqrt{a} \operatorname{atan}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right) - 2\sqrt{-a+b(cx)^n} \right) - b \left\{ \begin{array}{ll} -\sqrt{-a} (cx)^n & \text{for } b = 0 \\ -\frac{2(-a+b(cx)^n)^{\frac{3}{2}}}{3b} & \text{otherwise} \end{array} \right.}{n} & \text{for } n \neq 0 \\ (-a\sqrt{-a+b} + b\sqrt{-a+b}) \log(x) & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a+b*(c*x)**n)**(3/2)/x,x)
```

```
[Out] Piecewise(((a*(2*sqrt(a)*atan(sqrt(-a + b*(c*x)**n)/sqrt(a)) - 2*sqrt(-a +
b*(c*x)**n)) - b*Piecewise((-sqrt(-a)*(c*x)**n, Eq(b, 0)), (-2*(-a + b*(c*x)
)**n)**(3/2)/(3*b), True)))/n, Ne(n, 0)), ((-a*sqrt(-a + b) + b*sqrt(-a + b
))*log(x), True))
```

$$3.410 \quad \int \frac{\sqrt{-a+b(cx)^n}}{x} dx$$

Optimal. Leaf size=53

$$\frac{2\sqrt{b(cx)^n - a}}{n} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n}$$

Rubi [A] time = 0.04, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {367, 12, 266, 50, 63, 205}

$$\frac{2\sqrt{b(cx)^n - a}}{n} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-a + b*(c*x)^n]/x,x]

[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

Rule 12

Int[(a_)*(u_), x_Symbol] :=> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :=> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :=> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] :=> Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-a + b(cx)^n}}{x} dx &= \frac{\text{Subst}\left(\int \frac{c\sqrt{-a+bx^n}}{x} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{\sqrt{-a + bx^n}}{x} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{\sqrt{-a+bx}}{x} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2\sqrt{-a + b(cx)^n}}{n} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{n} \\
 &= \frac{2\sqrt{-a + b(cx)^n}}{n} - \frac{(2a) \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + b(cx)^n}\right)}{bn} \\
 &= \frac{2\sqrt{-a + b(cx)^n}}{n} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{n}
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 51, normalized size = 0.96

$$\frac{2\sqrt{b(cx)^n - a} - 2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-a + b*(c*x)^n]/x,x]

[Out] (2*Sqrt[-a + b*(c*x)^n] - 2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

IntegrateAlgebraic [A] time = 0.28, size = 53, normalized size = 1.00

$$\frac{2\sqrt{b(cx)^n - a}}{n} - \frac{2\sqrt{a} \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-a + b*(c*x)^n]/x,x]

[Out] (2*Sqrt[-a + b*(c*x)^n])/n - (2*Sqrt[a]*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/n

fricas [A] time = 0.97, size = 110, normalized size = 2.08

$$\left[\frac{\sqrt{-a} \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b - a} \sqrt{-a - 2a}}{(cx)^n}\right) + 2\sqrt{(cx)^n b - a}}{n}, -\frac{2\left(\sqrt{a} \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) - \sqrt{(cx)^n b - a}\right)}{n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="fricas")

[Out] $[(\sqrt{-a}) \cdot \log((c \cdot x)^n \cdot b - 2 \cdot \sqrt{(c \cdot x)^n \cdot b - a}) \cdot \sqrt{-a} - 2 \cdot a) / (c \cdot x)^n + 2 \cdot \sqrt{(c \cdot x)^n \cdot b - a}) / n, -2 \cdot (\sqrt{a}) \cdot \arctan(\sqrt{(c \cdot x)^n \cdot b - a} / \sqrt{a}) - \sqrt{(c \cdot x)^n \cdot b - a}) / n]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b - a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="giac")`

[Out] `integrate(sqrt((c*x)^n*b - a)/x, x)`

maple [A] time = 0.00, size = 46, normalized size = 0.87

$$-\frac{2\sqrt{a} \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{b(cx)^n - a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*(c*x)^n-a)^(1/2)/x,x)`

[Out] `-2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))*a^(1/2)/n+2*(b*(c*x)^n-a)^(1/2)/n`

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(cx)^n b - a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+b*(c*x)^n)^(1/2)/x,x, algorithm="maxima")`

[Out] `integrate(sqrt((c*x)^n*b - a)/x, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{b(cx)^n - a}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*(c*x)^n - a)^(1/2)/x,x)`

[Out] `int((b*(c*x)^n - a)^(1/2)/x, x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-a + b(cx)^n}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+b*(c*x)**n)**(1/2)/x,x)`

[Out] `Integral(sqrt(-a + b*(c*x)**n)/x, x)`

$$3.411 \quad \int \frac{1}{x\sqrt{-a+b(cx)^n}} dx$$

Optimal. Leaf size=32

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}} \right)}{\sqrt{a} n}$$

Rubi [A] time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {367, 12, 266, 63, 205}

$$\frac{2 \tan^{-1} \left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}} \right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[-a + b*(c*x)^n]),x]

[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]]/(Sqrt[a]*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[(d*x)/c]^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{-a+b(cx)^n}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{-a+bx^n}} dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx^n}} dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{n} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a+b(cx)^n}\right)}{bn} \\
&= \frac{2 \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{\sqrt{a} n}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 32, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[-a + b*(c*x)^n]), x]

[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)

IntegrateAlgebraic [A] time = 0.27, size = 32, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[-a + b*(c*x)^n]), x]

[Out] (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(Sqrt[a]*n)

fricas [A] time = 0.65, size = 80, normalized size = 2.50

$$\left[\frac{\sqrt{-a} \log\left(\frac{(cx)^n b - 2\sqrt{(cx)^n b - a} \sqrt{-a} - 2a}{(cx)^n}\right)}{an}, \frac{2 \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right)}{\sqrt{a} n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2), x, algorithm="fricas")

[Out] [-sqrt(-a)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n)/(a*n), 2*arctan(sqrt((c*x)^n*b - a)/sqrt(a))/(sqrt(a)*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b - a} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)

maple [A] time = 0.01, size = 27, normalized size = 0.84

$$\frac{2 \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{\sqrt{a} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n-a)^(1/2),x)

[Out] 2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/n/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^n b - a} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x)^n*b - a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x \sqrt{b(cx)^n - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*(c*x)^n - a)^(1/2)),x)

[Out] int(1/(x*(b*(c*x)^n - a)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{-a + b(cx)^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(-a + b*(c*x)**n)), x)

$$3.412 \quad \int \frac{1}{x(-a+b(cx)^n)^{3/2}} dx$$

Optimal. Leaf size=56

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{3/2}n} - \frac{2}{an\sqrt{b(cx)^n - a}}$$

Rubi [A] time = 0.04, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {367, 12, 266, 51, 63, 205}

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{3/2}n} - \frac{2}{an\sqrt{b(cx)^n - a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-a + b*(c*x)^n)^(3/2)), x]

[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(3/2)*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(-a + b(cx)^n)^{3/2}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{3/2}} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{1}{x(-a + bx^n)^{3/2}} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}} dx, x, (cx)^n\right)}{n} \\
 &= -\frac{2}{an\sqrt{-a + b(cx)^n}} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{an} \\
 &= -\frac{2}{an\sqrt{-a + b(cx)^n}} - \frac{2 \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + b(cx)^n}\right)}{abn} \\
 &= -\frac{2}{an\sqrt{-a + b(cx)^n}} - \frac{2 \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{3/2}n}
 \end{aligned}$$

Mathematica [C] time = 0.03, size = 44, normalized size = 0.79

$$-\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1 - \frac{b(cx)^n}{a}\right)}{an\sqrt{b(cx)^n - a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(-a + b*(c*x)^n)^(3/2)), x]

[Out] (-2*Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*(c*x)^n)/a])/(a*n*Sqrt[-a + b*(c*x)^n])

IntegrateAlgebraic [A] time = 0.28, size = 56, normalized size = 1.00

$$-\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{3/2}n} - \frac{2}{an\sqrt{b(cx)^n - a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(-a + b*(c*x)^n)^(3/2)), x]

[Out] -2/(a*n*Sqrt[-a + b*(c*x)^n]) - (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(3/2)*n)

fricas [A] time = 0.84, size = 175, normalized size = 3.12

$$\left[\frac{\left((cx)^n \sqrt{-a} b - \sqrt{-a} a\right) \log\left(\frac{(cx)^{n+2} \sqrt{(cx)^n b - a} \sqrt{-a - 2a}}{(cx)^n}\right) + 2 \sqrt{(cx)^n b - a} a}{(cx)^n a^2 b n - a^3 n}, -\frac{2 \left(\left((cx)^n \sqrt{a} b - a^{\frac{3}{2}}\right) \arctan\left(\frac{\sqrt{(cx)^n b - a}}{\sqrt{a}}\right) + \sqrt{(cx)^n b - a} a\right)}{(cx)^n a^2 b n - a^3 n} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2), x, algorithm="fricas")

[Out] $[-(((c*x)^n*\sqrt{-a}*b - \sqrt{-a})*a)*\log(((c*x)^n*b + 2*\sqrt{(c*x)^n*b - a})*\sqrt{-a} - 2*a)/(c*x)^n + 2*\sqrt{(c*x)^n*b - a})*a)/((c*x)^n*a^{2*b*n} - a^{3*n}), -2*(((c*x)^n*\sqrt{a}*b - a^{(3/2)})*\arctan(\sqrt{(c*x)^n*b - a}/\sqrt{a}) + \sqrt{(c*x)^n*b - a})*a)/((c*x)^n*a^{2*b*n} - a^{3*n})]$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b - a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="giac")

[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)

maple [A] time = 0.01, size = 49, normalized size = 0.88

$$-\frac{2 \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} n} - \frac{2}{\sqrt{b(cx)^n - a} an}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n-a)^(3/2),x)

[Out] $-2*\arctan((b*(c*x)^n-a)^{(1/2)}/a^{(1/2)})/a^{(3/2)}/n-2/a/n/(b*(c*x)^n-a)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b - a)^{\frac{3}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(3/2),x, algorithm="maxima")

[Out] integrate(1/(((c*x)^n*b - a)^(3/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x (b (cx)^n - a)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*(c*x)^n - a)^(3/2)),x)

[Out] int(1/(x*(b*(c*x)^n - a)^(3/2)), x)

sympy [A] time = 15.94, size = 44, normalized size = 0.79

$$-\frac{2}{an\sqrt{-a + b(cx)^n}} - \frac{2 \operatorname{atan}\left(\frac{\sqrt{-a + b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{3}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)**n)**(3/2),x)

[Out] $-2/(a*n*\sqrt{-a + b*(c*x)**n}) - 2*\operatorname{atan}(\sqrt{-a + b*(c*x)**n}/\sqrt{a})/(a^{(3/2)*n})$

$$3.413 \quad \int \frac{1}{x(-a+b(cx)^n)^{5/2}} dx$$

Optimal. Leaf size=81

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{5/2}n} + \frac{2}{a^2n\sqrt{b(cx)^n - a}} - \frac{2}{3an(b(cx)^n - a)^{3/2}}$$

Rubi [A] time = 0.06, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {367, 12, 266, 51, 63, 205}

$$\frac{2}{a^2n\sqrt{b(cx)^n - a}} + \frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{5/2}n} - \frac{2}{3an(b(cx)^n - a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(-a + b*(c*x)^n)^(5/2)),x]

[Out] -2/(3*a*n*(-a + b*(c*x)^n)^(3/2)) + 2/(a^2*n*Sqrt[-a + b*(c*x)^n]) + (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a,

b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{x(-a + b(cx)^n)^{5/2}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x(-a+bx^n)^{5/2}} dx, x, cx\right)}{c} \\
 &= \text{Subst}\left(\int \frac{1}{x(-a + bx^n)^{5/2}} dx, x, cx\right) \\
 &= \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{5/2}} dx, x, (cx)^n\right)}{n} \\
 &= -\frac{2}{3an(-a + b(cx)^n)^{3/2}} - \frac{\text{Subst}\left(\int \frac{1}{x(-a+bx)^{3/2}} dx, x, (cx)^n\right)}{an} \\
 &= -\frac{2}{3an(-a + b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a + b(cx)^n}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{-a+bx}} dx, x, (cx)^n\right)}{a^2n} \\
 &= -\frac{2}{3an(-a + b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a + b(cx)^n}} + \frac{2 \text{Subst}\left(\int \frac{1}{\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{-a + b(cx)^n}\right)}{a^2bn} \\
 &= -\frac{2}{3an(-a + b(cx)^n)^{3/2}} + \frac{2}{a^2n\sqrt{-a + b(cx)^n}} + \frac{2 \tan^{-1}\left(\frac{\sqrt{-a+b(cx)^n}}{\sqrt{a}}\right)}{a^{5/2}n}
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 46, normalized size = 0.57

$$-\frac{{}_2F_1\left(-\frac{3}{2}, 1; -\frac{1}{2}; 1 - \frac{b(cx)^n}{a}\right)}{3an(b(cx)^n - a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(-a + b*(c*x)^n)^(5/2)), x]

[Out] (-2*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*(c*x)^n)/a])/(3*a*n*(-a + b*(c*x)^n)^(3/2))

IntegrateAlgebraic [A] time = 0.34, size = 73, normalized size = 0.90

$$\frac{2 \tan^{-1}\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{5/2}n} - \frac{2(a - 3(b(cx)^n - a))}{3a^2n(b(cx)^n - a)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(-a + b*(c*x)^n)^(5/2)), x]

[Out] (-2*(a - 3*(-a + b*(c*x)^n)))/(3*a^2*n*(-a + b*(c*x)^n)^(3/2)) + (2*ArcTan[Sqrt[-a + b*(c*x)^n]/Sqrt[a]])/(a^(5/2)*n)

fricas [A] time = 0.96, size = 277, normalized size = 3.42

$$\left[\frac{3(2(cx)^n\sqrt{-a}ab - (cx)^{2n}\sqrt{-a}b^2 - \sqrt{-a}a^2)\log\left(\frac{(cx)^{2n}b - 2\sqrt{(cx)^{2n}b - a}\sqrt{-a} - 2a}{(cx)^n}\right) + 2(3(cx)^nab - 4a^2)\sqrt{(cx)^nb - a}}{3(2(cx)^na^4bn - (cx)^{2n}a^3b^2n - a^5n)}, \frac{2\left(3\left(2(cx)^na^2b - (cx)^{2n}\sqrt{ab^2 - a^2}\right)\arctan\left(\frac{\sqrt{(cx)^nb - a}}{\sqrt{a}}\right) - (3(cx)^nab - 4a^2)\sqrt{(cx)^nb - a}\right)}{3(2(cx)^na^4bn - (cx)^{2n}a^3b^2n - a^5n)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="fricas")

[Out] [-1/3*(3*(2*(c*x)^n*sqrt(-a)*a*b - (c*x)^(2*n)*sqrt(-a)*b^2 - sqrt(-a)*a^2)*log(((c*x)^n*b - 2*sqrt((c*x)^n*b - a)*sqrt(-a) - 2*a)/(c*x)^n) + 2*(3*(c*x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n), 2/3*(3*(2*(c*x)^n*a^(3/2)*b - (c*x)^(2*n)*sqrt(a)*b^2 - a^(5/2))*arctan(sqrt((c*x)^n*b - a)/sqrt(a)) - (3*(c*x)^n*a*b - 4*a^2)*sqrt((c*x)^n*b - a))/(2*(c*x)^n*a^4*b*n - (c*x)^(2*n)*a^3*b^2*n - a^5*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b - a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="giac")

[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)

maple [A] time = 0.01, size = 70, normalized size = 0.86

$$-\frac{2}{3(b(cx)^n - a)^{\frac{3}{2}} an} + \frac{2 \arctan\left(\frac{\sqrt{b(cx)^n - a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} n} + \frac{2}{\sqrt{b(cx)^n - a} a^2 n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*(c*x)^n-a)^(5/2),x)

[Out] -2/3/a/n/(b*(c*x)^n-a)^(3/2)+2*arctan((b*(c*x)^n-a)^(1/2)/a^(1/2))/a^(5/2)/n+2/a^2/n/(b*(c*x)^n-a)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((cx)^n b - a)^{\frac{5}{2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-a+b*(c*x)^n)^(5/2),x, algorithm="maxima")

[Out] integrate(1/(((c*x)^n*b - a)^(5/2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(b(cx)^n - a)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*(c*x)^n - a)^(5/2)),x)

[Out] int(1/(x*(b*(c*x)^n - a)^(5/2)), x)

sympy [A] time = 15.63, size = 63, normalized size = 0.78

$$-\frac{2}{3an(-a + b(cx)^n)^{\frac{3}{2}}} + \frac{2}{a^2 n \sqrt{-a + b(cx)^n}} + \frac{2 \operatorname{atan}\left(\frac{\sqrt{-a + b(cx)^n}}{\sqrt{a}}\right)}{a^{\frac{5}{2}} n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(-a+b*(c*x)**n)**(5/2),x)
```

```
[Out] -2/(3*a*n*(-a + b*(c*x)**n)**(3/2)) + 2/(a**2*n*sqrt(-a + b*(c*x)**n)) + 2*  
atan(sqrt(-a + b*(c*x)**n)/sqrt(a))/(a**(5/2)*n)
```

$$3.414 \quad \int \frac{1}{x\sqrt{a+bx}} dx$$

Optimal. Leaf size=23

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {63, 208}

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rubi steps

$$\begin{aligned} \int \frac{1}{x\sqrt{a+bx}} dx &= \frac{2 \operatorname{Subst}\left(\int \frac{1}{-\frac{a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+bx}\right)}{b} \\ &= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]

IntegrateAlgebraic [A] time = 0.00, size = 23, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*x]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/Sqrt[a]

fricas [A] time = 0.87, size = 56, normalized size = 2.43

$$\left[\frac{\log\left(\frac{bx-2\sqrt{bx+a}\sqrt{a+2a}}{x}\right)}{\sqrt{a}}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{bx+a}\sqrt{-a}}{a}\right)}{a} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="fricas")

[Out] [log((b*x - 2*sqrt(b*x + a)*sqrt(a) + 2*a)/x)/sqrt(a), 2*sqrt(-a)*arctan(sqrt(b*x + a)*sqrt(-a)/a)/a]

giac [A] time = 0.41, size = 21, normalized size = 0.91

$$\frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="giac")

[Out] 2*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a)

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(b*x+a)^(1/2),x)

[Out] -2*arctanh((b*x+a)^(1/2)/a^(1/2))/a^(1/2)

maxima [A] time = 1.97, size = 32, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{bx+a}-\sqrt{a}}{\sqrt{bx+a}+\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(b*x+a)^(1/2),x, algorithm="maxima")

[Out] log((sqrt(b*x + a) - sqrt(a))/(sqrt(b*x + a) + sqrt(a)))/sqrt(a)

mupad [B] time = 3.11, size = 17, normalized size = 0.74

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x)^(1/2)),x)`

[Out] `-(2*atanh((a + b*x)^(1/2)/a^(1/2)))/a^(1/2)`

sympy [A] time = 1.01, size = 24, normalized size = 1.04

$$\frac{2 \operatorname{asinh}\left(\frac{\sqrt{a}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(b*x+a)**(1/2),x)`

[Out] `-2*asinh(sqrt(a)/(sqrt(b)*sqrt(x)))/sqrt(a)`

$$3.415 \quad \int \frac{1}{x\sqrt{a+bx^m}} dx$$

Optimal. Leaf size=30

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+bx^m}}{\sqrt{a}} \right)}{\sqrt{a} m}$$

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {367, 12, 266, 63, 208}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+bx^m}}{\sqrt{a}} \right)}{\sqrt{a} m}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*x)^m]),x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]]/(Sqrt[a]*m)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(cx)^m}} dx &= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^m}} dx, x, cx\right)}{c} \\
&= \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^m}} dx, x, cx\right) \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (cx)^m\right)}{m} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b(cx)^m}\right)}{bm} \\
&= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a} m}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 30, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a} m}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*(c*x)^m]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)

IntegrateAlgebraic [A] time = 0.27, size = 30, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(cx)^m}}{\sqrt{a}}\right)}{\sqrt{a} m}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*x)^m]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*x)^m]/Sqrt[a]])/(Sqrt[a]*m)

fricas [A] time = 0.99, size = 78, normalized size = 2.60

$$\left[\frac{\log\left(\frac{(cx)^m b - 2\sqrt{(cx)^m b + a}\sqrt{a} + 2a}{(cx)^m}\right)}{\sqrt{a} m}, \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{(cx)^m b + a}\sqrt{-a}}{a}\right)}{am} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^m)^(1/2), x, algorithm="fricas")

[Out] [log(((c*x)^m*b - 2*sqrt((c*x)^m*b + a)*sqrt(a) + 2*a)/(c*x)^m)/(sqrt(a)*m), 2*sqrt(-a)*arctan(sqrt((c*x)^m*b + a)*sqrt(-a)/a)/(a*m)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b + a} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)

maple [A] time = 0.01, size = 25, normalized size = 0.83

$$-\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(cx)^m+a}}{\sqrt{a}}\right)}{\sqrt{a} m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(c*x)^m)^(1/2),x)

[Out] -2*arctanh((a+b*(c*x)^m)^(1/2)/a^(1/2))/m/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(cx)^m b + a} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)^m)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt((c*x)^m*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x \sqrt{a + b (cx)^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*x)^m)^(1/2)),x)

[Out] int(1/(x*(a + b*(c*x)^m)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b (cx)^m}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*x)**m)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*x)**m)), x)

$$3.416 \quad \int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}} dx$$

Optimal. Leaf size=37

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a} mn}$$

Rubi [A] time = 0.17, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {367, 12, 266, 63, 208}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a} mn}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]]/(Sqrt[a]*m*n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 367

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(c(dx)^m)^n}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^n}} dx, x, (dx)^m\right)}{m} \\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^n}} dx, x, c(dx)^m\right)}{cm} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}} dx, x, c(dx)^m\right)}{m} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (c(dx)^m)^n\right)}{mn} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{-\frac{a}{b} + x^2}{b}} dx, x, \sqrt{a+b(c(dx)^m)^n}\right)}{bmn} \\
&= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a} mn}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 37, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a} mn}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]]/(Sqrt[a]*m*n)

IntegrateAlgebraic [A] time = 0.41, size = 37, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(dx)^m)^n}}{\sqrt{a}}\right)}{\sqrt{a} mn}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*x)^m)^n]), x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*x)^m)^n]/Sqrt[a]]/(Sqrt[a]*m*n)

fricas [A] time = 0.84, size = 116, normalized size = 3.14

$$\left[\frac{\log\left(\left(b e^{(m \log(dx) + n \log(c))} - 2 \sqrt{b e^{(m \log(dx) + n \log(c))} + a} \sqrt{a} + 2a\right) e^{(-m \log(dx) - n \log(c))}\right)}{\sqrt{a} mn}, \frac{2 \sqrt{-a} \arctan\left(\frac{\sqrt{b e^{(m \log(dx) + n \log(c))} + a} \sqrt{-a}}{a}\right)}{amn} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2), x, algorithm="fricas")

[Out] [log((b*e^(m*n*log(d*x)) + n*log(c)) - 2*sqrt(b*e^(m*n*log(d*x)) + n*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*log(d*x) - n*log(c))/(sqrt(a)*m*n), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*log(d*x)) + n*log(c)) + a)*sqrt(-a)/a)/(a*m*n)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{((dx)^m c)^n b + a x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

maple [A] time = 0.01, size = 32, normalized size = 0.86

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b(c(dx)^m)^n + a}}{\sqrt{a}}\right)}{\sqrt{a} mn}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x)

[Out] -2*arctanh((a+b*(c*(d*x)^m)^n)^(1/2)/a^(1/2))/m/n/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{((dx)^m c)^n b + a x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)^m)^n)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(((d*x)^m*c)^n*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{1}{x \sqrt{a + b (c (dx)^m)^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)),x)

[Out] int(1/(x*(a + b*(c*(d*x)^m)^n)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b (c (dx)^m)^n}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*x)**m)**n)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*x)**m)**n)), x)

$$3.417 \quad \int \frac{1}{x \sqrt{a+b(c(d(ex)^m)^n)^p}} dx$$

Optimal. Leaf size=44

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}} \right)}{\sqrt{a} m n p}$$

Rubi [A] time = 0.37, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {367, 12, 266, 63, 208}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}} \right)}{\sqrt{a} m n p}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n]^p)],x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n]^p]/Sqrt[a])/(Sqrt[a]*m*n*p)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[((d*x)/c)^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\sqrt{a+b(c(d(ex)^m)^n)^p}} dx &= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(c(dx)^n)^p}} dx, x, (ex)^m\right)}{m} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+b(cx)^p}} dx, x, (d(ex)^m)^n\right)}{mn} \\
&= \frac{\text{Subst}\left(\int \frac{c}{x\sqrt{a+bx^p}} dx, x, c(d(ex)^m)^n\right)}{cmn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^p}} dx, x, c(d(ex)^m)^n\right)}{mn} \\
&= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}} dx, x, (c(d(ex)^m)^n)^p\right)}{mnp} \\
&= \frac{2 \text{Subst}\left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b(c(d(ex)^m)^n)^p}\right)}{bmnp} \\
&= -\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a} mnp}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 44, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a} mnp}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n]^p)], x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n]^p]/Sqrt[a])/(Sqrt[a]*m*n*p)

IntegrateAlgebraic [A] time = 0.67, size = 44, normalized size = 1.00

$$-\frac{2 \tanh^{-1}\left(\frac{\sqrt{a+b(c(d(ex)^m)^n)^p}}{\sqrt{a}}\right)}{\sqrt{a} mnp}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*(e*x)^m)^n]^p)], x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*x)^m)^n]^p]/Sqrt[a])/(Sqrt[a]*m*n*p)

fricas [A] time = 0.76, size = 147, normalized size = 3.34

$$\left[\frac{\log\left(\frac{be^{(mnp \log(ex)+np \log(d)+p \log(c))} - 2\sqrt{be^{(mnp \log(ex)+np \log(d)+p \log(c))} + a\sqrt{a} + 2a}}{a}\right) e^{(-mnp \log(ex)-np \log(d)-p \log(c))}}{\sqrt{a} mnp}, \frac{2\sqrt{-a} \arctan\left(\frac{\sqrt{be^{(mnp \log(ex)+np \log(d)+p \log(c))} + a\sqrt{-a}}}{a}\right)}{amnp} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="fricas")

[Out] [log((b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) - 2*sqrt(b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*log(e*x) - n*p*log(d) - p*log(c)))/(sqrt(a)*m*n*p), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*log(e*x) + n*p*log(d) + p*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(((e*x)^m*d)^n*c)^p*b + a)*x), x)

maple [A] time = 0.02, size = 39, normalized size = 0.89

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b\left(c\left(d\left(e x\right)^m\right)^n\right)^p+a}}{\sqrt{a}}\right)}{\sqrt{a} m n p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x)

[Out] -2*arctanh((a+b*(c*(d*(e*x)^m)^n)^p)^(1/2)/a^(1/2))/m/n/p/a^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left((ex)^m d\right)^n c\right)^p b + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*(e*x)^m)^n)^p)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(((e*x)^m*d)^n*c)^p*b + a)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x \sqrt{a + b \left(c \left(d \left(e x \right)^m \right)^n \right)^p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)),x)

[Out] int(1/(x*(a + b*(c*(d*(e*x)^m)^n)^p)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b \left(c \left(d \left(e x \right)^m \right)^n \right)^p}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(a+b*(c*(d*(e*x)**m)**n)**p)**(1/2),x)

[Out] Integral(1/(x*sqrt(a + b*(c*(d*(e*x)**m)**n)**p)), x)

$$3.418 \quad \int \frac{1}{x \sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}} dx$$

Optimal. Leaf size=51

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}}{\sqrt{a}} \right)}{\sqrt{a} m n p q}$$

Rubi [A] time = 0.66, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {367, 12, 266, 63, 208}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}}{\sqrt{a}} \right)}{\sqrt{a} m n p q}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q)],x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q]/Sqrt[a])/(Sqrt[a]*m*n*p*q)

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 367

Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*((c_.)*(x_))^(n_))^(p_.), x_Symbol] := Dist[1/c, Subst[Int[(d*x)/c]^m*(a + b*x^n)^p, x], x, c*x], x] /; FreeQ[{a, b, c, d, m, n, p}, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x \sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}} dx &= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+b(c(dx)^p)^q}} dx, x, (fx)^m \right)}{m} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+b(c(dx)^p)^q}} dx, x, (e(fx)^m)^n \right)}{mn} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+b(cx)^q}} dx, x, \left(d \left(e(fx)^m \right)^n \right)^p \right)}{mnp} \\
&= \frac{\text{Subst} \left(\int \frac{c}{x \sqrt{a+bx^q}} dx, x, c \left(d \left(e(fx)^m \right)^n \right)^p \right)}{cmnp} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+bx^q}} dx, x, c \left(d \left(e(fx)^m \right)^n \right)^p \right)}{mnp} \\
&= \frac{\text{Subst} \left(\int \frac{1}{x \sqrt{a+bx}} dx, x, \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q \right)}{mnpq} \\
&= \frac{2 \text{Subst} \left(\int \frac{1}{\frac{-a}{b} + \frac{x^2}{b}} dx, x, \sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q} \right)}{bmnpq} \\
&= \frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}}{\sqrt{a}} \right)}{\sqrt{a} mnpq}
\end{aligned}$$

Mathematica [A] time = 0.26, size = 51, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}}{\sqrt{a}} \right)}{\sqrt{a} mnpq}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q)],x]

[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q]/Sqrt[a])/(Sqrt[a]*m*n*p*q)

IntegrateAlgebraic [A] time = 0.90, size = 51, normalized size = 1.00

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{a+b \left(c \left(d \left(e(fx)^m \right)^n \right)^p \right)^q}}{\sqrt{a}} \right)}{\sqrt{a} mnpq}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[1/(x*Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q),x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*(c*(d*(e*(f*x)^m)^n)^p]^q/Sqrt[a]])/(Sqrt[a]*m*n*p*q)
```

fricas [A] time = 0.72, size = 182, normalized size = 3.57

$$\frac{\log\left(\frac{b e^{(mnpq \log(fx) + npq \log(e) + pq \log(d) + q \log(c))} - 2 \sqrt{b e^{(mnpq \log(fx) + npq \log(e) + pq \log(d) + q \log(c))} + a} \sqrt{a} + 2a}{\sqrt{a} mnpq}\right)}{2 \sqrt{-a} \arctan\left(\frac{\sqrt{b e^{(mnpq \log(fx) + npq \log(e) + pq \log(d) + q \log(c))} + a} \sqrt{-a}}{a mnpq}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) - 2*sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(a) + 2*a)*e^(-m*n*p*q*log(f*x) - n*p*q*log(e) - p*q*log(d) - q*log(c)))/(sqrt(a)*m*n*p*q), 2*sqrt(-a)*arctan(sqrt(b*e^(m*n*p*q*log(f*x) + n*p*q*log(e) + p*q*log(d) + q*log(c)) + a)*sqrt(-a)/a)/(a*m*n*p*q)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(\left(\left(\left(fx\right)^m e\right)^n d\right)^p c\right)^q b + ax}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt((((f*x)^m*e)^n*d)^p*c)^q*b + a)*x, x)
```

maple [A] time = 0.02, size = 46, normalized size = 0.90

$$\frac{2 \operatorname{arctanh}\left(\frac{\sqrt{b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q+a}}{\sqrt{a}}\right)}{\sqrt{a} mnpq}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)/a^(1/2))/m/n/p/q/a^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{x \sqrt{a + b\left(c\left(d\left(e\left(fx\right)^m\right)^n\right)^p\right)^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)`

[Out] `int(1/(x*(a + b*(c*(d*(e*(f*x)^m)^n)^p)^q)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x \sqrt{a + b \left(c \left(d \left(e \left(f x \right)^m \right)^n \right)^p \right)^q}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(a+b*(c*(d*(e*(f*x)**m)**n)**p)**q)**(1/2), x)`

[Out] `Integral(1/(x*sqrt(a + b*(c*(d*(e*(f*x)**m)**n)**p)**q)), x)`

$$3.419 \quad \int \frac{\sqrt{-1+\frac{1}{x^2}}(-1+x^2)^3}{x} dx$$

Optimal. Leaf size=76

$$-\frac{35}{48} \left(\frac{1}{x^2} - 1\right)^{3/2} x^2 + \frac{35}{16} \sqrt{\frac{1}{x^2} - 1} - \frac{35}{16} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right) - \frac{1}{6} \left(\frac{1}{x^2} - 1\right)^{7/2} x^6 - \frac{7}{24} \left(\frac{1}{x^2} - 1\right)^{5/2} x^4$$

Rubi [A] time = 0.03, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{6} \left(\frac{1}{x^2} - 1\right)^{7/2} x^6 - \frac{7}{24} \left(\frac{1}{x^2} - 1\right)^{5/2} x^4 - \frac{35}{48} \left(\frac{1}{x^2} - 1\right)^{3/2} x^2 + \frac{35}{16} \sqrt{\frac{1}{x^2} - 1} - \frac{35}{16} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]

[Out] (35*Sqrt[-1 + x^(-2)])/16 - (35*(-1 + x^(-2))^(3/2)*x^2)/48 - (7*(-1 + x^(-2))^(5/2)*x^4)/24 - ((-1 + x^(-2))^(7/2)*x^6)/6 - (35*ArcTan[Sqrt[-1 + x^(-2)]])/16

Rule 25

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] := Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 47

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p_, x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-1 + \frac{1}{x^2}} (-1 + x^2)^3}{x} dx &= - \int \left(-1 + \frac{1}{x^2}\right)^{7/2} x^5 dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{(-1 + x)^{7/2}}{x^4} dx, x, \frac{1}{x^2} \right) \\
 &= -\frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6 + \frac{7}{12} \text{Subst} \left(\int \frac{(-1 + x)^{5/2}}{x^3} dx, x, \frac{1}{x^2} \right) \\
 &= -\frac{7}{24} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{48} \text{Subst} \left(\int \frac{(-1 + x)^{3/2}}{x^2} dx, x, \frac{1}{x^2} \right) \\
 &= -\frac{35}{48} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6 + \frac{35}{32} \text{Subst} \left(\int \frac{(-1 + x)^{1/2}}{x} dx, x, \frac{1}{x^2} \right) \\
 &= \frac{35}{16} \sqrt{-1 + \frac{1}{x^2}} - \frac{35}{48} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6 \\
 &= \frac{35}{16} \sqrt{-1 + \frac{1}{x^2}} - \frac{35}{48} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6 \\
 &= \frac{35}{16} \sqrt{-1 + \frac{1}{x^2}} - \frac{35}{48} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{7}{24} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{1}{6} \left(-1 + \frac{1}{x^2}\right)^{7/2} x^6
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.45

$$\frac{\sqrt{\frac{1}{x^2} - 1} {}_2F_1\left(-\frac{7}{2}, -\frac{1}{2}; \frac{1}{2}; x^2\right)}{\sqrt{1 - x^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-7/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]
```

IntegrateAlgebraic [A] time = 0.07, size = 51, normalized size = 0.67

$$\frac{1}{48} \sqrt{\frac{1}{x^2} - 1} (8x^6 - 38x^4 + 87x^2 + 48) - \frac{35}{16} \tan^{-1} \left(\sqrt{\frac{1 - x^2}{x^2}} \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^3)/x,x]
```

```
[Out] (Sqrt[-1 + x^(-2)]*(48 + 87*x^2 - 38*x^4 + 8*x^6))/48 - (35*ArcTan[Sqrt[(1 - x^2)/x^2]])/16
```

fricas [A] time = 0.71, size = 55, normalized size = 0.72

$$\frac{1}{48} (8x^6 - 38x^4 + 87x^2 + 48) \sqrt{-\frac{x^2-1}{x^2}} - \frac{35}{8} \arctan\left(\frac{x\sqrt{-\frac{x^2-1}{x^2}} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")

[Out] 1/48*(8*x^6 - 38*x^4 + 87*x^2 + 48)*sqrt(-(x^2 - 1)/x^2) - 35/8*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)

giac [A] time = 0.39, size = 77, normalized size = 1.01

$$\frac{1}{48} (2(4x^2\text{sgn}(x) - 19\text{sgn}(x))x^2 + 87\text{sgn}(x))\sqrt{-x^2+1}x + \frac{35}{16} \arcsin(x)\text{sgn}(x) - \frac{x\text{sgn}(x)}{2(\sqrt{-x^2+1}-1)} + \frac{(\sqrt{-x^2+1}-1)\text{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/48*(2*(4*x^2*sgn(x) - 19*sgn(x))*x^2 + 87*sgn(x))*sqrt(-x^2 + 1)*x + 35/16*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

maple [A] time = 0.02, size = 83, normalized size = 1.09

$$\frac{\sqrt{-\frac{x^2-1}{x^2}} \left(-8(-x^2+1)^{\frac{3}{2}}x^4 + 30(-x^2+1)^{\frac{3}{2}}x^2 + 105\sqrt{-x^2+1}x^2 + 105x \arcsin(x) + 48(-x^2+1)^{\frac{3}{2}} \right)}{48\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x)

[Out] 1/48*(-(x^2-1)/x^2)^(1/2)*(-8*x^4*(-x^2+1)^(3/2)+30*x^2*(-x^2+1)^(3/2)+48*(-x^2+1)^(3/2)+105*x^2*(-x^2+1)^(1/2)+105*arcsin(x)*x)/(-x^2+1)^(1/2)

maxima [B] time = 2.07, size = 120, normalized size = 1.58

$$\frac{3}{2}x^2\sqrt{\frac{1}{x^2}-1} + \sqrt{\frac{1}{x^2}-1} - \frac{3\left(\frac{1}{x^2}-1\right)^{\frac{5}{2}} + 8\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - 3\sqrt{\frac{1}{x^2}-1}}{48\left(\left(\frac{1}{x^2}-1\right)^3 + 3\left(\frac{1}{x^2}-1\right)^2 + \frac{3}{x^2}-2\right)} + \frac{3\left(\left(\frac{1}{x^2}-1\right)^{\frac{3}{2}} - \sqrt{\frac{1}{x^2}-1}\right)}{8\left(\left(\frac{1}{x^2}-1\right)^2 + \frac{2}{x^2}-1\right)} - \frac{35}{16} \arctan\left(\sqrt{\frac{1}{x^2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^3*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] 3/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 1/48*(3*(1/x^2 - 1)^(5/2) + 8*(1/x^2 - 1)^(3/2) - 3*sqrt(1/x^2 - 1))/((1/x^2 - 1)^3 + 3*(1/x^2 - 1)^2 + 3/x^2 - 2) + 3/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) - 35/16*arctan(sqrt(1/x^2 - 1))

mupad [B] time = 3.50, size = 54, normalized size = 0.71

$$\sqrt{\frac{1}{x^2}-1} - \frac{35 \operatorname{atan}\left(\sqrt{\frac{1}{x^2}-1}\right)}{16} + \frac{19x^6\sqrt{\frac{1}{x^2}-1}}{16} + \frac{17x^6\left(\frac{1}{x^2}-1\right)^{3/2}}{6} + \frac{29x^6\left(\frac{1}{x^2}-1\right)^{5/2}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^3)/x,x)
```

```
[Out] (1/x^2 - 1)^(1/2) - (35*atan((1/x^2 - 1)^(1/2)))/16 + (19*x^6*(1/x^2 - 1)^(1/2))/16 + (17*x^6*(1/x^2 - 1)^(3/2))/6 + (29*x^6*(1/x^2 - 1)^(5/2))/16
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**3*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] Timed out
```

$$3.420 \quad \int \frac{\sqrt{-1 + \frac{1}{x^2}} (-1 + x^2)^2}{x} dx$$

Optimal. Leaf size=60

$$\frac{5}{8} \left(\frac{1}{x^2} - 1 \right)^{3/2} x^2 - \frac{15}{8} \sqrt{\frac{1}{x^2} - 1} + \frac{15}{8} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right) + \frac{1}{4} \left(\frac{1}{x^2} - 1 \right)^{5/2} x^4$$

Rubi [A] time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {25, 266, 47, 50, 63, 203}

$$\frac{1}{4} \left(\frac{1}{x^2} - 1 \right)^{5/2} x^4 + \frac{5}{8} \left(\frac{1}{x^2} - 1 \right)^{3/2} x^2 - \frac{15}{8} \sqrt{\frac{1}{x^2} - 1} + \frac{15}{8} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2)^2)/x,x]

[Out] (-15*Sqrt[-1 + x^(-2)])/8 + (5*(-1 + x^(-2))^(3/2)*x^2)/8 + ((-1 + x^(-2))^(5/2)*x^4)/4 + (15*ArcTan[Sqrt[-1 + x^(-2)]])/8

Rule 25

Int[(u_)*((a_) + (b_)*(x_)^(n_))^(m_)*((c_) + (d_)*(x_)^(q_))^(p_), x_Symbol] := Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 47

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-1 + \frac{1}{x^2}} (-1 + x^2)^2}{x} dx &= \int \left(-1 + \frac{1}{x^2}\right)^{5/2} x^3 dx \\
 &= -\left(\frac{1}{2} \operatorname{Subst}\left(\int \frac{(-1 + x)^{5/2}}{x^3} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{1}{4} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{5}{8} \operatorname{Subst}\left(\int \frac{(-1 + x)^{3/2}}{x^2} dx, x, \frac{1}{x^2}\right) \\
 &= \frac{5}{8} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 - \frac{15}{16} \operatorname{Subst}\left(\int \frac{\sqrt{-1 + x}}{x} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{15}{8} \sqrt{-1 + \frac{1}{x^2}} + \frac{5}{8} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{16} \operatorname{Subst}\left(\int \frac{1}{\sqrt{-1 + x}} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{15}{8} \sqrt{-1 + \frac{1}{x^2}} + \frac{5}{8} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8} \operatorname{Subst}\left(\int \frac{1}{1 + x^2} dx, x, \frac{1}{x^2}\right) \\
 &= -\frac{15}{8} \sqrt{-1 + \frac{1}{x^2}} + \frac{5}{8} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 + \frac{1}{4} \left(-1 + \frac{1}{x^2}\right)^{5/2} x^4 + \frac{15}{8} \tan^{-1}\left(\sqrt{-1 + \frac{1}{x^2}}\right)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 35, normalized size = 0.58

$$\frac{\sqrt{\frac{1}{x^2} - 1} {}_2F_1\left(-\frac{5}{2}, -\frac{1}{2}; \frac{1}{2}; x^2\right)}{\sqrt{1 - x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x^(-2)])*(-1 + x^2)^2)/x,x]

[Out] -((Sqrt[-1 + x^(-2)])*Hypergeometric2F1[-5/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2])

IntegrateAlgebraic [A] time = 0.08, size = 46, normalized size = 0.77

$$\frac{15}{8} \tan^{-1}\left(\sqrt{\frac{1 - x^2}{x^2}}\right) + \frac{1}{8} \sqrt{\frac{1}{x^2} - 1} (2x^4 - 9x^2 - 8)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)])*(-1 + x^2)^2)/x,x]

[Out] (Sqrt[-1 + x^(-2)])*(-8 - 9*x^2 + 2*x^4))/8 + (15*ArcTan[Sqrt[(1 - x^2)/x^2]])/8

fricas [A] time = 0.59, size = 50, normalized size = 0.83

$$\frac{1}{8} (2x^4 - 9x^2 - 8) \sqrt{-\frac{x^2 - 1}{x^2}} + \frac{15}{4} \arctan\left(\frac{x \sqrt{-\frac{x^2 - 1}{x^2}} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")

[Out] 1/8*(2*x^4 - 9*x^2 - 8)*sqrt(-(x^2 - 1)/x^2) + 15/4*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)

giac [A] time = 0.43, size = 67, normalized size = 1.12

$$\frac{1}{8} \left(2x^2 \operatorname{sgn}(x) - 9 \operatorname{sgn}(x) \right) \sqrt{-x^2 + 1} x - \frac{15}{8} \arcsin(x) \operatorname{sgn}(x) + \frac{x \operatorname{sgn}(x)}{2 \left(\sqrt{-x^2 + 1} - 1 \right)} - \frac{\left(\sqrt{-x^2 + 1} - 1 \right) \operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/8*(2*x^2*sgn(x) - 9*sgn(x))*sqrt(-x^2 + 1)*x - 15/8*arcsin(x)*sgn(x) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

maple [A] time = 0.01, size = 69, normalized size = 1.15

$$\frac{\sqrt{-\frac{x^2-1}{x^2}} \left(2 \left(-x^2 + 1 \right)^{\frac{3}{2}} x^2 + 15 \sqrt{-x^2 + 1} x^2 + 15x \arcsin(x) + 8 \left(-x^2 + 1 \right)^{\frac{3}{2}} \right)}{8 \sqrt{-x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x)

[Out] -1/8*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)*x^2+8*(-x^2+1)^(3/2)+15*(-x^2+1)^(1/2)*x^2+15*x*arcsin(x))/(-x^2+1)^(1/2)

maxima [A] time = 2.00, size = 67, normalized size = 1.12

$$-x^2 \sqrt{\frac{1}{x^2} - 1} - \sqrt{\frac{1}{x^2} - 1} - \frac{\left(\frac{1}{x^2} - 1 \right)^{\frac{3}{2}} - \sqrt{\frac{1}{x^2} - 1}}{8 \left(\left(\frac{1}{x^2} - 1 \right)^2 + \frac{2}{x^2} - 1 \right)} + \frac{15}{8} \arctan \left(\sqrt{\frac{1}{x^2} - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)^2*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] -x^2*sqrt(1/x^2 - 1) - sqrt(1/x^2 - 1) - 1/8*((1/x^2 - 1)^(3/2) - sqrt(1/x^2 - 1))/((1/x^2 - 1)^2 + 2/x^2 - 1) + 15/8*arctan(sqrt(1/x^2 - 1))

mupad [B] time = 3.36, size = 44, normalized size = 0.73

$$\frac{15 \operatorname{atan} \left(\sqrt{\frac{1}{x^2} - 1} \right)}{8} - \sqrt{\frac{1}{x^2} - 1} - \frac{7x^4 \sqrt{\frac{1}{x^2} - 1}}{8} - \frac{9x^4 \left(\frac{1}{x^2} - 1 \right)^{3/2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1)^2)/x,x)

[Out] (15*atan((1/x^2 - 1)^(1/2)))/8 - (1/x^2 - 1)^(1/2) - (7*x^4*(1/x^2 - 1)^(1/2))/8 - (9*x^4*(1/x^2 - 1)^(3/2))/8

sympy [A] time = 118.56, size = 60, normalized size = 1.00

$$\frac{x^4 \sqrt{-1 + \frac{1}{x^2}} \left(2 - \frac{1}{x^2} \right)}{8} - x^2 \sqrt{-1 + \frac{1}{x^2}} - \sqrt{-1 + \frac{1}{x^2}} + \frac{15 \operatorname{atan} \left(\sqrt{-1 + \frac{1}{x^2}} \right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)**2*(-1+1/x**2)**(1/2)/x,x)
```

```
[Out] x**4*sqrt(-1 + x**(-2))*(2 - 1/x**2)/8 - x**2*sqrt(-1 + x**(-2)) - sqrt(-1 + x**(-2)) + 15*atan(sqrt(-1 + x**(-2)))/8
```

$$3.421 \quad \int \frac{\sqrt{-1 + \frac{1}{x^2}}(-1 + x^2)}{x} dx$$

Optimal. Leaf size=44

$$-\frac{1}{2} \left(\frac{1}{x^2} - 1 \right)^{3/2} x^2 + \frac{3}{2} \sqrt{\frac{1}{x^2} - 1} - \frac{3}{2} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right)$$

Rubi [A] time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {25, 266, 47, 50, 63, 203}

$$-\frac{1}{2} \left(\frac{1}{x^2} - 1 \right)^{3/2} x^2 + \frac{3}{2} \sqrt{\frac{1}{x^2} - 1} - \frac{3}{2} \tan^{-1} \left(\sqrt{\frac{1}{x^2} - 1} \right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]

[Out] (3*Sqrt[-1 + x^(-2)])/2 - ((-1 + x^(-2))^(3/2)*x^2)/2 - (3*ArcTan[Sqrt[-1 + x^(-2)]])/2

Rule 25

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] := Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 47

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !IntegerQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) && IntLinearQ[a, b, c, d, m, n, x]

Rule 50

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Simp[(a + b*x)^(m + 1)*(c + d*x)^n/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a

, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b,
m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{-1 + \frac{1}{x^2}} (-1 + x^2)}{x} dx &= - \int \left(-1 + \frac{1}{x^2}\right)^{3/2} x dx \\
 &= \frac{1}{2} \text{Subst} \left(\int \frac{(-1 + x)^{3/2}}{x^2} dx, x, \frac{1}{x^2} \right) \\
 &= -\frac{1}{2} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 + \frac{3}{4} \text{Subst} \left(\int \frac{\sqrt{-1 + x}}{x} dx, x, \frac{1}{x^2} \right) \\
 &= \frac{3}{2} \sqrt{-1 + \frac{1}{x^2}} - \frac{1}{2} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{3}{4} \text{Subst} \left(\int \frac{1}{\sqrt{-1 + x} x} dx, x, \frac{1}{x^2} \right) \\
 &= \frac{3}{2} \sqrt{-1 + \frac{1}{x^2}} - \frac{1}{2} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{3}{2} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{-1 + \frac{1}{x^2}} \right) \\
 &= \frac{3}{2} \sqrt{-1 + \frac{1}{x^2}} - \frac{1}{2} \left(-1 + \frac{1}{x^2}\right)^{3/2} x^2 - \frac{3}{2} \tan^{-1} \left(\sqrt{-1 + \frac{1}{x^2}} \right)
 \end{aligned}$$

Mathematica [C] time = 0.01, size = 34, normalized size = 0.77

$$\frac{\sqrt{\frac{1}{x^2} - 1} {}_2F_1\left(-\frac{3}{2}, -\frac{1}{2}; \frac{1}{2}; x^2\right)}{\sqrt{1 - x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]

[Out] (Sqrt[-1 + x^(-2)]*Hypergeometric2F1[-3/2, -1/2, 1/2, x^2])/Sqrt[1 - x^2]

IntegrateAlgebraic [A] time = 0.05, size = 39, normalized size = 0.89

$$\frac{1}{2} \sqrt{\frac{1}{x^2} - 1} (x^2 + 2) - \frac{3}{2} \tan^{-1} \left(\sqrt{\frac{1 - x^2}{x^2}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x^(-2)]*(-1 + x^2))/x,x]

[Out] (Sqrt[-1 + x^(-2)]*(2 + x^2))/2 - (3*ArcTan[Sqrt[(1 - x^2)/x^2]])/2

fricas [A] time = 0.58, size = 43, normalized size = 0.98

$$\frac{1}{2} (x^2 + 2) \sqrt{-\frac{x^2 - 1}{x^2}} - 3 \arctan \left(\frac{x \sqrt{-\frac{x^2 - 1}{x^2}} - 1}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="fricas")

[Out] 1/2*(x^2 + 2)*sqrt(-(x^2 - 1)/x^2) - 3*arctan((x*sqrt(-(x^2 - 1)/x^2) - 1)/x)

giac [A] time = 0.38, size = 57, normalized size = 1.30

$$\frac{1}{2} \sqrt{-x^2 + 1} x \operatorname{sgn}(x) + \frac{3}{2} \arcsin(x) \operatorname{sgn}(x) - \frac{x \operatorname{sgn}(x)}{2(\sqrt{-x^2 + 1} - 1)} + \frac{(\sqrt{-x^2 + 1} - 1) \operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="giac")

[Out] 1/2*sqrt(-x^2 + 1)*x*sgn(x) + 3/2*arcsin(x)*sgn(x) - 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

maple [A] time = 0.01, size = 55, normalized size = 1.25

$$\frac{\sqrt{-\frac{x^2-1}{x^2}} \left(3\sqrt{-x^2+1} x^2 + 3x \arcsin(x) + 2(-x^2+1)^{\frac{3}{2}} \right)}{2\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)*(-1+1/x^2)^(1/2)/x,x)

[Out] 1/2*(-(x^2-1)/x^2)^(1/2)*(2*(-x^2+1)^(3/2)+3*(-x^2+1)^(1/2)*x^2+3*x*arcsin(x))/(-x^2+1)^(1/2)

maxima [A] time = 1.99, size = 30, normalized size = 0.68

$$\frac{1}{2} x^2 \sqrt{\frac{1}{x^2} - 1} + \sqrt{\frac{1}{x^2} - 1} - \frac{3}{2} \arctan\left(\sqrt{\frac{1}{x^2} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)*(-1+1/x^2)^(1/2)/x,x, algorithm="maxima")

[Out] 1/2*x^2*sqrt(1/x^2 - 1) + sqrt(1/x^2 - 1) - 3/2*arctan(sqrt(1/x^2 - 1))

mupad [B] time = 3.57, size = 30, normalized size = 0.68

$$\sqrt{\frac{1}{x^2} - 1} - \frac{3 \operatorname{atan}\left(\sqrt{\frac{1}{x^2} - 1}\right)}{2} + \frac{x^2 \sqrt{\frac{1}{x^2} - 1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1/x^2 - 1)^(1/2)*(x^2 - 1))/x,x)

[Out] (1/x^2 - 1)^(1/2) - (3*atan((1/x^2 - 1)^(1/2)))/2 + (x^2*(1/x^2 - 1)^(1/2))/2

sympy [A] time = 43.65, size = 39, normalized size = 0.89

$$\frac{x^2 \sqrt{-1 + \frac{1}{x^2}}}{2} + \sqrt{-1 + \frac{1}{x^2}} - \frac{3 \operatorname{atan}\left(\sqrt{-1 + \frac{1}{x^2}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)*(-1+1/x**2)**(1/2)/x,x)

[Out] x**2*sqrt(-1 + x**(-2))/2 + sqrt(-1 + x**(-2)) - 3*atan(sqrt(-1 + x**(-2)))/2

$$3.422 \quad \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)} dx$$

Optimal. Leaf size=9

$$\sqrt{\frac{1}{x^2} - 1}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {25, 261}

$$\sqrt{\frac{1}{x^2} - 1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]

Rule 25

Int[(u_)*((a_) + (b_)*(x_)^(n_))^(m_)*((c_) + (d_)*(x_)^(q_))^(p_), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)} dx &= - \int \frac{1}{\sqrt{-1 + \frac{1}{x^2}} x^3} dx \\ &= \sqrt{-1 + \frac{1}{x^2}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$\sqrt{\frac{1}{x^2} - 1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[-1 + x^(-2)]

IntegrateAlgebraic [A] time = 0.03, size = 15, normalized size = 1.67

$$\sqrt{\frac{1 - x^2}{x^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)),x]

[Out] Sqrt[(1 - x^2)/x^2]

fricas [A] time = 0.58, size = 12, normalized size = 1.33

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="fricas")

[Out] sqrt(-(x^2 - 1)/x^2)

giac [B] time = 0.40, size = 37, normalized size = 4.11

$$-\frac{x\operatorname{sgn}(x)}{2\left(\sqrt{-x^2+1}-1\right)} + \frac{\left(\sqrt{-x^2+1}-1\right)\operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="giac")

[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

maple [A] time = 0.00, size = 13, normalized size = 1.44

$$\sqrt{-\frac{x^2-1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+1/x^2)^(1/2)/x/(x^2-1),x)

[Out] (-(x^2-1)/x^2)^(1/2)

maxima [B] time = 0.58, size = 16, normalized size = 1.78

$$\frac{\sqrt{x+1}\sqrt{-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1),x, algorithm="maxima")

[Out] sqrt(x + 1)*sqrt(-x + 1)/x

mupad [B] time = 3.18, size = 14, normalized size = 1.56

$$\frac{\sqrt{1-x^2}}{|x|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)),x)

[Out] (1 - x^2)^(1/2)/abs(x)

sympy [A] time = 2.25, size = 8, normalized size = 0.89

$$\sqrt{-1 + \frac{1}{x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1),x)
```

```
[Out] sqrt(-1 + x**(-2))
```

$$3.423 \quad \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)^2} dx$$

Optimal. Leaf size=21

$$\frac{1}{\sqrt{\frac{1}{x^2} - 1}} - \sqrt{\frac{1}{x^2} - 1}$$

Rubi [A] time = 0.01, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {25, 266, 43}

$$\frac{1}{\sqrt{\frac{1}{x^2} - 1}} - \sqrt{\frac{1}{x^2} - 1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2),x]

[Out] 1/Sqrt[-1 + x^(-2)] - Sqrt[-1 + x^(-2)]

Rule 25

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] := Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)^2} dx &= \int \frac{1}{\left(-1 + \frac{1}{x^2}\right)^{3/2} x^5} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{x}{(-1 + x)^{3/2}} dx, x, \frac{1}{x^2}\right)\right) \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \left(\frac{1}{(-1 + x)^{3/2}} + \frac{1}{\sqrt{-1 + x}}\right) dx, x, \frac{1}{x^2}\right)\right) \\ &= \frac{1}{\sqrt{-1 + \frac{1}{x^2}}} - \sqrt{-1 + \frac{1}{x^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.14

$$\frac{\sqrt{\frac{1}{x^2} - 1} (1 - 2x^2)}{x^2 - 1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2), x]

[Out] (Sqrt[-1 + x^(-2)]*(1 - 2*x^2))/(-1 + x^2)

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.43

$$\frac{(1 - 2x^2) \sqrt{\frac{1-x^2}{x^2}}}{x^2 - 1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^2), x]

[Out] ((1 - 2*x^2)*Sqrt[(1 - x^2)/x^2])/(-1 + x^2)

fricas [A] time = 0.53, size = 28, normalized size = 1.33

$$-\frac{(2x^2 - 1) \sqrt{-\frac{x^2-1}{x^2}}}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="fricas")

[Out] -(2*x^2 - 1)*sqrt(-(x^2 - 1)/x^2)/(x^2 - 1)

giac [B] time = 0.38, size = 58, normalized size = 2.76

$$-\frac{\sqrt{-x^2 + 1} x \operatorname{sgn}(x)}{x^2 - 1} + \frac{x \operatorname{sgn}(x)}{2(\sqrt{-x^2 + 1} - 1)} - \frac{(\sqrt{-x^2 + 1} - 1) \operatorname{sgn}(x)}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="giac")

[Out] -sqrt(-x^2 + 1)*x*sgn(x)/(x^2 - 1) + 1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x

maple [A] time = 0.01, size = 29, normalized size = 1.38

$$-\frac{(2x^2 - 1) \sqrt{-\frac{x^2-1}{x^2}}}{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x)

[Out] -(2*x^2-1)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)

maxima [A] time = 0.73, size = 30, normalized size = 1.43

$$-\frac{(2x^2 - 1) \sqrt{x + 1} \sqrt{-x + 1}}{x^3 - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^2,x, algorithm="maxima")

[Out] -(2*x^2 - 1)*sqrt(x + 1)*sqrt(-x + 1)/(x^3 - x)

mupad [B] time = 3.11, size = 25, normalized size = 1.19

$$\frac{x \sqrt{\frac{1}{x^2} - 1} (2x^2 - 1)}{x - x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^2),x)

[Out] (x*(1/x^2 - 1)^(1/2)*(2*x^2 - 1))/(x - x^3)

sympy [A] time = 3.85, size = 20, normalized size = 0.95

$$-\sqrt{-1 + \frac{1}{x^2}} + \frac{1}{\sqrt{-1 + \frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**2,x)

[Out] -sqrt(-1 + x**(-2)) + 1/sqrt(-1 + x**(-2))

$$3.424 \quad \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)^3} dx$$

Optimal. Leaf size=34

$$\sqrt{\frac{1}{x^2} - 1} - \frac{2}{\sqrt{\frac{1}{x^2} - 1}} - \frac{1}{3\left(\frac{1}{x^2} - 1\right)^{3/2}}$$

Rubi [A] time = 0.01, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {25, 266, 43}

$$\sqrt{\frac{1}{x^2} - 1} - \frac{2}{\sqrt{\frac{1}{x^2} - 1}} - \frac{1}{3\left(\frac{1}{x^2} - 1\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3), x]

[Out] -1/(3*(-1 + x^(-2))^(3/2)) - 2/Sqrt[-1 + x^(-2)] + Sqrt[-1 + x^(-2)]

Rule 25

Int[(u_)*((a_) + (b_)*(x_)^(n_))^(m_)*((c_) + (d_)*(x_)^(q_))^(p_), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 43

Int[((a_) + (b_)*(x_)^(n_))*((c_) + (d_)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1 + \frac{1}{x^2}}}{x(-1 + x^2)^3} dx &= - \int \frac{1}{\left(-1 + \frac{1}{x^2}\right)^{5/2} x^7} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{x^2}{(-1 + x)^{5/2}} dx, x, \frac{1}{x^2} \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{(-1 + x)^{5/2}} + \frac{2}{(-1 + x)^{3/2}} + \frac{1}{\sqrt{-1 + x}} \right) dx, x, \frac{1}{x^2} \right) \\ &= -\frac{1}{3\left(-1 + \frac{1}{x^2}\right)^{3/2}} - \frac{2}{\sqrt{-1 + \frac{1}{x^2}}} + \sqrt{-1 + \frac{1}{x^2}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 0.94

$$\frac{\sqrt{\frac{1}{x^2} - 1} (8x^4 - 12x^2 + 3)}{3(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3), x]

[Out] (Sqrt[-1 + x^(-2)]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)

IntegrateAlgebraic [A] time = 0.04, size = 38, normalized size = 1.12

$$\frac{\sqrt{\frac{1-x^2}{x^2}} (8x^4 - 12x^2 + 3)}{3(x^2 - 1)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + x^(-2)]/(x*(-1 + x^2)^3), x]

[Out] (Sqrt[(1 - x^2)/x^2]*(3 - 12*x^2 + 8*x^4))/(3*(-1 + x^2)^2)

fricas [A] time = 0.54, size = 38, normalized size = 1.12

$$\frac{(8x^4 - 12x^2 + 3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^4 - 2x^2 + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="fricas")

[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(-(x^2 - 1)/x^2)/(x^4 - 2*x^2 + 1)

giac [B] time = 0.44, size = 68, normalized size = 2.00

$$-\frac{x\operatorname{sgn}(x)}{2(\sqrt{-x^2+1}-1)} + \frac{(\sqrt{-x^2+1}-1)\operatorname{sgn}(x)}{2x} - \frac{(5x^2\operatorname{sgn}(x)-6\operatorname{sgn}(x))x}{3(x^2-1)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="giac")

[Out] -1/2*x*sgn(x)/(sqrt(-x^2 + 1) - 1) + 1/2*(sqrt(-x^2 + 1) - 1)*sgn(x)/x - 1/3*(5*x^2*sgn(x) - 6*sgn(x))*x/((x^2 - 1)*sqrt(-x^2 + 1))

maple [A] time = 0.00, size = 34, normalized size = 1.00

$$\frac{(8x^4 - 12x^2 + 3)\sqrt{-\frac{x^2-1}{x^2}}}{3(x^2 - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x)

[Out] 1/3*(8*x^4-12*x^2+3)*(-(x^2-1)/x^2)^(1/2)/(x^2-1)^2

maxima [A] time = 0.64, size = 38, normalized size = 1.12

$$\frac{(8x^4 - 12x^2 + 3)\sqrt{x+1}\sqrt{-x+1}}{3(x^5 - 2x^3 + x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x^2)^(1/2)/x/(x^2-1)^3,x, algorithm="maxima")

[Out] 1/3*(8*x^4 - 12*x^2 + 3)*sqrt(x + 1)*sqrt(-x + 1)/(x^5 - 2*x^3 + x)

mupad [B] time = 3.09, size = 28, normalized size = 0.82

$$\frac{\sqrt{\frac{1}{x^2} - 1} (8x^4 - 12x^2 + 3)}{3(x^2 - 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x^2 - 1)^(1/2)/(x*(x^2 - 1)^3),x)

[Out] ((1/x^2 - 1)^(1/2)*(8*x^4 - 12*x^2 + 3))/(3*(x^2 - 1)^2)

sympy [A] time = 5.12, size = 34, normalized size = 1.00

$$\sqrt{-1 + \frac{1}{x^2}} - \frac{2}{\sqrt{-1 + \frac{1}{x^2}}} - \frac{1}{3\left(-1 + \frac{1}{x^2}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+1/x**2)**(1/2)/x/(x**2-1)**3,x)

[Out] sqrt(-1 + x**(-2)) - 2/sqrt(-1 + x**(-2)) - 1/(3*(-1 + x**(-2))**(3/2))

$$3.425 \quad \int \frac{\sqrt{1 + \frac{1}{x^2}} x}{(1 + x^2)^2} dx$$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2} + 1}}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2} + 1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]

[Out] 1/Sqrt[1 + x^(-2)]

Rule 25

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1 + \frac{1}{x^2}} x}{(1 + x^2)^2} dx &= \int \frac{1}{\left(1 + \frac{1}{x^2}\right)^{3/2} x^3} dx \\ &= \frac{1}{\sqrt{1 + \frac{1}{x^2}}} \end{aligned}$$

Mathematica [B] time = 0.01, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2} + 1} x^2}{x^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]

[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)

IntegrateAlgebraic [B] time = 0.03, size = 24, normalized size = 2.67

$$\frac{x^2 \sqrt{\frac{x^2+1}{x^2}}}{x^2 + 1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1 + x^(-2)]*x)/(1 + x^2)^2,x]

[Out] (x^2*Sqrt[(1 + x^2)/x^2])/(1 + x^2)

fricas [B] time = 0.50, size = 28, normalized size = 3.11

$$\frac{x^2 \sqrt{\frac{x^2+1}{x^2}} + x^2 + 1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="fricas")

[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)

giac [A] time = 0.34, size = 11, normalized size = 1.22

$$\frac{x \operatorname{sgn}(x)}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="giac")

[Out] x*sgn(x)/sqrt(x^2 + 1)

maple [B] time = 0.01, size = 23, normalized size = 2.56

$$\frac{\sqrt{\frac{x^2+1}{x^2}} x^2}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x)

[Out] 1/(x^2+1)*x^2*((x^2+1)/x^2)^(1/2)

maxima [A] time = 1.40, size = 11, normalized size = 1.22

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/x^2)^(1/2)/(x^2+1)^2,x, algorithm="maxima")

[Out] 1/sqrt((x^2 + 1)/x^2)

mupad [B] time = 3.11, size = 18, normalized size = 2.00

$$\frac{x^2 \sqrt{\frac{1}{x^2} + 1}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1/x^2 + 1)^(1/2))/(x^2 + 1)^2,x)
```

```
[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)
```

sympy [A] time = 2.83, size = 8, normalized size = 0.89

$$\frac{x}{\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+1/x**2)**(1/2)/(x**2+1)**2,x)
```

```
[Out] x/sqrt(x**2 + 1)
```


$$3.426 \quad \int \frac{1}{\sqrt{1 + \frac{1}{x^2}} x(1+x^2)} dx$$

Optimal. Leaf size=9

$$\frac{1}{\sqrt{\frac{1}{x^2} + 1}}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {25, 261}

$$\frac{1}{\sqrt{\frac{1}{x^2} + 1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] 1/Sqrt[1 + x^(-2)]

Rule 25

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1 + \frac{1}{x^2}} x(1+x^2)} dx &= \int \frac{1}{\left(1 + \frac{1}{x^2}\right)^{3/2} x^3} dx \\ &= \frac{1}{\sqrt{1 + \frac{1}{x^2}}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 20, normalized size = 2.22

$$\frac{\sqrt{\frac{1}{x^2} + 1} x^2}{x^2 + 1}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + x^(-2)]*x*(1 + x^2)),x]

[Out] (Sqrt[1 + x^(-2)]*x^2)/(1 + x^2)

IntegrateAlgebraic [B] time = 0.03, size = 24, normalized size = 2.67

$$\frac{x^2 \sqrt{\frac{x^2+1}{x^2}}}{x^2 + 1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[1 + x^(-2)])*x*(1 + x^2)),x]

[Out] (x^2*Sqrt[(1 + x^2)/x^2])/(1 + x^2)

fricas [B] time = 0.68, size = 28, normalized size = 3.11

$$\frac{x^2 \sqrt{\frac{x^2+1}{x^2}} + x^2 + 1}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="fricas")

[Out] (x^2*sqrt((x^2 + 1)/x^2) + x^2 + 1)/(x^2 + 1)

giac [B] time = 0.33, size = 20, normalized size = 2.22

$$\frac{1}{x^2 - \sqrt{x^4 + x^2 + 1}} - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="giac")

[Out] 1/(x^2 - sqrt(x^4 + x^2) + 1) - 1

maple [A] time = 0.00, size = 12, normalized size = 1.33

$$\frac{1}{\sqrt{\frac{x^2+1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(x^2+1)/(1+1/x^2)^(1/2),x)

[Out] 1/((x^2+1)/x^2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2 + 1)x\sqrt{\frac{1}{x^2} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+1)/(1+1/x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^2 + 1)*x*sqrt(1/x^2 + 1)), x)

mupad [B] time = 3.10, size = 18, normalized size = 2.00

$$\frac{x^2 \sqrt{\frac{1}{x^2} + 1}}{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(1/x^2 + 1)^(1/2)*(x^2 + 1)),x)

[Out] (x^2*(1/x^2 + 1)^(1/2))/(x^2 + 1)

sympy [A] time = 3.17, size = 10, normalized size = 1.11

$$\frac{1}{\sqrt{1 + \frac{1}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(x**2+1)/(1+1/x**2)**(1/2),x)
```

```
[Out] 1/sqrt(1 + x**(-2))
```

$$3.427 \quad \int \frac{x}{a+bx^2+\sqrt{a+bx^2}} dx$$

Optimal. Leaf size=18

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2155, 31}

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]

[Out] Log[1 + Sqrt[a + b*x^2]]/b

Rule 31

Int[((a_) + (b_.)*(x_))⁽⁻¹⁾, x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 2155

Int[(x_)^{(m_)/((c_) + (d_.)*(x_)^{(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] :> Dist[1/n, Subst[Int[x^{((m + 1)/n - 1)/(c + d*x + e*Sqrt[a + b*x])}], x], x, xⁿ], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[(m + 1)/n]}}

Rubi steps

$$\begin{aligned} \int \frac{x}{a+bx^2+\sqrt{a+bx^2}} dx &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{a+bx+\sqrt{a+bx}} dx, x, x^2\right) \\ &= \frac{\text{Subst}\left(\int \frac{1}{1+x} dx, x, \sqrt{a+bx^2}\right)}{b} \\ &= \frac{\log\left(1+\sqrt{a+bx^2}\right)}{b} \end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 1.00

$$\frac{\log\left(\sqrt{a+bx^2}+1\right)}{b}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2 + Sqrt[a + b*x^2]),x]

[Out] Log[1 + Sqrt[a + b*x^2]]/b

IntegrateAlgebraic [A] time = 0.04, size = 20, normalized size = 1.11

$$\frac{\log\left(b\sqrt{a+bx^2}+b\right)}{b}$$

maxima [A] time = 0.57, size = 16, normalized size = 0.89

$$\frac{\log\left(\sqrt{bx^2 + a} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*x^2+(b*x^2+a)^(1/2)),x, algorithm="maxima")

[Out] log(sqrt(b*x^2 + a) + 1)/b

mupad [B] time = 3.38, size = 26, normalized size = 1.44

$$\frac{\operatorname{atanh}\left(\sqrt{bx^2 + a}\right) + \frac{\ln(bx^2+a-1)}{2}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*x^2 + (a + b*x^2)^(1/2)),x)

[Out] (atanh((a + b*x^2)^(1/2)) + log(a + b*x^2 - 1)/2)/b

sympy [A] time = 3.39, size = 14, normalized size = 0.78

$$\frac{\log\left(\sqrt{a + bx^2} + 1\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(a+b*x**2+(b*x**2+a)**(1/2)),x)

[Out] log(sqrt(a + b*x**2) + 1)/b

$$3.428 \quad \int \frac{x}{x^2 - \sqrt[3]{x^2}} dx$$

Optimal. Leaf size=16

$$\frac{3}{4} \log\left(1 - (x^2)^{2/3}\right)$$

Rubi [A] time = 0.06, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {6715, 1593, 260}

$$\frac{3}{4} \log\left(1 - (x^2)^{2/3}\right)$$

Antiderivative was successfully verified.

[In] Int[x/(x^2 - (x^2)^(1/3)), x]

[Out] (3*Log[1 - (x^2)^(2/3)])/4

Rule 260

Int[(x_)^(m_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6715

Int[(u_)*(x_)^(m_), x_Symbol] :> Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionOfQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{x^2 - \sqrt[3]{x^2}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{-\sqrt[3]{x} + x} dx, x, x^2 \right) \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{(-1 + x^{2/3}) \sqrt[3]{x}} dx, x, x^2 \right) \\ &= \frac{3}{4} \log\left(1 - (x^2)^{2/3}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 1.00

$$\frac{3}{4} \log\left(1 - (x^2)^{2/3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(x^2 - (x^2)^(1/3)), x]

[Out] (3*Log[1 - (x^2)^(2/3)])/4

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 1.81

$$\frac{3}{4} \log\left(\sqrt[3]{x^2} - 1\right) + \frac{3}{4} \log\left(\sqrt[3]{x^2} + 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(x^2 - (x^2)^(1/3)),x]

[Out] (3*Log[-1 + (x^2)^(1/3)])/4 + (3*Log[1 + (x^2)^(1/3)])/4

fricas [B] time = 0.81, size = 32, normalized size = 2.00

$$-3 \log\left(\frac{(x^2)^{\frac{1}{3}}}{x}\right) + \frac{3}{4} \log\left(-\frac{x^2 - (x^2)^{\frac{1}{3}}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="fricas")

[Out] -3*log((x^2)^(1/3)/x) + 3/4*log(-(x^2 - (x^2)^(1/3))/x^2)

giac [A] time = 0.47, size = 16, normalized size = 1.00

$$\frac{3}{4} \log\left(\left|(x \operatorname{sgn}(x))^{\frac{1}{3}} x \operatorname{sgn}(x) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="giac")

[Out] 3/4*log(abs((x*sgn(x))^(1/3)*x*sgn(x) - 1))

maple [B] time = 0.05, size = 70, normalized size = 4.38

$$\frac{\ln\left((x^2)^{\frac{1}{3}} - 1\right)}{2} + \frac{\ln\left((x^2)^{\frac{1}{3}} + 1\right)}{2} - \frac{\ln\left((x^2)^{\frac{2}{3}} - (x^2)^{\frac{1}{3}} + 1\right)}{4} - \frac{\ln\left((x^2)^{\frac{2}{3}} + (x^2)^{\frac{1}{3}} + 1\right)}{4} + \frac{\ln(x^2 - 1)}{4} + \frac{\ln(x^2 + 1)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x^2-(x^2)^(1/3)),x)

[Out] 1/4*ln(x^2-1)+1/4*ln(x^2+1)+1/2*ln((x^2)^(1/3)-1)-1/4*ln((x^2)^(2/3)+(x^2)^(1/3)+1)-1/4*ln((x^2)^(2/3)-(x^2)^(1/3)+1)+1/2*ln((x^2)^(1/3)+1)

maxima [A] time = 0.64, size = 21, normalized size = 1.31

$$\frac{3}{4} \log\left((x^2)^{\frac{1}{3}} + 1\right) + \frac{3}{4} \log\left((x^2)^{\frac{1}{3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x^2-(x^2)^(1/3)),x, algorithm="maxima")

[Out] 3/4*log((x^2)^(1/3) + 1) + 3/4*log((x^2)^(1/3) - 1)

mupad [B] time = 3.34, size = 10, normalized size = 0.62

$$\frac{3 \ln\left((x^2)^{2/3} - 1\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-x/((x^2)^(1/3) - x^2), x)`

[Out] `(3*log((x^2)^(2/3) - 1))/4`

sympy [A] time = 0.22, size = 19, normalized size = 1.19

$$-\frac{\log(x)}{2} + \frac{3 \log\left(x^2 - \sqrt[3]{x^2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(x**2-(x**2)**(1/3)), x)`

[Out] `-log(x)/2 + 3*log(x**2 - (x**2)**(1/3))/4`

$$3.429 \quad \int x(1+x^2)^3 \sqrt{2+2x^2+x^4} dx$$

Optimal. Leaf size=44

$$\frac{1}{10}(x^2+1)^2(x^4+2x^2+2)^{3/2} - \frac{1}{15}(x^4+2x^2+2)^{3/2}$$

Rubi [A] time = 0.03, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1247, 692, 629}

$$\frac{1}{10}(x^2+1)^2(x^4+2x^2+2)^{3/2} - \frac{1}{15}(x^4+2x^2+2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*(1+x^2)^3*Sqrt[2+2*x^2+x^4],x]

[Out] -(2+2*x^2+x^4)^(3/2)/15 + ((1+x^2)^2*(2+2*x^2+x^4)^(3/2))/10

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 692

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p])) || OddQ[m])

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rubi steps

$$\begin{aligned} \int x(1+x^2)^3 \sqrt{2+2x^2+x^4} dx &= \frac{1}{2} \text{Subst} \left(\int (1+x)^3 \sqrt{2+2x+x^2} dx, x, x^2 \right) \\ &= \frac{1}{10} (1+x^2)^2 (2+2x^2+x^4)^{3/2} - \frac{1}{5} \text{Subst} \left(\int (1+x) \sqrt{2+2x+x^2} dx, x, x^2 \right) \\ &= -\frac{1}{15} (2+2x^2+x^4)^{3/2} + \frac{1}{10} (1+x^2)^2 (2+2x^2+x^4)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.68

$$\frac{1}{30}(x^4+2x^2+2)^{3/2}(3x^4+6x^2+1)$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4], x]

[Out] ((2 + 2*x^2 + x^4)^(3/2)*(1 + 6*x^2 + 3*x^4))/30

IntegrateAlgebraic [A] time = 0.28, size = 40, normalized size = 0.91

$$\frac{1}{30} \sqrt{x^4 + 2x^2 + 2} (3x^8 + 12x^6 + 19x^4 + 14x^2 + 2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(1 + x^2)^3*Sqrt[2 + 2*x^2 + x^4], x]

[Out] (Sqrt[2 + 2*x^2 + x^4]*(2 + 14*x^2 + 19*x^4 + 12*x^6 + 3*x^8))/30

fricas [A] time = 0.65, size = 36, normalized size = 0.82

$$\frac{1}{30} (3x^8 + 12x^6 + 19x^4 + 14x^2 + 2) \sqrt{x^4 + 2x^2 + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2), x, algorithm="fricas")

[Out] 1/30*(3*x^8 + 12*x^6 + 19*x^4 + 14*x^2 + 2)*sqrt(x^4 + 2*x^2 + 2)

giac [A] time = 0.35, size = 29, normalized size = 0.66

$$\frac{1}{10} (x^4 + 2x^2 + 2)^{\frac{5}{2}} - \frac{1}{6} (x^4 + 2x^2 + 2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2), x, algorithm="giac")

[Out] 1/10*(x^4 + 2*x^2 + 2)^(5/2) - 1/6*(x^4 + 2*x^2 + 2)^(3/2)

maple [A] time = 0.01, size = 27, normalized size = 0.61

$$\frac{(x^4 + 2x^2 + 2)^{\frac{3}{2}} (3x^4 + 6x^2 + 1)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2), x)

[Out] 1/30*(x^4+2*x^2+2)^(3/2)*(3*x^4+6*x^2+1)

maxima [A] time = 1.47, size = 49, normalized size = 1.11

$$\frac{1}{10} (x^4 + 2x^2 + 2)^{\frac{3}{2}} x^4 + \frac{1}{5} (x^4 + 2x^2 + 2)^{\frac{3}{2}} x^2 + \frac{1}{30} (x^4 + 2x^2 + 2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x^2+1)^3*(x^4+2*x^2+2)^(1/2), x, algorithm="maxima")

[Out] 1/10*(x^4 + 2*x^2 + 2)^(3/2)*x^4 + 1/5*(x^4 + 2*x^2 + 2)^(3/2)*x^2 + 1/30*(x^4 + 2*x^2 + 2)^(3/2)

mupad [B] time = 0.09, size = 26, normalized size = 0.59

$$\frac{(x^4 + 2x^2 + 2)^{\frac{3}{2}} (3x^4 + 6x^2 + 1)}{30}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^2 + 1)^3*(2*x^2 + x^4 + 2)^(1/2), x)`

[Out] `((2*x^2 + x^4 + 2)^(3/2)*(6*x^2 + 3*x^4 + 1))/30`

sympy [B] time = 0.65, size = 94, normalized size = 2.14

$$\frac{x^8\sqrt{x^4+2x^2+2}}{10} + \frac{2x^6\sqrt{x^4+2x^2+2}}{5} + \frac{19x^4\sqrt{x^4+2x^2+2}}{30} + \frac{7x^2\sqrt{x^4+2x^2+2}}{15} + \frac{\sqrt{x^4+2x^2+2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x**2+1)**3*(x**4+2*x**2+2)**(1/2), x)`

[Out] `x**8*sqrt(x**4 + 2*x**2 + 2)/10 + 2*x**6*sqrt(x**4 + 2*x**2 + 2)/5 + 19*x**4*sqrt(x**4 + 2*x**2 + 2)/30 + 7*x**2*sqrt(x**4 + 2*x**2 + 2)/15 + sqrt(x**4 + 2*x**2 + 2)/15`

$$3.430 \quad \int x^5 \sqrt{1-x^3} (1+x^9)^2 dx$$

Optimal. Leaf size=121

$$\frac{2}{51} (1-x^3)^{17/2} - \frac{14}{45} (1-x^3)^{15/2} + \frac{14}{13} (1-x^3)^{13/2} - \frac{74}{33} (1-x^3)^{11/2} + \frac{86}{27} (1-x^3)^{9/2} - \frac{22}{7} (1-x^3)^{7/2} + \frac{32}{15} (1-x^3)^{5/2}$$

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1821, 1620}

$$\frac{2}{51} (1-x^3)^{17/2} - \frac{14}{45} (1-x^3)^{15/2} + \frac{14}{13} (1-x^3)^{13/2} - \frac{74}{33} (1-x^3)^{11/2} + \frac{86}{27} (1-x^3)^{9/2} - \frac{22}{7} (1-x^3)^{7/2} + \frac{32}{15} (1-x^3)^{5/2} - \frac{8}{9} (1-x^3)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]

[Out] (-8*(1 - x^3)^(3/2))/9 + (32*(1 - x^3)^(5/2))/15 - (22*(1 - x^3)^(7/2))/7 + (86*(1 - x^3)^(9/2))/27 - (74*(1 - x^3)^(11/2))/33 + (14*(1 - x^3)^(13/2))/13 - (14*(1 - x^3)^(15/2))/45 + (2*(1 - x^3)^(17/2))/51

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rule 1821

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*SubstFor[x^n, Pq, x]*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && PolyQ[Pq, x^n] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int x^5 \sqrt{1-x^3} (1+x^9)^2 dx &= \frac{1}{3} \text{Subst} \left(\int \sqrt{1-x} x (1+x^3)^2 dx, x, x^3 \right) \\ &= \frac{1}{3} \text{Subst} \left(\int \left(4\sqrt{1-x} - 16(1-x)^{3/2} + 33(1-x)^{5/2} - 43(1-x)^{7/2} + 37(1-x)^{9/2} - \frac{8}{9}(1-x)^{3/2} + \frac{32}{15}(1-x)^{5/2} - \frac{22}{7}(1-x)^{7/2} + \frac{86}{27}(1-x)^{9/2} - \frac{74}{33}(1-x)^{11/2} \right) dx, x, x^3 \right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3} (45045x^{24} - 3003x^{21} - 3234x^{18} + 135702x^{15} - 19390x^{12} - 22160x^9 + 126561x^6 - 86507x^3 - 173014)}{2297295}$$

Antiderivative was successfully verified.

[In] Integrate[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]

[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295

IntegrateAlgebraic [A] time = 0.04, size = 57, normalized size = 0.47

$$\frac{2\sqrt{1-x^3} (45045x^{24} - 3003x^{21} - 3234x^{18} + 135702x^{15} - 19390x^{12} - 22160x^9 + 126561x^6 - 86507x^3 - 173014)}{2297295}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^5*Sqrt[1 - x^3]*(1 + x^9)^2,x]

[Out] (2*Sqrt[1 - x^3]*(-173014 - 86507*x^3 + 126561*x^6 - 22160*x^9 - 19390*x^12 + 135702*x^15 - 3234*x^18 - 3003*x^21 + 45045*x^24))/2297295

fricas [A] time = 0.53, size = 53, normalized size = 0.44

$$\frac{2}{2297295} (45045 x^{24} - 3003 x^{21} - 3234 x^{18} + 135702 x^{15} - 19390 x^{12} - 22160 x^9 + 126561 x^6 - 86507 x^3 - 173014) \sqrt{-x^3 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="fricas")

[Out] 2/2297295*(45045*x^24 - 3003*x^21 - 3234*x^18 + 135702*x^15 - 19390*x^12 - 22160*x^9 + 126561*x^6 - 86507*x^3 - 173014)*sqrt(-x^3 + 1)

giac [A] time = 0.37, size = 138, normalized size = 1.14

$$\frac{2}{51} (x^3 - 1)^8 \sqrt{-x^3 + 1} + \frac{14}{45} (x^3 - 1)^7 \sqrt{-x^3 + 1} + \frac{14}{13} (x^3 - 1)^6 \sqrt{-x^3 + 1} + \frac{74}{33} (x^3 - 1)^5 \sqrt{-x^3 + 1} + \frac{86}{27} (x^3 - 1)^4 \sqrt{-x^3 + 1} + \frac{22}{7} (x^3 - 1)^3 \sqrt{-x^3 + 1} + \frac{32}{15} (x^3 - 1)^2 \sqrt{-x^3 + 1} - \frac{8}{9} (-x^3 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="giac")

[Out] 2/51*(x^3 - 1)^8*sqrt(-x^3 + 1) + 14/45*(x^3 - 1)^7*sqrt(-x^3 + 1) + 14/13*(x^3 - 1)^6*sqrt(-x^3 + 1) + 74/33*(x^3 - 1)^5*sqrt(-x^3 + 1) + 86/27*(x^3 - 1)^4*sqrt(-x^3 + 1) + 22/7*(x^3 - 1)^3*sqrt(-x^3 + 1) + 32/15*(x^3 - 1)^2*sqrt(-x^3 + 1) - 8/9*(-x^3 + 1)^(3/2)

maple [A] time = 0.01, size = 58, normalized size = 0.48

$$\frac{2\sqrt{-x^3 + 1} (45045x^{21} + 42042x^{18} + 38808x^{15} + 174510x^{12} + 155120x^9 + 132960x^6 + 259521x^3 + 173014)(x - 1)(x^2 + x + 1)}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x)

[Out] 2/2297295*(-x^3+1)^(1/2)*(45045*x^21+42042*x^18+38808*x^15+174510*x^12+155120*x^9+132960*x^6+259521*x^3+173014)*(x-1)*(x^2+x+1)

maxima [A] time = 1.54, size = 89, normalized size = 0.74

$$\frac{2}{51} (-x^3 + 1)^{\frac{17}{2}} - \frac{14}{45} (-x^3 + 1)^{\frac{15}{2}} + \frac{14}{13} (-x^3 + 1)^{\frac{13}{2}} - \frac{74}{33} (-x^3 + 1)^{\frac{11}{2}} + \frac{86}{27} (-x^3 + 1)^{\frac{9}{2}} - \frac{22}{7} (-x^3 + 1)^{\frac{7}{2}} + \frac{32}{15} (-x^3 + 1)^{\frac{5}{2}} - \frac{8}{9} (-x^3 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5*(x^9+1)^2*(-x^3+1)^(1/2),x, algorithm="maxima")

[Out] 2/51*(-x^3 + 1)^(17/2) - 14/45*(-x^3 + 1)^(15/2) + 14/13*(-x^3 + 1)^(13/2) - 74/33*(-x^3 + 1)^(11/2) + 86/27*(-x^3 + 1)^(9/2) - 22/7*(-x^3 + 1)^(7/2) + 32/15*(-x^3 + 1)^(5/2) - 8/9*(-x^3 + 1)^(3/2)

mupad [B] time = 3.27, size = 124, normalized size = 1.02

$$\frac{84374x^6\sqrt{1-x^3}}{765765} - \frac{173014x^3\sqrt{1-x^3}}{2297295} - \frac{8864x^9\sqrt{1-x^3}}{459459} - \frac{1108x^{12}\sqrt{1-x^3}}{65637} + \frac{1436x^{15}\sqrt{1-x^3}}{12155} - \frac{28x^{18}\sqrt{1-x^3}}{9945} - \frac{2x^{21}\sqrt{1-x^3}}{765} + \frac{2x^{24}\sqrt{1-x^3}}{51} - \frac{346028\sqrt{1-x^3}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5*(1 - x^3)^(1/2)*(x^9 + 1)^2,x)

```
[Out] (84374*x^6*(1 - x^3)^(1/2))/765765 - (173014*x^3*(1 - x^3)^(1/2))/2297295 -
(8864*x^9*(1 - x^3)^(1/2))/459459 - (1108*x^12*(1 - x^3)^(1/2))/65637 + (1
436*x^15*(1 - x^3)^(1/2))/12155 - (28*x^18*(1 - x^3)^(1/2))/9945 - (2*x^21*
(1 - x^3)^(1/2))/765 + (2*x^24*(1 - x^3)^(1/2))/51 - (346028*(1 - x^3)^(1/2
))/2297295
```

sympy [A] time = 12.71, size = 133, normalized size = 1.10

$$\frac{2x^{24}\sqrt{1-x^3}}{51} - \frac{2x^{21}\sqrt{1-x^3}}{765} - \frac{28x^{18}\sqrt{1-x^3}}{9945} + \frac{1436x^{15}\sqrt{1-x^3}}{12155} - \frac{1108x^{12}\sqrt{1-x^3}}{65637} - \frac{8864x^9\sqrt{1-x^3}}{459459} + \frac{84374x^6\sqrt{1-x^3}}{765765} - \frac{173014x^3\sqrt{1-x^3}}{2297295} - \frac{346028\sqrt{1-x^3}}{2297295}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**9+1)**2*(-x**3+1)**(1/2),x)
```

```
[Out] 2*x**24*sqrt(1 - x**3)/51 - 2*x**21*sqrt(1 - x**3)/765 - 28*x**18*sqrt(1 -
x**3)/9945 + 1436*x**15*sqrt(1 - x**3)/12155 - 1108*x**12*sqrt(1 - x**3)/65
637 - 8864*x**9*sqrt(1 - x**3)/459459 + 84374*x**6*sqrt(1 - x**3)/765765 -
173014*x**3*sqrt(1 - x**3)/2297295 - 346028*sqrt(1 - x**3)/2297295
```

$$3.431 \quad \int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx$$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]),x]

[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx &= \int \frac{x}{(a+bx^2)^{3/2}} dx + \int \frac{x}{(1+x^2)\sqrt{a+bx^2}} dx \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2} \operatorname{Subst} \left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, x^2 \right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\operatorname{Subst} \left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx^2} \right)}{b} \\
&= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]), x]

[Out] (a - b + Sqrt[a - b]*b*Sqrt[a + b*x^2]*ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]])/(b*(-a + b)*Sqrt[a + b*x^2])

IntegrateAlgebraic [A] time = 0.08, size = 57, normalized size = 1.14

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]), x]

[Out] -(1/(b*Sqrt[a + b*x^2])) - ArcTan[(Sqrt[-a + b]*Sqrt[a + b*x^2])/(a - b)]/Sqrt[-a + b]

fricas [B] time = 0.79, size = 268, normalized size = 5.36

$$\left[\frac{(b^2x^2 + ab)\sqrt{a-b} \log\left(\frac{b^2x^4 + 2(4ab - 3b^2)x^2 - 4(bx^2 + 2a - b)\sqrt{bx^2 + a}\sqrt{a-b} + 8a^2 - 8ab + b^2}{x^4 + 2x^2 + 1}\right) - 4\sqrt{bx^2 + a}(a - b)}{4(a^2b - ab^2 + (ab^2 - b^3)x^2)}, \frac{(b^2x^2 + ab)\sqrt{-a + b} \arctan\left(-\frac{(bx^2 + 2a - b)\sqrt{bx^2 + a}\sqrt{-a + b}}{2((ab - b^2)x^2 + a^2 - ab)}\right) + 2\sqrt{bx^2 + a}(a - b)}{2(a^2b - ab^2 + (ab^2 - b^3)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/4*((b^2*x^2 + a*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2), -1/2*((b^2*x^2 + a*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*sqrt(b*x^2 + a)*(a - b))/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]

giac [A] time = 0.35, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)

maple [A] time = 0.03, size = 42, normalized size = 0.84

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)

[Out] -1/b/(b*x^2+a)^(1/2)+1/(b-a)^(1/2)*arctan((b*x^2+a)^(1/2)/(b-a)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 3.76, size = 42, normalized size = 0.84

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{1}{b\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*x^2)^(3/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)

[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(1/2))

sympy [A] time = 3.71, size = 49, normalized size = 0.98

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^2} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)

[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)

$$3.432 \quad \int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}} dx$$

Optimal. Leaf size=50

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.06, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {6, 571, 78, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 + a + x^2 + b*x^2))/((1 + x^2)*(a + b*x^2)^(3/2)),x]

[Out] -(1/(b*sqrt[a + b*x^2])) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 571

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x(1+a+x^2+bx^2)}{(1+x^2)(a+bx^2)^{3/2}} dx &= \int \frac{x(1+a+(1+b)x^2)}{(1+x^2)(a+bx^2)^{3/2}} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1+a+(1+b)x}{(1+x)(a+bx)^{3/2}} dx, x, x^2 \right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x)\sqrt{a+bx}} dx, x, x^2 \right) \\
&= -\frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst} \left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+bx^2} \right)}{b} \\
&= -\frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 71, normalized size = 1.42

$$\frac{b\sqrt{a-b}\sqrt{a+bx^2}\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)+a-b}{b(b-a)\sqrt{a+bx^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1+a+x^2+b*x^2))/((1+x^2)*(a+b*x^2)^(3/2)),x]

[Out] (a-b+Sqrt[a-b]*b*Sqrt[a+b*x^2]*ArcTanh[Sqrt[a+b*x^2]/Sqrt[a-b]])/(b*(-a+b)*Sqrt[a+b*x^2])

IntegrateAlgebraic [A] time = 0.07, size = 57, normalized size = 1.14

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{a+bx^2}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(1+a+x^2+b*x^2))/((1+x^2)*(a+b*x^2)^(3/2)),x]

[Out] -(1/(b*Sqrt[a+b*x^2])) - ArcTan[(Sqrt[-a+b]*Sqrt[a+b*x^2])/(a-b)]/Sqrt[-a+b]

fricas [B] time = 0.76, size = 268, normalized size = 5.36

$$\left[\frac{(b^2x^2+ab)\sqrt{a-b}\log\left(\frac{b^2x^4+2(4ab-3b^2)x^2-4(bx^2+2a-b)\sqrt{bx^2+a}\sqrt{a-b}+8a^2-8ab+b^2}{x^4+2x^2+1}\right)-4\sqrt{bx^2+a}(a-b)}{4(a^2b-ab^2+(ab^2-b^3)x^2)}, -\frac{(b^2x^2+ab)\sqrt{-a+b}\arctan\left(-\frac{(bx^2+2a-b)\sqrt{bx^2+a}\sqrt{-a+b}}{2((ab-b^2)x^2+a^2-ab)}\right)+2\sqrt{bx^2+a}(a-b)}{2(a^2b-ab^2+(ab^2-b^3)x^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="fricas")

[Out] [1/4*((b^2*x^2+a*b)*sqrt(a-b)*log((b^2*x^4+2*(4*a*b-3*b^2)*x^2-4*(b*x^2+2*a-b)*sqrt(b*x^2+a)*sqrt(a-b)+8*a^2-8*a*b+b^2)/(x^4+2*x^2+1))-4*sqrt(b*x^2+a)*(a-b))/(a^2*b-a*b^2+(a*b^2-b^3)*x^2), -1/2*((b^2*x^2+a*b)*sqrt(-a+b)*arctan(-1/2*(b*x^2+2*a-b)*sqrt(b

$*x^2 + a)*\sqrt{-a + b}/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*\sqrt{b*x^2 + a}*(a - b)/(a^2*b - a*b^2 + (a*b^2 - b^3)*x^2)]$

giac [A] time = 0.38, size = 41, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b)

maple [B] time = 0.02, size = 133, normalized size = 2.66

$$\frac{a \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}} - \frac{b \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{(a-b)\sqrt{-a+b}} + \frac{a}{(a-b)\sqrt{bx^2+a}} - \frac{b}{(a-b)\sqrt{bx^2+a}} - \frac{1}{\sqrt{bx^2+ab}} - \frac{1}{\sqrt{bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x)

[Out] $-1/(b*x^2+a)^{(1/2)} - 1/(b*x^2+a)^{(1/2)}/b - b/(-b+a)/(b*x^2+a)^{(1/2)} - b/(-b+a)/(-a+b)^{(1/2)} * \arctan((b*x^2+a)^{(1/2)}/(-a+b)^{(1/2)}) + a/(-b+a)/(b*x^2+a)^{(1/2)} + a/(-b+a)/(-a+b)^{(1/2)} * \arctan((b*x^2+a)^{(1/2)}/(-a+b)^{(1/2)})$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b*x^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(3/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see 'assume?' for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 4.52, size = 96, normalized size = 1.92

$$\frac{1}{\sqrt{bx^2+a}(a-b)} - \frac{a}{\sqrt{bx^2+a}(ab-b^2)} - \frac{a \operatorname{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{b \operatorname{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + b*x^2 + x^2 + 1))/((x^2 + 1)*(a + b*x^2)^(3/2)),x)

[Out] $1/((a + b*x^2)^{(1/2)}*(a - b)) - a/((a + b*x^2)^{(1/2)}*(a*b - b^2)) - (a*\operatorname{atanh}((a + b*x^2)^{(1/2)}/(a - b)^{(1/2)}))/((a - b)^{(3/2)} + (b*\operatorname{atanh}((a + b*x^2)^{(1/2)}/(a - b)^{(1/2)}))/((a - b)^{(3/2)})$

sympy [A] time = 79.83, size = 37, normalized size = 0.74

$$\frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{b\sqrt{a+bx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(3/2),x)
```

```
[Out] atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b) - 1/(b*sqrt(a + b*x**2))
```

$$3.433 \quad \int \left(\frac{x}{(a+bx^2)^{5/2}} + \frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx$$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 47, $\frac{\text{number of rules}}{\text{integrand size}} = 0.085$, Rules used = {261, 444, 63, 208}

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]), x]

[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 261

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 444

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{(a+bx^2)^{5/2}} + \frac{x}{(a+bx^2)^{3/2}} + \frac{x}{(1+x^2)\sqrt{a+bx^2}} \right) dx &= \int \frac{x}{(a+bx^2)^{5/2}} dx + \int \frac{x}{(a+bx^2)^{3/2}} dx + \int \frac{x}{(1+x^2)\sqrt{a+bx^2}} dx \\
&= -\frac{1}{3b(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}} dx \right) \\
&= -\frac{1}{3b(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \frac{\text{Subst} \left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}} dx \right)}{b} \\
&= -\frac{1}{3b(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.19, size = 63, normalized size = 0.93

$$\frac{-3a - 3bx^2 - 1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]), x]

[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]

IntegrateAlgebraic [A] time = 0.13, size = 70, normalized size = 1.03

$$\frac{-3a - 3bx^2 - 1}{3b(a+bx^2)^{3/2}} - \frac{\tan^{-1} \left(\frac{\sqrt{b-a}\sqrt{a+bx^2}}{a-b} \right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(a + b*x^2)^(5/2) + x/(a + b*x^2)^(3/2) + x/((1 + x^2)*Sqrt[a + b*x^2]), x]

[Out] (-1 - 3*a - 3*b*x^2)/(3*b*(a + b*x^2)^(3/2)) - ArcTan[(Sqrt[-a + b]*Sqrt[a + b*x^2])/(a - b)]/Sqrt[-a + b]

fricas [B] time = 0.94, size = 382, normalized size = 5.62

$$\frac{3(b^3x^4 + 2ab^2x^2 + a^2b)\sqrt{a-b} \log\left(\frac{b^2x^4 + 2(4ab-3b^2)x^2 - 4((a^2+2a-b)\sqrt{a-b} + \sqrt{a-b} + 8ab + a^2)}{x^4 + 2x^2 + 1}\right) - 4(3(ab-b^2)x^2 + 3a^2 - (3a+1)b+a)\sqrt{bx^2+a}}{12((ab^3-b^4)x^4 + a^2b - a^2b^2 + 2(a^2b^2 - ab^3)x^2)} - \frac{3(b^3x^4 + 2ab^2x^2 + a^2b)\sqrt{-a+b} \arctan\left(\frac{(bx^2+a)\sqrt{a-b}}{2((ab-b^2)x^2 + a^2 - ab)}\right) + 2(3(ab-b^2)x^2 + 3a^2 - (3a+1)b+a)\sqrt{bx^2+a}}{6((ab^3-b^4)x^4 + a^2b - a^2b^2 + 2(a^2b^2 - ab^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(a - b)*log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a)]/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 -

$a*b^3*x^2$), $-1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sqrt(-a + b)*arctan(-1/2*(b*x^2 + 2*a - b)*sqrt(b*x^2 + a)*sqrt(-a + b)/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*sqrt(b*x^2 + a)/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]$

giac [A] time = 0.35, size = 55, normalized size = 0.81

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{\sqrt{bx^2+a}b} - \frac{1}{3(bx^2+a)^{\frac{3}{2}}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(b*x^2 + a)/sqrt(-a + b))/sqrt(-a + b) - 1/(sqrt(b*x^2 + a)*b) - 1/3/((b*x^2 + a)^(3/2)*b)

maple [A] time = 0.02, size = 56, normalized size = 0.82

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{1}{3(bx^2+a)^{\frac{3}{2}}b} - \frac{1}{\sqrt{bx^2+a}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x)

[Out] -1/3/b/(b*x^2+a)^(3/2)-1/(b*x^2+a)^(1/2)/b+1/(-a+b)^(1/2)*arctan((b*x^2+a)^(1/2)/(-a+b)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(b*x^2+a)^(5/2)+x/(b*x^2+a)^(3/2)+x/(x^2+1)/(b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see 'assume?' for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 3.69, size = 56, normalized size = 0.82

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{1}{b\sqrt{bx^2+a}} - \frac{1}{3b(bx^2+a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a + b*x^2)^(3/2) + x/(a + b*x^2)^(5/2) + x/((x^2 + 1)*(a + b*x^2)^(1/2)),x)

[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - 1/(b*(a + b*x^2)^(1/2)) - 1/(3*b*(a + b*x^2)^(3/2))

sympy [A] time = 4.39, size = 97, normalized size = 1.43

$$\begin{cases} -\frac{1}{b\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} + \begin{cases} -\frac{1}{3ab\sqrt{a+bx^2}+3b^2x^2\sqrt{a+bx^2}} & \text{for } b \neq 0 \\ \frac{x^2}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} + \frac{\operatorname{atan}\left(\frac{\sqrt{a+bx^2}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(b*x**2+a)**(5/2)+x/(b*x**2+a)**(3/2)+x/(x**2+1)/(b*x**2+a)**(1/2),x)
```

```
[Out] Piecewise((-1/(b*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(3/2)), True)) +
Piecewise((-1/(3*a*b*sqrt(a + b*x**2) + 3*b**2*x**2*sqrt(a + b*x**2)), Ne(b, 0)), (x**2/(2*a**(5/2)), True)) + atan(sqrt(a + b*x**2)/sqrt(-a + b))/sqrt(-a + b)
```

$$3.434 \quad \int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} dx$$

Optimal. Leaf size=68

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.51, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {6, 6715, 897, 1261, 207}

$$-\frac{1}{b\sqrt{a+bx^2}} - \frac{1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[(x*(1 + a + a^2 + x^2 + a*x^2 + b*x^2 + 2*a*b*x^2 + b*x^4 + b^2*x^4))/((1 + x^2)*(a + b*x^2)^(5/2)),x]

[Out] -1/(3*b*(a + b*x^2)^(3/2)) - 1/(b*Sqrt[a + b*x^2]) - ArcTanh[Sqrt[a + b*x^2]/Sqrt[a - b]]/Sqrt[a - b]

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_) + (b_.)*(v_))^(p_.), x_Symbol] := Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 897

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := With[{q = Denominator[m]}, Dist[q/e, Subst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + (g*x^q)/e)^n*((c*d^2 - b*d*e + a*e^2)/e^2 - ((2*c*d - b*e)*x^q)/e^2 + (c*x^(2*q))/e^2]^p, x], x, (d + e*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && FractionQ[m]

Rule 1261

Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]

Rule 6715

Int[(u_)*(x_)^(m_.), x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), u, x], x], x, x^(m + 1)], x] /; FreeQ[m, x] && NeQ[m, -1] && FunctionQ[fQ[x^(m + 1), u, x]

Rubi steps

$$\begin{aligned}
\int \frac{x(1+a+a^2+x^2+ax^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} dx &= \int \frac{x(1+a+a^2+(1+a)x^2+bx^2+2abx^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} \\
&= \int \frac{x(1+a+a^2+2abx^2+(1+a+b)x^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} \\
&= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+bx^4+b^2x^4)}{(1+x^2)(a+bx^2)^{5/2}} \\
&= \int \frac{x(1+a+a^2+(1+a+b+2ab)x^2+(b+b^2)x)}{(1+x^2)(a+bx^2)^{5/2}} \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1+a+a^2+(1+a+b+2ab)x+(b+b^2)}{(1+x)(a+bx)^{5/2}} \right. \\
&\quad \left. \text{Subst} \left(\int \frac{\frac{(1+a+a^2)b^2-ab(1+a+b+2ab)+a^2(b+b^2)}{b^2} - \frac{(-b(1+a+b+2ab)+2a)}{b^2}}{x^4 \left(\frac{-a+b}{b} + \frac{x^2}{b} \right)} \right) \right. \\
&= \frac{\text{Subst} \left(\int \left(\frac{1}{x^4} + \frac{1}{x^2} + \frac{b}{-a+bx^2} \right) dx, x, \sqrt{a+bx^2} \right)}{b} \\
&= -\frac{1}{3b(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} + \text{Subst} \left(\int \frac{1}{-a+bx^2} \right. \\
&= -\frac{1}{3b(a+bx^2)^{3/2}} - \frac{1}{b\sqrt{a+bx^2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 63, normalized size = 0.93

$$\frac{-3a-3bx^2-1}{3b(a+bx^2)^{3/2}} - \frac{\tanh^{-1} \left(\frac{\sqrt{a+bx^2}}{\sqrt{a-b}} \right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Integrate[(x*(1+a+a^2+x^2+ax^2+bx^2+2*a*b*x^2+bx^4+b^2*x^4))/((1+x^2)*(a+bx^2)^(5/2)),x]

[Out] (-1-3*a-3*b*x^2)/(3*b*(a+bx^2)^(3/2))-ArcTanh[Sqrt[a+bx^2]/Sqrt[a-b]]/Sqrt[a-b]

IntegrateAlgebraic [A] time = 0.08, size = 70, normalized size = 1.03

$$\frac{-3a-3bx^2-1}{3b(a+bx^2)^{3/2}} - \frac{\tan^{-1} \left(\frac{\sqrt{b-a}\sqrt{a+bx^2}}{a-b} \right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*(1+a+a^2+x^2+ax^2+bx^2+2*a*b*x^2+bx^4+b^2*x^4))/((1+x^2)*(a+bx^2)^(5/2)),x]

[Out] $(-1 - 3a - 3bx^2)/(3b(a + bx^2)^{3/2}) - \text{ArcTan}[(\text{Sqrt}[-a + b] \cdot \text{Sqrt}[a + bx^2])/(a - b)]/\text{Sqrt}[-a + b]$

fricas [B] time = 0.69, size = 382, normalized size = 5.62

$$\frac{3(b^3x^4 + 2ab^2x^2 + a^2b)\sqrt{a-b} \log\left(\frac{(b^2x^2 + 2a - b)\sqrt{b^2x^2 + a} - 4(3(ab - b^2)x^2 + 3a^2 - (3a + 1)b + a)\sqrt{bx^2 + a}}{x^2 + 2x + 1}\right) - 4(3(ab - b^2)x^2 + 3a^2 - (3a + 1)b + a)\sqrt{bx^2 + a} - 3(b^3x^4 + 2ab^2x^2 + a^2b)\sqrt{-a + b} \arctan\left(\frac{(b^2x^2 + 2a - b)\sqrt{b^2x^2 + a}}{2((ab - b^2)x^2 + a^2 - ab)}\right) + 2(3(ab - b^2)x^2 + 3a^2 - (3a + 1)b + a)\sqrt{bx^2 + a}}{12((ab^3 - b^4)x^4 + a^3b - a^2b^2 + 2(a^2b^2 - ab^3)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(5/2),x, algorithm="fricas")

[Out] $[1/12*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{sqrt}(a - b)*\log((b^2*x^4 + 2*(4*a*b - 3*b^2)*x^2 - 4*(b*x^2 + 2*a - b)*\text{sqrt}(b*x^2 + a)*\text{sqrt}(a - b) + 8*a^2 - 8*a*b + b^2)/(x^4 + 2*x^2 + 1)) - 4*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*\text{sqrt}(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2), -1/6*(3*(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*\text{sqrt}(-a + b)*\text{arctan}(-1/2*(b*x^2 + 2*a - b)*\text{sqrt}(b*x^2 + a)*\text{sqrt}(-a + b))/((a*b - b^2)*x^2 + a^2 - a*b)) + 2*(3*(a*b - b^2)*x^2 + 3*a^2 - (3*a + 1)*b + a)*\text{sqrt}(b*x^2 + a))/((a*b^3 - b^4)*x^4 + a^3*b - a^2*b^2 + 2*(a^2*b^2 - a*b^3)*x^2)]$

giac [A] time = 0.46, size = 52, normalized size = 0.76

$$\frac{\arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right)}{\sqrt{-a+b}} - \frac{3bx^2 + 3a + 1}{3(bx^2 + a)^{3/2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(5/2),x, algorithm="giac")

[Out] $\arctan(\text{sqrt}(b*x^2 + a)/\text{sqrt}(-a + b))/\text{sqrt}(-a + b) - 1/3*(3*b*x^2 + 3*a + 1)/((b*x^2 + a)^{(3/2)*b})$

maple [B] time = 0.03, size = 314, normalized size = 4.62

$$\frac{a^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right) - 2ab \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right) + b^2 \arctan\left(\frac{\sqrt{bx^2+a}}{\sqrt{-a+b}}\right) - \frac{bx^2}{(bx^2+a)^{3/2}} + \frac{a^2}{(a-b)^2 \sqrt{bx^2+a}} + \frac{a^2}{3(a-b)(bx^2+a)^{3/2}} - \frac{2ab}{(a-b)^2 \sqrt{bx^2+a}} - \frac{2ab}{3(a-b)(bx^2+a)^{3/2}} + \frac{b^2}{(a-b)^2 \sqrt{bx^2+a}} + \frac{b^2}{3(a-b)(bx^2+a)^{3/2}} - \frac{x^2}{(bx^2+a)^{3/2}} - \frac{4a}{3(bx^2+a)^{3/2}} + \frac{b}{3(bx^2+a)^{3/2}} - \frac{a}{(bx^2+a)^{3/2}b} - \frac{1}{3(bx^2+a)^{3/2}b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(5/2),x)

[Out] $-x^2*b/(b*x^2+a)^{(3/2)} - x^2/(b*x^2+a)^{(3/2)} - 4/3*a/(b*x^2+a)^{(3/2)} - a/b/(b*x^2+a)^{(3/2)} + 1/3*b/(b*x^2+a)^{(3/2)} - 1/3/(b*x^2+a)^{(3/2)}/b + 1/(a-b)^2/(b*x^2+a)^{(1/2)}*a^2 - 2/(a-b)^2/(b*x^2+a)^{(1/2)}*a*b + 1/(a-b)^2/(b*x^2+a)^{(1/2)}*b^2 + 1/(a-b)^2/(-a+b)^{(1/2)}*\arctan((b*x^2+a)^{(1/2)}/(-a+b)^{(1/2)})*a^2 - 2/(a-b)^2/(-a+b)^{(1/2)}*\arctan((b*x^2+a)^{(1/2)}/(-a+b)^{(1/2)})*a*b + 1/(a-b)^2/(-a+b)^{(1/2)}*\arctan((b*x^2+a)^{(1/2)}/(-a+b)^{(1/2)})*b^2 + 1/3/(a-b)/(b*x^2+a)^{(3/2)}*a^2 - 2/3/(a-b)/(b*x^2+a)^{(3/2)}*a*b + 1/3/(a-b)/(b*x^2+a)^{(3/2)}*b^2$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b^2*x^4+b*x^4+2*a*b*x^2+a*x^2+b*x^2+a^2+x^2+a+1)/(x^2+1)/(b*x^2+a)^(5/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see 'assume?' for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 3.90, size = 50, normalized size = 0.74

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{bx^2+a}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} - \frac{bx^2 + a + \frac{1}{3}}{b(bx^2 + a)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a + a*x^2 + b*x^2 + b*x^4 + a^2 + x^2 + b^2*x^4 + 2*a*b*x^2 + 1))/(x^2 + 1)*(a + b*x^2)^(5/2)), x)

[Out] - atanh((a + b*x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) - (a + b*x^2 + 1/3)/(b*(a + b*x^2)^(3/2))

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(b**2*x**4+b*x**4+2*a*b*x**2+a*x**2+b*x**2+a**2+x**2+a+1)/(x**2+1)/(b*x**2+a)**(5/2), x)

[Out] Timed out

$$3.435 \quad \int \frac{1}{\sqrt{\sqrt{x}+x}} dx$$

Optimal. Leaf size=34

$$2\sqrt{x + \sqrt{x}} - 2 \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{x + \sqrt{x}}} \right)$$

Rubi [A] time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2010, 2013, 620, 206}

$$2\sqrt{x + \sqrt{x}} - 2 \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{x + \sqrt{x}}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[Sqrt[x] + x],x]

[Out] 2*Sqrt[Sqrt[x] + x] - 2*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 2010

Int[1/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[(-2*Sqrt[a*x^j + b*x^n])/(b*(n - 2)*x^(n - 1)), x] - Dist[(a*(2*n - j - 2))/(b*(n - 2)), Int[1/(x^(n - j)*Sqrt[a*x^j + b*x^n]), x], x] /; FreeQ[{a, b}, x] && LtQ[2*(n - 1), j, n]

Rule 2013

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\sqrt{x}+x}} dx &= 2\sqrt{\sqrt{x}+x} - \frac{1}{2} \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x}+x}} dx \\
&= 2\sqrt{\sqrt{x}+x} - \text{Subst}\left(\int \frac{1}{\sqrt{x+x^2}} dx, x, \sqrt{x}\right) \\
&= 2\sqrt{\sqrt{x}+x} - 2\text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right) \\
&= 2\sqrt{\sqrt{x}+x} - 2\text{tanh}^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt{x}+x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}} \left(1 - \frac{\sinh^{-1}(\sqrt[4]{x})}{\sqrt{\sqrt{x}+1}\sqrt[4]{x}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[Sqrt[x] + x], x]

[Out] 2*Sqrt[Sqrt[x] + x]*(1 - ArcSinh[x^(1/4)]/(Sqrt[1 + Sqrt[x]]*x^(1/4)))

IntegrateAlgebraic [A] time = 0.11, size = 37, normalized size = 1.09

$$2\sqrt{x+\sqrt{x}} + \log\left(-2\sqrt{x} + 2\sqrt{x+\sqrt{x}} - 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[Sqrt[x] + x], x]

[Out] 2*Sqrt[Sqrt[x] + x] + Log[-1 - 2*Sqrt[x] + 2*Sqrt[Sqrt[x] + x]]

fricas [A] time = 1.11, size = 39, normalized size = 1.15

$$2\sqrt{x+\sqrt{x}} + \frac{1}{2} \log\left(4\sqrt{x+\sqrt{x}}(2\sqrt{x}+1) - 8x - 8\sqrt{x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+x^(1/2))^(1/2), x, algorithm="fricas")

[Out] 2*sqrt(x + sqrt(x)) + 1/2*log(4*sqrt(x + sqrt(x))*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 1)

giac [A] time = 0.37, size = 27, normalized size = 0.79

$$2\sqrt{x+\sqrt{x}} + \log\left(-2\sqrt{x+\sqrt{x}} + 2\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+x^(1/2))^(1/2), x, algorithm="giac")

[Out] 2*sqrt(x + sqrt(x)) + log(-2*sqrt(x + sqrt(x)) + 2*sqrt(x) + 1)

maple [A] time = 0.01, size = 44, normalized size = 1.29

$$\frac{\sqrt{x + \sqrt{x}} \left(\ln \left(\sqrt{x} + \frac{1}{2} + \sqrt{x + \sqrt{x}} \right) - 2\sqrt{x + \sqrt{x}} \right)}{\sqrt{(\sqrt{x} + 1)\sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+x^(1/2))^(1/2),x)

[Out] $-(x+x^{1/2})^{1/2} * (-2*(x+x^{1/2})^{1/2} + \ln(x^{1/2} + 1/2 + (x+x^{1/2})^{1/2})) / (x^{1/2} * (x^{1/2} + 1))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x + sqrt(x)), x)

mupad [B] time = 3.33, size = 39, normalized size = 1.15

$$\frac{2\sqrt{x}(\sqrt{x} + 1) + x^{1/4} \operatorname{asin}(x^{1/4} 1i) \sqrt{\sqrt{x} + 1} 2i}{\sqrt{x + \sqrt{x}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + x^(1/2))^(1/2),x)

[Out] $(2*x^{1/2}*(x^{1/2} + 1) + x^{1/4}*\operatorname{asin}(x^{1/4}*1i)*(x^{1/2} + 1)^{1/2}*2i) / (x + x^{1/2})^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{x} + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+x**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(sqrt(x) + x), x)

$$3.436 \quad \int \sqrt{\sqrt{x} + x} dx$$

Optimal. Leaf size=74

$$\frac{2}{3}\sqrt{x+\sqrt{x}}x + \frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x} - \frac{\sqrt{x+\sqrt{x}}}{4} + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$, Rules used = {2004, 2018, 670, 640, 620, 206}

$$\frac{2}{3}\sqrt{x+\sqrt{x}}x + \frac{1}{6}\sqrt{x+\sqrt{x}}\sqrt{x} - \frac{\sqrt{x+\sqrt{x}}}{4} + \frac{1}{4}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sqrt[x] + x], x]

[Out] -Sqrt[Sqrt[x] + x]/4 + (Sqrt[x]*Sqrt[Sqrt[x] + x])/6 + (2*x*Sqrt[Sqrt[x] + x])/3 + ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 670

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 2004

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2018

```
Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt{x} + x} dx &= \frac{2}{3}x\sqrt{\sqrt{x} + x} + \frac{1}{6} \int \frac{\sqrt{x}}{\sqrt{\sqrt{x} + x}} dx \\
&= \frac{2}{3}x\sqrt{\sqrt{x} + x} + \frac{1}{3} \text{Subst} \left(\int \frac{x^2}{\sqrt{x + x^2}} dx, x, \sqrt{x} \right) \\
&= \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x} + x} + \frac{2}{3}x\sqrt{\sqrt{x} + x} - \frac{1}{4} \text{Subst} \left(\int \frac{x}{\sqrt{x + x^2}} dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x} + x} + \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x} + x} + \frac{2}{3}x\sqrt{\sqrt{x} + x} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{\sqrt{x + x^2}} dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x} + x} + \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x} + x} + \frac{2}{3}x\sqrt{\sqrt{x} + x} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt{x} + x}} \right) \\
&= -\frac{1}{4}\sqrt{\sqrt{x} + x} + \frac{1}{6}\sqrt{x}\sqrt{\sqrt{x} + x} + \frac{2}{3}x\sqrt{\sqrt{x} + x} + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{\sqrt{x} + x}} \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 51, normalized size = 0.69

$$\frac{1}{12}\sqrt{x + \sqrt{x}} \left(8x + 2\sqrt{x} + \frac{3 \sinh^{-1}(\sqrt[4]{x})}{\sqrt{\sqrt{x} + 1} \sqrt[4]{x}} - 3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sqrt[x] + x], x]

[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x + (3*ArcSinh[x^(1/4)])))/(Sqrt[1 + Sqrt[x]]*x^(1/4))/12

IntegrateAlgebraic [A] time = 0.14, size = 50, normalized size = 0.68

$$\frac{1}{12}\sqrt{x + \sqrt{x}} (8x + 2\sqrt{x} - 3) + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{x + \sqrt{x}}}{\sqrt{x}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[Sqrt[x] + x], x]

[Out] (Sqrt[Sqrt[x] + x]*(-3 + 2*Sqrt[x] + 8*x))/12 + ArcTanh[Sqrt[Sqrt[x] + x]/Sqrt[x]]/4

fricas [A] time = 1.90, size = 49, normalized size = 0.66

$$\frac{1}{12} (8x + 2\sqrt{x} - 3)\sqrt{x + \sqrt{x}} + \frac{1}{16} \log \left(4\sqrt{x + \sqrt{x}} (2\sqrt{x} + 1) + 8x + 8\sqrt{x} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/12*(8*x + 2*sqrt(x) - 3)*sqrt(x + sqrt(x)) + 1/16*log(4*sqrt(x + sqrt(x))
*(2*sqrt(x) + 1) + 8*x + 8*sqrt(x) + 1)

giac [A] time = 0.45, size = 43, normalized size = 0.58

$$\frac{1}{12} (2\sqrt{x}(4\sqrt{x} + 1) - 3)\sqrt{x + \sqrt{x}} - \frac{1}{8} \log\left(-2\sqrt{x + \sqrt{x}} + 2\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 3)*sqrt(x + sqrt(x)) - 1/8*log(-2*sqrt(x
+ sqrt(x)) + 2*sqrt(x) + 1)

maple [A] time = 0.00, size = 42, normalized size = 0.57

$$\frac{\ln\left(\sqrt{x} + \frac{1}{2} + \sqrt{x + \sqrt{x}}\right)}{8} + \frac{2(x + \sqrt{x})^{\frac{3}{2}}}{3} - \frac{(2\sqrt{x} + 1)\sqrt{x + \sqrt{x}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+x^(1/2))^(1/2),x)

[Out] 2/3*(x+x^(1/2))^(3/2)-1/4*(1+2*x^(1/2))*(x+x^(1/2))^(1/2)+1/8*ln(x^(1/2)+1/
2+(x+x^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(x)), x)

mupad [B] time = 3.16, size = 27, normalized size = 0.36

$$\frac{4x\sqrt{x + \sqrt{x}} {}_2F_1\left(-\frac{1}{2}, \frac{5}{2}; \frac{7}{2}; -\sqrt{x}\right)}{5\sqrt{\sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^(1/2))^(1/2),x)

[Out] (4*x*(x + x^(1/2))^(1/2)*hypergeom([-1/2, 5/2], 7/2, -x^(1/2)))/(5*(x^(1/2)
+ 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x**(1/2))**(1/2),x)

[Out] Integral(sqrt(sqrt(x) + x), x)

$$3.437 \quad \int \sqrt{-x} (\sqrt{-x} + x) dx$$

Optimal. Leaf size=19

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {14}

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-x]*(Sqrt[-x] + x),x]

[Out] (2*(-x)^(5/2))/5 - x^2/2

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \sqrt{-x} (\sqrt{-x} + x) dx &= \int (-(x)^{3/2} - x) dx \\ &= \frac{2}{5}(-x)^{5/2} - \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$\frac{2}{5}(-x)^{5/2} - \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-x]*(Sqrt[-x] + x),x]

[Out] (2*(-x)^(5/2))/5 - x^2/2

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 0.95

$$\frac{1}{10} (4\sqrt{-x} - 5) x^2$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-x]*(Sqrt[-x] + x),x]

[Out] ((-5 + 4*Sqrt[-x])*x^2)/10

fricas [A] time = 0.63, size = 16, normalized size = 0.84

$$\frac{2}{5} \sqrt{-x} x^2 - \frac{1}{2} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="fricas")

[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2

giac [A] time = 0.31, size = 16, normalized size = 0.84

$$\frac{2}{5} \sqrt{-x} x^2 - \frac{1}{2} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="giac")

[Out] 2/5*sqrt(-x)*x^2 - 1/2*x^2

maple [A] time = 0.00, size = 14, normalized size = 0.74

$$-\frac{x^2}{2} + \frac{2(-x)^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x)^(1/2)*(x+(-x)^(1/2)),x)

[Out] 2/5*(-x)^(5/2)-1/2*x^2

maxima [A] time = 0.67, size = 13, normalized size = 0.68

$$\frac{2}{5} (-x)^{\frac{5}{2}} - \frac{1}{2} x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)^(1/2)*(x+(-x)^(1/2)),x, algorithm="maxima")

[Out] 2/5*(-x)^(5/2) - 1/2*x^2

mupad [B] time = 0.03, size = 13, normalized size = 0.68

$$\frac{2(-x)^{5/2}}{5} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x)^(1/2)*(x + (-x)^(1/2)),x)

[Out] (2*(-x)^(5/2))/5 - x^2/2

sympy [C] time = 0.19, size = 14, normalized size = 0.74

$$\frac{2ix^{\frac{5}{2}}}{5} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x)**(1/2)*(x+(-x)**(1/2)),x)

[Out] 2*I*x**(5/2)/5 - x**2/2

$$3.438 \quad \int \frac{5 + \sqrt[4]{x}}{-6 + x} dx$$

Optimal. Leaf size=54

$$4\sqrt[4]{x} + 5\log(6 - x) - 2\sqrt[4]{6} \tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) - 2\sqrt[4]{6} \tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Rubi [A] time = 0.08, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1831, 260, 321, 212, 206, 203}

$$4\sqrt[4]{x} + 5\log(6 - x) - 2\sqrt[4]{6} \tan^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right) - 2\sqrt[4]{6} \tanh^{-1}\left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(5 + x^(1/4))/(-6 + x), x]

[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6^(1/4)] + 5*Log[6 - x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 321

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1831

Int[((Pq_)*((c_.)*(x_)^(m_.)))/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = Sum[((c*x)^(m + ii)*(Coeff[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(c^ii*(a + b*x^n)), {ii, 0, n/2 - 1}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && Expon[Pq, x] < n

Rubi steps

$$\begin{aligned}
\int \frac{5 + \sqrt[4]{x}}{-6 + x} dx &= 4 \operatorname{Subst} \left(\int \frac{x^3(5 + x)}{-6 + x^4} dx, x, \sqrt[4]{x} \right) \\
&= 4 \operatorname{Subst} \left(\int \left(\frac{5x^3}{-6 + x^4} + \frac{x^4}{-6 + x^4} \right) dx, x, \sqrt[4]{x} \right) \\
&= 4 \operatorname{Subst} \left(\int \frac{x^4}{-6 + x^4} dx, x, \sqrt[4]{x} \right) + 20 \operatorname{Subst} \left(\int \frac{x^3}{-6 + x^4} dx, x, \sqrt[4]{x} \right) \\
&= 4\sqrt[4]{x} + 5 \log(6 - x) + 24 \operatorname{Subst} \left(\int \frac{1}{-6 + x^4} dx, x, \sqrt[4]{x} \right) \\
&= 4\sqrt[4]{x} + 5 \log(6 - x) - (2\sqrt{6}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{6} - x^2} dx, x, \sqrt[4]{x} \right) - (2\sqrt{6}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{6} + x^2} dx, x, \sqrt[4]{x} \right) \\
&= 4\sqrt[4]{x} - 2\sqrt{6} \tan^{-1} \left(\frac{\sqrt[4]{x}}{\sqrt{6}} \right) - 2\sqrt{6} \tanh^{-1} \left(\frac{\sqrt[4]{x}}{\sqrt{6}} \right) + 5 \log(6 - x)
\end{aligned}$$

Mathematica [C] time = 0.08, size = 107, normalized size = 1.98

$$4\sqrt[4]{x} + (5 + \sqrt[4]{6}) \log(\sqrt[4]{6} - \sqrt[4]{x}) + (5 - i\sqrt[4]{6}) \log(\sqrt[4]{6} - i\sqrt[4]{x}) + (5 + i\sqrt[4]{6}) \log(\sqrt[4]{6} + i\sqrt[4]{x}) - (\sqrt[4]{6} - 5) \log(\sqrt[4]{x} + \sqrt[4]{6})$$

Antiderivative was successfully verified.

[In] Integrate[(5 + x^(1/4))/(-6 + x), x]

[Out] 4*x^(1/4) + (5 + 6^(1/4))*Log[6^(1/4) - x^(1/4)] + (5 - I*6^(1/4))*Log[6^(1/4) - I*x^(1/4)] + (5 + I*6^(1/4))*Log[6^(1/4) + I*x^(1/4)] - (-5 + 6^(1/4))*Log[6^(1/4) + x^(1/4)]

IntegrateAlgebraic [A] time = 0.12, size = 52, normalized size = 0.96

$$4\sqrt[4]{x} + 5 \log(x - 6) - 2\sqrt[4]{6} \tan^{-1} \left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}} \right) - 2\sqrt[4]{6} \tanh^{-1} \left(\frac{\sqrt[4]{x}}{\sqrt[4]{6}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(5 + x^(1/4))/(-6 + x), x]

[Out] 4*x^(1/4) - 2*6^(1/4)*ArcTan[x^(1/4)/6^(1/4)] - 2*6^(1/4)*ArcTanh[x^(1/4)/6^(1/4)] + 5*Log[-6 + x]

fricas [B] time = 0.74, size = 86, normalized size = 1.59

$$-(\frac{1}{6^{\frac{1}{4}}}-5) \log(2 \cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}) + (\frac{1}{6^{\frac{1}{4}}}+5) \log(-2 \cdot 6^{\frac{1}{4}}+2x^{\frac{1}{4}}) + 4 \cdot 6^{\frac{1}{4}} \arctan\left(\frac{1}{6} \cdot 6^{\frac{3}{4}} \sqrt{\sqrt{6} + \sqrt{x}} - \frac{1}{6} \cdot 6^{\frac{3}{4}} x^{\frac{1}{4}}\right) + 4x^{\frac{1}{4}} + 5 \log(4\sqrt{6} + 4\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x^(1/4))/(-6+x), x, algorithm="fricas")

[Out] -(6^(1/4) - 5)*log(2*6^(1/4) + 2*x^(1/4)) + (6^(1/4) + 5)*log(-2*6^(1/4) + 2*x^(1/4)) + 4*6^(1/4)*arctan(1/6*6^(3/4)*sqrt(sqrt(6) + sqrt(x)) - 1/6*6^(3/4)*x^(1/4)) + 4*x^(1/4) + 5*log(4*sqrt(6) + 4*sqrt(x))

giac [A] time = 0.49, size = 55, normalized size = 1.02

$$-2 \cdot 6^{\frac{1}{4}} \arctan \left(\frac{1}{6} \cdot 6^{\frac{3}{4}} x^{\frac{1}{4}} \right) - 6^{\frac{1}{4}} \log \left(6^{\frac{1}{4}} + x^{\frac{1}{4}} \right) + 6^{\frac{1}{4}} \log \left(\left| -6^{\frac{1}{4}} + x^{\frac{1}{4}} \right| \right) + 4x^{\frac{1}{4}} + 5 \log(|x - 6|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="giac")

[Out] $-2 \cdot 6^{1/4} \cdot \arctan(1/6 \cdot 6^{3/4} \cdot x^{1/4}) - 6^{1/4} \cdot \log(6^{1/4} + x^{1/4}) + 6^{1/4} \cdot \log(\text{abs}(-6^{1/4} + x^{1/4})) + 4 \cdot x^{1/4} + 5 \cdot \log(\text{abs}(x - 6))$

maple [A] time = 0.00, size = 52, normalized size = 0.96

$$-2 \cdot 6^{1/4} \arctan\left(\frac{6^{3/4} x^{1/4}}{6}\right) - 6^{1/4} \ln\left(\frac{x^{1/4} + 6^{1/4}}{x^{1/4} - 6^{1/4}}\right) + 5 \ln(x - 6) + 4x^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((5+x^(1/4))/(-6+x),x)

[Out] $4 \cdot x^{1/4} - 2 \cdot 6^{1/4} \cdot \arctan(1/6 \cdot x^{1/4} \cdot 6^{3/4}) - 6^{1/4} \cdot \ln((x^{1/4} + 6^{1/4}) / (x^{1/4} - 6^{1/4})) + 5 \cdot \ln(-6 + x)$

maxima [A] time = 1.33, size = 67, normalized size = 1.24

$$-2 \cdot 6^{1/4} \arctan\left(\frac{1}{6} \cdot 6^{3/4} x^{1/4}\right) + 6^{1/4} \log\left(-\frac{6^{1/4} - x^{1/4}}{6^{1/4} + x^{1/4}}\right) + 4x^{1/4} + 5 \log(\sqrt{6} + \sqrt{x}) + 5 \log(-\sqrt{6} + \sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x^(1/4))/(-6+x),x, algorithm="maxima")

[Out] $-2 \cdot 6^{1/4} \cdot \arctan(1/6 \cdot 6^{3/4} \cdot x^{1/4}) + 6^{1/4} \cdot \log(-(6^{1/4} - x^{1/4}) / (6^{1/4} + x^{1/4})) + 4 \cdot x^{1/4} + 5 \cdot \log(\text{sqrt}(6) + \text{sqrt}(x)) + 5 \cdot \log(-\text{sqrt}(6) + \text{sqrt}(x))$

mupad [B] time = 0.09, size = 162, normalized size = 3.00

$\ln(11520x^{1/4} - (6^{1/4} + 5)(2304x^{1/4} - 2304 \cdot 6^{1/4} + 11520) + 57600)(6^{1/4} + 5) - \ln((6^{1/4} - 5)(2304x^{1/4} + 2304 \cdot 6^{1/4} + 11520) + 57600)(6^{1/4} - 5) - \ln(11520x^{1/4} + (\sqrt{-6} - 5)(2304\sqrt{-6} + 2304x^{1/4} + 11520) + 57600)(\sqrt{-6} - 5) + \ln(11520x^{1/4} - (\sqrt{-6} + 5)(2304\sqrt{-6} - 2304x^{1/4} + 11520) + 57600)(\sqrt{-6} + 5) + 4x^{1/4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/4) + 5)/(x - 6),x)

[Out] $\log(11520x^{1/4} - (6^{1/4} + 5)(2304x^{1/4} - 2304 \cdot 6^{1/4} + 11520) + 57600)(6^{1/4} + 5) - \log((6^{1/4} - 5)(2304x^{1/4} + 2304 \cdot 6^{1/4} + 11520) + 11520x^{1/4} + 57600)(6^{1/4} - 5) - \log(11520x^{1/4} + ((-6^{1/2})^{1/2} - 5)(2304 \cdot (-6^{1/2})^{1/2} + 2304x^{1/4} + 11520) + 57600)((-6^{1/2})^{1/2} - 5) + \log(11520x^{1/4} - ((-6^{1/2})^{1/2} + 5)(2304x^{1/4} - 2304 \cdot (-6^{1/2})^{1/2} + 11520) + 57600)((-6^{1/2})^{1/2} + 5) + 4x^{1/4}$

sympy [A] time = 1.45, size = 100, normalized size = 1.85

$$4\sqrt[4]{x} + \sqrt[4]{6} \log(\sqrt[4]{x} - \sqrt[4]{6}) + 5 \log(\sqrt[4]{x} - \sqrt[4]{6}) - \sqrt[4]{6} \log(\sqrt[4]{x} + \sqrt[4]{6}) + 5 \log(\sqrt[4]{x} + \sqrt[4]{6}) + 5 \log(\sqrt{x} + \sqrt{6}) - 2\sqrt[4]{6} \operatorname{atan}\left(\frac{6^{3/4}\sqrt[4]{x}}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((5+x**(1/4))/(-6+x),x)

[Out] $4 \cdot x^{1/4} + 6^{1/4} \cdot \log(x^{1/4} - 6^{1/4}) + 5 \cdot \log(x^{1/4} - 6^{1/4}) - 6^{1/4} \cdot \log(x^{1/4} + 6^{1/4}) + 5 \cdot \log(x^{1/4} + 6^{1/4}) + 5 \cdot \log(\text{sqrt}(x) + \text{sqrt}(6)) - 2 \cdot 6^{1/4} \cdot \operatorname{atan}(6^{3/4} \cdot x^{1/4} / 6)$

$$3.439 \quad \int \frac{1}{4 + \sqrt{4-x} - x} dx$$

Optimal. Leaf size=14

$$-2 \log(\sqrt{4-x} + 1)$$

Rubi [A] time = 0.02, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {31}

$$-2 \log(\sqrt{4-x} + 1)$$

Antiderivative was successfully verified.

[In] Int[(4 + Sqrt[4 - x] - x)^(-1), x]

[Out] -2*Log[1 + Sqrt[4 - x]]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{4 + \sqrt{4-x} - x} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt{4-x} \right) \right) \\ &= -2 \log(1 + \sqrt{4-x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$-2 \log(\sqrt{4-x} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + Sqrt[4 - x] - x)^(-1), x]

[Out] -2*Log[1 + Sqrt[4 - x]]

IntegrateAlgebraic [A] time = 0.01, size = 14, normalized size = 1.00

$$-2 \log(\sqrt{4-x} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(4 + Sqrt[4 - x] - x)^(-1), x]

[Out] -2*Log[1 + Sqrt[4 - x]]

fricas [A] time = 1.12, size = 12, normalized size = 0.86

$$-2 \log(\sqrt{-x+4} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="fricas")

[Out] -2*log(sqrt(-x + 4) + 1)

giac [A] time = 0.31, size = 12, normalized size = 0.86

$$-2 \log\left(\sqrt{-x + 4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="giac")

[Out] -2*log(sqrt(-x + 4) + 1)

maple [A] time = 0.01, size = 18, normalized size = 1.29

$$-2 \operatorname{arctanh}\left(\sqrt{-x + 4}\right) - \ln(x - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4-x+(4-x)^(1/2)),x)

[Out] -ln(x-3)-2*arctanh((4-x)^(1/2))

maxima [A] time = 0.66, size = 12, normalized size = 0.86

$$-2 \log\left(\sqrt{-x + 4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-x+(4-x)^(1/2)),x, algorithm="maxima")

[Out] -2*log(sqrt(-x + 4) + 1)

mupad [B] time = 0.19, size = 12, normalized size = 0.86

$$-2 \ln\left(\sqrt{4 - x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((4 - x)^(1/2) - x + 4),x)

[Out] -2*log((4 - x)^(1/2) + 1)

sympy [B] time = 4.23, size = 32, normalized size = 2.29

$$\log\left(2\sqrt{4 - x}\right) - \log\left(2\sqrt{4 - x} + 2\right) - \log\left(x - \sqrt{4 - x} - 4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4-x+(4-x)**(1/2)),x)

[Out] log(2*sqrt(4 - x)) - log(2*sqrt(4 - x) + 2) - log(x - sqrt(4 - x) - 4)

$$3.440 \quad \int \frac{1}{1+x-\sqrt{2+x}} dx$$

Optimal. Leaf size=61

$$\frac{1}{5}(5-\sqrt{5})\log(-2\sqrt{x+2}-\sqrt{5}+1) + \frac{1}{5}(5+\sqrt{5})\log(-2\sqrt{x+2}+\sqrt{5}+1)$$

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {632, 31}

$$\frac{1}{5}(5-\sqrt{5})\log(-2\sqrt{x+2}-\sqrt{5}+1) + \frac{1}{5}(5+\sqrt{5})\log(-2\sqrt{x+2}+\sqrt{5}+1)$$

Antiderivative was successfully verified.

[In] Int[(1 + x - Sqrt[2 + x])^(-1), x]

[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{1+x-\sqrt{2+x}} dx &= 2 \text{Subst} \left(\int \frac{x}{-1-x+x^2} dx, x, \sqrt{2+x} \right) \\ &= \frac{1}{5}(5-\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{2+x} \right) + \frac{1}{5}(5+\sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{2+x} \right) \\ &= \frac{1}{5}(5-\sqrt{5})\log(1-\sqrt{5}-2\sqrt{2+x}) + \frac{1}{5}(5+\sqrt{5})\log(1+\sqrt{5}-2\sqrt{2+x}) \end{aligned}$$

Mathematica [A] time = 0.04, size = 58, normalized size = 0.95

$$\frac{1}{5}((5+\sqrt{5})\log(-2\sqrt{x+2}+\sqrt{5}+1) - (\sqrt{5}-5)\log(-2\sqrt{x+2}-\sqrt{5}+1))$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x - Sqrt[2 + x])^(-1), x]

[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[2 + x]]) + (5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5

IntegrateAlgebraic [A] time = 0.08, size = 59, normalized size = 0.97

$$\frac{1}{5}(5+\sqrt{5})\log(-2\sqrt{x+2}+\sqrt{5}+1) + \frac{1}{5}(5-\sqrt{5})\log(2\sqrt{x+2}+\sqrt{5}-1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x - Sqrt[2 + x])^(-1), x]

[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[2 + x]])/5 + ((5 - Sqrt[5])*Log[-1 + Sqrt[5] + 2*Sqrt[2 + x]])/5

fricas [A] time = 0.97, size = 63, normalized size = 1.03

$$\frac{1}{5} \sqrt{5} \log \left(\frac{2x^2 - \sqrt{5}(x+3) - (\sqrt{5}(2x+1) - 5)\sqrt{x+2} + 7x+3}{x^2 + x - 1} \right) + \log(x - \sqrt{x+2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x-(2+x)^(1/2)), x, algorithm="fricas")

[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 3) - (sqrt(5)*(2*x + 1) - 5)*sqrt(x + 2) + 7*x + 3)/(x^2 + x - 1)) + log(x - sqrt(x + 2) + 1)

giac [A] time = 0.33, size = 50, normalized size = 0.82

$$\frac{1}{5} \sqrt{5} \log \left(\frac{|\sqrt{5} + 2\sqrt{x+2} - 1|}{|\sqrt{5} - 2\sqrt{x+2} - 1|} \right) + \log(|x - \sqrt{x+2} + 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x-(2+x)^(1/2)), x, algorithm="giac")

[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 2) - 1)/abs(sqrt(5) + 2*sqrt(x + 2) - 1)) + log(abs(x - sqrt(x + 2) + 1))

maple [A] time = 0.01, size = 91, normalized size = 1.49

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2\sqrt{x+2}-1)\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2\sqrt{x+2}+1)\sqrt{5}}{5}\right)}{5} + \frac{\ln(x+1-\sqrt{x+2})}{2} - \frac{\ln(x+1+\sqrt{x+2})}{2} + \frac{\ln(x^2+x-1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x-(x+2)^(1/2)), x)

[Out] -1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(x^2+x-1)-1/2*ln(1+x+(x+2)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)+1)*5^(1/2))+1/2*ln(1+x-(x+2)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+2)^(1/2)-1)*5^(1/2))

maxima [A] time = 1.48, size = 46, normalized size = 0.75

$$\frac{1}{5} \sqrt{5} \log \left(-\frac{\sqrt{5} - 2\sqrt{x+2} + 1}{\sqrt{5} + 2\sqrt{x+2} - 1} \right) + \log(x - \sqrt{x+2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x-(2+x)^(1/2)), x, algorithm="maxima")

[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 2) + 1)/(sqrt(5) + 2*sqrt(x + 2) - 1)) + log(x - sqrt(x + 2) + 1)

mupad [B] time = 3.24, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+2} - \left(\frac{\sqrt{5}}{5} + 1\right)(2\sqrt{x+2} - 1)\right)\left(\frac{\sqrt{5}}{5} + 1\right) - \ln\left(2\sqrt{x+2} + \left(\frac{\sqrt{5}}{5} - 1\right)(2\sqrt{x+2} - 1)\right)\left(\frac{\sqrt{5}}{5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - (x + 2)^(1/2) + 1),x)`

[Out] `log(2*(x + 2)^(1/2) - (5^(1/2)/5 + 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 + 1) - log(2*(x + 2)^(1/2) + (5^(1/2)/5 - 1)*(2*(x + 2)^(1/2) - 1))*(5^(1/2)/5 - 1)`

sympy [A] time = 2.55, size = 94, normalized size = 1.54

$$4 \left\{ \begin{array}{l} \frac{\sqrt{5} \operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} \quad \text{for } \left(\sqrt{x+2}-\frac{1}{2}\right)^2 > \frac{5}{4} \\ \frac{\sqrt{5} \operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+2}-\frac{1}{2}\right)}{5}\right)}{10} \quad \text{for } \left(\sqrt{x+2}-\frac{1}{2}\right)^2 < \frac{5}{4} \end{array} \right\} + \log(x - \sqrt{x+2} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x-(2+x)**(1/2)),x)`

[Out] `4*Piecewise((-sqrt(5)*acoth(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x + 2) - 1/2)**2 > 5/4), (-sqrt(5)*atanh(2*sqrt(5)*(sqrt(x + 2) - 1/2)/5)/10, (sqrt(x + 2) - 1/2)**2 < 5/4)) + log(x - sqrt(x + 2) + 1)`

$$3.441 \quad \int \frac{1}{4+x+\sqrt{1+x}} dx$$

Optimal. Leaf size=37

$$\log(x + \sqrt{x+1} + 4) - \frac{2 \tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {634, 618, 204, 628}

$$\log(x + \sqrt{x+1} + 4) - \frac{2 \tan^{-1}\left(\frac{2\sqrt{x+1}+1}{\sqrt{11}}\right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x + Sqrt[1 + x])^(-1), x]

[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]]/Sqrt[11] + Log[4 + x + Sqrt[1 + x]])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned}
\int \frac{1}{4+x+\sqrt{1+x}} dx &= 2 \operatorname{Subst} \left(\int \frac{x}{3+x+x^2} dx, x, \sqrt{1+x} \right) \\
&= -\operatorname{Subst} \left(\int \frac{1}{3+x+x^2} dx, x, \sqrt{1+x} \right) + \operatorname{Subst} \left(\int \frac{1+2x}{3+x+x^2} dx, x, \sqrt{1+x} \right) \\
&= \log(4+x+\sqrt{1+x}) + 2 \operatorname{Subst} \left(\int \frac{1}{-11-x^2} dx, x, 1+2\sqrt{1+x} \right) \\
&= -\frac{2 \tan^{-1} \left(\frac{1+2\sqrt{1+x}}{\sqrt{11}} \right)}{\sqrt{11}} + \log(4+x+\sqrt{1+x})
\end{aligned}$$

Mathematica [A] time = 0.01, size = 37, normalized size = 1.00

$$\log(x + \sqrt{x+1} + 4) - \frac{2 \tan^{-1} \left(\frac{2\sqrt{x+1}+1}{\sqrt{11}} \right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x + Sqrt[1 + x])^(-1), x]

[Out] (-2*ArcTan[(1 + 2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x]]

IntegrateAlgebraic [A] time = 0.05, size = 40, normalized size = 1.08

$$\log(x + \sqrt{x+1} + 4) - \frac{2 \tan^{-1} \left(\frac{2\sqrt{x+1}}{\sqrt{11}} + \frac{1}{\sqrt{11}} \right)}{\sqrt{11}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(4 + x + Sqrt[1 + x])^(-1), x]

[Out] (-2*ArcTan[1/Sqrt[11] + (2*Sqrt[1 + x])/Sqrt[11]])/Sqrt[11] + Log[4 + x + Sqrt[1 + x]]

fricas [A] time = 0.61, size = 32, normalized size = 0.86

$$-\frac{2}{11} \sqrt{11} \arctan \left(\frac{2}{11} \sqrt{11} \sqrt{x+1} + \frac{1}{11} \sqrt{11} \right) + \log(x + \sqrt{x+1} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="fricas")

[Out] -2/11*sqrt(11)*arctan(2/11*sqrt(11)*sqrt(x + 1) + 1/11*sqrt(11)) + log(x + sqrt(x + 1) + 4)

giac [A] time = 0.37, size = 30, normalized size = 0.81

$$-\frac{2}{11} \sqrt{11} \arctan \left(\frac{1}{11} \sqrt{11} (2\sqrt{x+1} + 1) \right) + \log(x + \sqrt{x+1} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="giac")

[Out] -2/11*sqrt(11)*arctan(1/11*sqrt(11)*(2*sqrt(x + 1) + 1)) + log(x + sqrt(x + 1) + 4)

maple [B] time = 0.01, size = 93, normalized size = 2.51

$$-\frac{\sqrt{11} \arctan\left(\frac{(1+2\sqrt{x+1})\sqrt{11}}{11}\right)}{11} + \frac{\sqrt{11} \arctan\left(\frac{(2x+7)\sqrt{11}}{11}\right)}{11} - \frac{\sqrt{11} \arctan\left(\frac{(2\sqrt{x+1}-1)\sqrt{11}}{11}\right)}{11} + \frac{\ln(x+4+\sqrt{x+1})}{2} - \frac{\ln(x+4-\sqrt{x+1})}{2} + \frac{\ln(x^2+7x+15)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4+x+(x+1)^(1/2)),x)

[Out] $-1/2*\ln(x+4-(x+1)^{(1/2)})-1/11*11^{(1/2)}*\arctan(1/11*(2*(x+1)^{(1/2)}-1)*11^{(1/2)})+1/2*\ln(4+x+(x+1)^{(1/2)})-1/11*\arctan(1/11*(1+2*(x+1)^{(1/2)})*11^{(1/2)})*11^{(1/2)}+1/11*11^{(1/2)}*\arctan(1/11*(2*x+7)*11^{(1/2)})+1/2*\ln(x^2+7*x+15)$

maxima [A] time = 1.73, size = 30, normalized size = 0.81

$$-\frac{2}{11} \sqrt{11} \arctan\left(\frac{1}{11} \sqrt{11} (2 \sqrt{x+1} + 1)\right) + \log(x + \sqrt{x+1} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x+(1+x)^(1/2)),x, algorithm="maxima")

[Out] $-2/11*\sqrt{11}*\arctan(1/11*\sqrt{11}*(2*\sqrt{x+1}+1))+\log(x+\sqrt{x+1}+4)$

mupad [B] time = 0.07, size = 32, normalized size = 0.86

$$\ln(x + \sqrt{x+1} + 4) - \frac{2\sqrt{11} \operatorname{atan}\left(\frac{\sqrt{11}}{11} + \frac{2\sqrt{11}\sqrt{x+1}}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(x+1)^(1/2)+4),x)

[Out] $\log(x+(x+1)^{(1/2)}+4)-(2*11^{(1/2)}*\operatorname{atan}(11^{(1/2)}/11+(2*11^{(1/2)}*(x+1)^{(1/2)}/11))/11$

sympy [A] time = 2.31, size = 39, normalized size = 1.05

$$\log(x + \sqrt{x+1} + 4) - \frac{2\sqrt{11} \operatorname{atan}\left(\frac{2\sqrt{11}\left(\sqrt{x+1} + \frac{1}{2}\right)}{11}\right)}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(4+x+(1+x)**(1/2)),x)

[Out] $\log(x+\sqrt{x+1}+4)-2*\sqrt{11}*\operatorname{atan}(2*\sqrt{11}*(\sqrt{x+1}+1/2)/11)/11$

$$3.442 \quad \int \frac{1}{x - \sqrt{1+x}} dx$$

Optimal. Leaf size=61

$$\frac{1}{5} (5 - \sqrt{5}) \log(-2\sqrt{x+1} - \sqrt{5} + 1) + \frac{1}{5} (5 + \sqrt{5}) \log(-2\sqrt{x+1} + \sqrt{5} + 1)$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {632, 31}

$$\frac{1}{5} (5 - \sqrt{5}) \log(-2\sqrt{x+1} - \sqrt{5} + 1) + \frac{1}{5} (5 + \sqrt{5}) \log(-2\sqrt{x+1} + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 + x])^(-1), x]

[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{x - \sqrt{1+x}} dx &= 2 \text{Subst} \left(\int \frac{x}{-1 - x + x^2} dx, x, \sqrt{1+x} \right) \\ &= \frac{1}{5} (5 - \sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{1+x} \right) + \frac{1}{5} (5 + \sqrt{5}) \text{Subst} \left(\int \frac{1}{-\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{1+x} \right) \\ &= \frac{1}{5} (5 - \sqrt{5}) \log(1 - \sqrt{5} - 2\sqrt{1+x}) + \frac{1}{5} (5 + \sqrt{5}) \log(1 + \sqrt{5} - 2\sqrt{1+x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.95

$$\frac{1}{5} \left((5 + \sqrt{5}) \log(-2\sqrt{x+1} + \sqrt{5} + 1) - (\sqrt{5} - 5) \log(-2\sqrt{x+1} - \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[1 + x])^(-1), x]

[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] - 2*Sqrt[1 + x]]) + (5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5

IntegrateAlgebraic [A] time = 0.06, size = 59, normalized size = 0.97

$$\frac{1}{5} (5 + \sqrt{5}) \log(-2\sqrt{x+1} + \sqrt{5} + 1) + \frac{1}{5} (5 - \sqrt{5}) \log(2\sqrt{x+1} + \sqrt{5} - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[1 + x])^(-1), x]

[Out] ((5 + Sqrt[5])*Log[1 + Sqrt[5] - 2*Sqrt[1 + x]])/5 + ((5 - Sqrt[5])*Log[-1 + Sqrt[5] + 2*Sqrt[1 + x]])/5

fricas [A] time = 0.92, size = 64, normalized size = 1.05

$$\frac{1}{5} \sqrt{5} \log\left(\frac{2x^2 - \sqrt{5}(x+2) - (\sqrt{5}(2x-1) - 5)\sqrt{x+1} + 3x - 2}{x^2 - x - 1}\right) + \log(x - \sqrt{x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1+x)^(1/2)), x, algorithm="fricas")

[Out] 1/5*sqrt(5)*log((2*x^2 - sqrt(5)*(x + 2) - (sqrt(5)*(2*x - 1) - 5)*sqrt(x + 1) + 3*x - 2)/(x^2 - x - 1)) + log(x - sqrt(x + 1))

giac [A] time = 0.36, size = 49, normalized size = 0.80

$$\frac{1}{5} \sqrt{5} \log\left(\frac{|-\sqrt{5} + 2\sqrt{x+1} - 1|}{|\sqrt{5} + 2\sqrt{x+1} - 1|}\right) + \log(|x - \sqrt{x+1}|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1+x)^(1/2)), x, algorithm="giac")

[Out] 1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(x + 1) - 1)/abs(sqrt(5) + 2*sqrt(x + 1) - 1)) + log(abs(x - sqrt(x + 1)))

maple [A] time = 0.01, size = 91, normalized size = 1.49

$$-\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x-1)\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(1+2\sqrt{x+1})\sqrt{5}}{5}\right)}{5} - \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2\sqrt{x+1}-1)\sqrt{5}}{5}\right)}{5} + \frac{\ln(x - \sqrt{x+1})}{2} - \frac{\ln(x + \sqrt{x+1})}{2} + \frac{\ln(x^2 - x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-(x+1)^(1/2)), x)

[Out] 1/2*ln(x^2-x-1)-1/5*5^(1/2)*arctanh(1/5*(2*x-1)*5^(1/2))-1/2*ln(x+(x+1)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(1+2*(x+1)^(1/2))*5^(1/2))+1/2*ln(x-(x+1)^(1/2))-1/5*5^(1/2)*arctanh(1/5*(2*(x+1)^(1/2)-1)*5^(1/2))

maxima [A] time = 1.51, size = 45, normalized size = 0.74

$$\frac{1}{5} \sqrt{5} \log\left(-\frac{\sqrt{5} - 2\sqrt{x+1} + 1}{\sqrt{5} + 2\sqrt{x+1} - 1}\right) + \log(x - \sqrt{x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1+x)^(1/2)), x, algorithm="maxima")

[Out] 1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(x + 1) + 1)/(sqrt(5) + 2*sqrt(x + 1) - 1)) + log(x - sqrt(x + 1))

mupad [B] time = 0.12, size = 71, normalized size = 1.16

$$\ln\left(2\sqrt{x+1} - \left(\frac{\sqrt{5}}{5} + 1\right)(2\sqrt{x+1} - 1)\right)\left(\frac{\sqrt{5}}{5} + 1\right) - \ln\left(2\sqrt{x+1} + \left(\frac{\sqrt{5}}{5} - 1\right)(2\sqrt{x+1} - 1)\right)\left(\frac{\sqrt{5}}{5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - (x + 1)^(1/2)),x)`

[Out] $\log(2*(x + 1)^{(1/2)} - (5^{(1/2)}/5 + 1)*(2*(x + 1)^{(1/2)} - 1))*(5^{(1/2)}/5 + 1) - \log(2*(x + 1)^{(1/2)} + (5^{(1/2)}/5 - 1)*(2*(x + 1)^{(1/2)} - 1))*(5^{(1/2)}/5 - 1)$

sympy [A] time = 2.21, size = 92, normalized size = 1.51

$$4 \left\{ \begin{array}{l} \frac{\sqrt{5} \operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} \quad \text{for } \left(\sqrt{x+1}-\frac{1}{2}\right)^2 > \frac{5}{4} \\ \frac{\sqrt{5} \operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{x+1}-\frac{1}{2}\right)}{5}\right)}{10} \quad \text{for } \left(\sqrt{x+1}-\frac{1}{2}\right)^2 < \frac{5}{4} \end{array} \right\} + \log(x - \sqrt{x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-(1+x)**(1/2)),x)`

[Out] $4*\operatorname{Piecewise}\left(\left(-\sqrt{5}*\operatorname{acoth}\left(2*\sqrt{5}*\left(\sqrt{x+1}-\frac{1}{2}\right)/5\right)/10,\left(\sqrt{x+1}-\frac{1}{2}\right)**2 > 5/4\right),\left(-\sqrt{5}*\operatorname{atanh}\left(2*\sqrt{5}*\left(\sqrt{x+1}-\frac{1}{2}\right)/5\right)/10,\left(\sqrt{x+1}-\frac{1}{2}\right)**2 < 5/4\right)\right) + \log(x - \sqrt{x+1})$

$$3.443 \quad \int \frac{1}{x - \sqrt{2+x}} dx$$

Optimal. Leaf size=31

$$\frac{4}{3} \log(2 - \sqrt{x+2}) + \frac{2}{3} \log(\sqrt{x+2} + 1)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {632, 31}

$$\frac{4}{3} \log(2 - \sqrt{x+2}) + \frac{2}{3} \log(\sqrt{x+2} + 1)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[2 + x])^(-1), x]

[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{x - \sqrt{2+x}} dx &= 2 \text{Subst} \left(\int \frac{x}{-2 - x + x^2} dx, x, \sqrt{2+x} \right) \\ &= \frac{2}{3} \text{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt{2+x} \right) + \frac{4}{3} \text{Subst} \left(\int \frac{1}{-2+x} dx, x, \sqrt{2+x} \right) \\ &= \frac{4}{3} \log(2 - \sqrt{2+x}) + \frac{2}{3} \log(1 + \sqrt{2+x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$\frac{4}{3} \log(2 - \sqrt{x+2}) + \frac{2}{3} \log(\sqrt{x+2} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[2 + x])^(-1), x]

[Out] (4*Log[2 - Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3

IntegrateAlgebraic [A] time = 0.02, size = 29, normalized size = 0.94

$$\frac{4}{3} \log(\sqrt{x+2} - 2) + \frac{2}{3} \log(\sqrt{x+2} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[2 + x])^(-1),x]

[Out] (4*Log[-2 + Sqrt[2 + x]])/3 + (2*Log[1 + Sqrt[2 + x]])/3

fricas [A] time = 0.63, size = 21, normalized size = 0.68

$$\frac{2}{3} \log(\sqrt{x+2} + 1) + \frac{4}{3} \log(\sqrt{x+2} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="fricas")

[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)

giac [A] time = 0.39, size = 22, normalized size = 0.71

$$\frac{2}{3} \log(\sqrt{x+2} + 1) + \frac{4}{3} \log(|\sqrt{x+2} - 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="giac")

[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(abs(sqrt(x + 2) - 2))

maple [B] time = 0.02, size = 54, normalized size = 1.74

$$\frac{2 \ln(x-2)}{3} + \frac{2 \ln(-2 + \sqrt{x+2})}{3} + \frac{\ln(x+1)}{3} + \frac{\ln(1 + \sqrt{x+2})}{3} - \frac{\ln(\sqrt{x+2} - 1)}{3} - \frac{2 \ln(\sqrt{x+2} + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-(x+2)^(1/2)),x)

[Out] 2/3*ln(x-2)+1/3*ln(x+1)-1/3*ln((x+2)^(1/2)-1)+2/3*ln(-2+(x+2)^(1/2))-2/3*ln((x+2)^(1/2)+2)+1/3*ln(1+(x+2)^(1/2))

maxima [A] time = 0.59, size = 21, normalized size = 0.68

$$\frac{2}{3} \log(\sqrt{x+2} + 1) + \frac{4}{3} \log(\sqrt{x+2} - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2+x)^(1/2)),x, algorithm="maxima")

[Out] 2/3*log(sqrt(x + 2) + 1) + 4/3*log(sqrt(x + 2) - 2)

mupad [B] time = 3.08, size = 25, normalized size = 0.81

$$\frac{2 \ln\left(\frac{2\sqrt{x+2}}{3} + \frac{2}{3}\right)}{3} + \frac{4 \ln\left(\frac{4}{3} - \frac{2\sqrt{x+2}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x - (x + 2)^(1/2)),x)

[Out] (2*log((2*(x + 2)^(1/2))/3 + 2/3))/3 + (4*log(4/3 - (2*(x + 2)^(1/2))/3))/3

sympy [A] time = 2.43, size = 36, normalized size = 1.16

$$\log(x - \sqrt{x+2}) + \frac{\log(2\sqrt{x+2} - 4)}{3} - \frac{\log(2\sqrt{x+2} + 2)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x-(2+x)**(1/2)),x)
```

```
[Out] log(x - sqrt(x + 2)) + log(2*sqrt(x + 2) - 4)/3 - log(2*sqrt(x + 2) + 2)/3
```

$$3.444 \quad \int \frac{1}{-\sqrt{1-x}+x} dx$$

Optimal. Leaf size=65

$$\frac{1}{5}(5 - \sqrt{5}) \log(2\sqrt{1-x} - \sqrt{5} + 1) + \frac{1}{5}(5 + \sqrt{5}) \log(2\sqrt{1-x} + \sqrt{5} + 1)$$

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {632, 31}

$$\frac{1}{5}(5 - \sqrt{5}) \log(2\sqrt{1-x} - \sqrt{5} + 1) + \frac{1}{5}(5 + \sqrt{5}) \log(2\sqrt{1-x} + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[1 - x] + x)^(-1), x]

[Out] ((5 - Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*Sqrt[1 - x]])/5

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{1}{-\sqrt{1-x}+x} dx &= 2 \text{Subst} \left(\int \frac{x}{-1+x+x^2} dx, x, \sqrt{1-x} \right) \\ &= \frac{1}{5}(5 - \sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{1}{2} - \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{1-x} \right) + \frac{1}{5}(5 + \sqrt{5}) \text{Subst} \left(\int \frac{1}{\frac{1}{2} + \frac{\sqrt{5}}{2} + x} dx, x, \sqrt{1-x} \right) \\ &= \frac{1}{5}(5 - \sqrt{5}) \log(1 - \sqrt{5} + 2\sqrt{1-x}) + \frac{1}{5}(5 + \sqrt{5}) \log(1 + \sqrt{5} + 2\sqrt{1-x}) \end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.95

$$\frac{1}{5} \left((5 + \sqrt{5}) \log(2\sqrt{1-x} + \sqrt{5} + 1) - (\sqrt{5} - 5) \log(2\sqrt{1-x} - \sqrt{5} + 1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[1 - x] + x)^(-1), x]

[Out] (-((-5 + Sqrt[5])*Log[1 - Sqrt[5] + 2*Sqrt[1 - x]]) + (5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*Sqrt[1 - x]])/5

IntegrateAlgebraic [A] time = 0.05, size = 63, normalized size = 0.97

$$\frac{1}{5}(5 - \sqrt{5}) \log(-2\sqrt{1-x} + \sqrt{5} - 1) + \frac{1}{5}(5 + \sqrt{5}) \log(2\sqrt{1-x} + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-Sqrt[1 - x] + x)^(-1), x]

[Out] ((5 - Sqrt[5])*Log[-1 + Sqrt[5] - 2*Sqrt[1 - x]])/5 + ((5 + Sqrt[5])*Log[1 + Sqrt[5] + 2*Sqrt[1 - x]])/5

fricas [A] time = 0.97, size = 65, normalized size = 1.00

$$\frac{1}{5} \sqrt{5} \log \left(\frac{2x^2 + \sqrt{5}(x-2) - (\sqrt{5}(2x+1) + 5)\sqrt{-x+1} - 3x - 2}{x^2 + x - 1} \right) + \log(-x + \sqrt{-x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1-x)^(1/2)), x, algorithm="fricas")

[Out] 1/5*sqrt(5)*log((2*x^2 + sqrt(5)*(x - 2) - (sqrt(5)*(2*x + 1) + 5)*sqrt(-x + 1) - 3*x - 2)/(x^2 + x - 1)) + log(-x + sqrt(-x + 1))

giac [A] time = 0.35, size = 54, normalized size = 0.83

$$-\frac{1}{5} \sqrt{5} \log \left(\frac{|-\sqrt{5} + 2\sqrt{-x+1} + 1|}{\sqrt{5} + 2\sqrt{-x+1} + 1} \right) + \log(|-x + \sqrt{-x+1}|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1-x)^(1/2)), x, algorithm="giac")

[Out] -1/5*sqrt(5)*log(abs(-sqrt(5) + 2*sqrt(-x + 1) + 1)/(sqrt(5) + 2*sqrt(-x + 1) + 1)) + log(abs(-x + sqrt(-x + 1)))

maple [B] time = 0.01, size = 101, normalized size = 1.55

$$\frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2x+1)\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2\sqrt{-x+1}-1)\sqrt{5}}{5}\right)}{5} + \frac{\sqrt{5} \operatorname{arctanh}\left(\frac{(2\sqrt{-x+1}+1)\sqrt{5}}{5}\right)}{5} - \frac{\ln(-x - \sqrt{-x+1})}{2} + \frac{\ln(-x + \sqrt{-x+1})}{2} + \frac{\ln(x^2 + x - 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-(-x+1)^(1/2)), x)

[Out] 1/2*ln(x^2+x-1)+1/5*5^(1/2)*arctanh(1/5*(2*x+1)*5^(1/2))+1/2*ln(-x+(-x+1)^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)+1)*5^(1/2))-1/2*ln(-x-(-x+1)^(1/2))+1/5*5^(1/2)*arctanh(1/5*(2*(-x+1)^(1/2)-1)*5^(1/2))

maxima [A] time = 1.32, size = 51, normalized size = 0.78

$$-\frac{1}{5} \sqrt{5} \log \left(-\frac{\sqrt{5} - 2\sqrt{-x+1} - 1}{\sqrt{5} + 2\sqrt{-x+1} + 1} \right) + \log(-x + \sqrt{-x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(1-x)^(1/2)), x, algorithm="maxima")

[Out] -1/5*sqrt(5)*log(-(sqrt(5) - 2*sqrt(-x + 1) - 1)/(sqrt(5) + 2*sqrt(-x + 1) + 1)) + log(-x + sqrt(-x + 1))

mupad [B] time = 3.16, size = 79, normalized size = 1.22

$$\ln\left(2\sqrt{1-x} - \left(\frac{\sqrt{5}}{5} + 1\right)\left(2\sqrt{1-x} + 1\right)\right)\left(\frac{\sqrt{5}}{5} + 1\right) - \ln\left(2\sqrt{1-x} + \left(\frac{\sqrt{5}}{5} - 1\right)\left(2\sqrt{1-x} + 1\right)\right)\left(\frac{\sqrt{5}}{5} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - (1 - x)^(1/2)),x)`

[Out] $\log(2*(1 - x)^{(1/2)} - (5^{(1/2)}/5 + 1)*(2*(1 - x)^{(1/2)} + 1))*(5^{(1/2)}/5 + 1) - \log(2*(1 - x)^{(1/2)} + (5^{(1/2)}/5 - 1)*(2*(1 - x)^{(1/2)} + 1))*(5^{(1/2)}/5 - 1)$

sympy [A] time = 2.31, size = 92, normalized size = 1.42

$$-4 \left(\begin{array}{l} \left(\frac{\sqrt{5} \operatorname{acoth}\left(\frac{2\sqrt{5}\left(\sqrt{1-x} + \frac{1}{2}\right)}{5}\right)}{10} \right) \text{ for } \left(\sqrt{1-x} + \frac{1}{2}\right)^2 > \frac{5}{4} \\ \left(\frac{\sqrt{5} \operatorname{atanh}\left(\frac{2\sqrt{5}\left(\sqrt{1-x} + \frac{1}{2}\right)}{5}\right)}{10} \right) \text{ for } \left(\sqrt{1-x} + \frac{1}{2}\right)^2 < \frac{5}{4} \end{array} \right) + \log(x - \sqrt{1-x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-(1-x)**(1/2)),x)`

[Out] $-4*\operatorname{Piecewise}\left(\left(-\sqrt{5}\operatorname{acoth}\left(2*\sqrt{5}\left(\sqrt{1-x} + \frac{1}{2}\right)/5\right)/10, \left(\sqrt{1-x} + \frac{1}{2}\right)**2 > 5/4\right), \left(-\sqrt{5}\operatorname{atanh}\left(2*\sqrt{5}\left(\sqrt{1-x} + \frac{1}{2}\right)/5\right)/10, \left(\sqrt{1-x} + \frac{1}{2}\right)**2 < 5/4\right)\right) + \log(x - \sqrt{1-x})$

$$3.445 \quad \int \sqrt{1 + \sqrt{x} + x} dx$$

Optimal. Leaf size=62

$$\frac{2}{3}(x + \sqrt{x} + 1)^{3/2} - \frac{1}{4}(2\sqrt{x} + 1)\sqrt{x + \sqrt{x} + 1} - \frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x} + 1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {1341, 640, 612, 619, 215}

$$\frac{2}{3}(x + \sqrt{x} + 1)^{3/2} - \frac{1}{4}(2\sqrt{x} + 1)\sqrt{x + \sqrt{x} + 1} - \frac{3}{8}\sinh^{-1}\left(\frac{2\sqrt{x} + 1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[x] + x], x]

[Out] -((1 + 2*Sqrt[x])*Sqrt[1 + Sqrt[x] + x])/4 + (2*(1 + Sqrt[x] + x)^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/8

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_) + (e_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))]^p, x], x, x^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \sqrt{1 + \sqrt{x} + x} dx &= 2 \operatorname{Subst} \left(\int x \sqrt{1 + x + x^2} dx, x, \sqrt{x} \right) \\
&= \frac{2}{3} (1 + \sqrt{x} + x)^{3/2} - \operatorname{Subst} \left(\int \sqrt{1 + x + x^2} dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 + \sqrt{x} + x} + \frac{2}{3} (1 + \sqrt{x} + x)^{3/2} - \frac{3}{8} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 + x + x^2}} dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 + \sqrt{x} + x} + \frac{2}{3} (1 + \sqrt{x} + x)^{3/2} - \frac{1}{8} \sqrt{3} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{3}}} dx, x, 1 + 2\sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 + \sqrt{x} + x} + \frac{2}{3} (1 + \sqrt{x} + x)^{3/2} - \frac{3}{8} \sinh^{-1} \left(\frac{1 + 2\sqrt{x}}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.79

$$\frac{1}{24} \left(2\sqrt{x + \sqrt{x} + 1} (8x + 2\sqrt{x} + 5) - 9 \sinh^{-1} \left(\frac{2\sqrt{x} + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sqrt[x] + x], x]

[Out] (2*Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x) - 9*ArcSinh[(1 + 2*Sqrt[x])/Sqrt[3]])/24

IntegrateAlgebraic [A] time = 0.13, size = 57, normalized size = 0.92

$$\frac{1}{12} \sqrt{x + \sqrt{x} + 1} (8x + 2\sqrt{x} + 5) + \frac{3}{8} \log \left(-2\sqrt{x} + 2\sqrt{x + \sqrt{x} + 1} - 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[x] + x], x]

[Out] (Sqrt[1 + Sqrt[x] + x]*(5 + 2*Sqrt[x] + 8*x))/12 + (3*Log[-1 - 2*Sqrt[x] + 2*Sqrt[1 + Sqrt[x] + x]])/8

fricas [A] time = 1.28, size = 51, normalized size = 0.82

$$\frac{1}{12} (8x + 2\sqrt{x} + 5) \sqrt{x + \sqrt{x} + 1} + \frac{3}{16} \log \left(4\sqrt{x + \sqrt{x} + 1} (2\sqrt{x} + 1) - 8x - 8\sqrt{x} - 5 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+x^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/12*(8*x + 2*sqrt(x) + 5)*sqrt(x + sqrt(x) + 1) + 3/16*log(4*sqrt(x + sqrt(x) + 1)*(2*sqrt(x) + 1) - 8*x - 8*sqrt(x) - 5)

giac [A] time = 0.40, size = 45, normalized size = 0.73

$$\frac{1}{12} (2\sqrt{x} (4\sqrt{x} + 1) + 5) \sqrt{x + \sqrt{x} + 1} + \frac{3}{8} \log \left(2\sqrt{x + \sqrt{x} + 1} - 2\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+x^(1/2))^(1/2), x, algorithm="giac")

[Out] $1/12*(2*\sqrt{x})*(4*\sqrt{x} + 1) + 5)*\sqrt{x + \sqrt{x} + 1} + 3/8*\log(2*\sqrt{x + \sqrt{x} + 1} - 2*\sqrt{x} - 1)$

maple [A] time = 0.01, size = 42, normalized size = 0.68

$$-\frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x}+\frac{1}{2}\right)}{3}\right)}{8} + \frac{2\left(x+\sqrt{x}+1\right)^{\frac{3}{2}}}{3} - \frac{\left(2\sqrt{x}+1\right)\sqrt{x+\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x+x^(1/2))^(1/2),x)`

[Out] $2/3*(1+x+x^{(1/2)})^{(3/2)}-1/4*(2*x^{(1/2)}+1)*(1+x+x^{(1/2)})^{(1/2)}-3/8*\operatorname{arcsinh}(2/3*3^{(1/2)}*(x^{(1/2)}+1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x+x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x + sqrt(x) + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x + \sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^(1/2) + 1)^(1/2),x)`

[Out] `int((x + x^(1/2) + 1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{x} + x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x+x**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(sqrt(x) + x + 1), x)`

$$3.446 \quad \int \sqrt{1+x+\sqrt{1+x}} dx$$

Optimal. Leaf size=75

$$\frac{2}{3} \left(x + \sqrt{x+1} + 1 \right)^{3/2} - \frac{1}{4} \left(2\sqrt{x+1} + 1 \right) \sqrt{x + \sqrt{x+1} + 1} + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{x+1}}{\sqrt{x + \sqrt{x+1} + 1}} \right)$$

Rubi [A] time = 0.04, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1980, 640, 612, 620, 206}

$$\frac{2}{3} \left(x + \sqrt{x+1} + 1 \right)^{3/2} - \frac{1}{4} \left(2\sqrt{x+1} + 1 \right) \sqrt{x + \sqrt{x+1} + 1} + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{x+1}}{\sqrt{x + \sqrt{x+1} + 1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x + Sqrt[1 + x]],x]

[Out] (2*(1 + x + Sqrt[1 + x])^(3/2))/3 - (Sqrt[1 + x + Sqrt[1 + x]]*(1 + 2*Sqrt[1 + x]))/4 + ArcTanh[Sqrt[1 + x]/Sqrt[1 + x + Sqrt[1 + x]]]/4

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1980

Int[(u_)^(p_.)*((c_.)*(x_)^(m_.)), x_Symbol] := Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{1+x+\sqrt{1+x}} \, dx &= 2 \operatorname{Subst} \left(\int x \sqrt{x(1+x)} \, dx, x, \sqrt{1+x} \right) \\
&= 2 \operatorname{Subst} \left(\int x \sqrt{x+x^2} \, dx, x, \sqrt{1+x} \right) \\
&= \frac{2}{3} (1+x+\sqrt{1+x})^{3/2} - \operatorname{Subst} \left(\int \sqrt{x+x^2} \, dx, x, \sqrt{1+x} \right) \\
&= \frac{2}{3} (1+x+\sqrt{1+x})^{3/2} - \frac{1}{4} \sqrt{1+x+\sqrt{1+x}} (1+2\sqrt{1+x}) + \frac{1}{8} \operatorname{Subst} \left(\int \frac{1}{\sqrt{x+x^2}} \, dx, x, \sqrt{1+x} \right) \\
&= \frac{2}{3} (1+x+\sqrt{1+x})^{3/2} - \frac{1}{4} \sqrt{1+x+\sqrt{1+x}} (1+2\sqrt{1+x}) + \frac{1}{4} \operatorname{Subst} \left(\int \frac{1}{1-x^2} \, dx, x, \sqrt{1+x} \right) \\
&= \frac{2}{3} (1+x+\sqrt{1+x})^{3/2} - \frac{1}{4} \sqrt{1+x+\sqrt{1+x}} (1+2\sqrt{1+x}) + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{1+x}}{\sqrt{1+x+\sqrt{1+x}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 62, normalized size = 0.83

$$\frac{1}{12} \sqrt{x+\sqrt{x+1}} + 1 \left(8x + 2\sqrt{x+1} + \frac{3 \sinh^{-1}(\sqrt[4]{x+1})}{\sqrt[4]{x+1} \sqrt{\sqrt{x+1}+1}} + 5 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x + Sqrt[1 + x]], x]

[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(5 + 8*x + 2*Sqrt[1 + x] + (3*ArcSinh[(1 + x)^(1/4)])))/((1 + x)^(1/4)*Sqrt[1 + Sqrt[1 + x]])/12

IntegrateAlgebraic [A] time = 0.13, size = 62, normalized size = 0.83

$$\frac{1}{12} \sqrt{x+\sqrt{x+1}} + 1 \left(8(x+1) + 2\sqrt{x+1} - 3 \right) + \frac{1}{4} \tanh^{-1} \left(\frac{\sqrt{x+\sqrt{x+1}} + 1}{\sqrt{x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x + Sqrt[1 + x]], x]

[Out] (Sqrt[1 + x + Sqrt[1 + x]]*(-3 + 2*Sqrt[1 + x] + 8*(1 + x)))/12 + ArcTanh[Sqrt[1 + x + Sqrt[1 + x]]/Sqrt[1 + x]]/4

fricas [A] time = 1.47, size = 61, normalized size = 0.81

$$\frac{1}{12} (8x + 2\sqrt{x+1} + 5) \sqrt{x+\sqrt{x+1}} + 1 + \frac{1}{16} \log \left(-4\sqrt{x+\sqrt{x+1}} + 1 (2\sqrt{x+1} + 1) - 8x - 8\sqrt{x+1} - 9 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+(1+x)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/12*(8*x + 2*sqrt(x + 1) + 5)*sqrt(x + sqrt(x + 1) + 1) + 1/16*log(-4*sqrt(x + sqrt(x + 1) + 1)*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 9)

giac [A] time = 0.41, size = 55, normalized size = 0.73

$$\frac{1}{12} (2\sqrt{x+1} (4\sqrt{x+1} + 1) - 3) \sqrt{x+\sqrt{x+1}} + 1 - \frac{1}{8} \log \left(-2\sqrt{x+\sqrt{x+1}} + 1 + 2\sqrt{x+1} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/12*(2*sqrt(x + 1)*(4*sqrt(x + 1) + 1) - 3)*sqrt(x + sqrt(x + 1) + 1) - 1/8*log(-2*sqrt(x + sqrt(x + 1) + 1) + 2*sqrt(x + 1) + 1)

maple [A] time = 0.01, size = 55, normalized size = 0.73

$$\frac{\ln\left(\sqrt{x+1} + \frac{1}{2} + \sqrt{x+1 + \sqrt{x+1}}\right)}{8} + \frac{2(x+1 + \sqrt{x+1})^{\frac{3}{2}}}{3} - \frac{(1 + 2\sqrt{x+1})\sqrt{x+1 + \sqrt{x+1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x+(x+1)^(1/2))^(1/2),x)

[Out] 2/3*(1+x+(x+1)^(1/2))^(3/2)-1/4*(1+2*(x+1)^(1/2))*(1+x+(x+1)^(1/2))^(1/2)+1/8*ln((x+1)^(1/2)+1/2+(1+x+(x+1)^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1}} + 1 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + sqrt(x + 1) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x + \sqrt{x+1}} + 1 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + (x + 1)^(1/2) + 1)^(1/2),x)

[Out] int((x + (x + 1)^(1/2) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x+1}} + 1 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x+(1+x)**(1/2))**(1/2),x)

[Out] Integral(sqrt(x + sqrt(x + 1) + 1), x)

$$3.447 \quad \int \sqrt{\sqrt{-1+x} + x} dx$$

Optimal. Leaf size=68

$$\frac{2}{3} \left(x + \sqrt{x-1} \right)^{3/2} - \frac{1}{4} \left(2\sqrt{x-1} + 1 \right) \sqrt{x + \sqrt{x-1}} - \frac{3}{8} \sinh^{-1} \left(\frac{2\sqrt{x-1} + 1}{\sqrt{3}} \right)$$

Rubi [A] time = 0.04, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {640, 612, 619, 215}

$$\frac{2}{3} \left(x + \sqrt{x-1} \right)^{3/2} - \frac{1}{4} \left(2\sqrt{x-1} + 1 \right) \sqrt{x + \sqrt{x-1}} - \frac{3}{8} \sinh^{-1} \left(\frac{2\sqrt{x-1} + 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Sqrt[-1 + x] + x], x]

[Out] -((1 + 2*Sqrt[-1 + x])*Sqrt[Sqrt[-1 + x] + x])/4 + (2*(Sqrt[-1 + x] + x)^(3/2))/3 - (3*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/8

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt{-1+x}+x} dx &= 2 \operatorname{Subst} \left(\int x \sqrt{1+x+x^2} dx, x, \sqrt{-1+x} \right) \\
&= \frac{2}{3} \left(\sqrt{-1+x}+x \right)^{3/2} - \operatorname{Subst} \left(\int \sqrt{1+x+x^2} dx, x, \sqrt{-1+x} \right) \\
&= -\frac{1}{4} \left(1+2\sqrt{-1+x} \right) \sqrt{\sqrt{-1+x}+x} + \frac{2}{3} \left(\sqrt{-1+x}+x \right)^{3/2} - \frac{3}{8} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+x+x^2}} dx, x, \sqrt{-1+x} \right) \\
&= -\frac{1}{4} \left(1+2\sqrt{-1+x} \right) \sqrt{\sqrt{-1+x}+x} + \frac{2}{3} \left(\sqrt{-1+x}+x \right)^{3/2} - \frac{1}{8} \sqrt{3} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, \sqrt{-1+x} \right) \\
&= -\frac{1}{4} \left(1+2\sqrt{-1+x} \right) \sqrt{\sqrt{-1+x}+x} + \frac{2}{3} \left(\sqrt{-1+x}+x \right)^{3/2} - \frac{3}{8} \sinh^{-1} \left(\frac{1+2\sqrt{-1+x}}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.79

$$\frac{1}{24} \left(2\sqrt{x+\sqrt{x-1}} \left(8x+2\sqrt{x-1}-3 \right) - 9 \sinh^{-1} \left(\frac{2\sqrt{x-1}+1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sqrt[-1 + x] + x], x]

[Out] (2*Sqrt[Sqrt[-1 + x] + x]*(-3 + 2*Sqrt[-1 + x] + 8*x) - 9*ArcSinh[(1 + 2*Sqrt[-1 + x])/Sqrt[3]])/24

IntegrateAlgebraic [A] time = 0.13, size = 65, normalized size = 0.96

$$\frac{1}{12} \sqrt{x+\sqrt{x-1}} \left(8(x-1) + 2\sqrt{x-1} + 5 \right) + \frac{3}{8} \log \left(-2\sqrt{x-1} + 2\sqrt{x+\sqrt{x-1}} - 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[Sqrt[-1 + x] + x], x]

[Out] ((5 + 2*Sqrt[-1 + x] + 8*(-1 + x))*Sqrt[Sqrt[-1 + x] + x])/12 + (3*Log[-1 - 2*Sqrt[-1 + x] + 2*Sqrt[Sqrt[-1 + x] + x]])/8

fricas [A] time = 1.52, size = 59, normalized size = 0.87

$$\frac{1}{12} \left(8x+2\sqrt{x-1}-3 \right) \sqrt{x+\sqrt{x-1}} + \frac{3}{16} \log \left(-4\sqrt{x+\sqrt{x-1}} \left(2\sqrt{x-1}+1 \right) + 8x+8\sqrt{x-1}-3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/12*(8*x + 2*sqrt(x - 1) - 3)*sqrt(x + sqrt(x - 1)) + 3/16*log(-4*sqrt(x + sqrt(x - 1))*(2*sqrt(x - 1) + 1) + 8*x + 8*sqrt(x - 1) - 3)

giac [A] time = 0.40, size = 53, normalized size = 0.78

$$\frac{1}{12} \left(2\sqrt{x-1} \left(4\sqrt{x-1} + 1 \right) + 5 \right) \sqrt{x+\sqrt{x-1}} + \frac{3}{8} \log \left(2\sqrt{x+\sqrt{x-1}} - 2\sqrt{x-1} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="giac")

[Out] $1/12*(2*\sqrt{x-1}*(4*\sqrt{x-1}+1)+5)*\sqrt{x+\sqrt{x-1}}+3/8*\log(2*\sqrt{x+\sqrt{x-1}}-2*\sqrt{x-1}-1)$

maple [A] time = 0.01, size = 48, normalized size = 0.71

$$-\frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}+\frac{1}{2}\right)}{3}\right)}{8} + \frac{2(x+\sqrt{x-1})^{\frac{3}{2}}}{3} - \frac{(1+2\sqrt{x-1})\sqrt{x+\sqrt{x-1}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+(x-1)^(1/2))^(1/2),x)`

[Out] $2/3*(x+(x-1)^{(1/2)})^{(3/2)}-1/4*(1+2*(x-1)^{(1/2)})*(x+(x-1)^{(1/2)})^{(1/2)}-3/8*\operatorname{arcsinh}(2/3*3^{(1/2)}*((x-1)^{(1/2)}+1/2))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(-1+x)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x + sqrt(x - 1)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x+\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+(x-1)^(1/2))^(1/2),x)`

[Out] `int((x+(x-1)^(1/2))^(1/2),x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x+\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(-1+x)**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(x + sqrt(x - 1)), x)`

$$3.448 \quad \int \sqrt{2x + \sqrt{-1 + 2x}} dx$$

Optimal. Leaf size=80

$$\frac{1}{3} \left(2x + \sqrt{2x-1} \right)^{3/2} - \frac{1}{8} \left(2\sqrt{2x-1} + 1 \right) \sqrt{2x + \sqrt{2x-1}} - \frac{3}{16} \sinh^{-1} \left(\frac{2\sqrt{2x-1} + 1}{\sqrt{3}} \right)$$

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {640, 612, 619, 215}

$$\frac{1}{3} \left(2x + \sqrt{2x-1} \right)^{3/2} - \frac{1}{8} \left(2\sqrt{2x-1} + 1 \right) \sqrt{2x + \sqrt{2x-1}} - \frac{3}{16} \sinh^{-1} \left(\frac{2\sqrt{2x-1} + 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2*x + Sqrt[-1 + 2*x]], x]

[Out] (2*x + Sqrt[-1 + 2*x])^(3/2)/3 - (Sqrt[2*x + Sqrt[-1 + 2*x]]*(1 + 2*Sqrt[-1 + 2*x]))/8 - (3*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/16

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{2x + \sqrt{-1 + 2x}} dx &= \text{Subst} \left(\int x \sqrt{1 + x + x^2} dx, x, \sqrt{-1 + 2x} \right) \\
&= \frac{1}{3} \left(2x + \sqrt{-1 + 2x} \right)^{3/2} - \frac{1}{2} \text{Subst} \left(\int \sqrt{1 + x + x^2} dx, x, \sqrt{-1 + 2x} \right) \\
&= \frac{1}{3} \left(2x + \sqrt{-1 + 2x} \right)^{3/2} - \frac{1}{8} \sqrt{2x + \sqrt{-1 + 2x}} \left(1 + 2\sqrt{-1 + 2x} \right) - \frac{3}{16} \text{Subst} \left(\int \frac{1}{\sqrt{1 + x + x^2}} dx, x, \sqrt{-1 + 2x} \right) \\
&= \frac{1}{3} \left(2x + \sqrt{-1 + 2x} \right)^{3/2} - \frac{1}{8} \sqrt{2x + \sqrt{-1 + 2x}} \left(1 + 2\sqrt{-1 + 2x} \right) - \frac{1}{16} \sqrt{3} \text{Subst} \left(\int \frac{1}{\sqrt{1 + x + x^2}} dx, x, \sqrt{-1 + 2x} \right) \\
&= \frac{1}{3} \left(2x + \sqrt{-1 + 2x} \right)^{3/2} - \frac{1}{8} \sqrt{2x + \sqrt{-1 + 2x}} \left(1 + 2\sqrt{-1 + 2x} \right) - \frac{3}{16} \sinh^{-1} \left(\frac{1 + 2\sqrt{-1 + 2x}}{\sqrt{3}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 62, normalized size = 0.78

$$\frac{1}{48} \left(2\sqrt{2x + \sqrt{2x - 1}} \left(16x + 2\sqrt{2x - 1} - 3 \right) - 9 \sinh^{-1} \left(\frac{2\sqrt{2x - 1} + 1}{\sqrt{3}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2*x + Sqrt[-1 + 2*x]], x]

[Out] (2*Sqrt[2*x + Sqrt[-1 + 2*x]]*(-3 + 16*x + 2*Sqrt[-1 + 2*x]) - 9*ArcSinh[(1 + 2*Sqrt[-1 + 2*x])/Sqrt[3]])/48

IntegrateAlgebraic [A] time = 0.13, size = 79, normalized size = 0.99

$$\frac{1}{24} \sqrt{2x + \sqrt{2x - 1}} \left(8(2x - 1) + 2\sqrt{2x - 1} + 5 \right) + \frac{3}{16} \log \left(-2\sqrt{2x - 1} + 2\sqrt{2x + \sqrt{2x - 1}} - 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2*x + Sqrt[-1 + 2*x]], x]

[Out] (Sqrt[2*x + Sqrt[-1 + 2*x]]*(5 + 2*Sqrt[-1 + 2*x] + 8*(-1 + 2*x)))/24 + (3*Log[-1 - 2*Sqrt[-1 + 2*x] + 2*Sqrt[2*x + Sqrt[-1 + 2*x]]])/16

fricas [A] time = 1.39, size = 73, normalized size = 0.91

$$\frac{1}{24} \left(16x + 2\sqrt{2x - 1} - 3 \right) \sqrt{2x + \sqrt{2x - 1}} + \frac{3}{32} \log \left(-4\sqrt{2x + \sqrt{2x - 1}} \left(2\sqrt{2x - 1} + 1 \right) + 16x + 8\sqrt{2x - 1} - 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/24*(16*x + 2*sqrt(2*x - 1) - 3)*sqrt(2*x + sqrt(2*x - 1)) + 3/32*log(-4*sqrt(2*x + sqrt(2*x - 1))*(2*sqrt(2*x - 1) + 1) + 16*x + 8*sqrt(2*x - 1) - 3)

giac [A] time = 0.45, size = 67, normalized size = 0.84

$$\frac{1}{24} \left(2\sqrt{2x - 1} \left(4\sqrt{2x - 1} + 1 \right) + 5 \right) \sqrt{2x + \sqrt{2x - 1}} + \frac{3}{16} \log \left(2\sqrt{2x + \sqrt{2x - 1}} - 2\sqrt{2x - 1} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x+(-1+2*x)^(1/2))^(1/2), x, algorithm="giac")

[Out] $\frac{1}{24}*(2*\sqrt{2*x - 1}*(4*\sqrt{2*x - 1} + 1) + 5)*\sqrt{2*x + \sqrt{2*x - 1}} + \frac{3}{16}*\log(2*\sqrt{2*x + \sqrt{2*x - 1}}) - 2*\sqrt{2*x - 1} - 1)$

maple [A] time = 0.01, size = 60, normalized size = 0.75

$$-\frac{3 \operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{2x-1}+\frac{1}{2}\right)}{3}\right)}{16} + \frac{(2x + \sqrt{2x-1})^{\frac{3}{2}}}{3} - \frac{(1 + 2\sqrt{2x-1})\sqrt{2x + \sqrt{2x-1}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x+(2*x-1)^(1/2))^(1/2),x)`

[Out] $\frac{1}{3}*(2*x+(2*x-1)^{(1/2)})^{(3/2)} - \frac{1}{8}*(1+2*(2*x-1)^{(1/2)})*(2*x+(2*x-1)^{(1/2)})^{(1/2)} - \frac{3}{16}*\operatorname{arcsinh}\left(\frac{2}{3}*3^{(1/2)}*((2*x-1)^{(1/2)}+1/2)\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x + \sqrt{2x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(2*x + sqrt(2*x - 1)), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2x + \sqrt{2x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x + (2*x - 1)^(1/2))^(1/2),x)`

[Out] `int((2*x + (2*x - 1)^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2x + \sqrt{2x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2*x+(-1+2*x)**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(2*x + sqrt(2*x - 1)), x)`

$$3.449 \quad \int \sqrt{3x + \sqrt{-7 + 8x}} \, dx$$

Optimal. Leaf size=109

$$\frac{(-3(7-8x) + 8\sqrt{8x-7} + 21)^{3/2}}{72\sqrt{2}} - \frac{(3\sqrt{8x-7} + 4)\sqrt{-3(7-8x) + 8\sqrt{8x-7} + 21}}{36\sqrt{2}} - \frac{47 \sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Rubi [A] time = 0.07, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {640, 612, 619, 215}

$$\frac{(-3(7-8x) + 8\sqrt{8x-7} + 21)^{3/2}}{72\sqrt{2}} - \frac{(3\sqrt{8x-7} + 4)\sqrt{-3(7-8x) + 8\sqrt{8x-7} + 21}}{36\sqrt{2}} - \frac{47 \sinh^{-1}\left(\frac{3\sqrt{8x-7}+4}{\sqrt{47}}\right)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[3*x + Sqrt[-7 + 8*x]], x]

[Out] -((4 + 3*Sqrt[-7 + 8*x])*Sqrt[21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x]])/(36*Sqrt[2]) + (21 - 3*(7 - 8*x) + 8*Sqrt[-7 + 8*x])^(3/2)/(72*Sqrt[2]) - (47*ArcSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/(36*Sqrt[6])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned}
\int \sqrt{3x + \sqrt{-7 + 8x}} \, dx &= \frac{1}{4} \text{Subst} \left(\int x \sqrt{\frac{21}{8} + x + \frac{3x^2}{8}} \, dx, x, \sqrt{-7 + 8x} \right) \\
&= \frac{(21 - 3(7 - 8x) + 8\sqrt{-7 + 8x})^{3/2}}{72\sqrt{2}} - \frac{1}{3} \text{Subst} \left(\int \sqrt{\frac{21}{8} + x + \frac{3x^2}{8}} \, dx, x, \sqrt{-7 + 8x} \right) \\
&= -\frac{(4 + 3\sqrt{-7 + 8x}) \sqrt{21 - 3(7 - 8x) + 8\sqrt{-7 + 8x}}}{36\sqrt{2}} + \frac{(21 - 3(7 - 8x) + 8\sqrt{-7 + 8x})^3}{72\sqrt{2}} \\
&= -\frac{(4 + 3\sqrt{-7 + 8x}) \sqrt{21 - 3(7 - 8x) + 8\sqrt{-7 + 8x}}}{36\sqrt{2}} + \frac{(21 - 3(7 - 8x) + 8\sqrt{-7 + 8x})^3}{72\sqrt{2}} \\
&= -\frac{(4 + 3\sqrt{-7 + 8x}) \sqrt{21 - 3(7 - 8x) + 8\sqrt{-7 + 8x}}}{36\sqrt{2}} + \frac{(21 - 3(7 - 8x) + 8\sqrt{-7 + 8x})^3}{72\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 65, normalized size = 0.60

$$\frac{1}{216} \left(12\sqrt{3x + \sqrt{8x - 7}} (12x + \sqrt{8x - 7} - 4) - 47\sqrt{6} \sinh^{-1} \left(\frac{3\sqrt{8x - 7} + 4}{\sqrt{47}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[3*x + Sqrt[-7 + 8*x]], x]

[Out] (12*Sqrt[3*x + Sqrt[-7 + 8*x]]*(-4 + 12*x + Sqrt[-7 + 8*x]) - 47*Sqrt[6]*ArcSinh[(4 + 3*Sqrt[-7 + 8*x])/Sqrt[47]])/216

IntegrateAlgebraic [A] time = 0.32, size = 118, normalized size = 1.08

$$\frac{1}{144} \sqrt{3(8x - 7) + 8\sqrt{8x - 7} + 21} (3\sqrt{2}(8x - 7) + 2\sqrt{2}\sqrt{8x - 7} + 13\sqrt{2}) + \frac{47 \log(-3\sqrt{8x - 7} + \sqrt{3}\sqrt{3(8x - 7) + 8\sqrt{8x - 7} + 21} - 4)}{36\sqrt{6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[3*x + Sqrt[-7 + 8*x]], x]

[Out] (Sqrt[21 + 8*Sqrt[-7 + 8*x]] + 3*(-7 + 8*x))*(13*Sqrt[2] + 2*Sqrt[2]*Sqrt[-7 + 8*x] + 3*Sqrt[2]*(-7 + 8*x))/144 + (47*Log[-4 - 3*Sqrt[-7 + 8*x] + Sqrt[3]*Sqrt[21 + 8*Sqrt[-7 + 8*x] + 3*(-7 + 8*x)]])/(36*Sqrt[6])

fricas [A] time = 3.10, size = 101, normalized size = 0.93

$$\frac{1}{18} (12x + \sqrt{8x - 7} - 4) \sqrt{3x + \sqrt{8x - 7}} + \frac{47}{864} \sqrt{6} \log(-41472x^2 - 192(144x - 47)\sqrt{8x - 7} + 8(3\sqrt{6}(144x + 17)\sqrt{8x - 7} + 4\sqrt{6}(432x - 299))\sqrt{3x + \sqrt{8x - 7}} - 9792x + 30047)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 1/18*(12*x + sqrt(8*x - 7) - 4)*sqrt(3*x + sqrt(8*x - 7)) + 47/864*sqrt(6)*log(-41472*x^2 - 192*(144*x - 47)*sqrt(8*x - 7) + 8*(3*sqrt(6)*(144*x + 17)*sqrt(8*x - 7) + 4*sqrt(6)*(432*x - 299))*sqrt(3*x + sqrt(8*x - 7)) - 9792*x + 30047)

giac [A] time = 0.47, size = 88, normalized size = 0.81

$$\frac{1}{216} \sqrt{2} \left(3\sqrt{2}(\sqrt{8x - 7}(3\sqrt{8x - 7} + 2) + 13)\sqrt{3x + \sqrt{8x - 7}} + 47\sqrt{3} \log\left(-\sqrt{3}\left(\sqrt{3}\sqrt{8x - 7} - 2\sqrt{2}\sqrt{3x + \sqrt{8x - 7}}\right) - 4\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/216*sqrt(2)*(3*sqrt(2)*(sqrt(8*x - 7)*(3*sqrt(8*x - 7) + 2) + 13)*sqrt(3*x + sqrt(8*x - 7)) + 47*sqrt(3)*log(-sqrt(3)*(sqrt(3)*sqrt(8*x - 7) - 2*sqrt(2)*sqrt(3*x + sqrt(8*x - 7)))) - 4))

maple [A] time = 0.01, size = 67, normalized size = 0.61

$$\frac{47\sqrt{6} \operatorname{arcsinh}\left(\frac{3\sqrt{47}\left(\sqrt{8x-7}+\frac{4}{3}\right)}{47}\right)}{216} + \frac{(48x+16\sqrt{8x-7})^{\frac{3}{2}}}{288} - \frac{(12\sqrt{8x-7}+16)\sqrt{48x+16\sqrt{8x-7}}}{288}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+(-7+8*x)^(1/2))^(1/2),x)

[Out] 1/288*(48*x+16*(-7+8*x)^(1/2))^(3/2)-1/288*(12*(-7+8*x)^(1/2)+16)*(48*x+16*(-7+8*x)^(1/2))^(1/2)-47/216*6^(1/2)*arcsinh(3/47*47^(1/2)*((-7+8*x)^(1/2)+4/3))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x + \sqrt{8x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x+(-7+8*x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(3*x + sqrt(8*x - 7)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{3x + \sqrt{8x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x + (8*x - 7)^(1/2))^(1/2),x)

[Out] int((3*x + (8*x - 7)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{3x + \sqrt{8x - 7}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x+(-7+8*x)**(1/2))**(1/2),x)

[Out] Integral(sqrt(3*x + sqrt(8*x - 7)), x)

$$3.450 \quad \int \frac{1}{\sqrt{x+\sqrt{1+x}}} dx$$

Optimal. Leaf size=47

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {640, 621, 206}

$$2\sqrt{x+\sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1}+1}{2\sqrt{x+\sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x + Sqrt[1 + x]],x]

[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x+\sqrt{1+x}}} dx &= 2 \text{Subst} \left(\int \frac{x}{\sqrt{-1+x+x^2}} dx, x, \sqrt{1+x} \right) \\ &= 2\sqrt{x+\sqrt{1+x}} - \text{Subst} \left(\int \frac{1}{\sqrt{-1+x+x^2}} dx, x, \sqrt{1+x} \right) \\ &= 2\sqrt{x+\sqrt{1+x}} - 2 \text{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{1+2\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}} \right) \\ &= 2\sqrt{x+\sqrt{1+x}} - \tanh^{-1} \left(\frac{1+2\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 47, normalized size = 1.00

$$2\sqrt{x + \sqrt{x+1}} - \tanh^{-1}\left(\frac{2\sqrt{x+1} + 1}{2\sqrt{x + \sqrt{x+1}}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x + Sqrt[1 + x]], x]

[Out] 2*Sqrt[x + Sqrt[1 + x]] - ArcTanh[(1 + 2*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]

IntegrateAlgebraic [A] time = 0.09, size = 43, normalized size = 0.91

$$2\sqrt{x + \sqrt{x+1}} + \log\left(-2\sqrt{x+1} + 2\sqrt{x + \sqrt{x+1}} - 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x + Sqrt[1 + x]], x]

[Out] 2*Sqrt[x + Sqrt[1 + x]] + Log[-1 - 2*Sqrt[1 + x] + 2*Sqrt[x + Sqrt[1 + x]]]

fricas [A] time = 2.12, size = 47, normalized size = 1.00

$$2\sqrt{x + \sqrt{x+1}} + \frac{1}{2} \log\left(4\sqrt{x + \sqrt{x+1}}(2\sqrt{x+1} + 1) - 8x - 8\sqrt{x+1} - 5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(1+x)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 2*sqrt(x + sqrt(x + 1)) + 1/2*log(4*sqrt(x + sqrt(x + 1))*(2*sqrt(x + 1) + 1) - 8*x - 8*sqrt(x + 1) - 5)

giac [A] time = 0.43, size = 33, normalized size = 0.70

$$2\sqrt{x + \sqrt{x+1}} + \log\left(-2\sqrt{x + \sqrt{x+1}} + 2\sqrt{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(1+x)^(1/2))^(1/2), x, algorithm="giac")

[Out] 2*sqrt(x + sqrt(x + 1)) + log(-2*sqrt(x + sqrt(x + 1)) + 2*sqrt(x + 1) + 1)

maple [A] time = 0.01, size = 32, normalized size = 0.68

$$-\ln\left(\sqrt{x+1} + \frac{1}{2} + \sqrt{x + \sqrt{x+1}}\right) + 2\sqrt{x + \sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(x+1)^(1/2))^(1/2), x)

[Out] 2*(x+(x+1)^(1/2))^(1/2) - ln((x+1)^(1/2) + 1/2 + (x+(x+1)^(1/2))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{x+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x + sqrt(x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x + \sqrt{x + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (x + 1)^(1/2))^(1/2),x)

[Out] int(1/(x + (x + 1)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{x + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(1+x)**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(x + sqrt(x + 1)), x)

$$3.451 \quad \int \frac{1+x}{4+x+\sqrt{-9+6x}} dx$$

Optimal. Leaf size=67

$$x - 2\sqrt{3}\sqrt{2x-3} + 3 \log(x + \sqrt{3}\sqrt{2x-3} + 4) + 4\sqrt{6} \tan^{-1}\left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}}\right)$$

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1628, 634, 618, 204, 628}

$$x - 2\sqrt{3}\sqrt{2x-3} + 3 \log(x + \sqrt{3}\sqrt{2x-3} + 4) + 4\sqrt{6} \tan^{-1}\left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/(4 + x + Sqrt[-9 + 6*x]), x]

[Out] x - 2*Sqrt[3]*Sqrt[-3 + 2*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{1+x}{4+x+\sqrt{-9+6x}} dx &= \frac{1}{3} \text{Subst} \left(\int \frac{x(15+x^2)}{33+6x+x^2} dx, x, \sqrt{-9+6x} \right) \\
&= \frac{1}{3} \text{Subst} \left(\int \left(-6+x + \frac{18(11+x)}{33+6x+x^2} \right) dx, x, \sqrt{-9+6x} \right) \\
&= x - 2\sqrt{3}\sqrt{-3+2x} + 6 \text{Subst} \left(\int \frac{11+x}{33+6x+x^2} dx, x, \sqrt{-9+6x} \right) \\
&= x - 2\sqrt{3}\sqrt{-3+2x} + 3 \text{Subst} \left(\int \frac{6+2x}{33+6x+x^2} dx, x, \sqrt{-9+6x} \right) + 48 \text{Subst} \left(\int \frac{1}{33-} \right. \\
&= x - 2\sqrt{3}\sqrt{-3+2x} + 3 \log(4+x+\sqrt{3}\sqrt{-3+2x}) - 96 \text{Subst} \left(\int \frac{1}{-96-x^2} dx, x, 6 \right. \\
&= x - 2\sqrt{3}\sqrt{-3+2x} + 4\sqrt{6} \tan^{-1} \left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}} \right) + 3 \log(4+x+\sqrt{3}\sqrt{-3+2x})
\end{aligned}$$

Mathematica [A] time = 0.07, size = 56, normalized size = 0.84

$$x - 2\sqrt{6x-9} + 3 \log(x + \sqrt{6x-9} + 4) + 4\sqrt{6} \tan^{-1} \left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/(4 + x + Sqrt[-9 + 6*x]), x]

[Out] x - 2*Sqrt[-9 + 6*x] + 4*Sqrt[6]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] + 3*Log[4 + x + Sqrt[-9 + 6*x]]

IntegrateAlgebraic [A] time = 0.12, size = 85, normalized size = 1.27

$$\frac{1}{2}(2x-3) - 2\sqrt{3}\sqrt{2x-3} + 3 \log(x + \sqrt{3}\sqrt{2x-3} + 4) + 4\sqrt{6} \tan^{-1} \left(\frac{\sqrt{2x-3}}{2\sqrt{2}} + \frac{\sqrt{\frac{3}{2}}}{2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)/(4 + x + Sqrt[-9 + 6*x]), x]

[Out] -2*Sqrt[3]*Sqrt[-3 + 2*x] + (-3 + 2*x)/2 + 4*Sqrt[6]*ArcTan[Sqrt[3/2]/2 + Sqrt[-3 + 2*x]/(2*Sqrt[2])] + 3*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]

fricas [A] time = 0.63, size = 48, normalized size = 0.72

$$4\sqrt{6} \arctan \left(\frac{1}{12} \sqrt{6}\sqrt{6x-9} + \frac{1}{4} \sqrt{6} \right) + x - 2\sqrt{6x-9} + 3 \log(x + \sqrt{6x-9} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="fricas")

[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*sqrt(6*x - 9) + 1/4*sqrt(6)) + x - 2*sqrt(6*x - 9) + 3*log(x + sqrt(6*x - 9) + 4)

giac [A] time = 0.38, size = 49, normalized size = 0.73

$$4\sqrt{6} \arctan \left(\frac{1}{12} \sqrt{6}(\sqrt{6x-9} + 3) \right) + x - 2\sqrt{6x-9} + 3 \log(6x + 6\sqrt{6x-9} + 24) - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")

[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2

maple [A] time = 0.01, size = 52, normalized size = 0.78

$$x + 4\sqrt{6} \arctan\left(\frac{(2\sqrt{6x-9} + 6)\sqrt{6}}{24}\right) + 3 \ln(6x + 24 + 6\sqrt{6x-9}) - 2\sqrt{6x-9} - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(4+x+(-9+6*x)^(1/2)),x)

[Out] -2*(-9+6*x)^(1/2)-3/2+x+3*ln(24+6*x+6*(-9+6*x)^(1/2))+4*6^(1/2)*arctan(1/24*(2*(-9+6*x)^(1/2)+6)*6^(1/2))

maxima [A] time = 2.24, size = 49, normalized size = 0.73

$$4\sqrt{6} \arctan\left(\frac{1}{12}\sqrt{6}(\sqrt{6x-9} + 3)\right) + x - 2\sqrt{6x-9} + 3 \log(6x + 6\sqrt{6x-9} + 24) - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")

[Out] 4*sqrt(6)*arctan(1/12*sqrt(6)*(sqrt(6*x - 9) + 3)) + x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) - 3/2

mupad [B] time = 3.09, size = 102, normalized size = 1.52

$$x + 3 \ln((6\sqrt{6x-9} + (-3 + \sqrt{6} 2i)(2\sqrt{6x-9} + 6) + 66)(6\sqrt{6x-9} - (3 + \sqrt{6} 2i)(2\sqrt{6x-9} + 6) + 66)) + 4\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}\sqrt{6x-9}}{12} + \frac{\sqrt{6}}{4}\right) - 2\sqrt{6x-9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/(x + (6*x - 9)^(1/2) + 4),x)

[Out] x + 3*log((6*(6*x - 9)^(1/2) + (6^(1/2)*2i - 3)*(2*(6*x - 9)^(1/2) + 6) + 66)*(6*(6*x - 9)^(1/2) - (6^(1/2)*2i + 3)*(2*(6*x - 9)^(1/2) + 6) + 66)) + 4*6^(1/2)*atan((6^(1/2)*(6*x - 9)^(1/2))/12 + 6^(1/2)/4) - 2*(6*x - 9)^(1/2)

sympy [A] time = 38.42, size = 58, normalized size = 0.87

$$x - 2\sqrt{6x-9} + 3 \log(6x + 6\sqrt{6x-9} + 24) + 4\sqrt{6} \operatorname{atan}\left(\frac{\sqrt{6}(\sqrt{6x-9} + 3)}{12}\right) - \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(4+x+(-9+6*x)**(1/2)),x)

[Out] x - 2*sqrt(6*x - 9) + 3*log(6*x + 6*sqrt(6*x - 9) + 24) + 4*sqrt(6)*atan(sqrt(6)*(sqrt(6*x - 9) + 3)/12) - 3/2

$$3.452 \quad \int \frac{12-x}{4+x+\sqrt{-9+6x}} dx$$

Optimal. Leaf size=71

$$-x + 2\sqrt{3}\sqrt{2x-3} + 10 \log(x + \sqrt{3}\sqrt{2x-3} + 4) - 21\sqrt{\frac{3}{2}} \tan^{-1}\left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1628, 634, 618, 204, 628}

$$-x + 2\sqrt{3}\sqrt{2x-3} + 10 \log(x + \sqrt{3}\sqrt{2x-3} + 4) - 21\sqrt{\frac{3}{2}} \tan^{-1}\left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

[In] Int[(12 - x)/(4 + x + Sqrt[-9 + 6*x]),x]

[Out] -x + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1628

Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{12-x}{4+x+\sqrt{-9+6x}} dx &= -\left(\frac{1}{3} \text{Subst}\left(\int \frac{x(-63+x^2)}{33+6x+x^2} dx, x, \sqrt{-9+6x}\right)\right) \\
&= -\left(\frac{1}{3} \text{Subst}\left(\int \left(-6+x+\frac{6(33-10x)}{33+6x+x^2}\right) dx, x, \sqrt{-9+6x}\right)\right) \\
&= -x + 2\sqrt{3}\sqrt{-3+2x} - 2 \text{Subst}\left(\int \frac{33-10x}{33+6x+x^2} dx, x, \sqrt{-9+6x}\right) \\
&= -x + 2\sqrt{3}\sqrt{-3+2x} + 10 \text{Subst}\left(\int \frac{6+2x}{33+6x+x^2} dx, x, \sqrt{-9+6x}\right) - 126 \text{Subst}\left(\int \frac{1}{-96-x^2} dx, x, \sqrt{-9+6x}\right) \\
&= -x + 2\sqrt{3}\sqrt{-3+2x} + 10 \log(4+x+\sqrt{3}\sqrt{-3+2x}) + 252 \text{Subst}\left(\int \frac{1}{-96-x^2} dx, x, \sqrt{-9+6x}\right) \\
&= -x + 2\sqrt{3}\sqrt{-3+2x} - 21\sqrt{\frac{3}{2}} \tan^{-1}\left(\frac{3+\sqrt{3}\sqrt{-3+2x}}{2\sqrt{6}}\right) + 10 \log(4+x+\sqrt{3}\sqrt{-3+2x})
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.85

$$-x + 2\sqrt{6x-9} + 10 \log(x + \sqrt{6x-9} + 4) - 21\sqrt{\frac{3}{2}} \tan^{-1}\left(\frac{\sqrt{6x-9} + 3}{2\sqrt{6}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(12 - x)/(4 + x + Sqrt[-9 + 6*x]), x]

[Out] -x + 2*Sqrt[-9 + 6*x] - 21*Sqrt[3/2]*ArcTan[(3 + Sqrt[-9 + 6*x])/(2*Sqrt[6])] + 10*Log[4 + x + Sqrt[-9 + 6*x]]

IntegrateAlgebraic [A] time = 0.10, size = 87, normalized size = 1.23

$$\frac{1}{2}(3-2x) + 2\sqrt{3}\sqrt{2x-3} + 10 \log(x + \sqrt{3}\sqrt{2x-3} + 4) - 21\sqrt{\frac{3}{2}} \tan^{-1}\left(\frac{\sqrt{2x-3}}{2\sqrt{2}} + \frac{\sqrt{\frac{3}{2}}}{2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(12 - x)/(4 + x + Sqrt[-9 + 6*x]), x]

[Out] (3 - 2*x)/2 + 2*Sqrt[3]*Sqrt[-3 + 2*x] - 21*Sqrt[3/2]*ArcTan[Sqrt[3/2]/2 + Sqrt[-3 + 2*x]/(2*Sqrt[2])] + 10*Log[4 + x + Sqrt[3]*Sqrt[-3 + 2*x]]

fricas [A] time = 0.77, size = 59, normalized size = 0.83

$$-\frac{21}{2}\sqrt{3}\sqrt{2} \arctan\left(\frac{1}{12}\sqrt{3}\sqrt{2}\sqrt{6x-9} + \frac{1}{4}\sqrt{3}\sqrt{2}\right) - x + 2\sqrt{6x-9} + 10 \log(x + \sqrt{6x-9} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)), x, algorithm="fricas")

[Out] -21/2*sqrt(3)*sqrt(2)*arctan(1/12*sqrt(3)*sqrt(2)*sqrt(6*x - 9) + 1/4*sqrt(3)*sqrt(2)) - x + 2*sqrt(6*x - 9) + 10*log(x + sqrt(6*x - 9) + 4)

giac [A] time = 0.33, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6} \arctan\left(\frac{1}{12}\sqrt{6}(\sqrt{6x-9} + 3)\right) - x + 2\sqrt{6x-9} + 10 \log(6x + 6\sqrt{6x-9} + 24) + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="giac")

[Out] $-21/2\sqrt{6}\arctan(1/12\sqrt{6}(\sqrt{6x-9}+3)) - x + 2\sqrt{6x-9} + 10\log(6x + 6\sqrt{6x-9} + 24) + 3/2$

maple [A] time = 0.00, size = 54, normalized size = 0.76

$$-x - \frac{21\sqrt{6} \arctan\left(\frac{(2\sqrt{6x-9}+6)\sqrt{6}}{24}\right)}{2} + 10\ln(6x + 24 + 6\sqrt{6x-9}) + 2\sqrt{6x-9} + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((12-x)/(4+x+(6*x-9)^(1/2)),x)

[Out] $2*(6*x-9)^(1/2)+3/2-x+10*\ln(6*x+24+6*(6*x-9)^(1/2))-21/2*6^(1/2)*\arctan(1/2*4*(2*(6*x-9)^(1/2)+6)*6^(1/2))$

maxima [A] time = 2.08, size = 51, normalized size = 0.72

$$-\frac{21}{2}\sqrt{6}\arctan\left(\frac{1}{12}\sqrt{6}(\sqrt{6x-9}+3)\right) - x + 2\sqrt{6x-9} + 10\log(6x + 6\sqrt{6x-9} + 24) + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12-x)/(4+x+(-9+6*x)^(1/2)),x, algorithm="maxima")

[Out] $-21/2\sqrt{6}\arctan(1/12\sqrt{6}(\sqrt{6x-9}+3)) - x + 2\sqrt{6x-9} + 10\log(6x + 6\sqrt{6x-9} + 24) + 3/2$

mupad [B] time = 0.03, size = 118, normalized size = 1.66

$$2\sqrt{6x-9} + 10\ln\left(\left((2\sqrt{6x-9}+6)\left(-10 + \frac{\sqrt{2}\sqrt{3}21i}{4}\right) + 20\sqrt{6x-9} - 66\right)\left((2\sqrt{6x-9}+6)\left(10 + \frac{\sqrt{2}\sqrt{3}21i}{4}\right) - 20\sqrt{6x-9} + 66\right)\right) - x - \frac{21\sqrt{2}\sqrt{3}\operatorname{atan}\left(\frac{\sqrt{2}\sqrt{3}}{4} + \frac{\sqrt{2}\sqrt{3}\sqrt{6x-9}}{12}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x-12)/(x+(6*x-9)^(1/2)+4),x)

[Out] $10*\log(((2*(6*x-9)^(1/2)+6)*((2^(1/2)*3^(1/2)*21i)/4-10)+20*(6*x-9)^(1/2)-66)*((2*(6*x-9)^(1/2)+6)*((2^(1/2)*3^(1/2)*21i)/4+10)-20*(6*x-9)^(1/2)+66))-x+2*(6*x-9)^(1/2)-(21*2^(1/2)*3^(1/2)*\operatorname{atan}((2^(1/2)*3^(1/2))/4+(2^(1/2)*3^(1/2)*(6*x-9)^(1/2))/12))/2$

sympy [A] time = 73.42, size = 60, normalized size = 0.85

$$-x + 2\sqrt{6x-9} + 10\log(6x + 6\sqrt{6x-9} + 24) - \frac{21\sqrt{6}\operatorname{atan}\left(\frac{\sqrt{6}(\sqrt{6x-9}+3)}{12}\right)}{2} + \frac{3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((12-x)/(4+x+(-9+6*x)**(1/2)),x)

[Out] $-x + 2\sqrt{6x-9} + 10\log(6x + 6\sqrt{6x-9} + 24) - 21\sqrt{6}\operatorname{atan}(\sqrt{6}(\sqrt{6x-9}+3)/12)/2 + 3/2$

$$3.453 \quad \int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx$$

Optimal. Leaf size=52

$$\frac{2x^{3/2}}{3} + \sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{x}\right) - \sqrt{2} \tan^{-1}\left(\sqrt{2} \sqrt{x} + 1\right)$$

Rubi [A] time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1802, 827, 1162, 617, 204}

$$\frac{2x^{3/2}}{3} + \sqrt{2} \tan^{-1}\left(1 - \sqrt{2} \sqrt{x}\right) - \sqrt{2} \tan^{-1}\left(\sqrt{2} \sqrt{x} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^3)/(Sqrt[x]*(1 + x^2)), x]

[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 617

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c])] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 827

Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_) + (c_.)*(x_)^2)), x_Symbol] := Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 1162

Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] := With[{q = Rt[(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x^3}{\sqrt{x}(1+x^2)} dx &= \int \left(\sqrt{x} - \frac{1+x}{\sqrt{x}(1+x^2)} \right) dx \\
&= \frac{2x^{3/2}}{3} - \int \frac{1+x}{\sqrt{x}(1+x^2)} dx \\
&= \frac{2x^{3/2}}{3} - 2 \operatorname{Subst} \left(\int \frac{1+x^2}{1+x^4} dx, x, \sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3} - \operatorname{Subst} \left(\int \frac{1}{1-\sqrt{2}x+x^2} dx, x, \sqrt{x} \right) - \operatorname{Subst} \left(\int \frac{1}{1+\sqrt{2}x+x^2} dx, x, \sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3} - \sqrt{2} \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1-\sqrt{2}\sqrt{x} \right) + \sqrt{2} \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, 1+\sqrt{2}\sqrt{x} \right) \\
&= \frac{2x^{3/2}}{3} + \sqrt{2} \tan^{-1} \left(1-\sqrt{2}\sqrt{x} \right) - \sqrt{2} \tan^{-1} \left(1+\sqrt{2}\sqrt{x} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 52, normalized size = 1.00

$$\frac{2x^{3/2}}{3} + \sqrt{2} \tan^{-1} \left(1 - \sqrt{2} \sqrt{x} \right) - \sqrt{2} \tan^{-1} \left(\sqrt{2} \sqrt{x} + 1 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]

[Out] (2*x^(3/2))/3 + Sqrt[2]*ArcTan[1 - Sqrt[2]*Sqrt[x]] - Sqrt[2]*ArcTan[1 + Sqrt[2]*Sqrt[x]]

IntegrateAlgebraic [A] time = 0.06, size = 39, normalized size = 0.75

$$\frac{2x^{3/2}}{3} - \sqrt{2} \tan^{-1} \left(\frac{\frac{x}{\sqrt{2}} - \frac{1}{\sqrt{2}}}{\sqrt{x}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^3)/(Sqrt[x]*(1 + x^2)),x]

[Out] (2*x^(3/2))/3 - Sqrt[2]*ArcTan[(-1/Sqrt[2]) + x/Sqrt[2]]/Sqrt[x]]

fricas [A] time = 0.73, size = 23, normalized size = 0.44

$$\frac{2}{3} x^{\frac{3}{2}} - \sqrt{2} \arctan \left(\frac{\sqrt{2}(x-1)}{2\sqrt{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="fricas")

[Out] 2/3*x^(3/2) - sqrt(2)*arctan(1/2*sqrt(2)*(x - 1)/sqrt(x))

giac [A] time = 0.31, size = 46, normalized size = 0.88

$$\frac{2}{3} x^{\frac{3}{2}} - \sqrt{2} \arctan \left(\frac{1}{2} \sqrt{2} \left(\sqrt{2} + 2\sqrt{x} \right) \right) - \sqrt{2} \arctan \left(-\frac{1}{2} \sqrt{2} \left(\sqrt{2} - 2\sqrt{x} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3-1)/(x^2+1)/x^(1/2),x, algorithm="giac")

[Out] $\frac{2}{3}x^{3/2} - \sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right)$

maple [B] time = 0.01, size = 97, normalized size = 1.87

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2} \arctan\left(\sqrt{2}\sqrt{x} - 1\right) - \sqrt{2} \arctan\left(\sqrt{2}\sqrt{x} + 1\right) - \frac{\sqrt{2} \ln\left(\frac{x-\sqrt{2}\sqrt{x}+1}{x+\sqrt{2}\sqrt{x}+1}\right)}{4} - \frac{\sqrt{2} \ln\left(\frac{x+\sqrt{2}\sqrt{x}+1}{x-\sqrt{2}\sqrt{x}+1}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3-1)/(x^2+1)/x^(1/2), x)`

[Out] $\frac{2}{3}x^{3/2} - \arctan(1+2^{1/2}x^{1/2})2^{1/2} - \arctan(-1+2^{1/2}x^{1/2})2^{1/2} - \frac{1}{4}2^{1/2} \ln\left(\frac{(x+2^{1/2}x^{1/2}+1)}{(x-2^{1/2}x^{1/2}+1)}\right) - \frac{1}{4}2^{1/2} \ln\left(\frac{(x-2^{1/2}x^{1/2}+1)}{(x+2^{1/2}x^{1/2}+1)}\right)$

maxima [A] time = 1.81, size = 46, normalized size = 0.88

$$\frac{2}{3}x^{\frac{3}{2}} - \sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^3-1)/(x^2+1)/x^(1/2), x, algorithm="maxima")`

[Out] $\frac{2}{3}x^{3/2} - \sqrt{2} \arctan\left(\frac{1}{2}\sqrt{2}(\sqrt{2} + 2\sqrt{x})\right) - \sqrt{2} \arctan\left(-\frac{1}{2}\sqrt{2}(\sqrt{2} - 2\sqrt{x})\right)$

mupad [B] time = 3.17, size = 43, normalized size = 0.83

$$\frac{2x^{3/2}}{3} - \frac{\sqrt{2} \left(2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x}}{2} + \frac{\sqrt{2}x^{3/2}}{2}\right) + 2 \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{x}}{2}\right) \right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^3 - 1)/(x^(1/2)*(x^2 + 1)), x)`

[Out] $\frac{(2x^{3/2})}{3} - \frac{(2^{1/2}*(2*\operatorname{atan}((2^{1/2}x^{1/2}))/2 + (2^{1/2}x^{3/2})/2) + 2*\operatorname{atan}((2^{1/2}x^{1/2})/2))}{2}$

sympy [A] time = 0.77, size = 44, normalized size = 0.85

$$\frac{2x^{\frac{3}{2}}}{3} - \sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} - 1\right) - \sqrt{2} \operatorname{atan}\left(\sqrt{2}\sqrt{x} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**3-1)/(x**2+1)/x**(1/2), x)`

[Out] $2x^{3/2}/3 - \sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} - 1) - \sqrt{2} \operatorname{atan}(\sqrt{2}\sqrt{x} + 1)$

$$3.454 \quad \int \frac{1}{2\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx$$

Optimal. Leaf size=20

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {12, 619, 215}

$$-\sinh^{-1}\left(\frac{1-2\sqrt{x-1}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{2\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx &= \frac{1}{2} \int \frac{1}{\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx \\ &= \text{Subst} \left(\int \frac{1}{\sqrt{1-x+x^2}} dx, x, \sqrt{-1+x} \right) \\ &= \frac{\text{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, -1+2\sqrt{-1+x} \right)}{\sqrt{3}} \\ &= -\sinh^{-1} \left(\frac{1-2\sqrt{-1+x}}{\sqrt{3}} \right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 18, normalized size = 0.90

$$\sinh^{-1} \left(\frac{2\sqrt{x-1}-1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]

IntegrateAlgebraic [A] time = 0.11, size = 31, normalized size = 1.55

$$-\log\left(-2\sqrt{x-1} + 2\sqrt{x-\sqrt{x-1}} + 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(2*Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]

fricas [B] time = 1.29, size = 37, normalized size = 1.85

$$\frac{1}{2} \log\left(4\sqrt{x-\sqrt{x-1}}\left(2\sqrt{x-1}-1\right)+8x-8\sqrt{x-1}-3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/2*log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)

giac [A] time = 0.37, size = 25, normalized size = 1.25

$$-\log\left(2\sqrt{x-\sqrt{x-1}}-2\sqrt{x-1}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")

[Out] -log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)

maple [A] time = 0.01, size = 14, normalized size = 0.70

$$\operatorname{arcsinh}\left(\frac{2\sqrt{3}\left(\sqrt{x-1}-\frac{1}{2}\right)}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)

[Out] arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} \int \frac{1}{\sqrt{x-\sqrt{x-1}}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] 1/2*integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{2\sqrt{x-\sqrt{x-1}}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)

[Out] int(1/(2*(x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2), x)

[Out] Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)/2

$$3.455 \quad \int \frac{1+x^{7/2}}{1-x^2} dx$$

Optimal. Leaf size=43

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} - \log(1 - \sqrt{x}) + \frac{1}{2} \log(x+1) + \tan^{-1}(\sqrt{x})$$

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 0.72, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {1833, 275, 206, 302, 212, 203}

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x}) + \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(7/2))/(1 - x^2), x]

[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]] + ArcTanh[x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 275

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = GCD[m + 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]

Rule 302

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[PolynomialDivide[x^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && GtQ[m, 2*n - 1]

Rule 1833

Int[(Pq_)*((c_.)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)), x_Symbol] := Module[{q = Expon[Pq, x], j, k}, Int[Sum[((c*x)^(m + j)*Sum[Coeff[Pq, x, j + (k*n)/2]*x^((k*n)/2), {k, 0, (2*(q - j))/n + 1})*(a + b*x^n)^p)/c^j, {j, 0, n/2 - 1}], x] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n/2, 0] && !PolyQ[Pq, x^(n/2)]

Rubi steps

$$\begin{aligned}
\int \frac{1+x^{7/2}}{1-x^2} dx &= 2 \operatorname{Subst} \left(\int \frac{x(1+x^7)}{1-x^4} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \left(\frac{x}{1-x^4} + \frac{x^8}{1-x^4} \right) dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \frac{x}{1-x^4} dx, x, \sqrt{x} \right) + 2 \operatorname{Subst} \left(\int \frac{x^8}{1-x^4} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \left(-1-x^4 + \frac{1}{1-x^4} \right) dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, x \right) \\
&= -2\sqrt{x} - \frac{2x^{5/2}}{5} + \tanh^{-1}(x) + 2 \operatorname{Subst} \left(\int \frac{1}{1-x^4} dx, x, \sqrt{x} \right) \\
&= -2\sqrt{x} - \frac{2x^{5/2}}{5} + \tanh^{-1}(x) + \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{x} \right) \\
&= -2\sqrt{x} - \frac{2x^{5/2}}{5} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x}) + \tanh^{-1}(x)
\end{aligned}$$

Mathematica [C] time = 0.03, size = 67, normalized size = 1.56

$$-\frac{2x^{5/2}}{5} - 2\sqrt{x} + \left(\frac{1}{2} - \frac{i}{2}\right) \log(-\sqrt{x} + i) - \log(1 - \sqrt{x}) + \left(\frac{1}{2} + \frac{i}{2}\right) \log(\sqrt{x} + i)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x^(7/2))/(1 - x^2), x]

[Out] -2*Sqrt[x] - (2*x^(5/2))/5 + (1/2 - I/2)*Log[I - Sqrt[x]] - Log[1 - Sqrt[x]] + (1/2 + I/2)*Log[I + Sqrt[x]]

IntegrateAlgebraic [A] time = 0.04, size = 39, normalized size = 0.91

$$-\frac{2}{5}\sqrt{x}(x^2+5) - \log(\sqrt{x}-1) + \frac{1}{2}\log(x+1) + \tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x^(7/2))/(1 - x^2), x]

[Out] (-2*Sqrt[x]*(5 + x^2))/5 + ArcTan[Sqrt[x]] - Log[-1 + Sqrt[x]] + Log[1 + x]/2

fricas [A] time = 0.70, size = 29, normalized size = 0.67

$$-\frac{2}{5}(x^2+5)\sqrt{x} + \arctan(\sqrt{x}) + \frac{1}{2}\log(x+1) - \log(\sqrt{x}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(7/2))/(-x^2+1), x, algorithm="fricas")

[Out] -2/5*(x^2 + 5)*sqrt(x) + arctan(sqrt(x)) + 1/2*log(x + 1) - log(sqrt(x) - 1)

giac [A] time = 0.34, size = 30, normalized size = 0.70

$$-\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \arctan(\sqrt{x}) + \frac{1}{2}\log(x+1) - \log(|\sqrt{x}-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="giac")

[Out] $-2/5*x^{5/2} - 2*\sqrt{x} + \arctan(\sqrt{x}) + 1/2*\log(x + 1) - \log(\text{abs}(\sqrt{x} - 1))$

maple [A] time = 0.01, size = 34, normalized size = 0.79

$$-\frac{2x^{\frac{5}{2}}}{5} + \operatorname{arctanh}(x) + \arctan(\sqrt{x}) - \frac{\ln(\sqrt{x} - 1)}{2} + \frac{\ln(\sqrt{x} + 1)}{2} - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x^(7/2))/(-x^2+1),x)

[Out] $-2/5*x^{5/2} - 2*x^{1/2} - 1/2*\ln(x^{1/2} - 1) + 1/2*\ln(x^{1/2} + 1) + \arctan(x^{1/2}) + \operatorname{arctanh}(x)$

maxima [A] time = 1.80, size = 29, normalized size = 0.67

$$-\frac{2}{5}x^{\frac{5}{2}} - 2\sqrt{x} + \arctan(\sqrt{x}) + \frac{1}{2}\log(x + 1) - \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(7/2))/(-x^2+1),x, algorithm="maxima")

[Out] $-2/5*x^{5/2} - 2*\sqrt{x} + \arctan(\sqrt{x}) + 1/2*\log(x + 1) - \log(\sqrt{x} - 1)$

mupad [B] time = 3.11, size = 53, normalized size = 1.23

$$-\ln(10\sqrt{x} - 10) - 2\sqrt{x} - \frac{2x^{5/2}}{5} + \ln(1 + \sqrt{x}(-3 - i) - 3i) \left(\frac{1}{2} + \frac{1}{2}i\right) + \ln(1 + \sqrt{x}(-3 + i) + 3i) \left(\frac{1}{2} - \frac{1}{2}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^(7/2) + 1)/(x^2 - 1),x)

[Out] $\log((1 - 3i) - x^{1/2}*(3 + 1i))*(1/2 + 1i/2) - \log(10*x^{1/2} - 10) + \log((1 + 3i) - x^{1/2}*(3 - 1i))*(1/2 - 1i/2) - 2*x^{1/2} - (2*x^{5/2})/5$

sympy [A] time = 2.43, size = 36, normalized size = 0.84

$$-\frac{2x^{\frac{5}{2}}}{5} - 2\sqrt{x} - \log(\sqrt{x} - 1) + \frac{\log(x + 1)}{2} + \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x**(7/2))/(-x**2+1),x)

[Out] $-2*x^{5/2}/5 - 2*\sqrt{x} - \log(\sqrt{x} - 1) + \log(x + 1)/2 + \operatorname{atan}(\sqrt{x})$

$$3.456 \quad \int \frac{4+2x}{\sqrt[3]{-1+2x} + \sqrt{-1+2x}} dx$$

Optimal. Leaf size=116

$$\frac{1}{3}(2x-1)^{3/2} - \frac{3}{8}(2x-1)^{4/3} + \frac{3}{7}(2x-1)^{7/6} + \frac{3}{5}(2x-1)^{5/6} - \frac{3}{4}(2x-1)^{2/3} + 6\sqrt{2x-1} - 9\sqrt[3]{2x-1} + 18\sqrt[6]{2x-1} - x - 18 \log\left(\sqrt[6]{2x-1} + 1\right)$$

Rubi [A] time = 0.14, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1620}

$$\frac{1}{3}(2x-1)^{3/2} - \frac{3}{8}(2x-1)^{4/3} + \frac{3}{7}(2x-1)^{7/6} + \frac{3}{5}(2x-1)^{5/6} - \frac{3}{4}(2x-1)^{2/3} + 6\sqrt{2x-1} - 9\sqrt[3]{2x-1} + 18\sqrt[6]{2x-1} - x - 18 \log\left(\sqrt[6]{2x-1} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]), x]

[Out] -x + 18*(-1 + 2*x)^(1/6) - 9*(-1 + 2*x)^(1/3) + 6*Sqrt[-1 + 2*x] - (3*(-1 + 2*x)^(2/3))/4 + (3*(-1 + 2*x)^(5/6))/5 + (3*(-1 + 2*x)^(7/6))/7 - (3*(-1 + 2*x)^(4/3))/8 + (-1 + 2*x)^(3/2)/3 - 18*Log[1 + (-1 + 2*x)^(1/6)]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \frac{4+2x}{\sqrt[3]{-1+2x} + \sqrt{-1+2x}} dx &= 3 \text{Subst} \left(\int \frac{x^3(5+x^6)}{1+x} dx, x, \sqrt[6]{-1+2x} \right) \\ &= 3 \text{Subst} \left(\int \left(6 - 6x + 6x^2 - x^3 + x^4 - x^5 + x^6 - x^7 + x^8 - \frac{6}{1+x} \right) dx, x, \sqrt[6]{-1+2x} \right) \\ &= -x + 18\sqrt[6]{-1+2x} - 9\sqrt[3]{-1+2x} + 6\sqrt{-1+2x} - \frac{3}{4}(-1+2x)^{2/3} + \frac{3}{5}(-1+2x)^{5/6} \end{aligned}$$

Mathematica [A] time = 0.08, size = 127, normalized size = 1.09

$$2 \left(x \left(\frac{1}{3} \sqrt{2x-1} - \frac{3}{8} \sqrt[3]{2x-1} + \frac{3}{7} \sqrt[6]{2x-1} - \frac{1}{2} \right) + \frac{3}{10} (2x-1)^{5/6} - \frac{3}{8} (2x-1)^{2/3} + \frac{17}{6} \sqrt{2x-1} - \frac{69}{16} \sqrt[3]{2x-1} + \frac{123}{14} \sqrt[6]{2x-1} - 9 \log\left(\sqrt[6]{2x-1} + 1\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]), x]

[Out] 2*((123*(-1 + 2*x)^(1/6))/14 - (69*(-1 + 2*x)^(1/3))/16 + (17*Sqrt[-1 + 2*x])/6 - (3*(-1 + 2*x)^(2/3))/8 + (3*(-1 + 2*x)^(5/6))/10 + x*(-1/2 + (3*(-1 + 2*x)^(1/6))/7 - (3*(-1 + 2*x)^(1/3))/8 + Sqrt[-1 + 2*x]/3) - 9*Log[1 + (-1 + 2*x)^(1/6)])

IntegrateAlgebraic [A] time = 16.08, size = 98, normalized size = 0.84

$$-x + \frac{1}{3} \sqrt{2x-1} (2x+17) - \frac{3}{8} \sqrt[3]{2x-1} (2x+23) + \frac{3}{7} \sqrt[6]{2x-1} (2x+41) + \frac{3}{5} (2x-1)^{5/6} - \frac{3}{4} (2x-1)^{2/3} - 18 \log\left(\sqrt[6]{2x-1} + 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(4 + 2*x)/((-1 + 2*x)^(1/3) + Sqrt[-1 + 2*x]),x]

[Out] $-x - (3*(-1 + 2*x)^{(2/3)})/4 + (3*(-1 + 2*x)^{(5/6)})/5 + (\text{Sqrt}[-1 + 2*x]*(17 + 2*x))/3 - (3*(-1 + 2*x)^{(1/3)}*(23 + 2*x))/8 + (3*(-1 + 2*x)^{(1/6)}*(41 + 2*x))/7 - 18*\text{Log}[1 + (-1 + 2*x)^{(1/6)}]$

fricas [A] time = 0.64, size = 76, normalized size = 0.66

$$\frac{1}{3}(2x+17)\sqrt{2x-1} - \frac{3}{8}(2x+23)(2x-1)^{\frac{1}{3}} + \frac{3}{7}(2x+41)(2x-1)^{\frac{1}{6}} - x + \frac{3}{5}(2x-1)^{\frac{5}{6}} - \frac{3}{4}(2x-1)^{\frac{2}{3}} - 18 \log\left((2x-1)^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="fricas")

[Out] $1/3*(2*x + 17)*\text{sqrt}(2*x - 1) - 3/8*(2*x + 23)*(2*x - 1)^{(1/3)} + 3/7*(2*x + 41)*(2*x - 1)^{(1/6)} - x + 3/5*(2*x - 1)^{(5/6)} - 3/4*(2*x - 1)^{(2/3)} - 18*\text{log}((2*x - 1)^{(1/6)} + 1)$

giac [A] time = 0.39, size = 89, normalized size = 0.77

$$\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{3}{8}(2x-1)^{\frac{4}{3}} + \frac{3}{7}(2x-1)^{\frac{7}{6}} - x + \frac{3}{5}(2x-1)^{\frac{5}{6}} - \frac{3}{4}(2x-1)^{\frac{2}{3}} + 6\sqrt{2x-1} - 9(2x-1)^{\frac{1}{3}} + 18(2x-1)^{\frac{1}{6}} - 18 \log\left((2x-1)^{\frac{1}{6}} + 1\right) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="giac")

[Out] $1/3*(2*x - 1)^{(3/2)} - 3/8*(2*x - 1)^{(4/3)} + 3/7*(2*x - 1)^{(7/6)} - x + 3/5*(2*x - 1)^{(5/6)} - 3/4*(2*x - 1)^{(2/3)} + 6*\text{sqrt}(2*x - 1) - 9*(2*x - 1)^{(1/3)} + 18*(2*x - 1)^{(1/6)} - 18*\text{log}((2*x - 1)^{(1/6)} + 1) + 1/2$

maple [A] time = 0.00, size = 90, normalized size = 0.78

$$-x - 18 \ln\left(1 + (2x-1)^{\frac{1}{6}}\right) + \frac{(2x-1)^{\frac{3}{2}}}{3} - \frac{3(2x-1)^{\frac{4}{3}}}{8} + \frac{3(2x-1)^{\frac{7}{6}}}{7} + \frac{1}{2} + \frac{3(2x-1)^{\frac{5}{6}}}{5} - \frac{3(2x-1)^{\frac{2}{3}}}{4} + 6\sqrt{2x-1} - 9(2x-1)^{\frac{1}{3}} + 18(2x-1)^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4+2*x)/((2*x-1)^(1/3)+(2*x-1)^(1/2)),x)

[Out] $1/3*(2*x-1)^{(3/2)} - 3/8*(2*x-1)^{(4/3)} + 3/7*(2*x-1)^{(7/6)} - x + 1/2 + 3/5*(2*x-1)^{(5/6)} - 3/4*(2*x-1)^{(2/3)} + 6*(2*x-1)^{(1/2)} - 9*(2*x-1)^{(1/3)} + 18*(2*x-1)^{(1/6)} - 18*\ln(1+(2*x-1)^{(1/6)})$

maxima [A] time = 0.90, size = 89, normalized size = 0.77

$$\frac{1}{3}(2x-1)^{\frac{3}{2}} - \frac{3}{8}(2x-1)^{\frac{4}{3}} + \frac{3}{7}(2x-1)^{\frac{7}{6}} - x + \frac{3}{5}(2x-1)^{\frac{5}{6}} - \frac{3}{4}(2x-1)^{\frac{2}{3}} + 6\sqrt{2x-1} - 9(2x-1)^{\frac{1}{3}} + 18(2x-1)^{\frac{1}{6}} - 18 \log\left((2x-1)^{\frac{1}{6}} + 1\right) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+2*x)/((-1+2*x)^(1/3)+(-1+2*x)^(1/2)),x, algorithm="maxima")

[Out] $1/3*(2*x - 1)^{(3/2)} - 3/8*(2*x - 1)^{(4/3)} + 3/7*(2*x - 1)^{(7/6)} - x + 3/5*(2*x - 1)^{(5/6)} - 3/4*(2*x - 1)^{(2/3)} + 6*\text{sqrt}(2*x - 1) - 9*(2*x - 1)^{(1/3)} + 18*(2*x - 1)^{(1/6)} - 18*\text{log}((2*x - 1)^{(1/6)} + 1) + 1/2$

mupad [B] time = 0.13, size = 88, normalized size = 0.76

$$6\sqrt{2x-1} - 18 \ln\left((2x-1)^{1/6} + 1\right) - x - 9(2x-1)^{1/3} - \frac{3(2x-1)^{2/3}}{4} + \frac{(2x-1)^{3/2}}{3} + 18(2x-1)^{1/6} - \frac{3(2x-1)^{4/3}}{8} + \frac{3(2x-1)^{5/6}}{5} + \frac{3(2x-1)^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 4)/((2*x - 1)^(1/2) + (2*x - 1)^(1/3)),x)

```
[Out] 6*(2*x - 1)^(1/2) - 18*log((2*x - 1)^(1/6) + 1) - x - 9*(2*x - 1)^(1/3) - (
3*(2*x - 1)^(2/3))/4 + (2*x - 1)^(3/2)/3 + 18*(2*x - 1)^(1/6) - (3*(2*x - 1)
)^(4/3))/8 + (3*(2*x - 1)^(5/6))/5 + (3*(2*x - 1)^(7/6))/7
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$2 \left(\int \frac{x}{\sqrt[3]{2x-1} + \sqrt{2x-1}} dx + \int \frac{2}{\sqrt[3]{2x-1} + \sqrt{2x-1}} dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4+2*x)/((-1+2*x)**(1/3)+(-1+2*x)**(1/2)),x)
```

```
[Out] 2*(Integral(x/((2*x - 1)**(1/3) + sqrt(2*x - 1)), x) + Integral(2/((2*x - 1)
)**(1/3) + sqrt(2*x - 1)), x))
```

$$3.457 \quad \int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx$$

Optimal. Leaf size=83

$$\frac{8}{7} \left(\sqrt{\sqrt{x}+1}+2 \right)^{7/2} - \frac{48}{5} \left(\sqrt{\sqrt{x}+1}+2 \right)^{5/2} + \frac{88}{3} \left(\sqrt{\sqrt{x}+1}+2 \right)^{3/2} - 48 \sqrt{\sqrt{\sqrt{x}+1}+2}$$

Rubi [A] time = 0.06, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{8}{7} \left(\sqrt{\sqrt{x}+1}+2 \right)^{7/2} - \frac{48}{5} \left(\sqrt{\sqrt{x}+1}+2 \right)^{5/2} + \frac{88}{3} \left(\sqrt{\sqrt{x}+1}+2 \right)^{3/2} - 48 \sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]

[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2 + \sqrt{1 + \sqrt{x}}}} dx &= 2 \operatorname{Subst} \left(\int \frac{x}{\sqrt{2 + \sqrt{1 + x}}} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \frac{-1 + x}{\sqrt{2 + \sqrt{x}}} dx, x, 1 + \sqrt{x} \right) \\
&= 4 \operatorname{Subst} \left(\int \frac{x(-1 + x^2)}{\sqrt{2 + x}} dx, x, \sqrt{1 + \sqrt{x}} \right) \\
&= 4 \operatorname{Subst} \left(\int \left(-\frac{6}{\sqrt{2 + x}} + 11\sqrt{2 + x} - 6(2 + x)^{3/2} + (2 + x)^{5/2} \right) dx, x, \sqrt{1 + \sqrt{x}} \right) \\
&= -48\sqrt{2 + \sqrt{1 + \sqrt{x}}} + \frac{88}{3} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{3/2} - \frac{48}{5} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{5/2} + \frac{8}{7} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.70

$$\frac{8}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} \left(3\sqrt{x} \left(5\sqrt{\sqrt{x} + 1} - 12 \right) + 76\sqrt{\sqrt{x} + 1} - 280 \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]], x]

[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/105

IntegrateAlgebraic [A] time = 0.07, size = 72, normalized size = 0.87

$$\frac{8}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} \sqrt{\sqrt{x} + 1} (15\sqrt{x} + 76) - \frac{32}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} (9\sqrt{x} + 70)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]], x]

[Out] (-32*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(70 + 9*Sqrt[x]))/105 + (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]*(76 + 15*Sqrt[x]))/105

fricas [A] time = 1.00, size = 35, normalized size = 0.42

$$\frac{8}{105} \left((15\sqrt{x} + 76)\sqrt{\sqrt{x} + 1} - 36\sqrt{x} - 280 \right) \sqrt{\sqrt{\sqrt{x} + 1} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2), x, algorithm="fricas")

[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sqrt(x) + 1) + 2)

giac [A] time = 3.46, size = 82, normalized size = 0.99

$$\frac{8 \left(15 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{7}{2}} - 126 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{5}{2}} + 385 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{3}{2}} - 630 \sqrt{\sqrt{\sqrt{x} + 1} + 2} \right)}{105 \operatorname{sgn} \left(4 \left(\sqrt{x} + 1 \right)^2 - 8\sqrt{x} - 7 \right) \operatorname{sgn} (4x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")

[Out] $\frac{8}{105} \cdot (15 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{7/2} - 126 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{5/2} + 385 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{3/2} - 630 \cdot \sqrt{\sqrt{\sqrt{x} + 1} + 2}) / (\operatorname{sgn}(4 \cdot (\sqrt{x} + 1)^2 - 8 \cdot \sqrt{x} - 7) \cdot \operatorname{sgn}(4 \cdot x - 3))$

maple [A] time = 0.01, size = 54, normalized size = 0.65

$$\frac{88 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{3}{2}}}{3} - \frac{48 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{5}{2}}}{5} + \frac{8 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{7}{2}}}{7} - 48 \sqrt{2 + \sqrt{\sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)

[Out] $88/3 \cdot (2 + (x^{1/2} + 1)^{1/2})^{3/2} - 48/5 \cdot (2 + (x^{1/2} + 1)^{1/2})^{5/2} + 8/7 \cdot (2 + (x^{1/2} + 1)^{1/2})^{7/2} - 48 \cdot (2 + (x^{1/2} + 1)^{1/2})^{1/2}$

maxima [A] time = 0.90, size = 53, normalized size = 0.64

$$\frac{8}{7} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{7}{2}} - \frac{48}{5} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{5}{2}} + \frac{88}{3} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{3}{2}} - 48 \sqrt{\sqrt{\sqrt{x} + 1} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")

[Out] $8/7 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{7/2} - 48/5 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{5/2} + 88/3 \cdot (\sqrt{\sqrt{x} + 1} + 2)^{3/2} - 48 \cdot \sqrt{\sqrt{\sqrt{x} + 1} + 2}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x} + 1} + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)

[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x} + 1} + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)

$$3.458 \quad \int \sqrt{2 + \sqrt{4 + \sqrt{x}}} \, dx$$

Optimal. Leaf size=64

$$\frac{8}{9} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{9/2} - \frac{48}{7} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{7/2} + \frac{64}{5} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{5/2}$$

Rubi [A] time = 0.05, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{8}{9} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{9/2} - \frac{48}{7} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{7/2} + \frac{64}{5} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + Sqrt[4 + Sqrt[x]]], x]

[Out] (64*(2 + Sqrt[4 + Sqrt[x]])^(5/2))/5 - (48*(2 + Sqrt[4 + Sqrt[x]])^(7/2))/7 + (8*(2 + Sqrt[4 + Sqrt[x]])^(9/2))/9

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \sqrt{2 + \sqrt{4 + \sqrt{x}}} \, dx &= 2 \operatorname{Subst} \left(\int x \sqrt{2 + \sqrt{4 + x}} \, dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int \sqrt{2 + \sqrt{x}} (-4 + x) \, dx, x, 4 + \sqrt{x} \right) \\ &= 4 \operatorname{Subst} \left(\int x \sqrt{2 + x} (-4 + x^2) \, dx, x, \sqrt{4 + \sqrt{x}} \right) \\ &= 4 \operatorname{Subst} \left(\int (8(2 + x)^{3/2} - 6(2 + x)^{5/2} + (2 + x)^{7/2}) \, dx, x, \sqrt{4 + \sqrt{x}} \right) \\ &= \frac{64}{5} \left(2 + \sqrt{4 + \sqrt{x}} \right)^{5/2} - \frac{48}{7} \left(2 + \sqrt{4 + \sqrt{x}} \right)^{7/2} + \frac{8}{9} \left(2 + \sqrt{4 + \sqrt{x}} \right)^{9/2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 43, normalized size = 0.67

$$-\frac{8}{315} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{5/2} \left(130\sqrt{\sqrt{x} + 4} - 35\sqrt{x} - 244 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + Sqrt[4 + Sqrt[x]]], x]

[Out] (-8*(2 + Sqrt[4 + Sqrt[x]])^(5/2)*(-244 + 130*Sqrt[4 + Sqrt[x]] - 35*Sqrt[x]))/315

IntegrateAlgebraic [A] time = 0.08, size = 75, normalized size = 1.17

$$\frac{16}{315} \sqrt{\sqrt{\sqrt{x} + 4} + 2} \sqrt{\sqrt{x} + 4} (5\sqrt{x} - 32) + \frac{8}{315} \sqrt{\sqrt{\sqrt{x} + 4} + 2} (35x + 4\sqrt{x} - 128)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 + Sqrt[4 + Sqrt[x]]], x]

[Out] (16*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*Sqrt[4 + Sqrt[x]]*(-32 + 5*Sqrt[x]))/315 + (8*Sqrt[2 + Sqrt[4 + Sqrt[x]]]*(-128 + 4*Sqrt[x] + 35*x))/315

fricas [A] time = 0.65, size = 39, normalized size = 0.61

$$\frac{8}{315} \left(2(5\sqrt{x} - 32)\sqrt{\sqrt{x} + 4} + 35x + 4\sqrt{x} - 128 \right) \sqrt{\sqrt{\sqrt{x} + 4} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2), x, algorithm="fricas")

[Out] 8/315*(2*(5*sqrt(x) - 32)*sqrt(sqrt(x) + 4) + 35*x + 4*sqrt(x) - 128)*sqrt(sqrt(sqrt(x) + 4) + 2)

giac [B] time = 7.65, size = 268, normalized size = 4.19

$\frac{8}{315} \left(2(5\sqrt{x} - 32)\sqrt{\sqrt{x} + 4} + 35x + 4\sqrt{x} - 128 \right) \sqrt{\sqrt{\sqrt{x} + 4} + 2}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(4+x^(1/2))^(1/2))^(1/2), x, algorithm="giac")

[Out] 8/315*((35*(sqrt(sqrt(x) + 4) + 2)^(9/2) - 360*(sqrt(sqrt(x) + 4) + 2)^(7/2) + 1512*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 3360*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 5040*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) + 18*(5*(sqrt(sqrt(x) + 4) + 2)^(7/2) - 42*(sqrt(sqrt(x) + 4) + 2)^(5/2) + 140*(sqrt(sqrt(x) + 4) + 2)^(3/2) - 280*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 84*(3*(sqrt(sqrt(x) + 4) + 2)^(5/2) - 20*(sqrt(sqrt(x) + 4) + 2)^(3/2) + 60*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79) - 840*((sqrt(sqrt(x) + 4) + 2)^(3/2) - 6*sqrt(sqrt(sqrt(x) + 4) + 2))*sgn(4*(sqrt(x) + 4)^2 - 32*sqrt(x) - 79))*sgn(4*x - 15))

maple [A] time = 0.01, size = 41, normalized size = 0.64

$$\frac{64 \left(2 + \sqrt{\sqrt{x} + 4} \right)^{\frac{5}{2}}}{5} - \frac{48 \left(2 + \sqrt{\sqrt{x} + 4} \right)^{\frac{7}{2}}}{7} + \frac{8 \left(2 + \sqrt{\sqrt{x} + 4} \right)^{\frac{9}{2}}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+(x^(1/2)+4)^(1/2))^(1/2),x)`

[Out] $64/5*(2+(x^{1/2}+4)^{1/2})^{5/2}-48/7*(2+(x^{1/2}+4)^{1/2})^{7/2}+8/9*(2+(x^{1/2}+4)^{1/2})^{9/2}$

maxima [A] time = 0.88, size = 40, normalized size = 0.62

$$\frac{8}{9} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{\frac{9}{2}} - \frac{48}{7} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{\frac{7}{2}} + \frac{64}{5} \left(\sqrt{\sqrt{x} + 4} + 2 \right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+(4+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $8/9*(\text{sqrt}(\text{sqrt}(x) + 4) + 2)^{9/2} - 48/7*(\text{sqrt}(\text{sqrt}(x) + 4) + 2)^{7/2} + 64/5*(\text{sqrt}(\text{sqrt}(x) + 4) + 2)^{5/2}$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\sqrt{\sqrt{x} + 4} + 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^(1/2) + 4)^(1/2) + 2)^(1/2),x)`

[Out] `int(((x^(1/2) + 4)^(1/2) + 2)^(1/2), x)`

sympy [B] time = 2.51, size = 216, normalized size = 3.38

$$\frac{2\sqrt{2}\sqrt{x}\sqrt{\sqrt{x}+4}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{63\pi} - \frac{4\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} - \frac{\sqrt{2}x\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{9\pi} + \frac{64\sqrt{2}\sqrt{x}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi} + \frac{128\sqrt{2}\sqrt{\sqrt{\sqrt{x}+4}+2}\Gamma\left(-\frac{1}{4}\right)\Gamma\left(\frac{1}{4}\right)}{315\pi}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+(4+x**(1/2))**(1/2))**(1/2),x)`

[Out] $-2*\text{sqrt}(2)*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(x) + 4)*\text{sqrt}(\text{sqrt}(\text{sqrt}(x) + 4) + 2)*\text{gamma}(-1/4)*\text{gamma}(1/4)/(63*\text{pi}) - 4*\text{sqrt}(2)*\text{sqrt}(x)*\text{sqrt}(\text{sqrt}(\text{sqrt}(x) + 4) + 2)*\text{gamma}(-1/4)*\text{gamma}(1/4)/(315*\text{pi}) - \text{sqrt}(2)*x*\text{sqrt}(\text{sqrt}(\text{sqrt}(x) + 4) + 2)*\text{gamma}(-1/4)*\text{gamma}(1/4)/(9*\text{pi}) + 64*\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(x) + 4)*\text{sqrt}(\text{sqrt}(\text{sqrt}(x) + 4) + 2)*\text{gamma}(-1/4)*\text{gamma}(1/4)/(315*\text{pi}) + 128*\text{sqrt}(2)*\text{sqrt}(\text{sqrt}(\text{sqrt}(x) + 4) + 2)*\text{gamma}(-1/4)*\text{gamma}(1/4)/(315*\text{pi})$

$$3.459 \quad \int \sqrt{2 - \sqrt{4 + \sqrt{-9 + 5x}}} dx$$

Optimal. Leaf size=82

$$\frac{8}{45} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{9/2} - \frac{48}{35} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{7/2} + \frac{64}{25} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{5/2}$$

Rubi [A] time = 0.08, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {371, 1398, 772}

$$\frac{8}{45} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{9/2} - \frac{48}{35} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{7/2} + \frac{64}{25} \left(2 - \sqrt{\sqrt{5x-9} + 4}\right)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]

[Out] (64*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2))/25 - (48*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(7/2))/35 + (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(9/2))/45

Rule 371

Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \sqrt{2 - \sqrt{4 + \sqrt{-9 + 5x}}} dx &= \frac{2}{5} \text{Subst} \left(\int x \sqrt{2 - \sqrt{4 + x}} dx, x, \sqrt{-9 + 5x} \right) \\ &= \frac{2}{5} \text{Subst} \left(\int \sqrt{2 - \sqrt{x}} (-4 + x) dx, x, 4 + \sqrt{-9 + 5x} \right) \\ &= \frac{4}{5} \text{Subst} \left(\int \sqrt{2 - x} x (-4 + x^2) dx, x, \sqrt{4 + \sqrt{-9 + 5x}} \right) \\ &= \frac{4}{5} \text{Subst} \left(\int (-8(2 - x)^{3/2} + 6(2 - x)^{5/2} - (2 - x)^{7/2}) dx, x, \sqrt{4 + \sqrt{-9 + 5x}} \right) \\ &= \frac{64}{25} \left(2 - \sqrt{4 + \sqrt{-9 + 5x}}\right)^{5/2} - \frac{48}{35} \left(2 - \sqrt{4 + \sqrt{-9 + 5x}}\right)^{7/2} + \frac{8}{45} \left(2 - \sqrt{4 + \sqrt{-9 + 5x}}\right)^{9/2} \end{aligned}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.70

$$\frac{8 \left(2 - \sqrt{\sqrt{5x-9} + 4} \right)^{5/2} \left(35\sqrt{5x-9} + 130\sqrt{\sqrt{5x-9} + 4} + 244 \right)}{1575}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]], x]

[Out] (8*(2 - Sqrt[4 + Sqrt[-9 + 5*x]])^(5/2)*(244 + 35*Sqrt[-9 + 5*x] + 130*Sqrt[4 + Sqrt[-9 + 5*x]]))/1575

IntegrateAlgebraic [A] time = 0.09, size = 103, normalized size = 1.26

$$\frac{8(35(5x-9)+4\sqrt{5x-9}-128)\sqrt{2-\sqrt{\sqrt{5x-9}+4}}}{1575} - \frac{16\sqrt{\sqrt{5x-9}+4}(5\sqrt{5x-9}-32)\sqrt{2-\sqrt{\sqrt{5x-9}+4}}}{1575}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]], x]

[Out] (-16*Sqrt[4 + Sqrt[-9 + 5*x]]*(-32 + 5*Sqrt[-9 + 5*x])*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]])/1575 + (8*(-128 + 4*Sqrt[-9 + 5*x] + 35*(-9 + 5*x))*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]])/1575

fricas [A] time = 0.71, size = 57, normalized size = 0.70

$$-\frac{8}{1575} \left(2(5\sqrt{5x-9}-32)\sqrt{\sqrt{5x-9}+4} - 175x - 4\sqrt{5x-9} + 443 \right) \sqrt{-\sqrt{\sqrt{5x-9}+4}+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2), x, algorithm="fricas")

[Out] -8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt(5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)

giac [B] time = 7.17, size = 474, normalized size = 5.78

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2), x, algorithm="giac")

[Out] -8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 18*(5*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) + 42*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 140*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)

maple [A] time = 0.01, size = 59, normalized size = 0.72

$$\frac{64 \left(2 - \sqrt{4 + \sqrt{5x - 9}}\right)^{\frac{5}{2}}}{25} - \frac{48 \left(2 - \sqrt{4 + \sqrt{5x - 9}}\right)^{\frac{7}{2}}}{35} + \frac{8 \left(2 - \sqrt{4 + \sqrt{5x - 9}}\right)^{\frac{9}{2}}}{45}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)

[Out] 64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2))^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)

maxima [A] time = 0.87, size = 58, normalized size = 0.71

$$\frac{8}{45} \left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{9}{2}} - \frac{48}{35} \left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{7}{2}} + \frac{64}{25} \left(-\sqrt{\sqrt{5x-9}+4}+2\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")

[Out] 8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{2 - \sqrt{\sqrt{5x - 9} + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)

[Out] int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{2 - \sqrt{\sqrt{5x - 9} + 4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)

[Out] Integral(sqrt(2 - sqrt(sqrt(5*x - 9) + 4)), x)

$$3.460 \quad \int \frac{1}{\sqrt{2+\sqrt{1+\sqrt{x}}}} dx$$

Optimal. Leaf size=83

$$\frac{8}{7} \left(\sqrt{\sqrt{x}+1}+2 \right)^{7/2} - \frac{48}{5} \left(\sqrt{\sqrt{x}+1}+2 \right)^{5/2} + \frac{88}{3} \left(\sqrt{\sqrt{x}+1}+2 \right)^{3/2} - 48 \sqrt{\sqrt{\sqrt{x}+1}+2}$$

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {371, 1398, 772}

$$\frac{8}{7} \left(\sqrt{\sqrt{x}+1}+2 \right)^{7/2} - \frac{48}{5} \left(\sqrt{\sqrt{x}+1}+2 \right)^{5/2} + \frac{88}{3} \left(\sqrt{\sqrt{x}+1}+2 \right)^{3/2} - 48 \sqrt{\sqrt{\sqrt{x}+1}+2}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]
```

```
[Out] -48*Sqrt[2 + Sqrt[1 + Sqrt[x]]] + (88*(2 + Sqrt[1 + Sqrt[x]])^(3/2))/3 - (48*(2 + Sqrt[1 + Sqrt[x]])^(5/2))/5 + (8*(2 + Sqrt[1 + Sqrt[x]])^(7/2))/7
```

Rule 371

```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] := With[{c = Coefficient[v, x, 0], d = Coefficient[v, x, 1]}, Dist[1/d^(m + 1), Subst[Int[SimplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1398

```
Int[((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{2 + \sqrt{1 + \sqrt{x}}}} dx &= 2 \operatorname{Subst} \left(\int \frac{x}{\sqrt{2 + \sqrt{1 + x}}} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \frac{-1 + x}{\sqrt{2 + \sqrt{x}}} dx, x, 1 + \sqrt{x} \right) \\
&= 4 \operatorname{Subst} \left(\int \frac{x(-1 + x^2)}{\sqrt{2 + x}} dx, x, \sqrt{1 + \sqrt{x}} \right) \\
&= 4 \operatorname{Subst} \left(\int \left(-\frac{6}{\sqrt{2 + x}} + 11\sqrt{2 + x} - 6(2 + x)^{3/2} + (2 + x)^{5/2} \right) dx, x, \sqrt{1 + \sqrt{x}} \right) \\
&= -48\sqrt{2 + \sqrt{1 + \sqrt{x}}} + \frac{88}{3} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{3/2} - \frac{48}{5} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{5/2} + \frac{8}{7} \left(2 + \sqrt{1 + \sqrt{x}} \right)^{7/2}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 0.70

$$\frac{8}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} \left(3\sqrt{x} \left(5\sqrt{\sqrt{x} + 1} - 12 \right) + 76\sqrt{\sqrt{x} + 1} - 280 \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]

[Out] (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(-280 + 76*Sqrt[1 + Sqrt[x]] + 3*(-12 + 5*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/105

IntegrateAlgebraic [A] time = 0.00, size = 72, normalized size = 0.87

$$\frac{8}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} \sqrt{\sqrt{x} + 1} (15\sqrt{x} + 76) - \frac{32}{105} \sqrt{\sqrt{\sqrt{x} + 1} + 2} (9\sqrt{x} + 70)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[2 + Sqrt[1 + Sqrt[x]]],x]

[Out] (-32*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*(70 + 9*Sqrt[x]))/105 + (8*Sqrt[2 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]*(76 + 15*Sqrt[x]))/105

fricas [A] time = 0.91, size = 35, normalized size = 0.42

$$\frac{8}{105} \left((15\sqrt{x} + 76)\sqrt{\sqrt{x} + 1} - 36\sqrt{x} - 280 \right) \sqrt{\sqrt{\sqrt{x} + 1} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="fricas")

[Out] 8/105*((15*sqrt(x) + 76)*sqrt(sqrt(x) + 1) - 36*sqrt(x) - 280)*sqrt(sqrt(sqrt(x) + 1) + 2)

giac [A] time = 3.18, size = 82, normalized size = 0.99

$$\frac{8 \left(15 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{7}{2}} - 126 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{5}{2}} + 385 \left(\sqrt{\sqrt{x} + 1} + 2 \right)^{\frac{3}{2}} - 630 \sqrt{\sqrt{\sqrt{x} + 1} + 2} \right)}{105 \operatorname{sgn} \left(4 \left(\sqrt{x} + 1 \right)^2 - 8\sqrt{x} - 7 \right) \operatorname{sgn}(4x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="giac")

[Out] $8/105*(15*(\sqrt{\sqrt{x} + 1} + 2)^{7/2} - 126*(\sqrt{\sqrt{x} + 1} + 2)^{5/2} + 385*(\sqrt{\sqrt{x} + 1} + 2)^{3/2} - 630*\sqrt{\sqrt{\sqrt{x} + 1} + 2})/(\operatorname{sgn}(4*(\sqrt{x} + 1)^2 - 8*\sqrt{x} - 7)*\operatorname{sgn}(4*x - 3))$

maple [A] time = 0.01, size = 54, normalized size = 0.65

$$\frac{88 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{3}{2}}}{3} - \frac{48 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{5}{2}}}{5} + \frac{8 \left(2 + \sqrt{\sqrt{x} + 1}\right)^{\frac{7}{2}}}{7} - 48 \sqrt{2 + \sqrt{\sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2+(x^(1/2)+1)^(1/2))^(1/2),x)

[Out] $88/3*(2+(x^{1/2}+1)^{1/2})^{3/2}-48/5*(2+(x^{1/2}+1)^{1/2})^{5/2}+8/7*(2+(x^{1/2}+1)^{1/2})^{7/2}-48*(2+(x^{1/2}+1)^{1/2})^{1/2}$

maxima [A] time = 0.88, size = 53, normalized size = 0.64

$$\frac{8}{7} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{7}{2}} - \frac{48}{5} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{5}{2}} + \frac{88}{3} \left(\sqrt{\sqrt{x} + 1} + 2\right)^{\frac{3}{2}} - 48 \sqrt{\sqrt{\sqrt{x} + 1} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x^(1/2))^(1/2))^(1/2),x, algorithm="maxima")

[Out] $8/7*(\sqrt{\sqrt{x} + 1} + 2)^{7/2} - 48/5*(\sqrt{\sqrt{x} + 1} + 2)^{5/2} + 88/3*(\sqrt{\sqrt{x} + 1} + 2)^{3/2} - 48*\sqrt{\sqrt{\sqrt{x} + 1} + 2}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x} + 1} + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2),x)

[Out] int(1/((x^(1/2) + 1)^(1/2) + 2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\sqrt{\sqrt{x} + 1} + 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(2+(1+x**(1/2))**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(sqrt(sqrt(x) + 1) + 2), x)

$$3.461 \quad \int \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}} \, dx$$

Optimal. Leaf size=190

$$\frac{16}{17} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{17/2} - \frac{112}{15} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{15/2} + \frac{288}{13} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{13/2} - \frac{320}{11} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{11/2} + \frac{112}{9} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{9/2} - \frac{48}{7} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{7/2} + \frac{32}{5} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{5/2} - \frac{16}{17} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{1/2}$$

Rubi [A] time = 0.37, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1618, 1620}

$$\frac{16}{17} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{17/2} - \frac{112}{15} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{15/2} + \frac{288}{13} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{13/2} - \frac{320}{11} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{11/2} + \frac{112}{9} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{9/2} - \frac{48}{7} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{7/2} + \frac{32}{5} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{5/2} - \frac{16}{17} \left(\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1 \right)^{1/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]

[Out] (-32*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (48*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(7/2))/7 + (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 - (320*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (288*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(13/2))/13 - (112*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))/15 + (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17

Rule 1618

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
;/; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder
[Px, a + b*x, x], 0]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c,
d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}}} \, dx &= 2 \operatorname{Subst} \left(\int x \sqrt{1 + \sqrt{1 + \sqrt{1 + x}}} \, dx, x, \sqrt{x} \right) \\
&= 4 \operatorname{Subst} \left(\int x (-1 + x^2) \sqrt{1 + \sqrt{1 + x}} \, dx, x, \sqrt{1 + \sqrt{x}} \right) \\
&= 8 \operatorname{Subst} \left(\int x^3 \sqrt{1 + x} (-2 + x^2) (-1 + x^2) \, dx, x, \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right) \\
&= 8 \operatorname{Subst} \left(\int x^3 (1 + x)^{3/2} (2 - 2x - x^2 + x^3) \, dx, x, \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right) \\
&= 8 \operatorname{Subst} \left(\int (-2(1 + x)^{3/2} + 3(1 + x)^{5/2} + 7(1 + x)^{7/2} - 20(1 + x)^{9/2} + 18(1 + x)^{11/2}) \, dx, x, \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right) \\
&= -\frac{32}{5} \left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right)^{5/2} + \frac{48}{7} \left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right)^{7/2} + \frac{112}{9} \left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right)^{9/2} - \frac{128}{11} \left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right)^{11/2} + \frac{128}{13} \left(1 + \sqrt{1 + \sqrt{1 + \sqrt{x}}} \right)^{13/2}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 135, normalized size = 0.71

$$\frac{16 \left(\sqrt{\sqrt{x} + 1} + 1 \right)^{5/2} \left(231 \sqrt{x} \left(-377 \sqrt{\sqrt{x} + 1} + 195 \sqrt{x} + 365 \right) + 8 \left(252 \sqrt{x} + 1 \sqrt{\sqrt{x} + 1} + 8642 \sqrt{\sqrt{x} + 1} - 4865 \sqrt{x} - 8221 \right) \right)}{765765}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]], x]

[Out] (16*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2)*(8*(-8221 + 8642*Sqrt[1 + Sqrt[1 + Sqrt[x]]] - 4865*Sqrt[1 + Sqrt[x]] + 252*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]]) + 231*(365 - 377*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 195*Sqrt[1 + Sqrt[x]])*Sqrt[x]))/765765

IntegrateAlgebraic [A] time = 0.24, size = 195, normalized size = 1.03

$$\sqrt{x+1} \left(\frac{16 \sqrt{\sqrt{x}+1} (231\sqrt{x}-1304)}{765765} + \frac{16 \sqrt{\sqrt{x}+1} \sqrt{\sqrt{x}+1} (3003\sqrt{x}-4672)}{765765} \right) - \frac{128 \sqrt{\sqrt{x}+1} \sqrt{\sqrt{x}+1} (441\sqrt{x}-1094)}{765765} + \frac{16 \sqrt{\sqrt{x}+1} \sqrt{\sqrt{x}+1} (45045x+4613\sqrt{x}-28152)}{765765}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]], x]

[Out] ((16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-1304 + 231*Sqrt[x]))/765765 + (16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*(-4672 + 3003*Sqrt[x]))/765765)*Sqrt[1 + Sqrt[x]] - (128*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*(-1094 + 441*Sqrt[x]))/765765 + (16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-28152 + 4613*Sqrt[x] + 45045*x))/765765

fricas [A] time = 0.83, size = 76, normalized size = 0.40

$$\frac{16}{765765} \left((231\sqrt{x}-1304)\sqrt{\sqrt{x}+1} + (3003\sqrt{x}-4672)\sqrt{\sqrt{x}+1} - 3528\sqrt{x} + 8752 \right) \sqrt{\sqrt{\sqrt{x}+1}+1} + 45045x + 4613\sqrt{x} - 28152 \sqrt{\sqrt{\sqrt{x}+1}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+(1+(1+x^(1/2)))^(1/2))^(1/2))^(1/2), x, algorithm="fricas")

[Out] $16/765765*((231*\sqrt{x} - 1304)*\sqrt{\sqrt{x} + 1} + ((3003*\sqrt{x} - 4672)*\sqrt{\sqrt{x} + 1} - 3528*\sqrt{x} + 8752)*\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 45045*x + 4613*\sqrt{x} - 28152)*\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)$

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+(1+x^(1/2)))^(1/2))^(1/2))^(1/2),x, algorithm="giac")`

[Out] Timed out

maple [A] time = 0.01, size = 121, normalized size = 0.64

$$\frac{32\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{5}{2}}}{5} + \frac{48\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{7}{2}}}{7} + \frac{112\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{9}{2}}}{9} - \frac{320\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{11}{2}}}{11} + \frac{288\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{13}{2}}}{13} - \frac{112\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{15}{2}}}{15} + \frac{16\left(1+\sqrt{1+\sqrt{x}+1}\right)^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+(1+(x^(1/2)+1)^(1/2))^(1/2))^(1/2),x)`

[Out] $-32/5*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{5/2}+48/7*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{7/2}+112/9*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{9/2}-320/11*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{11/2}+288/13*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{13/2}-112/15*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{15/2}+16/17*(1+(1+(x^{1/2}+1)^{1/2})^{1/2})^{17/2}$

maxima [A] time = 0.92, size = 120, normalized size = 0.63

$$\frac{16}{17}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{17}{2}} - \frac{112}{15}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{15}{2}} + \frac{288}{13}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{13}{2}} - \frac{320}{11}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{11}{2}} + \frac{112}{9}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{9}{2}} + \frac{48}{7}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{7}{2}} - \frac{32}{5}\left(\sqrt{\sqrt{x}+1}+1\right)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+(1+x^(1/2)))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")`

[Out] $16/17*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{17/2} - 112/15*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{15/2} + 288/13*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{13/2} - 320/11*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{11/2} + 112/9*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{9/2} + 48/7*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{7/2} - 32/5*(\sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} + 1)^{5/2}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)`

[Out] `int((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x} + 1} + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1), x)`

$$3.462 \quad \int \sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}} \, dx$$

Optimal. Leaf size=233

$$\frac{4}{17} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{17/2} - \frac{56}{15} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{15/2} + \frac{300}{13} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{13/2} - \frac{760}{11} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{11/2} + \frac{480}{7} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{7/2} - \frac{136}{5} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{5/2} + \frac{16}{3} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{3/2}$$

Rubi [A] time = 0.38, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 1, integrand size = 25, number of rules / integrand size = 0.040, Rules used = {1620}

$$\frac{4}{17} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{17/2} - \frac{56}{15} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{15/2} + \frac{300}{13} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{13/2} - \frac{760}{11} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{11/2} + \frac{480}{7} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{7/2} - \frac{136}{5} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{5/2} + \frac{16}{3} \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]], x]
```

```
[Out] (-16*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (136*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(5/2))/5 - (480*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(7/2))/7 + (304*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (760*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (300*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(13/2))/13 - (56*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2))/15 + (4*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]
```

Rubi steps

$$\begin{aligned} \int \sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}} \, dx &= 2 \text{Subst} \left(\int x \sqrt{2 + \sqrt{3 + \sqrt{-1 + 2x}}} \, dx, x, \sqrt{x} \right) \\ &= \text{Subst} \left(\int x(1+x^2) \sqrt{2 + \sqrt{3+x}} \, dx, x, \sqrt{-1 + 2\sqrt{x}} \right) \\ &= 2 \text{Subst} \left(\int x \sqrt{2+x} (-3+x^2) (1+(-3+x^2)^2) \, dx, x, \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right) \\ &= 2 \text{Subst} \left(\int (-4\sqrt{2+x} + 34(2+x)^{3/2} - 120(2+x)^{5/2} + 228(2+x)^{7/2} - 190(2+x)^{9/2}) \, dx, x, \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right) \\ &= -\frac{16}{3} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{3/2} + \frac{136}{5} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{5/2} - \frac{480}{7} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{7/2} + \frac{304}{3} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{9/2} - \frac{760}{11} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{11/2} + \frac{300}{13} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{13/2} - \frac{56}{15} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{15/2} + \frac{4}{17} \left(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}} \right)^{17/2} \end{aligned}$$

Mathematica [A] time = 0.13, size = 183, normalized size = 0.79

$$\frac{8 \left(\sqrt{\sqrt{2\sqrt{x}-1}+3+2} \right)^{3/2} \left(7\sqrt{x} \left(2145\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3} + 1452\sqrt{2\sqrt{x}-1} + 3 - 4004\sqrt{2\sqrt{x}-1} - 3576 \right) + 4 \left(3843\sqrt{2\sqrt{x}-1}\sqrt{\sqrt{2\sqrt{x}-1}+3} - 2535\sqrt{2\sqrt{x}-1} + 3 - 4286\sqrt{2\sqrt{x}-1} - 9786 \right) \right)}{255255}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]

[Out] $(8*(2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}})^{3/2}*(4*(-9786 - 2535*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}) - 4286*\sqrt{-1 + 2\sqrt{x}} + 3843*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}})*\sqrt{-1 + 2\sqrt{x}}) + 7*(-3576 + 1452*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}) - 4004*\sqrt{-1 + 2\sqrt{x}} + 2145*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}})*\sqrt{-1 + 2\sqrt{x}})/255255$

IntegrateAlgebraic [A] time = 0.21, size = 209, normalized size = 0.90

$$\sqrt{2\sqrt{x}-1} \left(\frac{16\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\sqrt{2\sqrt{x}-1}+3(1001\sqrt{x}+6800)}{255255} - \frac{8\sqrt{\sqrt{2\sqrt{x}-1}+3}+2(847\sqrt{x}-1688)}{255255} \right) - \frac{256\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\sqrt{2\sqrt{x}-1}+3(49\sqrt{x}+619)}{85085} + \frac{8\sqrt{\sqrt{2\sqrt{x}-1}+3}+2(10010x-1281\sqrt{x}-41360)}{85085}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]

[Out] $((-8*\sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}})*(-1688 + 847*\sqrt{x}))/255255 + (16*\sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}})*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}]*(6800 + 1001*\sqrt{x}))/255255)*\sqrt{-1 + 2\sqrt{x}} - (256*\sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}})*\sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}]*(619 + 49*\sqrt{x}))/85085 + (8*\sqrt{2 + \sqrt{3 + \sqrt{-1 + 2\sqrt{x}}}})*(-41360 - 1281*\sqrt{x} + 10010*x))/85085$

fricas [A] time = 0.49, size = 85, normalized size = 0.36

$$-\frac{8}{255255} \left((847\sqrt{x}-1688)\sqrt{2\sqrt{x}-1} - 2 \left((1001\sqrt{x}+6800)\sqrt{2\sqrt{x}-1} - 2352\sqrt{x} - 29712 \right) \sqrt{\sqrt{2\sqrt{x}-1}+3} - 30030x + 3843\sqrt{x} + 124080 \right) \sqrt{\sqrt{2\sqrt{x}-1}+3} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")

[Out] $-8/255255*((847*\text{sqrt}(x) - 1688)*\text{sqrt}(2*\text{sqrt}(x) - 1) - 2*((1001*\text{sqrt}(x) + 6800)*\text{sqrt}(2*\text{sqrt}(x) - 1) - 2352*\text{sqrt}(x) - 29712)*\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) - 30030*x + 3843*\text{sqrt}(x) + 124080)*\text{sqrt}(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2))$

giac [A] time = 51.25, size = 271, normalized size = 1.16

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")

[Out] $4/255255*(15015*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{17/2} - 238238*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{15/2} + 1472625*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{13/2} - 4408950*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{11/2} + 6466460*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{9/2} - 4375800*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{7/2} + 1735734*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{5/2} - 340340*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{3/2}))*\text{sgn}(131072*x^{23} + 6029312*x^{22} + 131596288*x^{21} + 1823539200*x^{20} + 18092523520*x^{19} + 137313009664*x^{18} + 830934196224*x^{17} + 4121209913344*x^{16} + 17059018985472*x^{15} + 59571270234112*x^{14} + 176317166240256*x^{13} + 442104199109632*x^{12} + 934792487842816*x^{11} + 1653389259996160*x^{10} + 2419262240692992*x^9 + 2886578907966976*x^8 + 2756595188687360*x^7 + 2055315711024768*x^6 + 1156127428771360*x^5 + 466803251648192*x^4 + 125285938081152*x^3 + 19649836876032*x^2 + 1399854182400*x + 14929920000)$

maple [A] time = 0.02, size = 154, normalized size = 0.66

$$\frac{16(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^3}{3} + \frac{136(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^5}{5} - \frac{480(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^7}{7} + \frac{304(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^9}{3} - \frac{760(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^{11}}{11} + \frac{300(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^{13}}{13} - \frac{56(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^{15}}{15} + \frac{4(2+\sqrt{3+\sqrt{2\sqrt{x}-1}})^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+(3+(-1+2*x^(1/2)))^(1/2))^(1/2), x)`

[Out] $-16/3*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(3/2)}+136/5*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(5/2)}-480/7*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(7/2)}+304/3*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(9/2)}-760/11*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(11/2)}+300/13*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(13/2)}-56/15*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(15/2)}+4/17*(2+(3+(-1+2*x^{(1/2)))^{(1/2)})^{(1/2)})^{(17/2)}$

maxima [A] time = 0.93, size = 153, normalized size = 0.66

$$\frac{4}{17} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{17}{2}} - \frac{56}{15} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{15}{2}} + \frac{300}{13} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{13}{2}} - \frac{760}{11} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{11}{2}} + \frac{304}{3} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{9}{2}} - \frac{480}{7} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{7}{2}} + \frac{136}{5} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{5}{2}} - \frac{16}{3} \left(\sqrt{\sqrt{2\sqrt{x}-1+3+2}} \right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+(3+(-1+2*x^(1/2)))^(1/2))^(1/2), x, algorithm="maxima")`

[Out] $4/17*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(17/2)} - 56/15*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(15/2)} + 300/13*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(13/2)} - 760/11*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(11/2)} + 304/3*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(9/2)} - 480/7*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(7/2)} + 136/5*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(5/2)} - 16/3*(\text{sqrt}(\text{sqrt}(2*\text{sqrt}(x) - 1) + 3) + 2)^{(3/2)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1+3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)`

[Out] `int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1+3+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2), x)`

[Out] `Integral(sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2), x)`

$$3.463 \quad \int \sqrt{1 + \sqrt{1 + \sqrt{-1 + x}}} x dx$$

Optimal. Leaf size=160

$$\frac{8}{17} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{17/2} - \frac{56}{15} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{15/2} + \frac{144}{13} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{13/2} - \frac{160}{11} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{11/2} + \frac{8}{7} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{9/2} - \frac{24}{5} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{7/2} + \frac{16}{3} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{5/2}$$

Rubi [A] time = 0.28, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1618, 1620}

$$\frac{8}{17} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{17/2} - \frac{56}{15} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{15/2} + \frac{144}{13} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{13/2} - \frac{160}{11} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{11/2} + \frac{8}{7} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{9/2} - \frac{24}{5} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{7/2} + \frac{16}{3} \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]

[Out] (16*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (24*(1 + Sqrt[1 + Sqrt[-1 + x]])^(7/2))/7 + 8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (160*(1 + Sqrt[1 + Sqrt[-1 + x]])^(11/2))/11 + (144*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (56*(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (8*(1 + Sqrt[1 + Sqrt[-1 + x]])^(17/2))/17

Rule 1618

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 :> Int[PolynomialQuotient[Px, a + b*x, x]*(a + b*x)^(m + 1)*(c + d*x)^n, x]
 /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && EqQ[PolynomialRemainder[Px, a + b*x, x], 0]

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
 :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rubi steps

$$\begin{aligned} \int \sqrt{1 + \sqrt{1 + \sqrt{-1 + x}}} x dx &= 2 \text{Subst} \left(\int x (1 + x^2) \sqrt{1 + \sqrt{1 + x}} dx, x, \sqrt{-1 + x} \right) \\ &= 4 \text{Subst} \left(\int x \sqrt{1 + x} (-1 + x^2) \left(1 + (-1 + x^2)^2 \right) dx, x, \sqrt{1 + \sqrt{-1 + x}} \right) \\ &= 4 \text{Subst} \left(\int x (1 + x)^{3/2} (-2 + 2x + 2x^2 - 2x^3 - x^4 + x^5) dx, x, \sqrt{1 + \sqrt{-1 + x}} \right) \\ &= 4 \text{Subst} \left(\int (2(1 + x)^{3/2} - 3(1 + x)^{5/2} + 9(1 + x)^{7/2} - 20(1 + x)^{9/2} + 18(1 + x)^{11/2} - 8(1 + x)^{13/2}) dx, x, \sqrt{1 + \sqrt{-1 + x}} \right) \\ &= \frac{16}{5} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{5/2} - \frac{24}{7} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{7/2} + 8 \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{9/2} - \frac{160}{11} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{11/2} + \frac{144}{13} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{13/2} - \frac{56}{15} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{15/2} + \frac{8}{17} \left(1 + \sqrt{1 + \sqrt{-1 + x}} \right)^{17/2} \end{aligned}$$

Mathematica [A] time = 0.09, size = 103, normalized size = 0.64

$$\frac{8 \left(\sqrt{\sqrt{x-1} + 1} + 1 \right)^{5/2} \left(8 \left(84 \sqrt{x-1} \sqrt{\sqrt{x-1} + 1} - 3030 \sqrt{\sqrt{x-1} + 1} + 1715 \sqrt{x-1} + 2591 \right) + 77 \left(-377 \sqrt{\sqrt{x-1} + 1} + 195 \sqrt{x-1} + 365 \right) x \right)}{255255}$$

$(1 + 1)^{(5/2)} - 420 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(3/2)} + 315 * \text{sqrt}(\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1) * \text{sgn}(4 * (\text{sqrt}(x - 1) + 1)^2 - 8 * \text{sqrt}(x - 1) - 7) + 87516 * (5 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(7/2)} - 21 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(5/2)} + 35 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(3/2)} - 35 * \text{sqrt}(\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)) * \text{sgn}(4 * (\text{sqrt}(x - 1) + 1)^2 - 8 * \text{sqrt}(x - 1) - 7) - 102102 * (3 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(5/2)} - 10 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(3/2)} + 15 * \text{sqrt}(\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)) * \text{sgn}(4 * (\text{sqrt}(x - 1) + 1)^2 - 8 * \text{sqrt}(x - 1) - 7) - 510510 * ((\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(3/2)} - 3 * \text{sqrt}(\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)) * \text{sgn}(4 * (\text{sqrt}(x - 1) + 1)^2 - 8 * \text{sqrt}(x - 1) - 7)) * \text{sgn}(4 * x - 7)$

maple [A] time = 0.01, size = 107, normalized size = 0.67

$$\frac{16(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{5}{2}}}{5} - \frac{24(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{7}{2}}}{7} + 8(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{9}{2}} - \frac{160(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{11}{2}}}{11} + \frac{144(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{13}{2}}}{13} - \frac{56(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{15}{2}}}{15} + \frac{8(1 + \sqrt{1 + \sqrt{x-1}})^{\frac{17}{2}}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1+(1+(x-1)^(1/2))^(1/2))^(1/2), x)

[Out] $16/5 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(5/2)} - 24/7 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(7/2)} + 8 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(9/2)} - 160/11 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(11/2)} + 144/13 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(13/2)} - 56/15 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(15/2)} + 8/17 * (1 + (1 + (x - 1)^{(1/2)})^{(1/2)})^{(17/2)}$

maxima [A] time = 0.89, size = 106, normalized size = 0.66

$$\frac{8}{17} (\sqrt{\sqrt{x-1}+1})^{\frac{17}{2}} - \frac{56}{15} (\sqrt{\sqrt{x-1}+1})^{\frac{15}{2}} + \frac{144}{13} (\sqrt{\sqrt{x-1}+1})^{\frac{13}{2}} - \frac{160}{11} (\sqrt{\sqrt{x-1}+1})^{\frac{11}{2}} + 8 (\sqrt{\sqrt{x-1}+1})^{\frac{9}{2}} - \frac{24}{7} (\sqrt{\sqrt{x-1}+1})^{\frac{7}{2}} + \frac{16}{5} (\sqrt{\sqrt{x-1}+1})^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+(1+(-1+x)^(1/2))^(1/2))^(1/2), x, algorithm="maxima")

[Out] $8/17 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(17/2)} - 56/15 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(15/2)} + 144/13 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(13/2)} - 160/11 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(11/2)} + 8 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(9/2)} - 24/7 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(7/2)} + 16/5 * (\text{sqrt}(\text{sqrt}(x - 1) + 1) + 1)^{(5/2)}$

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{\sqrt{\sqrt{x-1}+1}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)

[Out] int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x \sqrt{\sqrt{\sqrt{x-1}+1}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+(1+(-1+x)**(1/2))**(1/2))**(1/2), x)

[Out] Integral(x*sqrt(sqrt(sqrt(x - 1) + 1) + 1), x)

$$3.464 \quad \int \frac{1}{\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx$$

Optimal. Leaf size=20

$$-2 \sinh^{-1} \left(\frac{1 - 2\sqrt{x-1}}{\sqrt{3}} \right)$$

Rubi [A] time = 0.08, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {619, 215}

$$-2 \sinh^{-1} \left(\frac{1 - 2\sqrt{x-1}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -2*ArcSinh[(1 - 2*Sqrt[-1 + x])/Sqrt[3]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-1+x} \sqrt{-\sqrt{-1+x}+x}} dx &= 2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1-x+x^2}} dx, x, \sqrt{-1+x} \right) \\ &= \frac{2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+\frac{x^2}{3}}} dx, x, -1+2\sqrt{-1+x} \right)}{\sqrt{3}} \\ &= -2 \sinh^{-1} \left(\frac{1 - 2\sqrt{-1+x}}{\sqrt{3}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 20, normalized size = 1.00

$$2 \sinh^{-1} \left(\frac{2\sqrt{x-1} - 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] 2*ArcSinh[(-1 + 2*Sqrt[-1 + x])/Sqrt[3]]

IntegrateAlgebraic [A] time = 0.13, size = 31, normalized size = 1.55

$$-2 \log \left(-2\sqrt{x-1} + 2\sqrt{x-\sqrt{x-1}} + 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-1 + x]*Sqrt[-Sqrt[-1 + x] + x]),x]

[Out] -2*Log[1 - 2*Sqrt[-1 + x] + 2*Sqrt[-Sqrt[-1 + x] + x]]

fricas [B] time = 1.31, size = 35, normalized size = 1.75

$$\log \left(4\sqrt{x-\sqrt{x-1}} \left(2\sqrt{x-1} - 1 \right) + 8x - 8\sqrt{x-1} - 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] log(4*sqrt(x - sqrt(x - 1))*(2*sqrt(x - 1) - 1) + 8*x - 8*sqrt(x - 1) - 3)

giac [A] time = 0.32, size = 25, normalized size = 1.25

$$-2 \log \left(2\sqrt{x-\sqrt{x-1}} - 2\sqrt{x-1} + 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="giac")

[Out] -2*log(2*sqrt(x - sqrt(x - 1)) - 2*sqrt(x - 1) + 1)

maple [A] time = 0.00, size = 16, normalized size = 0.80

$$2 \operatorname{arcsinh} \left(\frac{2\sqrt{3} \left(\sqrt{x-1} - \frac{1}{2} \right)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-1)^(1/2)/(x-(x-1)^(1/2))^(1/2),x)

[Out] 2*arcsinh(2/3*3^(1/2)*((x-1)^(1/2)-1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}} \sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-1+x)^(1/2)/(x-(-1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x - sqrt(x - 1))*sqrt(x - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{x-\sqrt{x-1}} \sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)`

[Out] `int(1/((x - (x - 1)^(1/2))^(1/2)*(x - 1)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1}\sqrt{x-\sqrt{x-1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)**(1/2)/(x-(-1+x)**(1/2))**(1/2), x)`

[Out] `Integral(1/(sqrt(x - 1)*sqrt(x - sqrt(x - 1))), x)`

$$3.465 \quad \int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx$$

Optimal. Leaf size=44

$$2\sqrt{x + \sqrt{2x-1} + 1} - \sqrt{2} \sinh^{-1}\left(\frac{\sqrt{2x-1} + 1}{\sqrt{2}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 52, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {640, 619, 215}

$$\sqrt{2} \sqrt{2x + 2\sqrt{2x-1} + 2} - \sqrt{2} \sinh^{-1}\left(\frac{\sqrt{2x-1} + 1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x + Sqrt[-1 + 2*x]], x]

[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[2]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^(p)), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{1+x+\sqrt{-1+2x}}} dx &= \text{Subst} \left(\int \frac{x}{\sqrt{\frac{3}{2} + x + \frac{x^2}{2}}} dx, x, \sqrt{-1+2x} \right) \\ &= \sqrt{2} \sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst} \left(\int \frac{1}{\sqrt{\frac{3}{2} + x + \frac{x^2}{2}}} dx, x, \sqrt{-1+2x} \right) \\ &= \sqrt{2} \sqrt{2+2x+2\sqrt{-1+2x}} - \text{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{2}}} dx, x, 1 + \sqrt{-1+2x} \right) \\ &= \sqrt{2} \sqrt{2+2x+2\sqrt{-1+2x}} - \sqrt{2} \sinh^{-1} \left(\frac{1 + \sqrt{-1+2x}}{\sqrt{2}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 44, normalized size = 1.00

$$2\sqrt{x + \sqrt{2x-1}} + 1 - \sqrt{2} \sinh^{-1}\left(\frac{\sqrt{2x-1} + 1}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]

[Out] 2*Sqrt[1 + x + Sqrt[-1 + 2*x]] - Sqrt[2]*ArcSinh[(1 + Sqrt[-1 + 2*x])/Sqrt[2]]

IntegrateAlgebraic [A] time = 0.14, size = 67, normalized size = 1.52

$$\sqrt{2}\sqrt{2x + 2\sqrt{2x-1}} + 2 + \sqrt{2} \log\left(-\sqrt{2x-1} + \sqrt{2x + 2\sqrt{2x-1}} + 2 - 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 + x + Sqrt[-1 + 2*x]],x]

[Out] Sqrt[2]*Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]] + Sqrt[2]*Log[-1 - Sqrt[-1 + 2*x] + Sqrt[2 + 2*x + 2*Sqrt[-1 + 2*x]]]

fricas [B] time = 1.79, size = 85, normalized size = 1.93

$$\frac{1}{4}\sqrt{2} \log\left(-8x^2 - 8(2x+1)\sqrt{2x-1} + 2\left(\sqrt{2}(2x+3)\sqrt{2x-1} + \sqrt{2}(6x-1)\right)\sqrt{x + \sqrt{2x-1}} + 1 - 24x + 7\right) + 2\sqrt{x + \sqrt{2x-1}} + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(2)*log(-8*x^2 - 8*(2*x + 1)*sqrt(2*x - 1) + 2*(sqrt(2)*(2*x + 3)*sqrt(2*x - 1) + sqrt(2)*(6*x - 1))*sqrt(x + sqrt(2*x - 1) + 1) - 24*x + 7) + 2*sqrt(x + sqrt(2*x - 1) + 1)

giac [A] time = 0.30, size = 49, normalized size = 1.11

$$\sqrt{2}\left(\sqrt{2x + 2\sqrt{2x-1}} + 2 + \log\left(\sqrt{2x + 2\sqrt{2x-1}} + 2 - \sqrt{2x-1} - 1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="giac")

[Out] sqrt(2)*(sqrt(2*x + 2*sqrt(2*x - 1) + 2) + log(sqrt(2*x + 2*sqrt(2*x - 1) + 2) - sqrt(2*x - 1) - 1))

maple [A] time = 0.01, size = 38, normalized size = 0.86

$$-\sqrt{2} \operatorname{arcsinh}\left(\frac{(1 + \sqrt{2x-1})\sqrt{2}}{2}\right) + \sqrt{4x + 4 + 4\sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x+(2*x-1)^(1/2))^(1/2),x)

[Out] (4*x+4+4*(2*x-1)^(1/2))^(1/2)-arcsinh(1/2*(1+(2*x-1)^(1/2))*2^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{2x - 1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x+(-1+2*x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(x + sqrt(2*x - 1) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x + \sqrt{2x - 1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2),x)

[Out] int(1/(x + (2*x - 1)^(1/2) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x + \sqrt{2x - 1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x+(-1+2*x)**(1/2))**(1/2),x)

[Out] Integral(1/sqrt(x + sqrt(2*x - 1) + 1), x)

$$3.466 \quad \int \frac{q+px}{\sqrt{b+ax}(f+\sqrt{b+ax})} dx$$

Optimal. Leaf size=54

$$-\frac{2(-aq+bp+f^2(-p))\log(\sqrt{ax+b}+f)}{a^2} - \frac{2fp\sqrt{ax+b}}{a^2} + \frac{px}{a}$$

Rubi [A] time = 0.38, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$, Rules used = {697}

$$-\frac{2(-aq+bp+f^2(-p))\log(\sqrt{ax+b}+f)}{a^2} - \frac{2fp\sqrt{ax+b}}{a^2} + \frac{px}{a}$$

Antiderivative was successfully verified.

[In] Int[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]

[Out] (p*x)/a - (2*f*p*Sqrt[b + a*x])/a^2 - (2*(b*p - f^2*p - a*q)*Log[f + Sqrt[b + a*x]])/a^2

Rule 697

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \int \frac{q+px}{\sqrt{b+ax}(f+\sqrt{b+ax})} dx &= \frac{2 \text{Subst}\left(\int \frac{-bp+aq+px^2}{f+x} dx, x, \sqrt{b+ax}\right)}{a^2} \\ &= \frac{2 \text{Subst}\left(\int \left(-fp+px + \frac{-bp+f^2p+aq}{f+x}\right) dx, x, \sqrt{b+ax}\right)}{a^2} \\ &= \frac{px}{a} - \frac{2fp\sqrt{b+ax}}{a^2} - \frac{2(bp-f^2p-aq)\log(f+\sqrt{b+ax})}{a^2} \end{aligned}$$

Mathematica [A] time = 0.08, size = 50, normalized size = 0.93

$$\frac{2(aq-bp+f^2p)\log(\sqrt{ax+b}+f)+p(ax-2f\sqrt{ax+b})}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]

[Out] (p*(a*x - 2*f*Sqrt[b + a*x]) + 2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b + a*x]])/a^2

IntegrateAlgebraic [A] time = 0.05, size = 58, normalized size = 1.07

$$\frac{2(aq-bp+f^2p)\log(\sqrt{ax+b}+f)}{a^2} + \frac{p\sqrt{ax+b}(\sqrt{ax+b}-2f)}{a^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(q + p*x)/(Sqrt[b + a*x]*(f + Sqrt[b + a*x])),x]

[Out] (p*Sqrt[b + a*x]*(-2*f + Sqrt[b + a*x]))/a^2 + (2*(-(b*p) + f^2*p + a*q)*Log[f + Sqrt[b + a*x]])/a^2

fricas [A] time = 0.59, size = 45, normalized size = 0.83

$$\frac{apx - 2\sqrt{ax+b}fp + 2((f^2 - b)p + aq)\log(f + \sqrt{ax+b})}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="fricas")

[Out] (a*p*x - 2*sqrt(a*x + b)*f*p + 2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b)))/a^2

giac [A] time = 0.32, size = 61, normalized size = 1.13

$$\frac{2(f^2p - bp + aq)\log(|f + \sqrt{ax+b}|)}{a^2} - \frac{2\sqrt{ax+b}a^2fp - (ax+b)a^2p}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="giac")

[Out] 2*(f^2*p - b*p + a*q)*log(abs(f + sqrt(a*x + b)))/a^2 - (2*sqrt(a*x + b)*a^2*f*p - (a*x + b)*a^2*p)/a^4

maple [A] time = 0.00, size = 80, normalized size = 1.48

$$\frac{2f^2p\ln(f + \sqrt{ax+b})}{a^2} + \frac{px}{a} + \frac{2q\ln(f + \sqrt{ax+b})}{a} - \frac{2bp\ln(f + \sqrt{ax+b})}{a^2} + \frac{bp}{a^2} - \frac{2\sqrt{ax+b}fp}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x)

[Out] p*x/a+1/a^2*b*p-2*f*p*(a*x+b)^(1/2)/a^2+2/a^2*ln(f+(a*x+b)^(1/2))*f^2*p+2/a*ln(f+(a*x+b)^(1/2))*q-2/a^2*ln(f+(a*x+b)^(1/2))*b*p

maxima [A] time = 0.88, size = 58, normalized size = 1.07

$$\frac{2((f^2-b)p+aq)\log(f+\sqrt{ax+b})}{a} - \frac{2\sqrt{ax+b}fp-(ax+b)p}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((p*x+q)/(a*x+b)^(1/2)/(f+(a*x+b)^(1/2)),x, algorithm="maxima")

[Out] (2*((f^2 - b)*p + a*q)*log(f + sqrt(a*x + b)))/a - (2*sqrt(a*x + b)*f*p - (a*x + b)*p)/a

mupad [B] time = 3.10, size = 50, normalized size = 0.93

$$\frac{\ln(f + \sqrt{b+ax})(2pf^2 + 2aq - 2bp)}{a^2} + \frac{px}{a} - \frac{2fp\sqrt{b+ax}}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((q + p*x)/((f + (b + a*x)^(1/2))*(b + a*x)^(1/2)),x)

[Out] $(\log(f + (b + a*x)^{1/2})*(2*a*q - 2*b*p + 2*f^2*p))/a^2 + (p*x)/a - (2*f*p*(b + a*x)^{1/2})/a^2$

sympy [A] time = 32.90, size = 99, normalized size = 1.83

$$-\frac{2fp\sqrt{ax+b}}{a^2} - \frac{2f(-aq+bp-f^2p) \begin{cases} \frac{1}{\sqrt{ax+b}} & \text{for } f=0 \\ \frac{\log\left(\frac{f}{\sqrt{ax+b}}+1\right)}{f} & \text{otherwise} \end{cases}}{a^2} + \frac{p(ax+b)}{a^2} + \frac{2(-aq+bp-f^2p)\log\left(\frac{1}{\sqrt{ax+b}}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((p*x+q)/(a*x+b)**(1/2)/(f+(a*x+b)**(1/2)),x)`

[Out] $-2*f*p*\sqrt{a*x + b}/a**2 - 2*f*(-a*q + b*p - f**2*p)*\text{Piecewise}((1/\sqrt{a*x + b}), \text{Eq}(f, 0)), (\log(f/\sqrt{a*x + b}) + 1)/f, \text{True}))/a**2 + p*(a*x + b)/a**2 + 2*(-a*q + b*p - f**2*p)*\log(1/\sqrt{a*x + b})/a**2$

$$3.467 \quad \int \sqrt{1 - \sqrt{x} - x} dx$$

Optimal. Leaf size=70

$$-\frac{2}{3}(-x - \sqrt{x} + 1)^{3/2} - \frac{1}{4}(2\sqrt{x} + 1)\sqrt{-x - \sqrt{x} + 1} - \frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x} + 1}{\sqrt{5}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1341, 640, 612, 619, 216}

$$-\frac{2}{3}(-x - \sqrt{x} + 1)^{3/2} - \frac{1}{4}(2\sqrt{x} + 1)\sqrt{-x - \sqrt{x} + 1} - \frac{5}{8}\sin^{-1}\left(\frac{2\sqrt{x} + 1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Sqrt[x] - x], x]

[Out] -((1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/4 - (2*(1 - Sqrt[x] - x)^(3/2))/3 - (5*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/8

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1341

Int[((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*x^(k*n) + c*x^(2*k*n))^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \sqrt{1 - \sqrt{x} - x} \, dx &= 2 \operatorname{Subst} \left(\int x \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= -\frac{2}{3} (1 - \sqrt{x} - x)^{3/2} - \operatorname{Subst} \left(\int \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} - \frac{2}{3} (1 - \sqrt{x} - x)^{3/2} - \frac{5}{8} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - x - x^2}} \, dx, x, \sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} - \frac{2}{3} (1 - \sqrt{x} - x)^{3/2} + \frac{1}{8} \sqrt{5} \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{5}}} \, dx, x, -1 - 2\sqrt{x} \right) \\
&= -\frac{1}{4} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} - \frac{2}{3} (1 - \sqrt{x} - x)^{3/2} - \frac{5}{8} \sin^{-1} \left(\frac{1 + 2\sqrt{x}}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 53, normalized size = 0.76

$$\frac{1}{12} \sqrt{-x - \sqrt{x} + 1} (8x + 2\sqrt{x} - 11) + \frac{5}{8} \sin^{-1} \left(\frac{-2\sqrt{x} - 1}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - Sqrt[x] - x], x]

[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 + (5*ArcSin[(-1 - 2*Sqrt[x])/Sqrt[5]])/8

IntegrateAlgebraic [A] time = 0.14, size = 64, normalized size = 0.91

$$\frac{1}{12} \sqrt{-x - \sqrt{x} + 1} (8x + 2\sqrt{x} - 11) - \frac{5}{4} \tan^{-1} \left(\frac{\sqrt{x}}{\sqrt{-x - \sqrt{x} + 1} - 1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[x] - x], x]

[Out] (Sqrt[1 - Sqrt[x] - x]*(-11 + 2*Sqrt[x] + 8*x))/12 - (5*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/4

fricas [A] time = 2.06, size = 84, normalized size = 1.20

$$\frac{1}{12} (8x + 2\sqrt{x} - 11) \sqrt{-x - \sqrt{x} + 1} + \frac{5}{16} \arctan \left(-\frac{(8x^2 - (16x^2 - 38x + 11)\sqrt{x} - 9x + 3)\sqrt{-x - \sqrt{x} + 1}}{4(4x^3 - 13x^2 + 7x - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x-x^(1/2))^(1/2)),x, algorithm="fricas")

[Out] 1/12*(8*x + 2*sqrt(x) - 11)*sqrt(-x - sqrt(x) + 1) + 5/16*arctan(-1/4*(8*x^2 - (16*x^2 - 38*x + 11)*sqrt(x) - 9*x + 3)*sqrt(-x - sqrt(x) + 1)/(4*x^3 - 13*x^2 + 7*x - 1))

giac [A] time = 0.35, size = 44, normalized size = 0.63

$$\frac{1}{12} (2\sqrt{x}(4\sqrt{x} + 1) - 11) \sqrt{-x - \sqrt{x} + 1} - \frac{5}{8} \arcsin \left(\frac{1}{5} \sqrt{5} (2\sqrt{x} + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="giac")

[Out] 1/12*(2*sqrt(x)*(4*sqrt(x) + 1) - 11)*sqrt(-x - sqrt(x) + 1) - 5/8*arcsin(1/5*sqrt(5)*(2*sqrt(x) + 1))

maple [A] time = 0.01, size = 50, normalized size = 0.71

$$-\frac{5 \arcsin\left(\frac{2\sqrt{5}\left(\sqrt{x}+\frac{1}{2}\right)}{5}\right)}{8} - \frac{2\left(-x-\sqrt{x}+1\right)^{\frac{3}{2}}}{3} + \frac{\left(-2\sqrt{x}-1\right)\sqrt{-x-\sqrt{x}+1}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x-x^(1/2))^(1/2),x)

[Out] -2/3*(1-x-x^(1/2))^(3/2)+1/4*(-2*x^(1/2)-1)*(1-x-x^(1/2))^(1/2)-5/8*arcsin(2/5*5^(1/2)*(x^(1/2)+1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x - \sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-x^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x - sqrt(x) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{1 - \sqrt{x} - x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^(1/2) - x)^(1/2),x)

[Out] int((1 - x^(1/2) - x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\sqrt{x} - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x-x**(1/2))**(1/2),x)

[Out] Integral(sqrt(-sqrt(x) - x + 1), x)

$$3.468 \quad \int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} dx$$

Optimal. Leaf size=19

$$x + 4\sqrt{x} + 2 \log(\sqrt{x} + 4)$$

Rubi [A] time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {28, 1397, 771}

$$x + 4\sqrt{x} + 2 \log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] Int[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]

[Out] 4*Sqrt[x] + x + 2*Log[4 + Sqrt[x]]

Rule 28

Int[(u_.)*((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] && EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]

Rule 771

Int[((d_.) + (e_.)*(x_)^(m_.))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))

Rule 1397

Int[((a_.) + (c_.)*(x_)^(n2_.)) + (b_.)*(x_)^(n_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g-1)*(d + e*x^(g*n))^q*(a + b*x^(g*n) + c*x^(2*g*n))^p, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{9+6\sqrt{x}+x}{4\sqrt{x}+x} dx &= \int \frac{(3+\sqrt{x})^2}{4\sqrt{x}+x} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{x(3+x)^2}{4x+x^2} dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int \left(2+x + \frac{1}{4+x} \right) dx, x, \sqrt{x} \right) \\ &= 4\sqrt{x} + x + 2 \log(4 + \sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 1.00

$$x + 4\sqrt{x} + 2 \log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] Integrate[(9 + 6*Sqrt[x] + x)/(4*Sqrt[x] + x),x]

[Out] $4\sqrt{x} + x + 2\log[4 + \sqrt{x}]$

IntegrateAlgebraic [A] time = 0.02, size = 24, normalized size = 1.26

$$\sqrt{x}(\sqrt{x} + 4) + 2\log(\sqrt{x} + 4)$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[(9 + 6*sqrt(x) + x)/(4*sqrt(x) + x), x]`

[Out] $(4 + \sqrt{x})\sqrt{x} + 2\log[4 + \sqrt{x}]$

fricas [A] time = 1.19, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)), x, algorithm="fricas")`

[Out] $x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$

giac [A] time = 0.39, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)), x, algorithm="giac")`

[Out] $x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$

maple [A] time = 0.00, size = 16, normalized size = 0.84

$$x + 2\ln(\sqrt{x} + 4) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((9+x+6*x^(1/2))/(x+4*x^(1/2)), x)`

[Out] $x + 2\ln(x^{1/2} + 4) + 4x^{1/2}$

maxima [A] time = 0.88, size = 15, normalized size = 0.79

$$x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9+x+6*x^(1/2))/(x+4*x^(1/2)), x, algorithm="maxima")`

[Out] $x + 4\sqrt{x} + 2\log(\sqrt{x} + 4)$

mupad [B] time = 3.04, size = 15, normalized size = 0.79

$$x + 2\ln(\sqrt{x} + 4) + 4\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + 6*x^(1/2) + 9)/(x + 4*x^(1/2)), x)`

[Out] $x + 2\log(x^{1/2} + 4) + 4x^{1/2}$

sympy [A] time = 0.17, size = 17, normalized size = 0.89

$$4\sqrt{x} + x + 2\log(\sqrt{x} + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((9+x+6*x**(1/2))/(x+4*x**(1/2)),x)
```

```
[Out] 4*sqrt(x) + x + 2*log(sqrt(x) + 4)
```

$$3.469 \quad \int \frac{6-8x^{7/2}}{5-9\sqrt{x}} dx$$

Optimal. Leaf size=77

$$\frac{80x^{7/2}}{567} + \frac{400x^{5/2}}{6561} + \frac{50000x^{3/2}}{1594323} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{56145628\sqrt{x}}{43046721} - \frac{280728140 \log(5-9\sqrt{x})}{387420489}$$

Rubi [A] time = 0.07, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1893, 190, 43, 266}

$$\frac{2x^4}{9} + \frac{80x^{7/2}}{567} + \frac{200x^3}{2187} + \frac{400x^{5/2}}{6561} + \frac{2500x^2}{59049} + \frac{50000x^{3/2}}{1594323} + \frac{125000x}{4782969} - \frac{56145628\sqrt{x}}{43046721} - \frac{280728140 \log(5-9\sqrt{x})}{387420489}$$

Antiderivative was successfully verified.

[In] Int[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]), x]

[Out] (-56145628*Sqrt[x])/43046721 + (125000*x)/4782969 + (50000*x^(3/2))/1594323 + (2500*x^2)/59049 + (400*x^(5/2))/6561 + (200*x^3)/2187 + (80*x^(7/2))/567 + (2*x^4)/9 - (280728140*Log[5 - 9*Sqrt[x]])/387420489

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1893

Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n])

Rubi steps

$$\begin{aligned}
\int \frac{6 - 8x^{7/2}}{5 - 9\sqrt{x}} dx &= \int \left(-\frac{6}{-5 + 9\sqrt{x}} + \frac{8x^{7/2}}{-5 + 9\sqrt{x}} \right) dx \\
&= -\left(6 \int \frac{1}{-5 + 9\sqrt{x}} dx \right) + 8 \int \frac{x^{7/2}}{-5 + 9\sqrt{x}} dx \\
&= -\left(12 \operatorname{Subst} \left(\int \frac{x}{-5 + 9x} dx, x, \sqrt{x} \right) \right) + 16 \operatorname{Subst} \left(\int \frac{x^8}{-5 + 9x} dx, x, \sqrt{x} \right) \\
&= -\left(12 \operatorname{Subst} \left(\int \left(\frac{1}{9} + \frac{5}{9(-5 + 9x)} \right) dx, x, \sqrt{x} \right) \right) + 16 \operatorname{Subst} \left(\int \left(\frac{78125}{43046721} + \frac{15625x}{4782969} + \frac{3125x^2}{531441} \right) dx, x, \sqrt{x} \right) \\
&= -\frac{56145628\sqrt{x}}{43046721} + \frac{125000x}{4782969} + \frac{50000x^{3/2}}{1594323} + \frac{2500x^2}{59049} + \frac{400x^{5/2}}{6561} + \frac{200x^3}{2187} + \frac{80x^{7/2}}{567} + \frac{2x^4}{9} - \frac{280728140}{387420489} \log(9\sqrt{x} - 5)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 66, normalized size = 0.86

$$\frac{2 \left(9(21257640x^{7/2} + 9185400x^{5/2} + 4725000x^{3/2} + 33480783x^4 + 13778100x^3 + 6378750x^2 + 3937500x - 19650969\sqrt{x}) - 982548490 \log(5 - 9\sqrt{x}) \right)}{2711943423}$$

Antiderivative was successfully verified.

[In] Integrate[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]), x]

[Out] (2*(9*(-196509698*Sqrt[x] + 3937500*x + 4725000*x^(3/2) + 6378750*x^2 + 9185400*x^(5/2) + 13778100*x^3 + 21257640*x^(7/2) + 33480783*x^4) - 982548490*Log[5 - 9*Sqrt[x]]))/2711943423

IntegrateAlgebraic [A] time = 0.03, size = 67, normalized size = 0.87

$$\frac{2\sqrt{x} \left(33480783x^{7/2} + 13778100x^{5/2} + 6378750x^{3/2} + 21257640x^3 + 9185400x^2 + 4725000x + 3937500\sqrt{x} - 19650969 \right)}{301327047} - \frac{280728140 \log(9\sqrt{x} - 5)}{387420489}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(6 - 8*x^(7/2))/(5 - 9*Sqrt[x]), x]

[Out] (2*Sqrt[x]*(-196509698 + 3937500*Sqrt[x] + 4725000*x + 6378750*x^(3/2) + 9185400*x^2 + 13778100*x^(5/2) + 21257640*x^3 + 33480783*x^(7/2)))/301327047 - (280728140*Log[-5 + 9*Sqrt[x]])/387420489

fricas [A] time = 0.59, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4 + \frac{200}{2187}x^3 + \frac{2500}{59049}x^2 + \frac{4}{301327047}(10628820x^3 + 4592700x^2 + 2362500x - 98254849)\sqrt{x} + \frac{125000}{4782969}x - \frac{280728140}{387420489} \log(9\sqrt{x} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="fricas")

[Out] 2/9*x^4 + 200/2187*x^3 + 2500/59049*x^2 + 4/301327047*(10628820*x^3 + 4592700*x^2 + 2362500*x - 98254849)*sqrt(x) + 125000/4782969*x - 280728140/387420489*log(9*sqrt(x) - 5)

giac [A] time = 0.35, size = 50, normalized size = 0.65

$$\frac{2}{9}x^4 + \frac{80}{567}x^3 + \frac{200}{2187}x^3 + \frac{400}{6561}x^2 + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^2 + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489} \log(|9\sqrt{x} - 5|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="giac")

[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 280728140/387420489*log(abs(9*sqrt(x) - 5))

maple [A] time = 0.00, size = 50, normalized size = 0.65

$$\frac{2x^4}{9} + \frac{80x^{\frac{7}{2}}}{567} + \frac{200x^3}{2187} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{2500x^2}{59049} + \frac{50000x^{\frac{3}{2}}}{1594323} + \frac{125000x}{4782969} - \frac{280728140 \ln(9\sqrt{x} - 5)}{387420489} - \frac{56145628\sqrt{x}}{43046721}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((6-8*x^(7/2))/(5-9*x^(1/2)),x)

[Out] 2/9*x^4+80/567*x^(7/2)+200/2187*x^3+400/6561*x^(5/2)+2500/59049*x^2+50000/1594323*x^(3/2)+125000/4782969*x-56145628/43046721*x^(1/2)-280728140/387420489*ln(-5+9*x^(1/2))

maxima [A] time = 0.88, size = 49, normalized size = 0.64

$$\frac{2}{9}x^4 + \frac{80}{567}x^{\frac{7}{2}} + \frac{200}{2187}x^3 + \frac{400}{6561}x^{\frac{5}{2}} + \frac{2500}{59049}x^2 + \frac{50000}{1594323}x^{\frac{3}{2}} + \frac{125000}{4782969}x - \frac{56145628}{43046721}\sqrt{x} - \frac{280728140}{387420489}\log(9\sqrt{x} - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6-8*x^(7/2))/(5-9*x^(1/2)),x, algorithm="maxima")

[Out] 2/9*x^4 + 80/567*x^(7/2) + 200/2187*x^3 + 400/6561*x^(5/2) + 2500/59049*x^2 + 50000/1594323*x^(3/2) + 125000/4782969*x - 56145628/43046721*sqrt(x) - 280728140/387420489*log(9*sqrt(x) - 5)

mupad [B] time = 0.05, size = 47, normalized size = 0.61

$$\frac{125000x}{4782969} - \frac{280728140 \ln\left(\sqrt{x} - \frac{5}{9}\right)}{387420489} + \frac{2500x^2}{59049} - \frac{56145628\sqrt{x}}{43046721} + \frac{200x^3}{2187} + \frac{2x^4}{9} + \frac{50000x^{3/2}}{1594323} + \frac{400x^{5/2}}{6561} + \frac{80x^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8*x^(7/2) - 6)/(9*x^(1/2) - 5),x)

[Out] (125000*x)/4782969 - (280728140*log(x^(1/2) - 5/9))/387420489 + (2500*x^2)/59049 - (56145628*x^(1/2))/43046721 + (200*x^3)/2187 + (2*x^4)/9 + (50000*x^(3/2))/1594323 + (400*x^(5/2))/6561 + (80*x^(7/2))/567

sympy [A] time = 2.32, size = 71, normalized size = 0.92

$$\frac{80x^{\frac{7}{2}}}{567} + \frac{400x^{\frac{5}{2}}}{6561} + \frac{50000x^{\frac{3}{2}}}{1594323} - \frac{56145628\sqrt{x}}{43046721} + \frac{2x^4}{9} + \frac{200x^3}{2187} + \frac{2500x^2}{59049} + \frac{125000x}{4782969} - \frac{280728140 \log\left(\sqrt{x} - \frac{5}{9}\right)}{387420489}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((6-8*x**(7/2))/(5-9*x**(1/2)),x)

[Out] 80*x**(7/2)/567 + 400*x**(5/2)/6561 + 50000*x**(3/2)/1594323 - 56145628*sqrt(x)/43046721 + 2*x**4/9 + 200*x**3/2187 + 2500*x**2/59049 + 125000*x/4782969 - 280728140*log(sqrt(x) - 5/9)/387420489

$$3.470 \quad \int \frac{\sqrt{1+x}(1+x^3)}{1+x^2} dx$$

Optimal. Leaf size=80

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} + (1-i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right) + (1+i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Rubi [B] time = 0.29, antiderivative size = 224, normalized size of antiderivative = 2.80, number of steps used = 16, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1625, 1629, 825, 12, 708, 1094, 634, 618, 204, 628}

$$\frac{2}{5}(x+1)^{5/2} - \frac{2}{3}(x+1)^{3/2} - 2\sqrt{x+1} - \frac{\log\left(x - \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right)}{2\sqrt{1+\sqrt{2}}} + \frac{\log\left(x + \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right)}{2\sqrt{1+\sqrt{2}}} - \sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})} - 2\sqrt{x+1}}{\sqrt{2(\sqrt{2}-1)}}\right) + \sqrt{1+\sqrt{2}} \tan^{-1}\left(\frac{2\sqrt{x+1} + \sqrt{2(1+\sqrt{2})}}{\sqrt{2(\sqrt{2}-1)}}\right)$$

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[1 + x]*(1 + x^3))/(1 + x^2), x]

[Out] -2*Sqrt[1 + x] - (2*(1 + x)^(3/2))/3 + (2*(1 + x)^(5/2))/5 - Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] - 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] + Sqrt[1 + Sqrt[2]]*ArcTan[(Sqrt[2*(1 + Sqrt[2])] + 2*Sqrt[1 + x])/Sqrt[2*(-1 + Sqrt[2])]] - Log[1 + Sqrt[2] + x - Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[1 + Sqrt[2]]) + Log[1 + Sqrt[2] + x + Sqrt[2*(1 + Sqrt[2])]*Sqrt[1 + x]]/(2*Sqrt[1 + Sqrt[2]])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[Rt[-b, 2]*x]/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 708

Int[1/(Sqrt[(d_) + (e_.)*(x_)])*((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[2*e, Subst[Int[1/(c*d^2 + a*e^2 - 2*c*d*x^2 + c*x^4), x], x, Sqrt[d + e*x]],

$x]$ /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0]

Rule 825

Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_)))/((a_.) + (c_.)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d + c*e*f)*x, x])/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]

Rule 1094

Int[((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int[(r - x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(r + x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && NegQ[b^2 - 4*a*c]

Rule 1625

Int[(Pq_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + 1)*PolynomialQuotient[Pq, d + e*x, x]*(a + c*x^2)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, d + e*x, x], 0]

Rule 1629

Int[(Pq_.)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x}(1+x^3)}{1+x^2} dx &= \int \frac{(1+x)^{3/2}(1-x+x^2)}{1+x^2} dx \\
&= \int \left((1+x)^{3/2} - \frac{x(1+x)^{3/2}}{1+x^2} \right) dx \\
&= \frac{2}{5}(1+x)^{5/2} - \int \frac{x(1+x)^{3/2}}{1+x^2} dx \\
&= -\frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int \frac{(-1+x)\sqrt{1+x}}{1+x^2} dx \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \int -\frac{2}{\sqrt{1+x}(1+x^2)} dx \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 2 \int \frac{1}{\sqrt{1+x}(1+x^2)} dx \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + 4 \operatorname{Subst} \left(\int \frac{1}{2-2x^2+x^4} dx, x, \sqrt{1+x} \right) \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\operatorname{Subst} \left(\int \frac{\sqrt{2(1+\sqrt{2})-x}}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+x} \right)}{\sqrt{1+\sqrt{2}}} + \dots \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} + \frac{\operatorname{Subst} \left(\int \frac{1}{\sqrt{2}-\sqrt{2(1+\sqrt{2})}x+x^2} dx, x, \sqrt{1+x} \right)}{\sqrt{2}} + \dots \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\log \left(1 + \sqrt{2} + x - \sqrt{2(1+\sqrt{2})} \sqrt{1+x} \right)}{2\sqrt{1+\sqrt{2}}} + \dots \\
&= -2\sqrt{1+x} - \frac{2}{3}(1+x)^{3/2} + \frac{2}{5}(1+x)^{5/2} - \frac{\tan^{-1} \left(\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+x}}{\sqrt{2(-1+\sqrt{2})}} \right)}{\sqrt{-1+\sqrt{2}}} + \frac{\tan^{-1} \left(\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+x}}{\sqrt{2(-1+\sqrt{2})}} \right)}{\sqrt{-1+\sqrt{2}}}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 68, normalized size = 0.85

$$\frac{2}{15} \sqrt{x+1} (3x^2 + x - 17) + (1-i)^{3/2} \tanh^{-1} \left(\frac{\sqrt{x+1}}{\sqrt{1-i}} \right) + (1+i)^{3/2} \tanh^{-1} \left(\frac{\sqrt{x+1}}{\sqrt{1+i}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1+x]*(1+x^3))/(1+x^2),x]

[Out] (2*Sqrt[1+x]*(-17+x+3*x^2))/15+(1-I)^(3/2)*ArcTanh[Sqrt[1+x]/Sqrt[1-I]]+(1+I)^(3/2)*ArcTanh[Sqrt[1+x]/Sqrt[1+I]]

IntegrateAlgebraic [A] time = 0.15, size = 89, normalized size = 1.11

$$\frac{2}{15} (3(x+1)^{5/2} - 5(x+1)^{3/2} - 15\sqrt{x+1}) + \sqrt{2+2i} \tan^{-1} \left(\sqrt{-\frac{1}{2}-\frac{i}{2}} \sqrt{x+1} \right) + \sqrt{2-2i} \tan^{-1} \left(\sqrt{-\frac{1}{2}+\frac{i}{2}} \sqrt{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[1+x]*(1+x^3))/(1+x^2),x]

[Out] (2*(-15*Sqrt[1 + x] - 5*(1 + x)^(3/2) + 3*(1 + x)^(5/2)))/15 + Sqrt[2 + 2*I]*ArcTan[Sqrt[-1/2 - I/2]*Sqrt[1 + x]] + Sqrt[2 - 2*I]*ArcTan[Sqrt[-1/2 + I/2]*Sqrt[1 + x]]

fricas [B] time = 0.60, size = 302, normalized size = 3.78

$\frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} + \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} - \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} + \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} - \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} + \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} - \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} + \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right) + \frac{1}{2} \sqrt{2} \sqrt{-1} \operatorname{arctan}\left(\frac{\sqrt{2} \sqrt{-1} \sqrt{1+x}}{\sqrt{2} \sqrt{-1} \sqrt{1+x} - \sqrt{2} \sqrt{-1} \sqrt{1+x}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="fricas")

[Out] -1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) + 1/8*8^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2)*log(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4) - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt(2)*sqrt(2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) - sqrt(2) - 1 - 1/2*8^(1/4)*sqrt(2)*sqrt(2*sqrt(2) + 4)*arctan(1/16*8^(3/4)*sqrt(2)*sqrt(-2*8^(1/4)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + 4*x + 4*sqrt(2) + 4)*sqrt(2*sqrt(2) + 4) - 1/8*8^(3/4)*sqrt(2)*sqrt(x + 1)*sqrt(2*sqrt(2) + 4) + sqrt(2) + 1 + 2/15*(3*x^2 + x - 17)*sqrt(x + 1)

giac [B] time = 1.71, size = 171, normalized size = 2.14

$\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + \sqrt{2+1} \operatorname{arctan}\left(\frac{2^{\frac{3}{4}}(2^{\frac{1}{4}}\sqrt{2+2\sqrt{x+1}})}{2\sqrt{-2+2}}\right) + \sqrt{2+1} \operatorname{arctan}\left(-\frac{2^{\frac{3}{4}}(2^{\frac{1}{4}}\sqrt{2+2\sqrt{x+1}})}{2\sqrt{-2+2}}\right) + \frac{1}{2}\sqrt{2-1} \log\left(2^{\frac{1}{4}}\sqrt{x+1}\sqrt{2+2+x+\sqrt{2+1}}\right) - \frac{1}{2}\sqrt{2-1} \log\left(-2^{\frac{1}{4}}\sqrt{x+1}\sqrt{2+2+x+\sqrt{2+1}}\right) - 2\sqrt{x+1}$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="giac")

[Out] 2/5*(x + 1)^(5/2) - 2/3*(x + 1)^(3/2) + sqrt(sqrt(2) + 1)*arctan(1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) + 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + sqrt(sqrt(2) + 1)*arctan(-1/2*2^(3/4)*(2^(1/4)*sqrt(sqrt(2) + 2) - 2*sqrt(x + 1))/sqrt(-sqrt(2) + 2)) + 1/2*sqrt(sqrt(2) - 1)*log(2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 1/2*sqrt(sqrt(2) - 1)*log(-2^(1/4)*sqrt(x + 1)*sqrt(sqrt(2) + 2) + x + sqrt(2) + 1) - 2*sqrt(x + 1)

maple [B] time = 0.05, size = 443, normalized size = 5.54

$\frac{(1+\sqrt{2})\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{2\sqrt{-2+2\sqrt{2}}} + \frac{(1+\sqrt{2})\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{2\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{(1+\sqrt{2})\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{2\sqrt{-2+2\sqrt{2}}} + \frac{(1+\sqrt{2})\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{2\sqrt{2}\operatorname{arctan}\left(\frac{\sqrt{2}\sqrt{1+x}}{\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{\sqrt{2+2\sqrt{2}}\sqrt{2}\ln\left(\frac{1+\sqrt{2}+\sqrt{2}\sqrt{1+x}}{1-\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{\sqrt{2+2\sqrt{2}}\ln\left(\frac{1+\sqrt{2}+\sqrt{2}\sqrt{1+x}}{1-\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{\sqrt{2+2\sqrt{2}}\ln\left(\frac{1+\sqrt{2}+\sqrt{2}\sqrt{1+x}}{1-\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{\sqrt{2+2\sqrt{2}}\ln\left(\frac{1+\sqrt{2}+\sqrt{2}\sqrt{1+x}}{1-\sqrt{2}+\sqrt{2}\sqrt{1+x}}\right)}{\sqrt{-2+2\sqrt{2}}} + \frac{2(1+\sqrt{2})}{\sqrt{-2+2\sqrt{2}}} + \frac{2(1+\sqrt{2})}{\sqrt{-2+2\sqrt{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+1)*(x+1)^(1/2)/(x^2+1),x)

[Out] 2/5*(x+1)^(5/2)-2/3*(x+1)^(3/2)-2*(x+1)^(1/2)-1/4*ln(1+x+2^(1/2)+(x+1)^(1/2))*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)+1/2*ln(1+x+2^(1/2)+(x+1)^(1/2))*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)+(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)+1/4*ln(1+x+2^(1/2)-(x+1)^(1/2))*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)*2^(1/2)-1/2*ln(1+x+2^(1/2)-(x+1)^(1/2))*(2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))^(1/2)+1/2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))*2^(1/2)-1/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*(2+2*2^(1/2))+2/(-2+2*2^(1/2))^(1/2)*arctan((2*(x+1)^(1/2)-(2+2*2^(1/2))^(1/2))/(-2+2*2^(1/2))^(1/2))*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x^3 + 1)\sqrt{x + 1}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+1)*(1+x)^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate((x^3 + 1)*sqrt(x + 1)/(x^2 + 1), x)

mupad [B] time = 0.10, size = 255, normalized size = 3.19

$$\frac{2(x+1)^{5/2}}{5} - \frac{2(x+1)^{3/2}}{3} - 2\sqrt{x+1} - \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{x+1}+64i}}{256\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{\frac{\sqrt{2}-1}{4}-64}}}} - \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{x+1}+64i}}{256\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{\frac{\sqrt{2}-1}{4}-64}}}}\right)\left(\sqrt{\frac{\sqrt{2}-1}{4}}^{2i} + \sqrt{\frac{\sqrt{2}-1}{4}}^{-2i}\right) + \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{x+1}+64i}}{256\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{\frac{\sqrt{2}-1}{4}+64}}}} + \frac{\sqrt{2}\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{x+1}+64i}}{256\sqrt{\frac{\sqrt{2}-1}{4}\sqrt{\frac{\sqrt{2}-1}{4}+64}}}}\right)\left(\sqrt{\frac{\sqrt{2}-1}{4}}^{2i} - \sqrt{\frac{\sqrt{2}-1}{4}}^{-2i}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^3 + 1)*(x + 1)^(1/2))/(x^2 + 1),x)

[Out] (2*(x + 1)^(5/2))/5 - (2*(x + 1)^(3/2))/3 - 2*(x + 1)^(1/2) - atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64) - (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) - 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i + (2^(1/2)/4 - 1/4)^(1/2)*2i) + atan((2^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64) + (2^(1/2)*(2^(1/2)/4 - 1/4)^(1/2)*(x + 1)^(1/2)*64i)/(256*(2^(1/2)/4 - 1/4)^(1/2)*(- 2^(1/2)/4 - 1/4)^(1/2) + 64))*((- 2^(1/2)/4 - 1/4)^(1/2)*2i - (2^(1/2)/4 - 1/4)^(1/2)*2i)

sympy [A] time = 12.15, size = 56, normalized size = 0.70

$$\frac{2(x+1)^{5/2}}{5} - \frac{2(x+1)^{3/2}}{3} - 2\sqrt{x+1} + 4\operatorname{RootSum}\left(512t^4 + 32t^2 + 1, \left(t \mapsto t \log\left(-128t^3 + \sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**3+1)*(1+x)**(1/2)/(x**2+1),x)

[Out] 2*(x + 1)**(5/2)/5 - 2*(x + 1)**(3/2)/3 - 2*sqrt(x + 1) + 4*RootSum(512*_t**4 + 32*_t**2 + 1, Lambda(_t, _t*log(-128*_t**3 + sqrt(x + 1))))

$$3.471 \quad \int \frac{\sqrt{-1-\sqrt{x}+x}}{(-1+x)\sqrt{x}} dx$$

Optimal. Leaf size=89

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x}-\sqrt{x}-1}\right) - 2 \tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x}-\sqrt{x}-1}\right) - \tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x}-\sqrt{x}-1}\right)$$

Rubi [A] time = 0.26, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {990, 621, 206, 1033, 724, 204}

$$\tan^{-1}\left(\frac{3-\sqrt{x}}{2\sqrt{x}-\sqrt{x}-1}\right) - 2 \tanh^{-1}\left(\frac{1-2\sqrt{x}}{2\sqrt{x}-\sqrt{x}-1}\right) - \tanh^{-1}\left(\frac{3\sqrt{x}+1}{2\sqrt{x}-\sqrt{x}-1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]), x]

[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 990

Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol] :> Dist[c/f, Int[1/Sqrt[a + b*x + c*x^2], x], x] - Dist[1/f, Int[(c*d - a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1033

```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1 - \sqrt{x} + x}}{(-1 + x)\sqrt{x}} dx &= 2 \operatorname{Subst} \left(\int \frac{\sqrt{-1 - x + x^2}}{-1 + x^2} dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{-1 - x + x^2}} dx, x, \sqrt{x} \right) - 2 \operatorname{Subst} \left(\int \frac{x}{(-1 + x^2)\sqrt{-1 - x + x^2}} dx, x, \sqrt{x} \right) \\ &= 4 \operatorname{Subst} \left(\int \frac{1}{4 - x^2} dx, x, \frac{-1 + 2\sqrt{x}}{\sqrt{-1 - \sqrt{x} + x}} \right) - \operatorname{Subst} \left(\int \frac{1}{(-1 + x)\sqrt{-1 - x + x^2}} dx, x, \sqrt{x} \right) \\ &= -2 \tanh^{-1} \left(\frac{1 - 2\sqrt{x}}{2\sqrt{-1 - \sqrt{x} + x}} \right) + 2 \operatorname{Subst} \left(\int \frac{1}{-4 - x^2} dx, x, \frac{-3 + \sqrt{x}}{\sqrt{-1 - \sqrt{x} + x}} \right) + 2 \operatorname{Subst} \left(\int \frac{1}{(-1 + x)\sqrt{-1 - x + x^2}} dx, x, \sqrt{x} \right) \\ &= \tan^{-1} \left(\frac{3 - \sqrt{x}}{2\sqrt{-1 - \sqrt{x} + x}} \right) - 2 \tanh^{-1} \left(\frac{1 - 2\sqrt{x}}{2\sqrt{-1 - \sqrt{x} + x}} \right) - \tanh^{-1} \left(\frac{1 + 3\sqrt{x}}{2\sqrt{-1 - \sqrt{x} + x}} \right) \end{aligned}$$

Mathematica [A] time = 0.06, size = 89, normalized size = 1.00

$$\tan^{-1} \left(\frac{3 - \sqrt{x}}{2\sqrt{x - \sqrt{x} - 1}} \right) - 2 \tanh^{-1} \left(\frac{1 - 2\sqrt{x}}{2\sqrt{x - \sqrt{x} - 1}} \right) - \tanh^{-1} \left(\frac{3\sqrt{x} + 1}{2\sqrt{x - \sqrt{x} - 1}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]), x]
```

```
[Out] ArcTan[(3 - Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - 2*ArcTanh[(1 - 2*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])] - ArcTanh[(1 + 3*Sqrt[x])/(2*Sqrt[-1 - Sqrt[x] + x])]
```

IntegrateAlgebraic [A] time = 0.20, size = 81, normalized size = 0.91

$$-2 \log \left(-2\sqrt{x} + 2\sqrt{x - \sqrt{x} - 1} + 1 \right) - 2 \tan^{-1} \left(-\sqrt{x} + \sqrt{x - \sqrt{x} - 1} + 1 \right) - 2 \tanh^{-1} \left(\sqrt{x} - \sqrt{x - \sqrt{x} - 1} + 1 \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1 - Sqrt[x] + x]/((-1 + x)*Sqrt[x]), x]
```

```
[Out] -2*ArcTan[1 - Sqrt[x] + Sqrt[-1 - Sqrt[x] + x]] - 2*ArcTanh[1 + Sqrt[x] - Sqrt[-1 - Sqrt[x] + x]] - 2*Log[1 - 2*Sqrt[x] + 2*Sqrt[-1 - Sqrt[x] + x]]
```

fricas [A] time = 6.64, size = 87, normalized size = 0.98

$$-\arctan \left(\frac{((x-4)\sqrt{x} - 2x + 3)\sqrt{x - \sqrt{x} - 1}}{2(x^2 - 3x + 1)} \right) + \log \left(\frac{8x^2 + 2((4x-5)\sqrt{x} + 2x - 1)\sqrt{x - \sqrt{x} - 1} - 17x - 2\sqrt{x} + 11}{x - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="fricas")

[Out] -arctan(1/2*((x - 4)*sqrt(x) - 2*x + 3)*sqrt(x - sqrt(x) - 1)/(x^2 - 3*x + 1)) + log(-(8*x^2 + 2*((4*x - 5)*sqrt(x) + 2*x - 1)*sqrt(x - sqrt(x) - 1) - 17*x - 2*sqrt(x) + 11)/(x - 1))

giac [A] time = 1.24, size = 81, normalized size = 0.91

$$-2 \operatorname{arctan}\left(\sqrt{x - \sqrt{x} - 1} - \sqrt{x} + 1\right) - \log\left(-\sqrt{x - \sqrt{x} - 1} + \sqrt{x} + 2\right) + \log\left(-\sqrt{x - \sqrt{x} - 1} + \sqrt{x}\right) - 2 \log\left(\left|2\sqrt{x - \sqrt{x} - 1} - 2\sqrt{x} + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="giac")

[Out] -2*arctan(sqrt(x - sqrt(x) - 1) - sqrt(x) + 1) - log(-sqrt(x - sqrt(x) - 1) + sqrt(x) + 2) + log(-sqrt(x - sqrt(x) - 1) + sqrt(x)) - 2*log(abs(2*sqrt(x - sqrt(x) - 1) - 2*sqrt(x) + 1))

maple [A] time = 0.02, size = 130, normalized size = 1.46

$$\operatorname{arctanh}\left(\frac{-3\sqrt{x}-1}{2\sqrt{-3\sqrt{x}+(\sqrt{x}+1)^2-2}}\right) - \operatorname{arctan}\left(\frac{\sqrt{x}-3}{2\sqrt{\sqrt{x}+(\sqrt{x}-1)^2-2}}\right) + \frac{3\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{-3\sqrt{x}+(\sqrt{x}+1)^2-2}\right)}{2} + \frac{\ln\left(\sqrt{x}-\frac{1}{2}+\sqrt{\sqrt{x}+(\sqrt{x}-1)^2-2}\right)}{2} + \sqrt{\sqrt{x}+(\sqrt{x}-1)^2-2} - \sqrt{-3\sqrt{x}+(\sqrt{x}+1)^2-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x-x^(1/2))^(1/2)/(x-1)/x^(1/2),x)

[Out] ((x^(1/2)-1)^2+x^(1/2)-2)^(1/2)+1/2*ln(x^(1/2)-1/2+((x^(1/2)-1)^2+x^(1/2)-2)^(1/2))-arctan(1/2*(x^(1/2)-3)/((x^(1/2)-1)^2+x^(1/2)-2)^(1/2))-((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/2)+3/2*ln(-1/2+x^(1/2)+((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/2))+arctanh(1/2*(-1-3*x^(1/2))/((x^(1/2)+1)^2-3*x^(1/2)-2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x - \sqrt{x} - 1}}{(x - 1)\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x^(1/2))^(1/2)/(-1+x)/x^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x - sqrt(x) - 1)/((x - 1)*sqrt(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x - \sqrt{x} - 1}}{\sqrt{x} (x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)),x)

[Out] int((x - x^(1/2) - 1)^(1/2)/(x^(1/2)*(x - 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\sqrt{x} + x - 1}}{\sqrt{x} (x - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x-x**(1/2))**(1/2)/(-1+x)/x**(1/2),x)

[Out] Integral(sqrt(-sqrt(x) + x - 1)/(sqrt(x)*(x - 1)), x)

$$3.472 \quad \int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}} dx$$

Optimal. Leaf size=61

$$3 \tanh^{-1} \left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}} \right) - \tan^{-1} \left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}} \right)$$

Rubi [A] time = 0.51, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {1033, 724, 206, 204}

$$3 \tanh^{-1} \left(\frac{1-3\sqrt{x+1}}{2\sqrt{x+\sqrt{x+1}}} \right) - \tan^{-1} \left(\frac{\sqrt{x+1}+3}{2\sqrt{x+\sqrt{x+1}}} \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]), x]

[Out] -ArcTan[(3 + Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] + 3*ArcTanh[(1 - 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 1033

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[-(a*c), 2]}, Dist[h/2 + (c*g)/(2*q), Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/2 - (c*g)/(2*q), Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[-(a*c)]

Rubi steps

$$\begin{aligned}
\int \frac{1+2\sqrt{1+x}}{x\sqrt{1+x}\sqrt{x+\sqrt{1+x}}} dx &= 2 \operatorname{Subst} \left(\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx, x, \sqrt{1+x} \right) \\
&= 3 \operatorname{Subst} \left(\int \frac{1}{(-1+x)\sqrt{-1+x+x^2}} dx, x, \sqrt{1+x} \right) + \operatorname{Subst} \left(\int \frac{1}{(1+x)\sqrt{-1+x}} dx, x, \sqrt{1+x} \right) \\
&= - \left(2 \operatorname{Subst} \left(\int \frac{1}{-4-x^2} dx, x, \frac{-3-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}} \right) \right) - 6 \operatorname{Subst} \left(\int \frac{1}{4-x^2} dx, x, \frac{-1-\sqrt{1+x}}{\sqrt{x+\sqrt{1+x}}} \right) \\
&= -\tan^{-1} \left(\frac{3+\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}} \right) + 3 \tanh^{-1} \left(\frac{1-3\sqrt{1+x}}{2\sqrt{x+\sqrt{1+x}}} \right)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$\tan^{-1} \left(\frac{-\sqrt{x+1}-3}{2\sqrt{x+\sqrt{x+1}}} \right) - 3 \tanh^{-1} \left(\frac{3\sqrt{x+1}-1}{2\sqrt{x+\sqrt{x+1}}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]), x]
[Out] ArcTan[(-3 - Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])] - 3*ArcTanh[(-1 + 3*Sqrt[1 + x])/(2*Sqrt[x + Sqrt[1 + x]])]
```

IntegrateAlgebraic [A] time = 0.25, size = 55, normalized size = 0.90

$$-2 \tan^{-1} \left(\sqrt{x+1} - \sqrt{x+\sqrt{x+1}} + 1 \right) - 6 \tanh^{-1} \left(-\sqrt{x+1} + \sqrt{x+\sqrt{x+1}} + 1 \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + 2*Sqrt[1 + x])/(x*Sqrt[1 + x]*Sqrt[x + Sqrt[1 + x]]), x]
[Out] -2*ArcTan[1 + Sqrt[1 + x] - Sqrt[x + Sqrt[1 + x]]] - 6*ArcTanh[1 - Sqrt[1 + x] + Sqrt[x + Sqrt[1 + x]]]
```

fricas [A] time = 4.39, size = 62, normalized size = 1.02

$$\arctan \left(\frac{2\sqrt{x+\sqrt{x+1}}(\sqrt{x+1}-3)}{x-8} \right) + 3 \log \left(\frac{2\sqrt{x+\sqrt{x+1}}(\sqrt{x+1}+1) - 3x - 2\sqrt{x+1} - 2}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2), x, algorithm="fricas")
[Out] arctan(2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) - 3)/(x - 8)) + 3*log((2*sqrt(x + sqrt(x + 1))*(sqrt(x + 1) + 1) - 3*x - 2*sqrt(x + 1) - 2)/x)
```

giac [A] time = 1.16, size = 65, normalized size = 1.07

$$2 \arctan \left(\sqrt{x+\sqrt{x+1}} - \sqrt{x+1} - 1 \right) - 3 \log \left(\left| \sqrt{x+\sqrt{x+1}} - \sqrt{x+1} + 2 \right| \right) + 3 \log \left(\left| \sqrt{x+\sqrt{x+1}} - \sqrt{x+1} \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorithm="giac")

[Out] 2*arctan(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) - 1) - 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1) + 2)) + 3*log(abs(sqrt(x + sqrt(x + 1)) - sqrt(x + 1)))

maple [A] time = 0.02, size = 68, normalized size = 1.11

$$-3 \operatorname{arctanh}\left(\frac{3\sqrt{x+1}-1}{2\sqrt{(\sqrt{x+1}-1)^2+3\sqrt{x+1}-2}}\right) + \operatorname{arctan}\left(\frac{-\sqrt{x+1}-3}{2\sqrt{(1+\sqrt{x+1})^2-\sqrt{x+1}-2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2*(x+1)^(1/2))/x/(x+1)^(1/2)/(x+(x+1)^(1/2))^(1/2),x)

[Out] -3*arctanh(1/2*(3*(x+1)^(1/2)-1)/(((x+1)^(1/2)-1)^2+3*(x+1)^(1/2)-2)^(1/2)) + arctan(1/2*(-(x+1)^(1/2)-3)/((1+(x+1)^(1/2))^2-(x+1)^(1/2)-2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{\sqrt{x+\sqrt{x+1}}\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+2*(1+x)^(1/2))/x/(1+x)^(1/2)/(x+(1+x)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate((2*sqrt(x + 1) + 1)/(sqrt(x + sqrt(x + 1))*sqrt(x + 1)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+\sqrt{x+1}}\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)),x)

[Out] int((2*(x + 1)^(1/2) + 1)/(x*(x + (x + 1)^(1/2))^(1/2)*(x + 1)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2\sqrt{x+1}+1}{x\sqrt{x+1}\sqrt{x+\sqrt{x+1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+2*(1+x)**(1/2))/x/(1+x)**(1/2)/(x+(1+x)**(1/2))**(1/2),x)

[Out] Integral((2*sqrt(x + 1) + 1)/(x*sqrt(x + 1)*sqrt(x + sqrt(x + 1))), x)

$$3.473 \quad \int \frac{1}{\sqrt{x} \sqrt{1+x}} dx$$

Optimal. Leaf size=8

$$2 \sinh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {54, 215}

$$2 \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] 2*ArcSinh[Sqrt[x]]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x} \sqrt{1+x}} dx &= 2 \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x} \right) \\ &= 2 \sinh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2 \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] 2*ArcSinh[Sqrt[x]]

IntegrateAlgebraic [B] time = 0.03, size = 18, normalized size = 2.25

$$-2 \log(\sqrt{x+1} - \sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*Sqrt[1 + x]),x]

[Out] -2*Log[-Sqrt[x] + Sqrt[1 + x]]

fricas [B] time = 0.67, size = 18, normalized size = 2.25

$$-\log(2\sqrt{x+1}\sqrt{x} - 2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")

[Out] -log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)

giac [B] time = 0.25, size = 14, normalized size = 1.75

$$-2 \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")

[Out] -2*log(sqrt(x + 1) - sqrt(x))

maple [B] time = 0.00, size = 28, normalized size = 3.50

$$\frac{\sqrt{(x+1)x} \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)}{\sqrt{x+1} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(x+1)^(1/2),x)

[Out] (x*(x+1))^(1/2)/x^(1/2)/(x+1)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))

maxima [B] time = 0.87, size = 27, normalized size = 3.38

$$\log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) - \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")

[Out] log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)

mupad [B] time = 0.16, size = 14, normalized size = 1.75

$$4 \operatorname{atanh}\left(\frac{\sqrt{x+1} - 1}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(x + 1)^(1/2)),x)

[Out] 4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))

sympy [A] time = 0.95, size = 26, normalized size = 3.25

$$\begin{cases} 2 \operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i \operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(1+x)**(1/2),x)

[Out] Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)), True))

$$3.474 \quad \int \frac{\sqrt{\frac{x}{1+x}}}{x} dx$$

Optimal. Leaf size=8

$$2 \sinh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1958, 54, 215}

$$2 \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x/(1 + x)]/x,x]

[Out] 2*ArcSinh[Sqrt[x]]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{x}{1+x}}}{x} dx &= \int \frac{1}{\sqrt{x} \sqrt{1+x}} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x} \right) \\ &= 2 \sinh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 8, normalized size = 1.00

$$2 \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x/(1 + x)]/x,x]

[Out] 2*ArcSinh[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.02, size = 14, normalized size = 1.75

$$2 \tanh^{-1} \left(\sqrt{\frac{x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x/(1 + x)]/x,x]

[Out] 2*ArcTanh[Sqrt[x/(1 + x)]]

fricas [B] time = 0.65, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}} + 1\right) - \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")

[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)

giac [B] time = 0.40, size = 22, normalized size = 2.75

$$-\log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right) \operatorname{sgn}(x + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")

[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)

maple [B] time = 0.02, size = 32, normalized size = 4.00

$$\frac{\sqrt{\frac{x}{x+1}} (x + 1) \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)}{\sqrt{(x + 1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x/(x+1))^(1/2)/x,x)

[Out] (x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))

maxima [B] time = 0.88, size = 27, normalized size = 3.38

$$\log\left(\sqrt{\frac{x}{x+1}} + 1\right) - \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")

[Out] log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)

mupad [B] time = 0.06, size = 12, normalized size = 1.50

$$2 \operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x/(x + 1))^(1/2)/x,x)

[Out] 2*atanh((x/(x + 1))^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x}{x+1}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x/(1+x))**(1/2)/x, x)
```

```
[Out] Integral(sqrt(x/(x + 1))/x, x)
```

$$3.475 \quad \int \frac{\sqrt{x}}{\sqrt{1+x}} dx$$

Optimal. Leaf size=22

$$\sqrt{x} \sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {50, 54, 215}

$$\sqrt{x} \sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/Sqrt[1 + x], x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{1+x}} dx &= \sqrt{x} \sqrt{1+x} - \frac{1}{2} \int \frac{1}{\sqrt{x} \sqrt{1+x}} dx \\ &= \sqrt{x} \sqrt{1+x} - \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x} \right) \\ &= \sqrt{x} \sqrt{1+x} - \sinh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}} \left(\sqrt{x} (x+1) - \sqrt{x+1} \sinh^{-1}(\sqrt{x}) \right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/Sqrt[1 + x], x]

[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]

IntegrateAlgebraic [A] time = 0.05, size = 30, normalized size = 1.36

$$\sqrt{x} \sqrt{x+1} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/Sqrt[1 + x], x]

[Out] Sqrt[x]*Sqrt[1 + x] + Log[-Sqrt[x] + Sqrt[1 + x]]

fricas [A] time = 0.61, size = 28, normalized size = 1.27

$$\sqrt{x+1} \sqrt{x} + \frac{1}{2} \log\left(2 \sqrt{x+1} \sqrt{x} - 2x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(1/2), x, algorithm="fricas")

[Out] sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)

giac [A] time = 0.39, size = 22, normalized size = 1.00

$$\sqrt{x+1} \sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(1/2), x, algorithm="giac")

[Out] sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))

maple [B] time = 0.00, size = 39, normalized size = 1.77

$$\sqrt{x+1} \sqrt{x} - \frac{\sqrt{(x+1)x} \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)}{2\sqrt{x+1} \sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x+1)^(1/2), x)

[Out] x^(1/2)*(x+1)^(1/2)-1/2*((x+1)*x)^(1/2)/(x+1)^(1/2)/x^(1/2)*ln(x+1/2+(x^2+x)^(1/2))

maxima [B] time = 0.87, size = 49, normalized size = 2.23

$$\frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x} - 1\right)} - \frac{1}{2} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} + 1\right) + \frac{1}{2} \log\left(\frac{\sqrt{x+1}}{\sqrt{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x)^(1/2), x, algorithm="maxima")

[Out] sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) + 1/2*log(sqrt(x + 1)/sqrt(x) - 1)

mupad [B] time = 3.72, size = 26, normalized size = 1.18

$$\sqrt{x} \sqrt{x+1} - 2 \operatorname{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1} - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x + 1)^(1/2),x)`

[Out] `x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))`

sympy [A] time = 1.48, size = 60, normalized size = 2.73

$$\begin{cases} -\operatorname{acosh}(\sqrt{x+1}) + \frac{(x+1)^{3/2}}{\sqrt{x}} - \frac{\sqrt{x+1}}{\sqrt{x}} & \text{for } |x+1| > 1 \\ i\sqrt{-x}\sqrt{x+1} + i\operatorname{asin}(\sqrt{x+1}) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(1+x)**(1/2),x)`

[Out] `Piecewise((-acosh(sqrt(x + 1)) + (x + 1)**(3/2)/sqrt(x) - sqrt(x + 1)/sqrt(x), Abs(x + 1) > 1), (I*sqrt(-x)*sqrt(x + 1) + I*asin(sqrt(x + 1)), True))`

$$3.476 \quad \int \sqrt{\frac{x}{1+x}} dx$$

Optimal. Leaf size=22

$$\sqrt{x} \sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1958, 50, 54, 215}

$$\sqrt{x} \sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x/(1 + x)],x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{x}{1+x}} dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}} dx \\ &= \sqrt{x} \sqrt{1+x} - \frac{1}{2} \int \frac{1}{\sqrt{x} \sqrt{1+x}} dx \\ &= \sqrt{x} \sqrt{1+x} - \text{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x} \right) \\ &= \sqrt{x} \sqrt{1+x} - \sinh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}} \left(\sqrt{x}(x+1) - \sqrt{x+1} \sinh^{-1}(\sqrt{x}) \right)}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x/(1 + x)], x]

[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 1.36

$$\sqrt{\frac{x}{x+1}}(x+1) - \tanh^{-1}\left(\sqrt{\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x/(1 + x)], x]

[Out] Sqrt[x/(1 + x)]*(1 + x) - ArcTanh[Sqrt[x/(1 + x)]]

fricas [B] time = 0.75, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2), x, algorithm="fricas")

[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x + 1)) - 1)

giac [B] time = 0.44, size = 35, normalized size = 1.59

$$\frac{1}{2} \log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right) \operatorname{sgn}(x+1) + \sqrt{x^2 + x} \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2), x, algorithm="giac")

[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x + 1)

maple [B] time = 0.01, size = 45, normalized size = 2.05

$$\frac{\sqrt{\frac{x}{x+1}}(x+1)\left(-\ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right) + 2\sqrt{x^2 + x}\right)}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(x+1)*x)^(1/2), x)

[Out] 1/2*(1/(x+1)*x)^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*x)^(1/2)

maxima [B] time = 0.82, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")

[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x + 1)) - 1)

mupad [B] time = 3.12, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x/(x + 1))^(1/2),x)

[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))**(1/2),x)

[Out] Integral(sqrt(x/(x + 1)), x)

$$3.477 \quad \int \frac{\sqrt{-1+x}}{x^2 \sqrt{1+x}} dx$$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]

[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 94

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 203

Int[((a_.) + (b_.)*(x_.)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-1+x}}{x^2 \sqrt{1+x}} dx &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x}x\sqrt{1+x}} dx \\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1+x}\sqrt{1+x}\right) \\ &= -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \tan^{-1}\left(\sqrt{-1+x}\sqrt{1+x}\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}} \left(-x^2 + \sqrt{x^2-1} x \tan^{-1}\left(\sqrt{x^2-1}\right) + 1\right)}{(x-1)x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]])/((-1 + x)*x)

IntegrateAlgebraic [A] time = 0.17, size = 41, normalized size = 1.14

$$-\frac{\sqrt{x-1}\sqrt{x+1}}{x} - 2 \tan^{-1}\left(x - \sqrt{x-1}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]

[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) - 2*ArcTan[x - Sqrt[-1 + x]*Sqrt[1 + x]]

fricas [A] time = 0.79, size = 39, normalized size = 1.08

$$\frac{2x \arctan(\sqrt{x+1}\sqrt{x-1} - x) - \sqrt{x+1}\sqrt{x-1} - x}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")

[Out] (2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x

giac [A] time = 0.33, size = 42, normalized size = 1.17

$$-\frac{8}{(\sqrt{x+1} - \sqrt{x-1})^4 + 4} - 2 \arctan\left(\frac{1}{2}(\sqrt{x+1} - \sqrt{x-1})^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")

[Out] -8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(x - 1))^2)

maple [A] time = 0.02, size = 43, normalized size = 1.19

$$\frac{\left(-x \arctan\left(\frac{1}{\sqrt{x^2-1}}\right) - \sqrt{x^2-1}\right) \sqrt{x-1} \sqrt{x+1}}{\sqrt{x^2-1} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)^(1/2)/x^2/(x+1)^(1/2),x)

[Out] (-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-1)^(1/2)

maxima [A] time = 1.98, size = 20, normalized size = 0.56

$$-\frac{\sqrt{x^2-1}}{x} - \arcsin\left(\frac{1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")

[Out] $-\sqrt{x^2 - 1}/x - \arcsin(1/\text{abs}(x))$

mupad [B] time = 5.09, size = 138, normalized size = 3.83

$$-\ln\left(\frac{(\sqrt{x-1}-i)^2}{(\sqrt{x+1}-1)^2}+1\right)1i + \ln\left(\frac{\sqrt{x-1}-i}{\sqrt{x+1}-1}\right)1i - \frac{\sqrt{x-1}-i}{4(\sqrt{x+1}-1)} - \frac{\frac{5(\sqrt{x-1}-i)^2}{(\sqrt{x+1}-1)^2}+1}{\frac{4(\sqrt{x-1}-i)^3}{(\sqrt{x+1}-1)^3} + \frac{4(\sqrt{x-1}-i)}{\sqrt{x+1}-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)`

[Out] $\log((x-1)^{1/2}-1i)/((x+1)^{1/2}-1)*1i - \log((x-1)^{1/2}-1i)^2/((x+1)^{1/2}-1)^2+1*1i - ((x-1)^{1/2}-1i)/(4*((x+1)^{1/2}-1)) - ((5*((x-1)^{1/2}-1i)^2)/((x+1)^{1/2}-1)^2+1)/((4*((x-1)^{1/2}-1i)^3)/((x+1)^{1/2}-1)^3 + (4*((x-1)^{1/2}-1i))/((x+1)^{1/2}-1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)**(1/2)/x**2/(1+x)**(1/2),x)`

[Out] `Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)`

$$3.478 \quad \int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$$

Optimal. Leaf size=36

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1958, 94, 92, 203}

$$\tan^{-1}\left(\sqrt{x-1}\sqrt{x+1}\right) - \frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]

[Out] -((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_.)]*Sqrt[(c_.) + (d_.)*(x_.)]*((e_.) + (f_.)*(x_.))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 94

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.)*((e_.) + (f_.)*(x_.))^(p_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx &= \int \frac{\sqrt{-1+x}}{x^2 \sqrt{1+x}} dx \\
&= -\frac{\sqrt{-1+x} \sqrt{1+x}}{x} + \int \frac{1}{\sqrt{-1+x} x \sqrt{1+x}} dx \\
&= -\frac{\sqrt{-1+x} \sqrt{1+x}}{x} + \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1+x} \sqrt{1+x} \right) \\
&= -\frac{\sqrt{-1+x} \sqrt{1+x}}{x} + \tan^{-1} \left(\sqrt{-1+x} \sqrt{1+x} \right)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 50, normalized size = 1.39

$$\frac{\sqrt{\frac{x-1}{x+1}} \left(-x^2 + \sqrt{x^2-1} x \tan^{-1} \left(\sqrt{x^2-1} \right) + 1 \right)}{(x-1)x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(1 - x^2 + x*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]]))/((-1 + x)*x)

IntegrateAlgebraic [A] time = 0.04, size = 45, normalized size = 1.25

$$2 \tan^{-1} \left(\sqrt{\frac{x-1}{x+1}} \right) - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} + 1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x)/(1 + x)]/x^2,x]

[Out] (-2*Sqrt[(-1 + x)/(1 + x)])/(1 + (-1 + x)/(1 + x)) + 2*ArcTan[Sqrt[(-1 + x)/(1 + x)]]

fricas [A] time = 0.79, size = 36, normalized size = 1.00

$$\frac{2x \arctan \left(\sqrt{\frac{x-1}{x+1}} \right) - (x+1) \sqrt{\frac{x-1}{x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="fricas")

[Out] (2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x

giac [A] time = 0.41, size = 51, normalized size = 1.42

$$-\frac{1}{2}(\pi - 2)\text{sgn}(x+1) + 2 \arctan \left(-x + \sqrt{x^2-1} \right) \text{sgn}(x+1) - \frac{2 \text{sgn}(x+1)}{\left(x - \sqrt{x^2-1} \right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-1+x)/(1+x))^(1/2)/x^2,x, algorithm="giac")

[Out] $-1/2*(\pi - 2)*\text{sgn}(x + 1) + 2*\arctan(-x + \sqrt{x^2 - 1})*\text{sgn}(x + 1) - 2*\text{sgn}(x + 1)/((x - \sqrt{x^2 - 1})^2 + 1)$

maple [B] time = 0.02, size = 59, normalized size = 1.64

$$\frac{\sqrt{\frac{x-1}{x+1}} (x+1) \left(-\sqrt{x^2-1} x^2 - x \arctan\left(\frac{1}{\sqrt{x^2-1}}\right) + (x^2-1)^{\frac{3}{2}} \right)}{\sqrt{(x-1)(x+1)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x-1)/(x+1))^(1/2)/x^2,x)`

[Out] $((x-1)/(x+1))^{1/2}*(x+1)*((x^2-1)^{3/2}-x^2*(x^2-1)^{1/2}-x*\arctan(1/(x^2-1)^{1/2}))/((x-1)*(x+1))^{1/2}/x$

maxima [A] time = 1.95, size = 41, normalized size = 1.14

$$-\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} + 1} + 2 \arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x-1)/(x+1))^(1/2)/x^2,x, algorithm="maxima")`

[Out] $-2*\sqrt{(x-1)/(x+1)}/((x-1)/(x+1) + 1) + 2*\arctan(\sqrt{(x-1)/(x+1)})$

mupad [B] time = 0.06, size = 41, normalized size = 1.14

$$2 \operatorname{atan}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x-1)/(x+1))^(1/2)/x^2,x)`

[Out] $2*\operatorname{atan}((x-1)/(x+1))^{1/2}) - (2*((x-1)/(x+1))^{1/2})/((x-1)/(x+1) + 1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((x-1)/(x+1))**(1/2)/x**2,x)`

[Out] `Integral(sqrt((x-1)/(x+1))/x**2, x)`

$$3.479 \quad \int \frac{\sqrt{-1+x} x^3}{\sqrt{1+x}} dx$$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1} - \frac{3}{8}\sqrt{x-1}\sqrt{x+1} + \frac{3}{8}\cosh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1} - \frac{3}{8}\sqrt{x-1}\sqrt{x+1} + \frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x], x]

[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 52

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b
- d, 0] && GtQ[a, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_
))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
+ b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 147

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m
+ 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2
*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m +
n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3
) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), In
t[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n},
x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}} dx &= \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{1}{4} \int \frac{(2-x)\sqrt{-1+x}x}{\sqrt{1+x}} dx \\
&= \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} - \frac{3}{8} \int \frac{\sqrt{-1+x}}{\sqrt{1+x}} dx \\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8} \int \frac{\sqrt{-1+x}}{\sqrt{1+x}} dx \\
&= -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2}\sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x} + \frac{3}{8} \cosh^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 76, normalized size = 1.10

$$\frac{\sqrt{\frac{x-1}{x+1}} \left(6x^5 - 8x^4 + 3x^3 - 8x^2 - 18\sqrt{1-x^2} \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 9x + 16 \right)}{24(x-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x], x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))

IntegrateAlgebraic [A] time = 0.16, size = 71, normalized size = 1.03

$$\frac{1}{24}\sqrt{x-1} \left(6(x+1)^{7/2} - 26(x+1)^{5/2} + 43(x+1)^{3/2} - 39\sqrt{x+1} \right) - \frac{3}{4} \log\left(\sqrt{x-1} - \sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x], x]

[Out] (Sqrt[-1 + x]*(-39*Sqrt[1 + x] + 43*(1 + x)^(3/2) - 26*(1 + x)^(5/2) + 6*(1 + x)^(7/2)))/24 - (3*Log[Sqrt[-1 + x] - Sqrt[1 + x]])/4

fricas [A] time = 0.99, size = 46, normalized size = 0.67

$$\frac{1}{24} \left(6x^3 - 8x^2 + 9x - 16 \right) \sqrt{x+1} \sqrt{x-1} - \frac{3}{8} \log\left(\sqrt{x+1} \sqrt{x-1} - x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2), x, algorithm="fricas")

[Out] 1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x + 1)*sqrt(x - 1) - x)

giac [A] time = 0.33, size = 47, normalized size = 0.68

$$\frac{1}{24} \left((2(3x-10)(x+1) + 43)(x+1) - 39 \right) \sqrt{x+1} \sqrt{x-1} - \frac{3}{4} \log\left(\sqrt{x+1} - \sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2), x, algorithm="giac")

[Out] 1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) - 3/4*log(sqrt(x + 1) - sqrt(x - 1))

maple [A] time = 0.01, size = 76, normalized size = 1.10

$$\frac{\sqrt{x-1} \sqrt{x+1} \left(6\sqrt{x^2-1} x^3 - 8\sqrt{x^2-1} x^2 + 9\sqrt{x^2-1} x + 9 \ln \left(x + \sqrt{x^2-1} \right) - 16\sqrt{x^2-1} \right)}{24\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x-1)^(1/2)/(x+1)^(1/2), x)

[Out] 1/24*(x-1)^(1/2)*(x+1)^(1/2)*(6*x^3*(x^2-1)^(1/2)-8*(x^2-1)^(1/2)*x^2+9*x*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2))-16*(x^2-1)^(1/2))/(x^2-1)^(1/2)

maxima [A] time = 0.88, size = 55, normalized size = 0.80

$$\frac{1}{4} (x^2 - 1)^{\frac{3}{2}} x - \frac{1}{3} (x^2 - 1)^{\frac{3}{2}} + \frac{5}{8} \sqrt{x^2 - 1} x - \sqrt{x^2 - 1} + \frac{3}{8} \log \left(2x + 2\sqrt{x^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(-1+x)^(1/2)/(1+x)^(1/2), x, algorithm="maxima")

[Out] 1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x^2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))

mupad [B] time = 12.86, size = 473, normalized size = 6.86

$$\frac{3 \operatorname{atanh} \left(\frac{\sqrt{x-1}}{\sqrt{x+1}} \right) + \frac{23(\sqrt{x-1})^3}{2(\sqrt{x+1})^3} - \frac{(\sqrt{x-1})^4 64i}{(\sqrt{x+1})^4} + \frac{333(\sqrt{x-1})^5}{2(\sqrt{x+1})^5} + \frac{(\sqrt{x-1})^6 256i}{3(\sqrt{x+1})^6} + \frac{671(\sqrt{x-1})^7}{2(\sqrt{x+1})^7} - \frac{(\sqrt{x-1})^8 128i}{3(\sqrt{x+1})^8} + \frac{671(\sqrt{x-1})^9}{2(\sqrt{x+1})^9} + \frac{(\sqrt{x-1})^{10} 256i}{3(\sqrt{x+1})^{10}} + \frac{333(\sqrt{x-1})^{11}}{2(\sqrt{x+1})^{11}} - \frac{(\sqrt{x-1})^{12} 64i}{(\sqrt{x+1})^{12}} + \frac{23(\sqrt{x-1})^{13}}{2(\sqrt{x+1})^{13}} - \frac{3(\sqrt{x-1})^{15}}{2(\sqrt{x+1})^{15}} - \frac{3(\sqrt{x-1})}{2(\sqrt{x+1})}}{1 + \frac{28(\sqrt{x-1})^4}{(\sqrt{x+1})^4} - \frac{56(\sqrt{x-1})^6}{(\sqrt{x+1})^6} + \frac{70(\sqrt{x-1})^8}{(\sqrt{x+1})^8} - \frac{56(\sqrt{x-1})^{10}}{(\sqrt{x+1})^{10}} + \frac{28(\sqrt{x-1})^{12}}{(\sqrt{x+1})^{12}} - \frac{8(\sqrt{x-1})^{14}}{(\sqrt{x+1})^{14}} + \frac{(\sqrt{x-1})^{16}}{(\sqrt{x+1})^{16}} - \frac{8(\sqrt{x-1})^2}{(\sqrt{x+1})^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2), x)

[Out] (3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1i)))/2 + ((23*((x - 1)^(1/2) - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x + 1)^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) + (((x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(1/2) - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3*((x + 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) - 1)^9) + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*((x - 1)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i)^12*64i)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1)^(1/2) - 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15) - (3*((x - 1)^(1/2) - 1i))/((2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) - 1i)^4)/((x + 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 - (56*((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(1/2) - 1i)^8)/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1)^(1/2) - 1)^10 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8*((x - 1)^(1/2) - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16/((x + 1)^(1/2) - 1)^16 + 1)

sympy [A] time = 13.85, size = 83, normalized size = 1.20

$$\frac{(x-1)^{\frac{7}{2}} \sqrt{x+1}}{4} + \frac{5(x-1)^{\frac{5}{2}} \sqrt{x+1}}{12} + \frac{11(x-1)^{\frac{3}{2}} \sqrt{x+1}}{24} - \frac{3\sqrt{x-1} \sqrt{x+1}}{8} + \frac{3 \operatorname{asinh} \left(\frac{\sqrt{2} \sqrt{x-1}}{2} \right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(-1+x)**(1/2)/(1+x)**(1/2), x)

[Out] (x - 1)**(7/2)*sqrt(x + 1)/4 + 5*(x - 1)**(5/2)*sqrt(x + 1)/12 + 11*(x - 1)**(3/2)*sqrt(x + 1)/24 - 3*sqrt(x - 1)*sqrt(x + 1)/8 + 3*asinh(sqrt(2)*sqrt(x - 1)/2)/4

$$3.480 \quad \int x^3 \sqrt{\frac{-1+x}{1+x}} dx$$

Optimal. Leaf size=69

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1} - \frac{3}{8}\sqrt{x-1}\sqrt{x+1} + \frac{3}{8}\cosh^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1958, 100, 147, 50, 52}

$$\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2 + \frac{1}{24}(7-2x)(x-1)^{3/2}\sqrt{x+1} - \frac{3}{8}\sqrt{x-1}\sqrt{x+1} + \frac{3}{8}\cosh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^3*Sqrt[(-1 + x)/(1 + x)], x]

[Out] (-3*Sqrt[-1 + x]*Sqrt[1 + x])/8 + ((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x])/24 + ((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (3*ArcCosh[x])/8

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 52

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[ArcCosh[(b*x)/a]/b, x] /; FreeQ[{a, b, c, d}, x] && EqQ[a + c, 0] && EqQ[b - d, 0] && GtQ[a, 0]

Rule 100

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m + n) + c*f*(m + p)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]

Rule 147

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))*((g_.) + (h_.)*(x_)), x_Symbol] :> -Simp[((a*d*f*h*(n + 2) + b*c*f*h*(m + 2) - b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d^2*(m + n + 2)*(m + n + 3)), x] + Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3)), Int[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rubi steps

$$\begin{aligned}
 \int x^3 \sqrt{\frac{-1+x}{1+x}} dx &= \int \frac{\sqrt{-1+x} x^3}{\sqrt{1+x}} dx \\
 &= \frac{1}{4}(-1+x)^{3/2} x^2 \sqrt{1+x} + \frac{1}{4} \int \frac{(2-x)\sqrt{-1+x} x}{\sqrt{1+x}} dx \\
 &= \frac{1}{24}(7-2x)(-1+x)^{3/2} \sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2} x^2 \sqrt{1+x} - \frac{3}{8} \int \frac{\sqrt{-1+x}}{\sqrt{1+x}} dx \\
 &= -\frac{3}{8} \sqrt{-1+x} \sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2} \sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2} x^2 \sqrt{1+x} + \frac{3}{8} \int \frac{\sqrt{-1+x}}{\sqrt{1+x}} dx \\
 &= -\frac{3}{8} \sqrt{-1+x} \sqrt{1+x} + \frac{1}{24}(7-2x)(-1+x)^{3/2} \sqrt{1+x} + \frac{1}{4}(-1+x)^{3/2} x^2 \sqrt{1+x} + \frac{3}{8} \cosh^{-1}(x)
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.10

$$\frac{\sqrt{\frac{x-1}{x+1}} \left(6x^5 - 8x^4 + 3x^3 - 8x^2 - 18\sqrt{1-x^2} \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) - 9x + 16 \right)}{24(x-1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3*Sqrt[(-1 + x)/(1 + x)], x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(16 - 9*x - 8*x^2 + 3*x^3 - 8*x^4 + 6*x^5 - 18*Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(24*(-1 + x))

IntegrateAlgebraic [A] time = 0.10, size = 85, normalized size = 1.23

$$\frac{\sqrt{\frac{x-1}{x+1}} \left(\frac{39(x-1)^3}{(x+1)^3} - \frac{31(x-1)^2}{(x+1)^2} + \frac{49(x-1)}{x+1} - 9 \right)}{12 \left(\frac{x-1}{x+1} - 1 \right)^4} + \frac{3}{4} \tanh^{-1} \left(\sqrt{\frac{x-1}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*Sqrt[(-1 + x)/(1 + x)], x]

[Out] (Sqrt[(-1 + x)/(1 + x)]*(-9 + (39*(-1 + x)^3)/(1 + x)^3 - (31*(-1 + x)^2)/(1 + x)^2 + (49*(-1 + x))/(1 + x)))/(12*(-1 + (-1 + x)/(1 + x))^4) + (3*ArcTanh[Sqrt[(-1 + x)/(1 + x)]])/4

fricas [A] time = 0.71, size = 64, normalized size = 0.93

$$\frac{1}{24} (6x^4 - 2x^3 + x^2 - 7x - 16) \sqrt{\frac{x-1}{x+1}} + \frac{3}{8} \log \left(\sqrt{\frac{x-1}{x+1}} + 1 \right) - \frac{3}{8} \log \left(\sqrt{\frac{x-1}{x+1}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2), x, algorithm="fricas")

[Out] 1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)

giac [A] time = 0.34, size = 62, normalized size = 0.90

$$-\frac{3}{8} \log\left(\left|-x + \sqrt{x^2 - 1}\right|\right) \operatorname{sgn}(x + 1) + \frac{1}{24} \left(\left(2(3x \operatorname{sgn}(x + 1) - 4 \operatorname{sgn}(x + 1))x + 9 \operatorname{sgn}(x + 1)\right)x - 16 \operatorname{sgn}(x + 1)\right) \sqrt{x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="giac")

[Out] -3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) - 4*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)

maple [A] time = 0.01, size = 79, normalized size = 1.14

$$\frac{\sqrt{\frac{x-1}{x+1}} (x+1) \left(6(x^2-1)^{\frac{3}{2}} x + 15\sqrt{x^2-1} x + 9 \ln\left(x + \sqrt{x^2-1}\right) - 8((x-1)(x+1))^{\frac{3}{2}} - 24\sqrt{x^2-1}\right)}{24\sqrt{(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((x-1)/(x+1))^(1/2),x)

[Out] 1/24*((x-1)/(x+1))^(1/2)*(x+1)*(6*x*(x^2-1)^(3/2)-8*((x-1)*(x+1))^(3/2)+15*(x^2-1)^(1/2)*x-24*(x^2-1)^(1/2)+9*ln(x+(x^2-1)^(1/2)))/((x-1)*(x+1))^(1/2)

maxima [B] time = 0.85, size = 138, normalized size = 2.00

$$\frac{39 \left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} - 31 \left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} + 49 \left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 9 \sqrt{\frac{x-1}{x+1}}}{12 \left(\frac{4(x-1)}{x+1} - \frac{6(x-1)^2}{(x+1)^2} + \frac{4(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} + \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} + 1\right) - \frac{3}{8} \log\left(\sqrt{\frac{x-1}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*((-1+x)/(1+x))^(1/2),x, algorithm="maxima")

[Out] -1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x - 1)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1)^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)

mupad [B] time = 0.05, size = 119, normalized size = 1.72

$$\frac{3 \operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4} - \frac{3\sqrt{\frac{x-1}{x+1}} - \frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12} + \frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12} - \frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2} - \frac{4(x-1)}{x+1} - \frac{4(x-1)^3}{(x+1)^3} + \frac{(x-1)^4}{(x+1)^4} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*((x - 1)/(x + 1))^(1/2),x)

[Out] (3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (49*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x - 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \sqrt{\frac{x-1}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*((-1+x)/(1+x))**(1/2),x)

[Out] Integral(x**3*sqrt((x - 1)/(x + 1)), x)

$$3.481 \quad \int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx$$

Optimal. Leaf size=15

$$2 \tan^{-1} \left(\sqrt{-\frac{x}{x+1}} \right)$$

Rubi [A] time = 0.01, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1960, 204}

$$2 \tan^{-1} \left(\sqrt{-\frac{x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-(x/(1 + x))]/x,x]

[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1960

Int[(x_)^(m_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(Simplify[(m + 1)/n] - 1)]/(b*e - d*x^q)^(Simplify[(m + 1)/n] + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && FractionQ[p] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{-\frac{x}{1+x}}}{x} dx &= - \left(2 \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{-\frac{x}{1+x}} \right) \right) \\ &= 2 \tan^{-1} \left(\sqrt{-\frac{x}{1+x}} \right) \end{aligned}$$

Mathematica [B] time = 0.02, size = 32, normalized size = 2.13

$$\frac{2\sqrt{-\frac{x}{x+1}}\sqrt{x+1}\sinh^{-1}(\sqrt{x})}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-(x/(1 + x))]/x,x]

[Out] (2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*ArcSinh[Sqrt[x]])/Sqrt[x]

IntegrateAlgebraic [A] time = 0.03, size = 15, normalized size = 1.00

$$2 \tan^{-1} \left(\sqrt{-\frac{x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-(x/(1 + x))]/x,x]

[Out] 2*ArcTan[Sqrt[-(x/(1 + x))]]

fricas [A] time = 2.00, size = 13, normalized size = 0.87

$$2 \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")

[Out] 2*arctan(sqrt(-x/(x + 1)))

giac [A] time = 0.30, size = 20, normalized size = 1.33

$$-\frac{1}{2} \pi \operatorname{sgn}(x+1) - \arcsin(2x+1) \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")

[Out] -1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)

maple [B] time = 0.02, size = 33, normalized size = 2.20

$$\frac{\sqrt{-\frac{x}{x+1}} (x+1) \ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right)}{\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/(x+1)*x)^(1/2)/x,x)

[Out] (-1/(x+1)*x)^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(x+1/2+(x^2+x)^(1/2))

maxima [A] time = 1.95, size = 13, normalized size = 0.87

$$2 \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")

[Out] 2*arctan(sqrt(-x/(x + 1)))

mupad [B] time = 0.18, size = 13, normalized size = 0.87

$$2 \operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x/(x + 1))^(1/2)/x,x)

[Out] 2*atan((-x/(x + 1))^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{-x}{x+1}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))**(1/2)/x,x)

[Out] Integral(sqrt(-x/(x + 1))/x, x)

$$3.482 \quad \int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx$$

Optimal. Leaf size=18

$$2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Rubi [A] time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1961, 204}

$$2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x), x]

[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 1961

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)*(u /. x -> (-a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r]/(b*e - d*x^q)^(1/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q), x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x} dx &= - \left(4 \text{Subst} \left(\int \frac{1}{-2-2x^2} dx, x, \sqrt{\frac{1-x}{1+x}} \right) \right) \\ &= 2 \tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 34, normalized size = 1.89

$$\frac{\sqrt{\frac{1-x}{x+1}} \sqrt{1-x^2} \sin^{-1}(x)}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x), x]

[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcSin[x])/(-1 + x)

IntegrateAlgebraic [A] time = 0.03, size = 18, normalized size = 1.00

$$2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 - x)/(1 + x)]/(-1 + x), x]

[Out] 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

fricas [A] time = 0.91, size = 15, normalized size = 0.83

$$2 \arctan \left(\sqrt{-\frac{x-1}{x+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x), x, algorithm="fricas")

[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))

giac [A] time = 0.31, size = 16, normalized size = 0.89

$$-\frac{1}{2} \pi \operatorname{sgn}(x+1) - \arcsin(x) \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x), x, algorithm="giac")

[Out] -1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)

maple [A] time = 0.02, size = 30, normalized size = 1.67

$$-\frac{\sqrt{-\frac{x-1}{x+1}} (x+1) \arcsin(x)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x-1)/(x+1))^(1/2)/(x-1), x)

[Out] -(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)

maxima [A] time = 2.54, size = 15, normalized size = 0.83

$$2 \arctan \left(\sqrt{-\frac{x-1}{x+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2)/(-1+x), x, algorithm="maxima")

[Out] 2*arctan(sqrt(-(x - 1)/(x + 1)))

mupad [B] time = 3.14, size = 15, normalized size = 0.83

$$2 \operatorname{atan} \left(\sqrt{-\frac{x-1}{x+1}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x - 1)/(x + 1))^(1/2)/(x - 1),x)
```

```
[Out] 2*atan((-x - 1)/(x + 1))^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{-x-1}{x+1}}}{x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((1-x)/(1+x))**(1/2)/(-1+x),x)
```

```
[Out] Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

$$3.483 \quad \int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx$$

Optimal. Leaf size=24

$$\frac{2 \tan^{-1} \left(\sqrt{\frac{a+bx}{c-bx}} \right)}{b}$$

Rubi [A] time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1961, 12, 203}

$$\frac{2 \tan^{-1} \left(\sqrt{\frac{a+bx}{c-bx}} \right)}{b}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]

[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1961

Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(u /. x -> (-a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r]/(b*e - d*x^q)^(1/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} dx &= (2b(a+c)) \text{Subst} \left(\int \frac{1}{b^2(a+c)(1+x^2)} dx, x, \sqrt{\frac{a+bx}{c-bx}} \right) \\ &= \frac{2 \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{a+bx}{c-bx}} \right)}{b} \\ &= \frac{2 \tan^{-1} \left(\sqrt{\frac{a+bx}{c-bx}} \right)}{b} \end{aligned}$$

Mathematica [B] time = 0.23, size = 93, normalized size = 3.88

$$\frac{2b\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\sin^{-1}\left(\frac{b\sqrt{c-bx}}{\sqrt{-b}\sqrt{-b(a+c)}}\right)}{(-b)^{3/2}\sqrt{-b(a+c)}\sqrt{\frac{a+bx}{a+c}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x), x]

[Out] (2*b*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcSin[(b*Sqrt[c - b*x])/(Sqrt[-b]*Sqrt[-(b*(a + c))])])/((-b)^(3/2)*Sqrt[-(b*(a + c))]*Sqrt[(a + b*x)/(a + c)])

IntegrateAlgebraic [A] time = 0.10, size = 24, normalized size = 1.00

$$\frac{2 \tan^{-1}\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x), x]

[Out] (2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b

fricas [A] time = 0.62, size = 24, normalized size = 1.00

$$\frac{2 \arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a), x, algorithm="fricas")

[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b

giac [A] time = 0.50, size = 41, normalized size = 1.71

$$\frac{\arcsin\left(-\frac{2bx+a-c}{a+c}\right)\operatorname{sgn}(-ab-bc)\operatorname{sgn}(bx-c)}{|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a), x, algorithm="giac")

[Out] -arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)

maple [B] time = 0.03, size = 85, normalized size = 3.54

$$\frac{(bx-c)\sqrt{-\frac{bx+a}{bx-c}}\arctan\left(\frac{\sqrt{b^2}(2bx+a-c)}{2\sqrt{-(bx+a)(bx-c)}b}\right)}{\sqrt{b^2}\sqrt{-(bx+a)(bx-c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a), x)

[Out] -arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-(b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)

maxima [A] time = 1.94, size = 24, normalized size = 1.00

$$\frac{2 \arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")

[Out] 2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b

mupad [B] time = 0.18, size = 36, normalized size = 1.50

$$-\frac{2\sqrt{-b} \operatorname{atanh}\left(\frac{\sqrt{-b} \sqrt{\frac{a+bx}{c-bx}}}{\sqrt{b}}\right)}{b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)

[Out] -(2*(-b)^(1/2)*atanh((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(3/2)))/b^(3/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)

[Out] Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)

$$3.484 \quad \int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx$$

Optimal. Leaf size=41

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right)}{\sqrt{b} \sqrt{d}}$$

Rubi [A] time = 0.07, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1961, 12, 208}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right)}{\sqrt{b} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x), x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x))]/Sqrt[b]])/(Sqrt[b]*Sqrt[d])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 1961

Int[(u_)^(r_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1)*(u /. x -> (- (a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r)/(b*e - d*x^q)^(1/n + 1), x], x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegerQ[r]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx &= (2(bc - ad)) \text{Subst} \left(\int \frac{1}{(bc - ad)(b - dx^2)} dx, x, \sqrt{\frac{a+bx}{c+dx}} \right) \\ &= 2 \text{Subst} \left(\int \frac{1}{b - dx^2} dx, x, \sqrt{\frac{a+bx}{c+dx}} \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right)}{\sqrt{b} \sqrt{d}} \end{aligned}$$

Mathematica [B] time = 0.08, size = 97, normalized size = 2.37

$$\frac{2\sqrt{bc-ad}\sqrt{\frac{a+bx}{c+dx}}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{a+bx}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x), x]

[Out] (2*Sqrt[b*c - a*d]*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[a + b*x])

IntegrateAlgebraic [A] time = 0.08, size = 41, normalized size = 1.00

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x), x]

[Out] (2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])]/Sqrt[b])/Sqrt[b]*Sqrt[d]

fricas [A] time = 2.27, size = 105, normalized size = 2.56

$$\left[\frac{\sqrt{bd} \log\left(2 bdx + bc + ad + 2 \sqrt{bd} (dx + c) \sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2 \sqrt{-bd} \arctan\left(\frac{\sqrt{-bd} (dx+c) \sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a), x, algorithm="fricas")

[Out] [sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/(d*x + c)))/(b*d*x + a*d))/(b*d)]

giac [B] time = 0.49, size = 74, normalized size = 1.80

$$\frac{\sqrt{bd} \log\left(\left|-2\left(\sqrt{bd}x - \sqrt{bdx^2 + bcx + adx + ac}\right)bd - \sqrt{bd}bc - \sqrt{bd}ad\right|\right) \operatorname{sgn}(dx + c)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a), x, algorithm="giac")

[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))*sgn(d*x + c)/(b*d)

maple [B] time = 0.03, size = 80, normalized size = 1.95

$$\frac{(dx + c) \sqrt{\frac{bx+a}{dx+c}} \ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\sqrt{bd}}{2\sqrt{bd}}\right)}{\sqrt{(bx+a)(dx+c)}\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)

[Out] $\ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))$
 $* (d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)$

maxima [A] time = 1.73, size = 59, normalized size = 1.44

$$\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")

[Out] $-\log((d*\sqrt{(b*x+a)/(d*x+c)})-\sqrt{b*d})/(d*\sqrt{(b*x+a)/(d*x+c)})$
 $+ \sqrt{b*d})/\sqrt{b*d}$

mupad [B] time = 0.20, size = 31, normalized size = 0.76

$$\frac{2 \operatorname{atanh}\left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b} \sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)

[Out] $(2*\operatorname{atanh}((d^(1/2))*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)

[Out] Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)

$$3.485 \quad \int \sqrt{-\frac{x}{1+x}} dx$$

Optimal. Leaf size=32

$$\sqrt{-\frac{x}{x+1}}(x+1) - \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Rubi [A] time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1959, 288, 204}

$$\sqrt{-\frac{x}{x+1}}(x+1) - \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-(x/(1 + x))], x]

[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]

Rule 204

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1959

Int[(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sqrt{-\frac{x}{1+x}} dx &= -\left(2 \text{Subst}\left(\int \frac{x^2}{(-1-x^2)^2} dx, x, \sqrt{-\frac{x}{1+x}}\right)\right) \\ &= \sqrt{-\frac{x}{1+x}}(1+x) + \text{Subst}\left(\int \frac{1}{-1-x^2} dx, x, \sqrt{-\frac{x}{1+x}}\right) \\ &= \sqrt{-\frac{x}{1+x}}(1+x) - \tan^{-1}\left(\sqrt{-\frac{x}{1+x}}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.34

$$\frac{\sqrt{-\frac{x}{x+1}}(\sqrt{x}(x+1) - \sqrt{x+1} \sinh^{-1}(\sqrt{x}))}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-(x/(1 + x))], x]

[Out] (Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 1.00

$$\sqrt{-\frac{x}{x+1}}(x+1) - \tan^{-1}\left(\sqrt{-\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-(x/(1 + x))], x]

[Out] Sqrt[-(x/(1 + x))]*(1 + x) - ArcTan[Sqrt[-(x/(1 + x))]]]

fricas [A] time = 0.60, size = 28, normalized size = 0.88

$$(x+1)\sqrt{-\frac{x}{x+1}} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2), x, algorithm="fricas")

[Out] (x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))

giac [A] time = 0.35, size = 36, normalized size = 1.12

$$\frac{1}{4}\pi\operatorname{sgn}(x+1) + \frac{1}{2}\arcsin(2x+1)\operatorname{sgn}(x+1) + \sqrt{-x^2-x}\operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2), x, algorithm="giac")

[Out] 1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x + 1)

maple [A] time = 0.01, size = 46, normalized size = 1.44

$$\frac{\sqrt{-\frac{x}{x+1}}(x+1)\left(-\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)+2\sqrt{x^2+x}\right)}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/(x+1)*x)^(1/2), x)

[Out] 1/2*(-1/(x+1)*x)^(1/2)*(x+1)*(-ln(x+1/2+(x^2+x)^(1/2))+2*(x^2+x)^(1/2))/((x+1)*x)^(1/2)

maxima [A] time = 1.80, size = 37, normalized size = 1.16

$$-\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x/(1+x))^(1/2), x, algorithm="maxima")

[Out] $-\sqrt{-x/(x+1)}/(x/(x+1)-1) - \arctan(\sqrt{-x/(x+1)})$

mupad [B] time = 3.13, size = 37, normalized size = 1.16

$$-\operatorname{atan}\left(\sqrt{-\frac{x}{x+1}}\right) - \frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x/(x+1))^(1/2),x)`

[Out] $-\operatorname{atan}((-x/(x+1))^{1/2}) - (-x/(x+1))^{1/2}/(x/(x+1)-1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{x}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x/(1+x))**(1/2),x)`

[Out] `Integral(sqrt(-x/(x+1)), x)`

$$3.486 \quad \int \sqrt{\frac{1-x}{1+x}} dx$$

Optimal. Leaf size=38

$$\sqrt{\frac{1-x}{x+1}}(x+1) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Rubi [A] time = 0.01, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1959, 288, 204}

$$\sqrt{\frac{1-x}{x+1}}(x+1) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{x+1}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x)/(1 + x)], x]

[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1959

Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-(a*e) + c*x^q)^(1/n - 1))/(b*e - d*x^q)^(1/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] & FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1-x}{1+x}} dx &= - \left(4 \text{Subst} \left(\int \frac{x^2}{(-1-x^2)^2} dx, x, \sqrt{\frac{1-x}{1+x}} \right) \right) \\ &= \sqrt{\frac{1-x}{1+x}}(1+x) + 2 \text{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \sqrt{\frac{1-x}{1+x}} \right) \\ &= \sqrt{\frac{1-x}{1+x}}(1+x) - 2 \tan^{-1} \left(\sqrt{\frac{1-x}{1+x}} \right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.76

$$\frac{\sqrt{\frac{1-x}{x+1}} \sqrt{x+1} \left(\sqrt{x+1}(x-1) + 2\sqrt{1-x} \sin^{-1} \left(\frac{\sqrt{1-x}}{\sqrt{2}} \right) \right)}{x-1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 - x)/(1 + x)], x]

[Out] (Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*((-1 + x)*Sqrt[1 + x] + 2*Sqrt[1 - x]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/(-1 + x)

IntegrateAlgebraic [A] time = 0.03, size = 38, normalized size = 1.00

$$\sqrt{\frac{1-x}{x+1}}(x+1) - 2 \tan^{-1}\left(\sqrt{\frac{1-x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 - x)/(1 + x)], x]

[Out] Sqrt[(1 - x)/(1 + x)]*(1 + x) - 2*ArcTan[Sqrt[(1 - x)/(1 + x)]]

fricas [A] time = 0.52, size = 32, normalized size = 0.84

$$(x+1)\sqrt{-\frac{x-1}{x+1}} - 2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2), x, algorithm="fricas")

[Out] (x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))

giac [A] time = 0.40, size = 29, normalized size = 0.76

$$\frac{1}{2} \pi \operatorname{sgn}(x+1) + \arcsin(x) \operatorname{sgn}(x+1) + \sqrt{-x^2+1} \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2), x, algorithm="giac")

[Out] 1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)

maple [A] time = 0.01, size = 39, normalized size = 1.03

$$\frac{\sqrt{-\frac{x-1}{x+1}}(x+1)\left(\arcsin(x) + \sqrt{-x^2+1}\right)}{\sqrt{-(x-1)(x+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((1-x)/(1+x))^(1/2), x)

[Out] (-x+1)/(x+1)^(1/2)*(x+1)/(-x+1)*(x+1)^(1/2)*((-x^2+1)^(1/2)+arcsin(x))

maxima [A] time = 1.85, size = 43, normalized size = 1.13

$$-\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2 \arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/(1+x))^(1/2), x, algorithm="maxima")

[Out] -2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(x + 1)))

mupad [B] time = 0.03, size = 43, normalized size = 1.13

$$-2 \operatorname{atan} \left(\sqrt{\frac{x-1}{x+1}} \right) - \frac{2 \sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x - 1)/(x + 1))^(1/2), x)`

[Out] `- 2*atan((-x - 1)/(x + 1))^(1/2)) - (2*(-x - 1)/(x + 1))^(1/2))/((x - 1)/(x + 1) - 1)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-x}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((1-x)/(1+x))**(1/2), x)`

[Out] `Integral(sqrt((1 - x)/(x + 1)), x)`

$$3.487 \quad \int \sqrt{\frac{a+x}{a-x}} dx$$

Optimal. Leaf size=42

$$2a \tan^{-1} \left(\sqrt{\frac{a+x}{a-x}} \right) - (a-x) \sqrt{\frac{a+x}{a-x}}$$

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1959, 288, 203}

$$2a \tan^{-1} \left(\sqrt{\frac{a+x}{a-x}} \right) - (a-x) \sqrt{\frac{a+x}{a-x}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + x)/(a - x)], x]

[Out] -((a - x)*Sqrt[(a + x)/(a - x)]) + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 288

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1959

Int[(((e_)*((a_) + (b_)*(x_)^(n_))))/(((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{a+x}{a-x}} dx &= (4a) \text{Subst} \left(\int \frac{x^2}{(1+x^2)^2} dx, x, \sqrt{\frac{a+x}{a-x}} \right) \\ &= - \left((a-x) \sqrt{\frac{a+x}{a-x}} \right) + (2a) \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{\frac{a+x}{a-x}} \right) \\ &= - \left((a-x) \sqrt{\frac{a+x}{a-x}} \right) + 2a \tan^{-1} \left(\sqrt{\frac{a+x}{a-x}} \right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 83, normalized size = 1.98

$$\frac{\sqrt{x-a} \sqrt{\frac{a+x}{a-x}} \left(2a^{3/2} \sqrt{\frac{a+x}{a}} \sinh^{-1} \left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}} \right) + \sqrt{x-a} (a+x) \right)}{a+x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(a + x)/(a - x)], x]

[Out] (Sqrt[-a + x]*Sqrt[(a + x)/(a - x)]*(Sqrt[-a + x]*(a + x) + 2*a^(3/2)*Sqrt[(a + x)/a]*ArcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/(a + x)

IntegrateAlgebraic [A] time = 0.04, size = 41, normalized size = 0.98

$$\sqrt{\frac{a+x}{a-x}}(x-a) + 2a \tan^{-1}\left(\sqrt{\frac{a+x}{a-x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(a + x)/(a - x)], x]

[Out] (-a + x)*Sqrt[(a + x)/(a - x)] + 2*a*ArcTan[Sqrt[(a + x)/(a - x)]]

fricas [A] time = 0.70, size = 38, normalized size = 0.90

$$2a \arctan\left(\sqrt{\frac{a+x}{a-x}}\right) - (a-x)\sqrt{\frac{a+x}{a-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a-x))^(1/2), x, algorithm="fricas")

[Out] 2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))

giac [A] time = 0.45, size = 36, normalized size = 0.86

$$a \arcsin\left(\frac{x}{a}\right) \operatorname{sgn}(a-x) \operatorname{sgn}(a) - \sqrt{a^2 - x^2} \operatorname{sgn}(a-x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a-x))^(1/2), x, algorithm="giac")

[Out] a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)

maple [A] time = 0.02, size = 64, normalized size = 1.52

$$\frac{\sqrt{-\frac{a+x}{-a+x}}(-a+x)\left(a \arctan\left(\frac{x}{\sqrt{a^2-x^2}}\right) - \sqrt{a^2-x^2}\right)}{\sqrt{-(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a+x)/(a-x))^(1/2), x)

[Out] -(-(a+x)/(-a+x))^(1/2)*(-a+x)*(a*arctan(x/(a^2-x^2)^(1/2))-(a^2-x^2)^(1/2))/(-(a+x)*(-a+x))^(1/2)

maxima [A] time = 2.01, size = 49, normalized size = 1.17

$$-2a \left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x} + 1} - \arctan\left(\sqrt{\frac{a+x}{a-x}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a-x))^(1/2), x, algorithm="maxima")

[Out] $-2*a*(\sqrt{(a+x)/(a-x)})/((a+x)/(a-x)+1) - \arctan(\sqrt{(a+x)/(a-x)})$

mupad [B] time = 3.17, size = 49, normalized size = 1.17

$$2a \operatorname{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2a\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((a+x)/(a-x))^(1/2),x)`

[Out] $2*a*\operatorname{atan}(((a+x)/(a-x))^{(1/2)}) - (2*a*((a+x)/(a-x))^{(1/2)})/((a+x)/(a-x)+1)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a+x}{a-x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((a+x)/(a-x))**(1/2),x)`

[Out] `Integral(sqrt((a+x)/(a-x)), x)`

$$3.488 \quad \int \sqrt{\frac{-a+x}{a+x}} dx$$

Optimal. Leaf size=41

$$\sqrt{\frac{a-x}{a+x}}(a+x) - 2a \tanh^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1959, 288, 206}

$$\sqrt{\frac{a-x}{a+x}}(a+x) - 2a \tanh^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-a + x)/(a + x)], x]

[Out] Sqrt[-((a - x)/(a + x))]*(a + x) - 2*a*ArcTanh[Sqrt[-((a - x)/(a + x))]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 288

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !IntegerQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1959

Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/(((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{-a+x}{a+x}} dx &= (4a) \text{Subst} \left(\int \frac{x^2}{(1-x^2)^2} dx, x, \sqrt{\frac{-a+x}{a+x}} \right) \\ &= \sqrt{\frac{a-x}{a+x}}(a+x) - (2a) \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{\frac{-a+x}{a+x}} \right) \\ &= \sqrt{\frac{a-x}{a+x}}(a+x) - 2a \tanh^{-1}\left(\sqrt{\frac{a-x}{a+x}}\right) \end{aligned}$$

Mathematica [A] time = 0.07, size = 78, normalized size = 1.90

$$\frac{\sqrt{\frac{x-a}{a+x}} \left(\sqrt{x-a}(a+x) - 2a^{3/2} \sqrt{\frac{a+x}{a}} \sinh^{-1}\left(\frac{\sqrt{x-a}}{\sqrt{2}\sqrt{a}}\right) \right)}{\sqrt{x-a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-a + x)/(a + x)], x]

[Out] (Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a^(3/2)*Sqrt[(a + x)/a]*ArcSinh[Sqrt[-a + x]/(Sqrt[2]*Sqrt[a])]))/Sqrt[-a + x]

IntegrateAlgebraic [A] time = 0.06, size = 39, normalized size = 0.95

$$\sqrt{\frac{x-a}{a+x}}(a+x) - 2a \tanh^{-1}\left(\sqrt{\frac{x-a}{a+x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-a + x)/(a + x)], x]

[Out] Sqrt[(-a + x)/(a + x)]*(a + x) - 2*a*ArcTanh[Sqrt[(-a + x)/(a + x)]]

fricas [A] time = 0.66, size = 58, normalized size = 1.41

$$-a \log\left(\sqrt{-\frac{a-x}{a+x}} + 1\right) + a \log\left(\sqrt{-\frac{a-x}{a+x}} - 1\right) + (a+x)\sqrt{-\frac{a-x}{a+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a+x))^(1/2), x, algorithm="fricas")

[Out] -a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (a + x)*sqrt(-(a - x)/(a + x))

giac [A] time = 0.31, size = 40, normalized size = 0.98

$$a \log\left(\left|-x + \sqrt{-a^2 + x^2}\right|\right) \operatorname{sgn}(a+x) + \sqrt{-a^2 + x^2} \operatorname{sgn}(a+x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a+x))^(1/2), x, algorithm="giac")

[Out] a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)

maple [A] time = 0.01, size = 60, normalized size = 1.46

$$\frac{\sqrt{\frac{-a+x}{a+x}}(a+x)\left(a \ln\left(x + \sqrt{-a^2 + x^2}\right) - \sqrt{-a^2 + x^2}\right)}{\sqrt{(a+x)(-a+x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a+x)/(a+x))^(1/2), x)

[Out] -((a+x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/((a+x)*(-a+x))^(1/2)

maxima [A] time = 0.91, size = 70, normalized size = 1.71

$$a \left(\frac{2\sqrt{\frac{a-x}{a+x}}}{\frac{a-x}{a+x} + 1} - \log\left(\sqrt{-\frac{a-x}{a+x}} + 1\right) + \log\left(\sqrt{-\frac{a-x}{a+x}} - 1\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((a+x)/(a+x))^(1/2), x, algorithm="maxima")

[Out] $a \cdot (2 \sqrt{-(a-x)/(a+x)}) / ((a-x)/(a+x) + 1) - \log(\sqrt{-(a-x)/(a+x)} + 1) + \log(\sqrt{-(a-x)/(a+x)} - 1)$

mupad [B] time = 0.05, size = 51, normalized size = 1.24

$$\frac{2a \sqrt{\frac{a-x}{a+x}}}{\frac{a-x}{a+x} + 1} - 2a \operatorname{atanh}\left(\sqrt{\frac{a-x}{a+x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a - x)/(a + x))^(1/2), x)`

[Out] $(2a \cdot (-a - x)/(a + x))^{(1/2)} / ((a - x)/(a + x) + 1) - 2a \cdot \operatorname{atanh}((-a - x)/(a + x))^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{-a+x}{a+x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a+x)/(a+x)**(1/2), x)`

[Out] `Integral(sqrt((-a + x)/(a + x)), x)`

$$3.489 \quad \int \sqrt{\frac{a+bx}{c+dx}} dx$$

Optimal. Leaf size=76

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

Rubi [A] time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1959, 288, 208}

$$\frac{(c+dx)\sqrt{\frac{a+bx}{c+dx}}}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(a + b*x)/(c + d*x)],x]

[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d - ((b*c - a*d)*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x))]/Sqrt[b]])/(Sqrt[b]*d^(3/2))

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 288

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(n-1)*(c*x)^(m-n+1)*(a+b*x^n)^(p+1))/(b*n*(p+1)), x] - Dist[(c^(n*(m-n+1)))/(b*n*(p+1)), Int[(c*x)^(m-n)*(a+b*x^n)^(p+1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m+1, n] && !IntegerQ[(m+n*(p+1)+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1959

Int[(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[(x^(q*(p+1) - 1)*(-a*e) + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && FractionQ[p] && IntegerQ[1/n]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{a+bx}{c+dx}} dx &= (2(bc-ad)) \text{Subst} \left(\int \frac{x^2}{(b-dx^2)^2} dx, x, \sqrt{\frac{a+bx}{c+dx}} \right) \\ &= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad) \text{Subst} \left(\int \frac{1}{b-dx^2} dx, x, \sqrt{\frac{a+bx}{c+dx}} \right)}{d} \\ &= \frac{\sqrt{\frac{a+bx}{c+dx}}(c+dx)}{d} - \frac{(bc-ad)\tanh^{-1}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.27, size = 123, normalized size = 1.62

$$\frac{\sqrt{\frac{a+bx}{c+dx}} \left(b\sqrt{d}(a+bx)(c+dx) - \sqrt{a+bx}(bc-ad)^{3/2} \sqrt{\frac{b(c+dx)}{bc-ad}} \sinh^{-1} \left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}} \right) \right)}{bd^{3/2}(a+bx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(a + b*x)/(c + d*x)], x]

[Out] (Sqrt[(a + b*x)/(c + d*x)]*(b*Sqrt[d]*(a + b*x)*(c + d*x) - (b*c - a*d)^(3/2)*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*d^(3/2)*(a + b*x))

IntegrateAlgebraic [A] time = 0.13, size = 75, normalized size = 0.99

$$\frac{(ad - bc) \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right)}{\sqrt{b} d^{3/2}} + \frac{(c + dx) \sqrt{\frac{a+bx}{c+dx}}}{d}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(a + b*x)/(c + d*x)], x]

[Out] (Sqrt[(a + b*x)/(c + d*x)]*(c + d*x))/d + ((-b*c) + a*d)*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]]/(Sqrt[b]*d^(3/2))

fricas [A] time = 0.62, size = 180, normalized size = 2.37

$$\left[\frac{(bc - ad)\sqrt{bd} \log\left(2bdx + bc + ad + 2\sqrt{bd}(dx + c)\sqrt{\frac{bx+a}{dx+c}}\right) - 2(bd^2x + bcd)\sqrt{\frac{bx+a}{dx+c}}}{2bd^2}, \frac{(bc - ad)\sqrt{-bd} \arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right) + (bd^2x + bcd)\sqrt{\frac{bx+a}{dx+c}}}{bd^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2), x, algorithm="fricas")

[Out] [-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a)/(d*x + c)))/(b*d*x + a*d) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x + c)))/(b*d^2)]

giac [A] time = 0.57, size = 119, normalized size = 1.57

$$\frac{\sqrt{bdx^2 + bcx + adx + ac} \operatorname{sgn}(dx + c)}{d} + \frac{(bc \operatorname{sgn}(dx + c) - ad \operatorname{sgn}(dx + c)) \sqrt{bd} \log\left(-2\left(\sqrt{bd}x - \sqrt{bdx^2 + bcx + adx + ac}\right)bd - \sqrt{bd}bc - \sqrt{bd}ad\right)}{2bd^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2), x, algorithm="giac")

[Out] sqrt(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d + 1/2*(b*c*sgn(d*x + c) - a*d*sgn(d*x + c))*sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d^2)

maple [B] time = 0.01, size = 152, normalized size = 2.00

$$\frac{\sqrt{\frac{bx+a}{dx+c}}(dx+c) \left(ad \ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\sqrt{bd}}{2\sqrt{bd}}\right) - bc \ln\left(\frac{2bdx+ad+bc+2\sqrt{(bx+a)(dx+c)}\sqrt{bd}}{2\sqrt{bd}}\right) + 2\sqrt{(bx+a)(dx+c)}\sqrt{bd} \right)}{2\sqrt{(bx+a)(dx+c)}\sqrt{bd}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((b*x+a)/(d*x+c))^(1/2),x)

[Out] $\frac{1}{2} \cdot \left(\frac{b*x+a}{d*x+c} \right)^{1/2} \cdot (d*x+c) \cdot \left(\ln \left(\frac{1}{2} \cdot (2*b*d*x+a*d+b*c+2*((b*x+a)*(d*x+c))^{1/2}) \cdot (b*d)^{1/2} \right) / (b*d)^{1/2} \right) \cdot a*d - \ln \left(\frac{1}{2} \cdot (2*b*d*x+a*d+b*c+2*((b*x+a)*(d*x+c))^{1/2}) \cdot (b*d)^{1/2} \right) / (b*d)^{1/2} \cdot b*c + 2 \cdot \left(\frac{b*x+a}{d*x+c} \right)^{1/2} \cdot (b*d)^{1/2} \right) / \left(\frac{b*x+a}{d*x+c} \right)^{1/2} / d / (b*d)^{1/2}$

maxima [A] time = 1.23, size = 118, normalized size = 1.55

$$\frac{(bc - ad) \sqrt{\frac{bx+a}{dx+c}}}{bd - \frac{(bx+a)d^2}{dx+c}} + \frac{(bc - ad) \log \left(\frac{d \sqrt{\frac{bx+a}{dx+c}} - \sqrt{bd}}{d \sqrt{\frac{bx+a}{dx+c}} + \sqrt{bd}} \right)}{2 \sqrt{bd} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")

[Out] $(b*c - a*d) \cdot \sqrt{(b*x + a)/(d*x + c)} / (b*d - (b*x + a) \cdot d^2 / (d*x + c)) + 1/2 \cdot (b*c - a*d) \cdot \log((d \cdot \sqrt{(b*x + a)/(d*x + c)} - \sqrt{b*d}) / (d \cdot \sqrt{(b*x + a)/(d*x + c)} + \sqrt{b*d})) / (\sqrt{b*d} \cdot d)$

mupad [B] time = 0.27, size = 90, normalized size = 1.18

$$\frac{\operatorname{atanh} \left(\frac{\sqrt{d} \sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}} \right) (ad - bc)}{\sqrt{b} d^{3/2}} + \frac{(ad - bc) \sqrt{\frac{a+bx}{c+dx}}}{bd \left(\frac{d(a+bx)}{b(c+dx)} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b*x)/(c + d*x))^(1/2),x)

[Out] $(\operatorname{atanh}((d^{1/2} \cdot ((a + b*x)/(c + d*x))^{1/2}) / b^{1/2}) \cdot (a*d - b*c)) / (b^{1/2} \cdot d^{3/2}) + ((a*d - b*c) \cdot ((a + b*x)/(c + d*x))^{1/2}) / (b*d \cdot ((d \cdot (a + b*x)) / (b \cdot (c + d*x)) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{a + bx}{c + dx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((b*x+a)/(d*x+c))^(1/2),x)

[Out] Integral(sqrt((a + b*x)/(c + d*x)), x)

$$3.490 \quad \int \sqrt{\frac{-1+x}{5+3x}} dx$$

Optimal. Leaf size=49

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5} - \frac{8 \sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1958, 50, 54, 215}

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5} - \frac{8 \sinh^{-1}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x)/(5 + 3*x)], x]

[Out] (Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(3*Sqrt[3])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_.) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.))^(p
_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-1+x}{5+3x}} dx &= \int \frac{\sqrt{-1+x}}{\sqrt{5+3x}} dx \\
&= \frac{1}{3} \sqrt{-1+x} \sqrt{5+3x} - \frac{4}{3} \int \frac{1}{\sqrt{-1+x} \sqrt{5+3x}} dx \\
&= \frac{1}{3} \sqrt{-1+x} \sqrt{5+3x} - \frac{8}{3} \operatorname{Subst} \left(\int \frac{1}{\sqrt{8+3x^2}} dx, x, \sqrt{-1+x} \right) \\
&= \frac{1}{3} \sqrt{-1+x} \sqrt{5+3x} - \frac{8 \sinh^{-1} \left(\frac{1}{2} \sqrt{\frac{3}{2}} \sqrt{-1+x} \right)}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 76, normalized size = 1.55

$$\frac{3(x-1)\sqrt{3x+5} - 8\sqrt{3}\sqrt{x-1} \sinh^{-1} \left(\frac{1}{2} \sqrt{\frac{3}{2}} \sqrt{x-1} \right)}{9\sqrt{\frac{x-1}{3x+5}} \sqrt{3x+5}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x)/(5 + 3*x)], x]

[Out] (3*(-1 + x)*Sqrt[5 + 3*x] - 8*Sqrt[3]*Sqrt[-1 + x]*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(9*Sqrt[(-1 + x)/(5 + 3*x)]*Sqrt[5 + 3*x])

IntegrateAlgebraic [A] time = 0.09, size = 56, normalized size = 1.14

$$\frac{1}{3} \sqrt{\frac{x-1}{3x+5}} (3x+5) - \frac{8 \tanh^{-1} \left(\sqrt{3} \sqrt{\frac{x-1}{3x+5}} \right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x)/(5 + 3*x)], x]

[Out] (Sqrt[(-1 + x)/(5 + 3*x)]*(5 + 3*x))/3 - (8*ArcTanh[Sqrt[3]*Sqrt[(-1 + x)/(5 + 3*x)]])/(3*Sqrt[3])

fricas [A] time = 0.64, size = 54, normalized size = 1.10

$$\frac{1}{3} (3x+5) \sqrt{\frac{x-1}{3x+5}} + \frac{4}{9} \sqrt{3} \log \left(\sqrt{3} (3x+5) \sqrt{\frac{x-1}{3x+5}} - 3x-1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-1+x)/(5+3*x))^(1/2), x, algorithm="fricas")

[Out] 1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) - 3*x - 1)

giac [B] time = 0.40, size = 74, normalized size = 1.51

$$-\frac{8}{9} \sqrt{3} \log(2) \operatorname{sgn}(3x+5) + \frac{4}{9} \sqrt{3} \log \left(\left| -\sqrt{3} \left(\sqrt{3}x - \sqrt{3x^2 + 2x - 5} \right) - 1 \right| \operatorname{sgn}(3x+5) + \frac{1}{3} \sqrt{3x^2 + 2x - 5} \operatorname{sgn}(3x+5) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((-1+x)/(5+3*x))^(1/2), x, algorithm="giac")

[Out] $-8/9*\sqrt{3}*\log(2)*\text{sgn}(3*x + 5) + 4/9*\sqrt{3}*\log(\text{abs}(-\sqrt{3}*(\sqrt{3}*x - \sqrt{3*x^2 + 2*x - 5}) - 1))*\text{sgn}(3*x + 5) + 1/3*\sqrt{3*x^2 + 2*x - 5}*\text{sgn}(3*x + 5)$

maple [B] time = 0.01, size = 76, normalized size = 1.55

$$\frac{\sqrt{\frac{x-1}{3x+5}} (3x+5) \left(4\sqrt{3} \ln \left(\sqrt{3} x + \frac{\sqrt{3}}{3} + \sqrt{3x^2 + 2x - 5} \right) - 3\sqrt{3x^2 + 2x - 5} \right)}{9\sqrt{(3x+5)(x-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x-1)/(5+3*x))^(1/2), x)`

[Out] $-1/9*((x-1)/(5+3*x))^{(1/2)}*(5+3*x)*(4*\ln(3^{(1/2)}*x+1/3*3^{(1/2)}+(3*x^2+2*x-5)^{(1/2)})*3^{(1/2)}-3*(3*x^2+2*x-5)^{(1/2)})/((5+3*x)*(x-1))^{(1/2)}$

maxima [B] time = 1.39, size = 80, normalized size = 1.63

$$\frac{4}{9} \sqrt{3} \log \left(\frac{\sqrt{3} - 3 \sqrt{\frac{x-1}{3x+5}}}{\sqrt{3} + 3 \sqrt{\frac{x-1}{3x+5}}} \right) - \frac{8 \sqrt{\frac{x-1}{3x+5}}}{3 \left(\frac{3(x-1)}{3x+5} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(5+3*x))^(1/2), x, algorithm="maxima"`

[Out] $4/9*\sqrt{3}*\log(-(\sqrt{3} - 3*\sqrt{(x-1)/(3*x+5)})/(\sqrt{3} + 3*\sqrt{(x-1)/(3*x+5)})) - 8/3*\sqrt{(x-1)/(3*x+5)}/(3*(x-1)/(3*x+5) - 1)$

mupad [B] time = 3.16, size = 57, normalized size = 1.16

$$-\frac{8\sqrt{3} \operatorname{atanh}\left(\sqrt{3} \sqrt{\frac{x-1}{3x+5}}\right)}{9} - \frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3x-3}{3x+5} - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x-1)/(3*x+5))^(1/2), x)`

[Out] $-(8*3^{(1/2)}*\operatorname{atanh}(3^{(1/2)}*((x-1)/(3*x+5))^{(1/2)}))/9 - (8*((x-1)/(3*x+5))^{(1/2)})/(3*((3*x-3)/(3*x+5) - 1))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x-1}{3x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+x)/(5+3*x)**(1/2), x)`

[Out] `Integral(sqrt((x-1)/(3*x+5)), x)`

$$3.491 \quad \int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx$$

Optimal. Leaf size=46

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x} - 12 \tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Rubi [A] time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1958, 94, 93, 204}

$$-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x} - 12 \tan^{-1}\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

```
[Out] -((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*x]]
```

Rule 93

```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

Rule 94

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1))/((m + 1)*(b*e - a*f)), x] - Dist[(n*(d*e - c*f))/((m + 1)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && !(SumSimplerQ[p, 1] && !SumSimplerQ[m, 1])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_.) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2} dx &= \int \frac{\sqrt{-1+5x}}{x^2 \sqrt{1+7x}} dx \\
&= -\frac{\sqrt{-1+5x} \sqrt{1+7x}}{x} + 6 \int \frac{1}{x \sqrt{-1+5x} \sqrt{1+7x}} dx \\
&= -\frac{\sqrt{-1+5x} \sqrt{1+7x}}{x} + 12 \operatorname{Subst} \left(\int \frac{1}{-1-x^2} dx, x, \frac{\sqrt{1+7x}}{\sqrt{-1+5x}} \right) \\
&= -\frac{\sqrt{-1+5x} \sqrt{1+7x}}{x} - 12 \tan^{-1} \left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 1.72

$$\frac{\sqrt{\frac{5x-1}{7x+1}} \left(12x\sqrt{7x+1} \tan^{-1} \left(\frac{\sqrt{5x-1}}{\sqrt{7x+1}} \right) - \sqrt{5x-1} (7x+1) \right)}{x\sqrt{5x-1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]

[Out] (Sqrt[(-1 + 5*x)/(1 + 7*x)]*(-(Sqrt[-1 + 5*x]*(1 + 7*x)) + 12*x*Sqrt[1 + 7*x]*ArcTan[Sqrt[-1 + 5*x]/Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x])

IntegrateAlgebraic [A] time = 0.04, size = 47, normalized size = 1.02

$$\frac{\sqrt{\frac{5x-1}{7x+1}} (-7x-1)}{x} + 12 \tan^{-1} \left(\sqrt{\frac{5x-1}{7x+1}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]

[Out] ((-1 - 7*x)*Sqrt[(-1 + 5*x)/(1 + 7*x)]/x + 12*ArcTan[Sqrt[(-1 + 5*x)/(1 + 7*x)]])

fricas [A] time = 0.48, size = 46, normalized size = 1.00

$$\frac{12x \arctan \left(\sqrt{\frac{5x-1}{7x+1}} \right) - (7x+1) \sqrt{\frac{5x-1}{7x+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")

[Out] (12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x + 1)))/x

giac [B] time = 0.40, size = 114, normalized size = 2.48

$$\left(\sqrt{35} - 12 \arctan \left(\frac{1}{7} \sqrt{35} \right) \right) \operatorname{sgn}(7x+1) + 12 \arctan \left(-\sqrt{35}x + \sqrt{35x^2 - 2x - 1} \right) \operatorname{sgn}(7x+1) - \frac{2 \left((\sqrt{35}x - \sqrt{35x^2 - 2x - 1}) \operatorname{sgn}(7x+1) + \sqrt{35} \operatorname{sgn}(7x+1) \right)}{(\sqrt{35}x - \sqrt{35x^2 - 2x - 1})^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")

[Out] $(\sqrt{35} - 12 \arctan(1/7 \sqrt{35})) \operatorname{sgn}(7x + 1) + 12 \arctan(-\sqrt{35}x + \sqrt{35x^2 - 2x - 1}) \operatorname{sgn}(7x + 1) - 2((\sqrt{35}x - \sqrt{35x^2 - 2x - 1}) \operatorname{sgn}(7x + 1) + \sqrt{35} \operatorname{sgn}(7x + 1)) / ((\sqrt{35}x - \sqrt{35x^2 - 2x - 1})^2 + 1)$

maple [B] time = 0.02, size = 103, normalized size = 2.24

$$\frac{\sqrt{\frac{5x-1}{7x+1}} (7x+1) \left(-35\sqrt{35x^2-2x-1} x^2 - 6x \arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right) + 2\sqrt{35x^2-2x-1} x + (35x^2-2x-1)^{\frac{3}{2}} \right)}{\sqrt{(5x-1)(7x+1)} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((−1+5*x)/(1+7*x))^(1/2)/x^2,x)`

[Out] $((-1+5x)/(1+7x))^{1/2} (1+7x) * ((35x^2-2x-1)^{3/2} - 35*(35x^2-2x-1)^{1/2} * x^2 + 2*(35x^2-2x-1)^{1/2} * x - 6 \arctan((x+1)/(35x^2-2x-1)^{1/2}) * x) / (((-1+5x)*(1+7x))^{1/2} / x)$

maxima [A] time = 1.43, size = 53, normalized size = 1.15

$$-\frac{12 \sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1} + 1} + 12 \arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((−1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")`

[Out] $-12 \sqrt{(5x-1)/(7x+1)} / ((5x-1)/(7x+1) + 1) + 12 \arctan(\sqrt{(5x-1)/(7x+1)})$

mupad [B] time = 3.24, size = 74, normalized size = 1.61

$$12 \operatorname{atan}\left(\frac{\sqrt{5} \sqrt{7} \sqrt{35} \sqrt{\frac{5x-1}{7x+1}}}{35}\right) - \frac{12 \sqrt{5} \sqrt{7} \sqrt{35} \sqrt{\frac{5x-1}{7x+1}}}{25 \left(\frac{7x-7}{7x+1} + \frac{7}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)`

[Out] $12 \operatorname{atan}((5^{1/2} * 7^{1/2} * 35^{1/2} * ((5x-1)/(7x+1))^{1/2}) / 35) - (12 * 5^{1/2} * 7^{1/2} * 35^{1/2} * ((5x-1)/(7x+1))^{1/2}) / (25 * ((7x-7)/(7x+1) + 7/5))$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(((−1+5*x)/(1+7*x))**(1/2)/x**2,x)`

[Out] `Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)`

$$3.492 \quad \int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)} dx$$

Optimal. Leaf size=20

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1962, 12, 383}

$$-\sqrt{\frac{1-x}{x+1}}(x+1)$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]

[Out] -(Sqrt[(1 - x)/(1 + x)]*(1 + x))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rule 1962

Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_) + (d_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{q = Denominator[p]}, Dist[(q*e*(b*c - a*d))/n, Subst[Int[SimplifyIntegrand[(x^(q*(p + 1) - 1)*(-a*e) + c*x^q)^(m + 1)/n - 1*(u /. x -> (-a*e) + c*x^q)^(1/n)/(b*e - d*x^q)^(1/n))^r]/(b*e - d*x^q)^((m + 1)/n + 1), x], x, ((e*(a + b*x^n))/(c + d*x^n))^(1/q)], x] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && IntegersQ[m, r]

Rubi steps

$$\begin{aligned} \int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)} dx &= -\left(4 \operatorname{Subst}\left(\int \frac{1-x^2}{2(1+x^2)^2} dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right) \\ &= -\left(2 \operatorname{Subst}\left(\int \frac{1-x^2}{(1+x^2)^2} dx, x, \sqrt{\frac{1-x}{1+x}}\right)\right) \\ &= -\sqrt{\frac{1-x}{1+x}}(1+x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.95

$$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]

[Out] (-1 + x)/Sqrt[(1 - x)/(1 + x)]

IntegrateAlgebraic [A] time = 0.03, size = 32, normalized size = 1.60

$$\frac{2\sqrt{\frac{1-x}{x+1}}}{\frac{1-x}{x+1} + 1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]

[Out] (-2*Sqrt[(1 - x)/(1 + x)])/(1 + (1 - x)/(1 + x))

fricas [A] time = 0.60, size = 17, normalized size = 0.85

$$-(x + 1)\sqrt{-\frac{x - 1}{x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="fricas")

[Out] -(x + 1)*sqrt(-(x - 1)/(x + 1))

giac [A] time = 0.42, size = 29, normalized size = 1.45

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}} + \frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="giac")

[Out] -2/(sqrt(-(x - 1)/(x + 1)) + 1/sqrt(-(x - 1)/(x + 1)))

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x - 1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+1)/((-x+1)/(x+1))^(1/2),x)

[Out] (x-1)/((-x-1)/(x+1))^(1/2)

maxima [A] time = 0.61, size = 27, normalized size = 1.35

$$\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((1-x)/(1+x))^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)

mupad [B] time = 0.06, size = 17, normalized size = 0.85

$$-\sqrt{-\frac{x-1}{x+1}}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/((-x - 1)/(x + 1))^(1/2)*(x + 1), x)

[Out] -((-x - 1)/(x + 1))^(1/2)*(x + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((1-x)/(1+x))**(1/2), x)

[Out] Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)

$$3.493 \quad \int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}} dx$$

Optimal. Leaf size=18

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {512, 514, 375, 74}

$$-(x+1)\sqrt{\frac{2}{x+1}-1}$$

Antiderivative was successfully verified.

[In] Int[x/((1+x)*Sqrt[-1+2/(1+x)]),x]

[Out] -((1+x)*Sqrt[-1+2/(1+x)])

Rule 74

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

Rule 375

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := -Subst[Int[((a + b/x^n)^p*(c + d/x^n)^q)/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

Rule 512

```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[v, x, 1]^(m + 1), Subst[Int[SimplifyIntegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m] && NeQ[v, x]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}} dx &= \text{Subst} \left(\int \frac{-1+x}{\sqrt{-1+\frac{2}{x}}} dx, x, 1+x \right) \\
&= \text{Subst} \left(\int \frac{1-\frac{1}{x}}{\sqrt{-1+\frac{2}{x}}} dx, x, 1+x \right) \\
&= -\text{Subst} \left(\int \frac{1-x}{x^2\sqrt{-1+2x}} dx, x, \frac{1}{1+x} \right) \\
&= -\left((1+x)\sqrt{-1+\frac{2}{1+x}} \right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{\frac{2}{x+1}-1}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((1+x)*Sqrt[-1+2/(1+x)]),x]

[Out] (-1+x)/Sqrt[-1+2/(1+x)]

IntegrateAlgebraic [A] time = 0.06, size = 21, normalized size = 1.17

$$(-x-1)\sqrt{\frac{1-x}{x+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/((1+x)*Sqrt[-1+2/(1+x)]),x]

[Out] (-1-x)*Sqrt[(1-x)/(1+x)]

fricas [A] time = 0.51, size = 17, normalized size = 0.94

$$-(x+1)\sqrt{-\frac{x-1}{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")

[Out] -(x+1)*sqrt(-(x-1)/(x+1))

giac [A] time = 0.26, size = 29, normalized size = 1.61

$$-\frac{2}{\sqrt{-\frac{x-1}{x+1}} + \frac{1}{\sqrt{-\frac{x-1}{x+1}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")

[Out] -2/(sqrt(-(x-1)/(x+1)) + 1/sqrt(-(x-1)/(x+1)))

maple [A] time = 0.00, size = 17, normalized size = 0.94

$$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(x+1)/(-1+2/(x+1))^(1/2),x)`

[Out] `(x-1)/(-(x-1)/(x+1))^(1/2)`

maxima [A] time = 0.61, size = 16, normalized size = 0.89

$$\frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")`

[Out] `sqrt(x + 1)*(x - 1)/sqrt(-x + 1)`

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/((x + 1)*(2/(x + 1) - 1))^(1/2)),x)`

[Out] `int(x/((x + 1)*(2/(x + 1) - 1))^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-\frac{x-1}{x+1}}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)`

[Out] `Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)`

$$3.494 \quad \int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx$$

Optimal. Leaf size=54

$$\sqrt{x+2}\sqrt{x+3} - \sinh^{-1}(\sqrt{x+2}) + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1958, 154, 157, 54, 215, 93, 207}

$$\sqrt{x+2}\sqrt{x+3} - \sinh^{-1}(\sqrt{x+2}) + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)$$

Antiderivative was successfully verified.

[In] Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]), x]

[Out] Sqrt[2 + x]*Sqrt[3 + x] - ArcSinh[Sqrt[2 + x]] + 2*Sqrt[2]*ArcTanh[(Sqrt[2]*Sqrt[2 + x])/Sqrt[3 + x]]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 93

Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x_)), x_Symbol] :> With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 154

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[(h*(a + b*x)^m*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(m + n + p + 2)), x] + Dist[1/(d*f*(m + n + p + 2)), Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2) + h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1)))]*x, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0] && IntegersQ[2*m, 2*n, 2*p]

Rule 157

Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/((a_.) + (b_.)*(x_)), x_Symbol] :> Dist[h/b, Int[(c + d*x)^n*(e + f*x)^p, x], x] + Dist[(b*g - a*h)/b, Int[(((c + d*x)^n*(e + f*x)^p)/(a + b*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]

Rule 207

Int[(((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 215

`Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]`

Rule 1958

`Int[(u_)*(((e_)*((a_) + (b_)*(x_)^(n_))))/((c_) + (d_)*(x_)^(n_)))^(p_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]`

Rubi steps

$$\begin{aligned}
 \int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}} dx &= \int \frac{x\sqrt{3+x}}{(1+x)\sqrt{2+x}} dx \\
 &= \sqrt{2+x}\sqrt{3+x} + \int \frac{-\frac{5}{2} - \frac{x}{2}}{(1+x)\sqrt{2+x}\sqrt{3+x}} dx \\
 &= \sqrt{2+x}\sqrt{3+x} - \frac{1}{2} \int \frac{1}{\sqrt{2+x}\sqrt{3+x}} dx - 2 \int \frac{1}{(1+x)\sqrt{2+x}\sqrt{3+x}} dx \\
 &= \sqrt{2+x}\sqrt{3+x} - 4 \operatorname{Subst}\left(\int \frac{1}{-1+2x^2} dx, x, \frac{\sqrt{2+x}}{\sqrt{3+x}}\right) - \operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{2+x}\right) \\
 &= \sqrt{2+x}\sqrt{3+x} - \sinh^{-1}(\sqrt{2+x}) + 2\sqrt{2} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)
 \end{aligned}$$

Mathematica [A] time = 0.07, size = 106, normalized size = 1.96

$$\frac{\sqrt{x+3}(x^2+5x+6) + 2\sqrt{2}\sqrt{x+2}\sqrt{-(x+3)^2} \tan^{-1}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{-x-3}}\right) - \sqrt{x+2}(x+3) \sinh^{-1}(\sqrt{x+2})}{\sqrt{\frac{x+2}{x+3}}(x+3)^{3/2}}$$

Antiderivative was successfully verified.

`[In] Integrate[x/((1+x)*Sqrt[(2+x)/(3+x)]), x]`

`[Out] (Sqrt[3+x]*(6+5*x+x^2) - Sqrt[2+x]*(3+x)*ArcSinh[Sqrt[2+x]] + 2*Sqrt[2]*Sqrt[2+x]*Sqrt[-(3+x)^2]*ArcTan[(Sqrt[2]*Sqrt[2+x])/Sqrt[-3-x]])/(Sqrt[(2+x)/(3+x)]*(3+x)^(3/2))`

IntegrateAlgebraic [A] time = 0.10, size = 72, normalized size = 1.33

$$-\frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1} - \tanh^{-1}\left(\sqrt{\frac{x+2}{x+3}}\right) + 2\sqrt{2} \tanh^{-1}\left(\sqrt{2}\sqrt{\frac{x+2}{x+3}}\right)$$

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[x/((1+x)*Sqrt[(2+x)/(3+x)]), x]`

`[Out] -(Sqrt[(2+x)/(3+x)]/(-1+(2+x)/(3+x))) - ArcTanh[Sqrt[(2+x)/(3+x)]] + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Sqrt[(2+x)/(3+x)]]`

fricas [B] time = 0.64, size = 83, normalized size = 1.54

$$(x+3)\sqrt{\frac{x+2}{x+3}} + \sqrt{2} \log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}} + 3x+7}{x+1}\right) - \frac{1}{2} \log\left(\sqrt{\frac{x+2}{x+3}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x+2}{x+3}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")

[Out] (x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2)/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)

giac [B] time = 0.30, size = 107, normalized size = 1.98

$$-\sqrt{2} \log \left(\frac{\left| -2\sqrt{2} + 4\sqrt{\frac{x+2}{x+3}} \right|}{2\left(\sqrt{2} + 2\sqrt{\frac{x+2}{x+3}}\right)} \right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3} - 1} - \frac{1}{2} \log \left(\sqrt{\frac{x+2}{x+3}} + 1 \right) + \frac{1}{2} \log \left(\left| \sqrt{\frac{x+2}{x+3}} - 1 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")

[Out] -sqrt(2)*log(1/2*abs(-2*sqrt(2) + 4*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(abs(sqrt((x + 2)/(x + 3)) - 1))

maple [A] time = 0.02, size = 81, normalized size = 1.50

$$\frac{(x + 2) \left(2\sqrt{2} \operatorname{arctanh} \left(\frac{(3x+7)\sqrt{2}}{4\sqrt{x^2+5x+6}} \right) - \ln \left(x + \frac{5}{2} + \sqrt{x^2 + 5x + 6} \right) + 2\sqrt{x^2 + 5x + 6} \right)}{2\sqrt{\frac{x+2}{x+3}} \sqrt{(x+3)(x+2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+1)/((x+2)/(x+3))^(1/2),x)

[Out] 1/2*(x+2)*(2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+2*(x^2+5*x+6)^(1/2)-ln(x+5/2+(x^2+5*x+6)^(1/2)))/((x+2)/(x+3))^(1/2)/((x+3)*(x+2))^(1/2)

maxima [B] time = 1.22, size = 103, normalized size = 1.91

$$-\sqrt{2} \log \left(-\frac{\sqrt{2} - 2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2} + 2\sqrt{\frac{x+2}{x+3}}} \right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3} - 1} - \frac{1}{2} \log \left(\sqrt{\frac{x+2}{x+3}} + 1 \right) + \frac{1}{2} \log \left(\sqrt{\frac{x+2}{x+3}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")

[Out] -sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)

mupad [B] time = 0.09, size = 62, normalized size = 1.15

$$2\sqrt{2} \operatorname{atanh} \left(\sqrt{2} \sqrt{\frac{x+2}{x+3}} \right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3} - 1} - \operatorname{atanh} \left(\sqrt{\frac{x+2}{x+3}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)

[Out] 2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)/((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{\frac{x+2}{x+3}} (x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)

[Out] Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)

$$3.495 \quad \int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx$$

Optimal. Leaf size=11

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {25, 261}

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]

Rule 25

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} dx &= \int \frac{1}{\left(1+\frac{1}{x}\right)^{3/2} x^2} dx \\ &= \frac{2}{\sqrt{1+\frac{1}{x}}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] 2/Sqrt[1 + x^(-1)]

IntegrateAlgebraic [A] time = 0.04, size = 19, normalized size = 1.73

$$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/(1 + x)^2,x]

[Out] (2*x*Sqrt[(1 + x)/x])/(1 + x)

fricas [A] time = 0.78, size = 17, normalized size = 1.55

$$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")

[Out] 2*x*sqrt((x + 1)/x)/(x + 1)

giac [B] time = 0.29, size = 23, normalized size = 2.09

$$\frac{2 \operatorname{sgn}(x)}{x - \sqrt{x^2 + x + 1}} - 2 \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")

[Out] 2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)

maple [A] time = 0.01, size = 18, normalized size = 1.64

$$\frac{2\sqrt{\frac{x+1}{x}}x}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/x)^(1/2)/(x+1)^2,x)

[Out] 2/(x+1)*x*((x+1)/x)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x} + 1}}{(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")

[Out] integrate(sqrt(1/x + 1)/(x + 1)^2, x)

mupad [B] time = 3.13, size = 15, normalized size = 1.36

$$\frac{2x\sqrt{\frac{1}{x} + 1}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/x + 1)^(1/2)/(x + 1)^2,x)
```

```
[Out] (2*x*(1/x + 1)^(1/2))/(x + 1)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1 + \frac{1}{x}}}{(x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

```
[Out] Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

$$3.496 \quad \int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=29

$$-\frac{\sqrt{\frac{1}{x}+1} \sqrt{x} \sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1448, 26, 53, 619, 216}

$$-\frac{\sqrt{\frac{1}{x}+1} \sqrt{x} \sin^{-1}(1-2x)}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2], x]

[Out] -((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])

Rule 26

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(- (b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 53

Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1448

Int[((d_) + (e_.)*(x_)^(mn_.))^(q_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Symbol] :> Dist[(e^IntPart[q]*(d + e*x^mn)^FracPart[q])/(x^(mn*FracPart[q])*(1 + d/(x^mn*e))^FracPart[q]), Int[x^(mn*q)*(1 + d/(x^mn*e))^q*(a + c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] && !IntegerQ[p] && !IntegerQ[q] && PosQ[n2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1 + \frac{1}{x}}}{\sqrt{1 - x^2}} dx &= \frac{\left(\sqrt{1 + \frac{1}{x}} \sqrt{x}\right) \int \frac{\sqrt{1+x}}{\sqrt{x} \sqrt{1-x^2}} dx}{\sqrt{1+x}} \\
&= \frac{\left(\sqrt{1 + \frac{1}{x}} \sqrt{x}\right) \int \frac{1}{\sqrt{1-x} \sqrt{x}} dx}{\sqrt{1+x}} \\
&= \frac{\left(\sqrt{1 + \frac{1}{x}} \sqrt{x}\right) \int \frac{1}{\sqrt{x-x^2}} dx}{\sqrt{1+x}} \\
&= -\frac{\left(\sqrt{1 + \frac{1}{x}} \sqrt{x}\right) \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right)}{\sqrt{1+x}} \\
&= -\frac{\sqrt{1 + \frac{1}{x}} \sqrt{x} \sin^{-1}(1-2x)}{\sqrt{1+x}}
\end{aligned}$$

Mathematica [A] time = 0.23, size = 41, normalized size = 1.41

$$-\tan^{-1}\left(\frac{\sqrt{\frac{x+1}{x}}(2x-1)\sqrt{1-x^2}}{2(x^2-1)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2], x]

[Out] -ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]

IntegrateAlgebraic [F] time = 2.54, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1 + \frac{1}{x}}}{\sqrt{1 - x^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 + x^(-1)]/Sqrt[1 - x^2], x]

fricas [A] time = 0.76, size = 34, normalized size = 1.17

$$-\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2x^2+x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] -arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x} + 1}}{\sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)

maple [A] time = 0.02, size = 40, normalized size = 1.38

$$\frac{\sqrt{\frac{x+1}{x}} \sqrt{-x^2+1} x \arcsin(2x-1)}{(x+1) \sqrt{-(x-1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)

[Out] ((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-(x-1)*x)^(1/2)*arcsin(2*x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)

[Out] int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)

$$3.497 \quad \int \frac{1}{x + \sqrt{3 - 2x - x^2}} dx$$

Optimal. Leaf size=180

$$-\frac{1}{2} \log \left(\frac{-\sqrt{3} \sqrt{-x^2 - 2x + 3} - x + 3}{x^2} \right) + \frac{1}{14} (7 + \sqrt{7}) \log \left(-\frac{\sqrt{3} (\sqrt{3} - \sqrt{-x^2 - 2x + 3})}{x} - \sqrt{7} + \sqrt{3} + 1 \right) + \frac{1}{14} (7 - \sqrt{7}) \log \left(-\frac{\sqrt{3} (\sqrt{3} - \sqrt{-x^2 - 2x + 3})}{x} + \sqrt{7} + \sqrt{3} + 1 \right) + \tan^{-1} \left(\frac{\sqrt{3} - \sqrt{-x^2 - 2x + 3}}{x} \right)$$

Rubi [A] time = 0.20, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1074, 632, 31, 635, 203, 260}

$$-\frac{1}{2} \log \left(\frac{-\sqrt{3} \sqrt{-x^2 - 2x + 3} - x + 3}{x^2} \right) + \frac{1}{14} (7 + \sqrt{7}) \log \left(-\frac{\sqrt{3} (\sqrt{3} - \sqrt{-x^2 - 2x + 3})}{x} - \sqrt{7} + \sqrt{3} + 1 \right) + \frac{1}{14} (7 - \sqrt{7}) \log \left(-\frac{\sqrt{3} (\sqrt{3} - \sqrt{-x^2 - 2x + 3})}{x} + \sqrt{7} + \sqrt{3} + 1 \right) + \tan^{-1} \left(\frac{\sqrt{3} - \sqrt{-x^2 - 2x + 3}}{x} \right)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-1), x]

[Out] ArcTan[(Sqrt[3] - Sqrt[3 - 2*x - x^2])/x] - Log[-((3 - x - Sqrt[3]*Sqrt[3 - 2*x - x^2])/x^2)]/2 + ((7 + Sqrt[7])*Log[1 + Sqrt[3] - Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14 + ((7 - Sqrt[7])*Log[1 + Sqrt[3] + Sqrt[7] - (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x])/14

Rule 31

Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 632

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && NiceSqrtQ[b^2 - 4*a*c]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1074

Int[((A_.) + (B_.)*(x_) + (C_.)*(x_)^2)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Symbol] := With[{q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Dist[1/q, Int[(A*c^2*d - a*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[(c*C*d^2 + b*B*d*f - A*c*d*f - a*C*d*f + a*A*f^2 - f*(B

c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, A, B, C}, x] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{1}{x + \sqrt{3 - 2x - x^2}} dx = 2 \operatorname{Subst} \left(\int \frac{\sqrt{3} - 2x - \sqrt{3}x^2}{(1 + x^2)(2 - \sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2)} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right)$$

$$= \frac{1}{16} \operatorname{Subst} \left(\int \frac{-6 + 2\sqrt{3}(2 - \sqrt{3}) - 4(1 + \sqrt{3}) - (-2\sqrt{3} + 2(2 - \sqrt{3}) + 4\sqrt{3}(1 + \sqrt{3}))}{1 + x^2} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right)$$

$$= -\left(\frac{1}{2} \left(\sqrt{\frac{3}{7}} (1 - \sqrt{7}) \right) \operatorname{Subst} \left(\int \frac{1}{1 + \sqrt{3} + \sqrt{7} + \sqrt{3}x} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right) \right)$$

$$= -\tan^{-1} \left(\frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right) - \frac{1}{2} \log \left(\frac{-3 + x + \sqrt{3} \sqrt{3 - 2x - x^2}}{x^2} \right) + \frac{1}{14} (7 + \sqrt{7})$$

Mathematica [A] time = 0.38, size = 197, normalized size = 1.09

$$\frac{1}{28} \left(-\sqrt{14(4 + \sqrt{7})} \operatorname{tanh}^{-1} \left(\frac{(\sqrt{7} - 1)x + \sqrt{7} + 7}{\sqrt{2(4 + \sqrt{7})} \sqrt{-x^2 - 2x + 3}} \right) - \sqrt{56 - 14\sqrt{7}} \operatorname{tanh}^{-1} \left(\frac{\sqrt{7}x + x + \sqrt{7} - 7}{\sqrt{2} \sqrt{(\sqrt{7} - 4)(x^2 + 2x - 3)}} \right) - \sqrt{7} \log(2x - \sqrt{7} + 1) + 7 \log(2x - \sqrt{7} + 1) + \sqrt{7} \log(2x + \sqrt{7} + 1) + 7 \log(2x + \sqrt{7} + 1) + 14 \sin^{-1} \left(\frac{x + 1}{2} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-1), x]

[Out] (14*ArcSin[(1 + x)/2] - Sqrt[14*(4 + Sqrt[7])]*ArcTanh[(7 + Sqrt[7] + (-1 + Sqrt[7])*x)/(Sqrt[2*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2])] - Sqrt[56 - 14*Sqrt[7])*ArcTanh[(-7 + Sqrt[7] + x + Sqrt[7]*x)/(Sqrt[2]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x + x^2)]]] + 7*Log[1 - Sqrt[7] + 2*x] - Sqrt[7]*Log[1 - Sqrt[7] + 2*x] + 7*Log[1 + Sqrt[7] + 2*x] + Sqrt[7]*Log[1 + Sqrt[7] + 2*x])/28

IntegrateAlgebraic [A] time = 0.40, size = 116, normalized size = 0.64

$$\frac{1}{14} (7 - \sqrt{7}) \log(-\sqrt{-x^2 - 2x + 3} + \sqrt{7}(x - 1) + 2x - 2) + \frac{1}{14} (7 + \sqrt{7}) \log(\sqrt{-x^2 - 2x + 3} + \sqrt{7}(x - 1) - 2x + 2) - \tan^{-1} \left(\frac{\sqrt{-x^2 - 2x + 3}}{x + 3} \right) - \frac{1}{2} \log(x - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-1), x]

[Out] -ArcTan[Sqrt[3 - 2*x - x^2]/(3 + x)] - Log[-1 + x]/2 + ((7 - Sqrt[7])*Log[-2 + Sqrt[7]*(-1 + x) + 2*x - Sqrt[3 - 2*x - x^2]])/14 + ((7 + Sqrt[7])*Log[-2 + Sqrt[7]*(-1 + x) - 2*x + Sqrt[3 - 2*x - x^2]])/14

fricas [B] time = 0.81, size = 372, normalized size = 2.07

$$\frac{1}{28} \sqrt{7} \operatorname{arctan} \left(\frac{24x^4 + 62x^3 - 153x^2 - 2\sqrt{7}(x^4 + x^3 - 45x^2 + 45x) - (14x^3 - 84x^2 + \sqrt{7}(8x^3 - 30x^2 + 27x - 27) + 126x) \sqrt{-x^2 - 2x + 3}}{4x^4 + 8x^3 - 8x^2 - 12x + 9} \right) - \frac{1}{28} \sqrt{7} \operatorname{arctan} \left(\frac{24x^4 + 62x^3 - 153x^2 - 2\sqrt{7}(x^4 + x^3 - 45x^2 + 45x) - (14x^3 - 84x^2 + \sqrt{7}(8x^3 - 30x^2 + 27x - 27) + 126x) \sqrt{-x^2 - 2x + 3}}{4x^4 + 8x^3 - 8x^2 - 12x + 9} \right) - \frac{1}{28} \sqrt{7} \log \left(\frac{2x + \sqrt{7}(2x + 1) + 2 + 4}{2x^2 + 2x - 3} \right) - \frac{1}{28} \sqrt{7} \log \left(\frac{2x + \sqrt{7}(2x + 1) + 2 + 4}{2x^2 + 2x - 3} \right) - \frac{1}{2} \log(x - 1) - \frac{1}{2} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2)),x, algorithm="fricas")

[Out] 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 + 2*sqrt(7)*(3*x^4 + x^3 - 45*x^2 + 45*x) - (14*x^3 - 84*x^2 + sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) + 1/56*sqrt(7)*log((24*x^4 + 62*x^3 - 153*x^2 - 2*sqrt(7)*(3*x^4 + x^3 - 45*x^2 + 45*x) + (14*x^3 - 84*x^2 - sqrt(7)*(8*x^3 - 30*x^2 + 27*x - 27) + 126*x)*sqrt(-x^2 - 2*x + 3) + 180*x - 135)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x + (3 - x^2 - 2*x)^(1/2)),x)
```

```
[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{x + \sqrt{-x^2 - 2x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x**2-2*x+3)**(1/2)),x)
```

```
[Out] Integral(1/(x + sqrt(-x**2 - 2*x + 3)), x)
```

$$3.498 \quad \int \frac{1}{\left(x + \sqrt{3-2x-x^2}\right)^2} dx$$

Optimal. Leaf size=172

$$\frac{2 \left(\frac{3(\sqrt{3-\sqrt{-x^2-2x+3}})}{x} - \sqrt{3} + 4 \right)}{7 \left(\frac{\sqrt{3}(\sqrt{3-\sqrt{-x^2-2x+3}})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3-\sqrt{-x^2-2x+3}})}{x} - \sqrt{3} + 2 \right)} + \frac{8 \tanh^{-1} \left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3} - \sqrt{3}x - x + 3}{\sqrt{7}x} \right)}{7\sqrt{7}}$$

Rubi [A] time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1660, 12, 618, 206}

$$\frac{2 \left(\frac{3(\sqrt{3-\sqrt{-x^2-2x+3}})}{x} - \sqrt{3} + 4 \right)}{7 \left(\frac{\sqrt{3}(\sqrt{3-\sqrt{-x^2-2x+3}})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3-\sqrt{-x^2-2x+3}})}{x} - \sqrt{3} + 2 \right)} + \frac{8 \tanh^{-1} \left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3} - \sqrt{3}x - x + 3}{\sqrt{7}x} \right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-2), x]

[Out] (2*(4 - Sqrt[3] + (3*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x))/(7*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)) + (8*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3 - 2*x - x^2])/(Sqrt[7]*x)]/(7*Sqrt[7]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(x + \sqrt{3 - 2x - x^2})^2} dx &= 2 \operatorname{Subst} \left(\int \frac{-\sqrt{3} + 2x + \sqrt{3} x^2}{(2 - \sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3} x^2)^2} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right) \\
 &= \frac{2 \left(4 - \sqrt{3} + \frac{3(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} \right)}{7 \left(2 - \sqrt{3} - \frac{2(1 + \sqrt{3})(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3 - 2x - x^2})^2}{x^2} \right)} - \frac{1}{14} \operatorname{Subst} \left(\int \frac{1}{2 - \sqrt{3} + 2x} dx \right) \\
 &= \frac{2 \left(4 - \sqrt{3} + \frac{3(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} \right)}{7 \left(2 - \sqrt{3} - \frac{2(1 + \sqrt{3})(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3 - 2x - x^2})^2}{x^2} \right)} + \frac{8}{7} \operatorname{Subst} \left(\int \frac{1}{2 - \sqrt{3} + 2x} dx \right) \\
 &= \frac{2 \left(4 - \sqrt{3} + \frac{3(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} \right)}{7 \left(2 - \sqrt{3} - \frac{2(1 + \sqrt{3})(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3 - 2x - x^2})^2}{x^2} \right)} - \frac{16}{7} \operatorname{Subst} \left(\int \frac{1}{28 - x^2} dx \right) \\
 &= \frac{2 \left(4 - \sqrt{3} + \frac{3(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} \right)}{7 \left(2 - \sqrt{3} - \frac{2(1 + \sqrt{3})(\sqrt{3} - \sqrt{3 - 2x - x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3 - 2x - x^2})^2}{x^2} \right)} + \frac{8 \tanh^{-1} \left(\frac{3 - x - \sqrt{3}x - \sqrt{3} \sqrt{3 - 2x - x^2}}{\sqrt{7}x} \right)}{7\sqrt{7}}
 \end{aligned}$$

Mathematica [A] time = 0.45, size = 306, normalized size = 1.78

$$\frac{1}{98} \left(\frac{7(3-8x)}{2x^2+2x-3} - \frac{14x-3\sqrt{-x^2-2x+3}}{2x^2+2x-3} - 2(1+\sqrt{7}) \frac{\sqrt{14}}{4+\sqrt{7}} \log \left(\sqrt{14(4+\sqrt{7})} \sqrt{-x^2-2x+3} - \sqrt{7}x + 7x + 7\sqrt{7} + 7 \right) - \frac{2}{3}(\sqrt{7}-1) \sqrt{14(4+\sqrt{7})} \log \left(-\sqrt{14} \sqrt{(\sqrt{7}-4)(x^2+2x-3)} + (7+\sqrt{7})x - 7\sqrt{7} + 7 \right) - 4\sqrt{7} \log(-2x + \sqrt{7} - 1) \right) + \frac{2}{3}(\sqrt{7}-1) \sqrt{14(4+\sqrt{7})} \log(2x - \sqrt{7} + 1) + 2(1+\sqrt{7}) \frac{\sqrt{14}}{4+\sqrt{7}} \log(2x + \sqrt{7} + 1) + 4\sqrt{7} \log(2x + \sqrt{7} + 1)$$

Warning: Unable to verify antiderivative.

```

[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-2), x]
[Out] ((7*(3 - 8*x))/(-3 + 2*x + 2*x^2) - (14*(-3 + x)*Sqrt[3 - 2*x - x^2])/(-3 + 2*x + 2*x^2) - 4*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + (2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x])/3 + 4*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 2*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 2*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[7])]*Sqrt[3 - 2*x - x^2]] - (2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[14]*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x + x^2)]])/3)/98
    
```

IntegrateAlgebraic [A] time = 0.39, size = 109, normalized size = 0.63

$$\frac{3 - 8x}{14(2x^2 + 2x - 3)} + \frac{(3 - x)\sqrt{-x^2 - 2x + 3}}{7(2x^2 + 2x - 3)} + \frac{8 \tanh^{-1} \left(\frac{\frac{\sqrt{-x^2 - 2x + 3}}{\sqrt{7}} - \frac{2x}{\sqrt{7}} + \frac{2}{\sqrt{7}}}{x - 1} \right)}{7\sqrt{7}}$$

Antiderivative was successfully verified.

```

[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-2), x]
    
```

[Out] $(3 - 8x)/(14(-3 + 2x + 2x^2)) + ((3 - x)\sqrt{3 - 2x - x^2})/(7(-3 + 2x + 2x^2)) + (8\text{ArcTanh}[(2/\sqrt{7} - (2x)/\sqrt{7} + \sqrt{3 - 2x - x^2})/\sqrt{7}]/(-1 + x))/(7\sqrt{7})$

fricas [A] time = 0.73, size = 171, normalized size = 0.99

$$\frac{2\sqrt{7}(2x^2 + 2x - 3)\log\left(\frac{x^4 + 44x^3 - \sqrt{7}(3x^3 + x^2 - 45x + 45)\sqrt{-x^2 - 2x + 3} + 26x^2 - 276x + 207}{4x^4 + 8x^3 - 8x^2 - 12x + 9}\right) + 4\sqrt{7}(2x^2 + 2x - 3)\log\left(\frac{2x^2 + \sqrt{7}(2x + 1) + 2x + 4}{2x^2 + 2x - 3}\right) - 14\sqrt{-x^2 - 2x + 3}(x - 3) - 56x + 21}{98(2x^2 + 2x - 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="fricas")

[Out] $1/98*(2*\sqrt{7}*(2*x^2 + 2*x - 3)*\log((x^4 + 44*x^3 - \sqrt{7}*(3*x^3 + x^2 - 45*x + 45)*\sqrt{-x^2 - 2*x + 3} + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) + 4*\sqrt{7}*(2*x^2 + 2*x - 3)*\log((2*x^2 + \sqrt{7}*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 14*\sqrt{-x^2 - 2*x + 3}*(x - 3) - 56*x + 21)/(2*x^2 + 2*x - 3)$

giac [B] time = 0.48, size = 350, normalized size = 2.03

$$-\frac{2}{49}\sqrt{7}\log\left(\frac{4x - 2\sqrt{7} + 2}{4x + 2\sqrt{7} + 2}\right) + \frac{2}{49}\sqrt{7}\log\left(\frac{-2\sqrt{7} + \frac{6(\sqrt{-x^2-2x+3}-2)}{x+1} + 4}{2\sqrt{7} + \frac{6(\sqrt{-x^2-2x+3}-2)}{x+1} + 4}\right) - \frac{2}{49}\sqrt{7}\log\left(\frac{-2\sqrt{7} + \frac{2(\sqrt{-x^2-2x+3}-2)}{x+1} - 4}{2\sqrt{7} + \frac{2(\sqrt{-x^2-2x+3}-2)}{x+1} - 4}\right) - \frac{8x - 3}{14(2x^2 + 2x - 3)} - \frac{8\left(\frac{5(\sqrt{-x^2-2x+3}-2)}{x+1} + \frac{26(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2} + \frac{11(\sqrt{-x^2-2x+3}-2)^3}{(x+1)^3} - 6\right)}{21\left(\frac{8(\sqrt{-x^2-2x+3}-2)}{x+1} + \frac{26(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2} + \frac{8(\sqrt{-x^2-2x+3}-2)^3}{(x+1)^3} - \frac{3(\sqrt{-x^2-2x+3}-2)^4}{(x+1)^4} - 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="giac")

[Out] $-2/49*\sqrt{7}*\log(\text{abs}(4*x - 2*\sqrt{7} + 2)/\text{abs}(4*x + 2*\sqrt{7} + 2)) + 2/49*\sqrt{7}*\log(\text{abs}(-2*\sqrt{7} + 6*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) + 4)/\text{abs}(2*\sqrt{7} + 6*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) + 4)) - 2/49*\sqrt{7}*\log(\text{abs}(-2*\sqrt{7} + 2*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) - 4)/\text{abs}(2*\sqrt{7} + 2*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) - 4)) - 1/14*(8*x - 3)/(2*x^2 + 2*x - 3) - 8/21*(5*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) + 26*(\sqrt{-x^2 - 2*x + 3} - 2)^2/(x + 1)^2 + 11*(\sqrt{-x^2 - 2*x + 3} - 2)^3/(x + 1)^3 - 6)/(8*(\sqrt{-x^2 - 2*x + 3} - 2)/(x + 1) + 26*(\sqrt{-x^2 - 2*x + 3} - 2)^2/(x + 1)^2 + 8*(\sqrt{-x^2 - 2*x + 3} - 2)^3/(x + 1)^3 - 3*(\sqrt{-x^2 - 2*x + 3} - 2)^4/(x + 1)^4 - 3)$

maple [B] time = 0.05, size = 1066, normalized size = 6.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(-x^2-2*x+3)^(1/2))^2,x)

[Out] $-3/28*(4*x+2)/(2*x^2+2*x-3)+4/49*7^(1/2)*\text{arctanh}(1/14*(4*x+2)*7^(1/2))+1/14*(-2*x+6)/(2*x^2+2*x-3)-2*(-1/14-1/14*7^(1/2))*(-1/4/(2+1/2*7^(1/2)))/(x+1/2+1/2*7^(1/2))*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(3/2)+1/8*(-1+7^(1/2))/(2+1/2*7^(1/2))*(1/2*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/2*(-1+7^(1/2))*\text{arcsin}(1/(2+1/2*7^(1/2))+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1)-(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*\text{arctanh}((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2)))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))-1/2/(2+1/2*7^(1/2))*(-1/4*(-2*x-2))*(-(x+1/2+1/2*7^(1/2))^2+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+2+1/2*7^(1/2))^(1/2)-1/8*(-8-2*7^(1/2))-(-1+7^(1/2))^2*\text{arcsin}(1/(2+1/2*7^(1/2))+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))+1/49*7^(1/2)*(1/4*(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2)+1/4*(-1+7^(1/2))*\text{arcsin}(1/(2+1/2*7^(1/2))+1/4*(-1+7^(1/2))^2)^(1/2)*(x+1))-1/2*(2+1/2*7^(1/2))/(1/2+1/2*7^(1/2))*\text{arctanh}((4+7^(1/2)+(-1+7^(1/2))*(x+1/2+1/2*7^(1/2)))/(1/2+1/2*7^(1/2)))/(-4*(x+1/2+1/2*7^(1/2))^2+4*(-1+7^(1/2))*(x+1/2+1/2*7^(1/2))+8+2*7^(1/2))^(1/2))-1/49*7^(1/2)$

2)*(1/4*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/4*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2))+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))-1/2*(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2)))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))-2*(-1/14+1/14*7^(1/2))*(-1/4/(2-1/2*7^(1/2)))/(x+1/2-1/2*7^(1/2))*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(3/2)+1/8*(-1-7^(1/2))/(2-1/2*7^(1/2))*(1/2*(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2)+1/2*(-1-7^(1/2))*arcsin(1/(2-1/2*7^(1/2))+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))-(2-1/2*7^(1/2))/(-1/2+1/2*7^(1/2))*arctanh((4-7^(1/2)+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2)))/(-1/2+1/2*7^(1/2)))/(-4*(x+1/2-1/2*7^(1/2))^2+4*(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+8-2*7^(1/2))^(1/2))-1/2/(2-1/2*7^(1/2))*(-1/4*(-2*x-2)*(-(x+1/2-1/2*7^(1/2))^2+(-1-7^(1/2))*(x+1/2-1/2*7^(1/2))+2-1/2*7^(1/2))^(1/2)-1/8*(-8+2*7^(1/2)-(-1-7^(1/2))^2)*arcsin(1/(2-1/2*7^(1/2))+1/4*(-1-7^(1/2))^2)^(1/2)*(x+1))))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2,x)

[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**2,x)

[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-2), x)

$$3.499 \quad \int \frac{1}{\left(x + \sqrt{3-2x-x^2}\right)^3} dx$$

Optimal. Leaf size=307

$$\frac{4 \left(\frac{(21+5\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - 5\sqrt{3} + 9 \right)}{21 \left(\frac{\sqrt{3}(\sqrt{3}-\sqrt{-x^2-2x+3})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - \sqrt{3} + 2 \right)^2} + \frac{2 \left(-\frac{(18+49\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - 43\sqrt{3} \right)}{147 \left(\frac{\sqrt{3}(\sqrt{3}-\sqrt{-x^2-2x+3})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - \sqrt{3} + 2 \right)^2}$$

Rubi [A] time = 0.25, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1660, 12, 618, 206}

$$\frac{4 \left(\frac{(21+5\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - 5\sqrt{3} + 9 \right)}{21 \left(\frac{\sqrt{3}(\sqrt{3}-\sqrt{-x^2-2x+3})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - \sqrt{3} + 2 \right)^2} + \frac{2 \left(-\frac{(18+49\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - 43\sqrt{3} + 18 \right)}{147 \left(\frac{\sqrt{3}(\sqrt{3}-\sqrt{-x^2-2x+3})^2}{x^2} - \frac{2(1+\sqrt{3})(\sqrt{3}-\sqrt{-x^2-2x+3})}{x} - \sqrt{3} + 2 \right)^2} + \frac{12 \tanh^{-1} \left(\frac{-\sqrt{3}\sqrt{-x^2-2x+3} - \sqrt{3}x + 3}{\sqrt{7}x} \right)}{49\sqrt{7}}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[3 - 2*x - x^2])^(-3), x]

[Out] (-4*(9 - 5*Sqrt[3] + ((21 + 5*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x) / (21*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (2*(18 - 43*Sqrt[3] - ((18 + 49*Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x) / (147*(2 - Sqrt[3] - (2*(1 + Sqrt[3])*(Sqrt[3] - Sqrt[3 - 2*x - x^2]))/x + (Sqrt[3]*(Sqrt[3] - Sqrt[3 - 2*x - x^2])^2)/x^2)^2) + (12*ArcTanh[(3 - x - Sqrt[3]*x - Sqrt[3]*Sqrt[3 - 2*x - x^2])/(Sqrt[7]*x)] / (49*Sqrt[7]))

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]] / (Rt[a, 2]*Rt[-b, 2])), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(p + 1)) / ((p + 1)*(b^2 - 4*a*c)), x] + Dist[1 / ((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(2*c*f - b*g), x], x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{(x + \sqrt{3 - 2x - x^2})^3} dx &= 2 \operatorname{Subst} \left(\int \frac{\sqrt{3} - 2x - 2x^3 - \sqrt{3}x^4}{(2 - \sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2)^3} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right) \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} - \frac{1}{28} \operatorname{Subst} \left(\int \frac{-\frac{8}{3}}{(2 - \sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2)^3} dx, x, \frac{-\sqrt{3} + \sqrt{3 - 2x - x^2}}{x} \right) \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} + \frac{2 \left(18 - 43\sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2 \right)}{147 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} + \frac{2 \left(18 - 43\sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2 \right)}{147 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} + \frac{2 \left(18 - 43\sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2 \right)}{147 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} + \frac{2 \left(18 - 43\sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2 \right)}{147 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} \\
 &= -\frac{4 \left(9 - 5\sqrt{3} + \frac{(21+5\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} \right)}{21 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2} + \frac{2 \left(18 - 43\sqrt{3} + 2(1 + \sqrt{3})x + \sqrt{3}x^2 \right)}{147 \left(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2} \right)^2}
 \end{aligned}$$

Mathematica [A] time = 1.11, size = 333, normalized size = 1.08

$$\frac{98(-12 + 11x)}{21(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2})^2} - \frac{7(37 - 24x)}{147(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2})^2} - \frac{14\sqrt{3}\sqrt{3-2x-x^2}(-15 - 83x + 58x^2 + 34x^3)}{147(2 - \sqrt{3} - \frac{2(1+\sqrt{3})(\sqrt{3} - \sqrt{3-2x-x^2})}{x} + \frac{\sqrt{3}(\sqrt{3} - \sqrt{3-2x-x^2})^2}{x^2})^2} - 12\sqrt{7}\sqrt{3-2x-x^2}\log[-1 + \sqrt{7} - 2x] + 2(-1 + \sqrt{7})\sqrt{3-2x-x^2}\sqrt{14(4 + \sqrt{7})}\log[1 - \sqrt{7} + 2x] + 12\sqrt{7}\sqrt{3-2x-x^2}\log[1 + \sqrt{7} + 2x] + 6(1 + \sqrt{7})\sqrt{3-2x-x^2}\sqrt{14(4 + \sqrt{7})}\log[1 + \sqrt{7} + 2x] - 6(1 + \sqrt{7})\sqrt{3-2x-x^2}\sqrt{14(4 + \sqrt{7})}\log[7 + 7\sqrt{7} + 7x - \sqrt{7}x + \sqrt{14(4 + \sqrt{7})}] * \log[7 - 7\sqrt{7} + (7 + \sqrt{7})x - \sqrt{7}x + \sqrt{14(4 + \sqrt{7})}] * (-3 + 2x + x^2)] / 1372$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x + Sqrt[3 - 2*x - x^2])^(-3), x]

[Out] ((98*(-12 + 11*x))/(-3 + 2*x + 2*x^2)^2 + (7*(37 - 24*x))/(-3 + 2*x + 2*x^2) - (14*Sqrt[3 - 2*x - x^2]*(-15 - 83*x + 58*x^2 + 34*x^3))/(-3 + 2*x + 2*x^2)^2 - 12*Sqrt[7]*Log[-1 + Sqrt[7] - 2*x] + 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[1 - Sqrt[7] + 2*x] + 12*Sqrt[7]*Log[1 + Sqrt[7] + 2*x] + 6*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[1 + Sqrt[7] + 2*x] - 6*(1 + Sqrt[7])*Sqrt[14/(4 + Sqrt[7])]*Log[7 + 7*Sqrt[7] + 7*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[7])])*Sqrt[3 - 2*x - x^2]] - 2*(-1 + Sqrt[7])*Sqrt[14*(4 + Sqrt[7])]*Log[7 - 7*Sqrt[7] + (7 + Sqrt[7])*x - Sqrt[7]*x + Sqrt[14*(4 + Sqrt[7])])*Sqrt[(-4 + Sqrt[7])*(-3 + 2*x + x^2)])/1372

IntegrateAlgebraic [A] time = 0.51, size = 129, normalized size = 0.42

$$\frac{12 \tanh^{-1}\left(\frac{\frac{\sqrt{-x^2-2x+3}}{\sqrt{7}} - \frac{2x}{\sqrt{7}} + \frac{2}{\sqrt{7}}}{x-1}\right)}{49\sqrt{7}} + \frac{-48x^3 + 26x^2 + 300x - 279}{196(2x^2 + 2x - 3)^2} + \frac{\sqrt{-x^2 - 2x + 3}(-34x^3 - 58x^2 + 83x + 15)}{98(2x^2 + 2x - 3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[3 - 2*x - x^2])^(-3), x]

[Out] (-279 + 300*x + 26*x^2 - 48*x^3)/(196*(-3 + 2*x + 2*x^2)^2) + (Sqrt[3 - 2*x - x^2]*(15 + 83*x - 58*x^2 - 34*x^3))/(98*(-3 + 2*x + 2*x^2)^2) + (12*ArcTanh[(2/Sqrt[7] - (2*x)/Sqrt[7] + Sqrt[3 - 2*x - x^2])/Sqrt[7]]/(-1 + x))/(49*Sqrt[7])

fricas [A] time = 0.81, size = 223, normalized size = 0.73

$$\frac{336x^3 - 6\sqrt{7}(4x^4 + 8x^3 - 8x^2 - 12x + 9) \log\left(\frac{x^4 + 4x^3 - \sqrt{7}(3x^3 + x^2 - 45x + 45)\sqrt{-x^2 - 2x + 3} + 26x^2 - 276x + 207}{4x^4 + 8x^3 - 8x^2 - 12x + 9}\right) - 12\sqrt{7}(4x^4 + 8x^3 - 8x^2 - 12x + 9) \log\left(\frac{2x^2 + \sqrt{7}(2x+1) + 2x+4}{2x^2 + 2x - 3}\right) - 182x^2 + 14(34x^3 + 58x^2 - 83x - 15)\sqrt{-x^2 - 2x + 3} - 2100x + 1953}{1372(4x^4 + 8x^3 - 8x^2 - 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="fricas")

[Out] -1/1372*(336*x^3 - 6*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((x^4 + 44*x^3 - sqrt(7)*(3*x^3 + x^2 - 45*x + 45)*sqrt(-x^2 - 2*x + 3) + 26*x^2 - 276*x + 207)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)) - 12*sqrt(7)*(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)*log((2*x^2 + sqrt(7)*(2*x + 1) + 2*x + 4)/(2*x^2 + 2*x - 3)) - 182*x^2 + 14*(34*x^3 + 58*x^2 - 83*x - 15)*sqrt(-x^2 - 2*x + 3) - 2100*x + 1953)/(4*x^4 + 8*x^3 - 8*x^2 - 12*x + 9)

giac [A] time = 0.56, size = 452, normalized size = 1.47

$$\frac{-\frac{3}{343}\sqrt{7}\log\left(\frac{4x-2\sqrt{7}+2}{4x+2\sqrt{7}+2}\right) + \frac{3}{343}\sqrt{7}\log\left(\frac{-2\sqrt{7} + \frac{6(\sqrt{-x^2-2x+3}-2)}{x+1} + 4}{2\sqrt{7} + \frac{6(\sqrt{-x^2-2x+3}-2)}{x+1} + 4}\right) - \frac{3}{343}\sqrt{7}\log\left(\frac{-2\sqrt{7} + \frac{2(\sqrt{-x^2-2x+3}-2)}{x+1} - 4}{2\sqrt{7} + \frac{2(\sqrt{-x^2-2x+3}-2)}{x+1} - 4}\right)}{196(2x^2 + 2x - 3)^2} + \frac{4\left(\frac{231(\sqrt{-x^2-2x+3}-2)}{x+1} + \frac{3286(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2} + \frac{4441(\sqrt{-x^2-2x+3}-2)^3}{(x+1)^3} + \frac{18906(\sqrt{-x^2-2x+3}-2)^4}{(x+1)^4} + \frac{12487(\sqrt{-x^2-2x+3}-2)^5}{(x+1)^5} + \frac{946(\sqrt{-x^2-2x+3}-2)^6}{(x+1)^6} + \frac{1977(\sqrt{-x^2-2x+3}-2)^7}{(x+1)^7} - 414\right)}{441\left(\frac{6(\sqrt{-x^2-2x+3}-2)}{x+1} + \frac{2(\sqrt{-x^2-2x+3}-2)^2}{(x+1)^2} + \frac{6(\sqrt{-x^2-2x+3}-2)^3}{(x+1)^3} - \frac{3(\sqrt{-x^2-2x+3}-2)^4}{(x+1)^4} - 3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="giac")

[Out] -3/343*sqrt(7)*log(abs(4*x - 2*sqrt(7) + 2)/abs(4*x + 2*sqrt(7) + 2)) + 3/343*sqrt(7)*log(abs(-2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)/abs(2*sqrt(7) + 6*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 4)) - 3/343*sqrt(7)*log(abs(-2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)/abs(2*sqrt(7) + 2*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) - 4)) - 1/196*(48*x^3 - 26*x^2 - 300*x + 279)/(2*x^2 + 2*x - 3)^2 + 4/441*(231*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 3286*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 - 4441*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 18906*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 12487*(sqrt(-x^2 - 2*x + 3) - 2)^5/(x + 1)^5 + 946*(sqrt(-x^2 - 2*x + 3) - 2)^6/(x + 1)^6 + 1977*(sqrt(-x^2 - 2*x + 3) - 2)^7/(x + 1)^7 - 414)/(8*(sqrt(-x^2 - 2*x + 3) - 2)/(x + 1) + 26*(sqrt(-x^2 - 2*x + 3) - 2)^2/(x + 1)^2 + 8*(sqrt(-x^2 - 2*x + 3) - 2)^3/(x + 1)^3 - 3*(sqrt(-x^2 - 2*x + 3) - 2)^4/(x + 1)^4 - 3)^2

maple [B] time = 0.07, size = 5984, normalized size = 19.49

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(-x^2-2*x+3)^(1/2))^3,x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-2*x+3)^(1/2))^3,x, algorithm="maxima")

[Out] integrate((x + sqrt(-x^2 - 2*x + 3))^(-3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3,x)

[Out] int(1/(x + (3 - x^2 - 2*x)^(1/2))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 2x + 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x**2-2*x+3)**(1/2))**3,x)

[Out] Integral((x + sqrt(-x**2 - 2*x + 3))**(-3), x)

$$3.500 \quad \int \frac{1}{x + \sqrt{-3 - 2x + x^2}} dx$$

Optimal. Leaf size=65

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + 2 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - \frac{3}{2} \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Rubi [A] time = 0.03, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + 2 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - \frac{3}{2} \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-1), x]

[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 2*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - (3*Log[x + Sqrt[-3 - 2*x + x^2]])/2

Rule 893

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))^(n_.)*((a_.) + (b_.)*(x_.) + (c_.)*(x_.^2))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_.) + (f_.)*Sqrt[(a_.) + (b_.)*(x_.) + (c_.)*(x_.^2)])^(n_.))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{x + \sqrt{-3 - 2x + x^2}} dx &= 2 \operatorname{Subst} \left(\int \frac{-3 - 2x + x^2}{x(-2 + 2x)^2} dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= 2 \operatorname{Subst} \left(\int \left(-\frac{1}{(-1 + x)^2} + \frac{1}{-1 + x} - \frac{3}{4x} \right) dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= -\frac{2}{1 - x - \sqrt{-3 - 2x + x^2}} + 2 \log\left(1 - x - \sqrt{-3 - 2x + x^2}\right) - \frac{3}{2} \log\left(x + \sqrt{-3 - 2x + x^2}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 59, normalized size = 0.91

$$2 \left(\frac{1}{\sqrt{x^2 - 2x - 3} + x - 1} + \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - \frac{3}{4} \log\left(\sqrt{x^2 - 2x - 3} + x\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-1), x]

[Out] $2*((-1 + x + \sqrt{-3 - 2x + x^2})^{-1}) + \text{Log}[1 - x - \sqrt{-3 - 2x + x^2}] - (3*\text{Log}[x + \sqrt{-3 - 2x + x^2}])/4)$

IntegrateAlgebraic [A] time = 0.31, size = 84, normalized size = 1.29

$$-\frac{1}{2}\sqrt{x^2-2x-3} - \frac{1}{2}\log(\sqrt{x^2-2x-3}-x-1) + 2\log(\sqrt{x^2-2x-3}+x+1) - \frac{3}{2}\log(\sqrt{x^2-2x-3}+3x+3) + \frac{x}{2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-1), x]

[Out] $x/2 - \sqrt{-3 - 2x + x^2}/2 - \text{Log}[-1 - x + \sqrt{-3 - 2x + x^2}]/2 + 2*\text{Log}[1 + x + \sqrt{-3 - 2x + x^2}] - (3*\text{Log}[3 + 3x + \sqrt{-3 - 2x + x^2}])/2$

fricas [A] time = 0.72, size = 77, normalized size = 1.18

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2-2x-3} - \frac{3}{4}\log(2x+3) - \frac{5}{4}\log(-x + \sqrt{x^2-2x-3} + 1) + \frac{3}{4}\log(-x + \sqrt{x^2-2x-3}) - \frac{3}{4}\log(-x + \sqrt{x^2-2x-3} - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="fricas")

[Out] $1/2*x - 1/2*\text{sqrt}(x^2 - 2*x - 3) - 3/4*\log(2*x + 3) - 5/4*\log(-x + \text{sqrt}(x^2 - 2*x - 3) + 1) + 3/4*\log(-x + \text{sqrt}(x^2 - 2*x - 3)) - 3/4*\log(-x + \text{sqrt}(x^2 - 2*x - 3) - 3)$

giac [A] time = 0.37, size = 81, normalized size = 1.25

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2-2x-3} - \frac{3}{4}\log(|2x+3|) - \frac{5}{4}\log(|-x + \sqrt{x^2-2x-3} + 1|) + \frac{3}{4}\log(|-x + \sqrt{x^2-2x-3}|) - \frac{3}{4}\log(|-x + \sqrt{x^2-2x-3} - 3|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="giac")

[Out] $1/2*x - 1/2*\text{sqrt}(x^2 - 2*x - 3) - 3/4*\log(\text{abs}(2*x + 3)) - 5/4*\log(\text{abs}(-x + \text{sqrt}(x^2 - 2*x - 3) + 1)) + 3/4*\log(\text{abs}(-x + \text{sqrt}(x^2 - 2*x - 3))) - 3/4*\log(\text{abs}(-x + \text{sqrt}(x^2 - 2*x - 3) - 3))$

maple [A] time = 0.01, size = 71, normalized size = 1.09

$$\frac{x}{2} + \frac{3 \operatorname{arctanh}\left(\frac{-\frac{10x}{3}-2}{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}\right)}{4} - \frac{3 \ln(2x+3)}{4} + \frac{5 \ln\left(x-1 + \sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}\right)}{4} - \frac{\sqrt{-20x+4\left(x+\frac{3}{2}\right)^2-21}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(x^2-2*x-3)^(1/2)),x)

[Out] $-1/4*(4*(x+3/2)^2-20*x-21)^(1/2)+5/4*\ln(-1+x+((x+3/2)^2-5*x-21/4)^(1/2))+3/4*\operatorname{arctanh}(2/3*(-3-5*x)/(4*(x+3/2)^2-20*x-21)^(1/2))+1/2*x-3/4*\ln(2*x+3)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{x^2 - 2x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x + sqrt(x^2 - 2*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\frac{x}{2} - \frac{3 \ln\left(x + \frac{3}{2}\right)}{4} - \int \frac{\sqrt{x^2 - 2x - 3}}{2x + 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (x^2 - 2*x - 3)^(1/2)), x)

[Out] x/2 - (3*log(x + 3/2))/4 - int((x^2 - 2*x - 3)^(1/2)/(2*x + 3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{x^2 - 2x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x**2-2*x-3)**(1/2)), x)

[Out] Integral(1/(x + sqrt(x**2 - 2*x - 3)), x)

$$3.501 \quad \int \frac{1}{\left(x + \sqrt{-3 - 2x + x^2}\right)^2} dx$$

Optimal. Leaf size=83

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + \frac{3}{2(\sqrt{x^2 - 2x - 3} + x)} + 4 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 4 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Rubi [A] time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + \frac{3}{2(\sqrt{x^2 - 2x - 3} + x)} + 4 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 4 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-2), x]

[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]))^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[(g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2)/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(x + \sqrt{-3 - 2x + x^2}\right)^2} dx &= 2 \text{Subst} \left(\int \frac{-3 - 2x + x^2}{x^2(-2 + 2x)^2} dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= 2 \text{Subst} \left(\int \left(-\frac{1}{(-1 + x)^2} + \frac{2}{-1 + x} - \frac{3}{4x^2} - \frac{2}{x} \right) dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= -\frac{2}{1 - x - \sqrt{-3 - 2x + x^2}} + \frac{3}{2(x + \sqrt{-3 - 2x + x^2})} + 4 \log\left(1 - x - \sqrt{-3 - 2x + x^2}\right) - 4 \log\left(x + \sqrt{-3 - 2x + x^2}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 79, normalized size = 0.95

$$\frac{2}{\sqrt{x^2 - 2x - 3} + x - 1} + \frac{3}{2(\sqrt{x^2 - 2x - 3} + x)} + 4 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 4 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-2), x]

[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(2*(x + Sqrt[-3 - 2*x + x^2])) + 4*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 4*Log[x + Sqrt[-3 - 2*x + x^2]]

IntegrateAlgebraic [A] time = 0.33, size = 73, normalized size = 0.88

$$\frac{\sqrt{x^2 - 2x - 3}(-x - 3)}{2x + 3} + \frac{4x^2 + 6x - 9}{4(2x + 3)} - 8 \tanh^{-1}\left(\frac{x + 1}{\sqrt{x^2 - 2x - 3} + 2x + 2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-2), x]

[Out] ((-3 - x)*Sqrt[-3 - 2*x + x^2])/(3 + 2*x) + (-9 + 6*x + 4*x^2)/(4*(3 + 2*x)) - 8*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])]

fricas [A] time = 0.85, size = 97, normalized size = 1.17

$$\frac{4x^2 - 8(2x + 3)\log(x^2 - \sqrt{x^2 - 2x - 3}(x + 1) - 3) - 8(2x + 3)\log(2x + 3) + 8(2x + 3)\log(-x + \sqrt{x^2 - 2x - 3}) - 4\sqrt{x^2 - 2x - 3}(x + 3) + 2x - 15}{4(2x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="fricas")

[Out] 1/4*(4*x^2 - 8*(2*x + 3)*log(x^2 - sqrt(x^2 - 2*x - 3)*(x + 1) - 3) - 8*(2*x + 3)*log(2*x + 3) + 8*(2*x + 3)*log(-x + sqrt(x^2 - 2*x - 3)) - 4*sqrt(x^2 - 2*x - 3)*(x + 3) + 2*x - 15)/(2*x + 3)

giac [B] time = 0.51, size = 143, normalized size = 1.72

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2 - 2x - 3} - \frac{3(5x - 5\sqrt{x^2 - 2x - 3} + 3)}{4((x - \sqrt{x^2 - 2x - 3})^2 + 3x - 3\sqrt{x^2 - 2x - 3})} - \frac{9}{4(2x + 3)} - 2\log(2x + 3) - 2\log(-x + \sqrt{x^2 - 2x - 3} + 1) + 2\log(-x + \sqrt{x^2 - 2x - 3}) - 2\log(-x + \sqrt{x^2 - 2x - 3} - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="giac")

[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 3/4*(5*x - 5*sqrt(x^2 - 2*x - 3) + 3)/((x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3)) - 9/4/(2*x + 3) - 2*log(abs(2*x + 3)) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 2*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 2*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))

maple [A] time = 0.02, size = 118, normalized size = 1.42

$$\frac{x}{2} + 2 \operatorname{arctanh}\left(\frac{-\frac{10x}{3} - 2}{\sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21}}\right) - 2 \ln(2x + 3) + 2 \ln\left(x - 1 + \sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}\right) - \frac{9}{4(2x + 3)} - \frac{2\sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21}}{3} - \frac{\left(-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}\right)^{\frac{3}{2}}}{3\left(x + \frac{3}{2}\right)} + \frac{(2x - 2)\sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(x^2-2*x-3)^(1/2))^2,x)

[Out] -2*ln(2*x+3)+1/2*x-9/4/(2*x+3)-2/3*(-20*x+4*(x+3/2)^2-21)^(1/2)+2*ln(x-1+(-5*x+(x+3/2)^2-21/4)^(1/2))+2*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/2)^2-21)^(1/2))-1/3/(x+3/2)*(-5*x+(x+3/2)^2-21/4)^(3/2)+1/6*(2*x-2)*(-5*x+(x+3/2)^2-21/4)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2,x)

[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**2,x)

[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-2), x)

$$3.502 \quad \int \frac{1}{\left(x + \sqrt{-3 - 2x + x^2}\right)^3} dx$$

Optimal. Leaf size=101

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + \frac{4}{\sqrt{x^2 - 2x - 3} + x} + \frac{3}{4(\sqrt{x^2 - 2x - 3} + x)^2} + 6 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 6 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Rubi [A] time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2116, 893}

$$-\frac{2}{-\sqrt{x^2 - 2x - 3} - x + 1} + \frac{4}{\sqrt{x^2 - 2x - 3} + x} + \frac{3}{4(\sqrt{x^2 - 2x - 3} + x)^2} + 6 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 6 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 2*x + x^2])^(-3), x]

[Out] -2/(1 - x - Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x + Sqrt[-3 - 2*x + x^2]]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2116

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2])^(n_))^(p_.), x_Symbol] :> Dist[2, Subst[Int[((g + h*x^n)^p*(d^2*e - (b*d - a*e)*f^2 - (2*d*e - b*f^2)*x + e*x^2))/(-2*d*e + b*f^2 + 2*e*x)^2, x], x, d + e*x + f*Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(x + \sqrt{-3 - 2x + x^2}\right)^3} dx &= 2 \operatorname{Subst} \left(\int \frac{-3 - 2x + x^2}{x^3(-2 + 2x)^2} dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= 2 \operatorname{Subst} \left(\int \left(-\frac{1}{(-1 + x)^2} + \frac{3}{-1 + x} - \frac{3}{4x^3} - \frac{2}{x^2} - \frac{3}{x} \right) dx, x, x + \sqrt{-3 - 2x + x^2} \right) \\ &= -\frac{2}{1 - x - \sqrt{-3 - 2x + x^2}} + \frac{3}{4(x + \sqrt{-3 - 2x + x^2})^2} + \frac{4}{x + \sqrt{-3 - 2x + x^2}} + 6 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 6 \log\left(\sqrt{x^2 - 2x - 3} + x\right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 97, normalized size = 0.96

$$\frac{2}{\sqrt{x^2 - 2x - 3} + x - 1} + \frac{4}{\sqrt{x^2 - 2x - 3} + x} + \frac{3}{4(\sqrt{x^2 - 2x - 3} + x)^2} + 6 \log\left(-\sqrt{x^2 - 2x - 3} - x + 1\right) - 6 \log\left(\sqrt{x^2 - 2x - 3} + x\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[-3 - 2*x + x^2])^(-3), x]

[Out] 2/(-1 + x + Sqrt[-3 - 2*x + x^2]) + 3/(4*(x + Sqrt[-3 - 2*x + x^2])^2) + 4/(x + Sqrt[-3 - 2*x + x^2]) + 6*Log[1 - x - Sqrt[-3 - 2*x + x^2]] - 6*Log[x + Sqrt[-3 - 2*x + x^2]]

IntegrateAlgebraic [A] time = 0.35, size = 86, normalized size = 0.85

$$\frac{\sqrt{x^2 - 2x - 3} (-4x^2 - 31x - 33)}{2(2x + 3)^2} - 12 \tanh^{-1} \left(\frac{x + 1}{\sqrt{x^2 - 2x - 3} + 2x + 2} \right) + \frac{16x^3 + 48x^2 - 108x - 189}{8(2x + 3)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[-3 - 2*x + x^2])^(-3), x]

[Out] ((-33 - 31*x - 4*x^2)*Sqrt[-3 - 2*x + x^2])/(2*(3 + 2*x)^2) + (-189 - 108*x + 48*x^2 + 16*x^3)/(8*(3 + 2*x)^2) - 12*ArcTanh[(1 + x)/(2 + 2*x + Sqrt[-3 - 2*x + x^2])]

fricas [A] time = 0.64, size = 129, normalized size = 1.28

$$\frac{8x^3 - 10x^2 - 12(4x^2 + 12x + 9) \log(x^2 - \sqrt{x^2 - 2x - 3}(x + 1) - 3) - 12(4x^2 + 12x + 9) \log(2x + 3) + 12(4x^2 + 12x + 9) \log(-x + \sqrt{x^2 - 2x - 3}) - 2(4x^2 + 31x + 33)\sqrt{x^2 - 2x - 3} - 156x - 171}{4(4x^2 + 12x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="fricas")

[Out] 1/4*(8*x^3 - 10*x^2 - 12*(4*x^2 + 12*x + 9)*log(x^2 - sqrt(x^2 - 2*x - 3))*(x + 1) - 3) - 12*(4*x^2 + 12*x + 9)*log(2*x + 3) + 12*(4*x^2 + 12*x + 9)*log(-x + sqrt(x^2 - 2*x - 3)) - 2*(4*x^2 + 31*x + 33)*sqrt(x^2 - 2*x - 3) - 156*x - 171)/(4*x^2 + 12*x + 9)

giac [B] time = 0.42, size = 184, normalized size = 1.82

$$\frac{1}{2}x - \frac{1}{2}\sqrt{x^2 - 2x - 3} - \frac{104(x - \sqrt{x^2 - 2x - 3})^3 + 315(x - \sqrt{x^2 - 2x - 3})^2 + 162x - 162\sqrt{x^2 - 2x - 3} + 27}{8((x - \sqrt{x^2 - 2x - 3})^2 + 3x - 3)\sqrt{x^2 - 2x - 3}} - \frac{9(16x + 21)}{8(2x + 3)^2} - 3 \log(2x + 3) - 3 \log(-x + \sqrt{x^2 - 2x - 3} + 1) + 3 \log(-x + \sqrt{x^2 - 2x - 3}) - 3 \log(-x + \sqrt{x^2 - 2x - 3} - 3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="giac")

[Out] 1/2*x - 1/2*sqrt(x^2 - 2*x - 3) - 1/8*(104*(x - sqrt(x^2 - 2*x - 3))^3 + 315*(x - sqrt(x^2 - 2*x - 3))^2 + 162*x - 162*sqrt(x^2 - 2*x - 3) + 27)/((x - sqrt(x^2 - 2*x - 3))^2 + 3*x - 3*sqrt(x^2 - 2*x - 3))^2 - 9/8*(16*x + 21)/(2*x + 3)^2 - 3*log(abs(2*x + 3)) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) + 1)) + 3*log(abs(-x + sqrt(x^2 - 2*x - 3))) - 3*log(abs(-x + sqrt(x^2 - 2*x - 3) - 3))

maple [A] time = 0.03, size = 146, normalized size = 1.45

$$\frac{x}{2} + 3 \operatorname{arctanh} \left(\frac{-\frac{10x}{3} - 2}{\sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21}} \right) - 3 \ln(2x + 3) + 3 \ln \left(x - 1 + \sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}} \right) - \frac{9}{2x + 3} + \frac{27}{8(2x + 3)^2} - \frac{\left(-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}\right)^{\frac{3}{2}}}{2\left(x + \frac{3}{2}\right)} - \sqrt{-20x + 4\left(x + \frac{3}{2}\right)^2 - 21} + \frac{(2x - 2)\sqrt{-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}}}{4} + \frac{\left(-5x + \left(x + \frac{3}{2}\right)^2 - \frac{21}{4}\right)^{\frac{3}{2}}}{4\left(x + \frac{3}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(x^2-2*x-3)^(1/2))^3,x)

[Out] -9/(2*x+3) - 3*ln(2*x+3) + 1/2*x + 27/8/(2*x+3)^2 - 1/2/(x+3/2)*(-5*x+(x+3/2)^2-21/4)^(3/2) - (-20*x+4*(x+3/2)^2-21)^(1/2) + 3*arctanh(2/3*(-5*x-3)/(-20*x+4*(x+3/2)^2-21)^(1/2)) + 1/4*(2*x-2)*(-5*x+(x+3/2)^2-21/4)^(1/2) + 3*ln(x-1+(-5*x+(x+3/2)^2-21/4)^(1/2)) + 1/4/(x+3/2)^2*(-5*x+(x+3/2)^2-21/4)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x^2-2*x-3)^(1/2))^3,x, algorithm="maxima")

[Out] integrate((x + sqrt(x^2 - 2*x - 3))^(-3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3,x)

[Out] int(1/(x + (x^2 - 2*x - 3)^(1/2))^3, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{x^2 - 2x - 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(x**2-2*x-3)**(1/2))**3,x)

[Out] Integral((x + sqrt(x**2 - 2*x - 3))**(-3), x)

$$3.503 \quad \int \frac{1}{x + \sqrt{-3 - 4x - x^2}} dx$$

Optimal. Leaf size=108

$$\frac{1}{2} \log(x+3) + \frac{1}{2} \log\left(\frac{\sqrt{-x-1}x + \sqrt{x+3}x + 3\sqrt{-x-1}}{(x+3)^{3/2}}\right) - \tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right) - \sqrt{2} \tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {12, 1023, 634, 618, 204, 628, 635, 203, 260}

$$\frac{1}{2} \log(x+3) + \frac{1}{2} \log\left(\frac{\sqrt{-x-1}x + \sqrt{x+3}x + 3\sqrt{-x-1}}{(x+3)^{3/2}}\right) - \tan^{-1}\left(\frac{\sqrt{-x-1}}{\sqrt{x+3}}\right) - \sqrt{2} \tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-1), x]

[Out] -ArcTan[Sqrt[-1 - x]/Sqrt[3 + x]] - Sqrt[2]*ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]] + Log[3 + x]/2 + Log[(3*Sqrt[-1 - x] + Sqrt[-1 - x]*x + x*Sqrt[3 + x])/(3 + x)^(3/2)]/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 635

```
Int[((d_) + (e_)*(x_))/((a_) + (c_)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]
```

Rule 1023

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*((d_) + (f_)*(x_)^2)), x_Symbol] := With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2]}, Dist[1/q, Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Dist[1/q, Int[Simp[b*h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2), x], x] /; NeQ[q, 0] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{x + \sqrt{-3 - 4x - x^2}} dx &= 2 \operatorname{Subst} \left(\int \frac{2x}{(1+x^2)(1-2x+3x^2)} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\ &= 4 \operatorname{Subst} \left(\int \frac{x}{(1+x^2)(1-2x+3x^2)} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\ &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{-2-2x}{1+x^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) + \frac{1}{2} \operatorname{Subst} \left(\int \frac{2+6x}{1-2x+3x^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\ &= \frac{1}{2} \operatorname{Subst} \left(\int \frac{-2+6x}{1-2x+3x^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) + 2 \operatorname{Subst} \left(\int \frac{1}{1-2x+3x^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\ &= -\tan^{-1} \left(\frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) + \frac{1}{2} \log(3+x) + \frac{1}{2} \log \left(\frac{3\sqrt{-1-x} + \sqrt{-1-x}x + x\sqrt{3+x}}{(3+x)^{3/2}} \right) \\ &= -\tan^{-1} \left(\frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) - \sqrt{2} \tan^{-1} \left(\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}} \right) + \frac{1}{2} \log(3+x) + \frac{1}{2} \log \left(\frac{3\sqrt{-1-x}}{(3+x)^{3/2}} \right) \end{aligned}$$

Mathematica [C] time = 0.42, size = 187, normalized size = 1.73

$$\frac{1}{4} \left(\log(2x^2 + 4x + 3) + i\sqrt{1-2i\sqrt{2}} \tanh^{-1} \left(\frac{i\sqrt{2}x + 2x + 2i\sqrt{2} + 2}{\sqrt{2-4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) - i\sqrt{1+2i\sqrt{2}} \tanh^{-1} \left(\frac{(2-i\sqrt{2})x - 2i\sqrt{2} + 2}{\sqrt{2+4i\sqrt{2}} \sqrt{-x^2-4x-3}} \right) + 2\sin^{-1}(x+2) - 2\sqrt{2} \tan^{-1}(\sqrt{2}(x+1)) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-1), x]
```

```
[Out] (2*ArcSin[2 + x] - 2*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + I*Sqrt[1 - (2*I)*Sqrt[2]]*ArcTanh[(2 + (2*I)*Sqrt[2] + 2*x + I*Sqrt[2]*x)/(Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) - I*Sqrt[1 + (2*I)*Sqrt[2]]*ArcTanh[(2 - (2*I)*Sqrt[2] + (2 - I*Sqrt[2])*x)/(Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]]) + Log[3 + 4*x + 2*x^2])/4
```

IntegrateAlgebraic [A] time = 0.34, size = 86, normalized size = 0.80

$$\frac{1}{2} \log\left(\sqrt{-x^2 - 4x - 3} + x\right) - \tan^{-1}\left(\frac{\sqrt{-x^2 - 4x - 3}}{x + 3}\right) - \sqrt{2} \tan^{-1}\left(\frac{\sqrt{2}x + \sqrt{2}}{\sqrt{-x^2 - 4x - 3} + x + 1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-1), x]

[Out] -ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - Sqrt[2]*ArcTan[(Sqrt[2] + Sqrt[2]*x)/(1 + x + Sqrt[-3 - 4*x - x^2])] + Log[x + Sqrt[-3 - 4*x - x^2]]/2

fricas [B] time = 0.63, size = 187, normalized size = 1.73

$$\frac{1}{2} \sqrt{2} \arctan(\sqrt{2}(x+1)) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{4} \sqrt{2} \arctan\left(\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2} \arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) + \frac{1}{4} \log(2x^2+4x+3) - \frac{1}{8} \log\left(\frac{-2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{8} \log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)), x, algorithm="fricas")

[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 1/4*log(2*x^2 + 4*x + 3) - 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/8*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)

giac [B] time = 0.45, size = 197, normalized size = 1.82

$$\frac{1}{2} \sqrt{2} \arctan(\sqrt{2}(x+1)) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}\right) + \frac{1}{2} \sqrt{2} \arctan\left(\frac{3(\sqrt{-x^2-4x-3}+1)}{x+2}\right) + \frac{1}{2} \arcsin(x+2) + \frac{1}{4} \log(2x^2+4x+3) + \frac{1}{4} \log\left(\frac{2(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2}\right) - \frac{1}{4} \log\left(\frac{2(\sqrt{-x^2-4x-3}+1)}{x+2} + \frac{3(\sqrt{-x^2-4x-3}+1)^2}{(x+2)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)), x, algorithm="giac")

[Out] -1/2*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) + 1)/(x + 2) + 1)) + 1/2*arcsin(x + 2) + 1/4*log(2*x^2 + 4*x + 3) + 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) + 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) + 1)^2/(x + 2)^2 + 1)

maple [B] time = 0.03, size = 370, normalized size = 3.43

$$\frac{\arcsin(x+2)}{2} + \frac{\sqrt{2} \arctan\left(\frac{3\sqrt{2}(x+1)}{x}\right)}{2} + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{2}}{6}\right)}{3 \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{\frac{x}{(-3/2)^2} + 1}} + \frac{\ln(2x^2+4x+3)}{4} + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \left(-\operatorname{arctanh}\left(\frac{3x}{(-3/2)\sqrt{\frac{3x^2}{(-3/2)^2} - 12}}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{2}}{6}\right)\right)}{12 \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{\frac{x}{(-3/2)^2} + 1}} + \frac{\sqrt{3} \sqrt{4} \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \left(\operatorname{arctanh}\left(\frac{3x}{(-3/2)\sqrt{\frac{3x^2}{(-3/2)^2} - 12}}\right) + \sqrt{2} \arctan\left(\frac{\sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{2}}{6}\right)\right)}{6 \sqrt{\frac{3x^2}{(-3/2)^2} - 12} \sqrt{\frac{x}{(-3/2)^2} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(-x^2-4*x-3)^(1/2)), x)

[Out] 1/2*arcsin(x+2)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)+1/3*3^(1/2)*4^(1/2)/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(x/(-3/2-x)+1)^2)^(1/2)/(x/(-3/2-x)+1)+1/4*ln(2*x^2+4*x+3)-1/2*2^(1/2)*arctan(1/4*(4+4*x)*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x + sqrt(-x^2 - 4*x - 3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2)),x)

[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x + \sqrt{-x^2 - 4x - 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x**2-4*x-3)**(1/2)),x)

[Out] Integral(1/(x + sqrt(-x**2 - 4*x - 3)), x)

$$3.504 \quad \int \frac{1}{\left(x + \sqrt{-3 - 4x - x^2}\right)^2} dx$$

Optimal. Leaf size=87

$$\frac{1 - \frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1} + \frac{\tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Rubi [A] time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {12, 638, 618, 204}

$$\frac{1 - \frac{\sqrt{-x-1}}{\sqrt{x+3}}}{-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1} + \frac{\tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-2), x]

[Out] (1 - Sqrt[-1 - x]/Sqrt[3 + x])/(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x]) + ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]]/Sqrt[2]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

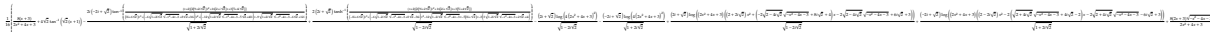
Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rubi steps

$$\begin{aligned}
\int \frac{1}{(x + \sqrt{-3 - 4x - x^2})^2} dx &= 2 \operatorname{Subst} \left(\int -\frac{2x}{(1 - 2x + 3x^2)^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\
&= - \left(4 \operatorname{Subst} \left(\int \frac{x}{(1 - 2x + 3x^2)^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \right) \\
&= \frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}} - \operatorname{Subst} \left(\int \frac{1}{1 - 2x + 3x^2} dx, x, \frac{\sqrt{-1-x}}{\sqrt{3+x}} \right) \\
&= \frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + 2 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, -2 + \frac{6\sqrt{-1-x}}{\sqrt{3+x}} \right) \\
&= \frac{1 - \frac{\sqrt{-1-x}}{\sqrt{3+x}}}{1 - \frac{3(1+x)}{3+x} - \frac{2\sqrt{-1-x}}{\sqrt{3+x}}} + \frac{\tan^{-1} \left(\frac{1 - \frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}} \right)}{\sqrt{2}}
\end{aligned}$$

Mathematica [C] time = 1.67, size = 881, normalized size = 10.13



Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-2), x]

[Out] ((8*(3 + x))/(3 + 4*x + 2*x^2) + (8*(3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(3 + 4*x + 2*x^2) + 4*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] - ((2*I)*(-2*I + Sqrt[2])*ArcTan[((2 + x)*(3*(5 + (4*I)*Sqrt[2])) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2)]/(12*I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 + (2*I)*Sqrt[2]] + (2*(2*I + Sqrt[2])*ArcTanh[((2 + x)*(3*(5*I + 4*Sqrt[2])) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*Sqrt[2])*x^2)]/(-5*(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*Sqrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 - (2*I)*Sqrt[2]] - ((-2*I + Sqrt[2])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] - ((2*I + Sqrt[2])*Log[4*(3 + 4*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] + ((2*I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 - (2*I)*Sqrt[2]] + ((-2*I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 + (2*I)*Sqrt[2]])/16

IntegrateAlgebraic [A] time = 0.37, size = 95, normalized size = 1.09

$$\frac{x + 3}{2(2x^2 + 4x + 3)} + \frac{(2x + 3)\sqrt{-x^2 - 4x - 3}}{2(2x^2 + 4x + 3)} + \frac{\tan^{-1} \left(\frac{\sqrt{2x+2}}{\sqrt{-x^2-4x-3+x+1}} \right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-2), x]

[Out] (3 + x)/(2*(3 + 4*x + 2*x^2)) + ((3 + 2*x)*Sqrt[-3 - 4*x - x^2])/(2*(3 + 4*x + 2*x^2)) + ArcTan[(Sqrt[2] + Sqrt[2]*x)/(1 + x + Sqrt[-3 - 4*x - x^2])]/Sqrt[2]

fricas [A] time = 0.70, size = 121, normalized size = 1.39

$$\frac{2\sqrt{2}(2x^2 + 4x + 3)\arctan(\sqrt{2}(x + 1)) - \sqrt{2}(2x^2 + 4x + 3)\arctan\left(\frac{\sqrt{2}(6x^2 + 20x + 15)\sqrt{-x^2 - 4x - 3}}{4(2x^3 + 11x^2 + 18x + 9)}\right) + 4\sqrt{-x^2 - 4x - 3}(2x + 3) + 4x + 12}{8(2x^2 + 4x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="fricas")

[Out] 1/8*(2*sqrt(2)*(2*x^2 + 4*x + 3)*arctan(sqrt(2)*(x + 1)) - sqrt(2)*(2*x^2 + 4*x + 3)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9)) + 4*sqrt(-x^2 - 4*x - 3)*(2*x + 3) + 4*x + 12)/(2*x^2 + 4*x + 3)

giac [B] time = 0.46, size = 263, normalized size = 3.02

$$\frac{1}{4}\sqrt{2}\arctan(\sqrt{2}(x+1)) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2}\right)+1\right) - \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}\right)+1\right) + \frac{x+3}{2(2x^2+4x+3)} - \frac{\frac{10(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{7(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} - \frac{2(\sqrt{-x^2-4x-3}-1)^3}{(x+2)^3} + 3}{3\left(\frac{8(\sqrt{-x^2-4x-3}-1)}{x+2} + \frac{14(\sqrt{-x^2-4x-3}-1)^2}{(x+2)^2} + \frac{8(\sqrt{-x^2-4x-3}-1)^3}{(x+2)^3} + \frac{3(\sqrt{-x^2-4x-3}-1)^4}{(x+2)^4} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="giac")

[Out] 1/4*sqrt(2)*arctan(sqrt(2)*(x + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*(x + 3)/(2*x^2 + 4*x + 3) - 1/3*(10*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 7*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 - 2*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)

maple [B] time = 0.09, size = 2407, normalized size = 27.67

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+(-x^2-4*x-3)^(1/2))^2,x)

[Out] -3/8*(4*x+4)/(2*x^2+4*x+3)+1/4*2^(1/2)*arctan(1/4*(4*x+4)*2^(1/2))-1/2*(-4*x-6)/(2*x^2+4*x+3)+1/72*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^2/(-x-3/2)^2-8*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))*x^2/(-x-3/2)^2+2*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))-6*(3/(-x-3/2)^2*x^2-12)^(1/2)-16*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)+1/36*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(7*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))/((1/(-x-3/2)*x+1)/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^(1/2)-2/9*3^(1/2)*4^(1/2)*(3/(-x-3/2)^2*x^2-12)^(1/2)*(3*2^(1/2)*arctan(1/6*(3/(-x-3/2)^2*x^2-12)^(1/2)*2^(1/2))*x^6/(-x-3/2)^6+2*ln((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6-2*ln((3/(-x-3/2)^2*x^2-12)^(1/2)*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6+4*arctanh(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^(1/2)*x))*x^6/(-x-3/2)^6+(3/(-x-3/2)^2*x^2-12)^(1/2)*x^5/(-x-3/2)^5-(3/(-x-3/2)^2*x^2-12)^(3/2)*

$$\begin{aligned}
& x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x^4/(-x-3/2)^4-36*2^{(1/2)}*\arctan \\
& (1/6*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})*x^2/(-x-3/2)^2-2*(3/(-x-3/2)^2*x^2 \\
& -12)^{(1/2)}*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x^2/(-x-3/2)^2-24* \\
& \ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^ \\
& 2*x^2-4))*x^2/(-x-3/2)^2+24*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)+1/(- \\
& x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2-48*\operatorname{arctanh}(3/(-x-3/2)/ \\
& (3/(-x-3/2)^2*x^2-12)^{(1/2)}*x)*x^2/(-x-3/2)^2-48*2^{(1/2)}*\arctan(1/6*(3/(-x- \\
& 3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})-8*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)+16*(\\
& 3/(-x-3/2)^2*x^2-12)^{(1/2)}-32*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/ \\
& (-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+32*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}* \\
& x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))-64*\operatorname{arctanh}(3/(-x-3/2)/ \\
& (3/(-x-3/2)^2*x^2-12)^{(1/2)}*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^{(1/2)} \\
& /((1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+2)/((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(\\
& -x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)+1/(-x-3 \\
& /2)^2*x^2-4)-2/9*3^{(1/2)}*4^{(1/2)}*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*(2^{(1/2)}*\arctan \\
& (1/6*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})+\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2 \\
& *x^2-12)^{(1/2)}*x))/((1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^{(1/2)}/(1/(-x-3 \\
& /2)*x+1)+1/18*3^{(1/2)}*4^{(1/2)}*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*(11*2^{(1/2)}*\arctan \\
& (1/6*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})*x^6/(-x-3/2)^6+8*\ln(((3/(-x-3/2) \\
& ^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^6/(\\
& -x-3/2)^6-8*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/ \\
& (1/(-x-3/2)^2*x^2-4))*x^6/(-x-3/2)^6+24*\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2*x^2 \\
& -12)^{(1/2)}*x)*x^6/(-x-3/2)^6+4*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x^5/(-x-3/2)^5- \\
& (3/(-x-3/2)^2*x^2-12)^{(3/2)}*x^2/(-x-3/2)^2+(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x^4/ \\
& (-x-3/2)^4-132*2^{(1/2)}*\arctan(1/6*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})*x^2/ \\
& (-x-3/2)^2-8*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x^3/(-x-3/2)^3-8*(3/(-x-3/2)^2*x^2 \\
& -12)^{(1/2)}*x^2/(-x-3/2)^2-96*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/ \\
& (-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x-3/2)^2+96*\ln(((3/(-x-3/2)^2* \\
& x^2-12)^{(1/2)}*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2)^2*x^2-4))*x^2/(-x- \\
& 3/2)^2-288*\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x)*x^2/(-x-3/2)^2 \\
& -176*2^{(1/2)}*\arctan(1/6*(3/(-x-3/2)^2*x^2-12)^{(1/2)}*2^{(1/2)})-32*(3/(-x-3/2) \\
& ^2*x^2-12)^{(1/2)}*x/(-x-3/2)+16*(3/(-x-3/2)^2*x^2-12)^{(1/2)}-128*\ln(((3/(-x-3 \\
& /2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/(1/(-x-3/2)^2*x^2-4))+12 \\
& 8*\ln(((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)/(1/(-x-3/2) \\
&)^2*x^2-4))-384*\operatorname{arctanh}(3/(-x-3/2)/(3/(-x-3/2)^2*x^2-12)^{(1/2)}*x))/((1/(-x- \\
& 3/2)^2*x^2-4)/(1/(-x-3/2)*x+1)^2)^{(1/2)}/(1/(-x-3/2)*x+1)/(1/(-x-3/2)^2*x^2+ \\
& 2)/((3/(-x-3/2)^2*x^2-12)^{(1/2)}*x/(-x-3/2)-1/(-x-3/2)^2*x^2+4)/((3/(-x-3/2) \\
& ^2*x^2-12)^{(1/2)}*x/(-x-3/2)+1/(-x-3/2)^2*x^2-4)
\end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 4x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^2,x, algorithm="maxima")

[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x + \sqrt{-x^2 - 4x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2,x)

[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^2, x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 4x - 3})^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**2,x)

[Out] Integral((x + sqrt(-x**2 - 4*x - 3))**(-2), x)

$$3.505 \quad \int \frac{1}{\left(x + \sqrt{-3-4x-x^2}\right)^3} dx$$

Optimal. Leaf size=149

$$-\frac{13 - \frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1\right)} - \frac{2\left(2 - \frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1\right)^2} - \frac{3 \tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.10, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {12, 1660, 638, 618, 204}

$$-\frac{13 - \frac{27\sqrt{-x-1}}{\sqrt{x+3}}}{18\left(-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1\right)} - \frac{2\left(2 - \frac{\sqrt{-x-1}}{\sqrt{x+3}}\right)}{9\left(-\frac{3(x+1)}{x+3} - \frac{2\sqrt{-x-1}}{\sqrt{x+3}} + 1\right)^2} - \frac{3 \tan^{-1}\left(\frac{1 - \frac{3\sqrt{-x-1}}{\sqrt{x+3}}}{\sqrt{2}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[-3 - 4*x - x^2])^(-3), x]

[Out] -(13 - (27*Sqrt[-1 - x])/Sqrt[3 + x])/(18*(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])) - (2*(2 - Sqrt[-1 - x])/Sqrt[3 + x])/(9*(1 - (3*(1 + x))/(3 + x) - (2*Sqrt[-1 - x])/Sqrt[3 + x])^2) - (3*ArcTan[(1 - (3*Sqrt[-1 - x])/Sqrt[3 + x])/Sqrt[2]])/(2*Sqrt[2])

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 638

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[((2*p + 3)*(2*c*d - b*e))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]

Rule 1660

Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x + c*x^2, x], x, 0]},

```
c*x^2, x], x, 1]}, Simp[((b*f - 2*a*g + (2*c*f - b*g)*x)*(a + b*x + c*x^2)^(
(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
- 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{(x + \sqrt{-3 - 4x - x^2})^3} dx = 2 \operatorname{Subst} \left(\int \frac{2x(1 + x^2)}{(1 - 2x + 3x^2)^3} dx, x, \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)$$

$$= 4 \operatorname{Subst} \left(\int \frac{x(1 + x^2)}{(1 - 2x + 3x^2)^3} dx, x, \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)$$

$$= -\frac{2 \left(2 - \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)}{9 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)^2} + \frac{1}{4} \operatorname{Subst} \left(\int \frac{\frac{56}{9} + \frac{16x}{3}}{(1 - 2x + 3x^2)^2} dx, x, \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)$$

$$= -\frac{13 - \frac{27\sqrt{-1 - x}}{\sqrt{3 + x}}}{18 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)} - \frac{2 \left(2 - \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)}{9 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)^2} + \frac{3}{2} \operatorname{Subst} \left(\int \frac{1}{1 - 2x + 3x^2} dx, x, \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)$$

$$= -\frac{13 - \frac{27\sqrt{-1 - x}}{\sqrt{3 + x}}}{18 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)} - \frac{2 \left(2 - \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)}{9 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)^2} - 3 \operatorname{Subst} \left(\int \frac{1}{-8 - x^2} dx, x, \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)$$

$$= -\frac{13 - \frac{27\sqrt{-1 - x}}{\sqrt{3 + x}}}{18 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)} - \frac{2 \left(2 - \frac{\sqrt{-1 - x}}{\sqrt{3 + x}} \right)}{9 \left(1 - \frac{3(1 + x)}{3 + x} - \frac{2\sqrt{-1 - x}}{\sqrt{3 + x}} \right)^2} - \frac{3 \tan^{-1} \left(\frac{1 - \frac{3\sqrt{-1 - x}}{\sqrt{3 + x}}}{\sqrt{2}} \right)}{2\sqrt{2}}$$

Mathematica [C] time = 2.46, size = 914, normalized size = 6.13



Antiderivative was successfully verified.

```
[In] Integrate[(x + Sqrt[-3 - 4*x - x^2])^(-3), x]
```

```
[Out] ((8*(-3 + 2*x))/(3 + 4*x + 2*x^2)^2 - (8*(2 + 3*x))/(3 + 4*x + 2*x^2) - (8*
Sqrt[-3 - 4*x - x^2]*(15 + 26*x + 22*x^2 + 8*x^3))/(3 + 4*x + 2*x^2)^2 - 12
*Sqrt[2]*ArcTan[Sqrt[2]*(1 + x)] + (6*(2 + I*Sqrt[2])*ArcTan[((2 + x)*(3*(5
+ (4*I)*Sqrt[2]) + 16*(2 + I*Sqrt[2])*x + 2*(9 + (2*I)*Sqrt[2])*x^2)))/(12*
I - 6*Sqrt[2] + (8*I + 6*Sqrt[2])*x^3 - 9*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 -
4*x - x^2] + x*(40*I - 5*Sqrt[2] - 12*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*
x - x^2]) + x^2*(36*I + 8*Sqrt[2] - 6*Sqrt[1 + (2*I)*Sqrt[2]]*Sqrt[-3 - 4*x
- x^2])))/Sqrt[1 + (2*I)*Sqrt[2]] - (6*(2*I + Sqrt[2])*ArcTanh[((2 + x)*(
3*(5*I + 4*Sqrt[2]) + 16*(2*I + Sqrt[2])*x + 2*(9*I + 2*Sqrt[2])*x^2)))/(-5*
(8*I + Sqrt[2])*x + (-8*I + 6*Sqrt[2])*x^3 - 12*Sqrt[1 - (2*I)*Sqrt[2]]*x*S
qrt[-3 - 4*x - x^2] + x^2*(-36*I + 8*Sqrt[2] - 6*Sqrt[1 - (2*I)*Sqrt[2]]*Sq
rt[-3 - 4*x - x^2]) - 3*(4*I + 2*Sqrt[2] + 3*Sqrt[1 - (2*I)*Sqrt[2]]*Sqrt[-
3 - 4*x - x^2])))/Sqrt[1 - (2*I)*Sqrt[2]] + (3*(-2*I + Sqrt[2])*Log[4*(3 +
4*x + 2*x^2)^2])/Sqrt[1 + (2*I)*Sqrt[2]] + (3*(2*I + Sqrt[2])*Log[4*(3 + 4
*x + 2*x^2)^2])/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(2*I + Sqrt[2])*Log[(3 + 4*x +
2*x^2)*(3 + (6*I)*Sqrt[2] + (2 + (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 - (4*I)*Sqr
```



```
t[2]]*Sqrt[-3 - 4*x - x^2] + x*(4 + (8*I)*Sqrt[2] - 2*Sqrt[2 - (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 - (2*I)*Sqrt[2]] - (3*(-2*I + Sqrt[2])*Log[(3 + 4*x + 2*x^2)*(3 - (6*I)*Sqrt[2] + (2 - (2*I)*Sqrt[2])*x^2 - 2*Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2] - 2*x*(-2 + (4*I)*Sqrt[2] + Sqrt[2 + (4*I)*Sqrt[2]]*Sqrt[-3 - 4*x - x^2])))/Sqrt[1 + (2*I)*Sqrt[2]])/32
```

IntegrateAlgebraic [A] time = 0.45, size = 120, normalized size = 0.81

$$-\frac{3 \tan^{-1}\left(\frac{\sqrt{2}x + \sqrt{2}}{\sqrt{-x^2 - 4x - 3} + x + 1}\right)}{2\sqrt{2}} + \frac{\sqrt{-x^2 - 4x - 3}(-8x^3 - 22x^2 - 26x - 15)}{4(2x^2 + 4x + 3)^2} + \frac{-6x^3 - 16x^2 - 15x - 9}{4(2x^2 + 4x + 3)^2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x + Sqrt[-3 - 4*x - x^2])^(-3), x]
[Out] (Sqrt[-3 - 4*x - x^2]*(-15 - 26*x - 22*x^2 - 8*x^3))/(4*(3 + 4*x + 2*x^2)^2) + (-9 - 15*x - 16*x^2 - 6*x^3)/(4*(3 + 4*x + 2*x^2)^2) - (3*ArcTan[(Sqrt[2] + Sqrt[2]*x)/(1 + x + Sqrt[-3 - 4*x - x^2])])/(2*Sqrt[2])
```

fricas [A] time = 0.71, size = 171, normalized size = 1.15

$$\frac{24x^3 + 6\sqrt{2}(4x^4 + 16x^3 + 28x^2 + 24x + 9)\arctan(\sqrt{2}(x+1)) - 3\sqrt{2}(4x^4 + 16x^3 + 28x^2 + 24x + 9)\arctan\left(\frac{\sqrt{2}(6x^2+20x+15)\sqrt{-x^2-4x-3}}{4(2x^2+11x^2+18x+9)}\right) + 64x^2 + 4(8x^3 + 22x^2 + 26x + 15)\sqrt{-x^2 - 4x - 3} + 60x + 36}{16(4x^4 + 16x^3 + 28x^2 + 24x + 9)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="fricas")
[Out] -1/16*(24*x^3 + 6*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(sqrt(2)*(x + 1)) - 3*sqrt(2)*(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2*x^3 + 11*x^2 + 18*x + 9)) + 64*x^2 + 4*(8*x^3 + 22*x^2 + 26*x + 15)*sqrt(-x^2 - 4*x - 3) + 60*x + 36)/(4*x^4 + 16*x^3 + 28*x^2 + 24*x + 9)
```

giac [B] time = 0.45, size = 367, normalized size = 2.46

$$\frac{-\frac{3}{8}\sqrt{2}\arctan(\sqrt{2}(x+1)) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3(\sqrt{-x^2-4x-3}-1)}{x+2} + 1\right)\right) + \frac{3}{8}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2} + 1\right)\right)}{4(2x^2+4x+3)^2} - \frac{618(\sqrt{-x^2-4x-3})}{1+2} + \frac{1547(\sqrt{-x^2-4x-3})^2}{(1+2)^2} + \frac{2362(\sqrt{-x^2-4x-3})^3}{(1+2)^3} + \frac{2223(\sqrt{-x^2-4x-3})^4}{(1+2)^4} + \frac{1174(\sqrt{-x^2-4x-3})^5}{(1+2)^5} + \frac{377(\sqrt{-x^2-4x-3})^6}{(1+2)^6} + \frac{6(\sqrt{-x^2-4x-3})^7}{(1+2)^7} + 117}{18\left(\frac{6(\sqrt{-x^2-4x-3})}{1+2} + \frac{14(\sqrt{-x^2-4x-3})^2}{(1+2)^2} + \frac{8(\sqrt{-x^2-4x-3})^3}{(1+2)^3} + \frac{3(\sqrt{-x^2-4x-3})^4}{(1+2)^4} + 3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="giac")
[Out] -3/8*sqrt(2)*arctan(sqrt(2)*(x + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) - 1/4*(6*x^3 + 16*x^2 + 15*x + 9)/(2*x^2 + 4*x + 3)^2 + 1/18*(618*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1547*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 2362*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 2223*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 1174*(sqrt(-x^2 - 4*x - 3) - 1)^5/(x + 2)^5 + 377*(sqrt(-x^2 - 4*x - 3) - 1)^6/(x + 2)^6 + 6*(sqrt(-x^2 - 4*x - 3) - 1)^7/(x + 2)^7 + 117)/(8*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 14*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 8*(sqrt(-x^2 - 4*x - 3) - 1)^3/(x + 2)^3 + 3*(sqrt(-x^2 - 4*x - 3) - 1)^4/(x + 2)^4 + 3)^2
```

maple [B] time = 0.28, size = 14529, normalized size = 97.51

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x+(-x^2-4*x-3)^(1/2))^3,x)
```

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{-x^2 - 4x - 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x^2-4*x-3)^(1/2))^3,x, algorithm="maxima")

[Out] integrate((x + sqrt(-x^2 - 4*x - 3))^(-3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x + \sqrt{-x^2 - 4x - 3})^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3,x)

[Out] int(1/(x + (- 4*x - x^2 - 3)^(1/2))^3, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x+(-x**2-4*x-3)**(1/2))**3,x)

[Out] Timed out

$$3.506 \quad \int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4} dx$$

Optimal. Leaf size=42

$$-\frac{1}{15}(-x^4-2x^3-x^2+1)^{3/2}(3x^4+6x^3+3x^2+2)$$

Rubi [A] time = 0.22, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1680, 12, 1247, 692, 629}

$$-\frac{1}{5}x^2(-x^4-2x^3-x^2+1)^{3/2}(x+1)^2 - \frac{2}{15}(-x^4-2x^3-x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^3*(1+x)^3*(1+2*x)*Sqrt[1-x^2-2*x^3-x^4],x]

[Out] (-2*(1-x^2-2*x^3-x^4)^(3/2))/15 - (x^2*(1+x)^2*(1-x^2-2*x^3-x^4)^(3/2))/5

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 692

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x, d/(4*e) + x] /; EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
\int x^3(1+x)^3(1+2x)\sqrt{1-x^2-2x^3-x^4} dx &= \text{Subst}\left(\int \frac{1}{128}x(-1+4x^2)^3\sqrt{15+8x^2-16x^4} dx, x, \frac{1}{2}+x\right) \\
&= \frac{1}{128}\text{Subst}\left(\int x(-1+4x^2)^3\sqrt{15+8x^2-16x^4} dx, x, \frac{1}{2}+x\right) \\
&= \frac{1}{256}\text{Subst}\left(\int (-1+4x)^3\sqrt{15+8x-16x^2} dx, x, \left(\frac{1}{2}+x\right)^2\right) \\
&= -\frac{1}{5}x^2(1+x)^2(1-x^2-2x^3-x^4)^{3/2} + \frac{1}{40}\text{Subst}\left(\int (-1+4x)\sqrt{15+8x-16x^2} dx, x, \left(\frac{1}{2}+x\right)^2\right) \\
&= -\frac{2}{15}(1-x^2-2x^3-x^4)^{3/2} - \frac{1}{5}x^2(1+x)^2(1-x^2-2x^3-x^4)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)$$

Antiderivative was successfully verified.

[In] Integrate[x^3*(1+x)^3*(1+2*x)*Sqrt[1-x^2-2*x^3-x^4],x]

[Out] (Sqrt[1-x^2-2*x^3-x^4]*(-2-x^2-2*x^3+2*x^4+12*x^5+18*x^6+12*x^7+3*x^8))/15

IntegrateAlgebraic [A] time = 0.08, size = 42, normalized size = 1.00

$$\frac{1}{15}(-3x^4-6x^3-3x^2-2)(-x^4-2x^3-x^2+1)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^3*(1+x)^3*(1+2*x)*Sqrt[1-x^2-2*x^3-x^4],x]

[Out] ((-2-3*x^2-6*x^3-3*x^4)*(1-x^2-2*x^3-x^4)^(3/2))/15

fricas [A] time = 0.68, size = 58, normalized size = 1.38

$$\frac{1}{15}(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*x^8+12*x^7+18*x^6+12*x^5+2*x^4-2*x^3-x^2-2)*sqrt(-x^4-2*x^3-x^2+1)

giac [A] time = 0.33, size = 58, normalized size = 1.38

$$\frac{1}{5}(x^4+2x^3+x^2-1)^2\sqrt{-x^4-2x^3-x^2+1}-\frac{1}{3}(-x^4-2x^3-x^2+1)^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2),x, algorithm="giac")

[Out] 1/5*(x^4+2*x^3+x^2-1)^2*sqrt(-x^4-2*x^3-x^2+1)-1/3*(-x^4-2*x^3-x^2+1)^(3/2)

maple [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{(x^2 + x + 1)(x^2 + x - 1)(3x^4 + 6x^3 + 3x^2 + 2)\sqrt{-x^4 - 2x^3 - x^2 + 1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(x+1)^3*(2*x+1)*(-x^4-2*x^3-x^2+1)^(1/2), x)

[Out] 1/15*(x^2+x+1)*(x^2+x-1)*(3*x^4+6*x^3+3*x^2+2)*(-x^4-2*x^3-x^2+1)^(1/2)

maxima [A] time = 0.83, size = 59, normalized size = 1.40

$$\frac{1}{15} (3x^8 + 12x^7 + 18x^6 + 12x^5 + 2x^4 - 2x^3 - x^2 - 2)\sqrt{x^2 + x + 1}\sqrt{-x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3*(1+x)^3*(1+2*x)*(-x^4-2*x^3-x^2+1)^(1/2), x, algorithm="maxima")

[Out] 1/15*(3*x^8 + 12*x^7 + 18*x^6 + 12*x^5 + 2*x^4 - 2*x^3 - x^2 - 2)*sqrt(x^2 + x + 1)*sqrt(-x^2 - x + 1)

mupad [B] time = 0.18, size = 38, normalized size = 0.90

$$\frac{(3x^4 + 6x^3 + 3x^2 + 2)(-x^4 - 2x^3 - x^2 + 1)^{3/2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3*(2*x + 1)*(x + 1)^3*(1 - 2*x^3 - x^4 - x^2)^(1/2), x)

[Out] -((3*x^2 + 6*x^3 + 3*x^4 + 2)*(1 - 2*x^3 - x^4 - x^2)^(3/2))/15

sympy [B] time = 0.70, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**3*(1+x)**3*(1+2*x)*(-x**4-2*x**3-x**2+1)**(1/2), x)

[Out] x**8*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**7*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 6*x**6*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 4*x**5*sqrt(-x**4 - 2*x**3 - x**2 + 1)/5 + 2*x**4*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*x**3*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - x**2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15 - 2*sqrt(-x**4 - 2*x**3 - x**2 + 1)/15

$$3.507 \quad \int (1 + 2x) (x + x^2)^3 \sqrt{1 - (x + x^2)^2} dx$$

Optimal. Leaf size=42

$$-\frac{1}{15} (-x^4 - 2x^3 - x^2 + 1)^{3/2} (3x^4 + 6x^3 + 3x^2 + 2)$$

Rubi [A] time = 0.24, antiderivative size = 59, normalized size of antiderivative = 1.40, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1593, 1680, 12, 1247, 692, 629}

$$-\frac{1}{5} x^2 (-x^4 - 2x^3 - x^2 + 1)^{3/2} (x + 1)^2 - \frac{2}{15} (-x^4 - 2x^3 - x^2 + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)*(x + x^2)^3*Sqrt[1 - (x + x^2)^2], x]

[Out] (-2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/15 - (x^2*(1 + x)^2*(1 - x^2 - 2*x^3 - x^4)^(3/2))/5

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 692

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(2*d*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(b*(m + 2*p + 1)), x] + Dist[(d^2*(m - 1)*(b^2 - 4*a*c))/(b^2*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && (IntegerQ[2*p] || (IntegerQ[m] && RationalQ[p]) || OddQ[m])

Rule 1247

Int[(x_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]

Rule 1593

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1680

Int[(Pq_)*(Q4_)^(p_), x_Symbol] := With[{a = Coeff[Q4, x, 0], b = Coeff[Q4, x, 1], c = Coeff[Q4, x, 2], d = Coeff[Q4, x, 3], e = Coeff[Q4, x, 4]}, Subst[Int[SimplifyIntegrand[(Pq /. x -> -(d/(4*e)) + x)*(a + d^4/(256*e^3) - (b*d)/(8*e) + (c - (3*d^2)/(8*e))*x^2 + e*x^4)^p, x], x], x, d/(4*e) + x] /;

EqQ[d^3 - 4*c*d*e + 8*b*e^2, 0] && NeQ[d, 0]] /; FreeQ[p, x] && PolyQ[Pq, x] && PolyQ[Q4, x, 4] && !IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \int (1+2x)(x+x^2)^3 \sqrt{1-(x+x^2)^2} dx &= \int x^3(1+x)^3(1+2x)\sqrt{1-(x+x^2)^2} dx \\
 &= \text{Subst}\left(\int \frac{1}{128}x(-1+4x^2)^3 \sqrt{15+8x^2-16x^4} dx, x, \frac{1}{2}+x\right) \\
 &= \frac{1}{128} \text{Subst}\left(\int x(-1+4x^2)^3 \sqrt{15+8x^2-16x^4} dx, x, \frac{1}{2}+x\right) \\
 &= \frac{1}{256} \text{Subst}\left(\int (-1+4x)^3 \sqrt{15+8x-16x^2} dx, x, \left(\frac{1}{2}+x\right)^2\right) \\
 &= -\frac{1}{5}x^2(1+x)^2(1-x^2-2x^3-x^4)^{3/2} + \frac{1}{40} \text{Subst}\left(\int (-1+4x)\sqrt{15+8x-16x^2} dx, x, \left(\frac{1}{2}+x\right)^2\right) \\
 &= -\frac{2}{15}(1-x^2-2x^3-x^4)^{3/2} - \frac{1}{5}x^2(1+x)^2(1-x^2-2x^3-x^4)^{3/2}
 \end{aligned}$$

Mathematica [A] time = 0.37, size = 62, normalized size = 1.48

$$\frac{1}{15}\sqrt{-x^4-2x^3-x^2+1}(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2)$$

Antiderivative was successfully verified.

[In] Integrate[(1+2*x)*(x+x^2)^3*Sqrt[1-(x+x^2)^2],x]

[Out] (Sqrt[1-x^2-2*x^3-x^4]*(-2-x^2-2*x^3+2*x^4+12*x^5+18*x^6+12*x^7+3*x^8))/15

IntegrateAlgebraic [A] time = 0.04, size = 30, normalized size = 0.71

$$\frac{1}{15}\left(-3(x^2+x)^2-2\right)\left(1-(x^2+x)^2\right)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1+2*x)*(x+x^2)^3*Sqrt[1-(x+x^2)^2],x]

[Out] ((-2-3*(x+x^2)^2)*(1-(x+x^2)^2)^(3/2))/15

fricas [A] time = 0.64, size = 58, normalized size = 1.38

$$\frac{1}{15}\left(3x^8+12x^7+18x^6+12x^5+2x^4-2x^3-x^2-2\right)\sqrt{-x^4-2x^3-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*x^8+12*x^7+18*x^6+12*x^5+2*x^4-2*x^3-x^2-2)*sqrt(-x^4-2*x^3-x^2+1)

giac [A] time = 0.39, size = 58, normalized size = 1.38

$$\frac{1}{5}\left(x^4+2x^3+x^2-1\right)^2\sqrt{-x^4-2x^3-x^2+1}-\frac{1}{3}\left(-x^4-2x^3-x^2+1\right)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="giac")

[Out] $\frac{1}{5}(x^4 + 2x^3 + x^2 - 1)^2 \sqrt{-x^4 - 2x^3 - x^2 + 1} - \frac{1}{3}(-x^4 - 2x^3 - x^2 + 1)^{3/2}$

maple [A] time = 0.01, size = 51, normalized size = 1.21

$$\frac{(x^2 + x + 1)(x^2 + x - 1)(3x^4 + 6x^3 + 3x^2 + 2)\sqrt{-x^4 - 2x^3 - x^2 + 1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x)

[Out] $\frac{1}{15}(x^2+x+1)(x^2+x-1)(3x^4+6x^3+3x^2+2)(-x^4-2x^3-x^2+1)^{1/2}$

maxima [A] time = 1.04, size = 59, normalized size = 1.40

$$\frac{1}{15}(3x^8 + 12x^7 + 18x^6 + 12x^5 + 2x^4 - 2x^3 - x^2 - 2)\sqrt{x^2 + x + 1}\sqrt{-x^2 - x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x^2+x)^3*(1-(x^2+x)^2)^(1/2),x, algorithm="maxima")

[Out] $\frac{1}{15}(3x^8 + 12x^7 + 18x^6 + 12x^5 + 2x^4 - 2x^3 - x^2 - 2)\sqrt{x^2 + x + 1}\sqrt{-x^2 - x + 1}$

mupad [B] time = 3.42, size = 51, normalized size = 1.21

$$\sqrt{1 - (x^2 + x)^2} \left(\frac{x^8}{5} + \frac{4x^7}{5} + \frac{6x^6}{5} + \frac{4x^5}{5} + \frac{2x^4}{15} - \frac{2x^3}{15} - \frac{x^2}{15} - \frac{2}{15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)*(1 - (x + x^2)^2)^(1/2)*(x + x^2)^3,x)

[Out] $(1 - (x + x^2)^2)^{1/2} \left(\frac{(2x^4)}{15} - \frac{(2x^3)}{15} - \frac{x^2}{15} + \frac{(4x^5)}{5} + \frac{(6x^6)}{5} + \frac{(4x^7)}{5} + \frac{x^8}{5} - \frac{2}{15} \right)$

sympy [B] time = 10.18, size = 182, normalized size = 4.33

$$\frac{x^8\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^7\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{6x^6\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{4x^5\sqrt{-x^4-2x^3-x^2+1}}{5} + \frac{2x^4\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2x^3\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{x^2\sqrt{-x^4-2x^3-x^2+1}}{15} - \frac{2\sqrt{-x^4-2x^3-x^2+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)*(x**2+x)**3*(1-(x**2+x)**2)**(1/2),x)

[Out] $x^{**8}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/5 + 4x^{**7}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/5 + 6x^{**6}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/5 + 4x^{**5}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/5 + 2x^{**4}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/15 - 2x^{**3}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/15 - x^{**2}\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/15 - 2\sqrt{-x^{**4} - 2x^{**3} - x^{**2} + 1}/15$

$$3.508 \quad \int \frac{\left(2\sqrt{3-x} + \frac{3}{\sqrt{1+x}}\right)^2}{x} dx$$

Optimal. Leaf size=56

$$-4x + 21 \log(x) - 9 \log(x+1) + 12 \sin^{-1}\left(\frac{1-x}{2}\right) - 24\sqrt{3} \tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Rubi [A] time = 0.21, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.370$, Rules used = {6742, 36, 29, 31, 105, 53, 619, 216, 93, 207}

$$-4x + 21 \log(x) - 9 \log(x+1) + 12 \sin^{-1}\left(\frac{1-x}{2}\right) - 24\sqrt{3} \tanh^{-1}\left(\frac{\sqrt{3}\sqrt{x+1}}{\sqrt{3-x}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x, x]

[Out] -4*x + 12*ArcSin[(1 - x)/2] - 24*Sqrt[3]*ArcTanh[(Sqrt[3]*Sqrt[1 + x])/Sqrt[3 - x]] + 21*Log[x] - 9*Log[1 + x]

Rule 29

Int[(x_)^(-1), x_Symbol] := Simp[Log[x], x]

Rule 31

Int[((a_) + (b_)*(x_))^(-1), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 36

Int[1/(((a_) + (b_)*(x_))*((c_) + (d_)*(x_))), x_Symbol] := Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 53

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Int[1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]

Rule 93

Int[(((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_))/((e_) + (f_)*(x_)), x_Symbol] := With[{q = Denominator[m]}, Dist[q, Subst[Int[x^(q*(m + 1) - 1)/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && LtQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]

Rule 105

Int[(((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_))/((e_) + (f_)*(x_)), x_Symbol] := Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

Rule 207

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Rt}[b, 2]*x]/\text{Rt}[-a, 2]]/\text{Rt}[-a, 2]*\text{Rt}[b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{LtQ}[a, 0] \ || \ \text{GtQ}[b, 0])$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_ + (b_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2]*x]/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}\{a, b\}, x \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rule 619

$\text{Int}[(a_ \cdot) + (b_ \cdot)(x_) + (c_ \cdot)(x_)^2]^{(p_)}, x_Symbol] \rightarrow \text{Dist}[1/(2*c*((-4*c)/(b^2 - 4*a*c))^{p}), \text{Subst}[\text{Int}[\text{Simp}[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; \text{FreeQ}\{a, b, c, p\}, x \ \&\& \ \text{GtQ}[4*a - b^2/c, 0]$

Rule 6742

$\text{Int}[u_, x_Symbol] \rightarrow \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$

Rubi steps

$$\begin{aligned} \int \frac{\left(2\sqrt{3-x} + \frac{3}{\sqrt{1+x}}\right)^2}{x} dx &= \int \left(-4 + \frac{12}{x} + \frac{9}{x(1+x)} + \frac{12\sqrt{3-x}}{x\sqrt{1+x}}\right) dx \\ &= -4x + 12 \log(x) + 9 \int \frac{1}{x(1+x)} dx + 12 \int \frac{\sqrt{3-x}}{x\sqrt{1+x}} dx \\ &= -4x + 12 \log(x) + 9 \int \frac{1}{x} dx - 9 \int \frac{1}{1+x} dx - 12 \int \frac{1}{\sqrt{3-x}\sqrt{1+x}} dx + 36 \int \frac{1}{\sqrt{3-x}} dx \\ &= -4x + 21 \log(x) - 9 \log(1+x) - 12 \int \frac{1}{\sqrt{3+2x-x^2}} dx + 72 \text{Subst} \left(\int \frac{1}{-1+3x^2} dx, \sqrt{3-x} \right) \\ &= -4x - 24\sqrt{3} \tanh^{-1} \left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}} \right) + 21 \log(x) - 9 \log(1+x) + 3 \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} dx, \sqrt{3-x} \right) \\ &= -4x + 12 \sin^{-1} \left(\frac{1-x}{2} \right) - 24\sqrt{3} \tanh^{-1} \left(\frac{\sqrt{3}\sqrt{1+x}}{\sqrt{3-x}} \right) + 21 \log(x) - 9 \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.11, size = 57, normalized size = 1.02

$$-4x + 21 \log(x) - 9 \log(x+1) + 24 \sin^{-1} \left(\frac{\sqrt{3-x}}{2} \right) - 24\sqrt{3} \tanh^{-1} \left(\frac{\sqrt{1-\frac{x}{3}}}{\sqrt{x+1}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x, x]

[Out] -4*x + 24*ArcSin[Sqrt[3 - x]/2] - 24*Sqrt[3]*ArcTanh[Sqrt[1 - x/3]/Sqrt[1 + x]] + 21*Log[x] - 9*Log[1 + x]

IntegrateAlgebraic [C] time = 0.58, size = 278, normalized size = 4.96

$-4(x+1) - 18 \log(\sqrt{3-x} - \sqrt{x+1} - 2) - (24+24) \log(\sqrt{3-x} - \sqrt{x+1}) - 18 \log(\sqrt{3-x} - \sqrt{x+1} + 2) + 3(7+4\sqrt{5}) \log(\sqrt{3-x} - \sqrt{x+1} + \sqrt{2-2\sqrt{5}}) - 3(4\sqrt{5}-7) \log(\sqrt{3-x} - \sqrt{x+1} + \sqrt{2+2\sqrt{5}}) + 3(7+4\sqrt{5}) \log(-\sqrt{3-x} + \sqrt{x+1} + \sqrt{2-2\sqrt{5}}) - 3(4\sqrt{5}-7) \log(-\sqrt{3-x} + \sqrt{x+1} + \sqrt{2+2\sqrt{5}})$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2*Sqrt[3 - x] + 3/Sqrt[1 + x])^2/x,x]

[Out] $-4*(1 + x) - 18*\text{Log}[-2 + \text{Sqrt}[3 - x] - I*\text{Sqrt}[1 + x]] - (24 + 24*I)*\text{Log}[\text{Sqrt}[3 - x] - I*\text{Sqrt}[1 + x]] - 18*\text{Log}[2 + \text{Sqrt}[3 - x] - I*\text{Sqrt}[1 + x]] + 3*(7 + 4*\text{Sqrt}[3])* \text{Log}[\text{Sqrt}[2 - (2*I)*\text{Sqrt}[3]] + \text{Sqrt}[3 - x] - I*\text{Sqrt}[1 + x]] - 3*(-7 + 4*\text{Sqrt}[3])* \text{Log}[\text{Sqrt}[2 + (2*I)*\text{Sqrt}[3]] + \text{Sqrt}[3 - x] - I*\text{Sqrt}[1 + x]] + 3*(7 + 4*\text{Sqrt}[3])* \text{Log}[\text{Sqrt}[2 - (2*I)*\text{Sqrt}[3]] - \text{Sqrt}[3 - x] + I*\text{Sqrt}[1 + x]] - 3*(-7 + 4*\text{Sqrt}[3])* \text{Log}[\text{Sqrt}[2 + (2*I)*\text{Sqrt}[3]] - \text{Sqrt}[3 - x] + I*\text{Sqrt}[1 + x]]$

fricas [A] time = 0.62, size = 81, normalized size = 1.45

$$6\sqrt{3} \log\left(-\frac{\sqrt{3}(x+3)\sqrt{x+1}\sqrt{-x+3}+x^2-6x-9}{x^2}\right) - 4x + 12 \arctan\left(\frac{\sqrt{x+1}(x-1)\sqrt{-x+3}}{x^2-2x-3}\right) - 9 \log(x+1) + 21 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="fricas")

[Out] $6*\text{sqrt}(3)*\text{log}(-(\text{sqrt}(3)*(x + 3)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 3) + x^2 - 6*x - 9)/x^2) - 4*x + 12*\text{arctan}(\text{sqrt}(x + 1)*(x - 1)*\text{sqrt}(-x + 3)/(x^2 - 2*x - 3)) - 9*\text{log}(x + 1) + 21*\text{log}(x)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-8,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-91.616423693]Warning, choosing root of [1,0,-8,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-15.8804557086]-9*ln(abs(-x-1))+21*ln(abs(x))+4*(-x+3)+36*ln(abs(-4*sqrt(3)+6*(2*sqrt(-x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3)))/abs(4*sqrt(3)+6*(2*sqrt(-x+3)/(-2*sqrt(x+1)+4)-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))))/sqrt(3)-24*(-1/2*pi-atan(sqrt(-x+3)*((-1/2*(-2*sqrt(x+1)+4)/sqrt(-x+3))^2-1)/(-2*sqrt(x+1)+4)))

maple [A] time = 0.02, size = 76, normalized size = 1.36

$$-4x + 21 \ln(x) - 9 \ln(x + 1) + \frac{12\sqrt{x+1}\sqrt{-x+3}\left(-\sqrt{3}\operatorname{arctanh}\left(\frac{(x+3)\sqrt{3}}{3\sqrt{-x^2+2x+3}}\right) - \arcsin\left(\frac{x}{2} - \frac{1}{2}\right)\right)}{\sqrt{-x^2+2x+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*(3-x)^(1/2)+3/(x+1)^(1/2))^2/x,x)

[Out] $-4*x + 21*\text{ln}(x) + 12*(x+1)^(1/2)*(3-x)^(1/2)/(-x^2+2*x+3)^(1/2)*(-\arcsin(-1/2+1/2*x)-3^(1/2)*\operatorname{arctanh}(1/3*(x+3)*3^(1/2)/(-x^2+2*x+3)^(1/2)))-9*\text{ln}(x+1)$

maxima [A] time = 0.96, size = 57, normalized size = 1.02

$$-12\sqrt{3} \log\left(\frac{2\sqrt{3}\sqrt{-x^2+2x+3}}{|x|} + \frac{6}{|x|} + 2\right) - 4x + 12 \arcsin\left(-\frac{1}{2}x + \frac{1}{2}\right) - 9 \log(x + 1) + 21 \log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(3-x)^(1/2)+3/(1+x)^(1/2))^2/x,x, algorithm="maxima")

[Out] -12*sqrt(3)*log(2*sqrt(3)*sqrt(-x^2 + 2*x + 3)/abs(x) + 6/abs(x) + 2) - 4*x + 12*arcsin(-1/2*x + 1/2) - 9*log(x + 1) + 21*log(x)

mupad [B] time = 7.91, size = 158, normalized size = 2.82

$$48 \operatorname{atan}\left(\frac{\sqrt{3-x}-4\sqrt{3}+3\sqrt{3}\sqrt{x+1}}{\sqrt{x+1}-3\sqrt{3}\sqrt{3-x}+8}\right) - 9 \ln(x+1) - 4x + 21 \ln(x) + 12\sqrt{3} \ln\left(\frac{6x-12\sqrt{x+1}+4\sqrt{3}\sqrt{3-x}+2\sqrt{3}\sqrt{x+1}\sqrt{3-x}-6}{3x+6\sqrt{3}\sqrt{3-x}-18}\right) - 12\sqrt{3} \ln\left(\frac{\sqrt{x+1}-1}{\sqrt{3}-\sqrt{3-x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3/(x + 1)^(1/2) + 2*(3 - x)^(1/2))^2/x,x)

[Out] 48*atan(((3 - x)^(1/2) - 4*3^(1/2) + 3*3^(1/2)*(x + 1)^(1/2))/((x + 1)^(1/2) - 3*3^(1/2)*(3 - x)^(1/2) + 8)) - 9*log(x + 1) - 4*x + 21*log(x) + 12*3^(1/2)*log((6*x - 12*(x + 1)^(1/2) + 4*3^(1/2)*(3 - x)^(1/2) + 2*3^(1/2)*(x + 1)^(1/2)*(3 - x)^(1/2) - 6)/(3*x + 6*3^(1/2)*(3 - x)^(1/2) - 18)) - 12*3^(1/2)*log(((x + 1)^(1/2) - 1)/(3^(1/2) - (3 - x)^(1/2)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2\sqrt{3-x}\sqrt{x+1} + 3)^2}{x(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*(3-x)**(1/2)+3/(1+x)**(1/2))**2/x,x)

[Out] Integral((2*sqrt(3 - x)*sqrt(x + 1) + 3)**2/(x*(x + 1)), x)

$$3.509 \quad \int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} dx$$

Optimal. Leaf size=65

$$\frac{1}{2}\sqrt{x^2+1}x + \sqrt{x^2+1} + \frac{\sqrt{x^2+1}}{x} - \log\left(\sqrt{x^2+1} + 1\right) - x - \frac{1}{x} - \frac{1}{2}\sinh^{-1}(x)$$

Rubi [A] time = 0.16, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {6742, 277, 215, 1591, 190, 43, 195}

$$\frac{1}{2}\sqrt{x^2+1}x + \sqrt{x^2+1} + \frac{\sqrt{x^2+1}}{x} - \log\left(\sqrt{x^2+1} + 1\right) - x - \frac{1}{x} - \frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]), x]

[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2 - Log[1 + Sqrt[1 + x^2]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c*x)^(m + 1)*(a + b*x^n)^p/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[

Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]

Rubi steps

$$\begin{aligned} \int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}} + \frac{x}{1+\sqrt{1+x^2}} + \frac{x^2}{1+\sqrt{1+x^2}} \right) dx \\ &= -\int \frac{1}{1+\sqrt{1+x^2}} dx + \int \frac{x}{1+\sqrt{1+x^2}} dx + \int \frac{x^2}{1+\sqrt{1+x^2}} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+\sqrt{x}} dx, x, 1+x^2 \right) + \int (-1+\sqrt{1+x^2}) dx - \int \left(-\frac{1}{x^2} + \frac{\sqrt{1+x^2}}{x^2} \right) dx \\ &= -\frac{1}{x} - x + \int \sqrt{1+x^2} dx - \int \frac{\sqrt{1+x^2}}{x^2} dx + \text{Subst} \left(\int \frac{x}{1+x} dx, x, \sqrt{1+x^2} \right) \\ &= -\frac{1}{x} - x + \frac{\sqrt{1+x^2}}{x} + \frac{1}{2} x \sqrt{1+x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} dx - \int \frac{1}{\sqrt{1+x^2}} dx + \text{Subst} \left(\int (1 + \frac{1}{x}) dx, x, \sqrt{1+x^2} \right) \\ &= -\frac{1}{x} - x + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} + \frac{1}{2} x \sqrt{1+x^2} - \frac{1}{2} \sinh^{-1}(x) - \log(1+\sqrt{1+x^2}) \end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 1.00

$$\frac{1}{2} \sqrt{x^2+1} x + \sqrt{x^2+1} + \frac{\sqrt{x^2+1}}{x} - \log(\sqrt{x^2+1}+1) - x - \frac{1}{x} - \frac{1}{2} \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]), x]

[Out] -x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2 - Log[1 + Sqrt[1 + x^2]]

IntegrateAlgebraic [A] time = 0.32, size = 71, normalized size = 1.09

$$\frac{-x^2-1}{x} + \frac{\sqrt{x^2+1}(x^2+2x+2)}{2x} + \frac{3}{2} \log(\sqrt{x^2+1}-x) - 2 \log(\sqrt{x^2+1}-x+1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]), x]

[Out] (-1 - x^2)/x + (Sqrt[1 + x^2]*(2 + 2*x + x^2))/(2*x) + (3*Log[-x + Sqrt[1 + x^2]])/2 - 2*Log[1 - x + Sqrt[1 + x^2]]

fricas [A] time = 0.77, size = 84, normalized size = 1.29

$$\frac{2x^2 + 2x \log(x) + 2x \log(-x + \sqrt{x^2+1} + 1) - x \log(-x + \sqrt{x^2+1}) - 2x \log(-x + \sqrt{x^2+1} - 1) - (x^2 + 2x + 2)\sqrt{x^2+1} - 2x + 2}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="fricas")

[Out] $-1/2*(2*x^2 + 2*x*\log(x) + 2*x*\log(-x + \sqrt{x^2 + 1}) + 1) - x*\log(-x + \sqrt{x^2 + 1}) - 2*x*\log(-x + \sqrt{x^2 + 1} - 1) - (x^2 + 2*x + 2)*\sqrt{x^2 + 1} - 2*x + 2)/x$

giac [A] time = 0.42, size = 89, normalized size = 1.37

$$\frac{1}{2}\sqrt{x^2+1}(x+2) - x - \frac{2}{(x-\sqrt{x^2+1})^2-1} - \frac{1}{x} + \frac{1}{2}\log(-x+\sqrt{x^2+1}) - \log(|x|) - \log(|-x+\sqrt{x^2+1}+1|) + \log(|-x+\sqrt{x^2+1}-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="giac")`

[Out] $1/2*\sqrt{x^2 + 1}*(x + 2) - x - 2/((x - \sqrt{x^2 + 1})^2 - 1) - 1/x + 1/2*\log(-x + \sqrt{x^2 + 1}) - \log(\text{abs}(x)) - \log(\text{abs}(-x + \sqrt{x^2 + 1} + 1)) + \log(\text{abs}(-x + \sqrt{x^2 + 1} - 1))$

maple [A] time = 0.01, size = 56, normalized size = 0.86

$$-x - \frac{\sqrt{x^2+1}x}{2} - \frac{\text{arcsinh}(x)}{2} - \text{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right) - \ln(x) - \frac{1}{x} + \frac{(x^2+1)^{\frac{3}{2}}}{x} + \sqrt{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)`

[Out] $-x-1/x-1/2*(x^2+1)^(1/2)*x-1/2*\text{arcsinh}(x)+(x^2+1)^(1/2)-\text{arctanh}(1/(x^2+1)^(1/2))-\ln(x)+1/x*(x^2+1)^(3/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2x - 5 \arctan\left(\frac{1}{2}x\right) + \int \frac{x^6 + x^5 - x^4}{3x^4 + 16x^2 + (x^4 + 8x^2 + 16)\sqrt{x^2 + 1} + 16} dx + \log(x^2 + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+x-1)/((x^2+1)^(1/2)+1),x, algorithm="maxima")`

[Out] $2*x - 5*\arctan(1/2*x) + \text{integrate}((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4 + 8*x^2 + 16)*\sqrt{x^2 + 1} + 16), x) + \log(x^2 + 4)$

mupad [B] time = 0.04, size = 55, normalized size = 0.85

$$\left(\frac{x}{2} + 1\right)\sqrt{x^2+1} - \frac{\text{asinh}(x)}{2} - \ln(x) - x + \frac{\sqrt{x^2+1}}{x} - \frac{1}{x} + \text{atan}\left(\sqrt{x^2+1}\right) \text{1i}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)`

[Out] $\text{atan}((x^2 + 1)^(1/2)*1i)*1i - x - \text{asinh}(x)/2 - \log(x) + (x/2 + 1)*(x^2 + 1)^(1/2) + (x^2 + 1)^(1/2)/x - 1/x$

sympy [A] time = 4.51, size = 63, normalized size = 0.97

$$\frac{x\sqrt{x^2+1}}{2} - x + \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1} + 1\right) - \frac{\text{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+x-1)/((x**2+1)**(1/2)+1),x)`

[Out] $x*\sqrt{x**2 + 1}/2 - x + x/\sqrt{x**2 + 1} + \sqrt{x**2 + 1} - \log(\sqrt{x**2 + 1} + 1) - \text{asinh}(x)/2 - 1/x + 1/(x*\sqrt{x**2 + 1})$

$$3.510 \quad \int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}} dx$$

Optimal. Leaf size=53

$$\frac{1}{12} \left(2x^3 + 6x^2 + (-2x^2 - 3x + 4) \sqrt{x^2 + 1} - 6 \log(\sqrt{x^2 + 1} + 1) - 3 \sinh^{-1}(x) \right)$$

Rubi [A] time = 0.21, antiderivative size = 101, normalized size of antiderivative = 1.91, number of steps used = 12, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {6742, 2117, 893, 195, 215, 261}

$$\frac{x^3}{6} + \frac{x^2}{2} - \frac{1}{4} \sqrt{x^2+1} x - \frac{1}{6} (x^2+1)^{3/2} + \frac{1}{2(\sqrt{x^2+1}+x)} + \frac{1}{2} \log(\sqrt{x^2+1}+x) - \log(\sqrt{x^2+1}+x+1) + \frac{x}{2} - \frac{1}{4} \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]), x]

[Out] x/2 + x^2/2 + x^3/6 - (x*Sqrt[1 + x^2])/4 - (1 + x^2)^(3/2)/6 + 1/(2*(x + Sqrt[1 + x^2])) - ArcSinh[x]/4 + Log[x + Sqrt[1 + x^2]]/2 - Log[1 + x + Sqrt[1 + x^2]]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 893

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegerQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))

Rule 2117

Int[((g_.) + (h_.)*((d_.) + (e_.)*(x_) + (f_.)*Sqrt[(a_) + (c_.)*(x_)^2]))^(n_.)^(p_.), x_Symbol] := Dist[1/(2*e), Subst[Int[((g + h*x^n)^p*(d^2 + a*f^2 - 2*d*x + x^2))/(d - x)^2, x], x, d + e*x + f*Sqrt[a + c*x^2]], x] /; FreeQ[{a, c, d, e, f, g, h, n}, x] && EqQ[e^2 - c*f^2, 0] && IntegerQ[p]

Rule 6742


```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{-1+x+x^2}{1+x+\sqrt{1+x^2}} dx &= \int \left(-\frac{1}{1+x+\sqrt{1+x^2}} + \frac{x}{1+x+\sqrt{1+x^2}} + \frac{x^2}{1+x+\sqrt{1+x^2}} \right) dx \\
&= -\int \frac{1}{1+x+\sqrt{1+x^2}} dx + \int \frac{x}{1+x+\sqrt{1+x^2}} dx + \int \frac{x^2}{1+x+\sqrt{1+x^2}} dx \\
&= -\left(\frac{1}{2} \text{Subst} \left(\int \frac{2-2x+x^2}{(1-x)^2 x} dx, x, 1+x+\sqrt{1+x^2} \right) \right) + \int \left(\frac{1}{2} + \frac{x}{2} - \frac{\sqrt{1+x^2}}{2} \right) dx + \\
&= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{2} \int \sqrt{1+x^2} dx - \frac{1}{2} \int x\sqrt{1+x^2} dx - \frac{1}{2} \text{Subst} \left(\int \left(\frac{1}{1-x} + \frac{1}{(-1+x)} \right) dx, \right. \\
&= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{4} x\sqrt{1+x^2} - \frac{1}{6} (1+x^2)^{3/2} + \frac{1}{2(x+\sqrt{1+x^2})} + \frac{1}{2} \log(x+\sqrt{1+x^2}) \\
&= \frac{x}{2} + \frac{x^2}{2} + \frac{x^3}{6} - \frac{1}{4} x\sqrt{1+x^2} - \frac{1}{6} (1+x^2)^{3/2} + \frac{1}{2(x+\sqrt{1+x^2})} - \frac{1}{4} \sinh^{-1}(x) + \frac{1}{2} \log
\end{aligned}$$

Mathematica [A] time = 0.15, size = 88, normalized size = 1.66

$$\frac{1}{12} \left(2x^3 + 6x^2 - 2(x^2+1)^{3/2} + 6 \left(\frac{1}{\sqrt{x^2+1}+x} + \log(\sqrt{x^2+1}+x) - 2 \log(\sqrt{x^2+1}+x+1) \right) - 3(\sqrt{x^2+1}x + \sinh^{-1}(x)) + 6x \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]), x]
```

```
[Out] (6*x + 6*x^2 + 2*x^3 - 2*(1 + x^2)^(3/2) - 3*(x*Sqrt[1 + x^2] + ArcSinh[x])
+ 6*((x + Sqrt[1 + x^2])^(-1) + Log[x + Sqrt[1 + x^2]] - 2*Log[1 + x + Sqr
t[1 + x^2]]))/12
```

IntegrateAlgebraic [A] time = 0.19, size = 72, normalized size = 1.36

$$\frac{1}{12} \sqrt{x^2+1} (-2x^2 - 3x + 4) + \frac{3}{4} \log(\sqrt{x^2+1} - x) - \log(\sqrt{x^2+1} - x + 1) + \frac{1}{6} (x^3 + 3x^2)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(-1 + x + x^2)/(1 + x + Sqrt[1 + x^2]), x]
```

```
[Out] ((4 - 3*x - 2*x^2)*Sqrt[1 + x^2])/12 + (3*x^2 + x^3)/6 + (3*Log[-x + Sqrt[1
+ x^2]])/4 - Log[1 - x + Sqrt[1 + x^2]]
```

fricas [A] time = 0.60, size = 78, normalized size = 1.47

$$\frac{1}{6} x^3 + \frac{1}{2} x^2 - \frac{1}{12} (2x^2 + 3x - 4) \sqrt{x^2+1} - \frac{1}{2} \log(x) - \frac{1}{2} \log(-x + \sqrt{x^2+1} + 1) + \frac{1}{4} \log(-x + \sqrt{x^2+1}) + \frac{1}{2} \log(-x + \sqrt{x^2+1} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)), x, algorithm="fricas")
```

```
[Out] 1/6*x^3 + 1/2*x^2 - 1/12*(2*x^2 + 3*x - 4)*sqrt(x^2 + 1) - 1/2*log(x) - 1/2
*log(-x + sqrt(x^2 + 1) + 1) + 1/4*log(-x + sqrt(x^2 + 1)) + 1/2*log(-x + s
qrt(x^2 + 1) - 1)
```

giac [A] time = 0.44, size = 80, normalized size = 1.51

$$\frac{1}{6}x^3 + \frac{1}{2}x^2 - \frac{1}{12}((2x+3)x-4)\sqrt{x^2+1} + \frac{1}{4}\log(-x+\sqrt{x^2+1}) - \frac{1}{2}\log(|x|) - \frac{1}{2}\log(|-x+\sqrt{x^2+1}+1|) + \frac{1}{2}\log(|-x+\sqrt{x^2+1}-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/6*x^3 + 1/2*x^2 - 1/12*((2*x + 3)*x - 4)*sqrt(x^2 + 1) + 1/4*log(-x + sqrt(x^2 + 1)) - 1/2*log(abs(x)) - 1/2*log(abs(-x + sqrt(x^2 + 1) + 1)) + 1/2*log(abs(-x + sqrt(x^2 + 1) - 1))

maple [A] time = 0.01, size = 58, normalized size = 1.09

$$\frac{x^3}{6} + \frac{x^2}{2} - \frac{\sqrt{x^2+1}x}{4} - \frac{\operatorname{arcsinh}(x)}{4} - \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)}{2} - \frac{\ln(x)}{2} + \frac{\sqrt{x^2+1}}{2} - \frac{(x^2+1)^{\frac{3}{2}}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x)

[Out] 1/2*x^2-1/2*ln(x)+1/6*x^3-1/4*(x^2+1)^(1/2)*x-1/4*arcsinh(x)+1/2*(x^2+1)^(1/2)-1/2*arctanh(1/(x^2+1)^(1/2))-1/6*(x^2+1)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2 - \frac{3}{56}\sqrt{7}\arctan\left(\frac{1}{7}\sqrt{7}(4x+3)\right) + \frac{1}{4}x + \int \frac{x^4+x^3-x^2}{4x^5+12x^4+19x^3+19x^2+(4x^4+12x^3+17x^2+12x+4)\sqrt{x^2+1}+12x+4} dx - \frac{7}{16}\log(2x^2+3x+2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+x-1)/(1+x+(x^2+1)^(1/2)),x, algorithm="maxima")

[Out] 1/4*x^2 - 3/56*sqrt(7)*arctan(1/7*sqrt(7)*(4*x + 3)) + 1/4*x + integrate((x^4 + x^3 - x^2)/(4*x^5 + 12*x^4 + 19*x^3 + 19*x^2 + (4*x^4 + 12*x^3 + 17*x^2 + 12*x + 4)*sqrt(x^2 + 1) + 12*x + 4), x) - 7/16*log(2*x^2 + 3*x + 2)

mupad [B] time = 0.04, size = 52, normalized size = 0.98

$$\frac{x^2}{2} - \frac{\ln(x)}{2} - \sqrt{x^2+1} \left(\frac{x^2}{6} + \frac{x}{4} - \frac{1}{3} \right) - \frac{\operatorname{asinh}(x)}{4} + \frac{x^3}{6} + \frac{\operatorname{atan}\left(\sqrt{x^2+1}\operatorname{li}\right)\operatorname{li}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^2 - 1)/(x + (x^2 + 1)^(1/2) + 1),x)

[Out] (atan((x^2 + 1)^(1/2)*1i)*1i)/2 - asinh(x)/4 - log(x)/2 - (x^2 + 1)^(1/2)*(x/4 + x^2/6 - 1/3) + x^2/2 + x^3/6

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 + x - 1}{x + \sqrt{x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+x-1)/(1+x+(x**2+1)**(1/2)),x)

[Out] Integral((x**2 + x - 1)/(x + sqrt(x**2 + 1) + 1), x)

$$3.511 \quad \int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} dx$$

Optimal. Leaf size=14

$$2\sqrt{x-1} + 2\log(x)$$

Rubi [A] time = 0.12, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6688}

$$2\sqrt{x-1} + 2\log(x)$$

Antiderivative was successfully verified.

[In] Int[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x), x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned} \int \frac{2\sqrt{-1+x}+x}{\sqrt{-1+x}x} dx &= \int \left(\frac{1}{\sqrt{-1+x}} + \frac{2}{x} \right) dx \\ &= 2\sqrt{-1+x} + 2\log(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 14, normalized size = 1.00

$$2\sqrt{x-1} + 2\log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x), x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

IntegrateAlgebraic [A] time = 0.02, size = 14, normalized size = 1.00

$$2\sqrt{x-1} + 2\log(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2*Sqrt[-1 + x] + x)/(Sqrt[-1 + x]*x), x]

[Out] 2*Sqrt[-1 + x] + 2*Log[x]

fricas [A] time = 0.61, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2), x, algorithm="fricas")

[Out] 2*sqrt(x - 1) + 2*log(x)

giac [A] time = 0.40, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(x - 1) + 2*log(x)

maple [A] time = 0.00, size = 13, normalized size = 0.93

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2*(x-1)^(1/2))/x/(x-1)^(1/2),x)

[Out] 2*ln(x)+2*(x-1)^(1/2)

maxima [A] time = 0.98, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)^(1/2))/x/(-1+x)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x - 1) + 2*log(x)

mupad [B] time = 3.39, size = 12, normalized size = 0.86

$$2\ln(x) + 2\sqrt{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2*(x - 1)^(1/2))/(x*(x - 1)^(1/2)),x)

[Out] 2*log(x) + 2*(x - 1)^(1/2)

sympy [A] time = 0.16, size = 12, normalized size = 0.86

$$2\sqrt{x-1} + 2\log(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+2*(-1+x)**(1/2))/x/(-1+x)**(1/2),x)

[Out] 2*sqrt(x - 1) + 2*log(x)

$$3.512 \quad \int (a + c\sqrt{x} + bx^{2/3})^2 dx$$

Optimal. Leaf size=61

$$a^2x + \frac{6}{5}abx^{5/3} + \frac{4}{3}acx^{3/2} + \frac{3}{7}b^2x^{7/3} + \frac{12}{13}bcx^{13/6} + \frac{c^2x^2}{2}$$

Rubi [A] time = 0.17, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6741, 6742}

$$a^2x + \frac{6}{5}abx^{5/3} + \frac{4}{3}acx^{3/2} + \frac{3}{7}b^2x^{7/3} + \frac{12}{13}bcx^{13/6} + \frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^2,x]

[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13/6))/13 + (3*b^2*x^(7/3))/7

Rule 6741

Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int (a + c\sqrt{x} + bx^{2/3})^2 dx &= 6 \text{Subst} \left(\int x^5 (a + x^3(c + bx))^2 dx, x, \sqrt[6]{x} \right) \\ &= 6 \text{Subst} \left(\int x^5 (a + cx^3 + bx^4)^2 dx, x, \sqrt[6]{x} \right) \\ &= 6 \text{Subst} \left(\int (a^2x^5 + 2acx^8 + 2abx^9 + c^2x^{11} + 2bcx^{12} + b^2x^{13}) dx, x, \sqrt[6]{x} \right) \\ &= a^2x + \frac{4}{3}acx^{3/2} + \frac{6}{5}abx^{5/3} + \frac{c^2x^2}{2} + \frac{12}{13}bcx^{13/6} + \frac{3}{7}b^2x^{7/3} \end{aligned}$$

Mathematica [A] time = 0.05, size = 61, normalized size = 1.00

$$a^2x + \frac{6}{5}abx^{5/3} + \frac{4}{3}acx^{3/2} + \frac{3}{7}b^2x^{7/3} + \frac{12}{13}bcx^{13/6} + \frac{c^2x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^2,x]

[Out] a^2*x + (4*a*c*x^(3/2))/3 + (6*a*b*x^(5/3))/5 + (c^2*x^2)/2 + (12*b*c*x^(13/6))/13 + (3*b^2*x^(7/3))/7

IntegrateAlgebraic [A] time = 0.04, size = 56, normalized size = 0.92

$$\frac{2730a^2x + 3276abx^{5/3} + 3640acx^{3/2} + 1170b^2x^{7/3} + 2520bcx^{13/6} + 1365c^2x^2}{2730}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c*Sqrt[x] + b*x^(2/3))^2,x]

[Out] (2730*a^2*x + 3640*a*c*x^(3/2) + 3276*a*b*x^(5/3) + 1365*c^2*x^2 + 2520*b*c*x^(13/6) + 1170*b^2*x^(7/3))/2730

fricas [A] time = 0.67, size = 43, normalized size = 0.70

$$\frac{3}{7}b^2x^{\frac{7}{3}} + \frac{12}{13}bcx^{\frac{13}{6}} + \frac{1}{2}c^2x^2 + \frac{6}{5}abx^{\frac{5}{3}} + \frac{4}{3}acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="fricas")

[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*a*c*x^(3/2) + a^2*x

giac [A] time = 0.43, size = 43, normalized size = 0.70

$$\frac{3}{7}b^2x^{\frac{7}{3}} + \frac{12}{13}bcx^{\frac{13}{6}} + \frac{1}{2}c^2x^2 + \frac{6}{5}abx^{\frac{5}{3}} + \frac{4}{3}acx^{\frac{3}{2}} + a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="giac")

[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + 6/5*a*b*x^(5/3) + 4/3*a*c*x^(3/2) + a^2*x

maple [A] time = 0.00, size = 46, normalized size = 0.75

$$\frac{3b^2x^{\frac{7}{3}}}{7} + \frac{c^2x^2}{2} + \frac{6abx^{\frac{5}{3}}}{5} + a^2x + 2\left(\frac{6bx^{\frac{13}{6}}}{13} + \frac{2ax^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^(2/3)+c*x^(1/2))^2,x)

[Out] 1/2*c^2*x^2+2*c*(6/13*b*x^(13/6)+2/3*a*x^(3/2))+a^2*x+3/7*b^2*x^(7/3)+6/5*a*b*x^(5/3)

maxima [A] time = 0.44, size = 45, normalized size = 0.74

$$\frac{3}{7}b^2x^{\frac{7}{3}} + \frac{12}{13}bcx^{\frac{13}{6}} + \frac{1}{2}c^2x^2 + a^2x + \frac{2}{15}\left(9bx^{\frac{5}{3}} + 10cx^{\frac{3}{2}}\right)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^2,x, algorithm="maxima")

[Out] 3/7*b^2*x^(7/3) + 12/13*b*c*x^(13/6) + 1/2*c^2*x^2 + a^2*x + 2/15*(9*b*x^(5/3) + 10*c*x^(3/2))*a

mupad [B] time = 3.36, size = 43, normalized size = 0.70

$$a^2x + \frac{3b^2x^{\frac{7}{3}}}{7} + \frac{c^2x^2}{2} + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{12bcx^{\frac{13}{6}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^(2/3) + c*x^(1/2))^2,x)

[Out] $a^2x + (3b^2x^{7/3})/7 + (c^2x^2)/2 + (6abx^{5/3})/5 + (4acx^{3/2})/3 + (12bcx^{13/6})/13$

sympy [A] time = 2.42, size = 60, normalized size = 0.98

$$a^2x + \frac{6abx^{\frac{5}{3}}}{5} + \frac{4acx^{\frac{3}{2}}}{3} + \frac{3b^2x^{\frac{7}{3}}}{7} + \frac{12bcx^{\frac{13}{6}}}{13} + \frac{c^2x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**(2/3)+c*x**(1/2))**2,x)

[Out] $a**2*x + 6*a*b*x**(5/3)/5 + 4*a*c*x**(3/2)/3 + 3*b**2*x**(7/3)/7 + 12*b*c*x**(13/6)/13 + c**2*x**2/2$

$$3.513 \quad \int (a + c\sqrt{x} + bx^{2/3})^3 dx$$

Optimal. Leaf size=114

$$a^3x + \frac{9}{5}a^2bx^{5/3} + 2a^2cx^{3/2} + \frac{9}{7}ab^2x^{7/3} + \frac{36}{13}abcx^{13/6} + \frac{3}{2}ac^2x^2 + \frac{b^3x^3}{3} + \frac{18}{17}b^2cx^{17/6} + \frac{9}{8}bc^2x^{8/3} + \frac{2}{5}c^3x^{5/2}$$

Rubi [A] time = 0.19, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6741, 6742}

$$\frac{9}{5}a^2bx^{5/3} + 2a^2cx^{3/2} + a^3x + \frac{9}{7}ab^2x^{7/3} + \frac{36}{13}abcx^{13/6} + \frac{3}{2}ac^2x^2 + \frac{18}{17}b^2cx^{17/6} + \frac{b^3x^3}{3} + \frac{9}{8}bc^2x^{8/3} + \frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(a + c*Sqrt[x] + b*x^(2/3))^3,x]

[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3

Rule 6741

Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v != u]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]

Rubi steps

$$\begin{aligned} \int (a + c\sqrt{x} + bx^{2/3})^3 dx &= 6 \text{Subst} \left(\int x^5 (a + x^3(c + bx))^3 dx, x, \sqrt[6]{x} \right) \\ &= 6 \text{Subst} \left(\int x^5 (a + cx^3 + bx^4)^3 dx, x, \sqrt[6]{x} \right) \\ &= 6 \text{Subst} \left(\int (a^3x^5 + 3a^2cx^8 + 3a^2bx^9 + 3ac^2x^{11} + 6abcx^{12} + 3ab^2x^{13} + c^3x^{14} + 3bc^2x^{15} + 3b^2cx^{16} + 3b^3x^{17}) dx, x, \sqrt[6]{x} \right) \\ &= a^3x + 2a^2cx^{3/2} + \frac{9}{5}a^2bx^{5/3} + \frac{3}{2}ac^2x^2 + \frac{36}{13}abcx^{13/6} + \frac{9}{7}ab^2x^{7/3} + \frac{2}{5}c^3x^{5/2} + \frac{9}{8}bc^2x^{8/3} + \frac{b^3x^3}{3} \end{aligned}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 1.00

$$a^3x + \frac{9}{5}a^2bx^{5/3} + 2a^2cx^{3/2} + \frac{9}{7}ab^2x^{7/3} + \frac{36}{13}abcx^{13/6} + \frac{3}{2}ac^2x^2 + \frac{b^3x^3}{3} + \frac{18}{17}b^2cx^{17/6} + \frac{9}{8}bc^2x^{8/3} + \frac{2}{5}c^3x^{5/2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c*Sqrt[x] + b*x^(2/3))^3,x]

[Out] a^3*x + 2*a^2*c*x^(3/2) + (9*a^2*b*x^(5/3))/5 + (3*a*c^2*x^2)/2 + (36*a*b*c*x^(13/6))/13 + (9*a*b^2*x^(7/3))/7 + (2*c^3*x^(5/2))/5 + (9*b*c^2*x^(8/3))/8 + (18*b^2*c*x^(17/6))/17 + (b^3*x^3)/3

IntegrateAlgebraic [A] time = 0.05, size = 103, normalized size = 0.90

$$\frac{185640a^3x + 334152a^2bx^{5/3} + 371280a^2cx^{3/2} + 238680ab^2x^{7/3} + 514080abcx^{13/6} + 278460ac^2x^2 + 61880b^3x^3 + 196560b^2cx^{17/6} + 208845bc^2x^{8/3} + 74256c^3x^{5/2}}{185640}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + c*Sqrt[x] + b*x^(2/3))^3,x]

[Out] (185640*a^3*x + 371280*a^2*c*x^(3/2) + 334152*a^2*b*x^(5/3) + 278460*a*c^2*x^2 + 514080*a*b*c*x^(13/6) + 238680*a*b^2*x^(7/3) + 74256*c^3*x^(5/2) + 208845*b*c^2*x^(8/3) + 196560*b^2*c*x^(17/6) + 61880*b^3*x^3)/185640

fricas [A] time = 1.54, size = 91, normalized size = 0.80

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{36}{13}abcx^{\frac{13}{6}} + \frac{3}{2}ac^2x^2 + a^3x + \frac{9}{40}(5bc^2x^2 + 8a^2bx)x^{\frac{2}{3}} + \frac{2}{5}(c^3x^2 + 5a^2cx)\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="fricas")

[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + a^3*x + 9/40*(5*b*c^2*x^2 + 8*a^2*b*x)*x^(2/3) + 2/5*(c^3*x^2 + 5*a^2*c*x)*sqrt(x)

giac [A] time = 0.34, size = 84, normalized size = 0.74

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{8}bc^2x^{\frac{8}{3}} + \frac{2}{5}c^3x^{\frac{5}{2}} + \frac{9}{7}ab^2x^{\frac{7}{3}} + \frac{36}{13}abcx^{\frac{13}{6}} + \frac{3}{2}ac^2x^2 + \frac{9}{5}a^2bx^{\frac{5}{3}} + 2a^2cx^{\frac{3}{2}} + a^3x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="giac")

[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) + 9/7*a*b^2*x^(7/3) + 36/13*a*b*c*x^(13/6) + 3/2*a*c^2*x^2 + 9/5*a^2*b*x^(5/3) + 2*a^2*c*x^(3/2) + a^3*x

maple [A] time = 0.00, size = 86, normalized size = 0.75

$$\frac{b^3x^3}{3} + \frac{2c^3x^{\frac{5}{2}}}{5} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{9a^2bx^{\frac{5}{3}}}{5} + a^3x + 3\left(\frac{3bx^{\frac{8}{3}}}{8} + \frac{ax^2}{2}\right)c^2 + 3\left(\frac{6b^2x^{\frac{17}{6}}}{17} + \frac{12abx^{\frac{13}{6}}}{13} + \frac{2a^2x^{\frac{3}{2}}}{3}\right)c$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^(2/3)+c*x^(1/2))^3,x)

[Out] 2/5*c^3*x^(5/2)+3*c^2*(3/8*b*x^(8/3)+1/2*a*x^2)+3*c*(6/17*b^2*x^(17/6)+12/13*a*b*x^(13/6)+2/3*a^2*x^(3/2))+a^3*x+1/3*b^3*x^3+9/5*a^2*b*x^(5/3)+9/7*a*b^2*x^(7/3)

maxima [A] time = 0.44, size = 85, normalized size = 0.75

$$\frac{1}{3}b^3x^3 + \frac{18}{17}b^2cx^{\frac{17}{6}} + \frac{9}{8}bc^2x^{\frac{8}{3}} + \frac{2}{5}c^3x^{\frac{5}{2}} + a^3x + \frac{1}{5}(9bx^{\frac{5}{3}} + 10cx^{\frac{3}{2}})a^2 + \frac{3}{182}(78b^2x^{\frac{7}{3}} + 168bcx^{\frac{13}{6}} + 91c^2x^2)a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(2/3)+c*x^(1/2))^3,x, algorithm="maxima")

[Out] 1/3*b^3*x^3 + 18/17*b^2*c*x^(17/6) + 9/8*b*c^2*x^(8/3) + 2/5*c^3*x^(5/2) + a^3*x + 1/5*(9*b*x^(5/3) + 10*c*x^(3/2))*a^2 + 3/182*(78*b^2*x^(7/3) + 168*b*c*x^(13/6) + 91*c^2*x^2)*a

mupad [B] time = 0.06, size = 84, normalized size = 0.74

$$a^3x + \frac{b^3x^3}{3} + \frac{2c^3x^{\frac{5}{2}}}{5} + \frac{9a^2bx^{\frac{5}{3}}}{5} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{3ac^2x^2}{2} + 2a^2cx^{\frac{3}{2}} + \frac{9bc^2x^{\frac{8}{3}}}{8} + \frac{18b^2cx^{\frac{17}{6}}}{17} + \frac{36abcx^{\frac{13}{6}}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a + b*x^(2/3) + c*x^(1/2))^3,x)`

[Out] $a^3x + (b^3x^3)/3 + (2c^3x^{5/2})/5 + (9a^2bx^{5/3})/5 + (9a^2cx^{7/3})/7 + (3ac^2x^2)/2 + 2a^2cx^{3/2} + (9b^2cx^{8/3})/8 + (18b^2c^2x^{17/6})/17 + (36abcx^{13/6})/13$

sympy [A] time = 3.45, size = 116, normalized size = 1.02

$$a^3x + \frac{9a^2bx^{\frac{5}{3}}}{5} + 2a^2cx^{\frac{3}{2}} + \frac{9ab^2x^{\frac{7}{3}}}{7} + \frac{36abcx^{\frac{13}{6}}}{13} + \frac{3ac^2x^2}{2} + \frac{b^3x^3}{3} + \frac{18b^2cx^{\frac{17}{6}}}{17} + \frac{9bc^2x^{\frac{8}{3}}}{8} + \frac{2c^3x^{\frac{5}{2}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*x**(2/3)+c*x**(1/2))**3,x)`

[Out] $a^3x + 9a^2bx^{5/3}/5 + 2a^2cx^{3/2} + 9a^2cx^{7/3}/7 + 36a^2b^2cx^{13/6}/13 + 3a^2c^2x^2/2 + b^3x^3/3 + 18b^2c^2x^{17/6}/17 + 9b^2c^2x^{8/3}/8 + 2c^3x^{5/2}/5$

$$3.514 \quad \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}} x^3} dx$$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3), x]

[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/Sqrt[a - b]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 514

Int[(x_)^(m_)*((c_) + (d_.)*(x_)^(mn_))^(q_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I

ntegerQ[p])

Rubi steps

$$\begin{aligned}
 \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}}x^3} dx &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}}x} dx \\
 &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}} dx, x, \frac{1}{x^2}\right) \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
 \end{aligned}$$

Mathematica [A] time = 0.06, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}(ax^2-bx^2+b)+bx\sqrt{ax^2-bx^2+b}\tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3), x]

[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqrt[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))]*x^2)

IntegrateAlgebraic [A] time = 0.09, size = 68, normalized size = 1.17

$$\frac{\sqrt{a+\frac{b}{x^2}}-b}{b} + \frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{ax^2-bx^2+b}{x^2}}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^2)/(Sqrt[a - b + b/x^2]*x^3), x]

[Out] Sqrt[a - b + b/x^2]/b + ArcTan[(Sqrt[-a + b]*Sqrt[(b + a*x^2 - b*x^2)/x^2])/(a - b)]/Sqrt[-a + b]

fricas [A] time = 1.28, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b}b \log\left(-2(a-b)x^2 - 2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}} - b\right) + 2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2(ab-b^2)}, \frac{\sqrt{-a+b}b \arctan\left(-\frac{\sqrt{-a+b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right) + (a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{2}(\sqrt{a-b} \cdot b \cdot \log(-2(a-b)x^2 - 2\sqrt{a-b}x^2\sqrt{(a-b)x^2+b}/x^2) - b) + 2(a-b)\sqrt{(a-b)x^2+b}/x^2)/(ab - b^2)$, $(\sqrt{(-a+b)b} \cdot \arctan(-\sqrt{-a+b}x^2\sqrt{(a-b)x^2+b}/x^2)/((a-b)x^2+b) + (a-b)\sqrt{(a-b)x^2+b}/x^2)/(ab - b^2)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{2,[0,2,1]%%}+%%{-2,[0,1,1]%%}+%%{2,[0,0,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{-2,[1,4,1]%%}+%%{2,[1,3,1]%%}+%%{4,[1,2,1]%%}+%%{-2,[1,1,1]%%}+%%{-2,[1,0,1]%%}+%%{1,[0,4,2]%%}+%%{-2,[0,3,2]%%}+%%{-1,[0,2,2]%%}+%%{2,[0,1,2]%%}+%%{1,[0,0,2]%%}] a t parameters values [86,-97,-82]Sign error (%%{b,0%%}+%%{-2*sqrt(a-b)*sqrt(b),1/2%%}+%%{2*(a-b),1%%}+%%{-a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(b))/b,3/2%%}+%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt(a-b)*sqrt(b))/(4*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached o r unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{ax^2 - bx^2 + b} \left(bx \ln \left(\sqrt{a-b} x + \sqrt{ax^2 - bx^2 + b} \right) + \sqrt{ax^2 - bx^2 + b} \sqrt{a-b} \right)}{\sqrt{\frac{ax^2 - bx^2 + b}{x^2}} \sqrt{a-b} b x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x)

[Out] $(ax^2 - bx^2 + b)^{1/2} (\ln((a-b)^{1/2} x + (ax^2 - bx^2 + b)^{1/2}) * b * x + (ax^2 - bx^2 + b)^{1/2} * (a-b)^{1/2}) / ((ax^2 - bx^2 + b) / x^2)^{1/2} / x^2 / (a-b)^{1/2} / b$

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a-b+b/x^2)^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 4.08, size = 46, normalized size = 0.79

$$\frac{\operatorname{atanh}\left(\frac{\sqrt{a-b+\frac{b}{x^2}}}{\sqrt{a-b}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a-b+\frac{b}{x^2}}}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 1)/(x^3*(a - b + b/x^2)^(1/2)), x)`

[Out] `atanh((a - b + b/x^2)^(1/2)/(a - b)^(1/2))/(a - b)^(1/2) + (a - b + b/x^2)^(1/2)/b`

sympy [A] time = 3.18, size = 70, normalized size = 1.21

$$\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)/x**3/(a-b+b/x**2)**(1/2), x)`

[Out] `-Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True))/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a - b))`

$$3.515 \quad \int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)}x^3} dx$$

Optimal. Leaf size=58

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Rubi [A] time = 0.14, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1978, 514, 446, 80, 63, 208}

$$\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))])*x^3], x]

[Out] Sqrt[a - b*(1 - x^(-2))]/b + ArcTanh[Sqrt[a - b*(1 - x^(-2))]/Sqrt[a - b]]/Sqrt[a - b]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 80

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 514

Int[(x_)^(m_)*((c_) + (d_.)*(x_)^(mn_))^(q_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I

ntegerQ[p])

Rule 1978

Int[(Pq_)*(u_)^(p_)*((c_)*(x_))^(m_), x_Symbol] := Int[(c*x)^m*Pq*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && PolyQ[Pq, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
 \int \frac{-1+x^2}{\sqrt{a+b\left(-1+\frac{1}{x^2}\right)x^3}} dx &= \int \frac{-1+x^2}{\sqrt{a-b+\frac{b}{x^2}x^3}} dx \\
 &= \int \frac{1-\frac{1}{x^2}}{\sqrt{a-b+\frac{b}{x^2}x}} dx \\
 &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1-x}{x\sqrt{a-b+bx}} dx, x, \frac{1}{x^2}\right)\right) \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{a-b+bx}} dx, x, \frac{1}{x^2}\right) \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a-b}{b}+\frac{x^2}{b}} dx, x, \sqrt{a+b\left(-1+\frac{1}{x^2}\right)}\right)}{b} \\
 &= \frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{b} + \frac{\tanh^{-1}\left(\frac{\sqrt{a-b\left(1-\frac{1}{x^2}\right)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 100, normalized size = 1.72

$$\frac{\sqrt{a-b}(ax^2-bx^2+b) + bx\sqrt{ax^2-bx^2+b} \tanh^{-1}\left(\frac{x\sqrt{a-b}}{\sqrt{x^2(a-b)+b}}\right)}{bx^2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))])*x^3, x]

[Out] (Sqrt[a - b]*(b + a*x^2 - b*x^2) + b*x*Sqrt[b + a*x^2 - b*x^2]*ArcTanh[(Sqrt[a - b]*x)/Sqrt[b + (a - b)*x^2]])/(Sqrt[a - b]*b*Sqrt[a + b*(-1 + x^(-2))])*x^2)

IntegrateAlgebraic [A] time = 0.10, size = 68, normalized size = 1.17

$$\frac{\sqrt{a+\frac{b}{x^2}-b}}{b} + \frac{\tan^{-1}\left(\frac{\sqrt{b-a}\sqrt{\frac{ax^2-bx^2+b}{x^2}}}{a-b}\right)}{\sqrt{b-a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^2)/(Sqrt[a + b*(-1 + x^(-2))])*x^3],x]

[Out] Sqrt[a - b + b/x^2]/b + ArcTan[(Sqrt[-a + b]*Sqrt[(b + a*x^2 - b*x^2)/x^2])/(a - b)]/Sqrt[-a + b]

fricas [A] time = 0.87, size = 180, normalized size = 3.10

$$\left[\frac{\sqrt{a-b} b \log\left(-2(a-b)x^2 - 2\sqrt{a-b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}} - b\right) + 2(a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{2(ab-b^2)}, \frac{\sqrt{-a+b} b \arctan\left(-\frac{\sqrt{-a+b}x^2\sqrt{\frac{(a-b)x^2+b}{x^2}}}{(a-b)x^2+b}\right) + (a-b)\sqrt{\frac{(a-b)x^2+b}{x^2}}}{ab-b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="fricas")

[Out] [1/2*(sqrt(a - b)*b*log(-2*(a - b)*x^2 - 2*sqrt(a - b)*x^2*sqrt(((a - b)*x^2 + b)/x^2) - b) + 2*(a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2), (sqrt(-a + b)*b*arctan(-sqrt(-a + b)*x^2*sqrt(((a - b)*x^2 + b)/x^2))/((a - b)*x^2 + b) + (a - b)*sqrt(((a - b)*x^2 + b)/x^2))/(a*b - b^2)]

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes constant sign by intervals (correct if the argument is real):Check [abs(t_n ostep)]Warning, choosing root of [1,0,%%{-2,[1,2,0]%%}+%%{-2,[1,0,0]%%}+%%{-2,[0,2,1]%%}+%%{-2,[0,1,1]%%}+%%{-2,[0,0,1]%%},0,%%{1,[2,4,0]%%}+%%{-2,[2,2,0]%%}+%%{1,[2,0,0]%%}+%%{-2,[1,4,1]%%}+%%{2,[1,3,1]%%}+%%{4,[1,2,1]%%}+%%{-2,[1,1,1]%%}+%%{-2,[1,0,1]%%}+%%{1,[0,4,2]%%}+%%{-2,[0,3,2]%%}+%%{-1,[0,2,2]%%}+%%{2,[0,1,2]%%}+%%{1,[0,0,2]%%}] a t parameters values [86,-97,-82]Sign error (%%{b,0%%}+%%{-2*sqrt(a-b)*sqrt(b),1/2%%}+%%{2*(a-b),1%%}+%%{-(a*sqrt(a-b)*sqrt(b)-b*sqrt(a-b)*sqrt(b))/b,3/2%%}+%%{-(-a^2*sqrt(a-b)*sqrt(b)+2*a*b*sqrt(a-b)*sqrt(b)-b^2*sqrt(a-b)*sqrt(b))/(4*b^2),5/2%%}+%%{undef,7/2%%})Limit: Max order reached o r unable to make series expansion Error: Bad Argument Value

maple [B] time = 0.02, size = 102, normalized size = 1.76

$$\frac{\sqrt{ax^2 - bx^2 + b} \left(bx \ln\left(\sqrt{a-b} x + \sqrt{ax^2 - bx^2 + b}\right) + \sqrt{ax^2 - bx^2 + b} \sqrt{a-b} \right)}{\sqrt{\frac{ax^2 - bx^2 + b}{x^2}} \sqrt{a-b} b x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x)

[Out] (a*x^2-b*x^2+b)^(1/2)*(b*x*ln((a-b)^(1/2)*x+(a*x^2-b*x^2+b)^(1/2)))+(a*x^2-b*x^2+b)^(1/2)*(a-b)^(1/2)/((a*x^2-b*x^2+b)/x^2)^(1/2)/(a-b)^(1/2)/b/x^2

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/x^3/(a+b*(-1+1/x^2))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a-4*b>0)', see `assume?` for more details)Is 4*a-4*b positive or negative?

mupad [B] time = 4.04, size = 62, normalized size = 1.07

$$\frac{\sqrt{a+b\left(\frac{1}{x^2}-1\right)}}{b} + \frac{\ln\left(x^2\left(2a-2b+2\sqrt{a-b}\sqrt{a+b\left(\frac{1}{x^2}-1\right)+\frac{b}{x^2}}\right)\right)}{2\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/(x^3*(a + b*(1/x^2 - 1))^(1/2)),x)

[Out] (a + b*(1/x^2 - 1))^(1/2)/b + log(x^2*(2*a - 2*b + 2*(a - b)^(1/2)*(a + b*(1/x^2 - 1))^(1/2) + b/x^2))/(2*(a - b)^(1/2))

sympy [A] time = 7.03, size = 70, normalized size = 1.21

$$\frac{\begin{cases} -\frac{1}{\sqrt{a}x^2} & \text{for } b = 0 \\ -\frac{2\sqrt{a-b+\frac{b}{x^2}}}{b} & \text{otherwise} \end{cases}}{2} - \frac{\operatorname{atan}\left(\frac{1}{\sqrt{-\frac{1}{a-b}}\sqrt{a-b+\frac{b}{x^2}}}\right)}{\sqrt{-\frac{1}{a-b}}(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2-1)/x**3/(a+b*(-1+1/x**2))**(1/2),x)

[Out] -Piecewise((-1/(sqrt(a)*x**2), Eq(b, 0)), (-2*sqrt(a - b + b/x**2)/b, True))/2 - atan(1/(sqrt(-1/(a - b))*sqrt(a - b + b/x**2)))/(sqrt(-1/(a - b))*(a - b))

$$3.516 \quad \int \frac{1+x}{(4+x^2)\sqrt{9+x^2}} dx$$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1010, 377, 203, 444, 63, 207}

$$\frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]), x]

[Out] ArcTan[(Sqrt[5]*x)/(2*Sqrt[9 + x^2])]/(2*Sqrt[5]) - ArcTanh[Sqrt[9 + x^2]/Sqrt[5]]/Sqrt[5]

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_.) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 377

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)/((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 444

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1010

Int[((g_.) + (h_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[g, Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Dist[h,

$\text{Int}[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; \text{FreeQ}[\{a, c, d, f, g, h, p, q\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{1+x}{(4+x^2)\sqrt{9+x^2}} dx &= \int \frac{1}{(4+x^2)\sqrt{9+x^2}} dx + \int \frac{x}{(4+x^2)\sqrt{9+x^2}} dx \\ &= \frac{1}{2} \text{Subst}\left(\int \frac{1}{(4+x)\sqrt{9+x}} dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{4+5x^2} dx, x, \frac{x}{\sqrt{9+x^2}}\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} + \text{Subst}\left(\int \frac{1}{-5+x^2} dx, x, \sqrt{9+x^2}\right) \\ &= \frac{\tan^{-1}\left(\frac{\sqrt{5}x}{2\sqrt{9+x^2}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{9+x^2}}{\sqrt{5}}\right)}{\sqrt{5}} \end{aligned}$$

Mathematica [C] time = 0.04, size = 64, normalized size = 1.21

$$\frac{(2+i)\tanh^{-1}\left(\frac{9-2ix}{\sqrt{5}\sqrt{x^2+9}}\right) + (2-i)\tanh^{-1}\left(\frac{9+2ix}{\sqrt{5}\sqrt{x^2+9}}\right)}{4\sqrt{5}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]), x]

[Out] -1/4*((2 + I)*ArcTanh[(9 - (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])] + (2 - I)*ArcTanh[(9 + (2*I)*x)/(Sqrt[5]*Sqrt[9 + x^2])])/Sqrt[5]

IntegrateAlgebraic [A] time = 0.65, size = 73, normalized size = 1.38

$$\frac{\tan^{-1}\left(\frac{x^2}{2\sqrt{5}} - \frac{\sqrt{x^2+9}x}{2\sqrt{5}} + \frac{2}{\sqrt{5}}\right)}{2\sqrt{5}} - \frac{\tanh^{-1}\left(\frac{\sqrt{x^2+9}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)/((4 + x^2)*Sqrt[9 + x^2]), x]

[Out] -1/2*ArcTan[2/Sqrt[5] + x^2/(2*Sqrt[5]) - (x*Sqrt[9 + x^2])/(2*Sqrt[5])]/Sqrt[5] - ArcTanh[Sqrt[9 + x^2]/Sqrt[5]]/Sqrt[5]

fricas [B] time = 0.56, size = 182, normalized size = 3.43

$\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}(x+\sqrt{5})}+\sqrt{5}x+9+\frac{1}{2}x+\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{5}\sqrt{5}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-\sqrt{x^2+9}(x-\sqrt{5})}-\sqrt{5}x+9+\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log(50x^2-50\sqrt{x^2+9}(x+\sqrt{5})+50\sqrt{5}x+450)-\frac{1}{10}\sqrt{5}\log(50x^2-50\sqrt{x^2+9}(x-\sqrt{5})-50\sqrt{5}x+450)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="fricas")

[Out] 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9))*(x + sqrt(5)) + sqrt(5)*x + 9) + 1/2*x + 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/5*sqrt(5)*arctan(1/2*sqrt(2)*sqrt(x^2 - sqrt(x^2 + 9))*(x - sqrt(5)) - sqrt(5)*x + 9) + 1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^2 + 9)*(x + sqrt(5)) + 50*sqrt(5)*x + 450) - 1/10*sqrt(5)*log(50*x^2 - 50*sqrt(x^2 + 9)*(x - sqrt(5)) - 50*sqrt(5)*x + 450)

giac [B] time = 0.44, size = 123, normalized size = 2.32

$$-\frac{1}{10}\sqrt{5}\arctan\left(\frac{1}{2}x-\frac{1}{2}\sqrt{5}-\frac{1}{2}\sqrt{x^2+9}\right)-\frac{1}{10}\sqrt{5}\arctan\left(-\frac{1}{2}x-\frac{1}{2}\sqrt{5}+\frac{1}{2}\sqrt{x^2+9}\right)+\frac{1}{10}\sqrt{5}\log\left(\left(x-\sqrt{x^2+9}\right)^2+2\sqrt{5}\left(x-\sqrt{x^2+9}\right)+9\right)-\frac{1}{10}\sqrt{5}\log\left(\left(x+\sqrt{x^2+9}\right)^2-2\sqrt{5}\left(x+\sqrt{x^2+9}\right)+9\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="giac")

[Out] -1/10*sqrt(5)*arctan(1/2*x - 1/2*sqrt(5) - 1/2*sqrt(x^2 + 9)) - 1/10*sqrt(5)*arctan(-1/2*x - 1/2*sqrt(5) + 1/2*sqrt(x^2 + 9)) + 1/10*sqrt(5)*log((x - sqrt(x^2 + 9))^2 + 2*sqrt(5)*(x - sqrt(x^2 + 9)) + 9) - 1/10*sqrt(5)*log((x + sqrt(x^2 + 9))^2 - 2*sqrt(5)*(x + sqrt(x^2 + 9)) + 9)

maple [A] time = 0.02, size = 39, normalized size = 0.74

$$-\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{\sqrt{x^2+9}\sqrt{5}}{5}\right)}{5}+\frac{\sqrt{5}\operatorname{arctan}\left(\frac{\sqrt{5}x}{2\sqrt{x^2+9}}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)/(x^2+4)/(x^2+9)^(1/2),x)

[Out] 1/10*arctan(1/2*x*5^(1/2)/(x^2+9)^(1/2))*5^(1/2)-1/5*arctanh(1/5*(x^2+9)^(1/2))*5^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{\sqrt{x^2+9}(x^2+4)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x^2+4)/(x^2+9)^(1/2),x, algorithm="maxima")

[Out] integrate((x + 1)/(sqrt(x^2 + 9)*(x^2 + 4)), x)

mupad [B] time = 3.59, size = 67, normalized size = 1.26

$$\sqrt{5}\left(\ln(x-2i)-\ln\left(\sqrt{5}\sqrt{x^2+9}+9+x2i\right)\right)\left(\frac{1}{10}-\frac{1}{20}i\right)+\sqrt{5}\left(\ln(x+2i)-\ln\left(\sqrt{5}\sqrt{x^2+9}+9-x2i\right)\right)\left(\frac{1}{10}+\frac{1}{20}i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)/((x^2 + 4)*(x^2 + 9)^(1/2)),x)

[Out] 5^(1/2)*(log(x - 2i) - log(x*2i + 5^(1/2)*(x^2 + 9)^(1/2) + 9))*(1/10 - 1i/20) + 5^(1/2)*(log(x + 2i) - log(5^(1/2)*(x^2 + 9)^(1/2) - x*2i + 9))*(1/10 + 1i/20)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x+1}{(x^2+4)\sqrt{x^2+9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)/(x**2+4)/(x**2+9)**(1/2),x)

[Out] Integral((x + 1)/((x**2 + 4)*sqrt(x**2 + 9)), x)

$$3.517 \quad \int x \left(1 + \sqrt{1 - x^2}\right) dx$$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{3} (1 - x^2)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {14, 261}

$$\frac{x^2}{2} - \frac{1}{3} (1 - x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*(1 + Sqrt[1 - x^2]),x]

[Out] x^2/2 - (1 - x^2)^(3/2)/3

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x \left(1 + \sqrt{1 - x^2}\right) dx &= \int \left(x + x\sqrt{1 - x^2}\right) dx \\ &= \frac{x^2}{2} + \int x\sqrt{1 - x^2} dx \\ &= \frac{x^2}{2} - \frac{1}{3} (1 - x^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{3} (1 - x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + Sqrt[1 - x^2]),x]

[Out] x^2/2 - (1 - x^2)^(3/2)/3

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 1.30

$$\frac{1}{3} \sqrt{1 - x^2} (x^2 - 1) + \frac{1}{2} (x^2 - 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(1 + Sqrt[1 - x^2]),x]

[Out] $(-1 + x^2)/2 + (\text{Sqrt}[1 - x^2]*(-1 + x^2))/3$

fricas [A] time = 0.78, size = 22, normalized size = 0.96

$$\frac{1}{2}x^2 + \frac{1}{3}(x^2 - 1)\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")`

[Out] $1/2*x^2 + 1/3*(x^2 - 1)*\text{sqrt}(-x^2 + 1)$

giac [A] time = 0.33, size = 18, normalized size = 0.78

$$\frac{1}{2}x^2 - \frac{1}{3}(-x^2 + 1)^{\frac{3}{2}} - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")`

[Out] $1/2*x^2 - 1/3*(-x^2 + 1)^{(3/2)} - 1/2$

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{x^2}{2} - \frac{(-x^2 + 1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+(-x^2+1)^(1/2)),x)`

[Out] $1/2*x^2 - 1/3*(-x^2 + 1)^{(3/2)}$

maxima [A] time = 0.43, size = 17, normalized size = 0.74

$$\frac{1}{2}x^2 - \frac{1}{3}(-x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] $1/2*x^2 - 1/3*(-x^2 + 1)^{(3/2)}$

mupad [B] time = 0.03, size = 23, normalized size = 1.00

$$\frac{x^2}{2} + \sqrt{1 - x^2} \left(\frac{x^2}{3} - \frac{1}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((1 - x^2)^(1/2) + 1),x)`

[Out] $x^2/2 + (1 - x^2)^{(1/2)}*(x^2/3 - 1/3)$

sympy [A] time = 0.19, size = 27, normalized size = 1.17

$$\frac{x^2\sqrt{1 - x^2}}{3} + \frac{x^2}{2} - \frac{\sqrt{1 - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(-x**2+1)**(1/2)),x)`

[Out] $x**2*\text{sqrt}(1 - x**2)/3 + x**2/2 - \text{sqrt}(1 - x**2)/3$

$$3.518 \quad \int x \left(1 + \sqrt{1-x} \sqrt{1+x}\right) dx$$

Optimal. Leaf size=23

$$\frac{x^2}{2} - \frac{1}{3} (1-x^2)^{3/2}$$

Rubi [A] time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {14, 261}

$$\frac{x^2}{2} - \frac{1}{3} (1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]

[Out] x^2/2 - (1 - x^2)^(3/2)/3

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int x \left(1 + \sqrt{1-x} \sqrt{1+x}\right) dx &= \int \left(x + x\sqrt{1-x^2}\right) dx \\ &= \frac{x^2}{2} + \int x\sqrt{1-x^2} dx \\ &= \frac{x^2}{2} - \frac{1}{3} (1-x^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 23, normalized size = 1.00

$$\frac{x^2}{2} - \frac{1}{3} (1-x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]

[Out] x^2/2 - (1 - x^2)^(3/2)/3

IntegrateAlgebraic [A] time = 0.14, size = 46, normalized size = 2.00

$$\frac{1}{2} \left((x+1)^2 - 2(x+1) \right) + \frac{1}{3} \sqrt{1-x} \left((x+1)^{5/2} - 2(x+1)^{3/2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(1 + Sqrt[1 - x]*Sqrt[1 + x]),x]

[Out] $(-2*(1 + x) + (1 + x)^2)/2 + (\text{Sqrt}[1 - x]*(-2*(1 + x)^{3/2} + (1 + x)^{5/2}))/3$

fricas [A] time = 0.72, size = 25, normalized size = 1.09

$$\frac{1}{2}x^2 + \frac{1}{3}(x^2 - 1)\sqrt{x+1}\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(1-x)^(1/2))*(1+x)^(1/2)),x, algorithm="fricas")`

[Out] $1/2*x^2 + 1/3*(x^2 - 1)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1)$

giac [B] time = 0.45, size = 54, normalized size = 2.35

$$\frac{1}{2}(x+1)^2 + \frac{1}{6}((2x-5)(x+1)+9)\sqrt{x+1}\sqrt{-x+1} + \frac{1}{2}\sqrt{x+1}(x-2)\sqrt{-x+1} - x - 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(1-x)^(1/2))*(1+x)^(1/2)),x, algorithm="giac")`

[Out] $1/2*(x + 1)^2 + 1/6*((2*x - 5)*(x + 1) + 9)*\text{sqrt}(x + 1)*\text{sqrt}(-x + 1) + 1/2*\text{sqrt}(x + 1)*(x - 2)*\text{sqrt}(-x + 1) - x - 1$

maple [A] time = 0.00, size = 26, normalized size = 1.13

$$\frac{x^2}{2} + \frac{\sqrt{x+1}\sqrt{-x+1}(x^2-1)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+(-x+1)^(1/2))*(x+1)^(1/2)),x)`

[Out] $1/3*(x+1)^(1/2)*(-x+1)^(1/2)*(x^2-1)+1/2*x^2$

maxima [A] time = 0.97, size = 17, normalized size = 0.74

$$\frac{1}{2}x^2 - \frac{1}{3}(-x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+(1-x)^(1/2))*(1+x)^(1/2)),x, algorithm="maxima")`

[Out] $1/2*x^2 - 1/3*(-x^2 + 1)^{3/2}$

mupad [B] time = 3.69, size = 35, normalized size = 1.52

$$\frac{x^2}{2} - \frac{\sqrt{1-x}\left(-\frac{x^3}{3} - \frac{x^2}{3} + \frac{x}{3} + \frac{1}{3}\right)}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*((1-x)^(1/2)*(x+1)^(1/2)+1),x)`

[Out] $x^2/2 - ((1-x)^(1/2)*(x/3 - x^2/3 - x^3/3 + 1/3))/(x+1)^(1/2)$

sympy [A] time = 92.60, size = 105, normalized size = 4.57

$$-x + \frac{(x+1)^2}{2} - 2 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} + \frac{\arcsin\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \text{ for } x \geq -1 \wedge x < 1 \right\} + 2 \left(\left\{ \frac{x\sqrt{1-x}\sqrt{x+1}}{4} - \frac{(1-x)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{6} + \frac{\arcsin\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right)}{2} \text{ for } x \geq -1 \wedge x < 1 \right\} \right)^{-1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+(1-x)**(1/2)*(1+x)**(1/2)),x)
```

```
[Out] -x + (x + 1)**2/2 - 2*Piecewise((x*sqrt(1 - x)*sqrt(x + 1)/4 + asin(sqrt(2)
*sqrt(x + 1)/2)/2, (x >= -1) & (x < 1))) + 2*Piecewise((x*sqrt(1 - x)*sqrt(
x + 1)/4 - (1 - x)**(3/2)*(x + 1)**(3/2)/6 + asin(sqrt(2)*sqrt(x + 1)/2)/2,
(x >= -1) & (x < 1))) - 1
```

$$3.519 \quad \int x \left(1 + \frac{1}{\sqrt{2+x} \sqrt{3+x}} \right) dx$$

Optimal. Leaf size=33

$$\frac{x^2}{2} + \sqrt{x+2} \sqrt{x+3} - 5 \sinh^{-1}(\sqrt{x+2})$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {14, 80, 54, 215}

$$\frac{x^2}{2} + \sqrt{x+2} \sqrt{x+3} - 5 \sinh^{-1}(\sqrt{x+2})$$

Antiderivative was successfully verified.

[In] Int[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]

[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]

Rule 14

Int[(u_)*((c_)*(x_))^(m_), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 54

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 80

Int[((a_) + (b_)*(x_))*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(d*f*(n + p + 2)), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int x \left(1 + \frac{1}{\sqrt{2+x} \sqrt{3+x}} \right) dx &= \int \left(x + \frac{x}{\sqrt{2+x} \sqrt{3+x}} \right) dx \\ &= \frac{x^2}{2} + \int \frac{x}{\sqrt{2+x} \sqrt{3+x}} dx \\ &= \frac{x^2}{2} + \sqrt{2+x} \sqrt{3+x} - \frac{5}{2} \int \frac{1}{\sqrt{2+x} \sqrt{3+x}} dx \\ &= \frac{x^2}{2} + \sqrt{2+x} \sqrt{3+x} - 5 \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{2+x} \right) \\ &= \frac{x^2}{2} + \sqrt{2+x} \sqrt{3+x} - 5 \sinh^{-1}(\sqrt{2+x}) \end{aligned}$$

Mathematica [A] time = 0.01, size = 33, normalized size = 1.00

$$\frac{x^2}{2} + \sqrt{x+2}\sqrt{x+3} - 5 \sinh^{-1}(\sqrt{x+2})$$

Antiderivative was successfully verified.

[In] Integrate[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]

[Out] x^2/2 + Sqrt[2 + x]*Sqrt[3 + x] - 5*ArcSinh[Sqrt[2 + x]]

IntegrateAlgebraic [A] time = 0.14, size = 51, normalized size = 1.55

$$\frac{1}{2} \left((x+3)^2 - 6(x+3) \right) + \sqrt{x+2}\sqrt{x+3} + 5 \log(\sqrt{x+2} - \sqrt{x+3})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*(1 + 1/(Sqrt[2 + x]*Sqrt[3 + x])),x]

[Out] Sqrt[2 + x]*Sqrt[3 + x] + (-6*(3 + x) + (3 + x)^2)/2 + 5*Log[Sqrt[2 + x] - Sqrt[3 + x]]

fricas [A] time = 0.73, size = 37, normalized size = 1.12

$$\frac{1}{2} x^2 + \sqrt{x+3}\sqrt{x+2} + \frac{5}{2} \log(2\sqrt{x+3}\sqrt{x+2} - 2x - 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/(2+x)^(1/2))/(3+x)^(1/2)),x, algorithm="fricas")

[Out] 1/2*x^2 + sqrt(x + 3)*sqrt(x + 2) + 5/2*log(2*sqrt(x + 3)*sqrt(x + 2) - 2*x - 5)

giac [A] time = 0.38, size = 39, normalized size = 1.18

$$\frac{1}{2} (x+3)^2 + \sqrt{x+3}\sqrt{x+2} - 3x + 5 \log(\sqrt{x+3} - \sqrt{x+2}) - 9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/(2+x)^(1/2))/(3+x)^(1/2)),x, algorithm="giac")

[Out] 1/2*(x + 3)^2 + sqrt(x + 3)*sqrt(x + 2) - 3*x + 5*log(sqrt(x + 3) - sqrt(x + 2)) - 9

maple [B] time = 0.01, size = 58, normalized size = 1.76

$$\frac{x^2}{2} - \frac{\sqrt{x+2}\sqrt{x+3} \left(5 \ln \left(x + \frac{5}{2} + \sqrt{x^2 + 5x + 6} \right) - 2\sqrt{x^2 + 5x + 6} \right)}{2\sqrt{x^2 + 5x + 6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1+1/(x+2)^(1/2))/(x+3)^(1/2)),x)

[Out] -1/2*(x+2)^(1/2)*(x+3)^(1/2)*(-2*(x^2+5*x+6)^(1/2)+5*ln(x+5/2+(x^2+5*x+6)^(1/2)))/(x^2+5*x+6)^(1/2)+1/2*x^2

maxima [A] time = 0.43, size = 36, normalized size = 1.09

$$\frac{1}{2} x^2 + \sqrt{x^2 + 5x + 6} - \frac{5}{2} \log(2x + 2\sqrt{x^2 + 5x + 6} + 5)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/(2+x)^(1/2))/(3+x)^(1/2)),x, algorithm="maxima")

[Out] 1/2*x^2 + sqrt(x^2 + 5*x + 6) - 5/2*log(2*x + 2*sqrt(x^2 + 5*x + 6) + 5)

mupad [B] time = 7.56, size = 180, normalized size = 5.45

$$\frac{\frac{10(\sqrt{x+2}-\sqrt{2})}{\sqrt{x+3}-\sqrt{3}} + \frac{10(\sqrt{x+2}-\sqrt{2})^3}{(\sqrt{x+3}-\sqrt{3})^3} - \frac{8\sqrt{6}(\sqrt{x+2}-\sqrt{2})^2}{(\sqrt{x+3}-\sqrt{3})^2}}{\frac{(\sqrt{x+2}-\sqrt{2})^4}{(\sqrt{x+3}-\sqrt{3})^4} - \frac{2(\sqrt{x+2}-\sqrt{2})^2}{(\sqrt{x+3}-\sqrt{3})^2} + 1}} - 10 \operatorname{atanh}\left(\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+3}-\sqrt{3}}\right) + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(1/((x + 2)^(1/2)*(x + 3)^(1/2)) + 1),x)

[Out] ((10*((x + 2)^(1/2) - 2^(1/2)))/((x + 3)^(1/2) - 3^(1/2)) + (10*((x + 2)^(1/2) - 2^(1/2))^3)/((x + 3)^(1/2) - 3^(1/2))^3 - (8*6^(1/2)*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2)/(((x + 2)^(1/2) - 2^(1/2))^4)/((x + 3)^(1/2) - 3^(1/2))^4 - (2*((x + 2)^(1/2) - 2^(1/2))^2)/((x + 3)^(1/2) - 3^(1/2))^2 + 1) - 10*atanh(((x + 2)^(1/2) - 2^(1/2))/((x + 3)^(1/2) - 3^(1/2))) + x^2/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(\sqrt{x+2}\sqrt{x+3}+1)}{\sqrt{x+2}\sqrt{x+3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(1+1/(2+x)**(1/2))/(3+x)**(1/2)),x)

[Out] Integral(x*(sqrt(x + 2)*sqrt(x + 3) + 1)/(sqrt(x + 2)*sqrt(x + 3)), x)

$$3.520 \quad \int \frac{x - \sqrt{x^6}}{x(1-x^4)} dx$$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.16, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[x^6])/(x*(1 - x^4)), x]

[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :=> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :=> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :=> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :=> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :=> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{x - \sqrt{x^6}}{x(1 - x^4)} dx &= \int \left(\frac{1}{1 - x^4} + \frac{\sqrt{x^6}}{x(-1 + x^4)} \right) dx \\
&= \int \frac{1}{1 - x^4} dx + \int \frac{\sqrt{x^6}}{x(-1 + x^4)} dx \\
&= \frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{1}{2} \int \frac{1}{1 + x^2} dx + \frac{\sqrt{x^6} \int \frac{x^2}{-1+x^4} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \int \frac{1}{1-x^2} dx}{2x^3} + \frac{\sqrt{x^6} \int \frac{1}{1+x^2} dx}{2x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 27, normalized size = 0.60

$$\frac{1}{2} \left(\frac{\sqrt{x^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[x^6])/(x*(1 - x^4)), x]

[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2

IntegrateAlgebraic [A] time = 1.25, size = 57, normalized size = 1.27

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{2} \tanh^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{4} \log(1 - x) + \frac{1}{4} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[x^6])/(x*(1 - x^4)), x]

[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 - x]/4 + Log[1 + x]/4

fricas [A] time = 1.06, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1), x, algorithm="fricas")

[Out] arctan(x)

giac [A] time = 0.40, size = 31, normalized size = 0.69

$$\frac{1}{2} (\operatorname{sgn}(x) + 1) \arctan(x) - \frac{1}{4} (\operatorname{sgn}(x) - 1) \log(|x + 1|) + \frac{1}{4} (\operatorname{sgn}(x) - 1) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/x/(-x^4+1), x, algorithm="giac")

[Out] $\frac{1}{2}*(\text{sgn}(x) + 1)*\arctan(x) - \frac{1}{4}*(\text{sgn}(x) - 1)*\log(\text{abs}(x + 1)) + \frac{1}{4}*(\text{sgn}(x) - 1)*\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\arctan(x)}{2} + \frac{\sqrt{x^6} (2 \arctan(x) + \ln(x - 1) - \ln(x + 1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x-(x^6)^(1/2))/x/(-x^4+1),x)`

[Out] $\frac{1}{4}*(x^6)^{(1/2)}*(\ln(x-1)-\ln(x+1)+2*\arctan(x))/x^3+\frac{1}{2}*\operatorname{arctanh}(x)+\frac{1}{2}*\arctan(x)$

maxima [A] time = 0.96, size = 2, normalized size = 0.04

$\arctan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x-(x^6)^(1/2))/x/(-x^4+1),x, algorithm="maxima")`

[Out] $\arctan(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x - \sqrt{x^6}}{x(x^4 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)),x)`

[Out] `int(-(x - (x^6)^(1/2))/(x*(x^4 - 1)), x)`

sympy [A] time = 0.11, size = 2, normalized size = 0.04

$\operatorname{atan}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x-(x**6)**(1/2))/x/(-x**4+1),x)`

[Out] $\operatorname{atan}(x)$

$$3.521 \quad \int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx$$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[x^6]/x)/(1 - x^4), x]

[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 6725

Int[(u_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1 - \frac{\sqrt{x^6}}{x}}{1 - x^4} dx &= \int \left(\frac{1}{1 - x^4} + \frac{\sqrt{x^6}}{x(-1 + x^4)} \right) dx \\
&= \int \frac{1}{1 - x^4} dx + \int \frac{\sqrt{x^6}}{x(-1 + x^4)} dx \\
&= \frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{1}{2} \int \frac{1}{1 + x^2} dx + \frac{\sqrt{x^6} \int \frac{x^2}{-1 + x^4} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \int \frac{1}{1 - x^2} dx}{2x^3} + \frac{\sqrt{x^6} \int \frac{1}{1 + x^2} dx}{2x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{1}{2} \left(\frac{\sqrt{x^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[x^6]/x)/(1 - x^4), x]

[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2

IntegrateAlgebraic [A] time = 1.30, size = 57, normalized size = 1.27

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{2} \tanh^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{4} \log(1 - x) + \frac{1}{4} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[x^6]/x)/(1 - x^4), x]

[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 - x]/4 + Log[1 + x]/4

fricas [A] time = 0.83, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="fricas")

[Out] arctan(x)

giac [A] time = 0.39, size = 31, normalized size = 0.69

$$\frac{1}{2} (\operatorname{sgn}(x) + 1) \arctan(x) - \frac{1}{4} (\operatorname{sgn}(x) - 1) \log(|x + 1|) + \frac{1}{4} (\operatorname{sgn}(x) - 1) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="giac")

[Out] $\frac{1}{2}(\operatorname{sgn}(x) + 1)\arctan(x) - \frac{1}{4}(\operatorname{sgn}(x) - 1)\log(\operatorname{abs}(x + 1)) + \frac{1}{4}(\operatorname{sgn}(x) - 1)\log(\operatorname{abs}(x - 1))$

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\arctan(x)}{2} + \frac{\sqrt{x^6} (2 \arctan(x) + \ln(x - 1) - \ln(x + 1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-(x^6)^(1/2)/x)/(-x^4+1),x)`

[Out] $\frac{1}{4}(x^6)^{1/2}(2\arctan(x)+\ln(x-1)-\ln(x+1))/x^3+1/2\operatorname{arctanh}(x)+1/2\arctan(x)$

maxima [A] time = 0.97, size = 2, normalized size = 0.04

$\arctan(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(x^6)^(1/2)/x)/(-x^4+1),x, algorithm="maxima")`

[Out] $\arctan(x)$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\frac{\sqrt{x^6} - 1}{x}}{x^4 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x^6)^(1/2)/x - 1)/(x^4 - 1),x)`

[Out] `int(((x^6)^(1/2)/x - 1)/(x^4 - 1), x)`

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$\operatorname{atan}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-(x**6)**(1/2)/x)/(-x**4+1),x)`

[Out] $\operatorname{atan}(x)$

$$3.522 \quad \int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.10, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1593, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[x^6])/(x - x^5), x]

[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x - \sqrt{x^6}}{x - x^5} dx &= \int \frac{x - \sqrt{x^6}}{x(1 - x^4)} dx \\
&= \int \left(\frac{1}{1 - x^4} + \frac{\sqrt{x^6}}{x(-1 + x^4)} \right) dx \\
&= \int \frac{1}{1 - x^4} dx + \int \frac{\sqrt{x^6}}{x(-1 + x^4)} dx \\
&= \frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{1}{2} \int \frac{1}{1 + x^2} dx + \frac{\sqrt{x^6} \int \frac{x^2}{-1 + x^4} dx}{x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \int \frac{1}{1 - x^2} dx}{2x^3} + \frac{\sqrt{x^6} \int \frac{1}{1 + x^2} dx}{2x^3} \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{1}{2} \left(\frac{\sqrt{x^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x - Sqrt[x^6])/(x - x^5), x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

IntegrateAlgebraic [A] time = 1.27, size = 57, normalized size = 1.27

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{2} \tanh^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{4} \log(1 - x) + \frac{1}{4} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x - Sqrt[x^6])/(x - x^5), x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

fricas [A] time = 1.42, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x-(x^6)^(1/2))/(-x^5+x), x, algorithm="fricas")
```

```
[Out] arctan(x)
```

giac [A] time = 0.36, size = 31, normalized size = 0.69

$$\frac{1}{2}(\operatorname{sgn}(x) + 1) \arctan(x) - \frac{1}{4}(\operatorname{sgn}(x) - 1) \log(|x + 1|) + \frac{1}{4}(\operatorname{sgn}(x) - 1) \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="giac")

[Out] 1/2*(sgn(x) + 1)*arctan(x) - 1/4*(sgn(x) - 1)*log(abs(x + 1)) + 1/4*(sgn(x) - 1)*log(abs(x - 1))

maple [A] time = 0.01, size = 35, normalized size = 0.78

$$\frac{\operatorname{arctanh}(x)}{2} + \frac{\arctan(x)}{2} + \frac{\sqrt{x^6} (2 \arctan(x) + \ln(x - 1) - \ln(x + 1))}{4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-(x^6)^(1/2))/(-x^5+x),x)

[Out] 1/4*(x^6)^(1/2)*(2*arctan(x)+ln(x-1)-ln(x+1))/x^3+1/2*arctanh(x)+1/2*arctan(x)

maxima [A] time = 0.98, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x^6)^(1/2))/(-x^5+x),x, algorithm="maxima")

[Out] arctan(x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x - \sqrt{x^6}}{x - x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - (x^6)^(1/2))/(x - x^5),x)

[Out] int((x - (x^6)^(1/2))/(x - x^5), x)

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-(x**6)**(1/2))/(-x**5+x),x)

[Out] atan(x)

$$3.523 \quad \int \frac{x}{x + \sqrt{x^6}} dx$$

Optimal. Leaf size=45

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Rubi [A] time = 0.13, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {6729, 1584, 6725, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3} + \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/(x + Sqrt[x^6]),x]

[Out] ArcTan[x]/2 + (Sqrt[x^6]*ArcTan[x])/(2*x^3) + ArcTanh[x]/2 - (Sqrt[x^6]*ArcTanh[x])/(2*x^3)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6725

```
Int[(u_)/((a_)+(b_)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_)/((a_)*(x_)^(m_) + (b_)*Sqrt[(c_)*(x_)^(n_)]), x_Symbol] := In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
c, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{x + \sqrt{x^6}} dx &= \int \frac{x(x - \sqrt{x^6})}{x^2 - x^6} dx \\
&= \int \frac{x - \sqrt{x^6}}{x(1 - x^4)} dx \\
&= \int \left(\frac{1}{1 - x^4} + \frac{\sqrt{x^6}}{x(-1 + x^4)} \right) dx \\
&= \int \frac{1}{1 - x^4} dx + \int \frac{\sqrt{x^6}}{x(-1 + x^4)} dx \\
&= \frac{1}{2} \int \frac{1}{1 - x^2} dx + \frac{1}{2} \int \frac{1}{1 + x^2} dx + \frac{\sqrt{x^6}}{x^3} \int \frac{x^2}{-1 + x^4} dx \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6}}{2x^3} \int \frac{1}{1 - x^2} dx + \frac{\sqrt{x^6}}{2x^3} \int \frac{1}{1 + x^2} dx \\
&= \frac{1}{2} \tan^{-1}(x) + \frac{\sqrt{x^6} \tan^{-1}(x)}{2x^3} + \frac{1}{2} \tanh^{-1}(x) - \frac{\sqrt{x^6} \tanh^{-1}(x)}{2x^3}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 0.60

$$\frac{1}{2} \left(\frac{\sqrt{x^6} (\tan^{-1}(x) - \tanh^{-1}(x))}{x^3} + \tan^{-1}(x) + \tanh^{-1}(x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x + Sqrt[x^6]), x]
```

```
[Out] (ArcTan[x] + (Sqrt[x^6]*(ArcTan[x] - ArcTanh[x]))/x^3 + ArcTanh[x])/2
```

IntegrateAlgebraic [A] time = 1.34, size = 57, normalized size = 1.27

$$-\frac{1}{2} \tan^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{2} \tanh^{-1} \left(\frac{\sqrt{x^6}}{x^4} \right) - \frac{1}{4} \log(1 - x) + \frac{1}{4} \log(x + 1) + \frac{1}{2} \tan^{-1}(x)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x + Sqrt[x^6]), x]
```

```
[Out] ArcTan[x]/2 - ArcTan[Sqrt[x^6]/x^4]/2 - ArcTanh[Sqrt[x^6]/x^4]/2 - Log[1 -
x]/4 + Log[1 + x]/4
```

fricas [A] time = 0.47, size = 2, normalized size = 0.04

arctan(x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="fricas")

[Out] arctan(x)

giac [A] time = 0.37, size = 12, normalized size = 0.27

$$\frac{\arctan\left(x\sqrt{\operatorname{sgn}(x)}\right)}{\sqrt{\operatorname{sgn}(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="giac")

[Out] arctan(x*sqrt(sgn(x)))/sqrt(sgn(x))

maple [A] time = 0.01, size = 27, normalized size = 0.60

$$\frac{\arctan\left(\sqrt{\frac{\sqrt{x^6}}{x^3}}x\right)}{\sqrt{\frac{\sqrt{x^6}}{x^3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+(x^6)^(1/2)),x)

[Out] 1/((x^6)^(1/2)/x^3)^(1/2)*arctan(((x^6)^(1/2)/x^3)^(1/2)*x)

maxima [A] time = 0.96, size = 2, normalized size = 0.04

$$\arctan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(x^6)^(1/2)),x, algorithm="maxima")

[Out] arctan(x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x}{x + \sqrt{x^6}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + (x^6)^(1/2)),x)

[Out] int(x/(x + (x^6)^(1/2)), x)

sympy [A] time = 0.10, size = 2, normalized size = 0.04

$$\operatorname{atan}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x+(x**6)**(1/2)),x)

[Out] atan(x)

$$3.524 \quad \int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx$$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.18, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] - Sqrt[x^3])/(x - x^3), x]

[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] := Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1593

$\text{Int}[(u_.)*((a_.)*(x_.)^{(p_.)} + (b_.)*(x_.)^{(q_.)})^{(n_.)}, x_Symbol] \rightarrow \text{Int}[u*x^{(n*p)}*(a + b*x^{(q-p)})^n, x] /;$ FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6725

$\text{Int}[(u_.)/((a_.) + (b_.)*(x_.)^{(n_.)}), x_Symbol] \rightarrow \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /;$ SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx &= \int \frac{\sqrt{x} - \sqrt{x^3}}{x(1 - x^2)} dx \\ &= \int \left(-\frac{1}{\sqrt{x}(-1 + x^2)} + \frac{\sqrt{x^3}}{x(-1 + x^2)} \right) dx \\ &= -\int \frac{1}{\sqrt{x}(-1 + x^2)} dx + \int \frac{\sqrt{x^3}}{x(-1 + x^2)} dx \\ &= -\left(2 \text{Subst} \left(\int \frac{1}{-1 + x^4} dx, x, \sqrt{x} \right) \right) + \frac{\sqrt{x^3} \int \frac{\sqrt{x}}{-1 + x^2} dx}{x^{3/2}} \\ &= \frac{(2\sqrt{x^3}) \text{Subst} \left(\int \frac{x^2}{-1 + x^4} dx, x, \sqrt{x} \right)}{x^{3/2}} + \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right) + \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{x} \right) \\ &= \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x}) - \frac{\sqrt{x^3} \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right)}{x^{3/2}} + \frac{\sqrt{x^3} \text{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{x} \right)}{x^{3/2}} \\ &= \tan^{-1}(\sqrt{x}) + \frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} + \tanh^{-1}(\sqrt{x}) - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 49, normalized size = 0.94

$$\frac{(x^{3/2} + \sqrt{x^3}) \tan^{-1}(\sqrt{x}) + (x^{3/2} - \sqrt{x^3}) \tanh^{-1}(\sqrt{x})}{x^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[x] - Sqrt[x^3])/(x - x^3), x]

[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt[x]])/x^(3/2)

IntegrateAlgebraic [A] time = 2.78, size = 39, normalized size = 0.75

$$\tan^{-1} \left(\frac{\sqrt{x^3}}{x} \right) - \tanh^{-1} \left(\frac{\sqrt{x^3}}{x} \right) + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x] - Sqrt[x^3])/(x - x^3), x]

[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3]/x]

fricas [A] time = 0.81, size = 6, normalized size = 0.12

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="fricas")

[Out] 2*arctan(sqrt(x))

giac [A] time = 0.32, size = 6, normalized size = 0.12

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="giac")

[Out] 2*arctan(sqrt(x))

maple [A] time = 0.01, size = 41, normalized size = 0.79

$$\operatorname{arctanh}(\sqrt{x}) + \arctan(\sqrt{x}) + \frac{\sqrt{x^3} (2 \arctan(\sqrt{x}) + \ln(\sqrt{x} - 1) - \ln(\sqrt{x} + 1))}{2x^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x)

[Out] arctanh(x^(1/2))+arctan(x^(1/2))+1/2*(x^3)^(1/2)*(ln(x^(1/2)-1)-ln(x^(1/2)+1)+2*arctan(x^(1/2)))/x^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\arctan(\sqrt{x}) - \int \frac{\sqrt{x}}{2(x+1)} dx + \int \frac{1}{4(\sqrt{x}+1)} dx + \int \frac{1}{4(\sqrt{x}-1)} dx + \frac{1}{2} \log(\sqrt{x}+1) - \frac{1}{2} \log(\sqrt{x}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/2)-(x^3)^(1/2))/(-x^3+x),x, algorithm="maxima")

[Out] arctan(sqrt(x)) - integrate(1/2*sqrt(x)/(x + 1), x) + integrate(1/4/(sqrt(x) + 1), x) + integrate(1/4/(sqrt(x) - 1), x) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$- \int \frac{\sqrt{x^3} - \sqrt{x}}{x - x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-((x^3)^(1/2) - x^(1/2))/(x - x^3),x)

[Out] -int(((x^3)^(1/2) - x^(1/2))/(x - x^3), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{x}}{x^3 - x} dx - \int \left(-\frac{\sqrt{x^3}}{x^3 - x} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(1/2)-(x**3)**(1/2))/(-x**3+x),x)

[Out] -Integral(sqrt(x)/(x**3 - x), x) - Integral(-sqrt(x**3)/(x**3 - x), x)

$$3.525 \quad \int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Optimal. Leaf size=52

$$\frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.13, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}} + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[x] + Sqrt[x^3])^(-1), x]

[Out] ArcTan[Sqrt[x]] + (Sqrt[x^3]*ArcTan[Sqrt[x]])/x^(3/2) + ArcTanh[Sqrt[x]] - (Sqrt[x^3]*ArcTanh[Sqrt[x]])/x^(3/2)

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 6729

```
Int[(u_.)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_.)]), x_Symbol] := In
t[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b,
c, m, n}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx &= \int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx \\
&= \int \frac{\sqrt{x} - \sqrt{x^3}}{x(1 - x^2)} dx \\
&= \int \left(-\frac{1}{\sqrt{x}(-1 + x^2)} + \frac{\sqrt{x^3}}{x(-1 + x^2)} \right) dx \\
&= -\int \frac{1}{\sqrt{x}(-1 + x^2)} dx + \int \frac{\sqrt{x^3}}{x(-1 + x^2)} dx \\
&= -\left(2 \operatorname{Subst} \left(\int \frac{1}{-1 + x^4} dx, x, \sqrt{x} \right) \right) + \frac{\sqrt{x^3} \int \frac{\sqrt{x}}{-1 + x^2} dx}{x^{3/2}} \\
&= \frac{(2\sqrt{x^3}) \operatorname{Subst} \left(\int \frac{x^2}{-1 + x^4} dx, x, \sqrt{x} \right)}{x^{3/2}} + \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{x} \right) \\
&= \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x}) - \frac{\sqrt{x^3} \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right)}{x^{3/2}} + \frac{\sqrt{x^3} \operatorname{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt{x} \right)}{x^{3/2}} \\
&= \tan^{-1}(\sqrt{x}) + \frac{\sqrt{x^3} \tan^{-1}(\sqrt{x})}{x^{3/2}} + \tanh^{-1}(\sqrt{x}) - \frac{\sqrt{x^3} \tanh^{-1}(\sqrt{x})}{x^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 0.94

$$\frac{(x^{3/2} + \sqrt{x^3}) \tan^{-1}(\sqrt{x}) + (x^{3/2} - \sqrt{x^3}) \tanh^{-1}(\sqrt{x})}{x^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[x] + Sqrt[x^3])^(-1), x]
```

```
[Out] ((x^(3/2) + Sqrt[x^3])*ArcTan[Sqrt[x]] + (x^(3/2) - Sqrt[x^3])*ArcTanh[Sqrt
[x]])/x^(3/2)
```

IntegrateAlgebraic [A] time = 2.58, size = 39, normalized size = 0.75

$$\tan^{-1}\left(\frac{\sqrt{x^3}}{x}\right) - \tanh^{-1}\left(\frac{\sqrt{x^3}}{x}\right) + \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[x] + Sqrt[x^3])^(-1), x]

[Out] ArcTan[Sqrt[x]] + ArcTan[Sqrt[x^3]/x] + ArcTanh[Sqrt[x]] - ArcTanh[Sqrt[x^3]/x]

fricas [A] time = 1.41, size = 6, normalized size = 0.12

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(x^3)^(1/2)), x, algorithm="fricas")

[Out] 2*arctan(sqrt(x))

giac [A] time = 0.32, size = 6, normalized size = 0.12

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(x^3)^(1/2)), x, algorithm="giac")

[Out] 2*arctan(sqrt(x))

maple [A] time = 0.01, size = 30, normalized size = 0.58

$$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{x^3}}{x^2}} \sqrt{x}\right)}{\sqrt{\frac{\sqrt{x^3}}{x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)+(x^3)^(1/2)), x)

[Out] 2/((x^3)^(1/2)/x^(3/2))^(1/2)*arctan(x^(1/2)*((x^3)^(1/2)/x^(3/2))^(1/2))

maxima [A] time = 0.99, size = 6, normalized size = 0.12

$$2 \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^(1/2)+(x^3)^(1/2)), x, algorithm="maxima")

[Out] 2*arctan(sqrt(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{x^3} + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((x^3)^(1/2) + x^(1/2)),x)
```

```
[Out] int(1/((x^3)^(1/2) + x^(1/2)), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(x**(1/2)+(x**3)**(1/2)),x)
```

```
[Out] Integral(1/(sqrt(x) + sqrt(x**3)), x)
```


$$3.526 \quad \int \frac{1}{\sqrt{-1+x} + \sqrt{(-1+x)^3}} dx$$

Optimal. Leaf size=68

$$\frac{\sqrt{(x-1)^3} \tan^{-1}(\sqrt{x-1})}{(x-1)^{3/2}} + \tan^{-1}(\sqrt{x-1}) - \frac{\sqrt{(x-1)^3} \tanh^{-1}(\sqrt{x-1})}{(x-1)^{3/2}} + \tanh^{-1}(\sqrt{x-1})$$

Rubi [A] time = 0.16, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {6729, 1593, 6725, 329, 212, 206, 203, 15, 298}

$$\frac{\sqrt{(x-1)^3} \tan^{-1}(\sqrt{x-1})}{(x-1)^{3/2}} + \tan^{-1}(\sqrt{x-1}) - \frac{\sqrt{(x-1)^3} \tanh^{-1}(\sqrt{x-1})}{(x-1)^{3/2}} + \tanh^{-1}(\sqrt{x-1})$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1), x]

[Out] ArcTan[Sqrt[-1 + x]] + (Sqrt[(-1 + x)^3]*ArcTan[Sqrt[-1 + x]])/(-1 + x)^(3/2) + ArcTanh[Sqrt[-1 + x]] - (Sqrt[(-1 + x)^3]*ArcTanh[Sqrt[-1 + x]])/(-1 + x)^(3/2)

Rule 15

Int[(u_)*((a_)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 298

Int[(x_)^2/((a_) + (b_)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_)*(x_)^(m_))*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F

ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rule 6729

Int[(u_)/((a_.)*(x_)^(m_.) + (b_.)*Sqrt[(c_.)*(x_)^(n_.)]), x_Symbol] := Int[(u*(a*x^m - b*Sqrt[c*x^n]))/(a^2*x^(2*m) - b^2*c*x^n), x] /; FreeQ[{a, b, c, m, n}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{-1+x} + \sqrt{(-1+x)^3}} dx &= \text{Subst} \left(\int \frac{1}{\sqrt{x} + \sqrt{x^3}} dx, x, -1+x \right) \\
 &= \text{Subst} \left(\int \frac{\sqrt{x} - \sqrt{x^3}}{x - x^3} dx, x, -1+x \right) \\
 &= \text{Subst} \left(\int \frac{\sqrt{x} - \sqrt{x^3}}{x(1-x^2)} dx, x, -1+x \right) \\
 &= \text{Subst} \left(\int \left(-\frac{1}{\sqrt{x}(-1+x^2)} + \frac{\sqrt{x^3}}{x(-1+x^2)} \right) dx, x, -1+x \right) \\
 &= -\text{Subst} \left(\int \frac{1}{\sqrt{x}(-1+x^2)} dx, x, -1+x \right) + \text{Subst} \left(\int \frac{\sqrt{x^3}}{x(-1+x^2)} dx, x, -1+x \right) \\
 &= -\left(2 \text{Subst} \left(\int \frac{1}{-1+x^4} dx, x, \sqrt{-1+x} \right) \right) + \frac{\sqrt{(-1+x)^3} \text{Subst} \left(\int \frac{\sqrt{x}}{-1+x^2} dx, x, -1+x \right)}{(-1+x)^{3/2}} \\
 &= \frac{(2\sqrt{(-1+x)^3}) \text{Subst} \left(\int \frac{x^2}{-1+x^4} dx, x, \sqrt{-1+x} \right)}{(-1+x)^{3/2}} + \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{-1+x} \right) \\
 &= \tan^{-1}(\sqrt{-1+x}) + \tanh^{-1}(\sqrt{-1+x}) - \frac{\sqrt{(-1+x)^3} \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{-1+x} \right)}{(-1+x)^{3/2}} \\
 &= \tan^{-1}(\sqrt{-1+x}) + \frac{\sqrt{(-1+x)^3} \tan^{-1}(\sqrt{-1+x})}{(-1+x)^{3/2}} + \tanh^{-1}(\sqrt{-1+x}) - \frac{\sqrt{(-1+x)^3} \tanh^{-1}(\sqrt{-1+x})}{(-1+x)^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.16, size = 64, normalized size = 0.94

$$\left(\frac{\sqrt{(x-1)^3}}{(x-1)^{3/2}} + 1 \right) \tan^{-1}(\sqrt{x-1}) + \frac{((x-1)^{3/2} - \sqrt{(x-1)^3}) \tanh^{-1}(\sqrt{x-1})}{(x-1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1), x]

[Out] (1 + Sqrt[(-1 + x)^3]/(-1 + x)^(3/2))*ArcTan[Sqrt[-1 + x]] + (((-1 + x)^(3/2) - Sqrt[(-1 + x)^3])*ArcTanh[Sqrt[-1 + x]])/(-1 + x)^(3/2)

IntegrateAlgebraic [A] time = 2.93, size = 67, normalized size = 0.99

$$\tan^{-1}\left(\frac{\sqrt{x^3 - 3x^2 + 3x - 1}}{x - 1}\right) - \tanh^{-1}\left(\frac{\sqrt{x^3 - 3x^2 + 3x - 1}}{x - 1}\right) + \tan^{-1}(\sqrt{x - 1}) + \tanh^{-1}(\sqrt{x - 1})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[-1 + x] + Sqrt[(-1 + x)^3])^(-1), x]

[Out] ArcTan[Sqrt[-1 + x]] + ArcTan[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)] + ArcTanh[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + 3*x - 3*x^2 + x^3]/(-1 + x)]

fricas [A] time = 0.50, size = 8, normalized size = 0.12

$$2 \arctan(\sqrt{x - 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)), x, algorithm="fricas")

[Out] 2*arctan(sqrt(x - 1))

giac [A] time = 0.41, size = 8, normalized size = 0.12

$$2 \arctan(\sqrt{x - 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)), x, algorithm="giac")

[Out] 2*arctan(sqrt(x - 1))

maple [A] time = 0.01, size = 40, normalized size = 0.59

$$\frac{2 \arctan\left(\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}} \sqrt{x-1}\right)}{\sqrt{\frac{\sqrt{(x-1)^3}}{(x-1)^{\frac{3}{2}}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x-1)^(1/2)+((x-1)^3)^(1/2)), x)

[Out] 2/(((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*arctan((((x-1)^3)^(1/2)/(x-1)^(3/2))^(1/2)*(x-1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$2\sqrt{x-1} - \int \frac{\sqrt{x-1}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)^(1/2)+((-1+x)^3)^(1/2)), x, algorithm="maxima")

[Out] 2*sqrt(x - 1) - integrate(sqrt(x - 1)/x, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{\sqrt{x-1} - \sqrt{(x-1)^3}}{(x-1)^3 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x - 1)^(1/2) + ((x - 1)^3)^(1/2)), x)

[Out] int(-((x - 1)^(1/2) - ((x - 1)^3)^(1/2))/((x - 1)^3 - x + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x-1} + \sqrt{(x-1)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1+x)**(1/2)+((-1+x)**3)**(1/2)), x)

[Out] Integral(1/(sqrt(x - 1) + sqrt((x - 1)**3)), x)

$$3.527 \quad \int \left(-\frac{3}{(4+5x)^2} - \frac{5+4x}{(4+5x)^2 \sqrt{1-x^2}} \right) dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {803}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 803

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && EqQ[c*d*f + a*e*g, 0]

Rubi steps

$$\begin{aligned} \int \left(-\frac{3}{(4+5x)^2} - \frac{5+4x}{(4+5x)^2 \sqrt{1-x^2}} \right) dx &= \frac{3}{5(4+5x)} - \int \frac{5+4x}{(4+5x)^2 \sqrt{1-x^2}} dx \\ &= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} \end{aligned}$$

Mathematica [A] time = 0.19, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2} + 3}{25x + 20}$$

Antiderivative was successfully verified.

[In] Integrate[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)

IntegrateAlgebraic [A] time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[-3/(4 + 5*x)^2 - (5 + 4*x)/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

fricas [A] time = 0.65, size = 25, normalized size = 0.81

$$\frac{25x + 20\sqrt{-x^2 + 1} + 32}{20(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)

giac [C] time = 0.44, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5x+4} + \frac{9}{(5x+4)^2} - 1}}{5 \operatorname{sgn}\left(\frac{1}{5x+4}\right)} + \frac{3}{5(5x+4)} - \frac{1}{5}i \operatorname{sgn}\left(\frac{1}{5x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4) - 1/5*I*sgn(1/(5*x + 4))

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}}}{5x + 4} + \frac{3}{5(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x)

[Out] 1/5/(x+4/5)*(-(x+4/5)^2+8/5*x+41/25)^(1/2)+3/5/(4+5*x)

maxima [A] time = 0.96, size = 27, normalized size = 0.87

$$\frac{\sqrt{-x^2 + 1}}{5x + 4} + \frac{3}{5(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/(4+5*x)^2+(-5-4*x)/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] sqrt(-x^2 + 1)/(5*x + 4) + 3/5/(5*x + 4)

mupad [B] time = 3.33, size = 19, normalized size = 0.61

$$\frac{\sqrt{1 - x^2} + \frac{3}{5}}{5x + 4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-3/(5*x + 4)^2 - (4*x + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)

[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4x}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx - \int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx - \int \frac{5}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-3/(4+5*x)**2+(-5-4*x)/(4+5*x)**2/(-x**2+1)**(1/2),x)

[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)

$$3.528 \quad \int \frac{-5-4x-3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}} dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Rubi [A] time = 0.29, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {6742, 731, 725, 206, 807}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 731

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]

Rule 807

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{-5 - 4x - 3\sqrt{1-x^2}}{(4+5x)^2\sqrt{1-x^2}} dx &= \int \left(-\frac{3}{(4+5x)^2} - \frac{5}{(4+5x)^2\sqrt{1-x^2}} - \frac{4x}{(4+5x)^2\sqrt{1-x^2}} \right) dx \\ &= \frac{3}{5(4+5x)} - 4 \int \frac{x}{(4+5x)^2\sqrt{1-x^2}} dx - 5 \int \frac{1}{(4+5x)^2\sqrt{1-x^2}} dx \\ &= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} \end{aligned}$$

Mathematica [A] time = 0.15, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2} + 3}{25x + 20}$$

Antiderivative was successfully verified.

[In] Integrate[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]), x]

[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)

IntegrateAlgebraic [A] time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-5 - 4*x - 3*Sqrt[1 - x^2])/((4 + 5*x)^2*Sqrt[1 - x^2]), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

fricas [A] time = 1.09, size = 25, normalized size = 0.81

$$\frac{25x + 20\sqrt{-x^2 + 1} + 32}{20(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)

giac [C] time = 0.59, size = 54, normalized size = 1.74

$$\frac{\sqrt{\frac{8}{5x+4} + \frac{9}{(5x+4)^2} - 1}}{5 \operatorname{sgn}\left(\frac{1}{5x+4}\right)} + \frac{3}{5(5x+4)} - \frac{1}{5}i \operatorname{sgn}\left(\frac{1}{5x+4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2), x, algorithm="giac")

[Out] 1/5*sqrt(8/(5*x + 4) + 9/(5*x + 4)^2 - 1)/sgn(1/(5*x + 4)) + 3/5/(5*x + 4) - 1/5*I*sgn(1/(5*x + 4))

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}}}{5x + 4} + \frac{3}{5(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-5-4*x-3*(-x^2+1)^(1/2))/(5*x+4)^2/(-x^2+1)^(1/2),x)

[Out] 1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)

maxima [A] time = 0.61, size = 25, normalized size = 0.81

$$\frac{5\sqrt{x+1}\sqrt{-x+1}+3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5-4*x-3*(-x^2+1)^(1/2))/(4+5*x)^2/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] 1/5*(5*sqrt(x + 1)*sqrt(-x + 1) + 3)/(5*x + 4)

mupad [B] time = 0.04, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(4*x + 3*(1 - x^2)^(1/2) + 5)/((5*x + 4)^2*(1 - x^2)^(1/2)),x)

[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{4x}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx - \int \frac{3\sqrt{1-x^2}}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx - \int \frac{5}{25x^2\sqrt{1-x^2} + 40x\sqrt{1-x^2} + 16\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-5-4*x-3*(-x**2+1)**(1/2))/(4+5*x)**2/(-x**2+1)**(1/2),x)

[Out] -Integral(4*x/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(3*sqrt(1 - x**2)/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x) - Integral(5/(25*x**2*sqrt(1 - x**2) + 40*x*sqrt(1 - x**2) + 16*sqrt(1 - x**2)), x)

$$3.529 \quad \int \frac{1}{(-5-4x)\sqrt{1-x^2}+3(1-x^2)} dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Rubi [A] time = 0.14, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 665

Int[((d_) + (e_.)*(x_)^m)*((a_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 725

Int[1/(((d_) + (e_.)*(x_)^m)*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 733

Int[((d_) + (e_.)*(x_)^m)*((a_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 735

Int[((d_) + (e_.)*(x_)^m)*((a_) + (c_.)*(x_)^2)^p, x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m, -1]

$Q[m + 2*p + 1, 0] \ \&\& \ (\ !\text{RationalQ}[m] \ || \ \text{LtQ}[m, 1]) \ \&\& \ !\text{ILtQ}[m + 2*p, 0] \ \&\& \ \text{IntQuadraticQ}[a, 0, c, d, e, m, p, x]$

Rule 844

$\text{Int}[(d_.) + (e_.)*(x_.))^{(m_.)}*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^{(p_.)}, x_Symbol] \ :> \ \text{Dist}[g/e, \ \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^p, x], x] + \ \text{Dist}[(e*f - d*g)/e, \ \text{Int}[(d + e*x)^m*(a + c*x^2)^p, x], x] \ /; \ \text{FreeQ}[\{a, c, d, e, f, g, m, p\}, x] \ \&\& \ \text{NeQ}[c*d^2 + a*e^2, 0] \ \&\& \ !\text{IGtQ}[m, 0]$

Rule 6742

$\text{Int}[u_, x_Symbol] \ :> \ \text{With}[\{v = \text{ExpandIntegrand}[u, x]\}, \ \text{Int}[v, x] \ /; \ \text{SumQ}[v]]$

Rubi steps

$$\begin{aligned} \int \frac{1}{(-5 - 4x)\sqrt{1 - x^2} + 3(1 - x^2)} dx &= \int \left(-\frac{3}{(4 + 5x)^2} + \frac{\sqrt{1 - x^2}}{18(-1 + x)} - \frac{\sqrt{1 - x^2}}{2(1 + x)} - \frac{5\sqrt{1 - x^2}}{(4 + 5x)^2} + \frac{20\sqrt{1 - x^2}}{9(4 + 5x)} \right) dx \\ &= \frac{3}{5(4 + 5x)} + \frac{1}{18} \int \frac{\sqrt{1 - x^2}}{-1 + x} dx - \frac{1}{2} \int \frac{\sqrt{1 - x^2}}{1 + x} dx + \frac{20}{9} \int \frac{\sqrt{1 - x^2}}{4 + 5x} dx - \frac{5}{9} \int \frac{\sqrt{1 - x^2}}{(4 + 5x)^2} dx \\ &= \frac{3}{5(4 + 5x)} + \frac{\sqrt{1 - x^2}}{4 + 5x} - \frac{1}{18} \int \frac{1}{\sqrt{1 - x^2}} dx + \frac{4}{9} \int \frac{5 + 4x}{(4 + 5x)\sqrt{1 - x^2}} dx - \frac{5}{9} \int \frac{\sqrt{1 - x^2}}{(4 + 5x)^2} dx \\ &= \frac{3}{5(4 + 5x)} + \frac{\sqrt{1 - x^2}}{4 + 5x} - \frac{5}{9} \sin^{-1}(x) + \frac{1}{5} \int \frac{1}{\sqrt{1 - x^2}} dx + \frac{16}{45} \int \frac{1}{\sqrt{1 - x^2}} dx \\ &= \frac{3}{5(4 + 5x)} + \frac{\sqrt{1 - x^2}}{4 + 5x} \end{aligned}$$

Mathematica [A] time = 0.09, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1 - x^2} + 3}{25x + 20}$$

Antiderivative was successfully verified.

[In] Integrate[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1), x]

[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)

IntegrateAlgebraic [A] time = 0.28, size = 31, normalized size = 1.00

$$\frac{\sqrt{1 - x^2}}{5x + 4} + \frac{3}{5(5x + 4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-5 - 4*x)*Sqrt[1 - x^2] + 3*(1 - x^2))^(-1), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

fricas [A] time = 0.82, size = 25, normalized size = 0.81

$$\frac{25x + 20\sqrt{-x^2 + 1} + 32}{20(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)

giac [B] time = 0.50, size = 68, normalized size = 2.19

$$\frac{\frac{5(\sqrt{-x^2+1}-1)}{x} - 4}{4\left(\frac{5(\sqrt{-x^2+1}-1)}{x} - \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 2\right)} + \frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)

maple [B] time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2} + \frac{41}{25}x}{9} + \frac{3}{5(5x+4)} + \frac{\sqrt{-2x - (x-1)^2 + 2}}{18} - \frac{\sqrt{2x - (x+1)^2 + 2}}{2} + \frac{5\left(\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x + \frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x)

[Out] 3/5/(5*x+4)+1/18*(-(x-1)^2-2*x+2)^(1/2)-1/2*(-(x+1)^2+2*x+2)^(1/2)+5/9/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)+5/9*x*(8/5*x-(x+4/5)^2+41/25)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + \sqrt{-x^2 + 1}(4x + 5) - 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x^2+3+(-5-4*x)*(-x^2+1)^(1/2)),x, algorithm="maxima")

[Out] -integrate(1/(3*x^2 + sqrt(-x^2 + 1)*(4*x + 5) - 3), x)

mupad [B] time = 0.07, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/((4*x + 5)*(1 - x^2)^(1/2) + 3*x^2 - 3),x)

[Out] ((1 - x^2)^(1/2) + 3/5)/(5*x + 4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3*x**2+3+(-5-4*x)*(-x**2+1)**(1/2)),x)

[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)

$$3.530 \quad \int \frac{1}{3-3x^2-5\sqrt{1-x^2}-4x\sqrt{1-x^2}} dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Rubi [A] time = 0.13, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6742, 665, 216, 733, 844, 725, 206, 735}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 665

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(2*c*d*p)/(e^2*(m + 2*p + 1)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 733

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 735

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] &&

IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1}{3 - 3x^2 - 5\sqrt{1-x^2} - 4x\sqrt{1-x^2}} dx &= \int \left(-\frac{3}{(4+5x)^2} + \frac{\sqrt{1-x^2}}{18(-1+x)} - \frac{\sqrt{1-x^2}}{2(1+x)} - \frac{5\sqrt{1-x^2}}{(4+5x)^2} + \frac{20\sqrt{1-x^2}}{9(4+5x)} \right) dx \\ &= \frac{3}{5(4+5x)} + \frac{1}{18} \int \frac{\sqrt{1-x^2}}{-1+x} dx - \frac{1}{2} \int \frac{\sqrt{1-x^2}}{1+x} dx + \frac{20}{9} \int \frac{\sqrt{1-x^2}}{4+5x} dx \\ &= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{18} \int \frac{1}{\sqrt{1-x^2}} dx + \frac{4}{9} \int \frac{5+4x}{(4+5x)\sqrt{1-x^2}} dx \\ &= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{5}{9} \sin^{-1}(x) + \frac{1}{5} \int \frac{1}{\sqrt{1-x^2}} dx + \frac{16}{45} \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} \end{aligned}$$

Mathematica [A] time = 0.06, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2} + 3}{25x + 20}$$

Antiderivative was successfully verified.

[In] Integrate[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1), x]

[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)

IntegrateAlgebraic [A] time = 0.28, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 - 3*x^2 - 5*Sqrt[1 - x^2] - 4*x*Sqrt[1 - x^2])^(-1), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

fricas [A] time = 0.82, size = 25, normalized size = 0.81

$$\frac{25x + 20\sqrt{-x^2 + 1} + 32}{20(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/20*(25*x + 20*sqrt(-x^2 + 1) + 32)/(5*x + 4)

giac [B] time = 0.37, size = 68, normalized size = 2.19

$$\frac{\frac{5(\sqrt{-x^2+1}-1)}{x} - 4}{4\left(\frac{5(\sqrt{-x^2+1}-1)}{x} - \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 2\right)} + \frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/4*(5*(sqrt(-x^2 + 1) - 1)/x - 4)/(5*(sqrt(-x^2 + 1) - 1)/x - 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 2) + 3/5/(5*x + 4)

maple [B] time = 0.04, size = 81, normalized size = 2.61

$$\frac{5\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}} x}{9} + \frac{3}{5(5x+4)} + \frac{\sqrt{-2x - (x-1)^2 + 2}}{18} - \frac{\sqrt{2x - (x+1)^2 + 2}}{2} + \frac{5\left(\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}\right)^{\frac{3}{2}}}{9\left(x + \frac{4}{5}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*(-x^2+1)^(1/2)*x),x)

[Out] 5/9*(8/5*x-(x+4/5)^2+41/25)^(1/2)*x+3/5/(5*x+4)+1/18*(-2*x-(x-1)^2+2)^(1/2)-1/2*(2*x-(x+1)^2+2)^(1/2)+5/9/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4\sqrt{-x^2 + 1}x + 5\sqrt{-x^2 + 1} - 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-3*x^2-5*(-x^2+1)^(1/2)-4*x*(-x^2+1)^(1/2)),x, algorithm="maxima")

[Out] -integrate(1/(3*x^2 + 4*sqrt(-x^2 + 1)*x + 5*sqrt(-x^2 + 1) - 3), x)

mupad [B] time = 0.05, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(4*x*(1-x^2)^(1/2)+3*x^2+5*(1-x^2)^(1/2)-3),x)

[Out] ((1-x^2)^(1/2)+3/5)/(5*x+4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{3x^2 + 4x\sqrt{1-x^2} + 5\sqrt{1-x^2} - 3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-3*x**2-5*(-x**2+1)**(1/2))-4*x*(-x**2+1)**(1/2)),x)
```

```
[Out] -Integral(1/(3*x**2 + 4*x*sqrt(1 - x**2) + 5*sqrt(1 - x**2) - 3), x)
```

$$3.531 \quad \int \frac{-1 + \sqrt{1-x^2}}{\sqrt{1-x^2} (2+x-2\sqrt{1-x^2})^2} dx$$

Optimal. Leaf size=31

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Rubi [A] time = 0.65, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 13, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.302$, Rules used = {6742, 277, 216, 266, 50, 63, 206, 733, 844, 725, 735, 264, 731}

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] Int[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2),x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 264

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c
*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] /; FreeQ[{a, b, c, m, n,
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 266

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
```

, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 731

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]

Rule 733

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 735

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{-1 + \sqrt{1-x^2}}{\sqrt{1-x^2} (2+x-2\sqrt{1-x^2})^2} dx &= \int \left(\frac{1}{(-2-x+2\sqrt{1-x^2})^2} - \frac{1}{\sqrt{1-x^2} (-2-x+2\sqrt{1-x^2})^2} \right) dx \\
&= \int \frac{1}{(-2-x+2\sqrt{1-x^2})^2} dx - \int \frac{1}{\sqrt{1-x^2} (-2-x+2\sqrt{1-x^2})^2} dx \\
&= - \int \left(\frac{1}{2x^2} - \frac{1}{x} + \frac{15}{2(4+5x)^2} + \frac{5}{4+5x} + \frac{1}{2x^2\sqrt{1-x^2}} - \frac{1}{x\sqrt{1-x^2}} + \frac{1}{2(4+5x)\sqrt{1-x^2}} \right) dx \\
&= \frac{3}{5(4+5x)} - \frac{1}{2} \int \frac{1}{x^2\sqrt{1-x^2}} dx + \frac{1}{2} \int \frac{\sqrt{1-x^2}}{x^2} dx - \frac{9}{2} \int \frac{1}{(4+5x)^2\sqrt{1-x^2}} dx \\
&= \frac{3}{5(4+5x)} + \sqrt{1-x^2} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, \frac{5+4x}{3\sqrt{1-x^2}} \right) \\
&= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} - \frac{1}{2} \sin^{-1}(x) + \frac{5}{3} \tanh^{-1} \left(\frac{5+4x}{3\sqrt{1-x^2}} \right) - \frac{3}{10} \int \frac{1}{\sqrt{1-x^2}} dx \\
&= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x} + \tanh^{-1} \left(\frac{5+4x}{3\sqrt{1-x^2}} \right) - \tanh^{-1}(\sqrt{1-x^2}) - \frac{6}{5} \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, \frac{5+4x}{3\sqrt{1-x^2}} \right) \\
&= \frac{3}{5(4+5x)} + \frac{\sqrt{1-x^2}}{4+5x}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 23, normalized size = 0.74

$$\frac{5\sqrt{1-x^2} + 3}{25x + 20}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2), x]

[Out] (3 + 5*Sqrt[1 - x^2])/(20 + 25*x)

IntegrateAlgebraic [A] time = 0.29, size = 31, normalized size = 1.00

$$\frac{\sqrt{1-x^2}}{5x+4} + \frac{3}{5(5x+4)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + Sqrt[1 - x^2])/(Sqrt[1 - x^2]*(2 + x - 2*Sqrt[1 - x^2])^2), x]

[Out] 3/(5*(4 + 5*x)) + Sqrt[1 - x^2]/(4 + 5*x)

fricas [A] time = 0.75, size = 25, normalized size = 0.81

$$\frac{25x + 20\sqrt{-x^2 + 1} + 32}{20(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] $1/20*(25*x + 20*\sqrt{-x^2 + 1} + 32)/(5*x + 4)$

giac [B] time = 0.62, size = 68, normalized size = 2.19

$$\frac{\frac{5(\sqrt{-x^2+1}-1)}{x} - 4}{4\left(\frac{5(\sqrt{-x^2+1}-1)}{x} - \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 2\right)} + \frac{3}{5(5x+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2), x, algorithm="giac")`

[Out] $1/4*(5*(\sqrt{-x^2 + 1} - 1)/x - 4)/(5*(\sqrt{-x^2 + 1} - 1)/x - 2*(\sqrt{-x^2 + 1} - 1)^2/x^2 - 2) + 3/5/(5*x + 4)$

maple [A] time = 0.01, size = 32, normalized size = 1.03

$$\frac{\sqrt{\frac{8x}{5} - \left(x + \frac{4}{5}\right)^2 + \frac{41}{25}}}{5x + 4} + \frac{3}{5(5x + 4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2), x)`

[Out] $1/5/(x+4/5)*(8/5*x-(x+4/5)^2+41/25)^(1/2)+3/5/(5*x+4)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-\frac{1}{56}\sqrt{7}\log\left(\frac{3x-2\sqrt{7}-2}{3x+2\sqrt{7}-2}\right) - \int \frac{100x^7 + 285x^6 + 264x^5 + 80x^4}{8(21x^9 + 278x^8 + 283x^7 - 2022x^6 - 3632x^5 + 2256x^4 + 7424x^3 + 1536x^2 - 8(9x^8 + 12x^7 - 101x^6 - 172x^5 + 284x^4 + 672x^3 + 64x^2 - 512x - 256)\sqrt{x+1}\sqrt{-x+1} - 4096x - 2048)}{dx} - \frac{1}{24}\log(x+2) + \frac{1}{16}\log(x+1) - \frac{1}{48}\log(x-1)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-1+(-x^2+1)^(1/2))/(2+x-2*(-x^2+1)^(1/2))^2/(-x^2+1)^(1/2), x, algorithm="maxima")`

[Out] $-1/56*\sqrt{7}*\log((3*x - 2*\sqrt{7} - 2)/(3*x + 2*\sqrt{7} - 2)) - \text{integrate}(-1/8*(100*x^7 + 285*x^6 + 264*x^5 + 80*x^4)/(21*x^9 + 278*x^8 + 283*x^7 - 2022*x^6 - 3632*x^5 + 2256*x^4 + 7424*x^3 + 1536*x^2 - 8*(9*x^8 + 12*x^7 - 101*x^6 - 172*x^5 + 284*x^4 + 672*x^3 + 64*x^2 - 512*x - 256)*\sqrt{x + 1}*\sqrt{-x + 1} - 4096*x - 2048), x) - 1/24*\log(x + 2) + 1/16*\log(x + 1) - 1/48*\log(x - 1)$

mupad [B] time = 3.32, size = 19, normalized size = 0.61

$$\frac{\sqrt{1-x^2} + \frac{3}{5}}{5x+4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((1-x^2)^(1/2)-1)/((1-x^2)^(1/2)*(x-2*(1-x^2)^(1/2)+2)^2), x)`

[Out] $((1-x^2)^(1/2)+3/5)/(5*x+4)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^2}-1}{\sqrt{-(x-1)(x+1)}\left(x-2\sqrt{1-x^2}+2\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+(-x**2+1)**(1/2))/(2+x-2*(-x**2+1)**(1/2))**2/(-x**2+1)**(1/2),x)
```

```
[Out] Integral((sqrt(1 - x**2) - 1)/(sqrt(-(x - 1)*(x + 1))*(x - 2*sqrt(1 - x**2) + 2)**2), x)
```

$$3.532 \quad \int \frac{a+bx^{-1+n}}{cx+dx^n} dx$$

Optimal. Leaf size=43

$$\frac{b \log(x)}{d} - \frac{(bc - ad) \log(cx^{1-n} + d)}{cd(1 - n)}$$

Rubi [A] time = 0.07, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1593, 514, 446, 72}

$$\frac{b \log(x)}{d} - \frac{(bc - ad) \log(cx^{1-n} + d)}{cd(1 - n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^(-1 + n))/(c*x + d*x^n), x]
```

```
[Out] (b*Log[x])/d - ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*d*(1 - n))
```

Rule 72

```
Int[((e_.) + (f_.)*(x_.)^(p_.))/(((a_.) + (b_.)*(x_.))*((c_.) + (d_.)*(x_.))),
  x_Symbol] := Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
  /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 446

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.),
  x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
  *(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
  b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 514

```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
  p_.), x_Symbol] := Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
  [{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !I
  ntegerQ[p])
```

Rule 1593

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x
  ^((n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
  PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a + bx^{-1+n}}{cx + dx^n} dx &= \int \frac{x^{-n} (a + bx^{-1+n})}{d + cx^{1-n}} dx \\
&= \int \frac{b + ax^{1-n}}{x(d + cx^{1-n})} dx \\
&= \frac{\text{Subst}\left(\int \frac{b+ax}{x(d+cx)} dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{\text{Subst}\left(\int \left(\frac{b}{dx} + \frac{-bc+ad}{d(d+cx)}\right) dx, x, x^{1-n}\right)}{1-n} \\
&= \frac{b \log(x)}{d} - \frac{(bc - ad) \log(d + cx^{1-n})}{cd(1-n)}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 0.88

$$\frac{\frac{(bc-ad) \log(cx^{1-n}+d)}{c(n-1)} + b \log(x)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x^(-1 + n))/(c*x + d*x^n), x]

[Out] (b*Log[x] + ((b*c - a*d)*Log[d + c*x^(1 - n)])/(c*(-1 + n)))/d

IntegrateAlgebraic [F] time = 0.17, size = 0, normalized size = 0.00

$$\int \frac{a + bx^{-1+n}}{cx + dx^n} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(a + b*x^(-1 + n))/(c*x + d*x^n), x]

[Out] Defer[IntegrateAlgebraic] [(a + b*x^(-1 + n))/(c*x + d*x^n), x]

fricas [A] time = 0.76, size = 44, normalized size = 1.02

$$\frac{(bc - ad) \log(cx + dx^n) + (adn - bc) \log(x)}{cdn - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n), x, algorithm="fricas")

[Out] ((b*c - a*d)*log(c*x + d*x^n) + (a*d*n - b*c)*log(x))/(c*d*n - c*d)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{bx^{n-1} + a}{cx + dx^n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n), x, algorithm="giac")

[Out] integrate((b*x^(n - 1) + a)/(c*x + d*x^n), x)

maple [A] time = 0.02, size = 73, normalized size = 1.70

$$\frac{an \ln(x)}{(n-1)c} - \frac{a \ln(cx + d e^{n \ln(x)})}{(n-1)c} - \frac{b \ln(x)}{(n-1)d} + \frac{b \ln(cx + d e^{n \ln(x)})}{(n-1)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*x^(n-1))/(c*x+d*x^n), x)

[Out] 1/c/(n-1)*ln(x)*a*n-1/d/(n-1)*ln(x)*b-1/c/(n-1)*ln(c*x+d*exp(n*ln(x)))*a+1/d/(n-1)*ln(c*x+d*exp(n*ln(x)))*b

maxima [B] time = 0.45, size = 85, normalized size = 1.98

$$b \left(\frac{\log(x)}{d} - \frac{n \log(x)}{d(n-1)} + \frac{\log\left(\frac{cx+dx^n}{d}\right)}{d(n-1)} \right) + a \left(\frac{n \log(x)}{c(n-1)} - \frac{\log\left(\frac{cx+dx^n}{d}\right)}{c(n-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x^(-1+n))/(c*x+d*x^n), x, algorithm="maxima")

[Out] b*(log(x)/d - n*log(x)/(d*(n - 1)) + log((c*x + d*x^n)/d)/(d*(n - 1))) + a*(n*log(x)/(c*(n - 1)) - log((c*x + d*x^n)/d)/(c*(n - 1)))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a + b x^{n-1}}{d x^n + c x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b*x^(n - 1))/(d*x^n + c*x), x)

[Out] int((a + b*x^(n - 1))/(d*x^n + c*x), x)

sympy [A] time = 10.18, size = 212, normalized size = 4.93

$$\left\{ \begin{array}{ll} \infty (a + b) \log(x) & \text{for } c = 0 \wedge d = 0 \wedge n = 1 \\ \frac{\frac{anx}{n^2x^n-nx^n} + \frac{bn^2x^n \log(x)}{n^2x^n-nx^n} - \frac{bnx^n \log(x)}{n^2x^n-nx^n} - \frac{bnx^n}{n^2x^n-nx^n}}{d} & \text{for } c = 0 \\ \frac{\frac{anx \log(x)}{nx-x} - \frac{ax \log(x)}{nx-x} + \frac{bx^n}{nx-x}}{c} & \text{for } d = 0 \\ \frac{(a+b) \log(x)}{c+d} & \text{for } n = 1 \\ \frac{adn \log(x)}{cdn-cd} - \frac{ad \log\left(x + \frac{dx^n}{c}\right)}{cdn-cd} - \frac{bc \log(x)}{cdn-cd} + \frac{bc \log\left(x + \frac{dx^n}{c}\right)}{cdn-cd} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*x**(-1+n))/(c*x+d*x**n), x)

[Out] Piecewise((zoo*(a + b)*log(x), Eq(c, 0) & Eq(d, 0) & Eq(n, 1)), ((-a*n*x/(n**2*x**n - n*x**n) + b*n**2*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n*log(x)/(n**2*x**n - n*x**n) - b*n*x**n/(n**2*x**n - n*x**n))/d, Eq(c, 0)), ((a*n*x*log(x)/(n*x - x) - a*x*log(x)/(n*x - x) + b*x**n/(n*x - x))/c, Eq(d, 0)), ((a + b)*log(x)/(c + d), Eq(n, 1)), (a*d*n*log(x)/(c*d*n - c*d) - a*d*log(x + d*x**n/c)/(c*d*n - c*d) - b*c*log(x)/(c*d*n - c*d) + b*c*log(x + d*x**n/c)/(c*d*n - c*d), True))

$$3.533 \quad \int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}} dx$$

Optimal. Leaf size=42

$$\frac{\sqrt{2x^2+1}}{2x} + x - \frac{1}{2x} - \frac{\sinh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Rubi [A] time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {6740, 6742, 277, 215}

$$\frac{\sqrt{2x^2+1}}{2x} + x - \frac{1}{2x} - \frac{\sinh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]

[Out] -1/(2*x) + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m+1)*(a+b*x^n)^p)/(c*(m+1)), x] - Dist[(b*n*p)/(c^n*(m+1)), Int[(c*x)^(m+n)*(a+b*x^n)^(p-1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m+n*p+n+1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 6740

Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[PolynomialInSubst[v, u, x]/(a+b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && PolynomialInQ[v, u, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}} dx &= \int \left(1 + \frac{1}{-1-\sqrt{1+2x^2}} \right) dx \\
&= x + \int \frac{1}{-1-\sqrt{1+2x^2}} dx \\
&= x + \int \left(\frac{1}{2x^2} - \frac{\sqrt{1+2x^2}}{2x^2} \right) dx \\
&= -\frac{1}{2x} + x - \frac{1}{2} \int \frac{\sqrt{1+2x^2}}{x^2} dx \\
&= -\frac{1}{2x} + x + \frac{\sqrt{1+2x^2}}{2x} - \int \frac{1}{\sqrt{1+2x^2}} dx \\
&= -\frac{1}{2x} + x + \frac{\sqrt{1+2x^2}}{2x} - \frac{\sinh^{-1}(\sqrt{2}x)}{\sqrt{2}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{\sqrt{2x^2+1}}{2x} + x - \frac{1}{2x} - \frac{\sinh^{-1}(\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]), x]

[Out] -1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]

IntegrateAlgebraic [A] time = 0.18, size = 60, normalized size = 1.43

$$\frac{2x^2-1}{2x} + \frac{\sqrt{2x^2+1}}{2x} + \frac{\log(\sqrt{2x^2+1}-\sqrt{2}x)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]), x]

[Out] (-1 + 2*x^2)/(2*x) + Sqrt[1 + 2*x^2]/(2*x) + Log[-(Sqrt[2]*x) + Sqrt[1 + 2*x^2]]/Sqrt[2]

fricas [A] time = 0.84, size = 44, normalized size = 1.05

$$\frac{\sqrt{2}x \log(\sqrt{2}x - \sqrt{2x^2+1}) + 2x^2 + \sqrt{2x^2+1} - 1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)), x, algorithm="fricas")

[Out] 1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1) - 1)/x

giac [A] time = 0.46, size = 57, normalized size = 1.36

$$\frac{1}{2} \sqrt{2} \log(-\sqrt{2}x + \sqrt{2x^2+1}) + x - \frac{\sqrt{2}}{(\sqrt{2}x - \sqrt{2x^2+1})^2 - 1} - \frac{1}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 1) - 1/2/x

maple [A] time = 0.01, size = 45, normalized size = 1.07

$$x - \sqrt{2x^2 + 1} x - \frac{\sqrt{2} \operatorname{arcsinh}(\sqrt{2} x)}{2} - \frac{1}{2x} + \frac{(2x^2 + 1)^{\frac{3}{2}}}{2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)

[Out] x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(2^(1/2)*x)*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{1}{\sqrt{2x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")

[Out] x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)

mupad [B] time = 3.39, size = 31, normalized size = 0.74

$$x - \frac{\sqrt{2} \operatorname{asinh}(\sqrt{2} x)}{2} + \frac{\sqrt{2} \sqrt{x^2 + \frac{1}{2}}}{2} - \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)

[Out] x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + 1}}{\sqrt{2x^2 + 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)

[Out] Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)

$$3.534 \quad \int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}} dx$$

Optimal. Leaf size=65

$$-\frac{1}{3}\sqrt{4x^2-1} + \frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}} + \frac{4x}{3} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Rubi [A] time = 0.13, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {6742, 444, 50, 63, 207, 388}

$$-\frac{1}{3}\sqrt{4x^2-1} + \frac{\tanh^{-1}\left(\sqrt{3}\sqrt{4x^2-1}\right)}{3\sqrt{3}} + \frac{4x}{3} - \frac{\tanh^{-1}\left(\sqrt{3}x\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-1 + 4*x^2]/(x + Sqrt[-1 + 4*x^2]), x]

[Out] (4*x)/3 - Sqrt[-1 + 4*x^2]/3 - ArcTanh[Sqrt[3]*x]/(3*Sqrt[3]) + ArcTanh[Sqrt[3]*Sqrt[-1 + 4*x^2]]/(3*Sqrt[3])

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 388

Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]

Rule 444

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-1+4x^2}}{x+\sqrt{-1+4x^2}} dx &= \int \left(-\frac{x\sqrt{-1+4x^2}}{-1+3x^2} + \frac{-1+4x^2}{-1+3x^2} \right) dx \\
&= -\int \frac{x\sqrt{-1+4x^2}}{-1+3x^2} dx + \int \frac{-1+4x^2}{-1+3x^2} dx \\
&= \frac{4x}{3} + \frac{1}{3} \int \frac{1}{-1+3x^2} dx - \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{-1+4x}}{-1+3x} dx, x, x^2 \right) \\
&= \frac{4x}{3} - \frac{1}{3} \sqrt{-1+4x^2} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{(-1+3x)\sqrt{-1+4x}} dx, x, x^2 \right) \\
&= \frac{4x}{3} - \frac{1}{3} \sqrt{-1+4x^2} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} - \frac{1}{12} \text{Subst} \left(\int \frac{1}{-\frac{1}{4} + \frac{3x^2}{4}} dx, x, \sqrt{-1+4x^2} \right) \\
&= \frac{4x}{3} - \frac{1}{3} \sqrt{-1+4x^2} - \frac{\tanh^{-1}(\sqrt{3}x)}{3\sqrt{3}} + \frac{\tanh^{-1}(\sqrt{3}\sqrt{-1+4x^2})}{3\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{9} \left(-3\sqrt{4x^2-1} + \sqrt{3} \tanh^{-1}(\sqrt{12x^2-3}) + 12x - \sqrt{3} \tanh^{-1}(\sqrt{3}x) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1+4*x^2]/(x+Sqrt[-1+4*x^2]),x]
```

```
[Out] (12*x - 3*Sqrt[-1+4*x^2] - Sqrt[3]*ArcTanh[Sqrt[3]*x] + Sqrt[3]*ArcTanh[Sqrt[-3+12*x^2]])/9
```

IntegrateAlgebraic [A] time = 0.27, size = 58, normalized size = 0.89

$$-\frac{1}{3}\sqrt{4x^2-1} - \frac{2 \tanh^{-1}\left(\frac{2x}{\sqrt{3}} - \frac{\sqrt{4x^2-1}}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{4x}{3}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[-1+4*x^2]/(x+Sqrt[-1+4*x^2]),x]
```

```
[Out] (4*x)/3 - Sqrt[-1+4*x^2]/3 - (2*ArcTanh[(2*x)/Sqrt[3] - Sqrt[-1+4*x^2]/Sqrt[3]])/(3*Sqrt[3])
```

fricas [A] time = 1.04, size = 80, normalized size = 1.23

$$\frac{1}{18} \sqrt{3} \log\left(\frac{6x^2 + \sqrt{3}\sqrt{4x^2-1} - 1}{3x^2-1}\right) + \frac{1}{18} \sqrt{3} \log\left(\frac{3x^2 - 2\sqrt{3}x + 1}{3x^2-1}\right) + \frac{4}{3}x - \frac{1}{3}\sqrt{4x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="fricas")
```

[Out] $\frac{1}{18}\sqrt{3}\log\left(\frac{(6x^2 + \sqrt{3})\sqrt{4x^2 - 1} - 1}{(3x^2 - 1)}\right) + \frac{1}{18}\sqrt{3}\log\left(\frac{(3x^2 - 2\sqrt{3})x + 1}{(3x^2 - 1)}\right) + \frac{4}{3}x - \frac{1}{3}\sqrt{4x^2 - 1} - 1$

giac [B] time = 0.57, size = 133, normalized size = 2.05

$$\frac{1}{18}\sqrt{3}\log\left(\frac{|6x - 2\sqrt{3}|}{|6x + 2\sqrt{3}|}\right) - \frac{1}{18}\sqrt{3}\log\left(\frac{\left|-12x - 4\sqrt{3} + 6\sqrt{4x^2 - 1} + \frac{6}{2x - \sqrt{4x^2 - 1}}\right|}{2\left(6x - 2\sqrt{3} - 3\sqrt{4x^2 - 1} - \frac{3}{2x - \sqrt{4x^2 - 1}}\right)}\right) + \frac{4}{3}x - \frac{1}{3}\sqrt{4x^2 - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="giac")`

[Out] $\frac{1}{18}\sqrt{3}\log\left(\frac{\text{abs}(6x - 2\sqrt{3})}{\text{abs}(6x + 2\sqrt{3})}\right) - \frac{1}{18}\sqrt{3}\log\left(\frac{-1/2\text{abs}(-12x - 4\sqrt{3} + 6\sqrt{4x^2 - 1} + 6/(2x - \sqrt{4x^2 - 1}))}{(6x - 2\sqrt{3} - 3\sqrt{4x^2 - 1} - 3/(2x - \sqrt{4x^2 - 1}))}\right) + \frac{4}{3}x - \frac{1}{3}\sqrt{4x^2 - 1}$

maple [B] time = 0.05, size = 262, normalized size = 4.03

$$\frac{4x}{3} - \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{3x^2+1}}\right)}{9} + \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(x-\frac{\sqrt{3}}{3}\right)}{\sqrt{3\left(x-\frac{\sqrt{3}}{3}\right)^2+1}}\right)}{18} + \frac{\sqrt{3}\operatorname{arctanh}\left(\frac{\sqrt{3}\left(x+\frac{\sqrt{3}}{3}\right)}{\sqrt{3\left(x+\frac{\sqrt{3}}{3}\right)^2+1}}\right)}{18} - \frac{\sqrt{3}\sqrt{4}\ln\left(\sqrt{4x+\sqrt{4x^2-1}}+\frac{\sqrt{3}}{3}\right)}{18} + \frac{\sqrt{3}\sqrt{4}\ln\left(\sqrt{4x+\sqrt{4x^2-1}}-\frac{\sqrt{3}}{3}\right)}{18} + \frac{\sqrt{36\left(x-\frac{\sqrt{3}}{3}\right)^2+24\left(x-\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}{18} + \frac{\sqrt{36\left(x+\frac{\sqrt{3}}{3}\right)^2-24\left(x+\frac{\sqrt{3}}{3}\right)\sqrt{3}+3}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x)`

[Out] $\frac{4}{3}x - \frac{1}{9}\operatorname{arctanh}\left(3^{1/2}x\right)3^{1/2} - \frac{1}{18}\left(36\left(x - \frac{1}{3}3^{1/2}\right)^2 + 24\left(x - \frac{1}{3}3^{1/2}\right)3^{1/2} + 3\right)^{1/2} - \frac{1}{18}3^{1/2}\ln\left(x4^{1/2} + \left(4\left(x - \frac{1}{3}3^{1/2}\right)^2 + 8/3\left(x - \frac{1}{3}3^{1/2}\right)3^{1/2} + 1/3\right)^{1/2}\right)4^{1/2} + \frac{1}{18}3^{1/2}\operatorname{arctanh}\left(\frac{3/2\left(2/3 + 8/3\left(x - \frac{1}{3}3^{1/2}\right)3^{1/2}\right)3^{1/2}}{\left(36\left(x - \frac{1}{3}3^{1/2}\right)^2 + 24\left(x - \frac{1}{3}3^{1/2}\right)3^{1/2} + 3\right)^{1/2}}\right) - \frac{1}{18}\left(36\left(x + \frac{1}{3}3^{1/2}\right)^2 - 24\left(x + \frac{1}{3}3^{1/2}\right)3^{1/2} + 3\right)^{1/2} + \frac{1}{18}3^{1/2}\ln\left(x4^{1/2} + \left(4\left(x + \frac{1}{3}3^{1/2}\right)^2 - 8/3\left(x + \frac{1}{3}3^{1/2}\right)3^{1/2} + 1/3\right)^{1/2}\right)4^{1/2} + \frac{1}{18}3^{1/2}\operatorname{arctanh}\left(\frac{3/2\left(2/3 - 8/3\left(x + \frac{1}{3}3^{1/2}\right)3^{1/2}\right)3^{1/2}}{\left(36\left(x + \frac{1}{3}3^{1/2}\right)^2 - 24\left(x + \frac{1}{3}3^{1/2}\right)3^{1/2} + 3\right)^{1/2}}\right)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$x - \int \frac{x}{\sqrt{2x+1}\sqrt{2x-1}+x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x^2-1)^(1/2)/(x+(4*x^2-1)^(1/2)),x, algorithm="maxima")`

[Out] `x - integrate(x/(sqrt(2*x + 1)*sqrt(2*x - 1) + x), x)`

mupad [B] time = 3.44, size = 60, normalized size = 0.92

$$\frac{4x}{3} + \frac{\sqrt{3}\ln\left(x - \frac{\sqrt{3}}{3}\right)}{18} - \frac{\sqrt{3}\ln\left(x + \frac{\sqrt{3}}{3}\right)}{18} + \frac{\sqrt{3}\operatorname{atanh}\left(\sqrt{3}\sqrt{4x^2 - 1}\right)}{9} - \frac{\sqrt{4x^2 - 1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((4*x^2 - 1)^(1/2)/(x + (4*x^2 - 1)^(1/2)),x)`

[Out] $\frac{(4x)}{3} + \frac{(3^{1/2})\log\left(x - 3^{1/2}/3\right)}{18} - \frac{(3^{1/2})\log\left(x + 3^{1/2}/3\right)}{18} + \frac{(3^{1/2})\operatorname{atanh}\left(3^{1/2}\sqrt{4x^2 - 1}\right)}{9} - \frac{(4x^2 - 1)^{1/2}}{3}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(2x-1)(2x+1)}}{x + \sqrt{4x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4*x**2-1)**(1/2)/(x+(4*x**2-1)**(1/2)),x)

[Out] Integral(sqrt((2*x - 1)*(2*x + 1))/(x + sqrt(4*x**2 - 1)), x)

$$3.535 \quad \int \frac{a+bx+cx^2}{(d+ex)^3 \sqrt{-1+x^2}} dx$$

Optimal. Leaf size=195

$$\frac{\sqrt{x^2-1} (ae^2 - bde + cd^2)}{2e(d^2 - e^2)(d + ex)^2} - \frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right) (-a(2d^2 + e^2) + 3bde - c(d^2 + 2e^2))}{2(d^2 - e^2)^{5/2}} + \frac{\sqrt{x^2-1} (c(d^3 - 4de^2) - e(3ade - b(d^2 + 2e^2)))}{2e(d^2 - e^2)^2(d + ex)^2}$$

Rubi [A] time = 0.21, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1651, 807, 725, 206}

$$\frac{\sqrt{x^2-1} (ae^2 - bde + cd^2)}{2e(d^2 - e^2)(d + ex)^2} + \frac{\sqrt{x^2-1} (c(d^3 - 4de^2) - e(3ade - b(d^2 + 2e^2)))}{2e(d^2 - e^2)^2(d + ex)} - \frac{\tanh^{-1}\left(\frac{dx+e}{\sqrt{x^2-1}\sqrt{d^2-e^2}}\right) (-a(2d^2 + e^2) + 3bde - c(d^2 + 2e^2))}{2(d^2 - e^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*x + c*x^2)/((d + e*x)^3*sqrt[-1 + x^2]),x]

[Out] -((c*d^2 - b*d*e + a*e^2)*sqrt[-1 + x^2])/(2*e*(d^2 - e^2)*(d + e*x)^2) + (c*(d^3 - 4*d*e^2) - e*(3*a*d*e - b*(d^2 + 2*e^2)))*sqrt[-1 + x^2])/(2*e*(d^2 - e^2)^2*(d + e*x)) - ((3*b*d*e - a*(2*d^2 + e^2) - c*(d^2 + 2*e^2))*ArcTanh[(e + d*x)/(sqrt[d^2 - e^2]*sqrt[-1 + x^2])])/(2*(d^2 - e^2)^(5/2))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 807

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]

Rule 1651

Int[(Pq_)*((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[Pq, d + e*x, x], R = PolynomialRemainder[Pq, d + e*x, x]}, Simp[(e*R*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p*ExpandToSum[(m + 1)*(c*d^2 + a*e^2)*Q + c*d*R*(m + 1) - c*e*R*(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, p}, x] && PolyQ[Pq, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[m, -1]

Rubi steps

$$\begin{aligned}
\int \frac{a + bx + cx^2}{(d + ex)^3 \sqrt{-1 + x^2}} dx &= -\frac{(cd^2 - bde + ae^2) \sqrt{-1 + x^2}}{2e(d^2 - e^2)(d + ex)^2} - \frac{\int \frac{-2(ad + cd - be) - \left(bd + \frac{cd^2}{e} - ae - 2ce \right) x}{(d + ex)^2 \sqrt{-1 + x^2}} dx}{2(d^2 - e^2)} \\
&= -\frac{(cd^2 - bde + ae^2) \sqrt{-1 + x^2}}{2e(d^2 - e^2)(d + ex)^2} + \frac{(c(d^3 - 4de^2) - e(3ade - b(d^2 + 2e^2))) \sqrt{-1 + x^2}}{2e(d^2 - e^2)^2(d + ex)} \\
&= -\frac{(cd^2 - bde + ae^2) \sqrt{-1 + x^2}}{2e(d^2 - e^2)(d + ex)^2} + \frac{(c(d^3 - 4de^2) - e(3ade - b(d^2 + 2e^2))) \sqrt{-1 + x^2}}{2e(d^2 - e^2)^2(d + ex)} \\
&= -\frac{(cd^2 - bde + ae^2) \sqrt{-1 + x^2}}{2e(d^2 - e^2)(d + ex)^2} + \frac{(c(d^3 - 4de^2) - e(3ade - b(d^2 + 2e^2))) \sqrt{-1 + x^2}}{2e(d^2 - e^2)^2(d + ex)}
\end{aligned}$$

Mathematica [A] time = 0.39, size = 240, normalized size = 1.23

$$\frac{1}{2} \left(\frac{\log(-\sqrt{x^2-1}\sqrt{d^2-e^2}+dx+e)(a(2d^2+e^2)-3bde+c(d^2+2e^2))}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}} + \frac{\log(d+ex)(a(2d^2+e^2)-3bde+c(d^2+2e^2))}{(d-e)^2(d+e)^2\sqrt{d^2-e^2}} + \frac{\sqrt{x^2-1}(ae(-4d^2-3dex+e^2)+b(2d^3+d^2ex+d^2+2e^3x)+cd(d^2x-3de-4e^2x))}{(d^2-e^2)^2(d+ex)^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]), x]

[Out] ((Sqrt[-1 + x^2]*(a*e*(-4*d^2 + e^2 - 3*d*e*x) + c*d*(-3*d*e + d^2*x - 4*e^2*x) + b*(2*d^3 + d*e^2 + d^2*e*x + 2*e^3*x)))/((d^2 - e^2)^2*(d + e*x)^2) + ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[d + e*x])/((d - e)^2*(d + e)^2*Sqrt[d^2 - e^2]) - ((-3*b*d*e + a*(2*d^2 + e^2) + c*(d^2 + 2*e^2))*Log[e + d*x - Sqrt[d^2 - e^2]*Sqrt[-1 + x^2]])/((d - e)^2*(d + e)^2*Sqrt[d^2 - e^2]))/2

IntegrateAlgebraic [A] time = 1.43, size = 239, normalized size = 1.23

$$\frac{\tan^{-1}\left(\frac{d-c\sqrt{x^2-1}+ex}{\sqrt{d^2-e^2}}\right)\left(2ad^2\sqrt{e^2-d^2}+ae^2\sqrt{e^2-d^2}-3bde\sqrt{e^2-d^2}+cd^2\sqrt{e^2-d^2}+2ce^2\sqrt{e^2-d^2}\right)}{(d-e)^3(d+e)^3} + \frac{\sqrt{x^2-1}(-4ad^2e-3ade^2x+ae^3+2bd^3+bd^2ex+bd^2e+2be^3x+cd^3x-3cd^2e-4cd^2x)}{2(d-e)^2(d+e)^2(d+ex)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a + b*x + c*x^2)/((d + e*x)^3*Sqrt[-1 + x^2]), x]

[Out] ((2*b*d^3 - 4*a*d^2*e - 3*c*d^2*e + b*d*e^2 + a*e^3 + c*d^3*x + b*d^2*e*x - 3*a*d*e^2*x - 4*c*d*e^2*x + 2*b*e^3*x)*Sqrt[-1 + x^2])/(2*(d - e)^2*(d + e)^2*(d + e*x)^2) + (((2*a*d^2*Sqrt[-d^2 + e^2] + c*d^2*Sqrt[-d^2 + e^2] - 3*b*d*e*Sqrt[-d^2 + e^2] + a*e^2*Sqrt[-d^2 + e^2] + 2*c*e^2*Sqrt[-d^2 + e^2])*ArcTan[(d + e*x - e*Sqrt[-1 + x^2])/Sqrt[-d^2 + e^2]])/((d - e)^3*(d + e)^3)

fricas [B] time = 0.60, size = 1174, normalized size = 6.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2), x, algorithm="fricas")

[Out] [1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 + ((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*sqrt(d^2 - e^2)*log((

$$\begin{aligned} & d^2x + d*e + \sqrt{d^2 - e^2}*(d*x + e) + (d^2 - e^2 + \sqrt{d^2 - e^2}*d)*\sqrt{x^2 - 1})/(e*x + d)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*\sqrt{x^2 - 1})/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^{10})*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x), 1/2*(c*d^7 + b*d^6*e - (3*a + 5*c)*d^5*e^2 + b*d^4*e^3 + (3*a + 4*c)*d^3*e^4 - 2*b*d^2*e^5 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x^2 - 2*((2*a + c)*d^4*e^2 - 3*b*d^3*e^3 + (a + 2*c)*d^2*e^4 + ((2*a + c)*d^2*e^4 - 3*b*d*e^5 + (a + 2*c)*e^6)*x^2 + 2*((2*a + c)*d^3*e^3 - 3*b*d^2*e^4 + (a + 2*c)*d*e^5)*x)*\sqrt{-d^2 + e^2}*\arctan(-(\sqrt{-d^2 + e^2})*\sqrt{x^2 - 1}*e - \sqrt{-d^2 + e^2}*(e*x + d))/(d^2 - e^2)) + 2*(c*d^6*e + b*d^5*e^2 - (3*a + 5*c)*d^4*e^3 + b*d^3*e^4 + (3*a + 4*c)*d^2*e^5 - 2*b*d*e^6)*x + (2*b*d^5*e^2 - (4*a + 3*c)*d^4*e^3 - b*d^3*e^4 + (5*a + 3*c)*d^2*e^5 - b*d*e^6 - a*e^7 + (c*d^5*e^2 + b*d^4*e^3 - (3*a + 5*c)*d^3*e^4 + b*d^2*e^5 + (3*a + 4*c)*d*e^6 - 2*b*e^7)*x)*\sqrt{x^2 - 1})/(d^8*e^2 - 3*d^6*e^4 + 3*d^4*e^6 - d^2*e^8 + (d^6*e^4 - 3*d^4*e^6 + 3*d^2*e^8 - e^{10})*x^2 + 2*(d^7*e^3 - 3*d^5*e^5 + 3*d^3*e^7 - d*e^9)*x)] \end{aligned}$$

giac [B] time = 0.49, size = 536, normalized size = 2.75

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="giac")

[Out] (2*a*d^2 + c*d^2 - 3*b*d*e + a*e^2 + 2*c*e^2)*arctan(-((x - sqrt(x^2 - 1))*e + d)/sqrt(-d^2 + e^2))/((d^4 - 2*d^2*e^2 + e^4)*sqrt(-d^2 + e^2)) + (2*c*d^4*(x - sqrt(x^2 - 1))^3*e + 2*c*d^5*(x - sqrt(x^2 - 1))^2 + 2*b*d^4*(x - sqrt(x^2 - 1))^2*e - 2*a*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 5*c*d^2*(x - sqrt(x^2 - 1))^3*e^3 - 6*a*d^3*(x - sqrt(x^2 - 1))^2*e^2 - 7*c*d^3*(x - sqrt(x^2 - 1))^2*e^2 + 2*c*d^4*(x - sqrt(x^2 - 1))*e + 3*b*d*(x - sqrt(x^2 - 1))^3*e^4 + 5*b*d^2*(x - sqrt(x^2 - 1))^2*e^3 + 4*b*d^3*(x - sqrt(x^2 - 1))*e^2 - a*(x - sqrt(x^2 - 1))^3*e^5 - 3*a*d*(x - sqrt(x^2 - 1))^2*e^4 - 4*c*d*(x - sqrt(x^2 - 1))^2*e^4 - 10*a*d^2*(x - sqrt(x^2 - 1))*e^3 - 11*c*d^2*(x - sqrt(x^2 - 1))*e^3 + c*d^3*e^2 + 2*b*(x - sqrt(x^2 - 1))^2*e^5 + 5*b*d*(x - sqrt(x^2 - 1))*e^4 + b*d^2*e^3 + a*(x - sqrt(x^2 - 1))*e^5 - 3*a*d*e^4 - 4*c*d*e^4 + 2*b*e^5)/(d^4*e^2 - 2*d^2*e^4 + e^6)*((x - sqrt(x^2 - 1))^2*e + 2*d*(x - sqrt(x^2 - 1)) + e)^2)

maple [B] time = 0.03, size = 1407, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x)

[Out] -1/2/e/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+1/2/e^2/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b*d-1/2/e^3/(d^2-e^2)/(x+d/e)^2*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*c*d^2-3/2*d/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*a+3/2/e*d^2/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b-3/2/e^2*d^3/(d^2-e^2)^2/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*c-3/2/e*d^2/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2))*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e)*a+3/2/e^2*d^3/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2))*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e)*b-3/2/e^3*d^4/(d^2-e^2)^2/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2))*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e)*c

```

2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c+1/2/e/(d^2-e^2)/((d^2-e^2)
/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)*((x+d
/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*a-3/2/e^2/(d^2-e^2)/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*b*d+5/2/e^3/(d^2-e
^2)/((d^2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^
2)^(1/2)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))*c*d^2-1/e/
(d^2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*b+2/e^2/(d^
2-e^2)/(x+d/e)*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2)*d*c-c/e^3/((d^
2-e^2)/e^2)^(1/2)*ln((2*(d^2-e^2)/e^2-2*d/e*(x+d/e)+2*((d^2-e^2)/e^2)^(1/2)
*((x+d/e)^2-2*d/e*(x+d/e)+(d^2-e^2)/e^2)^(1/2))/(x+d/e))

```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2+b*x+a)/(e*x+d)^3/(x^2-1)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(e-d>0)', see `assume?` for more det
ails)Is e-d positive, negative or zero?
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{cx^2 + bx + a}{\sqrt{x^2 - 1} (d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3),x)
```

```
[Out] int((a + b*x + c*x^2)/((x^2 - 1)^(1/2)*(d + e*x)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + bx + cx^2}{\sqrt{(x-1)(x+1)} (d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**3/(x**2-1)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/(sqrt((x - 1)*(x + 1))*(d + e*x)**3), x)
```

$$3.536 \quad \int \frac{1+2x^8}{x(1+x^8)^{3/2}} dx$$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || (EqQ[e, 0] || (EqQ[c, 0] || LtQ[p, n]))))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 446

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{1+2x^8}{x(1+x^8)^{3/2}} dx &= \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{x(1+x)^{3/2}} dx, x, x^8 \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^8 \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^8} \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4} \tanh^{-1} \left(\sqrt{1+x^8} \right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1} \left(\sqrt{x^8+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4

IntegrateAlgebraic [A] time = 0.04, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1} \left(\sqrt{x^8+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x^8)/(x*(1 + x^8)^(3/2)),x]

[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4

fricas [B] time = 0.75, size = 52, normalized size = 1.86

$$\frac{(x^8+1) \log(\sqrt{x^8+1}+1) - (x^8+1) \log(\sqrt{x^8+1}-1) + 2\sqrt{x^8+1}}{8(x^8+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="fricas")

[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) + 2*sqrt(x^8 + 1))/(x^8 + 1)

giac [A] time = 0.38, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8} \log(\sqrt{x^8+1}+1) + \frac{1}{8} \log(\sqrt{x^8+1}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="giac")

[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)

maple [A] time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^8+1)/x/(x^8+1)^(3/2),x)

[Out] -1/4/(x^8+1)^(1/2)+1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))

maxima [A] time = 0.96, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8} \log\left(\sqrt{x^8+1} + 1\right) + \frac{1}{8} \log\left(\sqrt{x^8+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^8+1)/x/(x^8+1)^(3/2),x, algorithm="maxima")

[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)

mupad [B] time = 3.85, size = 20, normalized size = 0.71

$$-\frac{\operatorname{atanh}\left(\sqrt{x^8+1}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^8 + 1)/(x*(x^8 + 1)^(3/2)),x)

[Out] - atanh((x^8 + 1)^(1/2))/4 - 1/(4*(x^8 + 1)^(1/2))

sympy [A] time = 22.80, size = 37, normalized size = 1.32

$$\frac{\log\left(\sqrt{x^8+1}-1\right)}{8} - \frac{\log\left(\sqrt{x^8+1}+1\right)}{8} - \frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**8+1)/x/(x**8+1)**(3/2),x)

[Out] log(sqrt(x**8 + 1) - 1)/8 - log(sqrt(x**8 + 1) + 1)/8 - 1/(4*sqrt(x**8 + 1))

$$3.537 \quad \int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx$$

Optimal. Leaf size=28

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Rubi [A] time = 0.06, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1586, 1593, 446, 78, 63, 207}

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}\left(\sqrt{x^8+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17), x]

[Out] -1/(4*Sqrt[1 + x^8]) - ArcTanh[Sqrt[1 + x^8]]/4

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 78

Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && (!LtQ[n, -1] || IntegerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 446

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1586

Int[(u_.)*(Px_)^(p_.)*(Qx_)^(q_.), x_Symbol] := Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&

PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+x^8}(1+2x^8)}{x+2x^9+x^{17}} dx &= \int \frac{1+2x^8}{\sqrt{1+x^8}(x+x^9)} dx \\
&= \int \frac{1+2x^8}{x(1+x^8)^{3/2}} dx \\
&= \frac{1}{8} \text{Subst} \left(\int \frac{1+2x}{x(1+x)^{3/2}} dx, x, x^8 \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{8} \text{Subst} \left(\int \frac{1}{x\sqrt{1+x}} dx, x, x^8 \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} + \frac{1}{4} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+x^8} \right) \\
&= -\frac{1}{4\sqrt{1+x^8}} - \frac{1}{4} \tanh^{-1}(\sqrt{1+x^8})
\end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}(\sqrt{x^8+1})$$

Antiderivative was successfully verified.

`[In] Integrate[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17), x]``[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4`**IntegrateAlgebraic [A]** time = 0.04, size = 28, normalized size = 1.00

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{4} \tanh^{-1}(\sqrt{x^8+1})$$

Antiderivative was successfully verified.

`[In] IntegrateAlgebraic[(Sqrt[1 + x^8]*(1 + 2*x^8))/(x + 2*x^9 + x^17), x]``[Out] -1/4*1/Sqrt[1 + x^8] - ArcTanh[Sqrt[1 + x^8]]/4`**fricas [B]** time = 0.76, size = 52, normalized size = 1.86

$$\frac{(x^8+1)\log(\sqrt{x^8+1}+1) - (x^8+1)\log(\sqrt{x^8+1}-1) + 2\sqrt{x^8+1}}{8(x^8+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x), x, algorithm="fricas")``[Out] -1/8*((x^8 + 1)*log(sqrt(x^8 + 1) + 1) - (x^8 + 1)*log(sqrt(x^8 + 1) - 1) + 2*sqrt(x^8 + 1))/(x^8 + 1)`**giac [A]** time = 0.50, size = 34, normalized size = 1.21

$$-\frac{1}{4\sqrt{x^8+1}} - \frac{1}{8} \log(\sqrt{x^8+1}+1) + \frac{1}{8} \log(\sqrt{x^8+1}-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="giac")

[Out] -1/4/sqrt(x^8 + 1) - 1/8*log(sqrt(x^8 + 1) + 1) + 1/8*log(sqrt(x^8 + 1) - 1)

maple [A] time = 0.03, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{\sqrt{x^8+1}-1}{\sqrt{x^8}}\right)}{4} - \frac{1}{4\sqrt{x^8+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x)

[Out] 1/4*ln(((x^8+1)^(1/2)-1)/(x^8)^(1/2))-1/4/(x^8+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(2x^8 + 1)\sqrt{x^8 + 1}}{x^{17} + 2x^9 + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^8+1)*(x^8+1)^(1/2)/(x^17+2*x^9+x),x, algorithm="maxima")

[Out] integrate((2*x^8 + 1)*sqrt(x^8 + 1)/(x^17 + 2*x^9 + x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{x^8 + 1} (2x^8 + 1)}{x^{17} + 2x^9 + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17),x)

[Out] int(((x^8 + 1)^(1/2)*(2*x^8 + 1))/(x + 2*x^9 + x^17), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x^8 + 1}{x(x^8 + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**8+1)*(x**8+1)**(1/2)/(x**17+2*x**9+x),x)

[Out] Integral((2*x**8 + 1)/(x*(x**8 + 1)**(3/2)), x)

$$3.538 \quad \int \left(1 - 9x^2 + \frac{x}{\sqrt{1-9x^2}} \right) dx$$

Optimal. Leaf size=22

$$-3x^3 - \frac{1}{9}\sqrt{1-9x^2} + x$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {261}

$$-3x^3 - \frac{1}{9}\sqrt{1-9x^2} + x$$

Antiderivative was successfully verified.

[In] Int[1 - 9*x^2 + x/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \left(1 - 9x^2 + \frac{x}{\sqrt{1-9x^2}} \right) dx &= x - 3x^3 + \int \frac{x}{\sqrt{1-9x^2}} dx \\ &= x - 3x^3 - \frac{1}{9}\sqrt{1-9x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$-3x^3 - \frac{1}{9}\sqrt{1-9x^2} + x$$

Antiderivative was successfully verified.

[In] Integrate[1 - 9*x^2 + x/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

IntegrateAlgebraic [A] time = 0.27, size = 22, normalized size = 1.00

$$-3x^3 - \frac{1}{9}\sqrt{1-9x^2} + x$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1 - 9*x^2 + x/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

fricas [A] time = 0.53, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="fricas")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

giac [A] time = 0.34, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="giac")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-3x^3 + x - \frac{\sqrt{-9x^2 + 1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-9*x^2+x/(-9*x^2+1)^(1/2),x)

[Out] x-3*x^3-1/9*(-9*x^2+1)^(1/2)

maxima [A] time = 0.58, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-9*x^2+x/(-9*x^2+1)^(1/2),x, algorithm="maxima")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

mupad [B] time = 0.04, size = 18, normalized size = 0.82

$$x - 3x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1 - 9*x^2)^(1/2) - 9*x^2 + 1,x)

[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3

sympy [A] time = 0.15, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{1 - 9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-9*x**2+x/(-9*x**2+1)**(1/2),x)

[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9

$$3.539 \quad \int \frac{x + (1 - 9x^2)^{3/2}}{\sqrt{1 - 9x^2}} dx$$

Optimal. Leaf size=22

$$-3x^3 - \frac{1}{9}\sqrt{1 - 9x^2} + x$$

Rubi [A] time = 0.08, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {6742, 261}

$$-3x^3 - \frac{1}{9}\sqrt{1 - 9x^2} + x$$

Antiderivative was successfully verified.

[In] Int[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 6742

Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{x + (1 - 9x^2)^{3/2}}{\sqrt{1 - 9x^2}} dx &= \int \left(1 - 9x^2 + \frac{x}{\sqrt{1 - 9x^2}} \right) dx \\ &= x - 3x^3 + \int \frac{x}{\sqrt{1 - 9x^2}} dx \\ &= x - 3x^3 - \frac{1}{9}\sqrt{1 - 9x^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 22, normalized size = 1.00

$$-3x^3 - \frac{1}{9}\sqrt{1 - 9x^2} + x$$

Antiderivative was successfully verified.

[In] Integrate[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

IntegrateAlgebraic [A] time = 0.26, size = 22, normalized size = 1.00

$$-3x^3 - \frac{1}{9}\sqrt{1 - 9x^2} + x$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + (1 - 9*x^2)^(3/2))/Sqrt[1 - 9*x^2], x]

[Out] x - 3*x^3 - Sqrt[1 - 9*x^2]/9

fricas [A] time = 0.64, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2), x, algorithm="fricas")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

giac [A] time = 0.57, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2), x, algorithm="giac")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

maple [A] time = 0.00, size = 19, normalized size = 0.86

$$-3x^3 + x - \frac{\sqrt{-9x^2 + 1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2), x)

[Out] -3*x^3+x-1/9*(-9*x^2+1)^(1/2)

maxima [A] time = 0.82, size = 18, normalized size = 0.82

$$-3x^3 + x - \frac{1}{9}\sqrt{-9x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-9*x^2+1)^(3/2))/(-9*x^2+1)^(1/2), x, algorithm="maxima")

[Out] -3*x^3 + x - 1/9*sqrt(-9*x^2 + 1)

mupad [B] time = 0.03, size = 18, normalized size = 0.82

$$x - 3x^3 - \frac{\sqrt{\frac{1}{9} - x^2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + (1 - 9*x^2)^(3/2))/(1 - 9*x^2)^(1/2), x)

[Out] x - 3*x^3 - (1/9 - x^2)^(1/2)/3

sympy [A] time = 1.17, size = 17, normalized size = 0.77

$$-3x^3 + x - \frac{\sqrt{1 - 9x^2}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-9*x**2+1)**(3/2))/(-9*x**2+1)**(1/2), x)

[Out] -3*x**3 + x - sqrt(1 - 9*x**2)/9

$$3.540 \quad \int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx$$

Optimal. Leaf size=17

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Rubi [A] time = 0.06, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2034, 629}

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] Int[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x], x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 2034

Int[(x_)^(m_.)*((b_.)*(x_)^(k_.) + (a_.)*(x_)^(j_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x^Simplify[k/n])^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, j, k, m, n, p, q}, x] && !IntegerQ[p] && NeQ[k, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[k/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rubi steps

$$\int \frac{(-3+2\sqrt{x})(-3\sqrt{x}+x)^{2/3}}{\sqrt{x}} dx = 2 \text{Subst} \left(\int (-3+2x)(-3x+x^2)^{2/3} dx, x, \sqrt{x} \right) \\ = \frac{6}{5} (-3\sqrt{x}+x)^{5/3}$$

Mathematica [A] time = 0.02, size = 17, normalized size = 1.00

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[((-3 + 2*Sqrt[x])*(-3*Sqrt[x] + x)^(2/3))/Sqrt[x], x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5

IntegrateAlgebraic [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((-3 + 2*sqrt[x])*(-3*sqrt[x] + x)^(2/3))/sqrt[x], x]

[Out] (6*(-3*sqrt[x] + x)^(5/3))/5

fricas [A] time = 0.57, size = 11, normalized size = 0.65

$$\frac{6}{5}(x - 3\sqrt{x})^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2), x, algorithm="fricas")

[Out] 6/5*(x - 3*sqrt(x))^(5/3)

giac [A] time = 0.37, size = 11, normalized size = 0.65

$$\frac{6}{5}(x - 3\sqrt{x})^{\frac{5}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2), x, algorithm="giac")

[Out] 6/5*(x - 3*sqrt(x))^(5/3)

maple [A] time = 0.01, size = 12, normalized size = 0.71

$$\frac{6(x - 3\sqrt{x})^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2), x)

[Out] 6/5*(x-3*x^(1/2))^(5/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x - 3\sqrt{x})^{\frac{2}{3}}(2\sqrt{x} - 3)}{\sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-3*x^(1/2))^(2/3)*(-3+2*x^(1/2))/x^(1/2), x, algorithm="maxima")

[Out] integrate((x - 3*sqrt(x))^(2/3)*(2*sqrt(x) - 3)/sqrt(x), x)

mupad [B] time = 3.70, size = 11, normalized size = 0.65

$$\frac{6(x - 3\sqrt{x})^{\frac{5}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x - 3*x^(1/2))^(2/3)*(2*x^(1/2) - 3))/x^(1/2), x)

[Out] (6*(x - 3*x^(1/2))^(5/3))/5

sympy [B] time = 1.27, size = 36, normalized size = 2.12

$$-\frac{18\sqrt{x}(-3\sqrt{x}+x)^{\frac{2}{3}}}{5} + \frac{6x(-3\sqrt{x}+x)^{\frac{2}{3}}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x-3*x**(1/2))**(2/3)*(-3+2*x**(1/2))/x**(1/2),x)

[Out] -18*sqrt(x)*(-3*sqrt(x) + x)**(2/3)/5 + 6*x*(-3*sqrt(x) + x)**(2/3)/5

$$3.541 \quad \int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}} dx$$

Optimal. Leaf size=17

$$\frac{6}{5}(x-3\sqrt{x})^{5/3}$$

Rubi [A] time = 0.05, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2043, 1631, 629}

$$\frac{6}{5}(x-3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] Int[(9 - 9*sqrt[x] + 2*x)/(-3*sqrt[x] + x)^(1/3), x]

[Out] (6*(-3*sqrt[x] + x)^(5/3))/5

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1631

Int[(Pq_)*((e_.)*(x_)^(m_.))*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[e, Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[PolynomialRemainder[Pq, b + c*x, x], 0]

Rule 2043

Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := With[{d = Denominator[n]}, Dist[d, Subst[Int[x^(d - 1)*(SubstFor[x^n, Pq, x] /. x -> x^(d*n))*(a*x^(d*j) + b*x^(d*n))^p, x], x, x^(1/d)], x] /; FreeQ[{a, b, j, n, p}, x] && PolyQ[Pq, x^n] && !IntegerQ[p] && NeQ[n, j] && RationalQ[j, n] && IntegerQ[j/n] && LtQ[-1, n, 1]

Rubi steps

$$\begin{aligned} \int \frac{9-9\sqrt{x}+2x}{\sqrt[3]{-3\sqrt{x}+x}} dx &= 2 \operatorname{Subst} \left(\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}} dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int (-3+2x)(-3x+x^2)^{2/3} dx, x, \sqrt{x} \right) \\ &= \frac{6}{5}(-3\sqrt{x}+x)^{5/3} \end{aligned}$$

Mathematica [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5}(x-3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] Integrate[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3), x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5

IntegrateAlgebraic [A] time = 0.04, size = 17, normalized size = 1.00

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(9 - 9*Sqrt[x] + 2*x)/(-3*Sqrt[x] + x)^(1/3), x]

[Out] (6*(-3*Sqrt[x] + x)^(5/3))/5

fricas [A] time = 0.59, size = 11, normalized size = 0.65

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3), x, algorithm="fricas")

[Out] 6/5*(x - 3*sqrt(x))^(5/3)

giac [A] time = 0.33, size = 11, normalized size = 0.65

$$\frac{6}{5} (x - 3\sqrt{x})^{5/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3), x, algorithm="giac")

[Out] 6/5*(x - 3*sqrt(x))^(5/3)

maple [C] time = 0.11, size = 125, normalized size = 7.35

$$\frac{43^2 \left(-\operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{11}{6}} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{11}{3}\right], \left[\frac{14}{3}, \frac{\sqrt{x}}{3}\right]\right) - 93^2 \left(-\operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{4}{3}} x^{\frac{4}{3}} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{8}{3}\right], \left[\frac{11}{3}, \frac{\sqrt{x}}{3}\right]\right) + 183^2 \left(-\operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)\right)^{\frac{5}{6}} x^{\frac{5}{6}} \operatorname{hypergeom}\left(\left[\frac{1}{3}, \frac{5}{3}\right], \left[\frac{8}{3}, \frac{\sqrt{x}}{3}\right]\right)}{11 \operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}} - 4 \operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}} + 5 \operatorname{signum}\left(\frac{\sqrt{x}}{3}-1\right)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((9+2*x-9*x^(1/2))/(x-3*x^(1/2))^(1/3), x)

[Out] 18/5*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(5/6)*hypergeom([1/3, 5/3], [8/3], 1/3*x^(1/2))+4/11*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(11/6)*hypergeom([1/3, 11/3], [14/3], 1/3*x^(1/2))-9/4*3^(2/3)/signum(-1+1/3*x^(1/2))^(1/3)*(-signum(-1+1/3*x^(1/2)))^(1/3)*x^(4/3)*hypergeom([1/3, 8/3], [11/3], 1/3*x^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x - 9\sqrt{x} + 9}{(x - 3\sqrt{x})^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x, algorithm="maxima")

[Out] integrate((2*x - 9*sqrt(x) + 9)/(x - 3*sqrt(x))^(1/3), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{2x - 9\sqrt{x} + 9}{(x - 3\sqrt{x})^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)`

[Out] `int((2*x - 9*x^(1/2) + 9)/(x - 3*x^(1/2))^(1/3), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{-9\sqrt{x} + 2x + 9}{\sqrt[3]{-3\sqrt{x} + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((9+2*x-9*x**(1/2))/(x-3*x**(1/2))**(1/3), x)`

[Out] `Integral((-9*sqrt(x) + 2*x + 9)/(-3*sqrt(x) + x)**(1/3), x)`

$$3.542 \quad \int \frac{1}{\sqrt{4-9x^2}} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {216}

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[4 - 9*x^2], x]

[Out] ArcSin[(3*x)/2]/3

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{4-9x^2}} dx = \frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Mathematica [A] time = 0.00, size = 10, normalized size = 1.00

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[4 - 9*x^2], x]

[Out] ArcSin[(3*x)/2]/3

IntegrateAlgebraic [C] time = 0.03, size = 24, normalized size = 2.40

$$\frac{1}{3} i \log\left(\sqrt{4-9x^2} - 3ix\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[4 - 9*x^2], x]

[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]

fricas [B] time = 0.70, size = 19, normalized size = 1.90

$$-\frac{2}{3} \arctan\left(\frac{\sqrt{-9x^2+4}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="fricas")

[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)

giac [A] time = 0.36, size = 6, normalized size = 0.60

$$\frac{1}{3} \arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="giac")

[Out] 1/3*arcsin(3/2*x)

maple [A] time = 0.00, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-9*x^2+4)^(1/2),x)

[Out] 1/3*arcsin(3/2*x)

maxima [A] time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3} \arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*x^2+4)^(1/2),x, algorithm="maxima")

[Out] 1/3*arcsin(3/2*x)

mupad [B] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(4 - 9*x^2)^(1/2),x)

[Out] asin((3*x)/2)/3

sympy [A] time = 0.15, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-9*x**2+4)**(1/2),x)

[Out] asin(3*x/2)/3

$$3.543 \quad \int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {41, 216}

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] ArcSin[(3*x)/2]/3

Rule 41

Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(m_.), x_Symbol] := Int[(a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{2-3x}\sqrt{2+3x}} dx &= \int \frac{1}{\sqrt{4-9x^2}} dx \\ &= \frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3} \sin^{-1}\left(\frac{3x}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] ArcSin[(3*x)/2]/3

IntegrateAlgebraic [B] time = 0.05, size = 24, normalized size = 2.40

$$-\frac{2}{3} \tan^{-1}\left(\frac{\sqrt{2-3x}}{\sqrt{3x+2}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[2 - 3*x]*Sqrt[2 + 3*x]),x]

[Out] $(-2*\text{ArcTan}[\text{Sqrt}[2 - 3*x]/\text{Sqrt}[2 + 3*x]])/3$

fricas [B] time = 0.61, size = 25, normalized size = 2.50

$$-\frac{2}{3} \arctan\left(\frac{\sqrt{3x+2}\sqrt{-3x+2}-2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="fricas")`

[Out] $-2/3*\arctan(1/3*(\text{sqrt}(3*x + 2)*\text{sqrt}(-3*x + 2) - 2)/x)$

giac [A] time = 0.37, size = 12, normalized size = 1.20

$$\frac{2}{3} \arcsin\left(\frac{1}{2} \sqrt{3x+2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="giac")`

[Out] $2/3*\arcsin(1/2*\text{sqrt}(3*x + 2))$

maple [B] time = 0.01, size = 34, normalized size = 3.40

$$\frac{\sqrt{(-3x+2)(3x+2)} \arcsin\left(\frac{3x}{2}\right)}{3\sqrt{-3x+2} \sqrt{3x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(2-3*x)^(1/2)/(3*x+2)^(1/2),x)`

[Out] $1/3*((2-3*x)*(3*x+2))^(1/2)/(2-3*x)^(1/2)/(3*x+2)^(1/2)*\arcsin(3/2*x)$

maxima [A] time = 1.93, size = 6, normalized size = 0.60

$$\frac{1}{3} \arcsin\left(\frac{3}{2} x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(2-3*x)^(1/2)/(2+3*x)^(1/2),x, algorithm="maxima")`

[Out] $1/3*\arcsin(3/2*x)$

mupad [B] time = 0.15, size = 32, normalized size = 3.20

$$-\frac{4 \operatorname{atan}\left(\frac{\sqrt{2}-\sqrt{2-3x}}{\sqrt{2}-\sqrt{3x+2}}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2 - 3*x)^(1/2)*(3*x + 2)^(1/2)),x)`

[Out] $-(4*\operatorname{atan}((2^(1/2) - (2 - 3*x)^(1/2))/(2^(1/2) - (3*x + 2)^(1/2))))/3$

sympy [B] time = 1.04, size = 51, normalized size = 5.10

$$\left\{ \begin{array}{l} -\frac{2i \operatorname{acosh}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} \quad \text{for } \frac{3|x+\frac{2}{3}|}{4} > 1 \\ \frac{2 \operatorname{asin}\left(\frac{\sqrt{3}\sqrt{x+\frac{2}{3}}}{2}\right)}{3} \quad \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2-3*x)**(1/2)/(2+3*x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(3)*sqrt(x + 2/3)/2)/3, 3*Abs(x + 2/3)/4 > 1), (2  
*asin(sqrt(3)*sqrt(x + 2/3)/2)/3, True))
```

$$3.544 \quad \int \frac{1}{\sqrt{(2-3x)(2+3x)}} dx$$

Optimal. Leaf size=10

$$\frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right)$$

Rubi [A] time = 0.00, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1972, 216}

$$\frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 1972

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(2-3x)(2+3x)}} dx &= \int \frac{1}{\sqrt{4-9x^2}} dx \\ &= \frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 10, normalized size = 1.00

$$\frac{1}{3} \sin^{-1} \left(\frac{3x}{2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] ArcSin[(3*x)/2]/3

IntegrateAlgebraic [C] time = 0.04, size = 24, normalized size = 2.40

$$\frac{1}{3} i \log \left(\sqrt{4-9x^2} - 3ix \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(2 - 3*x)*(2 + 3*x)],x]

[Out] (I/3)*Log[(-3*I)*x + Sqrt[4 - 9*x^2]]

fricas [B] time = 0.86, size = 19, normalized size = 1.90

$$-\frac{2}{3} \arctan\left(\frac{\sqrt{-9x^2 + 4} - 2}{3x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="fricas")

[Out] -2/3*arctan(1/3*(sqrt(-9*x^2 + 4) - 2)/x)

giac [A] time = 0.39, size = 6, normalized size = 0.60

$$\frac{1}{3} \arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="giac")

[Out] 1/3*arcsin(3/2*x)

maple [A] time = 0.01, size = 7, normalized size = 0.70

$$\frac{\arcsin\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-3*x+2)*(3*x+2))^(1/2),x)

[Out] 1/3*arcsin(3/2*x)

maxima [A] time = 1.96, size = 6, normalized size = 0.60

$$\frac{1}{3} \arcsin\left(\frac{3}{2}x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))^(1/2),x, algorithm="maxima")

[Out] 1/3*arcsin(3/2*x)

mupad [B] time = 0.01, size = 6, normalized size = 0.60

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-(3*x - 2)*(3*x + 2))^(1/2),x)

[Out] asin((3*x)/2)/3

sympy [A] time = 1.39, size = 7, normalized size = 0.70

$$\frac{\operatorname{asin}\left(\frac{3x}{2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((2-3*x)*(2+3*x))**(1/2),x)

[Out] asin(3*x/2)/3

$$3.545 \quad \int \frac{1}{\sqrt{15-2x-x^2}} dx$$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[15 - 2*x - x^2], x]

[Out] -ArcSin[(-1 - x)/4]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{15-2x-x^2}} dx &= -\left(\frac{1}{8} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} dx, x, -2-2x\right)\right) \\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[15 - 2*x - x^2], x]

[Out] -ArcSin[(-1 - x)/4]

IntegrateAlgebraic [A] time = 0.12, size = 23, normalized size = 1.92

$$-2 \tan^{-1}\left(\frac{\sqrt{-x^2-2x+15}}{x+5}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[15 - 2*x - x^2], x]

[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]

fricas [B] time = 0.74, size = 29, normalized size = 2.42

$$-\arctan\left(\frac{\sqrt{-x^2 - 2x + 15}(x + 1)}{x^2 + 2x - 15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-2*x+15)^(1/2), x, algorithm="fricas")

[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))

giac [A] time = 0.42, size = 6, normalized size = 0.50

$$\arcsin\left(\frac{1}{4}x + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-2*x+15)^(1/2), x, algorithm="giac")

[Out] arcsin(1/4*x + 1/4)

maple [A] time = 0.00, size = 7, normalized size = 0.58

$$\arcsin\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2-2*x+15)^(1/2), x)

[Out] arcsin(1/4+1/4*x)

maxima [A] time = 1.97, size = 8, normalized size = 0.67

$$-\arcsin\left(-\frac{1}{4}x - \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-2*x+15)^(1/2), x, algorithm="maxima")

[Out] -arcsin(-1/4*x - 1/4)

mupad [B] time = 3.12, size = 6, normalized size = 0.50

$$\operatorname{asin}\left(\frac{x}{4} + \frac{1}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(15 - x^2 - 2*x)^(1/2), x)

[Out] asin(x/4 + 1/4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 2x + 15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2-2*x+15)**(1/2), x)

[Out] Integral(1/sqrt(-x**2 - 2*x + 15), x)

$$3.546 \quad \int \frac{1}{\sqrt{3-x}\sqrt{5+x}} dx$$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {53, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]

[Out] -ArcSin[(-1 - x)/4]

Rule 53

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Int[1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3-x}\sqrt{5+x}} dx &= \int \frac{1}{\sqrt{15-2x-x^2}} dx \\ &= -\left(\frac{1}{8} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} dx, x, -2-2x\right)\right) \\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right) \end{aligned}$$

Mathematica [A] time = 0.02, size = 21, normalized size = 1.75

$$-2 \sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]

[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]

IntegrateAlgebraic [A] time = 0.04, size = 20, normalized size = 1.67

$$-2 \tan^{-1} \left(\frac{\sqrt{3-x}}{\sqrt{x+5}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[3 - x]*Sqrt[5 + x]),x]

[Out] -2*ArcTan[Sqrt[3 - x]/Sqrt[5 + x]]

fricas [B] time = 0.63, size = 29, normalized size = 2.42

$$-\arctan \left(\frac{\sqrt{x+5}(x+1)\sqrt{-x+3}}{x^2+2x-15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(x + 5)*(x + 1)*sqrt(-x + 3)/(x^2 + 2*x - 15))

giac [B] time = 0.33, size = 13, normalized size = 1.08

$$2 \arcsin \left(\frac{1}{4} \sqrt{2} \sqrt{x+5} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")

[Out] 2*arcsin(1/4*sqrt(2)*sqrt(x + 5))

maple [B] time = 0.01, size = 31, normalized size = 2.58

$$\frac{\sqrt{(-x+3)(x+5)} \arcsin \left(\frac{x}{4} + \frac{1}{4} \right)}{\sqrt{-x+3} \sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x+3)^(1/2)/(x+5)^(1/2),x)

[Out] ((-x+3)*(x+5))^(1/2)/(-x+3)^(1/2)/(x+5)^(1/2)*arcsin(1/4*x+1/4)

maxima [A] time = 1.96, size = 8, normalized size = 0.67

$$-\arcsin \left(-\frac{1}{4}x - \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-1/4*x - 1/4)

mupad [B] time = 3.43, size = 30, normalized size = 2.50

$$4 \operatorname{atan} \left(\frac{\sqrt{3} - \sqrt{3-x}}{\sqrt{x+5} - \sqrt{5}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((3 - x)^(1/2)*(x + 5)^(1/2)),x)
```

```
[Out] 4*atan((3^(1/2) - (3 - x)^(1/2))/((x + 5)^(1/2) - 5^(1/2)))
```

sympy [B] time = 1.02, size = 41, normalized size = 3.42

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{for } \frac{|x+5|}{8} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/4), Abs(x + 5)/8 > 1), (2*asin(sqrt(2)*sqrt(x + 5)/4), True))
```


$$3.547 \quad \int \frac{1}{\sqrt{(3-x)(5+x)}} dx$$

Optimal. Leaf size=12

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 619, 216}

$$-\sin^{-1}\left(\frac{1}{4}(-x-1)\right)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(3 - x)*(5 + x)], x]

[Out] -ArcSin[(-1 - x)/4]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(3-x)(5+x)}} dx &= \int \frac{1}{\sqrt{15-2x-x^2}} dx \\ &= -\left(\frac{1}{8} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{64}}} dx, x, -2-2x\right)\right) \\ &= -\sin^{-1}\left(\frac{1}{4}(-1-x)\right) \end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.75

$$-2 \sin^{-1}\left(\frac{\sqrt{3-x}}{2\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(3 - x)*(5 + x)], x]

[Out] -2*ArcSin[Sqrt[3 - x]/(2*Sqrt[2])]

IntegrateAlgebraic [A] time = 0.11, size = 23, normalized size = 1.92

$$-2 \tan^{-1} \left(\frac{\sqrt{-x^2 - 2x + 15}}{x + 5} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(3 - x)*(5 + x)], x]

[Out] -2*ArcTan[Sqrt[15 - 2*x - x^2]/(5 + x)]

fricas [B] time = 0.59, size = 29, normalized size = 2.42

$$- \arctan \left(\frac{\sqrt{-x^2 - 2x + 15}(x + 1)}{x^2 + 2x - 15} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3-x)*(5+x))^(1/2), x, algorithm="fricas")

[Out] -arctan(sqrt(-x^2 - 2*x + 15)*(x + 1)/(x^2 + 2*x - 15))

giac [A] time = 0.35, size = 6, normalized size = 0.50

$$\arcsin \left(\frac{1}{4}x + \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3-x)*(5+x))^(1/2), x, algorithm="giac")

[Out] arcsin(1/4*x + 1/4)

maple [A] time = 0.01, size = 7, normalized size = 0.58

$$\arcsin \left(\frac{x}{4} + \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-x+3)*(x+5))^(1/2), x)

[Out] arcsin(1/4*x+1/4)

maxima [A] time = 1.97, size = 8, normalized size = 0.67

$$- \arcsin \left(-\frac{1}{4}x - \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((3-x)*(5+x))^(1/2), x, algorithm="maxima")

[Out] -arcsin(-1/4*x - 1/4)

mupad [B] time = 3.37, size = 6, normalized size = 0.50

$$\operatorname{asin} \left(\frac{x}{4} + \frac{1}{4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-(x - 3)*(x + 5))^(1/2),x)
```

```
[Out] asin(x/4 + 1/4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(3-x)(x+5)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((3-x)*(5+x))**(1/2),x)
```

```
[Out] Integral(1/sqrt((3 - x)*(x + 5)), x)
```

$$3.548 \quad \int \frac{1}{\sqrt{-15-8x-x^2}} dx$$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[-15 - 8*x - x^2], x]

[Out] ArcSin[4 + x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\int \frac{1}{\sqrt{-15-8x-x^2}} dx = -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -8-2x\right)\right) = \sin^{-1}(4+x)$$

Mathematica [A] time = 0.01, size = 4, normalized size = 1.00

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[-15 - 8*x - x^2], x]

[Out] ArcSin[4 + x]

IntegrateAlgebraic [B] time = 0.11, size = 23, normalized size = 5.75

$$-2 \tan^{-1}\left(\frac{\sqrt{-x^2-8x-15}}{x+5}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[-15 - 8*x - x^2], x]

[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]

fricas [B] time = 0.77, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2 - 8x - 15}(x + 4)}{x^2 + 8x + 15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))

giac [A] time = 0.34, size = 4, normalized size = 1.00

$$\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="giac")

[Out] arcsin(x + 4)

maple [A] time = 0.00, size = 5, normalized size = 1.25

$$\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2-8*x-15)^(1/2),x)

[Out] arcsin(x+4)

maxima [A] time = 1.99, size = 8, normalized size = 2.00

$$-\arcsin(-x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2-8*x-15)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-x - 4)

mupad [B] time = 3.18, size = 4, normalized size = 1.00

$$\operatorname{asin}(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-8*x - x^2 - 15)^(1/2),x)

[Out] asin(x + 4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 - 8x - 15}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2-8*x-15)**(1/2),x)

[Out] Integral(1/sqrt(-x**2 - 8*x - 15), x)

$$3.549 \quad \int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx$$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {53, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]

[Out] ArcSin[4 + x]

Rule 53

Int[1/(Sqrt[(a_) + (b_)*(x_)]*Sqrt[(c_) + (d_)*(x_)]), x_Symbol] := Int[1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b + d, 0] && GtQ[a + c, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{-3-x}\sqrt{5+x}} dx &= \int \frac{1}{\sqrt{-15-8x-x^2}} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -8-2x\right)\right) \\ &= \sin^{-1}(4+x) \end{aligned}$$

Mathematica [B] time = 0.01, size = 18, normalized size = 4.50

$$-2 \sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]

[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]

IntegrateAlgebraic [B] time = 0.04, size = 20, normalized size = 5.00

$$-2 \tan^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{x+5}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[-3 - x]*Sqrt[5 + x]),x]

[Out] -2*ArcTan[Sqrt[-3 - x]/Sqrt[5 + x]]

fricas [B] time = 0.58, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{x+5}(x+4)\sqrt{-x-3}}{x^2+8x+15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(x + 5)*(x + 4)*sqrt(-x - 3)/(x^2 + 8*x + 15))

giac [B] time = 0.32, size = 13, normalized size = 3.25

$$2 \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{x+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="giac")

[Out] 2*arcsin(1/2*sqrt(2)*sqrt(x + 5))

maple [B] time = 0.01, size = 29, normalized size = 7.25

$$\frac{\sqrt{(-x-3)(x+5)} \arcsin(x+4)}{\sqrt{-x-3} \sqrt{x+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-3-x)^(1/2)/(x+5)^(1/2),x)

[Out] ((-3-x)*(x+5))^(1/2)/(-3-x)^(1/2)/(x+5)^(1/2)*arcsin(x+4)

maxima [A] time = 1.97, size = 8, normalized size = 2.00

$$-\arcsin(-x-4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-3-x)^(1/2)/(5+x)^(1/2),x, algorithm="maxima")

[Out] -arcsin(-x - 4)

mupad [B] time = 0.08, size = 33, normalized size = 8.25

$$4 \operatorname{atan}\left(\frac{-\sqrt{-x-3} + \sqrt{3} \operatorname{1i}}{\sqrt{x+5} - \sqrt{5}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-x-3)^(1/2)*(x+5)^(1/2)),x)

[Out] 4*atan((3^(1/2)*1i - (-x-3)^(1/2))/((x+5)^(1/2) - 5^(1/2)))

sympy [B] time = 1.02, size = 41, normalized size = 10.25

$$\begin{cases} -2i \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{for } \frac{|x+5|}{2} > 1 \\ 2 \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+5}}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-3-x)**(1/2)/(5+x)**(1/2),x)
```

```
[Out] Piecewise((-2*I*acosh(sqrt(2)*sqrt(x + 5)/2), Abs(x + 5)/2 > 1), (2*asin(sqrt(2)*sqrt(x + 5)/2), True))
```


$$3.550 \quad \int \frac{1}{\sqrt{(-3-x)(5+x)}} dx$$

Optimal. Leaf size=4

$$\sin^{-1}(x+4)$$

Rubi [A] time = 0.01, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1981, 619, 216}

$$\sin^{-1}(x+4)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(-3 - x)*(5 + x)], x]

[Out] ArcSin[4 + x]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(-3-x)(5+x)}} dx &= \int \frac{1}{\sqrt{-15-8x-x^2}} dx \\ &= -\left(\frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{4}}} dx, x, -8-2x\right)\right) \\ &= \sin^{-1}(4+x) \end{aligned}$$

Mathematica [B] time = 0.00, size = 18, normalized size = 4.50

$$-2 \sin^{-1}\left(\frac{\sqrt{-x-3}}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(-3 - x)*(5 + x)], x]

[Out] -2*ArcSin[Sqrt[-3 - x]/Sqrt[2]]

IntegrateAlgebraic [B] time = 0.11, size = 23, normalized size = 5.75

$$-2 \tan^{-1}\left(\frac{\sqrt{-x^2-8x-15}}{x+5}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(-3 - x)*(5 + x)],x]

[Out] -2*ArcTan[Sqrt[-15 - 8*x - x^2]/(5 + x)]

fricas [B] time = 0.95, size = 29, normalized size = 7.25

$$-\arctan\left(\frac{\sqrt{-x^2 - 8x - 15}(x + 4)}{x^2 + 8x + 15}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(-x^2 - 8*x - 15)*(x + 4)/(x^2 + 8*x + 15))

giac [A] time = 0.39, size = 4, normalized size = 1.00

$$\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="giac")

[Out] arcsin(x + 4)

maple [A] time = 0.01, size = 5, normalized size = 1.25

$$\arcsin(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-x-3)*(x+5))^(1/2),x)

[Out] arcsin(x+4)

maxima [A] time = 1.95, size = 8, normalized size = 2.00

$$-\arcsin(-x - 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3-x)*(5+x))^(1/2),x, algorithm="maxima")

[Out] -arcsin(-x - 4)

mupad [B] time = 3.36, size = 4, normalized size = 1.00

$$\operatorname{asin}(x + 4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-(x + 3)*(x + 5))^(1/2),x)

[Out] asin(x + 4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(-x - 3)(x + 5)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-3-x)*(5+x))**(1/2),x)

[Out] Integral(1/sqrt((-x - 3)*(x + 5)), x)

$$3.551 \quad \int (1 - \sqrt{x}) dx$$

Optimal. Leaf size=11

$$x - \frac{2x^{3/2}}{3}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[1 - Sqrt[x], x]

[Out] x - (2*x^(3/2))/3

Rubi steps

$$\int (1 - \sqrt{x}) dx = x - \frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[1 - Sqrt[x], x]

[Out] x - (2*x^(3/2))/3

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{3} (3x - 2x^{3/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1 - Sqrt[x], x]

[Out] (3*x - 2*x^(3/2))/3

fricas [A] time = 0.62, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2), x, algorithm="fricas")

[Out] -2/3*x^(3/2) + x

giac [A] time = 0.36, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="giac")

[Out] -2/3*x^(3/2) + x

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-x^(1/2),x)

[Out] x-2/3*x^(3/2)

maxima [A] time = 0.90, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="maxima")

[Out] -2/3*x^(3/2) + x

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1 - x^(1/2),x)

[Out] x - (2*x^(3/2))/3

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x

$$3.552 \quad \int \frac{1-x}{1+\sqrt{x}} dx$$

Optimal. Leaf size=11

$$x - \frac{2x^{3/2}}{3}$$

Rubi [A] time = 0.01, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1398, 26, 43}

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[(1 - x)/(1 + Sqrt[x]),x]

[Out] x - (2*x^(3/2))/3

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 1398

Int[((a_.) + (c_.)*(x_)^(n2_.))^(p_.)*((d_.) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{1-x}{1+\sqrt{x}} dx &= 2 \operatorname{Subst} \left(\int \frac{x(1-x^2)}{1+x} dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int (1-x)x dx, x, \sqrt{x} \right) \\ &= 2 \operatorname{Subst} \left(\int (x-x^2) dx, x, \sqrt{x} \right) \\ &= x - \frac{2x^{3/2}}{3} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x)/(1 + Sqrt[x]), x]

[Out] $x - (2x^{3/2})/3$

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{3} (3x - 2x^{3/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x)/(1 + Sqrt[x]), x]

[Out] $(3x - 2x^{3/2})/3$

fricas [A] time = 0.63, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{3/2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(1+x^(1/2)), x, algorithm="fricas")

[Out] $-2/3*x^{3/2} + x$

giac [A] time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{3/2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(1+x^(1/2)), x, algorithm="giac")

[Out] $-2/3*x^{3/2} + x$

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{3/2}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)/(x^(1/2)+1), x)

[Out] $-2/3*x^{3/2}+x$

maxima [A] time = 0.91, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{3/2} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)/(1+x^(1/2)), x, algorithm="maxima")

[Out] $-2/3*x^{3/2} + x$

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x - 1)/(x^(1/2) + 1),x)
```

```
[Out] x - (2*x^(3/2))/3
```

sympy [A] time = 0.15, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-x)/(1+x**(1/2)),x)
```

```
[Out] -2*x**(3/2)/3 + x
```

$$3.553 \quad \int \sqrt{\frac{1}{1-x^2}} dx$$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6720, 216}

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x^2)^(-1)], x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1}{1-x^2}} dx &= \left(\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \right) \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 - x^2)^(-1)], x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

IntegrateAlgebraic [A] time = 3.95, size = 38, normalized size = 1.41

$$-\sqrt{\frac{1}{1-x^2}} \sqrt{x^2-1} \log(\sqrt{x^2-1} - x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 - x^2)^(-1)],x]

[Out] -(Sqrt[(1 - x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])

fricas [A] time = 0.54, size = 26, normalized size = 0.96

$$2 \arctan\left(\frac{(x^2 - 1)\sqrt{-\frac{1}{x^2 - 1} + 1}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="fricas")

[Out] 2*arctan((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)

giac [A] time = 0.42, size = 10, normalized size = 0.37

$$-\arcsin(x)\operatorname{sgn}(x^2 - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="giac")

[Out] -arcsin(x)*sgn(x^2 - 1)

maple [A] time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2 - 1}} \sqrt{x^2 - 1} \ln(x + \sqrt{x^2 - 1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(-x^2+1))^(1/2),x)

[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{1}{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(-x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-1/(x^2 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-\frac{1}{x^2 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1/(x^2 - 1))^(1/2),x)

[Out] int((-1/(x^2 - 1))^(1/2), x)

sympy [A] time = 1.01, size = 7, normalized size = 0.26

$$\left\{ \operatorname{asin}(x) \text{ for } x > -1 \wedge x < 1 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(-x**2+1))**(1/2),x)

[Out] Piecewise((asin(x), (x > -1) & (x < 1)))

$$3.554 \quad \int \sqrt{\frac{1+x^2}{1-x^4}} dx$$

Optimal. Leaf size=27

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {6688, 6720, 216}

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x^2)/(1 - x^4)], x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rule 6720

Int[(u_)*((a_)*(v_)^(m_))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1+x^2}{1-x^4}} dx &= \int \sqrt{\frac{1}{1-x^2}} dx \\ &= \left(\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \right) \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.00

$$\sqrt{\frac{1}{1-x^2}} \sqrt{1-x^2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x^2)/(1 - x^4)], x]

[Out] Sqrt[(1 - x^2)^(-1)]*Sqrt[1 - x^2]*ArcSin[x]

IntegrateAlgebraic [A] time = 4.46, size = 38, normalized size = 1.41

$$-\sqrt{\frac{1}{1-x^2}} \sqrt{x^2-1} \log(\sqrt{x^2-1}-x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 + x^2)/(1 - x^4)],x]

[Out] -(Sqrt[(1 - x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])

fricas [A] time = 0.63, size = 26, normalized size = 0.96

$$2 \arctan\left(\frac{(x^2-1)\sqrt{-\frac{1}{x^2-1}+1}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="fricas")

[Out] 2*arctan(((x^2 - 1)*sqrt(-1/(x^2 - 1)) + 1)/x)

giac [A] time = 0.34, size = 10, normalized size = 0.37

$$-\arcsin(x)\operatorname{sgn}(x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="giac")

[Out] -arcsin(x)*sgn(x^2 - 1)

maple [A] time = 0.01, size = 30, normalized size = 1.11

$$\sqrt{-\frac{1}{x^2-1}} \sqrt{x^2-1} \ln(x + \sqrt{x^2-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2+1)/(-x^4+1))^(1/2),x)

[Out] (-1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-\frac{x^2+1}{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(-x^4+1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-(x^2 + 1)/(x^4 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{-\frac{x^2+1}{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2 + 1)/(x^4 - 1))^(1/2),x)

```
[Out] int((-x^2 + 1)/(x^4 - 1))^(1/2), x)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \sqrt{\frac{x^2 + 1}{1 - x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**2+1)/(-x**4+1))**(1/2),x)
```

```
[Out] Integral(sqrt((x**2 + 1)/(1 - x**4)), x)
```

$$3.555 \quad \int \sqrt{\frac{1}{-1+x^2}} dx$$

Optimal. Leaf size=25

$$\sqrt{1-x^2} \sqrt{\frac{1}{x^2-1}} \sin^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x^2)^(-1)], x]

[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]

Rule 206

Int[(a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1}{-1+x^2}} dx &= \left(\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \right) \int \frac{1}{\sqrt{-1+x^2}} dx \\ &= \left(\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \right) \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}} \right) \\ &= \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \tanh^{-1} \left(\frac{x}{\sqrt{-1+x^2}} \right) \end{aligned}$$

Mathematica [B] time = 0.03, size = 56, normalized size = 2.24

$$\frac{1}{2} \sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \left(\log \left(\frac{x}{\sqrt{x^2-1}} + 1 \right) - \log \left(1 - \frac{x}{\sqrt{x^2-1}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x^2)^(-1)], x]

[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]))/2

IntegrateAlgebraic [A] time = 3.96, size = 36, normalized size = 1.44

$$-\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \log(\sqrt{x^2-1} - x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^(-1)],x]

[Out] -(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])

fricas [A] time = 0.69, size = 14, normalized size = 0.56

$$-\log(-x + \sqrt{x^2-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(x^2-1))^(1/2),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 - 1))

giac [A] time = 0.36, size = 15, normalized size = 0.60

$$-\log\left(\left|-x + \sqrt{x^2-1}\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(x^2-1))^(1/2),x, algorithm="giac")

[Out] -log(abs(-x + sqrt(x^2 - 1)))

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \ln(x + \sqrt{x^2-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(x^2-1))^(1/2),x)

[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))

maxima [A] time = 0.88, size = 14, normalized size = 0.56

$$\log(2x + 2\sqrt{x^2-1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1/(x^2-1))^(1/2),x, algorithm="maxima")

[Out] log(2*x + 2*sqrt(x^2 - 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{1}{x^2-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(x^2 - 1))^(1/2),x)
```

```
[Out] int((1/(x^2 - 1))^(1/2), x)
```

sympy [A] time = 1.52, size = 15, normalized size = 0.60

$$\left\{ \log\left(x + \sqrt{x^2 - 1}\right) \text{ for } x > -1 \wedge x < 1 \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1/(x**2-1))**(1/2),x)
```

```
[Out] Piecewise((log(x + sqrt(x**2 - 1)), (x > -1) & (x < 1)))
```

$$3.556 \quad \int \sqrt{\frac{1+x^2}{-1+x^4}} dx$$

Optimal. Leaf size=25

$$\sqrt{1-x^2} \sqrt{\frac{1}{x^2-1}} \sin^{-1}(x)$$

Rubi [A] time = 0.02, antiderivative size = 33, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6688, 6720, 217, 206}

$$\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x^2)/(-1 + x^4)], x]

[Out] Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*ArcTanh[x/Sqrt[-1 + x^2]]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifierIntegrandQ[v, u, x]]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^p_, x_Symbol] := Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{1+x^2}{-1+x^4}} dx &= \int \sqrt{\frac{1}{-1+x^2}} dx \\ &= \left(\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \right) \int \frac{1}{\sqrt{-1+x^2}} dx \\ &= \left(\sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \right) \text{Subst} \left(\int \frac{1}{1-x^2} dx, x, \frac{x}{\sqrt{-1+x^2}} \right) \\ &= \sqrt{\frac{1}{-1+x^2}} \sqrt{-1+x^2} \tanh^{-1} \left(\frac{x}{\sqrt{-1+x^2}} \right) \end{aligned}$$

Mathematica [B] time = 0.00, size = 56, normalized size = 2.24

$$\frac{1}{2} \sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \left(\log\left(\frac{x}{\sqrt{x^2-1}} + 1\right) - \log\left(1 - \frac{x}{\sqrt{x^2-1}}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x^2)/(-1 + x^4)], x]

[Out] (Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*(-Log[1 - x/Sqrt[-1 + x^2]] + Log[1 + x/Sqrt[-1 + x^2]]))/2

IntegrateAlgebraic [A] time = 4.38, size = 36, normalized size = 1.44

$$-\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \log\left(\sqrt{x^2-1} - x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 + x^2)/(-1 + x^4)], x]

[Out] -(Sqrt[(-1 + x^2)^(-1)]*Sqrt[-1 + x^2]*Log[-x + Sqrt[-1 + x^2]])

fricas [A] time = 0.62, size = 14, normalized size = 0.56

$$-\log\left(-x + \sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(x^4-1))^(1/2), x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 - 1))

giac [A] time = 0.32, size = 21, normalized size = 0.84

$$-\log\left(\left|-x + \sqrt{x^2-1}\right|\right) \operatorname{sgn}(x^2-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(x^4-1))^(1/2), x, algorithm="giac")

[Out] -log(abs(-x + sqrt(x^2 - 1)))*sgn(x^2 - 1)

maple [A] time = 0.01, size = 28, normalized size = 1.12

$$\sqrt{\frac{1}{x^2-1}} \sqrt{x^2-1} \ln\left(x + \sqrt{x^2-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2+1)/(x^4-1))^(1/2), x)

[Out] (1/(x^2-1))^(1/2)*(x^2-1)^(1/2)*ln(x+(x^2-1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^2+1}{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2+1)/(x^4-1))^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt((x^2 + 1)/(x^4 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\frac{x^2 + 1}{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)

[Out] int(((x^2 + 1)/(x^4 - 1))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x^2 + 1}{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x**2+1)/(x**4-1))**(1/2), x)

[Out] Integral(sqrt((x**2 + 1)/(x**4 - 1)), x)

$$3.557 \quad \int \frac{1}{\sqrt{1-x}} dx$$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - x], x]

[Out] -2*Sqrt[1 - x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\sqrt{1-x}} dx = -2\sqrt{1-x}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - x], x]

[Out] -2*Sqrt[1 - x]

IntegrateAlgebraic [A] time = 0.01, size = 11, normalized size = 1.00

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 - x], x]

[Out] -2*Sqrt[1 - x]

fricas [A] time = 0.58, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(1/2), x, algorithm="fricas")

[Out] -2*sqrt(-x + 1)

giac [A] time = 0.33, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(1/2),x, algorithm="giac")

[Out] -2*sqrt(-x + 1)

maple [A] time = 0.00, size = 10, normalized size = 0.91

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x+1)^(1/2),x)

[Out] -2*(-x+1)^(1/2)

maxima [A] time = 0.89, size = 9, normalized size = 0.82

$$-2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)^(1/2),x, algorithm="maxima")

[Out] -2*sqrt(-x + 1)

mupad [B] time = 0.19, size = 9, normalized size = 0.82

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1 - x)^(1/2),x)

[Out] -2*(1 - x)^(1/2)

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-2\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x)**(1/2),x)

[Out] -2*sqrt(1 - x)

$$3.558 \quad \int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=11

$$-2\sqrt{1-x}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {26, 32}

$$-2\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x]/Sqrt[1 - x^2], x]

[Out] -2*Sqrt[1 - x]

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(- (b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 32

Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{\sqrt{1+x}}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1-x}} dx = -2\sqrt{1-x}$$

Mathematica [B] time = 0.02, size = 23, normalized size = 2.09

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x]/Sqrt[1 - x^2], x]

[Out] (2*(-1 + x)*Sqrt[1 + x])/Sqrt[1 - x^2]

IntegrateAlgebraic [B] time = 0.06, size = 26, normalized size = 2.36

$$-\frac{2\sqrt{2(x+1) - (x+1)^2}}{\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x]/Sqrt[1 - x^2], x]

[Out] (-2*Sqrt[2*(1 + x) - (1 + x)^2])/Sqrt[1 + x]

fricas [C] time = 0.76, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{-x^2+1}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-x^2 + 1)/sqrt(x + 1)

giac [A] time = 0.32, size = 15, normalized size = 1.36

$$2\sqrt{2} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(2) - 2*sqrt(-x + 1)

maple [B] time = 0.00, size = 20, normalized size = 1.82

$$\frac{2(x-1)\sqrt{x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)^(1/2)/(-x^2+1)^(1/2),x)

[Out] 2*(x-1)*(x+1)^(1/2)/(-x^2+1)^(1/2)

maxima [C] time = 0.92, size = 12, normalized size = 1.09

$$\frac{2(x-1)}{\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] 2*(x - 1)/sqrt(-x + 1)

mupad [B] time = 3.56, size = 16, normalized size = 1.45

$$-\frac{2\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(1/2)/(1 - x^2)^(1/2),x)

[Out] -(2*(1 - x^2)^(1/2))/(x + 1)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x+1}}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] Integral(sqrt(x + 1)/sqrt(-(x - 1)*(x + 1)), x)

$$3.559 \quad \int \frac{1}{\sqrt{1+x}} dx$$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x],x]

[Out] 2*Sqrt[1 + x]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{1}{\sqrt{1+x}} dx = 2\sqrt{1+x}$$

Mathematica [A] time = 0.00, size = 9, normalized size = 1.00

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x],x]

[Out] 2*Sqrt[1 + x]

IntegrateAlgebraic [A] time = 0.01, size = 9, normalized size = 1.00

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 + x],x]

[Out] 2*Sqrt[1 + x]

fricas [A] time = 0.66, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2),x, algorithm="fricas")

[Out] 2*sqrt(x + 1)

giac [A] time = 0.34, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(x + 1)

maple [A] time = 0.00, size = 8, normalized size = 0.89

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)^(1/2),x)

[Out] 2*(x+1)^(1/2)

maxima [A] time = 0.88, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x + 1)

mupad [B] time = 0.09, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x + 1)^(1/2),x)

[Out] 2*(x + 1)^(1/2)

sympy [A] time = 0.06, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)**(1/2),x)

[Out] 2*sqrt(x + 1)

$$3.560 \quad \int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=9

$$2\sqrt{x+1}$$

Rubi [A] time = 0.00, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 32}

$$2\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x]/Sqrt[1 - x^2], x]

[Out] 2*Sqrt[1 + x]

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(- (b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \frac{\sqrt{1-x}}{\sqrt{1-x^2}} dx = \int \frac{1}{\sqrt{1+x}} dx = 2\sqrt{1+x}$$

Mathematica [B] time = 0.02, size = 25, normalized size = 2.78

$$\frac{2\sqrt{1-x}(x+1)}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x]/Sqrt[1 - x^2], x]

[Out] (2*Sqrt[1 - x]*(1 + x))/Sqrt[1 - x^2]

IntegrateAlgebraic [B] time = 0.06, size = 32, normalized size = 3.56

$$\frac{2\sqrt{2(1-x) - (1-x)^2}}{\sqrt{1-x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x]/Sqrt[1 - x^2], x]

[Out] (2*Sqrt[2*(1 - x) - (1 - x)^2])/Sqrt[1 - x]

fricas [C] time = 0.88, size = 23, normalized size = 2.56

$$-\frac{2\sqrt{-x^2+1}\sqrt{-x+1}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(-x^2 + 1)*sqrt(-x + 1)/(x - 1)

giac [A] time = 0.43, size = 13, normalized size = 1.44

$$-2\sqrt{2} + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] -2*sqrt(2) + 2*sqrt(x + 1)

maple [B] time = 0.00, size = 22, normalized size = 2.44

$$\frac{2(x+1)\sqrt{-x+1}}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)/(-x^2+1)^(1/2),x)

[Out] 2*(x+1)*(-x+1)^(1/2)/(-x^2+1)^(1/2)

maxima [A] time = 0.90, size = 7, normalized size = 0.78

$$2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x + 1)

mupad [B] time = 3.64, size = 18, normalized size = 2.00

$$\frac{2\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(1/2)/(1 - x^2)^(1/2),x)

[Out] (2*(1 - x^2)^(1/2))/(1 - x)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2)/(-x**2+1)**(1/2),x)

[Out] Integral(sqrt(1 - x)/sqrt(-(x - 1)*(x + 1)), x)

$$3.561 \quad \int \sqrt{1-x} \, dx$$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x], x]

[Out] (-2*(1 - x)^(3/2))/3

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \sqrt{1-x} \, dx = -\frac{2}{3}(1-x)^{3/2}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x], x]

[Out] (-2*(1 - x)^(3/2))/3

IntegrateAlgebraic [A] time = 0.01, size = 13, normalized size = 1.00

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x], x]

[Out] (-2*(1 - x)^(3/2))/3

fricas [A] time = 0.56, size = 12, normalized size = 0.92

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2), x, algorithm="fricas")

[Out] 2/3*(x - 1)*sqrt(-x + 1)

giac [A] time = 0.35, size = 9, normalized size = 0.69

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2),x, algorithm="giac")

[Out] -2/3*(-x + 1)^(3/2)

maple [A] time = 0.00, size = 10, normalized size = 0.77

$$-\frac{2(-x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2),x)

[Out] -2/3*(-x+1)^(3/2)

maxima [A] time = 0.81, size = 9, normalized size = 0.69

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2),x, algorithm="maxima")

[Out] -2/3*(-x + 1)^(3/2)

mupad [B] time = 3.51, size = 9, normalized size = 0.69

$$-\frac{2(1-x)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(1/2),x)

[Out] -(2*(1 - x)^(3/2))/3

sympy [A] time = 0.06, size = 10, normalized size = 0.77

$$-\frac{2(1-x)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2),x)

[Out] -2*(1 - x)**(3/2)/3

$$3.562 \quad \int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} dx$$

Optimal. Leaf size=13

$$-\frac{2}{3}(1-x)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {26, 32}

$$-\frac{2}{3}(1-x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/Sqrt[1 + x], x]

[Out] (-2*(1 - x)^(3/2))/3

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 32

Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1+x}} dx &= \int \sqrt{1-x} dx \\ &= -\frac{2}{3}(1-x)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.92

$$\frac{2(x-1)\sqrt{1-x^2}}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 + x], x]

[Out] (2*(-1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 + x])

IntegrateAlgebraic [B] time = 0.06, size = 28, normalized size = 2.15

$$\frac{2(2(x+1) - (x+1)^2)^{3/2}}{3(x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 + x], x]

[Out] $(-2*(2*(1 + x) - (1 + x)^2)^{(3/2)})/(3*(1 + x)^{(3/2)})$

fricas [B] time = 0.71, size = 19, normalized size = 1.46

$$\frac{2\sqrt{-x^2+1}(x-1)}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")`

[Out] $2/3*\text{sqrt}(-x^2 + 1)*(x - 1)/\text{sqrt}(x + 1)$

giac [A] time = 0.33, size = 15, normalized size = 1.15

$$-\frac{2}{3}(-x+1)^{\frac{3}{2}} + \frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="giac")`

[Out] $-2/3*(-x + 1)^{(3/2)} + 4/3*\text{sqrt}(2)$

maple [B] time = 0.00, size = 20, normalized size = 1.54

$$\frac{2(x-1)\sqrt{-x^2+1}}{3\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)^(1/2)/(x+1)^(1/2),x)`

[Out] $2/3*(x-1)*(-x^2+1)^{(1/2)}/(x+1)^{(1/2)}$

maxima [A] time = 0.83, size = 12, normalized size = 0.92

$$\frac{2}{3}(x-1)\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")`

[Out] $2/3*(x - 1)*\text{sqrt}(-x + 1)$

mupad [B] time = 3.52, size = 20, normalized size = 1.54

$$\frac{\left(\frac{2x}{3} - \frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - x^2)^(1/2)/(x + 1)^(1/2),x)`

[Out] $((2*x)/3 - 2/3)*(1 - x^2)^{(1/2)}/(x + 1)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)**(1/2)/(1+x)**(1/2),x)`

[Out] `Integral(sqrt(-(x - 1)*(x + 1))/sqrt(x + 1), x)`

$$3.563 \quad \int \sqrt{1+x} \, dx$$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x], x]

[Out] (2*(1 + x)^(3/2))/3

Rule 32

Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\int \sqrt{1+x} \, dx = \frac{2}{3}(1+x)^{3/2}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x], x]

[Out] (2*(1 + x)^(3/2))/3

IntegrateAlgebraic [A] time = 0.01, size = 11, normalized size = 1.00

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x], x]

[Out] (2*(1 + x)^(3/2))/3

fricas [A] time = 0.58, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2), x, algorithm="fricas")

[Out] 2/3*(x + 1)^(3/2)

giac [A] time = 0.32, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2),x, algorithm="giac")

[Out] 2/3*(x + 1)^(3/2)

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$\frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)^(1/2),x)

[Out] 2/3*(x+1)^(3/2)

maxima [A] time = 0.88, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(1/2),x, algorithm="maxima")

[Out] 2/3*(x + 1)^(3/2)

mupad [B] time = 3.42, size = 7, normalized size = 0.64

$$\frac{2(x+1)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(1/2),x)

[Out] (2*(x + 1)^(3/2))/3

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$\frac{2(x+1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**(1/2),x)

[Out] 2*(x + 1)**(3/2)/3

$$3.564 \quad \int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx$$

Optimal. Leaf size=11

$$\frac{2}{3}(x+1)^{3/2}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 32}

$$\frac{2}{3}(x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x], x]

[Out] (2*(1 + x)^(3/2))/3

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 32

Int[((a_.) + (b_.)*(x_)^(m_)), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m + 1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^2}}{\sqrt{1-x}} dx &= \int \sqrt{1+x} dx \\ &= \frac{2}{3}(1+x)^{3/2} \end{aligned}$$

Mathematica [B] time = 0.02, size = 27, normalized size = 2.45

$$\frac{2(x+1)\sqrt{1-x^2}}{3\sqrt{1-x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x], x]

[Out] (2*(1 + x)*Sqrt[1 - x^2])/(3*Sqrt[1 - x])

IntegrateAlgebraic [B] time = 0.06, size = 34, normalized size = 3.09

$$\frac{2(2(1-x) - (1-x)^2)^{3/2}}{3(1-x)^{3/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 - x], x]

[Out] $(2*(2*(1-x) - (1-x)^2)^{(3/2)})/(3*(1-x)^{(3/2)})$

fricas [B] time = 0.84, size = 26, normalized size = 2.36

$$\frac{2\sqrt{-x^2+1}(x+1)\sqrt{-x+1}}{3(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="fricas")`

[Out] $-2/3*\text{sqrt}(-x^2+1)*(x+1)*\text{sqrt}(-x+1)/(x-1)$

giac [A] time = 0.33, size = 13, normalized size = 1.18

$$\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{4}{3}\sqrt{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="giac")`

[Out] $2/3*(x+1)^{(3/2)} - 4/3*\text{sqrt}(2)$

maple [B] time = 0.00, size = 22, normalized size = 2.00

$$\frac{2(x+1)\sqrt{-x^2+1}}{3\sqrt{-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)^(1/2)/(-x+1)^(1/2),x)`

[Out] $2/3*(x+1)*(-x^2+1)^{(1/2)/(-x+1)^{(1/2)}$

maxima [A] time = 0.92, size = 7, normalized size = 0.64

$$\frac{2}{3}(x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2)/(1-x)^(1/2),x, algorithm="maxima")`

[Out] $2/3*(x+1)^{(3/2)}$

mupad [B] time = 3.49, size = 22, normalized size = 2.00

$$\frac{\left(\frac{2x}{3} + \frac{2}{3}\right)\sqrt{1-x^2}}{\sqrt{1-x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-x^2)^(1/2)/(1-x)^(1/2),x)`

[Out] $((2*x)/3 + 2/3)*(1-x^2)^{(1/2)/(1-x)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{(x-1)(x+1)}}{\sqrt{1-x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)**(1/2)/(1-x)**(1/2),x)`

[Out] `Integral(sqrt(-(x-1)*(x+1))/sqrt(1-x), x)`

$$3.565 \quad \int \frac{\sqrt{2+3x}}{\sqrt{1+x}} dx$$

Optimal. Leaf size=35

$$\sqrt{x+1} \sqrt{3x+2} - \frac{\sinh^{-1}(\sqrt{3x+2})}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {50, 54, 215}

$$\sqrt{x+1} \sqrt{3x+2} - \frac{\sinh^{-1}(\sqrt{3x+2})}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + 3*x]/Sqrt[1 + x], x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2+3x}}{\sqrt{1+x}} dx &= \sqrt{1+x} \sqrt{2+3x} - \frac{1}{2} \int \frac{1}{\sqrt{1+x} \sqrt{2+3x}} dx \\ &= \sqrt{1+x} \sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{2+3x}\right)}{\sqrt{3}} \\ &= \sqrt{1+x} \sqrt{2+3x} - \frac{\sinh^{-1}(\sqrt{2+3x})}{\sqrt{3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}(3x+2) - \sqrt{9x+6} \sinh^{-1}(\sqrt{3x+2})}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + 3*x]/Sqrt[1 + x],x]

[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2 + 3*x])

IntegrateAlgebraic [A] time = 0.15, size = 53, normalized size = 1.51

$$\frac{\sqrt{3x+2}\sqrt{3x+3}}{\sqrt{3}} + \frac{\log(\sqrt{3x+3} - \sqrt{3x+2})}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 + 3*x]/Sqrt[1 + x],x]

[Out] (Sqrt[2 + 3*x]*Sqrt[3 + 3*x])/Sqrt[3] + Log[-Sqrt[2 + 3*x] + Sqrt[3 + 3*x]]/Sqrt[3]

fricas [A] time = 0.86, size = 52, normalized size = 1.49

$$\frac{1}{12} \sqrt{3} \log\left(-4\sqrt{3}(6x+5)\sqrt{3x+2}\sqrt{x+1} + 72x^2 + 120x + 49\right) + \sqrt{3x+2}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")

[Out] 1/12*sqrt(3)*log(-4*sqrt(3)*(6*x + 5)*sqrt(3*x + 2)*sqrt(x + 1) + 72*x^2 + 120*x + 49) + sqrt(3*x + 2)*sqrt(x + 1)

giac [A] time = 0.35, size = 39, normalized size = 1.11

$$\frac{1}{3} \sqrt{3} \left(\sqrt{3x+3}\sqrt{3x+2} + \log(\sqrt{3x+3} - \sqrt{3x+2}) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(3)*(sqrt(3*x + 3)*sqrt(3*x + 2) + log(sqrt(3*x + 3) - sqrt(3*x + 2)))

maple [B] time = 0.01, size = 67, normalized size = 1.91

$$-\frac{\sqrt{(x+1)(3x+2)}\sqrt{3}\ln\left(\frac{\left(\frac{3x+5}{2}\right)\sqrt{3}}{3} + \sqrt{3x^2+5x+2}\right)}{6\sqrt{3x+2}\sqrt{x+1}} + \sqrt{x+1}\sqrt{3x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x+2)^(1/2)/(x+1)^(1/2),x)

[Out] (x+1)^(1/2)*(3*x+2)^(1/2)-1/6*((x+1)*(3*x+2))^(1/2)/(3*x+2)^(1/2)/(x+1)^(1/2)*ln(1/3*(5/2+3*x)*3^(1/2)+(3*x^2+5*x+2)^(1/2))*3^(1/2)

maxima [A] time = 1.97, size = 41, normalized size = 1.17

$$-\frac{1}{6} \sqrt{3} \log\left(2\sqrt{3}\sqrt{3x^2+5x+2} + 6x + 5\right) + \sqrt{3x^2+5x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3*x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")

[Out] $-1/6*\sqrt{3}*\log(2*\sqrt{3}*\sqrt{3*x^2 + 5*x + 2} + 6*x + 5) + \sqrt{3*x^2 + 5*x + 2}$

mupad [B] time = 6.14, size = 172, normalized size = 4.91

$$\frac{2\sqrt{3} \operatorname{atanh}\left(\frac{\sqrt{3}(\sqrt{2}-\sqrt{3x+2})}{3(\sqrt{x+1}-1)}\right)}{3} - \frac{\frac{30(\sqrt{2}-\sqrt{3x+2})}{\sqrt{x+1}-1} + \frac{10(\sqrt{2}-\sqrt{3x+2})^3}{(\sqrt{x+1}-1)^3} + \frac{24\sqrt{2}(\sqrt{2}-\sqrt{3x+2})^2}{(\sqrt{x+1}-1)^2}}{\frac{(\sqrt{2}-\sqrt{3x+2})^4}{(\sqrt{x+1}-1)^4} - \frac{6(\sqrt{2}-\sqrt{3x+2})^2}{(\sqrt{x+1}-1)^2} + 9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3*x + 2)^(1/2)/(x + 1)^(1/2), x)`

[Out] $(2*3^{1/2}*\operatorname{atanh}((3^{1/2}*(2^{1/2}) - (3*x + 2)^{1/2}))/((3*((x + 1)^{1/2} - 1))))/3 - ((30*(2^{1/2}) - (3*x + 2)^{1/2}))/((x + 1)^{1/2} - 1) + (10*(2^{1/2} - (3*x + 2)^{1/2})^3)/((x + 1)^{1/2} - 1)^3 + (24*2^{1/2}*(2^{1/2} - (3*x + 2)^{1/2})^2)/((x + 1)^{1/2} - 1)^2)/((2^{1/2} - (3*x + 2)^{1/2})^4)/((x + 1)^{1/2} - 1)^4 - (6*(2^{1/2} - (3*x + 2)^{1/2})^2)/((x + 1)^{1/2} - 1)^2 + 9)$

sympy [A] time = 1.62, size = 97, normalized size = 2.77

$$\begin{cases} \frac{3(x+1)^{3/2}}{\sqrt{3x+2}} - \frac{\sqrt{x+1}}{\sqrt{3x+2}} - \frac{\sqrt{3} \operatorname{acosh}(\sqrt{3}\sqrt{x+1})}{3} & \text{for } 3|x+1| > 1 \\ i\sqrt{-3x-2}\sqrt{x+1} + \frac{\sqrt{3}i \operatorname{asin}(\sqrt{3}\sqrt{x+1})}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**(1/2)/(1+x)**(1/2), x)`

[Out] `Piecewise((3*(x + 1)**(3/2)/sqrt(3*x + 2) - sqrt(x + 1)/sqrt(3*x + 2) - sqrt(3)*acosh(sqrt(3)*sqrt(x + 1))/3, 3*Abs(x + 1) > 1), (I*sqrt(-3*x - 2)*sqrt(x + 1) + sqrt(3)*I*asin(sqrt(3)*sqrt(x + 1))/3, True))`

$$3.566 \quad \int \frac{\sqrt{1-x} \sqrt{2+3x}}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=35

$$\sqrt{x+1} \sqrt{3x+2} - \frac{\sinh^{-1}(\sqrt{3x+2})}{\sqrt{3}}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {26, 50, 54, 215}

$$\sqrt{x+1} \sqrt{3x+2} - \frac{\sinh^{-1}(\sqrt{3x+2})}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2],x]

[Out] Sqrt[1 + x]*Sqrt[2 + 3*x] - ArcSinh[Sqrt[2 + 3*x]]/Sqrt[3]

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 50

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 54

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x] /; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]

Rule 215

Int[1/Sqrt[(a_.) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}} dx &= \int \frac{\sqrt{2+3x}}{\sqrt{1+x}} dx \\
&= \sqrt{1+x}\sqrt{2+3x} - \frac{1}{2} \int \frac{1}{\sqrt{1+x}\sqrt{2+3x}} dx \\
&= \sqrt{1+x}\sqrt{2+3x} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{2+3x}\right)}{\sqrt{3}} \\
&= \sqrt{1+x}\sqrt{2+3x} - \frac{\sinh^{-1}(\sqrt{2+3x})}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 49, normalized size = 1.40

$$\frac{3\sqrt{x+1}(3x+2) - \sqrt{9x+6} \sinh^{-1}(\sqrt{3x+2})}{3\sqrt{3x+2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2], x]

[Out] (3*Sqrt[1 + x]*(2 + 3*x) - Sqrt[6 + 9*x]*ArcSinh[Sqrt[2 + 3*x]])/(3*Sqrt[2 + 3*x])

IntegrateAlgebraic [F] time = 2.62, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\sqrt{2+3x}}{\sqrt{1-x^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2], x]

[Out] Defer[IntegrateAlgebraic] [(Sqrt[1 - x]*Sqrt[2 + 3*x])/Sqrt[1 - x^2], x]

fricas [B] time = 0.90, size = 96, normalized size = 2.74

$$\frac{\sqrt{3}(x-1) \log\left(-\frac{72x^3+4\sqrt{3}\sqrt{-x^2+1}(6x+5)\sqrt{3x+2}\sqrt{-x+1}+48x^2-71x-49}{x-1}\right) - 12\sqrt{-x^2+1}\sqrt{3x+2}\sqrt{-x+1}}{12(x-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] 1/12*(sqrt(3)*(x - 1)*log(-(72*x^3 + 4*sqrt(3)*sqrt(-x^2 + 1)*(6*x + 5)*sqrt(3*x + 2)*sqrt(-x + 1) + 48*x^2 - 71*x - 49)/(x - 1)) - 12*sqrt(-x^2 + 1)*sqrt(3*x + 2)*sqrt(-x + 1))/(x - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)

maple [B] time = 0.01, size = 86, normalized size = 2.46

$$\frac{\sqrt{-x+1} \sqrt{3x+2} \sqrt{-x^2+1} \left(\sqrt{3} \ln \left(\sqrt{3} x + \frac{5\sqrt{3}}{6} + \sqrt{3x^2+5x+2} \right) - 6\sqrt{3x^2+5x+2} \right)}{6(x-1)\sqrt{3x^2+5x+2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)*(3*x+2)^(1/2)/(-x^2+1)^(1/2), x)

[Out] 1/6*(-x+1)^(1/2)*(3*x+2)^(1/2)*(-x^2+1)^(1/2)*(ln(5/6*3^(1/2)+3^(1/2)*x+(3*x^2+5*x+2)^(1/2))*3^(1/2)-6*(3*x^2+5*x+2)^(1/2))/(x-1)/(3*x^2+5*x+2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{3x+2}\sqrt{-x+1}}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)*(2+3*x)^(1/2)/(-x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(3*x + 2)*sqrt(-x + 1)/sqrt(-x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\sqrt{3x+2}\sqrt{1-x}}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)

[Out] int(((3*x + 2)^(1/2)*(1 - x)^(1/2))/(1 - x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x}\sqrt{3x+2}}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2)*(2+3*x)**(1/2)/(-x**2+1)**(1/2), x)

[Out] Integral(sqrt(1 - x)*sqrt(3*x + 2)/sqrt(-(x - 1)*(x + 1)), x)

$$3.567 \quad \int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} dx$$

Optimal. Leaf size=43

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {98, 21, 105, 41, 216, 92, 206}

$$\frac{4\sqrt{x+1}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^(3/2)/((1 - x)^(3/2)*x), x]

[Out] (4*Sqrt[1 + x])/Sqrt[1 - x] - ArcSin[x] - ArcTanh[Sqrt[1 - x]*Sqrt[1 + x]]

Rule 21

Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :> Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 41

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 98

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])

Rule 105

Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dist[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))

Rule 206

$\text{Int}[(a_ + (b_ \cdot)(x_)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(1 \cdot \text{ArcTanh}[\text{Rt}[-b, 2] \cdot x] / \text{Rt}[a, 2]) / (\text{Rt}[a, 2] \cdot \text{Rt}[-b, 2]), x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b] \ \&\& \ (\text{GtQ}[a, 0] \ || \ \text{LtQ}[b, 0])$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_ + (b_ \cdot)(x_)^2), x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[\text{Rt}[-b, 2] \cdot x] / \text{Sqrt}[a]] / \text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rubi steps

$$\begin{aligned} \int \frac{(1+x)^{3/2}}{(1-x)^{3/2}x} dx &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - 2 \int \frac{-\frac{1}{2} + \frac{x}{2}}{\sqrt{1-x}x\sqrt{1+x}} dx \\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} + \int \frac{\sqrt{1-x}}{x\sqrt{1+x}} dx \\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x}\sqrt{1+x}} dx + \int \frac{1}{\sqrt{1-x}x\sqrt{1+x}} dx \\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \int \frac{1}{\sqrt{1-x^2}} dx - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x}\sqrt{1+x}\right) \\ &= \frac{4\sqrt{1+x}}{\sqrt{1-x}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x}\sqrt{1+x}\right) \end{aligned}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 1.42

$$\frac{2\left(\sqrt{1-x^2} \sin^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + 2x + 2\right)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^(3/2)/((1 - x)^(3/2)*x), x]

[Out] (2*(2 + 2*x + Sqrt[1 - x^2]*ArcSin[Sqrt[1 - x]/Sqrt[2]]))/Sqrt[1 - x^2] - ArcTanh[Sqrt[1 - x^2]]

IntegrateAlgebraic [C] time = 0.23, size = 77, normalized size = 1.79

$$-\frac{4\sqrt{1-x}\sqrt{x+1}}{x-1} - 2i \log\left(\sqrt{1-x} - i\sqrt{x+1}\right) + 2i \tan^{-1}\left(x + i\sqrt{1-x}\sqrt{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)^(3/2)/((1 - x)^(3/2)*x), x]

[Out] (-4*Sqrt[1 - x]*Sqrt[1 + x])/(-1 + x) + (2*I)*ArcTan[x + I*Sqrt[1 - x]*Sqrt[1 + x]] - (2*I)*Log[Sqrt[1 - x] - I*Sqrt[1 + x]]

fricas [B] time = 0.71, size = 74, normalized size = 1.72

$$\frac{2(x-1) \arctan\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + (x-1) \log\left(\frac{\sqrt{x+1}\sqrt{-x+1}-1}{x}\right) + 4x - 4\sqrt{x+1}\sqrt{-x+1} - 4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="fricas")

[Out] (2*(x - 1)*arctan((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + (x - 1)*log((sqrt(x + 1)*sqrt(-x + 1) - 1)/x) + 4*x - 4*sqrt(x + 1)*sqrt(-x + 1) - 4)/(x - 1)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-93.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}] at parameters values [-17.8804557086]2*(-1/2*pi-atan(sqrt(x+1)*((-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1))^2-1)/(-2*sqrt(-x+1)+2*sqrt(2))))-ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+ln(abs(2*sqrt(x+1)/(-2*sqrt(-x+1)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+1)+2*sqrt(2))/sqrt(x+1)))+4*sqrt(x+1)*sqrt(-x+1)/(-x+1)

maple [A] time = 0.02, size = 70, normalized size = 1.63

$$\frac{\left(-x \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - x \arcsin(x) + \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) + \arcsin(x) - 4\sqrt{-x^2+1}\right) \sqrt{-x+1} \sqrt{x+1}}{(x-1)\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)^(3/2)/(-x+1)^(3/2)/x,x)

[Out] (-arcsin(x)*x-arctanh(1/(-x^2+1)^(1/2))*x+arcsin(x)+arctanh(1/(-x^2+1)^(1/2)))-4*(-x^2+1)^(1/2)*(-x+1)^(1/2)*(x+1)^(1/2)/(x-1)/(-x^2+1)^(1/2)

maxima [A] time = 1.99, size = 53, normalized size = 1.23

$$\frac{4x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^(3/2)/(1-x)^(3/2)/x,x, algorithm="maxima")

[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(x+1)^{3/2}}{x(1-x)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)

[Out] int((x + 1)^(3/2)/(x*(1 - x)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^{\frac{3}{2}}}{x(1-x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)/(1-x)**(3/2)/x,x)
```

```
[Out] Integral((x + 1)**(3/2)/(x*(1 - x)**(3/2)), x)
```

$$3.568 \quad \int \frac{(1+x)^3}{x(1-x^2)^{3/2}} dx$$

Optimal. Leaf size=35

$$\frac{4(x+1)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \sin^{-1}(x)$$

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1805, 844, 216, 266, 63, 206}

$$\frac{4(x+1)}{\sqrt{1-x^2}} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^3/(x*(1 - x^2)^(3/2)), x]

[Out] (4*(1 + x))/Sqrt[1 - x^2] - ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1805

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] := With[{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp

andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned} \int \frac{(1+x)^3}{x(1-x^2)^{3/2}} dx &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{-1+x}{x\sqrt{1-x^2}} dx \\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \int \frac{1}{\sqrt{1-x^2}} dx + \int \frac{1}{x\sqrt{1-x^2}} dx \\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-xx}} dx, x, x^2\right) \\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{1-x^2}\right) \\ &= \frac{4(1+x)}{\sqrt{1-x^2}} - \sin^{-1}(x) - \tanh^{-1}\left(\sqrt{1-x^2}\right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 47, normalized size = 1.34

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1-x^2\right) - \sqrt{1-x^2} \sin^{-1}(x) + 4x + 3}{\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^3/(x*(1 - x^2)^(3/2)), x]

[Out] (3 + 4*x - Sqrt[1 - x^2]*ArcSin[x] + Hypergeometric2F1[-1/2, 1, 1/2, 1 - x^2])/Sqrt[1 - x^2]

IntegrateAlgebraic [A] time = 0.25, size = 59, normalized size = 1.69

$$-\frac{4\sqrt{1-x^2}}{x-1} + 2 \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right) - 2 \tanh^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x)^3/(x*(1 - x^2)^(3/2)), x]

[Out] (-4*Sqrt[1 - x^2])/(-1 + x) + 2*ArcTan[Sqrt[1 - x^2]/(1 + x)] - 2*ArcTanh[Sqrt[1 - x^2]/(1 + x)]

fricas [B] time = 0.79, size = 63, normalized size = 1.80

$$\frac{2(x-1) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + (x-1) \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 4x - 4\sqrt{-x^2+1} - 4}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/x/(-x^2+1)^(3/2), x, algorithm="fricas")

[Out] (2*(x - 1)*arctan((sqrt(-x^2 + 1) - 1)/x) + (x - 1)*log((sqrt(-x^2 + 1) - 1)/x) + 4*x - 4*sqrt(-x^2 + 1) - 4)/(x - 1)

giac [A] time = 0.40, size = 44, normalized size = 1.26

$$\frac{8}{\frac{\sqrt{-x^2+1}-1}{x} + 1} - \arcsin(x) + \log\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="giac")

[Out] 8/((sqrt(-x^2 + 1) - 1)/x + 1) - arcsin(x) + log(-(sqrt(-x^2 + 1) - 1)/abs(x))

maple [A] time = 0.01, size = 41, normalized size = 1.17

$$\frac{4x}{\sqrt{-x^2+1}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \arcsin(x) + \frac{4}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+1)^3/x/(-x^2+1)^(3/2),x)

[Out] 4*x/(-x^2+1)^(1/2)-arcsin(x)+4/(-x^2+1)^(1/2)-arctanh(1/(-x^2+1)^(1/2))

maxima [A] time = 1.99, size = 53, normalized size = 1.51

$$\frac{4x}{\sqrt{-x^2+1}} + \frac{4}{\sqrt{-x^2+1}} - \arcsin(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)^3/x/(-x^2+1)^(3/2),x, algorithm="maxima")

[Out] 4*x/sqrt(-x^2 + 1) + 4/sqrt(-x^2 + 1) - arcsin(x) - log(2*sqrt(-x^2 + 1)/abs(x) + 2/abs(x))

mupad [B] time = 3.20, size = 37, normalized size = 1.06

$$\ln\left(\sqrt{\frac{1}{x^2}-1} - \sqrt{\frac{1}{x^2}}\right) - \operatorname{asin}(x) - \frac{4\sqrt{1-x^2}}{x-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 1)^3/(x*(1 - x^2)^(3/2)),x)

[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2)) - asin(x) - (4*(1 - x^2)^(1/2))/(x - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x+1)^3}{x(-(x-1)(x+1))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)**3/x/(-x**2+1)**(3/2),x)

[Out] Integral((x + 1)**3/(x*(-(x - 1)*(x + 1))**(3/2)), x)

$$3.569 \quad \int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} dx$$

Optimal. Leaf size=51

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

Rubi [A] time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {98, 21, 105, 41, 216, 92, 208}

$$\frac{4\sqrt{ax+1}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)),x]
```

```
[Out] (4*Sqrt[1 + a*x])/Sqrt[1 - a*x] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a*x]*Sqrt[1 + a*x]]
```

Rule 21

```
Int[(u_.)*((a_.) + (b_.)*(v_))^(m_.)*((c_.) + (d_.)*(v_))^(n_.), x_Symbol] :>
  Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
  && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x,
  a + b*x])
```

Rule 41

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[(
a*c + b*d*x^2)^m, x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[b*c + a*d, 0] && (
IntegerQ[m] || (GtQ[a, 0] && GtQ[c, 0]))
```

Rule 92

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_Symbol] :> Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x],
x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
2*b*d*e - f*(b*c + a*d), 0]
```

Rule 98

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[((b*c - a*d)*(a + b*x)^(m + 1)*(c + d*x)^(n - 1
))*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] + Dist[1/(b*(b*e - a*f)*(m
+ 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 2)*(e + f*x)^p*Simp[a*d*(d*e*(
n - 1) + c*f*(p + 1)) + b*c*(d*e*(m - n + 2) - c*f*(m + p + 2)) + d*(a*d*f*
(n + p) + b*(d*e*(m + 1) - c*f*(m + n + p + 1)))*x, x], x] /; FreeQ[{a,
b, c, d, e, f, p}, x] && LtQ[m, -1] && GtQ[n, 1] && (IntegersQ[2*m, 2*n, 2
*p] || IntegersQ[m, n + p] || IntegersQ[p, m + n])
```

Rule 105

```
Int[(((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.))/((e_.) + (f_.)*(x_
)), x_Symbol] :> Dist[b/f, Int[(a + b*x)^(m - 1)*(c + d*x)^n, x], x] - Dis
t[(b*e - a*f)/f, Int[((a + b*x)^(m - 1)*(c + d*x)^n)/(e + f*x), x], x] /; F
reeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[Simplify[m + n + 1], 0] && (GtQ[m
, 0] || (!RationalQ[m] && (SumSimplerQ[m, -1] || !SumSimplerQ[n, -1])))
```


Rule 208

$\text{Int}[(a_+ + (b_+)(x_+)^2)^{-1}, x_Symbol] \rightarrow \text{Simp}[(\text{Rt}[-(a/b), 2] * \text{ArcTanh}[x / \text{Rt}[-(a/b), 2]]) / a, x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{NegQ}[a/b]$

Rule 216

$\text{Int}[1/\text{Sqrt}[(a_+ + (b_+)(x_+)^2], x_Symbol] \rightarrow \text{Simp}[\text{ArcSin}[(\text{Rt}[-b, 2]*x)/\text{Sqrt}[a]]/\text{Rt}[-b, 2], x] /; \text{FreeQ}[\{a, b\}, x] \ \&\& \ \text{GtQ}[a, 0] \ \&\& \ \text{NegQ}[b]$

Rubi steps

$$\begin{aligned} \int \frac{(1+ax)^{3/2}}{x(1-ax)^{3/2}} dx &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \frac{2 \int \frac{-\frac{a}{2} + \frac{a^2x}{2}}{x\sqrt{1-ax}\sqrt{1+ax}} dx}{a} \\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} + \int \frac{\sqrt{1-ax}}{x\sqrt{1+ax}} dx \\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a \int \frac{1}{\sqrt{1-ax}\sqrt{1+ax}} dx + \int \frac{1}{x\sqrt{1-ax}\sqrt{1+ax}} dx \\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - a \int \frac{1}{\sqrt{1-a^2x^2}} dx - a \text{Subst}\left(\int \frac{1}{a-ax^2} dx, x, \sqrt{1-ax}\sqrt{1+ax}\right) \\ &= \frac{4\sqrt{1+ax}}{\sqrt{1-ax}} - \sin^{-1}(ax) - \tanh^{-1}\left(\sqrt{1-ax}\sqrt{1+ax}\right) \end{aligned}$$

Mathematica [A] time = 0.06, size = 72, normalized size = 1.41

$$\frac{2\left(\sqrt{1-a^2x^2} \sin^{-1}\left(\frac{\sqrt{1-ax}}{\sqrt{2}}\right) + 2ax + 2\right)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)), x]

[Out] (2*(2 + 2*a*x + Sqrt[1 - a^2*x^2])*ArcSin[Sqrt[1 - a*x]/Sqrt[2]])/Sqrt[1 - a^2*x^2] - ArcTanh[Sqrt[1 - a^2*x^2]]

IntegrateAlgebraic [C] time = 0.24, size = 90, normalized size = 1.76

$$-\frac{4\sqrt{1-ax}\sqrt{ax+1}}{ax-1} - 2i \log\left(\sqrt{1-ax} - i\sqrt{ax+1}\right) + 2i \tan^{-1}\left(ax + i\sqrt{1-ax}\sqrt{ax+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + a*x)^(3/2)/(x*(1 - a*x)^(3/2)), x]

[Out] (-4*Sqrt[1 - a*x]*Sqrt[1 + a*x])/(-1 + a*x) + (2*I)*ArcTan[a*x + I*Sqrt[1 - a*x]*Sqrt[1 + a*x]] - (2*I)*Log[Sqrt[1 - a*x] - I*Sqrt[1 + a*x]]

fricas [B] time = 0.83, size = 93, normalized size = 1.82

$$\frac{4ax + 2(ax-1) \arctan\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{ax}\right) + (ax-1) \log\left(\frac{\sqrt{ax+1}\sqrt{-ax+1}-1}{x}\right) - 4\sqrt{ax+1}\sqrt{-ax+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="fricas")

[Out] $(4ax + 2(a x - 1)\arctan(\sqrt{ax + 1}\sqrt{-ax + 1} - 1)/(ax)) + (ax - 1)\log(\sqrt{ax + 1}\sqrt{-ax + 1} - 1)/x - 4\sqrt{ax + 1}\sqrt{-ax + 1} - 4)/(ax - 1)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2,2]%%}] at parameters values [70,22] Warning, choosing root of [1,0,-4,0,%%{4,[2,2]%%}] at parameters values [42,56] $2*(-1/2\pi - \operatorname{atan}(\sqrt{ax+1}*((-1/2*(-2\sqrt{-ax+1}+2\sqrt{2}))/\sqrt{ax+1}))^2 - 1)/(-2\sqrt{-ax+1}+2\sqrt{2})) - \ln(\operatorname{abs}(2\sqrt{ax+1}/(-2\sqrt{-ax+1}+2\sqrt{2}))+2 - 1/2*(-2\sqrt{-ax+1}+2\sqrt{2}))/\sqrt{ax+1})) + \ln(\operatorname{abs}(2\sqrt{ax+1}/(-2\sqrt{-ax+1}+2\sqrt{2}))-2 - 1/2*(-2\sqrt{-ax+1}+2\sqrt{2}))/\sqrt{ax+1})) + 4\sqrt{ax+1}\sqrt{-ax+1}/(-ax+1)$

maple [C] time = 0.04, size = 134, normalized size = 2.63

$$\frac{(-ax \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) \operatorname{csgn}(a) - ax \operatorname{arctan}\left(\frac{ax \operatorname{csgn}(a)}{\sqrt{-(ax+1)(ax-1)}}\right) + \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) \operatorname{csgn}(a) + \operatorname{arctan}\left(\frac{ax \operatorname{csgn}(a)}{\sqrt{-(ax+1)(ax-1)}}\right) - 4\sqrt{-a^2x^2+1} \operatorname{csgn}(a))\sqrt{-ax+1}\sqrt{ax+1} \operatorname{csgn}(a)}{(ax-1)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x)

[Out] $(-\operatorname{arctanh}(1/(-a^2x^2+1)^{(1/2)}))\operatorname{csgn}(a)*x*a - \operatorname{arctan}(\operatorname{csgn}(a)*ax/(-ax+1)*(ax-1))^{(1/2)}*x*a + \operatorname{arctanh}(1/(-a^2x^2+1)^{(1/2)})*\operatorname{csgn}(a) - 4*(-a^2x^2+1)^{(1/2)}*\operatorname{csgn}(a) + \operatorname{arctan}(\operatorname{csgn}(a)*ax/(-ax+1)*(ax-1))^{(1/2)})*\operatorname{csgn}(a)*(-ax+1)^{(1/2)}*(ax+1)^{(1/2)}/(ax-1)/(-a^2x^2+1)^{(1/2)}$

maxima [A] time = 2.50, size = 65, normalized size = 1.27

$$\frac{4ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^(3/2)/x/(-a*x+1)^(3/2),x, algorithm="maxima")

[Out] $4ax/\sqrt{-a^2x^2+1} + 4/\sqrt{-a^2x^2+1} - \arcsin(ax) - \log(2\sqrt{-a^2x^2+1}/\operatorname{abs}(x) + 2/\operatorname{abs}(x))$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(ax+1)^{3/2}}{x(1-ax)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)),x)

[Out] int((a*x + 1)^(3/2)/(x*(1 - a*x)^(3/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^{\frac{3}{2}}}{x(-ax+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*x+1)**(3/2)/x/(-a*x+1)**(3/2),x)
```

```
[Out] Integral((a*x + 1)**(3/2)/(x*(-a*x + 1)**(3/2)), x)
```

$$3.570 \quad \int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}} dx$$

Optimal. Leaf size=45

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \sin^{-1}(ax)$$

Rubi [A] time = 0.09, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1805, 844, 216, 266, 63, 208}

$$\frac{4(ax+1)}{\sqrt{1-a^2x^2}} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)),x]

[Out] (4*(1 + a*x))/Sqrt[1 - a^2*x^2] - ArcSin[a*x] - ArcTanh[Sqrt[1 - a^2*x^2]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 1805

Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x] /; Fr

eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]

Rubi steps

$$\begin{aligned}
 \int \frac{(1+ax)^3}{x(1-a^2x^2)^{3/2}} dx &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \int \frac{-1+ax}{x\sqrt{1-a^2x^2}} dx \\
 &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - a \int \frac{1}{\sqrt{1-a^2x^2}} dx + \int \frac{1}{x\sqrt{1-a^2x^2}} dx \\
 &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{x\sqrt{1-a^2x}} dx, x, x^2 \right) \\
 &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \frac{\text{Subst} \left(\int \frac{1}{\frac{1}{a^2} - \frac{x^2}{a^2}} dx, x, \sqrt{1-a^2x^2} \right)}{a^2} \\
 &= \frac{4(1+ax)}{\sqrt{1-a^2x^2}} - \sin^{-1}(ax) - \tanh^{-1} \left(\sqrt{1-a^2x^2} \right)
 \end{aligned}$$

Mathematica [C] time = 0.04, size = 59, normalized size = 1.31

$$\frac{{}_2F_1\left(-\frac{1}{2}, 1; \frac{1}{2}; 1-a^2x^2\right) - \sqrt{1-a^2x^2} \sin^{-1}(ax) + 4ax + 3}{\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)), x]

[Out] (3 + 4*a*x - Sqrt[1 - a^2*x^2]*ArcSin[a*x] + Hypergeometric2F1[-1/2, 1, 1/2, 1 - a^2*x^2])/Sqrt[1 - a^2*x^2]

IntegrateAlgebraic [B] time = 0.41, size = 97, normalized size = 2.16

$$-\frac{4\sqrt{1-a^2x^2}}{ax-1} - \frac{\sqrt{-a^2} \log\left(\sqrt{1-a^2x^2} - \sqrt{-a^2}x\right)}{a} + 2 \tanh^{-1}\left(\sqrt{-a^2}x - \sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + a*x)^3/(x*(1 - a^2*x^2)^(3/2)), x]

[Out] (-4*Sqrt[1 - a^2*x^2])/(-1 + a*x) + 2*ArcTanh[Sqrt[-a^2]*x - Sqrt[1 - a^2*x^2]] - (Sqrt[-a^2]*Log[-(Sqrt[-a^2]*x) + Sqrt[1 - a^2*x^2]])/a

fricas [A] time = 0.95, size = 82, normalized size = 1.82

$$\frac{4ax + 2(ax-1) \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + (ax-1) \log\left(\frac{\sqrt{-a^2x^2+1}-1}{x}\right) - 4\sqrt{-a^2x^2+1} - 4}{ax-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2), x, algorithm="fricas")

[Out] (4*a*x + 2*(a*x - 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) + (a*x - 1)*log((sqrt(-a^2*x^2 + 1) - 1)/x) - 4*sqrt(-a^2*x^2 + 1) - 4)/(a*x - 1)

giac [B] time = 0.54, size = 87, normalized size = 1.93

$$\frac{a \arcsin(ax) \operatorname{sgn}(a)}{|a|} - \frac{a \log\left(\frac{|-2\sqrt{-a^2x^2+1}| |a| - 2a|}{2a^2|x|}\right)}{|a|} + \frac{8a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="giac")

[Out] -a*arcsin(a*x)*sgn(a)/abs(a) - a*log(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/abs(a) + 8*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a))

maple [A] time = 0.01, size = 75, normalized size = 1.67

$$\frac{4ax}{\sqrt{-a^2x^2+1}} - \frac{a \arctan\left(\frac{\sqrt{a^2} x}{\sqrt{-a^2x^2+1}}\right)}{\sqrt{a^2}} - \operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) + \frac{4}{\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x)

[Out] 4*a*x/(-a^2*x^2+1)^(1/2)-a/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+4/(-a^2*x^2+1)^(1/2)-arctanh(1/(-a^2*x^2+1)^(1/2))

maxima [A] time = 2.08, size = 65, normalized size = 1.44

$$\frac{4ax}{\sqrt{-a^2x^2+1}} + \frac{4}{\sqrt{-a^2x^2+1}} - \arcsin(ax) - \log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)^3/x/(-a^2*x^2+1)^(3/2),x, algorithm="maxima")

[Out] 4*a*x/sqrt(-a^2*x^2 + 1) + 4/sqrt(-a^2*x^2 + 1) - arcsin(a*x) - log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x))

mupad [B] time = 3.49, size = 82, normalized size = 1.82

$$\frac{4a\sqrt{1-a^2x^2}}{\left(x\sqrt{-a^2}-\frac{\sqrt{-a^2}}{a}\right)\sqrt{-a^2}} - \frac{a \operatorname{asinh}\left(x\sqrt{-a^2}\right)}{\sqrt{-a^2}} - \operatorname{atanh}\left(\sqrt{1-a^2x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x + 1)^3/(x*(1 - a^2*x^2)^(3/2)),x)

[Out] (4*a*(1 - a^2*x^2)^(1/2))/((x*(-a^2)^(1/2) - (-a^2)^(1/2)/a)*(-a^2)^(1/2)) - (a*asinh(x*(-a^2)^(1/2)))/(-a^2)^(1/2) - atanh(((1 - a^2*x^2)^(1/2)))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(ax+1)^3}{x(-ax-1)(ax+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x+1)**3/x/(-a**2*x**2+1)**(3/2),x)

[Out] Integral((a*x + 1)**3/(x*(-(a*x - 1)*(a*x + 1))**(3/2)), x)

$$3.571 \quad \int \frac{1}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 - x^2], x]

[Out] ArcSin[x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 - x^2], x]

[Out] ArcSin[x]

IntegrateAlgebraic [A] time = 0.03, size = 2, normalized size = 1.00

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 - x^2], x]

[Out] ArcSin[x]

fricas [B] time = 0.63, size = 18, normalized size = 9.00

$$-2 \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] -2*arctan((sqrt(-x^2 + 1) - 1)/x)

giac [A] time = 0.34, size = 2, normalized size = 1.00

$\arcsin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="giac")

[Out] arcsin(x)

maple [A] time = 0.00, size = 3, normalized size = 1.50

$\arcsin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)^(1/2),x)

[Out] arcsin(x)

maxima [A] time = 1.68, size = 2, normalized size = 1.00

$\arcsin(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)^(1/2),x, algorithm="maxima")

[Out] arcsin(x)

mupad [B] time = 0.01, size = 2, normalized size = 1.00

$\operatorname{asin}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1 - x^2)^(1/2),x)

[Out] asin(x)

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$\operatorname{asin}(x)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x**2+1)**(1/2),x)

[Out] asin(x)

$$3.572 \quad \int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$$

Optimal. Leaf size=2

$$\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {26, 216}

$$\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2]/Sqrt[1 - x^4], x]

[Out] ArcSin[x]

Rule 26

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx = \int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}(x)$$

Mathematica [B] time = 0.03, size = 32, normalized size = 16.00

$$-\tan^{-1}\left(\frac{x\sqrt{x^2+1}\sqrt{1-x^4}}{x^4-1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2]/Sqrt[1 - x^4], x]

[Out] -ArcTan[(x*Sqrt[1 + x^2]*Sqrt[1 - x^4])/(-1 + x^4)]

IntegrateAlgebraic [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 + x^2]/Sqrt[1 - x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 + x^2]/Sqrt[1 - x^4], x]

fricas [B] time = 0.83, size = 27, normalized size = 13.50

$$-\arctan\left(\frac{\sqrt{-x^4+1}\sqrt{x^2+1}}{x^3+x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")

[Out] -arctan(sqrt(-x^4 + 1)*sqrt(x^2 + 1)/(x^3 + x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)

maple [B] time = 0.02, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4+1} \arcsin(x)}{\sqrt{x^2+1} \sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2+1)^(1/2)/(-x^4+1)^(1/2),x)

[Out] 1/(x^2+1)^(1/2)*(-x^4+1)^(1/2)/(-x^2+1)^(1/2)*arcsin(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-x^4+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^2 + 1)/sqrt(-x^4 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{x^2+1}}{\sqrt{1-x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2),x)

[Out] int((x^2 + 1)^(1/2)/(1 - x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2+1}}{\sqrt{-(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**2+1)**(1/2)/(-x**4+1)**(1/2),x)

[Out] Integral(sqrt(x**2 + 1)/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)

$$3.573 \quad \int \frac{1}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x)$$

Mathematica [A] time = 0.00, size = 2, normalized size = 1.00

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + x^2],x]

[Out] ArcSinh[x]

IntegrateAlgebraic [B] time = 0.02, size = 16, normalized size = 8.00

$$-\log\left(\sqrt{x^2+1} - x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 + x^2],x]

[Out] -Log[-x + Sqrt[1 + x^2]]

fricas [B] time = 0.84, size = 14, normalized size = 7.00

$$-\log\left(-x + \sqrt{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^(1/2),x, algorithm="fricas")

[Out] -log(-x + sqrt(x^2 + 1))

giac [B] time = 0.43, size = 14, normalized size = 7.00

$$-\log\left(-x + \sqrt{x^2 + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^(1/2),x, algorithm="giac")

[Out] -log(-x + sqrt(x^2 + 1))

maple [A] time = 0.00, size = 3, normalized size = 1.50

$$\operatorname{arcsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2+1)^(1/2),x)

[Out] arcsinh(x)

maxima [A] time = 2.00, size = 2, normalized size = 1.00

$$\operatorname{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^2+1)^(1/2),x, algorithm="maxima")

[Out] arcsinh(x)

mupad [B] time = 0.03, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2 + 1)^(1/2),x)

[Out] asinh(x)

sympy [A] time = 0.14, size = 2, normalized size = 1.00

$$\operatorname{asinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**2+1)**(1/2),x)

[Out] asinh(x)

$$3.574 \quad \int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$$

Optimal. Leaf size=2

$$\sinh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 2, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {26, 215}

$$\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2]/Sqrt[1 - x^4], x]

[Out] ArcSinh[x]

Rule 26

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx = \int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}(x)$$

Mathematica [B] time = 0.02, size = 42, normalized size = 21.00

$$\log(1-x^2) - \log\left(x^3 + \sqrt{1-x^2}\sqrt{1-x^4} - x\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2]/Sqrt[1 - x^4], x]

[Out] Log[1 - x^2] - Log[-x + x^3 + Sqrt[1 - x^2]*Sqrt[1 - x^4]]

IntegrateAlgebraic [F] time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^4}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 - x^2]/Sqrt[1 - x^4], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^2]/Sqrt[1 - x^4], x]

fricas [B] time = 0.56, size = 81, normalized size = 40.50

$$-\frac{1}{2} \log\left(\frac{x^3 + \sqrt{-x^4 + 1} \sqrt{-x^2 + 1} - x}{x^3 - x}\right) + \frac{1}{2} \log\left(-\frac{x^3 - \sqrt{-x^4 + 1} \sqrt{-x^2 + 1} - x}{x^3 - x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="fricas")

[Out] -1/2*log((x^3 + sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x)) + 1/2*log(-(x^3 - sqrt(-x^4 + 1)*sqrt(-x^2 + 1) - x)/(x^3 - x))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1}}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)

maple [B] time = 0.01, size = 29, normalized size = 14.50

$$\frac{\sqrt{-x^4 + 1} \operatorname{arcsinh}(x)}{\sqrt{-x^2 + 1} \sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x)

[Out] 1/(-x^2+1)^(1/2)/(x^2+1)^(1/2)*(-x^4+1)^(1/2)*arcsinh(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^2 + 1}}{\sqrt{-x^4 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)^(1/2)/(-x^4+1)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + 1)/sqrt(-x^4 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.50

$$\int \frac{\sqrt{1 - x^2}}{\sqrt{1 - x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2),x)

[Out] int((1 - x^2)^(1/2)/(1 - x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{\sqrt{-(x-1)(x+1)(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)**(1/2)/(-x**4+1)**(1/2),x)

[Out] Integral(sqrt(-(x - 1)*(x + 1))/sqrt(-(x - 1)*(x + 1)*(x**2 + 1)), x)

$$3.575 \quad \int \sqrt{1-x^2} dx$$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x + \frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {195, 216}

$$\frac{1}{2}\sqrt{1-x^2}x + \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^2], x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \sqrt{1-x^2} dx &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 20, normalized size = 0.87

$$\frac{1}{2}\left(\sqrt{1-x^2}x + \sin^{-1}(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^2], x]

[Out] (x*Sqrt[1 - x^2] + ArcSin[x])/2

IntegrateAlgebraic [A] time = 0.06, size = 37, normalized size = 1.61

$$\frac{1}{2}x\sqrt{1-x^2} - \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2], x]

[Out] $(x*\text{Sqrt}[1 - x^2])/2 - \text{ArcTan}[\text{Sqrt}[1 - x^2]/(1 + x)]$

fricas [A] time = 0.62, size = 31, normalized size = 1.35

$$\frac{1}{2} \sqrt{-x^2 + 1} x - \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $1/2*\text{sqrt}(-x^2 + 1)*x - \text{arctan}((\text{sqrt}(-x^2 + 1) - 1)/x)$

giac [A] time = 0.36, size = 17, normalized size = 0.74

$$\frac{1}{2} \sqrt{-x^2 + 1} x + \frac{1}{2} \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2),x, algorithm="giac")`

[Out] $1/2*\text{sqrt}(-x^2 + 1)*x + 1/2*\arcsin(x)$

maple [A] time = 0.00, size = 18, normalized size = 0.78

$$\frac{\sqrt{-x^2 + 1} x}{2} + \frac{\arcsin(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-x^2+1)^(1/2),x)`

[Out] $1/2*\arcsin(x)+1/2*(-x^2+1)^(1/2)*x$

maxima [A] time = 1.93, size = 17, normalized size = 0.74

$$\frac{1}{2} \sqrt{-x^2 + 1} x + \frac{1}{2} \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $1/2*\text{sqrt}(-x^2 + 1)*x + 1/2*\arcsin(x)$

mupad [B] time = 0.03, size = 17, normalized size = 0.74

$$\frac{\text{asin}(x)}{2} + \frac{x \sqrt{1 - x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - x^2)^(1/2),x)`

[Out] $\text{asin}(x)/2 + (x*(1 - x^2)^(1/2))/2$

sympy [A] time = 0.20, size = 15, normalized size = 0.65

$$\frac{x \sqrt{1 - x^2}}{2} + \frac{\text{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**2+1)**(1/2),x)`

[Out] $x*\text{sqrt}(1 - x**2)/2 + \text{asin}(x)/2$

$$3.576 \quad \int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$$

Optimal. Leaf size=23

$$\frac{1}{2}\sqrt{1-x^2}x + \frac{1}{2}\sin^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {26, 195, 216}

$$\frac{1}{2}\sqrt{1-x^2}x + \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^4]/Sqrt[1 + x^2], x]

[Out] (x*Sqrt[1 - x^2])/2 + ArcSin[x]/2

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] := Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx &= \int \sqrt{1-x^2} dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{1}{2}x\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.04, size = 50, normalized size = 2.17

$$\frac{1}{2} \left(\frac{\sqrt{1-x^4}x}{\sqrt{x^2+1}} + \tan^{-1} \left(\frac{x\sqrt{x^2+1}}{\sqrt{1-x^4}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 + x^2], x]

[Out] $((x*\text{Sqrt}[1 - x^4])/ \text{Sqrt}[1 + x^2] + \text{ArcTan}[(x*\text{Sqrt}[1 + x^2])/ \text{Sqrt}[1 - x^4]]) / 2$

IntegrateAlgebraic [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1+x^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 - x^4]/Sqrt[1 + x^2], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^4]/Sqrt[1 + x^2], x]

fricas [B] time = 0.51, size = 60, normalized size = 2.61

$$\frac{\sqrt{-x^4+1} \sqrt{x^2+1} x - (x^2+1) \arctan\left(\frac{\sqrt{-x^4+1} \sqrt{x^2+1}}{x^3+x}\right)}{2(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2), x, algorithm="fricas")

[Out] $1/2*(\text{sqrt}(-x^4 + 1)*\text{sqrt}(x^2 + 1)*x - (x^2 + 1)*\arctan(\text{sqrt}(-x^4 + 1)*\text{sqrt}(x^2 + 1)/(x^3 + x)))/(x^2 + 1)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)

maple [B] time = 0.01, size = 42, normalized size = 1.83

$$\frac{\sqrt{-x^4+1} \left(\sqrt{-x^2+1} x + \arcsin(x) \right)}{2\sqrt{x^2+1} \sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^(1/2)/(x^2+1)^(1/2), x)

[Out] $1/2*(-x^4+1)^(1/2)/(x^2+1)^(1/2)*((-x^2+1)^(1/2)*x+\arcsin(x))/(-x^2+1)^(1/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 1)/sqrt(x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{1-x^4}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)

[Out] int((1 - x^4)^(1/2)/(x^2 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)**(1/2)/(x**2+1)**(1/2), x)

[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(x**2 + 1), x)

$$3.577 \quad \int \sqrt{1+x^2} dx$$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\sinh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {195, 215}

$$\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^2], x]

[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \sqrt{1+x^2} dx &= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} dx \\ &= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.86

$$\frac{1}{2}\left(\sqrt{x^2+1}x + \sinh^{-1}(x)\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^2], x]

[Out] (x*Sqrt[1 + x^2] + ArcSinh[x])/2

IntegrateAlgebraic [A] time = 0.03, size = 33, normalized size = 1.57

$$\frac{1}{2}x\sqrt{x^2+1} - \frac{1}{2}\log\left(\sqrt{x^2+1} - x\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x^2], x]

[Out] $(x\sqrt{1+x^2})/2 - \text{Log}[-x + \sqrt{1+x^2}]/2$

fricas [A] time = 0.59, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log(-x + \sqrt{x^2+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^(1/2),x, algorithm="fricas")`

[Out] $1/2*\text{sqrt}(x^2 + 1)*x - 1/2*\text{log}(-x + \text{sqrt}(x^2 + 1))$

giac [A] time = 0.37, size = 25, normalized size = 1.19

$$\frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\log(-x + \sqrt{x^2+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^(1/2),x, algorithm="giac")`

[Out] $1/2*\text{sqrt}(x^2 + 1)*x - 1/2*\text{log}(-x + \text{sqrt}(x^2 + 1))$

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$\frac{\sqrt{x^2+1}x}{2} + \frac{\text{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2+1)^(1/2),x)`

[Out] $1/2*\text{arcsinh}(x)+1/2*(x^2+1)^(1/2)*x$

maxima [A] time = 1.95, size = 15, normalized size = 0.71

$$\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\text{arsinh}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2+1)^(1/2),x, algorithm="maxima")`

[Out] $1/2*\text{sqrt}(x^2 + 1)*x + 1/2*\text{arcsinh}(x)$

mupad [B] time = 0.03, size = 15, normalized size = 0.71

$$\frac{\text{asinh}(x)}{2} + \frac{x\sqrt{x^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 + 1)^(1/2),x)`

[Out] $\text{asinh}(x)/2 + (x*(x^2 + 1)^(1/2))/2$

sympy [A] time = 0.19, size = 15, normalized size = 0.71

$$\frac{x\sqrt{x^2+1}}{2} + \frac{\text{asinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2+1)**(1/2),x)`

[Out] $x*\text{sqrt}(x**2 + 1)/2 + \text{asinh}(x)/2$

$$3.578 \quad \int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Optimal. Leaf size=21

$$\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\sinh^{-1}(x)$$

Rubi [A] time = 0.00, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {26, 195, 215}

$$\frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]

[Out] (x*Sqrt[1 + x^2])/2 + ArcSinh[x]/2

Rule 26

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-b^2/d)^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p])) || LtQ[Denominator[p + 1/n], Denominator[p]]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx &= \int \sqrt{1+x^2} dx \\ &= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} dx \\ &= \frac{1}{2}x\sqrt{1+x^2} + \frac{1}{2}\sinh^{-1}(x) \end{aligned}$$

Mathematica [B] time = 0.06, size = 70, normalized size = 3.33

$$\frac{1}{2} \left(\log(1-x^2) + \frac{\sqrt{1-x^4}x}{\sqrt{1-x^2}} - \log(x^3 + \sqrt{1-x^2}\sqrt{1-x^4} - x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x^4]/Sqrt[1 - x^2],x]

[Out] $((x*\text{Sqrt}[1 - x^4])/ \text{Sqrt}[1 - x^2] + \text{Log}[1 - x^2] - \text{Log}[-x + x^3 + \text{Sqrt}[1 - x^2]*\text{Sqrt}[1 - x^4]])/2$

IntegrateAlgebraic [F] time = 0.87, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[Sqrt[1 - x^4]/Sqrt[1 - x^2], x]

[Out] Defer[IntegrateAlgebraic][Sqrt[1 - x^4]/Sqrt[1 - x^2], x]

fricas [B] time = 0.56, size = 120, normalized size = 5.71

$$\frac{2\sqrt{-x^4+1}\sqrt{-x^2+1}x + (x^2-1)\log\left(\frac{x^3+\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right) - (x^2-1)\log\left(-\frac{x^3-\sqrt{-x^4+1}\sqrt{-x^2+1}-x}{x^3-x}\right)}{4(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2), x, algorithm="fricas")

[Out] $-1/4*(2*\text{sqrt}(-x^4 + 1)*\text{sqrt}(-x^2 + 1)*x + (x^2 - 1)*\log((x^3 + \text{sqrt}(-x^4 + 1)*\text{sqrt}(-x^2 + 1) - x)/(x^3 - x)) - (x^2 - 1)*\log(-x^3 - \text{sqrt}(-x^4 + 1)*\text{sqrt}(-x^2 + 1) - x)/(x^3 - x)))/(x^2 - 1)$

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)

maple [B] time = 0.01, size = 47, normalized size = 2.24

$$\frac{\sqrt{-x^4+1}\sqrt{-x^2+1}\left(\sqrt{x^2+1}x + \text{arcsinh}(x)\right)}{2(x^2-1)\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^(1/2)/(-x^2+1)^(1/2), x)

[Out] $-1/2*(-x^4+1)^(1/2)*(-x^2+1)^(1/2)*((x^2+1)^(1/2)*x+\text{arcsinh}(x))/(x^2-1)/(x^2+1)^(1/2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x^4+1}}{\sqrt{-x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^(1/2)/(-x^2+1)^(1/2), x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + 1)/sqrt(-x^2 + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{1-x^4}}{\sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)

[Out] int((1 - x^4)^(1/2)/(1 - x^2)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)(x^2+1)}}{\sqrt{-(x-1)(x+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)**(1/2)/(-x**2+1)**(1/2), x)

[Out] Integral(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))/sqrt(-(x - 1)*(x + 1)), x)

$$3.579 \quad \int \frac{1}{x - \sqrt{1+x^2}} dx$$

Optimal. Leaf size=28

$$-\frac{x^2}{2} - \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\sinh^{-1}(x)$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2106, 30, 195, 215}

$$-\frac{x^2}{2} - \frac{1}{2}\sqrt{x^2+1}x - \frac{1}{2}\sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 + x^2])^(-1), x]

[Out] -x^2/2 - (x*Sqrt[1 + x^2])/2 - ArcSinh[x]/2

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 195

Int[(a_) + (b_)*(x_)^(n_)^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2106

Int[(u_)/((d_)*(x_)^(n_) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x_Symbol] := -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{x - \sqrt{1+x^2}} dx &= - \int x dx - \int \sqrt{1+x^2} dx \\ &= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2} \int \frac{1}{\sqrt{1+x^2}} dx \\ &= -\frac{x^2}{2} - \frac{1}{2}x\sqrt{1+x^2} - \frac{1}{2}\sinh^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 38, normalized size = 1.36

$$\frac{1}{2} \log(x - \sqrt{x^2 + 1}) - \frac{1}{4(x - \sqrt{x^2 + 1})^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[1 + x^2])^(-1), x]

[Out] -1/4*1/(x - Sqrt[1 + x^2])^2 + Log[x - Sqrt[1 + x^2]]/2

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.43

$$-\frac{x^2}{2} - \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\log(\sqrt{x^2+1}-x)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[1 + x^2])^(-1), x]

[Out] -1/2*x^2 - (x*Sqrt[1 + x^2])/2 + Log[-x + Sqrt[1 + x^2]]/2

fricas [A] time = 0.52, size = 30, normalized size = 1.07

$$-\frac{1}{2}x^2 - \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\log(-x + \sqrt{x^2+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="fricas")

[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))

giac [A] time = 0.34, size = 30, normalized size = 1.07

$$-\frac{1}{2}x^2 - \frac{1}{2}\sqrt{x^2+1}x + \frac{1}{2}\log(-x + \sqrt{x^2+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="giac")

[Out] -1/2*x^2 - 1/2*sqrt(x^2 + 1)*x + 1/2*log(-x + sqrt(x^2 + 1))

maple [A] time = 0.00, size = 21, normalized size = 0.75

$$-\frac{x^2}{2} - \frac{\sqrt{x^2+1}x}{2} - \frac{\operatorname{arcsinh}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-(x^2+1)^(1/2)),x)

[Out] -1/2*x^2-1/2*arcsinh(x)-1/2*(x^2+1)^(1/2)*x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x - sqrt(x^2 + 1)), x)

mupad [B] time = 0.03, size = 20, normalized size = 0.71

$$-\frac{\operatorname{asinh}(x)}{2} - \frac{x\sqrt{x^2+1}}{2} - \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - (x^2 + 1)^(1/2)),x)`

[Out] `- asinh(x)/2 - (x*(x^2 + 1)^(1/2))/2 - x^2/2`

sympy [B] time = 0.35, size = 58, normalized size = 2.07

$$-\frac{x \operatorname{asinh}(x)}{2x - 2\sqrt{x^2 + 1}} + \frac{x}{2x - 2\sqrt{x^2 + 1}} + \frac{\sqrt{x^2 + 1} \operatorname{asinh}(x)}{2x - 2\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-(x**2+1)**(1/2)),x)`

[Out] `-x*asinh(x)/(2*x - 2*sqrt(x**2 + 1)) + x/(2*x - 2*sqrt(x**2 + 1)) + sqrt(x**2 + 1)*asinh(x)/(2*x - 2*sqrt(x**2 + 1))`

$$3.580 \quad \int \frac{1}{x - \sqrt{1-x^2}} dx$$

Optimal. Leaf size=37

$$\frac{1}{4} \log(1 - 2x^2) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \frac{1}{2} \sin^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 260, 402, 216, 377, 207}

$$\frac{1}{4} \log(1 - 2x^2) - \frac{1}{2} \tanh^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right) - \frac{1}{2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 - x^2])^(-1), x]

[Out] -ArcSin[x]/2 - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x - \sqrt{1-x^2}} dx &= \int \left(\frac{x}{-1+2x^2} + \frac{\sqrt{1-x^2}}{-1+2x^2} \right) dx \\
&= \int \frac{x}{-1+2x^2} dx + \int \frac{\sqrt{1-x^2}}{-1+2x^2} dx \\
&= \frac{1}{4} \log(1-2x^2) - \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}} dx + \frac{1}{2} \int \frac{1}{\sqrt{1-x^2}(-1+2x^2)} dx \\
&= -\frac{1}{2} \sin^{-1}(x) + \frac{1}{4} \log(1-2x^2) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right) \\
&= -\frac{1}{2} \sin^{-1}(x) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) + \frac{1}{4} \log(1-2x^2)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.00

$$\frac{1}{4} \log(1-2x^2) - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{1-x^2}} \right) - \frac{1}{2} \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[1 - x^2])^(-1), x]

[Out] -1/2*ArcSin[x] - ArcTanh[x/Sqrt[1 - x^2]]/2 + Log[1 - 2*x^2]/4

IntegrateAlgebraic [A] time = 0.07, size = 39, normalized size = 1.05

$$\frac{1}{2} \log(\sqrt{1-x^2} - x) + \tan^{-1} \left(\frac{\sqrt{1-x^2}}{x+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[1 - x^2])^(-1), x]

[Out] ArcTan[Sqrt[1 - x^2]/(1 + x)] + Log[-x + Sqrt[1 - x^2]]/2

fricas [B] time = 0.60, size = 84, normalized size = 2.27

$$\arctan \left(\frac{\sqrt{-x^2+1}-1}{x} \right) + \frac{1}{4} \log(2x^2-1) + \frac{1}{4} \log \left(-\frac{x^2 + \sqrt{-x^2+1}(x+1) - x - 1}{x^2} \right) - \frac{1}{4} \log \left(-\frac{x^2 - \sqrt{-x^2+1}(x-1) + x - 1}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(-x^2+1)^(1/2)), x, algorithm="fricas")

[Out] arctan((sqrt(-x^2 + 1) - 1)/x) + 1/4*log(2*x^2 - 1) + 1/4*log(-(x^2 + sqrt(-x^2 + 1)*(x + 1) - x - 1)/x^2) - 1/4*log(-(x^2 - sqrt(-x^2 + 1)*(x - 1) + x - 1)/x^2)

giac [B] time = 0.44, size = 140, normalized size = 3.78

$$-\frac{1}{4} \pi \operatorname{sgn}(x) - \frac{1}{2} \arctan \left(\frac{x \left(\frac{(\sqrt{-x^2+1}-1)^2}{x^2} - 1 \right)}{2(\sqrt{-x^2+1}-1)} \right) + \frac{1}{4} \log \left(\left| x + \frac{1}{2} \sqrt{2} \right| \right) + \frac{1}{4} \log \left(\left| x - \frac{1}{2} \sqrt{2} \right| \right) - \frac{1}{4} \log \left(\left| -\frac{x}{\sqrt{-x^2+1}-1} + \frac{\sqrt{-x^2+1}-1}{x} + 2 \right| \right) + \frac{1}{4} \log \left(\left| -\frac{x}{\sqrt{-x^2+1}-1} + \frac{\sqrt{-x^2+1}-1}{x} - 2 \right| \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(-x^2+1)^(1/2)), x, algorithm="giac")

[Out] $-1/4*\pi*\text{sgn}(x) - 1/2*\arctan(-1/2*x*((\sqrt{-x^2 + 1} - 1)^2/x^2 - 1)/(\sqrt{-x^2 + 1} - 1)) + 1/4*\log(\text{abs}(x + 1/2*\sqrt{2})) + 1/4*\log(\text{abs}(x - 1/2*\sqrt{2})) - 1/4*\log(\text{abs}(-x/(\sqrt{-x^2 + 1} - 1) + (\sqrt{-x^2 + 1} - 1)/x + 2)) + 1/4*\log(\text{abs}(-x/(\sqrt{-x^2 + 1} - 1) + (\sqrt{-x^2 + 1} - 1)/x - 2))$

maple [B] time = 0.05, size = 175, normalized size = 4.73

$$\frac{\operatorname{arctanh}\left(\frac{\left(1-\frac{\sqrt{2}}{2}\right)\sqrt{2}}{\sqrt{-4\left(\frac{x-\frac{\sqrt{2}}{2}\right)^2-4\left(\frac{x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}}\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{\left(1+\frac{\sqrt{2}}{2}\right)\sqrt{2}}{\sqrt{-4\left(\frac{x+\frac{\sqrt{2}}{2}\right)^2+4\left(\frac{x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}}\right)}{4} - \frac{\arcsin(x)}{2} + \frac{\ln(2x^2-1)}{4} - \frac{\sqrt{2}\sqrt{-4\left(\frac{x+\frac{\sqrt{2}}{2}\right)^2+4\left(\frac{x+\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}}{8} + \frac{\sqrt{2}\sqrt{-4\left(\frac{x-\frac{\sqrt{2}}{2}\right)^2-4\left(\frac{x-\frac{\sqrt{2}}{2}\right)\sqrt{2}+2}}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x-(-x^2+1)^(1/2)),x)`

[Out] $1/4*\ln(2*x^2-1)-1/8*2^{(1/2)}*(-4*(x+1/2*2^{(1/2)})^2+4*(x+1/2*2^{(1/2)})*2^{(1/2)}+2)^{(1/2)}-1/2*\arcsin(x)+1/4*\operatorname{arctanh}(((x+1/2*2^{(1/2)})*2^{(1/2)}+1)*2^{(1/2)}/(-4*(x+1/2*2^{(1/2)})^2+4*(x+1/2*2^{(1/2)})*2^{(1/2)}+2)^{(1/2)}))+1/8*2^{(1/2)}*(-4*(x-1/2*2^{(1/2)})^2-4*(x-1/2*2^{(1/2)})*2^{(1/2)}+2)^{(1/2)}-1/4*\operatorname{arctanh}((1-(x-1/2*2^{(1/2)})*2^{(1/2)})*2^{(1/2)}/(-4*(x-1/2*2^{(1/2)})^2-4*(x-1/2*2^{(1/2)})*2^{(1/2)}+2)^{(1/2)}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(1/(x - sqrt(-x^2 + 1)), x)`

mupad [B] time = 0.14, size = 105, normalized size = 2.84

$$\frac{\ln\left(x - \frac{\sqrt{2}}{2}\right)}{4} + \frac{\ln\left(x + \frac{\sqrt{2}}{2}\right)}{4} - \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x-1}{2}\right)_{1i}-\sqrt{1-x^2} 1i}{x-\frac{\sqrt{2}}{2}}\right)}{4} + \frac{\ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x+1}{2}\right)_{1i}+\sqrt{1-x^2} 1i}{x+\frac{\sqrt{2}}{2}}\right)}{4} - \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x - (1 - x^2)^(1/2)),x)`

[Out] $\log(x - 2^{(1/2)}/2)/4 + \log(x + 2^{(1/2)}/2)/4 - \log((2^{(1/2)}*((2^{(1/2)}*x)/2 - 1)*1i - (1 - x^2)^{(1/2)}*1i)/(x - 2^{(1/2)}/2))/4 + \log((2^{(1/2)}*((2^{(1/2)}*x)/2 + 1)*1i + (1 - x^2)^{(1/2)}*1i)/(x + 2^{(1/2)}/2))/4 - \operatorname{asin}(x)/2$

sympy [A] time = 0.17, size = 17, normalized size = 0.46

$$\frac{\log\left(x - \sqrt{1 - x^2}\right)}{2} - \frac{\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x-(-x**2+1)**(1/2)),x)`

[Out] `log(x - sqrt(1 - x**2))/2 - asin(x)/2`

$$3.581 \quad \int \frac{1}{x - \sqrt{1+2x^2}} dx$$

Optimal. Leaf size=40

$$-\frac{1}{2} \log(x^2 + 1) + \tanh^{-1}\left(\frac{x}{\sqrt{2x^2 + 1}}\right) - \sqrt{2} \sinh^{-1}(\sqrt{2}x)$$

Rubi [A] time = 0.04, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 260, 402, 215, 377, 206}

$$-\frac{1}{2} \log(x^2 + 1) + \tanh^{-1}\left(\frac{x}{\sqrt{2x^2 + 1}}\right) - \sqrt{2} \sinh^{-1}(\sqrt{2}x)$$

Antiderivative was successfully verified.

[In] Int[(x - Sqrt[1 + 2*x^2])^(-1), x]

[Out] -(Sqrt[2]*ArcSinh[Sqrt[2]*x]) + ArcTanh[x/Sqrt[1 + 2*x^2]] - Log[1 + x^2]/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 260

Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{1}{x - \sqrt{1 + 2x^2}} dx &= \int \left(-\frac{x}{1 + x^2} - \frac{\sqrt{1 + 2x^2}}{1 + x^2} \right) dx \\
&= -\int \frac{x}{1 + x^2} dx - \int \frac{\sqrt{1 + 2x^2}}{1 + x^2} dx \\
&= -\frac{1}{2} \log(1 + x^2) - 2 \int \frac{1}{\sqrt{1 + 2x^2}} dx + \int \frac{1}{(1 + x^2)\sqrt{1 + 2x^2}} dx \\
&= -\sqrt{2} \sinh^{-1}(\sqrt{2}x) - \frac{1}{2} \log(1 + x^2) + \text{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \frac{x}{\sqrt{1 + 2x^2}} \right) \\
&= -\sqrt{2} \sinh^{-1}(\sqrt{2}x) + \tanh^{-1} \left(\frac{x}{\sqrt{1 + 2x^2}} \right) - \frac{1}{2} \log(1 + x^2)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.85

$$\frac{1}{4} \left(-2 \log(x^2 + 1) - \log(3x^2 - 2\sqrt{2x^2 + 1}x + 1) + \log(3x^2 + 2\sqrt{2x^2 + 1}x + 1) - 4\sqrt{2} \sinh^{-1}(\sqrt{2}x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x - Sqrt[1 + 2*x^2])^(-1), x]

[Out] (-4*Sqrt[2]*ArcSinh[Sqrt[2]*x] - 2*Log[1 + x^2] - Log[1 + 3*x^2 - 2*x*Sqrt[1 + 2*x^2]] + Log[1 + 3*x^2 + 2*x*Sqrt[1 + 2*x^2]])/4

IntegrateAlgebraic [B] time = 0.14, size = 85, normalized size = 2.12

$$(1 + \sqrt{2}) \log(\sqrt{2}\sqrt{2x^2 + 1} - 2\sqrt{2x^2 + 1} + 2\sqrt{2}x - 2x) - \log(-2x^2 + \sqrt{2}\sqrt{2x^2 + 1}x + \sqrt{2} - 2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - Sqrt[1 + 2*x^2])^(-1), x]

[Out] (1 + Sqrt[2])*Log[-2*x + 2*Sqrt[2]*x - 2*Sqrt[1 + 2*x^2] + Sqrt[2]*Sqrt[1 + 2*x^2]] - Log[-2 + Sqrt[2] - 2*x^2 + Sqrt[2]*x*Sqrt[1 + 2*x^2]]

fricas [B] time = 0.55, size = 90, normalized size = 2.25

$$\sqrt{2} \log(\sqrt{2}x - \sqrt{2x^2 + 1}) - \frac{1}{2} \log(x^2 + 1) - \frac{1}{2} \log\left(\frac{2x^2 - \sqrt{2x^2 + 1}(x + 1) + x + 1}{x^2}\right) + \frac{1}{2} \log\left(\frac{2x^2 + \sqrt{2x^2 + 1}(x - 1) - x + 1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="fricas")

[Out] sqrt(2)*log(sqrt(2)*x - sqrt(2*x^2 + 1)) - 1/2*log(x^2 + 1) - 1/2*log((2*x^2 - sqrt(2*x^2 + 1)*(x + 1) + x + 1)/x^2) + 1/2*log((2*x^2 + sqrt(2*x^2 + 1)*(x - 1) - x + 1)/x^2)

giac [B] time = 0.41, size = 88, normalized size = 2.20

$$\sqrt{2} \log(-\sqrt{2}x + \sqrt{2x^2 + 1}) + \frac{1}{2} \log\left(\left(\sqrt{2}x - \sqrt{2x^2 + 1}\right)^2 + 2\sqrt{2} + 3\right) - \frac{1}{2} \log\left(\left(\sqrt{2}x - \sqrt{2x^2 + 1}\right)^2 - 2\sqrt{2} + 3\right) - \frac{1}{2} \log(x^2 + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")

[Out] sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 + 2*sqrt(2) + 3) - 1/2*log((sqrt(2)*x - sqrt(2*x^2 + 1))^2 - 2*sqrt(2) + 3) - 1/2*log(x^2 + 1)

maple [A] time = 0.01, size = 33, normalized size = 0.82

$$-\sqrt{2} \operatorname{arcsinh}(\sqrt{2} x) + \operatorname{arctanh}\left(\frac{x}{\sqrt{2x^2 + 1}}\right) - \frac{\ln(x^2 + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x-(2*x^2+1)^(1/2)),x)

[Out] arctanh(x/(2*x^2+1)^(1/2))-1/2*ln(x^2+1)-arcsinh(2^(1/2)*x)*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x - \sqrt{2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(x - sqrt(2*x^2 + 1)), x)

mupad [B] time = 3.50, size = 57, normalized size = 1.42

$$-\ln(x - i) - \frac{\ln\left(x - \frac{\sqrt{2}\sqrt{x^2 + \frac{1}{2}}}{2} + \frac{1}{2}i\right)}{2} + \frac{\ln\left(x + \frac{\sqrt{2}\sqrt{x^2 + \frac{1}{2}}}{2} - \frac{1}{2}i\right)}{2} - \sqrt{2} \operatorname{asinh}(\sqrt{2} x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x - (2*x^2 + 1)^(1/2)),x)

[Out] log(x + (2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1i/2)/2 - log(x - (2^(1/2)*(x^2 + 1/2)^(1/2))/2 + 1i/2)/2 - log(x - 1i) - 2^(1/2)*asinh(2^(1/2)*x)

sympy [A] time = 0.21, size = 27, normalized size = 0.68

$$-\log(x - \sqrt{2x^2 + 1}) - \sqrt{2} \operatorname{asinh}(\sqrt{2} x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x-(2*x**2+1)**(1/2)),x)

[Out] -log(x - sqrt(2*x**2 + 1)) - sqrt(2)*asinh(sqrt(2)*x)

$$3.582 \quad \int \frac{2x-x^3+x^2\sqrt{2-x^2}}{-2+2x^2} dx$$

Optimal. Leaf size=54

$$-\frac{x^2}{4} + \frac{1}{4}\sqrt{2-x^2}x + \frac{1}{4}\log(1-x^2) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Rubi [A] time = 0.13, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {6725, 260, 266, 43, 478, 12, 377, 207}

$$-\frac{x^2}{4} + \frac{1}{4}\sqrt{2-x^2}x + \frac{1}{4}\log(1-x^2) - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2), x]

[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x^2]/4

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_.))^(p_)/((c_) + (d_.)*(x_)^(n_.)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 478

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_)*((c_) + (d_.)*(x_)^(n_.))^(q_), x_Symbol] := Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)

$(c + d*x^n)^q / (b*(m + n*(p + q) + 1)), x] - \text{Dist}[e^n / (b*(m + n*(p + q) + 1)), \text{Int}[(e*x)^{m-n} * (a + b*x^n)^p * (c + d*x^n)^{q-1} * \text{Simp}[a*c*(m-n+1) + (a*d*(m-n+1) - n*q*(b*c - a*d))*x^n, x], x] /; \text{FreeQ}[\{a, b, c, d, e, p\}, x] \&\& \text{NeQ}[b*c - a*d, 0] \&\& \text{IGtQ}[n, 0] \&\& \text{GtQ}[q, 0] \&\& \text{GtQ}[m-n+1, 0] \&\& \text{IntBinomialQ}[a, b, c, d, e, m, n, p, q, x]$

Rule 6725

$\text{Int}[(u_)/(a_ + (b_)*(x_)^(n_)), x_Symbol] := \text{With}[\{v = \text{RationalFunctionExpand}[u/(a + b*x^n), x]\}, \text{Int}[v, x] /; \text{SumQ}[v]] /; \text{FreeQ}[\{a, b\}, x] \&\& \text{IGtQ}[n, 0]$

Rubi steps

$$\begin{aligned} \int \frac{2x - x^3 + x^2\sqrt{2-x^2}}{-2 + 2x^2} dx &= \int \left(\frac{x}{-1+x^2} - \frac{x^3}{2(-1+x^2)} + \frac{x^2\sqrt{2-x^2}}{2(-1+x^2)} \right) dx \\ &= -\left(\frac{1}{2} \int \frac{x^3}{-1+x^2} dx \right) + \frac{1}{2} \int \frac{x^2\sqrt{2-x^2}}{-1+x^2} dx + \int \frac{x}{-1+x^2} dx \\ &= \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{2} \log(1-x^2) - \frac{1}{4} \int -\frac{2}{\sqrt{2-x^2}(-1+x^2)} dx - \frac{1}{4} \text{Subst} \left(\int \frac{x}{-1+x^2} dx, x, x^2 \right) \\ &= \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{2} \log(1-x^2) - \frac{1}{4} \text{Subst} \left(\int \left(1 + \frac{1}{-1+x} \right) dx, x, x^2 \right) + \frac{1}{2} \int \frac{x}{\sqrt{2-x^2}} dx \\ &= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} + \frac{1}{4} \log(1-x^2) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \frac{x}{\sqrt{2-x^2}} \right) \\ &= -\frac{x^2}{4} + \frac{1}{4}x\sqrt{2-x^2} - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{2-x^2}} \right) + \frac{1}{4} \log(1-x^2) \end{aligned}$$

Mathematica [A] time = 0.06, size = 77, normalized size = 1.43

$$\frac{1}{4} \left(-x^2 + \sqrt{2-x^2}x + \log(1-x^2) - \log(\sqrt{2-x^2}-x+2) + \log(\sqrt{2-x^2}+x+2) + \log(1-x) - \log(x+1) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2), x]

[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 - x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4

IntegrateAlgebraic [C] time = 0.10, size = 72, normalized size = 1.33

$$-\frac{x^2}{4} + \frac{1}{4}\sqrt{2-x^2}x + \frac{1}{2} \log(\sqrt{2-x^2} - ix + (1-i)) + \tanh^{-1} \left(-(1+i)\sqrt{2-x^2} + (-1+i)x + 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2*x - x^3 + x^2*Sqrt[2 - x^2])/(-2 + 2*x^2), x]

[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 + ArcTanh[1 - (1 - I)*x - (1 + I)*Sqrt[2 - x^2]] + Log[(1 - I) - I*x + Sqrt[2 - x^2]]/2

fricas [A] time = 0.58, size = 67, normalized size = 1.24

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2x} + \frac{1}{4} \log(x^2-1) - \frac{1}{8} \log\left(-\frac{\sqrt{-x^2+2x}+1}{x^2}\right) + \frac{1}{8} \log\left(\frac{\sqrt{-x^2+2x}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="fricas")

[Out] $-1/4*x^2 + 1/4*\sqrt{-x^2 + 2}*x + 1/4*\log(x^2 - 1) - 1/8*\log(-(\sqrt{-x^2 + 2})*x + 1)/x^2) + 1/8*\log((\sqrt{-x^2 + 2})*x - 1)/x^2)$

giac [B] time = 0.44, size = 117, normalized size = 2.17

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log(|x^2-1|) - \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} + 2\right|\right) + \frac{1}{4}\log\left(\left|\frac{x}{\sqrt{2}-\sqrt{-x^2+2}} - \frac{\sqrt{2}-\sqrt{-x^2+2}}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="giac")

[Out] $-1/4*x^2 + 1/4*\sqrt{-x^2 + 2}*x + 1/4*\log(\text{abs}(x^2 - 1)) - 1/4*\log(\text{abs}(x/(\sqrt{2} - \sqrt{-x^2 + 2}) - (\sqrt{2} - \sqrt{-x^2 + 2})/x + 2)) + 1/4*\log(\text{abs}(x/(\sqrt{2} - \sqrt{-x^2 + 2}) - (\sqrt{2} - \sqrt{-x^2 + 2})/x - 2))$

maple [B] time = 0.02, size = 111, normalized size = 2.06

$$-\frac{x^2}{4} + \frac{\sqrt{-x^2+2}x}{4} - \frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4} + \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} + \frac{\sqrt{-2x-(x-1)^2+3}}{4} - \frac{\sqrt{2x-(x+1)^2+3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x)

[Out] $1/4*x*(-x^2+2)^(1/2)+1/4*(-(x-1)^2-2*x+3)^(1/2)-1/4*\operatorname{arctanh}(1/2*(4-2*x)/(-(x-1)^2-2*x+3)^(1/2))-1/4*(-(x+1)^2+2*x+3)^(1/2)+1/4*\operatorname{arctanh}(1/2*(4+2*x)/(-(x+1)^2+2*x+3)^(1/2))-1/4*x^2+1/4*\ln(x-1)+1/4*\ln(x+1)$

maxima [B] time = 2.03, size = 94, normalized size = 1.74

$$-\frac{1}{4}x^2 + \frac{1}{4}\sqrt{-x^2+2}x + \frac{1}{4}\log(x^2-1) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x+2|} + \frac{2}{|2x+2|} + 1\right) - \frac{1}{4}\log\left(\frac{2\sqrt{-x^2+2}}{|2x-2|} + \frac{2}{|2x-2|} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x-x^3+x^2*(-x^2+2)^(1/2))/(2*x^2-2),x, algorithm="maxima")

[Out] $-1/4*x^2 + 1/4*\sqrt{-x^2 + 2}*x + 1/4*\log(x^2 - 1) + 1/4*\log(2*\sqrt{-x^2 + 2}/\text{abs}(2*x + 2) + 2/\text{abs}(2*x + 2) + 1) - 1/4*\log(2*\sqrt{-x^2 + 2}/\text{abs}(2*x - 2) + 2/\text{abs}(2*x - 2) - 1)$

mupad [B] time = 3.33, size = 86, normalized size = 1.59

$$\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x1i+\sqrt{2-x^2}1i+2i}{x-1}\right)}{4} + \frac{\ln\left(\frac{x1i+\sqrt{2-x^2}1i+2i}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + x^2*(2 - x^2)^(1/2) - x^3)/(2*x^2 - 2),x)

[Out] $\log(x - 1)/4 + \log(x + 1)/4 - \log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))/4 + \log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/4 - x^2/4$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\left(-\frac{2x}{x^2-1}\right)dx + \int\frac{x^3}{x^2-1}dx + \int\left(-\frac{x^2\sqrt{2-x^2}}{x^2-1}\right)dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x-x**3+x**2*(-x**2+2)**(1/2))/(2*x**2-2),x)
```

```
[Out] -(Integral(-2*x/(x**2 - 1), x) + Integral(x**3/(x**2 - 1), x) + Integral(-x**2*sqrt(2 - x**2)/(x**2 - 1), x))/2
```

$$3.583 \quad \int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}} dx$$

Optimal. Leaf size=60

$$-\frac{x^2}{4} + \frac{1}{4}\sqrt{2-x^2}x - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right) + \frac{1}{4}\log(1-x) + \frac{1}{4}\log(x+1)$$

Rubi [A] time = 0.30, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {6742, 195, 216, 697, 402, 377, 207}

$$-\frac{x^2}{4} + \frac{1}{4}\sqrt{2-x^2}x - \frac{1}{2}\tanh^{-1}\left(\frac{x}{\sqrt{2-x^2}}\right) + \frac{1}{4}\log(1-x) + \frac{1}{4}\log(x+1)$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]),x]

[Out] -x^2/4 + (x*Sqrt[2 - x^2])/4 - ArcTanh[x/Sqrt[2 - x^2]]/2 + Log[1 - x]/4 + Log[1 + x]/4

Rule 195

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(x*(a + b*x^n)^p)/(n*p + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] && IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n], Denominator[p]])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 402

Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] := Dist[b/d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])

Rule 697

Int[((d_) + (e_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]

Rule 6742

```
Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{2-x^2}}{x-\sqrt{2-x^2}} dx &= \int \left(\frac{\sqrt{2-x^2}}{2} + \frac{2-x^2}{4(-1+x)} + \frac{2-x^2}{4(1+x)} + \frac{\sqrt{2-x^2}}{2(-1+x^2)} \right) dx \\ &= \frac{1}{4} \int \frac{2-x^2}{-1+x} dx + \frac{1}{4} \int \frac{2-x^2}{1+x} dx + \frac{1}{2} \int \sqrt{2-x^2} dx + \frac{1}{2} \int \frac{\sqrt{2-x^2}}{-1+x^2} dx \\ &= \frac{1}{4} x\sqrt{2-x^2} + \frac{1}{4} \int \left(-1 + \frac{1}{-1+x} - x \right) dx + \frac{1}{4} \int \left(1 - x + \frac{1}{1+x} \right) dx + \frac{1}{2} \int \frac{1}{\sqrt{2-x^2}(-1+x^2)} dx \\ &= -\frac{x^2}{4} + \frac{1}{4} x\sqrt{2-x^2} + \frac{1}{4} \log(1-x) + \frac{1}{4} \log(1+x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+x^2} dx, x, \frac{x}{\sqrt{2-x^2}} \right) \\ &= -\frac{x^2}{4} + \frac{1}{4} x\sqrt{2-x^2} - \frac{1}{2} \tanh^{-1} \left(\frac{x}{\sqrt{2-x^2}} \right) + \frac{1}{4} \log(1-x) + \frac{1}{4} \log(1+x) \end{aligned}$$

Mathematica [A] time = 0.09, size = 77, normalized size = 1.28

$$\frac{1}{4} \left(-x^2 + \sqrt{2-x^2} x + \log(1-x^2) - \log(\sqrt{2-x^2} - x + 2) + \log(\sqrt{2-x^2} + x + 2) + \log(1-x) - \log(x+1) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]), x]
```

```
[Out] (-x^2 + x*Sqrt[2 - x^2] + Log[1 - x] - Log[1 + x] + Log[1 - x^2] - Log[2 - x + Sqrt[2 - x^2]] + Log[2 + x + Sqrt[2 - x^2]])/4
```

IntegrateAlgebraic [C] time = 0.10, size = 72, normalized size = 1.20

$$-\frac{x^2}{4} + \frac{1}{4} \sqrt{2-x^2} x + \frac{1}{2} \log(\sqrt{2-x^2} - ix + (1-i)) + \tanh^{-1} \left(-(1+i)\sqrt{2-x^2} + (-1+i)x + 1 \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(x*Sqrt[2 - x^2])/(x - Sqrt[2 - x^2]), x]
```

```
[Out] -1/4*x^2 + (x*Sqrt[2 - x^2])/4 + ArcTanh[1 - (1 - I)*x - (1 + I)*Sqrt[2 - x^2]] + Log[(1 - I) - I*x + Sqrt[2 - x^2]]/2
```

fricas [A] time = 0.63, size = 67, normalized size = 1.12

$$-\frac{1}{4} x^2 + \frac{1}{4} \sqrt{-x^2 + 2x} + \frac{1}{4} \log(x^2 - 1) - \frac{1}{8} \log\left(-\frac{\sqrt{-x^2 + 2x} + 1}{x^2}\right) + \frac{1}{8} \log\left(\frac{\sqrt{-x^2 + 2x} - 1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)), x, algorithm="fricas")
```

```
[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(x^2 - 1) - 1/8*log(-(sqrt(-x^2 + 2)*x + 1)/x^2) + 1/8*log((sqrt(-x^2 + 2)*x - 1)/x^2)
```

giac [B] time = 0.51, size = 117, normalized size = 1.95

$$-\frac{1}{4} x^2 + \frac{1}{4} \sqrt{-x^2 + 2x} + \frac{1}{4} \log(|x^2 - 1|) - \frac{1}{4} \log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}} - \frac{\sqrt{2} - \sqrt{-x^2 + 2}}{x} + 2\right|\right) + \frac{1}{4} \log\left(\left|\frac{x}{\sqrt{2} - \sqrt{-x^2 + 2}} - \frac{\sqrt{2} - \sqrt{-x^2 + 2}}{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="giac")

[Out] -1/4*x^2 + 1/4*sqrt(-x^2 + 2)*x + 1/4*log(abs(x^2 - 1)) - 1/4*log(abs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x + 2)) + 1/4*log(abs(x/(sqrt(2) - sqrt(-x^2 + 2)) - (sqrt(2) - sqrt(-x^2 + 2))/x - 2))

maple [B] time = 0.02, size = 111, normalized size = 1.85

$$\frac{x^2}{4} + \frac{\sqrt{-x^2+2}x}{4} - \frac{\operatorname{arctanh}\left(\frac{-2x+4}{2\sqrt{-2x-(x-1)^2+3}}\right)}{4} + \frac{\operatorname{arctanh}\left(\frac{2x+4}{2\sqrt{2x-(x+1)^2+3}}\right)}{4} + \frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} + \frac{\sqrt{-2x-(x-1)^2+3}}{4} - \frac{\sqrt{2x-(x+1)^2+3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x)

[Out] 1/4*(-x^2+2)^(1/2)*x+1/4*(-2*x-(x-1)^2+3)^(1/2)-1/4*arctanh(1/2*(-2*x+4)/(-2*x-(x-1)^2+3)^(1/2))-1/4*(2*x-(x+1)^2+3)^(1/2)+1/4*arctanh(1/2*(2*x+4)/(2*x-(x+1)^2+3)^(1/2))-1/4*x^2+1/4*ln(x-1)+1/4*ln(x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{1}{2}x^2 - \int -\frac{x^2}{x - \sqrt{-x^2+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x^2+2)^(1/2)/(x-(-x^2+2)^(1/2)),x, algorithm="maxima")

[Out] -1/2*x^2 - integrate(-x^2/(x - sqrt(-x^2 + 2)), x)

mupad [B] time = 3.38, size = 86, normalized size = 1.43

$$\frac{\ln(x-1)}{4} + \frac{\ln(x+1)}{4} - \frac{\ln\left(\frac{-x1i+\sqrt{2-x^2}1i+2i}{x-1}\right)}{4} + \frac{\ln\left(\frac{x1i+\sqrt{2-x^2}1i+2i}{x+1}\right)}{4} + \frac{x\sqrt{2-x^2}}{4} - \frac{x^2}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(2 - x^2)^(1/2))/(x - (2 - x^2)^(1/2)),x)

[Out] log(x - 1)/4 + log(x + 1)/4 - log(((2 - x^2)^(1/2)*1i - x*1i + 2i)/(x - 1))/4 + log((x*1i + (2 - x^2)^(1/2)*1i + 2i)/(x + 1))/4 + (x*(2 - x^2)^(1/2))/(4 - x^2/4)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{2-x^2}}{x - \sqrt{2-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(-x**2+2)**(1/2)/(x-(-x**2+2)**(1/2)),x)

[Out] Integral(x*sqrt(2 - x**2)/(x - sqrt(2 - x**2)), x)

$$3.584 \quad \int \frac{x}{-x + \sqrt{2x - x^2}} dx$$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x - x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x - x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1 - x)$$

Rubi [A] time = 0.11, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {6742, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x - x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x - x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1 - x)$$

Antiderivative was successfully verified.

[In] Int[x/(-x + Sqrt[2*x - x^2]),x]

[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 685

Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && !LtQ[m, -1] && !(IGtQ[(m - 1)/2, 0] && (!IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]

Rule 688

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{x}{-x + \sqrt{2x - x^2}} dx &= \int \left(-\frac{1}{2} - \frac{1}{2(-1+x)} + \frac{\sqrt{2x-x^2}}{2(1-x)} \right) dx \\
&= -\frac{x}{2} - \frac{1}{2} \log(1-x) + \frac{1}{2} \int \frac{\sqrt{2x-x^2}}{1-x} dx \\
&= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \log(1-x) + \frac{1}{2} \int \frac{1}{(1-x)\sqrt{2x-x^2}} dx \\
&= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \log(1-x) - 2 \operatorname{Subst} \left(\int \frac{1}{-4+4x^2} dx, x, \sqrt{2x-x^2} \right) \\
&= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} + \frac{1}{2} \tanh^{-1} \left(\sqrt{2x-x^2} \right) - \frac{1}{2} \log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2} \left(-x - \sqrt{-(x-2)x} - \log(1-x) + \tanh^{-1} \left(\sqrt{-(x-2)x} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[x/(-x + Sqrt[2*x - x^2]),x]

[Out] (-x - Sqrt[-((-2 + x)*x)] + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2

IntegrateAlgebraic [C] time = 0.28, size = 58, normalized size = 1.14

$$-\frac{1}{2} \sqrt{2x-x^2} - \log \left(\sqrt{2x-x^2} + x - 2 \right) + \frac{1}{2} i(\pi + ix) + \frac{1}{2} \log(x-2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(-x + Sqrt[2*x - x^2]),x]

[Out] (I/2)*(Pi + I*x) - Sqrt[2*x - x^2]/2 + Log[-2 + x]/2 - Log[-2 + x + Sqrt[2*x - x^2]]

fricas [A] time = 0.42, size = 66, normalized size = 1.29

$$-\frac{1}{2} x - \frac{1}{2} \sqrt{-x^2 + 2x} - \frac{1}{2} \log(x-1) + \frac{1}{2} \log \left(\frac{x + \sqrt{-x^2 + 2x}}{x} \right) - \frac{1}{2} \log \left(-\frac{x - \sqrt{-x^2 + 2x}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="fricas")

[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)

giac [A] time = 0.43, size = 50, normalized size = 0.98

$$-\frac{1}{2} x - \frac{1}{2} \sqrt{-x^2 + 2x} - \frac{1}{2} \log \left(-\frac{2 \left(\sqrt{-x^2 + 2x} - 1 \right)}{|-2x + 2|} \right) - \frac{1}{2} \log(|x-1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="giac")

[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x + 2)) - 1/2*log(abs(x - 1))

maple [A] time = 0.00, size = 38, normalized size = 0.75

$$-\frac{x}{2} + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2} - \frac{\ln(x-1)}{2} - \frac{\sqrt{-(x-1)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-x+(-x^2+2*x)^(1/2)),x)

[Out] -1/2*x-1/2*ln(x-1)-1/2*(-(x-1)^2+1)^(1/2)+1/2*arctanh(1/(-(x-1)^2+1)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x - \sqrt{-x^2 + 2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x+(-x^2+2*x)^(1/2)),x, algorithm="maxima")

[Out] -integrate(x/(x - sqrt(-x^2 + 2*x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x}{x - \sqrt{2x - x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-x/(x - (2*x - x^2)^(1/2)),x)

[Out] int(-x/(x - (2*x - x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x}{x - \sqrt{-x^2 + 2x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-x+(-x**2+2*x)**(1/2)),x)

[Out] -Integral(x/(x - sqrt(-x**2 + 2*x)), x)

$$3.585 \quad \int \frac{x + \sqrt{2x - x^2}}{2 - 2x} dx$$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x - x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x - x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1 - x)$$

Rubi [A] time = 0.10, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x - x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x - x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1 - x)$$

Antiderivative was successfully verified.

[In] Int[(x + Sqrt[2*x - x^2])/(2 - 2*x),x]

[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 685

Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && !LtQ[m, -1] && !(IGtQ[(m - 1)/2, 0] && (!IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]

Rule 688

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{x + \sqrt{2x - x^2}}{2 - 2x} dx &= \int \left(-\frac{x}{2(-1+x)} + \frac{\sqrt{2x-x^2}}{2(1-x)} \right) dx \\
&= -\left(\frac{1}{2} \int \frac{x}{-1+x} dx \right) + \frac{1}{2} \int \frac{\sqrt{2x-x^2}}{1-x} dx \\
&= -\frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \int \left(1 + \frac{1}{-1+x} \right) dx + \frac{1}{2} \int \frac{1}{(1-x)\sqrt{2x-x^2}} dx \\
&= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \log(1-x) - 2 \operatorname{Subst} \left(\int \frac{1}{-4+4x^2} dx, x, \sqrt{2x-x^2} \right) \\
&= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} + \frac{1}{2} \tanh^{-1}(\sqrt{2x-x^2}) - \frac{1}{2} \log(1-x)
\end{aligned}$$

Mathematica [A] time = 0.05, size = 39, normalized size = 0.76

$$\frac{1}{2} \left(-x - \sqrt{-(x-2)x} - \log(1-x) + \tanh^{-1}(\sqrt{-(x-2)x}) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x + Sqrt[2*x - x^2])/(2 - 2*x), x]

[Out] (-x - Sqrt[-((-2 + x)*x)]) + ArcTanh[Sqrt[-((-2 + x)*x)]] - Log[1 - x])/2

IntegrateAlgebraic [C] time = 0.22, size = 58, normalized size = 1.14

$$-\frac{1}{2} \sqrt{2x-x^2} - \log\left(\sqrt{2x-x^2} + x - 2\right) + \frac{1}{2} i(\pi + ix) + \frac{1}{2} \log(x-2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x + Sqrt[2*x - x^2])/(2 - 2*x), x]

[Out] (I/2)*(Pi + I*x) - Sqrt[2*x - x^2]/2 + Log[-2 + x]/2 - Log[-2 + x + Sqrt[2*x - x^2]]

fricas [A] time = 0.40, size = 66, normalized size = 1.29

$$-\frac{1}{2} x - \frac{1}{2} \sqrt{-x^2 + 2x} - \frac{1}{2} \log(x-1) + \frac{1}{2} \log\left(\frac{x + \sqrt{-x^2 + 2x}}{x}\right) - \frac{1}{2} \log\left(-\frac{x - \sqrt{-x^2 + 2x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x), x, algorithm="fricas")

[Out] -1/2*x - 1/2*sqrt(-x^2 + 2*x) - 1/2*log(x - 1) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)

giac [A] time = 0.48, size = 50, normalized size = 0.98

$$-\frac{1}{2} x - \frac{1}{2} \sqrt{-x^2 + 2x} - \frac{1}{2} \log\left(-\frac{2\left(\sqrt{-x^2 + 2x} - 1\right)}{|-2x + 2|}\right) - \frac{1}{2} \log(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(-x^2+2*x)^(1/2))/(2-2*x), x, algorithm="giac")

[Out] $-1/2*x - 1/2*\sqrt{-x^2 + 2*x} - 1/2*\log(-2*(\sqrt{-x^2 + 2*x} - 1)/\text{abs}(-2*x + 2)) - 1/2*\log(\text{abs}(x - 1))$

maple [A] time = 0.01, size = 38, normalized size = 0.75

$$-\frac{x}{2} + \frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-1)^2+1}}\right)}{2} - \frac{\ln(x-1)}{2} - \frac{\sqrt{-(x-1)^2+1}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+(-x^2+2*x)^(1/2))/(2-2*x),x)`

[Out] $-1/2*x - 1/2*\ln(x-1) - 1/2*(-(x-1)^2+1)^{(1/2)} + 1/2*\operatorname{arctanh}(1/(-(x-1)^2+1)^{(1/2)})$

maxima [A] time = 1.98, size = 54, normalized size = 1.06

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{-x^2 + 2x} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2 + 2x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(-x^2+2*x)^(1/2))/(2-2*x),x, algorithm="maxima")`

[Out] $-1/2*x - 1/2*\sqrt{-x^2 + 2*x} - 1/2*\log(x - 1) + 1/2*\log(2*\sqrt{-x^2 + 2*x}/\text{abs}(x - 1) + 2/\text{abs}(x - 1))$

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{x + \sqrt{2x - x^2}}{2x - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-(x + (2*x - x^2)^(1/2))/(2*x - 2),x)`

[Out] `int(-(x + (2*x - x^2)^(1/2))/(2*x - 2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{x}{x-1} dx + \int \frac{\sqrt{-x^2+2x}}{x-1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+(-x**2+2*x)**(1/2))/(2-2*x),x)`

[Out] $-(\text{Integral}(x/(x - 1), x) + \text{Integral}(\sqrt{-x**2 + 2*x}/(x - 1), x))/2$

$$3.586 \quad \int \frac{\sqrt{2-x} \sqrt{x+x}}{2-2x} dx$$

Optimal. Leaf size=51

$$-\frac{1}{2}\sqrt{2x-x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x-x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1-x)$$

Rubi [A] time = 0.15, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {6688, 2115, 6742, 43, 685, 688, 207}

$$-\frac{1}{2}\sqrt{2x-x^2} + \frac{1}{2} \tanh^{-1}\left(\sqrt{2x-x^2}\right) - \frac{x}{2} - \frac{1}{2} \log(1-x)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[2 - x]*Sqrt[x] + x)/(2 - 2*x),x]

[Out] -x/2 - Sqrt[2*x - x^2]/2 + ArcTanh[Sqrt[2*x - x^2]]/2 - Log[1 - x]/2

Rule 43

Int[((a_.) + (b_.)*(x_.))^(m_.)*((c_.) + (d_.)*(x_.))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 685

Int[((d_) + (e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[(d*p*(b^2 - 4*a*c))/(b*e*(m + 2*p + 1)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] && !LtQ[m, -1] && !(IGtQ[(m - 1)/2, 0] && (!IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]

Rule 688

Int[1/(((d_) + (e_.)*(x_.))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rule 2115

Int[((u_) + (f_.)*((j_.) + (k_.)*Sqrt[v_]))^(n_.)*((g_.) + (h_.)*(x_.))^(m_.), x_Symbol] := Int[(g + h*x)^m*(ExpandToSum[u + f*j, x] + f*k*Sqrt[ExpandToSum[v, x]])^n, x] /; FreeQ[{f, g, h, j, k, m, n}, x] && LinearQ[u, x] && QuadraticQ[v, x] && !(LinearMatchQ[u, x] && QuadraticMatchQ[v, x] && (EqQ[j, 0] || EqQ[f, 1])) && EqQ[(Coefficient[u, x, 1]*g - h*(Coefficient[u, x, 0] + f*j))^2 - f^2*k^2*(Coefficient[v, x, 2]*g^2 - Coefficient[v, x, 1]*g*h + Coefficient[v, x, 0]*h^2), 0]

Rule 6688

`Int[u_, x_Symbol] :=> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]`

Rule 6742

`Int[u_, x_Symbol] :=> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]`

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{2-x}\sqrt{x} + x}{2-2x} dx &= \int \frac{x + \sqrt{-((-2+x)x)}}{2-2x} dx \\
 &= \int \frac{x + \sqrt{2x-x^2}}{2-2x} dx \\
 &= \int \left(-\frac{x}{2(-1+x)} + \frac{\sqrt{2x-x^2}}{2(1-x)} \right) dx \\
 &= -\left(\frac{1}{2} \int \frac{x}{-1+x} dx \right) + \frac{1}{2} \int \frac{\sqrt{2x-x^2}}{1-x} dx \\
 &= -\frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \int \left(1 + \frac{1}{-1+x} \right) dx + \frac{1}{2} \int \frac{1}{(1-x)\sqrt{2x-x^2}} dx \\
 &= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} - \frac{1}{2} \log(1-x) - 2 \operatorname{Subst} \left(\int \frac{1}{-4+4x^2} dx, x, \sqrt{2x-x^2} \right) \\
 &= -\frac{x}{2} - \frac{1}{2} \sqrt{2x-x^2} + \frac{1}{2} \tanh^{-1} \left(\sqrt{2x-x^2} \right) - \frac{1}{2} \log(1-x)
 \end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.76

$$\frac{1}{2} \left(-x - \sqrt{-((x-2)x)} - \log(1-x) + \tanh^{-1} \left(\sqrt{-((x-2)x)} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[2-x]*Sqrt[x] + x)/(2-2*x), x]

[Out] (-x - Sqrt[-((-2+x)*x)] + ArcTanh[Sqrt[-((-2+x)*x)]] - Log[1-x])/2

IntegrateAlgebraic [C] time = 0.12, size = 76, normalized size = 1.49

$$-\frac{x}{2} - \frac{1}{2} \sqrt{2-x}\sqrt{x} - \log \left(\sqrt{2-x} - i\sqrt{x} + (1-i) \right) - 2 \tanh^{-1} \left((-1-i)\sqrt{2-x} - (1-i)\sqrt{x} + 1 \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(Sqrt[2-x]*Sqrt[x] + x)/(2-2*x), x]

[Out] -1/2*(Sqrt[2-x]*Sqrt[x]) - x/2 - 2*ArcTanh[1 - (1+I)*Sqrt[2-x] - (1-I)*Sqrt[x]] - Log[(1-I) + Sqrt[2-x] - I*Sqrt[x]]

fricas [A] time = 0.40, size = 64, normalized size = 1.25

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="fricas")

[Out] $-\frac{1}{2}x - \frac{1}{2}\sqrt{x}\sqrt{-x+2} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2}\log\left(\frac{-x-\sqrt{x}\sqrt{-x+2}}{x}\right)$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-16.8804557086]-1/2*(x+ln(abs(x-1))+sqrt(x)*sqrt(-x+2)-ln(abs(2*sqrt(x)/(-2*sqrt(-x+2))+2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))+ln(abs(2*sqrt(x)/(-2*sqrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x))))

maple [A] time = 0.01, size = 51, normalized size = 1.00

$$-\frac{x}{2} - \frac{\ln(x-1)}{2} - \frac{\sqrt{-x+2} \left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-2)x}}\right) + \sqrt{-(x-2)x} \right) \sqrt{x}}{2\sqrt{-(x-2)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x)

[Out] $-\frac{1}{2}(2-x)^{1/2}x^{1/2}/(-x(x-2))^{1/2}*((-x(x-2))^{1/2}-\operatorname{arctanh}(1/(-x(x-2))^{1/2}))-1/2x-1/2\ln(x-1)$

maxima [A] time = 1.97, size = 54, normalized size = 1.06

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{-x^2+2x} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+(2-x)^(1/2)*x^(1/2))/(2-2*x),x, algorithm="maxima")

[Out] $-\frac{1}{2}x - \frac{1}{2}\sqrt{-x^2+2x} - \frac{1}{2}\log(x-1) + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{\operatorname{abs}(x-1)} + \frac{2}{\operatorname{abs}(x-1)}\right)$

mupad [B] time = 4.78, size = 56, normalized size = 1.10

$$\operatorname{atanh}\left(\frac{\sqrt{x}(\sqrt{2}-\sqrt{2-x})}{x+\sqrt{2}\sqrt{2-x}-2}\right) - \frac{\ln(x-1)}{2} - \frac{x}{2} - \frac{\sqrt{x}\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x+x^(1/2)*(2-x)^(1/2))/(2*x-2),x)

[Out] $\operatorname{atanh}\left(\frac{x^{1/2}(2^{1/2}-(2-x)^{1/2})}{x+2^{1/2}(2-x)^{1/2}-2}\right) - \log(x-1)/2 - x/2 - (x^{1/2}(2-x)^{1/2})/2$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{x}{x-1} dx + \int \frac{\sqrt{x}\sqrt{2-x}}{x-1} dx}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x+(2-x)**(1/2)*x**(1/2))/(2-2*x),x)
```

```
[Out] -(Integral(x/(x - 1), x) + Integral(sqrt(x)*sqrt(2 - x)/(x - 1), x))/2
```

$$3.587 \quad \int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx$$

Optimal. Leaf size=54

$$-\frac{x}{2} - \frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{1}{2}\log(1-x) + \frac{1}{2}\tanh^{-1}(\sqrt{2-x}\sqrt{x})$$

Rubi [A] time = 0.05, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2105, 101, 12, 92, 206, 43}

$$-\frac{x}{2} - \frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{1}{2}\log(1-x) + \frac{1}{2}\tanh^{-1}(\sqrt{2-x}\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(Sqrt[2 - x] - Sqrt[x]), x]

[Out] -(Sqrt[2 - x]*Sqrt[x])/2 - x/2 + ArcTanh[Sqrt[2 - x]*Sqrt[x]]/2 - Log[1 - x]/2

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])]

Rule 92

Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_))), x_Symbol] := Dist[b*f, Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sqrt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d*e - f*(b*c + a*d), 0]

Rule 101

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_Symbol] := Simp[((a + b*x)^m*(c + d*x)^n*(e + f*x)^(p + 1))/(f*(m + n + p + 1)), x] - Dist[1/(f*(m + n + p + 1)), Int[(a + b*x)^(m - 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[c*m*(b*e - a*f) + a*n*(d*e - c*f) + (d*m*(b*e - a*f) + b*n*(d*e - c*f))*x, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && GtQ[m, 0] && GtQ[n, 0] && NeQ[m + n + p + 1, 0] && (IntegersQ[2*m, 2*n, 2*p] || (IntegersQ[m, n + p] || IntegersQ[p, m + n]))

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2105

Int[(u_)/((e_.)*Sqrt[(a_.) + (b_.)*(x_)] + (f_.)*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dist[e, Int[(u*Sqrt[a + b*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2

) \sqrt{x}), x], x] - Dist[f, Int[(u*Sqrt[c + d*x])/(a*e^2 - c*f^2 + (b*e^2 - d*f^2)*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a*e^2 - c*f^2, 0] && NeQ[b*e^2 - d*f^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x}}{\sqrt{2-x}-\sqrt{x}} dx &= \int \frac{\sqrt{2-x}\sqrt{x}}{2-2x} dx + \int \frac{x}{2-2x} dx \\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x} + \frac{1}{2} \int \frac{2}{(2-2x)\sqrt{2-x}\sqrt{x}} dx + \int \left(-\frac{1}{2} - \frac{1}{2(-1+x)}\right) dx \\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} - \frac{1}{2} \log(1-x) + \int \frac{1}{(2-2x)\sqrt{2-x}\sqrt{x}} dx \\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} - \frac{1}{2} \log(1-x) + 2 \text{Subst}\left(\int \frac{1}{4-4x^2} dx, x, \sqrt{2-x}\sqrt{x}\right) \\ &= -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{x}{2} + \frac{1}{2} \tanh^{-1}(\sqrt{2-x}\sqrt{x}) - \frac{1}{2} \log(1-x) \end{aligned}$$

Mathematica [A] time = 0.14, size = 82, normalized size = 1.52

$$\frac{1}{2} \left(-x - \sqrt{-(x-2)x} - \log(1-\sqrt{x}) - \log(\sqrt{x}+1) + \tanh^{-1}\left(\frac{2-\sqrt{x}}{\sqrt{2-x}}\right) - \tanh^{-1}\left(\frac{\sqrt{x}+2}{\sqrt{2-x}}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[x]/(Sqrt[2-x]-Sqrt[x]),x]

[Out] (-x - Sqrt[-((x-2)*x)] + ArcTanh[(2 - Sqrt[x])/Sqrt[2-x]] - ArcTanh[(2 + Sqrt[x])/Sqrt[2-x]] - Log[1 - Sqrt[x]] - Log[1 + Sqrt[x]])/2

IntegrateAlgebraic [C] time = 0.12, size = 76, normalized size = 1.41

$$-\frac{x}{2} - \frac{1}{2}\sqrt{2-x}\sqrt{x} - \log(\sqrt{2-x} - i\sqrt{x} + (1-i)) - 2 \tanh^{-1}((-1-i)\sqrt{2-x} - (1-i)\sqrt{x} + 1)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(Sqrt[2-x]-Sqrt[x]),x]

[Out] -1/2*(Sqrt[2-x]*Sqrt[x]) - x/2 - 2*ArcTanh[1 - (1 + I)*Sqrt[2-x] - (1 - I)*Sqrt[x]] - Log[(1 - I) + Sqrt[2-x] - I*Sqrt[x]]

fricas [A] time = 0.39, size = 64, normalized size = 1.19

$$-\frac{1}{2}x - \frac{1}{2}\sqrt{x}\sqrt{-x+2} - \frac{1}{2} \log(x-1) + \frac{1}{2} \log\left(\frac{x+\sqrt{x}\sqrt{-x+2}}{x}\right) - \frac{1}{2} \log\left(-\frac{x-\sqrt{x}\sqrt{-x+2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="fricas")

[Out] -1/2*x - 1/2*sqrt(x)*sqrt(-x + 2) - 1/2*log(x - 1) + 1/2*log((x + sqrt(x)*sqrt(-x + 2))/x) - 1/2*log(-(x - sqrt(x)*sqrt(-x + 2))/x)

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-92.616423693]Warning, choosing root of [1,0,-4,0,%%{4,[2]%%}+%%{-8,[1]%%}+%%{4,[0]%%}] at parameters values [-16.8804557086]2*(-x/4-1/4*ln(abs(x-1))-1/4*sqrt(x)*sqrt(-x+2)+1/4*ln(abs(2*sqrt(x)/(-2*sqrt(-x+2)+2*sqrt(2))+2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x)))-1/4*ln(abs(2*sqrt(x)/(-2*sqrt(-x+2)+2*sqrt(2))-2-1/2*(-2*sqrt(-x+2)+2*sqrt(2))/sqrt(x))))

maple [A] time = 0.01, size = 51, normalized size = 0.94

$$-\frac{x}{2} - \frac{\ln(x-1)}{2} - \frac{\sqrt{-x+2} \left(-\operatorname{arctanh}\left(\frac{1}{\sqrt{-(x-2)x}}\right) + \sqrt{-(x-2)x} \right) \sqrt{x}}{2\sqrt{-(x-2)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/((-x+2)^(1/2)-x^(1/2)),x)

[Out] -1/2*(-x+2)^(1/2)*x^(1/2)/((-x+2)*x)^(1/2)*((-x+2)*x)^(1/2)-arctanh(1/((-x+2)*x)^(1/2))-1/2*x-1/2*ln(x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{x}}{\sqrt{x} - \sqrt{-x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x, algorithm="maxima")

[Out] -integrate(sqrt(x)/(sqrt(x) - sqrt(-x + 2)), x)

mupad [B] time = 0.06, size = 56, normalized size = 1.04

$$\operatorname{atanh}\left(\frac{\sqrt{x}(\sqrt{2}-\sqrt{2-x})}{x+\sqrt{2}\sqrt{2-x}-2}\right) - \frac{\ln(x-1)}{2} - \frac{x}{2} - \frac{\sqrt{x}\sqrt{2-x}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/((2-x)^(1/2)-x^(1/2)),x)

[Out] atanh((x^(1/2)*(2^(1/2)-(2-x)^(1/2)))/(x+2^(1/2)*(2-x)^(1/2)-2))-log(x-1)/2-x/2-(x^(1/2)*(2-x)^(1/2))/2

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x}}{-\sqrt{x} + \sqrt{2-x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/((2-x)**(1/2)-x**(1/2)),x)

[Out] Integral(sqrt(x)/(-sqrt(x) + sqrt(2 - x)), x)

$$3.588 \quad \int \frac{1}{((1+x)(-1+x^2))^{2/3}} dx$$

Optimal. Leaf size=27

$$-\frac{3(1-x^2)}{2(-(x+1)(1-x^2))^{2/3}}$$

Rubi [A] time = 0.03, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2067, 2064, 37}

$$-\frac{3(1-x)(x+1)}{2(x^3+x^2-x-1)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[((1 + x)*(-1 + x^2))^(2/3), x]

[Out] (-3*(1 - x)*(1 + x))/(2*(-1 - x + x^2 + x^3)^(2/3))

Rule 37

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -1]

Rule 2064

Int[((a_.) + (b_.)*(x_) + (d_.)*(x_)^3)^(p_), x_Symbol] := Dist[(a + b*x + d*x^3)^p/((3*a - b*x)^p*(3*a + 2*b*x)^(2*p)), Int[(3*a - b*x)^p*(3*a + 2*b*x)^(2*p), x], x] /; FreeQ[{a, b, d, p}, x] && EqQ[4*b^3 + 27*a^2*d, 0] && !IntegerQ[p]

Rule 2067

Int[(P3_)^(p_), x_Symbol] := With[{a = Coeff[P3, x, 0], b = Coeff[P3, x, 1], c = Coeff[P3, x, 2], d = Coeff[P3, x, 3]}, Subst[Int[Simp[(2*c^3 - 9*b*c*d + 27*a*d^2)/(27*d^2) - ((c^2 - 3*b*d)*x)/(3*d) + d*x^3, x]^p, x], x, x + c/(3*d)] /; NeQ[c, 0] /; FreeQ[p, x] && PolyQ[P3, x, 3]

Rubi steps

$$\begin{aligned} \int \frac{1}{((1+x)(-1+x^2))^{2/3}} dx &= \text{Subst} \left(\int \frac{1}{\left(-\frac{16}{27} - \frac{4x}{3} + x^3\right)^{2/3}} dx, x, \frac{1}{3} + x \right) \\ &= \frac{(32\sqrt[3]{2}(-1-x)^{4/3}(-1+x)^{2/3}) \text{Subst} \left(\int \frac{1}{\left(-\frac{16}{9} - \frac{8x}{3}\right)^{4/3} \left(-\frac{16}{9} + \frac{4x}{3}\right)^{2/3}} dx, x, \frac{1}{3} + x \right)}{9(-1-x+x^2+x^3)^{2/3}} \\ &= -\frac{3(1-x)(1+x)}{2(-1-x+x^2+x^3)^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.85

$$\frac{3(x-1)(x+1)}{2((x-1)(x+1)^2)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x)*(-1 + x^2))^(-2/3), x]

[Out] (3*(-1 + x)*(1 + x))/(2*((-1 + x)*(1 + x)^2)^(2/3))

IntegrateAlgebraic [A] time = 3.34, size = 24, normalized size = 0.89

$$\frac{3\sqrt[3]{x^3 + x^2 - x - 1}}{2(x + 1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + x)*(-1 + x^2))^(-2/3), x]

[Out] (3*(-1 - x + x^2 + x^3)^(1/3))/(2*(1 + x))

fricas [A] time = 0.40, size = 20, normalized size = 0.74

$$\frac{3(x^3 + x^2 - x - 1)^{1/3}}{2(x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)*(x^2-1))^(2/3), x, algorithm="fricas")

[Out] 3/2*(x^3 + x^2 - x - 1)^(1/3)/(x + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x^2 - 1)(x + 1))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)*(x^2-1))^(2/3), x, algorithm="giac")

[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)

maple [A] time = 0.00, size = 20, normalized size = 0.74

$$\frac{3(x-1)(x+1)}{2((x+1)(x^2-1))^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x+1)*(x^2-1))^(2/3), x)

[Out] 3/2*(x-1)*(x+1)/((x+1)*(x^2-1))^(2/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x^2 - 1)(x + 1))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)*(x^2-1))^(2/3),x, algorithm="maxima")

[Out] integrate(((x^2 - 1)*(x + 1))^(-2/3), x)

mupad [B] time = 3.42, size = 20, normalized size = 0.74

$$\frac{3 \left((x^2 - 1) (x + 1) \right)^{1/3}}{2 (x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 - 1)*(x + 1))^(2/3),x)

[Out] (3*((x^2 - 1)*(x + 1))^(1/3))/(2*(x + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left((x + 1) (x^2 - 1) \right)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)*(x**2-1))**(2/3),x)

[Out] Integral(((x + 1)*(x**2 - 1))**(-2/3), x)

$$3.589 \quad \int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}} dx$$

Optimal. Leaf size=14

$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

Rubi [A] time = 0.15, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {6719, 449}

$$-\frac{2x}{\sqrt{x(x^2+1)}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]), x]

[Out] (-2*x)/Sqrt[x*(1 + x^2)]

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rule 6719

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] := Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^2}{(1+x^2)\sqrt{x(1+x^2)}} dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right) \int \frac{-1+x^2}{\sqrt{x(1+x^2)^{3/2}} dx}{\sqrt{x(1+x^2)}} \\ &= -\frac{2x}{\sqrt{x(1+x^2)}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 12, normalized size = 0.86

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]), x]

[Out] (-2*x)/Sqrt[x + x^3]

IntegrateAlgebraic [A] time = 0.12, size = 18, normalized size = 1.29

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x*(1 + x^2)]), x]

[Out] (-2*Sqrt[x + x^3])/(1 + x^2)

fricas [A] time = 0.48, size = 16, normalized size = 1.14

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2), x, algorithm="fricas")

[Out] -2*sqrt(x^3 + x)/(x^2 + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2), x, algorithm="giac")

[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)

maple [A] time = 0.01, size = 13, normalized size = 0.93

$$-\frac{2x}{\sqrt{(x^2+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2), x)

[Out] -2*x/(x*(x^2+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{(x^2+1)x(x^2+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x*(x^2+1))^(1/2), x, algorithm="maxima")

[Out] integrate((x^2 - 1)/(sqrt((x^2 + 1)*x)*(x^2 + 1)), x)

mupad [B] time = 3.38, size = 138, normalized size = 9.86

$$-\frac{2x}{\sqrt{x^3+x}} - \frac{\sqrt{1-x} \operatorname{li}\left(\frac{1}{2} + \frac{x \operatorname{li}}{2} E\left(\operatorname{asin}(\sqrt{1-x} \operatorname{li})\right)\right)}{\sqrt{x^3+x}} + \frac{\sqrt{1-x} \operatorname{li}\left(\frac{1}{2} + \frac{x \operatorname{li}}{2} F\left(\operatorname{asin}(\sqrt{1-x} \operatorname{li})\right)\right)}{\sqrt{x^3+x}} - \frac{\sqrt{1-x} \operatorname{li}\sqrt{1+x} \operatorname{li}\sqrt{-x} \operatorname{li} E\left(\operatorname{asin}(\sqrt{-x} \operatorname{li})\right) - 1}{\sqrt{x^3+x}} \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2 - 1)/((x*(x^2 + 1))^(1/2)*(x^2 + 1)),x)`

[Out] $((1 - x*1i)^{(1/2)}*((x*1i)/2 + 1/2)^{(1/2)}*\text{ellipticF}(\text{asin}((1 - x*1i)^{(1/2)}), 1/2)*(x*1i)^{(1/2)*2i}/(x + x^3)^{(1/2)} - ((1 - x*1i)^{(1/2)}*((x*1i)/2 + 1/2)^{(1/2)}*\text{ellipticE}(\text{asin}((1 - x*1i)^{(1/2)}), 1/2)*(x*1i)^{(1/2)*2i}/(x + x^3)^{(1/2)} - (2*x)/(x + x^3)^{(1/2)} - ((1 - x*1i)^{(1/2)}*(x*1i + 1)^{(1/2)}*(-x*1i)^{(1/2)}*\text{ellipticE}(\text{asin}((-x*1i)^{(1/2)}), -1)*1i)/(x + x^3)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2-1)/(x**2+1)/(x*(x**2+1))**(1/2),x)`

[Out] `Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)`

$$3.590 \quad \int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} dx$$

Optimal. Leaf size=12

$$-\frac{2x}{\sqrt{x^3+x}}$$

Rubi [A] time = 0.07, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2056, 449}

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]), x]

[Out] (-2*x)/Sqrt[x + x^3]

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rule 2056

Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m*FracPart[p]))*Distrib[1/x^m, P]^FracPart[p]], Int[u*x^(m*p)*Distrib[1/x^m, P]^p, x] /; FreeQ[p, x] && !IntegerQ[p] && SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]

Rubi steps

$$\begin{aligned} \int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} dx &= \frac{\left(\sqrt{x}\sqrt{1+x^2}\right) \int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}} dx}{\sqrt{x+x^3}} \\ &= -\frac{2x}{\sqrt{x+x^3}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.00

$$-\frac{2x}{\sqrt{x^3+x}}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]), x]

[Out] (-2*x)/Sqrt[x + x^3]

IntegrateAlgebraic [A] time = 0.11, size = 18, normalized size = 1.50

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x^2)/((1 + x^2)*Sqrt[x + x^3]),x]

[Out] (-2*Sqrt[x + x^3])/(1 + x^2)

fricas [A] time = 0.40, size = 16, normalized size = 1.33

$$-\frac{2\sqrt{x^3+x}}{x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="fricas")

[Out] -2*sqrt(x^3 + x)/(x^2 + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="giac")

[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)

maple [A] time = 0.00, size = 11, normalized size = 0.92

$$-\frac{2x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x)

[Out] -2*x/(x^3+x)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2-1}{\sqrt{x^3+x}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2-1)/(x^2+1)/(x^3+x)^(1/2),x, algorithm="maxima")

[Out] integrate((x^2 - 1)/(sqrt(x^3 + x)*(x^2 + 1)), x)

mupad [B] time = 0.05, size = 10, normalized size = 0.83

$$-\frac{2x}{\sqrt{x^3+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 - 1)/((x^2 + 1)*(x + x^3)^(1/2)),x)

[Out] -(2*x)/(x + x^3)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(x-1)(x+1)}{\sqrt{x(x^2+1)}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2-1)/(x**2+1)/(x**3+x)**(1/2),x)
```

```
[Out] Integral((x - 1)*(x + 1)/(sqrt(x*(x**2 + 1))*(x**2 + 1)), x)
```

$$3.591 \quad \int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx$$

Optimal. Leaf size=36

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Rubi [A] time = 0.14, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6718, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x(x^2+1)}}}{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2), x]

[Out] (2*x*Sqrt[(1 - x^2)^2/(x*(1 + x^2))])/(1 - x^2)

Rule 449

Int[((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.)*((c_.) + (d_.)*(x_.)^(n_.)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rule 6718

Int[(u_.)*((a_.)*(v_.)^(m_.)*(w_.)^(n_.)*(z_.)^(q_.))^(p_.), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m*w^n*z^q)^FracPart[p]]/(v^(m*FracPart[p])*w^(n*FracPart[p])*z^(q*FracPart[p])), Int[u*v^(m*p)*w^(n*p)*z^(p*q), x], x] /; FreeQ[{a, m, n, p, q}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x] && !FreeQ[z, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}}}{1+x^2} dx &= \frac{\left(\sqrt{x} \sqrt{\frac{(-1+x^2)^2}{x(1+x^2)}} \sqrt{1+x^2} \right) \int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}} dx}{-1+x^2} \\ &= \frac{2x\sqrt{\frac{(1-x^2)^2}{x(1+x^2)}}}{1-x^2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2), x]

[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)

IntegrateAlgebraic [A] time = 10.53, size = 29, normalized size = 0.81

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^2/(x*(1 + x^2))]/(1 + x^2), x]

[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)

fricas [A] time = 0.40, size = 30, normalized size = 0.83

$$-\frac{2x\sqrt{\frac{x^4-2x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="fricas")

[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="giac")

[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)

maple [A] time = 0.01, size = 34, normalized size = 0.94

$$-\frac{2\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}} x}{(x-1)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1), x)

[Out] -2*x/(x-1)/(x+1)*((x^2-1)^2/x/(x^2+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="maxima")

[Out] integrate(sqrt((x^2 - 1)^2/((x^2 + 1)*x))/(x^2 + 1), x)

mupad [B] time = 3.49, size = 48, normalized size = 1.33

$$\frac{(2x^3 + 2x) \sqrt{\frac{1}{x^2+1}} \sqrt{(x^2-1)^2} \sqrt{\frac{1}{x}}}{(x^2-1)(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2 - 1)^2/(x*(x^2 + 1)))^(1/2)/(x^2 + 1), x)

[Out] -((2*x + 2*x^3)*(1/(x^2 + 1))^(1/2)*((x^2 - 1)^2)^(1/2)*(1/x)^(1/2))/((x^2 - 1)*(x^2 + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x**2-1)**2/x/(x**2+1))**(1/2)/(x**2+1), x)

[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)

$$3.592 \quad \int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2} dx$$

Optimal. Leaf size=33

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Rubi [A] time = 0.19, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6719, 2056, 449}

$$\frac{2x\sqrt{\frac{(1-x^2)^2}{x^3+x}}}{1-x^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2), x]

[Out] (2*x*Sqrt[(1 - x^2)^2/(x + x^3)])/(1 - x^2)

Rule 449

Int[((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rule 2056

Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m*FracPart[p])*Distrib[1/x^m, P]^FracPart[p]), Int[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x] /; FreeQ[p, x] && !IntegerQ[p] && SumQ[P] && EveryQ[BinomialQ[#1, x] &, P] && !PolyQ[P, x, 2]

Rule 6719

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{(-1+x^2)^2}{x+x^3}}}{1+x^2} dx &= \frac{\left(\sqrt{\frac{(-1+x^2)^2}{x+x^3}} \sqrt{x+x^3}\right) \int \frac{-1+x^2}{(1+x^2)\sqrt{x+x^3}} dx}{-1+x^2} \\ &= \frac{\left(\sqrt{x} \sqrt{1+x^2} \sqrt{\frac{(-1+x^2)^2}{x+x^3}}\right) \int \frac{-1+x^2}{\sqrt{x}(1+x^2)^{3/2}} dx}{-1+x^2} \\ &= \frac{2x\sqrt{\frac{(1-x^2)^2}{x+x^3}}}{1-x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2), x]

[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)

IntegrateAlgebraic [A] time = 10.11, size = 29, normalized size = 0.88

$$-\frac{2x\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2-1}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x^2)^2/(x + x^3)]/(1 + x^2), x]

[Out] (-2*x*Sqrt[(-1 + x^2)^2/(x + x^3)])/(-1 + x^2)

fricas [A] time = 0.47, size = 30, normalized size = 0.91

$$-\frac{2x\sqrt{\frac{x^4-2x^2+1}{x^3+x}}}{x^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1), x, algorithm="fricas")

[Out] -2*x*sqrt((x^4 - 2*x^2 + 1)/(x^3 + x))/(x^2 - 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1), x, algorithm="giac")

[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)

maple [A] time = 0.01, size = 34, normalized size = 1.03

$$-\frac{2\sqrt{\frac{(x^2-1)^2}{(x^2+1)x}}x}{(x-1)(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1), x)

[Out] -2/(x-1)/(x+1)*((x^2-1)^2/(x^2+1)/x)^(1/2)*x

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x^2-1)^2}{x^3+x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^2-1)^2/(x^3+x))^(1/2)/(x^2+1),x, algorithm="maxima")

[Out] integrate(sqrt((x^2 - 1)^2/(x^3 + x))/(x^2 + 1), x)

mupad [B] time = 3.49, size = 43, normalized size = 1.30

$$\frac{\sqrt{\frac{1}{x^3+x}} (2x^3 + 2x) \sqrt{(x^2-1)^2}}{(x^2-1)(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^2 - 1)^2/(x + x^3))^(1/2)/(x^2 + 1),x)

[Out] -((1/(x + x^3))^(1/2)*(2*x + 2*x^3)*((x^2 - 1)^2)^(1/2))/((x^2 - 1)*(x^2 + 1))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x-1)^2(x+1)^2}{x^3+x}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x**2-1)**2/(x**3+x))**(1/2)/(x**2+1),x)

[Out] Integral(sqrt((x - 1)**2*(x + 1)**2/(x**3 + x))/(x**2 + 1), x)

$$3.593 \quad \int \frac{1}{\sqrt{a + \frac{b}{x^2}} \sqrt{c + dx^2}} dx$$

Optimal. Leaf size=70

$$\frac{\sqrt{ax^2 + b} \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{ax^2 + b}}{\sqrt{a} \sqrt{c + dx^2}}\right)}{\sqrt{a} \sqrt{d} x \sqrt{a + \frac{b}{x^2}}}$$

Rubi [A] time = 0.07, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {435, 444, 63, 217, 206}

$$\frac{\sqrt{ax^2 + b} \tanh^{-1}\left(\frac{\sqrt{d} \sqrt{ax^2 + b}}{\sqrt{a} \sqrt{c + dx^2}}\right)}{\sqrt{a} \sqrt{d} x \sqrt{a + \frac{b}{x^2}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]),x]

[Out] (Sqrt[b + a*x^2]*ArcTanh[(Sqrt[d]*Sqrt[b + a*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(Sqrt[a]*Sqrt[d]*Sqrt[a + b/x^2]*x)

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 435

Int[((c_) + (d_.)*(x_)^(mn_.))^(q_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[(x^(n*FracPart[q])*(c + d/x^n)^FracPart[q])/(d + c*x^n)^FracPart[q], Int[((a + b*x^n)^p*(d + c*x^n)^q)/x^(n*q), x], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && EqQ[mn, -n] && !IntegerQ[q] && !IntegerQ[p]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)*((c_) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a + \frac{b}{x^2}} \sqrt{c + dx^2}} dx &= \frac{\sqrt{b + ax^2} \int \frac{x}{\sqrt{b+ax^2} \sqrt{c+dx^2}} dx}{\sqrt{a + \frac{b}{x^2}} x} \\
&= \frac{\sqrt{b + ax^2} \operatorname{Subst} \left(\int \frac{1}{\sqrt{b+ax} \sqrt{c+dx}} dx, x, x^2 \right)}{2\sqrt{a + \frac{b}{x^2}} x} \\
&= \frac{\sqrt{b + ax^2} \operatorname{Subst} \left(\int \frac{1}{\sqrt{c - \frac{bd}{a} + \frac{dx^2}{a}}} dx, x, \sqrt{b + ax^2} \right)}{a\sqrt{a + \frac{b}{x^2}} x} \\
&= \frac{\sqrt{b + ax^2} \operatorname{Subst} \left(\int \frac{1}{1 - \frac{dx^2}{a}} dx, x, \frac{\sqrt{b+ax^2}}{\sqrt{c+dx^2}} \right)}{a\sqrt{a + \frac{b}{x^2}} x} \\
&= \frac{\sqrt{b + ax^2} \tanh^{-1} \left(\frac{\sqrt{d} \sqrt{b+ax^2}}{\sqrt{a} \sqrt{c+dx^2}} \right)}{\sqrt{a} \sqrt{d} \sqrt{a + \frac{b}{x^2}} x}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 105, normalized size = 1.50

$$\frac{x\sqrt{a + \frac{b}{x^2}} \sqrt{c + dx^2} \sinh^{-1} \left(\frac{\sqrt{d} \sqrt{ax^2+b}}{\sqrt{ac-bd}} \right)}{\sqrt{d} \sqrt{ax^2 + b} \sqrt{ac - bd} \sqrt{\frac{a(c+dx^2)}{ac-bd}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]), x]

[Out] (Sqrt[a + b/x^2]*x*Sqrt[c + d*x^2]*ArcSinh[(Sqrt[d]*Sqrt[b + a*x^2])/Sqrt[a*c - b*d]])/(Sqrt[d]*Sqrt[a*c - b*d]*Sqrt[b + a*x^2]*Sqrt[(a*(c + d*x^2))/(a*c - b*d)])

IntegrateAlgebraic [A] time = 0.73, size = 68, normalized size = 0.97

$$\frac{x\sqrt{a + \frac{b}{x^2}} \tanh^{-1} \left(\frac{\sqrt{a} \sqrt{c+dx^2}}{\sqrt{d} \sqrt{ax^2+b}} \right)}{\sqrt{a} \sqrt{d} \sqrt{ax^2 + b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[a + b/x^2]*Sqrt[c + d*x^2]), x]

[Out] (Sqrt[a + b/x^2]*x*ArcTanh[(Sqrt[a]*Sqrt[c + d*x^2])/((Sqrt[d]*Sqrt[b + a*x^2]))])/(Sqrt[a]*Sqrt[d]*Sqrt[b + a*x^2])

fricas [A] time = 0.46, size = 208, normalized size = 2.97

$$\left[\frac{\sqrt{ad} \log \left(8a^2d^2x^4 + a^2c^2 + 6abcd + b^2d^2 + 8(a^2cd + abd^2)x^2 + 4(2adx^3 + (ac + bd)x)\sqrt{dx^2 + c} \sqrt{ad} \sqrt{\frac{ax^2+b}{x^2}} \right)}{4ad}, -\frac{\sqrt{-ad} \arctan \left(\frac{(2adx^3 + (ac + bd)x)\sqrt{dx^2 + c} \sqrt{-ad} \sqrt{\frac{ax^2+b}{x^2}}}{2(a^2d^2x^4 + abcd + (a^2cd + abd^2)x^2)} \right)}{2ad} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*sqrt(a*d)*log(8*a^2*d^2*x^4 + a^2*c^2 + 6*a*b*c*d + b^2*d^2 + 8*(a^2*c*d + a*b*d^2)*x^2 + 4*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(a*d)*sqrt((a*x^2 + b)/x^2))/(a*d), -1/2*sqrt(-a*d)*arctan(1/2*(2*a*d*x^3 + (a*c + b*d)*x)*sqrt(d*x^2 + c)*sqrt(-a*d)*sqrt((a*x^2 + b)/x^2)/(a^2*d^2*x^4 + a*b*c*d + (a^2*c*d + a*b*d^2)*x^2))/(a*d)]
```

```
giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00
```

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(t_n
ostep)]Warning, integration of abs or sign assumes constant sign by interva
ls (correct if the argument is real):Check [sign(t_nostep)]Warning, choosin
g root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] a
t parameters values [-22,93,91]Warning, choosing root of [1,0,%%{-2,[1,0,2
]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [31,-21,
88]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%
%%{1,[2,0,4]%%}] at parameters values [76,-66,66]Warning, choosing root of
[1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at paramet
ers values [5,-23,79]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-
4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-88,9,6]Warning,
choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]
%%}] at parameters values [-69,-8,31]Warning, choosing root of [1,0,%%{-2
,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [
2,90.2102860468,-92]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-
4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-17,26.2119182013,
64]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%
%%{1,[2,0,4]%%}] at parameters values [89,29.4664394325,-51]Warning, choos
ing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}]
at parameters values [22,45.1969879479,76]Warning, choosing root of [1,0,%
%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters val
ues [-63,68.7323710029,13]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}
+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [43,85.975885
5961,12]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%}
,0,%%{1,[2,0,4]%%}] at parameters values [72,95.2558838762,11]Warning, c
hoosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%
}] at parameters values [-9,91.3720739307,81]Warning, choosing root of [1
,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters
values [15,23.9552401127,-50]Warning, choosing root of [1,0,%%{-2,[1,0,2]
%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [93,60.82
46789905,-18]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,
0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-88,63.3562821955,93]Warn
ing, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2
,0,4]%%}] at parameters values [-11,92.3620133325,-60]Warning, choosing ro
ot of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at pa
rameters values [-88,51.1034068516,72]Warning, choosing root of [1,0,%%{-2
,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [
-90,71.1075269701,48]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-
4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-47,53.5483433446
,-60]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0
,%%{1,[2,0,4]%%}] at parameters values [70,89.9395644632,-32]Warning, cho
```

osing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-74,8.0407431256,-16]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [85,16.2654368887,-14]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-77,83.0705981795,31]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [76,41.5291932677,-48]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-47,25.6140411007,23]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [32,76.6146142203,-62]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [66,99.6590219955,-30]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-50,17.713730142,-44]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [45,14.5515509131,62]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [44,88.8429926978,11]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-19,42.8964279308,-92]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [67,19.2648137459,-22]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [9,87.7979063494,94]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [95,98.9582961812,-92]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [52,79.2538507222,22]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-43,16.7638230952,21]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [71,61.8959259251,-83]Warning, choosing root of [1,0,%%{-2,[1,0,2]%%}+%%{-4,[0,1,0]%%},0,%%{1,[2,0,4]%%}] at parameters values [-30,23.6526960679,40]sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument ValueWarning, choosing root of [1,0,%%{-2,[1,0,0]%%}+%%{-2,[0,1,2]%%}+%%{-2,[0,1,0]%%},0,%%{1,[2,0,0]%%}+%%{2,[1,1,2]%%}+%%{-2,[1,1,0]%%}+%%{1,[0,2,4]%%}+%%{-2,[0,2,2]%%}+%%{1,[0,2,0]%%}] at parameters values [0,56.2346625305,0]Warning, choosing root of [1,0,%%{-2,[1,0,0]%%}+%%{-2,[0,1,2]%%}+%%{-2,[0,1,0]%%},0,%%{1,[2,0,0]%%}+%%{2,[1,1,2]%%}+%%{-2,[1,1,0]%%}+%%{1,[0,2,4]%%}+%%{-2,[0,2,2]%%}+%%{1,[0,2,0]%%}] at parameters values [-43,9.82222589385,18]Warning, choosing root of [1,0,%%{-2,[1,0,0]%%}+%%{-2,[0,1,2]%%}+%%{-2,[0,1,0]%%},0,%%{1,[2,0,0]%%}+%%{2,[1,1,2]%%}+%%{-2,[1,1,0]%%}+%%{1,[0,2,4]%%}+%%{-2,[0,2,2]%%}+%%{1,[0,2,0]%%}] at parameters values [95,6.65142845921,-60]Warning, choosing root of [1,0,%%{-2,[1,0,0]%%}+%%{-2,[0,1,2]%%}+%%{-2,[0,1,0]%%},0,%%{1,[2,0,0]%%}+%%{2,[1,1,2]%%}+%%{-2,[1,1,0]%%}+%%{1,[0,2,4]%%}+%%{-2,[0,2,2]%%}+%%{1,[0,2,0]%%}] at parameters values [65,77.8863785956,-16]Warning, choosing root of [1,0,%%{-2,[1,0,0]%%}+%%{-2,[0,1,2]%%}+%%{-2,[0,1,0]%%},0,%%{1,[2,0,0]%%}+%%{2,[1,1,2]%%}+%%{-2,[1,1,0]%%}+%%{1,[0,2,4]%%}+%%{-2,[0,2,2]%%}+%%{1,[0,2,0]%%}] at parameters values [-81,85.3390313801,-19]gen.cc:simplify/tmp.type!=_EXT Error: Bad Argument ValueEvaluation time: 21.47Done

maple [B] time = 0.05, size = 117, normalized size = 1.67

$$\frac{(ax^2 + b)\sqrt{dx^2 + c} \ln\left(\frac{2adx^2 + ac + bd + 2\sqrt{ad}x^4 + acx^2 + bdx^2 + bc}{2\sqrt{ad}}\right)}{2\sqrt{\frac{ax^2 + b}{x^2}} \sqrt{ad} \sqrt{adx^4 + acx^2 + bdx^2 + bc} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x)

[Out] 1/2/((a*x^2+b)/x^2)^(1/2)/x*(a*x^2+b)*ln(1/2*(2*a*d*x^2+2*(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)*(a*d)^(1/2)+a*c+b*d)/(a*d)^(1/2))*(d*x^2+c)^(1/2)/(a*d)^(1/2)/(a*d*x^4+a*c*x^2+b*d*x^2+b*c)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{dx^2 + c} \sqrt{a + \frac{b}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x^2)^(1/2)/(d*x^2+c)^(1/2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(d*x^2 + c)*sqrt(a + b/x^2)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} \sqrt{dx^2 + c}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)),x)

[Out] int(1/((a + b/x^2)^(1/2)*(c + d*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \frac{b}{x^2}} \sqrt{c + dx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+b/x**2)**(1/2)/(d*x**2+c)**(1/2),x)

[Out] Integral(1/(sqrt(a + b/x**2)*sqrt(c + d*x**2)), x)

$$3.594 \quad \int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)} dx$$

Optimal. Leaf size=83

$$\frac{2\sqrt{x^4-2x^2} \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}} - \frac{\sqrt{x^4-2x^2} \tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Rubi [A] time = 0.15, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {2056, 571, 83, 63, 203}

$$\frac{2\sqrt{x^4-2x^2} \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}} - \frac{\sqrt{x^4-2x^2} \tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)), x]

[Out] (2*Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) - (Sqrt[-2*x^2 + x^4]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 83

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 571

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]

Rule 2056

Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m*FracPart[p]))*Distrib[1/x^m, P]^FracPart[p]], Int[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x] /; FreeQ[p, x] && !IntegerQ[p] && SumQ[P] && EveryQ[BinomialQ[#1, x] &, P] && !PolyQ[P, x, 2]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{-2x^2 + x^4}}{(-1 + x^2)(2 + x^2)} dx &= \frac{\sqrt{-2x^2 + x^4} \int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)} dx}{x\sqrt{-2 + x^2}} \\
&= \frac{\sqrt{-2x^2 + x^4} \operatorname{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)} dx, x, x^2\right)}{2x\sqrt{-2 + x^2}} \\
&= -\frac{\sqrt{-2x^2 + x^4} \operatorname{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)} dx, x, x^2\right)}{6x\sqrt{-2 + x^2}} + \frac{(2\sqrt{-2x^2 + x^4}) \operatorname{Subst}\left(\int \frac{1}{\sqrt{-2+x}(2+x)} dx, x, x^2\right)}{3x\sqrt{-2 + x^2}} \\
&= -\frac{\sqrt{-2x^2 + x^4} \operatorname{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{-2 + x^2}\right)}{3x\sqrt{-2 + x^2}} + \frac{(4\sqrt{-2x^2 + x^4}) \operatorname{Subst}\left(\int \frac{1}{4+x^2} dx, x, \sqrt{-2 + x^2}\right)}{3x\sqrt{-2 + x^2}} \\
&= \frac{2\sqrt{-2x^2 + x^4} \tan^{-1}\left(\frac{1}{2}\sqrt{-2 + x^2}\right)}{3x\sqrt{-2 + x^2}} - \frac{\sqrt{-2x^2 + x^4} \tan^{-1}\left(\sqrt{-2 + x^2}\right)}{3x\sqrt{-2 + x^2}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 52, normalized size = 0.63

$$\frac{x\sqrt{x^2 - 2} \left(2 \tan^{-1}\left(\frac{2}{\sqrt{x^2 - 2}}\right) + \tan^{-1}\left(\sqrt{x^2 - 2}\right)\right)}{3\sqrt{x^2(x^2 - 2)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)), x]

[Out] -1/3*(x*Sqrt[-2 + x^2]*(2*ArcTan[2/Sqrt[-2 + x^2]] + ArcTan[Sqrt[-2 + x^2]]))/Sqrt[x^2*(-2 + x^2)]

IntegrateAlgebraic [A] time = 0.09, size = 42, normalized size = 0.51

$$\frac{1}{3} \tan^{-1}\left(\frac{x}{\sqrt{x^4 - 2x^2}}\right) - \frac{2}{3} \tan^{-1}\left(\frac{2x}{\sqrt{x^4 - 2x^2}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-2*x^2 + x^4]/((-1 + x^2)*(2 + x^2)), x]

[Out] ArcTan[x/Sqrt[-2*x^2 + x^4]]/3 - (2*ArcTan[(2*x)/Sqrt[-2*x^2 + x^4]])/3

fricas [A] time = 0.43, size = 38, normalized size = 0.46

$$-\frac{1}{3} \arctan\left(\frac{\sqrt{x^4 - 2x^2}}{x}\right) + \frac{2}{3} \arctan\left(\frac{\sqrt{x^4 - 2x^2}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2), x, algorithm="fricas")

[Out] -1/3*arctan(sqrt(x^4 - 2*x^2)/x) + 2/3*arctan(1/2*sqrt(x^4 - 2*x^2)/x)

giac [C] time = 0.36, size = 46, normalized size = 0.55

$$\frac{1}{3} \left(\arctan\left(i\sqrt{2}\right) - 2 \arctan\left(\frac{1}{2}i\sqrt{2}\right) \right) \operatorname{sgn}(x) + \frac{2}{3} \arctan\left(\frac{1}{2}\sqrt{x^2 - 2}\right) \operatorname{sgn}(x) - \frac{1}{3} \arctan\left(\sqrt{x^2 - 2}\right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="giac")

[Out] 1/3*(arctan(I*sqrt(2)) - 2*arctan(1/2*I*sqrt(2)))*sgn(x) + 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x)

maple [A] time = 0.03, size = 63, normalized size = 0.76

$$\frac{\sqrt{x^4 - 2x^2} \left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right) - \arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right) - 4 \arctan\left(\frac{\sqrt{x^2-2}}{2}\right) \right)}{6\sqrt{x^2 - 2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x)

[Out] -1/6*(x^4-2*x^2)^(1/2)*(arctan((x-2)/(x^2-2)^(1/2))-arctan((x+2)/(x^2-2)^(1/2)))-4*arctan(1/2*(x^2-2)^(1/2))/x/(x^2-2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^4 - 2x^2}}{(x^2 + 2)(x^2 - 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^4-2*x^2)^(1/2)/(x^2-1)/(x^2+2),x, algorithm="maxima")

[Out] integrate(sqrt(x^4 - 2*x^2)/((x^2 + 2)*(x^2 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4 - 2x^2}}{(x^2 - 1)(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)),x)

[Out] int((x^4 - 2*x^2)^(1/2)/((x^2 - 1)*(x^2 + 2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x^2(x^2 - 2)}}{(x - 1)(x + 1)(x^2 + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**4-2*x**2)**(1/2)/(x**2-1)/(x**2+2),x)

[Out] Integral(sqrt(x**2*(x**2 - 2))/((x - 1)*(x + 1)*(x**2 + 2)), x)

$$3.595 \quad \int \frac{\sqrt{1 - \frac{1}{(-1+x^2)^2}}}{2-x^2} dx$$

Optimal. Leaf size=47

$$\frac{(1-x^2) \sqrt{1 - \frac{1}{(1-x^2)^2}} \tan^{-1}(\sqrt{x^2-2})}{x\sqrt{x^2-2}}$$

Rubi [A] time = 0.46, antiderivative size = 73, normalized size of antiderivative = 1.55, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {6722, 6725, 1990, 1146, 21, 261, 444, 50, 63, 203}

$$\frac{(1-x^2) \sqrt{x^4-2x^2} \sqrt{1 - \frac{1}{(1-x^2)^2}} \tan^{-1}(\sqrt{x^2-2})}{x\sqrt{x^2-2} \sqrt{(x^2-1)^2-1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2), x]

[Out] ((1 - x^2)*Sqrt[-2*x^2 + x^4]*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x^2]*Sqrt[-1 + (-1 + x^2)^2])

Rule 21

Int[(u_)*((a_) + (b_)*(v_))^(m_)*((c_) + (d_)*(v_))^(n_), x_Symbol] := Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[b*c - a*d, 0] && IntegerQ[m] && (!IntegerQ[n] || SimplerQ[c + d*x, a + b*x])

Rule 50

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&

NeQ[p, -1]

Rule 444

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n + 1, 0]

Rule 1146

Int[((d_) + (e_.)*(x_)^2)^(q_.)*((b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] := Dist[(b*x^2 + c*x^4)^FracPart[p]/(x^(2*FracPart[p]))*(b + c*x^2)^FracPart[p], Int[x^(2*p)*(d + e*x^2)^q*(b + c*x^2)^p, x], x] /; FreeQ[{b, c, d, e, p, q}, x] && !IntegerQ[p]

Rule 1990

Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] := Int[ExpandToSum[u, x]^q*ExpandToSum[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] && !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])

Rule 6722

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[(a + b*v^n)^FracPart[p]/(v^(n*FracPart[p]))*(b + a/v^n)^FracPart[p], Int[u*v^(n*p)*(b + a/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x] && !LinearQ[v, x]

Rule 6725

Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := With[{v = RationalFunctionExpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1 - \frac{1}{(-1+x^2)^2}}}{2-x^2} dx &= \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{\sqrt{-1+(-1+x^2)^2}}{(2-x^2)(-1+x^2)} dx}{\sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \left(\frac{\sqrt{-1+(-1+x^2)^2}}{2-x^2} + \frac{\sqrt{-1+(-1+x^2)^2}}{-1+x^2} \right) dx}{\sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{\sqrt{-1+(-1+x^2)^2}}{2-x^2} dx}{\sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{\sqrt{-1+(-1+x^2)^2}}{-1+x^2} dx}{\sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{\sqrt{-2x^2+x^4}}{2-x^2} dx}{\sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2) \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{\sqrt{-2x^2+x^4}}{-1+x^2} dx}{\sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{x\sqrt{-2+x^2}}{2-x^2} dx}{x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}} + \frac{\left((-1+x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \int \frac{x\sqrt{-2+x^2}}{-1+x^2} dx}{x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \text{Subst} \left(\int \frac{\sqrt{-2+x}}{-1+x} dx, x, x^2 \right)}{2x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}} - \frac{\left((-1+x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \text{Subst} \left(\int \frac{1}{\sqrt{-2+x}(-1+x)} dx, x, x^2 \right)}{2x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}} \\
&= \frac{\left((-1+x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(-1+x^2)^2}} \right) \text{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{-2+x^2} \right)}{x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}} \\
&= \frac{(1-x^2) \sqrt{-2x^2+x^4} \sqrt{1 - \frac{1}{(1-x^2)^2}} \tan^{-1} \left(\sqrt{-2+x^2} \right)}{x\sqrt{-2+x^2} \sqrt{-1+(-1+x^2)^2}}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 91, normalized size = 1.94

$$\frac{1}{2} \tan^{-1} \left(\frac{(x-1)(x+1)(x+2) \sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x(x^2-2)} \right) - \frac{1}{2} \tan^{-1} \left(\frac{(x-2)(x-1)(x+1) \sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}}{x(x^2-2)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2), x]

[Out] $-1/2*\text{ArcTan}[\frac{(-2+x)*(-1+x)*(1+x)*\text{Sqrt}[(x^2*(-2+x^2))/(-1+x^2)^2]}{(x*(-2+x^2))}] + \text{ArcTan}[\frac{(-1+x)*(1+x)*(2+x)*\text{Sqrt}[(x^2*(-2+x^2))/(-1+x^2)^2]}{(x*(-2+x^2))}]/2$

IntegrateAlgebraic [A] time = 5.57, size = 44, normalized size = 0.94

$$\frac{(x^2 - 1) \sqrt{1 - \frac{1}{(x^2 - 1)^2}} \tan^{-1}(\sqrt{x^2 - 2})}{x \sqrt{x^2 - 2}}$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2), x]`

[Out] $-\frac{((-1+x^2)*\text{Sqrt}[1 - (-1+x^2)^{-2}]*\text{ArcTan}[\text{Sqrt}[-2+x^2]])}{(x*\text{Sqrt}[-2+x^2])}$

fricas [A] time = 0.40, size = 36, normalized size = 0.77

$$-\arctan\left(\frac{(x^2 - 1) \sqrt{\frac{x^4 - 2x^2}{x^4 - 2x^2 + 1}}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2), x, algorithm="fricas")`

[Out] $-\arctan((x^2 - 1)*\text{sqrt}((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)$

giac [A] time = 0.34, size = 18, normalized size = 0.38

$$-\arctan(\sqrt{x^2 - 2}) \text{sgn}(x^3 - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2), x, algorithm="giac")`

[Out] $-\arctan(\text{sqrt}(x^2 - 2))*\text{sgn}(x^3 - x)$

maple [A] time = 0.03, size = 63, normalized size = 1.34

$$\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}} (x^2 - 1) \left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right) - \arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right) \right)}{2\sqrt{x^2 - 2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2), x)`

[Out] $-1/2*(x^2*(x^2-2)/(x^2-1)^2)^{(1/2)}*(x^2-1)*(\arctan((x-2)/(x^2-2)^{(1/2)})-\arctan((x+2)/(x^2-2)^{(1/2)}))/x/(x^2-2)^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-\frac{1}{(x^2-1)^2} + 1}}{x^2 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2), x, algorithm="maxima")`

[Out] -integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int -\frac{\sqrt{1 - \frac{1}{(x^2-1)^2}}}{x^2 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)

[Out] int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1} - \frac{2x^2}{x^4-2x^2+1}}}{x^2 - 2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2), x)

[Out] -Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**2 - 2), x)

$$3.596 \quad \int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2} dx$$

Optimal. Leaf size=123

$$\frac{(1-x^2) \sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}} - \frac{2(1-x^2) \sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Rubi [A] time = 0.29, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {6719, 2056, 571, 83, 63, 203}

$$\frac{(1-x^2) \sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\sqrt{x^2-2}\right)}{3x\sqrt{x^2-2}} - \frac{2(1-x^2) \sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2), x]

[Out] (-2*(1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]/2])/(3*x*Sqrt[-2 + x^2]) + ((1 - x^2)*Sqrt[-((2*x^2 - x^4)/(1 - x^2)^2)]*ArcTan[Sqrt[-2 + x^2]])/(3*x*Sqrt[-2 + x^2])

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 83

Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(a + b*x), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[(e + f*x)^(p - 1)/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 571

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, f, m, n, p, q, r}, x] && EqQ[m - n + 1, 0]

Rule 2056

Int[(u_.)*(P_)^(p_.), x_Symbol] :> With[{m = MinimumMonomialExponent[P, x]}, Dist[P^FracPart[p]/(x^(m*FracPart[p]))*Distrib[1/x^m, P]^FracPart[p]], Int[u*x^(m*p)*Distrib[1/x^m, P]^p, x], x] /; FreeQ[p, x] && !IntegerQ[p] &&

SumQ[P] && EveryQ[BinomialQ[#1, x] & , P] && !PolyQ[P, x, 2]

Rule 6719

Int[(u_.)*((a_.)*(v_)^(m_.)*(w_)^(n_.))^p_], x_Symbol] := Dist[(a^IntPart[p]*(a*v^m*w^n)^FracPart[p])/(v^(m*FracPart[p])*w^(n*FracPart[p])), Int[u*v^(m*p)*w^(n*p), x], x] /; FreeQ[{a, m, n, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !FreeQ[w, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}}{2+x^2} dx &= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \int \frac{\sqrt{-2x^2+x^4}}{(-1+x^2)(2+x^2)} dx}{\sqrt{-2x^2+x^4}} \\ &= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \int \frac{x\sqrt{-2+x^2}}{(-1+x^2)(2+x^2)} dx}{x\sqrt{-2+x^2}} \\ &= \frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \text{Subst}\left(\int \frac{\sqrt{-2+x}}{(-1+x)(2+x)} dx, x, x^2\right)}{2x\sqrt{-2+x^2}} \\ &= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \text{Subst}\left(\int \frac{1}{\sqrt{-2+x}(-1+x)} dx, x, x^2\right)}{6x\sqrt{-2+x^2}} + \frac{\left(2(-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \text{Subst}\left(\int \frac{1}{1+x} dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} \\ &= -\frac{\left((-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \text{Subst}\left(\int \frac{1}{1+x} dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{\left(4(-1+x^2)\sqrt{\frac{-2x^2+x^4}{(-1+x^2)^2}}\right) \text{Subst}\left(\int \frac{1}{1+x} dx, x, \sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} \\ &= -\frac{2(1-x^2)\sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{\frac{-2x^2-x^4}{(1-x^2)^2}} \tan^{-1}\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} \end{aligned}$$

Mathematica [A] time = 0.03, size = 70, normalized size = 0.57

$$\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}} (x^2-1) \left(2 \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right) - \tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{3x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2), x]

[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*(2*ArcTan[Sqrt[-2 + x^2]/2] - ArcTan[Sqrt[-2 + x^2]]))/(3*x*Sqrt[-2 + x^2])

IntegrateAlgebraic [A] time = 11.12, size = 71, normalized size = 0.58

$$\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}} (x^2-1) \left(\frac{2}{3} \tan^{-1}\left(\frac{\sqrt{x^2-2}}{2}\right) - \frac{1}{3} \tan^{-1}\left(\sqrt{x^2-2}\right)\right)}{x\sqrt{x^2-2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-2*x^2 + x^4)/(-1 + x^2)^2]/(2 + x^2), x]

[Out] (Sqrt[(x^2*(-2 + x^2))/(-1 + x^2)^2]*(-1 + x^2)*((2*ArcTan[Sqrt[-2 + x^2]/2])/3 - ArcTan[Sqrt[-2 + x^2]]/3))/(x*Sqrt[-2 + x^2])

fricas [A] time = 0.41, size = 74, normalized size = 0.60

$$-\frac{1}{3} \arctan\left(\frac{(x^2 - 1)\sqrt{\frac{x^4 - 2x^2}{x^4 - 2x^2 + 1}}}{x}\right) + \frac{2}{3} \arctan\left(\frac{(x^2 - 1)\sqrt{\frac{x^4 - 2x^2}{x^4 - 2x^2 + 1}}}{2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2), x, algorithm="fricas")

[Out] -1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arctan(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)

giac [A] time = 0.41, size = 39, normalized size = 0.32

$$\frac{2}{3} \arctan\left(\frac{1}{2} \sqrt{x^2 - 2}\right) \operatorname{sgn}(x^3 - x) - \frac{1}{3} \arctan\left(\sqrt{x^2 - 2}\right) \operatorname{sgn}(x^3 - x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2), x, algorithm="giac")

[Out] 2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn(x^3 - x)

maple [A] time = 0.03, size = 75, normalized size = 0.61

$$\frac{\sqrt{\frac{(x^2-2)x^2}{(x^2-1)^2}} (x^2 - 1) \left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right) - \arctan\left(\frac{x+2}{\sqrt{x^2-2}}\right) - 4 \arctan\left(\frac{\sqrt{x^2-2}}{2}\right) \right)}{6\sqrt{x^2-2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2), x)

[Out] -1/6*((x^2-2)/(x^2-1)^2*x^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-arctan((x+2)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{x^4-2x^2}{(x^2-1)^2}}}{x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x^4-2*x^2)/(x^2-1)^2)^(1/2)/(x^2+2), x, algorithm="maxima")

[Out] integrate(sqrt((x^4 - 2*x^2)/(x^2 - 1)^2)/(x^2 + 2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\frac{2x^2-x^4}{(x^2-1)^2}}}{x^2+2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

```
[Out] int((-2*x^2 - x^4)/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((x**4-2*x**2)/(x**2-1)**2)**(1/2)/(x**2+2), x)
```

```
[Out] Timed out
```

$$3.597 \quad \int \left(1 + \frac{2x}{1+x^2}\right)^{5/2} dx$$

Optimal. Leaf size=133

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3(x^2+1)} - \frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1) - \frac{(3x+4)(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1} + \frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1} \operatorname{arcsinh}\left(\sqrt{\frac{2x}{x^2+1}+1}\right)}{x+1}$$

Rubi [A] time = 0.07, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6723, 970, 739, 819, 780, 215}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)^3}{3(x^2+1)} - \frac{4}{3}(1-2x)\sqrt{\frac{2x}{x^2+1}+1}(x+1) - \frac{(3x+4)(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1} + \frac{5\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1} \sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + (2*x)/(1 + x^2))^(5/2), x]

[Out] (-4*(1 - 2*x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]/3 - ((1 - x)*(1 + x)^3*Sqrt[1 + (2*x)/(1 + x^2)]/(3*(1 + x^2)) - ((4 + 3*x)*(1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x)))/(1 + x) + (5*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/ (1 + x)

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 739

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a*e - c*d*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/((p + 1)*(-2*a*c)), Int[(d + e*x)^(m - 2)*Simp[a*e^2*(m - 1) - c*d^2*(2*p + 3) - d*c*e*(m + 2*p + 2)*x, x]*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 819

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*(a*(e*f + d*g) - (c*d*f - a*e*g)*x))/(2*a*c*(p + 1)), x] - Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && (EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) || !ILtQ[m + 2*p + 3, 0])

Rule 970

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_
Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x
^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a,
b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]
```

Rule 6723

```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] := Dist[(a +
b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), In
t[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !Intege
rQ[p] && ILtQ[n, 0] && BinomialQ[v, x]
```

Rubi steps

$$\begin{aligned} \int \left(1 + \frac{2x}{1+x^2}\right)^{5/2} dx &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(1+2x+x^2)^{5/2}}{(1+x^2)^{5/2}} dx}{\sqrt{1+2x+x^2}} \\ &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(2+2x)^5}{(1+x^2)^{5/2}} dx}{16(2+2x)} \\ &= -\frac{(1-x)(1+x)^3 \sqrt{1 + \frac{2x}{1+x^2}}}{3(1+x^2)} + \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(24-8x)(2+2x)^3}{(1+x^2)^{3/2}} dx}{48(2+2x)} \\ &= -\frac{4}{3}(1-2x)(1+x) \sqrt{1 + \frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3 \sqrt{1 + \frac{2x}{1+x^2}}}{3(1+x^2)} + \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(4+3x)(1+x^2) \sqrt{1+x^2}}{1+x} dx}{48(2+2x)} \\ &= -\frac{4}{3}(1-2x)(1+x) \sqrt{1 + \frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3 \sqrt{1 + \frac{2x}{1+x^2}}}{3(1+x^2)} - \frac{(4+3x)(1+x^2) \sqrt{1+x^2}}{1+x} \\ &= -\frac{4}{3}(1-2x)(1+x) \sqrt{1 + \frac{2x}{1+x^2}} - \frac{(1-x)(1+x)^3 \sqrt{1 + \frac{2x}{1+x^2}}}{3(1+x^2)} - \frac{(4+3x)(1+x^2) \sqrt{1+x^2}}{1+x} \end{aligned}$$

Mathematica [A] time = 0.10, size = 64, normalized size = 0.48

$$\frac{(x+1) \left(3x^4 - 8x^3 - 18x^2 + 15(x^2+1)^{3/2} \sinh^{-1}(x) - 12x - 17\right)}{3\sqrt{\frac{(x+1)^2}{x^2+1}} (x^2+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + (2*x)/(1 + x^2))^(5/2), x]
```

```
[Out] ((1 + x)*(-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4 + 15*(1 + x^2)^(3/2)*ArcSinh[x]))/(3*Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2)^2)
```

IntegrateAlgebraic [A] time = 6.91, size = 80, normalized size = 0.60

$$\frac{(x+1) \left(\frac{3x^4 - 8x^3 - 18x^2 - 12x - 17}{3(x^2+1)^{3/2}} - 5 \log\left(\sqrt{x^2+1} - x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(5/2), x]

[Out] ((1 + x)*((-17 - 12*x - 18*x^2 - 8*x^3 + 3*x^4)/(3*(1 + x^2)^(3/2)) - 5*Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

fricas [A] time = 0.40, size = 117, normalized size = 0.88

$$\frac{8x^3 + 8x^2 + 15(x^3 + x^2 + x + 1) \log\left(-\frac{x^2 - (x^2 + 1)\sqrt{\frac{x^2 + 2x + 1}{x^2 + 1}} + x}{x + 1}\right) - (3x^4 - 8x^3 - 18x^2 - 12x - 17)\sqrt{\frac{x^2 + 2x + 1}{x^2 + 1}} + 8x + 8}{3(x^3 + x^2 + x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(5/2), x, algorithm="fricas")

[Out] -1/3*(8*x^3 + 8*x^2 + 15*(x^3 + x^2 + x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (3*x^4 - 8*x^3 - 18*x^2 - 12*x - 17)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 8*x + 8)/(x^3 + x^2 + x + 1)

giac [A] time = 0.47, size = 86, normalized size = 0.65

$$(\sqrt{2} + 5 \log(\sqrt{2} + 1))\operatorname{sgn}(x + 1) - 5 \log(-x + \sqrt{x^2 + 1})\operatorname{sgn}(x + 1) + \frac{((3x\operatorname{sgn}(x + 1) - 8\operatorname{sgn}(x + 1))x - 18\operatorname{sgn}(x + 1))x - 12\operatorname{sgn}(x + 1))x - 17\operatorname{sgn}(x + 1)}{3(x^2 + 1)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(5/2), x, algorithm="giac")

[Out] (sqrt(2) + 5*log(sqrt(2) + 1))*sgn(x + 1) - 5*log(-x + sqrt(x^2 + 1))*sgn(x + 1) + 1/3*(((3*x*sgn(x + 1) - 8*sgn(x + 1))*x - 18*sgn(x + 1))*x - 12*sgn(x + 1))*x - 17*sgn(x + 1))/(x^2 + 1)^(3/2)

maple [A] time = 0.02, size = 62, normalized size = 0.47

$$\frac{\left(\frac{x^2 + 2x + 1}{x^2 + 1}\right)^{\frac{5}{2}} (x^2 + 1) \left(3x^4 - 8x^3 - 18x^2 - 12x + 15(x^2 + 1)^{\frac{3}{2}} \operatorname{arcsinh}(x) - 17\right)}{3(x + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2/(x^2+1)*x)^(5/2), x)

[Out] 1/3*((x^2+2*x+1)/(x^2+1))^(5/2)/(x+1)^5*(x^2+1)*(15*arcsinh(x)*(x^2+1)^(3/2)+3*x^4-8*x^3-18*x^2-12*x-17)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2 + 1} + 1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(5/2), x, algorithm="maxima")

[Out] integrate((2*x/(x^2 + 1) + 1)^(5/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(\frac{2x}{x^2 + 1} + 1\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

```
[Out] int(((2*x)/(x^2 + 1) + 1)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2 + 1} + 1 \right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+2*x/(x**2+1))**(5/2), x)
```

```
[Out] Integral((2*x/(x**2 + 1) + 1)**(5/2), x)
```

$$3.598 \quad \int \left(1 + \frac{2x}{1+x^2}\right)^{3/2} dx$$

Optimal. Leaf size=90

$$-\left((1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1)\right) - \frac{x(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1} + \frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {6723, 970, 739, 517, 388, 215}

$$-(1-x)\sqrt{\frac{2x}{x^2+1}+1}(x+1) - \frac{x(x^2+1)\sqrt{\frac{2x}{x^2+1}+1}}{x+1} + \frac{3\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}\sinh^{-1}(x)}{x+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + (2*x)/(1 + x^2))^(3/2), x]

[Out] -((1 - x)*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (x*(1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (3*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)

Rule 215

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 388

Int[((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(d*x*(a + b*x^n)^(p+1)/(b*(n*(p+1)+1)), x] - Dist[(a*d - b*c*(n*(p+1)+1))/(b*(n*(p+1)+1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p+1)+1, 0]

Rule 517

Int[(u_)*((c_) + (d_)*(x_)^(n_))^(q_)*((a1_) + (b1_)*(x_)^(non2_))^(p_)*((a2_) + (b2_)*(x_)^(non2_))^(p_), x_Symbol] := Int[u*(a1*a2 + b1*b2*x^n)^p*(c + d*x^n)^q, x] /; FreeQ[{a1, b1, a2, b2, c, d, n, p, q}, x] && EqQ[non2, n/2] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]))

Rule 739

Int[((d_) + (e_)*(x_))^(m_)*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m-1)*(a*e - c*d*x)*(a + c*x^2)^(p+1))/(2*a*c*(p+1)), x] + Dist[1/((p+1)*(-2*a*c)), Int[(d + e*x)^(m-2)*Simp[a*e^2*(m-1) - c*d^2*(2*p+3) - d*c*e*(m+2*p+2)*x, x]*(a + c*x^2)^(p+1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 970

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 6723

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_.)*(x_)^(m_.))^(p_), x_Symbol] := Dist[(a + b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x]

Rubi steps

$$\begin{aligned}
 \int \left(1 + \frac{2x}{1+x^2}\right)^{3/2} dx &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(1+2x+x^2)^{3/2}}{(1+x^2)^{3/2}} dx}{\sqrt{1+2x+x^2}} \\
 &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(2+2x)^3}{(1+x^2)^{3/2}} dx}{4(2+2x)} \\
 &= -\left((1-x)(1+x)\sqrt{1 + \frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{(8-8x)(2+2x)}{\sqrt{1+x^2}} dx}{4(2+2x)} \\
 &= -\left((1-x)(1+x)\sqrt{1 + \frac{2x}{1+x^2}}\right) + \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{16-16x^2}{\sqrt{1+x^2}} dx}{4(2+2x)} \\
 &= -\left((1-x)(1+x)\sqrt{1 + \frac{2x}{1+x^2}}\right) - \frac{x(1+x^2)\sqrt{1 + \frac{2x}{1+x^2}}}{1+x} + \frac{\left(6\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{1}{\sqrt{1+x^2}} dx}{2+2x} \\
 &= -\left((1-x)(1+x)\sqrt{1 + \frac{2x}{1+x^2}}\right) - \frac{x(1+x^2)\sqrt{1 + \frac{2x}{1+x^2}}}{1+x} + \frac{3\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}} \sinh^{-1}(x)}{1+x}
 \end{aligned}$$

Mathematica [A] time = 0.04, size = 44, normalized size = 0.49

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}} \left(x^2 + 3\sqrt{x^2+1} \sinh^{-1}(x) - 2x - 1\right)}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (2*x)/(1 + x^2))^(3/2), x]

[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(-1 - 2*x + x^2 + 3*Sqrt[1 + x^2]*ArcSinh[x]))/(1 + x)

IntegrateAlgebraic [A] time = 6.40, size = 65, normalized size = 0.72

$$\frac{(x+1) \left(\frac{x^2-2x-1}{\sqrt{x^2+1}} - 3 \log\left(\sqrt{x^2+1} - x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(3/2), x]

[Out] ((1 + x)*((-1 - 2*x + x^2)/Sqrt[1 + x^2] - 3*Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

fricas [A] time = 0.40, size = 83, normalized size = 0.92

$$\frac{3(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)-\left(x^2-2x-1\right)\sqrt{\frac{x^2+2x+1}{x^2+1}}+2x+2}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="fricas")

[Out] -(3*(x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (x^2 - 2*x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 2*x + 2)/(x + 1)

giac [A] time = 0.37, size = 67, normalized size = 0.74

$$-\left(\sqrt{2}-3\log\left(\sqrt{2}+1\right)\right)\operatorname{sgn}(x+1)-3\log\left(-x+\sqrt{x^2+1}\right)\operatorname{sgn}(x+1)+\frac{\left(x\operatorname{sgn}(x+1)-2\operatorname{sgn}(x+1)\right)x-\operatorname{sgn}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="giac")

[Out] -(sqrt(2) - 3*log(sqrt(2) + 1))*sgn(x + 1) - 3*log(-x + sqrt(x^2 + 1))*sgn(x + 1) + ((x*sgn(x + 1) - 2*sgn(x + 1))*x - sgn(x + 1))/sqrt(x^2 + 1)

maple [A] time = 0.02, size = 49, normalized size = 0.54

$$\frac{\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}(x^2+1)\left(x^2-2x+3\sqrt{x^2+1}\operatorname{arcsinh}(x)-1\right)}{(x+1)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2/(x^2+1)*x)^(3/2),x)

[Out] ((x^2+2*x+1)/(x^2+1))^(3/2)/(x+1)^3*(x^2+1)*(3*arcsinh(x)*(x^2+1)^(1/2)+x^2-2*x-1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")

[Out] integrate((2*x/(x^2 + 1) + 1)^(3/2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(\frac{2x}{x^2+1}+1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*x)/(x^2 + 1) + 1)^(3/2),x)

[Out] int(((2*x)/(x^2 + 1) + 1)^(3/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(\frac{2x}{x^2 + 1} + 1 \right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x**2+1))**(3/2), x)

[Out] Integral((2*x/(x**2 + 1) + 1)**(3/2), x)

$$3.599 \quad \int \sqrt{1 + \frac{2x}{1+x^2}} dx$$

Optimal. Leaf size=61

$$\frac{\sqrt{\frac{2x}{x^2+1} + 1} (x^2 + 1)}{x + 1} + \frac{\sqrt{\frac{2x}{x^2+1} + 1} \sqrt{x^2 + 1} \sinh^{-1}(x)}{x + 1}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {6723, 970, 641, 215}

$$\frac{\sqrt{\frac{2x}{x^2+1} + 1} (x^2 + 1)}{x + 1} + \frac{\sqrt{\frac{2x}{x^2+1} + 1} \sqrt{x^2 + 1} \sinh^{-1}(x)}{x + 1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + (2*x)/(1 + x^2)], x]

[Out] ((1 + x^2)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x) + (Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]*ArcSinh[x])/(1 + x)

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 641

Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]

Rule 970

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 6723

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))*(x_)^(m_.))^(p_), x_Symbol] := Dist[(a + b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{1 + \frac{2x}{1+x^2}} dx &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{\sqrt{1+2x+x^2}}{\sqrt{1+x^2}} dx}{\sqrt{1+2x+x^2}} \\
&= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{2+2x}{\sqrt{1+x^2}} dx}{2+2x} \\
&= \frac{(1+x^2) \sqrt{1 + \frac{2x}{1+x^2}}}{1+x} + \frac{\left(2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{1}{\sqrt{1+x^2}} dx}{2+2x} \\
&= \frac{(1+x^2) \sqrt{1 + \frac{2x}{1+x^2}}}{1+x} + \frac{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}} \sinh^{-1}(x)}{1+x}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 40, normalized size = 0.66

$$\frac{\sqrt{\frac{(x+1)^2}{x^2+1}} \left(x^2 + \sqrt{x^2+1} \sinh^{-1}(x) + 1\right)}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)], x]

[Out] (Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2 + Sqrt[1 + x^2]*ArcSinh[x]))/(1 + x)

IntegrateAlgebraic [A] time = 5.31, size = 56, normalized size = 0.92

$$\frac{(x+1) \left(\sqrt{x^2+1} - \log\left(\sqrt{x^2+1} - x\right)\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + (2*x)/(1 + x^2)], x]

[Out] ((1 + x)*(Sqrt[1 + x^2] - Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

fricas [A] time = 0.40, size = 75, normalized size = 1.23

$$\frac{(x+1) \log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right) - (x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2), x, algorithm="fricas")

[Out] -((x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)

giac [A] time = 0.48, size = 49, normalized size = 0.80

$$-\left(\sqrt{2} - \log\left(\sqrt{2} + 1\right)\right) \operatorname{sgn}(x+1) - \log\left(-x + \sqrt{x^2+1}\right) \operatorname{sgn}(x+1) + \sqrt{x^2+1} \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="giac")

[Out] -(sqrt(2) - log(sqrt(2) + 1))*sgn(x + 1) - log(-x + sqrt(x^2 + 1))*sgn(x + 1) + sqrt(x^2 + 1)*sgn(x + 1)

maple [A] time = 0.01, size = 42, normalized size = 0.69

$$\frac{\sqrt{\frac{x^2+2x+1}{x^2+1}} \sqrt{x^2+1} \left(\operatorname{arcsinh}(x) + \sqrt{x^2+1} \right)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2/(x^2+1)*x)^(1/2),x)

[Out] ((x^2+2*x+1)/(x^2+1))^(1/2)/(x+1)*(x^2+1)^(1/2)*((x^2+1)^(1/2)+arcsinh(x))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{2x}{x^2+1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(2*x/(x^2 + 1) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{\frac{2x}{x^2+1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*x)/(x^2 + 1) + 1)^(1/2),x)

[Out] int(((2*x)/(x^2 + 1) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{2x}{x^2+1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x**2+1))**(1/2),x)

[Out] Integral(sqrt(2*x/(x**2 + 1) + 1), x)

$$3.600 \quad \int \frac{1}{\sqrt{1 + \frac{2x}{1+x^2}}} dx$$

Optimal. Leaf size=109

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}} - \frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}} - \frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {6723, 970, 735, 844, 215, 725, 206}

$$\frac{x+1}{\sqrt{\frac{2x}{x^2+1}+1}} - \frac{(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}} - \frac{\sqrt{2}(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}+1}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[1 + (2*x)/(1 + x^2)], x]

[Out] (1 + x)/Sqrt[1 + (2*x)/(1 + x^2)] - ((1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]) - (Sqrt[2]*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 735

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 2*p + 1)), x] + Dist[(2*p)/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[a*e - c*d*x, x]*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 844

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 970

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_)*((d_) + (f_)*(x_)^2)^(q_), x_Symbol] := Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 6723

Int[(u_)*((a_) + (b_)*(v_)^(n_)*(x_)^(m_))^(p_), x_Symbol] := Dist[(a + b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x]

Rubi steps

$$\begin{aligned}
 \int \frac{1}{\sqrt{1 + \frac{2x}{1+x^2}}} dx &= \frac{\sqrt{1+2x+x^2} \int \frac{\sqrt{1+x^2}}{\sqrt{1+2x+x^2}} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\
 &= \frac{(2+2x) \int \frac{\sqrt{1+x^2}}{2+2x} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\
 &= \frac{1+x}{\sqrt{1 + \frac{2x}{1+x^2}}} + \frac{(2+2x) \int \frac{2-2x}{(2+2x)\sqrt{1+x^2}} dx}{2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\
 &= \frac{1+x}{\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(2+2x) \int \frac{1}{\sqrt{1+x^2}} dx}{2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} + \frac{(2(2+2x)) \int \frac{1}{(2+2x)\sqrt{1+x^2}} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\
 &= \frac{1+x}{\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(1+x) \sinh^{-1}(x)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(2(2+2x)) \text{Subst}\left(\int \frac{1}{8-x^2} dx, x, \frac{2-2x}{\sqrt{1+x^2}}\right)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\
 &= \frac{1+x}{\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(1+x) \sinh^{-1}(x)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} - \frac{\sqrt{2}(1+x) \tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}}
 \end{aligned}$$

Mathematica [A] time = 0.03, size = 72, normalized size = 0.66

$$\frac{(x+1) \left(\sqrt{x^2+1} - \sqrt{2} \tanh^{-1} \left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}} \right) - \sinh^{-1}(x) \right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[1 + (2*x)/(1 + x^2)], x]

[Out] ((1 + x)*(Sqrt[1 + x^2] - ArcSinh[x] - Sqrt[2]*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2]])))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

IntegrateAlgebraic [A] time = 4.76, size = 91, normalized size = 0.83

$$\frac{(x+1) \left(\sqrt{x^2+1} + \log \left(\sqrt{x^2+1} - x \right) + 2\sqrt{2} \tanh^{-1} \left(-\frac{\sqrt{x^2+1}}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}} \right) \right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[1 + (2*x)/(1 + x^2)],x]

[Out] ((1 + x)*(Sqrt[1 + x^2] + 2*Sqrt[2]*ArcTanh[1/Sqrt[2] + x/Sqrt[2] - Sqrt[1 + x^2]/Sqrt[2]] + Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

fricas [A] time = 0.40, size = 142, normalized size = 1.30

$$\frac{\sqrt{2}(x+1)\log\left(-\frac{x^2+\sqrt{2}(x^2-1)+(2x^2+\sqrt{2}(x^2+1)+2)\sqrt{\frac{x^2+2x+1}{x^2+1}}-1}{x^2+2x+1}\right)+(x+1)\log\left(-\frac{x^2-(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}+x}{x+1}\right)+(x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="fricas")

[Out] (sqrt(2)*(x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + (x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)))/(x + 1)

giac [A] time = 0.57, size = 88, normalized size = 0.81

$$\frac{\sqrt{2}\log\left(\frac{|-2x-2\sqrt{2}+2\sqrt{x^2+1}-2|}{|-2x+2\sqrt{2}+2\sqrt{x^2+1}-2|}\right)}{\operatorname{sgn}(x+1)} + \frac{\log(-x + \sqrt{x^2+1})}{\operatorname{sgn}(x+1)} + \frac{\sqrt{x^2+1}}{\operatorname{sgn}(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="giac")

[Out] sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + 1) - 2))/sgn(x + 1) + log(-x + sqrt(x^2 + 1))/sgn(x + 1) + sqrt(x^2 + 1)/sgn(x + 1)

maple [A] time = 0.04, size = 79, normalized size = 0.72

$$\frac{x+1}{\sqrt{\frac{(x+1)^2}{x^2+1}}} + \frac{\left(-\operatorname{arcsinh}(x) - \sqrt{2}\operatorname{arctanh}\left(\frac{(-2x+2)\sqrt{2}}{4\sqrt{-2x+(x+1)^2}}\right)\right)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+2/(x^2+1)*x)^(1/2),x)

[Out] 1/((x+1)^2/(x^2+1))^(1/2)*(x+1)+(-arcsinh(x)-2^(1/2)*arctanh(1/4*(2-2*x)*2^(1/2)/((x+1)^2-2*x)^(1/2)))/((x+1)^2/(x^2+1))^(1/2)/(x^2+1)^(1/2)*(x+1)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(1/2),x, algorithm="maxima")

[Out] integrate(1/sqrt(2*x/(x^2 + 1) + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)

[Out] int(1/((2*x)/(x^2 + 1) + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{2x}{x^2+1} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x**2+1))**(1/2), x)

[Out] Integral(1/sqrt(2*x/(x**2 + 1) + 1), x)

$$3.601 \quad \int \frac{1}{\left(1 + \frac{2x}{1+x^2}\right)^{3/2}} dx$$

Optimal. Leaf size=144

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}}+1} - \frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}}+1} - \frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}}+1} - \frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}}+1}$$

Rubi [A] time = 0.08, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {6723, 970, 733, 813, 844, 215, 725, 206}

$$\frac{3(x+2)}{2\sqrt{\frac{2x}{x^2+1}}+1} - \frac{x^2+1}{2(x+1)\sqrt{\frac{2x}{x^2+1}}+1} - \frac{3(x+1)\sinh^{-1}(x)}{\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}}+1} - \frac{9(x+1)\tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{x^2+1}}\right)}{2\sqrt{2}\sqrt{x^2+1}\sqrt{\frac{2x}{x^2+1}}+1}$$

Antiderivative was successfully verified.

[In] Int[(1 + (2*x)/(1 + x^2))^(3/2), x]

[Out] (3*(2 + x))/(2*Sqrt[1 + (2*x)/(1 + x^2)]) - (1 + x^2)/(2*(1 + x)*Sqrt[1 + (2*x)/(1 + x^2)]) - (3*(1 + x)*ArcSinh[x])/(Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)]) - (9*(1 + x)*ArcTanh[(1 - x)/(Sqrt[2]*Sqrt[1 + x^2])])/(2*Sqrt[2]*Sqrt[1 + x^2]*Sqrt[1 + (2*x)/(1 + x^2)])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 733

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((d + e*x)^(m + 1)*(a + c*x^2)^p)/(e*(m + 1)), x] - Dist[(2*c*p)/(e*(m + 1)), Int[x*(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] || LtQ[m, -1]) && NeQ[m, -1] && !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, 0, c, d, e, m, p, x]

Rule 813

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[((d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1) + e*g*(m + 1)*x)*(a + c*x^2)^p)/(e^2*(m + 1)*(m + 2*p + 2)), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],

$x], x] /; \text{FreeQ}[\{a, c, d, e, f, g, m\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& \text{RationalQ}[p] \&\& p > 0 \&\& (\text{LtQ}[m, -1] \mid\mid \text{EqQ}[p, 1] \mid\mid (\text{IntegerQ}[p] \&\& !\text{RationalQ}[m])) \&\& \text{NeQ}[m, -1] \&\& !\text{LtQ}[m + 2*p + 1, 0] \&\& (\text{IntegerQ}[m] \mid\mid \text{IntegerQ}[p] \mid\mid \text{IntegersQ}[2*m, 2*p])$

Rule 844

$\text{Int}[(d_.) + (e_.)*(x_.)^{(m_.)}*((f_.) + (g_.)*(x_.)^{(a_.) + (c_.)*(x_.)^2})^{(p_.)}, x_Symbol] := \text{Dist}[g/e, \text{Int}[(d + e*x)^{(m + 1)}*(a + c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + c*x^2)^p, x], x] /; \text{FreeQ}[\{a, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[c*d^2 + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rule 970

$\text{Int}[(a_.) + (b_.)*(x_.) + (c_.)*(x_.)^2]^{(p_.)}*((d_.) + (f_.)*(x_.)^2)^{(q_.)}, x_Symbol] := \text{Dist}[(a + b*x + c*x^2)^{\text{FracPart}[p]} / ((4*c)^{\text{IntPart}[p]}*(b + 2*c*x)^{(2*\text{FracPart}[p])}), \text{Int}[(b + 2*c*x)^{(2*p)}*(d + f*x^2)^q, x], x] /; \text{FreeQ}[\{a, b, c, d, f, p, q\}, x] \&\& \text{EqQ}[b^2 - 4*a*c, 0] \&\& !\text{IntegerQ}[p]$

Rule 6723

$\text{Int}[(u_.)*((a_.) + (b_.)*(v_.)^{(n_.)}*(x_.)^{(m_.)})^{(p_.)}, x_Symbol] := \text{Dist}[(a + b*x^m*v^n)^{\text{FracPart}[p]} / (v^{(n*\text{FracPart}[p])}*(b*x^m + a/v^n)^{\text{FracPart}[p]}), \text{Int}[u*v^{(n*p)}*(b*x^m + a/v^n)^p, x], x] /; \text{FreeQ}[\{a, b, m, p\}, x] \&\& !\text{IntegerQ}[p] \&\& \text{ILtQ}[n, 0] \&\& \text{BinomialQ}[v, x]$

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(1 + \frac{2x}{1+x^2}\right)^{3/2}} dx &= \frac{\sqrt{1+2x+x^2} \int \frac{(1+x^2)^{3/2}}{(1+2x+x^2)^{3/2}} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= \frac{(4(2+2x)) \int \frac{(1+x^2)^{3/2}}{(2+2x)^3} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= -\frac{1+x^2}{2(1+x)\sqrt{1 + \frac{2x}{1+x^2}}} + \frac{(3(2+2x)) \int \frac{x\sqrt{1+x^2}}{(2+2x)^2} dx}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= \frac{3(2+x)}{2\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(3(2+2x)) \int \frac{-4+8x}{(2+2x)\sqrt{1+x^2}} dx}{8\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= \frac{3(2+x)}{2\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(3(2+2x)) \int \frac{1}{\sqrt{1+x^2}} dx}{2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} + \frac{(9(2+2x)) \int \frac{1}{(2+2x)\sqrt{1+x^2}} dx}{2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= \frac{3(2+x)}{2\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{3(1+x) \sinh^{-1}(x)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} - \frac{(9(2+2x)) \text{Subst}\left(\int \frac{1}{8-x^2} dx, x\right)}{2\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \\ &= \frac{3(2+x)}{2\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{1+x^2}{2(1+x)\sqrt{1 + \frac{2x}{1+x^2}}} - \frac{3(1+x) \sinh^{-1}(x)}{\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} - \frac{9(1+x) \tanh^{-1}\left(\frac{1-x}{\sqrt{2}\sqrt{1+x^2}}\right)}{2\sqrt{2}\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}} \end{aligned}$$

Mathematica [A] time = 0.12, size = 95, normalized size = 0.66

$$\frac{(x+1)\left(2\sqrt{x^2+1}(2x^2+9x+5)+9\sqrt{2}(x+1)^2 \tanh^{-1}\left(\frac{x-1}{\sqrt{2}\sqrt{x^2+1}}\right)-12(x+1)^2 \sinh^{-1}(x)\right)}{4\left(\frac{(x+1)^2}{x^2+1}\right)^{3/2}(x^2+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + (2*x)/(1 + x^2))^(-3/2), x]

[Out] ((1 + x)*(2*Sqrt[1 + x^2]*(5 + 9*x + 2*x^2) - 12*(1 + x)^2*ArcSinh[x] + 9*Sqrt[2]*(1 + x)^2*ArcTanh[(-1 + x)/(Sqrt[2]*Sqrt[1 + x^2])]))/(4*((1 + x)^2/(1 + x^2))^(3/2)*(1 + x^2)^(3/2))

IntegrateAlgebraic [A] time = 5.94, size = 112, normalized size = 0.78

$$\frac{(x+1)\left(\frac{\sqrt{x^2+1}(2x^2+9x+5)}{2(x+1)^2} + 3 \log\left(\sqrt{x^2+1} - x\right) + \frac{9 \tanh^{-1}\left(-\frac{\sqrt{x^2+1}}{\sqrt{2}} + \frac{x}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)}{\sqrt{2}}\right)}{\sqrt{\frac{(x+1)^2}{x^2+1}} \sqrt{x^2+1}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + (2*x)/(1 + x^2))^(-3/2), x]

[Out] ((1 + x)*((Sqrt[1 + x^2]*(5 + 9*x + 2*x^2))/(2*(1 + x)^2) + (9*ArcTanh[1/Sqrt[2] + x/Sqrt[2] - Sqrt[1 + x^2]/Sqrt[2]])/Sqrt[2] + 3*Log[-x + Sqrt[1 + x^2]]))/(Sqrt[(1 + x)^2/(1 + x^2)]*Sqrt[1 + x^2])

fricas [A] time = 0.42, size = 205, normalized size = 1.42

$$\frac{10x^3 + 9\sqrt{2}(x^3 + 3x^2 + 3x + 1) \log\left(\frac{x^2 + \sqrt{2}(x^2-1) + (2x^2 + \sqrt{2}(x^2+1)+2)\sqrt{\frac{x^2+2x+1}{x^2+1}} - 1}{x^2+2x+1}\right) + 30x^2 + 12(x^3 + 3x^2 + 3x + 1) \log\left(\frac{x^2 - (x^2+1)\sqrt{\frac{x^2+2x+1}{x^2+1}} + x}{x+1}\right) + 2(2x^4 + 9x^3 + 7x^2 + 9x + 5)\sqrt{\frac{x^2+2x+1}{x^2+1}} + 30x + 10}{4(x^3 + 3x^2 + 3x + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(3/2), x, algorithm="fricas")

[Out] 1/4*(10*x^3 + 9*sqrt(2)*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 + sqrt(2)*(x^2 - 1) + (2*x^2 + sqrt(2)*(x^2 + 1) + 2)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) - 1)/(x^2 + 2*x + 1)) + 30*x^2 + 12*(x^3 + 3*x^2 + 3*x + 1)*log(-(x^2 - (x^2 + 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x)/(x + 1)) + 2*(2*x^4 + 9*x^3 + 7*x^2 + 9*x + 5)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + 30*x + 10)/(x^3 + 3*x^2 + 3*x + 1)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x/(x^2+1))^(3/2), x, algorithm="giac")

[Out] integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)

maple [A] time = 0.02, size = 218, normalized size = 1.51

$$\frac{(x+1)\left(-\left(x^2+1\right)^{\frac{3}{2}}x^3-6\sqrt{x^2+1}x^2-24x^2\operatorname{arcsinh}(x)+18\sqrt{2}x^2\operatorname{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)+\left(x^2+1\right)^{\frac{3}{2}}x^2+6\sqrt{x^2+1}x^2-48x\operatorname{arcsinh}(x)+36\sqrt{2}x\operatorname{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)+\left(x^2+1\right)^{\frac{3}{2}}x+5\left(x^2+1\right)^{\frac{3}{2}}x+30\sqrt{x^2+1}x-24\operatorname{arcsinh}(x)+18\sqrt{2}\operatorname{arctanh}\left(\frac{(x-1)\sqrt{2}}{2\sqrt{x^2+1}}\right)-\left(x^2+1\right)^{\frac{3}{2}}+3\left(x^2+1\right)^{\frac{3}{2}}+18\sqrt{x^2+1}\right)}{8\left(\frac{x^2+2x+1}{x^2+1}\right)^{\frac{3}{2}}\left(x^2+1\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(1+2/(x^2+1)*x)^(3/2),x)`

[Out] $\frac{1}{8} \left(\frac{x^2+2x+1}{x^2+1} \right)^{3/2} (x+1) \left(\frac{x^2+1}{2} \right)^{5/2} x - (x^2+1)^{3/2} x^3 - (x^2+1)^{5/2} + (x^2+1)^{3/2} x^2 + 18 \cdot 2^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \frac{x-1}{x^2+1} \right)^{1/2} x^2 + 5 x (x^2+1)^{3/2} - 6 (x^2+1)^{1/2} x^3 + 36 \cdot 2^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \frac{x-1}{x^2+1} \right)^{1/2} x + 3 (x^2+1)^{3/2} + 6 (x^2+1)^{1/2} x^2 - 24 \operatorname{arcsinh}(x) x^2 + 18 \cdot 2^{1/2} \operatorname{arctanh} \left(\frac{1}{2} \frac{x-1}{x^2+1} \right)^{1/2} + 30 (x^2+1)^{1/2} x - 48 \operatorname{arcsinh}(x) x + 18 (x^2+1)^{1/2} - 24 \operatorname{arcsinh}(x) \right) / (x^2+1)^{3/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+2*x/(x^2+1))^(3/2),x, algorithm="maxima")`

[Out] `integrate((2*x/(x^2 + 1) + 1)^(-3/2), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x)/(x^2 + 1) + 1)^(3/2),x)`

[Out] `int(1/((2*x)/(x^2 + 1) + 1)^(3/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(\frac{2x}{x^2+1} + 1\right)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+2*x/(x**2+1))**(3/2),x)`

[Out] `Integral((2*x/(x**2 + 1) + 1)**(-3/2), x)`

$$3.602 \quad \int \frac{\sqrt{1 + \frac{2x}{1+x^2}}}{1+x^2} dx$$

Optimal. Leaf size=28

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Rubi [A] time = 0.12, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6723, 970, 637}

$$-\frac{(1-x)\sqrt{\frac{2x}{x^2+1}+1}}{x+1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2), x]

[Out] -(((1 - x)*Sqrt[1 + (2*x)/(1 + x^2)])/(1 + x))

Rule 637

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-(a*e) + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]

Rule 970

Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p])), Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] && !IntegerQ[p]

Rule 6723

Int[(u_.)*((a_.) + (b_.)*(v_)^(n_)*(x_)^(m_.))^(p_), x_Symbol] :> Dist[(a + b*x^m*v^n)^FracPart[p]/(v^(n*FracPart[p])*(b*x^m + a/v^n)^FracPart[p]), Int[u*v^(n*p)*(b*x^m + a/v^n)^p, x], x] /; FreeQ[{a, b, m, p}, x] && !IntegerQ[p] && ILtQ[n, 0] && BinomialQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1 + \frac{2x}{1+x^2}}}{1+x^2} dx &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{\sqrt{1+2x+x^2}}{(1+x^2)^{3/2}} dx}{\sqrt{1+2x+x^2}} \\ &= \frac{\left(\sqrt{1+x^2} \sqrt{1 + \frac{2x}{1+x^2}}\right) \int \frac{2+2x}{(1+x^2)^{3/2}} dx}{2+2x} \\ &= -\frac{(1-x)\sqrt{1 + \frac{2x}{1+x^2}}}{1+x} \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.93

$$\frac{(x-1)\sqrt{\frac{(x+1)^2}{x^2+1}}}{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2), x]

[Out] ((-1 + x)*Sqrt[(1 + x)^2/(1 + x^2)])/(1 + x)

IntegrateAlgebraic [A] time = 5.49, size = 31, normalized size = 1.11

$$\frac{(x-1)(x+1)}{\sqrt{\frac{(x+1)^2}{x^2+1}}(x^2+1)}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + (2*x)/(1 + x^2)]/(1 + x^2), x]

[Out] ((-1 + x)*(1 + x))/(Sqrt[(1 + x)^2/(1 + x^2)]*(1 + x^2))

fricas [A] time = 0.42, size = 31, normalized size = 1.11

$$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}} + x + 1}{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="fricas")

[Out] ((x - 1)*sqrt((x^2 + 2*x + 1)/(x^2 + 1)) + x + 1)/(x + 1)

giac [A] time = 0.39, size = 30, normalized size = 1.07

$$\sqrt{2} \operatorname{sgn}(x+1) + \frac{x \operatorname{sgn}(x+1) - \operatorname{sgn}(x+1)}{\sqrt{x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="giac")

[Out] sqrt(2)*sgn(x + 1) + (x*sgn(x + 1) - sgn(x + 1))/sqrt(x^2 + 1)

maple [A] time = 0.01, size = 28, normalized size = 1.00

$$\frac{(x-1)\sqrt{\frac{x^2+2x+1}{x^2+1}}}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+2/(x^2+1)*x)^(1/2)/(x^2+1), x)

[Out] (x-1)/(x+1)*((x^2+2*x+1)/(x^2+1))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{2x}{x^2+1} + 1}}{x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x^2+1))^(1/2)/(x^2+1), x, algorithm="maxima")

[Out] integrate(sqrt(2*x/(x^2 + 1) + 1)/(x^2 + 1), x)

mupad [B] time = 3.52, size = 23, normalized size = 0.82

$$\frac{\sqrt{\frac{2x}{x^2+1} + 1} (x-1)}{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((2*x)/(x^2 + 1) + 1)^(1/2)/(x^2 + 1), x)

[Out] (((2*x)/(x^2 + 1) + 1)^(1/2)*(x - 1))/(x + 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{(x+1)^2}{x^2+1}}}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x/(x**2+1))**(1/2)/(x**2+1), x)

[Out] Integral(sqrt((x + 1)**2/(x**2 + 1))/(x**2 + 1), x)

$$3.603 \quad \int \frac{\sqrt{bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Rubi [A] time = 0.11, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {2132, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4], x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2132

Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0]

Rubi steps

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}} dx = \text{Subst} \left(\int \frac{1}{1 - 2bx^2} dx, x, \frac{x}{\sqrt{bx^2 + \sqrt{a + b^2x^4}}} \right) \\ = \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [A] time = 0.02, size = 47, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}+bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4], x]

[Out] ArcTanh[(Sqrt[2]*Sqrt[b]*x)/Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

IntegrateAlgebraic [C] time = 0.43, size = 81, normalized size = 1.72

$$\frac{\log\left(i\sqrt{b}\sqrt{a+b^2x^4} + i\sqrt{2}bx\sqrt{\sqrt{a+b^2x^4}+bx^2} + ib^{3/2}x^2\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4], x]

[Out] Log[I*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + I*Sqrt[2]*b*x*Sqrt[b*x^2 + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

fricas [A] time = 1.80, size = 135, normalized size = 2.87

$$\left[\frac{\sqrt{2} \log\left(4b^2x^4 + 4\sqrt{b^2x^4 + a}bx^2 + 2\left(\sqrt{2}b^{3/2}x^3 + \sqrt{2}\sqrt{b^2x^4 + a}\sqrt{bx}\right)\sqrt{bx^2 + \sqrt{b^2x^4 + a} + a}\right)}{4\sqrt{b}}, -\frac{1}{2}\sqrt{2}\sqrt{-\frac{1}{b}} \arctan\left(\frac{\sqrt{2}\sqrt{bx^2 + \sqrt{b^2x^4 + a}}\sqrt{-\frac{1}{b}}}{2x}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2), x, algorithm="fricas")

[Out] [1/4*sqrt(2)*log(4*b^2*x^4 + 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^(3/2)*x^3 + sqrt(2)*sqrt(b^2*x^4 + a)*sqrt(b)*x)*sqrt(b*x^2 + sqrt(b^2*x^4 + a) + a)/sqrt(b), -1/2*sqrt(2)*sqrt(-1/b)*arctan(1/2*sqrt(2)*sqrt(b*x^2 + sqrt(b^2*x^4 + a))*sqrt(-1/b)/x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2), x, algorithm="giac")

[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2), x)

[Out] int((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2), x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + \sqrt{b^2x^4 + a}}}{\sqrt{b^2x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2 x^4 + a} + b x^2}}{\sqrt{b^2 x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)

[Out] int(((a + b^2*x^4)^(1/2) + b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)

[Out] Integral(sqrt(b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)

$$3.604 \quad \int \frac{\sqrt{-bx^2 + \sqrt{a+b^2x^4}}}{\sqrt{a+b^2x^4}} dx$$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Rubi [A] time = 0.11, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2132, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2132

Int[Sqrt[(c_.)*(x_)^2 + (d_.)*Sqrt[(a_) + (b_.)*(x_)^4]]/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := Dist[d, Subst[Int[1/(1 - 2*c*x^2), x], x, x/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[c^2 - b*d^2, 0]

Rubi steps

$$\int \frac{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}} dx = \text{Subst}\left(\int \frac{1}{1 + 2bx^2} dx, x, \frac{x}{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}\right)$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Mathematica [A] time = 0.02, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\sqrt{b}x}{\sqrt{\sqrt{a+b^2x^4}-bx^2}}\right)}{\sqrt{2}\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] ArcTan[(Sqrt[2]*Sqrt[b]*x)/Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]]/(Sqrt[2]*Sqrt[b])

IntegrateAlgebraic [C] time = 0.46, size = 82, normalized size = 1.71

$$\frac{i \log \left(i \sqrt{b} \sqrt{a + b^2 x^4} + \sqrt{2} b x \sqrt{\sqrt{a + b^2 x^4} - b x^2} - i b^{3/2} x^2 \right)}{\sqrt{2} \sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]]/Sqrt[a + b^2*x^4],x]

[Out] (I*Log[(-I)*b^(3/2)*x^2 + I*Sqrt[b]*Sqrt[a + b^2*x^4] + Sqrt[2]*b*x*Sqrt[-(b*x^2) + Sqrt[a + b^2*x^4]])/(Sqrt[2]*Sqrt[b])

fricas [A] time = 1.77, size = 146, normalized size = 3.04

$$\left[\frac{1}{4} \sqrt{2} \sqrt{-\frac{1}{b}} \log \left(4 b^2 x^4 - 4 \sqrt{b^2 x^4 + a} b x^2 + 2 \left(\sqrt{2} b^2 x^3 \sqrt{-\frac{1}{b}} - \sqrt{2} \sqrt{b^2 x^4 + a} b x \sqrt{-\frac{1}{b}} \right) \sqrt{-b x^2 + \sqrt{b^2 x^4 + a} + a} \right), -\frac{\sqrt{2} \arctan \left(\frac{\sqrt{2} \sqrt{-b x^2 + \sqrt{b^2 x^4 + a}}}{2 \sqrt{b} x} \right)}{2 \sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="fricas")

[Out] [1/4*sqrt(2)*sqrt(-1/b)*log(4*b^2*x^4 - 4*sqrt(b^2*x^4 + a)*b*x^2 + 2*(sqrt(2)*b^2*x^3*sqrt(-1/b) - sqrt(2)*sqrt(b^2*x^4 + a)*b*x*sqrt(-1/b))*sqrt(-b*x^2 + sqrt(b^2*x^4 + a) + a), -1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/(sqrt(b)*x))/sqrt(b)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b x^2 + \sqrt{b^2 x^4 + a}}}{\sqrt{b^2 x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)

maple [F(-2)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b x^2 + \sqrt{b^2 x^4 + a}}}{\sqrt{b^2 x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)

[Out] int((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-b x^2 + \sqrt{b^2 x^4 + a}}}{\sqrt{b^2 x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x^2+(b^2*x^4+a)^(1/2))^(1/2)/(b^2*x^4+a)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-b*x^2 + sqrt(b^2*x^4 + a))/sqrt(b^2*x^4 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{b^2 x^4 + a} - b x^2}}{\sqrt{b^2 x^4 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2),x)

[Out] int(((a + b^2*x^4)^(1/2) - b*x^2)^(1/2)/(a + b^2*x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-bx^2 + \sqrt{a + b^2x^4}}}{\sqrt{a + b^2x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-b*x**2+(b**2*x**4+a)**(1/2))**(1/2)/(b**2*x**4+a)**(1/2),x)

[Out] Integral(sqrt(-b*x**2 + sqrt(a + b**2*x**4))/sqrt(a + b**2*x**4), x)

$$3.605 \quad \int \frac{\sqrt{2x^2 + \sqrt{3+4x^4}}}{(c+dx)\sqrt{3+4x^4}} dx$$

Optimal. Leaf size=169

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

Rubi [A] time = 0.27, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2133, 725, 204, 206}

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) \tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{-\sqrt{3}d^2+2ic^2}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{\sqrt{\sqrt{3}d^2+2ic^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]

[Out] ((1/2 - I/2)*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2]])/Sqrt[(2*I)*c^2 - Sqrt[3]*d^2] - ((1/2 + I/2)*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (2*I)*x^2]])/Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 2133

Int[(((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^4]])/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ[a, 0]

Rubi steps

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)\sqrt{3 + 4x^4}} dx = \left(\frac{1}{2} - \frac{i}{2}\right) \int \frac{1}{(c + dx)\sqrt{\sqrt{3} - 2ix^2}} dx + \left(\frac{1}{2} + \frac{i}{2}\right) \int \frac{1}{(c + dx)\sqrt{\sqrt{3} + 2ix^2}} dx$$

$$= \left(-\frac{1}{2} - \frac{i}{2}\right) \text{Subst} \left(\int \frac{1}{2ic^2 + \sqrt{3}d^2 - x^2} dx, x, \frac{\sqrt{3}d - 2icx}{\sqrt{\sqrt{3} + 2ix^2}} \right) + \left(-\frac{1}{2} + \frac{i}{2}\right) \text{Subst} \left(\int \frac{1}{2ic^2 + \sqrt{3}d^2 - x^2} dx, x, \frac{\sqrt{3}d + 2icx}{\sqrt{\sqrt{3} - 2ix^2}} \right)$$

$$= \frac{\left(\frac{1}{2} - \frac{i}{2}\right) \tan^{-1} \left(\frac{\sqrt{3}d + 2icx}{\sqrt{2ic^2 - \sqrt{3}d^2} \sqrt{\sqrt{3} - 2ix^2}} \right)}{\sqrt{2ic^2 - \sqrt{3}d^2}} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) \tanh^{-1} \left(\frac{\sqrt{3}d - 2icx}{\sqrt{2ic^2 + \sqrt{3}d^2} \sqrt{\sqrt{3} + 2ix^2}} \right)}{\sqrt{2ic^2 + \sqrt{3}d^2}}$$

Mathematica [F] time = 0.11, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)\sqrt{3 + 4x^4}} dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]

[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]

IntegrateAlgebraic [C] time = 1.17, size = 364, normalized size = 2.15

$$c\text{RootSum} \left[\#1^4 d^2 - 8\#1^2 c^2 - 6\#1^2 d^2 - 24\#1 c^2 + 9d^2 \&, \frac{\#1^2 \left(-\log(-\#1 + \sqrt{4x^4 + 3} + 2x^2 + 2\sqrt{4x^4 + 3 + 2x^2}) \right) - 3 \log(-\#1 + \sqrt{4x^4 + 3} + 2x^2 + 2\sqrt{4x^4 + 3 + 2x^2})}{\#1^3 d^2 - 6\#1^2 c^2 - 3\#1 d^2 - 6c^2} \right] - \frac{\sqrt{-4c^2 + 3d^2} \tan^{-1} \left(\frac{d\sqrt{4x^4 + 3} + 2x^2}{\sqrt{-4c^2 + 3d^2}} \right)}{\sqrt{4c^2 + 3d^2}} + \frac{\sqrt{4c^2 + 3d^2} \tan^{-1} \left(\frac{d\sqrt{4x^4 + 3} + 2x^2}{\sqrt{4c^2 + 3d^2}} \right)}{\sqrt{4c^2 + 3d^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)*Sqrt[3 + 4*x^4]), x]

[Out] -((Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 - Sqrt[4*c^4 + 3*d^4]])/Sqrt[4*c^4 + 3*d^4]) + (Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]]*ArcTan[(d*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]])/Sqrt[-2*c^2 + Sqrt[4*c^4 + 3*d^4]])/Sqrt[4*c^4 + 3*d^4] + c*RootSum[9*d^2 - 24*c^2*#1 - 6*d^2*#1^2 - 8*c^2*#1^3 + d^2*#1^4 &, (-3*Log[2*x^2 + Sqrt[3 + 4*x^4]] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1] - Log[2*x^2 + Sqrt[3 + 4*x^4]] + 2*x*Sqrt[2*x^2 + Sqrt[3 + 4*x^4]] - #1]*#1^2)/(-6*c^2 - 3*d^2*#1 - 6*c^2*#1^2 + d^2*#1^3) &]

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(dx + c)\sqrt{4x^4 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)

[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)/(4*x^4+3)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)),x)

[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)\sqrt{4x^4 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)/(4*x**4+3)**(1/2),x)

[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/((c + d*x)*sqrt(4*x**4 + 3)), x)

$$3.606 \quad \int \frac{\sqrt{2x^2 + \sqrt{3+4x^4}}}{(c+dx)^2 \sqrt{3+4x^4}} dx$$

Optimal. Leaf size=268

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(-\sqrt{3}d^2 + 2ic^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(\sqrt{3}d^2 + 2ic^2)(c + dx)} + \frac{(1+i)c \tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{(-\sqrt{3}d^2 + 2ic^2)^{3/2}} + \frac{(1-i)c \tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{(\sqrt{3}d^2 + 2ic^2)^{3/2}}$$

Rubi [A] time = 0.31, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2133, 731, 725, 204, 206}

$$\frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(-\sqrt{3}d^2 + 2ic^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(\sqrt{3}d^2 + 2ic^2)(c + dx)} + \frac{(1+i)c \tan^{-1}\left(\frac{\sqrt{3}d+2icx}{\sqrt{\sqrt{3}-2ix^2}\sqrt{-\sqrt{3}d^2+2ic^2}}\right)}{(-\sqrt{3}d^2 + 2ic^2)^{3/2}} + \frac{(1-i)c \tanh^{-1}\left(\frac{\sqrt{3}d-2icx}{\sqrt{\sqrt{3}+2ix^2}\sqrt{\sqrt{3}d^2+2ic^2}}\right)}{(\sqrt{3}d^2 + 2ic^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]),x]

[Out] ((1/2 - I/2)*d*Sqrt[Sqrt[3] - (2*I)*x^2])/(((2*I)*c^2 - Sqrt[3]*d^2)*(c + d*x)) - ((1/2 + I/2)*d*Sqrt[Sqrt[3] + (2*I)*x^2])/(((2*I)*c^2 + Sqrt[3]*d^2)*(c + d*x)) + ((1 + I)*c*ArcTan[(Sqrt[3]*d + (2*I)*c*x)/(Sqrt[(2*I)*c^2 - Sqrt[3]*d^2]*Sqrt[Sqrt[3] - (2*I)*x^2]])/(((2*I)*c^2 - Sqrt[3]*d^2)^(3/2) + ((1 - I)*c*ArcTanh[(Sqrt[3]*d - (2*I)*c*x)/(Sqrt[(2*I)*c^2 + Sqrt[3]*d^2]*Sqrt[Sqrt[3] + (2*I)*x^2]])/(((2*I)*c^2 + Sqrt[3]*d^2)^(3/2))

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]]/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 725

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ[{a, c, d, e}, x]

Rule 731

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/((m + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d)/(c*d^2 + a*e^2), Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[m + 2*p + 3, 0]

Rule 2133

Int((((c_.) + (d_.)*(x_))^(m_.)*Sqrt[(b_.)*(x_)^2 + Sqrt[(a_) + (e_.)*(x_)^4]]/Sqrt[(a_) + (e_.)*(x_)^4], x_Symbol] :> Dist[(1 - I)/2, Int[(c + d*x)^m/Sqrt[Sqrt[a] - I*b*x^2], x], x] + Dist[(1 + I)/2, Int[(c + d*x)^m/Sqrt[Sqrt[a] + I*b*x^2], x], x] /; FreeQ[{a, b, c, d, m}, x] && EqQ[e, b^2] && GtQ

[a, 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2 \sqrt{3 + 4x^4}} dx &= \left(\frac{1}{2} - \frac{i}{2}\right) \int \frac{1}{(c + dx)^2 \sqrt{\sqrt{3} - 2ix^2}} dx + \left(\frac{1}{2} + \frac{i}{2}\right) \int \frac{1}{(c + dx)^2 \sqrt{\sqrt{3} + 2ix^2}} dx \\
&= \frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(2ic^2 - \sqrt{3} d^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(2ic^2 + \sqrt{3} d^2)(c + dx)} + \frac{((1 + i)c) \int \frac{1}{(c + dx) \sqrt{\sqrt{3} + 2ix^2}} dx}{2c^2 - i\sqrt{3} d^2} + \dots \\
&= \frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(2ic^2 - \sqrt{3} d^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(2ic^2 + \sqrt{3} d^2)(c + dx)} + \frac{((1 + i)c) \operatorname{Subst}\left(\int \frac{1}{2ic^2 + \sqrt{3} d^2 - x^2}\right)}{2c^2 - i\sqrt{3} d^2} \\
&= \frac{\left(\frac{1}{2} - \frac{i}{2}\right) d \sqrt{\sqrt{3} - 2ix^2}}{(2ic^2 - \sqrt{3} d^2)(c + dx)} - \frac{\left(\frac{1}{2} + \frac{i}{2}\right) d \sqrt{\sqrt{3} + 2ix^2}}{(2ic^2 + \sqrt{3} d^2)(c + dx)} + \frac{(1 + i)c \tan^{-1}\left(\frac{\sqrt{3} d + 2icx}{\sqrt{2ic^2 - \sqrt{3} d^2} \sqrt{\sqrt{3} - 2ix^2}}\right)}{(2ic^2 - \sqrt{3} d^2)^{3/2}}
\end{aligned}$$

Mathematica [F] time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{3 + 4x^4}}}{(c + dx)^2 \sqrt{3 + 4x^4}} dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]

[Out] Integrate[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]

IntegrateAlgebraic [C] time = 5.17, size = 1609, normalized size = 6.00

result too large to display

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2*x^2 + Sqrt[3 + 4*x^4]]/((c + d*x)^2*Sqrt[3 + 4*x^4]), x]

[Out] $(-d(-6c^3 - 6cd^2x^2 - 16c^3x^4) \sqrt{2x^2 + \sqrt{3 + 4x^4}}) - d$
 $x(6c^2d + 6d^3x^2 + 16c^2dx^4) \sqrt{2x^2 + \sqrt{3 + 4x^4}} + \sqrt{3 + 4x^4}(-d(-3cd^2 - 8c^3x^2) \sqrt{2x^2 + \sqrt{3 + 4x^4}}) - d$
 $x(3d^3 + 8c^2dx^2) \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / (4(4c^4 + 3d^4)$
 $x^2(-c^2 + d^2x^2) \sqrt{3 + 4x^4} + (4c^4 + 3d^4)(-c^2 + d^2x^2)(3$
 $+ 8x^4) + (8c^5 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 -$
 $\sqrt{4c^4 + 3d^4}]]) / ((4c^4 + 3d^4)^{3/2} \sqrt{-2c^2 - \sqrt{4c^4 + 3d^4}}) - (6cd^4 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 - \sqrt{4c^4 + 3d^4}}]) / ((4c^4 + 3d^4)^{3/2} \sqrt{-2c^2 - \sqrt{4c^4 + 3d^4}}) + (4c^3 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 - \sqrt{4c^4 + 3d^4}}]) / ((4c^4 + 3d^4) \sqrt{-2c^2 - \sqrt{4c^4 + 3d^4}}) - (8c^5 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}]) / ((4c^4 + 3d^4)^{3/2} \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}) + (6cd^4 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}]) / ((4c^4 + 3d^4)^{3/2} \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}) + (4c^3 \operatorname{ArcTan}[(d \sqrt{2x^2 + \sqrt{3 + 4x^4}}) / \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}]) / ((4c^4 + 3d^4) \sqrt{-2c^2 + \sqrt{4c^4 + 3d^4}}) - \operatorname{RootSum}[9d^2 - 24c$

$c^2\#1 - 6*d^2\#1^2 - 8*c^2\#1^3 + d^2\#1^4$ & , $(128*c^4*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1] + 3*d^4*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1] + 16*c^2*d^2*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1 + d^4*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1^2)/(6*c^2 + 3*d^2\#1 + 6*c^2\#1^2 - d^2\#1^3)$ &]/d^4 + RootSum[9*d^2 - 24*c^2\#1 - 6*d^2\#1^2 - 8*c^2\#1^3 + d^2\#1^4 & , (512*c^8*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1] + 408*c^4*d^4*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1] + 9*d^8*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1] + 64*c^6*d^2*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1 + 36*c^2*d^6*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1 + 8*c^4*d^4*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1^2 + 3*d^8*\text{Log}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] + 2*x*\text{Sqrt}[2*x^2 + \text{Sqrt}[3 + 4*x^4]] - \#1]*\#1^2)/(6*c^2 + 3*d^2\#1 + 6*c^2\#1^2 - d^2\#1^3) &]/(d^4*(4*c^4 + 3*d^4))

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(dx + c)^2 \sqrt{4x^4 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)

[Out] int((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3}(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x^2+(4*x^4+3)^(1/2))^(1/2)/(d*x+c)^2/(4*x^4+3)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(2*x^2 + sqrt(4*x^4 + 3))/(sqrt(4*x^4 + 3)*(d*x + c)^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{\sqrt{4x^4 + 3} (c + dx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)

[Out] int((2*x^2 + (4*x^4 + 3)^(1/2))^(1/2)/((4*x^4 + 3)^(1/2)*(c + d*x)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^2 + \sqrt{4x^4 + 3}}}{(c + dx)^2 \sqrt{4x^4 + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2*x**2+(4*x**4+3)**(1/2))**(1/2)/(d*x+c)**2/(4*x**4+3)**(1/2), x)

[Out] Integral(sqrt(2*x**2 + sqrt(4*x**4 + 3))/(c + d*x)**2*sqrt(4*x**4 + 3)), x)

$$3.607 \quad \int \frac{-4+x}{(1+\sqrt[3]{x})\sqrt{x}} dx$$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 30\sqrt[6]{x} + 30 \tan^{-1}(\sqrt[6]{x})$$

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {1840, 1620, 50, 63, 203}

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 30\sqrt[6]{x} + 30 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Int[(-4 + x)/((1 + x^(1/3))*Sqrt[x]),x]

[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 1620

Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[Expon[Px, x], 2]

Rule 1840

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g*(m + 1) - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, b, m, p}, x] && PolyQ[Pq, x] && FractionQ[n]

Rubi steps

$$\begin{aligned}
\int \frac{-4+x}{(1+\sqrt[3]{x})\sqrt{x}} dx &= 3 \operatorname{Subst} \left(\int \frac{\sqrt{x}(-4+x^3)}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= 3 \operatorname{Subst} \left(\int \left(\sqrt{x} - x^{3/2} + x^{5/2} - \frac{5\sqrt{x}}{1+x} \right) dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 15 \operatorname{Subst} \left(\int \frac{\sqrt{x}}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= -30\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 15 \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}(1+x)} dx, x, \sqrt[3]{x} \right) \\
&= -30\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 30 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt[6]{x} \right) \\
&= -30\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 30 \tan^{-1}(\sqrt[6]{x})
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 30\sqrt[6]{x} + 30 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Integrate[(-4 + x)/((1 + x^(1/3))*Sqrt[x]), x]

[Out] -30*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 30*ArcTan[x^(1/6)]

IntegrateAlgebraic [A] time = 0.04, size = 42, normalized size = 1.02

$$\frac{2}{35} (15x^{7/6} - 21x^{5/6} + 35\sqrt{x} - 525\sqrt[6]{x}) + 30 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-4 + x)/((1 + x^(1/3))*Sqrt[x]), x]

[Out] (2*(-525*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 30*ArcTan[x^(1/6)]

fricas [A] time = 1.07, size = 25, normalized size = 0.61

$$\frac{6}{7} (x - 35)x^{1/6} - \frac{6}{5} x^{5/6} + 2\sqrt{x} + 30 \arctan(x^{1/6})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2), x, algorithm="fricas")

[Out] 6/7*(x - 35)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 30*arctan(x^(1/6))

giac [A] time = 0.40, size = 27, normalized size = 0.66

$$\frac{6}{7} x^{7/6} - \frac{6}{5} x^{5/6} + 2\sqrt{x} - 30x^{1/6} + 30 \arctan(x^{1/6})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-4+x)/(1+x^(1/3))/x^(1/2), x, algorithm="giac")

[Out] $6/7*x^{(7/6)} - 6/5*x^{(5/6)} + 2*\text{sqrt}(x) - 30*x^{(1/6)} + 30*\text{arctan}(x^{(1/6)})$

maple [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7} + 30 \arctan\left(x^{\frac{1}{6}}\right) - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 30x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x-4)/(1+x^(1/3))/x^(1/2), x)`

[Out] $-30*x^{(1/6)}-6/5*x^{(5/6)}+6/7*x^{(7/6)}+30*\text{arctan}(x^{(1/6)})+2*x^{(1/2)}$

maxima [A] time = 2.34, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 30x^{\frac{1}{6}} + 30 \arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+x)/(1+x^(1/3))/x^(1/2), x, algorithm="maxima")`

[Out] $6/7*x^{(7/6)} - 6/5*x^{(5/6)} + 2*\text{sqrt}(x) - 30*x^{(1/6)} + 30*\text{arctan}(x^{(1/6)})$

mupad [B] time = 3.37, size = 27, normalized size = 0.66

$$30 \operatorname{atan}\left(x^{1/6}\right) + 2\sqrt{x} - 30x^{1/6} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x - 4)/(x^(1/2)*(x^(1/3) + 1)), x)`

[Out] $30*\text{atan}(x^{(1/6)}) + 2*x^{(1/2)} - 30*x^{(1/6)} - (6*x^{(5/6)})/5 + (6*x^{(7/6)})/7$

sympy [A] time = 11.87, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 30\sqrt[6]{x} + 2\sqrt{x} + 30 \operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-4+x)/(1+x**(1/3))/x**(1/2), x)`

[Out] $6*x^{(7/6)}/7 - 6*x^{(5/6)}/5 - 30*x^{(1/6)} + 2*\text{sqrt}(x) + 30*\text{atan}(x^{(1/6)})$

$$3.608 \quad \int \frac{1+\sqrt{x}}{x^{5/6}+x^{7/6}} dx$$

Optimal. Leaf size=26

$$3\sqrt[3]{x} - 3 \log(\sqrt[3]{x} + 1) + 6 \tan^{-1}(\sqrt[6]{x})$$

Rubi [A] time = 0.04, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1593, 1819, 1810, 635, 203, 260}

$$3\sqrt[3]{x} - 3 \log(\sqrt[3]{x} + 1) + 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)),x]

[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[Rt[b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1810

Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 1819

Int[(Pq_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^p, x_Symbol] := Dist[1/(m + 1), Subst[Int[SubstFor[x^(m + 1), Pq, x]*(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[{a, b, m, n, p}, x] && NeQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && PolyQ[Pq, x^(m + 1)]

Rubi steps

$$\begin{aligned}
\int \frac{1 + \sqrt{x}}{x^{5/6} + x^{7/6}} dx &= \int \frac{1 + \sqrt{x}}{(1 + \sqrt[3]{x})x^{5/6}} dx \\
&= 6 \operatorname{Subst} \left(\int \frac{1 + x^3}{1 + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 6 \operatorname{Subst} \left(\int \left(x + \frac{1-x}{1+x^2} \right) dx, x, \sqrt[6]{x} \right) \\
&= 3\sqrt[3]{x} + 6 \operatorname{Subst} \left(\int \frac{1-x}{1+x^2} dx, x, \sqrt[6]{x} \right) \\
&= 3\sqrt[3]{x} + 6 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt[6]{x} \right) - 6 \operatorname{Subst} \left(\int \frac{x}{1+x^2} dx, x, \sqrt[6]{x} \right) \\
&= 3\sqrt[3]{x} + 6 \tan^{-1} \left(\sqrt[6]{x} \right) - 3 \log \left(1 + \sqrt[3]{x} \right)
\end{aligned}$$

Mathematica [C] time = 0.02, size = 38, normalized size = 1.46

$$3\sqrt[3]{x} + (-3 - 3i) \log(-\sqrt[6]{x} + i) - (3 - 3i) \log(\sqrt[6]{x} + i)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)), x]

[Out] 3*x^(1/3) - (3 + 3*I)*Log[I - x^(1/6)] - (3 - 3*I)*Log[I + x^(1/6)]

IntegrateAlgebraic [A] time = 0.05, size = 26, normalized size = 1.00

$$3\sqrt[3]{x} - 3 \log(\sqrt[3]{x} + 1) + 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[x])/(x^(5/6) + x^(7/6)), x]

[Out] 3*x^(1/3) + 6*ArcTan[x^(1/6)] - 3*Log[1 + x^(1/3)]

fricas [A] time = 0.86, size = 20, normalized size = 0.77

$$3x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)), x, algorithm="fricas")

[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)

giac [A] time = 0.35, size = 20, normalized size = 0.77

$$3x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)), x, algorithm="giac")

[Out] 3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)

maple [A] time = 0.01, size = 21, normalized size = 0.81

$$6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \ln\left(x^{\frac{1}{3}} + 1\right) + 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)+1)/(x^(5/6)+x^(7/6)),x)`

[Out] `3*x^(1/3)+6*arctan(x^(1/6))-3*ln(1+x^(1/3))`

maxima [A] time = 2.29, size = 20, normalized size = 0.77

$$3x^{\frac{1}{3}} + 6 \arctan\left(x^{\frac{1}{6}}\right) - 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/2))/(x^(5/6)+x^(7/6)),x, algorithm="maxima")`

[Out] `3*x^(1/3) + 6*arctan(x^(1/6)) - 3*log(x^(1/3) + 1)`

mupad [B] time = 3.36, size = 22, normalized size = 0.85

$$6 \operatorname{atan}\left(x^{1/6}\right) - 3 \ln\left(36x^{1/3} + 36\right) + 3x^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2) + 1)/(x^(5/6) + x^(7/6)),x)`

[Out] `6*atan(x^(1/6)) - 3*log(36*x^(1/3) + 36) + 3*x^(1/3)`

sympy [A] time = 3.66, size = 24, normalized size = 0.92

$$3\sqrt[3]{x} - 3 \log\left(\sqrt[3]{x} + 1\right) + 6 \operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x**(1/2))/(x**(5/6)+x**(7/6)),x)`

[Out] `3*x**(1/3) - 3*log(x**(1/3) + 1) + 6*atan(x**(1/6))`

$$3.609 \quad \int \frac{1+\sqrt{x}}{(1+\sqrt[3]{x})\sqrt{x}} dx$$

Optimal. Leaf size=42

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3 \log(\sqrt[3]{x} + 1) - 6 \tan^{-1}(\sqrt[6]{x})$$

Rubi [A] time = 0.15, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6688, 1593, 1802, 635, 203, 260}

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3 \log(\sqrt[3]{x} + 1) - 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]),x]

[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/3)]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 260

Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_.)), x_Symbol] := Simp[Log[RemoveContent[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]

Rule 635

Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] := Dist[d, Int[1/(a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && !NiceSqrtQ[-(a*c)]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^n, x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1802

Int[(Pq_)*((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]

Rule 6688

Int[u_, x_Symbol] := With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned}
\int \frac{1 + \sqrt{x}}{(1 + \sqrt[3]{x})\sqrt{x}} dx &= \int \frac{1 + \frac{1}{\sqrt{x}}}{1 + \sqrt[3]{x}} dx \\
&= 6 \operatorname{Subst} \left(\int \frac{x^2 + x^5}{1 + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 6 \operatorname{Subst} \left(\int \frac{x^2(1 + x^3)}{1 + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 6 \operatorname{Subst} \left(\int \left(1 - x + x^3 - \frac{1 - x}{1 + x^2} \right) dx, x, \sqrt[6]{x} \right) \\
&= 6\sqrt[6]{x} - 3\sqrt[3]{x} + \frac{3x^{2/3}}{2} - 6 \operatorname{Subst} \left(\int \frac{1 - x}{1 + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 6\sqrt[6]{x} - 3\sqrt[3]{x} + \frac{3x^{2/3}}{2} - 6 \operatorname{Subst} \left(\int \frac{1}{1 + x^2} dx, x, \sqrt[6]{x} \right) + 6 \operatorname{Subst} \left(\int \frac{x}{1 + x^2} dx, x, \sqrt[6]{x} \right) \\
&= 6\sqrt[6]{x} - 3\sqrt[3]{x} + \frac{3x^{2/3}}{2} - 6 \tan^{-1}(\sqrt[6]{x}) + 3 \log(1 + \sqrt[3]{x})
\end{aligned}$$

Mathematica [C] time = 0.02, size = 54, normalized size = 1.29

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + (3 + 3i) \log(-\sqrt[6]{x} + i) + (3 - 3i) \log(\sqrt[6]{x} + i)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]), x]

[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 + (3 + 3*I)*Log[-sqrt[6]{x} + i] + (3 - 3*I)*Log[sqrt[6]{x} + i]

IntegrateAlgebraic [A] time = 2.31, size = 42, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - 3\sqrt[3]{x} + 6\sqrt[6]{x} + 3 \log(\sqrt[3]{x} + 1) - 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[x])/((1 + x^(1/3))*Sqrt[x]), x]

[Out] 6*x^(1/6) - 3*x^(1/3) + (3*x^(2/3))/2 - 6*ArcTan[x^(1/6)] + 3*Log[1 + x^(1/3)]

fricas [A] time = 1.26, size = 30, normalized size = 0.71

$$\frac{3}{2} x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \arctan\left(x^{\frac{1}{6}}\right) + 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2), x, algorithm="fricas")

[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1)

giac [A] time = 0.34, size = 30, normalized size = 0.71

$$\frac{3}{2} x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \arctan\left(x^{\frac{1}{6}}\right) + 3 \log\left(x^{\frac{1}{3}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="giac")

[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1)

maple [A] time = 0.02, size = 48, normalized size = 1.14

$$-6 \arctan\left(x^{\frac{1}{6}}\right) + \ln(x+1) + 2 \ln\left(x^{\frac{1}{3}}+1\right) - \ln\left(x^{\frac{2}{3}}-x^{\frac{1}{3}}+1\right) + \frac{3x^{\frac{2}{3}}}{2} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)+1)/(x^(1/3)+1)/x^(1/2),x)

[Out] ln(x+1)+3/2*x^(2/3)-ln(x^(2/3)-x^(1/3)+1)+2*ln(x^(1/3)+1)-3*x^(1/3)+6*x^(1/6)-6*arctan(x^(1/6))

maxima [A] time = 2.39, size = 30, normalized size = 0.71

$$\frac{3}{2}x^{\frac{2}{3}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6 \arctan\left(x^{\frac{1}{6}}\right) + 3 \log\left(x^{\frac{1}{3}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/2))/(1+x^(1/3))/x^(1/2),x, algorithm="maxima")

[Out] 3/2*x^(2/3) - 3*x^(1/3) + 6*x^(1/6) - 6*arctan(x^(1/6)) + 3*log(x^(1/3) + 1)

mupad [B] time = 0.03, size = 42, normalized size = 1.00

$$\frac{3x^{2/3}}{2} + 3 \ln\left(\left(-6 + x^{1/6} 6i\right)\left(6 + x^{1/6} 6i\right)\right) - 3x^{1/3} - 6 \operatorname{atan}\left(x^{1/6}\right) + 6x^{1/6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2) + 1)/(x^(1/2)*(x^(1/3) + 1)),x)

[Out] 3*log((x^(1/6)*6i - 6)*(x^(1/6)*6i + 6)) - 6*atan(x^(1/6)) - 3*x^(1/3) + (3*x^(2/3))/2 + 6*x^(1/6)

sympy [A] time = 23.13, size = 39, normalized size = 0.93

$$6\sqrt[6]{x} + \frac{3x^{\frac{2}{3}}}{2} - 3\sqrt[3]{x} + 3 \log\left(\sqrt[3]{x}+1\right) - 6 \operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x**(1/2))/(1+x**(1/3))/x**(1/2),x)

[Out] 6*x**(1/6) + 3*x**(2/3)/2 - 3*x**(1/3) + 3*log(x**(1/3) + 1) - 6*atan(x**(1/6))

$$3.610 \quad \int \frac{\sqrt{2 + \frac{b}{x^2}}}{b + 2x^2} dx$$

Optimal. Leaf size=20

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {25, 335, 215}

$$-\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 + b/x^2]/(b + 2*x^2), x]

[Out] -(ArcCsch[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])

Rule 25

Int[(u_.)*((a_.) + (b_.)*(x_)^(n_.))^(m_.)*((c_.) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 215

Int[1/Sqrt[(a_.) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 335

Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_.))^(p_.), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2 + \frac{b}{x^2}}}{b + 2x^2} dx &= \int \frac{1}{\sqrt{2 + \frac{b}{x^2}} x^2} dx \\ &= -\operatorname{Subst}\left(\int \frac{1}{\sqrt{2 + bx^2}} dx, x, \frac{1}{x}\right) \\ &= -\frac{\operatorname{csch}^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.01, size = 48, normalized size = 2.40

$$\frac{x\sqrt{\frac{b}{x^2} + 2} \tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 + b/x^2]/(b + 2*x^2), x]

[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*x^2]))

IntegrateAlgebraic [B] time = 3.52, size = 48, normalized size = 2.40

$$\frac{x\sqrt{\frac{b}{x^2} + 2} \tanh^{-1}\left(\frac{\sqrt{b+2x^2}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{b+2x^2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 + b/x^2]/(b + 2*x^2), x]

[Out] -((Sqrt[2 + b/x^2]*x*ArcTanh[Sqrt[b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[b + 2*x^2]))

fricas [B] time = 1.22, size = 75, normalized size = 3.75

$$\left[\frac{\log\left(\frac{x^2 - \sqrt{b}x\sqrt{\frac{2x^2+b}{x^2} + b}}{x^2}\right)}{2\sqrt{b}}, \frac{\sqrt{-b} \arctan\left(\frac{\sqrt{-b}x\sqrt{\frac{2x^2+b}{x^2}}}{2x^2+b}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b), x, algorithm="fricas")

[Out] [1/2*log(-(x^2 - sqrt(b)*x*sqrt((2*x^2 + b)/x^2) + b)/x^2)/sqrt(b), sqrt(-b)*arctan(sqrt(-b)*x*sqrt((2*x^2 + b)/x^2)/(2*x^2 + b))/b]

giac [B] time = 0.35, size = 44, normalized size = 2.20

$$\frac{\arctan\left(\frac{\sqrt{2x^2+b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}} - \frac{\arctan\left(\frac{\sqrt{b}}{\sqrt{-b}}\right) \operatorname{sgn}(x)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b), x, algorithm="giac")

[Out] arctan(sqrt(2*x^2 + b)/sqrt(-b))*sgn(x)/sqrt(-b) - arctan(sqrt(b)/sqrt(-b))*sgn(x)/sqrt(-b)

maple [B] time = 0.01, size = 50, normalized size = 2.50

$$\frac{\sqrt{\frac{2x^2+b}{x^2}} x \ln\left(\frac{2b+2\sqrt{2x^2+b}\sqrt{b}}{x}\right)}{\sqrt{2x^2+b}\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+b/x^2)^(1/2)/(2*x^2+b), x)

[Out] -((2*x^2+b)/x^2)^(1/2)*x/(2*x^2+b)^(1/2)/b^(1/2)*ln(2*(b^(1/2)*(2*x^2+b)^(1/2)+b)/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2} + 2}}{2x^2 + b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+b/x^2)^(1/2)/(2*x^2+b),x, algorithm="maxima")

[Out] integrate(sqrt(b/x^2 + 2)/(2*x^2 + b), x)

mupad [B] time = 3.46, size = 17, normalized size = 0.85

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{b}}{2x}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b/x^2 + 2)^(1/2)/(b + 2*x^2),x)

[Out] -asinh((2^(1/2)*b^(1/2))/(2*x))/b^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\frac{b}{x^2} + 2}}{b + 2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+b/x**2)**(1/2)/(2*x**2+b),x)

[Out] Integral(sqrt(b/x**2 + 2)/(b + 2*x**2), x)

$$3.611 \quad \int \frac{\sqrt{2 - \frac{b}{x^2}}}{-b + 2x^2} dx$$

Optimal. Leaf size=20

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {25, 335, 216}

$$-\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[2 - b/x^2]/(-b + 2*x^2), x]

[Out] -(ArcCsc[(Sqrt[2]*x)/Sqrt[b]]/Sqrt[b])

Rule 25

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(q_.))^(p_.), x_Symbol] :> Dist[(d/a)^p, Int[(u*(a + b*x^n)^(m + p))/x^(n*p), x], x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[q, -n] && IntegerQ[p] && EqQ[a*c - b*d, 0] && !(IntegerQ[m] && NegQ[n])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 335

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{2 - \frac{b}{x^2}}}{-b + 2x^2} dx &= \int \frac{1}{\sqrt{2 - \frac{b}{x^2}} x^2} dx \\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{2 - bx^2}} dx, x, \frac{1}{x}\right) \\ &= -\frac{\csc^{-1}\left(\frac{\sqrt{2}x}{\sqrt{b}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.01, size = 52, normalized size = 2.60

$$\frac{x\sqrt{2 - \frac{b}{x^2}} \tan^{-1}\left(\frac{\sqrt{2x^2 - b}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{2x^2 - b}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[2 - b/x^2]/(-b + 2*x^2), x]

[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x^2])

IntegrateAlgebraic [B] time = 3.58, size = 52, normalized size = 2.60

$$\frac{x\sqrt{2 - \frac{b}{x^2}} \tan^{-1}\left(\frac{\sqrt{2x^2 - b}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{2x^2 - b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[2 - b/x^2]/(-b + 2*x^2), x]

[Out] (Sqrt[2 - b/x^2]*x*ArcTan[Sqrt[-b + 2*x^2]/Sqrt[b]])/(Sqrt[b]*Sqrt[-b + 2*x^2])

fricas [B] time = 1.08, size = 84, normalized size = 4.20

$$\left[\frac{\sqrt{-b} \log\left(-\frac{x^2 - \sqrt{-b}x\sqrt{\frac{2x^2 - b}{x^2}} - b}{x^2}\right)}{2b}, \frac{\arctan\left(\frac{\sqrt{b}x\sqrt{\frac{2x^2 - b}{x^2}}}{2x^2 - b}\right)}{\sqrt{b}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b), x, algorithm="fricas")

[Out] [-1/2*sqrt(-b)*log(-(x^2 - sqrt(-b)*x*sqrt((2*x^2 - b)/x^2) - b)/x^2)/b, -arctan(sqrt(b)*x*sqrt((2*x^2 - b)/x^2)/(2*x^2 - b))/sqrt(b)]

giac [B] time = 0.42, size = 40, normalized size = 2.00

$$\frac{\arctan\left(\frac{\sqrt{2x^2 - b}}{\sqrt{b}}\right) \operatorname{sgn}(x)}{\sqrt{b}} - \frac{\arctan\left(\frac{\sqrt{-b}}{\sqrt{b}}\right) \operatorname{sgn}(x)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b), x, algorithm="giac")

[Out] arctan(sqrt(2*x^2 - b)/sqrt(b))*sgn(x)/sqrt(b) - arctan(sqrt(-b)/sqrt(b))*sgn(x)/sqrt(b)

maple [B] time = 0.01, size = 62, normalized size = 3.10

$$-\frac{\sqrt{\frac{2x^2 - b}{x^2}} x \ln\left(\frac{-2b + 2\sqrt{-b}\sqrt{2x^2 - b}}{x}\right)}{\sqrt{2x^2 - b}\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2-b/x^2)^(1/2)/(2*x^2-b), x)

[Out] -((2*x^2-b)/x^2)^(1/2)*x/(2*x^2-b)^(1/2)/(-b)^(1/2)*ln(2*((-b)^(1/2)*(2*x^2-b)^(1/2)-b)/x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{b}{x^2} + 2}}{2x^2 - b} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-b/x^2)^(1/2)/(2*x^2-b),x, algorithm="maxima")

[Out] integrate(sqrt(-b/x^2 + 2)/(2*x^2 - b), x)

mupad [B] time = 3.47, size = 21, normalized size = 1.05

$$-\frac{\operatorname{asinh}\left(\frac{\sqrt{2}\sqrt{-b}}{2x}\right)}{\sqrt{-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(2 - b/x^2)^(1/2)/(b - 2*x^2),x)

[Out] -asinh((2^(1/2)*(-b)^(1/2))/(2*x))/(-b)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\frac{b}{x^2} + 2}}{-b + 2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2-b/x**2)**(1/2)/(2*x**2-b),x)

[Out] Integral(sqrt(-b/x**2 + 2)/(-b + 2*x**2), x)

$$3.612 \quad \int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex} dx$$

Optimal. Leaf size=121

$$-\frac{\sqrt{ad^2 + ce^2} \tanh^{-1}\left(\frac{ad - \frac{ce}{x}}{\sqrt{a + \frac{c}{x^2}} \sqrt{ad^2 + ce^2}}\right)}{de} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a + \frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + \frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Rubi [A] time = 0.16, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.526$, Rules used = {1444, 1475, 896, 266, 63, 208, 844, 217, 206, 725}

$$-\frac{\sqrt{ad^2 + ce^2} \tanh^{-1}\left(\frac{ad - \frac{ce}{x}}{\sqrt{a + \frac{c}{x^2}} \sqrt{ad^2 + ce^2}}\right)}{de} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}}{x\sqrt{a + \frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + \frac{c}{x^2}}}{\sqrt{a}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2]/(d + e*x), x]

[Out] (Sqrt[a]*ArcTanh[Sqrt[a + c/x^2]/Sqrt[a]])/e - (Sqrt[a*d^2 + c*e^2]*ArcTanh[(a*d - (c*e)/x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[a + c/x^2]])/(d*e) - (Sqrt[c]*ArcTanh[Sqrt[c]/(Sqrt[a + c/x^2]*x)])/d

Rule 63

Int[((a_.) + (b_.)*(x_)^(m_))*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[Rt[-b, 2]*x]/Rt[a, 2])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 208

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 217

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Subst[Int[1/(1 - b*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 725

Int[1/(((d_) + (e_.)*(x_)*)Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] := -Subst[Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ

[{a, c, d, e}, x]

Rule 844

Int[((d_.) + (e_.)*(x_.))^(m_.)*((f_.) + (g_.)*(x_.))*((a_.) + (c_.)*(x_.)^2)^(p_.), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] && !IGtQ[m, 0]

Rule 896

Int[((a_.) + (c_.)*(x_.)^2)^(p_.)/(((d_.) + (e_.)*(x_.))*((f_.) + (g_.)*(x_.))), x_Symbol] := Dist[(c*d^2 + a*e^2)/(e*(e*f - d*g)), Int[(a + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int[(Simp[c*d*f + a*e*g - c*(e*f - d*g)*x, x]*(a + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && FractionQ[p] && GtQ[p, 0]

Rule 1444

Int[((d_.) + (e_.)*(x_.)^(mn_.))^(q_.)*((a_.) + (c_.)*(x_.)^(n2_.))^(p_.), x_Symbol] := Int[x^(mn*q)*(e + d/x^mn)^q*(a + c*x^n2)^p, x] /; FreeQ[{a, c, d, e, mn, p}, x] && EqQ[n2, -2*mn] && IntegerQ[q] && (PosQ[n2] || !IntegerQ[p])

Rule 1475

Int[(x_.)^(m_.)*((a_.) + (c_.)*(x_.)^(n2_.))^(p_.)*((d_.) + (e_.)*(x_.)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex} dx &= \int \frac{\sqrt{a + \frac{c}{x^2}}}{\left(e + \frac{d}{x}\right)x} dx \\
 &= -\text{Subst}\left(\int \frac{\sqrt{a + cx^2}}{x(e + dx)} dx, x, \frac{1}{x}\right) \\
 &= \frac{\text{Subst}\left(\int \frac{ad - cex}{(e + dx)\sqrt{a + cx^2}} dx, x, \frac{1}{x}\right)}{e} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a + cx^2}} dx, x, \frac{1}{x}\right)}{e} \\
 &= -\frac{c \text{Subst}\left(\int \frac{1}{\sqrt{a + cx^2}} dx, x, \frac{1}{x}\right)}{d} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a + cx}} dx, x, \frac{1}{x^2}\right)}{2e} + \left(\frac{ad}{e} + \frac{ce}{d}\right) \text{Subst}\left(\int \frac{1}{(e + dx)} dx, x, \frac{1}{x}\right) \\
 &= -\frac{c \text{Subst}\left(\int \frac{1}{1 - cx^2} dx, x, \frac{1}{\sqrt{a + \frac{c}{x^2}}x}\right)}{d} - \frac{a \text{Subst}\left(\int \frac{1}{-\frac{a}{c} + \frac{x^2}{c}} dx, x, \sqrt{a + \frac{c}{x^2}}\right)}{ce} + \left(-\frac{ad}{e} - \frac{ce}{d}\right) \text{Subst}\left(\int \frac{1}{(e + dx)} dx, x, \frac{1}{x}\right) \\
 &= \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a + \frac{c}{x^2}}}{\sqrt{a}}\right)}{e} - \frac{\sqrt{ad^2 + ce^2} \tanh^{-1}\left(\frac{ad - \frac{ce}{x}}{\sqrt{ad^2 + ce^2} \sqrt{a + \frac{c}{x^2}}}\right)}{de} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}}{\sqrt{a + \frac{c}{x^2}}x}\right)}{d}
 \end{aligned}$$

Mathematica [A] time = 0.10, size = 136, normalized size = 1.12

$$\frac{x\sqrt{a + \frac{c}{x^2}} \left(\sqrt{ad^2 + ce^2} \tanh^{-1} \left(\frac{ce - adx}{\sqrt{ax^2 + c} \sqrt{ad^2 + ce^2}} \right) + \sqrt{a} d \tanh^{-1} \left(\frac{\sqrt{a} x}{\sqrt{ax^2 + c}} \right) - \sqrt{c} e \tanh^{-1} \left(\frac{\sqrt{ax^2 + c}}{\sqrt{c}} \right) \right)}{de\sqrt{ax^2 + c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2]/(d + e*x), x]

[Out] (Sqrt[a + c/x^2]*x*(Sqrt[a]*d*ArcTanh[(Sqrt[a]*x)/Sqrt[c + a*x^2]] + Sqrt[a]*d^2 + c*e^2)*ArcTanh[(c*e - a*d*x)/(Sqrt[a*d^2 + c*e^2]*Sqrt[c + a*x^2])] - Sqrt[c]*e*ArcTanh[Sqrt[c + a*x^2]/Sqrt[c]])/(d*e*Sqrt[c + a*x^2])

IntegrateAlgebraic [A] time = 4.59, size = 205, normalized size = 1.69

$$\frac{x\sqrt{a + \frac{c}{x^2}} \left(-\frac{2\sqrt{-ad^2 - ce^2} \tan^{-1} \left(-\frac{e\sqrt{ax^2 + c}}{\sqrt{-ad^2 - ce^2}} + \frac{\sqrt{a}ex}{\sqrt{-ad^2 - ce^2}} + \frac{\sqrt{a}d}{\sqrt{-ad^2 - ce^2}} \right)}{de} + \frac{2\sqrt{c} \tanh^{-1} \left(\frac{\sqrt{a}x}{\sqrt{c}} - \frac{\sqrt{ax^2 + c}}{\sqrt{c}} \right)}{d} - \frac{\sqrt{a} \log(\sqrt{ax^2 + c} - \sqrt{a}x)}{e} \right)}{\sqrt{ax^2 + c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c/x^2]/(d + e*x), x]

[Out] (Sqrt[a + c/x^2]*x*((-2*Sqrt[-(a*d^2) - c*e^2]*ArcTan[(Sqrt[a]*d)/Sqrt[-(a*d^2) - c*e^2]] + (Sqrt[a]*e*x)/Sqrt[-(a*d^2) - c*e^2] - (e*Sqrt[c + a*x^2])/Sqrt[-(a*d^2) - c*e^2]))/(d*e) + (2*Sqrt[c]*ArcTanh[(Sqrt[a]*x)/Sqrt[c] - Sqrt[c + a*x^2]/Sqrt[c]])/d - (Sqrt[a]*Log[-(Sqrt[a]*x) + Sqrt[c + a*x^2]])/e)/Sqrt[c + a*x^2]

fricas [A] time = 2.52, size = 1532, normalized size = 12.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2)^(1/2)/(e*x+d), x, algorithm="fricas")

[Out] [1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(c)*e*log(-(a*x^2 - 2*sqrt(c)*x*sqrt((a*x^2 + c)/x^2) + 2*c)/x^2) - 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*sqrt((a*x^2 + c)/x^2) - c) + sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - 2*sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x^2 + c)) - sqrt(a*d^2 + c*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*c^2*e^2 - (2*a^2*d^2 + a*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e)

```
*c*e^2)*x^2 + 2*(a*d*x^2 - c*e*x)*sqrt(a*d^2 + c*e^2)*sqrt((a*x^2 + c)/x^2)
)/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*e*arctan(sqrt(-c)*x*sq
rt((a*x^2 + c)/x^2)/(a*x^2 + c)) + sqrt(a)*d*log(-2*a*x^2 - 2*sqrt(a)*x^2*s
qrt((a*x^2 + c)/x^2) - c) + 2*sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)
)*sqrt(-a*d^2 - c*e^2)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 +
a*c*e^2)*x^2)))/(d*e), -(sqrt(-a)*d*arctan(sqrt(-a)*x^2*sqrt((a*x^2 + c)/x
^2)/(a*x^2 + c)) - sqrt(-c)*e*arctan(sqrt(-c)*x*sqrt((a*x^2 + c)/x^2)/(a*x
^2 + c)) - sqrt(-a*d^2 - c*e^2)*arctan((a*d*x^2 - c*e*x)*sqrt(-a*d^2 - c*e^2
)*sqrt((a*x^2 + c)/x^2)/(a*c*d^2 + c^2*e^2 + (a^2*d^2 + a*c*e^2)*x^2)))/(d*
e)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
constant sign by intervals (correct if the argument is real):Check [abs(x)]
Error: Bad Argument Type
```

maple [B] time = 0.03, size = 244, normalized size = 2.02

$$\frac{\sqrt{\frac{ax^2+c}{x^2}} \left(a d^2 \ln \left(\frac{-2adx+2ce+2\sqrt{ax^2+c}\sqrt{\frac{ad^2+ce^2}{e^2}}}{ex+d} \right) + c e^2 \ln \left(\frac{-2adx+2ce+2\sqrt{ax^2+c}\sqrt{\frac{ad^2+ce^2}{e^2}}}{ex+d} \right) + \sqrt{\frac{ad^2+ce^2}{e^2}} \sqrt{a} \operatorname{de} \ln \left(\frac{ax+\sqrt{ax^2+c}\sqrt{a}}{\sqrt{a}} \right) - \sqrt{\frac{ad^2+ce^2}{e^2}} \sqrt{c} e^2 \ln \left(\frac{2c+2\sqrt{ax^2+c}\sqrt{c}}{x} \right) \right) x}{\sqrt{ax^2+c}\sqrt{\frac{ad^2+ce^2}{e^2}} d e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+c/x^2)^(1/2)/(e*x+d),x)
```

```
[Out] ((a*x^2+c)/x^2)^(1/2)*x*(a^(1/2)*d*ln(((a*x^2+c)^(1/2)*a^(1/2)+a*x)/a^(1/2)
)*e*((a*d^2+c*e^2)/e^2)^(1/2)-((a*d^2+c*e^2)/e^2)^(1/2)*c^(1/2)*ln(2*(c^(1/
2)*(a*x^2+c)^(1/2)+c)/x)*e^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/2
)*e-a*x*d+c*e)/(e*x+d))*a*d^2+ln(2*((a*x^2+c)^(1/2)*((a*d^2+c*e^2)/e^2)^(1/
2)*e-a*x*d+c*e)/(e*x+d))*c*e^2/(a*x^2+c)^(1/2)/d/e^2/((a*d^2+c*e^2)/e^2)^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+c/x^2)^(1/2)/(e*x+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a + c/x^2)/(e*x + d), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c/x^2)^(1/2)/(d + e*x),x)
```

```
[Out] int((a + c/x^2)^(1/2)/(d + e*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{c}{x^2}}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x**2)**(1/2)/(e*x+d),x)

[Out] Integral(sqrt(a + c/x**2)/(d + e*x), x)

$$3.613 \quad \int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{d + ex} dx$$

Optimal. Leaf size=181

$$\frac{\sqrt{ad^2 - e(bd - ce)} \tanh^{-1}\left(\frac{2ad + \frac{bd - 2ce}{x} - be}{2\sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{ad^2 - e(bd - ce)}}\right)}{de} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{e}$$

Rubi [A] time = 0.27, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 24, number of rules / integrand size = 0.292, Rules used = {1443, 1474, 895, 724, 206, 843, 621}

$$\frac{\sqrt{ad^2 - e(bd - ce)} \tanh^{-1}\left(\frac{2ad + \frac{bd - 2ce}{x} - be}{2\sqrt{a + \frac{b}{x} + \frac{c}{x^2}} \sqrt{ad^2 - e(bd - ce)}}\right)}{de} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b + \frac{2c}{x}}{2\sqrt{c} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{d} + \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a} \sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c/x^2 + b/x]/(d + e*x), x]

[Out] (Sqrt[a]*ArcTanh[(2*a + b/x)/(2*Sqrt[a]*Sqrt[a + c/x^2 + b/x]))/e - (Sqrt[c]*ArcTanh[(b + (2*c)/x)/(2*Sqrt[c]*Sqrt[a + c/x^2 + b/x]))/d - (Sqrt[a*d^2 - e*(b*d - c*e)]*ArcTanh[(2*a*d - b*e + (b*d - 2*c*e)/x]/(2*Sqrt[a*d^2 - e*(b*d - c*e)]*Sqrt[a + c/x^2 + b/x]))/(d*e)

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IGtQ[m, 0]

Rule 895

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))), x_Symbol] := Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), Int[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), Int

```
t[(Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*(a + b*x + c*x^2)^(p - 1))/(f + g*x), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p] && GtQ[p, 0]
```

Rule 1443

```
Int[((d_) + (e_)*(x_)^(mn_))^(q_)*((a_) + (b_)*(x_)^(n_) + (c_)*(x_)^(n2_))^(p_), x_Symbol] := Int[((e + d*x^n)^q*(a + b*x^n + c*x^(2*n))^p)/x^(n*q), x] /; FreeQ[{a, b, c, d, e, n, p}, x] && EqQ[n2, 2*n] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] || !IntegerQ[p])
```

Rule 1474

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{d + ex} dx &= \int \frac{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}{\left(e + \frac{d}{x}\right)x} dx \\ &= -\text{Subst}\left(\int \frac{\sqrt{a + bx + cx^2}}{x(e + dx)} dx, x, \frac{1}{x}\right) \\ &= \frac{\text{Subst}\left(\int \frac{ad - be - cex}{(e + dx)\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x}\right)}{e} - \frac{a \text{Subst}\left(\int \frac{1}{x\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x}\right)}{e} \\ &= -\frac{c \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x}\right)}{d} + \frac{(2a) \text{Subst}\left(\int \frac{1}{4a - x^2} dx, x, \frac{2a + \frac{b}{x}}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{e} + \left(-b + \frac{ad}{e} + \frac{ce}{d}\right) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x}\right) \\ &= \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{e} - \frac{(2c) \text{Subst}\left(\int \frac{1}{4c - x^2} dx, x, \frac{b + \frac{2c}{x}}{\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{d} + \left(2\left(b - \frac{ad}{e} - \frac{ce}{d}\right)\right) \text{Subst}\left(\int \frac{1}{\sqrt{a + bx + cx^2}} dx, x, \frac{1}{x}\right) \\ &= \frac{\sqrt{a} \tanh^{-1}\left(\frac{2a + \frac{b}{x}}{2\sqrt{a}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{e} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b + \frac{2c}{x}}{2\sqrt{c}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{d} - \frac{\sqrt{ad^2 - e(bd - ce)} \tanh^{-1}\left(\frac{bd - ce - d^2x}{2\sqrt{ad^2 - e(bd - ce)}\sqrt{a + \frac{c}{x^2} + \frac{b}{x}}}\right)}{de} \end{aligned}$$

Mathematica [A] time = 0.28, size = 189, normalized size = 1.04

$$\frac{x\sqrt{a + \frac{bx+c}{x^2}} \left(\sqrt{ad^2 - bde + ce^2} \tanh^{-1}\left(\frac{2adx + bd - bex - 2ce}{2\sqrt{x(ax+b)+c}\sqrt{ad^2 - bde + ce^2}}\right) - \sqrt{a}d \tanh^{-1}\left(\frac{2ax+b}{2\sqrt{a}\sqrt{x(ax+b)+c}}\right) + \sqrt{c}e \tanh^{-1}\left(\frac{bx+2c}{2\sqrt{c}\sqrt{x(ax+b)+c}}\right) \right)}{de\sqrt{x(ax+b)+c}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c/x^2 + b/x]/(d + e*x), x]

[Out] -((x*Sqrt[a + (c + b*x)/x^2]*(-(Sqrt[a]*d*ArcTanh[(b + 2*a*x)/(2*Sqrt[a]*Sqrt[c + x*(b + a*x)])]) + Sqrt[c]*e*ArcTanh[(2*c + b*x)/(2*Sqrt[c]*Sqrt[c +

$x*(b + a*x)]]) + \text{Sqrt}[a*d^2 - b*d*e + c*e^2]*\text{ArcTanh}[(b*d - 2*c*e + 2*a*d*x - b*e*x)/(2*\text{Sqrt}[a*d^2 - b*d*e + c*e^2]*\text{Sqrt}[c + x*(b + a*x)])])]/(d*e*\text{Sqrt}[c + x*(b + a*x)])]$

IntegrateAlgebraic [A] time = 6.52, size = 244, normalized size = 1.35

$$x\sqrt{a + \frac{bx+c}{x^2}} \left(-\frac{2\sqrt{-ad^2+bde-ce^2} \tan^{-1}\left(\frac{e\sqrt{ax^2+bx+c}}{\sqrt{-ad^2+bde-ce^2}} + \frac{\sqrt{a}ex}{\sqrt{-ad^2+bde-ce^2}} + \frac{\sqrt{a}d}{\sqrt{-ad^2+bde-ce^2}}\right)}{de} + \frac{2\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{a}x}{\sqrt{c}} - \frac{\sqrt{ax^2+bx+c}}{\sqrt{c}}\right)}{d} - \frac{\sqrt{a} \log\left(-2\sqrt{a}e\sqrt{ax^2+bx+c}+2aex+be\right)}{e} \right) / \sqrt{x(ax+b)+c}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a + c/x^2 + b/x]/(d + e*x),x]

[Out] $(x*\text{Sqrt}[a + (c + b*x)/x^2]*((-2*\text{Sqrt}[-(a*d^2) + b*d*e - c*e^2]*\text{ArcTan}[(\text{Sqrt}[a]*d)/\text{Sqrt}[-(a*d^2) + b*d*e - c*e^2] + (\text{Sqrt}[a]*e*x)/\text{Sqrt}[-(a*d^2) + b*d*e - c*e^2] - (e*\text{Sqrt}[c + b*x + a*x^2])/(\text{Sqrt}[-(a*d^2) + b*d*e - c*e^2])])/(d*e) + (2*\text{Sqrt}[c]*\text{ArcTanh}[(\text{Sqrt}[a]*x)/\text{Sqrt}[c] - \text{Sqrt}[c + b*x + a*x^2]/\text{Sqrt}[c]])/d - (\text{Sqrt}[a]*\text{Log}[b*e + 2*a*e*x - 2*\text{Sqrt}[a]*e*\text{Sqrt}[c + b*x + a*x^2]])/e)/\text{Sqrt}[c + x*(b + a*x)]$

fricas [A] time = 76.22, size = 2411, normalized size = 13.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="fricas")

[Out] $[1/2*(\text{sqrt}(a)*d*\log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\text{sqrt}(a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)) + \text{sqrt}(c)*e*\log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/x^2 + \text{sqrt}(a*d^2 - b*d*e + c*e^2)*\log(((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*\text{sqrt}(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*\text{sqrt}(-a)*d*\arctan(1/2*(2*a*x^2 + b*x)*\text{sqrt}(-a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - \text{sqrt}(c)*e*\log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/x^2 - \text{sqrt}(a*d^2 - b*d*e + c*e^2)*\log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*\text{sqrt}(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(\text{sqrt}(a)*d*\log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\text{sqrt}(a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)) + \text{sqrt}(c)*e*\log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/x^2 - 2*\text{sqrt}(-a*d^2 + b*d*e - c*e^2)*\arctan(-1/2*\text{sqrt}(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e), -1/2*(2*\text{sqrt}(-a)*d*\arctan(1/2*(2*a*x^2 + b*x)*\text{sqrt}(-a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a^2*x^2 + a*b*x + a*c)) - \text{sqrt}(c)*e*\log(-(8*b*c*x + (b^2 + 4*a*c)*x^2 + 8*c^2 - 4*(b*x^2 + 2*c*x)*\text{sqrt}(c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)))/x^2 + 2*\text{sqrt}(-a*d^2 + b*d*e - c*e^2)*\arctan(-1/2*\text{sqrt}(-a*d^2 + b*d*e - c*e^2)*((2*a*d - b*e)*x^2 + (b*d - 2*c*e)*x)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a*c*d^2 - b*c*d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b*c*e^2)*x)))/(d*e), 1/2*(2*\text{sqrt}(-c)*e*\arctan(1/2*(b*x^2 + 2*c*x)*\text{sqrt}(-c)*\text{sqrt}((a*x^2 + b*x + c)/x^2)/(a*c*x^2 + b*c*x + c^2)) + \text{sqrt}(a)*d*\log(-8*a^2*x^2 - 8*a*b*x - b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\text{sqrt}(a)*\text{sqrt}((a*x^2 + b*x + c)/x^2)) + \text{sqrt}(a*d^2 - b*d*e + c*e^2)*\log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3*b^2 + 4*a*c)*d*e)*x + 4*\text{sqrt}(a*d^2 - b*d*e + c*e^2)*((2*a*d - b*e)*x^2 + ($

$$\begin{aligned} & (b*d - 2*c*e)*x)*\sqrt{(a*x^2 + b*x + c)/x^2)} / (e^2*x^2 + 2*d*e*x + d^2)) / (d \\ & *e), -1/2*(2*\sqrt{-a}*d*\arctan(1/2*(2*a*x^2 + b*x)*\sqrt{-a}*\sqrt{(a*x^2 + b \\ & *x + c)/x^2)} / (a^2*x^2 + a*b*x + a*c)) - 2*\sqrt{-c}*e*\arctan(1/2*(b*x^2 + 2* \\ & c*x)*\sqrt{-c}*\sqrt{(a*x^2 + b*x + c)/x^2)} / (a*c*x^2 + b*c*x + c^2)) - \sqrt{a} \\ & *d^2 - b*d*e + c*e^2)*\log((8*b*c*d*e - 8*c^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*a \\ & ^2*d^2 - 8*a*b*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*a*b*d^2 + 4*b*c*e^2 - (3 \\ & *b^2 + 4*a*c)*d*e)*x + 4*\sqrt{a*d^2 - b*d*e + c*e^2}*((2*a*d - b*e)*x^2 + (\\ & b*d - 2*c*e)*x)*\sqrt{(a*x^2 + b*x + c)/x^2)} / (e^2*x^2 + 2*d*e*x + d^2)) / (d \\ & *e), 1/2*(2*\sqrt{-c}*e*\arctan(1/2*(b*x^2 + 2*c*x)*\sqrt{-c}*\sqrt{(a*x^2 + b*x \\ & + c)/x^2)} / (a*c*x^2 + b*c*x + c^2)) + \sqrt{a}*d*\log(-8*a^2*x^2 - 8*a*b*x - \\ & b^2 - 4*a*c - 4*(2*a*x^2 + b*x)*\sqrt{a}*\sqrt{(a*x^2 + b*x + c)/x^2}) - 2*s \\ & \sqrt{-a*d^2 + b*d*e - c*e^2}*\arctan(-1/2*\sqrt{-a*d^2 + b*d*e - c*e^2}*((2*a* \\ & d - b*e)*x^2 + (b*d - 2*c*e)*x)*\sqrt{(a*x^2 + b*x + c)/x^2)} / (a*c*d^2 - b*c* \\ & d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b* \\ & c*e^2)*x)) / (d*e), -(\sqrt{-a}*d*\arctan(1/2*(2*a*x^2 + b*x)*\sqrt{-a}*\sqrt{(a \\ & *x^2 + b*x + c)/x^2)} / (a^2*x^2 + a*b*x + a*c)) - \sqrt{-c}*e*\arctan(1/2*(b*x^ \\ & 2 + 2*c*x)*\sqrt{-c}*\sqrt{(a*x^2 + b*x + c)/x^2)} / (a*c*x^2 + b*c*x + c^2)) + \\ & \sqrt{-a*d^2 + b*d*e - c*e^2}*\arctan(-1/2*\sqrt{-a*d^2 + b*d*e - c*e^2}*((2*a \\ & *d - b*e)*x^2 + (b*d - 2*c*e)*x)*\sqrt{(a*x^2 + b*x + c)/x^2)} / (a*c*d^2 - b*c \\ & *d*e + c^2*e^2 + (a^2*d^2 - a*b*d*e + a*c*e^2)*x^2 + (a*b*d^2 - b^2*d*e + b \\ & *c*e^2)*x)) / (d*e)] \end{aligned}$$

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
 UT:sage2:=int(sage0,x):;OUTPUT:Warning, integration of abs or sign assumes
 constant sign by intervals (correct if the argument is real):Check [abs(x)]
 Error: Bad Argument Type

maple [B] time = 0.04, size = 397, normalized size = 2.19

$$\frac{\sqrt{\frac{a^2+bx+c}{e^2}} \left(-\frac{3}{2} a^2 \ln \left(\frac{-2ad+bc-bd+2e+2\sqrt{a^2+bx+c} \sqrt{\frac{a^2-3bd+c^2}{e^2}}}{e^2} \right) + \sqrt{a} \operatorname{Ei} \ln \left(\frac{-2ad+bc-bd+2e+2\sqrt{a^2+bx+c} \sqrt{\frac{a^2-3bd+c^2}{e^2}}}{e^2} \right) - \sqrt{a} c^2 \ln \left(\frac{-2ad+bc-bd+2e+2\sqrt{a^2+bx+c} \sqrt{\frac{a^2-3bd+c^2}{e^2}}}{e^2} \right) - \sqrt{\frac{a^2-3bd+c^2}{e^2}} \operatorname{Ei} \ln \left(\frac{2a+2e+2\sqrt{a^2+bx+c} \sqrt{e}}{2\sqrt{e}} \right) + \sqrt{\frac{a^2-3bd+c^2}{e^2}} \sqrt{a} \sqrt{e}^2 \ln \left(\frac{bx+2e+2\sqrt{a^2+bx+c} \sqrt{e}}{e} \right) \right)}{\sqrt{a^2+bx+c} \sqrt{\frac{a^2-3bd+c^2}{e^2}} \sqrt{a} d e^2} x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+c/x^2+b/x)^(1/2)/(e*x+d),x)

[Out]
$$\begin{aligned} & -((a*x^2+b*x+c)/x^2)^{(1/2)}*x*((a*d^2-b*d*e+c*e^2)/e^2)^{(1/2)}*c^{(1/2)}*a^{(1/2)} \\ & *2*\ln((2*c+b*x+2*c^{(1/2)}*(a*x^2+b*x+c)^{(1/2)})/x)*e^2-((a*d^2-b*d*e+c*e^2)/e \\ & ^2)^{(1/2)}*\ln(1/2*(2*(a*x^2+b*x+c)^{(1/2)}*a^{(1/2)}+2*a*x+b)/a^{(1/2)})*a*d*e-a^{(\\ & 3/2)}*\ln((2*(a*x^2+b*x+c)^{(1/2)}*((a*d^2-b*d*e+c*e^2)/e^2)^{(1/2)}*e-2*a*d*x+x* \\ & b*e-b*d+2*c*e)/(e*x+d))*d^2+a^{(1/2)}*\ln((2*(a*x^2+b*x+c)^{(1/2)}*((a*d^2-b*d*e \\ & +c*e^2)/e^2)^{(1/2)}*e-2*a*d*x+x*b*e-b*d+2*c*e)/(e*x+d))*b*d*e-a^{(1/2)}*\ln((2* \\ & (a*x^2+b*x+c)^{(1/2)}*((a*d^2-b*d*e+c*e^2)/e^2)^{(1/2)}*e-2*a*d*x+x*b*e-b*d+2*c \\ & *e)/(e*x+d))*c*e^2)/(a*x^2+b*x+c)^{(1/2)}/d/e^2/a^{(1/2)}/((a*d^2-b*d*e+c*e^2)/ \\ & e^2)^{(1/2)} \end{aligned}$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x^2+b/x)^(1/2)/(e*x+d),x, algorithm="maxima")

[Out] integrate(sqrt(a + b/x + c/x^2)/(e*x + d), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)

[Out] int((a + b/x + c/x^2)^(1/2)/(d + e*x), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + \frac{b}{x} + \frac{c}{x^2}}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+c/x**2+b/x)**(1/2)/(e*x+d), x)

[Out] Integral(sqrt(a + b/x + c/x**2)/(d + e*x), x)

$$3.614 \quad \int \frac{\sqrt[6]{x} + \sqrt[5]{x^3}}{\sqrt{x}} dx$$

Optimal. Leaf size=26

$$\frac{3x^{2/3}}{2} + \frac{10}{11} \sqrt[5]{x^3} \sqrt{x}$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {14, 15, 30}

$$\frac{3x^{2/3}}{2} + \frac{10}{11} \sqrt[5]{x^3} \sqrt{x}$$

Antiderivative was successfully verified.

[In] Int[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] (3*x^(2/3))/2 + (10*Sqrt[x]*(x^3)^(1/5))/11

Rule 14

Int[(u_)*((c_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_) + (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 30

Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[6]{x} + \sqrt[5]{x^3}}{\sqrt{x}} dx &= \int \left(\frac{1}{\sqrt[3]{x}} + \frac{\sqrt[5]{x^3}}{\sqrt{x}} \right) dx \\ &= \frac{3x^{2/3}}{2} + \int \frac{\sqrt[5]{x^3}}{\sqrt{x}} dx \\ &= \frac{3x^{2/3}}{2} + \frac{\sqrt[5]{x^3} \int \sqrt[10]{x} dx}{x^{3/5}} \\ &= \frac{3x^{2/3}}{2} + \frac{10}{11} \sqrt{x} \sqrt[5]{x^3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2} + \frac{10}{11} \sqrt[5]{x^3} \sqrt{x}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] $(3x^{2/3})/2 + (10\sqrt{x}(x^3)^{1/5})/11$

IntegrateAlgebraic [A] time = 6.64, size = 26, normalized size = 1.00

$$\frac{3x^{2/3}}{2} + \frac{10}{11}\sqrt[5]{x^3}\sqrt{x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(1/6) + (x^3)^(1/5))/Sqrt[x],x]

[Out] $(3x^{2/3})/2 + (10\sqrt{x}(x^3)^{1/5})/11$

fricas [A] time = 0.41, size = 16, normalized size = 0.62

$$\frac{10}{11}(x^3)^{1/5}\sqrt{x} + \frac{3}{2}x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="fricas")

[Out] $10/11*(x^3)^{1/5}*sqrt(x) + 3/2*x^{2/3}$

giac [A] time = 0.51, size = 11, normalized size = 0.42

$$\frac{10}{11}x^{11/10} + \frac{3}{2}x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="giac")

[Out] $10/11*x^{11/10} + 3/2*x^{2/3}$

maple [A] time = 0.00, size = 17, normalized size = 0.65

$$\frac{3x^{2/3}}{2} + \frac{10(x^3)^{1/5}\sqrt{x}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/6)+(x^3)^(1/5))/x^(1/2),x)

[Out] $3/2*x^{2/3}+10/11*(x^3)^{1/5}*x^{1/2}$

maxima [A] time = 0.96, size = 16, normalized size = 0.62

$$\frac{10}{11}(x^3)^{1/5}\sqrt{x} + \frac{3}{2}x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/6)+(x^3)^(1/5))/x^(1/2),x, algorithm="maxima")

[Out] $10/11*(x^3)^{1/5}*sqrt(x) + 3/2*x^{2/3}$

mupad [B] time = 3.54, size = 16, normalized size = 0.62

$$\frac{10\sqrt{x}(x^3)^{1/5}}{11} + \frac{3x^{2/3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^3)^(1/5) + x^(1/6))/x^(1/2),x)
```

```
[Out] (10*x^(1/2)*(x^3)^(1/5))/11 + (3*x^(2/3))/2
```

```
sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00
```

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**(1/6)+(x**3)**(1/5))/x**(1/2),x)
```

```
[Out] Timed out
```

$$3.615 \quad \int \frac{2+x}{\sqrt{4x-x^2}} dx$$

Optimal. Leaf size=26

$$-\sqrt{4x-x^2} - 4 \sin^{-1}\left(1 - \frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {640, 619, 216}

$$-\sqrt{4x-x^2} - 4 \sin^{-1}\left(1 - \frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[(2 + x)/Sqrt[4*x - x^2], x]

[Out] -Sqrt[4*x - x^2] - 4*ArcSin[1 - x/2]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\begin{aligned} \int \frac{2+x}{\sqrt{4x-x^2}} dx &= -\sqrt{4x-x^2} + 4 \int \frac{1}{\sqrt{4x-x^2}} dx \\ &= -\sqrt{4x-x^2} - \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} dx, x, 4-2x\right) \\ &= -\sqrt{4x-x^2} - 4 \sin^{-1}\left(1 - \frac{x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.03, size = 27, normalized size = 1.04

$$-\sqrt{-((x-4)x)} - 8 \sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(2 + x)/Sqrt[4*x - x^2], x]

[Out] -Sqrt[-((-4 + x)*x)] - 8*ArcSin[Sqrt[1 - x/4]]

IntegrateAlgebraic [A] time = 0.20, size = 36, normalized size = 1.38

$$-\sqrt{4x - x^2} - 8 \tan^{-1}\left(\frac{\sqrt{4x - x^2}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(2 + x)/Sqrt[4*x - x^2], x]

[Out] -Sqrt[4*x - x^2] - 8*ArcTan[Sqrt[4*x - x^2]/x]

fricas [A] time = 0.41, size = 32, normalized size = 1.23

$$-\sqrt{-x^2 + 4x} - 8 \arctan\left(\frac{\sqrt{-x^2 + 4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-x^2+4*x)^(1/2), x, algorithm="fricas")

[Out] -sqrt(-x^2 + 4*x) - 8*arctan(sqrt(-x^2 + 4*x)/x)

giac [A] time = 0.61, size = 22, normalized size = 0.85

$$-\sqrt{-x^2 + 4x} + 4 \arcsin\left(\frac{1}{2}x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-x^2+4*x)^(1/2), x, algorithm="giac")

[Out] -sqrt(-x^2 + 4*x) + 4*arcsin(1/2*x - 1)

maple [A] time = 0.01, size = 23, normalized size = 0.88

$$4 \arcsin\left(\frac{x}{2} - 1\right) - \sqrt{-x^2 + 4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+2)/(-x^2+4*x)^(1/2), x)

[Out] 4*arcsin(1/2*x-1)-(-x^2+4*x)^(1/2)

maxima [A] time = 1.93, size = 22, normalized size = 0.85

$$-\sqrt{-x^2 + 4x} - 4 \arcsin\left(-\frac{1}{2}x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+x)/(-x^2+4*x)^(1/2), x, algorithm="maxima")

[Out] -sqrt(-x^2 + 4*x) - 4*arcsin(-1/2*x + 1)

mupad [B] time = 3.53, size = 22, normalized size = 0.85

$$4 \operatorname{asin}\left(\frac{x}{2} - 1\right) - \sqrt{4x - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 2)/(4*x - x^2)^(1/2), x)

```
[Out] 4*asin(x/2 - 1) - (4*x - x^2)^(1/2)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{x+2}{\sqrt{-x(x-4)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2+x)/(-x**2+4*x)**(1/2),x)
```

```
[Out] Integral((x + 2)/sqrt(-x*(x - 4)), x)
```

$$3.616 \quad \int \frac{3+x}{\sqrt[3]{6x+x^2}} dx$$

Optimal. Leaf size=15

$$\frac{3}{4} (x^2 + 6x)^{2/3}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {629}

$$\frac{3}{4} (x^2 + 6x)^{2/3}$$

Antiderivative was successfully verified.

[In] Int[(3 + x)/(6*x + x^2)^(1/3), x]

[Out] (3*(6*x + x^2)^(2/3))/4

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{3+x}{\sqrt[3]{6x+x^2}} dx = \frac{3}{4} (6x+x^2)^{2/3}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 0.87

$$\frac{3}{4} (x(x+6))^{2/3}$$

Antiderivative was successfully verified.

[In] Integrate[(3 + x)/(6*x + x^2)^(1/3), x]

[Out] (3*(x*(6 + x))^(2/3))/4

IntegrateAlgebraic [A] time = 0.02, size = 13, normalized size = 0.87

$$\frac{3}{4} (x(x+6))^{2/3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(3 + x)/(6*x + x^2)^(1/3), x]

[Out] (3*(x*(6 + x))^(2/3))/4

fricas [A] time = 0.44, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^2 + 6x)^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3+x)/(x^2+6*x)^(1/3), x, algorithm="fricas")

[Out] $3/4*(x^2 + 6*x)^{(2/3)}$

giac [A] time = 0.43, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^2 + 6x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="giac")`

[Out] $3/4*(x^2 + 6*x)^{(2/3)}$

maple [A] time = 0.00, size = 16, normalized size = 1.07

$$\frac{3(x+6)x}{4(x^2+6x)^{\frac{1}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+3)/(x^2+6*x)^(1/3),x)`

[Out] $3/4*x*(x+6)/(x^2+6*x)^{(1/3)}$

maxima [A] time = 0.86, size = 11, normalized size = 0.73

$$\frac{3}{4} (x^2 + 6x)^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+x)/(x^2+6*x)^(1/3),x, algorithm="maxima")`

[Out] $3/4*(x^2 + 6*x)^{(2/3)}$

mupad [B] time = 3.56, size = 9, normalized size = 0.60

$$\frac{3(x(x+6))^{2/3}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x+3)/(6*x+x^2)^(1/3),x)`

[Out] $(3*(x*(x+6))^{(2/3)})/4$

sympy [A] time = 0.16, size = 12, normalized size = 0.80

$$\frac{3(x^2+6x)^{\frac{2}{3}}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3+x)/(x**2+6*x)**(1/3),x)`

[Out] $3*(x**2+6*x)**(2/3)/4$

$$3.617 \quad \int \frac{4+x}{(6x-x^2)^{3/2}} dx$$

Optimal. Leaf size=22

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {636}

$$-\frac{12-7x}{9\sqrt{6x-x^2}}$$

Antiderivative was successfully verified.

[In] Int[(4 + x)/(6*x - x^2)^(3/2), x]

[Out] -(12 - 7*x)/(9*Sqrt[6*x - x^2])

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rubi steps

$$\int \frac{4+x}{(6x-x^2)^{3/2}} dx = -\frac{12-7x}{9\sqrt{6x-x^2}}$$

Mathematica [A] time = 0.01, size = 19, normalized size = 0.86

$$\frac{7x-12}{9\sqrt{-((x-6)x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(4 + x)/(6*x - x^2)^(3/2), x]

[Out] (-12 + 7*x)/(9*Sqrt[-((-6 + x)*x)])

IntegrateAlgebraic [A] time = 0.22, size = 30, normalized size = 1.36

$$\frac{(12-7x)\sqrt{6x-x^2}}{9(x-6)x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(4 + x)/(6*x - x^2)^(3/2), x]

[Out] ((12 - 7*x)*Sqrt[6*x - x^2])/(9*(-6 + x)*x)

fricas [A] time = 0.40, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2+6x}(7x-12)}{9(x^2-6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="fricas")

[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)

giac [A] time = 0.52, size = 27, normalized size = 1.23

$$-\frac{\sqrt{-x^2 + 6x}(7x - 12)}{9(x^2 - 6x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="giac")

[Out] -1/9*sqrt(-x^2 + 6*x)*(7*x - 12)/(x^2 - 6*x)

maple [A] time = 0.00, size = 23, normalized size = 1.05

$$\frac{(x - 6)(7x - 12)x}{9(-x^2 + 6x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+4)/(-x^2+6*x)^(3/2),x)

[Out] -1/9*x*(x-6)*(-12+7*x)/(-x^2+6*x)^(3/2)

maxima [A] time = 0.88, size = 28, normalized size = 1.27

$$\frac{7x}{9\sqrt{-x^2 + 6x}} - \frac{4}{3\sqrt{-x^2 + 6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(-x^2+6*x)^(3/2),x, algorithm="maxima")

[Out] 7/9*x/sqrt(-x^2 + 6*x) - 4/3/sqrt(-x^2 + 6*x)

mupad [B] time = 3.50, size = 18, normalized size = 0.82

$$\frac{7x - 12}{9\sqrt{6x - x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + 4)/(6*x - x^2)^(3/2),x)

[Out] (7*x - 12)/(9*(6*x - x^2)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x + 4}{(-x(x - 6))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4+x)/(-x**2+6*x)**(3/2),x)

[Out] Integral((x + 4)/(-x*(x - 6))**(3/2), x)

$$3.618 \quad \int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] ArcTan[Sqrt[2*x + x^2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 688

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)\sqrt{2x+x^2}} dx &= 4 \text{Subst} \left(\int \frac{1}{4+4x^2} dx, x, \sqrt{2x+x^2} \right) \\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)*Sqrt[2*x + x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[2 + x]*ArcTan[Sqrt[x]/Sqrt[2 + x]])/Sqrt[x*(2 + x)]

IntegrateAlgebraic [A] time = 0.19, size = 18, normalized size = 1.50

$$-2 \tan^{-1}\left(\frac{\sqrt{x^2+2x}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[2*x + x^2]), x]

[Out] -2*ArcTan[Sqrt[2*x + x^2]/x]

fricas [A] time = 0.41, size = 17, normalized size = 1.42

$$2 \arctan\left(-x + \sqrt{x^2 + 2x - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="fricas")

[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)

giac [A] time = 0.59, size = 17, normalized size = 1.42

$$2 \arctan\left(-x + \sqrt{x^2 + 2x - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="giac")

[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)

maple [A] time = 0.01, size = 13, normalized size = 1.08

$$- \arctan\left(\frac{1}{\sqrt{(x+1)^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)/(x^2+2*x)^(1/2), x)

[Out] -arctan(1/((x+1)^2-1)^(1/2))

maxima [A] time = 1.94, size = 9, normalized size = 0.75

$$- \arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="maxima")

[Out] -arcsin(1/abs(x + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2 + 2x} (x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)

[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x**2+2*x)**(1/2), x)

[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)

$$3.619 \quad \int \frac{1}{(1+2x)\sqrt{x+x^2}} dx$$

Optimal. Leaf size=12

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {688, 203}

$$\tan^{-1}\left(2\sqrt{x^2+x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] ArcTan[2*Sqrt[x + x^2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 688

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+2x)\sqrt{x+x^2}} dx &= 4 \text{Subst} \left(\int \frac{1}{2+8x^2} dx, x, \sqrt{x+x^2} \right) \\ &= \tan^{-1}\left(2\sqrt{x+x^2}\right) \end{aligned}$$

Mathematica [B] time = 0.02, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+1}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+1}}\right)}{\sqrt{x(x+1)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[1 + x]*ArcTan[Sqrt[x]/Sqrt[1 + x]])/Sqrt[x*(1 + x)]

IntegrateAlgebraic [A] time = 0.21, size = 16, normalized size = 1.33

$$-2 \tan^{-1}\left(\frac{\sqrt{x^2+x}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 + 2*x)*Sqrt[x + x^2]),x]

[Out] -2*ArcTan[Sqrt[x + x^2]/x]

fricas [A] time = 0.41, size = 17, normalized size = 1.42

$$2 \arctan\left(-2x + 2\sqrt{x^2 + x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="fricas")

[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)

giac [A] time = 0.62, size = 17, normalized size = 1.42

$$2 \arctan\left(-2x + 2\sqrt{x^2 + x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")

[Out] 2*arctan(-2*x + 2*sqrt(x^2 + x) - 1)

maple [A] time = 0.01, size = 15, normalized size = 1.25

$$-\arctan\left(\frac{1}{\sqrt{4\left(x + \frac{1}{2}\right)^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x+1)/(x^2+x)^(1/2),x)

[Out] -arctan(1/(4*(x+1/2)^2-1)^(1/2))

maxima [A] time = 1.93, size = 11, normalized size = 0.92

$$-\arcsin\left(\frac{1}{|2x + 1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")

[Out] -arcsin(1/abs(2*x + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{(2x + 1)\sqrt{x^2 + x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x + 1)*(x + x^2)^(1/2)),x)

[Out] int(1/((2*x + 1)*(x + x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+1)}(2x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+2*x)/(x**2+x)**(1/2),x)
```

```
[Out] Integral(1/(sqrt(x*(x + 1))*(2*x + 1)), x)
```


$$3.620 \quad \int \frac{-1+x}{\sqrt{2x-x^2}} dx$$

Optimal. Leaf size=15

$$-\sqrt{2x-x^2}$$

Rubi [A] time = 0.00, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {629}

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/Sqrt[2*x - x^2], x]

[Out] -Sqrt[2*x - x^2]

Rule 629

Int[((d_) + (e_)*(x_))*((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{-1+x}{\sqrt{2x-x^2}} dx = -\sqrt{2x-x^2}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 0.80

$$-\sqrt{-((x-2)x)}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/Sqrt[2*x - x^2], x]

[Out] -Sqrt[-((-2 + x)*x)]

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.00

$$-\sqrt{2x-x^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x)/Sqrt[2*x - x^2], x]

[Out] -Sqrt[2*x - x^2]

fricas [A] time = 0.40, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(-x^2+2*x)^(1/2), x, algorithm="fricas")

[Out] -sqrt(-x^2 + 2*x)

giac [A] time = 0.60, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="giac")

[Out] -sqrt(-x^2 + 2*x)

maple [A] time = 0.00, size = 17, normalized size = 1.13

$$\frac{(x-2)x}{\sqrt{-x^2+2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(-x^2+2*x)^(1/2),x)

[Out] (x-2)*x/(-x^2+2*x)^(1/2)

maxima [A] time = 0.87, size = 13, normalized size = 0.87

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")

[Out] -sqrt(-x^2 + 2*x)

mupad [B] time = 3.64, size = 10, normalized size = 0.67

$$-\sqrt{-x(x-2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 1)/(2*x - x^2)^(1/2),x)

[Out] -(-x*(x - 2))^(1/2)

sympy [A] time = 0.14, size = 10, normalized size = 0.67

$$-\sqrt{-x^2 + 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/(-x**2+2*x)**(1/2),x)

[Out] -sqrt(-x**2 + 2*x)

$$3.621 \quad \int \frac{\sqrt{x-x^2}}{1+x} dx$$

Optimal. Leaf size=54

$$\sqrt{x-x^2} + \sqrt{2} \tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right) - \frac{3}{2} \sin^{-1}(1-2x)$$

Rubi [A] time = 0.04, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {734, 843, 619, 216, 724, 204}

$$\sqrt{x-x^2} + \sqrt{2} \tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right) - \frac{3}{2} \sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x - x^2]/(1 + x), x]

[Out] Sqrt[x - x^2] - (3*ArcSin[1 - 2*x])/2 + Sqrt[2]*ArcTan[(1 - 3*x)/(2*Sqrt[2]*Sqrt[x - x^2])]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 734

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((d + e*x)^(m + 1)*(a + b*x + c*x^2)^p)/(e*(m + 2*p + 1)), x] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && (!RationalQ[m] || LtQ[m, 1]) && !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 843

Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +

$c*x^2)^p, x], x] + \text{Dist}[(e*f - d*g)/e, \text{Int}[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, g, m, p\}, x] \&\& \text{NeQ}[b^2 - 4*a*c, 0] \&\& \text{NeQ}[c*d^2 - b*d*e + a*e^2, 0] \&\& !\text{IGtQ}[m, 0]$

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{x-x^2}}{1+x} dx &= \sqrt{x-x^2} - \frac{1}{2} \int \frac{1-3x}{(1+x)\sqrt{x-x^2}} dx \\ &= \sqrt{x-x^2} + \frac{3}{2} \int \frac{1}{\sqrt{x-x^2}} dx - 2 \int \frac{1}{(1+x)\sqrt{x-x^2}} dx \\ &= \sqrt{x-x^2} - \frac{3}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right) + 4 \text{Subst}\left(\int \frac{1}{-8-x^2} dx, x, \frac{-1+3x}{\sqrt{x-x^2}}\right) \\ &= \sqrt{x-x^2} - \frac{3}{2} \sin^{-1}(1-2x) + \sqrt{2} \tan^{-1}\left(\frac{1-3x}{2\sqrt{2}\sqrt{x-x^2}}\right) \end{aligned}$$

Mathematica [A] time = 0.08, size = 95, normalized size = 1.76

$$\sqrt{-(x-1)x} - \frac{3\sqrt{-(x-1)x} \sin^{-1}(\sqrt{1-x})}{\sqrt{1-x}\sqrt{x}} + \frac{2\sqrt{2}\sqrt{-(x-1)x} \tanh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{x-1}\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x - x^2]/(1 + x), x]

[Out] Sqrt[-((-1 + x)*x)] - (3*Sqrt[-((-1 + x)*x)]*ArcSin[Sqrt[1 - x]])/(Sqrt[1 - x]*Sqrt[x]) + (2*Sqrt[2]*Sqrt[-((-1 + x)*x)]*ArcTanh[Sqrt[-1 + x]]/(Sqrt[2]*Sqrt[x]))/(Sqrt[-1 + x]*Sqrt[x])

IntegrateAlgebraic [A] time = 0.21, size = 58, normalized size = 1.07

$$\sqrt{x-x^2} - 3 \tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right) + 2\sqrt{2} \tan^{-1}\left(\frac{\sqrt{x-x^2}}{\sqrt{2}x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x - x^2]/(1 + x), x]

[Out] Sqrt[x - x^2] - 3*ArcTan[Sqrt[x - x^2]/x] + 2*Sqrt[2]*ArcTan[Sqrt[x - x^2]/(Sqrt[2]*x)]

fricas [A] time = 0.42, size = 49, normalized size = 0.91

$$2\sqrt{2} \arctan\left(\frac{\sqrt{2}\sqrt{-x^2+x}}{2x}\right) + \sqrt{-x^2+x} - 3 \arctan\left(\frac{\sqrt{-x^2+x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+x)^(1/2)/(1+x), x, algorithm="fricas")

[Out] 2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + x)/x) + sqrt(-x^2 + x) - 3*arctan(sqrt(-x^2 + x)/x)

giac [A] time = 0.50, size = 53, normalized size = 0.98

$$2\sqrt{2} \arctan\left(\frac{1}{4}\sqrt{2}\left(\frac{3\left(2\sqrt{-x^2+x}-1\right)}{2x-1}-1\right)\right) + \sqrt{-x^2+x} + \frac{3}{2} \arcsin(2x-1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="giac")

[Out] 2*sqrt(2)*arctan(1/4*sqrt(2)*(3*(2*sqrt(-x^2 + x) - 1)/(2*x - 1) - 1)) + sqrt(-x^2 + x) + 3/2*arcsin(2*x - 1)

maple [A] time = 0.01, size = 54, normalized size = 1.00

$$\frac{3 \arcsin(2x - 1)}{2} - \sqrt{2} \arctan\left(\frac{(3x - 1)\sqrt{2}}{4\sqrt{3x - (x + 1)^2 + 1}}\right) + \sqrt{3x - (x + 1)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+x)^(1/2)/(x+1),x)

[Out] -(x+1)^2+3*x+1)^(1/2)+3/2*arcsin(2*x-1)-2^(1/2)*arctan(1/4*(3*x-1)*2^(1/2)/(-(x+1)^2+3*x+1)^(1/2))

maxima [A] time = 1.91, size = 42, normalized size = 0.78

$$-\sqrt{2} \arcsin\left(\frac{3x}{|x+1|} - \frac{1}{|x+1|}\right) + \sqrt{-x^2 + x} + \frac{3}{2} \arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+x)^(1/2)/(1+x),x, algorithm="maxima")

[Out] -sqrt(2)*arcsin(3*x/abs(x + 1) - 1/abs(x + 1)) + sqrt(-x^2 + x) + 3/2*arcsin(2*x - 1)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x - x^2}}{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - x^2)^(1/2)/(x + 1),x)

[Out] int((x - x^2)^(1/2)/(x + 1), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x(x-1)}}{x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+x)**(1/2)/(1+x),x)

[Out] Integral(sqrt(-x*(x - 1))/(x + 1), x)

$$3.622 \quad \int \sqrt{\sqrt[4]{x} + x} dx$$

Optimal. Leaf size=59

$$\frac{2}{3} \sqrt{x + \sqrt[4]{x} x} + \frac{1}{3} \sqrt{x + \sqrt[4]{x}} \sqrt[4]{x} - \frac{1}{3} \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{x + \sqrt[4]{x}}} \right)$$

Rubi [A] time = 0.07, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {2004, 2018, 2024, 2029, 206}

$$\frac{2}{3} \sqrt{x + \sqrt[4]{x} x} + \frac{1}{3} \sqrt{x + \sqrt[4]{x}} \sqrt[4]{x} - \frac{1}{3} \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{x + \sqrt[4]{x}}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^(1/4) + x], x]

[Out] (x^(1/4)*Sqrt[x^(1/4) + x])/3 + (2*x*Sqrt[x^(1/4) + x])/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]/3

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2004

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> Simp[(x*(a*x^j + b*x^n)^p)/(n*p + 1), x] + Dist[(a*(n - j)*p)/(n*p + 1), Int[x^j*(a*x^j + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] && !IntegerQ[p] && LtQ[0, j, n] && GtQ[p, 0] && NeQ[n*p + 1, 0]

Rule 2018

Int[(x_)^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]

Rule 2024

Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^p], x_Symbol] :> Simp[(c^(n - 1)*(c*x)^(m - n + 1)*(a*x^j + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[(a*c^(n - j)*(m + j*p - n + j + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x] && !IntegerQ[p] && LtQ[0, j, n] && (IntegerQ[j, n] || GtQ[c, 0]) && GtQ[m + j*p + 1 - n + j, 0] && NeQ[m + n*p + 1, 0]

Rule 2029

Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Dist[-2/(n - j), Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]], x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]

Rubi steps

$$\begin{aligned}
\int \sqrt{\sqrt[4]{x} + x} dx &= \frac{2}{3}x\sqrt{\sqrt[4]{x} + x} + \frac{1}{4} \int \frac{\sqrt[4]{x}}{\sqrt{\sqrt[4]{x} + x}} dx \\
&= \frac{2}{3}x\sqrt{\sqrt[4]{x} + x} + \text{Subst}\left(\int \frac{x^4}{\sqrt{x + x^4}} dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x} + x} + \frac{2}{3}x\sqrt{\sqrt[4]{x} + x} - \frac{1}{2}\text{Subst}\left(\int \frac{x}{\sqrt{x + x^4}} dx, x, \sqrt[4]{x}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x} + x} + \frac{2}{3}x\sqrt{\sqrt[4]{x} + x} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{1 - x^2} dx, x, \frac{\sqrt{x}}{\sqrt{\sqrt[4]{x} + x}}\right) \\
&= \frac{1}{3}\sqrt[4]{x}\sqrt{\sqrt[4]{x} + x} + \frac{2}{3}x\sqrt{\sqrt[4]{x} + x} - \frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{\sqrt[4]{x} + x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 57, normalized size = 0.97

$$\frac{3x^{5/4} - \sqrt{x^{3/4} + 1}\sqrt[8]{x} \sinh^{-1}(x^{3/8}) + 2x^2 + \sqrt{x}}{3\sqrt{x + \sqrt[4]{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^(1/4) + x], x]

[Out] (Sqrt[x] + 3*x^(5/4) + 2*x^2 - Sqrt[1 + x^(3/4)]*x^(1/8)*ArcSinh[x^(3/8)])/(3*Sqrt[x^(1/4) + x])

IntegrateAlgebraic [A] time = 0.43, size = 47, normalized size = 0.80

$$\frac{1}{3}\sqrt{x + \sqrt[4]{x}}(2x + \sqrt[4]{x}) - \frac{1}{3}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x + \sqrt[4]{x}}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^(1/4) + x], x]

[Out] (Sqrt[x^(1/4) + x]*(x^(1/4) + 2*x))/3 - ArcTanh[Sqrt[x]/Sqrt[x^(1/4) + x]]/3

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/4)+x)^(1/2), x, algorithm="fricas")

[Out] Timed out

giac [A] time = 4.57, size = 45, normalized size = 0.76

$$\frac{1}{3}\sqrt{x + x^{1/4}x^{1/4}}(2x^{3/4} + 1) - \frac{1}{6}\log\left(\sqrt{\frac{1}{x^{3/4}} + 1} + 1\right) + \frac{1}{6}\log\left(\left|\sqrt{\frac{1}{x^{3/4}} + 1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="giac")

[Out] 1/3*sqrt(x + x^(1/4))*x^(1/4)*(2*x^(3/4) + 1) - 1/6*log(sqrt(1/x^(3/4) + 1) + 1) + 1/6*log(abs(sqrt(1/x^(3/4) + 1) - 1))

maple [C] time = 0.10, size = 342, normalized size = 5.80

$$\frac{2\sqrt{x+x^{\frac{1}{4}}}}{3} + \frac{\sqrt{x+x^{\frac{1}{4}}}}{3} + \frac{\left(\frac{1}{2}-i\sqrt{3}\right)\sqrt{\left(\frac{3+i\sqrt{3}}{2}\right)^{\frac{1}{4}}}}{\sqrt{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}} \left(x^{\frac{1}{4}}+1\right)^2 \sqrt{\frac{\frac{1}{4}-\frac{1}{2}+i\sqrt{3}}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}} \sqrt{\frac{\frac{1}{4}-\frac{1}{2}-i\sqrt{3}}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}} \left(-\operatorname{EllipticF}\left(\sqrt{\frac{\left(\frac{3+i\sqrt{3}}{2}\right)^{\frac{1}{4}}}}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}, \sqrt{\frac{\left(\frac{3+i\sqrt{3}}{2}\right)^{\frac{1}{4}}\left(\frac{1}{2}-i\sqrt{3}\right)}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}}\right)\right) + \operatorname{EllipticPi}\left(\sqrt{\frac{\left(\frac{3+i\sqrt{3}}{2}\right)^{\frac{1}{4}}}}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}, \frac{1+i\sqrt{3}}{2}, \sqrt{\frac{\left(\frac{3+i\sqrt{3}}{2}\right)^{\frac{1}{4}}\left(\frac{1}{2}-i\sqrt{3}\right)}{\left(\frac{1}{2}+i\sqrt{3}\right)^{\frac{1}{4}}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/4)+x)^(1/2),x)

[Out] 2/3*x*(x^(1/4)+x)^(1/2)+1/3*x^(1/4)*(x^(1/4)+x)^(1/2)+(-1/2-1/2*I*3^(1/2))*((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2)))/(x^(1/4)+1)^(1/2)*(x^(1/4)+1)^2*(-(x^(1/4)-1/2+1/2*I*3^(1/2))/(1/2-1/2*I*3^(1/2)))/(x^(1/4)+1)^(1/2)*(-(x^(1/4)-1/2-1/2*I*3^(1/2))/(1/2+1/2*I*3^(1/2)))/(x^(1/4)+1)^(1/2)/(3/2+1/2*I*3^(1/2))/(x^(1/4)*x^(1/4)+1)*(x^(1/4)-1/2+1/2*I*3^(1/2))*(x^(1/4)-1/2-1/2*I*3^(1/2))^(1/2)*(-EllipticF(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2)))/(x^(1/4)+1)^(1/2),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2)))/(-1/2+1/2*I*3^(1/2)))/(-3/2-1/2*I*3^(1/2)))^(1/2))+EllipticPi(((3/2+1/2*I*3^(1/2))*x^(1/4)/(1/2+1/2*I*3^(1/2)))/(x^(1/4)+1)^(1/2),(1/2+1/2*I*3^(1/2))/(3/2+1/2*I*3^(1/2)),((-3/2+1/2*I*3^(1/2))*(-1/2-1/2*I*3^(1/2)))/(-1/2+1/2*I*3^(1/2)))/(-3/2-1/2*I*3^(1/2)))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + x^{\frac{1}{4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^(1/4)+x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x + x^(1/4)), x)

mupad [B] time = 3.53, size = 27, normalized size = 0.46

$$\frac{8x\sqrt{x+x^{\frac{1}{4}}}}{9\sqrt{x^{\frac{3}{4}}+1}} {}_2F_1\left(-\frac{1}{2}, \frac{3}{2}; \frac{5}{2}; -x^{\frac{3}{4}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^(1/4))^(1/2),x)

[Out] (8*x*(x + x^(1/4))^(1/2)*hypergeom([-1/2, 3/2], 5/2, -x^(3/4)))/(9*(x^(3/4) + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[4]{x} + x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x**(1/4)+x)**(1/2),x)

[Out] Integral(sqrt(x**(1/4) + x), x)

3.623 $\int \sqrt{x + x^{3/2}} dx$

Optimal. Leaf size=59

$$\frac{4(x^{3/2} + x)^{3/2}}{7\sqrt{x}} - \frac{16(x^{3/2} + x)^{3/2}}{35x} + \frac{32(x^{3/2} + x)^{3/2}}{105x^{3/2}}$$

Rubi [A] time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {2002, 2016, 2014}

$$\frac{4(x^{3/2} + x)^{3/2}}{7\sqrt{x}} - \frac{16(x^{3/2} + x)^{3/2}}{35x} + \frac{32(x^{3/2} + x)^{3/2}}{105x^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x + x^(3/2)], x]

[Out] (32*(x + x^(3/2))^(3/2))/(105*x^(3/2)) - (16*(x + x^(3/2))^(3/2))/(35*x) + (4*(x + x^(3/2))^(3/2))/(7*Sqrt[x])

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(m + j*p + 1)), x] - Dist[(b*(m + n*p + n - j + 1))/(a*c^(n - j)*(m + j*p + 1)), Int[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{x + x^{3/2}} dx &= \frac{4(x + x^{3/2})^{3/2}}{7\sqrt{x}} - \frac{4}{7} \int \frac{\sqrt{x + x^{3/2}}}{\sqrt{x}} dx \\ &= -\frac{16(x + x^{3/2})^{3/2}}{35x} + \frac{4(x + x^{3/2})^{3/2}}{7\sqrt{x}} + \frac{8}{35} \int \frac{\sqrt{x + x^{3/2}}}{x} dx \\ &= \frac{32(x + x^{3/2})^{3/2}}{105x^{3/2}} - \frac{16(x + x^{3/2})^{3/2}}{35x} + \frac{4(x + x^{3/2})^{3/2}}{7\sqrt{x}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 39, normalized size = 0.66

$$\frac{4(\sqrt{x} + 1)(15x - 12\sqrt{x} + 8)\sqrt{x^{3/2} + x}}{105\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x + x^(3/2)], x]

[Out] (4*(1 + Sqrt[x])*(8 - 12*Sqrt[x] + 15*x)*Sqrt[x + x^(3/2)])/(105*Sqrt[x])

IntegrateAlgebraic [A] time = 0.03, size = 39, normalized size = 0.66

$$\frac{4\sqrt{x^{3/2} + x}(15x^{3/2} + 3x - 4\sqrt{x} + 8)}{105\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x + x^(3/2)], x]

[Out] (4*Sqrt[x + x^(3/2)]*(8 - 4*Sqrt[x] + 3*x + 15*x^(3/2)))/(105*Sqrt[x])

fricas [A] time = 0.85, size = 30, normalized size = 0.51

$$\frac{4(15x^2 + (3x + 8)\sqrt{x} - 4x)\sqrt{x^2 + x}}{105x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(3/2))^(1/2), x, algorithm="fricas")

[Out] 4/105*(15*x^2 + (3*x + 8)*sqrt(x) - 4*x)*sqrt(x^(3/2) + x)/x

giac [A] time = 0.40, size = 33, normalized size = 0.56

$$\frac{4}{105} \left(15(\sqrt{x} + 1)^{\frac{7}{2}} - 42(\sqrt{x} + 1)^{\frac{5}{2}} + 35(\sqrt{x} + 1)^{\frac{3}{2}} - 8 \right) \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(3/2))^(1/2), x, algorithm="giac")

[Out] 4/105*(15*(sqrt(x) + 1)^(7/2) - 42*(sqrt(x) + 1)^(5/2) + 35*(sqrt(x) + 1)^(3/2) - 8)*sgn(x)

maple [A] time = 0.01, size = 28, normalized size = 0.47

$$\frac{4\sqrt{x^{\frac{3}{2}} + x}(\sqrt{x} + 1)(15x - 12\sqrt{x} + 8)}{105\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x+x^(3/2))^(1/2), x)

[Out] 4/105*(x+x^(3/2))^(1/2)*(x^(1/2)+1)*(15*x-12*x^(1/2)+8)/x^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x^(3/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(x^(3/2) + x), x)

mupad [B] time = 3.54, size = 27, normalized size = 0.46

$$\frac{2x\sqrt{x+x^{3/2}} {}_2F_1\left(-\frac{1}{2}, 3; 4; -\sqrt{x}\right)}{3\sqrt{\sqrt{x}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x + x^(3/2))^(1/2),x)

[Out] (2*x*(x + x^(3/2))^(1/2)*hypergeom([-1/2, 3], 4, -x^(1/2)))/(3*(x^(1/2) + 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x+x**(3/2))**(1/2),x)

[Out] Integral(sqrt(x**(3/2) + x), x)

3.624 $\int x\sqrt{x+x^{3/2}} dx$

Optimal. Leaf size=94

$$\frac{4}{11}\sqrt{x}(x^{3/2}+x)^{3/2} + \frac{64(x^{3/2}+x)^{3/2}}{231\sqrt{x}} - \frac{256(x^{3/2}+x)^{3/2}}{1155x} + \frac{512(x^{3/2}+x)^{3/2}}{3465x^{3/2}} - \frac{32}{99}(x^{3/2}+x)^{3/2}$$

Rubi [A] time = 0.09, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2016, 2002, 2014}

$$\frac{4}{11}\sqrt{x}(x^{3/2}+x)^{3/2} + \frac{64(x^{3/2}+x)^{3/2}}{231\sqrt{x}} - \frac{256(x^{3/2}+x)^{3/2}}{1155x} + \frac{512(x^{3/2}+x)^{3/2}}{3465x^{3/2}} - \frac{32}{99}(x^{3/2}+x)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x*Sqrt[x + x^(3/2)], x]

[Out] (-32*(x + x^(3/2))^(3/2))/99 + (512*(x + x^(3/2))^(3/2))/(3465*x^(3/2)) - (256*(x + x^(3/2))^(3/2))/(1155*x) + (64*(x + x^(3/2))^(3/2))/(231*Sqrt[x]) + (4*Sqrt[x]*(x + x^(3/2))^(3/2))/11

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p+1)/(a*(j*p+1)*x^(j-1)), x] - Dist[(b*(n*p+n-j+1))/(a*(j*p+1)), Int[x^(n-j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p+n-j+1)/(n-j)], 0] && NeQ[j*p+1, 0]

Rule 2014

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(n-j)*(p+1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m+n*p+n-j+1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rule 2016

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(c^(j-1)*(c*x)^(m-j+1)*(a*x^j + b*x^n)^(p+1))/(a*(m+j*p+1)), x] - Dist[(b*(m+n*p+n-j+1))/(a*c^(n-j)*(m+j*p+1)), Int[(c*x)^(m+n-j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m+n*p+n-j+1)/(n-j)], 0] && NeQ[m+j*p+1, 0] && (IntegersQ[j, n] || GtQ[c, 0])

Rubi steps

$$\begin{aligned}
\int x\sqrt{x+x^{3/2}} dx &= \frac{4}{11}\sqrt{x}(x+x^{3/2})^{3/2} - \frac{8}{11}\int\sqrt{x}\sqrt{x+x^{3/2}} dx \\
&= -\frac{32}{99}(x+x^{3/2})^{3/2} + \frac{4}{11}\sqrt{x}(x+x^{3/2})^{3/2} + \frac{16}{33}\int\sqrt{x+x^{3/2}} dx \\
&= -\frac{32}{99}(x+x^{3/2})^{3/2} + \frac{64(x+x^{3/2})^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}(x+x^{3/2})^{3/2} - \frac{64}{231}\int\frac{\sqrt{x+x^{3/2}}}{\sqrt{x}} dx \\
&= -\frac{32}{99}(x+x^{3/2})^{3/2} - \frac{256(x+x^{3/2})^{3/2}}{1155x} + \frac{64(x+x^{3/2})^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}(x+x^{3/2})^{3/2} + \frac{128}{1155}\int\frac{\sqrt{x+x^{3/2}}}{x} dx \\
&= -\frac{32}{99}(x+x^{3/2})^{3/2} + \frac{512(x+x^{3/2})^{3/2}}{3465x^{3/2}} - \frac{256(x+x^{3/2})^{3/2}}{1155x} + \frac{64(x+x^{3/2})^{3/2}}{231\sqrt{x}} + \frac{4}{11}\sqrt{x}(x+x^{3/2})^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 51, normalized size = 0.54

$$\frac{4(\sqrt{x}+1)\sqrt{x^{3/2}+x}(-280x^{3/2}+315x^2+240x-192\sqrt{x}+128)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[x*Sqrt[x + x^(3/2)], x]

[Out] (4*(1 + Sqrt[x])*Sqrt[x + x^(3/2)]*(128 - 192*Sqrt[x] + 240*x - 280*x^(3/2) + 315*x^2))/(3465*Sqrt[x])

IntegrateAlgebraic [A] time = 0.04, size = 51, normalized size = 0.54

$$\frac{4\sqrt{x^{3/2}+x}(315x^{5/2}-40x^{3/2}+35x^2+48x-64\sqrt{x}+128)}{3465\sqrt{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x*Sqrt[x + x^(3/2)], x]

[Out] (4*Sqrt[x + x^(3/2)]*(128 - 64*Sqrt[x] + 48*x - 40*x^(3/2) + 35*x^2 + 315*x^(5/2)))/(3465*Sqrt[x])

fricas [A] time = 0.87, size = 40, normalized size = 0.43

$$\frac{4(315x^3 - 40x^2 + (35x^2 + 48x + 128)\sqrt{x} - 64x)\sqrt{x^{\frac{3}{2}} + x}}{3465x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x+x^(3/2))^(1/2), x, algorithm="fricas")

[Out] 4/3465*(315*x^3 - 40*x^2 + (35*x^2 + 48*x + 128)*sqrt(x) - 64*x)*sqrt(x^(3/2) + x)/x

giac [A] time = 0.38, size = 51, normalized size = 0.54

$$\frac{4}{3465}\left(315(\sqrt{x}+1)^{\frac{11}{2}}-1540(\sqrt{x}+1)^{\frac{9}{2}}+2970(\sqrt{x}+1)^{\frac{7}{2}}-2772(\sqrt{x}+1)^{\frac{5}{2}}+1155(\sqrt{x}+1)^{\frac{3}{2}}-128\right)\operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(x+x^(3/2))^(1/2), x, algorithm="giac")

[Out] $4/3465*(315*(\sqrt{x} + 1)^{(11/2)} - 1540*(\sqrt{x} + 1)^{(9/2)} + 2970*(\sqrt{x} + 1)^{(7/2)} - 2772*(\sqrt{x} + 1)^{(5/2)} + 1155*(\sqrt{x} + 1)^{(3/2)} - 128)*\operatorname{sgn}(x)$

maple [A] time = 0.00, size = 38, normalized size = 0.40

$$\frac{4\sqrt{x^{\frac{3}{2}} + x} (\sqrt{x} + 1) (315x^2 - 280x^{\frac{3}{2}} + 240x - 192\sqrt{x} + 128)}{3465\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x^(3/2)+x)^(1/2),x)`

[Out] $4/3465*(x^{(3/2)}+x)^{(1/2)}*(x^{(1/2)}+1)*(315*x^2-280*x^{(3/2)}+240*x-192*x^{(1/2)}+128)/x^{(1/2)}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^{\frac{3}{2}} + x} x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x+x^(3/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x^(3/2) + x)*x, x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int x \sqrt{x + x^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(x + x^(3/2))^(1/2),x)`

[Out] `int(x*(x + x^(3/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{x^{\frac{3}{2}} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(x+x**(3/2))**(1/2),x)`

[Out] `Integral(x*sqrt(x**(3/2) + x), x)`

$$3.625 \quad \int (1 - x^2) \sqrt{\frac{1}{2-x^2}} dx$$

Optimal. Leaf size=18

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6720, 383}

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]

[Out] x/(2*Sqrt[(2 - x^2)^(-1)])

Rule 383

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Simp[(c*x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d - b*c*(n*(p + 1) + 1), 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int (1 - x^2) \sqrt{\frac{1}{2-x^2}} dx &= \left(\sqrt{\frac{1}{2-x^2}} \sqrt{2-x^2} \right) \int \frac{1-x^2}{\sqrt{2-x^2}} dx \\ &= \frac{x}{2\sqrt{\frac{1}{2-x^2}}} \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{x}{2\sqrt{\frac{1}{2-x^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^2)*Sqrt[(2 - x^2)^(-1)],x]

[Out] x/(2*Sqrt[(2 - x^2)^(-1)])

IntegrateAlgebraic [A] time = 0.05, size = 23, normalized size = 1.28

$$-\frac{1}{2}x\sqrt{\frac{1}{2-x^2}}(x^2-2)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x^2)*Sqrt[(2 - x^2)^(-1)], x]

[Out] -1/2*(x*Sqrt[(2 - x^2)^(-1)]*(-2 + x^2))

fricas [A] time = 1.01, size = 20, normalized size = 1.11

$$-\frac{1}{2}(x^3 - 2x)\sqrt{-\frac{1}{x^2 - 2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2), x, algorithm="fricas")

[Out] -1/2*(x^3 - 2*x)*sqrt(-1/(x^2 - 2))

giac [A] time = 0.48, size = 18, normalized size = 1.00

$$-\frac{1}{2}\sqrt{-x^2 + 2}x\operatorname{sgn}(x^2 - 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2), x, algorithm="giac")

[Out] -1/2*sqrt(-x^2 + 2)*x*sgn(x^2 - 2)

maple [A] time = 0.01, size = 20, normalized size = 1.11

$$-\frac{(x^2 - 2)\sqrt{-\frac{1}{x^2 - 2}}x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)*(1/(-x^2+2))^(1/2), x)

[Out] -1/2*(x^2-2)*x*(-1/(x^2-2))^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int (x^2 - 1)\sqrt{-\frac{1}{x^2 - 2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)*(1/(-x^2+2))^(1/2), x, algorithm="maxima")

[Out] -integrate((x^2 - 1)*sqrt(-1/(x^2 - 2)), x)

mupad [B] time = 3.50, size = 19, normalized size = 1.06

$$-\frac{x(x^2 - 2)\sqrt{-\frac{1}{x^2 - 2}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)*(-1/(x^2 - 2))^(1/2), x)

[Out] -(x*(x^2 - 2)*(-1/(x^2 - 2))^(1/2))/2

sympy [B] time = 0.50, size = 26, normalized size = 1.44

$$-\frac{x^3\sqrt{\frac{1}{2-x^2}}}{2} + x\sqrt{\frac{1}{2-x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)*(1/(-x**2+2))**(1/2),x)
```

```
[Out] -x**3*sqrt(1/(2 - x**2))/2 + x*sqrt(1/(2 - x**2))
```

3.626 $\int \sqrt{x^2 + x^3 - x^4} dx$

Optimal. Leaf size=107

$$-\frac{\sqrt{-x^4 + x^3 + x^2}(1 - 2x)}{8x} - \frac{(-x^2 + x + 1)\sqrt{-x^4 + x^3 + x^2}}{3x} - \frac{5\sqrt{-x^4 + x^3 + x^2} \sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2 + x + 1}}$$

Rubi [A] time = 0.03, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {1903, 640, 612, 619, 216}

$$-\frac{\sqrt{-x^4 + x^3 + x^2}(1 - 2x)}{8x} - \frac{(-x^2 + x + 1)\sqrt{-x^4 + x^3 + x^2}}{3x} - \frac{5\sqrt{-x^4 + x^3 + x^2} \sin^{-1}\left(\frac{1-2x}{\sqrt{5}}\right)}{16x\sqrt{-x^2 + x + 1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + x^3 - x^4], x]

[Out] -((1 - 2*x)*Sqrt[x^2 + x^3 - x^4])/(8*x) - ((1 + x - x^2)*Sqrt[x^2 + x^3 - x^4])/(3*x) - (5*Sqrt[x^2 + x^3 - x^4]*ArcSin[(1 - 2*x)/Sqrt[5]])/(16*x*Sqrt[1 + x - x^2])

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 1903

Int[Sqrt[(b_.)*(x_)^(n_.) + (a_.)*(x_)^(q_.) + (c_.)*(x_)^(r_.)], x_Symbol] := Dist[Sqrt[a*x^q + b*x^n + c*x^(2*n - q)]/(x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))]), Int[x^(q/2)*Sqrt[a + b*x^(n - q) + c*x^(2*(n - q))], x], x] /; FreeQ[{a, b, c, n, q}, x] && EqQ[r, 2*n - q] && PosQ[n - q]

Rubi steps

$$\begin{aligned}
\int \sqrt{x^2 + x^3 - x^4} dx &= \frac{\sqrt{x^2 + x^3 - x^4} \int x\sqrt{1+x-x^2} dx}{x\sqrt{1+x-x^2}} \\
&= -\frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} + \frac{\sqrt{x^2+x^3-x^4} \int \sqrt{1+x-x^2} dx}{2x\sqrt{1+x-x^2}} \\
&= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} + \frac{(5\sqrt{x^2+x^3-x^4}) \int \frac{1}{\sqrt{1+x-x^2}}}{16x\sqrt{1+x-x^2}} \\
&= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} - \frac{(\sqrt{5}\sqrt{x^2+x^3-x^4}) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1+x-x^2}} \right)}{16x\sqrt{1+x-x^2}} \\
&= -\frac{(1-2x)\sqrt{x^2+x^3-x^4}}{8x} - \frac{(1+x-x^2)\sqrt{x^2+x^3-x^4}}{3x} - \frac{5\sqrt{x^2+x^3-x^4} \sin^{-1} \left(\frac{1-2x}{\sqrt{5}} \right)}{16x\sqrt{1+x-x^2}}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 84, normalized size = 0.79

$$\frac{\sqrt{-x^4 + x^3 + x^2} \left(2\sqrt{x^2 - x - 1} (8x^2 - 2x - 11) - 15 \tanh^{-1} \left(\frac{2x-1}{2\sqrt{x^2-x-1}} \right) \right)}{48x\sqrt{x^2 - x - 1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2 + x^3 - x^4], x]

[Out] (Sqrt[x^2 + x^3 - x^4]*(2*Sqrt[-1 - x + x^2]*(-11 - 2*x + 8*x^2) - 15*ArcTanh[(-1 + 2*x)/(2*Sqrt[-1 - x + x^2])]))/(48*x*Sqrt[-1 - x + x^2])

IntegrateAlgebraic [A] time = 0.18, size = 63, normalized size = 0.59

$$\frac{(8x^2 - 2x - 11)\sqrt{-x^4 + x^3 + x^2}}{24x} - \frac{5}{8} \tan^{-1} \left(\frac{\sqrt{-x^4 + x^3 + x^2} - x}{x^2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^2 + x^3 - x^4], x]

[Out] ((-11 - 2*x + 8*x^2)*Sqrt[x^2 + x^3 - x^4])/(24*x) - (5*ArcTan[(-x + Sqrt[x^2 + x^3 - x^4])/x^2])/8

fricas [A] time = 0.60, size = 62, normalized size = 0.58

$$\frac{15x \arctan \left(-\frac{x - \sqrt{-x^4 + x^3 + x^2}}{x^2} \right) - \sqrt{-x^4 + x^3 + x^2} (8x^2 - 2x - 11) + 11x}{24x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^3+x^2)^(1/2), x, algorithm="fricas")

[Out] -1/24*(15*x*arctan(-(x - sqrt(-x^4 + x^3 + x^2))/x^2) - sqrt(-x^4 + x^3 + x^2)*(8*x^2 - 2*x - 11) + 11*x)/x

giac [A] time = 0.43, size = 60, normalized size = 0.56

$$\frac{1}{48} \left(15 \arcsin \left(\frac{1}{5} \sqrt{5} \right) + 22 \right) \operatorname{sgn}(x) + \frac{5}{16} \arcsin \left(\frac{1}{5} \sqrt{5} (2x-1) \right) \operatorname{sgn}(x) + \frac{1}{24} (2(4x \operatorname{sgn}(x) - \operatorname{sgn}(x))x - 11 \operatorname{sgn}(x)) \sqrt{-x^2 + x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="giac")

[Out] 1/48*(15*arcsin(1/5*sqrt(5)) + 22)*sgn(x) + 5/16*arcsin(1/5*sqrt(5)*(2*x - 1))*sgn(x) + 1/24*(2*(4*x*sgn(x) - sgn(x))*x - 11*sgn(x))*sqrt(-x^2 + x + 1)

maple [A] time = 0.01, size = 81, normalized size = 0.76

$$\frac{\sqrt{-x^4 + x^3 + x^2} \left(-12\sqrt{-x^2 + x + 1} x - 15 \arcsin\left(\frac{(2x-1)\sqrt{5}}{5}\right) + 16(-x^2 + x + 1)^{\frac{3}{2}} + 6\sqrt{-x^2 + x + 1} \right)}{48\sqrt{-x^2 + x + 1} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+x^3+x^2)^(1/2),x)

[Out] -1/48*(-x^4+x^3+x^2)^(1/2)*(16*(-x^2+x+1)^(3/2)-12*x*(-x^2+x+1)^(1/2)+6*(-x^2+x+1)^(1/2)-15*arcsin(1/5*(2*x-1)*5^(1/2)))/x/(-x^2+x+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^3 + x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+x^3+x^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^4 + x^3 + x^2), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{-x^4 + x^3 + x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3 - x^4)^(1/2),x)

[Out] int((x^2 + x^3 - x^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^4 + x^3 + x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+x**3+x**2)**(1/2),x)

[Out] Integral(sqrt(-x**4 + x**3 + x**2), x)

$$3.627 \quad \int \frac{1}{\sqrt{(a^2+x^2)^3}} dx$$

Optimal. Leaf size=25

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6720, 191}

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a^2 + x^2)^3],x]

[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])

Rule 191

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1))/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]

Rule 6720

Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a*v^m)^FracPart[p])/v^(m*FracPart[p]), Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p}, x] && !IntegerQ[p] && !FreeQ[v, x] && !(EqQ[a, 1] && EqQ[m, 1]) && !(EqQ[v, x] && EqQ[m, 1])

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(a^2+x^2)^3}} dx &= \frac{(a^2+x^2)^{3/2} \int \frac{1}{(a^2+x^2)^{3/2}} dx}{\sqrt{(a^2+x^2)^3}} \\ &= \frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 1.00

$$\frac{x(a^2+x^2)}{a^2\sqrt{(a^2+x^2)^3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a^2 + x^2)^3],x]

[Out] (x*(a^2 + x^2))/(a^2*Sqrt[(a^2 + x^2)^3])

IntegrateAlgebraic [A] time = 0.06, size = 41, normalized size = 1.64

$$\frac{x\sqrt{a^6 + 3a^4x^2 + 3a^2x^4 + x^6}}{a^2(a^2 + x^2)^2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a^2 + x^2)^3], x]

[Out] (x*Sqrt[a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6])/(a^2*(a^2 + x^2)^2)

fricas [B] time = 0.80, size = 64, normalized size = 2.56

$$\frac{a^4 + 2a^2x^2 + x^4 + \sqrt{a^6 + 3a^4x^2 + 3a^2x^4 + x^6}x}{a^6 + 2a^4x^2 + a^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+x^2)^3)^(1/2), x, algorithm="fricas")

[Out] (a^4 + 2*a^2*x^2 + x^4 + sqrt(a^6 + 3*a^4*x^2 + 3*a^2*x^4 + x^6)*x)/(a^6 + 2*a^4*x^2 + a^2*x^4)

giac [A] time = 0.42, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2 + x^2} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+x^2)^3)^(1/2), x, algorithm="giac")

[Out] x/(sqrt(a^2 + x^2)*a^2)

maple [A] time = 0.00, size = 24, normalized size = 0.96

$$\frac{(a^2 + x^2)x}{\sqrt{(a^2 + x^2)^3} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a^2+x^2)^3)^(1/2), x)

[Out] x*(a^2+x^2)/a^2/((a^2+x^2)^3)^(1/2)

maxima [A] time = 0.90, size = 14, normalized size = 0.56

$$\frac{x}{\sqrt{a^2 + x^2} a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a^2+x^2)^3)^(1/2), x, algorithm="maxima")

[Out] x/(sqrt(a^2 + x^2)*a^2)

mupad [B] time = 3.51, size = 25, normalized size = 1.00

$$\frac{x\sqrt{(a^2 + x^2)^3}}{a^2(a^2 + x^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a^2 + x^2)^3)^(1/2), x)`

[Out] `(x*((a^2 + x^2)^3)^(1/2))/(a^2*(a^2 + x^2)^2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a^2 + x^2)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((a**2+x**2)**3)**(1/2), x)`

[Out] `Integral(1/sqrt((a**2 + x**2)**3), x)`

$$3.628 \quad \int \frac{\sqrt{x}}{1+\sqrt{x}+x} dx$$

Optimal. Leaf size=42

$$2\sqrt{x} - \log(x + \sqrt{x} + 1) - \frac{2 \tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1357, 703, 634, 618, 204, 628}

$$2\sqrt{x} - \log(x + \sqrt{x} + 1) - \frac{2 \tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[x]/(1 + Sqrt[x] + x), x]
```

```
[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] + x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 703

```
Int[((d_.) + (e_.)*(x_)^m)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(e*(d + e*x)^(m - 1))/(c*(m - 1)), x] + Dist[1/c, Int[((d + e*x)^(m - 2)*Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x])/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 1357

```
Int[(x_)^m*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
```


], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
 \int \frac{\sqrt{x}}{1 + \sqrt{x} + x} dx &= 2 \operatorname{Subst} \left(\int \frac{x^2}{1 + x + x^2} dx, x, \sqrt{x} \right) \\
 &= 2\sqrt{x} + 2 \operatorname{Subst} \left(\int \frac{-1 - x}{1 + x + x^2} dx, x, \sqrt{x} \right) \\
 &= 2\sqrt{x} - \operatorname{Subst} \left(\int \frac{1}{1 + x + x^2} dx, x, \sqrt{x} \right) - \operatorname{Subst} \left(\int \frac{1 + 2x}{1 + x + x^2} dx, x, \sqrt{x} \right) \\
 &= 2\sqrt{x} - \log(1 + \sqrt{x} + x) + 2 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2\sqrt{x} \right) \\
 &= 2\sqrt{x} - \frac{2 \tan^{-1} \left(\frac{1 + 2\sqrt{x}}{\sqrt{3}} \right)}{\sqrt{3}} - \log(1 + \sqrt{x} + x)
 \end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.00

$$2\sqrt{x} - \log(x + \sqrt{x} + 1) - \frac{2 \tan^{-1} \left(\frac{2\sqrt{x} + 1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(1 + Sqrt[x] + x), x]

[Out] 2*Sqrt[x] - (2*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] + x]

IntegrateAlgebraic [A] time = 0.04, size = 45, normalized size = 1.07

$$2\sqrt{x} - \log(x + \sqrt{x} + 1) - \frac{2 \tan^{-1} \left(\frac{2\sqrt{x}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(1 + Sqrt[x] + x), x]

[Out] 2*Sqrt[x] - (2*ArcTan[1/Sqrt[3] + (2*Sqrt[x])/Sqrt[3]])/Sqrt[3] - Log[1 + Sqrt[x] + x]

fricas [A] time = 0.75, size = 35, normalized size = 0.83

$$-\frac{2}{3} \sqrt{3} \arctan \left(\frac{2}{3} \sqrt{3} \sqrt{x} + \frac{1}{3} \sqrt{3} \right) + 2\sqrt{x} - \log(x + \sqrt{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x+x^(1/2)), x, algorithm="fricas")

[Out] -2/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + 2*sqrt(x) - log(x + sqrt(x) + 1)

giac [A] time = 0.39, size = 33, normalized size = 0.79

$$-\frac{2}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2\sqrt{x} + 1) \right) + 2\sqrt{x} - \log(x + \sqrt{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="giac")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt(x) + 1)

maple [A] time = 0.00, size = 34, normalized size = 0.81

$$-\frac{2\sqrt{3} \arctan\left(\frac{(2\sqrt{x}+1)\sqrt{3}}{3}\right)}{3} - \ln(x + \sqrt{x} + 1) + 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x+x^(1/2)+1),x)

[Out] -ln(x+x^(1/2)+1)-2/3*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))*3^(1/2)+2*x^(1/2)

maxima [A] time = 2.01, size = 33, normalized size = 0.79

$$-\frac{2}{3}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x} + 1)\right) + 2\sqrt{x} - \log(x + \sqrt{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x+x^(1/2)),x, algorithm="maxima")

[Out] -2/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + 2*sqrt(x) - log(x + sqrt(x) + 1)

mupad [B] time = 3.45, size = 35, normalized size = 0.83

$$2\sqrt{x} - \frac{2\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - \ln(x + \sqrt{x} + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x + x^(1/2) + 1),x)

[Out] 2*x^(1/2) - (2*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - log(x + x^(1/2) + 1)

sympy [A] time = 0.25, size = 49, normalized size = 1.17

$$2\sqrt{x} - \log(4\sqrt{x} + 4x + 4) - \frac{2\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(1+x+x**(1/2)),x)

[Out] 2*sqrt(x) - log(4*sqrt(x) + 4*x + 4) - 2*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3

$$3.629 \quad \int \frac{x}{1+\sqrt{x}+x} dx$$

Optimal. Leaf size=32

$$x - 2\sqrt{x} + \frac{4 \tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1357, 701, 618, 204}

$$x - 2\sqrt{x} + \frac{4 \tan^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Int[x/(1 + Sqrt[x] + x),x]

[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 701

Int[((d_.) + (e_.)*(x_)^m)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Int[PolynomialDivide[(d + e*x)^m, a + b*x + c*x^2, x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && IGtQ[m, 1] && (NeQ[d, 0] || GtQ[m, 2])

Rule 1357

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_.))^p_., x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]

Rubi steps

$$\begin{aligned}
\int \frac{x}{1 + \sqrt{x} + x} dx &= 2 \operatorname{Subst} \left(\int \frac{x^3}{1 + x + x^2} dx, x, \sqrt{x} \right) \\
&= 2 \operatorname{Subst} \left(\int \left(-1 + x + \frac{1}{1 + x + x^2} \right) dx, x, \sqrt{x} \right) \\
&= -2\sqrt{x} + x + 2 \operatorname{Subst} \left(\int \frac{1}{1 + x + x^2} dx, x, \sqrt{x} \right) \\
&= -2\sqrt{x} + x - 4 \operatorname{Subst} \left(\int \frac{1}{-3 - x^2} dx, x, 1 + 2\sqrt{x} \right) \\
&= -2\sqrt{x} + x + \frac{4 \tan^{-1} \left(\frac{1+2\sqrt{x}}{\sqrt{3}} \right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 32, normalized size = 1.00

$$x - 2\sqrt{x} + \frac{4 \tan^{-1} \left(\frac{2\sqrt{x}+1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(1 + Sqrt[x] + x), x]

[Out] -2*Sqrt[x] + x + (4*ArcTan[(1 + 2*Sqrt[x])/Sqrt[3]])/Sqrt[3]

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 1.25

$$\sqrt{x} (\sqrt{x} - 2) + \frac{4 \tan^{-1} \left(\frac{2\sqrt{x}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(1 + Sqrt[x] + x), x]

[Out] (-2 + Sqrt[x])*Sqrt[x] + (4*ArcTan[1/Sqrt[3] + (2*Sqrt[x])/Sqrt[3]])/Sqrt[3]

fricas [A] time = 0.80, size = 27, normalized size = 0.84

$$\frac{4}{3} \sqrt{3} \arctan \left(\frac{2}{3} \sqrt{3} \sqrt{x} + \frac{1}{3} \sqrt{3} \right) + x - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x+x^(1/2)), x, algorithm="fricas")

[Out] 4/3*sqrt(3)*arctan(2/3*sqrt(3)*sqrt(x) + 1/3*sqrt(3)) + x - 2*sqrt(x)

giac [A] time = 0.36, size = 25, normalized size = 0.78

$$\frac{4}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2\sqrt{x} + 1) \right) + x - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x+x^(1/2)), x, algorithm="giac")

[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)

maple [A] time = 0.00, size = 26, normalized size = 0.81

$$x + \frac{4\sqrt{3} \arctan\left(\frac{(2\sqrt{x}+1)\sqrt{3}}{3}\right)}{3} - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x+x^(1/2)+1),x)

[Out] x+4/3*3^(1/2)*arctan(1/3*(2*x^(1/2)+1)*3^(1/2))-2*x^(1/2)

maxima [A] time = 2.13, size = 25, normalized size = 0.78

$$\frac{4}{3}\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}(2\sqrt{x}+1)\right) + x - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x+x^(1/2)),x, algorithm="maxima")

[Out] 4/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*sqrt(x) + 1)) + x - 2*sqrt(x)

mupad [B] time = 0.04, size = 27, normalized size = 0.84

$$x + \frac{4\sqrt{3} \operatorname{atan}\left(\frac{\sqrt{3}}{3} + \frac{2\sqrt{3}\sqrt{x}}{3}\right)}{3} - 2\sqrt{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x + x^(1/2) + 1),x)

[Out] x + (4*3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^(1/2))/3))/3 - 2*x^(1/2)

sympy [A] time = 0.23, size = 37, normalized size = 1.16

$$-2\sqrt{x} + x + \frac{4\sqrt{3} \operatorname{atan}\left(\frac{2\sqrt{3}\sqrt{x}}{3} + \frac{\sqrt{3}}{3}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x+x**(1/2)),x)

[Out] -2*sqrt(x) + x + 4*sqrt(3)*atan(2*sqrt(3)*sqrt(x)/3 + sqrt(3)/3)/3

$$3.630 \quad \int \frac{1}{\sqrt{x}(1+\sqrt{x}+x)^{7/2}} dx$$

Optimal. Leaf size=76

$$\frac{512(2\sqrt{x}+1)}{405\sqrt{x+\sqrt{x}+1}} + \frac{64(2\sqrt{x}+1)}{135(x+\sqrt{x}+1)^{3/2}} + \frac{4(2\sqrt{x}+1)}{15(x+\sqrt{x}+1)^{5/2}}$$

Rubi [A] time = 0.02, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1352, 614, 613}

$$\frac{512(2\sqrt{x}+1)}{405\sqrt{x+\sqrt{x}+1}} + \frac{64(2\sqrt{x}+1)}{135(x+\sqrt{x}+1)^{3/2}} + \frac{4(2\sqrt{x}+1)}{15(x+\sqrt{x}+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(1+Sqrt[x]+x)^(7/2)),x]

[Out] (4*(1+2*Sqrt[x]))/(15*(1+Sqrt[x]+x)^(5/2)) + (64*(1+2*Sqrt[x]))/(135*(1+Sqrt[x]+x)^(3/2)) + (512*(1+2*Sqrt[x]))/(405*Sqrt[1+Sqrt[x]+x])

Rule 613

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] := Simp[(-2*(b + 2*c*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 614

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(b^2 - 4*a*c)), x] - Dist[(2*c*(2*p + 3))/((p + 1)*(b^2 - 4*a*c)), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && IntegerQ[4*p]

Rule 1352

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{x}(1+\sqrt{x}+x)^{7/2}} dx &= 2 \operatorname{Subst} \left(\int \frac{1}{(1+x+x^2)^{7/2}} dx, x, \sqrt{x} \right) \\
&= \frac{4(1+2\sqrt{x})}{15(1+\sqrt{x}+x)^{5/2}} + \frac{32}{15} \operatorname{Subst} \left(\int \frac{1}{(1+x+x^2)^{5/2}} dx, x, \sqrt{x} \right) \\
&= \frac{4(1+2\sqrt{x})}{15(1+\sqrt{x}+x)^{5/2}} + \frac{64(1+2\sqrt{x})}{135(1+\sqrt{x}+x)^{3/2}} + \frac{256}{135} \operatorname{Subst} \left(\int \frac{1}{(1+x+x^2)^{3/2}} dx, x, \sqrt{x} \right) \\
&= \frac{4(1+2\sqrt{x})}{15(1+\sqrt{x}+x)^{5/2}} + \frac{64(1+2\sqrt{x})}{135(1+\sqrt{x}+x)^{3/2}} + \frac{512(1+2\sqrt{x})}{405\sqrt{1+\sqrt{x}+x}}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 49, normalized size = 0.64

$$\frac{4(2\sqrt{x}+1)(256x^{3/2}+128x^2+432x+304\sqrt{x}+203)}{405(x+\sqrt{x}+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(1+Sqrt[x]+x)^(7/2)),x]

[Out] (4*(1+2*Sqrt[x])*(203+304*Sqrt[x]+432*x+256*x^(3/2)+128*x^2))/(405*(1+Sqrt[x]+x)^(5/2))

IntegrateAlgebraic [A] time = 0.32, size = 49, normalized size = 0.64

$$\frac{4(2\sqrt{x}+1)(256x^{3/2}+128x^2+432x+304\sqrt{x}+203)}{405(x+\sqrt{x}+1)^{5/2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*(1+Sqrt[x]+x)^(7/2)),x]

[Out] (4*(1+2*Sqrt[x])*(203+304*Sqrt[x]+432*x+256*x^(3/2)+128*x^2))/(405*(1+Sqrt[x]+x)^(5/2))

fricas [A] time = 0.92, size = 95, normalized size = 1.25

$$\frac{4(128x^5+272x^4+455x^3+232x^2-(256x^5+736x^4+1366x^3+1427x^2+839x+101)\sqrt{x}-128x-203)\sqrt{x+\sqrt{x}+1}}{405(x^6+3x^5+6x^4+7x^3+6x^2+3x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="fricas")

[Out] -4/405*(128*x^5+272*x^4+455*x^3+232*x^2-(256*x^5+736*x^4+1366*x^3+1427*x^2+839*x+101)*sqrt(x)-128*x-203)*sqrt(x+sqrt(x)+1)/(x^6+3*x^5+6*x^4+7*x^3+6*x^2+3*x+1)

giac [A] time = 0.43, size = 45, normalized size = 0.59

$$\frac{4(2(8(2(4\sqrt{x}(2\sqrt{x}+5)+35)\sqrt{x}+65)\sqrt{x}+355)\sqrt{x}+203)}{405(x+\sqrt{x}+1)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="giac")

[Out] 4/405*(2*(8*(2*(4*sqrt(x))*(2*sqrt(x) + 5) + 35)*sqrt(x) + 65)*sqrt(x) + 355)*sqrt(x) + 203)/(x + sqrt(x) + 1)^(5/2)

maple [A] time = 0.00, size = 53, normalized size = 0.70

$$\frac{\frac{8\sqrt{x}}{15} + \frac{4}{15}}{(x + \sqrt{x} + 1)^{\frac{5}{2}}} + \frac{\frac{128\sqrt{x}}{135} + \frac{64}{135}}{(x + \sqrt{x} + 1)^{\frac{3}{2}}} + \frac{\frac{1024\sqrt{x}}{405} + \frac{512}{405}}{\sqrt{x + \sqrt{x} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^(1/2)/(x+x^(1/2)+1)^(7/2),x)

[Out] 4/15*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(5/2)+64/135*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(3/2)+512/405*(2*x^(1/2)+1)/(x+x^(1/2)+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + \sqrt{x} + 1)^{\frac{7}{2}} \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^(1/2)/(1+x+x^(1/2))^(7/2),x, algorithm="maxima")

[Out] integrate(1/((x + sqrt(x) + 1)^(7/2)*sqrt(x)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{x} (x + \sqrt{x} + 1)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)),x)

[Out] int(1/(x^(1/2)*(x + x^(1/2) + 1)^(7/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x} (\sqrt{x} + x + 1)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x**(1/2)/(1+x+x**(1/2))**(7/2),x)

[Out] Integral(1/(sqrt(x)*(sqrt(x) + x + 1)**(7/2)), x)

$$3.631 \quad \int \frac{-1+x}{1+\sqrt{1+x^2}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{x^2+1}}{x} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1} + 1\right) - \frac{1}{x} - \sinh^{-1}(x)$$

Rubi [A] time = 0.09, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {6742, 277, 215, 1591, 190, 43}

$$\frac{\sqrt{x^2+1}}{x} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1} + 1\right) - \frac{1}{x} - \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(-1 + x)/(1 + Sqrt[1 + x^2]), x]

[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x^2]]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 277

Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[((c*x)^(m + 1)*(a + b*x^n)^p)/(c*(m + 1)), x] - Dist[(b*n*p)/(c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && GtQ[p, 0] && LtQ[m, -1] && !ILtQ[(m + n*p + n + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1591

Int[((a_.) + (b_.)*(Pq_)^(n_.))^(p_.)*(Qr_), x_Symbol] := With[{q = Expon[Pq, x], r = Expon[Qr, x]}, Dist[Coeff[Qr, x, r]/(q*Coeff[Pq, x, q]), Subst[Int[(a + b*x^n)^p, x], x, Pq], x] /; EqQ[r, q - 1] && EqQ[Coeff[Qr, x, r]*D[Pq, x], q*Coeff[Pq, x, q]*Qr]] /; FreeQ[{a, b, n, p}, x] && PolyQ[Pq, x] && PolyQ[Qr, x]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
\int \frac{-1+x}{1+\sqrt{1+x^2}} dx &= \int \left(-\frac{1}{1+\sqrt{1+x^2}} + \frac{x}{1+\sqrt{1+x^2}} \right) dx \\
&= -\int \frac{1}{1+\sqrt{1+x^2}} dx + \int \frac{x}{1+\sqrt{1+x^2}} dx \\
&= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1+\sqrt{x}} dx, x, 1+x^2 \right) - \int \left(-\frac{1}{x^2} + \frac{\sqrt{1+x^2}}{x^2} \right) dx \\
&= -\frac{1}{x} - \int \frac{\sqrt{1+x^2}}{x^2} dx + \text{Subst} \left(\int \frac{x}{1+x} dx, x, \sqrt{1+x^2} \right) \\
&= -\frac{1}{x} + \frac{\sqrt{1+x^2}}{x} - \int \frac{1}{\sqrt{1+x^2}} dx + \text{Subst} \left(\int \left(1 + \frac{1}{-1-x} \right) dx, x, \sqrt{1+x^2} \right) \\
&= -\frac{1}{x} + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} - \sinh^{-1}(x) - \log(1+\sqrt{1+x^2})
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 1.00

$$\frac{\sqrt{x^2+1}}{x} + \sqrt{x^2+1} - \log(\sqrt{x^2+1} + 1) - \frac{1}{x} - \sinh^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]), x]

[Out] -x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 + x^2]]

IntegrateAlgebraic [A] time = 0.19, size = 41, normalized size = 0.89

$$\frac{\sqrt{x^2+1}(x+1)}{x} - 4 \tanh^{-1}\left(2\sqrt{x^2+1} - 2x + 1\right) - \frac{1}{x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(-1 + x)/(1 + Sqrt[1 + x^2]), x]

[Out] -x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]

fricas [A] time = 0.71, size = 64, normalized size = 1.39

$$\frac{x \log\left(2x^2 - \sqrt{x^2+1}(2x+1) + x+1\right) - x \log(x) - x \log\left(-x + \sqrt{x^2+1} + 1\right) + \sqrt{x^2+1}(x+1) + x-1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/((x^2+1)^(1/2)+1), x, algorithm="fricas")

[Out] (x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sqrt(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x

giac [A] time = 0.43, size = 79, normalized size = 1.72

$$\sqrt{x^2+1} - \frac{2}{(x-\sqrt{x^2+1})^2-1} - \frac{1}{x} + \log(-x + \sqrt{x^2+1}) - \log(|x|) - \log\left(\left|-x + \sqrt{x^2+1} + 1\right|\right) + \log\left(\left|-x + \sqrt{x^2+1} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="giac")

[Out] sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x^2 + 1) - 1))

maple [A] time = 0.00, size = 53, normalized size = 1.15

$$-\sqrt{x^2 + 1} x - \operatorname{arcsinh}(x) - \operatorname{arctanh}\left(\frac{1}{\sqrt{x^2 + 1}}\right) - \ln(x) - \frac{1}{x} + \frac{(x^2 + 1)^{\frac{3}{2}}}{x} + \sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x-1)/(1+(x^2+1)^(1/2)),x)

[Out] -1/x+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^(1/2))-ln(x)+(x^2+1)^(3/2)/x-(x^2+1)^(1/2)*x-arcsinh(x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{4}x^2 - \frac{1}{2}x - \int \frac{x^3 - x^2}{2(x^2 + 2\sqrt{x^2 + 1} + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/((x^2+1)^(1/2)+1),x, algorithm="maxima")

[Out] 1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x)

mupad [B] time = 0.04, size = 46, normalized size = 1.00

$$\sqrt{x^2 + 1} - \ln(x) - \operatorname{asinh}(x) + \frac{\sqrt{x^2 + 1}}{x} - \frac{1}{x} + \operatorname{atan}\left(\sqrt{x^2 + 1} \operatorname{li}\right) \operatorname{li}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 1)/((x^2 + 1)^(1/2) + 1),x)

[Out] atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 + 1)^(1/2)/x - 1/x

sympy [A] time = 2.98, size = 48, normalized size = 1.04

$$\frac{x}{\sqrt{x^2 + 1}} + \sqrt{x^2 + 1} - \log\left(\sqrt{x^2 + 1} + 1\right) - \operatorname{asinh}(x) - \frac{1}{x} + \frac{1}{x\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)/((x**2+1)**(1/2)+1),x)

[Out] x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1/x + 1/(x*sqrt(x**2 + 1))

$$3.632 \quad \int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}} dx$$

Optimal. Leaf size=20

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Rubi [A] time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {651}

$$\frac{3\sqrt[3]{x^2-1}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)), x]

[Out] (3*(-1 + x^2)^(1/3))/(2*(1 + x)^(2/3))

Rule 651

Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^(m*(a + c*x^2)^(p + 1)))/(2*c*d*(p + 1)), x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rubi steps

$$\int \frac{1}{(1+x)^{2/3}(-1+x^2)^{2/3}} dx = \frac{3\sqrt[3]{-1+x^2}}{2(1+x)^{2/3}}$$

Mathematica [A] time = 0.03, size = 23, normalized size = 1.15

$$\frac{3(x-1)\sqrt[3]{x+1}}{2(x^2-1)^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)), x]

[Out] (3*(-1 + x)*(1 + x)^(1/3))/(2*(-1 + x^2)^(2/3))

IntegrateAlgebraic [A] time = 0.40, size = 26, normalized size = 1.30

$$\frac{3\sqrt[3]{(x+1)^2-2(x+1)}}{2(x+1)^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 + x)^(2/3)*(-1 + x^2)^(2/3)), x]

[Out] (3*(-2*(1 + x) + (1 + x)^2)^(1/3))/(2*(1 + x)^(2/3))

fricas [A] time = 0.73, size = 14, normalized size = 0.70

$$\frac{3(x^2-1)^{\frac{1}{3}}}{2(x+1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="fricas")

[Out] 3/2*(x^2 - 1)^(1/3)/(x + 1)^(2/3)

giac [A] time = 0.36, size = 13, normalized size = 0.65

$$\frac{3}{2} \left(-\frac{2}{x+1} + 1 \right)^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="giac")

[Out] 3/2*(-2/(x + 1) + 1)^(1/3)

maple [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{3(x-1)(x+1)^{\frac{1}{3}}}{2(x^2-1)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)^(2/3)/(x^2-1)^(2/3),x)

[Out] 3/2*(x-1)*(x+1)^(1/3)/(x^2-1)^(2/3)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^2-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)^(2/3)/(x^2-1)^(2/3),x, algorithm="maxima")

[Out] integrate(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{(x^2-1)^{\frac{2}{3}}(x+1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)),x)

[Out] int(1/((x^2 - 1)^(2/3)*(x + 1)^(2/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{((x-1)(x+1))^{\frac{2}{3}}(x+1)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)**(2/3)/(x**2-1)**(2/3),x)

[Out] Integral(1/(((x - 1)*(x + 1))**(2/3)*(x + 1)**(2/3)), x)

$$3.633 \quad \int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx$$

Optimal. Leaf size=35

$$\frac{1}{5}x(1-x^6)^{2/3} - \frac{(1-x^6)^{2/3}}{5x^5}$$

Rubi [C] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {245, 364}

$$x {}_2F_1\left(-\frac{2}{3}, \frac{1}{6}; \frac{7}{6}; x^6\right) - \frac{{}_2F_1\left(-\frac{5}{6}, -\frac{2}{3}; \frac{1}{6}; x^6\right)}{5x^5}$$

Antiderivative was successfully verified.

[In] Int[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6, x]

[Out] -Hypergeometric2F1[-5/6, -2/3, 1/6, x^6]/(5*x^5) + x*Hypergeometric2F1[-2/3, 1/6, 7/6, x^6]

Rule 245

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[a^p*x*Hypergeometric2F1[-p, 1/n, 1/n + 1, -(b*x^n)/a], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] || GtQ[a, 0])

Rule 364

Int[((c_.)*(x_)^(m_.))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a^p*(c*x)^(m+1)*Hypergeometric2F1[-p, (m+1)/n, (m+1)/n + 1, -(b*x^n)/a])]/(c*(m+1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILtQ[p, 0] || GtQ[a, 0])

Rubi steps

$$\begin{aligned} \int \left((1-x^6)^{2/3} + \frac{(1-x^6)^{2/3}}{x^6} \right) dx &= \int (1-x^6)^{2/3} dx + \int \frac{(1-x^6)^{2/3}}{x^6} dx \\ &= -\frac{{}_2F_1\left(-\frac{5}{6}, -\frac{2}{3}; \frac{1}{6}; x^6\right)}{5x^5} + x {}_2F_1\left(-\frac{2}{3}, \frac{1}{6}; \frac{7}{6}; x^6\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 0.51

$$-\frac{(1-x^6)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6, x]

[Out] -1/5*(1 - x^6)^(5/3)/x^5

IntegrateAlgebraic [A] time = 0.19, size = 18, normalized size = 0.51

$$\frac{(1-x^6)^{5/3}}{5x^5}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x^6)^(2/3) + (1 - x^6)^(2/3)/x^6,x]

[Out] -1/5*(1 - x^6)^(5/3)/x^5

fricas [A] time = 0.69, size = 19, normalized size = 0.54

$$\frac{(x^6-1)(-x^6+1)^{2/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="fricas")

[Out] 1/5*(x^6 - 1)*(-x^6 + 1)^(2/3)/x^5

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (-x^6+1)^{2/3} + \frac{(-x^6+1)^{2/3}}{x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="giac")

[Out] integrate((-x^6 + 1)^(2/3) + (-x^6 + 1)^(2/3)/x^6, x)

maple [A] time = 0.01, size = 35, normalized size = 1.00

$$\frac{(-x^6+1)^{2/3}(x^2-x+1)(x^2+x+1)(x+1)(x-1)}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x)

[Out] 1/5*(-x^6+1)^(2/3)*(x^2-x+1)*(x^2+x+1)*(x+1)/x^5*(x-1)

maxima [A] time = 2.15, size = 38, normalized size = 1.09

$$\frac{(x^6-1)(x^2+x+1)^{2/3}(-x^2+x-1)^{2/3}(x+1)^{2/3}(x-1)^{2/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^6+1)^(2/3)+(-x^6+1)^(2/3)/x^6,x, algorithm="maxima")

[Out] 1/5*(x^6 - 1)*(x^2 + x + 1)^(2/3)*(-x^2 + x - 1)^(2/3)*(x + 1)^(2/3)*(x - 1)^(2/3)/x^5

mupad [B] time = 3.59, size = 14, normalized size = 0.40

$$\frac{(1-x^6)^{5/3}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1 - x^6)^(2/3)/x^6 + (1 - x^6)^(2/3), x)`

[Out] `-(1 - x^6)^(5/3)/(5*x^5)`

sympy [C] time = 1.14, size = 68, normalized size = 1.94

$$\frac{x\Gamma\left(\frac{1}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{2}{3}, \frac{1}{6} \\ \frac{7}{6} \end{matrix} \middle| x^6 e^{2i\pi}\right)}{6\Gamma\left(\frac{7}{6}\right)} + \frac{\Gamma\left(-\frac{5}{6}\right) {}_2F_1\left(\begin{matrix} -\frac{5}{6}, -\frac{2}{3} \\ \frac{1}{6} \end{matrix} \middle| x^6 e^{2i\pi}\right)}{6x^5\Gamma\left(\frac{1}{6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-x**6+1)**(2/3)+(-x**6+1)**(2/3)/x**6, x)`

[Out] `x*gamma(1/6)*hyper((-2/3, 1/6), (7/6,), x**6*exp_polar(2*I*pi))/(6*gamma(7/6)) + gamma(-5/6)*hyper((-5/6, -2/3), (1/6,), x**6*exp_polar(2*I*pi))/(6*x**5*gamma(1/6))`

$$3.634 \quad \int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}} dx$$

Optimal. Leaf size=15

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Rubi [A] time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$, Rules used = {12, 449}

$$\frac{x^m}{\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Int[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]

[Out] x^m/Sqrt[a + b*x^n]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]

Rule 449

Int[((e_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_)), x_Symbol] := Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)), x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[a*d*(m + 1) - b*c*(m + n*(p + 1) + 1), 0] && NeQ[m, -1]

Rubi steps

$$\begin{aligned} \int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}} dx &= \frac{1}{2} \int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{(a+bx^n)^{3/2}} dx \\ &= \frac{x^m}{\sqrt{a+bx^n}} \end{aligned}$$

Mathematica [C] time = 0.18, size = 111, normalized size = 7.40

$$\frac{x^m \sqrt{\frac{bx^n}{a} + 1} \left(b(2m-n)x^n {}_2F_1\left(\frac{3}{2}, \frac{m+n}{n}; \frac{m}{n} + 2; -\frac{bx^n}{a}\right) + 2a(m+n) {}_2F_1\left(\frac{3}{2}, \frac{m}{n}; \frac{m+n}{n}; -\frac{bx^n}{a}\right) \right)}{2a(m+n)\sqrt{a+bx^n}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)),x]

[Out] (x^m*Sqrt[1 + (b*x^n)/a]*(2*a*(m + n)*Hypergeometric2F1[3/2, m/n, (m + n)/n, -((b*x^n)/a)] + b*(2*m - n)*x^n*Hypergeometric2F1[3/2, (m + n)/n, 2 + m/n, -((b*x^n)/a)])/(2*a*(m + n)*Sqrt[a + b*x^n])

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{-1+m}(2am+b(2m-n)x^n)}{2(a+bx^n)^{3/2}} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)), x]

[Out] Defer[IntegrateAlgebraic] [(x^(-1 + m)*(2*a*m + b*(2*m - n)*x^n))/(2*(a + b*x^n)^(3/2)), x]

fricas [A] time = 0.59, size = 16, normalized size = 1.07

$$\frac{xx^{m-1}}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2), x, algorithm="fricas")

[Out] x*x^(m - 1)/sqrt(b*x^n + a)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b(2m - n)x^n + 2am)x^{m-1}}{2(bx^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2), x, algorithm="giac")

[Out] integrate(1/2*(b*(2*m - n)*x^n + 2*a*m)*x^(m - 1)/(b*x^n + a)^(3/2), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{(2am + (2m - n)bx^n)x^{m-1}}{2(bx^n + a)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2), x)

[Out] int(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(b*x^n+a)^(3/2), x)

maxima [A] time = 1.13, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{bx^n + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*x^(-1+m)*(2*a*m+b*(2*m-n)*x^n)/(a+b*x^n)^(3/2), x, algorithm="maxima")

[Out] x^m/sqrt(b*x^n + a)

mupad [B] time = 3.68, size = 13, normalized size = 0.87

$$\frac{x^m}{\sqrt{a + bx^n}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(m - 1)*(2*a*m + b*x^n*(2*m - n)))/(2*(a + b*x^n)^(3/2)), x)

[Out] $x^m/(a + b*x^n)^{1/2}$

sympy [C] time = 98.05, size = 100, normalized size = 6.67

$$\frac{m x^m \Gamma\left(\frac{m}{n}\right) {}_2F_1\left(\frac{3}{2}, \frac{m}{n} \left| \frac{b x^n e^{i\pi}}{a} \right. \right)}{\sqrt{a} n \Gamma\left(\frac{m}{n} + 1\right)} + \frac{b x^m x^n (2m - n) \Gamma\left(\frac{m}{n} + 1\right) {}_2F_1\left(\frac{3}{2}, \frac{m}{n} + 1 \left| \frac{b x^n e^{i\pi}}{a} \right. \right)}{2 a^{\frac{3}{2}} n \Gamma\left(\frac{m}{n} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2*x**(-1+m)*(2*a*m+b*(2*m-n)*x**n)/(a+b*x**n)**(3/2),x)

[Out] $m*x**m*\text{gamma}(m/n)*\text{hyper}((3/2, m/n), (m/n + 1,), b*x**n*\text{exp_polar}(I*\text{pi})/a)/(\text{sqrt}(a)*n*\text{gamma}(m/n + 1)) + b*x**m*x**n*(2*m - n)*\text{gamma}(m/n + 1)*\text{hyper}((3/2, m/n + 1), (m/n + 2,), b*x**n*\text{exp_polar}(I*\text{pi})/a)/(2*a**(3/2)*n*\text{gamma}(m/n + 2))$

$$3.635 \quad \int \frac{x-2x^3}{\sqrt{2+3x}} dx$$

Optimal. Leaf size=53

$$-\frac{4}{567}(3x+2)^{7/2} + \frac{8}{135}(3x+2)^{5/2} - \frac{10}{81}(3x+2)^{3/2} - \frac{4}{81}\sqrt{3x+2}$$

Rubi [A] time = 0.02, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1593, 772}

$$-\frac{4}{567}(3x+2)^{7/2} + \frac{8}{135}(3x+2)^{5/2} - \frac{10}{81}(3x+2)^{3/2} - \frac{4}{81}\sqrt{3x+2}$$

Antiderivative was successfully verified.

[In] Int[(x - 2*x^3)/Sqrt[2 + 3*x], x]

[Out] (-4*Sqrt[2 + 3*x])/81 - (10*(2 + 3*x)^(3/2))/81 + (8*(2 + 3*x)^(5/2))/135 - (4*(2 + 3*x)^(7/2))/567

Rule 772

Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned} \int \frac{x-2x^3}{\sqrt{2+3x}} dx &= \int \frac{x(1-2x^2)}{\sqrt{2+3x}} dx \\ &= \int \left(-\frac{2}{27\sqrt{2+3x}} - \frac{5}{9}\sqrt{2+3x} + \frac{4}{9}(2+3x)^{3/2} - \frac{2}{27}(2+3x)^{5/2} \right) dx \\ &= -\frac{4}{81}\sqrt{2+3x} - \frac{10}{81}(2+3x)^{3/2} + \frac{8}{135}(2+3x)^{5/2} - \frac{4}{567}(2+3x)^{7/2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 28, normalized size = 0.53

$$\frac{2\sqrt{3x+2} (270x^3 - 216x^2 - 123x + 164)}{2835}$$

Antiderivative was successfully verified.

[In] Integrate[(x - 2*x^3)/Sqrt[2 + 3*x], x]

[Out] (-2*Sqrt[2 + 3*x]*(164 - 123*x - 216*x^2 + 270*x^3))/2835

IntegrateAlgebraic [A] time = 0.03, size = 40, normalized size = 0.75

$$\frac{2\sqrt{3x+2} (10(3x+2)^3 - 84(3x+2)^2 + 175(3x+2) + 70)}{2835}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x - 2*x^3)/Sqrt[2 + 3*x],x]

[Out] (-2*Sqrt[2 + 3*x]*(70 + 175*(2 + 3*x) - 84*(2 + 3*x)^2 + 10*(2 + 3*x)^3))/2835

fricas [A] time = 0.62, size = 24, normalized size = 0.45

$$-\frac{2}{2835} (270x^3 - 216x^2 - 123x + 164) \sqrt{3x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="fricas")

[Out] -2/2835*(270*x^3 - 216*x^2 - 123*x + 164)*sqrt(3*x + 2)

giac [A] time = 0.34, size = 37, normalized size = 0.70

$$-\frac{4}{567} (3x + 2)^{\frac{7}{2}} + \frac{8}{135} (3x + 2)^{\frac{5}{2}} - \frac{10}{81} (3x + 2)^{\frac{3}{2}} - \frac{4}{81} \sqrt{3x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="giac")

[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/81*sqrt(3*x + 2)

maple [A] time = 0.00, size = 25, normalized size = 0.47

$$\frac{2(270x^3 - 216x^2 - 123x + 164) \sqrt{3x + 2}}{2835}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-2*x^3+x)/(3*x+2)^(1/2),x)

[Out] -2/2835*(270*x^3-216*x^2-123*x+164)*(3*x+2)^(1/2)

maxima [A] time = 0.58, size = 37, normalized size = 0.70

$$-\frac{4}{567} (3x + 2)^{\frac{7}{2}} + \frac{8}{135} (3x + 2)^{\frac{5}{2}} - \frac{10}{81} (3x + 2)^{\frac{3}{2}} - \frac{4}{81} \sqrt{3x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x^3+x)/(2+3*x)^(1/2),x, algorithm="maxima")

[Out] -4/567*(3*x + 2)^(7/2) + 8/135*(3*x + 2)^(5/2) - 10/81*(3*x + 2)^(3/2) - 4/81*sqrt(3*x + 2)

mupad [B] time = 0.04, size = 37, normalized size = 0.70

$$\frac{8(3x + 2)^{5/2}}{135} - \frac{10(3x + 2)^{3/2}}{81} - \frac{4\sqrt{3x + 2}}{81} - \frac{4(3x + 2)^{7/2}}{567}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x - 2*x^3)/(3*x + 2)^(1/2),x)

[Out] (8*(3*x + 2)^(5/2))/135 - (10*(3*x + 2)^(3/2))/81 - (4*(3*x + 2)^(1/2))/81 - (4*(3*x + 2)^(7/2))/567

sympy [A] time = 11.73, size = 46, normalized size = 0.87

$$-\frac{4(3x+2)^{\frac{7}{2}}}{567} + \frac{8(3x+2)^{\frac{5}{2}}}{135} - \frac{10(3x+2)^{\frac{3}{2}}}{81} - \frac{4\sqrt{3x+2}}{81}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-2*x**3+x)/(2+3*x)**(1/2),x)

[Out] -4*(3*x + 2)**(7/2)/567 + 8*(3*x + 2)**(5/2)/135 - 10*(3*x + 2)**(3/2)/81 - 4*sqrt(3*x + 2)/81

$$3.636 \quad \int \frac{1}{\sqrt[4]{1+x} + \sqrt{1+x}} dx$$

Optimal. Leaf size=31

$$2\sqrt{x+1} - 4\sqrt[4]{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {2012, 1593, 266, 43}

$$2\sqrt{x+1} - 4\sqrt[4]{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[((1 + x)^(1/4) + Sqrt[1 + x])^(-1), x]

[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LtQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2012

Int[((a_.)*(u_)^(j_.) + (b_.)*(u_)^(n_.))^(p_), x_Symbol] := Dist[1/Coefficient[u, x, 1], Subst[Int[(a*x^j + b*x^n)^p, x], x, u], x] /; FreeQ[{a, b, j, n, p}, x] && LinearQ[u, x] && NeQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt[4]{1+x} + \sqrt{1+x}} dx &= \text{Subst}\left(\int \frac{1}{\sqrt[4]{x} + \sqrt{x}} dx, x, 1+x\right) \\ &= \text{Subst}\left(\int \frac{1}{(1 + \sqrt[4]{x})\sqrt[4]{x}} dx, x, 1+x\right) \\ &= 4 \text{Subst}\left(\int \frac{x^2}{1+x} dx, x, \sqrt[4]{1+x}\right) \\ &= 4 \text{Subst}\left(\int \left(-1 + x + \frac{1}{1+x}\right) dx, x, \sqrt[4]{1+x}\right) \\ &= -4\sqrt[4]{1+x} + 2\sqrt{1+x} + 4\log\left(1 + \sqrt[4]{1+x}\right) \end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.00

$$2\sqrt{x+1} - 4\sqrt[4]{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x)^(1/4) + Sqrt[1 + x])^(-1), x]

[Out] -4*(1 + x)^(1/4) + 2*Sqrt[1 + x] + 4*Log[1 + (1 + x)^(1/4)]

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 1.00

$$2\sqrt[4]{x+1} \left(\sqrt[4]{x+1} - 2\right) + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[((1 + x)^(1/4) + Sqrt[1 + x])^(-1), x]

[Out] 2*(1 + x)^(1/4)*(-2 + (1 + x)^(1/4)) + 4*Log[1 + (1 + x)^(1/4)]

fricas [A] time = 0.59, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4(x+1)^{\frac{1}{4}} + 4\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)

giac [A] time = 0.32, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4(x+1)^{\frac{1}{4}} + 4\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="giac")

[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)

maple [A] time = 0.02, size = 26, normalized size = 0.84

$$4\ln\left(1 + (x+1)^{\frac{1}{4}}\right) - 4(x+1)^{\frac{1}{4}} + 2\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x+1)^(1/4)+(x+1)^(1/2)),x)

[Out] -4*(x+1)^(1/4)+4*ln(1+(x+1)^(1/4))+2*(x+1)^(1/2)

maxima [A] time = 0.65, size = 25, normalized size = 0.81

$$2\sqrt{x+1} - 4(x+1)^{\frac{1}{4}} + 4\log\left((x+1)^{\frac{1}{4}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)^(1/4)+(1+x)^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(x + 1) - 4*(x + 1)^(1/4) + 4*log((x + 1)^(1/4) + 1)

mupad [B] time = 0.07, size = 25, normalized size = 0.81

$$4 \ln \left((x+1)^{1/4} + 1 \right) + 2 \sqrt{x+1} - 4(x+1)^{1/4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((x + 1)^(1/2) + (x + 1)^(1/4)), x)

[Out] 4*log((x + 1)^(1/4) + 1) + 2*(x + 1)^(1/2) - 4*(x + 1)^(1/4)

sympy [A] time = 0.24, size = 27, normalized size = 0.87

$$-4\sqrt[4]{x+1} + 2\sqrt{x+1} + 4\log\left(\sqrt[4]{x+1} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1+x)**(1/4)+(1+x)**(1/2)), x)

[Out] -4*(x + 1)**(1/4) + 2*sqrt(x + 1) + 4*log((x + 1)**(1/4) + 1)

$$3.637 \quad \int \frac{1+2x}{\sqrt{x+x^2}} dx$$

Optimal. Leaf size=11

$$2\sqrt{x^2+x}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {629}

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/Sqrt[x + x^2], x]

[Out] 2*Sqrt[x + x^2]

Rule 629

Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Simp[(d*(a + b*x + c*x^2)^(p + 1))/(b*(p + 1)), x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rubi steps

$$\int \frac{1+2x}{\sqrt{x+x^2}} dx = 2\sqrt{x+x^2}$$

Mathematica [A] time = 0.01, size = 11, normalized size = 1.00

$$2\sqrt{x(x+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + 2*x)/Sqrt[x + x^2], x]

[Out] 2*Sqrt[x*(1 + x)]

IntegrateAlgebraic [A] time = 0.02, size = 11, normalized size = 1.00

$$2\sqrt{x^2+x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 2*x)/Sqrt[x + x^2], x]

[Out] 2*Sqrt[x + x^2]

fricas [A] time = 0.70, size = 9, normalized size = 0.82

$$2\sqrt{x^2+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x)^(1/2), x, algorithm="fricas")

[Out] 2*sqrt(x^2 + x)

giac [A] time = 0.36, size = 9, normalized size = 0.82

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="giac")

[Out] 2*sqrt(x^2 + x)

maple [A] time = 0.00, size = 14, normalized size = 1.27

$$\frac{2(x+1)x}{\sqrt{x^2+x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)/(x^2+x)^(1/2),x)

[Out] 2*(x+1)*x/(x^2+x)^(1/2)

maxima [A] time = 0.63, size = 9, normalized size = 0.82

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+x)^(1/2),x, algorithm="maxima")

[Out] 2*sqrt(x^2 + x)

mupad [B] time = 3.52, size = 9, normalized size = 0.82

$$2\sqrt{x(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/(x + x^2)^(1/2),x)

[Out] 2*(x*(x + 1))^(1/2)

sympy [A] time = 0.14, size = 8, normalized size = 0.73

$$2\sqrt{x^2 + x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+x)**(1/2),x)

[Out] 2*sqrt(x**2 + x)

$$3.638 \quad \int \frac{1}{2\sqrt{x}(1+x)} dx$$

Optimal. Leaf size=6

$$\tan^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {12, 63, 203}

$$\tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[1/(2*Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]

Rule 12

Int[(a_)*(u_), x_Symbol] := Dist[a, Int[u, x], x] /; FreeQ[a, x] && !MatchQ[u, (b_)*(v_)] /; FreeQ[b, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{2\sqrt{x}(1+x)} dx &= \frac{1}{2} \int \frac{1}{\sqrt{x}(1+x)} dx \\ &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{x}\right) \\ &= \tan^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[1/(2*Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.01, size = 6, normalized size = 1.00

$$\tan^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(2*Sqrt[x]*(1 + x)),x]

[Out] ArcTan[Sqrt[x]]

fricas [A] time = 0.88, size = 4, normalized size = 0.67

$$\arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="fricas")

[Out] arctan(sqrt(x))

giac [A] time = 0.33, size = 4, normalized size = 0.67

$$\arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(x))

maple [A] time = 0.00, size = 5, normalized size = 0.83

$$\arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/2/(x+1)/x^(1/2),x)

[Out] arctan(x^(1/2))

maxima [A] time = 1.39, size = 4, normalized size = 0.67

$$\arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(1+x)/x^(1/2),x, algorithm="maxima")

[Out] arctan(sqrt(x))

mupad [B] time = 0.14, size = 4, normalized size = 0.67

$$\operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(2*x^(1/2)*(x + 1)),x)

[Out] atan(x^(1/2))

sympy [A] time = 0.21, size = 5, normalized size = 0.83

$$\operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/2/(1+x)/x**(1/2),x)

[Out] atan(sqrt(x))

$$3.639 \quad \int \frac{1}{x\sqrt{6x-x^2}} dx$$

Optimal. Leaf size=20

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {650}

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

[In] Int[1/(x*Sqrt[6*x - x^2]),x]

[Out] -Sqrt[6*x - x^2]/(3*x)

Rule 650

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(e*(d + e*x)^m*(a + b*x + c*x^2)^(p + 1))/((p + 1)*(2*c*d - b*e)), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[p] && EqQ[m + 2*p + 2, 0]

Rubi steps

$$\int \frac{1}{x\sqrt{6x-x^2}} dx = -\frac{\sqrt{6x-x^2}}{3x}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-(x-6)x}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*Sqrt[6*x - x^2]),x]

[Out] (-6 + x)/(3*Sqrt[-((-6 + x)*x)])

IntegrateAlgebraic [A] time = 0.10, size = 20, normalized size = 1.00

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*Sqrt[6*x - x^2]),x]

[Out] -1/3*Sqrt[6*x - x^2]/x

fricas [A] time = 0.46, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6x}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="fricas")

[Out] -1/3*sqrt(-x^2 + 6*x)/x

giac [A] time = 0.41, size = 25, normalized size = 1.25

$$\frac{2}{3 \left(\frac{\sqrt{-x^2+6x}-3}{x-3} - 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="giac")

[Out] 2/3/((sqrt(-x^2 + 6*x) - 3)/(x - 3) - 1)

maple [A] time = 0.00, size = 17, normalized size = 0.85

$$\frac{x-6}{3\sqrt{-x^2+6x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-x^2+6*x)^(1/2),x)

[Out] 1/3*(x-6)/(-x^2+6*x)^(1/2)

maxima [A] time = 1.43, size = 16, normalized size = 0.80

$$-\frac{\sqrt{-x^2+6x}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x^2+6*x)^(1/2),x, algorithm="maxima")

[Out] -1/3*sqrt(-x^2 + 6*x)/x

mupad [B] time = 3.51, size = 16, normalized size = 0.80

$$-\frac{\sqrt{6x-x^2}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(6*x - x^2)^(1/2)),x)

[Out] -(6*x - x^2)^(1/2)/(3*x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{-x(x-6)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(-x**2+6*x)**(1/2),x)

[Out] Integral(1/(x*sqrt(-x*(x - 6))), x)

$$3.640 \quad \int (1 + \sqrt{x}) \sqrt{x} dx$$

Optimal. Leaf size=17

$$\frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

Rubi [A] time = 0.00, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {14}

$$\frac{x^2}{2} + \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x])*Sqrt[x], x]

[Out] (2*x^(3/2))/3 + x^2/2

Rule 14

Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int (1 + \sqrt{x}) \sqrt{x} dx &= \int (\sqrt{x} + x) dx \\ &= \frac{2x^{3/2}}{3} + \frac{x^2}{2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 17, normalized size = 1.00

$$\frac{2x^{3/2}}{3} + \frac{x^2}{2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[x])*Sqrt[x], x]

[Out] (2*x^(3/2))/3 + x^2/2

IntegrateAlgebraic [A] time = 0.01, size = 18, normalized size = 1.06

$$\frac{1}{6} (3\sqrt{x} + 4) x^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + Sqrt[x])*Sqrt[x], x]

[Out] ((4 + 3*Sqrt[x])*x^(3/2))/6

fricas [A] time = 0.43, size = 11, normalized size = 0.65

$$\frac{1}{2} x^2 + \frac{2}{3} x^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="fricas")

[Out] 1/2*x^2 + 2/3*x^(3/2)

giac [A] time = 0.35, size = 11, normalized size = 0.65

$$\frac{1}{2}x^2 + \frac{2}{3}x^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="giac")

[Out] 1/2*x^2 + 2/3*x^(3/2)

maple [A] time = 0.00, size = 12, normalized size = 0.71

$$\frac{x^2}{2} + \frac{2x^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)+1)*x^(1/2),x)

[Out] 2/3*x^(3/2)+1/2*x^2

maxima [B] time = 0.73, size = 26, normalized size = 1.53

$$\frac{1}{2}(\sqrt{x} + 1)^4 - \frac{4}{3}(\sqrt{x} + 1)^3 + (\sqrt{x} + 1)^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(1+x^(1/2)),x, algorithm="maxima")

[Out] 1/2*(sqrt(x) + 1)^4 - 4/3*(sqrt(x) + 1)^3 + (sqrt(x) + 1)^2

mupad [B] time = 0.02, size = 11, normalized size = 0.65

$$\frac{x^2}{2} + \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)*(x^(1/2) + 1),x)

[Out] x^2/2 + (2*x^(3/2))/3

sympy [A] time = 0.14, size = 12, normalized size = 0.71

$$\frac{2x^{\frac{3}{2}}}{3} + \frac{x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)*(1+x**(1/2)),x)

[Out] 2*x**(3/2)/3 + x**2/2

$$3.641 \quad \int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx$$

Optimal. Leaf size=19

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Rubi [A] time = 0.00, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {14}

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[x])/x^(1/3), x]

[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7

Rule 14

Int[(u_)*((c_.)*(x_.))^(m_.), x_Symbol] := Int[ExpandIntegrand[(c*x)^m*u, x], x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_ + (b_.)*(v_)) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{\sqrt[3]{x}} dx &= \int \left(\frac{1}{\sqrt[3]{x}} - \sqrt[6]{x} \right) dx \\ &= \frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7} \end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$\frac{3x^{2/3}}{2} - \frac{6x^{7/6}}{7}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[x])/x^(1/3), x]

[Out] (3*x^(2/3))/2 - (6*x^(7/6))/7

IntegrateAlgebraic [A] time = 0.02, size = 19, normalized size = 1.00

$$-\frac{3}{14} (4x^{7/6} - 7x^{2/3})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[x])/x^(1/3), x]

[Out] (-3*(-7*x^(2/3) + 4*x^(7/6)))/14

fricas [A] time = 0.51, size = 11, normalized size = 0.58

$$-\frac{6}{7} x^{7/6} + \frac{3}{2} x^{2/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="fricas")

[Out] -6/7*x^(7/6) + 3/2*x^(2/3)

giac [A] time = 0.37, size = 11, normalized size = 0.58

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="giac")

[Out] -6/7*x^(7/6) + 3/2*x^(2/3)

maple [A] time = 0.00, size = 12, normalized size = 0.63

$$-\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x^(1/2))/x^(1/3),x)

[Out] 3/2*x^(2/3)-6/7*x^(7/6)

maxima [A] time = 0.48, size = 11, normalized size = 0.58

$$-\frac{6}{7}x^{\frac{7}{6}} + \frac{3}{2}x^{\frac{2}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/x^(1/3),x, algorithm="maxima")

[Out] -6/7*x^(7/6) + 3/2*x^(2/3)

mupad [B] time = 0.02, size = 12, normalized size = 0.63

$$-\frac{3x^{2/3}(4\sqrt{x}-7)}{14}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^(1/2) - 1)/x^(1/3),x)

[Out] -(3*x^(2/3)*(4*x^(1/2) - 7))/14

sympy [A] time = 1.53, size = 15, normalized size = 0.79

$$-\frac{6x^{\frac{7}{6}}}{7} + \frac{3x^{\frac{2}{3}}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x**(1/2))/x**(1/3),x)

[Out] -6*x**(7/6)/7 + 3*x**(2/3)/2

$$3.642 \quad \int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx$$

Optimal. Leaf size=41

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \tan^{-1}(\sqrt[6]{x})$$

Rubi [A] time = 0.01, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {341, 50, 63, 203}

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(1 + x^(1/3)), x]

[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 341

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denomi
nator[n]}, Dist[k, Subst[Int[x^(k*(m + 1) - 1)*(a + b*x^(k*n))^p, x], x, x^(
1/k)], x] /; FreeQ[{a, b, m, p}, x] && FractionQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{1+\sqrt[3]{x}} dx &= 3 \operatorname{Subst} \left(\int \frac{x^{7/2}}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= \frac{6x^{7/6}}{7} - 3 \operatorname{Subst} \left(\int \frac{x^{5/2}}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= -\frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3 \operatorname{Subst} \left(\int \frac{x^{3/2}}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} - 3 \operatorname{Subst} \left(\int \frac{\sqrt{x}}{1+x} dx, x, \sqrt[3]{x} \right) \\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 3 \operatorname{Subst} \left(\int \frac{1}{\sqrt{x}(1+x)} dx, x, \sqrt[3]{x} \right) \\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6 \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt[6]{x} \right) \\
&= -6\sqrt[6]{x} + 2\sqrt{x} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7} + 6 \tan^{-1}(\sqrt[6]{x})
\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.00

$$\frac{6x^{7/6}}{7} - \frac{6x^{5/6}}{5} + 2\sqrt{x} - 6\sqrt[6]{x} + 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(1 + x^(1/3)), x]

[Out] -6*x^(1/6) + 2*Sqrt[x] - (6*x^(5/6))/5 + (6*x^(7/6))/7 + 6*ArcTan[x^(1/6)]

IntegrateAlgebraic [A] time = 0.03, size = 42, normalized size = 1.02

$$\frac{2}{35} (15x^{7/6} - 21x^{5/6} + 35\sqrt{x} - 105\sqrt[6]{x}) + 6 \tan^{-1}(\sqrt[6]{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(1 + x^(1/3)), x]

[Out] (2*(-105*x^(1/6) + 35*Sqrt[x] - 21*x^(5/6) + 15*x^(7/6)))/35 + 6*ArcTan[x^(1/6)]

fricas [A] time = 0.45, size = 25, normalized size = 0.61

$$\frac{6}{7}(x-7)x^{\frac{1}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} + 6 \arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x^(1/3)), x, algorithm="fricas")

[Out] 6/7*(x - 7)*x^(1/6) - 6/5*x^(5/6) + 2*sqrt(x) + 6*arctan(x^(1/6))

giac [A] time = 0.38, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6 \arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(1+x^(1/3)), x, algorithm="giac")

[Out] $6/7*x^{(7/6)} - 6/5*x^{(5/6)} + 2*\text{sqrt}(x) - 6*x^{(1/6)} + 6*\text{arctan}(x^{(1/6)})$

maple [A] time = 0.00, size = 28, normalized size = 0.68

$$\frac{6x^{\frac{7}{6}}}{7} + 6 \arctan\left(x^{\frac{1}{6}}\right) - \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 6x^{\frac{1}{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^(1/3)+1),x)`

[Out] $-6*x^{(1/6)}-6/5*x^{(5/6)}+6/7*x^{(7/6)}+6*\text{arctan}(x^{(1/6)})+2*x^{(1/2)}$

maxima [A] time = 1.63, size = 27, normalized size = 0.66

$$\frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 6x^{\frac{1}{6}} + 6 \arctan\left(x^{\frac{1}{6}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)/(1+x^(1/3)),x, algorithm="maxima")`

[Out] $6/7*x^{(7/6)} - 6/5*x^{(5/6)} + 2*\text{sqrt}(x) - 6*x^{(1/6)} + 6*\text{arctan}(x^{(1/6)})$

mupad [B] time = 0.03, size = 27, normalized size = 0.66

$$6 \operatorname{atan}\left(x^{1/6}\right) + 2\sqrt{x} - 6x^{1/6} - \frac{6x^{5/6}}{5} + \frac{6x^{7/6}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)/(x^(1/3) + 1),x)`

[Out] $6*\operatorname{atan}(x^{(1/6)}) + 2*x^{(1/2)} - 6*x^{(1/6)} - (6*x^{(5/6)})/5 + (6*x^{(7/6)})/7$

sympy [A] time = 3.36, size = 37, normalized size = 0.90

$$\frac{6x^{\frac{7}{6}}}{7} - \frac{6x^{\frac{5}{6}}}{5} - 6\sqrt[6]{x} + 2\sqrt{x} + 6 \operatorname{atan}\left(\sqrt[6]{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)/(1+x**(1/3)),x)`

[Out] $6*x^{(7/6)}/7 - 6*x^{(5/6)}/5 - 6*x^{(1/6)} + 2*\text{sqrt}(x) + 6*\operatorname{atan}(x^{(1/6)})$

$$3.643 \quad \int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx$$

Optimal. Leaf size=67

$$6\sqrt[3]{\sqrt{x}+1} + 3\log\left(1 - \sqrt[3]{\sqrt{x}+1}\right) - \frac{\log(x)}{2} - 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Rubi [A] time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {266, 50, 57, 618, 204, 31}

$$6\sqrt[3]{\sqrt{x}+1} + 3\log\left(1 - \sqrt[3]{\sqrt{x}+1}\right) - \frac{\log(x)}{2} - 2\sqrt{3} \tan^{-1}\left(\frac{2\sqrt[3]{\sqrt{x}+1}+1}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[x])^(1/3)/x,x]

[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3]] + 3*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[x]/2

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 57

Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :> With[{q = Rt[(b*c - a*d)/b, 3]}, -Simp[Log[RemoveContent[a + b*x, x]]/(2*b*q^2), x] + (-Dist[3/(2*b*q), Subst[Int[1/(q^2 + q*x + x^2), x], x, (c + d*x)^(1/3)], x] - Dist[3/(2*b*q^2), Subst[Int[1/(q - x), x], x, (c + d*x)^(1/3)], x])]/; FreeQ[{a, b, c, d}, x] && PosQ[(b*c - a*d)/b]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 266

Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\sqrt[3]{1+\sqrt{x}}}{x} dx &= 2 \operatorname{Subst} \left(\int \frac{\sqrt[3]{1+x}}{x} dx, x, \sqrt{x} \right) \\ &= 6\sqrt[3]{1+\sqrt{x}} + 2 \operatorname{Subst} \left(\int \frac{1}{x(1+x)^{2/3}} dx, x, \sqrt{x} \right) \\ &= 6\sqrt[3]{1+\sqrt{x}} - \frac{\log(x)}{2} - 3 \operatorname{Subst} \left(\int \frac{1}{1-x} dx, x, \sqrt[3]{1+\sqrt{x}} \right) - 3 \operatorname{Subst} \left(\int \frac{1}{1+x+x^2} dx, x, \sqrt[3]{1+\sqrt{x}} \right) \\ &= 6\sqrt[3]{1+\sqrt{x}} + 3 \log \left(1 - \sqrt[3]{1+\sqrt{x}} \right) - \frac{\log(x)}{2} + 6 \operatorname{Subst} \left(\int \frac{1}{-3-x^2} dx, x, 1 + 2\sqrt[3]{1+\sqrt{x}} \right) \\ &= 6\sqrt[3]{1+\sqrt{x}} - 2\sqrt{3} \tan^{-1} \left(\frac{1 + 2\sqrt[3]{1+\sqrt{x}}}{\sqrt{3}} \right) + 3 \log \left(1 - \sqrt[3]{1+\sqrt{x}} \right) - \frac{\log(x)}{2} \end{aligned}$$

Mathematica [A] time = 0.02, size = 88, normalized size = 1.31

$$6\sqrt[3]{\sqrt{x}+1} + 2 \log \left(1 - \sqrt[3]{\sqrt{x}+1} \right) - \log \left((\sqrt{x}+1)^{2/3} + \sqrt[3]{\sqrt{x}+1} + 1 \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{\sqrt{x}+1} + 1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sqrt[x])^(1/3)/x, x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[(1 + 2*(1 + Sqrt[x])^(1/3))/Sqrt[3]] + 2*Log[1 - (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + Sqrt[x])^(2/3)]
```

IntegrateAlgebraic [A] time = 0.05, size = 89, normalized size = 1.33

$$6\sqrt[3]{\sqrt{x}+1} + 2 \log \left(\sqrt[3]{\sqrt{x}+1} - 1 \right) - \log \left((\sqrt{x}+1)^{2/3} + \sqrt[3]{\sqrt{x}+1} + 1 \right) - 2\sqrt{3} \tan^{-1} \left(\frac{2\sqrt[3]{\sqrt{x}+1}}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + Sqrt[x])^(1/3)/x, x]
```

```
[Out] 6*(1 + Sqrt[x])^(1/3) - 2*Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(1 + Sqrt[x])^(1/3))/Sqrt[3]] + 2*Log[-1 + (1 + Sqrt[x])^(1/3)] - Log[1 + (1 + Sqrt[x])^(1/3) + (1 + Sqrt[x])^(2/3)]
```

fricas [A] time = 0.46, size = 65, normalized size = 0.97

$$-2\sqrt{3} \arctan \left(\frac{2}{3}\sqrt{3}(\sqrt{x}+1)^{\frac{1}{3}} + \frac{1}{3}\sqrt{3} \right) + 6(\sqrt{x}+1)^{\frac{1}{3}} - \log \left((\sqrt{x}+1)^{\frac{2}{3}} + (\sqrt{x}+1)^{\frac{1}{3}} + 1 \right) + 2 \log \left((\sqrt{x}+1)^{\frac{1}{3}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x^(1/2))^(1/3)/x,x, algorithm="fricas")
```


[Out] $-2\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}\left(\sqrt{x}+1\right)^{1/3}+1\right)+6\left(\sqrt{x}+1\right)^{1/3}-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)$

giac [A] time = 0.63, size = 64, normalized size = 0.96

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{1/3}+1\right)\right)+6\left(\sqrt{x}+1\right)^{1/3}-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/2))^(1/3)/x,x, algorithm="giac")`

[Out] $-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{1/3}+1\right)\right)+6\left(\sqrt{x}+1\right)^{1/3}-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+2\log\left(\left|\left(\sqrt{x}+1\right)^{1/3}-1\right|\right)$

maple [A] time = 0.01, size = 64, normalized size = 0.96

$$-2\sqrt{3}\arctan\left(\frac{\left(1+2\left(\sqrt{x}+1\right)^{1/3}\right)\sqrt{3}}{3}\right)+2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)-\ln\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+6\left(\sqrt{x}+1\right)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)+1)^(1/3)/x,x)`

[Out] $6\left(x^{1/2}+1\right)^{1/3}+2\ln\left(\left(x^{1/2}+1\right)^{1/3}-1\right)-\ln\left(\left(x^{1/2}+1\right)^{2/3}+\left(x^{1/2}+1\right)^{1/3}+1\right)-2\arctan\left(\frac{1}{3}\left(1+2\left(x^{1/2}+1\right)^{1/3}\right)\sqrt{3}\right)$

maxima [A] time = 1.72, size = 63, normalized size = 0.94

$$-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{1/3}+1\right)\right)+6\left(\sqrt{x}+1\right)^{1/3}-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x^(1/2))^(1/3)/x,x, algorithm="maxima")`

[Out] $-2\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2\left(\sqrt{x}+1\right)^{1/3}+1\right)\right)+6\left(\sqrt{x}+1\right)^{1/3}-\log\left(\left(\sqrt{x}+1\right)^{2/3}+\left(\sqrt{x}+1\right)^{1/3}+1\right)+2\log\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)$

mupad [B] time = 3.83, size = 73, normalized size = 1.09

$$2\ln\left(\left(\sqrt{x}+1\right)^{1/3}-1\right)+6\left(\sqrt{x}+1\right)^{1/3}+\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}-\frac{\sqrt{3}1i}{2}\right)\left(-1+\sqrt{3}1i\right)-\ln\left(\left(\sqrt{x}+1\right)^{1/3}+\frac{1}{2}+\frac{\sqrt{3}1i}{2}\right)\left(1+\sqrt{3}1i\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^(1/2)+1)^(1/3)/x,x)`

[Out] $2\log\left(\left(x^{1/2}+1\right)^{1/3}-1\right)+6\left(x^{1/2}+1\right)^{1/3}+\log\left(\left(x^{1/2}+1\right)^{1/3}-\left(3^{1/2}1i\right)/2+1/2\right)\left(3^{1/2}1i-1\right)-\log\left(\left(3^{1/2}1i\right)/2+\left(x^{1/2}+1\right)^{1/3}+1/2\right)\left(3^{1/2}1i+1\right)$

sympy [C] time = 1.05, size = 39, normalized size = 0.58

$$\frac{2\sqrt{x}\Gamma\left(-\frac{1}{3}\right) {}_2F_1\left(\begin{matrix} -\frac{1}{3}, -\frac{1}{3} \\ \frac{2}{3} \end{matrix} \middle| \frac{e^{i\pi}}{\sqrt{x}}\right)}{\Gamma\left(\frac{2}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x**(1/2))**(1/3)/x,x)
```

```
[Out] -2*x**(1/6)*gamma(-1/3)*hyper((-1/3, -1/3), (2/3,), exp_polar(I*pi)/sqrt(x)
)/gamma(2/3)
```

$$3.644 \quad \int (1 - \sqrt{x}) dx$$

Optimal. Leaf size=11

$$x - \frac{2x^{3/2}}{3}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Int[1 - Sqrt[x], x]

[Out] x - (2*x^(3/2))/3

Rubi steps

$$\int (1 - \sqrt{x}) dx = x - \frac{2x^{3/2}}{3}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{2x^{3/2}}{3}$$

Antiderivative was successfully verified.

[In] Integrate[1 - Sqrt[x], x]

[Out] x - (2*x^(3/2))/3

IntegrateAlgebraic [A] time = 0.00, size = 15, normalized size = 1.36

$$\frac{1}{3} (3x - 2x^{3/2})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1 - Sqrt[x], x]

[Out] (3*x - 2*x^(3/2))/3

fricas [A] time = 0.44, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2), x, algorithm="fricas")

[Out] -2/3*x^(3/2) + x

giac [A] time = 0.33, size = 7, normalized size = 0.64

$$-\frac{2}{3} x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="giac")

[Out] -2/3*x^(3/2) + x

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-x^(1/2),x)

[Out] -2/3*x^(3/2)+x

maxima [A] time = 0.88, size = 7, normalized size = 0.64

$$-\frac{2}{3}x^{\frac{3}{2}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/2),x, algorithm="maxima")

[Out] -2/3*x^(3/2) + x

mupad [B] time = 0.00, size = 7, normalized size = 0.64

$$x - \frac{2x^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1 - x^(1/2),x)

[Out] x - (2*x^(3/2))/3

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{2x^{\frac{3}{2}}}{3} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x**(1/2),x)

[Out] -2*x**(3/2)/3 + x

$$3.645 \quad \int (1 - \sqrt[4]{x}) dx$$

Optimal. Leaf size=11

$$x - \frac{4x^{5/4}}{5}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Int[1 - x^(1/4), x]

[Out] x - (4*x^(5/4))/5

Rubi steps

$$\int (1 - \sqrt[4]{x}) dx = x - \frac{4x^{5/4}}{5}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[1 - x^(1/4), x]

[Out] x - (4*x^(5/4))/5

IntegrateAlgebraic [A] time = 0.01, size = 15, normalized size = 1.36

$$\frac{1}{5} (5x - 4x^{5/4})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1 - x^(1/4), x]

[Out] (5*x - 4*x^(5/4))/5

fricas [A] time = 0.43, size = 7, normalized size = 0.64

$$-\frac{4}{5} x^{5/4} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/4), x, algorithm="fricas")

[Out] -4/5*x^(5/4) + x

giac [A] time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5} x^{5/4} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/4),x, algorithm="giac")

[Out] -4/5*x^(5/4) + x

maple [A] time = 0.00, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1-x^(1/4),x)

[Out] x-4/5*x^(5/4)

maxima [A] time = 0.62, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x^(1/4),x, algorithm="maxima")

[Out] -4/5*x^(5/4) + x

mupad [B] time = 0.02, size = 7, normalized size = 0.64

$$x - \frac{4x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1 - x^(1/4),x)

[Out] x - (4*x^(5/4))/5

sympy [A] time = 0.06, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1-x**(1/4),x)

[Out] -4*x**(5/4)/5 + x

$$3.646 \quad \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx$$

Optimal. Leaf size=11

$$x - \frac{4x^{5/4}}{5}$$

Rubi [A] time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {26}

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sqrt[x])/(1 + x^(1/4)),x]

[Out] x - (4*x^(5/4))/5

Rule 26

Int[(u_.)*((a_) + (b_.)*(x_)^(n_.))^(m_.)*((c_) + (d_.)*(x_)^(j_.))^(p_.), x_Symbol] :> Dist[(-(b^2/d))^m, Int[u/(a - b*x^n)^m, x], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && EqQ[j, 2*n] && EqQ[p, -m] && EqQ[b^2*c + a^2*d, 0] && GtQ[a, 0] && LtQ[d, 0]

Rubi steps

$$\begin{aligned} \int \frac{1-\sqrt{x}}{1+\sqrt[4]{x}} dx &= \int (1 - \sqrt[4]{x}) dx \\ &= x - \frac{4x^{5/4}}{5} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 1.00

$$x - \frac{4x^{5/4}}{5}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - Sqrt[x])/(1 + x^(1/4)),x]

[Out] x - (4*x^(5/4))/5

IntegrateAlgebraic [A] time = 0.02, size = 15, normalized size = 1.36

$$\frac{1}{5} (5x - 4x^{5/4})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - Sqrt[x])/(1 + x^(1/4)),x]

[Out] (5*x - 4*x^(5/4))/5

fricas [A] time = 0.43, size = 7, normalized size = 0.64

$$-\frac{4}{5} x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="fricas")

[Out] -4/5*x^(5/4) + x

giac [A] time = 0.35, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="giac")

[Out] -4/5*x^(5/4) + x

maple [C] time = 0.01, size = 44, normalized size = 4.00

$$-\frac{4x^{\frac{5}{4}}}{5} + x - \ln(x-1) - \ln(\sqrt{x}-1) + \ln(\sqrt{x}+1) + 2\ln\left(x^{\frac{1}{4}}+1\right) + 2\ln\left(x^{\frac{1}{4}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x^(1/2))/(x^(1/4)+1),x)

[Out] -4/5*x^(5/4)+x+2*ln(x^(1/4)+1)+2*ln(x^(1/4)-1)-ln(x-1)-ln(x^(1/2)-1)+ln(x^(1/2)+1)

maxima [A] time = 0.61, size = 7, normalized size = 0.64

$$-\frac{4}{5}x^{\frac{5}{4}} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^(1/2))/(1+x^(1/4)),x, algorithm="maxima")

[Out] -4/5*x^(5/4) + x

mupad [B] time = 0.03, size = 7, normalized size = 0.64

$$x - \frac{4x^{5/4}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^(1/2) - 1)/(x^(1/4) + 1),x)

[Out] x - (4*x^(5/4))/5

sympy [A] time = 4.36, size = 8, normalized size = 0.73

$$-\frac{4x^{\frac{5}{4}}}{5} + x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x**(1/2))/(1+x**(1/4)),x)

[Out] -4*x**(5/4)/5 + x

$$3.647 \quad \int \frac{1}{\sqrt{(a+bx)(c+dx)}} dx$$

Optimal. Leaf size=61

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1981, 621, 206}

$$\frac{\tanh^{-1}\left(\frac{ad+bc+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(ad+bc)+ac+bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x)*(c + d*x)], x]

[Out] ArcTanh[(b*c + a*d + 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x + b*d*x^2])]/(Sqrt[b]*Sqrt[d])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(a+bx)(c+dx)}} dx &= \int \frac{1}{\sqrt{ac + (bc+ad)x + bdx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{4bd - x^2} dx, x, \frac{bc+ad+2bdx}{\sqrt{ac+(bc+ad)x+bdx^2}}\right) \\ &= \frac{\tanh^{-1}\left(\frac{bc+ad+2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc+ad)x+bdx^2}}\right)}{\sqrt{b}\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.06, size = 95, normalized size = 1.56

$$\frac{2\sqrt{a+bx}\sqrt{bc-ad}\sqrt{\frac{b(c+dx)}{bc-ad}}\sinh^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{bc-ad}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a + b*x)*(c + d*x)],x]

[Out] (2*Sqrt[b*c - a*d]*Sqrt[a + b*x]*Sqrt[(b*(c + d*x))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c - a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c + d*x)])

IntegrateAlgebraic [B] time = 0.44, size = 189, normalized size = 3.10

$$\frac{\sqrt{bd} \log(a^2 d^2 + 8bdx\sqrt{bd} \sqrt{x(ad+bc) + ac + bdx^2} - 2abcd - 4abd^2 x + b^2 c^2 - 4b^2 cd x - 8b^2 d^2 x^2)}{2bd} - \frac{\tanh^{-1}\left(\frac{2\sqrt{b} \sqrt{d} x \sqrt{bd}}{ad+bc} - \frac{2\sqrt{b} \sqrt{d} \sqrt{x(ad+bc) + ac + bdx^2}}{ad+bc}\right)}{\sqrt{b} \sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a + b*x)*(c + d*x)],x]

[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[d]*Sqrt[b*d]*x)/(b*c + a*d) - (2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c + a*d)*x + b*d*x^2])/(b*c + a*d)]/(Sqrt[b]*Sqrt[d])) - (Sqrt[b*d]*Log[b^2*c^2 - 2*a*b*c*d + a^2*d^2 - 4*b^2*c*d*x - 4*a*b*d^2*x - 8*b^2*d^2*x^2 + 8*b*d*Sqrt[b*d]*x*Sqrt[a*c + (b*c + a*d)*x + b*d*x^2]])/(2*b*d)

fricas [A] time = 0.44, size = 192, normalized size = 3.15

$$\left[\frac{\sqrt{bd} \log(8b^2 d^2 x^2 + b^2 c^2 + 6abcd + a^2 d^2 + 4\sqrt{bd} x^2 + ac + (bc + ad)x(2bdx + bc + ad)\sqrt{bd} + 8(b^2 cd + abd^2)x)}{2bd}, -\frac{\sqrt{-bd} \arctan\left(\frac{\sqrt{bd} x^2 + ac + (bc + ad)x(2bdx + bc + ad)\sqrt{-bd}}{2(b^2 d^2 x^2 + abd^2 + b^2 cd + abd^2)x}\right)}{bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 4*sqrt(b*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(b*d) + 8*(b^2*c*d + a*b*d^2)*x)/(b*d), -sqrt(-b*d)*arctan(1/2*sqrt(b*d*x^2 + a*c + (b*c + a*d)*x)*(2*b*d*x + b*c + a*d)*sqrt(-b*d)/(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x))/(b*d)]

giac [A] time = 0.72, size = 68, normalized size = 1.11

$$\frac{\sqrt{bd} \log\left(\left|-2\left(\sqrt{bd} x - \sqrt{bdx^2 + bcx + adx + ac}\right)bd - \sqrt{bd} bc - \sqrt{bd} ad\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="giac")

[Out] -sqrt(b*d)*log(abs(-2*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))*b*d - sqrt(b*d)*b*c - sqrt(b*d)*a*d))/(b*d)

maple [A] time = 0.01, size = 49, normalized size = 0.80

$$\frac{\ln\left(\frac{bdx + \frac{1}{2}ad + \frac{1}{2}bc}{\sqrt{bd}} + \sqrt{bd} x^2 + ac + (ad + bc)x}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)*(d*x+c))^(1/2),x)

[Out] ln((1/2*a*d+1/2*b*c+b*d*x)/(b*d)^(1/2)+(a*c+(a*d+b*c)*x+b*d*x^2)^(1/2))/(b*d)^(1/2)

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)*(c + d*x))^(1/2),x)

[Out] int(1/((a + b*x)*(c + d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)(c+dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(d*x+c))**(1/2),x)

[Out] Integral(1/sqrt((a + b*x)*(c + d*x)), x)

$$3.648 \quad \int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$$

Optimal. Leaf size=65

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Rubi [A] time = 0.02, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1981, 621, 204}

$$-\frac{\tan^{-1}\left(\frac{-ad+bc-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a + b*x)*(c - d*x)], x]

[Out] -(ArcTan[(b*c - a*d - 2*b*d*x)/(2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x - b*d*x^2]])/(Sqrt[b]*Sqrt[d])

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 621

Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1981

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && QquadraticQ[u, x] && !QuadraticMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx &= \int \frac{1}{\sqrt{ac + (bc-ad)x - bdx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{-4bd - x^2} dx, x, \frac{bc-ad-2bdx}{\sqrt{ac + (bc-ad)x - bdx^2}}\right) \\ &= -\frac{\tan^{-1}\left(\frac{bc-ad-2bdx}{2\sqrt{b}\sqrt{d}\sqrt{ac+(bc-ad)x-bdx^2}}\right)}{\sqrt{b}\sqrt{d}} \end{aligned}$$

Mathematica [A] time = 0.08, size = 94, normalized size = 1.45

$$\frac{2\sqrt{a+bx}\sqrt{ad+bc}\sqrt{\frac{b(c-dx)}{ad+bc}}\sin^{-1}\left(\frac{\sqrt{d}\sqrt{a+bx}}{\sqrt{ad+bc}}\right)}{b\sqrt{d}\sqrt{(a+bx)(c-dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a + b*x)*(c - d*x)],x]

[Out] (2*Sqrt[b*c + a*d]*Sqrt[a + b*x]*Sqrt[(b*(c - d*x))/(b*c + a*d)]*ArcSin[(Sqrt[d]*Sqrt[a + b*x])/Sqrt[b*c + a*d]])/(b*Sqrt[d]*Sqrt[(a + b*x)*(c - d*x)])

IntegrateAlgebraic [B] time = 0.43, size = 198, normalized size = 3.05

$$\frac{\sqrt{-bd} \log(a^2 d^2 - 8bdx\sqrt{-bd}\sqrt{x(bc-ad)+ac-bdx^2} + 2abcd - 4abd^2x + b^2c^2 + 4b^2cdx - 8b^2d^2x^2)}{2bd} - \frac{\tan^{-1}\left(\frac{2\sqrt{b}\sqrt{d}x\sqrt{-bd}}{bc-ad} - \frac{2\sqrt{b}\sqrt{d}\sqrt{x(bc-ad)+ac-bdx^2}}{bc-ad}\right)}{\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a + b*x)*(c - d*x)],x]

[Out] -(ArcTan[(2*Sqrt[b]*Sqrt[d]*Sqrt[-(b*d)]*x)/(b*c - a*d) - (2*Sqrt[b]*Sqrt[d]*Sqrt[a*c + (b*c - a*d)*x - b*d*x^2])/(b*c - a*d)]/(Sqrt[b]*Sqrt[d])) + (Sqrt[-(b*d)]*Log[b^2*c^2 + 2*a*b*c*d + a^2*d^2 + 4*b^2*c*d*x - 4*a*b*d^2*x - 8*b^2*d^2*x^2 - 8*b*d*Sqrt[-(b*d)]*x*Sqrt[a*c + (b*c - a*d)*x - b*d*x^2]])/(2*b*d)

fricas [A] time = 0.44, size = 202, normalized size = 3.11

$$\left[\frac{\sqrt{-bd} \log(8b^2d^2x^2 + b^2c^2 - 6abcd + a^2d^2 - 4\sqrt{-bd}x^2 + ac + (bc-ad)x(2bdx - bc + ad)\sqrt{-bd} - 8(b^2cd - abd^2)x)}{2bd}, -\frac{\sqrt{bd} \arctan\left(\frac{\sqrt{-bd}x^2 + ac + (bc-ad)x(2bdx - bc + ad)\sqrt{bd}}{2(b^2d^2x^2 - abcd - (b^2cd - abd^2)x)}\right)}{bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/2*sqrt(-b*d)*log(8*b^2*d^2*x^2 + b^2*c^2 - 6*a*b*c*d + a^2*d^2 - 4*sqrt(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(-b*d) - 8*(b^2*c*d - a*b*d^2)*x)/(b*d), -sqrt(b*d)*arctan(1/2*sqrt(-b*d*x^2 + a*c + (b*c - a*d)*x)*(2*b*d*x - b*c + a*d)*sqrt(b*d)/(b^2*d^2*x^2 - a*b*c*d - (b^2*c*d - a*b*d^2)*x))/(b*d)]

giac [A] time = 0.74, size = 59, normalized size = 0.91

$$\frac{\log\left(\left|bc - ad + 2\sqrt{-bd}\left(\sqrt{-bd}x - \sqrt{-bdx^2 + bcx - adx + ac}\right)\right|\right)}{\sqrt{-bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="giac")

[Out] -log(abs(b*c - a*d + 2*sqrt(-b*d)*(sqrt(-b*d)*x - sqrt(-b*d*x^2 + b*c*x - a*d*x + a*c))))/sqrt(-b*d)

maple [A] time = 0.01, size = 55, normalized size = 0.85

$$\frac{\arctan\left(\frac{\sqrt{bd}\left(x - \frac{-ad+bc}{2bd}\right)}{\sqrt{-bd}x^2 + ac + (-ad+bc)x}\right)}{\sqrt{bd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x+a)*(-d*x+c))^(1/2),x)

[Out] 1/(b*d)^(1/2)*arctan((b*d)^(1/2)*(x-1/2*(-a*d+b*c)/b/d)/(a*c+(-a*d+b*c)*x-b*d*x^2)^(1/2))

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(-d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(a*d-b*c>0)', see `assume?` for more details)Is a*d-b*c zero or nonzero?

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x)*(c - d*x))^(1/2),x)

[Out] int(1/((a + b*x)*(c - d*x))^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{(a+bx)(c-dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x+a)*(-d*x+c))**(1/2),x)

[Out] Integral(1/sqrt((a + b*x)*(c - d*x)), x)

$$3.649 \quad \int \frac{1}{\sqrt{x}(1-x^2)} dx$$

Optimal. Leaf size=13

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {329, 212, 206, 203}

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[x]*(1 - x^2)),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x}(1-x^2)} dx &= 2 \operatorname{Subst} \left(\int \frac{1}{1-x^4} dx, x, \sqrt{x} \right) \\ &= \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{x} \right) \\ &= \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[x]*(1 - x^2)),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[x]*(1 - x^2)),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

fricas [B] time = 0.42, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="fricas")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

giac [B] time = 0.39, size = 22, normalized size = 1.69

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(|\sqrt{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="giac")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}(\sqrt{x}) + \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x^2+1)/x^(1/2),x)

[Out] arctan(x^(1/2))+arctanh(x^(1/2))

maxima [B] time = 1.01, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-x^2+1)/x^(1/2),x, algorithm="maxima")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

mupad [B] time = 3.34, size = 9, normalized size = 0.69

$$\operatorname{atan}(\sqrt{x}) + \operatorname{atanh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(-1/(x^(1/2)*(x^2 - 1)),x)`

[Out] `atan(x^(1/2)) + atanh(x^(1/2))`

sympy [B] time = 0.39, size = 26, normalized size = 2.00

$$-\frac{\log(\sqrt{x} - 1)}{2} + \frac{\log(\sqrt{x} + 1)}{2} + \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-x**2+1)/x**(1/2),x)`

[Out] `-log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))`

$$3.650 \quad \int \frac{\sqrt{x}}{x-x^3} dx$$

Optimal. Leaf size=13

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1584, 329, 212, 206, 203}

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]/(x - x^3),x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 212

Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] && !GtQ[a/b, 0]

Rule 329

Int[((c_.)*(x_)^(m_))*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := With[{k = Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n)))/c^n]^p, x], x, (c*x)^(1/k)], x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]

Rule 1584

Int[(u_.)*(x_)^(m_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(m + n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, m, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{x}}{x-x^3} dx &= \int \frac{1}{\sqrt{x}(1-x^2)} dx \\
&= 2 \operatorname{Subst} \left(\int \frac{1}{1-x^4} dx, x, \sqrt{x} \right) \\
&= \operatorname{Subst} \left(\int \frac{1}{1-x^2} dx, x, \sqrt{x} \right) + \operatorname{Subst} \left(\int \frac{1}{1+x^2} dx, x, \sqrt{x} \right) \\
&= \tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})
\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]/(x - x^3), x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.03, size = 13, normalized size = 1.00

$$\tan^{-1}(\sqrt{x}) + \tanh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]/(x - x^3), x]

[Out] ArcTan[Sqrt[x]] + ArcTanh[Sqrt[x]]

fricas [B] time = 0.42, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-x^3+x), x, algorithm="fricas")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

giac [B] time = 0.30, size = 22, normalized size = 1.69

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(|\sqrt{x} - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-x^3+x), x, algorithm="giac")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(abs(sqrt(x) - 1))

maple [A] time = 0.01, size = 10, normalized size = 0.77

$$\operatorname{arctanh}(\sqrt{x}) + \arctan(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(-x^3+x), x)

[Out] arctan(x^(1/2))+arctanh(x^(1/2))

maxima [B] time = 1.01, size = 21, normalized size = 1.62

$$\arctan(\sqrt{x}) + \frac{1}{2} \log(\sqrt{x} + 1) - \frac{1}{2} \log(\sqrt{x} - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)/(-x^3+x),x, algorithm="maxima")

[Out] arctan(sqrt(x)) + 1/2*log(sqrt(x) + 1) - 1/2*log(sqrt(x) - 1)

mupad [B] time = 0.03, size = 9, normalized size = 0.69

$$\operatorname{atan}(\sqrt{x}) + \operatorname{atanh}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x - x^3),x)

[Out] atan(x^(1/2)) + atanh(x^(1/2))

sympy [B] time = 0.54, size = 26, normalized size = 2.00

$$-\frac{\log(\sqrt{x} - 1)}{2} + \frac{\log(\sqrt{x} + 1)}{2} + \operatorname{atan}(\sqrt{x})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**(1/2)/(-x**3+x),x)

[Out] -log(sqrt(x) - 1)/2 + log(sqrt(x) + 1)/2 + atan(sqrt(x))

$$3.651 \quad \int \frac{x}{2 - \sqrt{3} + (1 + \sqrt{3})x + x^2} dx$$

Optimal. Leaf size=72

$$\frac{1}{2} \log(x^2 + (1 + \sqrt{3})x - \sqrt{3} + 2) + \sqrt{\frac{1}{23}(13 + 8\sqrt{3})} \tanh^{-1}\left(\frac{2x + \sqrt{3} + 1}{\sqrt{2(3\sqrt{3} - 2)}}\right)$$

Rubi [A] time = 0.10, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {634, 618, 206, 628}

$$\frac{1}{2} \log(x^2 + (1 + \sqrt{3})x - \sqrt{3} + 2) + \sqrt{\frac{1}{23}(13 + 8\sqrt{3})} \tanh^{-1}\left(\frac{2x + \sqrt{3} + 1}{\sqrt{2(3\sqrt{3} - 2)}}\right)$$

Antiderivative was successfully verified.

[In] Int[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2), x]

[Out] Sqrt[(13 + 8*Sqrt[3])/23]*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[2*(-2 + 3*Sqrt[3])]] + Log[2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2]/2

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rubi steps

$$\begin{aligned} \int \frac{x}{2 - \sqrt{3} + (1 + \sqrt{3})x + x^2} dx &= \frac{1}{2} \int \frac{1 + \sqrt{3} + 2x}{2 - \sqrt{3} + (1 + \sqrt{3})x + x^2} dx + \frac{1}{2} (-1 - \sqrt{3}) \int \frac{1}{2 - \sqrt{3} + (1 + \sqrt{3})x + x^2} dx \\ &= \frac{1}{2} \log(2 - \sqrt{3} + (1 + \sqrt{3})x + x^2) + (1 + \sqrt{3}) \operatorname{Subst} \left(\int \frac{1}{-2(2 - 3\sqrt{3}) - x^2} dx \right) \\ &= \sqrt{\frac{1}{23}(13 + 8\sqrt{3})} \tanh^{-1} \left(\frac{1 + \sqrt{3} + 2x}{\sqrt{2(-2 + 3\sqrt{3})}} \right) + \frac{1}{2} \log(2 - \sqrt{3} + (1 + \sqrt{3})x + x^2) \end{aligned}$$

Mathematica [A] time = 0.10, size = 72, normalized size = 1.00

$$\frac{1}{2} \log(x^2 + \sqrt{3}x + x - \sqrt{3} + 2) + \frac{(1 + \sqrt{3}) \tanh^{-1} \left(\frac{2x + \sqrt{3} + 1}{\sqrt{6\sqrt{3} - 4}} \right)}{\sqrt{6\sqrt{3} - 4}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2), x]

[Out] ((1 + Sqrt[3])*ArcTanh[(1 + Sqrt[3] + 2*x)/Sqrt[-4 + 6*Sqrt[3]]])/Sqrt[-4 + 6*Sqrt[3]] + Log[2 - Sqrt[3] + x + Sqrt[3]*x + x^2]/2

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{2 - \sqrt{3} + (1 + \sqrt{3})x + x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2), x]

[Out] IntegrateAlgebraic[x/(2 - Sqrt[3] + (1 + Sqrt[3])*x + x^2), x]

fricas [A] time = 0.43, size = 100, normalized size = 1.39

$$\frac{1}{46} \sqrt{23} \sqrt{8\sqrt{3} + 13} \log \left(-\frac{\sqrt{23} \sqrt{8\sqrt{3} + 13} (5\sqrt{3} - 11) - 46x - 23\sqrt{3} - 23}{\sqrt{23} \sqrt{8\sqrt{3} + 13} (5\sqrt{3} - 11) + 46x + 23\sqrt{3} + 23} \right) + \frac{1}{2} \log(x^2 + x(\sqrt{3} + 1) - \sqrt{3} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="fricas")

[Out] 1/46*sqrt(23)*sqrt(8*sqrt(3) + 13)*log(-(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*sqrt(3) - 11) - 46*x - 23*sqrt(3) - 23)/(sqrt(23)*sqrt(8*sqrt(3) + 13)*(5*sqrt(3) - 11) + 46*x + 23*sqrt(3) + 23)) + 1/2*log(x^2 + x*(sqrt(3) + 1) - sqrt(3) + 2)

giac [A] time = 0.43, size = 80, normalized size = 1.11

$$-\frac{(\sqrt{3} + 1) \log \left(\frac{|2x + \sqrt{3} - \sqrt{6\sqrt{3} - 4} + 1|}{|2x + \sqrt{3} + \sqrt{6\sqrt{3} - 4} + 1|} \right)}{2\sqrt{6\sqrt{3} - 4}} + \frac{1}{2} \log(|x^2 + x(\sqrt{3} + 1) - \sqrt{3} + 2|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="giac")

[Out] $-\frac{1}{2}(\sqrt{3} + 1)\log(\text{abs}(2x + \sqrt{3}) - \sqrt{6\sqrt{3} - 4}) + 1/\text{abs}(2x + \sqrt{3} + \sqrt{6\sqrt{3} - 4}) + 1/2\log(\text{abs}(x^2 + x(\sqrt{3} + 1) - \sqrt{3} + 2))$

maple [A] time = 0.02, size = 82, normalized size = 1.14

$$\frac{\operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}} + \frac{\sqrt{3} \operatorname{arctanh}\left(\frac{2x+1+\sqrt{3}}{\sqrt{-4+6\sqrt{3}}}\right)}{\sqrt{-4+6\sqrt{3}}} + \frac{\ln(x^2 + \sqrt{3}x + x - \sqrt{3} + 2)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x)

[Out] $\frac{1}{2}\ln(3^{(1/2)}*x+x^2-3^{(1/2)}+x+2)+1/(-4+6*3^{(1/2)})^{(1/2)}*\operatorname{arctanh}((1+2*x+3^{(1/2)})/(-4+6*3^{(1/2)})^{(1/2)})+1/(-4+6*3^{(1/2)})^{(1/2)}*\operatorname{arctanh}((1+2*x+3^{(1/2)})/(-4+6*3^{(1/2)})^{(1/2)})*3^{(1/2)}$

maxima [A] time = 1.00, size = 77, normalized size = 1.07

$$-\frac{(\sqrt{3} + 1)\log\left(\frac{2x + \sqrt{3} - \sqrt{6\sqrt{3} - 4} + 1}{2x + \sqrt{3} + \sqrt{6\sqrt{3} - 4} + 1}\right)}{2\sqrt{6\sqrt{3} - 4}} + \frac{1}{2}\log(x^2 + x(\sqrt{3} + 1) - \sqrt{3} + 2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x^2-3^(1/2)+x*(1+3^(1/2))),x, algorithm="maxima")

[Out] $-\frac{1}{2}(\sqrt{3} + 1)\log((2x + \sqrt{3}) - \sqrt{6\sqrt{3} - 4}) + 1/(2x + \sqrt{3} + \sqrt{6\sqrt{3} - 4}) + 1/2\log(x^2 + x(\sqrt{3} + 1) - \sqrt{3} + 2)$

mupad [B] time = 4.23, size = 233, normalized size = 3.24

$$\ln\left(x - \left(\frac{\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)} + \frac{\sqrt{3}\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)}}{2}}{(\sqrt{3}-1)(\sqrt{3}+7)} + \frac{1}{2}\right)(2x + \sqrt{3} + 1)\right) - \ln\left(x + \left(\frac{\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)} + \frac{\sqrt{3}\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)}}{2}}{(\sqrt{3}-1)(\sqrt{3}+7)} - \frac{1}{2}\right)(2x + \sqrt{3} + 1)\right) - \frac{1}{2}\left(\frac{\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)} + \frac{\sqrt{3}\sqrt{(\sqrt{3}-1)(\sqrt{3}+7)}}{2}}{(\sqrt{3}-1)(\sqrt{3}+7)} - \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x*(3^(1/2) + 1) - 3^(1/2) + x^2 + 2),x)

[Out] $\log(x - (((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)}/2 + (3^{(1/2)}*((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)})/2)/((3^{(1/2)} - 1)*(3^{(1/2)} + 7)) + 1/2)*(2*x + 3^{(1/2)} + 1))*(((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)}/2 + (3^{(1/2)}*((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)})/2)/((3^{(1/2)} - 1)*(3^{(1/2)} + 7)) + 1/2) - \log(x + (((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)}/2 + (3^{(1/2)}*((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)})/2)/((3^{(1/2)} - 1)*(3^{(1/2)} + 7)) - 1/2)*(2*x + 3^{(1/2)} + 1))*(((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)}/2 + (3^{(1/2)}*((3^{(1/2)} - 1)*(3^{(1/2)} + 7))^{(1/2)})/2)/((3^{(1/2)} - 1)*(3^{(1/2)} + 7)) - 1/2)$

sympy [B] time = 1.55, size = 202, normalized size = 2.81

$$\left(\frac{1}{2} - \frac{\sqrt{11+64\sqrt{3}}}{2(-31+12\sqrt{3})}\right)\log\left(x - \frac{287\sqrt{3}}{11+64\sqrt{3}} + \frac{1}{2} - \frac{\sqrt{11+64\sqrt{3}}}{2(-31+12\sqrt{3})}\right) + \left(\frac{269}{214+139\sqrt{3}} + \frac{459\sqrt{3}}{214+139\sqrt{3}} + \frac{521}{11+64\sqrt{3}}\right) + \left(\frac{\sqrt{11+64\sqrt{3}}}{2(-31+12\sqrt{3})} + \frac{1}{2}\right)\log\left(x - \frac{287\sqrt{3}}{11+64\sqrt{3}} + \frac{\sqrt{11+64\sqrt{3}}}{2(-31+12\sqrt{3})} + \frac{1}{2}\right) + \left(\frac{269}{214+139\sqrt{3}} + \frac{459\sqrt{3}}{214+139\sqrt{3}} + \frac{521}{11+64\sqrt{3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(2+x**2-3**(1/2)+x*(1+3**(1/2))),x)

```
[Out] (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*log(x - 287*sqrt(3)/(1
1 + 64*sqrt(3)) + (1/2 - sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3))))*(269
/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 + 64*sqrt
(3))) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3)))) + 1/2)*log(x - 287*sq
rt(3)/(11 + 64*sqrt(3)) + (sqrt(11 + 64*sqrt(3))/(2*(-31 + 12*sqrt(3)))) + 1
/2)*(269/(214 + 139*sqrt(3)) + 459*sqrt(3)/(214 + 139*sqrt(3))) + 521/(11 +
64*sqrt(3)))
```


$$3.652 \quad \int \sqrt{x^2 + x^3} dx$$

Optimal. Leaf size=37

$$\frac{2(x^3 + x^2)^{3/2}}{5x^2} - \frac{4(x^3 + x^2)^{3/2}}{15x^3}$$

Rubi [A] time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {2002, 2014}

$$\frac{2(x^3 + x^2)^{3/2}}{5x^2} - \frac{4(x^3 + x^2)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2 + x^3], x]

[Out] (-4*(x^2 + x^3)^(3/2))/(15*x^3) + (2*(x^2 + x^3)^(3/2))/(5*x^2)

Rule 2002

Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := Simp[(a*x^j + b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Dist[(b*(n*p + n - j + 1))/(a*(j*p + 1)), Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n - j)], 0] && NeQ[j*p + 1, 0]

Rule 2014

Int[((c_.)*(x_)^(m_.))*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] := -Simp[(c^(j - 1)*(c*x)^(m - j + 1)*(a*x^j + b*x^n)^(p + 1))/(a*(n - j)*(p + 1)), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])

Rubi steps

$$\begin{aligned} \int \sqrt{x^2 + x^3} dx &= \frac{2(x^2 + x^3)^{3/2}}{5x^2} - \frac{2}{5} \int \frac{\sqrt{x^2 + x^3}}{x} dx \\ &= -\frac{4(x^2 + x^3)^{3/2}}{15x^3} + \frac{2(x^2 + x^3)^{3/2}}{5x^2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 23, normalized size = 0.62

$$\frac{2(x^2(x+1))^{3/2}(3x-2)}{15x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2 + x^3], x]

[Out] (2*(x^2*(1 + x))^(3/2)*(-2 + 3*x))/(15*x^3)

IntegrateAlgebraic [A] time = 0.03, size = 23, normalized size = 0.62

$$\frac{2(3x-2)(x^3+x^2)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x^2 + x^3], x]

[Out] (2*(-2 + 3*x)*(x^2 + x^3)^(3/2))/(15*x^3)

fricas [A] time = 0.40, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}(3x^2+x-2)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2)^(1/2), x, algorithm="fricas")

[Out] 2/15*sqrt(x^3 + x^2)*(3*x^2 + x - 2)/x

giac [A] time = 0.38, size = 48, normalized size = 1.30

$$\frac{2}{15} \left(3(x+1)^{\frac{5}{2}} - 10(x+1)^{\frac{3}{2}} + 15\sqrt{x+1} \right) \operatorname{sgn}(x) + \frac{2}{3} \left((x+1)^{\frac{3}{2}} - 3\sqrt{x+1} \right) \operatorname{sgn}(x) + \frac{4}{15} \operatorname{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2)^(1/2), x, algorithm="giac")

[Out] 2/15*(3*(x + 1)^(5/2) - 10*(x + 1)^(3/2) + 15*sqrt(x + 1))*sgn(x) + 2/3*((x + 1)^(3/2) - 3*sqrt(x + 1))*sgn(x) + 4/15*sgn(x)

maple [A] time = 0.00, size = 23, normalized size = 0.62

$$\frac{2(x+1)(3x-2)\sqrt{x^3+x^2}}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^3+x^2)^(1/2), x)

[Out] 2/15*(x+1)*(3*x-2)*(x^3+x^2)^(1/2)/x

maxima [A] time = 0.45, size = 15, normalized size = 0.41

$$\frac{2}{15} (3x^2 + x - 2)\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^3+x^2)^(1/2), x, algorithm="maxima")

[Out] 2/15*(3*x^2 + x - 2)*sqrt(x + 1)

mupad [B] time = 3.51, size = 22, normalized size = 0.59

$$\frac{2\sqrt{x^3+x^2}(3x^2+x-2)}{15x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2 + x^3)^(1/2), x)

[Out] (2*(x^2 + x^3)^(1/2)*(x + 3*x^2 - 2))/(15*x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x^3+x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+x**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x**3 + x**2), x)
```

$$3.653 \quad \int \frac{1}{(1+x)\sqrt{2x+x^2}} dx$$

Optimal. Leaf size=12

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Rubi [A] time = 0.01, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {688, 203}

$$\tan^{-1}\left(\sqrt{x^2+2x}\right)$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)*Sqrt[2*x+x^2]),x]

[Out] ArcTan[Sqrt[2*x+x^2]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 688

Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[4*c, Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[2*c*d - b*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x)\sqrt{2x+x^2}} dx &= 4 \text{Subst} \left(\int \frac{1}{4+4x^2} dx, x, \sqrt{2x+x^2} \right) \\ &= \tan^{-1}\left(\sqrt{2x+x^2}\right) \end{aligned}$$

Mathematica [B] time = 0.01, size = 37, normalized size = 3.08

$$\frac{2\sqrt{x}\sqrt{x+2}\tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+2}}\right)}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1+x)*Sqrt[2*x+x^2]),x]

[Out] (2*Sqrt[x]*Sqrt[2+x]*ArcTan[Sqrt[x]/Sqrt[2+x]])/Sqrt[x*(2+x)]

IntegrateAlgebraic [A] time = 0.00, size = 18, normalized size = 1.50

$$-2 \tan^{-1}\left(\frac{\sqrt{x^2+2x}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 + x)*Sqrt[2*x + x^2]), x]

[Out] -2*ArcTan[Sqrt[2*x + x^2]/x]

fricas [A] time = 0.40, size = 17, normalized size = 1.42

$$2 \arctan\left(-x + \sqrt{x^2 + 2x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="fricas")

[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)

giac [A] time = 0.46, size = 17, normalized size = 1.42

$$2 \arctan\left(-x + \sqrt{x^2 + 2x} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="giac")

[Out] 2*arctan(-x + sqrt(x^2 + 2*x) - 1)

maple [A] time = 0.00, size = 13, normalized size = 1.08

$$- \arctan\left(\frac{1}{\sqrt{(x+1)^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x+1)/(x^2+2*x)^(1/2), x)

[Out] -arctan(1/((x+1)^2-1)^(1/2))

maxima [A] time = 0.97, size = 9, normalized size = 0.75

$$- \arcsin\left(\frac{1}{|x+1|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x^2+2*x)^(1/2), x, algorithm="maxima")

[Out] -arcsin(1/abs(x + 1))

mupad [F] time = 0.00, size = -1, normalized size = -0.08

$$\int \frac{1}{\sqrt{x^2 + 2x} (x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)

[Out] int(1/((2*x + x^2)^(1/2)*(x + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(x+2)}(x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1+x)/(x**2+2*x)**(1/2), x)

[Out] Integral(1/(sqrt(x*(x + 2))*(x + 1)), x)

$$3.654 \quad \int \sqrt{1 - \sqrt{x} - x} \sqrt{x} dx$$

Optimal. Leaf size=95

$$-\frac{1}{2}\sqrt{x}(-x-\sqrt{x}+1)^{3/2} + \frac{5}{12}(-x-\sqrt{x}+1)^{3/2} + \frac{9}{32}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1} + \frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Rubi [A] time = 0.05, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1357, 742, 640, 612, 619, 216}

$$-\frac{1}{2}\sqrt{x}(-x-\sqrt{x}+1)^{3/2} + \frac{5}{12}(-x-\sqrt{x}+1)^{3/2} + \frac{9}{32}(2\sqrt{x}+1)\sqrt{-x-\sqrt{x}+1} + \frac{45}{64}\sin^{-1}\left(\frac{2\sqrt{x}+1}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - Sqrt[x] - x]*Sqrt[x],x]

[Out] (9*(1 + 2*Sqrt[x])*Sqrt[1 - Sqrt[x] - x])/32 + (5*(1 - Sqrt[x] - x)^(3/2))/12 - ((1 - Sqrt[x] - x)^(3/2)*Sqrt[x])/2 + (45*ArcSin[(1 + 2*Sqrt[x])/Sqrt[5]])/64

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 742

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[1/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m + 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && If[RationalQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]

Rule 1357

```
Int[(x_)^(m_)*((a_) + (c_)*(x_)^(n2_) + (b_)*(x_)^(n_))^(p_), x_Symbol
] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x + c*x^2)^p, x
], x, x^n], x] /; FreeQ[{a, b, c, m, n, p}, x] && EqQ[n2, 2*n] && NeQ[b^2 -
4*a*c, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{1 - \sqrt{x} - x} \sqrt{x} \, dx &= 2 \operatorname{Subst} \left(\int x^2 \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= -\frac{1}{2} (1 - \sqrt{x} - x)^{3/2} \sqrt{x} - \frac{1}{2} \operatorname{Subst} \left(\int \left(-1 + \frac{5x}{2} \right) \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= \frac{5}{12} (1 - \sqrt{x} - x)^{3/2} - \frac{1}{2} (1 - \sqrt{x} - x)^{3/2} \sqrt{x} + \frac{9}{8} \operatorname{Subst} \left(\int \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= \frac{9}{32} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} + \frac{5}{12} (1 - \sqrt{x} - x)^{3/2} - \frac{1}{2} (1 - \sqrt{x} - x)^{3/2} \sqrt{x} + \frac{45}{64} \operatorname{Subst} \left(\int \sqrt{1 - x - x^2} \, dx, x, \sqrt{x} \right) \\
&= \frac{9}{32} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} + \frac{5}{12} (1 - \sqrt{x} - x)^{3/2} - \frac{1}{2} (1 - \sqrt{x} - x)^{3/2} \sqrt{x} - \frac{1}{64} (9\sqrt{5} \operatorname{ArcSin} \left(\frac{-2\sqrt{x} - 1}{\sqrt{5}} \right) + 45 \operatorname{ArcSin} \left(\frac{\sqrt{x}}{\sqrt{-x - \sqrt{x} + 1}} \right)) \\
&= \frac{9}{32} (1 + 2\sqrt{x}) \sqrt{1 - \sqrt{x} - x} + \frac{5}{12} (1 - \sqrt{x} - x)^{3/2} - \frac{1}{2} (1 - \sqrt{x} - x)^{3/2} \sqrt{x} + \frac{45}{64} \operatorname{ArcSin} \left(\frac{-2\sqrt{x} - 1}{\sqrt{5}} \right)
\end{aligned}$$

Mathematica [A] time = 0.04, size = 60, normalized size = 0.63

$$\frac{1}{96} \sqrt{-x - \sqrt{x} + 1} (48x^{3/2} + 8x - 34\sqrt{x} + 67) - \frac{45}{64} \sin^{-1} \left(\frac{-2\sqrt{x} - 1}{\sqrt{5}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sqrt[x] - x]*Sqrt[x], x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (45*ArcSin[(-1 - 2*Sqrt[x])/Sqrt[5]])/64
```

IntegrateAlgebraic [A] time = 0.19, size = 71, normalized size = 0.75

$$\frac{1}{96} \sqrt{-x - \sqrt{x} + 1} (48x^{3/2} + 8x - 34\sqrt{x} + 67) + \frac{45}{32} \tan^{-1} \left(\frac{\sqrt{x}}{\sqrt{-x - \sqrt{x} + 1} - 1} \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[Sqrt[1 - Sqrt[x] - x]*Sqrt[x], x]
```

```
[Out] (Sqrt[1 - Sqrt[x] - x]*(67 - 34*Sqrt[x] + 8*x + 48*x^(3/2)))/96 + (45*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - Sqrt[x] - x])])/32
```

fricas [A] time = 1.53, size = 89, normalized size = 0.94

$$\frac{1}{96} (2(24x - 17)\sqrt{x} + 8x + 67) \sqrt{-x - \sqrt{x} + 1} - \frac{45}{128} \arctan \left(\frac{(8x^2 - (16x^2 - 38x + 11)\sqrt{x} - 9x + 3)\sqrt{-x - \sqrt{x} + 1}}{4(4x^3 - 13x^2 + 7x - 1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1/2)*(1-x-x^(1/2))^(1/2), x, algorithm="fricas")
```

[Out] $\frac{1}{96} * (2 * (24 * x - 17) * \sqrt{x} + 8 * x + 67) * \sqrt{-x - \sqrt{x} + 1} - \frac{45}{128} * \arctan(-\frac{1}{4} * (8 * x^2 - (16 * x^2 - 38 * x + 11) * \sqrt{x} - 9 * x + 3) * \sqrt{-x - \sqrt{x} + 1}) / (4 * x^3 - 13 * x^2 + 7 * x - 1)$

giac [A] time = 0.39, size = 51, normalized size = 0.54

$$\frac{1}{96} \left(2 \left(4 \sqrt{x} (6 \sqrt{x} + 1) - 17 \right) \sqrt{x} + 67 \right) \sqrt{-x - \sqrt{x} + 1} + \frac{45}{64} \arcsin \left(\frac{1}{5} \sqrt{5} (2 \sqrt{x} + 1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{96} * (2 * (4 * \sqrt{x} * (6 * \sqrt{x} + 1) - 17) * \sqrt{x} + 67) * \sqrt{-x - \sqrt{x} + 1} + \frac{45}{64} * \arcsin(\frac{1}{5} * \sqrt{5} * (2 * \sqrt{x} + 1))$

maple [A] time = 0.01, size = 67, normalized size = 0.71

$$\frac{45 \arcsin \left(\frac{2\sqrt{5} \left(\sqrt{x} + \frac{1}{2} \right)}{5} \right)}{64} - \frac{(-x - \sqrt{x} + 1)^{\frac{3}{2}} \sqrt{x}}{2} + \frac{5(-x - \sqrt{x} + 1)^{\frac{3}{2}}}{12} - \frac{9(-2\sqrt{x} - 1) \sqrt{-x - \sqrt{x} + 1}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(-x-x^(1/2)+1)^(1/2),x)`

[Out] $-\frac{1}{2} * (-x - x^{1/2} + 1)^{3/2} * x^{1/2} + \frac{5}{12} * (-x - x^{1/2} + 1)^{3/2} - \frac{9}{32} * (-2 * x^{1/2} - 1) * (-x - x^{1/2} + 1)^{1/2} + \frac{45}{64} * \arcsin(\frac{2}{5} * 5^{1/2} * (x^{1/2} + \frac{1}{2}))$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{-x - \sqrt{x} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(1-x-x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x)*sqrt(-x - sqrt(x) + 1), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \sqrt{1 - \sqrt{x} - x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(1 - x^(1/2) - x)^(1/2),x)`

[Out] `int(x^(1/2)*(1 - x^(1/2) - x)^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{-\sqrt{x} - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(1-x-x**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(x)*sqrt(-sqrt(x) - x + 1), x)`

$$3.655 \quad \int \sqrt[3]{1 + \sqrt{-3 + x}} \, dx$$

Optimal. Leaf size=35

$$\frac{6}{7}(\sqrt{x-3} + 1)^{7/3} - \frac{3}{2}(\sqrt{x-3} + 1)^{4/3}$$

Rubi [A] time = 0.01, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {247, 190, 43}

$$\frac{6}{7}(\sqrt{x-3} + 1)^{7/3} - \frac{3}{2}(\sqrt{x-3} + 1)^{4/3}$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[-3 + x])^(1/3), x]

[Out] (-3*(1 + Sqrt[-3 + x])^(4/3))/2 + (6*(1 + Sqrt[-3 + x])^(7/3))/7

Rule 43

Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] := Int [ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{1 + \sqrt{-3 + x}} \, dx &= \text{Subst} \left(\int \sqrt[3]{1 + \sqrt{x}} \, dx, x, -3 + x \right) \\ &= 2 \text{Subst} \left(\int x \sqrt[3]{1 + x} \, dx, x, \sqrt{-3 + x} \right) \\ &= 2 \text{Subst} \left(\int \left(-\sqrt[3]{1 + x} + (1 + x)^{4/3} \right) dx, x, \sqrt{-3 + x} \right) \\ &= -\frac{3}{2} (1 + \sqrt{-3 + x})^{4/3} + \frac{6}{7} (1 + \sqrt{-3 + x})^{7/3} \end{aligned}$$

Mathematica [A] time = 0.01, size = 28, normalized size = 0.80

$$\frac{3}{14}(\sqrt{x-3} + 1)^{4/3} (4\sqrt{x-3} - 3)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[-3 + x])^(1/3), x]

[Out] $(3*(1 + \text{Sqrt}[-3 + x])^{(4/3)}*(-3 + 4*\text{Sqrt}[-3 + x]))/14$

IntegrateAlgebraic [A] time = 0.02, size = 28, normalized size = 0.80

$$\frac{3}{14} (\sqrt{x-3} + 1)^{4/3} (4\sqrt{x-3} - 3)$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[(1 + Sqrt[-3 + x])^(1/3), x]`

[Out] $(3*(1 + \text{Sqrt}[-3 + x])^{(4/3)}*(-3 + 4*\text{Sqrt}[-3 + x]))/14$

fricas [A] time = 0.42, size = 21, normalized size = 0.60

$$\frac{3}{14} (4x + \sqrt{x-3} - 15)(\sqrt{x-3} + 1)^{1/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-3+x)^(1/2))^(1/3), x, algorithm="fricas")`

[Out] $3/14*(4*x + \text{sqrt}(x - 3) - 15)*(\text{sqrt}(x - 3) + 1)^{(1/3)}$

giac [A] time = 0.32, size = 23, normalized size = 0.66

$$\frac{6}{7} (\sqrt{x-3} + 1)^{7/3} - \frac{3}{2} (\sqrt{x-3} + 1)^{4/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-3+x)^(1/2))^(1/3), x, algorithm="giac")`

[Out] $6/7*(\text{sqrt}(x - 3) + 1)^{(7/3)} - 3/2*(\text{sqrt}(x - 3) + 1)^{(4/3)}$

maple [A] time = 0.00, size = 24, normalized size = 0.69

$$-\frac{3(1 + \sqrt{x-3})^{4/3}}{2} + \frac{6(1 + \sqrt{x-3})^{7/3}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+(x-3)^(1/2))^(1/3), x)`

[Out] $-3/2*(1+(x-3)^(1/2))^(4/3)+6/7*(1+(x-3)^(1/2))^(7/3)$

maxima [A] time = 0.43, size = 23, normalized size = 0.66

$$\frac{6}{7} (\sqrt{x-3} + 1)^{7/3} - \frac{3}{2} (\sqrt{x-3} + 1)^{4/3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-3+x)^(1/2))^(1/3), x, algorithm="maxima")`

[Out] $6/7*(\text{sqrt}(x - 3) + 1)^{(7/3)} - 3/2*(\text{sqrt}(x - 3) + 1)^{(4/3)}$

mupad [B] time = 3.51, size = 16, normalized size = 0.46

$$(x-3) {}_2F_1\left(-\frac{1}{3}, 2; 3; -\sqrt{x-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(((x - 3)^(1/2) + 1)^(1/3), x)`

[Out] `(x - 3)*hypergeom([-1/3, 2], 3, -(x - 3)^(1/2))`

sympy [B] time = 1.17, size = 184, normalized size = 5.26

$$\frac{12(x-3)^{\frac{7}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} - \frac{6(x-3)^{\frac{5}{2}}\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{9(x-3)^{\frac{5}{2}}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{15(x-3)^3\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} - \frac{9(x-3)^2\sqrt[3]{\sqrt{x-3}+1}}{14(x-3)^{\frac{5}{2}}+14(x-3)^2} + \frac{9(x-3)^2}{14(x-3)^{\frac{5}{2}}+14(x-3)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+(-3+x)**(1/2))**(1/3), x)`

[Out] `12*(x - 3)**(7/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 6*(x - 3)**(5/2)*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x - 3)**(5/2)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 15*(x - 3)**3*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) - 9*(x - 3)**2*(sqrt(x - 3) + 1)**(1/3)/(14*(x - 3)**(5/2) + 14*(x - 3)**2) + 9*(x - 3)**2/(14*(x - 3)**(5/2) + 14*(x - 3)**2)`

$$3.656 \quad \int \frac{1}{\sqrt{3+\sqrt{-1+2x}}} dx$$

Optimal. Leaf size=37

$$\frac{2}{3} \left(\sqrt{2x-1} + 3 \right)^{3/2} - 6\sqrt{\sqrt{2x-1} + 3}$$

Rubi [A] time = 0.01, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {247, 190, 43}

$$\frac{2}{3} \left(\sqrt{2x-1} + 3 \right)^{3/2} - 6\sqrt{\sqrt{2x-1} + 3}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[3 + Sqrt[-1 + 2*x]], x]

[Out] -6*Sqrt[3 + Sqrt[-1 + 2*x]] + (2*(3 + Sqrt[-1 + 2*x])^(3/2))/3

Rule 43

Int[((a_.) + (b_.)*(x_)^(m_.))*((c_.) + (d_.)*(x_)^(n_.)), x_Symbol] := Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && (!IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])

Rule 190

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(1/n - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] && IntegerQ[1/n]

Rule 247

Int[((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] := Dist[1/Coefficient[v, x, 1], Subst[Int[(a + b*x^n)^p, x], x, v], x] /; FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && NeQ[v, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{3+\sqrt{-1+2x}}} dx &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{3+\sqrt{x}}} dx, x, -1+2x \right) \\ &= \text{Subst} \left(\int \frac{x}{\sqrt{3+x}} dx, x, \sqrt{-1+2x} \right) \\ &= \text{Subst} \left(\int \left(-\frac{3}{\sqrt{3+x}} + \sqrt{3+x} \right) dx, x, \sqrt{-1+2x} \right) \\ &= -6\sqrt{3+\sqrt{-1+2x}} + \frac{2}{3} \left(3+\sqrt{-1+2x} \right)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 0.81

$$\frac{2}{3} \left(\sqrt{2x-1} - 6 \right) \sqrt{\sqrt{2x-1} + 3}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[3 + Sqrt[-1 + 2*x]], x]

[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3

IntegrateAlgebraic [A] time = 0.02, size = 30, normalized size = 0.81

$$\frac{2}{3} \left(\sqrt{2x-1} - 6 \right) \sqrt{\sqrt{2x-1} + 3}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[3 + Sqrt[-1 + 2*x]], x]

[Out] (2*(-6 + Sqrt[-1 + 2*x])*Sqrt[3 + Sqrt[-1 + 2*x]])/3

fricas [A] time = 0.43, size = 22, normalized size = 0.59

$$\frac{2}{3} \sqrt{\sqrt{2x-1} + 3} \left(\sqrt{2x-1} - 6 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2), x, algorithm="fricas")

[Out] 2/3*sqrt(sqrt(2*x - 1) + 3)*(sqrt(2*x - 1) - 6)

giac [A] time = 0.34, size = 27, normalized size = 0.73

$$\frac{2}{3} \left(\sqrt{2x-1} + 3 \right)^{\frac{3}{2}} - 6 \sqrt{\sqrt{2x-1} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2), x, algorithm="giac")

[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)

maple [A] time = 0.00, size = 28, normalized size = 0.76

$$\frac{2 \left(3 + \sqrt{2x-1} \right)^{\frac{3}{2}}}{3} - 6 \sqrt{3 + \sqrt{2x-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(3+(2*x-1)^(1/2))^(1/2), x)

[Out] 2/3*(3+(2*x-1)^(1/2))^(3/2)-6*(3+(2*x-1)^(1/2))^(1/2)

maxima [A] time = 0.43, size = 27, normalized size = 0.73

$$\frac{2}{3} \left(\sqrt{2x-1} + 3 \right)^{\frac{3}{2}} - 6 \sqrt{\sqrt{2x-1} + 3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(3+(-1+2*x)^(1/2))^(1/2), x, algorithm="maxima")

[Out] 2/3*(sqrt(2*x - 1) + 3)^(3/2) - 6*sqrt(sqrt(2*x - 1) + 3)

mupad [B] time = 3.58, size = 24, normalized size = 0.65

$$\frac{\sqrt{3} (2x-1) {}_2F_1\left(\frac{1}{2}, 2; 3; -\frac{\sqrt{2x-1}}{3}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((2*x - 1)^(1/2) + 3)^(1/2),x)`

[Out] $(3^{1/2}*(2*x - 1)*\text{hypergeom}([1/2, 2], 3, -(2*x - 1)^{1/2}/3))/6$

sympy [B] time = 1.16, size = 265, normalized size = 7.16

$$-\frac{6\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^2} + \frac{36\sqrt{2}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^2} + \frac{4\sqrt{3}\left(x-\frac{1}{2}\right)^3\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^2} - \frac{36\sqrt{3}\left(x-\frac{1}{2}\right)^2\sqrt{\sqrt{2}\sqrt{x-\frac{1}{2}}+3}}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^2} + \frac{108\left(x-\frac{1}{2}\right)^2}{3\sqrt{6}\left(x-\frac{1}{2}\right)^{\frac{5}{2}}+9\sqrt{3}\left(x-\frac{1}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(3+(-1+2*x)**(1/2))**(1/2),x)`

[Out] $-6*\sqrt{6}*(x - 1/2)**(5/2)*\sqrt{\sqrt{2}*\sqrt{x - 1/2} + 3}/(3*\sqrt{6}*(x - 1/2)**(5/2) + 9*\sqrt{3}*(x - 1/2)**2) + 36*\sqrt{2}*(x - 1/2)**(5/2)/(3*\sqrt{6}*(x - 1/2)**(5/2) + 9*\sqrt{3}*(x - 1/2)**2) + 4*\sqrt{3}*(x - 1/2)**3*\sqrt{\sqrt{2}*\sqrt{x - 1/2} + 3}/(3*\sqrt{6}*(x - 1/2)**(5/2) + 9*\sqrt{3}*(x - 1/2)**2) - 36*\sqrt{3}*(x - 1/2)**2*\sqrt{\sqrt{2}*\sqrt{x - 1/2} + 3}/(3*\sqrt{6}*(x - 1/2)**(5/2) + 9*\sqrt{3}*(x - 1/2)**2) + 108*(x - 1/2)**2/(3*\sqrt{6}*(x - 1/2)**(5/2) + 9*\sqrt{3}*(x - 1/2)**2)$

$$3.657 \quad \int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx$$

Optimal. Leaf size=29

$$-\sqrt{1-x} (2 - \sqrt{x}) - \sin^{-1}(\sqrt{x})$$

Rubi [A] time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {1398, 785, 780, 216}

$$-\sqrt{1-x} (2 - \sqrt{x}) - \sin^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x]/(1 + Sqrt[x]), x]

[Out] -((2 - Sqrt[x])*Sqrt[1 - x]) - ArcSin[Sqrt[x]]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 780

Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 785

Int[(x_)*((d_) + (e_.)*(x_)^(m_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0] && EqQ[m, -1] && !ILtQ[p - 1/2, 0]

Rule 1398

Int[((a_) + (c_.)*(x_)^(n2_.))^p_)*((d_) + (e_.)*(x_)^(n_))^q_, x_Symbol] :> With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{1+\sqrt{x}} dx &= 2 \text{Subst} \left(\int \frac{x\sqrt{1-x^2}}{1+x} dx, x, \sqrt{x} \right) \\ &= 2 \text{Subst} \left(\int \frac{(1-x)x}{\sqrt{1-x^2}} dx, x, \sqrt{x} \right) \\ &= - \left((2 - \sqrt{x}) \sqrt{1-x} \right) - \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, \sqrt{x} \right) \\ &= - \left((2 - \sqrt{x}) \sqrt{1-x} \right) - \sin^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 0.90

$$(\sqrt{x} - 2)\sqrt{1-x} - \sin^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x]/(1 + Sqrt[x]), x]

[Out] (-2 + Sqrt[x])*Sqrt[1 - x] - ArcSin[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.18, size = 40, normalized size = 1.38

$$\sqrt{1-x}(\sqrt{x}-2) + 2 \tan^{-1}\left(\frac{\sqrt{1-x}}{\sqrt{x}+1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 + Sqrt[x]), x]

[Out] (-2 + Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[1 - x]/(1 + Sqrt[x])]

fricas [A] time = 0.44, size = 33, normalized size = 1.14

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1+x^(1/2)), x, algorithm="fricas")

[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arctan(sqrt(-x + 1)/sqrt(x))

giac [A] time = 0.41, size = 29, normalized size = 1.00

$$\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} + \arcsin(\sqrt{-x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1+x^(1/2)), x, algorithm="giac")

[Out] sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) + arcsin(sqrt(-x + 1))

maple [B] time = 0.01, size = 48, normalized size = 1.66

$$-\frac{\sqrt{-x+1}(\arcsin(2x-1) - 2\sqrt{-(x-1)x})\sqrt{x}}{2\sqrt{-(x-1)x}} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)/(x^(1/2)+1), x)

[Out] -1/2*(-x+1)^(1/2)*x^(1/2)*(-2*(-x*(x-1))^(1/2)+arcsin(2*x-1))/(-x*(x-1))^(1/2)-2*(-x+1)^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x+1}}{\sqrt{x}+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1+x^(1/2)),x, algorithm="maxima")

[Out] integrate(sqrt(-x + 1)/(sqrt(x) + 1), x)

mupad [B] time = 3.90, size = 39, normalized size = 1.34

$$\sqrt{x} \sqrt{1-x} - 2\sqrt{1-x} - 2 \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1 - x)^(1/2)/(x^(1/2) + 1),x)

[Out] x^(1/2)*(1 - x)^(1/2) - 2*(1 - x)^(1/2) - 2*atan(x^(1/2)/((1 - x)^(1/2) - 1))

sympy [C] time = 1.79, size = 32, normalized size = 1.10

$$i\sqrt{x} \sqrt{x-1} - 2i\sqrt{x-1} + i \operatorname{asinh}\left(\sqrt{x-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2)/(1+x**(1/2)),x)

[Out] I*sqrt(x)*sqrt(x - 1) - 2*I*sqrt(x - 1) + I*asinh(sqrt(x - 1))

$$3.658 \quad \int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx$$

Optimal. Leaf size=25

$$\sin^{-1}(\sqrt{x}) - (\sqrt{x} + 2)\sqrt{1-x}$$

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1398, 785, 780, 216}

$$\sin^{-1}(\sqrt{x}) - (\sqrt{x} + 2)\sqrt{1-x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - x]/(1 - Sqrt[x]), x]

[Out] -((2 + Sqrt[x])*Sqrt[1 - x]) + ArcSin[Sqrt[x]]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[Rt[-b, 2]*x]/Sqrt[a]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 780

Int[((d_) + (e_)*(x_))*((f_) + (g_)*(x_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[(((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*(a + c*x^2)^(p + 1))/(2*c*(p + 1)*(2*p + 3)), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p + 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && !LeQ[p, -1]

Rule 785

Int[(x_)*((d_) + (e_)*(x_)^(m_))*((a_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[d^m*e^m, Int[(x*(a + c*x^2)^(m + p))/(a*e + c*d*x)^m, x], x] /; FreeQ[{a, c, d, e, p}, x] && EqQ[c*d^2 + a*e^2, 0] && !IntegerQ[p] && ILtQ[m, 0] && EqQ[m, -1] && !ILtQ[p - 1/2, 0]

Rule 1398

Int[((a_) + (c_)*(x_)^(n2_))^(p_)*((d_) + (e_)*(x_)^(n_))^(q_), x_Symbol] := With[{g = Denominator[n]}, Dist[g, Subst[Int[x^(g - 1)*(d + e*x^(g*n))^q*(a + c*x^(2*g*n))^p, x], x, x^(1/g)], x] /; FreeQ[{a, c, d, e, p, q}, x] && EqQ[n2, 2*n] && FractionQ[n]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{1-x}}{1-\sqrt{x}} dx &= 2 \text{Subst} \left(\int \frac{x\sqrt{1-x^2}}{1-x} dx, x, \sqrt{x} \right) \\ &= - \left(2 \text{Subst} \left(\int \frac{(-1-x)x}{\sqrt{1-x^2}} dx, x, \sqrt{x} \right) \right) \\ &= - \left((2 + \sqrt{x})\sqrt{1-x} \right) + \text{Subst} \left(\int \frac{1}{\sqrt{1-x^2}} dx, x, \sqrt{x} \right) \\ &= - \left((2 + \sqrt{x})\sqrt{1-x} \right) + \sin^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.03, size = 26, normalized size = 1.04

$$\sqrt{1-x}(-\sqrt{x}-2) + \sin^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - x]/(1 - Sqrt[x]), x]

[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + ArcSin[Sqrt[x]]

IntegrateAlgebraic [A] time = 0.18, size = 42, normalized size = 1.68

$$\sqrt{1-x}(-\sqrt{x}-2) + 2 \tan^{-1}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x]/(1 - Sqrt[x]), x]

[Out] (-2 - Sqrt[x])*Sqrt[1 - x] + 2*ArcTan[Sqrt[x]/(-1 + Sqrt[1 - x])]

fricas [A] time = 0.42, size = 36, normalized size = 1.44

$$-\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arctan\left(\frac{\sqrt{-x+1}}{\sqrt{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1-x^(1/2)), x, algorithm="fricas")

[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arctan(sqrt(-x + 1)/sqrt(x))

giac [A] time = 0.40, size = 32, normalized size = 1.28

$$-\sqrt{x}\sqrt{-x+1} - 2\sqrt{-x+1} - \arcsin(\sqrt{-x+1})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1-x^(1/2)), x, algorithm="giac")

[Out] -sqrt(x)*sqrt(-x + 1) - 2*sqrt(-x + 1) - arcsin(sqrt(-x + 1))

maple [B] time = 0.01, size = 48, normalized size = 1.92

$$\frac{\sqrt{-x+1}(\arcsin(2x-1) - 2\sqrt{-(x-1)x})\sqrt{x}}{2\sqrt{-(x-1)x}} - 2\sqrt{-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+1)^(1/2)/(1-x^(1/2)), x)

[Out] -2*(-x+1)^(1/2)+1/2*(-x+1)^(1/2)*(arcsin(2*x-1)-2*(-(x-1)*x)^(1/2))/(-(x-1)*x)^(1/2)*x^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{-x+1}}{\sqrt{x}-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)^(1/2)/(1-x^(1/2)),x, algorithm="maxima")

[Out] -integrate(sqrt(-x + 1)/(sqrt(x) - 1), x)

mupad [B] time = 3.65, size = 40, normalized size = 1.60

$$2 \operatorname{atan}\left(\frac{\sqrt{x}}{\sqrt{1-x}-1}\right) - 2\sqrt{1-x} - \sqrt{x}\sqrt{1-x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(1 - x)^(1/2)/(x^(1/2) - 1),x)

[Out] 2*atan(x^(1/2)/((1 - x)^(1/2) - 1)) - 2*(1 - x)^(1/2) - x^(1/2)*(1 - x)^(1/2)

sympy [A] time = 3.77, size = 87, normalized size = 3.48

$$2 \left\{ \begin{array}{ll} -\sqrt{1-x} + \frac{i \operatorname{acosh}(\sqrt{1-x})}{2} - \frac{i(1-x)^{3/2}}{2\sqrt{-x}} + \frac{i\sqrt{1-x}}{2\sqrt{-x}} & \text{for } |x-1| > 1 \\ \frac{\sqrt{x}\sqrt{1-x}}{2} - \sqrt{1-x} + \frac{\operatorname{asin}(\sqrt{1-x})}{2} & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x)**(1/2)/(1-x**(1/2)),x)

[Out] 2*Piecewise((-sqrt(1 - x) + I*acosh(sqrt(1 - x))/2 - I*(1 - x)**(3/2)/(2*sqrt(-x)) + I*sqrt(1 - x)/(2*sqrt(-x)), Abs(x - 1) > 1), (sqrt(x)*sqrt(1 - x)/2 - sqrt(1 - x) + asin(sqrt(1 - x))/2, True))

$$3.659 \quad \int \frac{x}{x - \sqrt{1+x^2}} dx$$

Optimal. Leaf size=21

$$-\frac{x^3}{3} - \frac{1}{3}(x^2 + 1)^{3/2}$$

Rubi [A] time = 0.02, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {2106, 30, 261}

$$-\frac{x^3}{3} - \frac{1}{3}(x^2 + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x/(x - Sqrt[1 + x^2]), x]

[Out] -x^3/3 - (1 + x^2)^(3/2)/3

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && NeQ[m, -1]

Rule 261

Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 2106

Int[(u_)/((d_)*(x_)^(n_) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x_Symbol] := -Dist[b/(a*d), Int[u*x^n, x], x] + Dist[1/(a*c), Int[u*Sqrt[a + b*x^(2*n)], x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2*n] && EqQ[b*c^2 - d^2, 0]

Rubi steps

$$\begin{aligned} \int \frac{x}{x - \sqrt{1+x^2}} dx &= - \int x^2 dx - \int x\sqrt{1+x^2} dx \\ &= -\frac{x^3}{3} - \frac{1}{3}(1+x^2)^{3/2} \end{aligned}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$-\frac{x^3}{3} - \frac{1}{3}(x^2 + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x/(x - Sqrt[1 + x^2]), x]

[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3

IntegrateAlgebraic [A] time = 0.10, size = 21, normalized size = 1.00

$$-\frac{x^3}{3} - \frac{1}{3}(x^2 + 1)^{3/2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(x - Sqrt[1 + x^2]),x]

[Out] -1/3*x^3 - (1 + x^2)^(3/2)/3

fricas [A] time = 0.41, size = 15, normalized size = 0.71

$$-\frac{1}{3}x^3 - \frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="fricas")

[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)

giac [A] time = 0.34, size = 15, normalized size = 0.71

$$-\frac{1}{3}x^3 - \frac{1}{3}(x^2 + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="giac")

[Out] -1/3*x^3 - 1/3*(x^2 + 1)^(3/2)

maple [A] time = 0.00, size = 16, normalized size = 0.76

$$-\frac{x^3}{3} - \frac{(x^2 + 1)^{\frac{3}{2}}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x-(x^2+1)^(1/2)),x)

[Out] -1/3*x^3-1/3*(x^2+1)^(3/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(x/(x - sqrt(x^2 + 1)), x)

mupad [B] time = 0.04, size = 22, normalized size = 1.05

$$-\sqrt{x^2 + 1} \left(\frac{x^2}{3} + \frac{1}{3} \right) - \frac{x^3}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x - (x^2 + 1)^(1/2)),x)

[Out] - (x^2 + 1)^(1/2)*(x^2/3 + 1/3) - x^3/3

sympy [B] time = 0.37, size = 56, normalized size = 2.67

$$\frac{2x^2}{3x - 3\sqrt{x^2 + 1}} - \frac{x\sqrt{x^2 + 1}}{3x - 3\sqrt{x^2 + 1}} + \frac{1}{3x - 3\sqrt{x^2 + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(x**2+1)**(1/2)),x)
```

```
[Out] 2*x**2/(3*x - 3*sqrt(x**2 + 1)) - x*sqrt(x**2 + 1)/(3*x - 3*sqrt(x**2 + 1))  
+ 1/(3*x - 3*sqrt(x**2 + 1))
```

$$3.660 \quad \int \frac{x}{x - \sqrt{1-x^2}} dx$$

Optimal. Leaf size=65

$$\frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}} + \frac{x}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

Rubi [A] time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {2107, 321, 206, 444, 50, 63, 207}

$$\frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}\left(\sqrt{2}\sqrt{1-x^2}\right)}{2\sqrt{2}} + \frac{x}{2} - \frac{\tanh^{-1}\left(\sqrt{2}x\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[x/(x - Sqrt[1 - x^2]),x]

[Out] x/2 + Sqrt[1 - x^2]/2 - ArcTanh[Sqrt[2]*x]/(2*Sqrt[2]) - ArcTanh[Sqrt[2]*Sqrt[1 - x^2]]/(2*Sqrt[2])

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[
b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```


Rule 444

```
Int[(x_)^(m_)*((a_) + (b_)*(x_)^(n_))^(p_)*((c_) + (d_)*(x_)^(n_))^(q_
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 2107

```
Int[(x_)^(m_)/((d_)*(x_)^(n_) + (c_)*Sqrt[(a_) + (b_)*(x_)^(p_)]), x
_Symbol] := -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{x - \sqrt{1-x^2}} dx &= -\int \frac{x^2}{1-2x^2} dx - \int \frac{x\sqrt{1-x^2}}{1-2x^2} dx \\ &= \frac{x}{2} - \frac{1}{2} \int \frac{1}{1-2x^2} dx - \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1-x}}{1-2x} dx, x, x^2 \right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}(\sqrt{2}x)}{2\sqrt{2}} - \frac{1}{4} \text{Subst} \left(\int \frac{1}{(1-2x)\sqrt{1-x}} dx, x, x^2 \right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}(\sqrt{2}x)}{2\sqrt{2}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{-1+2x^2} dx, x, \sqrt{1-x^2} \right) \\ &= \frac{x}{2} + \frac{\sqrt{1-x^2}}{2} - \frac{\tanh^{-1}(\sqrt{2}x)}{2\sqrt{2}} - \frac{\tanh^{-1}(\sqrt{2}\sqrt{1-x^2})}{2\sqrt{2}} \end{aligned}$$

Mathematica [A] time = 0.05, size = 54, normalized size = 0.83

$$\frac{1}{4} \left(2 \left(\sqrt{1-x^2} + x \right) - \sqrt{2} \tanh^{-1} \left(\sqrt{2-2x^2} \right) - \sqrt{2} \tanh^{-1} \left(\sqrt{2}x \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(x - Sqrt[1 - x^2]), x]
```

```
[Out] (2*(x + Sqrt[1 - x^2]) - Sqrt[2]*ArcTanh[Sqrt[2]*x] - Sqrt[2]*ArcTanh[Sqrt[
2 - 2*x^2]])/4
```

IntegrateAlgebraic [A] time = 0.20, size = 53, normalized size = 0.82

$$\frac{\sqrt{1-x^2}}{2} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}x}{\sqrt{1-x^2}-x-1}\right)}{\sqrt{2}} + \frac{x}{2}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[x/(x - Sqrt[1 - x^2]), x]
```

```
[Out] x/2 + Sqrt[1 - x^2]/2 + ArcTanh[(Sqrt[2]*x)/(-1 - x + Sqrt[1 - x^2])]/Sqrt[
2]
```

fricas [B] time = 0.40, size = 97, normalized size = 1.49

$$\frac{1}{8} \sqrt{2} \log \left(\frac{6x^2 - 2\sqrt{2}(2x^2 - 3) + 2\sqrt{-x^2 + 1}(3\sqrt{2} - 4) - 9}{2x^2 - 1} \right) + \frac{1}{8} \sqrt{2} \log \left(\frac{2x^2 - 2\sqrt{2}x + 1}{2x^2 - 1} \right) + \frac{1}{2}x + \frac{1}{2}\sqrt{-x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/8*sqrt(2)*log((6*x^2 - 2*sqrt(2)*(2*x^2 - 3) + 2*sqrt(-x^2 + 1)*(3*sqrt(2) - 4) - 9)/(2*x^2 - 1)) + 1/8*sqrt(2)*log((2*x^2 - 2*sqrt(2)*x + 1)/(2*x^2 - 1)) + 1/2*x + 1/2*sqrt(-x^2 + 1)

giac [B] time = 0.57, size = 105, normalized size = 1.62

$$\frac{1}{8} \sqrt{2} \log \left(\left| \frac{4x - 2\sqrt{2}}{4x + 2\sqrt{2}} \right| \right) - \frac{1}{8} \sqrt{2} \log \left(\left| \frac{-4\sqrt{2} + \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 6}{4\sqrt{2} + \frac{2(\sqrt{-x^2+1}-1)^2}{x^2} - 6} \right| \right) + \frac{1}{2}x + \frac{1}{2}\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/8*sqrt(2)*log(abs(4*x - 2*sqrt(2))/abs(4*x + 2*sqrt(2))) - 1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)/abs(4*sqrt(2) + 2*(sqrt(-x^2 + 1) - 1)^2/x^2 - 6)) + 1/2*x + 1/2*sqrt(-x^2 + 1)

maple [B] time = 0.02, size = 175, normalized size = 2.69

$$\frac{x}{2} - \frac{\sqrt{2} \operatorname{arctanh}(\sqrt{2}x)}{4} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\left(1 - \left(x \frac{\sqrt{2}}{2}\right)\sqrt{2}\right)\sqrt{2}}{\sqrt{-4\left(x - \frac{\sqrt{2}}{2}\right)^2 - 4\left(x - \frac{\sqrt{2}}{2}\right)\sqrt{2} + 2}}\right)}{8} - \frac{\sqrt{2} \operatorname{arctanh}\left(\frac{\left(\left(x + \frac{\sqrt{2}}{2}\right)\sqrt{2} + 1\right)\sqrt{2}}{\sqrt{-4\left(x + \frac{\sqrt{2}}{2}\right)^2 + 4\left(x + \frac{\sqrt{2}}{2}\right)\sqrt{2} + 2}}\right)}{8} + \frac{\sqrt{-4\left(x + \frac{\sqrt{2}}{2}\right)^2 + 4\left(x + \frac{\sqrt{2}}{2}\right)\sqrt{2} + 2}}{8} + \frac{\sqrt{-4\left(x - \frac{\sqrt{2}}{2}\right)^2 - 4\left(x - \frac{\sqrt{2}}{2}\right)\sqrt{2} + 2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x-(-x^2+1)^(1/2)),x)

[Out] 1/2*x-1/4*arctanh(2^(1/2)*x)*2^(1/2)+1/8*(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh(((x+1/2*2^(1/2))*2^(1/2)+1)*2^(1/2)/(-4*(x+1/2*2^(1/2))^2+4*(x+1/2*2^(1/2))*2^(1/2)+2)^(1/2))+1/8*(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2)-1/8*2^(1/2)*arctanh((1-(x-1/2*2^(1/2))*2^(1/2))*2^(1/2)/(-4*(x-1/2*2^(1/2))^2-4*(x-1/2*2^(1/2))*2^(1/2)+2)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{-x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(-x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(x/(x - sqrt(-x^2 + 1)), x)

mupad [B] time = 3.50, size = 127, normalized size = 1.95

$$\frac{x}{2} - \frac{\sqrt{2} \ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x-1}{2}\right)_{1i}-\sqrt{1-x^2} 1i}{x-\frac{\sqrt{2}}{2}}\right)}{8} - \frac{\sqrt{2} \ln\left(\frac{\sqrt{2}\left(\frac{\sqrt{2}x+1}{2}\right)_{1i}+\sqrt{1-x^2} 1i}{x+\frac{\sqrt{2}}{2}}\right)}{8} + \frac{\sqrt{2} \ln\left(x - \frac{\sqrt{2}}{2}\right)}{8} - \frac{\sqrt{2} \ln\left(x + \frac{\sqrt{2}}{2}\right)}{8} + \frac{\sqrt{1-x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x - (1 - x^2)^(1/2)),x)

```
[Out] x/2 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 - 1)*1i - (1 - x^2)^(1/2)*1i)/(x
- 2^(1/2)/2)))/8 - (2^(1/2)*log((2^(1/2)*((2^(1/2)*x)/2 + 1)*1i + (1 - x^2
)^(1/2)*1i)/(x + 2^(1/2)/2)))/8 + (2^(1/2)*log(x - 2^(1/2)/2))/8 - (2^(1/2)
*log(x + 2^(1/2)/2))/8 + (1 - x^2)^(1/2)/2
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{1 - x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(x-(-x**2+1)**(1/2)),x)
```

```
[Out] Integral(x/(x - sqrt(1 - x**2)), x)
```

$$3.661 \quad \int \frac{x}{x - \sqrt{1+2x^2}} dx$$

Optimal. Leaf size=31

$$-\sqrt{2x^2+1} + \tan^{-1}\left(\sqrt{2x^2+1}\right) - x + \tan^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {2107, 321, 203, 444, 50, 63}

$$-\sqrt{2x^2+1} + \tan^{-1}\left(\sqrt{2x^2+1}\right) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x/(x - Sqrt[1 + 2*x^2]),x]

[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
+ 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] := Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
1, 0]
```

Rule 2107

```
Int[(x_)^(m_.)/((d_.)*(x_)^(n_.) + (c_.)*Sqrt[(a_.) + (b_.)*(x_)^(p_.)]), x
_Symbol] := -Dist[d, Int[x^(m + n)/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x], x]
+ Dist[c, Int[(x^m*Sqrt[a + b*x^(2*n)])/(a*c^2 + (b*c^2 - d^2)*x^(2*n)), x]
, x] /; FreeQ[{a, b, c, d, m, n}, x] && EqQ[p, 2*n] && NeQ[b*c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{x}{x - \sqrt{1 + 2x^2}} dx &= - \int \frac{x^2}{1 + x^2} dx - \int \frac{x\sqrt{1 + 2x^2}}{1 + x^2} dx \\ &= -x - \frac{1}{2} \text{Subst} \left(\int \frac{\sqrt{1 + 2x}}{1 + x} dx, x, x^2 \right) + \int \frac{1}{1 + x^2} dx \\ &= -x - \sqrt{1 + 2x^2} + \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{(1 + x)\sqrt{1 + 2x}} dx, x, x^2 \right) \\ &= -x - \sqrt{1 + 2x^2} + \tan^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\frac{1}{2} + \frac{x^2}{2}} dx, x, \sqrt{1 + 2x^2} \right) \\ &= -x - \sqrt{1 + 2x^2} + \tan^{-1}(x) + \tan^{-1}(\sqrt{1 + 2x^2}) \end{aligned}$$

Mathematica [A] time = 0.04, size = 31, normalized size = 1.00

$$-\sqrt{2x^2 + 1} + \tan^{-1}(\sqrt{2x^2 + 1}) - x + \tan^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x/(x - Sqrt[1 + 2*x^2]), x]

[Out] -x - Sqrt[1 + 2*x^2] + ArcTan[x] + ArcTan[Sqrt[1 + 2*x^2]]

IntegrateAlgebraic [A] time = 0.26, size = 62, normalized size = 2.00

$$-\sqrt{2x^2 + 1} + 2 \tan^{-1} \left(-\sqrt{2} \sqrt{2x^2 + 1} - \sqrt{2x^2 + 1} + \sqrt{2}x + 2x \right) - x$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(x - Sqrt[1 + 2*x^2]), x]

[Out] -x - Sqrt[1 + 2*x^2] + 2*ArcTan[2*x + Sqrt[2]*x - Sqrt[1 + 2*x^2] - Sqrt[2]*Sqrt[1 + 2*x^2]]

fricas [A] time = 0.42, size = 41, normalized size = 1.32

$$-x - \sqrt{2x^2 + 1} + \arctan(x) - \arctan \left(-\frac{x^2 - \sqrt{2x^2 + 1} + 1}{x^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(2*x^2+1)^(1/2)), x, algorithm="fricas")

[Out] -x - sqrt(2*x^2 + 1) + arctan(x) - arctan(-(x^2 - sqrt(2*x^2 + 1) + 1)/x^2)

giac [B] time = 0.46, size = 63, normalized size = 2.03

$$-\frac{1}{2} \pi - x - \sqrt{2x^2 + 1} + \arctan(x) + \arctan \left(-\frac{\left(\sqrt{2}x - \sqrt{2x^2 + 1} \right)^2 + 1}{2 \left(\sqrt{2}x - \sqrt{2x^2 + 1} \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="giac")

[Out] $-1/2\pi - x - \sqrt{2x^2 + 1} + \arctan(x) + \arctan(-1/2*((\sqrt{2})x - \sqrt{2x^2 + 1})^2 + 1)/(\sqrt{2}x - \sqrt{2x^2 + 1})$

maple [A] time = 0.01, size = 28, normalized size = 0.90

$$-x + \arctan(x) + \arctan\left(\sqrt{2x^2 + 1}\right) - \sqrt{2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x-(2*x^2+1)^(1/2)),x)

[Out] $-x + \arctan(x) + \arctan((2x^2 + 1)^{1/2}) - (2x^2 + 1)^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(2*x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(x/(x - sqrt(2*x^2 + 1)), x)

mupad [B] time = 0.19, size = 64, normalized size = 2.06

$$-x - \sqrt{2} \sqrt{x^2 + \frac{1}{2}} - \ln(x - i) 1i + \frac{\ln\left(x - \frac{\sqrt{2} \sqrt{x^2 + \frac{1}{2}}}{2} + \frac{1}{2}i\right) 1i}{2} + \frac{\ln\left(x + \frac{\sqrt{2} \sqrt{x^2 + \frac{1}{2}}}{2} - \frac{1}{2}i\right) 1i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x - (2*x^2 + 1)^(1/2)),x)

[Out] $(\log(x - (2^{1/2}*(x^2 + 1/2)^{1/2}))/2 + 1i/2)*1i)/2 - \log(x - 1i)*1i - x + (\log(x + (2^{1/2}*(x^2 + 1/2)^{1/2}))/2 - 1i/2)*1i)/2 - 2^{1/2}*(x^2 + 1/2)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x - \sqrt{2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x-(2*x**2+1)**(1/2)),x)

[Out] Integral(x/(x - sqrt(2*x**2 + 1)), x)

$$3.662 \quad \int \sqrt{x} \sqrt{\sqrt{x} + x} dx$$

Optimal. Leaf size=82

$$\frac{1}{2}\sqrt{x}(x+\sqrt{x})^{3/2} - \frac{5}{12}(x+\sqrt{x})^{3/2} + \frac{5}{32}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Rubi [A] time = 0.04, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.353$, Rules used = {2018, 670, 640, 612, 620, 206}

$$\frac{1}{2}\sqrt{x}(x+\sqrt{x})^{3/2} - \frac{5}{12}(x+\sqrt{x})^{3/2} + \frac{5}{32}(2\sqrt{x}+1)\sqrt{x+\sqrt{x}} - \frac{5}{32}\tanh^{-1}\left(\frac{\sqrt{x}}{\sqrt{x+\sqrt{x}}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x]*Sqrt[Sqrt[x] + x], x]

[Out] (5*(1 + 2*Sqrt[x])*Sqrt[Sqrt[x] + x])/32 - (5*(Sqrt[x] + x)^(3/2))/12 + (Sqrt[x]*(Sqrt[x] + x)^(3/2))/2 - (5*ArcTanh[Sqrt[x]/Sqrt[Sqrt[x] + x]]/32

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 612

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 640

Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(a + b*x + c*x^2)^(p + 1))/(2*c*(p + 1)), x] + Dist[(2*c*d - b*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[2*c*d - b*e, 0] && NeQ[p, -1]

Rule 670

Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Simp[(e*(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1))/(c*(m + 2*p + 1)), x] + Dist[((m + p)*(2*c*d - b*e))/(c*(m + 2*p + 1)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2*p]

Rule 2018

```
Int[(x_)^(m_)*((a_)*(x_)^(j_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := Dist
[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a*x^Simplify[j/n] + b*x)^p, x]
, x, x^n], x] /; FreeQ[{a, b, j, m, n, p}, x] && !IntegerQ[p] && NeQ[n, j]
&& IntegerQ[Simplify[j/n]] && IntegerQ[Simplify[(m + 1)/n]] && NeQ[n^2, 1]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{x} \sqrt{\sqrt{x} + x} dx &= 2 \operatorname{Subst} \left(\int x^2 \sqrt{x + x^2} dx, x, \sqrt{x} \right) \\
&= \frac{1}{2} \sqrt{x} (\sqrt{x} + x)^{3/2} - \frac{5}{4} \operatorname{Subst} \left(\int x \sqrt{x + x^2} dx, x, \sqrt{x} \right) \\
&= -\frac{5}{12} (\sqrt{x} + x)^{3/2} + \frac{1}{2} \sqrt{x} (\sqrt{x} + x)^{3/2} + \frac{5}{8} \operatorname{Subst} \left(\int \sqrt{x + x^2} dx, x, \sqrt{x} \right) \\
&= \frac{5}{32} (1 + 2\sqrt{x}) \sqrt{\sqrt{x} + x} - \frac{5}{12} (\sqrt{x} + x)^{3/2} + \frac{1}{2} \sqrt{x} (\sqrt{x} + x)^{3/2} - \frac{5}{64} \operatorname{Subst} \left(\int \frac{1}{\sqrt{x + x^2}} dx, x, \sqrt{x} \right) \\
&= \frac{5}{32} (1 + 2\sqrt{x}) \sqrt{\sqrt{x} + x} - \frac{5}{12} (\sqrt{x} + x)^{3/2} + \frac{1}{2} \sqrt{x} (\sqrt{x} + x)^{3/2} - \frac{5}{32} \operatorname{Subst} \left(\int \frac{1}{1 - x^2} dx, x, \sqrt{x} \right) \\
&= \frac{5}{32} (1 + 2\sqrt{x}) \sqrt{\sqrt{x} + x} - \frac{5}{12} (\sqrt{x} + x)^{3/2} + \frac{1}{2} \sqrt{x} (\sqrt{x} + x)^{3/2} - \frac{5}{32} \tanh^{-1} \left(\frac{\sqrt{x}}{\sqrt{\sqrt{x} + x}} \right)
\end{aligned}$$

Mathematica [A] time = 0.07, size = 58, normalized size = 0.71

$$\frac{1}{96} \sqrt{x + \sqrt{x}} \left(48x^{3/2} + 8x - 10\sqrt{x} - \frac{15 \sinh^{-1}(\sqrt[4]{x})}{\sqrt{\sqrt{x} + 1} \sqrt[4]{x}} + 15 \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x]*Sqrt[Sqrt[x] + x], x]

[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2) - (15*ArcSinh[x^(1/4)])/Sqrt[1 + Sqrt[x]]*x^(1/4)))/96

IntegrateAlgebraic [A] time = 0.15, size = 57, normalized size = 0.70

$$\frac{1}{96} \sqrt{x + \sqrt{x}} (48x^{3/2} + 8x - 10\sqrt{x} + 15) - \frac{5}{32} \tanh^{-1} \left(\frac{\sqrt{x + \sqrt{x}}}{\sqrt{x}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x]*Sqrt[Sqrt[x] + x], x]

[Out] (Sqrt[Sqrt[x] + x]*(15 - 10*Sqrt[x] + 8*x + 48*x^(3/2)))/96 - (5*ArcTanh[Sqrt[Sqrt[x] + x]/Sqrt[x]])/32

fricas [A] time = 0.98, size = 54, normalized size = 0.66

$$\frac{1}{96} (2(24x - 5)\sqrt{x} + 8x + 15)\sqrt{x + \sqrt{x}} + \frac{5}{128} \log \left(4\sqrt{x + \sqrt{x}}(2\sqrt{x} + 1) - 8x - 8\sqrt{x} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{96}*(2*(24*x - 5)*\sqrt{x} + 8*x + 15)*\sqrt{x + \sqrt{x}} + \frac{5}{128}*\log(4*\sqrt{x + \sqrt{x}}*(2*\sqrt{x} + 1) - 8*x - 8*\sqrt{x} - 1)$

giac [A] time = 0.44, size = 50, normalized size = 0.61

$$\frac{1}{96} \left(2 \left(4 \sqrt{x} (6 \sqrt{x} + 1) - 5 \right) \sqrt{x + \sqrt{x}} + \frac{5}{64} \log \left(-2 \sqrt{x + \sqrt{x}} + 2 \sqrt{x} + 1 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="giac")`

[Out] $\frac{1}{96}*(2*(4*\sqrt{x}*(6*\sqrt{x} + 1) - 5)*\sqrt{x} + 15)*\sqrt{x + \sqrt{x}} + \frac{5}{64}*\log(-2*\sqrt{x + \sqrt{x}} + 2*\sqrt{x} + 1)$

maple [A] time = 0.00, size = 54, normalized size = 0.66

$$-\frac{5 \ln \left(\sqrt{x} + \frac{1}{2} + \sqrt{x + \sqrt{x}} \right)}{64} + \frac{(x + \sqrt{x})^{\frac{3}{2}} \sqrt{x}}{2} - \frac{5(x + \sqrt{x})^{\frac{3}{2}}}{12} + \frac{5(2\sqrt{x} + 1) \sqrt{x + \sqrt{x}}}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(x+x^(1/2))^(1/2),x)`

[Out] $\frac{1}{2}*x^{(1/2)}*(x+x^{(1/2)})^{(3/2)} - \frac{5}{12}*(x+x^{(1/2)})^{(3/2)} + \frac{5}{32}*(2*x^{(1/2)}+1)*(x+x^{(1/2)})^{(1/2)} - \frac{5}{64}*\ln(x^{(1/2)}+1/2+(x+x^{(1/2)})^{(1/2)})$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x + \sqrt{x}} \sqrt{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^(1/2)*(x+x^(1/2))^(1/2),x, algorithm="maxima")`

[Out] `integrate(sqrt(x + sqrt(x))*sqrt(x), x)`

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x} \sqrt{x + \sqrt{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(x + x^(1/2))^(1/2),x)`

[Out] `int(x^(1/2)*(x + x^(1/2))^(1/2), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x} \sqrt{\sqrt{x} + x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(x+x**(1/2))**(1/2),x)`

[Out] `Integral(sqrt(x)*sqrt(sqrt(x) + x), x)`

$$3.663 \quad \int \frac{1 + \sqrt[3]{x}}{1 + \sqrt{x}} dx$$

Optimal. Leaf size=74

$$\frac{6x^{5/6}}{5} + 2\sqrt{x} - 3\sqrt[3]{x} - 4\log(\sqrt[6]{x} + 1) - \log(\sqrt[3]{x} - \sqrt[6]{x} + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {1593, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{6x^{5/6}}{5} + 2\sqrt{x} - 3\sqrt[3]{x} - 4\log(\sqrt[6]{x} + 1) - \log(\sqrt[3]{x} - \sqrt[6]{x} + 1) - 2\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[6]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(1/3))/(1 + Sqrt[x]),x]

[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1874

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q
+ C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q -
C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*
A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
&& GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \frac{1 + \sqrt[3]{x}}{1 + \sqrt{x}} dx &= 6 \operatorname{Subst} \left(\int \frac{x^5 + x^7}{1 + x^3} dx, x, \sqrt[6]{x} \right) \\
&= 6 \operatorname{Subst} \left(\int \frac{x^5(1 + x^2)}{1 + x^3} dx, x, \sqrt[6]{x} \right) \\
&= 6 \operatorname{Subst} \left(\int \left(-x + x^2 + x^4 + \frac{(1-x)x}{1+x^3} \right) dx, x, \sqrt[6]{x} \right) \\
&= -3\sqrt[3]{x} + 2\sqrt{x} + \frac{6x^{5/6}}{5} + 6 \operatorname{Subst} \left(\int \frac{(1-x)x}{1+x^3} dx, x, \sqrt[6]{x} \right) \\
&= -3\sqrt[3]{x} + 2\sqrt{x} + \frac{6x^{5/6}}{5} + 2 \operatorname{Subst} \left(\int \frac{2-x}{1-x+x^2} dx, x, \sqrt[6]{x} \right) - 4 \operatorname{Subst} \left(\int \frac{1}{1+x} dx, x, \sqrt[6]{x} \right) \\
&= -3\sqrt[3]{x} + 2\sqrt{x} + \frac{6x^{5/6}}{5} - 4 \log(1 + \sqrt[6]{x}) + 3 \operatorname{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, \sqrt[6]{x} \right) - \operatorname{Subst} \left(\int \frac{-1}{1-x} dx, x, \sqrt[6]{x} \right) \\
&= -3\sqrt[3]{x} + 2\sqrt{x} + \frac{6x^{5/6}}{5} - 4 \log(1 + \sqrt[6]{x}) - \log(1 - \sqrt[6]{x} + \sqrt[3]{x}) - 6 \operatorname{Subst} \left(\int \frac{1}{-3-x^2} dx, x, \sqrt[6]{x} \right) \\
&= -3\sqrt[3]{x} + 2\sqrt{x} + \frac{6x^{5/6}}{5} - 2\sqrt{3} \tan^{-1} \left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}} \right) - 4 \log(1 + \sqrt[6]{x}) - \log(1 - \sqrt[6]{x} + \sqrt[3]{x})
\end{aligned}$$

Mathematica [A] time = 0.04, size = 74, normalized size = 1.00

$$\frac{6x^{5/6}}{5} + 2\sqrt{x} - 3\sqrt[3]{x} - 4 \log(\sqrt[6]{x} + 1) - \log(\sqrt[3]{x} - \sqrt[6]{x} + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{1-2\sqrt[6]{x}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + Sqrt[x]), x]
```

```
[Out] -3*x^(1/3) + 2*Sqrt[x] + (6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[(1 - 2*x^(1/6))/S
qrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

IntegrateAlgebraic [A] time = 0.06, size = 80, normalized size = 1.08

$$\frac{1}{5} (6x^{5/6} + 10\sqrt{x} - 15\sqrt[3]{x}) - 4 \log(\sqrt[6]{x} + 1) - \log(\sqrt[3]{x} - \sqrt[6]{x} + 1) - 2\sqrt{3} \tan^{-1} \left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[6]{x}}{\sqrt{3}} \right)$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(1 + x^(1/3))/(1 + Sqrt[x]), x]
```

```
[Out] (-15*x^(1/3) + 10*Sqrt[x] + 6*x^(5/6))/5 - 2*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*
x^(1/6))/Sqrt[3]] - 4*Log[1 + x^(1/6)] - Log[1 - x^(1/6) + x^(1/3)]
```

fricas [A] time = 0.42, size = 57, normalized size = 0.77

$$2\sqrt{3} \arctan\left(\frac{2}{3}\sqrt{3}x^{\frac{1}{6}} - \frac{1}{3}\sqrt{3}\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="fricas")

[Out] 2*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/6) - 1/3*sqrt(3)) + 6/5*x^(5/6) + 2*sqrt(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)

giac [A] time = 0.46, size = 55, normalized size = 0.74

$$2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="giac")

[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)

maple [A] time = 0.01, size = 56, normalized size = 0.76

$$2\sqrt{3} \arctan\left(\frac{\left(2x^{\frac{1}{6}} - 1\right)\sqrt{3}}{3}\right) - 4\ln\left(x^{\frac{1}{6}} + 1\right) - \ln\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) + \frac{6x^{\frac{5}{6}}}{5} + 2\sqrt{x} - 3x^{\frac{1}{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3)+1)/(x^(1/2)+1),x)

[Out] 6/5*x^(5/6)+2*x^(1/2)-3*x^(1/3)-ln(x^(1/3)-x^(1/6)+1)+2*3^(1/2)*arctan(1/3*(2*x^(1/6)-1)*3^(1/2))-4*ln(x^(1/6)+1)

maxima [A] time = 0.97, size = 55, normalized size = 0.74

$$2\sqrt{3} \arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{\frac{1}{6}} - 1\right)\right) + \frac{6}{5}x^{\frac{5}{6}} + 2\sqrt{x} - 3x^{\frac{1}{3}} - \log\left(x^{\frac{1}{3}} - x^{\frac{1}{6}} + 1\right) - 4\log\left(x^{\frac{1}{6}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/2)),x, algorithm="maxima")

[Out] 2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/6) - 1)) + 6/5*x^(5/6) + 2*sqrt(x) - 3*x^(1/3) - log(x^(1/3) - x^(1/6) + 1) - 4*log(x^(1/6) + 1)

mupad [B] time = 3.41, size = 95, normalized size = 1.28

$$2\sqrt{x} + \ln\left((-1 + \sqrt{3}i)(27 + \sqrt{3}9i) + 36x^{1/6} + 36\right) - \ln\left((1 + \sqrt{3}i)(-27 + \sqrt{3}9i) + 36x^{1/6} + 36\right) + (1 + \sqrt{3}i) - 4\ln(36x^{1/6} + 36) - 3x^{1/3} + \frac{6x^{5/6}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3) + 1)/(x^(1/2) + 1),x)

[Out] log((3^(1/2)*1i - 1)*(3^(1/2)*9i + 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i - 1) - 4*log(36*x^(1/6) + 36) - log((3^(1/2)*1i + 1)*(3^(1/2)*9i - 27) + 36*x^(1/6) + 36)*(3^(1/2)*1i + 1) + 2*x^(1/2) - 3*x^(1/3) + (6*x^(5/6))/5

sympy [C] time = 3.78, size = 155, normalized size = 2.09

$$\frac{16x^{\frac{5}{6}}\Gamma\left(\frac{8}{3}\right)}{5\Gamma\left(\frac{11}{3}\right)} - \frac{8\sqrt[3]{x}\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{11}{3}\right)} + 2\sqrt{x} - 2\log(\sqrt{x} + 1) - \frac{16e^{-\frac{2i\pi}{3}}\log\left(-\sqrt[3]{x}e^{\frac{i\pi}{3}} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16\log\left(-\sqrt[3]{x}e^{i\pi} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)} - \frac{16e^{\frac{2i\pi}{3}}\log\left(-\sqrt[3]{x}e^{\frac{5i\pi}{3}} + 1\right)\Gamma\left(\frac{8}{3}\right)}{3\Gamma\left(\frac{11}{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x**(1/3))/(1+x**(1/2)),x)

[Out] 16*x**(5/6)*gamma(8/3)/(5*gamma(11/3)) - 8*x**(1/3)*gamma(8/3)/gamma(11/3) + 2*sqrt(x) - 2*log(sqrt(x) + 1) - 16*exp(-2*I*pi/3)*log(-x**(1/6)*exp_polar(I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*log(-x**(1/6)*exp_polar(I*pi) + 1)*gamma(8/3)/(3*gamma(11/3)) - 16*exp(2*I*pi/3)*log(-x**(1/6)*exp_polar(5*I*pi/3) + 1)*gamma(8/3)/(3*gamma(11/3))

$$3.664 \quad \int \frac{1 + \sqrt[3]{x}}{1 + \sqrt[4]{x}} dx$$

Optimal. Leaf size=115

$$\frac{12x^{13/12}}{13} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} + \frac{12x^{7/12}}{7} - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} + 12\sqrt[12]{x} - 8\log(\sqrt[12]{x} + 1) - 2\log(\sqrt[6]{x} - \sqrt[12]{x} + 1) + 4\sqrt{3} \tan^{-1}$$

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {1593, 1836, 1887, 1874, 31, 634, 618, 204, 628}

$$\frac{12x^{13/12}}{13} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} + \frac{12x^{7/12}}{7} - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} + 12\sqrt[12]{x} - 8\log(\sqrt[12]{x} + 1) - 2\log(\sqrt[6]{x} - \sqrt[12]{x} + 1) + 4\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + x^(1/3))/(1 + x^(1/4)),x]

[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*Sqrt[3]*ArcTan[(1 - 2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(n_), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 628

Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Simp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]

Rule 634

Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]

Rule 1593

Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 1836

```
Int[(Pq_)*((c_)*(x_))^(m_)*((a_) + (b_)*(x_)^(n_))^(p_), x_Symbol] := With[{q = Expon[Pq, x]}, With[{Pqq = Coeff[Pq, x, q]}, Dist[1/(b*(m + q + n*p + 1)), Int[(c*x)^m*ExpandToSum[b*(m + q + n*p + 1)*(Pq - Pqq*x^q) - a*Pqq*(m + q - n + 1)*x^(q - n), x]*(a + b*x^n)^p, x], x] + Simp[(Pqq*(c*x)^(m + q - n + 1)*(a + b*x^n)^(p + 1))/(b*c^(q - n + 1)*(m + q + n*p + 1)), x]] /; NeQ[m + q + n*p + 1, 0] && q - n >= 0 && (IntegerQ[2*p] || IntegerQ[p + (q + 1)/(2*n)])] /; FreeQ[{a, b, c, m, p}, x] && PolyQ[Pq, x] && IGtQ[n, 0]
```

Rule 1874

```
Int[(P2_)/((a_) + (b_)*(x_)^3), x_Symbol] := With[{A = Coeff[P2, x, 0], B = Coeff[P2, x, 1], C = Coeff[P2, x, 2], q = (a/b)^(1/3)}, Dist[(q*(A - B*q + C*q^2))/(3*a), Int[1/(q + x), x], x] + Dist[q/(3*a), Int[(q*(2*A + B*q - C*q^2) - (A - B*q - 2*C*q^2)*x)/(q^2 - q*x + x^2), x], x] /; NeQ[a*B^3 - b*A^3, 0] && NeQ[A - B*q + C*q^2, 0]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2] && GtQ[a/b, 0]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_)*(x_)^(n_)), x_Symbol] := Int[ExpandIntegrand[Pq/(a + b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rubi steps

$$\int \frac{1 + \sqrt[3]{x}}{1 + \sqrt[4]{x}} dx = 12 \text{Subst} \left(\int \frac{x^{11} + x^{15}}{1 + x^3} dx, x, \sqrt[12]{x} \right)$$

$$= 12 \text{Subst} \left(\int \frac{x^{11} (1 + x^4)}{1 + x^3} dx, x, \sqrt[12]{x} \right)$$

$$= \frac{12x^{13/12}}{13} + \frac{12}{13} \text{Subst} \left(\int \frac{(13 - 13x)x^{11}}{1 + x^3} dx, x, \sqrt[12]{x} \right)$$

$$= \frac{12x^{13/12}}{13} + \frac{12}{13} \text{Subst} \left(\int \left(13 + 13x^2 - 13x^3 - 13x^5 + 13x^6 + 13x^8 - 13x^9 - \frac{13(1 + x^2)}{1 + x^3} \right) dx, x \right)$$

$$= 12 \sqrt[12]{x} + 4 \sqrt[4]{x} - 3 \sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 12 \text{Subst} \left(\int \frac{1 + x^2}{1 + x^3} dx, x \right)$$

$$= 12 \sqrt[12]{x} + 4 \sqrt[4]{x} - 3 \sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 4 \text{Subst} \left(\int \frac{1 + x}{1 - x + x^2} dx, x \right)$$

$$= 12 \sqrt[12]{x} + 4 \sqrt[4]{x} - 3 \sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8 \log(1 + \sqrt[12]{x}) - 2 \text{Subst} \left(\int \frac{1 + x}{1 - x + x^2} dx, x \right)$$

$$= 12 \sqrt[12]{x} + 4 \sqrt[4]{x} - 3 \sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} - 8 \log(1 + \sqrt[12]{x}) - 2 \log \left(\frac{1 - x + x^2}{1 + \sqrt[12]{x}} \right)$$

$$= 12 \sqrt[12]{x} + 4 \sqrt[4]{x} - 3 \sqrt[3]{x} - 2\sqrt{x} + \frac{12x^{7/12}}{7} + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + \frac{12x^{13/12}}{13} + 4\sqrt{3} \tan^{-1} \left(\frac{1 - 2 \sqrt[12]{x}}{\sqrt{3}} \right)$$

Mathematica [A] time = 0.09, size = 123, normalized size = 1.07

$$\frac{12x^{13/12}}{13} - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} + \frac{12x^{7/12}}{7} - 2\sqrt{x} - 3\sqrt[3]{x} + 4\sqrt[4]{x} + 12\sqrt[12]{x} + 4 \left(\sqrt[3]{-1} - 1 \right) \log \left(\sqrt[3]{-1} - \sqrt[12]{x} \right) - 4 \left(1 + (-1)^{2/3} \right) \log \left(-\sqrt[12]{x} - (-1)^{2/3} \right) - 8 \log(\sqrt[12]{x} + 1)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^(1/3))/(1 + x^(1/4)), x]
[Out] 12*x^(1/12) + 4*x^(1/4) - 3*x^(1/3) - 2*Sqrt[x] + (12*x^(7/12))/7 + (4*x^(3/4))/3 - (6*x^(5/6))/5 + (12*x^(13/12))/13 + 4*(-1 + (-1)^(1/3))*Log[(-1)^(1/3) - (1 + x^(1/3))/sqrt(3)]
```

$$\frac{1}{3} - x^{(1/12)}] - 4*(1 + (-1)^{(2/3)})*\text{Log}[-(-1)^{(2/3)} - x^{(1/12)}] - 8*\text{Log}[1 + x^{(1/12)}]$$

IntegrateAlgebraic [A] time = 0.06, size = 115, normalized size = 1.00

$$\frac{1260x^{13/12} - 1638x^{5/6} + 1820x^{3/4} + 2340x^{7/12} - 2730\sqrt{x} - 4095\sqrt[3]{x} + 5460\sqrt[4]{x} + 16380\sqrt[12]{x}}{1365} - 8\log\left(\frac{\sqrt[12]{x}}{\sqrt[12]{x}+1}\right) - 2\log\left(\frac{\sqrt[6]{x}}{\sqrt[6]{x}-\sqrt[12]{x}+1}\right) + 4\sqrt{3}\tan^{-1}\left(\frac{1}{\sqrt{3}} - \frac{2\sqrt[12]{x}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + x^(1/3))/(1 + x^(1/4)), x]

[Out] (16380*x^(1/12) + 5460*x^(1/4) - 4095*x^(1/3) - 2730*Sqrt[x] + 2340*x^(7/12) + 1820*x^(3/4) - 1638*x^(5/6) + 1260*x^(13/12))/1365 + 4*Sqrt[3]*ArcTan[1/Sqrt[3] - (2*x^(1/12))/Sqrt[3]] - 8*Log[1 + x^(1/12)] - 2*Log[1 - x^(1/12) + x^(1/6)]

fricas [A] time = 0.42, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{2}{3}\sqrt{3}x^{1/12} - \frac{1}{3}\sqrt{3}\right) + \frac{12}{13}(x+13)x^{1/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} + \frac{12}{7}x^{7/12} - 2\sqrt{x} - 3x^{1/3} + 4x^{1/4} - 2\log\left(x^{1/6} - x^{1/12} + 1\right) - 8\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/4)), x, algorithm="fricas")

[Out] -4*sqrt(3)*arctan(2/3*sqrt(3)*x^(1/12) - 1/3*sqrt(3)) + 12/13*(x + 13)*x^(1/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)

giac [A] time = 0.43, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{1/12} - 1\right)\right) + \frac{12}{13}x^{13/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} + \frac{12}{7}x^{7/12} - 2\sqrt{x} - 3x^{1/3} + 4x^{1/4} + 12x^{1/12} - 2\log\left(x^{1/6} - x^{1/12} + 1\right) - 8\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/4)), x, algorithm="giac")

[Out] -4*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^(1/12) - 1)) + 12/13*x^(13/12) - 6/5*x^(5/6) + 4/3*x^(3/4) + 12/7*x^(7/12) - 2*sqrt(x) - 3*x^(1/3) + 4*x^(1/4) + 12*x^(1/12) - 2*log(x^(1/6) - x^(1/12) + 1) - 8*log(x^(1/12) + 1)

maple [A] time = 0.01, size = 81, normalized size = 0.70

$$\frac{12x^{13/12}}{13} - 4\sqrt{3}\arctan\left(\frac{(2x^{1/12} - 1)\sqrt{3}}{3}\right) - 8\ln\left(x^{1/12} + 1\right) - 2\ln\left(x^{1/6} - x^{1/12} + 1\right) - \frac{6x^{5/6}}{5} + \frac{4x^{3/4}}{3} + \frac{12x^{7/12}}{7} - 2\sqrt{x} - 3x^{1/3} + 4x^{1/4} + 12x^{1/12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/3)+1)/(x^(1/4)+1), x)

[Out] 12/13*x^(13/12)-6/5*x^(5/6)+4/3*x^(3/4)+12/7*x^(7/12)-2*x^(1/2)-3*x^(1/3)+4*x^(1/4)+12*x^(1/12)-2*ln(x^(1/6)-x^(1/12)+1)-4*3^(1/2)*arctan(1/3*(2*x^(1/12)-1)*3^(1/2))-8*ln(x^(1/12)+1)

maxima [A] time = 0.96, size = 80, normalized size = 0.70

$$-4\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^{1/12} - 1\right)\right) + \frac{12}{13}x^{13/12} - \frac{6}{5}x^{5/6} + \frac{4}{3}x^{3/4} + \frac{12}{7}x^{7/12} - 2\sqrt{x} - 3x^{1/3} + 4x^{1/4} + 12x^{1/12} - 2\log\left(x^{1/6} - x^{1/12} + 1\right) - 8\log\left(x^{1/12} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x^(1/3))/(1+x^(1/4)), x, algorithm="maxima")

[Out] $-4\sqrt{3}\arctan(1/3\sqrt{3})(2x^{1/12} - 1) + 12/13x^{13/12} - 6/5x^{5/6} + 4/3x^{3/4} + 12/7x^{7/12} - 2\sqrt{x} - 3x^{1/3} + 4x^{1/4} + 12x^{1/12} - 2\log(x^{1/6} - x^{1/12} + 1) - 8\log(x^{1/12} + 1)$

mupad [B] time = 0.09, size = 130, normalized size = 1.13

$$4x^{1/4} + \ln((-2 + \sqrt{3}2i)(54 - 36x^{1/12} + \sqrt{3}18i) - 144x^{1/12} + 144)(-2 + \sqrt{3}2i) - \ln((2 + \sqrt{3}2i)(36x^{1/12} - 54 + \sqrt{3}18i) - 144x^{1/12} + 144)(2 + \sqrt{3}2i) - 2\sqrt{x} - 3x^{1/3} - 8\ln(144x^{1/12} + 144) + \frac{4x^{3/4}}{3} - \frac{6x^{5/6}}{5} + 12x^{7/12} + \frac{12x^{13/12}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((x^{1/3} + 1)/(x^{1/4} + 1), x)$

[Out] $\log((3^{1/2}2i - 2)(3^{1/2}18i - 36x^{1/12} + 54) - 144x^{1/12} + 144) * (3^{1/2}2i - 2) - 8\log(144x^{1/12} + 144) - \log((3^{1/2}2i + 2)(3^{1/2}18i + 36x^{1/12} - 54) - 144x^{1/12} + 144) * (3^{1/2}2i + 2) - 2x^{1/2} - 3x^{1/3} + 4x^{1/4} + (4x^{3/4})/3 - (6x^{5/6})/5 + 12x^{1/12} + (12x^{7/12})/7 + (12x^{13/12})/13$

sympy [C] time = 5.43, size = 221, normalized size = 1.92

$$\frac{64x^{13/12}\Gamma(\frac{16}{3})}{13\Gamma(\frac{19}{3})} + \frac{64x^{7/12}\Gamma(\frac{16}{3})}{7\Gamma(\frac{19}{3})} + \frac{64\sqrt[3]{x}\Gamma(\frac{16}{3})}{\Gamma(\frac{19}{3})} - \frac{32x^{5/6}\Gamma(\frac{16}{3})}{5\Gamma(\frac{19}{3})} + \frac{4x^{3/4}}{3} + 4\sqrt[3]{x} - \frac{16\sqrt[3]{x}\Gamma(\frac{16}{3})}{\Gamma(\frac{19}{3})} - 2\sqrt{x} - 4\log(\sqrt[3]{x} + 1) + \frac{64e^{-\frac{\pi}{3}}\log(-\sqrt[3]{x}e^{\frac{\pi}{3}} + 1)\Gamma(\frac{16}{3})}{3\Gamma(\frac{19}{3})} - \frac{64\log(-\sqrt[3]{x}e^{\pi} + 1)\Gamma(\frac{16}{3})}{3\Gamma(\frac{19}{3})} + \frac{64e^{\frac{\pi}{3}}\log(-\sqrt[3]{x}e^{\frac{\pi}{3}} + 1)\Gamma(\frac{16}{3})}{3\Gamma(\frac{19}{3})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((1+x^{1/3})/(1+x^{1/4}), x)$

[Out] $64x^{13/12}\text{gamma}(16/3)/(13\text{gamma}(19/3)) + 64x^{7/12}\text{gamma}(16/3)/(7\text{gamma}(19/3)) + 64x^{1/12}\text{gamma}(16/3)/\text{gamma}(19/3) - 32x^{5/6}\text{gamma}(16/3)/(5\text{gamma}(19/3)) + 4x^{3/4}/3 + 4x^{1/4} - 16x^{1/3}\text{gamma}(16/3)/\text{gamma}(19/3) - 2\sqrt{x} - 4\log(x^{1/4} + 1) + 64\exp(-I\pi/3)\log(-x^{1/12}\exp_{\text{polar}}(I\pi/3) + 1)\text{gamma}(16/3)/(3\text{gamma}(19/3)) - 64\log(-x^{1/12}\exp_{\text{polar}}(I\pi) + 1)\text{gamma}(16/3)/(3\text{gamma}(19/3)) + 64\exp(I\pi/3)\log(-x^{1/12}\exp_{\text{polar}}(5I\pi/3) + 1)\text{gamma}(16/3)/(3\text{gamma}(19/3))$

$$3.665 \quad \int \frac{x^2}{-1+x^2+\sqrt{1-x^2}} dx$$

Optimal. Leaf size=4

$$x + \sin^{-1}(x)$$

Rubi [A] time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2156, 8, 216}

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2156

Int[(u_.)/((c_) + (d_.)*(x_)^(n_) + (e_.)*Sqrt[(a_) + (b_.)*(x_)^(n_)]), x_Symbol] := Dist[c, Int[u/(c^2 - a*e^2 + c*d*x^n), x], x] - Dist[a*e, Int[u/((c^2 - a*e^2 + c*d*x^n)*Sqrt[a + b*x^n]), x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[b*c - a*d, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{-1+x^2+\sqrt{1-x^2}} dx &= -\int -1 dx + \int \frac{1}{\sqrt{1-x^2}} dx \\ &= x + \sin^{-1}(x) \end{aligned}$$

Mathematica [A] time = 0.06, size = 4, normalized size = 1.00

$$x + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] x + ArcSin[x]

IntegrateAlgebraic [B] time = 0.28, size = 22, normalized size = 5.50

$$2 \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}-1}\right) + x$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(-1 + x^2 + Sqrt[1 - x^2]),x]

[Out] $x + 2 \operatorname{ArcTan}\left[\frac{x}{-1 + \sqrt{1 - x^2}}\right]$

fricas [B] time = 0.40, size = 20, normalized size = 5.00

$$x - 2 \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="fricas")`

[Out] $x - 2 \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$

giac [A] time = 0.33, size = 4, normalized size = 1.00

$$x + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="giac")`

[Out] $x + \arcsin(x)$

maple [B] time = 0.02, size = 51, normalized size = 12.75

$$x + \operatorname{arctanh}(x) + \arcsin(x) + \frac{\ln(x-1)}{2} - \frac{\ln(x+1)}{2} - \frac{\sqrt{-2x - (x-1)^2 + 2}}{2} + \frac{\sqrt{2x - (x+1)^2 + 2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(-1+x^2+(-x^2+1)^(1/2)),x)`

[Out] $x + 1/2 \ln(x-1) - 1/2 \ln(x+1) + \operatorname{arctanh}(x) - 1/2 \sqrt{-2x - (x-1)^2 + 2} + \arcsin(x) + 1/2 \sqrt{2x - (x+1)^2 + 2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2 + \sqrt{-x^2 + 1} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(-1+x^2+(-x^2+1)^(1/2)),x, algorithm="maxima")`

[Out] `integrate(x^2/(x^2 + sqrt(-x^2 + 1) - 1), x)`

mupad [B] time = 0.03, size = 4, normalized size = 1.00

$$x + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(x^2 + (1 - x^2)^(1/2) - 1),x)`

[Out] $x + \operatorname{asin}(x)$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{x^2 + \sqrt{1 - x^2} - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(-1+x**2+(-x**2+1)**(1/2)),x)`

[Out] `Integral(x**2/(x**2 + sqrt(1 - x**2) - 1), x)`

$$3.666 \quad \int \sqrt{\frac{1+x}{x}} dx$$

Optimal. Leaf size=22

$$\sqrt{\frac{1}{x} + 1} x + \tanh^{-1} \left(\sqrt{\frac{1}{x} + 1} \right)$$

Rubi [A] time = 0.01, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1972, 242, 47, 63, 207}

$$\sqrt{\frac{1}{x} + 1} x + \tanh^{-1} \left(\sqrt{\frac{1}{x} + 1} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x)/x], x]

[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1+x}{x}} dx &= \int \sqrt{1 + \frac{1}{x}} dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x^2} dx, x, \frac{1}{x}\right) \\
&= \sqrt{1 + \frac{1}{x}} x - \frac{1}{2} \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \frac{1}{x}\right) \\
&= \sqrt{1 + \frac{1}{x}} x - \text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1 + \frac{1}{x}}\right) \\
&= \sqrt{1 + \frac{1}{x}} x + \tanh^{-1}\left(\sqrt{1 + \frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 22, normalized size = 1.00

$$\sqrt{\frac{1}{x} + 1} x + \tanh^{-1}\left(\sqrt{\frac{1}{x} + 1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x)/x], x]

[Out] Sqrt[1 + x^(-1)]*x + ArcTanh[Sqrt[1 + x^(-1)]]

IntegrateAlgebraic [A] time = 0.04, size = 26, normalized size = 1.18

$$\sqrt{\frac{x+1}{x}} x + \tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 + x)/x], x]

[Out] x*Sqrt[(1 + x)/x] + ArcTanh[Sqrt[(1 + x)/x]]

fricas [B] time = 0.43, size = 40, normalized size = 1.82

$$x\sqrt{\frac{x+1}{x}} + \frac{1}{2} \log\left(\sqrt{\frac{x+1}{x}} + 1\right) - \frac{1}{2} \log\left(\sqrt{\frac{x+1}{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2), x, algorithm="fricas")

[Out] x*sqrt((x + 1)/x) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x) - 1)

giac [A] time = 0.39, size = 31, normalized size = 1.41

$$-\frac{1}{2} \log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right) \text{sgn}(x) + \sqrt{x^2 + x} \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2), x, algorithm="giac")

[Out] -1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + sqrt(x^2 + x)*sgn(x)

maple [B] time = 0.00, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}} \left(\ln \left(x + \frac{1}{2} + \sqrt{x^2 + x} \right) + 2\sqrt{x^2 + x} \right) x}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)/x)^(1/2),x)

[Out] 1/2*((x+1)/x)^(1/2)*x*(2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*x)^(1/2)

maxima [B] time = 0.44, size = 50, normalized size = 2.27

$$\frac{\sqrt{\frac{x+1}{x}}}{\frac{x+1}{x} - 1} + \frac{1}{2} \log \left(\sqrt{\frac{x+1}{x}} + 1 \right) - \frac{1}{2} \log \left(\sqrt{\frac{x+1}{x}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2),x, algorithm="maxima")

[Out] sqrt((x + 1)/x)/((x + 1)/x - 1) + 1/2*log(sqrt((x + 1)/x) + 1) - 1/2*log(sqrt((x + 1)/x) - 1)

mupad [B] time = 3.39, size = 18, normalized size = 0.82

$$\operatorname{atanh} \left(\sqrt{\frac{1}{x} + 1} \right) + x \sqrt{\frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x + 1)/x)^(1/2),x)

[Out] atanh((1/x + 1)^(1/2)) + x*(1/x + 1)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x+1}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)**(1/2),x)

[Out] Integral(sqrt((x + 1)/x), x)

$$3.667 \quad \int \sqrt{\frac{1-x}{x}} dx$$

Optimal. Leaf size=24

$$\sqrt{\frac{1}{x}-1}x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Rubi [A] time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1972, 242, 47, 63, 203}

$$\sqrt{\frac{1}{x}-1}x - \tan^{-1}\left(\sqrt{\frac{1}{x}-1}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 - x)/x], x]

[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{1-x}{x}} dx &= \int \sqrt{-1 + \frac{1}{x}} dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{-1+x}}{x^2} dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1 + \frac{1}{x}} x - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{-1+x} x} dx, x, \frac{1}{x}\right) \\
&= \sqrt{-1 + \frac{1}{x}} x - \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \sqrt{-1 + \frac{1}{x}}\right) \\
&= \sqrt{-1 + \frac{1}{x}} x - \tan^{-1}\left(\sqrt{-1 + \frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$\sqrt{\frac{1}{x} - 1} x - \tan^{-1}\left(\sqrt{\frac{1}{x} - 1}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 - x)/x], x]

[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[-1 + x^(-1)]]

IntegrateAlgebraic [A] time = 0.04, size = 28, normalized size = 1.17

$$\sqrt{\frac{1}{x} - 1} x - \tan^{-1}\left(\sqrt{\frac{1-x}{x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 - x)/x], x]

[Out] Sqrt[-1 + x^(-1)]*x - ArcTan[Sqrt[(1 - x)/x]]

fricas [A] time = 0.41, size = 26, normalized size = 1.08

$$x\sqrt{-\frac{x-1}{x}} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/x)^(1/2), x, algorithm="fricas")

[Out] x*sqrt(-(x - 1)/x) - arctan(sqrt(-(x - 1)/x))

giac [A] time = 0.40, size = 28, normalized size = 1.17

$$\frac{1}{4} \pi \text{sgn}(x) + \frac{1}{2} \arcsin(2x - 1) \text{sgn}(x) + \sqrt{-x^2 + x} \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/x)^(1/2), x, algorithm="giac")

[Out] 1/4*pi*sgn(x) + 1/2*arcsin(2*x - 1)*sgn(x) + sqrt(-x^2 + x)*sgn(x)

maple [A] time = 0.01, size = 40, normalized size = 1.67

$$\frac{\sqrt{-\frac{x-1}{x}} \left(\arcsin(2x-1) + 2\sqrt{-x^2+x} \right) x}{2\sqrt{-(x-1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x-1)/x)^(1/2), x)

[Out] 1/2*(-(x-1)/x)^(1/2)*x*(2*(-x^2+x)^(1/2)+arcsin(2*x-1))/(-(x-1)*x)^(1/2)

maxima [A] time = 0.97, size = 37, normalized size = 1.54

$$-\frac{\sqrt{-\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \arctan\left(\sqrt{-\frac{x-1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/x)^(1/2), x, algorithm="maxima")

[Out] -sqrt(-(x - 1)/x)/((x - 1)/x - 1) - arctan(sqrt(-(x - 1)/x))

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$x\sqrt{\frac{1}{x}-1} - \operatorname{atan}\left(\sqrt{\frac{1}{x}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x - 1)/x)^(1/2), x)

[Out] x*(1/x - 1)^(1/2) - atan((1/x - 1)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{1-x}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1-x)/x)**(1/2), x)

[Out] Integral(sqrt((1 - x)/x), x)

$$3.668 \quad \int \sqrt{\frac{-1+x}{x}} dx$$

Optimal. Leaf size=24

$$\sqrt{x-1} \sqrt{x} - \sinh^{-1}(\sqrt{x-1})$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.17, number of steps used = 5, number of rules used = 5, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1972, 242, 47, 63, 206}

$$\sqrt{\frac{x-1}{x}} x - \tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(-1 + x)/x], x]

[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] && !(IntegerQ[n] && !Intege
rQ[m]) && !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 242

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]
```

Rule 1972

```
Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && B
inomialQ[u, x] && !BinomialMatchQ[u, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\frac{-1+x}{x}} dx &= \int \sqrt{1-\frac{1}{x}} dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2} dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}} x + \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-x}x} dx, x, \frac{1}{x}\right) \\
&= \sqrt{\frac{-1+x}{x}} x - \text{Subst}\left(\int \frac{1}{1-x^2} dx, x, \sqrt{\frac{-1+x}{x}}\right) \\
&= \sqrt{\frac{-1+x}{x}} x - \tanh^{-1}\left(\sqrt{\frac{-1+x}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.58

$$\frac{\sqrt{x}(x-1) + \sqrt{1-x} \sin^{-1}(\sqrt{1-x})}{\sqrt{x-1}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(-1 + x)/x], x]

[Out] ((-1 + x)*Sqrt[x] + Sqrt[1 - x]*ArcSin[Sqrt[1 - x]])/Sqrt[-1 + x]

IntegrateAlgebraic [A] time = 0.04, size = 28, normalized size = 1.17

$$\sqrt{\frac{x-1}{x}} x - \tanh^{-1}\left(\sqrt{\frac{x-1}{x}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(-1 + x)/x], x]

[Out] Sqrt[(-1 + x)/x]*x - ArcTanh[Sqrt[(-1 + x)/x]]

fricas [B] time = 0.42, size = 40, normalized size = 1.67

$$x\sqrt{\frac{x-1}{x}} - \frac{1}{2} \log\left(\sqrt{\frac{x-1}{x}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x-1}{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2), x, algorithm="fricas")

[Out] x*sqrt((x - 1)/x) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x) - 1)

giac [A] time = 0.38, size = 35, normalized size = 1.46

$$\frac{1}{2} \log\left(\left|-2x + 2\sqrt{x^2 - x} + 1\right|\right) \text{sgn}(x) + \sqrt{x^2 - x} \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2), x, algorithm="giac")

[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))*sgn(x) + sqrt(x^2 - x)*sgn(x)

maple [B] time = 0.01, size = 45, normalized size = 1.88

$$\frac{\sqrt{\frac{x-1}{x}} \left(\ln \left(x - \frac{1}{2} + \sqrt{x^2 - x} \right) - 2\sqrt{x^2 - x} \right) x}{2\sqrt{(x-1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x-1)/x)^(1/2),x)

[Out] -1/2*((x-1)/x)^(1/2)*x*(-2*(x^2-x)^(1/2)+ln(x-1/2+(x^2-x)^(1/2)))/((x-1)*x)^(1/2)

maxima [B] time = 0.43, size = 51, normalized size = 2.12

$$-\frac{\sqrt{\frac{x-1}{x}}}{\frac{x-1}{x}-1} - \frac{1}{2} \log \left(\sqrt{\frac{x-1}{x}} + 1 \right) + \frac{1}{2} \log \left(\sqrt{\frac{x-1}{x}} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x-1)/x)^(1/2),x, algorithm="maxima")

[Out] -sqrt((x - 1)/x)/((x - 1)/x - 1) - 1/2*log(sqrt((x - 1)/x) + 1) + 1/2*log(sqrt((x - 1)/x) - 1)

mupad [B] time = 0.03, size = 24, normalized size = 1.00

$$x \sqrt{1 - \frac{1}{x}} - \operatorname{atanh} \left(\sqrt{1 - \frac{1}{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x - 1)/x)^(1/2),x)

[Out] x*(1 - 1/x)^(1/2) - atanh((1 - 1/x)^(1/2))

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x-1}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((x-1)/x)**(1/2),x)

[Out] Integral(sqrt((x - 1)/x), x)

$$3.669 \quad \int \frac{\sqrt{\frac{1+x}{x}}}{x} dx$$

Optimal. Leaf size=24

$$2 \tanh^{-1} \left(\sqrt{\frac{1}{x} + 1} \right) - 2\sqrt{\frac{1}{x} + 1}$$

Rubi [A] time = 0.02, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1973, 266, 50, 63, 207}

$$2 \tanh^{-1} \left(\sqrt{\frac{1}{x} + 1} \right) - 2\sqrt{\frac{1}{x} + 1}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(1 + x)/x]/x,x]

[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]

Rule 50

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n + 2, 0] && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 266

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]

Rule 1973

Int[(u_)^(p_.)*((c_.)*(x_))^(m_.), x_Symbol] := Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\frac{1+x}{x}}}{x} dx &= \int \frac{\sqrt{1+\frac{1}{x}}}{x} dx \\
&= -\text{Subst}\left(\int \frac{\sqrt{1+x}}{x} dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - \text{Subst}\left(\int \frac{1}{x\sqrt{1+x}} dx, x, \frac{1}{x}\right) \\
&= -2\sqrt{1+\frac{1}{x}} - 2\text{Subst}\left(\int \frac{1}{-1+x^2} dx, x, \sqrt{1+\frac{1}{x}}\right) \\
&= -2\sqrt{1+\frac{1}{x}} + 2 \tanh^{-1}\left(\sqrt{1+\frac{1}{x}}\right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 24, normalized size = 1.00

$$2 \tanh^{-1}\left(\sqrt{\frac{1}{x}+1}\right) - 2\sqrt{\frac{1}{x}+1}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(1 + x)/x]/x,x]

[Out] -2*Sqrt[1 + x^(-1)] + 2*ArcTanh[Sqrt[1 + x^(-1)]]

IntegrateAlgebraic [A] time = 0.03, size = 28, normalized size = 1.17

$$2 \tanh^{-1}\left(\sqrt{\frac{x+1}{x}}\right) - 2\sqrt{\frac{x+1}{x}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(1 + x)/x]/x,x]

[Out] -2*Sqrt[(1 + x)/x] + 2*ArcTanh[Sqrt[(1 + x)/x]]

fricas [A] time = 0.42, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}} + 1\right) - \log\left(\sqrt{\frac{x+1}{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="fricas")

[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)

giac [A] time = 0.38, size = 38, normalized size = 1.58

$$-\log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right) \text{sgn}(x) + \frac{2 \text{sgn}(x)}{x - \sqrt{x^2 + x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="giac")

[Out] -log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x) + 2*sgn(x)/(x - sqrt(x^2 + x))

maple [B] time = 0.01, size = 60, normalized size = 2.50

$$\frac{\sqrt{\frac{x+1}{x}} \left(-x^2 \ln \left(x + \frac{1}{2} + \sqrt{x^2 + x} \right) - 2\sqrt{x^2 + x} x^2 + 2(x^2 + x)^{\frac{3}{2}} \right)}{\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x+1)/x)^(1/2)/x,x)

[Out] -((x+1)/x)^(1/2)/x*(2*(x^2+x)^(3/2)-2*(x^2+x)^(1/2)*x^2-ln(x+1/2+(x^2+x)^(1/2))*x^2)/((x+1)*x)^(1/2)

maxima [A] time = 0.44, size = 38, normalized size = 1.58

$$-2\sqrt{\frac{x+1}{x}} + \log\left(\sqrt{\frac{x+1}{x}} + 1\right) - \log\left(\sqrt{\frac{x+1}{x}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)^(1/2)/x,x, algorithm="maxima")

[Out] -2*sqrt((x + 1)/x) + log(sqrt((x + 1)/x) + 1) - log(sqrt((x + 1)/x) - 1)

mupad [B] time = 0.04, size = 20, normalized size = 0.83

$$2 \operatorname{atanh}\left(\sqrt{\frac{1}{x} + 1}\right) - 2\sqrt{\frac{1}{x} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((x + 1)/x)^(1/2)/x,x)

[Out] 2*atanh((1/x + 1)^(1/2)) - 2*(1/x + 1)^(1/2)

sympy [A] time = 3.23, size = 32, normalized size = 1.33

$$-2\sqrt{1 + \frac{1}{x}} - \log\left(\sqrt{1 + \frac{1}{x}} - 1\right) + \log\left(\sqrt{1 + \frac{1}{x}} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((1+x)/x)**(1/2)/x,x)

[Out] -2*sqrt(1 + 1/x) - log(sqrt(1 + 1/x) - 1) + log(sqrt(1 + 1/x) + 1)

$$3.670 \quad \int \sqrt{\frac{x}{1+x}} dx$$

Optimal. Leaf size=22

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Rubi [A] time = 0.00, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1958, 50, 54, 215}

$$\sqrt{x}\sqrt{x+1} - \sinh^{-1}(\sqrt{x})$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x/(1 + x)], x]

[Out] Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ[
m, 0] && (!IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 54

```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] := Dis
t[2/Sqrt[b], Subst[Int[1/Sqrt[b*c - a*d + d*x^2], x], x, Sqrt[a + b*x]], x]
/; FreeQ[{a, b, c, d}, x] && GtQ[b*c - a*d, 0] && GtQ[b, 0]
```

Rule 215

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[(Rt[b, 2]*x)/Sqr
t[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 1958

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.))))/((c_.) + (d_.)*(x_)^(n_.))^(p
_), x_Symbol] := Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{x}{1+x}} dx &= \int \frac{\sqrt{x}}{\sqrt{1+x}} dx \\ &= \sqrt{x}\sqrt{1+x} - \frac{1}{2} \int \frac{1}{\sqrt{x}\sqrt{1+x}} dx \\ &= \sqrt{x}\sqrt{1+x} - \text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}} dx, x, \sqrt{x}\right) \\ &= \sqrt{x}\sqrt{1+x} - \sinh^{-1}(\sqrt{x}) \end{aligned}$$

Mathematica [A] time = 0.02, size = 42, normalized size = 1.91

$$\frac{\sqrt{\frac{x}{x+1}} (\sqrt{x}(x+1) - \sqrt{x+1} \sinh^{-1}(\sqrt{x}))}{\sqrt{x}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x/(1 + x)], x]

[Out] (Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]]))/Sqrt[x]

IntegrateAlgebraic [A] time = 0.00, size = 30, normalized size = 1.36

$$\sqrt{\frac{x}{x+1}}(x+1) - \tanh^{-1}\left(\sqrt{\frac{x}{x+1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x/(1 + x)], x]

[Out] Sqrt[x/(1 + x)]*(1 + x) - ArcTanh[Sqrt[x/(1 + x)]]

fricas [B] time = 0.41, size = 42, normalized size = 1.91

$$(x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2), x, algorithm="fricas")

[Out] (x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x + 1)) - 1)

giac [B] time = 0.39, size = 35, normalized size = 1.59

$$\frac{1}{2} \log\left(\left|-2x + 2\sqrt{x^2 + x} - 1\right|\right) \operatorname{sgn}(x+1) + \sqrt{x^2 + x} \operatorname{sgn}(x+1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2), x, algorithm="giac")

[Out] 1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x + 1)

maple [B] time = 0.01, size = 43, normalized size = 1.95

$$\frac{\sqrt{\frac{x}{x+1}}(x+1)\left(\ln\left(x + \frac{1}{2} + \sqrt{x^2 + x}\right) - 2\sqrt{x^2 + x}\right)}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1/(x+1)*x)^(1/2), x)

[Out] -1/2*(1/(x+1)*x)^(1/2)*(x+1)*(-2*(x^2+x)^(1/2)+ln(x+1/2+(x^2+x)^(1/2)))/((x+1)*x)^(1/2)

maxima [B] time = 0.44, size = 51, normalized size = 2.32

$$-\frac{\sqrt{\frac{x}{x+1}}}{x+1} - \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} + 1\right) + \frac{1}{2} \log\left(\sqrt{\frac{x}{x+1}} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))^(1/2),x, algorithm="maxima")

[Out] -sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x + 1)) - 1)

mupad [B] time = 0.04, size = 35, normalized size = 1.59

$$-\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x/(x + 1))^(1/2),x)

[Out] - atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\frac{x}{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x/(1+x))**(1/2),x)

[Out] Integral(sqrt(x/(x + 1)), x)

$$3.671 \quad \int \frac{1}{\sqrt{\frac{-1-x}{x}}} dx$$

Optimal. Leaf size=29

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right) - x\sqrt{-\frac{x+1}{x}}$$

Rubi [A] time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1972, 242, 51, 63, 204}

$$\tan^{-1}\left(\sqrt{-\frac{x+1}{x}}\right) - x\sqrt{-\frac{x+1}{x}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(-1 - x)/x],x]

[Out] -(x*Sqrt[-((1 + x)/x)]) + ArcTan[Sqrt[-((1 + x)/x)]]

Rule 51

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := Simp[((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x]] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d, m, n, x]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 242

Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] := -Subst[Int[(a + b/x^n)^p/x^2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0]

Rule 1972

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{\frac{-1-x}{x}}} dx &= \int \frac{1}{\sqrt{-1 - \frac{1}{x}}} dx \\
&= -\text{Subst} \left(\int \frac{1}{\sqrt{-1 - x x^2}} dx, x, \frac{1}{x} \right) \\
&= -x \sqrt{-\frac{1+x}{x}} + \frac{1}{2} \text{Subst} \left(\int \frac{1}{\sqrt{-1 - x x}} dx, x, \frac{1}{x} \right) \\
&= -x \sqrt{-\frac{1+x}{x}} - \text{Subst} \left(\int \frac{1}{-1 - x^2} dx, x, \sqrt{-\frac{1+x}{x}} \right) \\
&= -x \sqrt{-\frac{1+x}{x}} + \tan^{-1} \left(\sqrt{-\frac{1+x}{x}} \right)
\end{aligned}$$

Mathematica [A] time = 0.01, size = 43, normalized size = 1.48

$$\frac{\sqrt{x}(x+1) - \sqrt{x+1} \sinh^{-1}(\sqrt{x})}{\sqrt{x} \sqrt{-\frac{x+1}{x}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(-1 - x)/x], x]

[Out] (Sqrt[x]*(1 + x) - Sqrt[1 + x]*ArcSinh[Sqrt[x]])/(Sqrt[x]*Sqrt[-((1 + x)/x)])

IntegrateAlgebraic [A] time = 0.03, size = 31, normalized size = 1.07

$$\tan^{-1} \left(\sqrt{\frac{-x-1}{x}} \right) - \sqrt{\frac{-x-1}{x}} x$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(-1 - x)/x], x]

[Out] -(Sqrt[(-1 - x)/x]*x) + ArcTan[Sqrt[(-1 - x)/x]]

fricas [A] time = 0.44, size = 25, normalized size = 0.86

$$-x \sqrt{-\frac{x+1}{x}} + \arctan \left(\sqrt{-\frac{x+1}{x}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1-x)/x)^(1/2), x, algorithm="fricas")

[Out] -x*sqrt(-(x + 1)/x) + arctan(sqrt(-(x + 1)/x))

giac [A] time = 0.46, size = 35, normalized size = 1.21

$$\frac{1}{4} \pi \text{sgn}(x) - \frac{\arcsin(2x+1)}{2 \text{sgn}(x)} - \frac{\sqrt{-x^2-x}}{\text{sgn}(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-1-x)/x)^(1/2), x, algorithm="giac")

[Out] $\frac{1}{4}\pi\operatorname{sgn}(x) - \frac{1}{2}\arcsin(2x + 1)/\operatorname{sgn}(x) - \sqrt{-x^2 - x}/\operatorname{sgn}(x)$

maple [A] time = 0.01, size = 44, normalized size = 1.52

$$\frac{(x + 1) \left(\arcsin(2x + 1) + 2\sqrt{-x^2 - x} \right)}{2\sqrt{-\frac{x+1}{x}} \sqrt{-(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((-1-x)/x)^(1/2), x)`

[Out] $\frac{1}{2}(x+1)(2\sqrt{-x^2-x} + \arcsin(2x+1))/(-x+1)/(-x+1)^{1/2}/(-x+1)^{1/2}$

maxima [A] time = 0.97, size = 35, normalized size = 1.21

$$-\frac{\sqrt{-\frac{x+1}{x}}}{\frac{x+1}{x} - 1} + \arctan\left(\sqrt{-\frac{x+1}{x}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1-x)/x)^(1/2), x, algorithm="maxima")`

[Out] $-\sqrt{-(x+1)/x}/((x+1)/x - 1) + \arctan(\sqrt{-(x+1)/x})$

mupad [B] time = 3.37, size = 23, normalized size = 0.79

$$\operatorname{atan}\left(\sqrt{-\frac{1}{x} - 1}\right) - x\sqrt{-\frac{1}{x} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-(x+1)/x)^(1/2), x)`

[Out] $\operatorname{atan}((-1/x - 1)^{1/2}) - x(-1/x - 1)^{1/2}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{-x-1}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-1-x)/x)**(1/2), x)`

[Out] `Integral(1/sqrt((-x - 1)/x), x)`

3.672 $\int \sqrt{(4-x)x} dx$

Optimal. Leaf size=33

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x) - 2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Rubi [A] time = 0.01, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1979, 612, 619, 216}

$$-\frac{1}{2}\sqrt{4x-x^2}(2-x) - 2\sin^{-1}\left(1-\frac{x}{2}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[(4 - x)*x], x]

[Out] -((2 - x)*Sqrt[4*x - x^2])/2 - 2*ArcSin[1 - x/2]

Rule 216

Int[1/Sqrt[(a_) + (b_)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 612

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Simp[((b + 2*c*x)*(a + b*x + c*x^2)^p)/(2*c*(2*p + 1)), x] - Dist[(p*(b^2 - 4*a*c))/(2*c*(2*p + 1)), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]

Rule 619

Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \sqrt{(4-x)x} dx &= \int \sqrt{4x-x^2} dx \\ &= -\frac{1}{2}(2-x)\sqrt{4x-x^2} + 2 \int \frac{1}{\sqrt{4x-x^2}} dx \\ &= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{x^2}{16}}} dx, x, 4-2x\right) \\ &= -\frac{1}{2}(2-x)\sqrt{4x-x^2} - 2\sin^{-1}\left(1-\frac{x}{2}\right) \end{aligned}$$

Mathematica [A] time = 0.05, size = 32, normalized size = 0.97

$$\frac{1}{2}(x-2)\sqrt{-(x-4)x} - 4\sin^{-1}\left(\sqrt{1-\frac{x}{4}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[(4 - x)*x], x]

[Out] $((-2 + x) \sqrt{-((-4 + x) * x)}) / 2 - 4 \operatorname{ArcSin}[\sqrt{1 - x/4}]$

IntegrateAlgebraic [A] time = 0.12, size = 41, normalized size = 1.24

$$\frac{1}{2}(x-2)\sqrt{4x-x^2} - 4 \tan^{-1}\left(\frac{\sqrt{4x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[(4 - x)*x], x]

[Out] $((-2 + x) \sqrt{4 * x - x^2}) / 2 - 4 \operatorname{ArcTan}[\sqrt{4 * x - x^2} / x]$

fricas [A] time = 0.41, size = 35, normalized size = 1.06

$$\frac{1}{2} \sqrt{-x^2 + 4x} (x - 2) - 4 \arctan\left(\frac{\sqrt{-x^2 + 4x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((4-x)*x)^(1/2), x, algorithm="fricas")

[Out] $1/2 * \operatorname{sqrt}(-x^2 + 4 * x) * (x - 2) - 4 * \operatorname{arctan}(\operatorname{sqrt}(-x^2 + 4 * x) / x)$

giac [A] time = 0.39, size = 25, normalized size = 0.76

$$\frac{1}{2} \sqrt{-x^2 + 4x} (x - 2) + 2 \arcsin\left(\frac{1}{2} x - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((4-x)*x)^(1/2), x, algorithm="giac")

[Out] $1/2 * \operatorname{sqrt}(-x^2 + 4 * x) * (x - 2) + 2 * \operatorname{arcsin}(1/2 * x - 1)$

maple [A] time = 0.01, size = 28, normalized size = 0.85

$$2 \arcsin\left(\frac{x}{2} - 1\right) - \frac{(-2x + 4) \sqrt{-x^2 + 4x}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((4-x)*x)^(1/2), x)

[Out] $-1/4 * (-2 * x + 4) * (-x^2 + 4 * x)^(1/2) + 2 * \operatorname{arcsin}(1/2 * x - 1)$

maxima [A] time = 1.02, size = 36, normalized size = 1.09

$$\frac{1}{2} \sqrt{-x^2 + 4x} x - \sqrt{-x^2 + 4x} - 2 \arcsin\left(-\frac{1}{2} x + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(((4-x)*x)^(1/2), x, algorithm="maxima")

[Out] $1/2 * \operatorname{sqrt}(-x^2 + 4 * x) * x - \operatorname{sqrt}(-x^2 + 4 * x) - 2 * \operatorname{arcsin}(-1/2 * x + 1)$

mupad [B] time = 3.47, size = 26, normalized size = 0.79

$$2 \operatorname{asin}\left(\frac{x}{2} - 1\right) + \left(\frac{x}{2} - 1\right) \sqrt{4x - x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(x - 4))^(1/2), x)
```

```
[Out] 2*asin(x/2 - 1) + (x/2 - 1)*(4*x - x^2)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{x(4-x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(((4-x)*x)**(1/2), x)
```

```
[Out] Integral(sqrt(x*(4 - x)), x)
```


$$3.673 \quad \int \frac{1}{\sqrt{(1-x)x}} dx$$

Optimal. Leaf size=8

$$-\sin^{-1}(1-2x)$$

Rubi [A] time = 0.00, antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1979, 619, 216}

$$-\sin^{-1}(1-2x)$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(1-x)*x],x]

[Out] -ArcSin[1-2*x]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 619

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] := Dist[1/(2*c*((-4*c)/(b^2 - 4*a*c))^p), Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x, b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{(1-x)x}} dx &= \int \frac{1}{\sqrt{x-x^2}} dx \\ &= -\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}} dx, x, 1-2x\right) \\ &= -\sin^{-1}(1-2x) \end{aligned}$$

Mathematica [A] time = 0.01, size = 12, normalized size = 1.50

$$-2 \sin^{-1}\left(\sqrt{1-x}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(1-x)*x],x]

[Out] -2*ArcSin[Sqrt[1-x]]

IntegrateAlgebraic [B] time = 0.10, size = 18, normalized size = 2.25

$$-2 \tan^{-1}\left(\frac{\sqrt{x-x^2}}{x}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(1 - x)*x], x]

[Out] -2*ArcTan[Sqrt[x - x^2]/x]

fricas [B] time = 0.41, size = 16, normalized size = 2.00

$$-2 \arctan\left(\frac{\sqrt{-x^2 + x}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)^(1/2), x, algorithm="fricas")

[Out] -2*arctan(sqrt(-x^2 + x)/x)

giac [A] time = 0.39, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)^(1/2), x, algorithm="giac")

[Out] arcsin(2*x - 1)

maple [A] time = 0.01, size = 7, normalized size = 0.88

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((-x+1)*x)^(1/2), x)

[Out] arcsin(2*x-1)

maxima [A] time = 0.96, size = 6, normalized size = 0.75

$$\arcsin(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)^(1/2), x, algorithm="maxima")

[Out] arcsin(2*x - 1)

mupad [B] time = 0.01, size = 6, normalized size = 0.75

$$\operatorname{asin}(2x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(-x*(x - 1))^(1/2), x)

[Out] asin(2*x - 1)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(1-x)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((1-x)*x)**(1/2), x)

[Out] Integral(1/sqrt(x*(1 - x)), x)

$$3.674 \quad \int \frac{x}{(x(2+x))^{3/2}} dx$$

Optimal. Leaf size=13

$$\frac{x}{\sqrt{x^2 + 2x}}$$

Rubi [A] time = 0.01, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1980, 636}

$$\frac{x}{\sqrt{x^2 + 2x}}$$

Antiderivative was successfully verified.

[In] Int[x/(x*(2 + x))^(3/2), x]

[Out] x/Sqrt[2*x + x^2]

Rule 636

Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[(-2*(b*d - 2*a*e + (2*c*d - b*e)*x))/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0]

Rule 1980

Int[(u_)^(p_.)*((c_.)*(x_)^(m_.), x_Symbol] :> Int[(c*x)^m*ExpandToSum[u, x]^p, x] /; FreeQ[{c, m, p}, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{x}{(x(2+x))^{3/2}} dx &= \int \frac{x}{(2x+x^2)^{3/2}} dx \\ &= \frac{x}{\sqrt{2x+x^2}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x(x+2)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/(x*(2 + x))^(3/2), x]

[Out] x/Sqrt[x*(2 + x)]

IntegrateAlgebraic [A] time = 0.15, size = 17, normalized size = 1.31

$$\frac{\sqrt{x^2 + 2x}}{x + 2}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x/(x*(2 + x))^(3/2), x]

[Out] Sqrt[2*x + x^2]/(2 + x)

fricas [A] time = 0.46, size = 18, normalized size = 1.38

$$\frac{x + \sqrt{x^2 + 2x} + 2}{x + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="fricas")

[Out] (x + sqrt(x^2 + 2*x) + 2)/(x + 2)

giac [A] time = 0.33, size = 16, normalized size = 1.23

$$\frac{2}{x - \sqrt{(x + 2)x} + 2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="giac")

[Out] 2/(x - sqrt((x + 2)*x) + 2)

maple [A] time = 0.00, size = 15, normalized size = 1.15

$$\frac{(x + 2)x^2}{((x + 2)x)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x*(x+2))^(3/2),x)

[Out] x^2*(x+2)/(x*(x+2))^(3/2)

maxima [A] time = 0.43, size = 11, normalized size = 0.85

$$\frac{x}{\sqrt{x^2 + 2x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x*(2+x))^(3/2),x, algorithm="maxima")

[Out] x/sqrt(x^2 + 2*x)

mupad [B] time = 3.52, size = 13, normalized size = 1.00

$$\frac{\sqrt{x(x+2)}}{x+2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(x*(x + 2))^(3/2),x)

[Out] (x*(x + 2))^(1/2)/(x + 2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(x(x+2))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(x*(2+x))**(3/2),x)

[Out] Integral(x/(x*(x + 2))**(3/2), x)

$$3.675 \quad \int \frac{\sqrt{1+\frac{1}{x}}}{1-x^2} dx$$

Optimal. Leaf size=22

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\frac{1}{x} + 1}}{\sqrt{2}} \right)$$

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1446, 1469, 627, 63, 207}

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\frac{1}{x} + 1}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + x^(-1)]/(1 - x^2), x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]

Rule 63

Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] := With[{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b + (d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 627

Int[((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] := Int[(d + e*x)^(m + p)*(a/d + (c*x)/e)^p, x] /; FreeQ[{a, c, d, e, m, p}, x] && EqQ[c*d^2 + a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && IntegerQ[m + p]))

Rule 1446

Int[((a_.) + (c_.)*(x_)^(mn2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Int[((d + e*x^n)^q*(c + a*x^(2*n))^p)/x^(2*n*p), x] /; FreeQ[{a, c, d, e, n, q}, x] && EqQ[mn2, -2*n] && IntegerQ[p]

Rule 1469

Int[(x_)^(m_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_.))^(q_.), x_Symbol] := Dist[1/n, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^n], x] /; FreeQ[{a, c, d, e, m, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[Simplify[m - n + 1], 0]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1 + \frac{1}{x}}}{1 - x^2} dx &= \int \frac{\sqrt{1 + \frac{1}{x}}}{\left(-1 + \frac{1}{x^2}\right) x^2} dx \\
&= -\text{Subst} \left(\int \frac{\sqrt{1+x}}{-1+x^2} dx, x, \frac{1}{x} \right) \\
&= -\text{Subst} \left(\int \frac{1}{(-1+x)\sqrt{1+x}} dx, x, \frac{1}{x} \right) \\
&= -\left(2 \text{Subst} \left(\int \frac{1}{-2+x^2} dx, x, \sqrt{1+\frac{1}{x}} \right) \right) \\
&= \sqrt{2} \tanh^{-1} \left(\frac{\sqrt{1+\frac{1}{x}}}{\sqrt{2}} \right)
\end{aligned}$$

Mathematica [A] time = 0.03, size = 22, normalized size = 1.00

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\frac{1}{x} + 1}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + x^(-1)]/(1 - x^2), x]

[Out] Sqrt[2]*ArcTanh[Sqrt[1 + x^(-1)]/Sqrt[2]]

IntegrateAlgebraic [A] time = 0.07, size = 24, normalized size = 1.09

$$\sqrt{2} \tanh^{-1} \left(\frac{\sqrt{\frac{x+1}{x}}}{\sqrt{2}} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 + x^(-1)]/(1 - x^2), x]

[Out] Sqrt[2]*ArcTanh[Sqrt[(1 + x)/x]/Sqrt[2]]

fricas [A] time = 0.42, size = 33, normalized size = 1.50

$$\frac{1}{2} \sqrt{2} \log \left(-\frac{2\sqrt{2}x\sqrt{\frac{x+1}{x}} + 3x + 1}{x - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1), x, algorithm="fricas")

[Out] 1/2*sqrt(2)*log(-(2*sqrt(2)*x*sqrt((x + 1)/x) + 3*x + 1)/(x - 1))

giac [B] time = 0.53, size = 73, normalized size = 3.32

$$\frac{1}{2} \sqrt{2} \log \left(\frac{\sqrt{2} - 1}{\sqrt{2} + 1} \right) \text{sgn}(x) - \frac{1}{2} \sqrt{2} \log \left(\frac{|-2x - 2\sqrt{2} + 2\sqrt{x^2 + x} + 2|}{|-2x + 2\sqrt{2} + 2\sqrt{x^2 + x} + 2|} \right) \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="giac")

[Out] 1/2*sqrt(2)*log((sqrt(2) - 1)/(sqrt(2) + 1))*sgn(x) - 1/2*sqrt(2)*log(abs(-2*x - 2*sqrt(2) + 2*sqrt(x^2 + x) + 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x^2 + x) + 2))*sgn(x)

maple [B] time = 0.02, size = 41, normalized size = 1.86

$$\frac{\sqrt{\frac{x+1}{x}} \sqrt{2} x \operatorname{arctanh}\left(\frac{(3x+1)\sqrt{2}}{4\sqrt{x^2+x}}\right)}{2\sqrt{(x+1)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+1/x)^(1/2)/(-x^2+1),x)

[Out] 1/2*((x+1)/x)^(1/2)*x/((x+1)*x)^(1/2)*2^(1/2)*arctanh(1/4*(3*x+1)*2^(1/2)/(x^2+x)^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{\frac{1}{x} + 1}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)^(1/2)/(-x^2+1),x, algorithm="maxima")

[Out] -integrate(sqrt(1/x + 1)/(x^2 - 1), x)

mupad [B] time = 3.58, size = 17, normalized size = 0.77

$$\sqrt{2} \operatorname{atanh}\left(\frac{\sqrt{2} \sqrt{\frac{1}{x} + 1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(1/x + 1)^(1/2)/(x^2 - 1),x)

[Out] 2^(1/2)*atanh((2^(1/2)*(1/x + 1)^(1/2))/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$- \int \frac{\sqrt{1 + \frac{1}{x}}}{x^2 - 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+1/x)**(1/2)/(-x**2+1),x)

[Out] -Integral(sqrt(1 + 1/x)/(x**2 - 1), x)

$$3.676 \quad \int \frac{1}{1 + \sqrt{5} - x^2 + \sqrt{5}x^2} dx$$

Optimal. Leaf size=24

$$\frac{1}{2} \tan^{-1} \left(\sqrt{\frac{1}{2}} (3 - \sqrt{5}) x \right)$$

Rubi [A] time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6, 203}

$$\frac{1}{2} \tan^{-1} \left(\sqrt{\frac{1}{2}} (3 - \sqrt{5}) x \right)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1), x]

[Out] ArcTan[Sqrt[(3 - Sqrt[5])/2]*x]/2

Rule 6

Int[(u_.)*((w_.) + (a_.)*(v_.) + (b_.)*(v_.))^(p_.), x_Symbol] :> Int[u*((a + b)*v + w)^p, x] /; FreeQ[{a, b}, x] && !FreeQ[v, x]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rubi steps

$$\begin{aligned} \int \frac{1}{1 + \sqrt{5} - x^2 + \sqrt{5}x^2} dx &= \int \frac{1}{1 + \sqrt{5} + (-1 + \sqrt{5})x^2} dx \\ &= \frac{1}{2} \tan^{-1} \left(\sqrt{\frac{1}{2}} (3 - \sqrt{5}) x \right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 39, normalized size = 1.62

$$\frac{1}{4} i \log(-2ix + \sqrt{5} + 1) - \frac{1}{4} i \log(2ix + \sqrt{5} + 1)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1), x]

[Out] (I/4)*Log[1 + Sqrt[5] - (2*I)*x] - (I/4)*Log[1 + Sqrt[5] + (2*I)*x]

IntegrateAlgebraic [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{1 + \sqrt{5} - x^2 + \sqrt{5}x^2} dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1), x]

[Out] IntegrateAlgebraic[(1 + Sqrt[5] - x^2 + Sqrt[5]*x^2)^(-1), x]

fricas [A] time = 0.48, size = 11, normalized size = 0.46

$$\frac{1}{2} \arctan\left(\frac{1}{2}x(\sqrt{5}-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="fricas")

[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))

giac [A] time = 0.36, size = 13, normalized size = 0.54

$$\frac{1}{2} \arctan\left(\frac{2x}{\sqrt{5}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="giac")

[Out] 1/2*arctan(2*x/(sqrt(5) + 1))

maple [B] time = 0.03, size = 32, normalized size = 1.33

$$\frac{4 \arctan\left(\frac{4x}{2+2\sqrt{5}}\right)}{(\sqrt{5}-1)(2+2\sqrt{5})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1-x^2+5^(1/2)+5^(1/2)*x^2), x)

[Out] 4/(5^(1/2)-1)/(2+2*5^(1/2))*arctan(4*x/(2+2*5^(1/2)))

maxima [A] time = 0.98, size = 11, normalized size = 0.46

$$\frac{1}{2} \arctan\left(\frac{1}{2}x(\sqrt{5}-1)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x^2+5^(1/2)+x^2*5^(1/2)),x, algorithm="maxima")

[Out] 1/2*arctan(1/2*x*(sqrt(5) - 1))

mupad [B] time = 0.10, size = 45, normalized size = 1.88

$$\frac{\sqrt{2} \operatorname{atan}\left(\frac{\sqrt{2}x(\sqrt{5}+1)}{4\left(\frac{\sqrt{5}+1}{4}\right)\sqrt{\sqrt{5}+3}}\right)(\sqrt{5}+1)}{4\sqrt{\sqrt{5}+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(5^(1/2) + 5^(1/2)*x^2 - x^2 + 1), x)

[Out] (2^(1/2)*atan((2^(1/2)*x*(5^(1/2) + 1))/(4*(5^(1/2)/4 + 1/4)*(5^(1/2) + 3)^(1/2)))*(5^(1/2) + 1))/(4*(5^(1/2) + 3)^(1/2))

sympy [A] time = 0.58, size = 15, normalized size = 0.62

$$-\frac{\operatorname{atan}\left(x\left(\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1-x**2+5**(1/2)+x**2*5**(1/2)),x)

[Out] -atan(x*(1/2 - sqrt(5)/2))/2

$$3.677 \quad \int \frac{1}{\sqrt{ax+bx^2}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*x + b*x^2],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{ax+bx^2}} dx &= 2 \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.02, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*x + b*x^2],x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.15, size = 34, normalized size = 1.21

$$\frac{\log\left(-2\sqrt{b}\sqrt{ax+bx^2} + a + 2bx\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a*x + b*x^2],x]

[Out] -(Log[a + 2*b*x - 2*Sqrt[b]*Sqrt[a*x + b*x^2]]/Sqrt[b])

fricas [A] time = 0.45, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2 + ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="fricas")

[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]

giac [A] time = 0.47, size = 35, normalized size = 1.25

$$-\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="giac")

[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)

maple [A] time = 0.00, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx + \frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^2+a*x)^(1/2),x)

[Out] ln((1/2*a+b*x)/b^(1/2)+(b*x^2+a*x)^(1/2))/b^(1/2)

maxima [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^2+a*x)^(1/2),x, algorithm="maxima")

[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)

mupad [B] time = 3.47, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2} + bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*x + b*x^2)^(1/2),x)
```

```
[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)
```

```
sympy [F] time = 0.00, size = 0, normalized size = 0.00
```

$$\int \frac{1}{\sqrt{ax + bx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(b*x**2+a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*x + b*x**2), x)
```

$$3.678 \quad \int \frac{1}{\sqrt{x(a+bx)}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[x*(a + b*x)],x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{x(a+bx)}} dx &= \int \frac{1}{\sqrt{ax+bx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[x*(a + b*x)],x]

[Out] $(2\sqrt{a}\sqrt{x}\sqrt{1 + (b*x)/a}*\text{ArcSinh}[(\sqrt{b}*\sqrt{x})/\sqrt{a}])/(\sqrt{b}*\sqrt{x*(a + b*x)})$

IntegrateAlgebraic [A] time = 0.00, size = 34, normalized size = 1.21

$$\frac{\log\left(-2\sqrt{b}\sqrt{ax + bx^2} + a + 2bx\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[x*(a + b*x)],x]

[Out] $-(\text{Log}[a + 2*b*x - 2*\text{Sqrt}[b]*\text{Sqrt}[a*x + b*x^2]]/\text{Sqrt}[b])$

fricas [A] time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2 + ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="fricas")

[Out] $[\log(2*b*x + a + 2*\text{sqrt}(b*x^2 + a*x)*\text{sqrt}(b))/\text{sqrt}(b), -2*\text{sqrt}(-b)*\arctan(\text{sqrt}(b*x^2 + a*x)*\text{sqrt}(-b)/(b*x))/b]$

giac [A] time = 0.41, size = 35, normalized size = 1.25

$$\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="giac")

[Out] $-\log(\text{abs}(-2*(\text{sqrt}(b)*x - \text{sqrt}(b*x^2 + a*x))*\text{sqrt}(b) - a))/\text{sqrt}(b)$

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx + \frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(b*x+a))^(1/2),x)

[Out] $1/b^{(1/2)}*\ln((b*x+1/2*a)/b^{(1/2)}+(b*x^2+a*x)^{(1/2)})$

maxima [A] time = 0.43, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x*(b*x+a))^(1/2),x, algorithm="maxima")

[Out] $\log(2bx + a + 2\sqrt{bx^2 + ax})\sqrt{b}/\sqrt{b}$

mupad [B] time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2} + bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(x*(a + b*x))^(1/2), x)`

[Out] $\log((a/2 + bx)/b^{(1/2)} + (ax + bx^2)^{(1/2)})/b^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x(a + bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x*(b*x+a))**(1/2), x)`

[Out] `Integral(1/sqrt(x*(a + b*x)), x)`

$$3.679 \quad \int \frac{1}{\sqrt{\left(b + \frac{a}{x}\right)x^2}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(b + a/x)*x^2], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\left(b + \frac{a}{x}\right)x^2}} dx &= \int \frac{1}{\sqrt{ax + bx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{x}{\sqrt{ax + bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(b + a/x)*x^2], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 1.96

$$\frac{2\sqrt{x}\sqrt{a+bx}\log(\sqrt{a+bx}-\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(b + a/x)*x^2], x]

[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

fricas [A] time = 0.43, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x+b)*x^2)^(1/2), x, algorithm="fricas")

[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]

giac [A] time = 0.42, size = 35, normalized size = 1.25

$$\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x+b)*x^2)^(1/2), x, algorithm="giac")

[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b+a/x)*x^2)^(1/2), x)

[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))

maxima [A] time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x+b)*x^2)^(1/2),x, algorithm="maxima")

[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)

mupad [B] time = 3.59, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^2*(b + a/x))^(1/2),x)

[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^2\left(\frac{a}{x} + b\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x+b)*x**2)**(1/2),x)

[Out] Integral(1/sqrt(x**2*(a/x + b)), x)

$$3.680 \quad \int \frac{1}{\sqrt{\left(\frac{a}{x^2} + \frac{b}{x}\right)x^3}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a/x^2 + b/x)*x^3], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\left(\frac{a}{x^2} + \frac{b}{x}\right)x^3}} dx &= \int \frac{1}{\sqrt{ax + bx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1 - bx^2} dx, x, \frac{x}{\sqrt{ax + bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a/x^2 + b/x)*x^3], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 1.96

$$\frac{2\sqrt{x}\sqrt{a+bx}\log(\sqrt{a+bx}-\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a/x^2 + b/x)*x^3], x]

[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

fricas [A] time = 0.44, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2 + ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2), x, algorithm="fricas")

[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]

giac [A] time = 0.50, size = 35, normalized size = 1.25

$$\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2), x, algorithm="giac")

[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx + \frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a/x^2+b/x)*x^3)^(1/2), x)

[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))

maxima [A] time = 0.45, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x^2+b/x)*x^3)^(1/2),x, algorithm="maxima")

[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)

mupad [B] time = 3.53, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3*(a/x^2 + b/x))^(1/2),x)

[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{x^3\left(\frac{a}{x^2} + \frac{b}{x}\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((a/x**2+b/x)*x**3)**(1/2),x)

[Out] Integral(1/sqrt(x**3*(a/x**2 + b/x)), x)

$$3.681 \quad \int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a*x^2 + b*x^3)/x], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx &= \int \frac{1}{\sqrt{ax+bx^2}} dx \\ &= 2 \text{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a*x^2 + b*x^3)/x], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.04, size = 55, normalized size = 1.96

$$\frac{2\sqrt{x}\sqrt{a+bx}\log(\sqrt{a+bx}-\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a*x^2 + b*x^3)/x], x]

[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

fricas [A] time = 0.46, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2+ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2), x, algorithm="fricas")

[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]

giac [A] time = 0.56, size = 35, normalized size = 1.25

$$\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2), x, algorithm="giac")

[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx+\frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^3+a*x^2)/x)^(1/2), x)

[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))

maxima [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^3+a*x^2)/x)^(1/2),x, algorithm="maxima")

[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)

mupad [B] time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x^2 + b*x^3)/x)^(1/2),x)

[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^2+bx^3}{x}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x**3+a*x**2)/x)**(1/2),x)

[Out] Integral(1/sqrt((a*x**2 + b*x**3)/x), x)

$$3.682 \quad \int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx$$

Optimal. Leaf size=28

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Rubi [A] time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[(a*x^3 + b*x^4)/x^2], x]

[Out] (2*ArcTanh[(Sqrt[b]*x)/Sqrt[a*x + b*x^2]])/Sqrt[b]

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx &= \int \frac{1}{\sqrt{ax+bx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1-bx^2} dx, x, \frac{x}{\sqrt{ax+bx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}x}{\sqrt{ax+bx^2}}\right)}{\sqrt{b}} \end{aligned}$$

Mathematica [B] time = 0.00, size = 57, normalized size = 2.04

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[(a*x^3 + b*x^4)/x^2], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.03, size = 55, normalized size = 1.96

$$\frac{2\sqrt{x}\sqrt{a+bx}\log(\sqrt{a+bx}-\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{x(a+bx)}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[(a*x^3 + b*x^4)/x^2], x]

[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[x*(a + b*x)])

fricas [A] time = 0.44, size = 62, normalized size = 2.21

$$\left[\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}, -\frac{2\sqrt{-b}\arctan\left(\frac{\sqrt{bx^2 + ax}\sqrt{-b}}{bx}\right)}{b} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2), x, algorithm="fricas")

[Out] [log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b), -2*sqrt(-b)*arctan(sqrt(b*x^2 + a*x)*sqrt(-b)/(b*x))/b]

giac [A] time = 0.47, size = 35, normalized size = 1.25

$$\frac{\log\left(\left|-2\left(\sqrt{b}x - \sqrt{bx^2 + ax}\right)\sqrt{b} - a\right|\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2), x, algorithm="giac")

[Out] -log(abs(-2*(sqrt(b)*x - sqrt(b*x^2 + a*x))*sqrt(b) - a))/sqrt(b)

maple [A] time = 0.01, size = 29, normalized size = 1.04

$$\frac{\ln\left(\frac{bx + \frac{a}{2}}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((b*x^4+a*x^3)/x^2)^(1/2), x)

[Out] 1/b^(1/2)*ln((b*x+1/2*a)/b^(1/2)+(b*x^2+a*x)^(1/2))

maxima [A] time = 0.44, size = 27, normalized size = 0.96

$$\frac{\log\left(2bx + a + 2\sqrt{bx^2 + ax}\sqrt{b}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x^4+a*x^3)/x^2)^(1/2),x, algorithm="maxima")

[Out] log(2*b*x + a + 2*sqrt(b*x^2 + a*x)*sqrt(b))/sqrt(b)

mupad [B] time = 3.52, size = 28, normalized size = 1.00

$$\frac{\ln\left(\frac{\frac{a}{2}+bx}{\sqrt{b}} + \sqrt{bx^2 + ax}\right)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a*x^3 + b*x^4)/x^2)^(1/2),x)

[Out] log((a/2 + b*x)/b^(1/2) + (a*x + b*x^2)^(1/2))/b^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\frac{ax^3+bx^4}{x^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((b*x**4+a*x**3)/x**2)**(1/2),x)

[Out] Integral(1/sqrt((a*x**3 + b*x**4)/x**2), x)

$$3.683 \quad \int \frac{1}{\sqrt{acx+bcx^2}} dx$$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[a*c*x + b*c*x^2], x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{acx+bcx^2}} dx &= 2 \text{Subst} \left(\int \frac{1}{1-bcx^2} dx, x, \frac{x}{\sqrt{acx+bcx^2}} \right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.02, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[a*c*x + b*c*x^2], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[c*x*(a + b*x)])

IntegrateAlgebraic [B] time = 0.26, size = 133, normalized size = 3.32

$$\frac{\sqrt{bc} \log\left(a^2c + 8bx\sqrt{bc}\sqrt{acx+bcx^2} - 4abcx - 8b^2cx^2\right)}{2bc} - \frac{\tanh^{-1}\left(\frac{2\sqrt{b}x\sqrt{bc}}{a\sqrt{c}} - \frac{2\sqrt{b}\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[a*c*x + b*c*x^2], x]

[Out] $-(\text{ArcTanh}[(2*\text{Sqrt}[b]*\text{Sqrt}[b*c]*x)/(a*\text{Sqrt}[c]) - (2*\text{Sqrt}[b]*\text{Sqrt}[a*c*x + b*c*x^2])/(a*\text{Sqrt}[c])]/(\text{Sqrt}[b]*\text{Sqrt}[c])) - (\text{Sqrt}[b*c]*\text{Log}[a^2*c - 4*a*b*c*x - 8*b^2*c*x^2 + 8*b*\text{Sqrt}[b*c]*x*\text{Sqrt}[a*c*x + b*c*x^2]])/(2*b*c)$

fricas [A] time = 0.43, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc} \log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{bc}, -\frac{2\sqrt{-bc} \arctan\left(\frac{\sqrt{bcx^2 + acx}\sqrt{-bc}}{bcx}\right)}{bc} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*c*x^2+a*c*x)^(1/2), x, algorithm="fricas")

[Out] $[\text{sqrt}(b*c)*\log(2*b*c*x + a*c + 2*\text{sqrt}(b*c*x^2 + a*c*x)*\text{sqrt}(b*c))/(b*c), -2*\text{sqrt}(-b*c)*\arctan(\text{sqrt}(b*c*x^2 + a*c*x)*\text{sqrt}(-b*c)/(b*c*x))/(b*c)]$

giac [A] time = 0.50, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc} \log\left(\left|-2\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)b - \sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*c*x^2+a*c*x)^(1/2), x, algorithm="giac")

[Out] $-\text{sqrt}(b*c)*\log(\text{abs}(-2*(\text{sqrt}(b*c)*x - \text{sqrt}(b*c*x^2 + a*c*x))*b - \text{sqrt}(b*c)*a))/(b*c)$

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx + \frac{1}{2}ac}{\sqrt{bc}} + \sqrt{bcx^2 + acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*c*x^2+a*c*x)^(1/2), x)

[Out] $\ln\left(\frac{1}{2}a*c + b*c*x\right)/(b*c)^{(1/2)} + (b*c*x^2 + a*c*x)^{(1/2)}/(b*c)^{(1/2)}$

maxima [A] time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*c*x^2+a*c*x)^(1/2), x, algorithm="maxima")

[Out] $\log(2*b*c*x + a*c + 2*\text{sqrt}(b*c*x^2 + a*c*x)*\text{sqrt}(b*c))/\text{sqrt}(b*c)$

mupad [B] time = 3.90, size = 33, normalized size = 0.82

$$\frac{\ln\left(ac + 2\sqrt{bc}\sqrt{cx(a+bx)} + 2bcx\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(a*c*x + b*c*x^2)^(1/2), x)`

[Out] `log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{acx + bcx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*c*x**2+a*c*x)**(1/2), x)`

[Out] `Integral(1/sqrt(a*c*x + b*c*x**2), x)`

$$3.684 \quad \int \frac{1}{\sqrt{c(ax+bx^2)}} dx$$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c*(a*x + b*x^2)],x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{c(ax+bx^2)}} dx &= \int \frac{1}{\sqrt{acx+bcx^2}} dx \\ &= 2 \operatorname{Subst} \left(\int \frac{1}{1-bcx^2} dx, x, \frac{x}{\sqrt{acx+bcx^2}} \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{c} x}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a} \sqrt{x} \sqrt{\frac{bx}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}{\sqrt{b} \sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[c*(a*x + b*x^2)], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[c*x*(a + b*x)])

IntegrateAlgebraic [B] time = 0.24, size = 133, normalized size = 3.32

$$\frac{\sqrt{bc} \log\left(a^2c + 8bx\sqrt{bc} \sqrt{acx + bcx^2} - 4abcx - 8b^2cx^2\right)}{2bc} - \frac{\tanh^{-1}\left(\frac{2\sqrt{b}x\sqrt{bc}}{a\sqrt{c}} - \frac{2\sqrt{b}\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[c*(a*x + b*x^2)], x]

[Out] -(ArcTanh[(2*Sqrt[b]*Sqrt[b*c]*x)/(a*Sqrt[c]) - (2*Sqrt[b]*Sqrt[a*c*x + b*c*x^2))/(a*Sqrt[c])])/(Sqrt[b]*Sqrt[c]) - (Sqrt[b*c]*Log[a^2*c - 4*a*b*c*x - 8*b^2*c*x^2 + 8*b*Sqrt[b*c]*x*Sqrt[a*c*x + b*c*x^2]])/(2*b*c)

fricas [A] time = 0.47, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc} \log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{bc}, -\frac{2\sqrt{-bc} \arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(b*x^2+a*x))^(1/2), x, algorithm="fricas")

[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]

giac [A] time = 0.57, size = 50, normalized size = 1.25

$$\frac{\sqrt{bc} \log\left(\left|-2\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)b - \sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(b*x^2+a*x))^(1/2), x, algorithm="giac")

[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a))/(b*c)

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}} + \sqrt{bcx^2 + acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*(b*x^2+a*x))^(1/2), x)

[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))

maxima [A] time = 0.44, size = 36, normalized size = 0.90

$$\frac{\log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(b*x^2+a*x))^(1/2),x, algorithm="maxima")

[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)

mupad [B] time = 3.56, size = 33, normalized size = 0.82

$$\frac{\ln(ac + 2\sqrt{bc}\sqrt{cx(a+bx)} + 2bcx)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*(a*x + b*x^2))^(1/2),x)

[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c(ax + bx^2)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(b*x**2+a*x))**(1/2),x)

[Out] Integral(1/sqrt(c*(a*x + b*x**2)), x)

$$3.685 \quad \int \frac{1}{\sqrt{cx(a+bx)}} dx$$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{cx}}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{cx}}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c*x*(a + b*x)],x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] :> Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{cx(a+bx)}} dx &= \int \frac{1}{\sqrt{acx+bcx^2}} dx \\ &= 2 \text{Subst} \left(\int \frac{1}{1-bcx^2} dx, x, \frac{x}{\sqrt{acx+bcx^2}} \right) \\ &= \frac{2 \tanh^{-1} \left(\frac{\sqrt{b} \sqrt{cx}}{\sqrt{acx+bcx^2}} \right)}{\sqrt{b} \sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a} \sqrt{x} \sqrt{\frac{bx}{a} + 1} \sinh^{-1} \left(\frac{\sqrt{b} \sqrt{x}}{\sqrt{a}} \right)}{\sqrt{b} \sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[c*x*(a + b*x)],x]

[Out] $(2\sqrt{a}\sqrt{x}\sqrt{1+(b*x)/a}\operatorname{ArcSinh}[(\sqrt{b}\sqrt{x})/\sqrt{a}])/(S\sqrt{b}\sqrt{c*x*(a+b*x)})$

IntegrateAlgebraic [B] time = 0.24, size = 133, normalized size = 3.32

$$\frac{\sqrt{bc} \log\left(a^2c + 8bx\sqrt{bc} \sqrt{acx + bcx^2} - 4abcx - 8b^2cx^2\right)}{2bc} - \frac{\tanh^{-1}\left(\frac{2\sqrt{b}x\sqrt{bc}}{a\sqrt{c}} - \frac{2\sqrt{b}\sqrt{acx+bcx^2}}{a\sqrt{c}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[c*x*(a + b*x)],x]

[Out] $-(\operatorname{ArcTanh}[(2\sqrt{b}\sqrt{b*c}x)/(a\sqrt{c}) - (2\sqrt{b}\sqrt{a*c*x + b*c*x^2})/(a\sqrt{c})])/(S\sqrt{b}\sqrt{c}) - (S\sqrt{b*c}*\operatorname{Log}[a^2*c - 4*a*b*c*x - 8*b^2*c*x^2 + 8*b*S\sqrt{b*c}*x*S\sqrt{a*c*x + b*c*x^2}])/(2*b*c)$

fricas [A] time = 0.45, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc} \log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{bc}, -\frac{2\sqrt{-bc} \arctan\left(\frac{\sqrt{bcx^2 + acx}\sqrt{-bc}}{bcx}\right)}{bc} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="fricas")

[Out] $[S\sqrt{b*c}*\log(2*b*c*x + a*c + 2*S\sqrt{b*c*x^2 + a*c*x}*S\sqrt{b*c})/(b*c), -2*S\sqrt{-b*c}*\arctan(S\sqrt{b*c*x^2 + a*c*x}*S\sqrt{-b*c})/(b*c*x)]$

giac [A] time = 0.49, size = 50, normalized size = 1.25

$$-\frac{\sqrt{bc} \log\left(\left|-2\left(\sqrt{bc}x - \sqrt{bcx^2 + acx}\right)b - \sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="giac")

[Out] $-S\sqrt{b*c}*\log(\operatorname{abs}(-2*(S\sqrt{b*c}*x - S\sqrt{b*c*x^2 + a*c*x})*b - S\sqrt{b*c}*a))/(b*c)$

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx + \frac{1}{2}ac}{\sqrt{bc}} + \sqrt{bcx^2 + acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x*(b*x+a))^(1/2),x)

[Out] $1/(b*c)^(1/2)*\ln((b*c*x + 1/2*a*c)/(b*c)^(1/2) + (b*c*x^2 + a*c*x)^(1/2))$

maxima [A] time = 0.46, size = 36, normalized size = 0.90

$$\frac{\log\left(2bcx + ac + 2\sqrt{bcx^2 + acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x*(b*x+a))^(1/2),x, algorithm="maxima")

[Out] $\log(2*b*c*x + a*c + 2*\sqrt{b*c*x^2 + a*c*x}*\sqrt{b*c})/\sqrt{b*c}$

mupad [B] time = 3.47, size = 33, normalized size = 0.82

$$\frac{\ln\left(ac + 2\sqrt{bc}\sqrt{cx(a+bx)} + 2bcx\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x*(a + b*x))^(1/2),x)

[Out] $\log(ac + 2*(b*c)^{(1/2)}*(c*x*(a + b*x))^{(1/2)} + 2*b*c*x)/(b*c)^{(1/2)}$

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx(a+bx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*x*(b*x+a))**(1/2),x)

[Out] Integral(1/sqrt(c*x*(a + b*x)), x)

$$3.686 \quad \int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}} dx$$

Optimal. Leaf size=40

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Rubi [A] time = 0.02, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {1979, 620, 206}

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/Sqrt[c*(b + a/x)*x^2], x]

[Out] (2*ArcTanh[(Sqrt[b]*Sqrt[c]*x)/Sqrt[a*c*x + b*c*x^2]])/(Sqrt[b]*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 620

Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] := Dist[2, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]

Rule 1979

Int[(u_)^(p_), x_Symbol] := Int[ExpandToSum[u, x]^p, x] /; FreeQ[p, x] && GeneralizedBinomialQ[u, x] && !GeneralizedBinomialMatchQ[u, x]

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{c\left(b+\frac{a}{x}\right)x^2}} dx &= \int \frac{1}{\sqrt{acx+bcx^2}} dx \\ &= 2 \operatorname{Subst}\left(\int \frac{1}{1-bcx^2} dx, x, \frac{x}{\sqrt{acx+bcx^2}}\right) \\ &= \frac{2 \tanh^{-1}\left(\frac{\sqrt{b}\sqrt{c}x}{\sqrt{acx+bcx^2}}\right)}{\sqrt{b}\sqrt{c}} \end{aligned}$$

Mathematica [A] time = 0.00, size = 58, normalized size = 1.45

$$\frac{2\sqrt{a}\sqrt{x}\sqrt{\frac{bx}{a}+1}\sinh^{-1}\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/Sqrt[c*(b + a/x)*x^2], x]

[Out] (2*Sqrt[a]*Sqrt[x]*Sqrt[1 + (b*x)/a]*ArcSinh[(Sqrt[b]*Sqrt[x])/Sqrt[a]])/(Sqrt[b]*Sqrt[c*x*(a + b*x)])

IntegrateAlgebraic [A] time = 0.11, size = 56, normalized size = 1.40

$$\frac{2\sqrt{x}\sqrt{a+bx}\log(\sqrt{a+bx}-\sqrt{b}\sqrt{x})}{\sqrt{b}\sqrt{cx(a+bx)}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/Sqrt[c*(b + a/x)*x^2], x]

[Out] (-2*Sqrt[x]*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]])/(Sqrt[b]*Sqrt[c*x*(a + b*x)])

fricas [A] time = 0.43, size = 87, normalized size = 2.18

$$\left[\frac{\sqrt{bc}\log\left(2bcx+ac+2\sqrt{bcx^2+acx}\sqrt{bc}\right)}{bc}, -\frac{2\sqrt{-bc}\arctan\left(\frac{\sqrt{bcx^2+acx}\sqrt{-bc}}{bcx}\right)}{bc} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(a/x+b)*x^2)^(1/2), x, algorithm="fricas")

[Out] [sqrt(b*c)*log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/(b*c), -2*sqrt(-b*c)*arctan(sqrt(b*c*x^2 + a*c*x)*sqrt(-b*c)/(b*c*x))/(b*c)]

giac [A] time = 0.52, size = 50, normalized size = 1.25

$$\frac{\sqrt{bc}\log\left(\left|-2\left(\sqrt{bc}x-\sqrt{bcx^2+acx}\right)b-\sqrt{bc}a\right|\right)}{bc}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(a/x+b)*x^2)^(1/2), x, algorithm="giac")

[Out] -sqrt(b*c)*log(abs(-2*(sqrt(b*c)*x - sqrt(b*c*x^2 + a*c*x))*b - sqrt(b*c)*a))/(b*c)

maple [A] time = 0.01, size = 37, normalized size = 0.92

$$\frac{\ln\left(\frac{bcx+\frac{1}{2}ac}{\sqrt{bc}}+\sqrt{bcx^2+acx}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*(b+a/x)*x^2)^(1/2), x)

[Out] 1/(b*c)^(1/2)*ln((b*c*x+1/2*a*c)/(b*c)^(1/2)+(b*c*x^2+a*c*x)^(1/2))

maxima [A] time = 0.45, size = 36, normalized size = 0.90

$$\frac{\log\left(2bcx+ac+2\sqrt{bcx^2+acx}\sqrt{bc}\right)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(a/x+b)*x^2)^(1/2),x, algorithm="maxima")

[Out] log(2*b*c*x + a*c + 2*sqrt(b*c*x^2 + a*c*x)*sqrt(b*c))/sqrt(b*c)

mupad [B] time = 3.65, size = 33, normalized size = 0.82

$$\frac{\ln(ac + 2\sqrt{bc}\sqrt{cx(a+bx)} + 2bcx)}{\sqrt{bc}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c*x^2*(b + a/x))^(1/2),x)

[Out] log(a*c + 2*(b*c)^(1/2)*(c*x*(a + b*x))^(1/2) + 2*b*c*x)/(b*c)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{cx^2\left(\frac{a}{x} + b\right)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c*(a/x+b)*x**2)**(1/2),x)

[Out] Integral(1/sqrt(c*x**2*(a/x + b)), x)

$$3.687 \quad \int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} \, dx$$

Optimal. Leaf size=63

$$\frac{1}{4} \sqrt{-x^2 + \sqrt{x^2 - 1} x + 1} (\sqrt{x^2 - 1} + 3x) + \frac{3 \sin^{-1}(x - \sqrt{x^2 - 1})}{4\sqrt{2}}$$

Rubi [F] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} \, dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]

[Out] Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]

Rubi steps

$$\int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} \, dx = \int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} \, dx$$

Mathematica [F] time = 0.03, size = 0, normalized size = 0.00

$$\int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} \, dx$$

Verification is not applicable to the result.

[In] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]

[Out] Integrate[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]

IntegrateAlgebraic [B] time = 0.94, size = 133, normalized size = 2.11

$$\frac{3x^3 + (x^2 - 1)^{3/2} - 3x}{4\sqrt{x^2 - 1} \sqrt{-x^2 + \sqrt{x^2 - 1} x + 1} x + 4(x^2 - 1) \sqrt{-x^2 + \sqrt{x^2 - 1} x + 1}} - \frac{3 \tan^{-1}\left(\frac{\sqrt{2} \sqrt{x^2 - 1}}{\sqrt{-x^2 + \sqrt{x^2 - 1} x + 1}}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x]

[Out] (-3*x + 3*x^3 + (-1 + x^2)^(3/2))/(4*x*Sqrt[-1 + x^2]*Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]] + 4*(-1 + x^2)*Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]) - (3*ArcTan[(Sqrt[2]*Sqrt[-1 + x^2])/Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]]])/(4*Sqrt[2])

fricas [A] time = 0.98, size = 68, normalized size = 1.08

$$\frac{1}{4} \sqrt{-x^2 + \sqrt{x^2 - 1} x + 1} (3x + \sqrt{x^2 - 1}) + \frac{3}{8} \sqrt{2} \arctan\left(\frac{\sqrt{2} \sqrt{-x^2 + \sqrt{x^2 - 1} x + 1}}{2 \sqrt{x^2 - 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] 1/4*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)*(3*x + sqrt(x^2 - 1)) + 3/8*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(-x^2 + sqrt(x^2 - 1)*x + 1)/sqrt(x^2 - 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)

maple [F] time = 0.02, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)

[Out] int((1-x^2+(x^2-1)^(1/2)*x)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + \sqrt{x^2 - 1}x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x^2+x*(x^2-1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(-x^2 + sqrt(x^2 - 1)*x + 1), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \sqrt{x\sqrt{x^2 - 1} - x^2 + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2),x)

[Out] int((x*(x^2 - 1)^(1/2) - x^2 + 1)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-x^2 + x\sqrt{x^2 - 1} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1-x**2+x*(x**2-1)**(1/2))**(1/2),x)

[Out] Integral(sqrt(-x**2 + x*sqrt(x**2 - 1) + 1), x)

$$3.688 \quad \int \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}} dx$$

Optimal. Leaf size=66

$$\frac{1}{2} \left(\sqrt{x} + 3\sqrt{x+1} \right) \sqrt{\sqrt{x} \sqrt{x+1} - x} - \frac{3 \sin^{-1}(\sqrt{x} - \sqrt{x+1})}{2\sqrt{2}}$$

Rubi [F] time = 0.14, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}} dx$$

Verification is not applicable to the result.

[In] Int[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x], x]

[Out] 2*Defer[Subst][Defer[Int][Sqrt[1 - x^2 + x*Sqrt[-1 + x^2]], x], x, Sqrt[1 + x]]

Rubi steps

$$\int \frac{\sqrt{-x + \sqrt{x} \sqrt{1+x}}}{\sqrt{1+x}} dx = 2 \text{Subst} \left(\int \sqrt{1 - x^2 + x\sqrt{-1 + x^2}} dx, x, \sqrt{1+x} \right)$$

Mathematica [B] time = 0.53, size = 180, normalized size = 2.73

$$\frac{(x+1)(2x-2\sqrt{x+1}\sqrt{x}+1)^2 \left(2\sqrt{\sqrt{x}\sqrt{x+1}-x}(-2x+2\sqrt{x+1}\sqrt{x}-3) + 3\sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2} \log \left(2\sqrt{\sqrt{x}\sqrt{x+1}-x} + \sqrt{-4x+4\sqrt{x+1}\sqrt{x}-2} \right) \right)}{4(\sqrt{x+1}-\sqrt{x})^3(x-\sqrt{x+1}\sqrt{x}+1)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x], x]

[Out] -1/4*((1 + x)*(1 + 2*x - 2*Sqrt[x]*Sqrt[1 + x])^2*(2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]*(-3 - 2*x + 2*Sqrt[x]*Sqrt[1 + x]) + 3*Sqrt[-2 - 4*x + 4*Sqrt[x]*Sqrt[1 + x]]*Log[2*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] + Sqrt[-2 - 4*x + 4*Sqrt[x]*Sqrt[1 + x]]])/((-Sqrt[x] + Sqrt[1 + x])^3*(1 + x - Sqrt[x]*Sqrt[1 + x])^2)

IntegrateAlgebraic [A] time = 1.00, size = 130, normalized size = 1.97

$$\frac{x^{3/2} + 3(x+1)^{3/2} - 3\sqrt{x+1}}{2\sqrt{\sqrt{x}\sqrt{x+1}-x}x + 2\sqrt{x+1}\sqrt{\sqrt{x}\sqrt{x+1}-x}\sqrt{x}} - \frac{3 \tan^{-1} \left(\frac{\sqrt{2}\sqrt{x}}{\sqrt{\sqrt{x}\sqrt{x+1}-x}} \right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]/Sqrt[1 + x], x]

[Out] (x^(3/2) - 3*Sqrt[1 + x] + 3*(1 + x)^(3/2))/(2*x*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]] + 2*Sqrt[x]*Sqrt[1 + x]*Sqrt[-x + Sqrt[x]*Sqrt[1 + x]]) - (3*ArcTan[(Sqrt[2]*Sqrt[x])/Sqrt[-x + Sqrt[x]*Sqrt[1 + x]])/(2*Sqrt[2])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(sqrt(x+1)*sqrt(x)-x)/sqrt(x+1),x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x+\sqrt{x+1}\sqrt{x}}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)

[Out] int((-x+(x+1)^(1/2)*x^(1/2))^(1/2)/(x+1)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x+1}\sqrt{x}-x}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x+x^(1/2)*(1+x)^(1/2))^(1/2)/(1+x)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(sqrt(x+1)*sqrt(x)-x)/sqrt(x+1),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^(1/2)*(x+1)^(1/2)-x)^(1/2)/(x+1)^(1/2),x)

[Out] int((x^(1/2)*(x+1)^(1/2)-x)^(1/2)/(x+1)^(1/2),x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\sqrt{x}\sqrt{x+1}-x}}{\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x+x**(1/2)*(1+x)**(1/2))**(1/2)/(1+x)**(1/2),x)

[Out] Integral(sqrt(sqrt(x)*sqrt(x+1)-x)/sqrt(x+1),x)

$$3.689 \quad \int -\frac{x+2\sqrt{1+x^2}}{x+x^3+\sqrt{1+x^2}} dx$$

Optimal. Leaf size=78

$$\sqrt{2(\sqrt{5}-1)} \tanh^{-1}\left(\sqrt{2+\sqrt{5}}(\sqrt{x^2+1}+x)\right) - \sqrt{2(1+\sqrt{5})} \tan^{-1}\left(\sqrt{\sqrt{5}-2}(\sqrt{x^2+1}+x)\right)$$

Rubi [B] time = 0.57, antiderivative size = 319, normalized size of antiderivative = 4.09, number of steps used = 25, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {6742, 261, 1130, 203, 207, 1251, 824, 707, 1093, 1247, 699, 1279}

$$-\frac{\sqrt{2}(\sqrt{5}-1) \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{x^2+1}\right) - \sqrt{\frac{2}{5(\sqrt{5}-1)}} \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} \sqrt{x^2+1}\right) + \sqrt{\frac{2}{5}(1+\sqrt{5})} \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x^2+1}\right) - \sqrt{\frac{2}{5(1+\sqrt{5})}} \tanh^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} \sqrt{x^2+1}\right) - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right) - 2 \sqrt{\frac{2}{5(1+\sqrt{5})}} \tan^{-1}\left(\sqrt{\frac{2}{1+\sqrt{5}}} x\right) + \sqrt{\frac{1}{10}(\sqrt{5}-1)} \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right) - 2 \sqrt{\frac{2}{5(\sqrt{5}-1)}} \tanh^{-1}\left(\sqrt{\frac{2}{\sqrt{5}-1}} x\right)}$$

Antiderivative was successfully verified.

```
[In] Int[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])),x]
```

```
[Out] -2*Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[(1 + Sqrt[5])/10]*ArcTan[Sqrt[2/(1 + Sqrt[5])]*x] - Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - Sqrt[(2*(-1 + Sqrt[5]))/5]*ArcTan[Sqrt[2/(-1 + Sqrt[5])]*Sqrt[1 + x^2]] - 2*Sqrt[2/(5*(-1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] + Sqrt[(-1 + Sqrt[5])/10]*ArcTanh[Sqrt[2/(-1 + Sqrt[5])]*x] - Sqrt[2/(5*(1 + Sqrt[5]))]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[1 + x^2]] + Sqrt[(2*(1 + Sqrt[5]))/5]*ArcTanh[Sqrt[2/(1 + Sqrt[5])]*Sqrt[1 + x^2]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])
```

Rule 261

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]
```

Rule 699

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 707

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[2*e, Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] := Simp[(g*(d + e*x)^m)/(c*m), x] + Dist[1/c, Int[
((d + e*x)^(m - 1)*Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x])/(a +
b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] := With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] := Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
+ q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1247

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] := Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1251

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] := Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*
(a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 6742

```
Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\begin{aligned}
\int -\frac{x + 2\sqrt{1+x^2}}{x + x^3 + \sqrt{1+x^2}} dx &= -\int \left(\frac{x}{x + x^3 + \sqrt{1+x^2}} + \frac{2\sqrt{1+x^2}}{x + x^3 + \sqrt{1+x^2}} \right) dx \\
&= -\left(2 \int \frac{\sqrt{1+x^2}}{x + x^3 + \sqrt{1+x^2}} dx \right) - \int \frac{x}{x + x^3 + \sqrt{1+x^2}} dx \\
&= -\left(2 \int \left(1 + \frac{x\sqrt{1+x^2}}{-1+x^2+x^4} - \frac{x^2(1+x^2)}{-1+x^2+x^4} \right) dx \right) - \int \left(\frac{x}{\sqrt{1+x^2}} + \frac{x^2}{-1+x^2+x^4} \right) dx \\
&= -2x - 2 \int \frac{x\sqrt{1+x^2}}{-1+x^2+x^4} dx + 2 \int \frac{x^2(1+x^2)}{-1+x^2+x^4} dx - \int \frac{x}{\sqrt{1+x^2}} dx - \int \frac{x^2}{-1+x^2+x^4} dx \\
&= -\sqrt{1+x^2} + \frac{1}{2} \text{Subst} \left(\int \frac{x\sqrt{1+x}}{-1+x+x^2} dx, x, x^2 \right) + 2 \int \frac{1}{-1+x^2+x^4} dx + \frac{1}{10} (-5 - \sqrt{10}) \\
&= -\sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) + \sqrt{\frac{1}{10}(-1+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{-1+\sqrt{5}}} x \right) \\
&= -2\sqrt{\frac{2}{5(1+\sqrt{5})}} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - 2\sqrt{\frac{1}{10}(-1+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{-1+\sqrt{5}}} x \right) \\
&= -2\sqrt{\frac{2}{5(1+\sqrt{5})}} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \sqrt{\frac{1}{10}(-1+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{-1+\sqrt{5}}} x \right) \\
&= -2\sqrt{\frac{2}{5(1+\sqrt{5})}} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \sqrt{\frac{1}{10}(1+\sqrt{5})} \tan^{-1} \left(\sqrt{\frac{2}{1+\sqrt{5}}} x \right) - \sqrt{\frac{1}{10}(-1+\sqrt{5})} \tanh^{-1} \left(\sqrt{\frac{2}{-1+\sqrt{5}}} x \right)
\end{aligned}$$

Mathematica [F] time = 0.43, size = 34, normalized size = 0.44

$$-\int \frac{2\sqrt{x^2+1} + x}{x^3 + \sqrt{x^2+1} + x} dx$$

Antiderivative was successfully verified.

[In] Integrate[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])), x]

[Out] -Integrate[(x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2]), x]

IntegrateAlgebraic [A] time = 0.67, size = 105, normalized size = 1.35

$$-\sqrt{2(1+\sqrt{5})} \tan^{-1} \left(\sqrt{2+\sqrt{5}} x - \sqrt{2+\sqrt{5}} \sqrt{x^2+1} \right) - \sqrt{2(\sqrt{5}-1)} \tanh^{-1} \left(\sqrt{\sqrt{5}-2} x - \sqrt{\sqrt{5}-2} \sqrt{x^2+1} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[-((x + 2*Sqrt[1 + x^2])/(x + x^3 + Sqrt[1 + x^2])), x]

[Out] -(Sqrt[2*(1 + Sqrt[5])]*ArcTan[Sqrt[2 + Sqrt[5]]*x - Sqrt[2 + Sqrt[5]]*Sqrt[1 + x^2]] - Sqrt[2*(-1 + Sqrt[5])]*ArcTanh[Sqrt[-2 + Sqrt[5]]*x - Sqrt[-2 + Sqrt[5]]*Sqrt[1 + x^2]])

fricas [B] time = 0.94, size = 383, normalized size = 4.91

sqrt(5)+1)atan(1/2*sqrt(2+sqrt(5))*x - sqrt(2+sqrt(5))*sqrt(x^2+1)) - sqrt(2*(sqrt(5)-1))*atanh(sqrt(-2+sqrt(5))*x - sqrt(-2+sqrt(5))*sqrt(x^2+1))

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="fricas")
[Out] sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/4*sqrt(2)*sqrt(4*x^4 + 4*x^2 + sqrt(5))*(2*x^2 + 1) - 2*(2*x^3 + sqrt(5)*x + x)*sqrt(x^2 + 1) + 1)*(sqrt(2)*x + sqrt(2)*sqrt(x^2 + 1))*sqrt(sqrt(5) + 1) - 1/2*sqrt(2)*sqrt(x^2 + 1)*sqrt(sqrt(5) + 1) + sqrt(2)*sqrt(sqrt(5) + 1)*arctan(1/8*sqrt(4*x^2 + 2*sqrt(5) + 2)*(sqrt(5)*sqrt(2) - sqrt(2))*sqrt(sqrt(5) + 1) - 1/4*(sqrt(5)*sqrt(2)*x - sqrt(2)*x)*sqrt(sqrt(5) + 1)) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*sqrt(x^2 + 1)*x + (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(4*x^2 - 4*sqrt(x^2 + 1)*x - (sqrt(5)*sqrt(2)*x - sqrt(x^2 + 1)*(sqrt(5)*sqrt(2) + sqrt(2)) + sqrt(2)*x)*sqrt(sqrt(5) - 1) + 4) - 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x + sqrt(2)*sqrt(sqrt(5) - 1)) + 1/4*sqrt(2)*sqrt(sqrt(5) - 1)*log(2*x - sqrt(2)*sqrt(sqrt(5) - 1))
```

giac [B] time = 0.86, size = 218, normalized size = 2.79

$$\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x-\sqrt{x^2+1}+\frac{1}{x-\sqrt{x^2+1}}}{\sqrt{2\sqrt{5}-2}}\right)-\frac{1}{2}\sqrt{2\sqrt{5}+2}\arctan\left(\frac{x}{\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}+\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(x+\sqrt{\frac{1}{2}\sqrt{5}+\frac{1}{2}}\right)+\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(x-\sqrt{\frac{1}{2}\sqrt{5}-\frac{1}{2}}\right)-\frac{1}{4}\sqrt{2\sqrt{5}-2}\log\left(-x+\sqrt{x^2+1}-\sqrt{2\sqrt{5}+2}-\frac{1}{x-\sqrt{x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="giac")
[Out] -1/2*sqrt(2*sqrt(5) + 2)*arctan(-(x - sqrt(x^2 + 1) + 1/(x - sqrt(x^2 + 1)))/sqrt(2*sqrt(5) - 2)) - 1/2*sqrt(2*sqrt(5) + 2)*arctan(x/sqrt(1/2*sqrt(5) + 1/2)) + 1/4*sqrt(2*sqrt(5) - 2)*log(-x + sqrt(x^2 + 1) + sqrt(2*sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x + sqrt(1/2*sqrt(5) - 1/2))) + 1/4*sqrt(2*sqrt(5) - 2)*log(abs(x - sqrt(1/2*sqrt(5) - 1/2))) - 1/4*sqrt(2*sqrt(5) - 2)*log(abs(-x + sqrt(x^2 + 1) - sqrt(2*sqrt(5) + 2) - 1/(x - sqrt(x^2 + 1))))
```

maple [B] time = 0.20, size = 438, normalized size = 5.62

$$\frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{2\sqrt{5}\sqrt{2+\sqrt{5}}\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{5}+\frac{\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{x}{\sqrt{2+2\sqrt{5}}}\right)}{\sqrt{2+2\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{\sqrt{5}\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}+\frac{3\sqrt{5}\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{10\sqrt{2+\sqrt{5}}}+\frac{\operatorname{arctanh}\left(\frac{x+\sqrt{5}}{\sqrt{2+\sqrt{5}}}\right)}{2\sqrt{2+\sqrt{5}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x)
[Out] -5^(1/2)/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-1/(2+2*5^(1/2))^(1/2)*arctan(2*x/(2+2*5^(1/2))^(1/2))-5^(1/2)/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))+1/(-2+2*5^(1/2))^(1/2)*arctanh(2*x/(-2+2*5^(1/2))^(1/2))-1/2*(x^2+1)^(1/2)-1/2*x-1/2/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+1/2*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/((x^2+1)^(1/2)-x)+3/10*5^(1/2)/(-2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-1/2/(-2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))+3/10*5^(1/2)/(2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+1/2/(2+5^(1/2))^(1/2)*arctan(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))+2/5*5^(1/2)*(-2+5^(1/2))^(1/2)*arctanh(((x^2+1)^(1/2)-x)/(-2+5^(1/2))^(1/2))-2/5*(2+5^(1/2))^(1/2)*5^(1/2)*arctan(((x^2+1)^(1/2)-x)/(2+5^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-x - \frac{1}{2} \arctan(x) + \int \frac{2x^6 + 3x^4 - x^2 - 1}{2\left(x^6 + 2x^4 + 2x^2 + 2(x^3 + x)\sqrt{x^2 + 1} + 1\right)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x-2*(x^2+1)^(1/2))/(x+x^3+(x^2+1)^(1/2)),x, algorithm="maxima")
```


[Out] $-x - 1/2 \arctan(x) + \text{integrate}(1/2*(2*x^6 + 3*x^4 - x^2 - 1)/(x^6 + 2*x^4 + 2*x^2 + 2*(x^3 + x)*\text{sqrt}(x^2 + 1) + 1), x)$

mupad [B] time = 4.38, size = 649, normalized size = 8.32

$$\frac{\frac{\sqrt{\frac{2x^2+1}{x^2+1}} \arctan\left(\frac{\sqrt{\frac{2x^2+1}{x^2+1}}}{\sqrt{\frac{2x^2+1}{x^2+1}}}\right) + \frac{\sqrt{\frac{2x^2+1}{x^2+1}} \arctan\left(\frac{\sqrt{\frac{2x^2+1}{x^2+1}}}{\sqrt{\frac{2x^2+1}{x^2+1}}}\right)}{\sqrt{\frac{2x^2+1}{x^2+1}} \sqrt{\frac{2x^2+1}{x^2+1}}} + \frac{\sqrt{\frac{2x^2+1}{x^2+1}} \arctan\left(\frac{\sqrt{\frac{2x^2+1}{x^2+1}}}{\sqrt{\frac{2x^2+1}{x^2+1}}}\right) + \frac{\sqrt{\frac{2x^2+1}{x^2+1}} \arctan\left(\frac{\sqrt{\frac{2x^2+1}{x^2+1}}}{\sqrt{\frac{2x^2+1}{x^2+1}}}\right)}{\sqrt{\frac{2x^2+1}{x^2+1}} \sqrt{\frac{2x^2+1}{x^2+1}}}{\sqrt{\frac{2x^2+1}{x^2+1}} \sqrt{\frac{2x^2+1}{x^2+1}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}(-(x + 2*(x^2 + 1)^{(1/2)})/(x + (x^2 + 1)^{(1/2)} + x^3), x)$

[Out] $(\log(x + (2^{(1/2)}*(5^{(1/2)} - 1)^{(1/2)})/2)*(5^{(1/2)}/2 - 5/2))/(2*(5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(5^{(1/2)}/2 - 1/2)^{(3/2)}) - (\log(x - (2^{(1/2)}*(5^{(1/2)} - 1)^{(1/2)})/2)*(5^{(1/2)}/2 - 5/2))/(2*(5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(5^{(1/2)}/2 - 1/2)^{(3/2)}) + (\log(x - (2^{(1/2)}*(-5^{(1/2)} - 1)^{(1/2)})/2)*(5^{(1/2)}/2 + 5/2))/(2*(-5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(-5^{(1/2)}/2 - 1/2)^{(3/2)}) - (\log(x + (2^{(1/2)}*(-5^{(1/2)} - 1)^{(1/2)})/2)*(5^{(1/2)}/2 + 5/2))/(2*(-5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(-5^{(1/2)}/2 - 1/2)^{(3/2)}) - ((\log(x - (2^{(1/2)}*(5^{(1/2)} - 1)^{(1/2)})/2) - \log((2^{(1/2)}*x*(5^{(1/2)} - 1)^{(1/2)})/2 + (2^{(1/2)}*(x^2 + 1)^{(1/2)}*(5^{(1/2)} + 1)^{(1/2)})/2 + 1))*((5^{(1/2)}/2 - 1/2)^{(1/2)} + 2*(5^{(1/2)}/2 - 1/2)^{(3/2)}))/((2*(5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(5^{(1/2)}/2 - 1/2)^{(3/2)})*(5^{(1/2)}/2 + 1/2)^{(1/2)}) - ((\log(x + (2^{(1/2)}*(5^{(1/2)} - 1)^{(1/2)})/2) - \log((2^{(1/2)}*(x^2 + 1)^{(1/2)}*(5^{(1/2)} + 1)^{(1/2)})/2 - (2^{(1/2)}*x*(5^{(1/2)} - 1)^{(1/2)})/2 + 1))*((5^{(1/2)}/2 - 1/2)^{(1/2)} + 2*(5^{(1/2)}/2 - 1/2)^{(3/2)}))/((2*(5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(5^{(1/2)}/2 - 1/2)^{(3/2)})*(5^{(1/2)}/2 + 1/2)^{(1/2)}) + ((\log((2^{(1/2)}*(x^2 + 1)^{(1/2)}*(1 - 5^{(1/2)})^{(1/2)})/2 - (2^{(1/2)}*x*(-5^{(1/2)} - 1)^{(1/2)})/2 + 1) - \log(x + (2^{(1/2)}*(-5^{(1/2)} - 1)^{(1/2)})/2))*((-5^{(1/2)}/2 - 1/2)^{(1/2)} + 2*(-5^{(1/2)}/2 - 1/2)^{(3/2)}))/((2*(-5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(-5^{(1/2)}/2 - 1/2)^{(3/2)})*(1/2 - 5^{(1/2)}/2)^{(1/2)}) + ((\log((2^{(1/2)}*x*(-5^{(1/2)} - 1)^{(1/2)})/2 + (2^{(1/2)}*(x^2 + 1)^{(1/2)}*(1 - 5^{(1/2)})^{(1/2)})/2 + 1) - \log(x - (2^{(1/2)}*(-5^{(1/2)} - 1)^{(1/2)})/2))*((-5^{(1/2)}/2 - 1/2)^{(1/2)} + 2*(-5^{(1/2)}/2 - 1/2)^{(3/2)}))/((2*(-5^{(1/2)}/2 - 1/2)^{(1/2)} + 4*(-5^{(1/2)}/2 - 1/2)^{(3/2)})*(1/2 - 5^{(1/2)}/2)^{(1/2)})$

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-x-2*(x**2+1)**(1/2))/(x+x**3+(x**2+1)**(1/2)), x)$

[Out] Timed out

$$3.690 \quad \int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}} dx$$

Optimal. Leaf size=126

$$-\sqrt{\frac{1}{2}}(1+\sqrt{5}) \tan^{-1}\left(\frac{2\sqrt{5}-(5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})}\sqrt{x^2+2x+2}}\right) - \sqrt{\frac{1}{2}}(\sqrt{5}-1) \tanh^{-1}\left(\frac{(5-\sqrt{5})x+2\sqrt{5}}{\sqrt{10(\sqrt{5}-1)}\sqrt{x^2+2x+2}}\right)$$

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1036, 1030, 207, 203}

$$-\sqrt{\frac{1}{2}}(1+\sqrt{5}) \tan^{-1}\left(\frac{2\sqrt{5}-(5+\sqrt{5})x}{\sqrt{10(1+\sqrt{5})}\sqrt{x^2+2x+2}}\right) - \sqrt{\frac{1}{2}}(\sqrt{5}-1) \tanh^{-1}\left(\frac{(5-\sqrt{5})x+2\sqrt{5}}{\sqrt{10(\sqrt{5}-1)}\sqrt{x^2+2x+2}}\right)$$

Antiderivative was successfully verified.

[In] Int[(1 + 2*x)/((1 + x^2)*Sqrt[2 + 2*x + x^2]), x]

[Out] -(Sqrt[(1 + Sqrt[5])/2]*ArcTan[(2*Sqrt[5] - (5 + Sqrt[5])*x)/(Sqrt[10*(1 + Sqrt[5]))*Sqrt[2 + 2*x + x^2]]) - Sqrt[(-1 + Sqrt[5])/2]*ArcTanh[(2*Sqrt[5] + (5 - Sqrt[5])*x)/(Sqrt[10*(-1 + Sqrt[5]))*Sqrt[2 + 2*x + x^2]])]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 1030

Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*a*g*h, Subst[Int[1/Simp[2*a^2*g*h*c + a*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]

Rule 1036

Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Dist[1/(2*q), Int[Simp[-(a*h*e) - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && NegQ[-(a*c)]

Rubi steps

$$\int \frac{1+2x}{(1+x^2)\sqrt{2+2x+x^2}} dx = -\frac{\int \frac{-5-\sqrt{5}-2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}} dx}{2\sqrt{5}} + \frac{\int \frac{-5+\sqrt{5}+2\sqrt{5}x}{(1+x^2)\sqrt{2+2x+x^2}} dx}{2\sqrt{5}}$$

$$= (2(5-\sqrt{5})) \operatorname{Subst}\left(\int \frac{1}{20(1-\sqrt{5})+2x^2} dx, x, \frac{2\sqrt{5}+(5-\sqrt{5})x}{\sqrt{2+2x+x^2}}\right) + (2(5+\sqrt{5})) \operatorname{Subst}\left(\int \frac{1}{20(1+\sqrt{5})+2x^2} dx, x, \frac{2\sqrt{5}+(5+\sqrt{5})x}{\sqrt{2+2x+x^2}}\right)$$

$$= -\sqrt{\frac{1}{2}}(1+\sqrt{5}) \tan^{-1}\left(\frac{2\sqrt{5}-(5+\sqrt{5})x}{\sqrt{10}(1+\sqrt{5})\sqrt{2+2x+x^2}}\right) - \sqrt{\frac{1}{2}}(-1+\sqrt{5}) \tan^{-1}\left(\frac{2\sqrt{5}-(5-\sqrt{5})x}{\sqrt{10}(1-\sqrt{5})\sqrt{2+2x+x^2}}\right)$$

Mathematica [C] time = 0.04, size = 87, normalized size = 0.69

$$\frac{1}{2}i\left(\sqrt{1+2i} \tanh^{-1}\left(\frac{(1+i)x+(2+i)}{\sqrt{1+2i}\sqrt{x^2+2x+2}}\right) - \sqrt{1-2i} \tanh^{-1}\left(\frac{(2-2i)x+(4-2i)}{2\sqrt{1-2i}\sqrt{x^2+2x+2}}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[(1+2*x)/((1+x^2)*Sqrt[2+2*x+x^2]),x]

[Out] (I/2)*(Sqrt[1+2*I]*ArcTanh[((2+I)+(1+I)*x)/(Sqrt[1+2*I]*Sqrt[2+2*x+x^2]]) - Sqrt[1-2*I]*ArcTanh[((4-2*I)+(2-2*I)*x)/(2*Sqrt[1-2*I]*Sqrt[2+2*x+x^2]])

IntegrateAlgebraic [C] time = 0.24, size = 97, normalized size = 0.77

$$\operatorname{RootSum}\left[\#1^4-8\#1+8\&, \frac{\#1^2 \log(-\#1+\sqrt{x^2+2x+2}-x) - \#1 \log(-\#1+\sqrt{x^2+2x+2}-x) - \log(-\#1+\sqrt{x^2+2x+2}-x)}{\#1^3-2}\right]\&$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1+2*x)/((1+x^2)*Sqrt[2+2*x+x^2]),x]

[Out] RootSum[8-8*#1+#1^4 &, (-Log[-x+Sqrt[2+2*x+x^2]-#1]-Log[-x+Sqrt[2+2*x+x^2]-#1]*#1+Log[-x+Sqrt[2+2*x+x^2]-#1]*#1^2)/(-2+#1^3) &]

fricas [B] time = 0.73, size = 770, normalized size = 6.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="fricas")

[Out] 1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5)+5)*arctan(1/200*sqrt(20*x^2-20*sqrt(x^2+2*x+2)*x-(2*5^(3/4)*sqrt(2)*sqrt(x^2+2*x+2)-5^(1/4)*(sqrt(5)*sqrt(2)*(2*x+1)-5*sqrt(2))))*sqrt(sqrt(5)+5)+20*x+10*sqrt(5)+30)*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2)-sqrt(2))+2*5^(3/4)*sqrt(2))*sqrt(sqrt(5)+5)+10*sqrt(10)*(sqrt(5)+3))+1/10*sqrt(5)*(sqrt(5)*(2*x+1)+5)+1/2*sqrt(5)*x+1/20*(5^(3/4)*(sqrt(5)*sqrt(2)*x-sqrt(2)*(x-2))-sqrt(x^2+2*x+2)*(5^(3/4)*(sqrt(5)*sqrt(2)-sqrt(2))+2*5^(3/4)*sqrt(2))+5^(1/4)*(sqrt(5)*sqrt(2)*(2*x+1)+5*sqrt(2)))*sqrt(sqrt(5)+5)-1/2*sqrt(x^2+2*x+2)*(sqrt(5)+3)+1/2*x+1)+1/5*5^(3/4)*sqrt(2)*sqrt(sqrt(5)+5)*arctan(1/200*sqrt(20*x^2-20*sqrt(x^2+2*x+2)*x+(2*5^(3/4)*sqrt(2)*sqrt(x^2+2*x+2)-5^(1/4)*(sqrt(5)*sqrt(2)*(2*x+1)-5*sqrt(2))))*sqrt(sqrt(5)+5)+20*x+10*sqrt(5)+30)*(sqrt(10)*(5^(3/4)*(sqrt(5)*sqrt(2)-sqrt(2))+2*5^(3/4)*sqrt(2))*sqrt(sqrt(5)+5)-10*sqrt(10)*(sqrt(5)+3))-1/10*sqrt(5)*(sqrt(5)*(2*x+1)+5)-1/2*sqrt(5)*x

$$+ 1/20*(5^{3/4}*(\sqrt{5}*\sqrt{2}*x - \sqrt{2}*(x - 2)) - \sqrt{x^2 + 2*x + 2}) * (5^{3/4}*(\sqrt{5}*\sqrt{2} - \sqrt{2}) + 2*5^{3/4}*\sqrt{2}) + 5^{1/4}*(\sqrt{5}*\sqrt{2}*(2*x + 1) + 5*\sqrt{2}))*\sqrt{\sqrt{5} + 5} + 1/2*\sqrt{x^2 + 2*x + 2}*(\sqrt{5} + 3) - 1/2*x - 1) + 1/40*5^{1/4}*(\sqrt{5}*\sqrt{2} - 5*\sqrt{2})*\sqrt{\sqrt{5} + 5}*\log(2*x^2 - 2*\sqrt{x^2 + 2*x + 2}*x + 1/10*(2*5^{3/4}*\sqrt{2})*\sqrt{x^2 + 2*x + 2} - 5^{1/4}*(\sqrt{5}*\sqrt{2}*(2*x + 1) - 5*\sqrt{2}))*\sqrt{\sqrt{5} + 5} + 2*x + \sqrt{5} + 3) - 1/40*5^{1/4}*(\sqrt{5}*\sqrt{2} - 5*\sqrt{2})*\sqrt{\sqrt{5} + 5}*\log(2*x^2 - 2*\sqrt{x^2 + 2*x + 2}*x - 1/10*(2*5^{3/4}*\sqrt{2})*\sqrt{x^2 + 2*x + 2} - 5^{1/4}*(\sqrt{5}*\sqrt{2}*(2*x + 1) - 5*\sqrt{2}))*\sqrt{\sqrt{5} + 5} + 2*x + \sqrt{5} + 3)$$

giac [B] time = 0.72, size = 444, normalized size = 3.52

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="giac")

[Out] $1/4*\sqrt{2*\sqrt{5} - 2}*\log(256*(\sqrt{5}*(x - \sqrt{x^2 + 2*x + 2}) - 2*x + \sqrt{5})*\sqrt{\sqrt{5} - 2} + \sqrt{5} + 2*\sqrt{x^2 + 2*x + 2} - 2*\sqrt{\sqrt{5} - 2} - 2)^2 + 256*(\sqrt{5}*(x - \sqrt{x^2 + 2*x + 2}) - 2*x - \sqrt{5} + 2*\sqrt{x^2 + 2*x + 2} + \sqrt{\sqrt{5} - 2} + 2)^2) - 1/4*\sqrt{2*\sqrt{5} - 2}*\log(256*(\sqrt{5}*(x - \sqrt{x^2 + 2*x + 2}) - 2*x - \sqrt{5})*\sqrt{\sqrt{5} - 2} + \sqrt{5} + 2*\sqrt{x^2 + 2*x + 2} + 2*\sqrt{\sqrt{5} - 2} - 2)^2 + 256*(\sqrt{5}*(x - \sqrt{x^2 + 2*x + 2}) - 2*x - \sqrt{5} + 2*\sqrt{x^2 + 2*x + 2} - \sqrt{\sqrt{5} - 2} + 2)^2) + 1/4*(\pi + 4*\arctan(1/2*(x - \sqrt{x^2 + 2*x + 2}))* (2*\sqrt{5}*\sqrt{\sqrt{5} - 2} + \sqrt{5} + 4*\sqrt{\sqrt{5} - 2} + 3) + 3/2*\sqrt{5}*\sqrt{\sqrt{5} - 2} + 1/2*\sqrt{5} + 7/2*\sqrt{\sqrt{5} - 2} + 3/2))*\sqrt{2*\sqrt{5} - 2}/(\sqrt{5} - 1) - 1/4*(\pi + 4*\arctan(-1/2*(x - \sqrt{x^2 + 2*x + 2}))* (2*\sqrt{5}*\sqrt{\sqrt{5} - 2} - \sqrt{5} + 4*\sqrt{\sqrt{5} - 2} - 3) - 3/2*\sqrt{5}*\sqrt{\sqrt{5} - 2} + 1/2*\sqrt{5} - 7/2*\sqrt{\sqrt{5} - 2} + 3/2))*\sqrt{2*\sqrt{5} - 2}/(\sqrt{5} - 1)$

maple [B] time = 0.11, size = 753, normalized size = 5.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x+1)/(x^2+1)/(x^2+2*x+2)^(1/2),x)

[Out] $-1/2*(10*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2-2*5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+10*2*5^{1/2})^{1/2}*(5*\arctan(1/80*(-22+10*5^{1/2}))^{1/2}*((5-5^{1/2})*(2*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+5^{1/2}+3))^{1/2}*(11*5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+25*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+4*5^{1/2}+10)*(-1/2*5^{1/2}+1/2+x)/(-1/2*5^{1/2}-1/2-x)*(5^{1/2}-5)/((-1/2*5^{1/2}+1/2+x)^4/(-1/2*5^{1/2}-1/2-x)^4+3*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+1))*(-10+10*5^{1/2})^{1/2}*(-22+10*5^{1/2})^{1/2}+3*\arctan(1/80*(-22+10*5^{1/2}))^{1/2}*((5-5^{1/2})*(2*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+5^{1/2}+3))^{1/2}*(11*5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+25*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+4*5^{1/2}+10)*(-1/2*5^{1/2}+1/2+x)/(-1/2*5^{1/2}-1/2-x)*(5^{1/2}-5)/((-1/2*5^{1/2}+1/2+x)^4/(-1/2*5^{1/2}-1/2-x)^4+3*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+1))*(-10+10*5^{1/2})^{1/2}*5^{1/2}*(-22+10*5^{1/2})^{1/2}+20*\operatorname{arctanh}((10*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2-2*5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+10+2*5^{1/2})^{1/2}/(-10+10*5^{1/2})^{1/2})*5^{1/2}-60*\operatorname{arctanh}((10*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2-2*5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2+10+2*5^{1/2})^{1/2}/(-10+10*5^{1/2})^{1/2}))/(-2*(5^{1/2}*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2-5*(-1/2*5^{1/2}+1/2+x)^2/(-1/2*5^{1/2}-1/2-x)^2-5^{1/2}-5)/((-1/2*5^{1/2}+1/2+x)/(-1/2*5^{1/2}-1/2-x)^2-5^{1/2}-5)/((-1/2*5^{1/2}+1/2+x)/(-1/2*5^{1/2}-1/2-x)^2-5^{1/2}-5))$

$(\sqrt{2-x+1})^2)^{1/2} / ((-1/2 \cdot 5^{1/2} + 1/2 + x) / (-1/2 \cdot 5^{1/2} - 1/2 - x + 1) / (5^{1/2} - 5) / (-10 + 10 \cdot 5^{1/2}))^{1/2}$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{\sqrt{x^2+2x+2}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x^2+1)/(x^2+2*x+2)^(1/2),x, algorithm="maxima")

[Out] integrate((2*x + 1)/(sqrt(x^2 + 2*x + 2)*(x^2 + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)

[Out] int((2*x + 1)/((x^2 + 1)*(2*x + x^2 + 2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+2x+2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+2*x)/(x**2+1)/(x**2+2*x+2)**(1/2),x)

[Out] Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + 2*x + 2)), x)

$$3.691 \quad \int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} dx$$

Optimal. Leaf size=22

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Rubi [A] time = 0.06, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {2128, 203}

$$\tan^{-1}\left(\frac{x}{\sqrt{\sqrt{x^4+1}-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcTan[x/Sqrt[-x^2 + Sqrt[1 + x^4]]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 2128

```
Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x_)^(n_.))^p_.]), x_Symbol] := Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2/n]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{(1+x^4)\sqrt{-x^2+\sqrt{1+x^4}}} dx &= \text{Subst}\left(\int \frac{1}{1+x^2} dx, x, \frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right) \\ &= \tan^{-1}\left(\frac{x}{\sqrt{-x^2+\sqrt{1+x^4}}}\right) \end{aligned}$$

Mathematica [A] time = 1.06, size = 24, normalized size = 1.09

$$\cot^{-1}\left(\frac{\sqrt{\sqrt{x^4+1}-x^2}}{x}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]
```

```
[Out] ArcCot[Sqrt[-x^2 + Sqrt[1 + x^4]]/x]
```

IntegrateAlgebraic [C] time = 0.35, size = 96, normalized size = 4.36

$$i \tanh^{-1} \left(\sqrt{2} x^4 + \frac{\sqrt{x^4 + 1} \left(-2x^2 + i\sqrt{2} x \sqrt{\sqrt{x^4 + 1} - x^2} \right)}{\sqrt{2}} - i\sqrt{\sqrt{x^4 + 1} - x^2} x^3 + \sqrt{2} \right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((1 + x^4)*Sqrt[-x^2 + Sqrt[1 + x^4]]),x]

[Out] I*ArcTanh[Sqrt[2] + Sqrt[2]*x^4 - I*x^3*Sqrt[-x^2 + Sqrt[1 + x^4]] + (Sqrt[1 + x^4]*(-2*x^2 + I*Sqrt[2]*x*Sqrt[-x^2 + Sqrt[1 + x^4]]))/Sqrt[2]]

fricas [B] time = 1.50, size = 62, normalized size = 2.82

$$-\frac{1}{4} \arctan \left(\frac{4 \left(10x^7 - 6x^3 + (7x^5 - x)\sqrt{x^4 + 1} \right) \sqrt{-x^2 + \sqrt{x^4 + 1}}}{17x^8 - 46x^4 + 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="fricas")

[Out] -1/4*arctan(4*(10*x^7 - 6*x^3 + (7*x^5 - x)*sqrt(x^4 + 1))*sqrt(-x^2 + sqrt(x^4 + 1))/(17*x^8 - 46*x^4 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 1)\sqrt{-x^2 + \sqrt{x^4 + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 1)\sqrt{-x^2 + \sqrt{x^4 + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)

[Out] int(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^4 + 1)\sqrt{-x^2 + \sqrt{x^4 + 1}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^4+1)/(-x^2+(x^4+1)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((x^4 + 1)*sqrt(-x^2 + sqrt(x^4 + 1))), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\sqrt{\sqrt{x^4+1}-x^2} (x^4+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)

[Out] int(1/(((x^4 + 1)^(1/2) - x^2)^(1/2)*(x^4 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2 + \sqrt{x^4+1}} (x^4+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x**4+1)/(-x**2+(x**4+1)**(1/2))**(1/2), x)

[Out] Integral(1/(sqrt(-x**2 + sqrt(x**4 + 1))*(x**4 + 1)), x)

$$3.692 \quad \int \frac{1}{(a+bx^4)\sqrt{cx^2+d}\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Rubi [A] time = 0.14, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2128, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}+cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]

[Out] ArcTanh[(Sqrt[c]*x)/Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 2128

Int[1/(((a_) + (b_.)*(x_)^(n_.))*Sqrt[(c_.)*(x_)^2 + (d_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.)]), x_Symbol] :> Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2/n]

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{cx^2+d}\sqrt{a+bx^4}} dx = \frac{\text{Subst}\left(\int \frac{1}{1-cx^2} dx, x, \frac{x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{cx^2+d\sqrt{a+bx^4}}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.49, size = 50, normalized size = 1.25

$$\frac{\sqrt{-\frac{1}{c}} \cot^{-1}\left(\frac{\sqrt{-\frac{1}{c}} \sqrt{d\sqrt{a+bx^4}+cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]

[Out] (Sqrt[-c^(-1)]*ArcCot[(Sqrt[-c^(-1)]*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]])/x])/a
IntegrateAlgebraic [A] time = 1.02, size = 42, normalized size = 1.05

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d\sqrt{a+bx^4}+cx^2}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x^4)*Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]),x]

[Out] ArcTanh[Sqrt[c*x^2 + d*Sqrt[a + b*x^4]]/(Sqrt[c]*x)]/(a*Sqrt[c])

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)

[Out] int(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)*sqrt(c*x^2 + sqrt(b*x^4 + a)*d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^4 + a) \sqrt{d\sqrt{bx^4 + a} + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)

[Out] int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4) \sqrt{cx^2 + d\sqrt{a + bx^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x**4+a)/(c*x**2+d*(b*x**4+a)**(1/2))**(1/2), x)

[Out] Integral(1/((a + b*x**4)*sqrt(c*x**2 + d*sqrt(a + b*x**4))), x)

$$3.693 \quad \int \frac{1}{(a+bx^4)\sqrt{-cx^2+d}\sqrt{a+bx^4}} dx$$

Optimal. Leaf size=41

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Rubi [A] time = 0.14, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2128, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{d\sqrt{a+bx^4}-cx^2}}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] Int[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]

[Out] ArcTan[(Sqrt[c]*x)/Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]]/(a*Sqrt[c])

Rule 203

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 2128

Int[1/(((a_) + (b_)*(x_)^(n_))*Sqrt[(c_)*(x_)^2 + (d_)*((a_) + (b_)*(x_)^(n_))^(p_)]), x_Symbol] := Dist[1/a, Subst[Int[1/(1 - c*x^2), x], x, x/Sqrt[c*x^2 + d*(a + b*x^n)^(2/n)]], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[p, 2/n]

Rubi steps

$$\int \frac{1}{(a+bx^4)\sqrt{-cx^2+d}\sqrt{a+bx^4}} dx = \frac{\text{Subst}\left(\int \frac{1}{1+cx^2} dx, x, \frac{x}{\sqrt{-cx^2+d}\sqrt{a+bx^4}}\right)}{a}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{c}x}{\sqrt{-cx^2+d}\sqrt{a+bx^4}}\right)}{a\sqrt{c}}$$

Mathematica [A] time = 0.56, size = 47, normalized size = 1.15

$$\frac{\sqrt{\frac{1}{c}} \cot^{-1}\left(\frac{\sqrt{\frac{1}{c}} \sqrt{d\sqrt{a+bx^4}-cx^2}}{x}\right)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]

[Out] $(\sqrt{c^{-1}}) \cdot \text{ArcCot}\left[\frac{(\sqrt{c^{-1}}) \cdot \sqrt{-(c \cdot x^2) + d \cdot \sqrt{a + b \cdot x^4}}}{x}\right] / a$

IntegrateAlgebraic [A] time = 1.03, size = 44, normalized size = 1.07

$$-\frac{\tan^{-1}\left(\frac{\sqrt{d\sqrt{a+bx^4}-cx^2}}{\sqrt{c}x}\right)}{a\sqrt{c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/((a + b*x^4)*Sqrt[-(c*x^2) + d*Sqrt[a + b*x^4]]),x]

[Out] $-(\text{ArcTan}[\sqrt{-(c \cdot x^2) + d \cdot \sqrt{a + b \cdot x^4}}] / (\sqrt{c} \cdot x)) / (a \cdot \sqrt{c})$

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="fricas")

[Out] Timed out

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{-cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="giac")

[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)

maple [F] time = 0.08, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{-cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)

[Out] int(1/(b*x^4+a)/(-c*x^2+(b*x^4+a)^(1/2)*d)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(bx^4 + a)\sqrt{-cx^2 + \sqrt{bx^4 + a}d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(b*x^4+a)/(-c*x^2+d*(b*x^4+a)^(1/2))^(1/2),x, algorithm="maxima")

[Out] integrate(1/((b*x^4 + a)*sqrt(-c*x^2 + sqrt(b*x^4 + a)*d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{(bx^4 + a) \sqrt{d \sqrt{bx^4 + a} - cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)),x)`

[Out] `int(1/((a + b*x^4)*(d*(a + b*x^4)^(1/2) - c*x^2)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + bx^4) \sqrt{-cx^2 + d \sqrt{a + bx^4}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(b*x**4+a)/(-c*x**2+d*(b*x**4+a)**(1/2))**(1/2),x)`

[Out] `Integral(1/((a + b*x**4)*sqrt(-c*x**2 + d*sqrt(a + b*x**4))), x)`

$$3.694 \quad \int \frac{a - cx^4}{\sqrt{a + bx^2 + cx^4} (ad + aex^2 + cdx^4)} dx$$

Optimal. Leaf size=54

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Rubi [A] time = 0.25, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2112, 208}

$$\frac{\tanh^{-1}\left(\frac{x\sqrt{bd-ae}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Antiderivative was successfully verified.

[In] Int[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]

[Out] ArcTanh[(Sqrt[b*d - a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[b*d - a*e])

Rule 208

Int[((a_) + (b_)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]

Rule 2112

Int[((u_)*((A_) + (B_)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v, x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Coeff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0] /; FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]

Rubi steps

$$\int \frac{a - cx^4}{\sqrt{a + bx^2 + cx^4} (ad + aex^2 + cdx^4)} dx = a \operatorname{Subst}\left(\int \frac{1}{ad - (abd - a^2e)x^2} dx, x, \frac{x}{\sqrt{a + bx^2 + cx^4}}\right) = \frac{\tanh^{-1}\left(\frac{\sqrt{bd-ae}x}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd-ae}}$$

Mathematica [C] time = 1.96, size = 419, normalized size = 7.76

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2c^2}{\sqrt{b^2-4ac}}} \sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}} + 1 \left(-\Pi\left(\frac{(b+\sqrt{b^2-4ac})d}{ae-\sqrt{a}\sqrt{a^2-4c^2}}; i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)\right) \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} - \Pi\left(\frac{(b+\sqrt{b^2-4ac})d}{ae+\sqrt{a}\sqrt{a^2-4c^2}}; i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)\right) \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}} \right) + F\left(i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)\right) \frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}}{\sqrt{2d}\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]

[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))

- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e - Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(a*e + Sqrt[a]*Sqrt[-4*c*d^2 + a*e^2]), I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])]/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*d*Sqrt[a + b*x^2 + c*x^4])

IntegrateAlgebraic [A] time = 22.78, size = 54, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae-bd}}{\sqrt{d}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae-bd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a - c*x^4)/(Sqrt[a + b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)), x]

[Out] ArcTan[(Sqrt[-(b*d) + a*e]*x)/(Sqrt[d]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[-(b*d) + a*e])

fricas [A] time = 37.19, size = 305, normalized size = 5.65

$$\left[\log\left(\frac{c^2 d^2 x^8 + 2(4 b c d^2 - 3 a c d e) x^6 - (8 a b d e - a^2 e^2 - 2(4 b^2 + a c) d^2) x^4 + a^2 d^2 + 2(4 a b d^2 - 3 a^2 d e) x^2 + 4(c d x^5 + (2 b d - a e) x^3 + a d x) \sqrt{c x^4 + b x^2 + a} \sqrt{b d^2 - a d e}}{c^2 d^2 x^8 + 2 a c d e x^6 + 2 a^2 d e x^2 + (2 a c d^2 + a^2 e^2) x^4 + a^2 d^2}\right), \frac{\sqrt{-b d^2 + a d e} \arctan\left(\frac{2 \sqrt{c x^4 + b x^2 + a} \sqrt{-b d^2 + a d e} x}{c d x^4 + (2 b d - a e) x^2 + a d}\right)}{2(b d^2 - a d e)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [1/4*log(-(c^2*d^2*x^8 + 2*(4*b*c*d^2 - 3*a*c*d*e)*x^6 - (8*a*b*d*e - a^2*e^2 - 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 + 2*(4*a*b*d^2 - 3*a^2*d*e)*x^2 + 4*(c*d*x^5 + (2*b*d - a*e)*x^3 + a*d*x)*sqrt(c*x^4 + b*x^2 + a)*sqrt(b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2)/sqrt(b*d^2 - a*d*e), -1/2*sqrt(-b*d^2 + a*d*e)*arctan(2*sqrt(c*x^4 + b*x^2 + a)*sqrt(-b*d^2 + a*d*e)*x/(c*d*x^4 + (2*b*d - a*e)*x^2 + a*d))/(b*d^2 - a*d*e)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)), x)

maple [C] time = 0.09, size = 514, normalized size = 9.52

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2), x)


```
[Out] -1/4/d*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_alpha^2*c*d+a*e)*(-1/(_alpha^2/d*(-a*e+b*d))^(1/2)*arctanh(1/2*(2*_alpha^2*c*x^2+_alpha^2*b+b*x^2+2*a)/(_alpha^2/d*(-a*e+b*d))^(1/2)/(c*x^4+b*x^2+a)^(1/2))+1/a/d*2^(1/2)*_alpha*(_alpha^2*c*d+a*e)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(2+b*x^2/a-1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)*(2+b*x^2/a+1/a*x^2*(-4*a*c+b^2)^(1/2))^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*( _alpha^2*(-4*a*c+b^2)^(1/2)*c*d+_alpha^2*b*c*d+(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)),_alpha=RootOf(_Z^4*c*d+_Z^2*a*e+a*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 + b*x^2 + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a - cx^4}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{a}{ad\sqrt{a+bx^2+cx^4} + aex^2\sqrt{a+bx^2+cx^4} + cdx^4\sqrt{a+bx^2+cx^4}} \right) dx - \int \frac{cx^4}{ad\sqrt{a+bx^2+cx^4} + aex^2\sqrt{a+bx^2+cx^4} + cdx^4\sqrt{a+bx^2+cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt(a + b*x**2 + c*x**4) + a*e*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

$$3.695 \quad \int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4} (ad+aex^2+cdx^4)} dx$$

Optimal. Leaf size=53

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Rubi [A] time = 0.26, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 44, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

[In] Int[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]

[Out] ArcTan[(Sqrt[b*d + a*e]*x)/(Sqrt[d]*Sqrt[a - b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[b*d + a*e])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2112

Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v, x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Coeff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0] /; FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]

Rubi steps

$$\int \frac{a-cx^4}{\sqrt{a-bx^2+cx^4} (ad+aex^2+cdx^4)} dx = a \text{Subst} \left(\int \frac{1}{ad - (-abd - a^2e)x^2} dx, x, \frac{x}{\sqrt{a-bx^2+cx^4}} \right) = \frac{\tan^{-1}\left(\frac{\sqrt{bd+ae}x}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{bd+ae}}$$

Mathematica [C] time = 1.45, size = 416, normalized size = 7.85

$$\frac{i\sqrt{\frac{4cx^2}{\sqrt{b^2-4ac}-b}+2}\sqrt{1-\frac{2cx^2}{\sqrt{b^2-4ac}+b}}\left(-\Pi\left(\frac{(b-\sqrt{b^2-4ac})d}{\sqrt{a}\sqrt{a^2-4cd^2-ae}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)\right)\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}-\Pi\left(\frac{(\sqrt{b^2-4ac}-b)d}{a+\sqrt{a}\sqrt{a^2-4cd^2}};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)\right)\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)+F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}x\right)\right)\frac{b-\sqrt{b^2-4ac}}{b+\sqrt{b^2-4ac}}\right)}{2d\sqrt{\frac{c}{\sqrt{b^2-4ac}-b}}\sqrt{a-bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - c*x^4)/(Sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)),x]

[Out] ((I/2)*Sqrt[2 + (4*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 - (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(-b + Sqrt[b^2 - 4*a*c])]]*x], (b - Sqrt[b^2 - 4*a*c])/(b + Sqrt[b^2 - 4*a*c])) - EllipticPi[(

$(b - \sqrt{b^2 - 4ac})d / (-ae + \sqrt{a} \sqrt{-4cd^2 + ae^2})$, $I \operatorname{ArcSinh}[\sqrt{2} \sqrt{c/(-b + \sqrt{b^2 - 4ac})}] x$, $(b - \sqrt{b^2 - 4ac}) / (b + \sqrt{b^2 - 4ac}) - \operatorname{EllipticPi}[\frac{(-b + \sqrt{b^2 - 4ac})d}{ae + \sqrt{a} \sqrt{-4cd^2 + ae^2}}]$, $I \operatorname{ArcSinh}[\sqrt{2} \sqrt{c/(-b + \sqrt{b^2 - 4ac})}] x$, $(b - \sqrt{b^2 - 4ac}) / (b + \sqrt{b^2 - 4ac}) / (\sqrt{c/(-b + \sqrt{b^2 - 4ac})} d \sqrt{a - bx^2 + cx^4})$

IntegrateAlgebraic [A] time = 22.67, size = 53, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae+bd}}{\sqrt{d}\sqrt{a-bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{ae+bd}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(a - c*x^4)/(sqrt[a - b*x^2 + c*x^4]*(a*d + a*e*x^2 + c*d*x^4)), x]

[Out] ArcTan[(sqrt[b*d + a*e]*x)/(sqrt[d]*sqrt[a - b*x^2 + c*x^4])]/(sqrt[d]*sqrt[b*d + a*e])

fricas [A] time = 35.75, size = 304, normalized size = 5.74

$$\left[\frac{\sqrt{-bd^2 - ade} \log\left(-\frac{c^2 d^2 x^8 - 2(4bcd^2 + 3acde)x^6 + (8abde + a^2 e^2 + 2(4b^2 + ac)d^2)x^4 + a^2 d^2 - 2(4abd^2 + 3a^2 de)x^2 + 4(cdx^5 - (2bd + ae)x^3 + adx)\sqrt{cx^4 - bx^2 + a}\sqrt{-bd^2 - ade}}{c^2 d^2 x^8 + 2acdex^6 + 2a^2 dex^2 + (2acd^2 + a^2 e^2)x^4 + a^2 d^2}\right)}{4(bd^2 + ade)}, \operatorname{arctan}\left(\frac{2\sqrt{cx^4 - bx^2 + a}\sqrt{bd^2 + ade}x}{cdx^4 - (2bd + ae)x^2 + ad}\right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2), x, algorithm="fricas")

[Out] [-1/4*sqrt(-b*d^2 - a*d*e)*log(-(c^2*d^2*x^8 - 2*(4*b*c*d^2 + 3*a*c*d*e)*x^6 + (8*a*b*d*e + a^2*e^2 + 2*(4*b^2 + a*c)*d^2)*x^4 + a^2*d^2 - 2*(4*a*b*d^2 + 3*a^2*d*e)*x^2 + 4*(c*d*x^5 - (2*b*d + a*e)*x^3 + a*d*x)*sqrt(c*x^4 - b*x^2 + a)*sqrt(-b*d^2 - a*d*e))/(c^2*d^2*x^8 + 2*a*c*d*e*x^6 + 2*a^2*d*e*x^2 + (2*a*c*d^2 + a^2*e^2)*x^4 + a^2*d^2)/(b*d^2 + a*d*e), 1/2*arctan(2*sqrt(c*x^4 - b*x^2 + a)*sqrt(b*d^2 + a*d*e)*x/(c*d*x^4 - (2*b*d + a*e)*x^2 + a*d))/sqrt(b*d^2 + a*d*e)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 - bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2), x, algorithm="giac")

[Out] integrate(-(c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)), x)

maple [C] time = 0.08, size = 517, normalized size = 9.75

$$\frac{a \left(\operatorname{RootOf}(cdx^4 + aex^2 + ad)^2 x^2 - 2 \right) \sqrt{\frac{2 \operatorname{RootOf}(cdx^4 + aex^2 + ad) \operatorname{RootOf}(cdx^4 + aex^2 + ad) \sqrt{cdx^4 - bx^2 + a}}{cdx^4 + aex^2 + ad}}}{4 \sqrt{2} \operatorname{RootOf}(cdx^4 + aex^2 + ad) \operatorname{RootOf}(cdx^4 + aex^2 + ad)} + \frac{2 \sqrt{cdx^4 - bx^2 + a} \operatorname{EllipticF}\left(\frac{\sqrt{cdx^4 - bx^2 + a}}{2}, \frac{\sqrt{cdx^4 - bx^2 + a}}{2}\right)}{4 \sqrt{cdx^4 - bx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2), x)

```
[Out] -1/4/d*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(b+(-4*a*c+b^2)^(1/2))
/a*x^2)^(1/2)*(4+2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/
2)*EllipticF(1/2*x*2^(1/2)*((b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4-2*b*(-b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/4*a/d*sum((-_alpha^2*e-2*d)/_alpha/(2*_a
lpha^2*c*d+a*e)*(-1/(-_alpha^2*(a*e+b*d)/d)^(1/2)*arctanh(1/2*(2*_alpha^2*c
*x^2-_alpha^2*b-b*x^2+2*a)/(-_alpha^2*(a*e+b*d)/d)^(1/2)/(c*x^4-b*x^2+a)^(1
/2))+1/a/d*2^(1/2)*_alpha*(alpha^2*c*d+a*e)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(2-1/a*b*x^2-(-4*a*c+b^2)^(1/2)/a*x^2)^(1/2)*(2-1/a*b*x^2+(-4*a*c+b^2)^(
1/2)/a*x^2)^(1/2)/(c*x^4-b*x^2+a)^(1/2)*EllipticPi(1/2*x*2^(1/2)*((b+(-4*a*
c+b^2)^(1/2))/a)^(1/2),-1/2*(-(-4*a*c+b^2)^(1/2)*_alpha^2*c*d+_alpha^2*b*c*
d-(-4*a*c+b^2)^(1/2)*a*e+a*b*e)/a/d/c,(-1/2*(-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))/a)^(1/2)),_alpha=RootOf(_Z^4*c*d+_Z^2*a*
e+a*d))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 - bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^4+a*e*x^2+a*d)/(c*x^4-b*x^2+a)^(1/2),x, algorit
hm="maxima")
```

```
[Out] -integrate((c*x^4 - a)/((c*d*x^4 + a*e*x^2 + a*d)*sqrt(c*x^4 - b*x^2 + a)),
x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{a - cx^4}{(cdx^4 + aex^2 + ad)\sqrt{cx^4 - bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((a - c*x^4)/((a*d + a*e*x^2 + c*d*x^4)*(a - b*x^2 + c*x^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{a}{ad\sqrt{a-bx^2+cx^4} + aex^2\sqrt{a-bx^2+cx^4} + cdx^4\sqrt{a-bx^2+cx^4}} \right) dx - \int \frac{cx^4}{ad\sqrt{a-bx^2+cx^4} + aex^2\sqrt{a-bx^2+cx^4} + cdx^4\sqrt{a-bx^2+cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x**4+a)/(c*d*x**4+a*e*x**2+a*d)/(c*x**4-b*x**2+a)**(1/2),x)
```

```
[Out] -Integral(-a/(a*d*sqrt(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*
x**4) + c*d*x**4*sqrt(a - b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*sqrt
(a - b*x**2 + c*x**4) + a*e*x**2*sqrt(a - b*x**2 + c*x**4) + c*d*x**4*sqrt(
a - b*x**2 + c*x**4)), x)
```

$$3.696 \quad \int \frac{1}{\sqrt{5-2x+x^2}(8+x^3)} dx$$

Optimal. Leaf size=84

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}} + \frac{1}{12} \tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Rubi [A] time = 0.12, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2074, 724, 206, 1025, 982, 203, 1024, 207}

$$-\frac{\tan^{-1}\left(\frac{1-x}{\sqrt{3}\sqrt{x^2-2x+5}}\right)}{4\sqrt{3}} - \frac{\tanh^{-1}\left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}}\right)}{12\sqrt{13}} + \frac{1}{12} \tanh^{-1}\left(\sqrt{x^2-2x+5}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)), x]

[Out] -ArcTan[(1 - x)/(Sqrt[3]*Sqrt[5 - 2*x + x^2])]/(4*Sqrt[3]) - ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])]/(12*Sqrt[13]) + ArcTanh[Sqrt[5 - 2*x + x^2]]/12

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 207

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a, 0] || GtQ[b, 0])

Rule 724

Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] := Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]

Rule 982

Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] := Dist[-2*e, Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0]

Rule 1024

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := Dist[-2*g, Subst[Int[1/(b*d - a*e - b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && EqQ[h*e - 2*g*f, 0]
```

Rule 1025

```
Int[((g_) + (h_)*(x_))/(((a_) + (b_)*(x_) + (c_)*(x_)^2)*Sqrt[(d_) + (e_)*(x_) + (f_)*(x_)^2]), x_Symbol] := -Dist[(h*e - 2*g*f)/(2*f), Int[1/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Dist[h/(2*f), Int[(e + 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_), x_Symbol] := With[{PP = Factor[P]}, Int[ExpandIntegrand[PP^p*Q^q, x], x] /; !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] && PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\sqrt{5-2x+x^2} (8+x^3)} dx &= \int \left(\frac{1}{12(2+x)\sqrt{5-2x+x^2}} + \frac{4-x}{12(4-2x+x^2)\sqrt{5-2x+x^2}} \right) dx \\ &= \frac{1}{12} \int \frac{1}{(2+x)\sqrt{5-2x+x^2}} dx + \frac{1}{12} \int \frac{4-x}{(4-2x+x^2)\sqrt{5-2x+x^2}} dx \\ &= -\left(\frac{1}{24} \int \frac{-2+2x}{(4-2x+x^2)\sqrt{5-2x+x^2}} dx \right) - \frac{1}{6} \text{Subst} \left(\int \frac{1}{52-x^2} dx, x, \frac{14-6x}{\sqrt{5-2x+x^2}} \right) \\ &= -\frac{\tanh^{-1} \left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}} \right)}{12\sqrt{13}} - \frac{1}{6} \text{Subst} \left(\int \frac{1}{-2+2x^2} dx, x, \sqrt{5-2x+x^2} \right) + \text{Subst} \left(\int \frac{1}{52-x^2} dx, x, \frac{14-6x}{\sqrt{5-2x+x^2}} \right) \\ &= \frac{\tan^{-1} \left(\frac{-2+2x}{2\sqrt{3}\sqrt{5-2x+x^2}} \right)}{4\sqrt{3}} - \frac{\tanh^{-1} \left(\frac{7-3x}{\sqrt{13}\sqrt{5-2x+x^2}} \right)}{12\sqrt{13}} + \frac{1}{12} \tanh^{-1} \left(\sqrt{5-2x+x^2} \right) \end{aligned}$$

Mathematica [C] time = 0.33, size = 160, normalized size = 1.90

$$\frac{1}{312} \left(-2\sqrt{13} \tanh^{-1} \left(\frac{7-3x}{\sqrt{13}\sqrt{x^2-2x+5}} \right) - 13 \left((\sqrt{3}+i) \tan^{-1} \left(\frac{-2\sqrt{-1}x+4x+5i\sqrt{3}+1}{\sqrt{2-2i\sqrt{3}}\sqrt{x^2-2x+5}} \right) + (\sqrt{3}-i) \tan^{-1} \left(\frac{2(2+(-1)^{2/3})x-5i\sqrt{3}+1}{\sqrt{2+2i\sqrt{3}}\sqrt{x^2-2x+5}} \right) \right) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)), x]
```

```
[Out] (-13*((I + Sqrt[3])*ArcTan[(1 + (5*I)*Sqrt[3] + 4*x - 2*(-1)^(1/3)*x)/(Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x^2]]) + (-I + Sqrt[3])*ArcTan[(1 - (5*I)*Sqrt[3] + 2*(2 + (-1)^(2/3))*x)/(Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[5 - 2*x + x^2]]) - 2*Sqrt[13]*ArcTanh[(7 - 3*x)/(Sqrt[13]*Sqrt[5 - 2*x + x^2])])/312
```

IntegrateAlgebraic [A] time = 0.46, size = 119, normalized size = 1.42

$$-\frac{\tan^{-1} \left(\frac{x^2}{\sqrt{3}} - \frac{(x-1)\sqrt{x^2-2x+5}}{\sqrt{3}} - \frac{2x}{\sqrt{3}} + \frac{4}{\sqrt{3}} \right)}{4\sqrt{3}} + \frac{1}{12} \tanh^{-1} \left(\sqrt{x^2-2x+5} \right) + \frac{\tanh^{-1} \left(-\frac{\sqrt{x^2-2x+5}}{\sqrt{13}} + \frac{x}{\sqrt{13}} + \frac{2}{\sqrt{13}} \right)}{6\sqrt{13}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(Sqrt[5 - 2*x + x^2]*(8 + x^3)),x]

[Out] $-\frac{1}{4} \operatorname{ArcTan}\left[\frac{4}{\sqrt{3}} - \frac{2x}{\sqrt{3}} + \frac{x^2}{\sqrt{3}} - \frac{(-1+x)\sqrt{5-2x+x^2}}{\sqrt{3}}\right] / \sqrt{3} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{5-2x+x^2}}{12} + \frac{\operatorname{ArcTanh}\left[\frac{2}{\sqrt{13}} + \frac{x}{\sqrt{13}} - \frac{\sqrt{5-2x+x^2}}{\sqrt{13}}\right]}{6\sqrt{13}}\right]}{12} + \frac{\operatorname{ArcTanh}\left[\frac{2}{\sqrt{13}} + \frac{x}{\sqrt{13}} - \frac{\sqrt{5-2x+x^2}}{\sqrt{13}}\right]}{6\sqrt{13}}$

fricas [B] time = 0.98, size = 154, normalized size = 1.83

$\frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}(x-2) + \frac{1}{3} \sqrt{3} \sqrt{x^2-2x+5}\right) - \frac{1}{12} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3}x + \frac{1}{3} \sqrt{3} \sqrt{x^2-2x+5}\right) + \frac{1}{156} \sqrt{13} \log\left(\frac{-\sqrt{13}(3x-7) + \sqrt{x^2-2x+5}(3\sqrt{13}+13) + 9x-21}{x+2}\right) + \frac{1}{24} \log\left(x^2 - \sqrt{x^2-2x+5}(x-2) - 3x+6\right) - \frac{1}{24} \log\left(x^2 - \sqrt{x^2-2x+5}x - x+4\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="fricas")

[Out] $\frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3} \sqrt{3}(x-2) + \frac{1}{3} \sqrt{3} \sqrt{3} \sqrt{x^2-2x+5}\right) - \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3} \sqrt{3}x + \frac{1}{3} \sqrt{3} \sqrt{3} \sqrt{x^2-2x+5}\right) + \frac{1}{156} \sqrt{13} \log\left(-\frac{\sqrt{13}(3x-7) + \sqrt{x^2-2x+5}(3\sqrt{13}+13) + 9x-21}{x+2}\right) + \frac{1}{24} \log\left(x^2 - \sqrt{x^2-2x+5}(x-2) - 3x+6\right) - \frac{1}{24} \log\left(x^2 - \sqrt{x^2-2x+5}x - x+4\right)$

giac [B] time = 0.59, size = 164, normalized size = 1.95

$-\frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3}(x - \sqrt{x^2-2x+5})\right) + \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3}(x - \sqrt{x^2-2x+5} - 2)\right) + \frac{1}{156} \sqrt{13} \log\left(\frac{-2x-2\sqrt{13}+2\sqrt{x^2-2x+5}-4}{-2x+2\sqrt{13}+2\sqrt{x^2-2x+5}-4}\right) + \frac{1}{24} \log\left((x - \sqrt{x^2-2x+5})^2 - 4x + 4\sqrt{x^2-2x+5} + 7\right) - \frac{1}{24} \log\left((x - \sqrt{x^2-2x+5})^2 + 3\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+8)/(x^2-2*x+5)^(1/2),x, algorithm="giac")

[Out] $-\frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3} \sqrt{3}(x - \sqrt{x^2-2x+5})\right) + \frac{1}{12} \sqrt{3} \arctan\left(-\frac{1}{3} \sqrt{3} \sqrt{3}(x - \sqrt{x^2-2x+5} - 2)\right) + \frac{1}{156} \sqrt{13} \log\left(\frac{\operatorname{abs}(-2x - 2\sqrt{13} + 2\sqrt{x^2-2x+5} - 4)}{\operatorname{abs}(-2x + 2\sqrt{13} + 2\sqrt{x^2-2x+5} - 4)}\right) + \frac{1}{24} \log\left((x - \sqrt{x^2-2x+5})^2 - 4x + 4\sqrt{x^2-2x+5} + 7\right) - \frac{1}{24} \log\left((x - \sqrt{x^2-2x+5})^2 + 3\right)$

maple [A] time = 0.03, size = 69, normalized size = 0.82

$$\frac{\operatorname{arctanh}\left(\sqrt{x^2-2x+5}\right)}{12} - \frac{\sqrt{13} \operatorname{arctanh}\left(\frac{(-6x+14)\sqrt{13}}{26\sqrt{-6x+(x+2)^2+1}}\right)}{156} + \frac{\sqrt{3} \operatorname{arctan}\left(\frac{\sqrt{3}(2x-2)}{6\sqrt{x^2-2x+5}}\right)}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^3+8)/(x^2-2*x+5)^(1/2),x)

[Out] $-\frac{1}{156} 13^{1/2} \operatorname{arctanh}\left(\frac{1}{26} (14-6x) 13^{1/2} / ((x+2)^2-6x+1)^{1/2}\right) + \frac{1}{12} \operatorname{arctanh}\left(\frac{\sqrt{x^2-2x+5}}{6\sqrt{x^2-2x+5}}\right) + \frac{1}{12} 13^{1/2} \operatorname{arctan}\left(\frac{1}{6} 13^{1/2} / (x^2-2x+5)^{1/2}\right) * (2x-2)$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3+8)\sqrt{x^2-2x+5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(x^3+8)*sqrt(x^2-2*x+5),x, algorithm="maxima")

[Out] integrate(1/((x^3+8)*sqrt(x^2-2*x+5)),x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{(x^3 + 8) \sqrt{x^2 - 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)`

[Out] `int(1/((x^3 + 8)*(x^2 - 2*x + 5)^(1/2)), x)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x + 2)(x^2 - 2x + 4) \sqrt{x^2 - 2x + 5}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(x**3+8)/(x**2-2*x+5)**(1/2), x)`

[Out] `Integral(1/((x + 2)*(x**2 - 2*x + 4)*sqrt(x**2 - 2*x + 5)), x)`

$$3.697 \quad \int \sqrt{\frac{x^2}{1+x^2}} dx$$

Optimal. Leaf size=20

$$\frac{\sqrt{x^2} \sqrt{x^2 + 1}}{x}$$

Rubi [A] time = 0.00, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1958, 15, 261}

$$\frac{\sqrt{x^2} \sqrt{x^2 + 1}}{x}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x^2/(1 + x^2)],x]

[Out] (Sqrt[x^2]*Sqrt[1 + x^2])/x

Rule 15

Int[(u_.)*((a_.)*(x_)^(n_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a*x^n)^FracPart[m])/x^(n*FracPart[m]), Int[u*x^(m*n), x], x] /; FreeQ[{a, m, n}, x] && !IntegerQ[m]

Rule 261

Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] && NeQ[p, -1]

Rule 1958

Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_Symbol] :> Int[(u*(e*(a + b*x^n))^p)/(c + d*x^n)^p, x] /; FreeQ[{a, b, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - (a*d)/b, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{\frac{x^2}{1+x^2}} dx &= \int \frac{\sqrt{x^2}}{\sqrt{1+x^2}} dx \\ &= \frac{\sqrt{x^2} \int \frac{x}{\sqrt{1+x^2}} dx}{x} \\ &= \frac{\sqrt{x^2} \sqrt{1+x^2}}{x} \end{aligned}$$

Mathematica [A] time = 0.01, size = 17, normalized size = 0.85

$$\frac{x}{\sqrt{\frac{x^2}{x^2+1}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x^2/(1 + x^2)],x]

[Out] $x/\text{Sqrt}[x^2/(1 + x^2)]$

IntegrateAlgebraic [A] time = 4.61, size = 20, normalized size = 1.00

$$\frac{\sqrt{x^2} \sqrt{x^2 + 1}}{x}$$

Antiderivative was successfully verified.

[In] `IntegrateAlgebraic[Sqrt[x^2/(1 + x^2)], x]`

[Out] $(\text{Sqrt}[x^2] * \text{Sqrt}[1 + x^2])/x$

fricas [A] time = 0.99, size = 22, normalized size = 1.10

$$\frac{(x^2 + 1) \sqrt{\frac{x^2}{x^2 + 1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2/(x^2+1))^(1/2), x, algorithm="fricas")`

[Out] $(x^2 + 1) * \text{sqrt}(x^2/(x^2 + 1))/x$

giac [A] time = 0.44, size = 15, normalized size = 0.75

$$\sqrt{x^2 + 1} \text{sgn}(x) - \text{sgn}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2/(x^2+1))^(1/2), x, algorithm="giac")`

[Out] $\text{sqrt}(x^2 + 1) * \text{sgn}(x) - \text{sgn}(x)$

maple [A] time = 0.00, size = 23, normalized size = 1.15

$$\frac{(x^2 + 1) \sqrt{\frac{x^2}{x^2 + 1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2/(x^2+1))^(1/2), x)`

[Out] $(x^2 + 1)/x * (x^2/(x^2 + 1))^(1/2)$

maxima [A] time = 1.08, size = 7, normalized size = 0.35

$$\sqrt{x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2/(x^2+1))^(1/2), x, algorithm="maxima")`

[Out] $\text{sqrt}(x^2 + 1)$

mupad [B] time = 3.41, size = 13, normalized size = 0.65

$$\frac{\sqrt{x^4 + x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x^2/(x^2 + 1))^(1/2),x)`

[Out] $(x^2 + x^4)^{1/2}/x$

sympy [B] time = 0.44, size = 36, normalized size = 1.80

$$x\sqrt{x^2} \sqrt{\frac{1}{x^2+1}} + \frac{\sqrt{x^2} \sqrt{\frac{1}{x^2+1}}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x**2/(x**2+1))**(1/2),x)`

[Out] $x*\text{sqrt}(x**2)*\text{sqrt}(1/(x**2 + 1)) + \text{sqrt}(x**2)*\text{sqrt}(1/(x**2 + 1))/x$

$$3.698 \quad \int \frac{ef - ef x^2}{(ad + bdx + adx^2) \sqrt{a + bx + cx^2 + bx^3 + ax^4}} dx$$

Optimal. Leaf size=88

$$\frac{ef \tan^{-1} \left(\frac{x(4a^2 - 2ac + b^2) + abx^2 + ab}{2a\sqrt{2a-c} \sqrt{ax^4 + a + bx^3 + bx + cx^2}} \right)}{ad\sqrt{2a-c}}$$

Rubi [A] time = 0.25, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 52, $\frac{\text{number of rules}}{\text{integrand size}} = 0.019$, Rules used = {2084}

$$\frac{ef \tan^{-1} \left(\frac{x(4a^2 - 2ac + b^2) + abx^2 + ab}{2a\sqrt{2a-c} \sqrt{ax^4 + a + bx^3 + bx + cx^2}} \right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

[In] Int[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]

[Out] (e*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a - c]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]])/(a*Sqrt[2*a - c]*d)

Rule 2084

Int[((f_) + (g_)*(x_)^2)/(((d_) + (e_)*(x_) + (d_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2 + (b_)*(x_)^3 + (a_)*(x_)^4]), x_Symbol] := Simp[(a*f*ArcTan[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[a^2*(2*a - c), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]])]/(d*Rt[a^2*(2*a - c), 2]), x] / ; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] && PosQ[a^2*(2*a - c)]

Rubi steps

$$\int \frac{ef - ef x^2}{(ad + bdx + adx^2) \sqrt{a + bx + cx^2 + bx^3 + ax^4}} dx = \frac{ef \tan^{-1} \left(\frac{ab + (4a^2 + b^2 - 2ac)x + abx^2}{2a\sqrt{2a-c} \sqrt{a + bx + cx^2 + bx^3 + ax^4}} \right)}{a\sqrt{2a-c}d}$$

Mathematica [C] time = 6.52, size = 13884, normalized size = 157.77

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]

[Out] Result too large to show

IntegrateAlgebraic [A] time = 0.86, size = 83, normalized size = 0.94

$$\frac{2ef \tan^{-1} \left(\frac{\sqrt{a}x\sqrt{2a-c}}{-\sqrt{a}\sqrt{ax^4 + a + bx^3 + bx + cx^2 + ax^2 + a + bx}} \right)}{ad\sqrt{2a-c}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]),x]
```

```
[Out] (-2*e*f*ArcTan[(Sqrt[a]*Sqrt[2*a - c]*x)/(a + b*x + a*x^2 - Sqrt[a]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4])])/(a*Sqrt[2*a - c]*d)
```

fricas [A] time = 9.01, size = 324, normalized size = 3.68

$$\left[\frac{\sqrt{-2a+cef} \log\left(\frac{2ab^2x^3+2ab^3x-(8a^4-4a^2b^2-4a^2c)x^4-8a^4+a^2b^2+4a^2c+(16a^4+10a^2b^2+b^4+8a^2c^2-4(6a^3+ab^2)x)^2-4(a^2bx^2+a^2b+(4a^3+ab^2-2a^2c)x)\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{-2a+c}}{a^2x^4+2abx^3+2abx+(2a^2+b^2)x^2+a^2}\right)}{2(2a^2-ac)d}, \frac{\sqrt{2a-c} \operatorname{arctan}\left(\frac{2\sqrt{ax^4+bx^3+cx^2+bx+a}\sqrt{2a-c}}{abx^2+ab+(4a^2+b^2-2ac)x}\right)}{(2a^2-ac)d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x - (8*a^4 - a^2*b^2 - 4*a^3*c)*x^4 - 8*a^4 + a^2*b^2 + 4*a^3*c + (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2*c^2 - 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b + (4*a^3 + a*b^2 - 2*a^2*c)*x)*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(-2*a + c))/(a^2*x^4 + 2*a*b*x^3 + 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/(2*a^2 - a*c)*d, -sqrt(2*a - c)*e*f*arctan(2*sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*sqrt(2*a - c)*a/(a*b*x^2 + a*b + (4*a^2 + b^2 - 2*a*c)*x))/(2*a^2 - a*c)*d]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a} (adx^2 + bdx + ad)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-(e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2 + b*d*x + a*d)), x)
```

maple [C] time = 0.16, size = 242984, normalized size = 2761.18

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{efx^2 - ef}{\sqrt{ax^4 + bx^3 + cx^2 + bx + a} (adx^2 + bdx + ad)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e*f*x^2+e*f)/(a*d*x^2+b*d*x+a*d)/(a*x^4+b*x^3+c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate((e*f*x^2 - e*f)/(sqrt(a*x^4 + b*x^3 + c*x^2 + b*x + a)*(a*d*x^2 + b*d*x + a*d)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{ef - ef x^2}{(ad x^2 + bdx + ad) \sqrt{ax^4 + bx^3 + cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)

[Out] int((e*f - e*f*x^2)/((a*d + b*d*x + a*d*x^2)*(a + b*x + a*x^4 + b*x^3 + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ef \left(\int \frac{x^2}{ax^2 \sqrt{ax^4 + a + bx^3 + bx + cx^2} + a \sqrt{ax^4 + a + bx^3 + bx + cx^2} + bx \sqrt{ax^4 + a + bx^3 + bx + cx^2}} dx + \int \left(-\frac{1}{ax^2 \sqrt{ax^4 + a + bx^3 + bx + cx^2} + a \sqrt{ax^4 + a + bx^3 + bx + cx^2} + bx \sqrt{ax^4 + a + bx^3 + bx + cx^2}} \right) dx \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*f*x**2+e*f)/(a*d*x**2+b*d*x+a*d)/(a*x**4+b*x**3+c*x**2+b*x+a)**(1/2), x)

[Out] -e*f*(Integral(x**2/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + a*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2) + b*x*sqrt(a*x**4 + a + b*x**3 + b*x + c*x**2)), x))/d

3.699 $\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}} dx$

Optimal. Leaf size=88

$$\frac{ef \tanh^{-1}\left(\frac{-x(4a^2 + 2ac + b^2) + abx^2 + ab}{2a\sqrt{2a+c}\sqrt{-ax^4 - a + bx^3 + bx + cx^2}}\right)}{ad\sqrt{2a+c}}$$

Rubi [A] time = 0.33, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.018$, Rules used = {2085}

$$\frac{ef \tanh^{-1}\left(\frac{-x(4a^2 + 2ac + b^2) + abx^2 + ab}{2a\sqrt{2a+c}\sqrt{-ax^4 - a + bx^3 + bx + cx^2}}\right)}{ad\sqrt{2a+c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*f - e*f*x^2)/((-a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4]),x]
[Out] (e*f*ArcTanh[(a*b - (4*a^2 + b^2 + 2*a*c)*x + a*b*x^2)/(2*a*Sqrt[2*a + c]*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4]])/(a*Sqrt[2*a + c]*d)
```

Rule 2085

```
Int[((f_) + (g_)*(x_)^2)/(((d_) + (e_)*(x_) + (d_)*(x_)^2)*Sqrt[(a_) + (b_)*(x_) + (c_)*(x_)^2 + (b_)*(x_)^3 + (a_)*(x_)^4]], x_Symbol] := -Simp[(a*f*ArcTanh[(a*b + (4*a^2 + b^2 - 2*a*c)*x + a*b*x^2)/(2*Rt[-(a^2*(2*a - c)), 2]*Sqrt[a + b*x + c*x^2 + b*x^3 + a*x^4]])/(d*Rt[-(a^2*(2*a - c)), 2]), x] /; FreeQ[{a, b, c, d, e, f, g}, x] && EqQ[b*d - a*e, 0] && EqQ[f + g, 0] && NegQ[a^2*(2*a - c)]
```

Rubi steps

$$\int \frac{ef - efx^2}{(-ad + bdx - adx^2)\sqrt{-a + bx + cx^2 + bx^3 - ax^4}} dx = \frac{ef \tanh^{-1}\left(\frac{ab - (4a^2 + b^2 + 2ac)x + abx^2}{2a\sqrt{2a+c}\sqrt{-a + bx + cx^2 + bx^3 - ax^4}}\right)}{a\sqrt{2a+c}d}$$

Mathematica [C] time = 6.55, size = 15147, normalized size = 172.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*f - e*f*x^2)/((-a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4]),x]
[Out] Result too large to show
```

IntegrateAlgebraic [C] time = 15.36, size = 536, normalized size = 6.09

$$\frac{ef(\sqrt{a}\sqrt{2a+c-b})\sqrt{-2d^2-2i\sqrt{a}b\sqrt{2a+c}-ac+b^2}\tan^{-1}\left(\frac{i\sqrt{2d^2-2i\sqrt{a}b\sqrt{2a+c}-ac+b^2}}{-i\sqrt{a}\sqrt{2a+c-b}\sqrt{2a+c}+\sqrt{2}\sqrt{a}x+\sqrt{a}\sqrt{c}}\right)}{ad\sqrt{2a+c}(2d^2+ac+b^2)} - \frac{ef(\sqrt{a}\sqrt{2a+c+b})\sqrt{-2d^2+2i\sqrt{a}b\sqrt{2a+c}-ac+b^2}\tan^{-1}\left(\frac{i\sqrt{2d^2+2i\sqrt{a}b\sqrt{2a+c}-ac+b^2}}{-i\sqrt{a}\sqrt{2a+c+b}\sqrt{2a+c}+\sqrt{2}\sqrt{a}x+\sqrt{a}\sqrt{c}}\right)}{ad\sqrt{2a+c}(2d^2+ac+b^2)} - \frac{\sqrt{-a}ef\tan^{-1}\left(\frac{2\sqrt{a}b\sqrt{2a+c}}{2d^2+4a^2d^2+2d^2+(2\sqrt{a}a^2+2\sqrt{a})\sqrt{-a}\sqrt{2a+c}+b^2}\right)}{a^{3/2}d\sqrt{2a+c}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(e*f - e*f*x^2)/((-a*d) + b*d*x - a*d*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4],x]

[Out] -(((((-I)*b + Sqrt[a]*Sqrt[2*a + c])*Sqrt[-2*a^2 + b^2 - a*c - (2*I)*Sqrt[a]*b*Sqrt[2*a + c]]*e*f*ArcTan[(Sqrt[-2*a^2 + b^2 - a*c - (2*I)*Sqrt[a]*b*Sqrt[2*a + c]]*x)/(Sqrt[-a]*Sqrt[a] + Sqrt[-a]*Sqrt[a]*x^2 - Sqrt[a]*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*(2*a^2 + b^2 + a*c)*d) - ((I*b + Sqrt[a]*Sqrt[2*a + c])*Sqrt[-2*a^2 + b^2 - a*c + (2*I)*Sqrt[a]*b*Sqrt[2*a + c]]*e*f*ArcTan[(Sqrt[-2*a^2 + b^2 - a*c + (2*I)*Sqrt[a]*b*Sqrt[2*a + c]]*x)/(Sqrt[-a]*Sqrt[a] + Sqrt[-a]*Sqrt[a]*x^2 - Sqrt[a]*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a*Sqrt[2*a + c]*(2*a^2 + b^2 + a*c)*d) - (Sqrt[-a]*e*f*ArcTan[(2*Sqrt[a]*b*Sqrt[2*a + c]*x^2)/(2*a^2 - a*b*x + 4*a^2*x^2 - b^2*x^2 - a*b*x^3 + 2*a^2*x^4 + (2*Sqrt[-a]*a + 2*Sqrt[-a]*a*x^2)*Sqrt[-a + b*x + c*x^2 + b*x^3 - a*x^4])])/(a^(3/2)*Sqrt[2*a + c]*d)

fricas [A] time = 9.27, size = 331, normalized size = 3.76

$$\left| \frac{\sqrt{2a+ce} \log\left(\frac{2ab^3x^3+2ab^2x+(8a^4-a^2b^2+4a^2c)x^4+8a^4-a^2b^2+4a^2c-(16a^4+10a^2b^2+b^4+8a^2c^2+4(6a^3+ab^2)c)x^2-4(a^2bx^2+a^2b-(4a^3+ab^2+2a^2c)x)\sqrt{-ax^4+bx^3+cx^2+bx-a}\sqrt{2a+c}}{a^2x^3-2abx^2-2abx+(2a^2+b^2)x^2+a^2}\right)}{2(2a^2+ac)d} \right|, \dots \frac{\sqrt{-2a-ce} \operatorname{arctan}\left(\frac{2\sqrt{-ax^4+bx^3+cx^2+bx-a}\sqrt{2a+c}}{abx^2+ab-(4a^2+b^2+2ac)x}\right)}{(2a^2+ac)d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2*a + c)*e*f*log((2*a*b^3*x^3 + 2*a*b^3*x + (8*a^4 - a^2*b^2 + 4*a^3*c)*x^4 + 8*a^4 - a^2*b^2 + 4*a^3*c - (16*a^4 + 10*a^2*b^2 + b^4 + 8*a^2*c^2 + 4*(6*a^3 + a*b^2)*c)*x^2 - 4*(a^2*b*x^2 + a^2*b - (4*a^3 + a*b^2 + 2*a^2*c)*x)*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*sqrt(2*a + c))/(a^2*x^4 - 2*a*b*x^3 - 2*a*b*x + (2*a^2 + b^2)*x^2 + a^2))/((2*a^2 + a*c)*d), -sqrt(-2*a - c)*e*f*arctan(2*sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*a*sqrt(-2*a - c)/(a*b*x^2 + a*b - (4*a^2 + b^2 + 2*a*c)*x))/((2*a^2 + a*c)*d)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a} (adx^2 - bdx + ad)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x, algorithm="giac")

[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2 - b*d*x + a*d)), x)

maple [C] time = 0.16, size = 269221, normalized size = 3059.33

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x)

[Out] result too large to display

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{efx^2 - ef}{\sqrt{-ax^4 + bx^3 + cx^2 + bx - a} (adx^2 - bdx + ad)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*f*x^2+e*f)/(-a*d*x^2+b*d*x-a*d)/(-a*x^4+b*x^3+c*x^2+b*x-a)^(1/2),x, algorithm="maxima")

[Out] integrate((e*f*x^2 - e*f)/(sqrt(-a*x^4 + b*x^3 + c*x^2 + b*x - a)*(a*d*x^2 - b*d*x + a*d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int -\frac{ef - ef x^2}{(ad x^2 - bdx + ad) \sqrt{-ax^4 + bx^3 + cx^2 + bx - a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*x^2)^(1/2)),x)

[Out] int(-(e*f - e*f*x^2)/((a*d - b*d*x + a*d*x^2)*(b*x - a - a*x^4 + b*x^3 + c*x^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{ef \left(\int \frac{x^2}{ax^2 \sqrt{-ax^4 - a + bx^3 + bx + cx^2} + a \sqrt{-ax^4 - a + bx^3 + bx + cx^2} - bx \sqrt{-ax^4 - a + bx^3 + bx + cx^2}} dx + \int \left(-\frac{1}{ax^2 \sqrt{-ax^4 - a + bx^3 + bx + cx^2} + a \sqrt{-ax^4 - a + bx^3 + bx + cx^2} - bx \sqrt{-ax^4 - a + bx^3 + bx + cx^2}} \right) dx \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e*f*x**2+e*f)/(-a*d*x**2+b*d*x-a*d)/(-a*x**4+b*x**3+c*x**2+b*x-a)**(1/2),x)

[Out] e*f*(Integral(x**2/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2)), x) + Integral(-1/(a*x**2*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) + a*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2) - b*x*sqrt(-a*x**4 - a + b*x**3 + b*x + c*x**2)), x))/d

$$3.700 \quad \int \frac{\sqrt{ax^2+bx}\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{2} b \sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Rubi [A] time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 59, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2130, 215}

$$\frac{\sqrt{2} b \sinh^{-1}\left(\frac{b\sqrt{\frac{a^2x^2}{b^2}-\frac{a}{b^2}}+ax}{\sqrt{a}}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]), x]

[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSinh[Rt[b, 2]*x]/Sqrt[a]]/Rt[b, 2], x /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2130

Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 + x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]

Rubi steps

$$\int \frac{\sqrt{ax^2+bx}\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx = \frac{(\sqrt{2} b) \text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}} dx, x, ax + b\sqrt{-\frac{a}{b^2} + \frac{a^2x^2}{b^2}}\right)}{a}$$

$$= \frac{\sqrt{2} b \sinh^{-1}\left(\frac{ax+b\sqrt{-\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}}\right)}{\sqrt{a}}$$

Mathematica [B] time = 1.12, size = 148, normalized size = 3.22

$$\frac{\sqrt{2} x \sqrt{ax \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right) \left(bx \sqrt{\frac{a(ax^2-1)}{b^2}} + ax^2 - 1 \right) \tanh^{-1} \left(\frac{\sqrt{ax \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right)}}{\sqrt{2} ax} \right)}}{\sqrt{\frac{a(ax^2-1)}{b^2}} \left(x \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right) \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]])*(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]])/(Sqrt[2]*a*x)]/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(3/2))

IntegrateAlgebraic [A] time = 3.48, size = 88, normalized size = 1.91

$$\frac{\sqrt{2} b \log \left(b \left(-\sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} \right) + \sqrt{2} \sqrt{a} \sqrt{bx \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} + ax^2 - ax} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]),x]

[Out] -((Sqrt[2]*b*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a]*Sqrt[a*x^2 + b*x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]])/Sqrt[a])

fricas [A] time = 25.05, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2} b \log \left(-4 a x^2 - 4 b x \sqrt{\frac{a^2 x^2 - a}{b^2}} - 2 \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}} \left(\sqrt{2} \sqrt{a} x + \frac{\sqrt{2} b \sqrt{\frac{a^2 x^2 - a}{b^2}}}{\sqrt{a}} \right) + 1 \right)}{2 \sqrt{a}}, -\sqrt{2} b \sqrt{-\frac{1}{a}} \arctan \left(\frac{\sqrt{2} \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}} \sqrt{-\frac{1}{a}}}{2 x} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a/b^2)*x), x)

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/2),x)

[Out] int((a*x^2+b*x*(-a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(-a/b^2+a^2/b^2*x^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{ax^2 + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x^2+b*x*(-a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(a*x^2 + sqrt(a^2*x^2/b^2 - a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 - a/b^2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{ax^2 + bx \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}}}{x \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a/b^2)^(1/2)),x)

[Out] int((a*x^2 + b*x*((a^2*x^2)/b^2 - a/b^2)^(1/2))^(1/2)/(x*((a^2*x^2)/b^2 - a/b^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x \left(ax + b \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} \right)}}{x \sqrt{\frac{a(ax^2-1)}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*x**2+b*x*(-a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(-a/b**2+a**2*x**2/b**2)**(1/2),x)

[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2 - 1)/b**2)), x)

$$3.701 \quad \int \frac{\sqrt{-ax^2+bx}\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{2} b \sin^{-1} \left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Rubi [A] time = 0.62, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.034$, Rules used = {2130, 216}

$$\frac{\sqrt{2} b \sin^{-1} \left(\frac{ax-b\sqrt{\frac{a^2x^2}{b^2}+\frac{a}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2130

Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 + x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]

Rubi steps

$$\int \frac{\sqrt{-ax^2+bx}\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{x\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}} dx = \frac{(\sqrt{2} b) \text{Subst} \left(\int \frac{1}{\sqrt{1-\frac{x^2}{a}}} dx, x, -ax + b\sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}} \right)}{a}$$

$$= \frac{\sqrt{2} b \sin^{-1} \left(\frac{ax-b\sqrt{\frac{a}{b^2}+\frac{a^2x^2}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Mathematica [B] time = 1.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2} b^2 \sqrt{\frac{a(ax^2+1)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(ax^2+1)}{b^2}} \right)} \sqrt{x \left(b \sqrt{\frac{a(ax^2+1)}{b^2}} - ax \right)} \tanh^{-1} \left(\frac{ax \left(ax - b \sqrt{\frac{a(ax^2+1)}{b^2}} \right)}{\sqrt{2} ax} \right)}{a^2 \left(-bx^2 \sqrt{\frac{a(ax^2+1)}{b^2}} + ax^3 + x \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]), x]

[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)]/(a^2*(x + a*x^3 - b*x^2*Sqrt[(a*(1 + a*x^2))/b^2])))

IntegrateAlgebraic [B] time = 13.76, size = 135, normalized size = 2.93

$$\frac{\sqrt{2} b \sqrt{x \left(b \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} - ax \right)} \sqrt{x \left(a^2(-x) - ab \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} \right)} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{x \left(a^2(-x) - ab \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} \right)}}{\sqrt{a}} \right)}{a^{3/2} x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[-(a*x^2) + b*x*Sqrt[a/b^2 + (a^2*x^2)/b^2]]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]), x]

[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*Sqrt[x*(-(a^2*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*ArcTanh[(Sqrt[2]*Sqrt[x*(-(a^2*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])])/Sqrt[a]])/(a^(3/2)*x)

fricas [A] time = 22.68, size = 161, normalized size = 3.50

$$\left[\frac{1}{2} \sqrt{2} b \sqrt{-\frac{1}{a}} \log \left(4ax^2 - 4bx \sqrt{\frac{a^2 x^2 + a}{b^2}} + 2 \sqrt{-ax^2 + bx \sqrt{\frac{a^2 x^2 + a}{b^2}}} \left(\sqrt{2} ax \sqrt{-\frac{1}{a}} - \sqrt{2} b \sqrt{-\frac{1}{a}} \sqrt{\frac{a^2 x^2 + a}{b^2}} \right) + 1 \right), -\frac{\sqrt{2} b \arctan \left(\frac{\sqrt{2} \sqrt{-ax^2 + bx \sqrt{\frac{a^2 x^2 + a}{b^2}}}}{2 \sqrt{a} x} \right)}{\sqrt{a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2), x, algorithm="fricas")

[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1, -sqrt(2)*b*arctan(1/2*sqrt(2)*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 + a/b^2)*x), x)
```

maple [F] time = 0.09, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2),x)
```

```
[Out] int((-a*x^2+b*x*(a/b^2+a^2/b^2*x^2)^(1/2))^(1/2)/x/(a/b^2+a^2/b^2*x^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-ax^2 + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}} bx}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x^2+b*x*(a/b^2+a^2*x^2/b^2)^(1/2))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-a*x^2 + sqrt(a^2*x^2/b^2 + a/b^2)*b*x)/(sqrt(a^2*x^2/b^2 + a/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{bx \sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}} - ax^2}}{x \sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)/b^2)^(1/2)),x)
```

```
[Out] int((b*x*(a/b^2 + (a^2*x^2)/b^2)^(1/2) - a*x^2)^(1/2)/(x*(a/b^2 + (a^2*x^2)/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x \left(ax - b \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}} \right)}}{x \sqrt{\frac{a(x^2+1)}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a*x**2+b*x*(a/b**2+a**2*x**2/b**2)**(1/2))**(1/2)/x/(a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2 + 1)/b**2)), x)
```


$$3.702 \quad \int \frac{\sqrt{x \left(ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{2} b \sinh^{-1} \left(\frac{b \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} + ax}{\sqrt{a}} \right)}{\sqrt{a}}$$

Rubi [A] time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 58, $\frac{\text{number of rules}}{\text{integrand size}} = 0.052$, Rules used = {2131, 2130, 215}

$$\frac{\sqrt{2} b \sinh^{-1} \left(\frac{b \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} + ax}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*b*ArcSinh[(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]

Rule 215

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[(Rt[b, 2]*x)/Sqrt[a]]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]

Rule 2130

Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 + x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]

Rule 2131

Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c*e + a, 0]

Rubi steps

$$\int \frac{\sqrt{x \left(ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx = \int \frac{\sqrt{ax^2 + bx \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

$$= \frac{(\sqrt{2} b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 + \frac{x^2}{a}}} dx, x, ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}{\sqrt{a}}$$

$$= \frac{\sqrt{2} b \sinh^{-1} \left(\frac{ax + b \sqrt{-\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Mathematica [B] time = 0.14, size = 148, normalized size = 3.22

$$\frac{\sqrt{2} x \sqrt{ax \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right) \left(bx \sqrt{\frac{a(ax^2-1)}{b^2}} + ax^2 - 1 \right) \tanh^{-1} \left(\frac{\sqrt{ax \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right)}}{\sqrt{2} ax} \right)}}{\sqrt{\frac{a(ax^2-1)}{b^2}} \left(x \left(b \sqrt{\frac{a(ax^2-1)}{b^2}} + ax \right) \right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*x*Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]])*(-1 + a*x^2 + b*x*Sqrt[(a*(-1 + a*x^2))/b^2])*ArcTanh[Sqrt[a*x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]])]/(Sqrt[2]*a*x)]/(Sqrt[(a*(-1 + a*x^2))/b^2]*(x*(a*x + b*Sqrt[(a*(-1 + a*x^2))/b^2]))^(3/2))

IntegrateAlgebraic [A] time = 3.45, size = 87, normalized size = 1.89

$$\frac{\sqrt{2} b \log \left(b \left(-\sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} \right) + \sqrt{2} \sqrt{a} \sqrt{x \left(b \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} + ax \right) - ax} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]]]/(x*Sqrt[-(a/b^2) + (a^2*x^2)/b^2]),x]

[Out] -((Sqrt[2]*b*Log[-(a*x) - b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2] + Sqrt[2]*Sqrt[a]*Sqrt[x*(a*x + b*Sqrt[-(a/b^2) + (a^2*x^2)/b^2])])]/Sqrt[a])

fricas [A] time = 24.44, size = 161, normalized size = 3.50

$$\left[\frac{\sqrt{2} b \log \left(-4 a x^2 - 4 b x \sqrt{\frac{a^2 x^2 - a}{b^2}} - 2 \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}} \left(\sqrt{2} \sqrt{a} x + \frac{\sqrt{2} b \sqrt{\frac{a^2 x^2 - a}{b^2}}}{\sqrt{a}} \right) + 1 \right)}{2 \sqrt{a}}, -\sqrt{2} b \sqrt{-\frac{1}{a}} \arctan \left(\frac{\sqrt{2} \sqrt{a x^2 + b x \sqrt{\frac{a^2 x^2 - a}{b^2}}} \sqrt{-\frac{1}{a}}}{2 x} \right) \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="fricas")

[Out] [1/2*sqrt(2)*b*log(-4*a*x^2 - 4*b*x*sqrt((a^2*x^2 - a)/b^2) - 2*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*(sqrt(2)*sqrt(a)*x + sqrt(2)*b*sqrt((a^2*x^2 - a)/b^2)/sqrt(a)) + 1)/sqrt(a), -sqrt(2)*b*sqrt(-1/a)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + b*x*sqrt((a^2*x^2 - a)/b^2))*sqrt(-1/a)/x)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a/b^2)*x), x)

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2),x)

[Out] int((x*(a*x+b*(a^2/b^2*x^2-a/b^2)^(1/2)))^(1/2)/x/(a^2/b^2*x^2-a/b^2)^(1/2),x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(ax + \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} b\right)x}}{\sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x*(a*x+b*(-a/b^2+a^2*x^2/b^2)^(1/2)))^(1/2)/x/(-a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt((a*x + sqrt(a^2*x^2/b^2 - a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 - a/b^2)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{x \left(ax + b \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}} \right)}}{x \sqrt{\frac{a^2x^2}{b^2} - \frac{a}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a/b^2)^(1/2)),x)
```

```
[Out] int((x*(a*x + b*((a^2*x^2)/b^2 - a/b^2)^(1/2)))^(1/2)/(x*((a^2*x^2)/b^2 - a/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{x \left(ax + b \sqrt{\frac{a^2 x^2}{b^2} - \frac{a}{b^2}} \right)}}{x \sqrt{\frac{a(ax^2-1)}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(a*x+(-a/b**2+a**2*x**2/b**2)**(1/2)*b))**(1/2)/x/(-a/b**2+a**2*x**2/b**2)**(1/2),x)
```

```
[Out] Integral(sqrt(x*(a*x + b*sqrt(a**2*x**2/b**2 - a/b**2)))/(x*sqrt(a*(a*x**2 - 1)/b**2)), x)
```

$$3.703 \quad \int \frac{\sqrt{x \left(-ax + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

Optimal. Leaf size=46

$$\frac{\sqrt{2} b \sin^{-1} \left(\frac{ax - b \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Rubi [A] time = 1.17, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 57, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2131, 2130, 216}

$$\frac{\sqrt{2} b \sin^{-1} \left(\frac{ax - b \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*b*ArcSin[(a*x - b*Sqrt[a/b^2 + (a^2*x^2)/b^2])/Sqrt[a]])/Sqrt[a]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 2130

Int[Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)*Sqrt[(c_) + (d_.)*(x_)^2]]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Dist[(Sqrt[2]*b)/a, Subst[Int[1/Sqrt[1 + x^2/a], x], x, a*x + b*Sqrt[c + d*x^2]], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c + a, 0]

Rule 2131

Int[Sqrt[(e_.)*(x_)*((a_.)*(x_) + (b_.)*Sqrt[(c_) + (d_.)*(x_)^2])]/((x_)*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] := Int[Sqrt[a*e*x^2 + b*e*x*Sqrt[c + d*x^2]]/(x*Sqrt[c + d*x^2]), x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[a^2 - b^2*d, 0] && EqQ[b^2*c*e + a, 0]

Rubi steps

$$\int \frac{\sqrt{x \left(-ax + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx = \int \frac{\sqrt{-ax^2 + bx \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}}{x \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}} dx$$

$$= \frac{(\sqrt{2}b) \operatorname{Subst} \left(\int \frac{1}{\sqrt{1 - \frac{x^2}{a}}} dx, x, -ax + b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}} \right)}{a}$$

$$= \frac{\sqrt{2}b \sin^{-1} \left(\frac{ax - b \sqrt{\frac{a}{b^2} + \frac{a^2 x^2}{b^2}}}{\sqrt{a}} \right)}{\sqrt{a}}$$

Mathematica [B] time = 0.19, size = 161, normalized size = 3.50

$$\frac{\sqrt{2} b^2 \sqrt{\frac{a(ax^2+1)}{b^2}} \sqrt{ax \left(ax - b \sqrt{\frac{a(ax^2+1)}{b^2}} \right)} \sqrt{x \left(b \sqrt{\frac{a(ax^2+1)}{b^2}} - ax \right)} \tanh^{-1} \left(\frac{\sqrt{ax \left(ax - b \sqrt{\frac{a(ax^2+1)}{b^2}} \right)}}{\sqrt{2} ax} \right)}{a^2 \left(-bx^2 \sqrt{\frac{a(ax^2+1)}{b^2}} + ax^3 + x \right)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]),x]

[Out] (Sqrt[2]*b^2*Sqrt[(a*(1 + a*x^2))/b^2]*Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]*Sqrt[x*(-(a*x) + b*Sqrt[(a*(1 + a*x^2))/b^2])]*ArcTanh[Sqrt[a*x*(a*x - b*Sqrt[(a*(1 + a*x^2))/b^2])]/(Sqrt[2]*a*x)]/(a^2*(x + a*x^3 - b*x^2*Sqrt[(a*(1 + a*x^2))/b^2])))

IntegrateAlgebraic [B] time = 12.50, size = 135, normalized size = 2.93

$$\frac{\sqrt{2} b \sqrt{x \left(b \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} - ax \right)} \sqrt{x \left(a^2(-x) - ab \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} \right)} \tanh^{-1} \left(\frac{\sqrt{2} \sqrt{x \left(a^2(-x) - ab \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} \right)}}{\sqrt{a}} \right)}{a^{3/2} x}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]/(x*Sqrt[a/b^2 + (a^2*x^2)/b^2]),x]

[Out] -((Sqrt[2]*b*Sqrt[x*(-(a*x) + b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*Sqrt[x*(-(a^2*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])]*ArcTanh[(Sqrt[2]*Sqrt[x*(-(a^2*x) - a*b*Sqrt[a/b^2 + (a^2*x^2)/b^2])])/Sqrt[a]]/(a^(3/2)*x))

fricas [A] time = 22.63, size = 161, normalized size = 3.50

$$\left[\frac{1}{2} \sqrt{2} b \sqrt{-\frac{1}{a}} \log \left(4ax^2 - 4bx \sqrt{\frac{a^2 x^2 + a}{b^2}} + 2 \sqrt{-ax^2 + bx \sqrt{\frac{a^2 x^2 + a}{b^2}}} \left(\sqrt{2} ax \sqrt{-\frac{1}{a}} - \sqrt{2} b \sqrt{-\frac{1}{a}} \sqrt{\frac{a^2 x^2 + a}{b^2}} \right) + 1 \right), -\frac{\sqrt{2} b \arctan \left(\frac{\sqrt{2} \sqrt{-ax^2 + bx \sqrt{\frac{a^2 x^2 + a}{b^2}}}}{2 \sqrt{ax}} \right)}{\sqrt{a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(2)*b*sqrt(-1/a)*log(4*a*x^2 - 4*b*x*sqrt((a^2*x^2 + a)/b^2) + 2*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))*(sqrt(2)*a*x*sqrt(-1/a) - sqrt(2)*b*sqrt(-1/a)*sqrt((a^2*x^2 + a)/b^2)) + 1, -sqrt(2)*b*arctan(1/2*sqrt(2)*sqrt(-a*x^2 + b*x*sqrt((a^2*x^2 + a)/b^2))/(sqrt(a)*x))/sqrt(a)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}b}\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 + a/b^2)*x), x)
```

maple [F] time = 0.07, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{\left(-ax + \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}b}\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2),x)
```

```
[Out] int((x*((a^2/b^2*x^2+a/b^2)^(1/2)*b-a*x))^(1/2)/x/(a^2/b^2*x^2+a/b^2)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(ax - \sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}b}\right)x}}{\sqrt{\frac{a^2x^2}{b^2} + \frac{a}{b^2}x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*(b*(a/b^2+a^2*x^2/b^2)^(1/2)-a*x))^(1/2)/x/(a/b^2+a^2*x^2/b^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-(a*x - sqrt(a^2*x^2/b^2 + a/b^2)*b)*x)/(sqrt(a^2*x^2/b^2 + a/b^2)*x), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{-x \left(ax - b \sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}} \right)}}{x \sqrt{\frac{a}{b^2} + \frac{a^2x^2}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2))^(1/2))/(x*(a/b^2 + (a^2*x^2)/b^2)^(1/2)), x)
```

```
[Out] int((-x*(a*x - b*(a/b^2 + (a^2*x^2)/b^2))^(1/2))/(x*(a/b^2 + (a^2*x^2)/b^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-x \left(ax - b \sqrt{\frac{a^2 x^2}{b^2} + \frac{a}{b^2}} \right)}}{x \sqrt{\frac{a(ax^2+1)}{b^2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x*((a/b**2+a**2*x**2/b**2)**(1/2)*b-a*x))**(1/2)/x/(a/b**2+a**2*x**2/b**2)**(1/2), x)
```

```
[Out] Integral(sqrt(-x*(a*x - b*sqrt(a**2*x**2/b**2 + a/b**2)))/(x*sqrt(a*(a*x**2 + 1)/b**2)), x)
```


$$3.704 \quad \int \frac{-\sqrt{-4+x} - 4\sqrt{-1+x} + \sqrt{-4+x}x + \sqrt{-1+x}x}{(1 + \sqrt{-4+x} + \sqrt{-1+x})(4 - 5x + x^2)} dx$$

Optimal. Leaf size=19

$$2 \log\left(\sqrt{x-4} + \sqrt{x-1} + 1\right)$$

Rubi [A] time = 0.56, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 66, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {6688, 1586, 6684}

$$2 \log\left(\sqrt{x-4} + \sqrt{x-1} + 1\right)$$

Antiderivative was successfully verified.

[In] Int[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]

[Out] 2*Log[1 + Sqrt[-4 + x] + Sqrt[-1 + x]]

Rule 1586

Int[(u_)*(Px_)^(p_)*(Qx_)^(q_), x_Symbol] :> Int[u*PolynomialQuotient[Px, Qx, x]^p*Qx^(p + q), x] /; FreeQ[q, x] && PolyQ[Px, x] && PolyQ[Qx, x] && EqQ[PolynomialRemainder[Px, Qx, x], 0] && IntegerQ[p] && LtQ[p*q, 0]

Rule 6684

Int[(u_)/(y_), x_Symbol] :> With[{q = DerivativeDivides[y, u, x]}, Simp[q*Log[RemoveContent[y, x]], x] /; !FalseQ[q]]

Rule 6688

Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; SimplifyIntegrandQ[v, u, x]]

Rubi steps

$$\begin{aligned} \int \frac{-\sqrt{-4+x} - 4\sqrt{-1+x} + \sqrt{-4+x}x + \sqrt{-1+x}x}{(1 + \sqrt{-4+x} + \sqrt{-1+x})(4 - 5x + x^2)} dx &= \int \frac{\sqrt{-1+x}(-4 + \sqrt{-4+x}\sqrt{-1+x} + x)}{(1 + \sqrt{-4+x} + \sqrt{-1+x})(4 - 5x + x^2)} dx \\ &= \int \frac{-4 + \sqrt{-4+x}\sqrt{-1+x} + x}{(1 + \sqrt{-4+x} + \sqrt{-1+x})(-4+x)\sqrt{-1+x}} dx \\ &= 2 \log\left(1 + \sqrt{-4+x} + \sqrt{-1+x}\right) \end{aligned}$$

Mathematica [B] time = 1.38, size = 75, normalized size = 3.95

$$\frac{1}{2} \log\left(-5x - 4\sqrt{x-4}\sqrt{x-1} + 17\right) + \frac{1}{2} \log\left(-2x - 2\sqrt{x-4}\sqrt{x-1} + 5\right) - \tanh^{-1}\left(\sqrt{x-4}\right) + \tanh^{-1}\left(\frac{\sqrt{x-1}}{2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(-Sqrt[-4 + x] - 4*Sqrt[-1 + x] + Sqrt[-4 + x]*x + Sqrt[-1 + x]*x)/((1 + Sqrt[-4 + x] + Sqrt[-1 + x])*(4 - 5*x + x^2)),x]

[Out] $-\text{ArcTanh}[\text{Sqrt}[-4 + x]] + \text{ArcTanh}[\text{Sqrt}[-1 + x]/2] + \text{Log}[17 - 4\text{Sqrt}[-4 + x] * \text{Sqrt}[-1 + x] - 5*x]/2 + \text{Log}[5 - 2\text{Sqrt}[-4 + x]*\text{Sqrt}[-1 + x] - 2*x]/2$

IntegrateAlgebraic [A] time = 0.43, size = 27, normalized size = 1.42

$$4 \tanh^{-1} \left(-\frac{2\sqrt{x-4}}{3} + \frac{2\sqrt{x-1}}{3} + 1 \right)$$

Antiderivative was successfully verified.

[In] $\text{IntegrateAlgebraic}[(-\text{Sqrt}[-4 + x] - 4*\text{Sqrt}[-1 + x] + \text{Sqrt}[-4 + x]*x + \text{Sqrt}[-1 + x]*x)/((1 + \text{Sqrt}[-4 + x] + \text{Sqrt}[-1 + x])*(4 - 5*x + x^2)), x]$

[Out] $4*\text{ArcTanh}[1 - (2*\text{Sqrt}[-4 + x])/3] + (2*\text{Sqrt}[-1 + x])/3]$

fricas [B] time = 0.88, size = 96, normalized size = 5.05

$-\frac{1}{2} \log(-(4x-11)\sqrt{x-1}\sqrt{x-4}+4x^2-21x+23) + \frac{1}{2} \log(\sqrt{x-1}\sqrt{x-4}-x+7) + \frac{1}{2} \log(x-5) + \frac{1}{2} \log(\sqrt{x-1}+2) - \frac{1}{2} \log(\sqrt{x-1}-2) - \frac{1}{2} \log(\sqrt{x-4}+1) + \frac{1}{2} \log(\sqrt{x-4}-1)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-(-4+x)^{(1/2)}+x*(-4+x)^{(1/2)}-4*(-1+x)^{(1/2)}+x*(-1+x)^{(1/2)})/(x^2-5*x+4)/(1+(-4+x)^{(1/2)}+(-1+x)^{(1/2)}), x, \text{algorithm}="fricas")$

[Out] $-1/2*\log(-(4*x - 11)*\text{sqrt}(x - 1)*\text{sqrt}(x - 4) + 4*x^2 - 21*x + 23) + 1/2*\log(\text{sqrt}(x - 1)*\text{sqrt}(x - 4) - x + 7) + 1/2*\log(x - 5) + 1/2*\log(\text{sqrt}(x - 1) + 2) - 1/2*\log(\text{sqrt}(x - 1) - 2) - 1/2*\log(\text{sqrt}(x - 4) + 1) + 1/2*\log(\text{sqrt}(x - 4) - 1)$

giac [B] time = 0.62, size = 58, normalized size = 3.05

$-\log(\sqrt{x-1} - \sqrt{x-4} + 1) - \log(\sqrt{x-1} - \sqrt{x-4}) + \log(\sqrt{x-1} + 2) + \log(|-\sqrt{x-1} + \sqrt{x-4} - 3|)$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((-(-4+x)^{(1/2)}+x*(-4+x)^{(1/2)}-4*(-1+x)^{(1/2)}+x*(-1+x)^{(1/2)})/(x^2-5*x+4)/(1+(-4+x)^{(1/2)}+(-1+x)^{(1/2)}), x, \text{algorithm}="giac")$

[Out] $-\log(\text{sqrt}(x - 1) - \text{sqrt}(x - 4) + 1) - \log(\text{sqrt}(x - 1) - \text{sqrt}(x - 4)) + \log(\text{sqrt}(x - 1) + 2) + \log(\text{abs}(-\text{sqrt}(x - 1) + \text{sqrt}(x - 4) - 3))$

maple [B] time = 0.06, size = 147, normalized size = 7.74

$\frac{7\sqrt{x-4}\sqrt{x-1}\text{arctanh}\left(\frac{5x-17}{4\sqrt{x^2-5x+4}}\right) + \frac{\ln(x-5)}{2} - \frac{\ln(-2+\sqrt{x-1})}{2} + \frac{\ln(-1+\sqrt{x-4})}{2} - \frac{\ln(1+\sqrt{x-4})}{2} + \frac{\ln(\sqrt{x-1}+2)}{2} + \frac{\sqrt{x-4}\sqrt{x-1}\left(-5\text{arctanh}\left(\frac{5x-17}{4\sqrt{x^2-5x+4}}\right) + 2\ln\left(x-\frac{5}{2}+\sqrt{x^2-5x+4}\right)\right)}{4\sqrt{x^2-5x+4}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((- (x-4)^{(1/2)}+x*(x-4)^{(1/2)}-4*(x-1)^{(1/2)}+x*(x-1)^{(1/2)})/(x^2-5*x+4)/(1+(x-4)^{(1/2)}+(x-1)^{(1/2)}), x)$

[Out] $1/2*\ln(x-5)+1/2*\ln(-1+(x-4)^{(1/2)})-1/2*\ln(1+(x-4)^{(1/2)})-1/2*\ln(-2+(x-1)^{(1/2)})+1/2*\ln((x-1)^{(1/2)}+2)+7/4*(x-4)^{(1/2)}*(x-1)^{(1/2)}/(x^2-5*x+4)^{(1/2)}*\text{arctanh}(1/4*(-17+5*x)/(x^2-5*x+4)^{(1/2)})+1/4*(x-4)^{(1/2)}*(x-1)^{(1/2)}*(2*\ln(-5/2+x+(x^2-5*x+4)^{(1/2)})-5*\text{arctanh}(1/4*(-17+5*x)/(x^2-5*x+4)^{(1/2)}))/((x^2-5*x+4)^{(1/2)})$

maxima [B] time = 0.65, size = 94, normalized size = 4.95

$\frac{1}{2} \log(x-1) + \frac{1}{2} \log\left(\frac{2x^2+2((x-1)\sqrt{x-4}+2x-6)\sqrt{x-1}+2(2x-3)\sqrt{x-4}-7x+3}{2((x-1)\sqrt{x-4}+2x-6)}\right) + \frac{1}{2} \log\left(\frac{(x-1)\sqrt{x-4}+2x-6}{x-1}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-4+x)^(1/2)+x*(-4+x)^(1/2)-4*(-1+x)^(1/2)+x*(-1+x)^(1/2))/(x^2-5*x+4)/(1+(-4+x)^(1/2)+(-1+x)^(1/2)),x, algorithm="maxima")

[Out] 1/2*log(x - 1) + 1/2*log(1/2*(2*x^2 + 2*(x - 1)*sqrt(x - 4) + 2*x - 6)*sqrt(x - 1) + 2*(2*x - 3)*sqrt(x - 4) - 7*x + 3)/((x - 1)*sqrt(x - 4) + 2*x - 6) + 1/2*log(((x - 1)*sqrt(x - 4) + 2*x - 6)/(x - 1))

mupad [B] time = 6.10, size = 132, normalized size = 6.95

$$\frac{\ln(x-5)}{2} + 2 \operatorname{atanh}\left(\frac{\sqrt{x-1}-\sqrt{3}}{\sqrt{x-4}}\right) + \frac{7 \operatorname{atanh}\left(\frac{4(\sqrt{x-1}-\sqrt{3})}{\left(\frac{(\sqrt{x-1}-\sqrt{3})^2}{x-4}+1\right)\sqrt{x-4}}\right)}{2} - \frac{5 \operatorname{atanh}\left(\frac{194400(\sqrt{x-1}-\sqrt{3})}{\left(\frac{48600(\sqrt{x-1}-\sqrt{3})^2}{x-4}+48600\right)\sqrt{x-4}}\right)}{2} - \operatorname{atanh}(\sqrt{x-4}) + \operatorname{atanh}\left(\frac{\sqrt{x-1}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x*(x - 1)^(1/2) + x*(x - 4)^(1/2) - 4*(x - 1)^(1/2) - (x - 4)^(1/2))/(x^2 - 5*x + 4)*((x - 1)^(1/2) + (x - 4)^(1/2) + 1)),x)

[Out] log(x - 5)/2 + 2*atanh(((x - 1)^(1/2) - 3^(1/2))/(x - 4)^(1/2)) + (7*atanh(4*((x - 1)^(1/2) - 3^(1/2)))/(((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 1)*(x - 4)^(1/2)))/2 - (5*atanh((194400*((x - 1)^(1/2) - 3^(1/2)))/((48600*((x - 1)^(1/2) - 3^(1/2))^2/(x - 4) + 48600)*(x - 4)^(1/2))))/2 - atanh((x - 4)^(1/2)) + atanh((x - 1)^(1/2)/2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-(-4+x)**(1/2)+x*(-4+x)**(1/2)-4*(-1+x)**(1/2)+x*(-1+x)**(1/2))/(x**2-5*x+4)/(1+(-4+x)**(1/2)+(-1+x)**(1/2)),x)

[Out] Timed out

$$3.705 \quad \int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}} dx$$

Optimal. Leaf size=90

$$-\frac{\log(1-(x+1)^3)}{6\sqrt[3]{3}} + \frac{\log(\sqrt[3]{3}(x+1) - \sqrt{(x+1)^3+2})}{2\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{\frac{2\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}+1}{\sqrt{3}}\right)}{3^{5/6}}$$

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.37, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {433, 431, 377, 200, 31, 634, 617, 204, 628}

$$\frac{\log\left(1 - \frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}} + \frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}} + 1\right)}{6\sqrt[3]{3}} - \frac{\tan^{-1}\left(\frac{2(x+1)}{\sqrt[3]{3}\sqrt[3]{(x+1)^3+2}} + \frac{1}{\sqrt{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

[In] Int[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]

[Out] -(ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))]/3^(5/6)) + Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(3*3^(1/3)) - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]/(6*3^(1/3))

Rule 31

Int[((a_) + (b_.)*(x_))^(n_), x_Symbol] := Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]

Rule 200

Int[((a_) + (b_.)*(x_)^3)^(n_), x_Symbol] := Dist[1/(3*Rt[a, 3]^2), Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; FreeQ[{a, b}, x]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(n_), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 431

Int[((a_.) + (b_.)*(u_)^(n_))^(p_.)*((c_.) + (d_.)*(u_)^(n_))^(q_.), x_Symbol] := Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*x^n)^p*(c + d*x^n)^q, x], x, u], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[u, x] && NeQ[u, x]

Rule 433

```
Int[(u_)^(p_)*(v_)^(q_)*(x_)^(m_), x_Symbol] := Int[NormalizePseudoBinomial[x^(m/p)*u, x]^p*NormalizePseudoBinomial[v, x]^q, x] /; FreeQ[{p, q}, x]
&& IntegersQ[p, m/p] && PseudoBinomialPairQ[x^(m/p)*u, v, x]
```

Rule 617

```
Int[((a_) + (b_)*(x_) + (c_)*(x_)^2)^(-1), x_Symbol] := With[{q = 1 - 4*Simplify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b], x] /; RationalQ[q] && (EqQ[q^2, 1] || !RationalQ[b^2 - 4*a*c]) /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Simplify[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_) + (e_)*(x_))/((a_) + (b_)*(x_) + (c_)*(x_)^2), x_Symbol] := Dist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x(3+3x+x^2)\sqrt[3]{3+3x+3x^2+x^3}} dx &= \int \frac{1}{(-1+(1+x)^3)\sqrt[3]{2+(1+x)^3}} dx \\
&= \text{Subst}\left(\int \frac{1}{(-1+x^3)\sqrt[3]{2+x^3}} dx, x, 1+x\right) \\
&= \text{Subst}\left(\int \frac{1}{-1+3x^3} dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \\
&= \frac{1}{3} \text{Subst}\left(\int \frac{1}{-1+\sqrt[3]{3}x} dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) + \frac{1}{3} \text{Subst}\left(\int \frac{1}{1+x} dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \\
&= \frac{\log\left(1 - \frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{1}{2} \text{Subst}\left(\int \frac{1}{1+\sqrt[3]{3}x+3^{2/3}x^2} dx, x, \frac{1+x}{\sqrt[3]{2+(1+x)^3}}\right) \\
&= \frac{\log\left(1 - \frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1 + \frac{3^{2/3}(1+x)^2}{(2+(1+x)^3)^{2/3}} + \frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{6\sqrt[3]{3}} + \frac{\log\left(1 + \frac{3^{2/3}(1+x)}{(2+(1+x)^3)^{2/3}}\right)}{6\sqrt[3]{3}} \\
&= -\frac{\tan^{-1}\left(\frac{1 + \frac{2\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}}{\sqrt{3}}\right)}{3^{5/6}} + \frac{\log\left(1 - \frac{\sqrt[3]{3}(1+x)}{\sqrt[3]{2+(1+x)^3}}\right)}{3\sqrt[3]{3}} - \frac{\log\left(1 + \frac{3^{2/3}(1+x)}{(2+(1+x)^3)^{2/3}}\right)}{6\sqrt[3]{3}}
\end{aligned}$$

Mathematica [A] time = 0.16, size = 120, normalized size = 1.33

$$\frac{\sqrt{3} \left(2 \log\left(1 - \frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}}\right) - \log\left(\frac{3^{2/3}(x+1)^2}{((x+1)^3+2)^{2/3}} + \frac{\sqrt[3]{3}(x+1)}{\sqrt[3]{(x+1)^3+2}} + 1\right) \right) - 6 \tan^{-1}\left(\frac{2(x+1)}{\sqrt[3]{3} \sqrt[3]{(x+1)^3+2}} + \frac{1}{\sqrt{3}}\right)}{6 \cdot 3^{5/6}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]

[Out] (-6*ArcTan[1/Sqrt[3] + (2*(1 + x))/(3^(1/6)*(2 + (1 + x)^3)^(1/3))] + Sqrt[3]*(2*Log[1 - (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)] - Log[1 + (3^(2/3)*(1 + x)^2)/(2 + (1 + x)^3)^(2/3) + (3^(1/3)*(1 + x))/(2 + (1 + x)^3)^(1/3)]])/(6*3^(5/6))

IntegrateAlgebraic [B] time = 0.45, size = 189, normalized size = 2.10

$$\frac{\log\left(-\sqrt[3]{x^3+3x^2+3x+3}+\sqrt[3]{3x+3}\right)}{3\sqrt[3]{3}}-\frac{\log\left(3^{2/3}x^2+(x^3+3x^2+3x+3)^{2/3}+(\sqrt[3]{3}x+\sqrt[3]{3})\sqrt[3]{x^3+3x^2+3x+3}+2\cdot 3^{2/3}x+3^{2/3}\right)}{6\sqrt[3]{3}}+\frac{\tan^{-1}\left(\frac{\sqrt{3}\sqrt[3]{x^3+3x^2+3x+3}}{\sqrt[3]{x^3+3x^2+3x+3}+2\sqrt[3]{3}x+2\sqrt[3]{3}}\right)}{3^{5/6}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[1/(x*(3 + 3*x + x^2)*(3 + 3*x + 3*x^2 + x^3)^(1/3)),x]

[Out] ArcTan[(Sqrt[3]*(3 + 3*x + 3*x^2 + x^3)^(1/3))/(2*3^(1/3) + 2*3^(1/3)*x + (3 + 3*x + 3*x^2 + x^3)^(1/3))]/3^(5/6) + Log[3^(1/3) + 3^(1/3)*x - (3 + 3*x + 3*x^2 + x^3)^(1/3)]/(3*3^(1/3)) - Log[3^(2/3) + 2*3^(2/3)*x + 3^(2/3)*x^2 + (3^(1/3) + 3^(1/3)*x)*(3 + 3*x + 3*x^2 + x^3)^(1/3) + (3 + 3*x + 3*x^2 + x^3)^(2/3)]/(6*3^(1/3))

fricas [B] time = 20.99, size = 458, normalized size = 5.09

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="fricas")

[Out] -1/54*3^(2/3)*log((3*3^(2/3)*(7*x^4 + 28*x^3 + 42*x^2 + 30*x + 9)*(x^3 + 3*x^2 + 3*x + 3)^(2/3) + 3^(1/3)*(31*x^6 + 186*x^5 + 465*x^4 + 666*x^3 + 603*x^2 + 324*x + 81) + 9*(5*x^5 + 25*x^4 + 50*x^3 + 54*x^2 + 33*x + 9)*(x^3 + 3*x^2 + 3*x + 3)^(1/3))/(x^6 + 6*x^5 + 15*x^4 + 18*x^3 + 9*x^2)) + 1/27*3^(2/3)*log((2*3^(2/3)*(x^3 + 3*x^2 + 3*x) - 9*3^(1/3)*(x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 2*x + 1) + 9*(x^3 + 3*x^2 + 3*x + 3)^(2/3)*(x + 1))/(x^3 + 3*x^2 + 3*x)) - 1/9*3^(1/6)*arctan(1/3*3^(1/6)*(12*3^(2/3)*(7*x^7 + 49*x^6 + 147*x^5 + 240*x^4 + 225*x^3 + 117*x^2 + 27*x)*(x^3 + 3*x^2 + 3*x + 3)^(2/3) - 3^(1/3)*(127*x^9 + 1143*x^8 + 4572*x^7 + 11070*x^6 + 18414*x^5 + 22032*x^4 + 18900*x^3 + 11178*x^2 + 4131*x + 729) - 18*(31*x^8 + 248*x^7 + 868*x^6 + 1782*x^5 + 2400*x^4 + 2196*x^3 + 1332*x^2 + 486*x + 81)*(x^3 + 3*x^2 + 3*x + 3)^(1/3))/(251*x^9 + 2259*x^8 + 9036*x^7 + 21546*x^6 + 34398*x^5 + 38556*x^4 + 30348*x^3 + 16038*x^2 + 5103*x + 729))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 3x^2 + 3x + 3)^{\frac{1}{3}}(x^2 + 3x + 3)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="giac")

[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)

maple [C] time = 13.67, size = 2515, normalized size = 27.94

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

2002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)*x+5196569877*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x^3+374182374*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x^3-2269837425*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)-6809512275*(x^3+3*x^2+3*x+3)^(2/3)*x-6809512275*(x^3+3*x^2+3*x+3)^(2/3)-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x^2-91932017904*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)*x-15322002984*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(2/3)-2269837425*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x^2-4539674850*RootOf(_Z^3-9)^2*(x^3+3*x^2+3*x+3)^(1/3)*x-45966008952*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)*(x^3+3*x^2+3*x+3)^(1/3)+1122547122*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3*x+15589709631*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2*x-36375989139*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)^2*RootOf(_Z^3-9)^2-2619276618*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)*RootOf(_Z^3-9)^3)/x/(x^2+3*x+3)*RootOf(RootOf(_Z^3-9)^2+9*_Z*RootOf(_Z^3-9)+81*_Z^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(x^3 + 3x^2 + 3x + 3)^{\frac{1}{3}}(x^2 + 3x + 3)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x^2+3*x+3)/(x^3+3*x^2+3*x+3)^(1/3),x, algorithm="maxima")

[Out] integrate(1/((x^3 + 3*x^2 + 3*x + 3)^(1/3)*(x^2 + 3*x + 3)*x), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x(x^2 + 3x + 3)(x^3 + 3x^2 + 3x + 3)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)),x)

[Out] int(1/(x*(3*x + x^2 + 3)*(3*x + 3*x^2 + x^3 + 3)^(1/3)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x(x^2 + 3x + 3)\sqrt[3]{x^3 + 3x^2 + 3x + 3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(x**2+3*x+3)/(x**3+3*x**2+3*x+3)**(1/3),x)

[Out] Integral(1/(x*(x**2 + 3*x + 3)*(x**3 + 3*x**2 + 3*x + 3)**(1/3)), x)

$$3.706 \quad \int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx$$

Optimal. Leaf size=103

$$-\frac{\log(-x^3 + 2(1-x)^3 + 1)}{2 \cdot 2^{2/3}} + \frac{3 \log(\sqrt[3]{1-x^3} + \sqrt[3]{2}(1-x))}{2 \cdot 2^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{1 - 2\sqrt[3]{2}(1-x)}{\sqrt[3]{1-x^3}}\right)}{2^{2/3}}$$

Rubi [F] time = 0.53, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Int[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]

[Out] -(x*Hypergeometric2F1[1/3, 2/3, 4/3, x^3]) - (1 + I*Sqrt[3])*Defer[Int][1/((-1 - I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x] - (1 - I*Sqrt[3])*Defer[Int][1/((-1 + I*Sqrt[3] + 2*x)*(1 - x^3)^(2/3)), x]

Rubi steps

$$\begin{aligned} \int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx &= \int \left(-\frac{1}{(1-x^3)^{2/3}} + \frac{2-x}{(1-x+x^2)(1-x^3)^{2/3}} \right) dx \\ &= -\int \frac{1}{(1-x^3)^{2/3}} dx + \int \frac{2-x}{(1-x+x^2)(1-x^3)^{2/3}} dx \\ &= -x {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) + \int \left(\frac{-1-i\sqrt{3}}{(-1-i\sqrt{3}+2x)(1-x^3)^{2/3}} + \frac{-1+i\sqrt{3}}{(-1+i\sqrt{3}+2x)(1-x^3)^{2/3}} \right) dx \\ &= -x {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; x^3\right) + (-1-i\sqrt{3}) \int \frac{1}{(-1-i\sqrt{3}+2x)(1-x^3)^{2/3}} dx + (-1+i\sqrt{3}) \int \frac{1}{(-1+i\sqrt{3}+2x)(1-x^3)^{2/3}} dx \end{aligned}$$

Mathematica [F] time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{1-x^2}{(1-x+x^2)(1-x^3)^{2/3}} dx$$

Verification is not applicable to the result.

[In] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]

[Out] Integrate[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)), x]

IntegrateAlgebraic [B] time = 1.29, size = 214, normalized size = 2.08

$$\frac{\log\left(2(1-x^3)^{2/3} + 2^{2/3}x^2 + 2^{2/3}x + 2^{2/3}\right)}{2^{2/3}} + \frac{\sqrt{3} \tan^{-1}\left(\frac{\sqrt{3}(1-x^3)^{2/3}}{(1-x^3)^{2/3} - 2^{2/3}x^2 - 2^{2/3}x - 2^{2/3}}\right)}{2^{2/3}} - \frac{\log\left(-\sqrt[3]{2}x^4 - 2\sqrt[3]{2}x^3 - 2(1-x^3)^{4/3} - 3\sqrt[3]{2}x^2 + (2^{2/3}x^2 + 2^{2/3}x + 2^{2/3})(1-x^3)^{2/3} - 2\sqrt[3]{2}x - \sqrt[3]{2}\right)}{2 \cdot 2^{2/3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 - x^2)/((1 - x + x^2)*(1 - x^3)^(2/3)),x]

[Out] (Sqrt[3]*ArcTan[(Sqrt[3]*(1 - x^3)^(2/3))/(-2^(2/3) - 2^(2/3)*x - 2^(2/3)*x^2 + (1 - x^3)^(2/3))]/2^(2/3) + Log[2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2 + 2*(1 - x^3)^(2/3)]/2^(2/3) - Log[-2^(1/3) - 2*2^(1/3)*x - 3*2^(1/3)*x^2 - 2*2^(1/3)*x^3 - 2^(1/3)*x^4 + (2^(2/3) + 2^(2/3)*x + 2^(2/3)*x^2)*(1 - x^3)^(2/3) - 2*(1 - x^3)^(4/3)]/(2*2^(2/3))

fricas [B] time = 11.27, size = 289, normalized size = 2.81

$$\frac{1}{6} 4^{\frac{1}{2}} \sqrt{5} \arctan\left(\frac{4^{\frac{1}{2}} \sqrt{5} (2 - 4^{\frac{1}{2}} (x^5 - 3x^2 + 3x - 1)(-x^2 + 1)^{\frac{1}{2}} + 4^{\frac{1}{2}} (x^5 - 4x^2 + 5x^2 - 4x + 1)(-x^2 + 1)^{\frac{1}{2}} + 4^{\frac{1}{2}} (x^5 - 7x^2 + 10x^2 - 7x^2 - 7x + 1))}{6(3x^5 - 9x^5 + 6x^5 - x^5 + 6x^2 - 9x + 3)}}\right) - \frac{1}{24} 4^{\frac{1}{2}} \log\left(\frac{2 \cdot 4^{\frac{1}{2}} (-x^2 + 1)^{\frac{1}{2}} (x^2 - 3x + 1) - 4^{\frac{1}{2}} (x^5 - 3x^2 + 1) - 8(-x^2 + 1)^{\frac{1}{2}} (x^2 - 1)}{x^4 - 2x^3 + 3x^2 - 2x + 1}}\right) + \frac{1}{12} 4^{\frac{1}{2}} \log\left(\frac{4^{\frac{1}{2}} (-x^2 + 1)^{\frac{1}{2}} (x - 1) - 4^{\frac{1}{2}} (x^2 - x + 1) - 2(-x^2 + 1)^{\frac{1}{2}}}{x^2 - x + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="fricas")

[Out] -1/6*4^(1/6)*sqrt(3)*arctan(1/6*4^(1/6)*sqrt(3)*(2*4^(2/3)*(x^5 - x^4 - 3*x^3 + 3*x^2 + x - 1)*(-x^3 + 1)^(1/3) + 4*(x^4 - 4*x^3 + 5*x^2 - 4*x + 1)*(-x^3 + 1)^(2/3) + 4^(1/3)*(x^6 - 7*x^5 + 10*x^4 - 7*x^3 + 10*x^2 - 7*x + 1))/(3*x^6 - 9*x^5 + 6*x^4 - x^3 + 6*x^2 - 9*x + 3)) - 1/24*4^(2/3)*log((2*4^(1/3)*(-x^3 + 1)^(2/3)*(x^2 - 3*x + 1) - 4^(2/3)*(x^4 - 3*x^2 + 1) - 8*(-x^3 + 1)^(1/3)*(x^2 - x))/(x^4 - 2*x^3 + 3*x^2 - 2*x + 1)) + 1/12*4^(2/3)*log(-(4^(2/3)*(-x^3 + 1)^(1/3)*(x - 1) - 4^(1/3)*(x^2 - x + 1) - 2*(-x^3 + 1)^(2/3))/(x^2 - x + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{x^2 - 1}{(-x^3 + 1)^{\frac{2}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="giac")

[Out] integrate(-(x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)

maple [C] time = 8.03, size = 1026, normalized size = 9.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x)

[Out] 1/2*RootOf(_Z^3-2)*ln(-(2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2+RootOf(_Z^3-2)^2*x^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2-RootOf(_Z^3-2)^2*x-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x-4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(-x^3+1)^(1/3)*x+RootOf(_Z^3-2)^2+2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*(-x^3+1)^(1/3))/(x^2-x+1))-1/2*ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-x^3+1)^(2/3))/(x^2-x+1))*RootOf(_Z^3-2)-ln((2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-x^3+1)^(2/3))/(x^2-x+1))

2)*RootOf(_Z^3-2)^4*x+4*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)^2*RootOf(_Z^3-2)^3*x+2*(-x^3+1)^(2/3)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)^2-RootOf(_Z^3-2)^2*x^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x^2+3*RootOf(_Z^3-2)^2*x+6*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)*x-2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)*x-RootOf(_Z^3-2)^2-2*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)*RootOf(_Z^3-2)+2*(-x^3+1)^(1/3)*RootOf(_Z^3-2)+2*(-x^3+1)^(2/3))/(x^2-x+1)*RootOf(RootOf(_Z^3-2)^2+2*_Z*RootOf(_Z^3-2)+4*_Z^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2 - 1}{(-x^3 + 1)^{\frac{2}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^2+1)/(x^2-x+1)/(-x^3+1)^(2/3),x, algorithm="maxima")

[Out] -integrate((x^2 - 1)/((-x^3 + 1)^(2/3)*(x^2 - x + 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2 - 1}{(1 - x^3)^{\frac{2}{3}}(x^2 - x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-(x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)),x)

[Out] -int((x^2 - 1)/((1 - x^3)^(2/3)*(x^2 - x + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2(1-x^3)^{\frac{2}{3}} - x(1-x^3)^{\frac{2}{3}} + (1-x^3)^{\frac{2}{3}}} dx - \int \left(-\frac{1}{x^2(1-x^3)^{\frac{2}{3}} - x(1-x^3)^{\frac{2}{3}} + (1-x^3)^{\frac{2}{3}}} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**2+1)/(x**2-x+1)/(-x**3+1)**(2/3),x)

[Out] -Integral(x**2/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**(2/3)), x) - Integral(-1/(x**2*(1 - x**3)**(2/3) - x*(1 - x**3)**(2/3) + (1 - x**3)**(2/3)), x)

$$3.707 \quad \int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)} dx$$

Optimal. Leaf size=49

$$-\frac{1}{4} \tan^{-1} \left(\frac{x^2+1}{x\sqrt{x^4-1}} \right) - \frac{1}{4} \tanh^{-1} \left(\frac{1-x^2}{x\sqrt{x^4-1}} \right)$$

Rubi [C] time = 0.12, antiderivative size = 47, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {490, 1211, 222, 1699, 206, 203}

$$\left(\frac{1}{8} + \frac{i}{8} \right) \tanh^{-1} \left(\frac{(1+i)x}{\sqrt{x^4-1}} \right) - \left(\frac{1}{8} + \frac{i}{8} \right) \tanh^{-1} \left(\frac{(1+i)x}{\sqrt{x^4-1}} \right)$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[-1 + x^4]*(1 + x^4)),x]

[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 + I/8)*ArcTanh[((1 + I)*x)/Sqrt[-1 + x^4]]

Rule 203

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])

Rule 206

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(1*ArcTanh[(Rt[-b, 2]*x)/Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])

Rule 222

Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] := With[{q = Rt[-(a*b), 2]}, Simp[(Sqrt[-a + q*x^2]*Sqrt[(a + q*x^2)/q]*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2])/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]), x] /; IntegerQ[q] /; FreeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]

Rule 490

Int[(x_)^2/(((a_) + (b_.)*(x_)^4)*Sqrt[(c_) + (d_.)*(x_)^4]), x_Symbol] := With[{r = Numerator[Rt[-(a/b), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/((r + s*x^2)*Sqrt[c + d*x^4]), x], x] - Dist[s/(2*b), Int[1/((r - s*x^2)*Sqrt[c + d*x^4]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]

Rule 1211

Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[1/(2*d), Int[1/Sqrt[a + c*x^4], x], x] + Dist[1/(2*d), Int[(d - e*x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0]

Rule 1699

Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] := Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4]]

]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^2 - a*e^2, 0] && EqQ[B*d + A*e, 0]

Rubi steps

$$\begin{aligned} \int \frac{x^2}{\sqrt{-1+x^4}(1+x^4)} dx &= -\left(\frac{1}{2} \int \frac{1}{(i-x^2)\sqrt{-1+x^4}} dx\right) + \frac{1}{2} \int \frac{1}{(i+x^2)\sqrt{-1+x^4}} dx \\ &= -\left(\frac{1}{4}i \int \frac{i-x^2}{(i+x^2)\sqrt{-1+x^4}} dx\right) + \frac{1}{4}i \int \frac{i+x^2}{(i-x^2)\sqrt{-1+x^4}} dx \\ &= \frac{1}{4} \text{Subst}\left(\int \frac{1}{i-2x^2} dx, x, \frac{x}{\sqrt{-1+x^4}}\right) - \frac{1}{4} \text{Subst}\left(\int \frac{1}{i+2x^2} dx, x, \frac{x}{\sqrt{-1+x^4}}\right) \\ &= \left(-\frac{1}{8} - \frac{i}{8}\right) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) + \left(\frac{1}{8} + \frac{i}{8}\right) \tanh^{-1}\left(\frac{(1+i)x}{\sqrt{-1+x^4}}\right) \end{aligned}$$

Mathematica [C] time = 0.02, size = 46, normalized size = 0.94

$$\frac{x^3 \sqrt{1-x^4} F_1\left(\frac{3}{4}; \frac{1}{2}, 1; \frac{7}{4}; x^4, -x^4\right)}{3\sqrt{x^4-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[x^2/(Sqrt[-1 + x^4]*(1 + x^4)), x]

[Out] (x^3*Sqrt[1 - x^4]*AppellF1[3/4, 1/2, 1, 7/4, x^4, -x^4])/(3*Sqrt[-1 + x^4])

IntegrateAlgebraic [C] time = 0.27, size = 53, normalized size = 1.08

$$\left(\frac{1}{8} - \frac{i}{8}\right) \tan^{-1}\left(\frac{\left(\frac{1}{2} + \frac{i}{2}\right)\sqrt{x^4-1}}{x}\right) - \left(\frac{1}{8} + \frac{i}{8}\right) \tan^{-1}\left(\frac{(1+i)x}{\sqrt{x^4-1}}\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x^2/(Sqrt[-1 + x^4]*(1 + x^4)), x]

[Out] (-1/8 - I/8)*ArcTan[((1 + I)*x)/Sqrt[-1 + x^4]] + (1/8 - I/8)*ArcTan[((1/2 + I/2)*Sqrt[-1 + x^4])/x]

fricas [A] time = 1.30, size = 51, normalized size = 1.04

$$\frac{1}{4} \arctan\left(\frac{\sqrt{x^4-1}x}{x^2+1}\right) + \frac{1}{8} \log\left(\frac{x^4+2x^2+2\sqrt{x^4-1}x-1}{x^4+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2), x, algorithm="fricas")

[Out] 1/4*arctan(sqrt(x^4 - 1)*x/(x^2 + 1)) + 1/8*log((x^4 + 2*x^2 + 2*sqrt(x^4 - 1)*x - 1)/(x^4 + 1))

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^4+1)\sqrt{x^4-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="giac")

[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)

maple [B] time = 0.03, size = 88, normalized size = 1.80

$$-\frac{\arctan\left(-\frac{\sqrt{x^4-1}}{x} + 1\right)}{8} + \frac{\arctan\left(\frac{\sqrt{x^4-1}}{x} + 1\right)}{8} + \frac{\ln\left(\frac{\frac{\sqrt{x^4-1}}{x} + \frac{x^4-1}{2x^2} + 1}{-\frac{\sqrt{x^4-1}}{x} + \frac{x^4-1}{2x^2} + 1}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(x^4+1)/(x^4-1)^(1/2),x)

[Out] 1/8*arctan((x^4-1)^(1/2)/x+1)-1/8*arctan(-(x^4-1)^(1/2)/x+1)+1/16*ln((1/2*(x^4-1)/x^2+(x^4-1)^(1/2)/x+1)/(1/2*(x^4-1)/x^2-(x^4-1)^(1/2)/x+1))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(x^4 + 1)\sqrt{x^4 - 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(x^4+1)/(x^4-1)^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/((x^4 + 1)*sqrt(x^4 - 1)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^2}{\sqrt{x^4 - 1} (x^4 + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)),x)

[Out] int(x^2/((x^4 - 1)^(1/2)*(x^4 + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)(x+1)(x^2+1)}(x^4+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(x**4+1)/(x**4-1)**(1/2),x)

[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**4 + 1)), x)

$$3.708 \quad \int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}} dx$$

Optimal. Leaf size=80

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Rubi [A] time = 0.45, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 46, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2112, 205}

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 2112

Int[((u_)*((A_) + (B_.)*(x_)^4))/Sqrt[v_], x_Symbol] :> With[{a = Coeff[v, x, 0], b = Coeff[v, x, 2], c = Coeff[v, x, 4], d = Coeff[1/u, x, 0], e = Coeff[1/u, x, 2], f = Coeff[1/u, x, 4]}, Dist[A, Subst[Int[1/(d - (b*d - a*e)*x^2), x], x, x/Sqrt[v]], x] /; EqQ[a*B + A*c, 0] && EqQ[c*d - a*f, 0] /; FreeQ[{A, B}, x] && PolyQ[v, x^2, 2] && PolyQ[1/u, x^2, 2]

Rubi steps

$$\int \frac{a-cx^4}{(ae+cdx^2)(d+ex^2)\sqrt{a+bx^2+cx^4}} dx = a \operatorname{Subst}\left(\int \frac{1}{ade - (abde - a(cd^2 + ae^2))x^2} dx, x, \frac{x}{\sqrt{a+bx^2+cx^4}}\right) = \frac{\tan^{-1}\left(\frac{\sqrt{cd^2-bde+ae^2}x}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{cd^2-bde+ae^2}}$$

Mathematica [C] time = 0.91, size = 383, normalized size = 4.79

$$\frac{i\sqrt{\frac{\sqrt{b^2-4ac}+b+2cx^2}{\sqrt{b^2-4ac}+b}}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}}+1\left(-\Pi\left(\frac{(b+\sqrt{b^2-4ac})^d}{2ac};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)x\right)^{\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}}\right)-\Pi\left(\frac{(b-\sqrt{b^2-4ac})^d}{2cd};i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)x\right)^{\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}}\right)+F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\right)x\right)^{\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}de\sqrt{\frac{c}{\sqrt{b^2-4ac}+b}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x]

[Out] (I*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])*(EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b

```
+ Sqrt[b^2 - 4*a*c]])*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]))
- EllipticPi[((b + Sqrt[b^2 - 4*a*c])*d)/(2*a*e), I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c]])*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c
])] - EllipticPi[((b + Sqrt[b^2 - 4*a*c])*e)/(2*c*d), I*ArcSinh[Sqrt[2]*Sqrt
[c/(b + Sqrt[b^2 - 4*a*c]])*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a
*c])))])/(Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c]])*d*e*Sqrt[a + b*x^2 + c*x^4
])
```

IntegrateAlgebraic [A] time = 25.01, size = 80, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{x\sqrt{ae^2-bde+cd^2}}{\sqrt{d}\sqrt{e}\sqrt{a+bx^2+cx^4}}\right)}{\sqrt{d}\sqrt{e}\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] IntegrateAlgebraic[(a - c*x^4)/((a*e + c*d*x^2)*(d + e*x^2)*Sqrt[a + b*x^2
+ c*x^4]),x]
[Out] ArcTan[(Sqrt[c*d^2 - b*d*e + a*e^2]*x)/(Sqrt[d]*Sqrt[e]*Sqrt[a + b*x^2 + c*
x^4])]/(Sqrt[d]*Sqrt[e]*Sqrt[c*d^2 - b*d*e + a*e^2])
```

fricas [A] time = 120.39, size = 472, normalized size = 5.90

$$\left[\frac{\sqrt{-cd^3e + bd^2e^2 - ade^3} \log\left(\frac{c^2d^2e^3 - 2(3c^2d^2e - 4bd^2e^2 + 3acd^3)e^3 + a^2d^2e^2 + (c^2d^4 - 8bd^2e - 8abd^3 + a^2e^4 + 4(2b^2 + ac)d^2e^2)x^2 - 2(3acd^2 - 4abd^2e + 3a^2d^3)x^2 + 4(cde^2 + ade - (cd^2 - 2bde + ae^2)x^2)\sqrt{-cd^3e + bd^2e^2 - ade^3}\sqrt{cx^4 + bx^2 + a}}{c^2d^2e^3 + 2(c^2d^2e + acd^3)e^3 + a^2d^2e^2 + (c^2d^4 + 4acd^2 + a^2e^4)x^2 + 2(acd^2e + a^2d^3)x^2}\right)}{4(cd^3e - bd^2e^2 + ade^3)} \right], \arctan\left(\frac{2\sqrt{cd^3e - bd^2e^2 + ade^3}\sqrt{cx^4 + bx^2 + a}}{cdx^4 + ade - (cd^2 - 2bde + ae^2)x^2}\right) \Bigg]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*log(-(c^2*d^2*e^2*x^8 - 2*(3*c^2
*d^3*e - 4*b*c*d^2*e^2 + 3*a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c^2*d^4 - 8*b*c*
d^3*e - 8*a*b*d*e^3 + a^2*e^4 + 4*(2*b^2 + a*c)*d^2*e^2)*x^4 - 2*(3*a*c*d^3
*e - 4*a*b*d^2*e^2 + 3*a^2*d*e^3)*x^2 + 4*(c*d*e*x^5 + a*d*e*x - (c*d^2 - 2
*b*d*e + a*e^2)*x^3)*sqrt(-c*d^3*e + b*d^2*e^2 - a*d*e^3)*sqrt(c*x^4 + b*x^
2 + a))/(c^2*d^2*e^2*x^8 + 2*(c^2*d^3*e + a*c*d*e^3)*x^6 + a^2*d^2*e^2 + (c
^2*d^4 + 4*a*c*d^2*e^2 + a^2*e^4)*x^4 + 2*(a*c*d^3*e + a^2*d*e^3)*x^2)/(c*
d^3*e - b*d^2*e^2 + a*d*e^3), 1/2*arctan(2*sqrt(c*d^3*e - b*d^2*e^2 + a*d*e
^3)*sqrt(c*x^4 + b*x^2 + a)*x/(c*d*e*x^4 + a*d*e - (c*d^2 - 2*b*d*e + a*e^
2)*x^2))/sqrt(c*d^3*e - b*d^2*e^2 + a*d*e^3)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a}(cdx^2 + ae)(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(-(c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)
), x)
```

maple [C] time = 0.09, size = 555, normalized size = 6.94

$$\frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \operatorname{EllipticF}\left(\frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}, \frac{\sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}\right)}{4 \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{cx^4 + bx^2 + a} dx} + \frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \operatorname{EllipticPi}\left(\frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}, \frac{\sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}, \frac{2a}{(-a\sqrt{-4ac^2d^2})} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}\right)}{\sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{cx^4 + bx^2 + a} dx} + \frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \operatorname{EllipticPi}\left(\frac{\sqrt{2} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}, \frac{\sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}}{2}, \frac{2ad}{(-a\sqrt{-4ac^2d^2})} \sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4}\right)}{\sqrt{\frac{3(-a\sqrt{-4ac^2d^2})^2}{4} + 4} \sqrt{cx^4 + bx^2 + a} dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)

[Out]
$$-1/4/d/e^{2^{1/2}}/((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*(-2*(-b+(-4*a*c+b^2)^{1/2})/a*x^2+4)^{1/2}*(2*(b+(-4*a*c+b^2)^{1/2})/a*x^2+4)^{1/2}/(c*x^4+b*x^2+a)^{1/2}*EllipticF(1/2*2^{1/2}*((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*x,1/2*(2*(b+(-4*a*c+b^2)^{1/2})/a*b/c-4)^{1/2})+1/d/e^{2^{1/2}}/(-1/a*b+1/a*(-4*a*c+b^2)^{1/2})^{1/2}*(1+1/2/a*b*x^2-1/2*(-4*a*c+b^2)^{1/2}/a*x^2)^{1/2}*(1+1/2/a*b*x^2+1/2*(-4*a*c+b^2)^{1/2}/a*x^2)^{1/2}/(c*x^4+b*x^2+a)^{1/2}*EllipticPi(1/2*2^{1/2}*((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*x,-2/(-b+(-4*a*c+b^2)^{1/2})*c*d/e,(-1/2*(b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2})+1/e/d*2^{1/2}/(-1/a*b+1/a*(-4*a*c+b^2)^{1/2})^{1/2}*(1+1/2/a*b*x^2-1/2*(-4*a*c+b^2)^{1/2}/a*x^2)^{1/2}*(1+1/2/a*b*x^2+1/2*(-4*a*c+b^2)^{1/2}/a*x^2)^{1/2}/(c*x^4+b*x^2+a)^{1/2}*EllipticPi(1/2*2^{1/2}*((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*x,-2/(-b+(-4*a*c+b^2)^{1/2})*a*e/d,(-1/2*(b+(-4*a*c+b^2)^{1/2})/a)^{1/2}*2^{1/2}/((-b+(-4*a*c+b^2)^{1/2})/a)^{1/2})$$

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{cx^4 - a}{\sqrt{cx^4 + bx^2 + a} (cdx^2 + ae)(ex^2 + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x^4+a)/(c*d*x^2+a*e)/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")

[Out] -integrate((c*x^4 - a)/(sqrt(c*x^4 + b*x^2 + a)*(c*d*x^2 + a*e)*(e*x^2 + d)), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a - cx^4}{(ex^2 + d)(cdx^2 + ae)\sqrt{cx^4 + bx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)

[Out] int((a - c*x^4)/((d + e*x^2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \left(\frac{a}{ade\sqrt{a+bx^2+cx^4} + ae^2x^2\sqrt{a+bx^2+cx^4} + cd^2x^2\sqrt{a+bx^2+cx^4} + cdex^4\sqrt{a+bx^2+cx^4}} \right) dx - \int \frac{cx^4}{ade\sqrt{a+bx^2+cx^4} + ae^2x^2\sqrt{a+bx^2+cx^4} + cd^2x^2\sqrt{a+bx^2+cx^4} + cdex^4\sqrt{a+bx^2+cx^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-c*x**4+a)/(c*d*x**2+a*e)/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)

[Out] -Integral(-a/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x) - Integral(c*x**4/(a*d*e*sqrt(a + b*x**2 + c*x**4) + a*e**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d**2*x**2*sqrt(a + b*x**2 + c*x**4) + c*d*e*x**4*sqrt(a + b*x**2 + c*x**4)), x)

$$3.709 \quad \int \left(x + \frac{1-x^2}{1+x} \right) dx$$

Optimal. Leaf size=1

x

Rubi [A] time = 0.00, antiderivative size = 1, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

x

Antiderivative was successfully verified.

[In] Int[x + (1 - x^2)/(1 + x), x]

[Out] x

Rubi steps

$$\int \left(x + \frac{1-x^2}{1+x} \right) dx = x$$

Mathematica [A] time = 0.00, size = 1, normalized size = 1.00

x

Antiderivative was successfully verified.

[In] Integrate[x + (1 - x^2)/(1 + x), x]

[Out] x

IntegrateAlgebraic [B] time = 0.03, size = 19, normalized size = 19.00

$$\frac{x^2}{2} - \frac{1}{2}(1-x)^2$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[x + (1 - x^2)/(1 + x), x]

[Out] -1/2*(1 - x)^2 + x^2/2

fricas [A] time = 0.78, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+(-x^2+1)/(1+x), x, algorithm="fricas")

[Out] x

giac [A] time = 0.35, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+(-x²+1)/(1+x),x, algorithm="giac")

[Out] x

maple [A] time = 0.00, size = 2, normalized size = 2.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x+(-x²+1)/(x+1),x)

[Out] x

maxima [A] time = 0.46, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+(-x²+1)/(1+x),x, algorithm="maxima")

[Out] x

mupad [B] time = 0.00, size = 1, normalized size = 1.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x - (x² - 1)/(x + 1),x)

[Out] x

sympy [A] time = 0.06, size = 0, normalized size = 0.00

x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x+(-x**2+1)/(1+x),x)

[Out] x

$$3.710 \quad \int \frac{1}{\frac{1}{x} + \sqrt{1-x^2}} dx$$

Optimal. Leaf size=42

$$\sin^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

Rubi [C] time = 0.41, antiderivative size = 122, normalized size of antiderivative = 2.90, number of steps used = 12, number of rules used = 9, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {6742, 1107, 618, 204, 1293, 216, 1174, 377, 205}

$$-\frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^(-1) + Sqrt[1 - x^2])^(-1), x]

[Out] ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((I - Sqrt[3])/(I + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] := Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] := Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] := Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] := Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1107

Int[(x_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol] := Dist[1/2, Subst[Int[(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, p}, x]

Rule 1174

```
Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol]
:> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b - r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]
```

Rule 1293

```
Int[(((f_)*(x_))^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol]
:> Dist[(e*f^2)/c, Int[(f*x)^(m-2)*(d + e*x^2)^(q-1), x], x] - Dist[f^2/c, Int[((f*x)^(m-2)*(d + e*x^2)^(q-1)*Simp[a*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 6742

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\frac{1}{x} + \sqrt{1-x^2}} dx &= \int \left(\frac{x}{1-x^2+x^4} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4} \right) dx \\ &= \int \frac{x}{1-x^2+x^4} dx - \int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4} dx \\ &= \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) + \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{1}{\sqrt{1-x^2}(1-x^2+x^4)} dx \\ &= \sin^{-1}(x) + \frac{(2i) \int \frac{1}{\sqrt{1-x^2}(-1-i\sqrt{3}+2x^2)} dx}{\sqrt{3}} - \frac{(2i) \int \frac{1}{\sqrt{1-x^2}(-1+i\sqrt{3}+2x^2)} dx}{\sqrt{3}} - \text{Subst} \left(\int \frac{1}{-3-x} dx, x, x^2 \right) \\ &= \sin^{-1}(x) + \frac{\tan^{-1} \left(\frac{-1+2x^2}{\sqrt{3}} \right)}{\sqrt{3}} - \frac{(2i) \text{Subst} \left(\int \frac{1}{-1+i\sqrt{3}-(-1-i\sqrt{3})x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right)}{\sqrt{3}} + \frac{(2i) \text{Subst} \left(\int \frac{1}{-1-i\sqrt{3}-(-1+i\sqrt{3})x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right)}{\sqrt{3}} \\ &= \sin^{-1}(x) - \frac{\tan^{-1} \left(\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{\sqrt{\frac{i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}{x} \right)}{\sqrt{3}} + \frac{\tan^{-1} \left(\frac{-1+2x^2}{\sqrt{3}} \right)}{\sqrt{3}} \end{aligned}$$

Mathematica [C] time = 3.96, size = 1932, normalized size = 46.00

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[(x^(-1) + Sqrt[1 - x^2])^(-1), x]
```

```
[Out] (24*ArcSin[x] - (2*(-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3])*x + I*(7*I + Sqrt[3])*x^2)]/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))]/Sqrt[(1 - I*Sqrt[3])/6] + (2*(I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] - 8*Sqrt[3])*x + (7 + I*Sqrt[3])*x^2)]/(-6 + (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 - (2*I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sq
```

```

rt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]))/Sqrt[(1 + I*Sqrt[3])/6]
- (2*(I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3]
)*x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]
]*Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x
^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/S
qrt[(1 + I*Sqrt[3])/6] + (2*(1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*
I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*
x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt
[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)
]*Sqrt[3]]*Sqrt[1 - x^2])))/Sqrt[(1 - I*Sqrt[3])/6] - (4*I)*Sqrt[3]*Log[-1/
2 - (I/2)*Sqrt[3] + x^2] + (4*I)*Sqrt[3]*Log[(I/2)*(I + Sqrt[3]) + x^2] - (
I*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 - I*Sqrt[3])/6] +
(I*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6]
+ ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 + I*Sqrt[3])/6]
+ ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/Sqrt[(1 - I*Sqrt[3])/
6] + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[
2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*S
qrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1
- x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])])
)/Sqrt[(1 - I*Sqrt[3])/6] - (I*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqr
t[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sq
rt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 -
(6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sq
rt[3]]*Sqrt[1 - x^2])))/Sqrt[(1 - I*Sqrt[3])/6] + ((1 - I*Sqrt[3])*Log[-3*I
+ Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]
] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sq
rt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt
[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/Sqrt[(1 + I*Sqrt[3])/6] + (I
*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)
]*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 -
x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) +
I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/Sqrt[(1
+ I*Sqrt[3])/6])/24

```

IntegrateAlgebraic [C] time = 0.33, size = 74, normalized size = 1.76

$$i \log(\sqrt{1-x^2} - ix) + \frac{2i \tanh^{-1}\left(-\frac{2ix^2}{\sqrt{3}} + \frac{2\sqrt{1-x^2}x}{\sqrt{3}} + \frac{2+i}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x^(-1) + Sqrt[1 - x^2])^(-1), x]

[Out] ((2*I)*ArcTanh[(2 + I)/Sqrt[3] - ((2*I)*x^2)/Sqrt[3] + (2*x*Sqrt[1 - x^2])/Sqrt[3]])/Sqrt[3] + I*Log[(-I)*x + Sqrt[1 - x^2]]

fricas [A] time = 0.40, size = 73, normalized size = 1.74

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{\sqrt{3} (2x^2 - 1) \sqrt{-x^2 + 1}}{3(x^3 - x)}\right) - 2 \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="fricas")

[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)

giac [B] time = 0.42, size = 193, normalized size = 4.60

$$\frac{1}{2} \pi \operatorname{sgn}(x) - \frac{1}{6} \sqrt{3} \left[\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}}{x} + \frac{(\sqrt{-x^2+1})^2}{x^2} - 1 \right)}{3(\sqrt{-x^2+1}-1)} \right) \right] - \frac{1}{6} \sqrt{3} \left[\pi \operatorname{sgn}(x) + 2 \arctan \left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}}{x} - \frac{(\sqrt{-x^2+1})^2}{x^2} + 1 \right)}{3(\sqrt{-x^2+1}-1)} \right) \right] + \frac{1}{3} \sqrt{3} \arctan \left(\frac{1}{3} \sqrt{3} (2x^2-1) \right) + \arctan \left(\frac{x \left(\frac{\sqrt{-x^2+1}}{x^2} - 1 \right)}{2(\sqrt{-x^2+1}-1)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="giac")

[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))

maple [C] time = 0.04, size = 234, normalized size = 5.57

$$-2 \arctan \left(\frac{-1 + \sqrt{-x^2+1}}{x} \right) + \frac{\sqrt{3} \arctan \left(\frac{(2x^2-1)\sqrt{3}}{3} \right)}{3} - i\sqrt{3} \ln \left(\frac{(-1-i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1 \right) - i\sqrt{3} \ln \left(\frac{(1-i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1 \right) + i\sqrt{3} \ln \left(\frac{(1+i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1 \right) + i\sqrt{3} \ln \left(\frac{(-1+i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x+(-x^2+1)^(1/2)),x)

[Out] 1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)-2*arctan((-1+(-x^2+1)^(1/2))/x)+1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(I*3^(1/2)-1)*(-1+(-x^2+1)^(1/2))/x-1)-1/6*I*3^(1/2)*ln((-1+(-x^2+1)^(1/2))^2/x^2+(-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x-1)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-x^2+1} + \frac{1}{x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x+(-x^2+1)^(1/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-x^2 + 1) + 1/x), x)

mupad [B] time = 3.92, size = 549, normalized size = 13.07

$$\operatorname{asin}(x) - \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} + \frac{1}{2}} \right)}{\sqrt{1 - \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2}} \sqrt{\sqrt{3} - 4 \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 + 11}} + \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} - \frac{1}{2}} \right)}{\sqrt{1 - \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2}} \sqrt{-\sqrt{3} + 4 \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 + 11}} - \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} + \frac{1}{2}} \right)}{\sqrt{1 - \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2}} \sqrt{-\sqrt{3} + 4 \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 + 11}} + \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} - \frac{1}{2}} \right)}{\sqrt{3 - 4 \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 + 11}} - \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} + \frac{1}{2}} \right)}{\sqrt{3 - 4 \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 + 11}} + \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} - \frac{1}{2}} \right)}{\sqrt{1 - \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2}} \sqrt{\sqrt{3} - 4 \left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 + 11}} - \frac{\ln \left(\frac{\left(\frac{\sqrt{\frac{3}{2}} + \frac{1}{2} \right)^2 - \sqrt{-x^2+1}}{\sqrt{\frac{3}{2}} + \frac{1}{2}} \right)}{\sqrt{1 - \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2}} \sqrt{-\sqrt{3} + 4 \left(\frac{\sqrt{\frac{3}{2}} - \frac{1}{2} \right)^2 + 11}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1/x + (1 - x^2)^(1/2)),x)

[Out] asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 - 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)

```

- (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 +
  1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/
  2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2))/((1 - (3^(1/2)/2
  + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2)
  )/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (
  log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(
  1/2) + 1i)

```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{x\sqrt{1-x^2} + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(1/x+(-x**2+1)**(1/2)),x)

[Out] Integral(x/(x*sqrt(1 - x**2) + 1), x)

$$3.711 \quad \int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx$$

Optimal. Leaf size=42

$$\sin^{-1}(x) - \frac{\tan^{-1}\left(\frac{1+4x\sqrt{1-x^2}}{\sqrt{3}(1-2x^2)}\right)}{\sqrt{3}}$$

Rubi [C] time = 0.36, antiderivative size = 149, normalized size of antiderivative = 3.55, number of steps used = 13, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {6742, 1293, 216, 1174, 377, 205, 1251, 773, 618, 204}

$$\frac{x^2}{2} - \frac{\tan^{-1}\left(\frac{1-2x^2}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{x}{\sqrt{\frac{-\sqrt{3}+i}{\sqrt{3}+i}}\sqrt{1-x^2}}\right)}{\sqrt{3}} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{-\sqrt{3}+i}{\sqrt{3}+i}}x}{\sqrt{1-x^2}}\right)}{\sqrt{3}} + \frac{1}{4}(1-x)^2 + \frac{1}{4}(x+1)^2 + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]

[Out] (1 - x)^2/4 - x^2/2 + (1 + x)^2/4 + ArcSin[x] - ArcTan[(1 - 2*x^2)/Sqrt[3]]/Sqrt[3] - ArcTan[x/(Sqrt[-((1 - Sqrt[3])/(1 + Sqrt[3]))]*Sqrt[1 - x^2])]/Sqrt[3] - ArcTan[(Sqrt[-((1 - Sqrt[3])/(1 + Sqrt[3]))]*x)/Sqrt[1 - x^2]]/Sqrt[3]

Rule 204

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[a, 0] || LtQ[b, 0])

Rule 205

Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]

Rule 216

Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqrt[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]

Rule 377

Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Subst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]

Rule 618

Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[Int[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 773

Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> Simp[(e*g*x)/c, x] + Dist[1/c, Int[(c*d*f - a*e*g + (c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}

, f, g}, x] && NeQ[b^2 - 4*a*c, 0]

Rule 1174

Int[((d_) + (e_)*(x_)^2)^(q_)/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[(2*c)/r, Int[(d + e*x^2)^q/(b - r + 2*c*x^2), x], x] - Dist[(2*c)/r, Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && !IntegerQ[q]

Rule 1251

Int[(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_)*((a_) + (b_)*(x_)^2 + (c_)*(x_)^4)^(p_), x_Symbol] := Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && IntegerQ[(m - 1)/2]

Rule 1293

Int[((f_)*(x_)^(m_)*((d_) + (e_)*(x_)^2)^(q_))/((a_) + (b_)*(x_)^2 + (c_)*(x_)^4), x_Symbol] := Dist[(e*f^2)/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^2/c, Int[((f*x)^(m - 2)*(d + e*x^2)^(q - 1)*Simp[a*e - (c*d - b*e)*x^2, x])/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1] && LeQ[m, 3]

Rule 6742

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned} \int \frac{x\sqrt{1-x^2}}{x-x^3+\sqrt{1-x^2}} dx &= \int \left(\frac{1}{2}(-1+x) + \frac{1+x}{2} - \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4} + \frac{x^3(1-x^2)}{1-x^2+x^4} \right) dx \\ &= \frac{1}{4}(1-x)^2 + \frac{1}{4}(1+x)^2 - \int \frac{x^2\sqrt{1-x^2}}{1-x^2+x^4} dx + \int \frac{x^3(1-x^2)}{1-x^2+x^4} dx \\ &= \frac{1}{4}(1-x)^2 + \frac{1}{4}(1+x)^2 + \frac{1}{2} \text{Subst} \left(\int \frac{(1-x)x}{1-x+x^2} dx, x, x^2 \right) + \int \frac{1}{\sqrt{1-x^2}} dx - \int \frac{1}{\sqrt{1-x^2}} dx \\ &= \frac{1}{4}(1-x)^2 - \frac{x^2}{2} + \frac{1}{4}(1+x)^2 + \sin^{-1}(x) + \frac{1}{2} \text{Subst} \left(\int \frac{1}{1-x+x^2} dx, x, x^2 \right) + \frac{(2i) \int \frac{1}{\sqrt{1-x^2}} dx}{\sqrt{3}} \\ &= \frac{1}{4}(1-x)^2 - \frac{x^2}{2} + \frac{1}{4}(1+x)^2 + \sin^{-1}(x) - \frac{(2i) \text{Subst} \left(\int \frac{1}{-1+i\sqrt{3}-(-1-i\sqrt{3})x^2} dx, x, \frac{x}{\sqrt{1-x^2}} \right)}{\sqrt{3}} \\ &= \frac{1}{4}(1-x)^2 - \frac{x^2}{2} + \frac{1}{4}(1+x)^2 + \sin^{-1}(x) - \frac{\tan^{-1} \left(\frac{x}{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}} \right)}{\sqrt{3}} - \frac{\tan^{-1} \left(\frac{\sqrt{\frac{-i-\sqrt{3}}{i+\sqrt{3}}}\sqrt{1-x^2}}{\sqrt{1-x^2}} \right)}{\sqrt{3}} + \end{aligned}$$

Mathematica [C] time = 0.93, size = 1910, normalized size = 45.48

result too large to display

Antiderivative was successfully verified.

[In] Integrate[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]

[Out] ArcSin[x] - ((-I + Sqrt[3])*ArcTan[(x*(-7 - I*Sqrt[3] + 8*Sqrt[3]*x + I*(7*I + Sqrt[3])*x^2))/(-6 - (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 - 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (2*I)*x^2*(9*I + Sqrt[3] + I*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2])) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/(2*Sqrt[6 - (6*I)*Sqrt[3]]) + ((I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] - 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(-6 + (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + x^2*(-18 - (2*I)*Sqrt[3] - 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/(2*Sqrt[6 + (6*I)*Sqrt[3]]) - ((I + Sqrt[3])*ArcTan[(x*(7 - I*Sqrt[3] + 8*Sqrt[3]*x + (7 + I*Sqrt[3])*x^2))/(6 - (2*I)*Sqrt[3] + 3*(I + Sqrt[3])*x^3 + 2*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 + I*Sqrt[3] + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(-3*I + 11*Sqrt[3] + 2*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/(2*Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*ArcTanh[(x*(7*I - Sqrt[3] + (8*I)*Sqrt[3]*x + (7*I + Sqrt[3])*x^2))/(6 + (2*I)*Sqrt[3] + 3*(-I + Sqrt[3])*x^3 + 2*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + 2*x^2*(9 - I*Sqrt[3] + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3*I + 11*Sqrt[3] + 2*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])))/(2*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/2)*Log[-1/2 - (I/2)*Sqrt[3] + x^2])/Sqrt[3] + ((I/2)*Log[(I/2)*(I + Sqrt[3]) + x^2])/Sqrt[3] - ((I/4)*(-I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 - (6*I)*Sqrt[3]] + ((I/4)*(I + Sqrt[3])*Log[16*(1 + Sqrt[3]*x + x^2)^2])/Sqrt[6 + (6*I)*Sqrt[3]] + ((1 + I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/(4*Sqrt[6 - (6*I)*Sqrt[3]]) + ((1 - I*Sqrt[3])*Log[(4 - 4*Sqrt[3]*x + 4*x^2)^2])/(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((1 + I*Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 + (5*I)*Sqrt[3] + (3*I)*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x^3*(3*I + 3*Sqrt[3] + Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]/(4*Sqrt[6 - (6*I)*Sqrt[3]]) - ((I/4)*(-I + Sqrt[3])*Log[3*I + Sqrt[3] - (-I + Sqrt[3])*x^4 + (2*I)*Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2] + (5*I)*x^2*(2 + Sqrt[2 - (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 - (3*I)*Sqrt[3] - I*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x*(-3*I + 5*Sqrt[3] + 3*Sqrt[6 - (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]/Sqrt[6 - (6*I)*Sqrt[3]] + ((1 - I*Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x*(3 - (5*I)*Sqrt[3] - (3*I)*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) - I*x^3*(-3*I + 3*Sqrt[3] + Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]/(4*Sqrt[6 + (6*I)*Sqrt[3]]) + ((I/4)*(I + Sqrt[3])*Log[-3*I + Sqrt[3] - (I + Sqrt[3])*x^4 - (2*I)*Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2] - (5*I)*x^2*(2 + Sqrt[2 + (2*I)*Sqrt[3]]*Sqrt[1 - x^2]) + x^3*(3 + (3*I)*Sqrt[3] + I*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2]) + I*x*(3*I + 5*Sqrt[3] + 3*Sqrt[6 + (6*I)*Sqrt[3]]*Sqrt[1 - x^2])]/Sqrt[6 + (6*I)*Sqrt[3]])

IntegrateAlgebraic [C] time = 0.29, size = 74, normalized size = 1.76

$$i \log\left(\sqrt{1-x^2} - ix\right) + \frac{2i \tanh^{-1}\left(-\frac{2ix^2}{\sqrt{3}} + \frac{2\sqrt{1-x^2}x}{\sqrt{3}} + \frac{2+i}{\sqrt{3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(x*Sqrt[1 - x^2])/(x - x^3 + Sqrt[1 - x^2]),x]

[Out] ((2*I)*ArcTanh[(2 + I)/Sqrt[3] - ((2*I)*x^2)/Sqrt[3] + (2*x*Sqrt[1 - x^2])/Sqrt[3])/Sqrt[3] + I*Log[(-I)*x + Sqrt[1 - x^2]]

fricas [A] time = 0.40, size = 73, normalized size = 1.74

$$\frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{\sqrt{3} (2x^2 - 1) \sqrt{-x^2 + 1}}{3(x^3 - x)}\right) - 2 \arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)*sqrt(-x^2 + 1)/(x^3 - x)) - 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

giac [B] time = 0.58, size = 193, normalized size = 4.60

$$\frac{1}{2} \pi \operatorname{sgn}(x) - \frac{1}{6} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}-1}{x} + \frac{(\sqrt{-x^2+1})^2}{x^2} - 1\right)}{3(\sqrt{-x^2+1}-1)}\right) \right) - \frac{1}{6} \sqrt{3} \left(\pi \operatorname{sgn}(x) + 2 \arctan\left(\frac{\sqrt{3} x \left(\frac{\sqrt{-x^2+1}-1}{x} - \frac{(\sqrt{-x^2+1})^2}{x^2} + 1\right)}{3(\sqrt{-x^2+1}-1)}\right) \right) + \frac{1}{3} \sqrt{3} \arctan\left(\frac{1}{3} \sqrt{3} (2x^2 - 1)\right) + \arctan\left(\frac{x \left(\frac{(\sqrt{-x^2+1})^2}{x^2} - 1\right)}{2(\sqrt{-x^2+1}-1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="giac")
```

```
[Out] 1/2*pi*sgn(x) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(-1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x + (sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))) - 1/6*sqrt(3)*(pi*sgn(x) + 2*arctan(1/3*sqrt(3)*x*((sqrt(-x^2 + 1) - 1)/x - (sqrt(-x^2 + 1) - 1)^2/x^2 + 1)/(sqrt(-x^2 + 1) - 1))) + 1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 - 1)) + arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

maple [C] time = 0.06, size = 234, normalized size = 5.57

$$-2 \arctan\left(\frac{-1 + \sqrt{-x^2 + 1}}{x}\right) + \frac{\sqrt{3} \arctan\left(\frac{(2x^2 - 1)\sqrt{3}}{3}\right)}{3} - \frac{i\sqrt{3} \ln\left(\frac{(1+i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1\right)}{6} - \frac{i\sqrt{3} \ln\left(\frac{(1-i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1\right)}{6} + \frac{i\sqrt{3} \ln\left(\frac{(1+i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1\right)}{6} + \frac{i\sqrt{3} \ln\left(\frac{(1-i\sqrt{3})(-1+\sqrt{-x^2+1})}{x} + \frac{(-1+\sqrt{-x^2+1})^2}{x^2} - 1\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x)
```

```
[Out] -2*arctan((-1+(-x^2+1)^(1/2))/x)+1/3*3^(1/2)*arctan(1/3*(2*x^2-1)*3^(1/2))-1/6*I*3^(1/2)*ln((-1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2/x^2-1)-1/6*I*3^(1/2)*ln((1-I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((1+I*3^(1/2))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2/x^2-1)+1/6*I*3^(1/2)*ln((I*3^(1/2)-1))*(-1+(-x^2+1)^(1/2))/x+(-1+(-x^2+1)^(1/2))^2/x^2-1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{1}{2} x^2 + \int \frac{x^4 - x^2}{x^3 - x - \sqrt{x+1} \sqrt{-x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(x-x^3+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

```
[Out] 1/2*x^2 + integrate(-(x^4 - x^2)/(x^3 - x - sqrt(x + 1)*sqrt(-x + 1)), x)
```

mupad [B] time = 3.89, size = 549, normalized size = 13.07

$$\operatorname{asin}(x) - \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} - \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}} + \frac{\ln\left(\frac{\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2 + \sqrt{-x^2+1}}{\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}}\right)}{\sqrt{1-\left(\frac{x-\frac{1}{2}}{\sqrt{x+\frac{1}{2}}}\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((x*(1 - x^2)^(1/2))/(x - x^3 + (1 - x^2)^(1/2)),x)`

[Out] `asin(x) - log((((x*(3^(1/2)/2 + 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(3^(1/2)/2 - x + 1i/2)))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + log((((x*(3^(1/2)/2 - 1i/2) - 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - 3^(1/2)/2 + 1i/2)))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - log((((x*(3^(1/2)/2 - 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 - 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 - 1i/2)))/((1 - (3^(1/2)/2 - 1i/2)^2)^(1/2)*(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)) - (log(x - 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) - (log(x + 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 + 1i/2))/(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i) + log((((x*(3^(1/2)/2 + 1i/2) + 1)*1i)/(1 - (3^(1/2)/2 + 1i/2)^2)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + 3^(1/2)/2 + 1i/2)))/((1 - (3^(1/2)/2 + 1i/2)^2)^(1/2)*(3^(1/2) - 4*(3^(1/2)/2 + 1i/2)^3 + 1i)) + (log(x - 3^(1/2)/2 + 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i) + (log(x + 3^(1/2)/2 - 1i/2)*(3^(1/2)/2 - 1i/2))/(4*(3^(1/2)/2 - 1i/2)^3 - 3^(1/2) + 1i)`

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x\sqrt{1-x^2}}{x^3 - x - \sqrt{1-x^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x**2+1)**(1/2)/(x-x**3+(-x**2+1)**(1/2)),x)`

[Out] `-Integral(x*sqrt(1 - x**2)/(x**3 - x - sqrt(1 - x**2)), x)`

$$3.712 \quad \int (1 + x + x^2 + x^3)^{-n} (1 - x^4)^n dx$$

Optimal. Leaf size=34

$$\frac{(1-x)(x^3+x^2+x+1)^{-n}(1-x^4)^n}{n+1}$$

Rubi [F] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (1 + x + x^2 + x^3)^{-n} (1 - x^4)^n dx$$

Verification is not applicable to the result.

[In] Int[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] Defer[Int][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]

Rubi steps

$$\int (1 + x + x^2 + x^3)^{-n} (1 - x^4)^n dx = \int (1 + x + x^2 + x^3)^{-n} (1 - x^4)^n dx$$

Mathematica [A] time = 0.04, size = 31, normalized size = 0.91

$$\frac{(x-1)(x^3+x^2+x+1)^{-n}(1-x^4)^n}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] ((-1 + x)*(1 - x^4)^n)/((1 + n)*(1 + x + x^2 + x^3)^n)

IntegrateAlgebraic [F] time = 0.11, size = 0, normalized size = 0.00

$$\int (1 + x + x^2 + x^3)^{-n} (1 - x^4)^n dx$$

Verification is not applicable to the result.

[In] IntegrateAlgebraic[(1 - x^4)^n/(1 + x + x^2 + x^3)^n,x]

[Out] Defer[IntegrateAlgebraic][(1 - x^4)^n/(1 + x + x^2 + x^3)^n, x]

fricas [A] time = 0.74, size = 31, normalized size = 0.91

$$\frac{(-x^4+1)^n(x-1)}{(x^3+x^2+x+1)^n(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="fricas")

[Out] (-x^4 + 1)^n*(x - 1)/((x^3 + x^2 + x + 1)^n*(n + 1))

giac [B] time = 0.46, size = 81, normalized size = 2.38

$$\frac{\frac{x e^{(n \log(x^3+x^2+x+1)+n \log(-x+1))}}{(x^3+x^2+x+1)^n} - \frac{e^{(n \log(x^3+x^2+x+1)+n \log(-x+1))}}{(x^3+x^2+x+1)^n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="giac")

[Out] (x*e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n - e^(n*log(x^3 + x^2 + x + 1) + n*log(-x + 1))/(x^3 + x^2 + x + 1)^n)/(n + 1)

maple [A] time = 0.00, size = 32, normalized size = 0.94

$$\frac{(x-1)(-x^4+1)^n(x^3+x^2+x+1)^{-n}}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-x^4+1)^n/((x^3+x^2+x+1)^n),x)

[Out] (x-1)/(n+1)*(-x^4+1)^n/((x^3+x^2+x+1)^n)

maxima [A] time = 1.09, size = 16, normalized size = 0.47

$$\frac{(x-1)(-x+1)^n}{n+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x^4+1)^n/((x^3+x^2+x+1)^n),x, algorithm="maxima")

[Out] (x - 1)*(-x + 1)^n/(n + 1)

mupad [B] time = 3.45, size = 31, normalized size = 0.91

$$\frac{(1-x^4)^n(x-1)}{(n+1)(x^3+x^2+x+1)^n}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1-x^4)^n/(x+x^2+x^3+1)^n,x)

[Out] ((1-x^4)^n*(x-1))/((n+1)*(x+x^2+x^3+1)^n)

sympy [A] time = 72.39, size = 73, normalized size = 2.15

$$\begin{cases} \frac{x(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} - \frac{(1-x^4)^n}{n(x^3+x^2+x+1)^n+(x^3+x^2+x+1)^n} & \text{for } n \neq -1 \\ -\log(x-1) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-x**4+1)**n/((x**3+x**2+x+1)**n),x)

[Out] Piecewise((x*(1-x**4)**n/(n*(x**3+x**2+x+1)**n+(x**3+x**2+x+1)**n) - (1-x**4)**n/(n*(x**3+x**2+x+1)**n+(x**3+x**2+x+1)**n), Ne(n,-1)), (-log(x-1), True))

$$3.713 \int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}} dx$$

Optimal. Leaf size=177

$$\log(20738073600000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 95105071448000000b^4c^8x^4 + 21641687369515008000b^3c^9x^5 + 32462531054272512000b^2c^{10}x^6 + 149587343098087735296c^{12}x^8 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4} \cdot (12203125b^6c^4 + 7920000b^5c^5x + 38880000b^4c^6x^2 + 110592000b^3c^7x^3 + 1990656000b^2c^8x^4 + 12230590464c^{10}x^6)) / (18432c^2)$$

Rubi [A] time = 0.09, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$, Rules used = {2082}

log(23462531054272512000b^10c^10 + 21641687369515008000b^9c^10 + 95105071448000000b^8c^10 + 2583100705996800000b^7c^10 + 597005697024000000b^6c^10 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4} \cdot (199656000b^4c^4 + 110592000b^3c^4x + 38880000b^2c^4x^2 + 7920000b^1c^4x^3 + 12230590464b^0c^4x^5) + 20738073600000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 95105071448000000b^4c^8x^4 + 21641687369515008000b^3c^9x^5 + 32462531054272512000b^2c^{10}x^6 + 149587343098087735296c^{12}x^8 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4} \cdot (12203125b^6c^4 + 7920000b^5c^5x + 38880000b^4c^6x^2 + 110592000b^3c^7x^3 + 1990656000b^2c^8x^4 + 12230590464c^{10}x^6)) / (18432c^2)

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4], x]
```

```
[Out] Log[207380736000000000*b^8*c^4 + 597005697024000000*b^6*c^6*x^2 + 2583100705996800000*b^5*c^7*x^3 + 951050714480640000*b^4*c^8*x^4 + 21641687369515008000*b^3*c^9*x^5 + 32462531054272512000*b^2*c^10*x^6 + 149587343098087735296*c^12*x^8 + 5308416*Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(12203125*b^6*c^4 + 7920000*b^5*c^5*x + 38880000*b^4*c^6*x^2 + 110592000*b^3*c^7*x^3 + 1990656000*b^2*c^8*x^4 + 12230590464*c^10*x^6)]/(18432*c^2)
```

Rule 2082

```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :
> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2])*x], D[Px, x], x]*Sqrt[a + b*x + c*x^2 + e*x^4])]/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && EqQ[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4}} dx = \frac{\log(20738073600000000b^8c^4 + 597005697024000000b^6c^6x^2 + 2583100705996800000b^5c^7x^3 + 95105071448000000b^4c^8x^4 + 21641687369515008000b^3c^9x^5 + 32462531054272512000b^2c^{10}x^6 + 149587343098087735296c^{12}x^8 + 5308416\sqrt{-44375b^4 + 576000b^3cx + 576000b^2c^2x^2 + 5308416c^4x^4} \cdot (12203125b^6c^4 + 7920000b^5c^5x + 38880000b^4c^6x^2 + 110592000b^3c^7x^3 + 1990656000b^2c^8x^4 + 12230590464c^{10}x^6))}{18432c^2}$$

Mathematica [C] time = 6.13, size = 1671, normalized size = 9.44

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Sqrt[-44375*b^4 + 576000*b^3*c*x + 576000*b^2*c^2*x^2 + 5308416*c^4*x^4], x]
```

```
[Out] (2*(x - (b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 2, 0]))/c)^2*(-((b*EllipticF[ArcSin[Sqrt[((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 1, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 4, 0])]/((c*x - b*Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 2, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 1, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 4, 0])))], -(((Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 3, 0])*(Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 2, 0] - Root[-44375 + 576000*#1 + 576000*#1^2 + 5308416*#1^4 &, 4, 0])))]
```



```
349208943630483456000000000000*b^9*c^11*x^4 - 212746443517280334643200000000
00*b^8*c^12*x^5 - 31911966527592050196480000000000*b^7*c^13*x^6 - 147050341
759144167305379840000000*b^5*c^15*x^8 + Sqrt[-44375*b^4 + 576000*b^3*c*x +
576000*b^2*c^2*x^2 + 5308416*c^4*x^4]*(6368060768256000000000000000*b^11*c^7
+ 4132961129594880000000000000*b^10*c^8*x + 2028908190892032000000000000*b^9*
c^9*x^2 + 57711166318706688000000000000*b^8*c^10*x^3 + 103880099373672038400
00000000*b^7*c^11*x^4 + 638239330551841003929600000000*b^5*c^13*x^6)]/c^2
```

fricas [A] time = 1.33, size = 164, normalized size = 0.93

log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44375*b^4))/c^2

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x, algorithm="fricas")

[Out] 1/18432*log(28179280429056*c^8*x^8 + 6115295232000*b^2*c^6*x^6 + 4076863488000*b^3*c^5*x^5 + 179159040000*b^4*c^4*x^4 + 486604800000*b^5*c^3*x^3 + 112464000000*b^6*c^2*x^2 + 3906640625*b^8 + (12230590464*c^6*x^6 + 1990656000*b^2*c^4*x^4 + 1105920000*b^3*c^3*x^3 + 38880000*b^4*c^2*x^2 + 79200000*b^5*c*x + 12203125*b^6)*sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44375*b^4))/c^2

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416 c^4 x^4 + 576000 b^2 c^2 x^2 + 576000 b^3 c x - 44375 b^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x, algorithm="giac")

[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44375*b^4), x)

maple [C] time = 0.74, size = 1597, normalized size = 9.02

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x)

[Out] 1/1152*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)*((5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))^2*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=3)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))^2*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)*(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=1)*b/c)/(x-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))^2*(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=4)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c)/(5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c-5/48*RootOf(_Z^4+10*_Z^2+96*_Z-71,index=2)*b/c))^2

) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (c^4 * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c) * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 3) * b / c) * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c)) ^ (1 / 2) * (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c * EllipticF(((5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c) * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c)) ^ (1 / 2), ((5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 3) * b / c) * (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 3) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c)) ^ (1 / 2)) + (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c) * EllipticPi(((5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c) * (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (x - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c)) ^ (1 / 2), (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c), ((5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 3) * b / c) * (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 1) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 3) * b / c) / (5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 2) * b / c - 5 / 48 * RootOf(_Z^4 + 10 * _Z^2 + 96 * _Z - 71, index = 4) * b / c)) ^ (1 / 2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{5308416 c^4 x^4 + 576000 b^2 c^2 x^2 + 576000 b^3 c x - 44375 b^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(5308416*c^4*x^4+576000*b^2*c^2*x^2+576000*b^3*c*x-44375*b^4)^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(5308416*c^4*x^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x - 44375*b^4), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(1/2),x)

[Out] int(x/(5308416*c^4*x^4 - 44375*b^4 + 576000*b^2*c^2*x^2 + 576000*b^3*c*x)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{-44375 b^4 + 576000 b^3 c x + 576000 b^2 c^2 x^2 + 5308416 c^4 x^4}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(5308416*c**4*x**4+576000*b**2*c**2*x**2+576000*b**3*c*x-44375*b**4)**(1/2),x)

[Out] Integral(x/sqrt(-44375*b**4 + 576000*b**3*c*x + 576000*b**2*c**2*x**2 + 5308416*c**4*x**4), x)

$$3.714 \quad \int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} dx$$

Optimal. Leaf size=100

$$\frac{1}{16} \log \left(4096x^8 + 8192x^7 + 12288x^6 + 19456x^5 + 17024x^4 + 13440x^3 + 9280x^2 + \sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9} \right)$$

Rubi [B] time = 0.14, antiderivative size = 243, normalized size of antiderivative = 2.43, number of steps used = 2, number of rules used = 2, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2083, 2082}

$\frac{1}{16} \log(4096x^8 + 8192x^7 + 12288x^6 + 19456x^5 + 17024x^4 + 13440x^3 + 9280x^2 + \sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9})$

Antiderivative was successfully verified.

[In] Int[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4], x]

[Out] Log[921 + 2864*x + 9280*x^2 + 13440*x^3 + 17024*x^4 + 19456*x^5 + 12288*x^6 + 8192*x^7 + 4096*x^8 + 179*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 44*x*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 744*x^2*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 1280*x^3*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 960*x^4*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 768*x^5*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] + 512*x^6*Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4]]/16

Rule 2082

Int[(x_)/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (e_.)*(x_)^4], x_Symbol] :> With[{Px = (1*(33*b^2*c + 6*a*c^2 + 40*a^2*e))/320 - (22*a*c*e*x^2)/5 + (22*b*c*e*x^3)/15 + (1*e*(5*c^2 + 4*a*e)*x^4)/4 + (4*b*e^2*x^5)/3 + 2*c*e^2*x^6 + e^3*x^8}, Simp[(1*Log[Px + Dist[1/(8*Rt[e, 2]*x), D[Px, x], x]*Sqrt[a + b*x + c*x^2 + e*x^4]])/(8*Rt[e, 2]), x]] /; FreeQ[{a, b, c, e}, x] && EqQ[71*c^2 + 100*a*e, 0] && EqQ[1152*c^3 - 125*b^2*e, 0]

Rule 2083

Int[((A_) + (B_.)*(x_))/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2 + (d_.)*(x_)^3 + (e_.)*(x_)^4], x_Symbol] := Dist[B, Subst[Int[x/Sqrt[(-3*d^4 + 16*c*d^2*e - 64*b*d*e^2 + 256*a*e^3)/(256*e^3) + ((d^3 - 4*c*d*e + 8*b*e^2)*x)/(8*e^2) - ((3*d^2 - 8*c*e)*x^2)/(8*e) + e*x^4], x], x, d/(4*e) + x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && EqQ[B*d - 4*A*e, 0] && EqQ[d*(141*d^3 - 752*c*d*e - 400*b*e^2) + 16*e^2*(71*c^2 + 100*a*e), 0] && EqQ[144*(3*d^2 - 8*c*e)^3 + 125*(d^3 - 4*c*d*e + 8*b*e^2)^2, 0]

Rubi steps

$$\int \frac{1+4x}{\sqrt{9+120x+64x^2+64x^3+64x^4}} dx = 4 \operatorname{Subst} \left(\int \frac{x}{\sqrt{-\frac{71}{4} + 96x + 40x^2 + 64x^4}} dx, x, \frac{1}{4} + x \right) = \frac{1}{16} \log \left(921 + 2864x + 9280x^2 + 13440x^3 + 17024x^4 + 19456x^5 + \dots \right)$$

Mathematica [C] time = 6.10, size = 2787, normalized size = 27.87

Result too large to show

Warning: Unable to verify antiderivative.

$0\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 2, 0]) * (-\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 1, 0] + \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 3, 0])) * (\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 1, 0] - \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0]) * \text{Sqrt}[(x - \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0]) / ((x - \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 2, 0]) * (-\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 1, 0] + \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0]))] * \text{Sqrt}(((x - \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 1, 0]) * (-\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 2, 0] + \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0])) / ((x - \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 2, 0]) * (-\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 1, 0] + \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0])))) / (\text{Sqrt}[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] * (-\text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 2, 0] + \text{Root}[9 + 120\#1 + 64\#1^2 + 64\#1^3 + 64\#1^4 \& , 4, 0]))$

IntegrateAlgebraic [A] time = 5.89, size = 100, normalized size = 1.00

$$-\frac{1}{16} \log\left(-4096x^8 - 8192x^7 - 12288x^6 - 19456x^5 - 17024x^4 - 13440x^3 - 9280x^2 + \sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9} (512x^6 + 768x^5 + 960x^4 + 1280x^3 + 744x^2 + 444x + 179) - 2864x - 921\right)$$

Antiderivative was successfully verified.

[In] IntegrateAlgebraic[(1 + 4*x)/Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4], x]

[Out] -1/16*Log[-921 - 2864*x - 9280*x^2 - 13440*x^3 - 17024*x^4 - 19456*x^5 - 12288*x^6 - 8192*x^7 - 4096*x^8 + Sqrt[9 + 120*x + 64*x^2 + 64*x^3 + 64*x^4] * (179 + 444*x + 744*x^2 + 1280*x^3 + 960*x^4 + 768*x^5 + 512*x^6)]

fricas [A] time = 1.18, size = 97, normalized size = 0.97

$$\frac{1}{16} \log\left(-4096x^8 - 8192x^7 - 12288x^6 - 19456x^5 - 17024x^4 - 13440x^3 - 9280x^2 - (512x^6 + 768x^5 + 960x^4 + 1280x^3 + 744x^2 + 444x + 179)\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9} - 2864x - 921\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2), x, algorithm="fricas")

[Out] 1/16*log(-4096*x^8 - 8192*x^7 - 12288*x^6 - 19456*x^5 - 17024*x^4 - 13440*x^3 - 9280*x^2 - (512*x^6 + 768*x^5 + 960*x^4 + 1280*x^3 + 744*x^2 + 444*x + 179)*sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9) - 2864*x - 921)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 1}{\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2), x, algorithm="giac")

[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)

maple [C] time = 1.12, size = 2992, normalized size = 29.92

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x+1)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2), x)

[Out] 1/4*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9, index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9, index=4))*((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9, index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9, index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9, index=1))

Of(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*EllipticPi(((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2))*(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)))/(x-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)))^(1/2),(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)),((1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3))*(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=1)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=3)))/(1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=2)-1/2*RootOf(4*_Z^4+8*_Z^3+16*_Z^2+60*_Z+9,index=4)))^(1/2)))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 1}{\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(64*x^4+64*x^3+64*x^2+120*x+9)^(1/2),x, algorithm="maxima")

[Out] integrate((4*x + 1)/sqrt(64*x^4 + 64*x^3 + 64*x^2 + 120*x + 9), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{4x + 1}{\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2),x)

[Out] int((4*x + 1)/(120*x + 64*x^2 + 64*x^3 + 64*x^4 + 9)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{4x + 1}{\sqrt{64x^4 + 64x^3 + 64x^2 + 120x + 9}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4*x)/(64*x**4+64*x**3+64*x**2+120*x+9)**(1/2),x)

[Out] Integral((4*x + 1)/sqrt(64*x**4 + 64*x**3 + 64*x**2 + 120*x + 9), x)

Chapter 4

Appendix

Local contents

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4.1 Download section

The following zip files contain the raw integrals used in this test.

This is a subset of Rubi test suite thanks to Albert Rich, which includes only the algebraic integrals with elementray optimal antiderivatives. It also includes a subset of a test file provided thanks to Sam Blake.

Mathematica format Mathematica_syntax_CAS_integration_elementary_version.zip

Maple and Mupad format Maple_syntax_CAS_integration_elementary_version.zip

Sympy format SYMPY_syntax_CAS_integration_elementary_version.zip

Sage math format SAGE_syntax_CAS_integration_elementary_version.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
```

```

(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
            If[ElementaryFunctionQ[Head[expn]],
              Max[3, ExpnType[expn[[1]]]],
              If[SpecialFunctionQ[Head[expn]],
                Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
                If[HypergeometricFunctionQ[Head[expn]],
                  Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
                  If[AppellFunctionQ[Head[expn]],
                    Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
                    If[Head[expn]===RootSum,
                      Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
                      If[Head[expn]===Integrate || Head[expn]===Int,
                        Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
                        9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,
    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

```

```

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

```

```

HypergeometricFunctionQ[func_] :=

```

```
MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#                    if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#                    see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);

    if debug then
        print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
ExpnType_optimal);
    fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F";
end if;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
end if;
```

```

if is_contains_complex(result) then
  if is_contains_complex(optimal) then
    if debug then
      print("both result and optimal complex");
    fi;
    #both result and optimal complex
    if leaf_count_result<=2*leaf_count_optimal then
      return "A";
    else
      return "B";
    end if
  else #result contains complex but optimal is not
    if debug then
      print("result contains complex but optimal is not");
    fi;
    return "C";
  end if
else # result do not contain complex
  # this assumes optimal do not as well
  if debug then
    print("result do not contain complex, this assumes optimal do
not as well");
  fi;
  if leaf_count_result<=2*leaf_count_optimal then
    if debug then
      print("leaf_count_result<=2*leaf_count_optimal");
    fi;
    return "A";
  else
    if debug then
      print("leaf_count_result>2*leaf_count_optimal");
    fi;
    return "B";
  end if
end if
else #ExpnType(result) > ExpnType(optimal)
  if debug then
    print("ExpnType(result) > ExpnType(optimal)");
  fi;
  return "C";
end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function

```

```

# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func,[
    exp,log,ln,
    sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
    erf,erfc,erfi,
    FresnelS,FresnelC,
    Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

```



```

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

4.2.3 Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
  #sympy do not have leaf count function. This is approximation
  return round(1.7*count_ops(expr))

def is_sqrt(expr):
  if isinstance(expr,Pow):
    if expr.args[1] == Rational(1,2):
      return True
    else:
      return False
  else:
    return False

def is_elementary_function(func):
  return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                 asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                 asinh,acosh,atanh,acoth,asech,acsch
                 ]

def is_special_function(func):
  return func in [ erf,erfc,erfi,
                 fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                 gamma,loggamma,digamma,zeta,polylog,LambertW,
                 elliptic_f,elliptic_e,elliptic_pi,exp_polar
                 ]

def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):

```

```

try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
    else:
        return False

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+`') or
    type(expn,'*`)
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
    expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
    ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function

```

```

def grade_antiderivative(result,optimal):

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```

4.2.4 SageMath grading function

```

#Dec 24, 2019. Nasser: Ported original Maple grading function by
#           Albert Rich to use with Sagemath. This is used to
#           grade Fracas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#           'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False

```

```

else:
    return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
sinh_integral'
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M',
        hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):

```

```

        return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
            return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
        return False

    except AttributeError as error:
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
    elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(4,m1) #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))
        return max(5,m1) #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

    return max(6,m1)    #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"

```